Title: What is the standard model of particle physics trying to tell us?

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URL: http://pirsa.org/16110084

Abstract: <p>I will argue that the standard model contains a rather strong hint that -- instead of being simply an ordinary continuous 4D manifold -- spacetime is actually the product of a 4D manifold and a certain discrete/finite 6D space (i.e. there are 6D discrete/finite "extra dimensions"). I will introduce this idea and the evidence for it in simple way, and then discuss various outstanding puzzles and future directions.</p>

What is the standard model of particle physics trying to tell us?

Latham Boyle

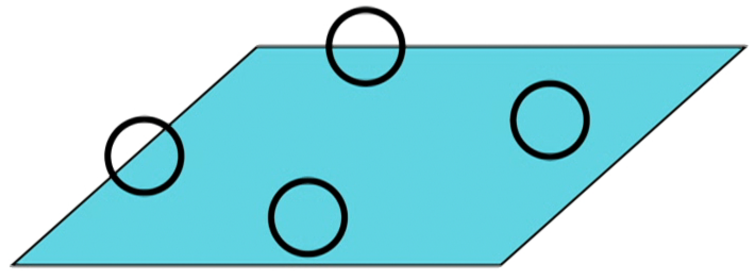
(Perimeter)

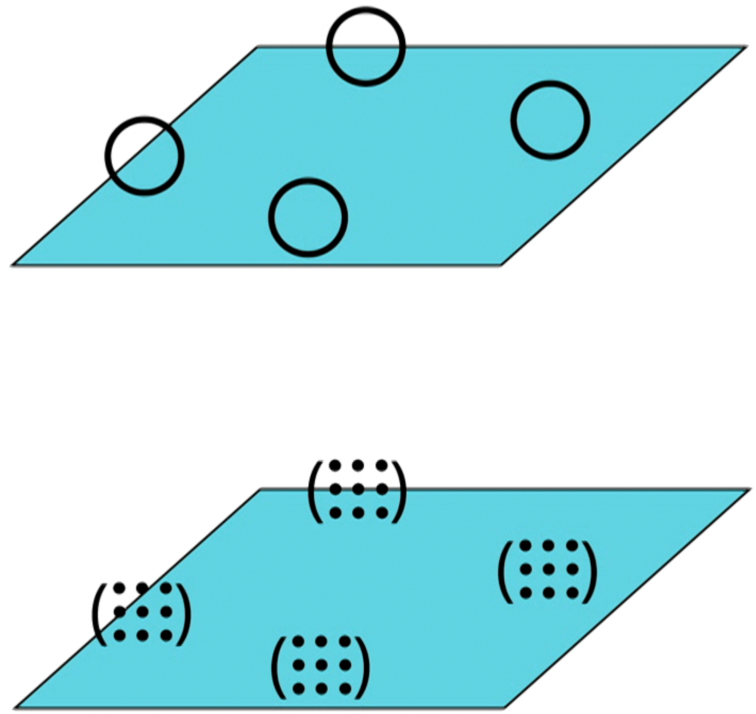
builds on work of

Barrett, Bizi, Brouder, Besnard, Chamseddine, Connes, Dubois-Violette, Kerner, Lott, Madore, Marcolli

based on arXiv:1604.00847 w/ S. Farnsworth

(and another in prep)





The EFT perspective: the basic input

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Step 1:	$SU(3)$	$SU(2)$	$U(1)$

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$$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

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Step 1:		$SU(3)$	$SU(2)$	$U(1)$
Step 2:	q_L^i	3		
	u_R^i	3		
	d_R^i	3		
	l_L^i			
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$\psi(x)$

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$$\psi(x) \to \psi_A(x) = \left(\begin{array}{cccc|cccc} & & & & \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ & & & & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ & & & & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ & & & & \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \\ \hline \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} & & & & \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} & & & & \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} & & & & \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} & & & & \end{array}\right)$$

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$$z = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$(a, b \in \mathbb{R})$$

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$$z = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \qquad q = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \qquad q_\lambda = \begin{pmatrix} \lambda & 0 \\ 0 & \bar{\lambda} \end{pmatrix}$$

$$(a, b \in \mathbb{R}) \qquad (\alpha, \beta \in \mathbb{C}) \qquad (\lambda \in \mathbb{C})$$

	EFT input:				**NCG input:**
Step 1:		$SU(3)$	$SU(2)$	$U(1)$	
Step 2:	q_L^i	3	2	1/6	
	u_R^i	3	1	2/3	
	d_R^i	3	1	$-1/3$	
	l_L^i	1	2	$-1/2$	
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NCG input:

$$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$$

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$$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$$

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$$\left(\begin{array}{c:c|c:c} q & & & \\ \hdashline & q_\lambda & & \\ \hline & & m & \\ \hdashline & & & \lambda \end{array}\right)$$

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$$\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix} \quad \begin{pmatrix} & \begin{matrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \end{matrix} \\ \begin{matrix} \bar{\psi}_{11} & \bar{\psi}_{21} & \bar{\psi}_{31} & \bar{\psi}_{41} \\ \bar{\psi}_{12} & \bar{\psi}_{22} & \bar{\psi}_{32} & \bar{\psi}_{42} \\ \bar{\psi}_{13} & \bar{\psi}_{23} & \bar{\psi}_{33} & \bar{\psi}_{43} \\ \bar{\psi}_{14} & \bar{\psi}_{24} & \bar{\psi}_{34} & \bar{\psi}_{44} \end{matrix} & \end{pmatrix}$$

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NCG input:

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Step 3: Gauge and Higgs bosons from covariance of D

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- $A_\mu^{k\prime} \;=\; A_\mu^k + f_{ij}^{\;\;k}\alpha^i A_\mu^j - (\partial_\mu \alpha^k) \qquad ([T_i, T_j] = f_{ij}^{\;\;k} T_k)$

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$$D = D_1 \hat{\otimes} 1 + 1 \hat{\otimes} D_2$$

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$$\delta_l = \left(\begin{array}{c|c|c|c} 0_{2\times 2} & Y_l & & \\ \hline Y_l^\dagger & 0_{2\times 2} & & \\ \hline & & 0_{3\times 3} & \\ \hline & & & 1 \end{array}\right)$$

$$\delta_m = \left(\begin{array}{c|c|c|c} & & & \\ \hline & & & \begin{matrix} \bar{\mu} \\ 0 \end{matrix} \\ \hline & & & \\ \hline & \mu \quad 0 & & \end{array}\right)$$

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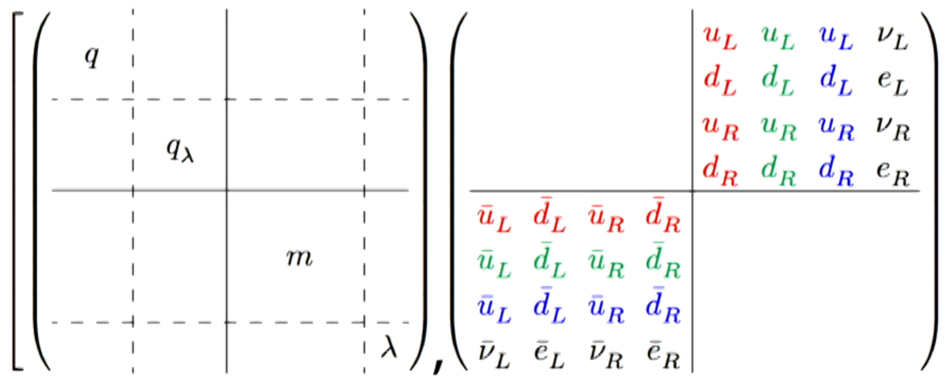
	EFT input:				**NCG input:**
Step 1:		$SU(3)$	$SU(2)$	$U(1)$	$\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$
Step 2:	q_L^i	3	2	1/6	
	u_R^i	3	1	2/3	
	d_R^i	3	1	$-1/3$	
	l_L^i	1	2	$-1/2$	
	ν_R^i	1	1	0	
	e_R^i	1	1	-1	
Step 3:	h	1	2	1/2	Gauge and Higgs bosons from covariance of D

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NCG input:

Step 1: $\{m, q, \lambda\} \quad \in \quad M_3(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}$

Step 2:

$$\left[\begin{pmatrix} q & & & \\ & q_\lambda & & \\ & & m & \\ & & & \lambda \end{pmatrix}, \begin{pmatrix} & & & & u_L & u_L & u_L & \nu_L \\ & & & & d_L & d_L & d_L & e_L \\ & & & & u_R & u_R & u_R & \nu_R \\ & & & & d_R & d_R & d_R & e_R \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{u}_L & \bar{d}_L & \bar{u}_R & \bar{d}_R & & & & \\ \bar{\nu}_L & \bar{e}_L & \bar{\nu}_R & \bar{e}_R & & & & \end{pmatrix} \right]$$

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Gauge and Higgs bosons from covariance of D

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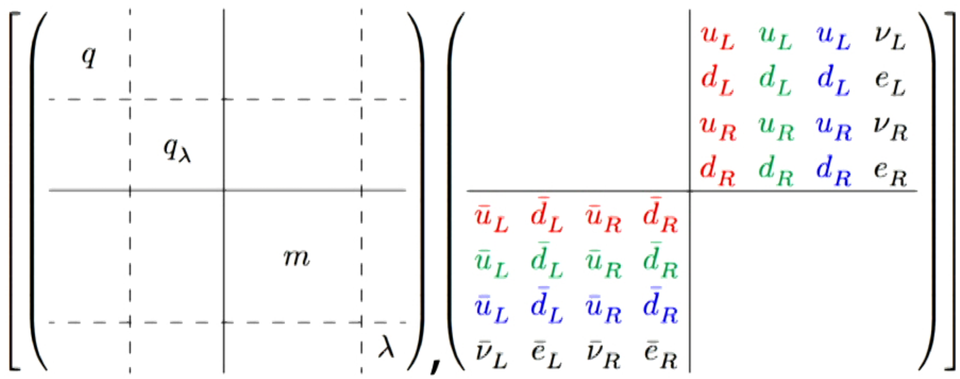

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$$S_{\text{fermion}} = \langle \psi | D | \psi \rangle$$

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- Recently: Associative → Jordan!

Some topics for the future

- A better bosonic action (F^2?)
- Relation to fine-tuning problems?
- Relation to SO(10) grand unification?
- Relation to exception Jordan algebra?