Title: Spatial symmetry breaking in FQH states and beyond, when geometry meets topology

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Abstract: <p>In this talk, I would introduce spontaneous nematicity in the background of fractional quantum Hall fluids where symmetry breaking phenomenon intertwined with topological phase of matter. The resulting nematic FQH state is characterized by an order parameter that represents these quadrupolar fluctuations, which play the role of fluctuations of the local geometry of the quantum fluid. We demonstrate that the low-energy effective theory of the nematic order parameter has z=2 dynamical scaling exponent, due to a Berry phase term of the order parameter, which is related to the nondissipative Hall viscosity. By investigating the spectrum of collective excitations, we demonstrate that the mass gap of the Girvin-MacDonald-Platzman mode collapses at the isotropic-nematic quantum phase transition. An interesting feature of the nematic phase is that it has topological defects carrying nontrivial braiding statistics and fractional charge inherited from the topological fluid nature. In addition, I would also mention the decorated nodal line condensation in pair density wave SC, where the topological phase emerges concurrently with symmetry recovery by decorated defect condensation.</p>

Spatial symmetry breaking in FQH states and beyond

----- when geometry meets topology

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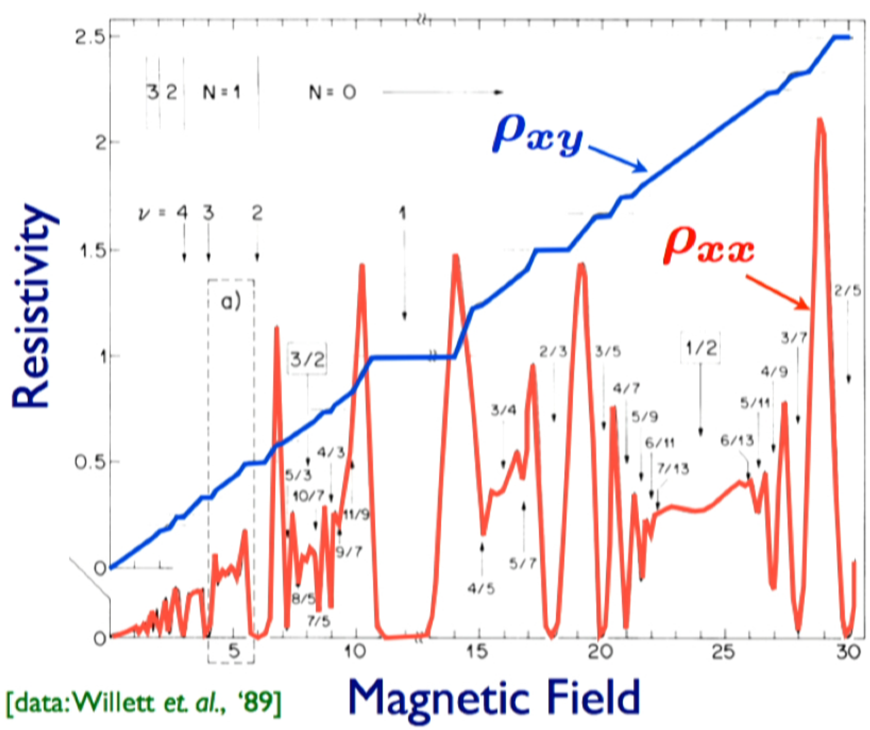
Can symmetry breaking be intertwined (co-exist) with topological phase?

1) Does the topological quasi-particle intertwine with the broken symmetry?

2) Is the disorder-order transition affected by the topological nature?

3) Interplay between symmetry defect VS topological quasi-particle ?

Answer: Spontaneous Nematic FQH states ?
(rotation symmetry breaking in the FQH region)



Fractional quantum Hall effect

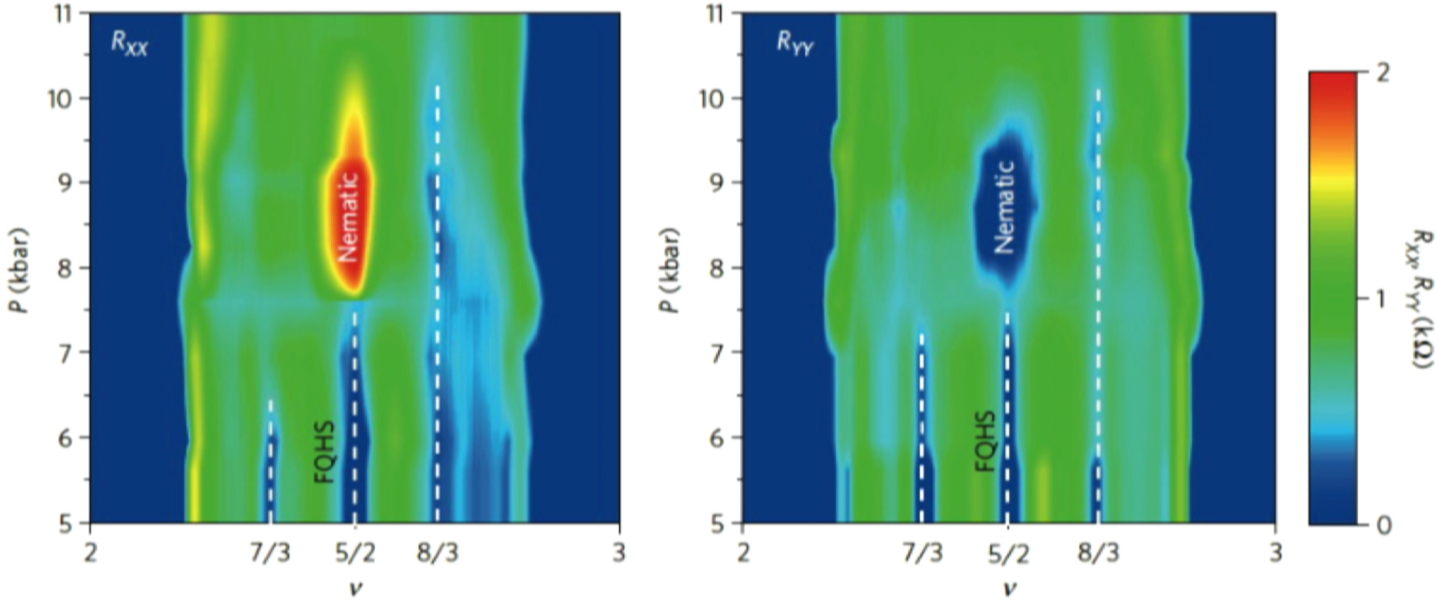
Partial filled LL, strong interaction

At certain filling, incompressible, topological order

FQH liquid Vs Broken symmetry

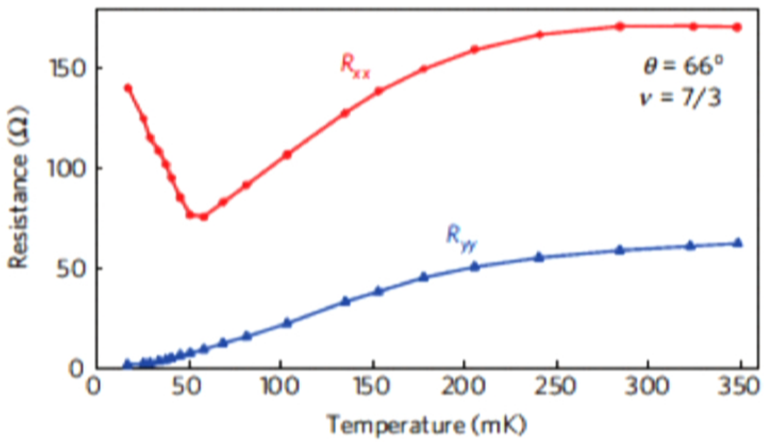
Competition Vs Coexsistence?

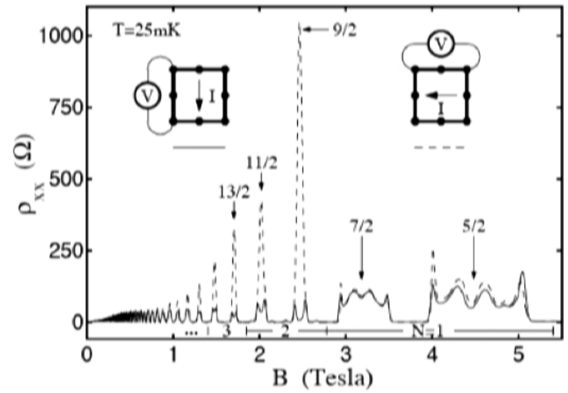
Possible competition between nematic and Pfaffian?

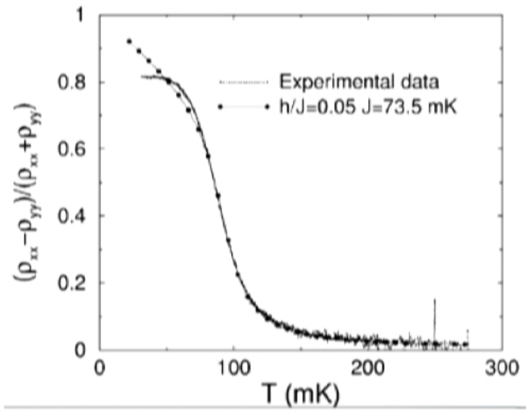


Nature Physics **12**, 191 (2016)

Nature Physics **7**, 845 (2011)







Nematics phase coexist with FQHE?

Why nematic?

Regard nematic order as a dynamical frame?

$$\boldsymbol{Q} = \begin{pmatrix} M_1 & M_2 \\ M_2 & -M_1 \end{pmatrix} \xrightarrow{\text{Nematics distort local geometry}} g^{ij} = e^i_a \delta^{ab} e^j_b$$

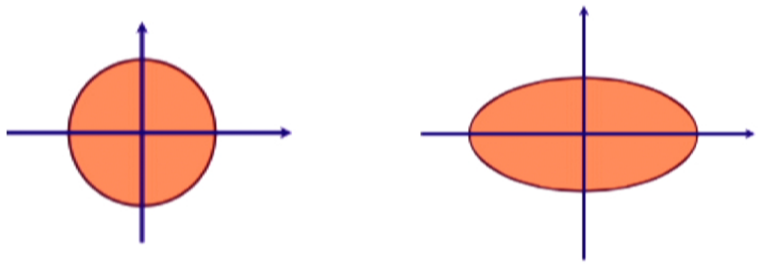
Interplay with geometry & topology?

Wen-Zee shift for FQHE, Gravitational anomaly

◆ Spontaneous Nematicity from interaction?

$$F^{s,a}_{pp'} = \sum_{\ell} F^{s,a}_{\ell} \cos(\theta \ell)$$

l = 2 Pomeranchuk deformation



Quadrupolar interaction for nematics (Oganesyan et al. 2001)

$$V_{quadr}(r) = \int dr \int dr' F_2(r - r') Tr[Q(r)Q(r')]$$

$$Q(r) = \Psi^\dagger(r) \begin{pmatrix} D_x^2 - D_y^2 & 2D_x D_y \\ 2D_x D_y & D_y^2 - D_x^2 \end{pmatrix} \Psi(r)$$

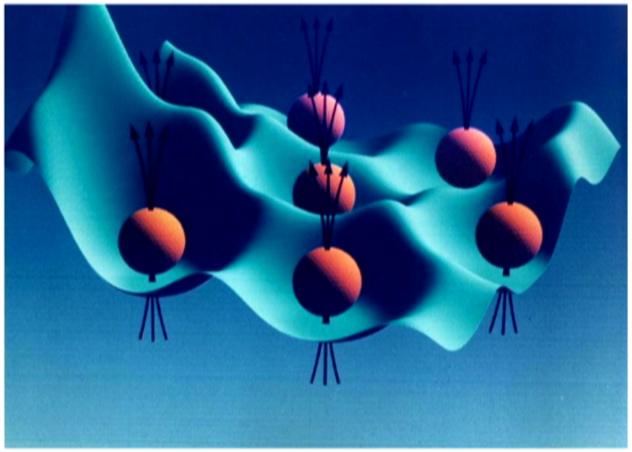
$$F_2(\boldsymbol{q}) = \frac{F_2}{1 + \kappa \boldsymbol{q}^2}$$

Chern-Simons theory for FQHE at 1/3 filling

$$\mathcal{S} = \int d^2x dt \left[\Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e} \left(\boldsymbol{D}\Psi(x)\right)^\dagger \cdot \left(\boldsymbol{D}\Psi(x)\right) \right]$$
$$- \frac{1}{32\pi^2} \int d^2x' d^2x dt \, V(|\boldsymbol{x} - \boldsymbol{x}'|) \delta b(x) \delta b(x')$$
$$+ \frac{1}{8\pi} \int d^2x dt \, \epsilon^{\mu\nu\lambda} \delta a_\mu \partial_\nu \delta a_\lambda$$

Attach two flux to fermion

$$D_\mu = \partial_\mu + i(A_\mu + a_\mu)$$



Composite fermion theory with nematic instability

$$\mathcal{S} = \int d^2x dt \left[\Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e} \left(\boldsymbol{D}\Psi(x)\right)^\dagger \cdot \left(\boldsymbol{D}\Psi(x)\right)\right]$$
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Composite fermion theory with nematic instability

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$$+ \frac{1}{8\pi} \int d^2x dt \, \epsilon^{\mu\nu\lambda} \delta a_\mu \partial_\nu \delta a_\lambda$$
$$+ \int d^2x dt \Big[\frac{1}{4F_2 m_e^2} \boldsymbol{M}^2 + \frac{\kappa}{4F_2 m_e^2} \sum_{i=1,2} |\boldsymbol{\nabla} M_i|^2$$
$$+ \frac{M_1}{m_e} \Psi^\dagger (D_x^2 - D_y^2) \Psi + \frac{M_2}{m_e} \Psi^\dagger (D_x D_y + D_y D_x) \Psi \Big]$$

Nematics couple with Stress tensor

$$S_6 = -\alpha \int d^2x dt \Psi^\dagger \left(\frac{-\boldsymbol{D}^2}{2m_e} - \frac{\bar{\rho}\pi}{m_e} \right)^3 \Psi$$

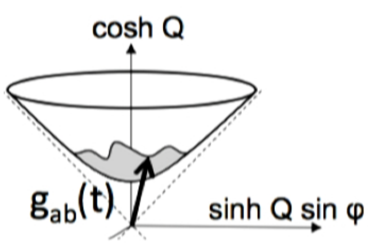
Essential for continuous transition

Nematic phase transition

$$\mathcal{L}_M = \frac{\epsilon^{ij}\bar{\rho}}{2(1+4\alpha\bar{\omega}_c^2)^2} M_i \partial_0 M_j - r\boldsymbol{M}^2 - \frac{\bar{\kappa}}{2}(\boldsymbol{\nabla} M_i)^2 - \frac{u}{4}(\boldsymbol{M}^2)^2.$$

r<0,nematic FQH

$$\boldsymbol{Q} = \begin{pmatrix} M_1 & M_2 \\ M_2 & -M_1 \end{pmatrix}$$



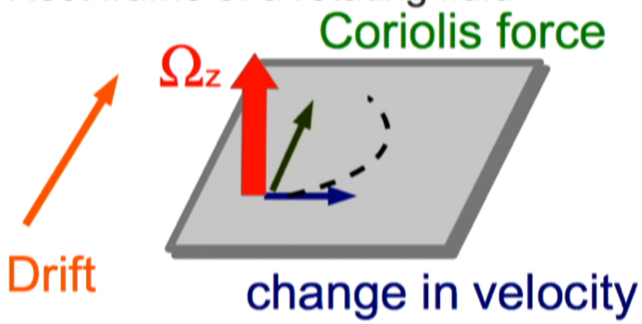
Berry phase term indicates area enclosed by a trajectory of the order parameter

✓ Z=2 Lifshitz criticality, Berry phase term

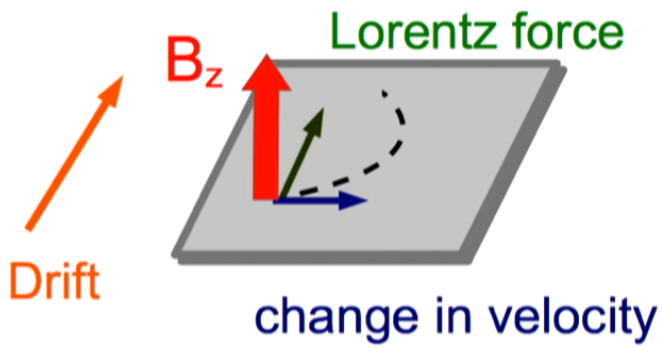
✓ Expected in all Chiral phases!

universal

Rest frame of a rotating fluid



$$T^{i0} = \zeta_H \epsilon^{ij} \partial_t g_{jt}$$



$$J^i = \sigma_H \epsilon^{ij} E_j, \quad \sigma_H = \frac{nec}{B}$$

Hall viscosity- Coriolis force

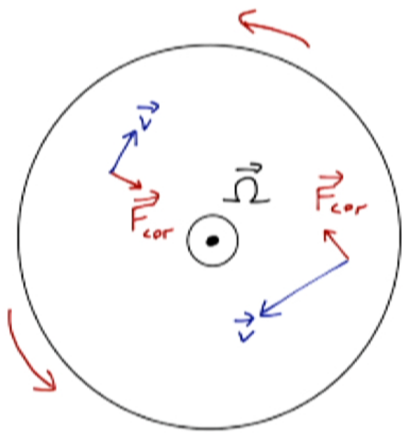
Hall conductivity –Lorenz force

Hall viscosity

displacement $x = x + u_a$ metric $g_{ab} = \partial_a u_b + \partial_b u_a$

Velocity field $v_a = \partial_0 u_a$

Shear stress $f_x = \eta v_y \rightarrow T_{xx} = \eta \partial_0 g_{yx} = \eta \partial_x v_y$



$$f_i = \eta \epsilon^{ij} v_j$$

Rotation frame

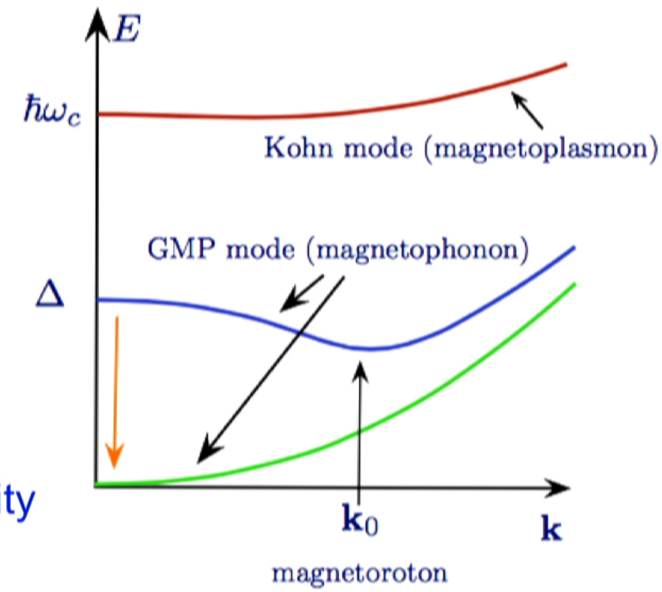
Force perpendicular to velocity

Collective excitation across phase transition?

Kohn mode:
remains gapped!

Laughlin quasiparticle: remain gapped, topological quantity remains!

GMP mode at long wave length:
Mixed with nematic fluctuation
Drop down when approaching criticality



Spectrum of collective modes of the Laughlin $\nu=1/3$ FQH state

Question answered

1) Spontanuous Nematic FQH states?
Yes, Quadrupolar interaction

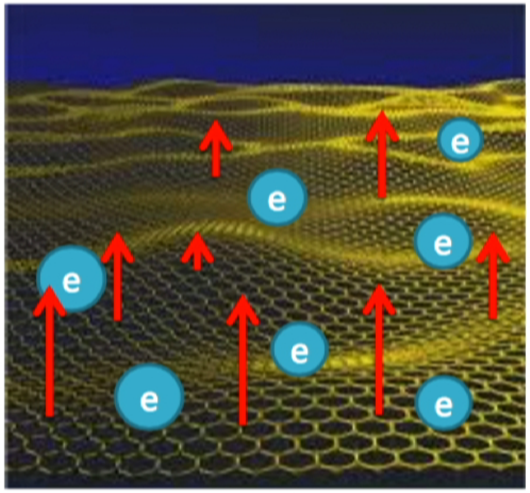
2) How FQH physics affect the nematic transition?
Berry phase term, related with Hall viscosity.
Change dynamical exponent to Z=2.
GMP mode is soft near criticality.

Next step:

How does nematic degree of freedom interplay with topological degree of freedom?

Nematic as a dynamical metric?

Nematic coupling with Gauge field?



Nematic is a dynamical metric

Modify local geometry

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2m} Q_{\mu\nu}$$

$$Q_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_1 & M_2 \\ 0 & M_2 & -M_1 \end{pmatrix}$$

$f_{\mu\nu} g^{\mu\alpha} f_{\alpha\beta} g^{\beta\nu}$ ⟶ Enter into the Maxwell term!

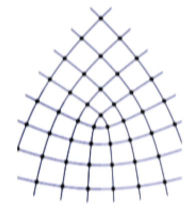
Nematic Gauge coupling - The Wen-Zee effect

$$\mathcal{L}_{wz} = \frac{1}{4\pi}\epsilon^{\mu\nu\rho}\omega^Q_\mu \partial_\nu(\delta a_\rho + \delta A_\rho)$$

Nematic vortex -> Wen-Zee type coupling

$$\omega^Q_0 = \frac{\epsilon^{ij}}{(1+4\alpha\bar{\omega}_c^2)^2} \boxed{M_i \partial_0 M_j,}$$

→ Nematic vortex



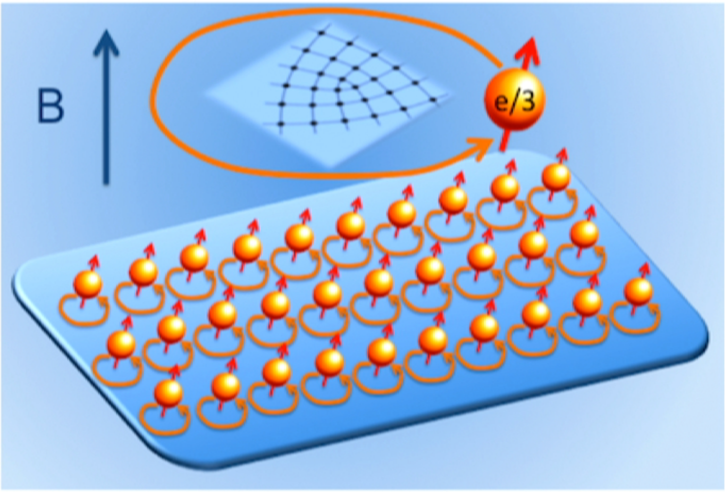
$$\omega^Q_x = \frac{\epsilon^{ij}}{(1+4\alpha\bar{\omega}_c^2)^2} M_i \partial_x M_j - t(\partial_x M_2 - \partial_y M_1)$$

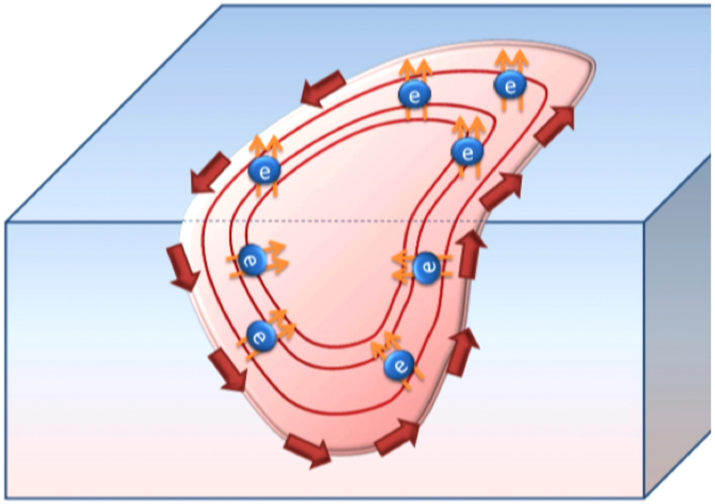
$$\omega^Q_y = \frac{\epsilon^{ij}}{(1+4\alpha\bar{\omega}_c^2)^2} M_i \partial_y M_j + \boxed{t(\partial_x M_1 + \partial_y M_2)}$$

→ Electron quadrupole

$$\partial_x \omega^Q_y - \partial_y \omega^Q_x \propto \frac{1}{2}\sqrt{g}R$$

→ Nematic vortex density, dynamical curvature





- Nematic vortex(disclinations) current couple with the EM gauge field, charge not quantized
- Nematic disclinations change the local “particle density”
- Disclination can have fractional statistics with quasiparticle

- Gravitational Chern-Simons term, self statistics of disclination!

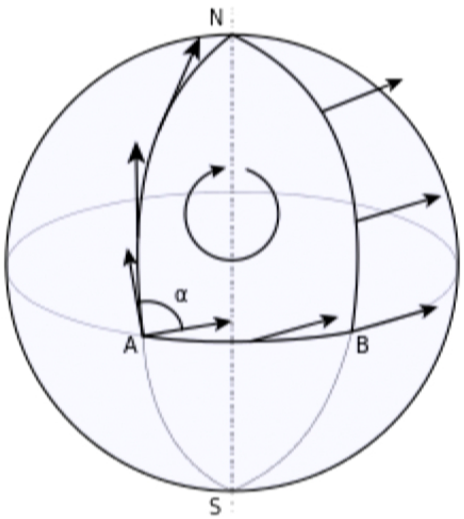
$$\frac{1}{24\pi}\omega \wedge d\omega$$

Define background metric $g^{ij} = e_a^i \delta^{ab} e_b^j$ $e_1^1 = -e_2^2 = -e_1$ $e_2^1 = -e_1^2 = -e_2$

Metric couple with stress tensor

$$e_2 \Psi^\dagger(x) 2 D_x^* D_y \Psi(x) + e_1 \Psi^\dagger(x)(D_x^* D_x - D_y^* D_y)\Psi(x)$$

The fermion is spinless, but the composite fermion is not!



Flux attachment -> spin connection?

$$D_i = \partial_i + iA_i + ia_i + iS\omega_i$$

Q: Are dynamical metric(nematic) VS background metric equivalent?

- Dynamical metric couple with stress energy tensor
- Background metric couple with stress energy tensor + carrys spin connection when attaching flux!

✓ Both the CF and spin connection of flux contribute to the Hall viscosity/ orbital spin

✓ The dynamical metric (nematic fluctuation) is different from background metric

Summary

✓ Nematic phase and phase transition can be very exotic in topological phases!
✓ This origins from the novel interplay between geometry and topology!

Nematic theory in other topological phases?

✓Nematic transition in QAH state (You-Fradkin 2013)
Always contain a Berry phase term, related with odd viscosity transport
Z=2 Lifshitz transition
✓ Nematics in 3d TSC/TI? (You-You 2016)
Nematic (vortex) disclination couple with Axion string!
Inherited from Gravitational Theta term, Intertwined between broken symmetry and topological quantity

Start from anisotropic topological state
-> breaks spatial symmetry while keep the topological quantity unchanged

Can we go from the opposite trend?

Start from spatial symmetry broken state (stripe, nematic)
-> restore the symmetry concurrently drive the system into novel topological phases?

Stripe SC-> Nematic SC -> isotropic SC

Stripe Superconductor $\Delta = |\Delta| \cos(\boldsymbol{Qr})$

Can boson SPT state emerge during the stripe melting process?

How does the topological degree of freedom intertwine with spatial symmetry?

Is the phase transition theory affected by SPT nature ?

Stripe SC-> Nematic SC -> isotropic SC

Stripe Superconductor $\Delta = |\Delta| \cos(\boldsymbol{Q}\boldsymbol{r})$

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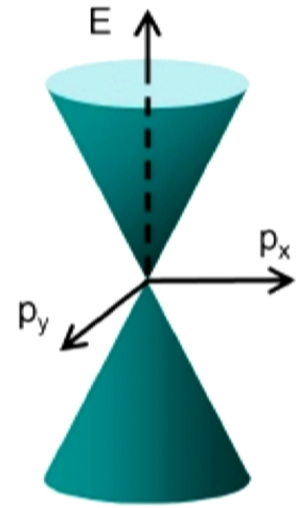
4 Dirac cones, intra-cone SC

$$H = \Psi^{\dagger}_{\boldsymbol{k}}(\sigma_x k_x + \sigma_z k_y)\Psi_{-\boldsymbol{k}}$$

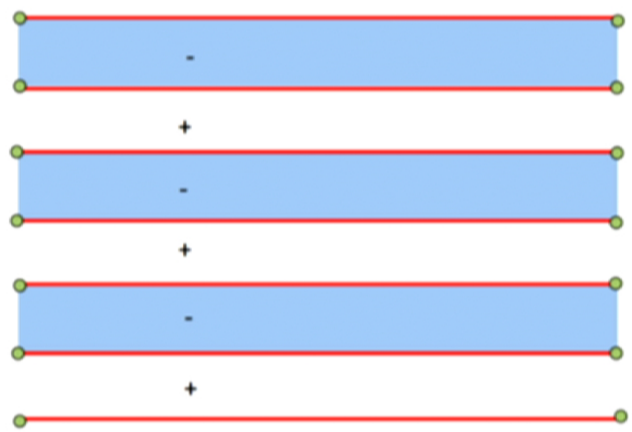
$$\Delta \;=\; \Psi^{\dagger}_{\uparrow}\Psi^{\dagger}_{\downarrow} - \Psi^{\dagger}_{\downarrow}\Psi^{\dagger}_{\uparrow}$$

Pairing, Gap the Dirac cone



$$H = \chi^{T}_{\boldsymbol{k}}(\sigma_x k_x + \sigma_z k_y + O_2\sigma_y\tau_x + O_1\sigma_y\tau_z)\chi_{-\boldsymbol{k}}$$
$$\Delta = O_1 + iO_2 \qquad (2.$$

$$\Delta \;=\; |\Delta|\cos(\boldsymbol{Q}\boldsymbol{r})$$



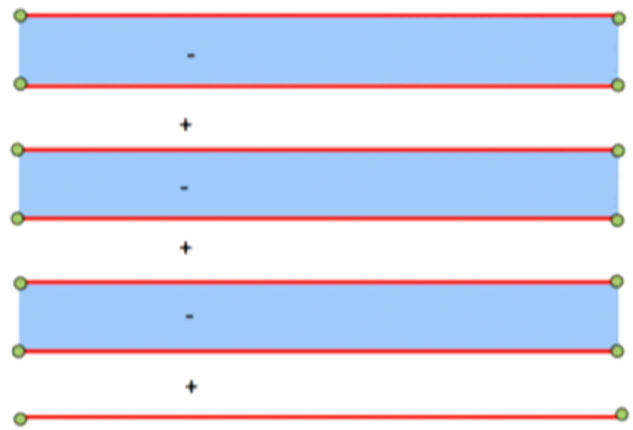
✔ Stripe SC configuration
✔ Contains Nodal lines
✔ Nodal line carries helical Majorana mode * 4

We have 4 helical Majorana modes ->couple them, get gapped bosonic chain

$$\mathcal{L} = \frac{1}{g}(\partial_\rho \vec{n})^2 + \frac{i2\pi}{\Omega^2}\epsilon^{ijk}\epsilon^{\mu\nu}n^i\partial_\mu n^j \partial_\nu n^k$$

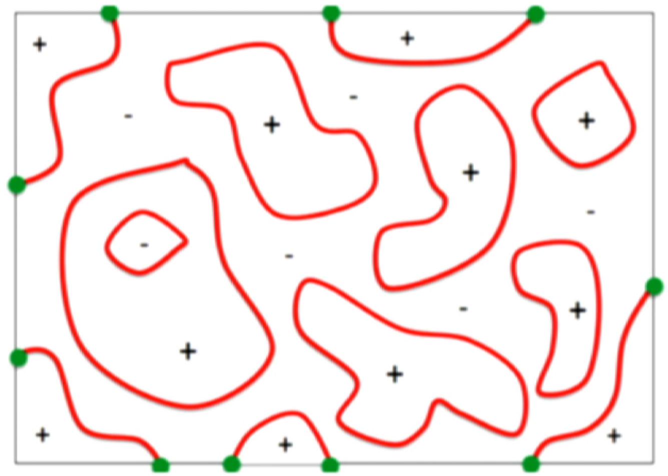
✔ Equivalent to the Haldane Chain, gapped
✔ 1d SPT state protected by $\mathcal{T}$

✔ Haldane Chain align with the nodal line



Melting the stripe-> condensation of "nodal line decorated with Haldane chain"

Condensation of nodal loops
n_4: scalar field, pairing
After stripe melting, n_4 disordered



Z_2(nodal line)* O(3) (Haldane chain) ~ O(4)

$$\mathcal{L} = \frac{1}{g}(\partial_\rho \vec{n})^2 + \frac{i2\pi}{\Omega^3}\epsilon^{ijkl}\epsilon^{\mu\nu\rho} n^i \partial_\mu n^j \partial_\nu n^k \partial_\rho n^l$$

$$\mathcal{T} : n_{1,2,3} \to -n_{1,2,3}; \; Z_2 : n_{1,2,3,4} \to -n_{1,2,3,4}$$

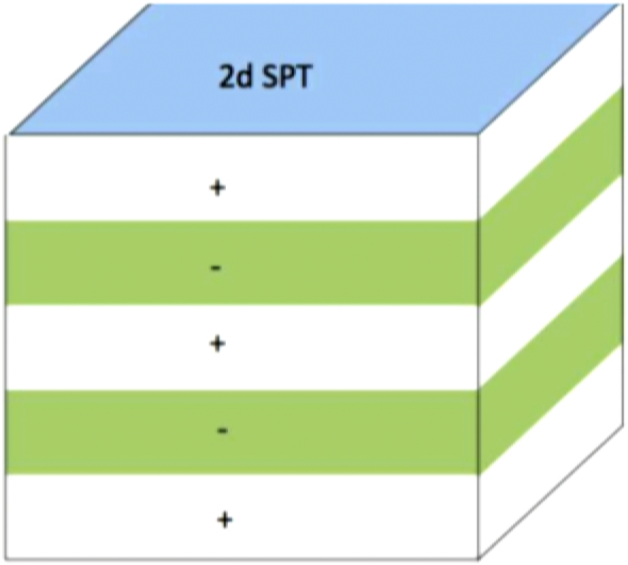
3d PDW melting -> topological order?

8 Weyl cones + Intra-cone SC

$$H = \chi^T_{i,\boldsymbol{k}}(i\partial_x\sigma^{10300} + i\partial_y\sigma^{30300} + i\partial_z\sigma^{22300} + O_1\sigma^{21000}$$
$$+ n_1\sigma^{02212} + n_2\sigma^{02232} + n_3\sigma^{02220} + n_4\sigma^{02100})\chi_{i,-\boldsymbol{k}}$$
$$\Delta = O_1 = |O_1|\cos(qz)$$

Slab configuration

Nodal plane -> 8 Majorana cone in 2d -> couple them to O(3) rotor(n_4=m) -> boson SPT in the nodal plane protected by Z_2= topological paramagnetic state

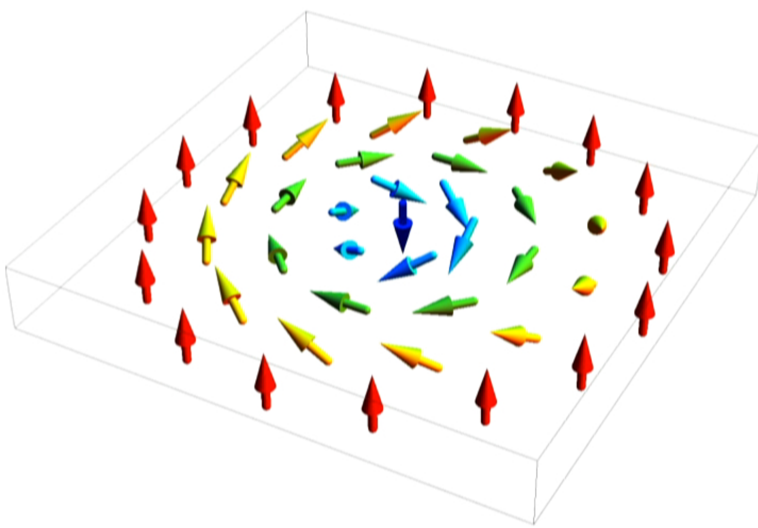


Wave-function inside the nodal plane

$$|GS\rangle = \int D[\vec{n}] e^{i\frac{2\pi}{\Omega^2}\int dx^2 \epsilon^{ijk} n_i \partial_x n_j \partial_y n_k} |\vec{n}\rangle$$

$$|\vec{n}\rangle = (n_1, n_2, n_3)$$

$$Z_2 : (n_1, n_2, n_3, O_2) \rightarrow -(n_1, n_2, n_3, O_2)$$



Skyrmion condensate:
a coherent sum of all skyrmion configurations

the sign factor counts the number of skyrmions

Topological paramagnetic state protected by Z_2 symmetry

Effective theory after nodal membrane condensation?

$$|GS\rangle = \int D[\vec{n}] e^{i\frac{2\pi}{\Omega^3}\int dx^3 \epsilon^{ijkl}\tilde{n}_i \partial_x \tilde{n}_j \partial_y \tilde{n}_k \partial_z \tilde{n}_l} |\vec{\tilde{n}}\rangle$$

$$|\vec{\tilde{n}}\rangle = (n_1, n_2, n_3, O_1)$$

Membrane condensate (O_1)
Membrane decorated with topological paramagnetic state. All close membrane in the GS.
Flux loop condensate (Skyrmion flux)
Flux loop decorated with fluctuating domain wall of O_1. All close loop in the GS

$$B_k = \epsilon^{ijk}\epsilon^{abc} n_a \partial_i n_b \partial_j n_c = \epsilon^{ijk}\partial_i a_j$$

Topological Excitations?
Open membrane- loop excitation (half SC vortex)
Open string- monopole excitation (monopole of the O(3) vector)

They have mutual \pi statistics! = Boson Z_2 Toric code in 3d!

$$\mathcal{L} = \sum_{a=1}^{5} \frac{1}{g}(\partial_\mu N_a)^2 + \frac{i2\pi}{\Omega^4}\epsilon^{ijklm} N^i \partial_x N^j \partial_y N^k \partial_z N^l \partial_t N^m$$
$$\vec{N} = (O_1, O_2, n_1, n_2, n_3)$$
$$\mathcal{T} : n_{1,2,3} \to n_{1,2,3}; \ Z_2 : (O_1, O_2) \to -(O_1, O_2) \qquad (5.9)$$

Gauging the Z_2 symmetry from SPT.
The PDW provides a deconfined half SC vortex (bound with half dislocation)

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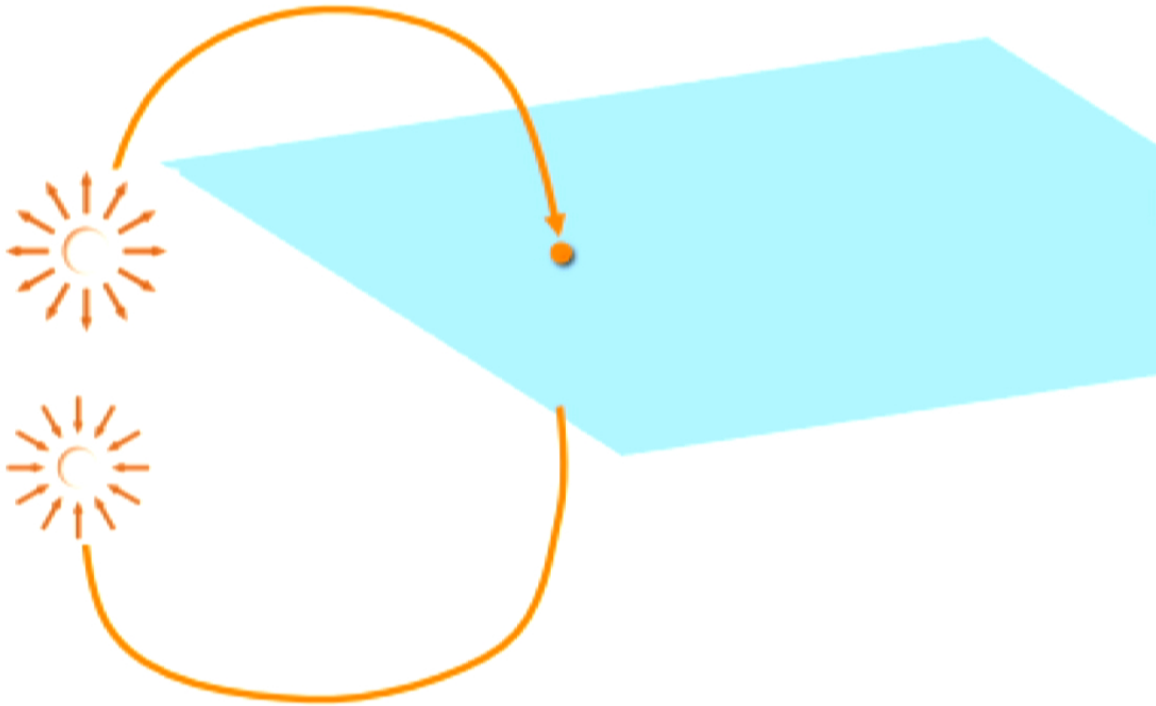

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Gauging the Z_2 symmetry from SPT.
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Spatial symmetry breaking ⟷ Topological phases

- ✔ Melting the stripe phase -> nontrivial SPT or topological order, nodal line/plane carries topological theta term

- ✔ The decoration of the nodal line provides protected entanglement and edge modes

Thank you !