Title: Quantum spaces are modular (continued)

Date: Nov 21, 2016 03:30 PM

URL: http://pirsa.org/16110083

Abstract:

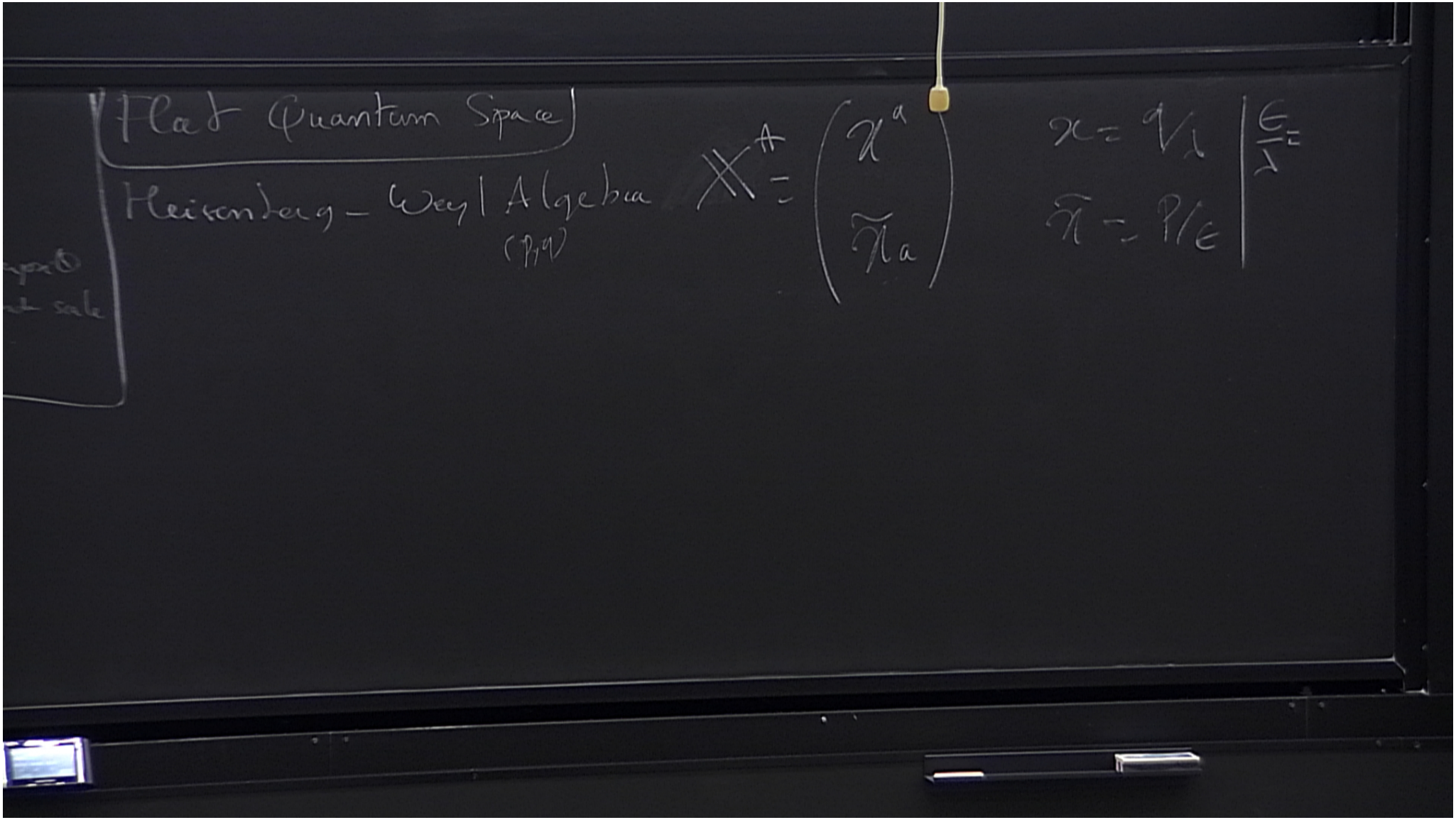
Nodular space II

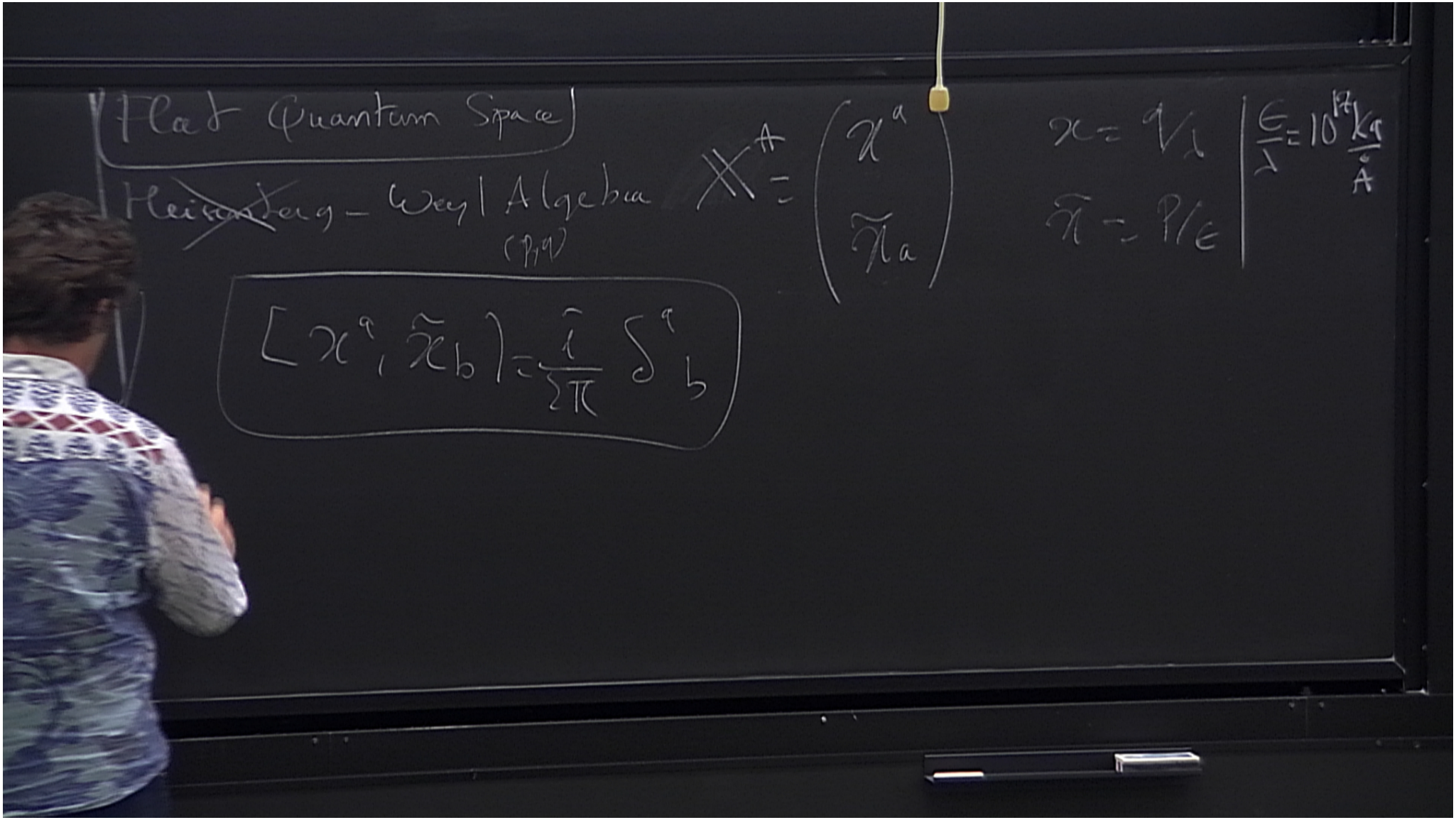
: Quantum is the Fundamental than classical.

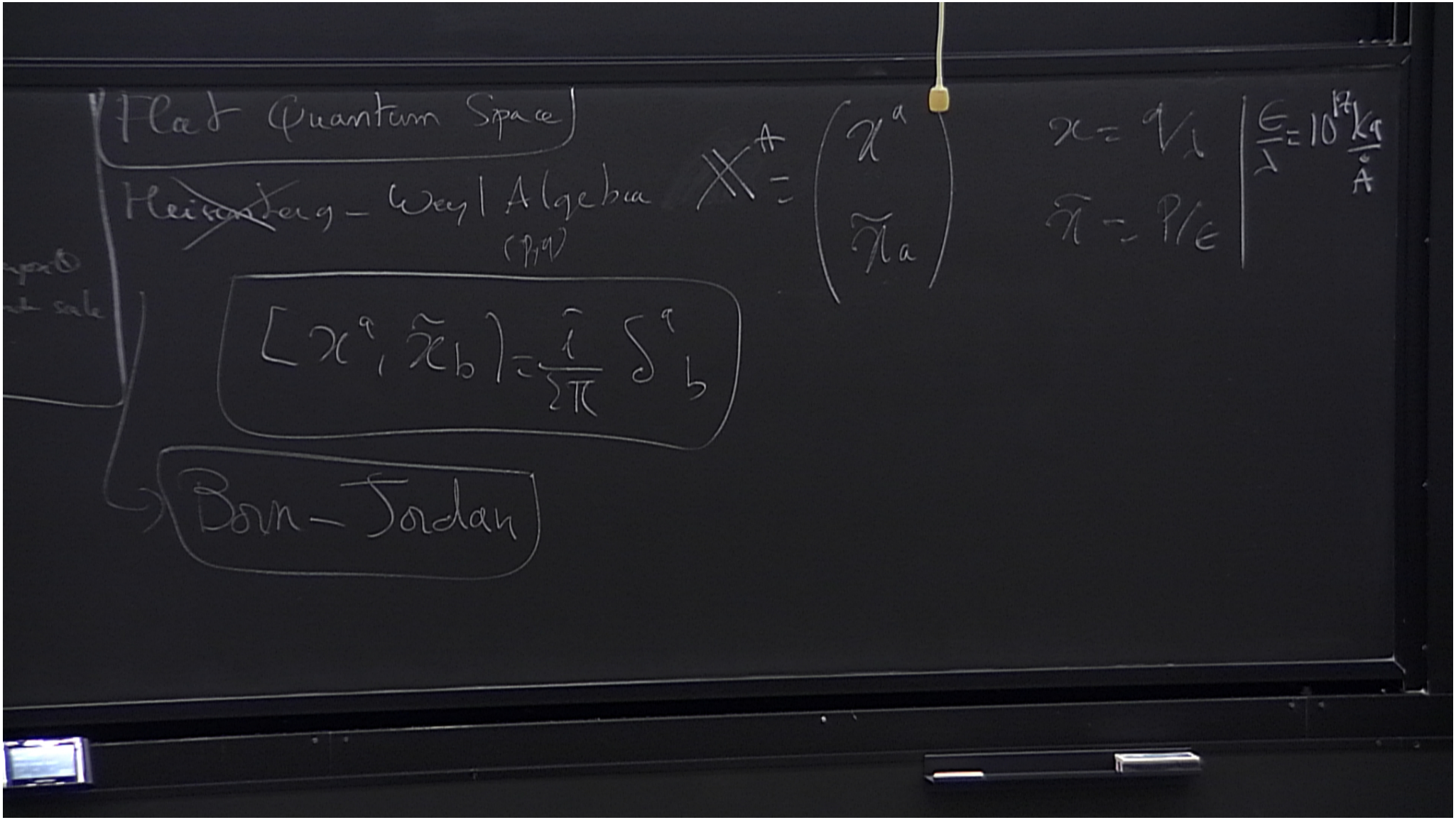
: Special Relativity + Quantum needs a new physical principle → SiRta Superposition
SiRta Fundamental scale

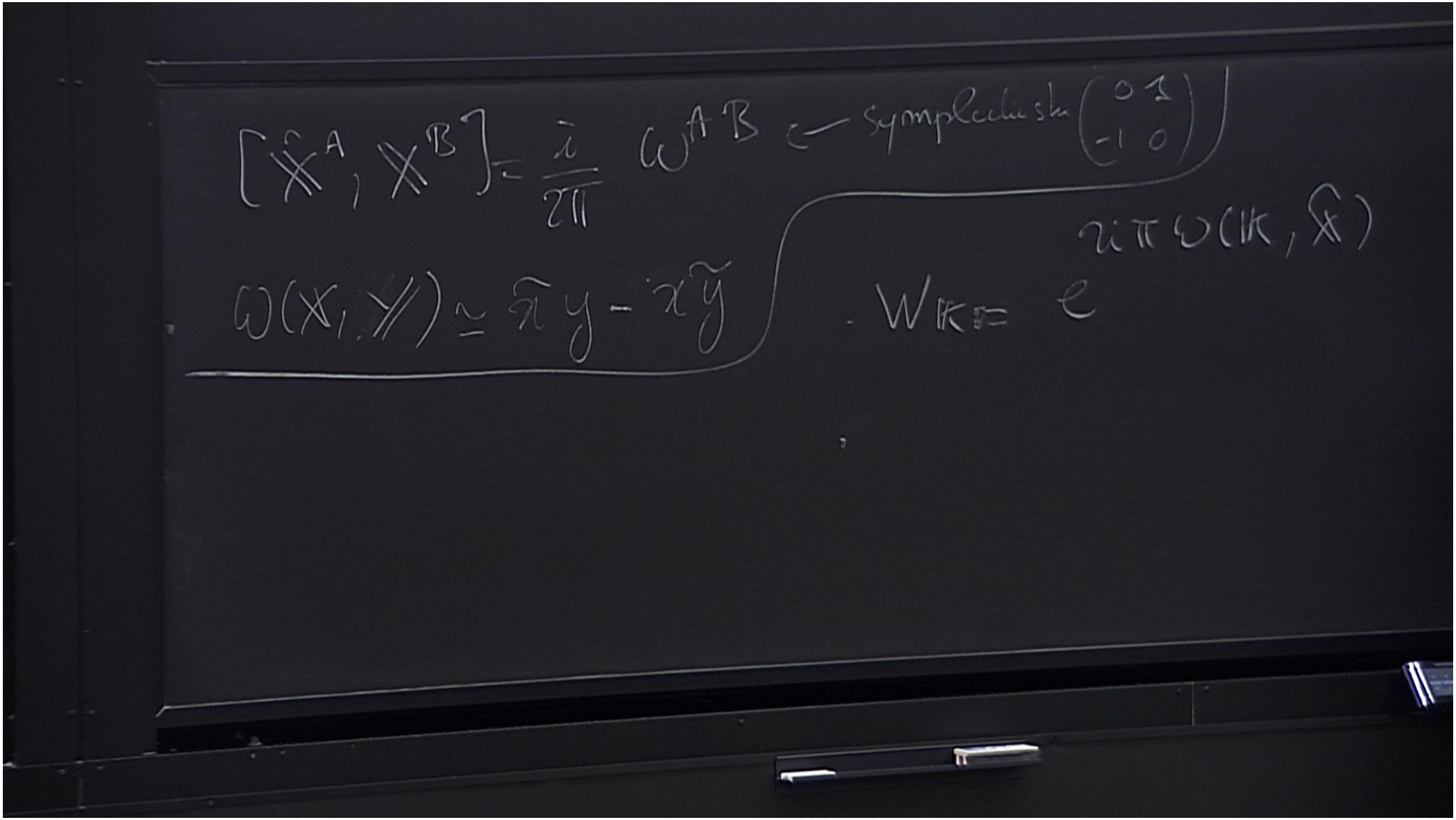
Modular Space II

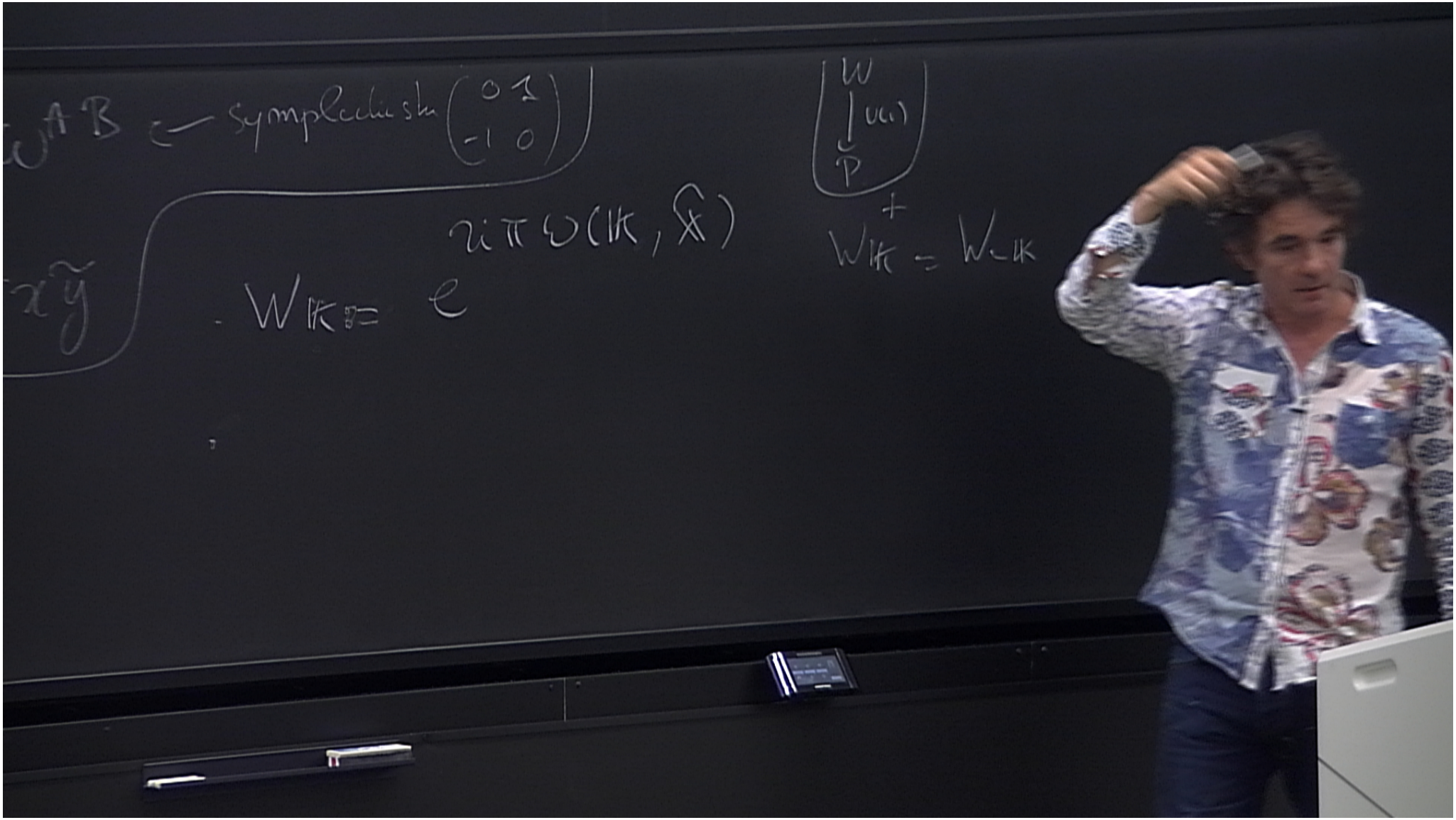
- Quantum is more Fundamental than classical.
- Special Relativity + Quantum needs a new physical principle → SR to Superposition / SR to Fundamental scale
- Space is a Polarisation
  ↪ Commutative * subalgebra

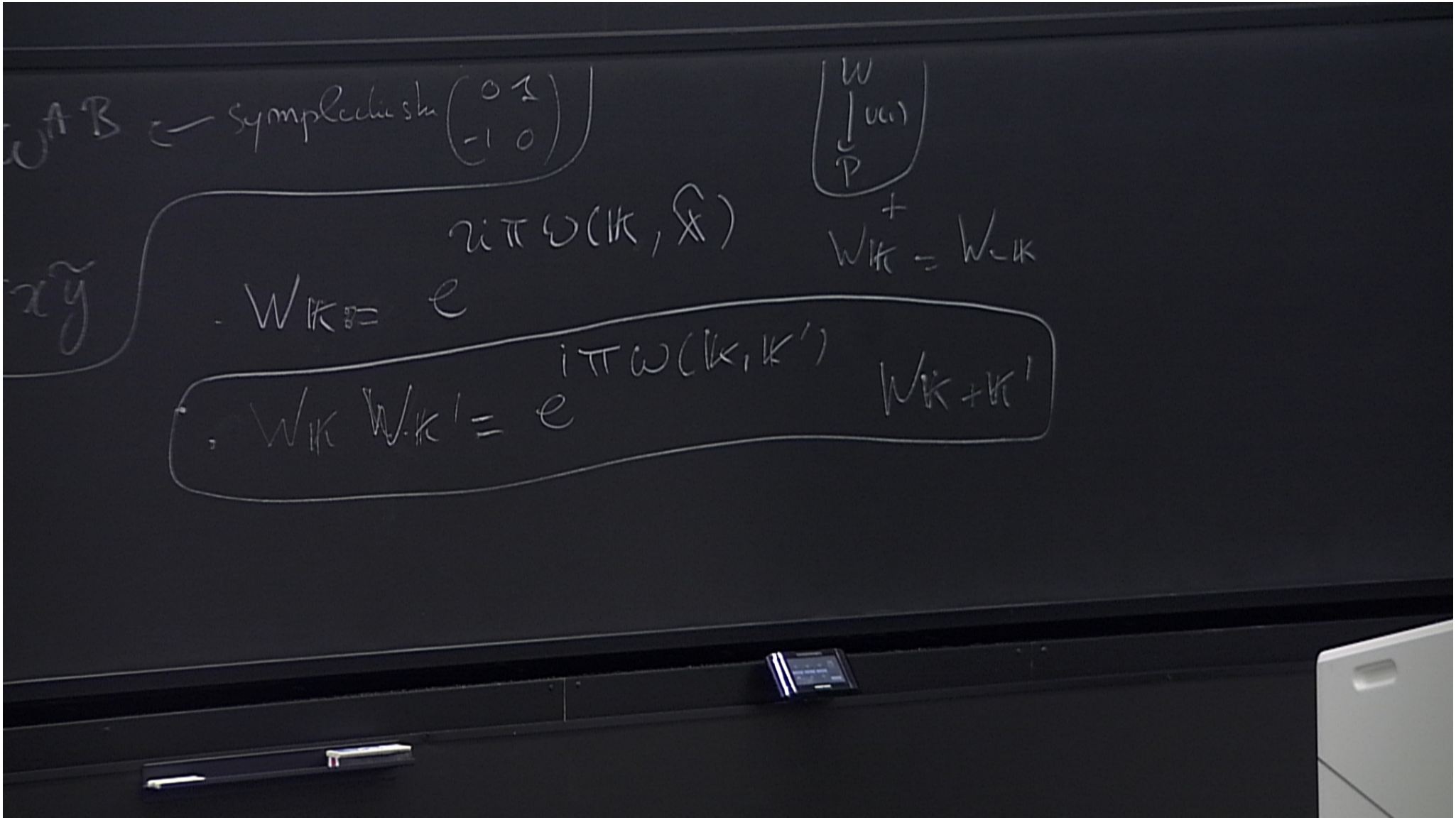

Flat Quantum Space
Heisenberg - Weyl Algebra
(P,Q)
$\hat{X}^A = \begin{pmatrix} \hat{x}^a \\ \hat{\pi}_a \end{pmatrix}$
$x = q/\lambda$
$\hat{\pi} = P/\epsilon$
$\frac{\epsilon}{\lambda}$

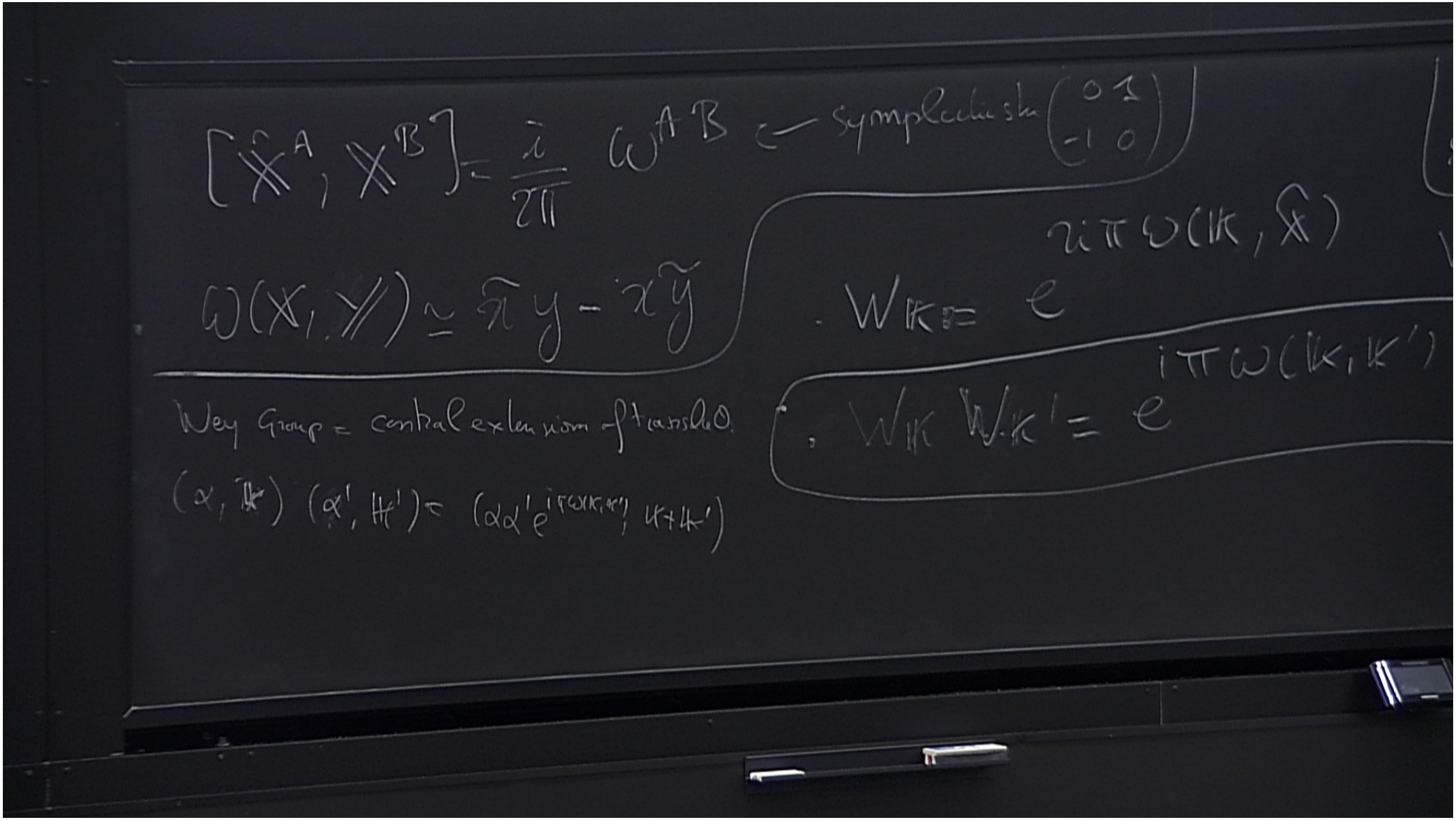

Flat Quantum Space
Heisenberg - Weyl Algebra
(P,Q)
$[x^a, \tilde{x}_b] = \frac{i}{\hbar} \delta^a_b$
$\hat{X}^A = \begin{pmatrix} x^a \\ \tilde{x}_a \end{pmatrix}$
$x = q/\lambda$
$\tilde{x} = p/\epsilon$
$\frac{\epsilon}{\lambda} = 10^{17} \frac{kg}{A}$

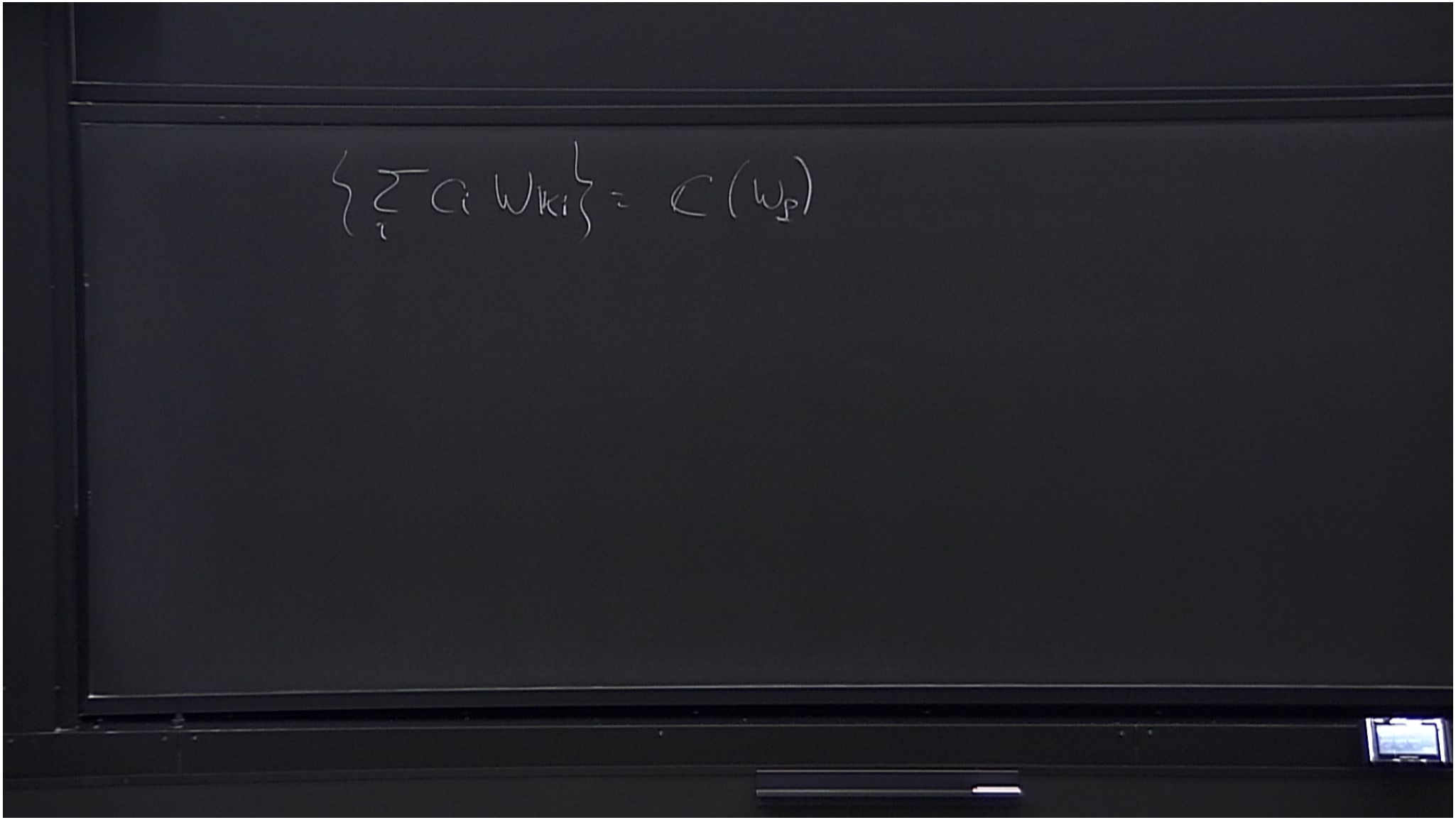

Flat Quantum Space
Heisenberg - Weyl Algebra
(P,Q)
$\hat{X}^A = \begin{pmatrix} x^a \\ \tilde{\pi}_a \end{pmatrix}$
$x = q/\lambda$
$\tilde{\pi} = p/\epsilon$
$\frac{\epsilon}{\lambda} = 10^{19} \frac{kg}{A}$
$[x^a, \tilde{x}_b] = \frac{i}{\lambda\pi} \delta^a{}_b$
Born-Jordan

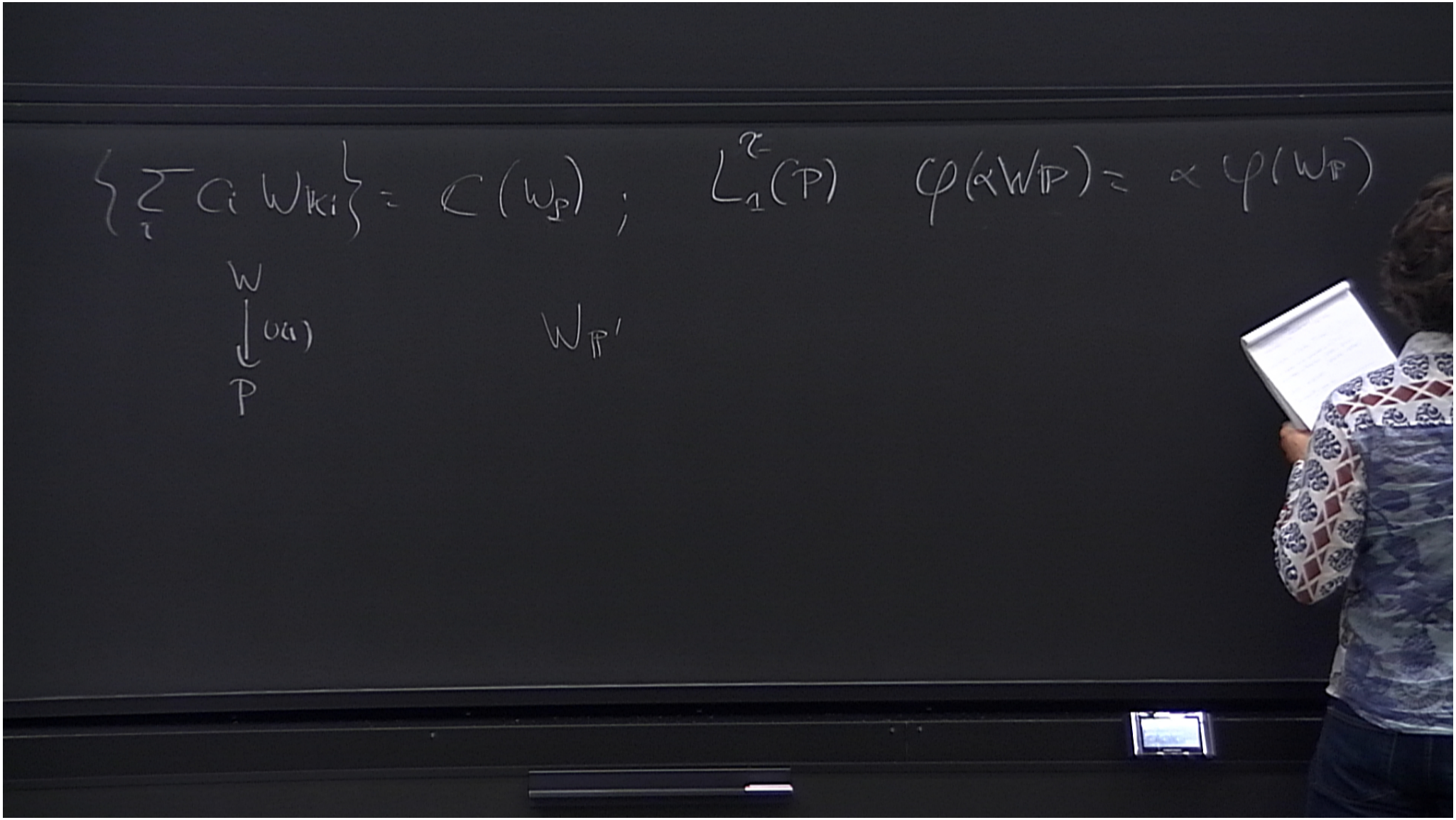

$[\hat{X}^A, \hat{X}^B] = \frac{i}{2\pi}\,\omega^{AB}$ ← symplectic $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$\omega(X, Y) = \tilde{x}y - x\tilde{y}$
$W_K = e^{2i\pi\omega(K, \hat{x})}$


symplectic
$W_{\mathbb{K}} = e^{2i\pi\omega(\mathbb{K},\mathbb{K})}$
$W_{\mathbb{K}}^{+} = W_{-\mathbb{K}}$


ω^{AB} ← symplectic $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$x\tilde{y}$
$W_k = e^{i\pi\omega(k,\hat{x})}$
$W_k W_{k'} = e^{i\pi\omega(k,k')} W_{k+k'}$
$W_k^+ = W_{-k}$


$[\hat{X}^A, \hat{X}^B] = \frac{i}{2\pi}\omega^{AB}$ ← symplectic $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$\omega(X, Y) = \tilde{x}y - x\tilde{y}$
$W_K := e^{2i\pi\omega(K, \hat{X})}$
$W_K W_{K'} = e^{i\pi\omega(K, K')}$
Weyl Group = central extension of translat.
$(\alpha, K)(\alpha', K') = (\alpha\alpha' e^{i\pi\omega(K,K')}, K+K')$


$\{\sum_i c_i w_{ki}\} = \mathcal{C}(w_s)$

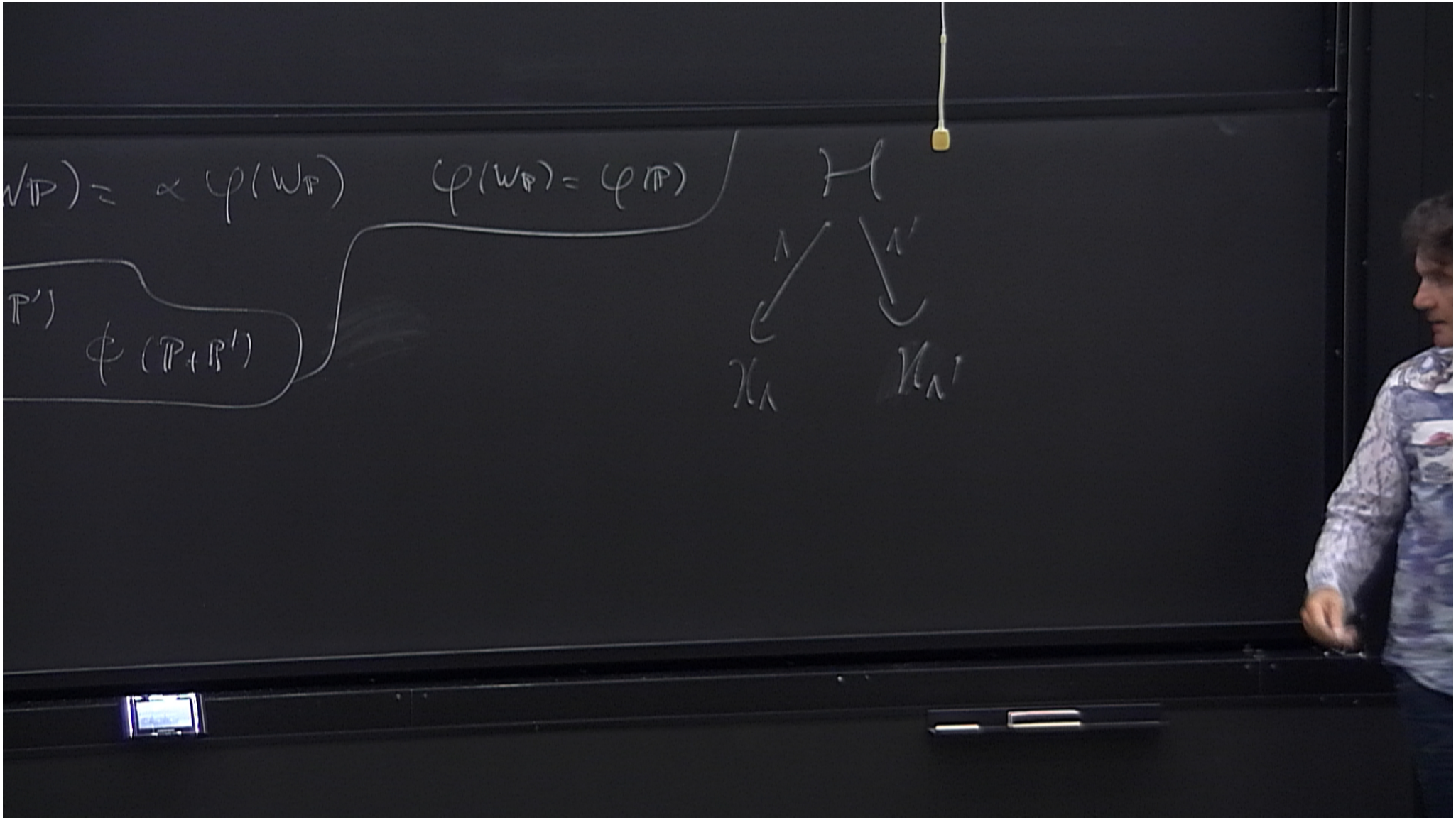

$\{\sum_i c_i W_{ki}\} = C(W_j)$;
$L_1^2(P)$
$\varphi(\alpha W_P) = \alpha \varphi(W_P)$
W
$\downarrow \omega$
P
$W_{P'}$

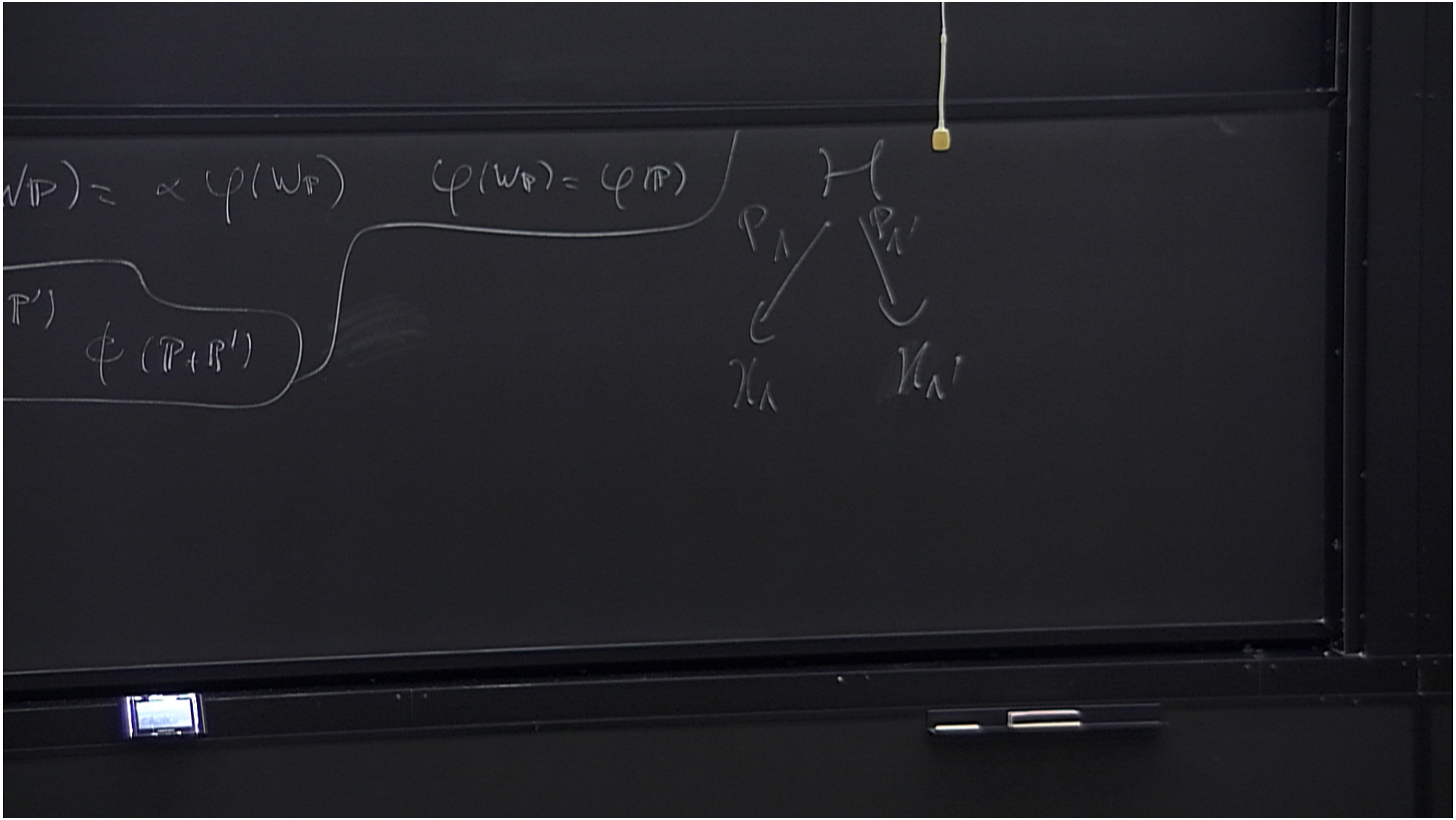
$\} = \mathcal{C}(W_P)$; $L_1^{\mathcal{C}}(\mathcal{P})$ $\varphi(\alpha W_{\mathcal{P}}) = \alpha\, \varphi(W_{\mathcal{P}})$ $\varphi(W_{\mathcal{P}}) = \varphi(\mathcal{P})$

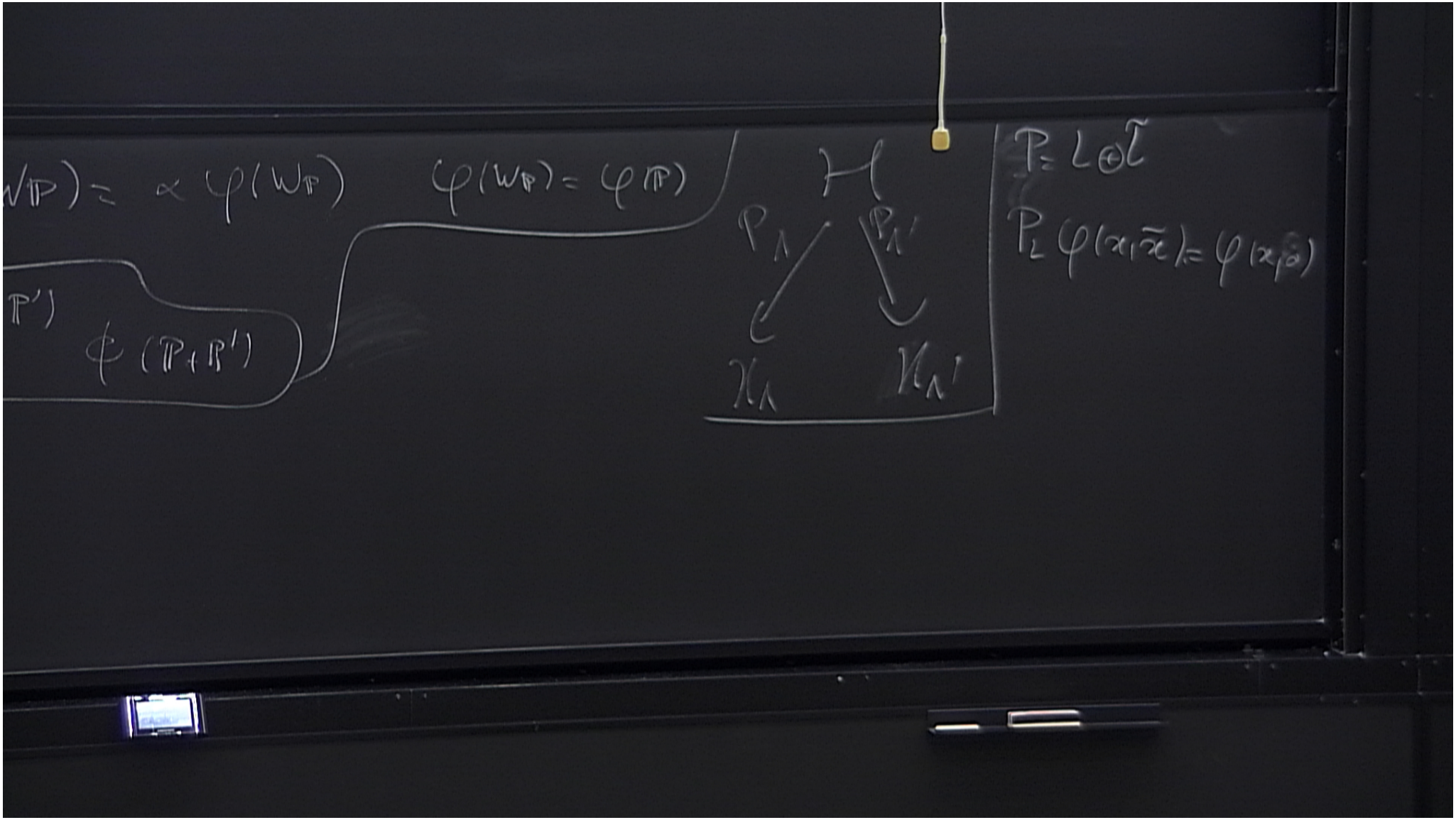
$$W_{\mathcal{P}'}\, \phi(\mathcal{P}) = e^{i\omega(\mathcal{P},\mathcal{P}')}\, \phi(\mathcal{P}+\mathcal{P}')$$

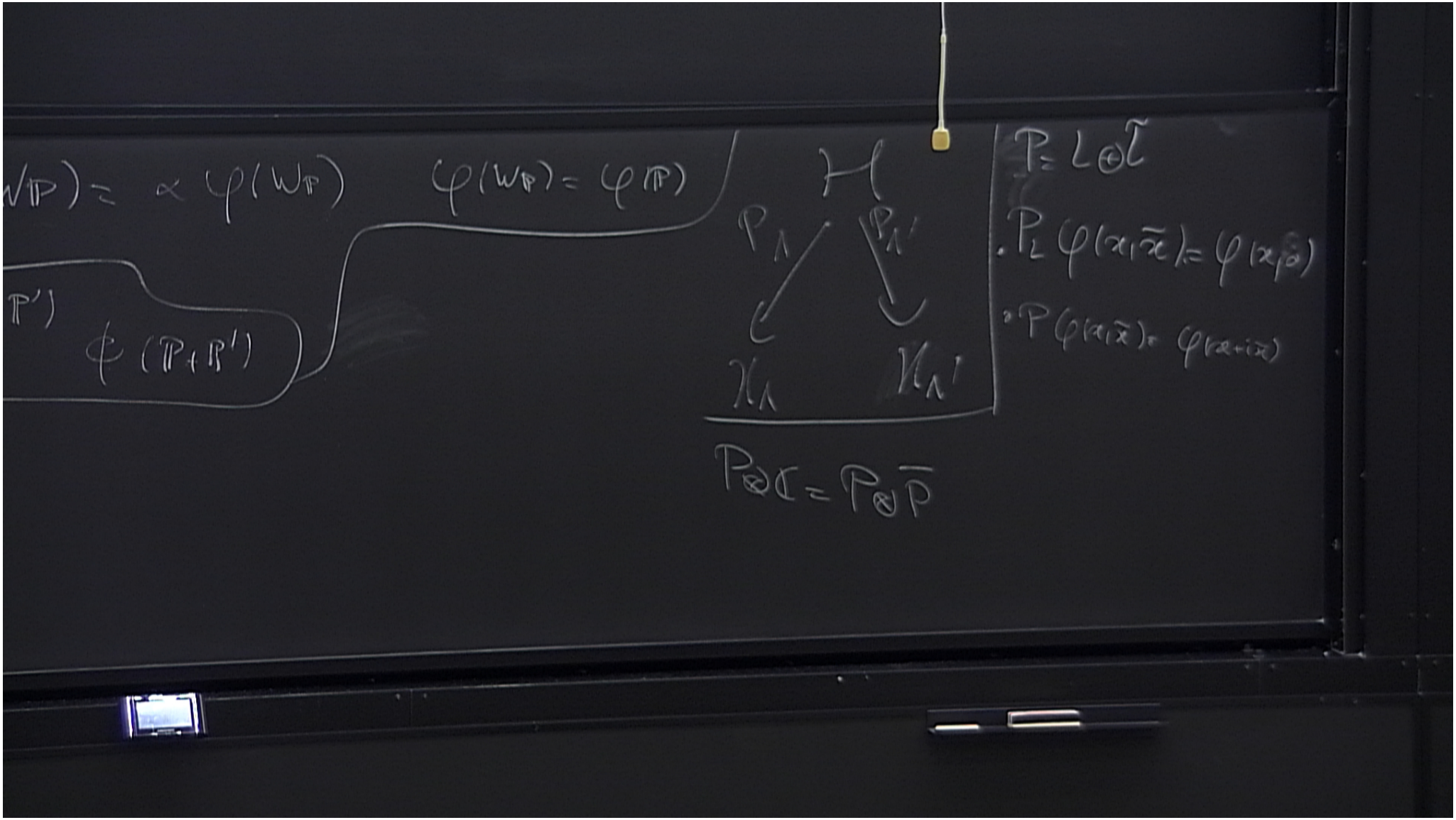
$\} = \mathcal{C}(W_P)$; $\mathcal{H} = L^2_1(\mathbb{P})$ $\varphi(\alpha W_P) = \alpha \varphi(W_P)$ $\varphi(W_P) = \varphi(\mathbb{P})$

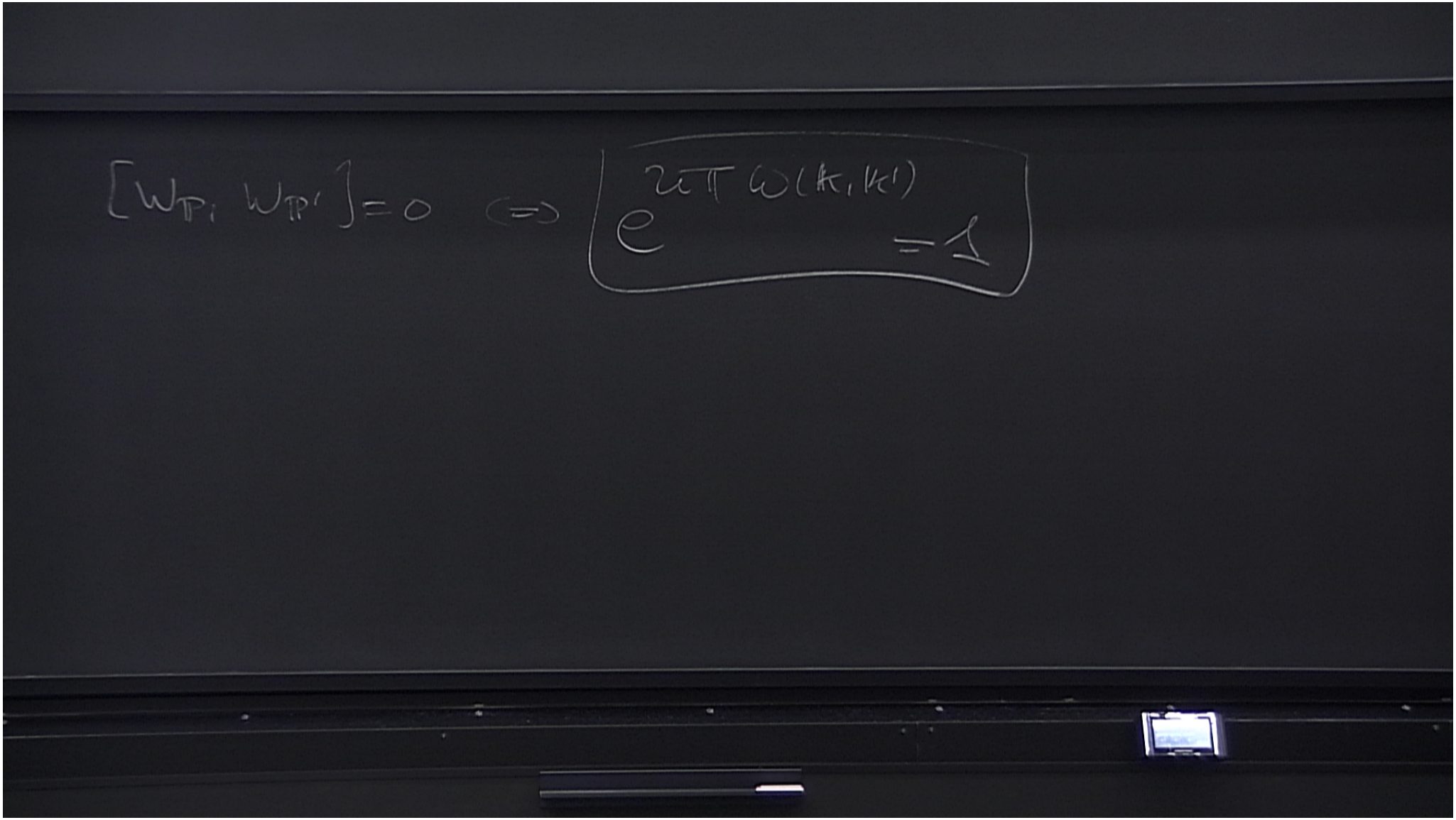
$$W_{P'} \phi(\mathbb{P}) = e^{i\omega(P,P')} \phi(\mathbb{P}+P')$$


$\varphi(W_{\mathbb{P}}) = \alpha\,\varphi(W_{\mathbb{P}})$
$\varphi(W_{\mathbb{P}}) = \varphi(\mathbb{P})$
$\mathbb{P}')$
$\varphi(\mathbb{P}+\mathbb{P}')$
$\mathcal{H}($
χ_λ
$\chi_{\lambda'}$




$\varphi(W_{\mathbb{P}}) = \propto \varphi(W_{\mathbb{P}})$
$\varphi(W_{\mathbb{P}}) = \varphi(\mathbb{P})$
$\mathbb{P}'$
$\varphi(\mathbb{P}+\mathbb{P}')$
$\mathcal{H}($
P_Λ
$P_{\Lambda'}$
χ_Λ
$\chi_{\Lambda'}$
$P = L \oplus \tilde{L}$
$P_L \varphi(x+\tilde{x}) = \varphi(x+0)$


$\varphi(W_{\mathbb{P}}) = \alpha\,\varphi(W_{\mathbb{P}})$
$\varphi(W_{\mathbb{P}}) = \varphi(\mathbb{P})$
$\mathbb{P}')$
$\varphi(\mathbb{P}+\mathbb{P}')$
$\mathcal{H}($
P_Λ
$P_{\Lambda'}$
$\mathcal{H}_\Lambda$
$\mathcal{H}_{\Lambda'}$
$P \otimes \mathbb{C} = P \otimes \bar{P}$
$P = L \oplus \tilde{L}$
$\cdot P_L\,\varphi(x,\tilde{x}) = \varphi(x,0)$
$\cdot P\,\varphi(x,\tilde{x}) = \varphi(x+i\tilde{x})$


$[W_{TP}, W_{TP'}] = 0 \iff e^{2\pi T\,\omega(k,k')} = 1$

$[W_P, W_{P'}] = 0 \iff e^{2\pi i \omega(k,k')} = 1$

Maximally commuting $\Leftrightarrow$ Integral Self-dual Lattices of P
& subalgebra of W_P

$(\lambda, P) \to \lambda + P$

$= 0 \iff e^{2\pi i\, \omega(k,k')} = 1$

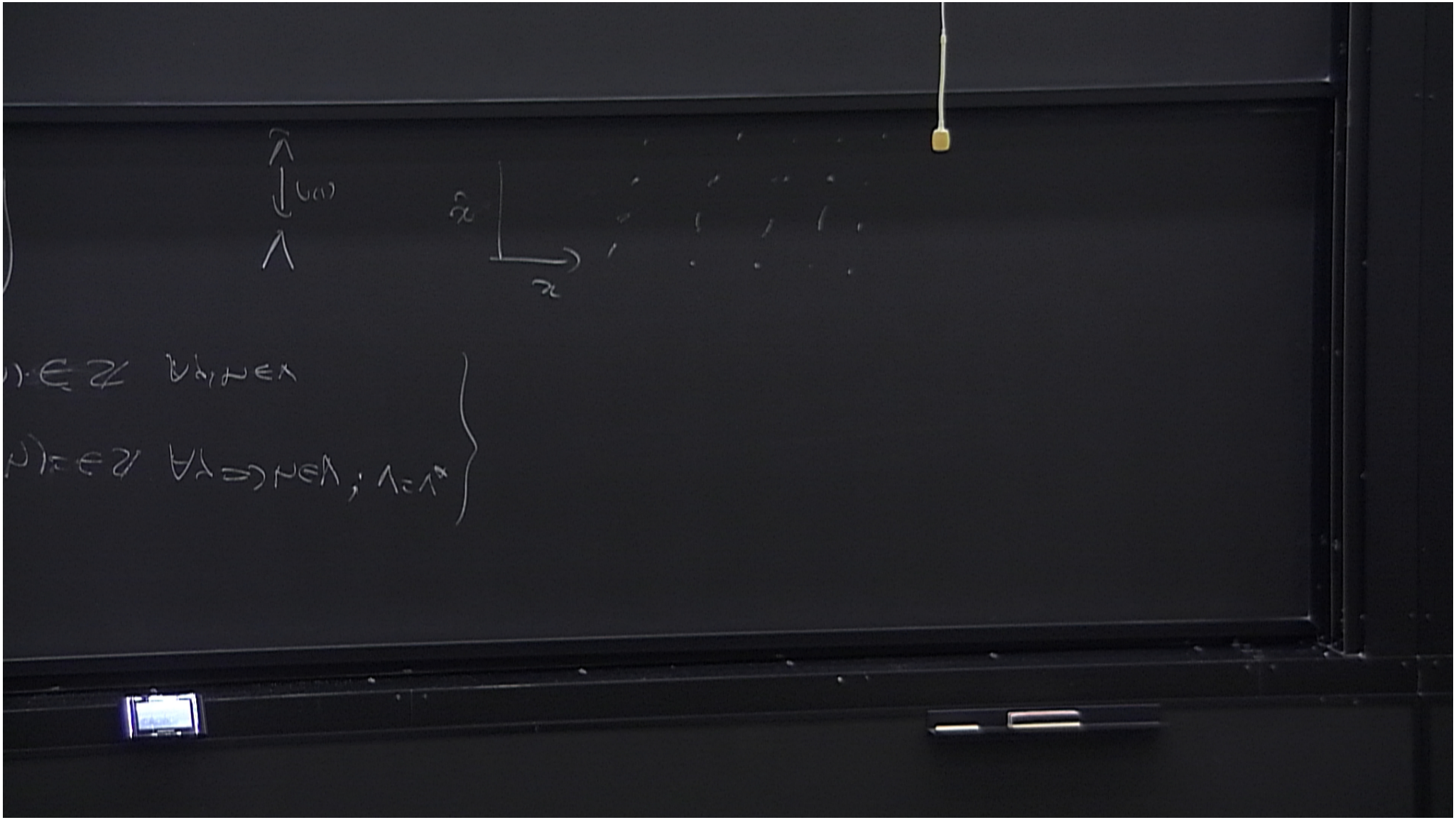
$\Lambda \supseteq \Lambda' \supseteq \Lambda$

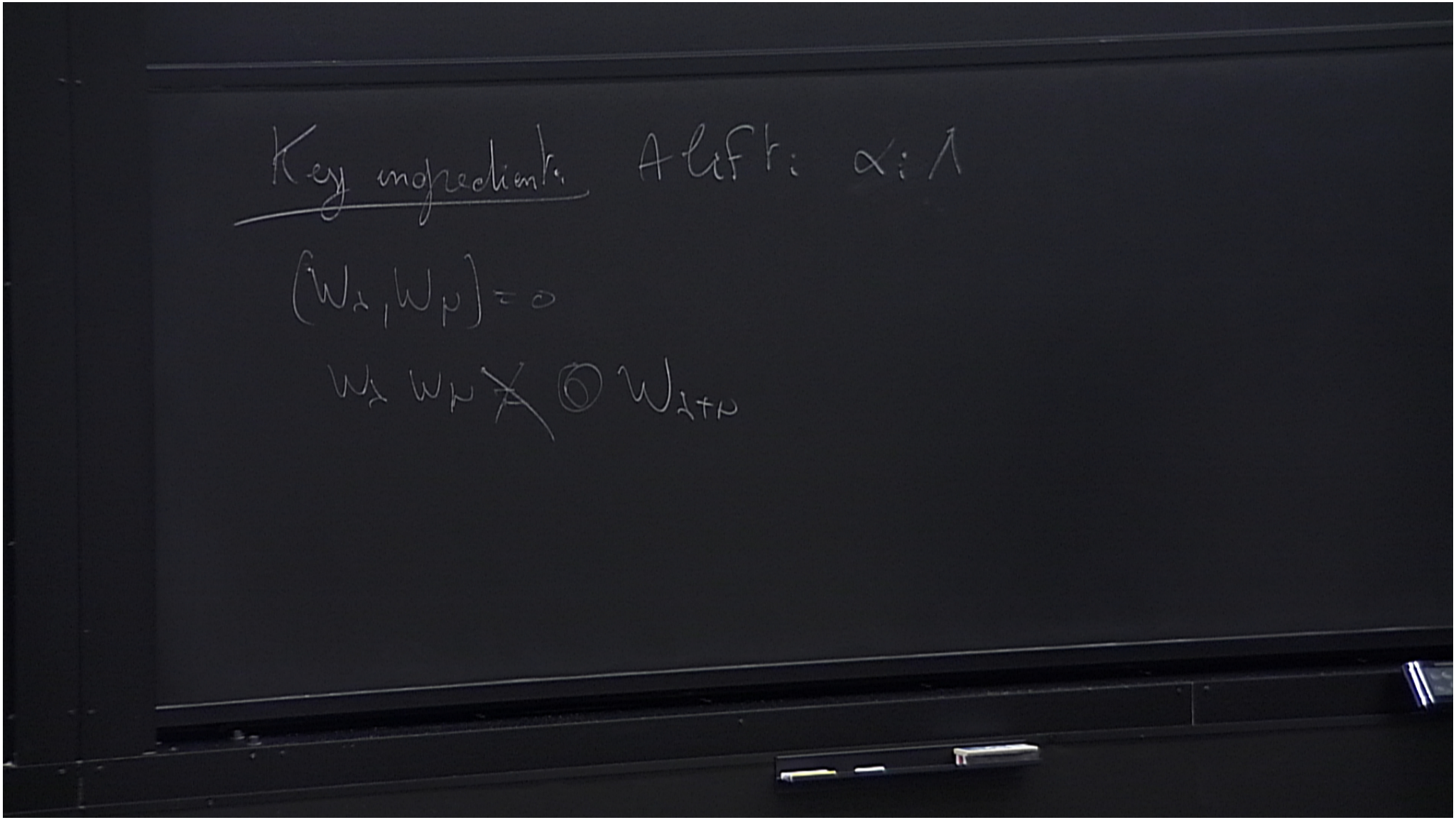
$\iff$ Integral $\cdot\ \omega(\lambda,\mu) \in \mathbb{Z} \quad \forall \lambda,\mu \in \Lambda$

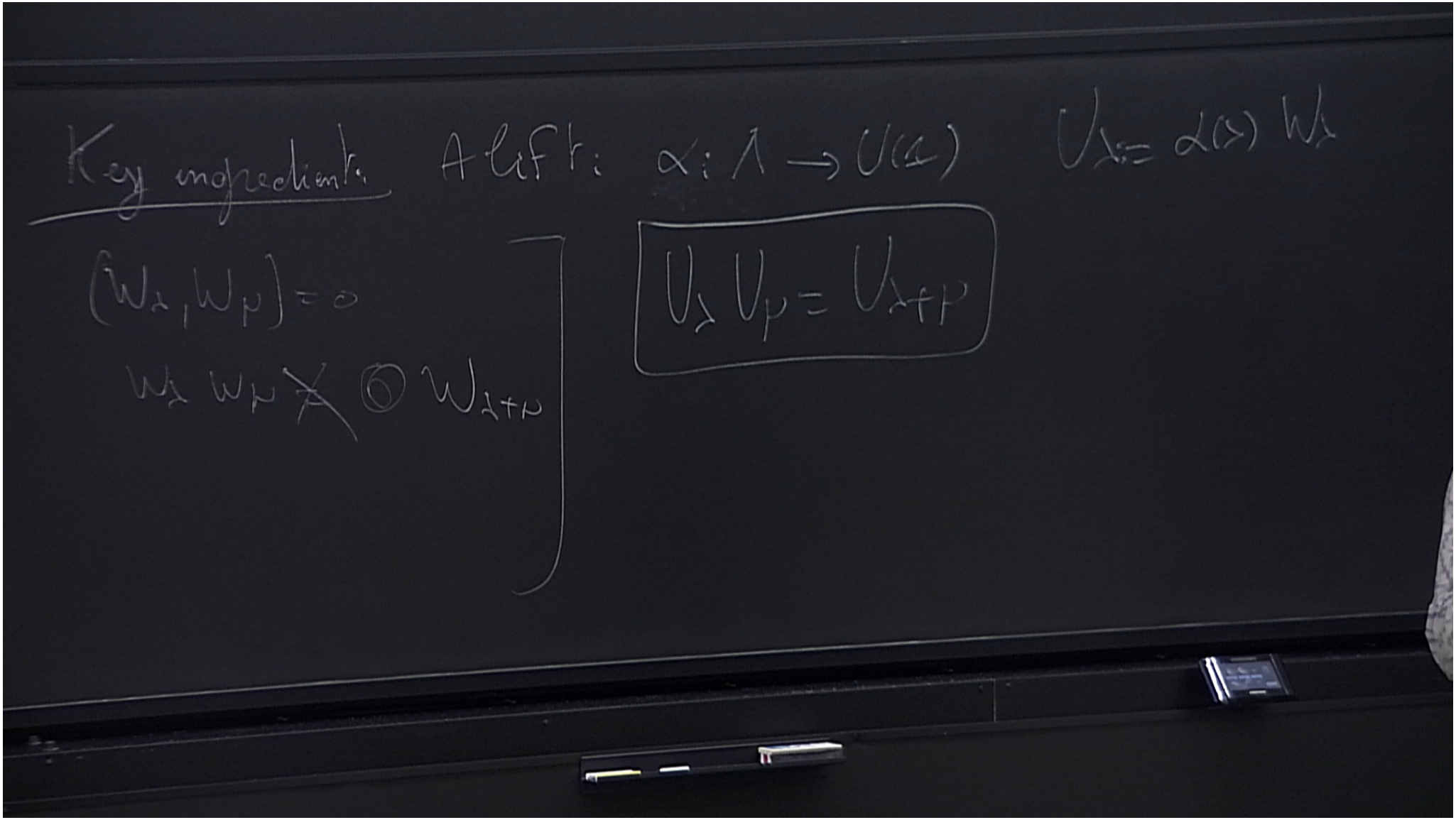
Self-dual $\cdot\ \omega(\lambda,\mu) = \in \mathbb{Z} \quad \forall \lambda \Rightarrow \mu \in \Lambda\,;\ \Lambda = \Lambda^*$

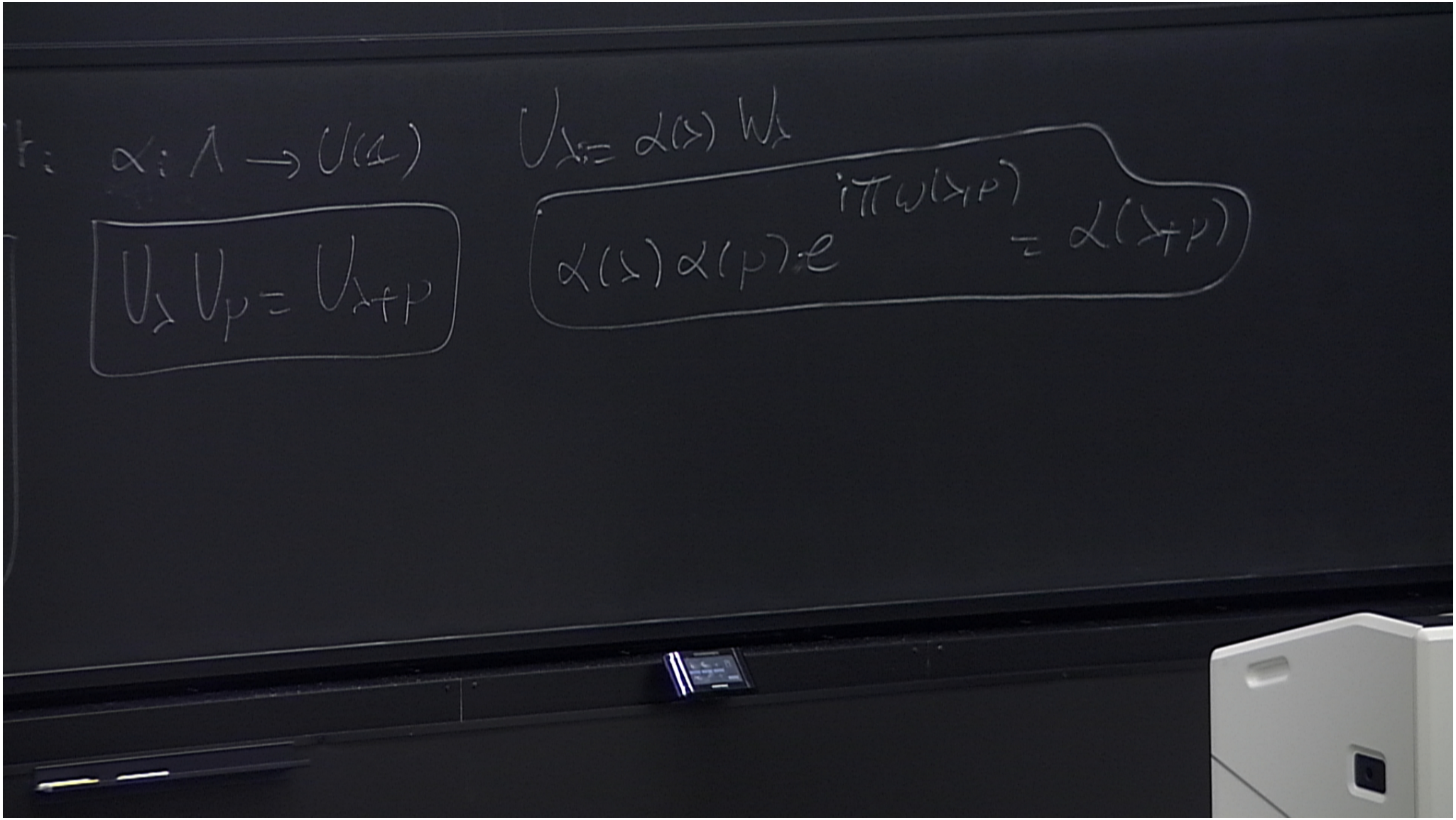
Lattices of $\underline{P}$

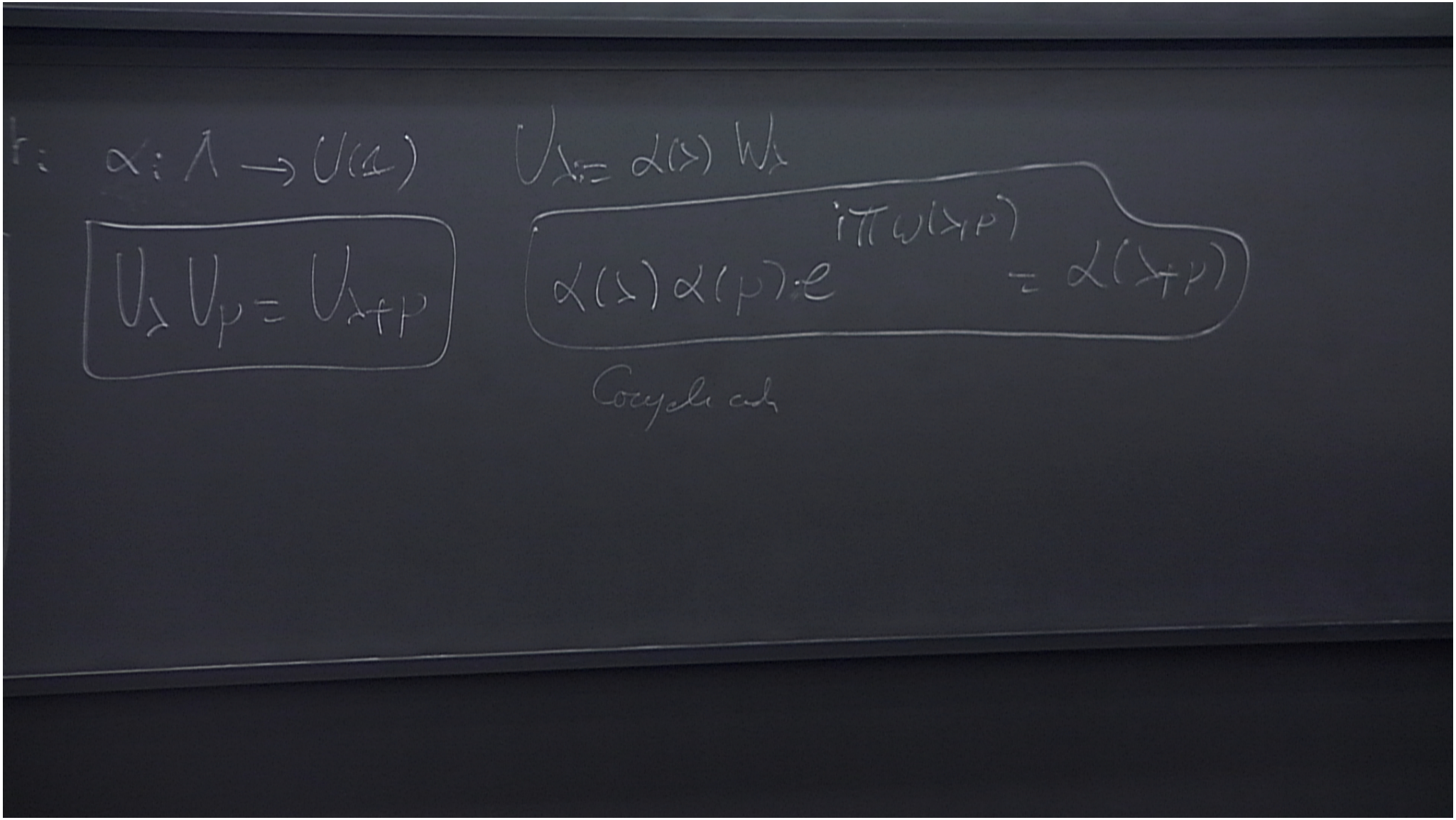
$(\lambda,\mu) \to \lambda + \mu$

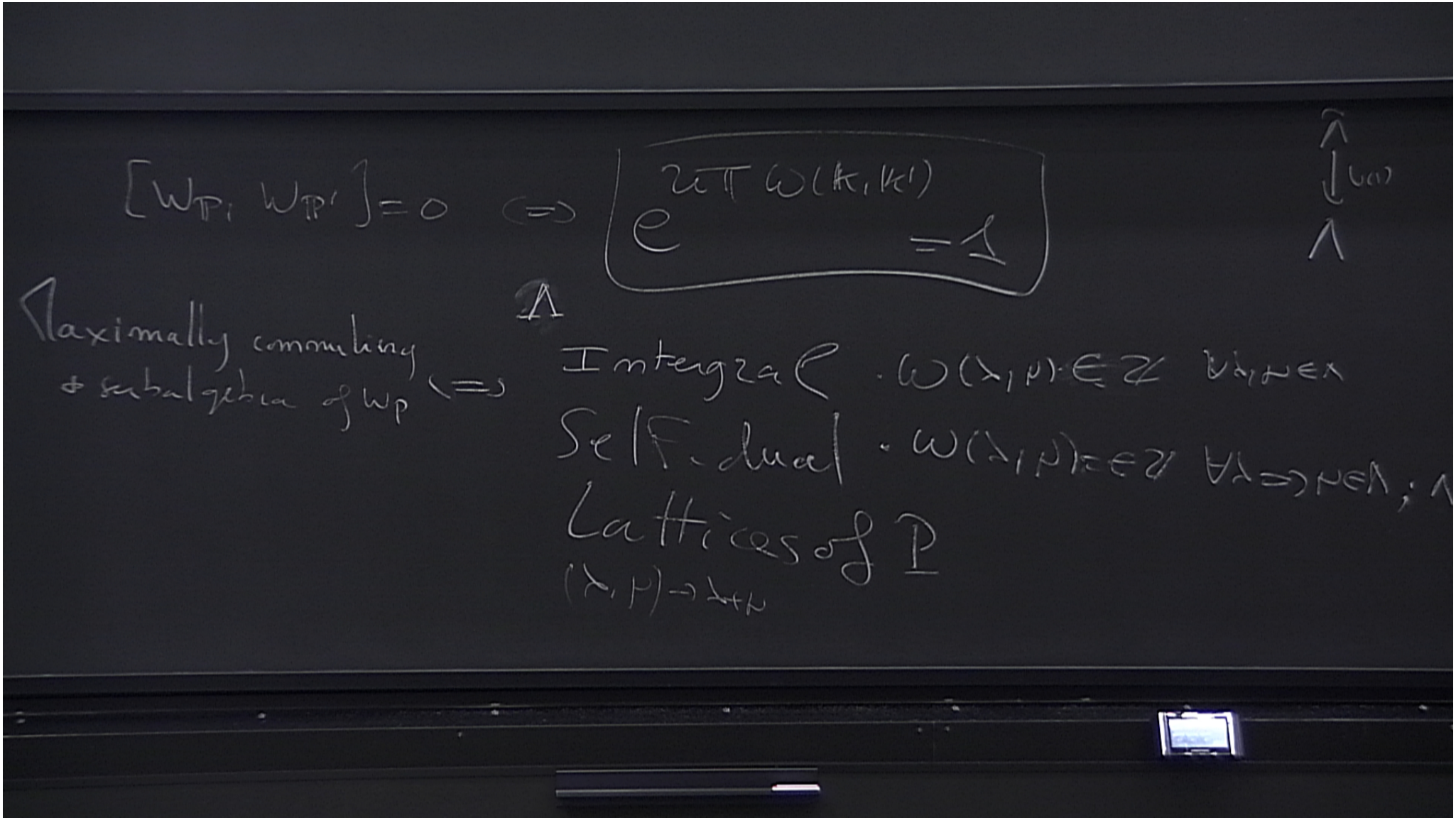


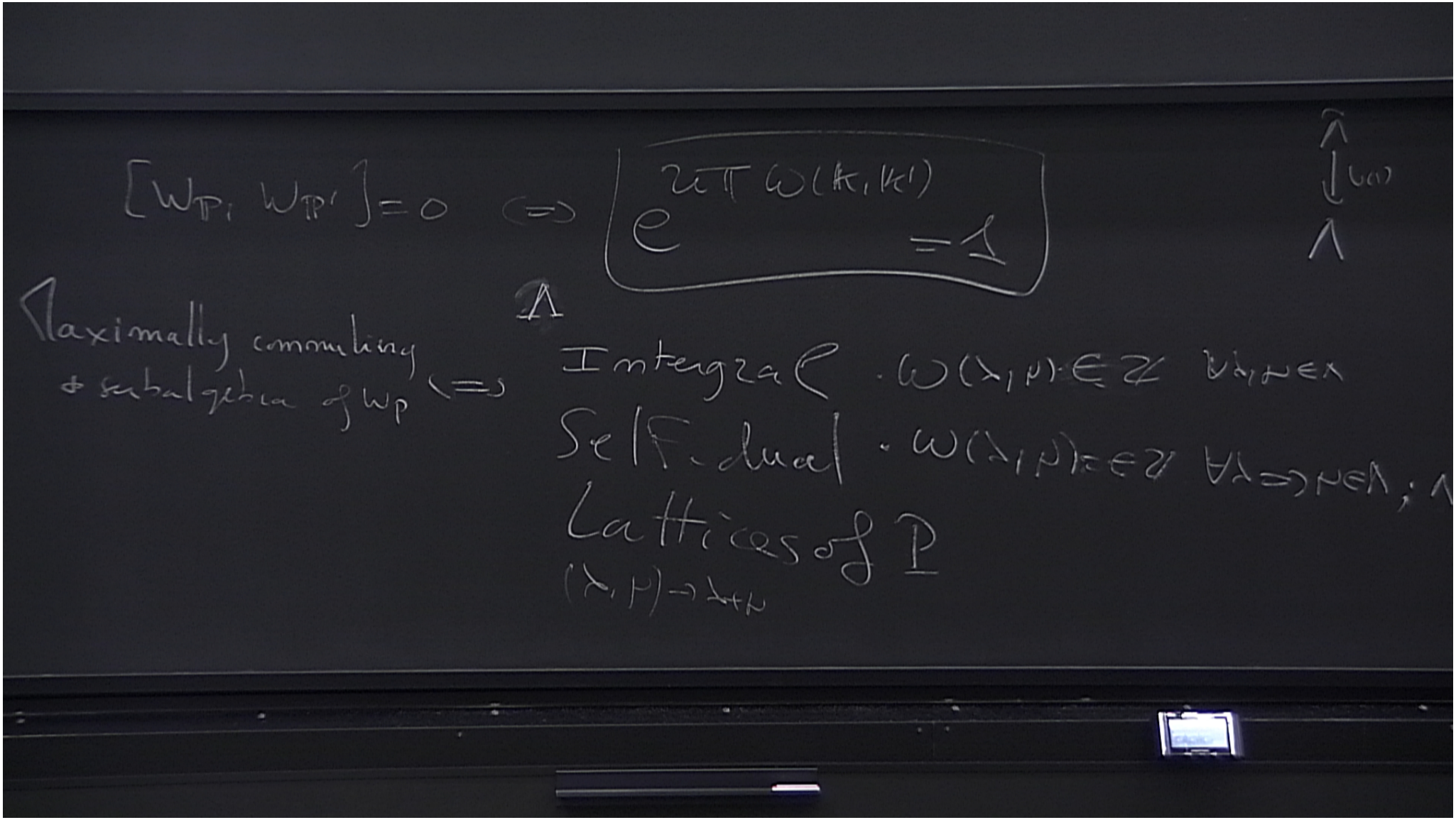

Key ingredient: A lift: $\alpha : \Lambda$
$(W_1, W_N) = 0$
$W_1 \; W_N \not\subset \mathcal{O} \; W_{1+N}$

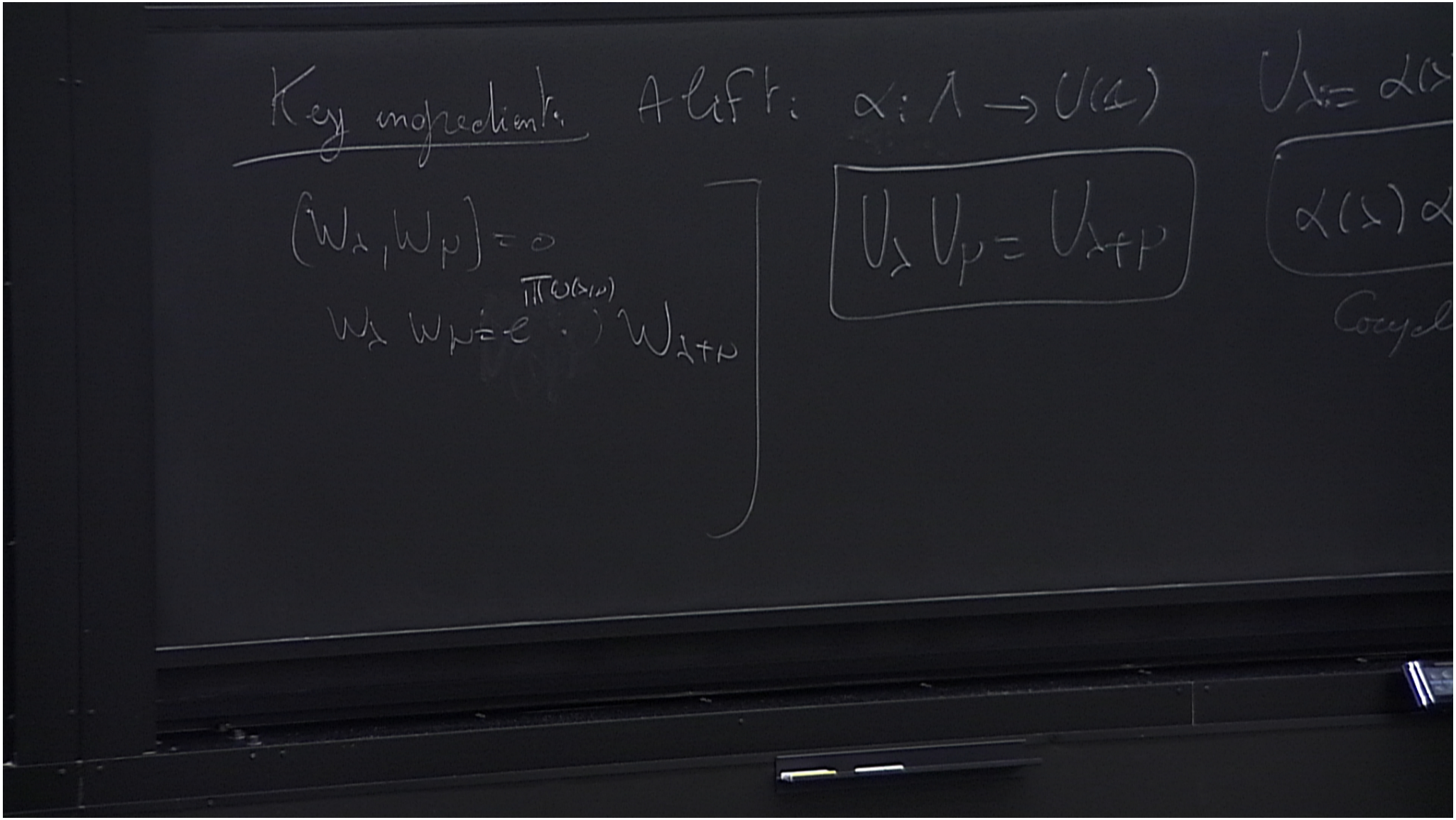

Key ingredients: A lift: $\alpha : \Lambda \to U(\mathcal{A})$ $U_\lambda := \alpha(\lambda) W_\lambda$
$(W_\lambda, W_\mu) = 0$
$W_\lambda W_\mu \not\subset \mathcal{O} W_{\lambda+\mu}$
$U_\lambda U_\mu = U_{\lambda+\mu}$

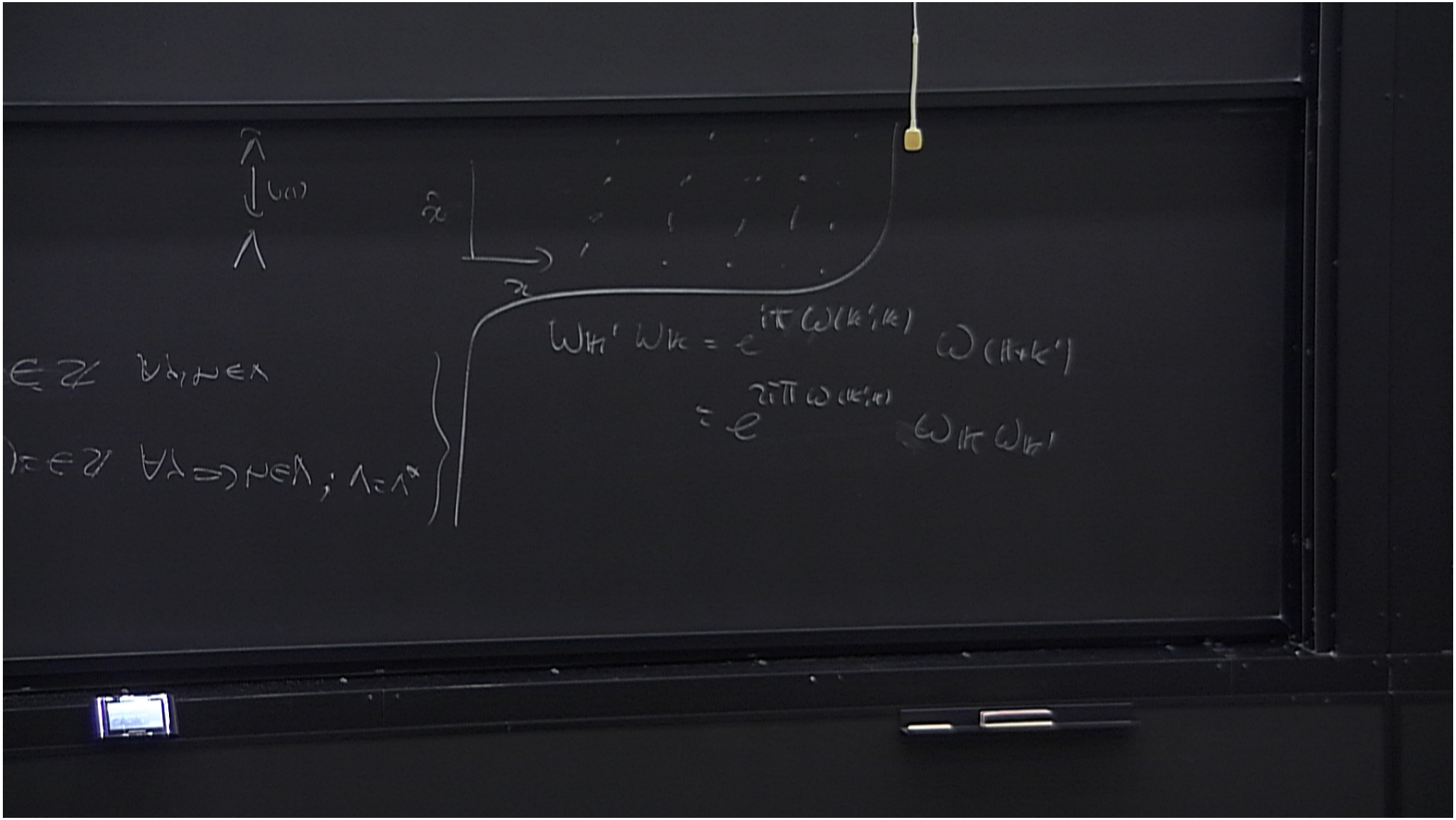

$\alpha : \Lambda \to U(1)$
$U_\Lambda = \alpha(\Lambda)\, W_\Lambda$
$U_\Lambda U_P = U_{\Lambda+P}$
$\alpha(\Lambda)\alpha(P) = e^{i\pi\omega(\Lambda,P)}\,\alpha(\Lambda+P)$

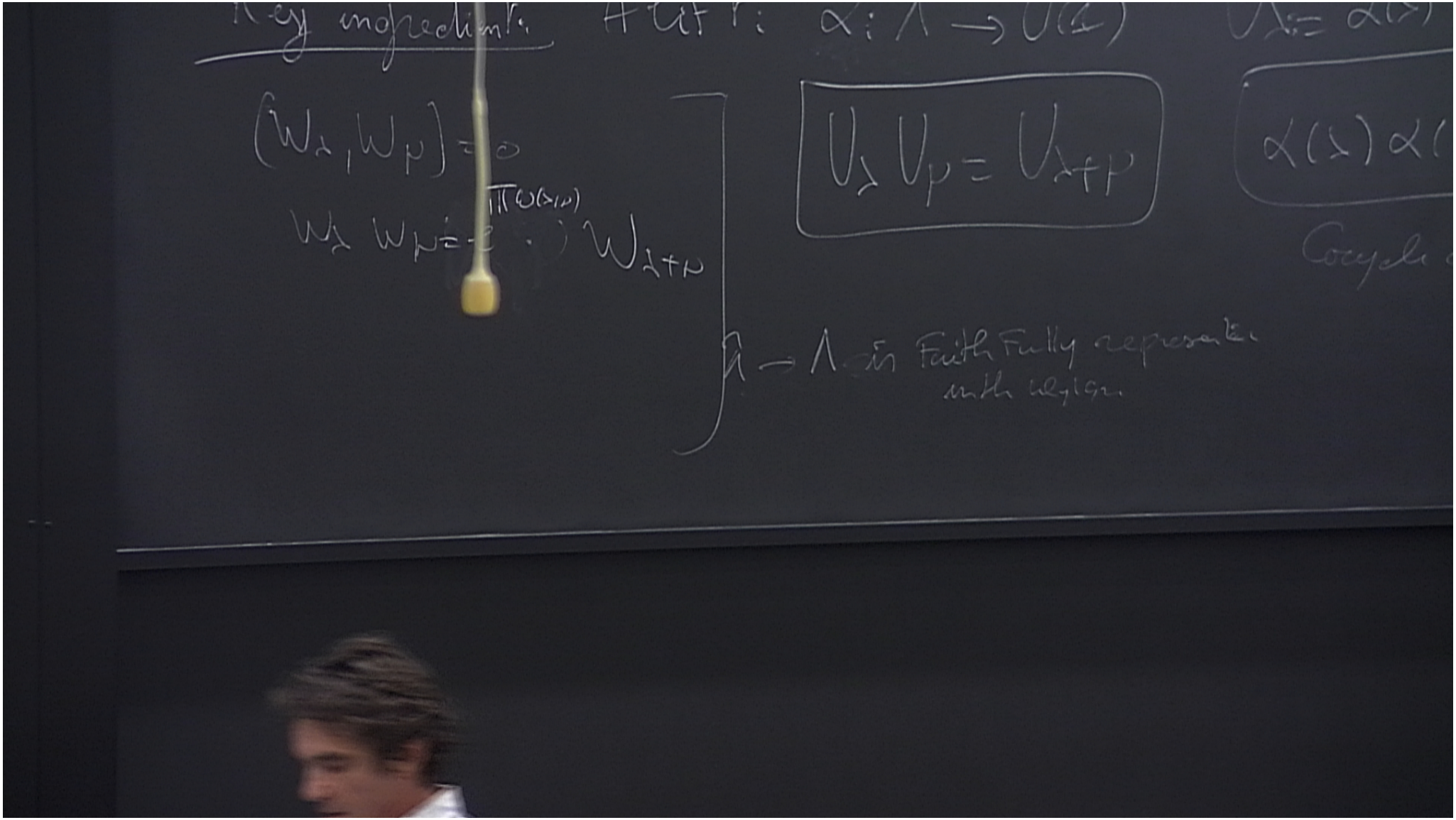

$\alpha : \Lambda \to U(1)$
$U_\lambda = \alpha(\lambda) W_\lambda$
$U_\lambda U_\mu = U_{\lambda+\mu}$
$\alpha(\lambda)\alpha(\mu) e^{i\pi\omega(\lambda,\mu)} = \alpha(\lambda+\mu)$
Cocycle cond

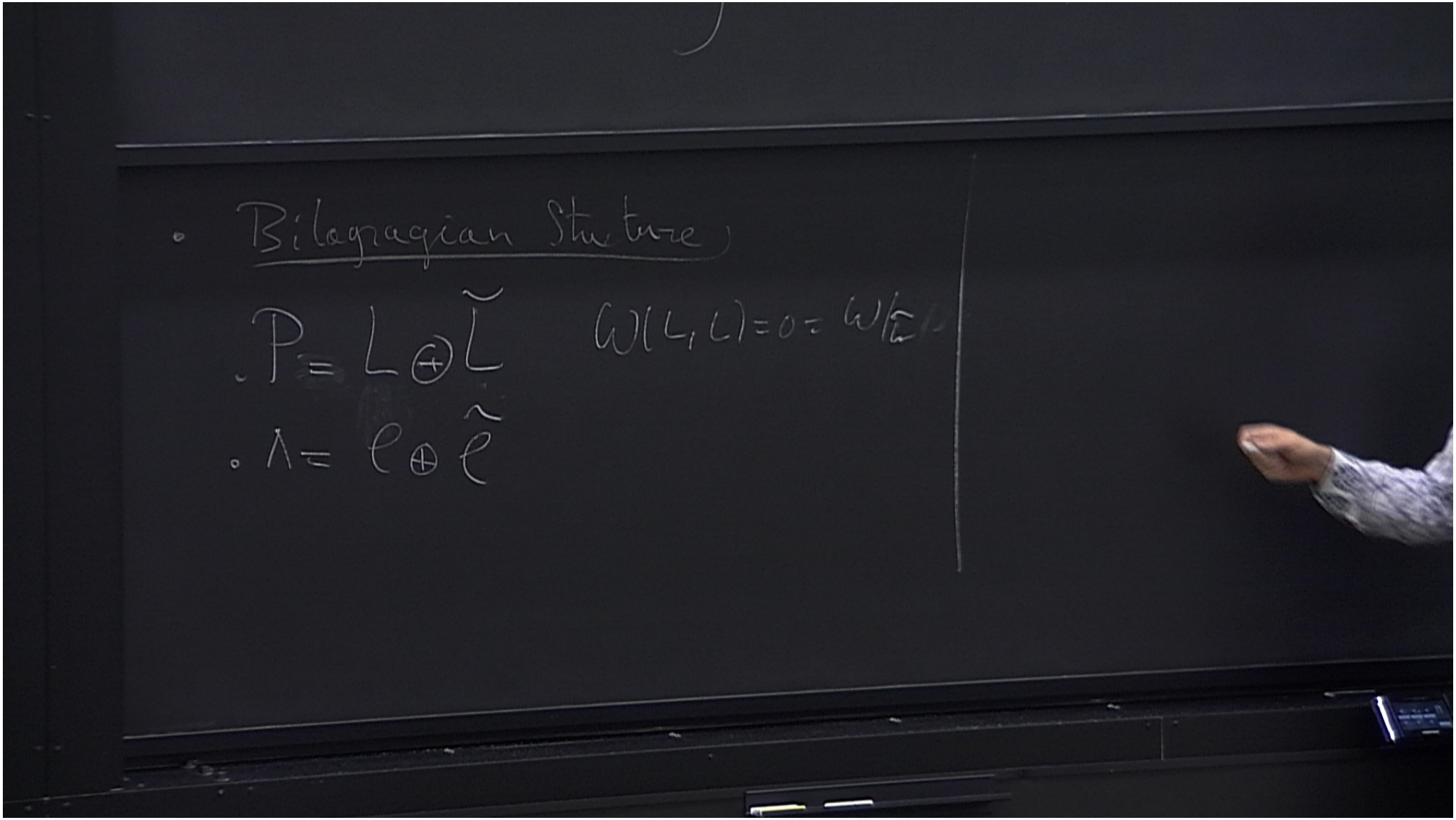

$[W_P, W_{P'}] = 0 \iff e^{2\pi i \omega(k,k')} = 1$
$\omega(\cdot,\cdot)$
Maximally commuting
& subalgebra of W_P $\iff$
Λ
Integral $\cdot\ \omega(\lambda,\mu) \in \mathbb{Z}$ $\forall \lambda,\mu \in \Lambda$
Self-dual $\cdot\ \omega(\lambda,\mu) \in \mathbb{Z}$ $\forall \lambda \Rightarrow \mu \in \Lambda$;
Lattices of P
$(\lambda,\mu) \to \lambda+\mu$


$[W_P, W_{P'}] = 0 \iff e^{2\pi i \omega(k,k')} = 1$
ω
Maximally commuting subalgebra of W_P
$\Leftarrow \Lambda \Rightarrow$
Integral $\cdot \omega(\lambda, \mu) \in \mathbb{Z} \quad \forall \lambda, \mu \in \Lambda$
Self-dual $\cdot \omega(\lambda, \mu) = 0 \in \mathbb{Z} \quad \forall \lambda \Rightarrow \mu \in \Lambda$;
Lattices of P
$(\lambda, \mu) \to \lambda + \mu$

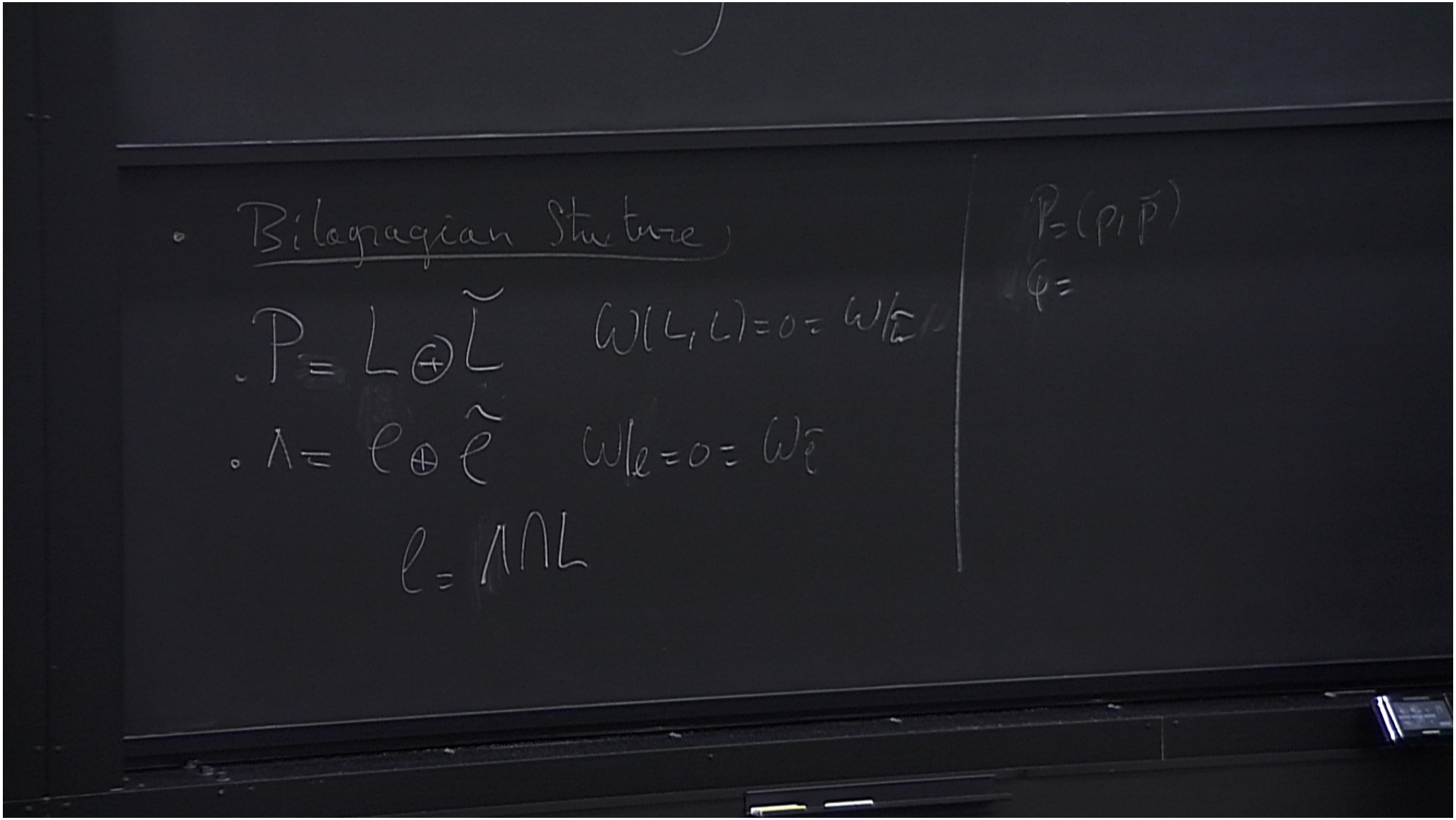

Key ingredients: A lift: $\alpha : \Lambda \to U(\mathcal{A})$
$U_\lambda = \alpha(\lambda$
$(W_\lambda, W_\mu) = 0$
$W_\lambda W_\mu = e^{i\pi\omega(\lambda,\mu)} W_{\lambda+\mu}$
$U_\lambda U_\mu = U_{\lambda+\mu}$
$\alpha(\lambda)\alpha$
Cocycl

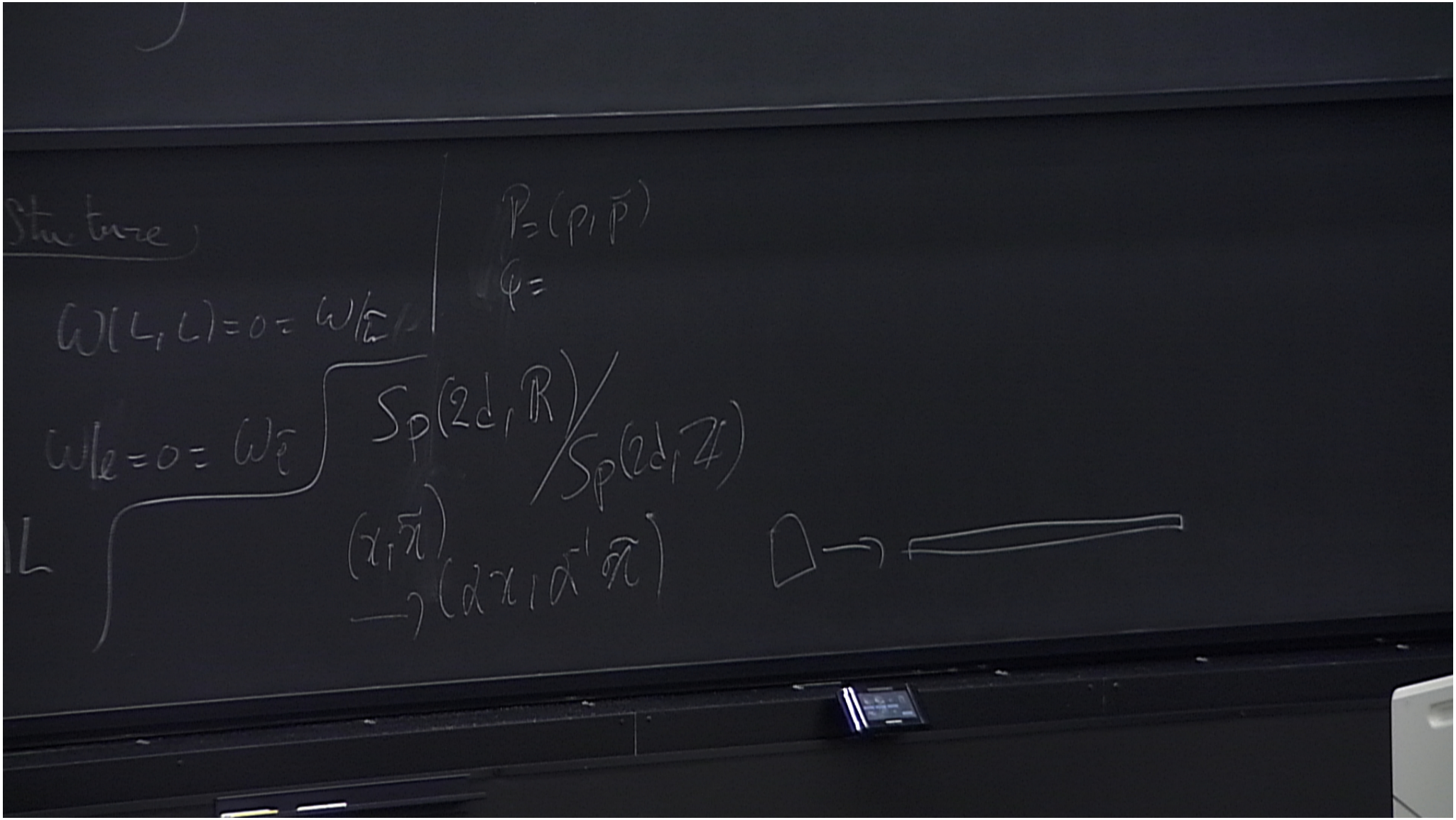

$W_{k+k'} W_k = e^{i\pi\omega(k';k)} \omega(k+k')$
$= e^{2i\pi\omega(k';k)} W_k W_{k'}$

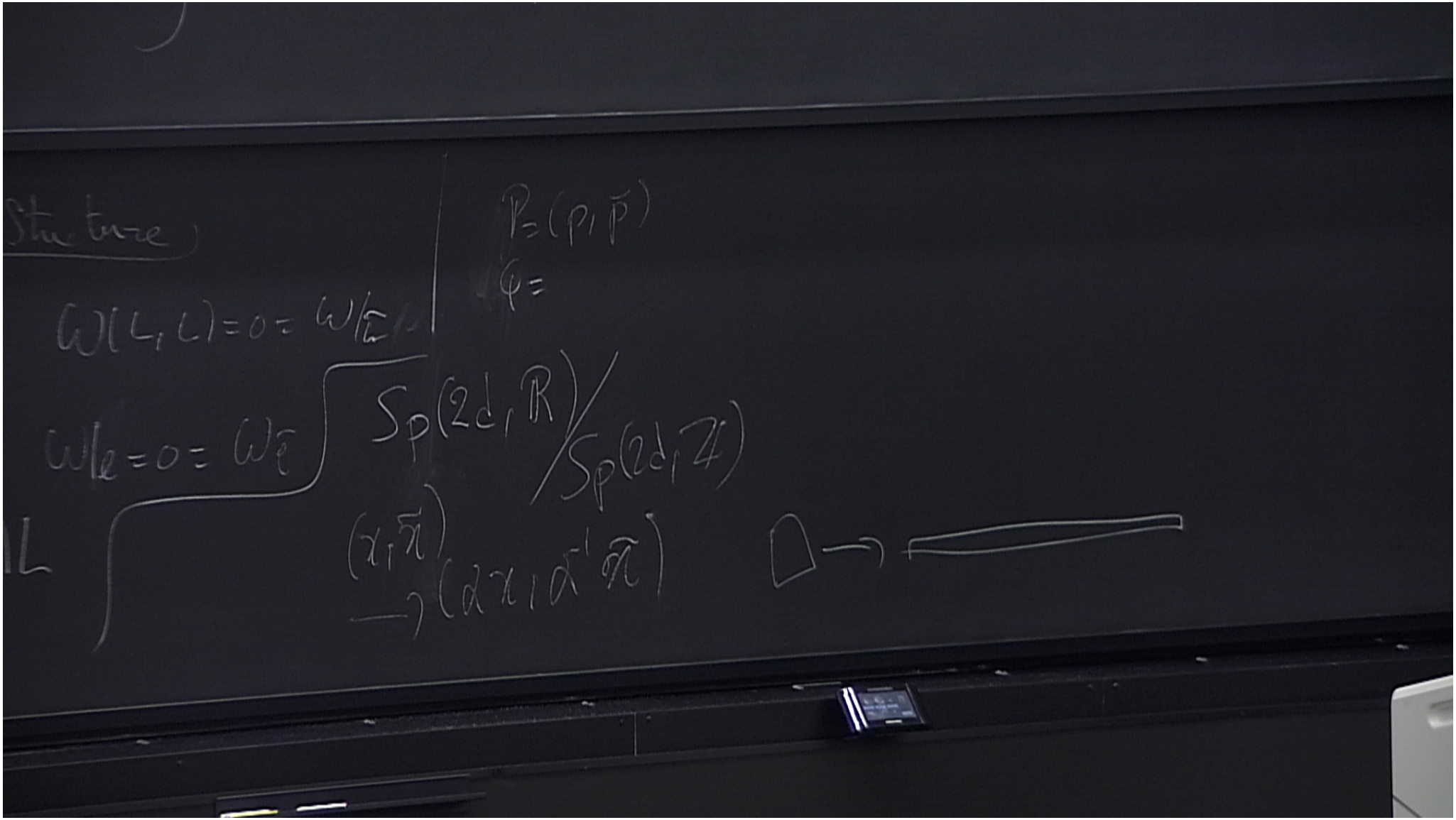

Key ingredients
$(W_\lambda, W_\mu) = 0$
$W_\lambda W_\mu = \pi(\cdot) W_{\lambda+\mu}$
$U_\lambda U_\mu = U_{\lambda+\mu}$
$\alpha(\lambda)\alpha($
Cocycle
Λ is Faith Fully represented

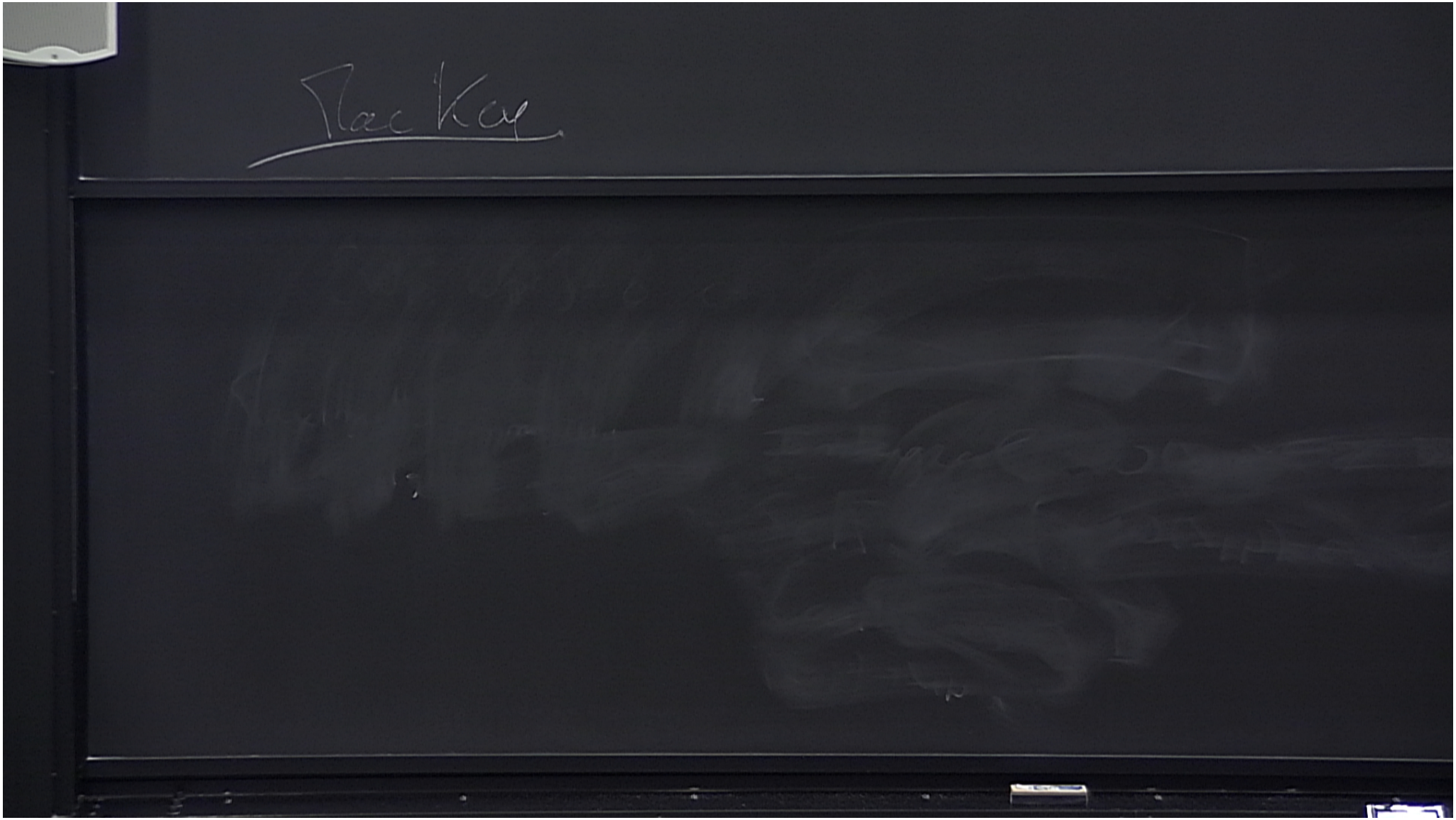

• Bilagrangian Structure
• $P = L \oplus \tilde{L}$ $\omega(L, L) = 0 = \omega(\tilde{L}, \tilde{L})$
• $\Lambda = \ell \oplus \tilde{\ell}$

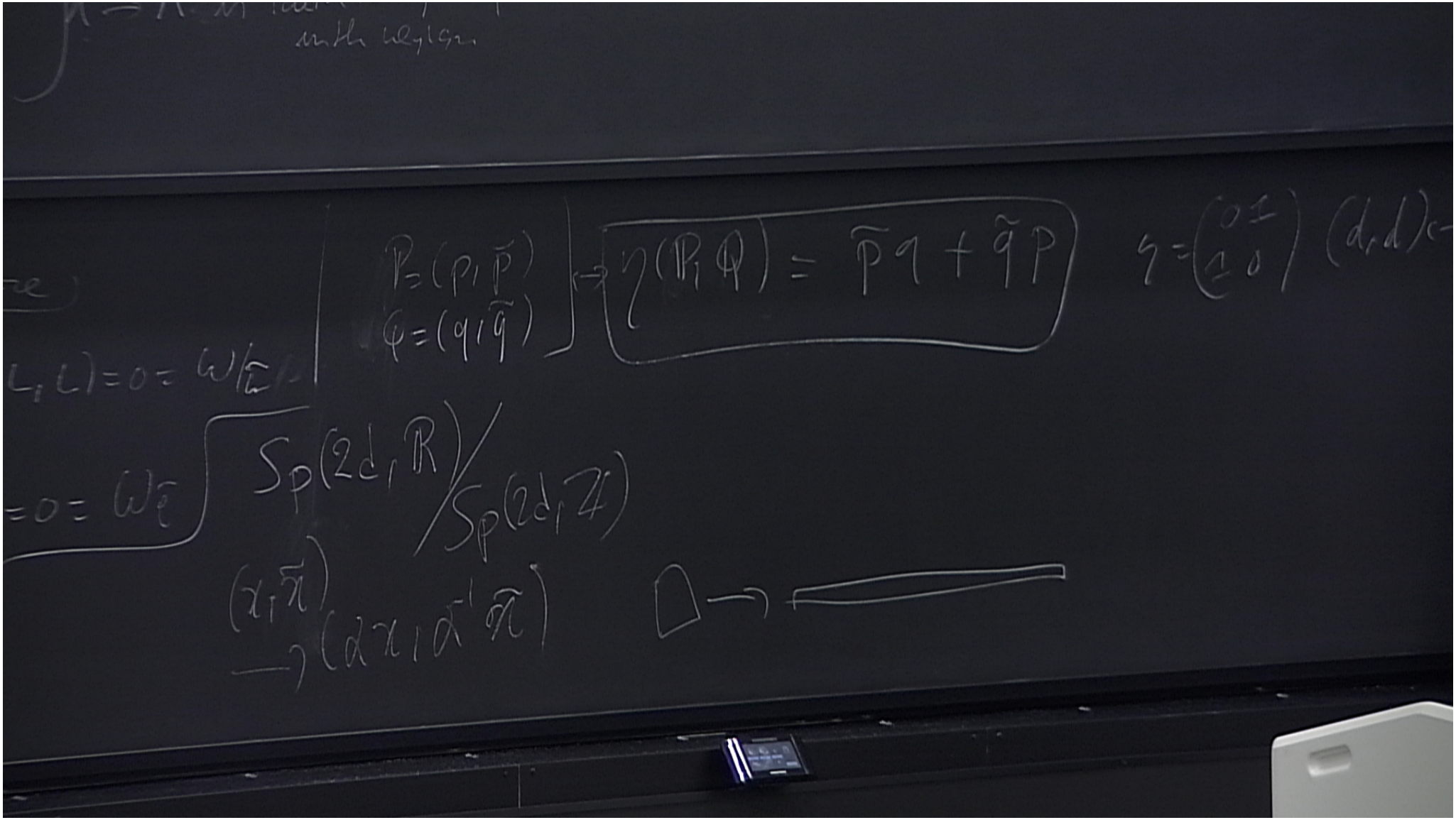

P=(ρ, p⃗)
φ=

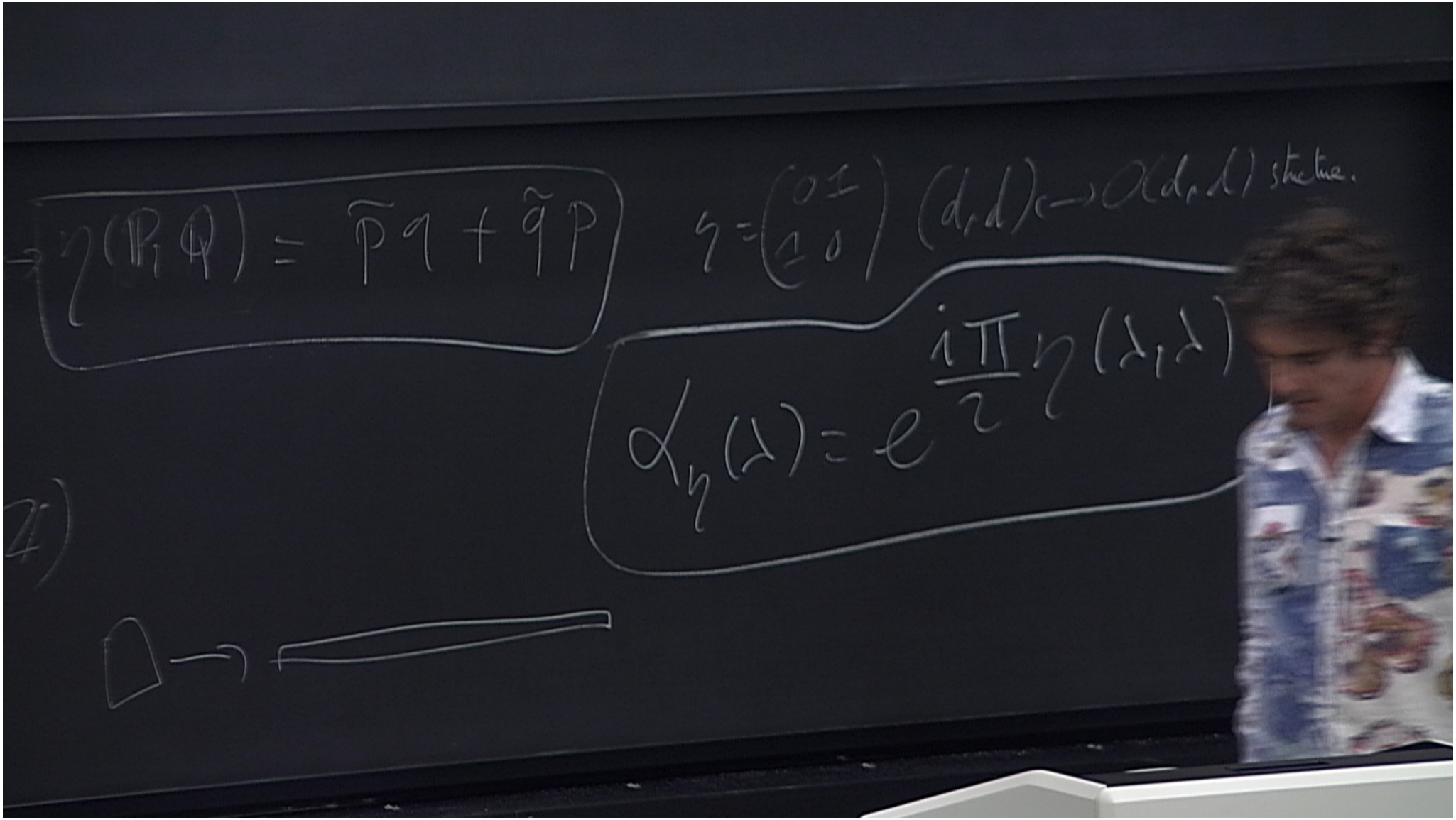

Bilagrangian Structure
$P = L \oplus \tilde{L}$ $\omega(L, L) = 0 = \omega(\tilde{L}, \tilde{L})$
$\Lambda = \ell \oplus \tilde{\ell}$ $\omega(\ell) = 0 = \omega(\tilde{\ell})$
$\ell = \Lambda \cap L$
$P = (p, \tilde{p})$
$\varphi =$

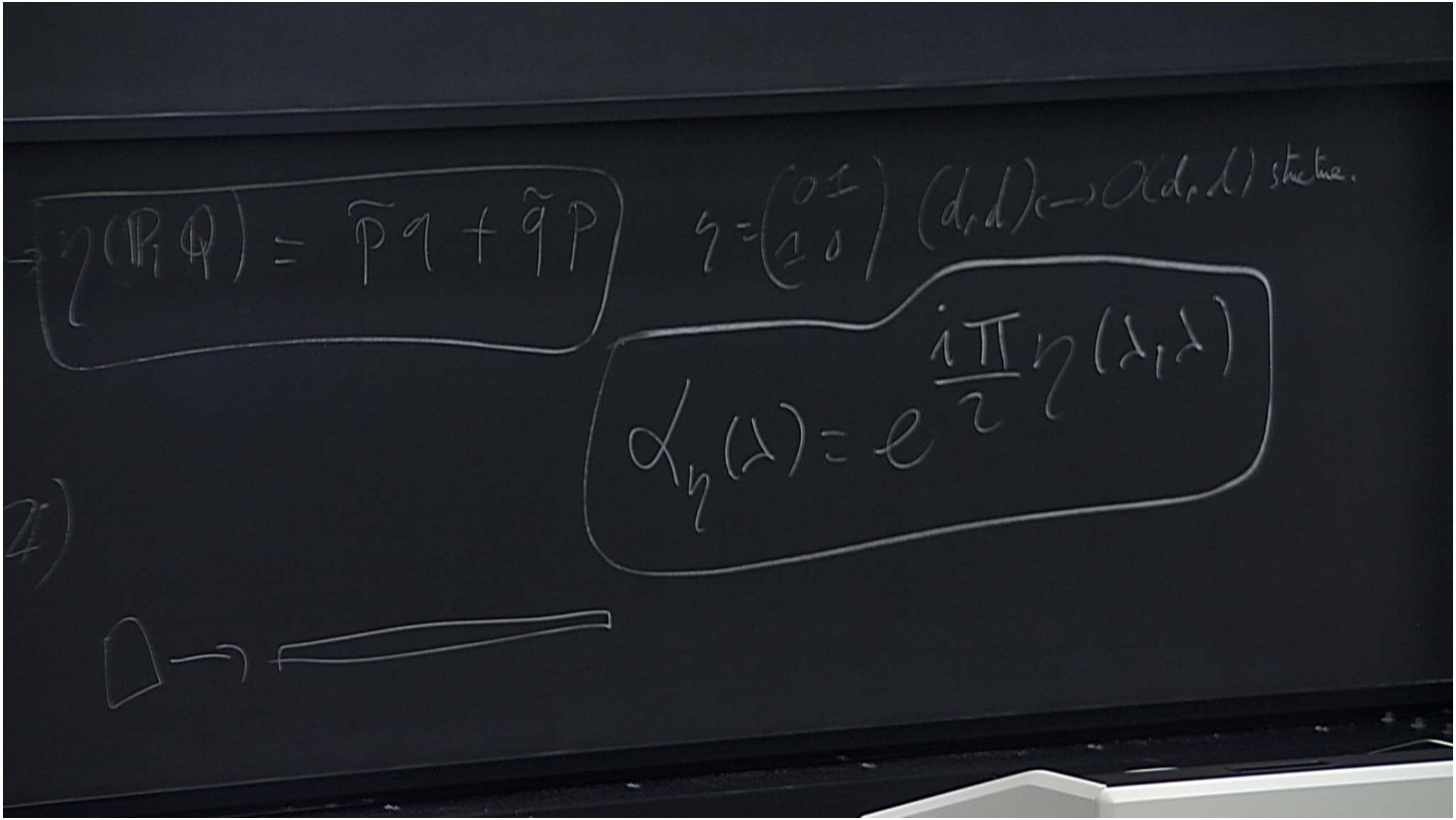

Structure
$W(L,L)=0=W_{\vec{L}}$
$W_{\vec{L}}=0=W_{\vec{L}}$
$\sqrt{Sp(2d,R)/Sp(2d,Z)}$
$(x,\vec{x})$
$\to(\alpha x,\alpha^{-1}\vec{x})$
$R=(p,\vec{P})$
$\varphi=$

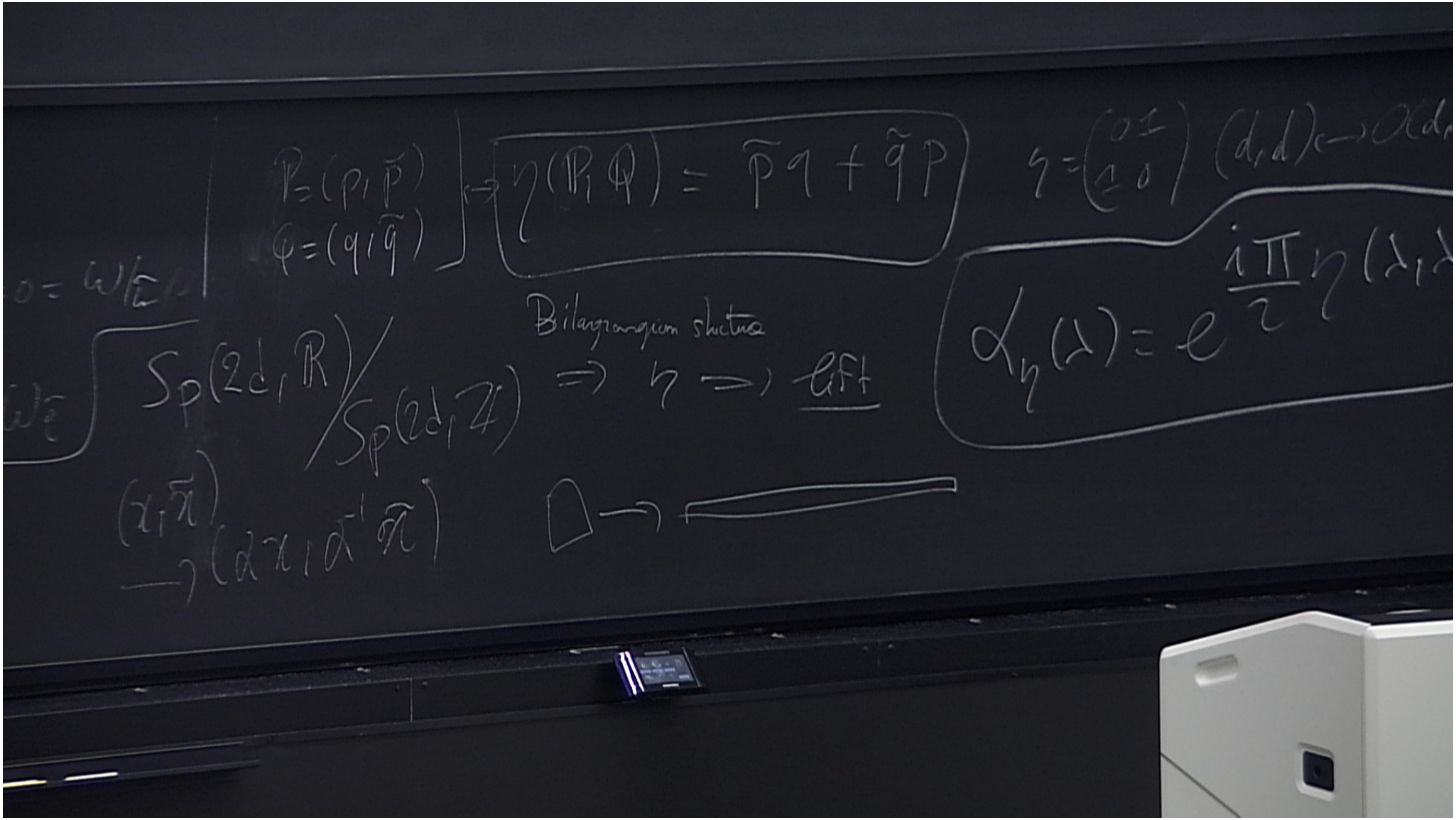

Structure
$W(L,L)=0=W_{\vec{L}=0}$
$W_{\vec{L}=0}=W_{\vec{L}}\sqrt{Sp(2d,R)/Sp(2d,Z)}$
$R=(P,\vec{P})$
$\varphi=$
$(x,\vec{x})$
$\to(\alpha x,\alpha^{-1}\vec{x})$

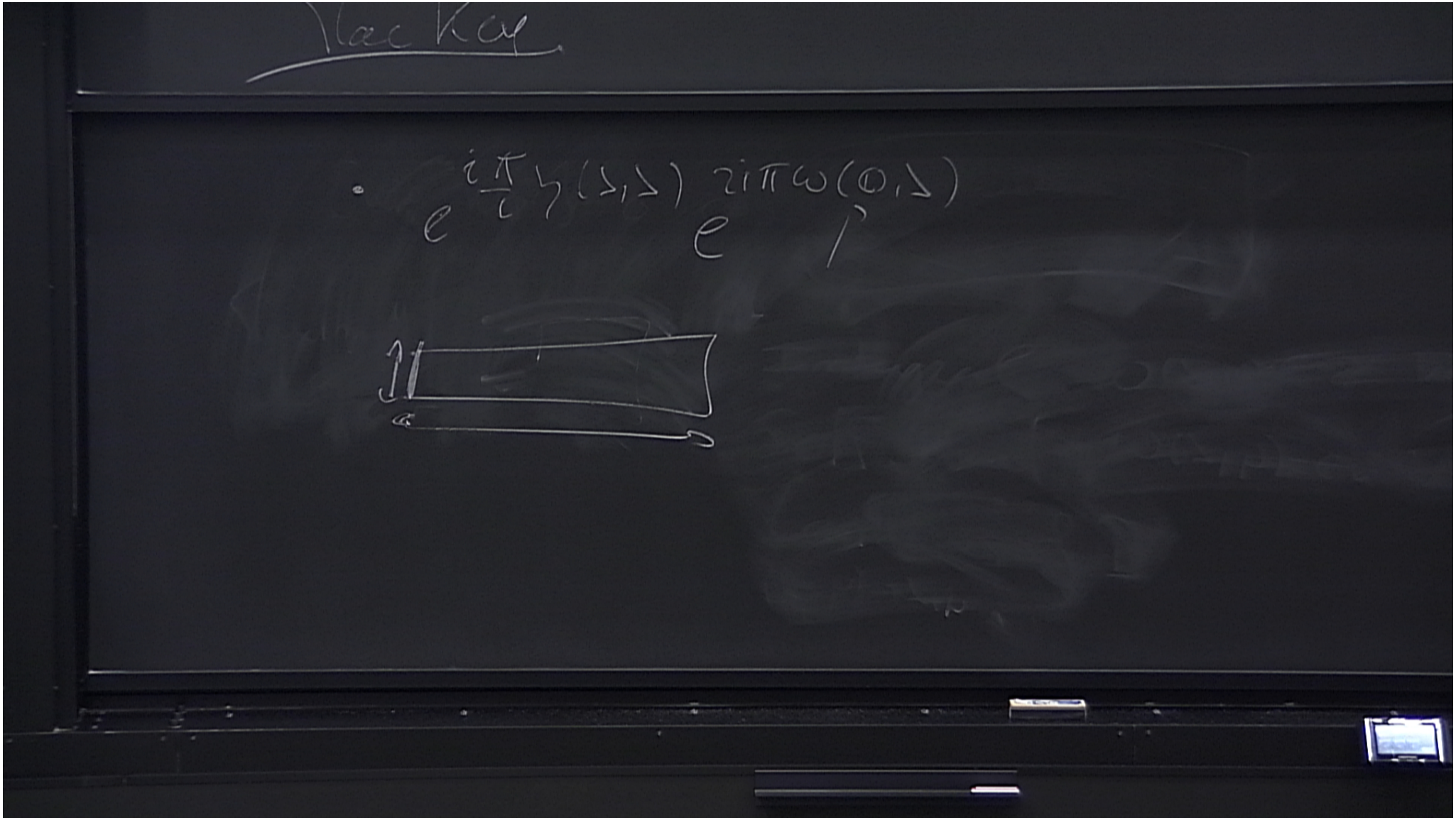


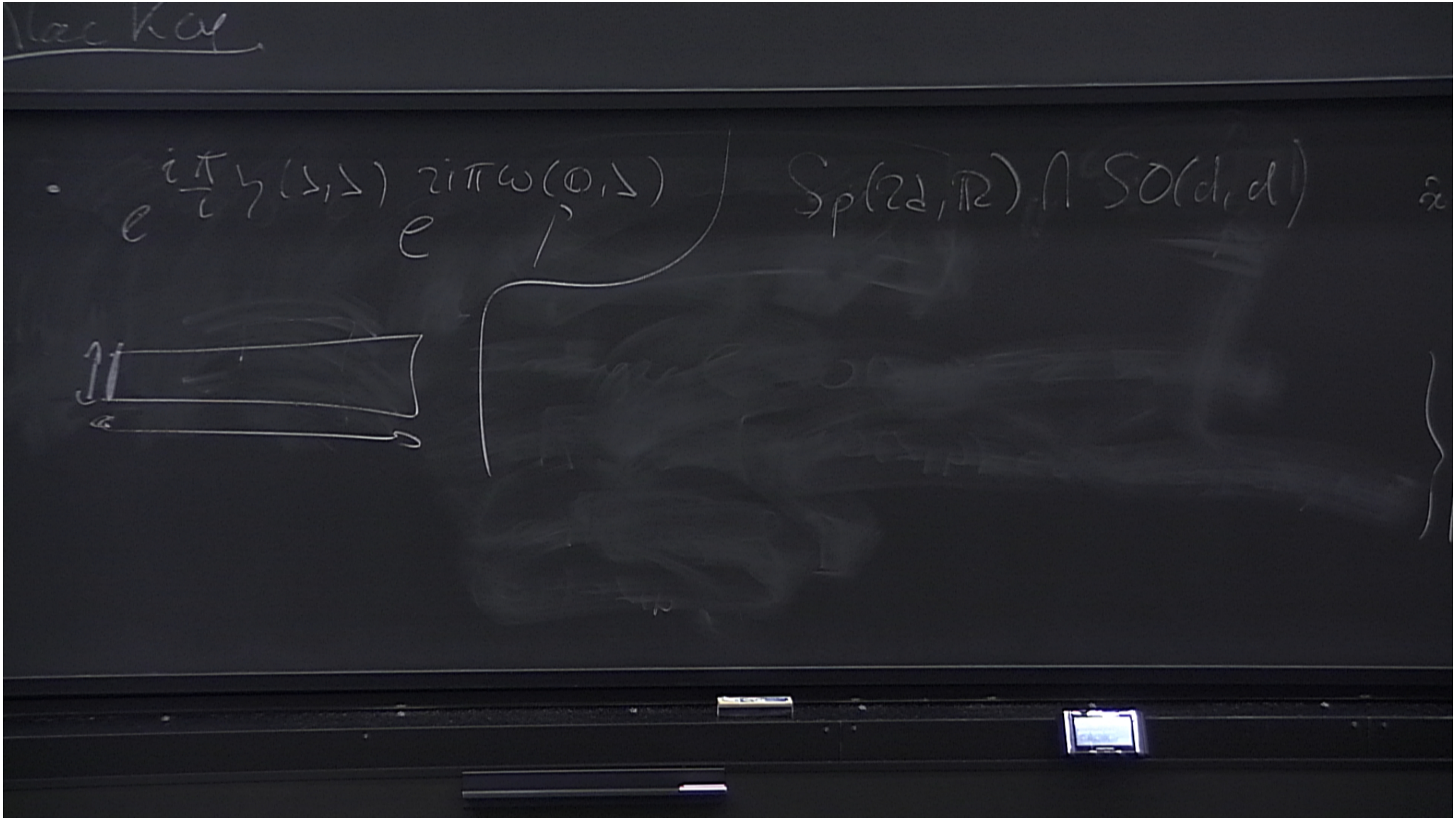

$P=(p,\tilde{p})$
$Q=(q,\tilde{q})$
$\gamma(P,Q)=\tilde{p}q+\tilde{q}p$
$Sp(2d,\mathbb{R})/Sp(2d,\mathbb{Z})$
$(x,\vec{\pi})$
$\to(\alpha x,\alpha^{-1}\vec{\pi})$

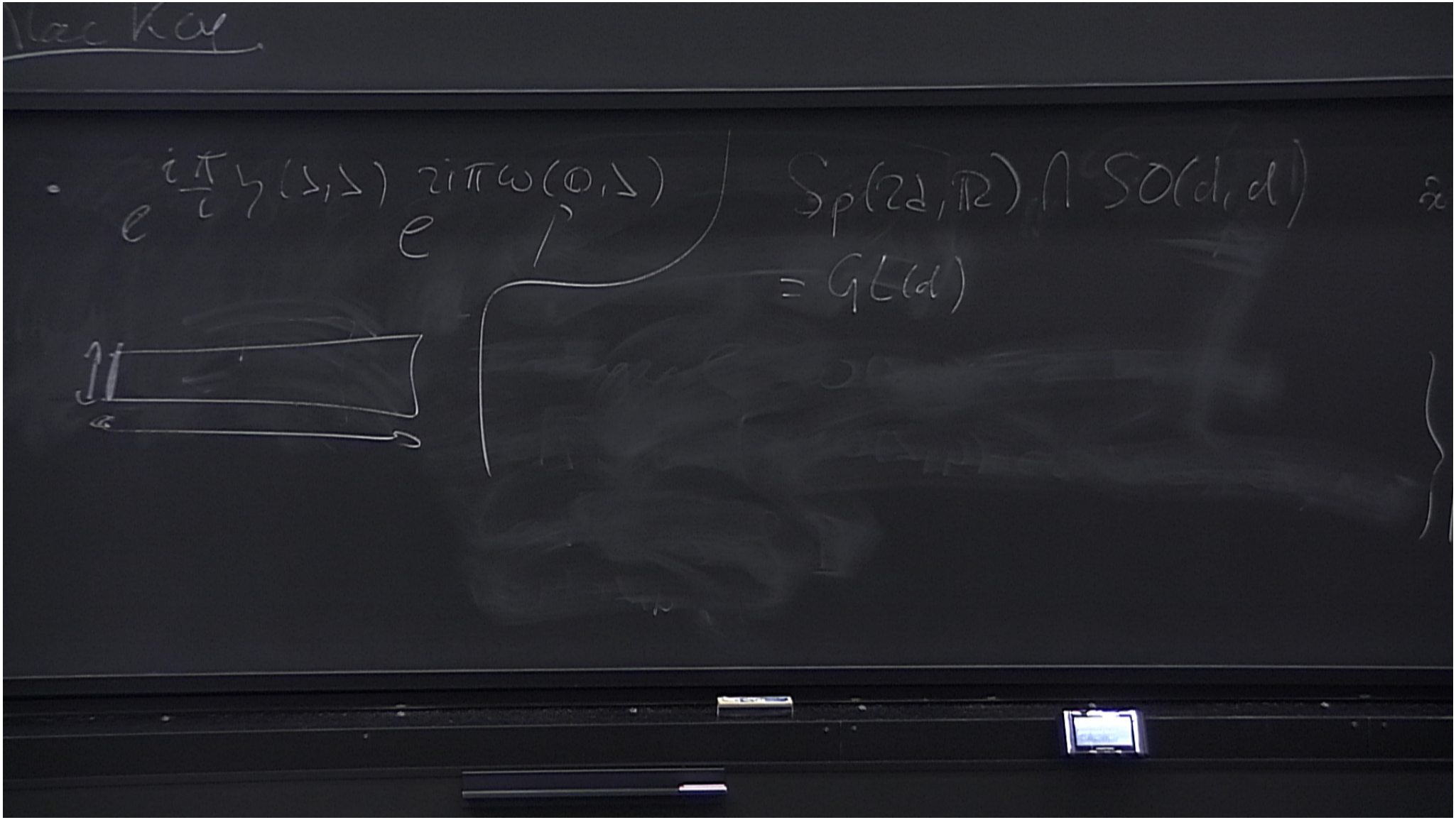

$\eta(P,Q) = \tilde{p}q + \tilde{q}p$
$\eta = \begin{pmatrix} 0 & 1 \\ \pm 1 & 0 \end{pmatrix}$ $(d,d) \hookrightarrow O(d,d)$ structure.
$\alpha_\eta(\lambda) = e^{\frac{i\pi}{2}\eta(\lambda,\lambda)}$

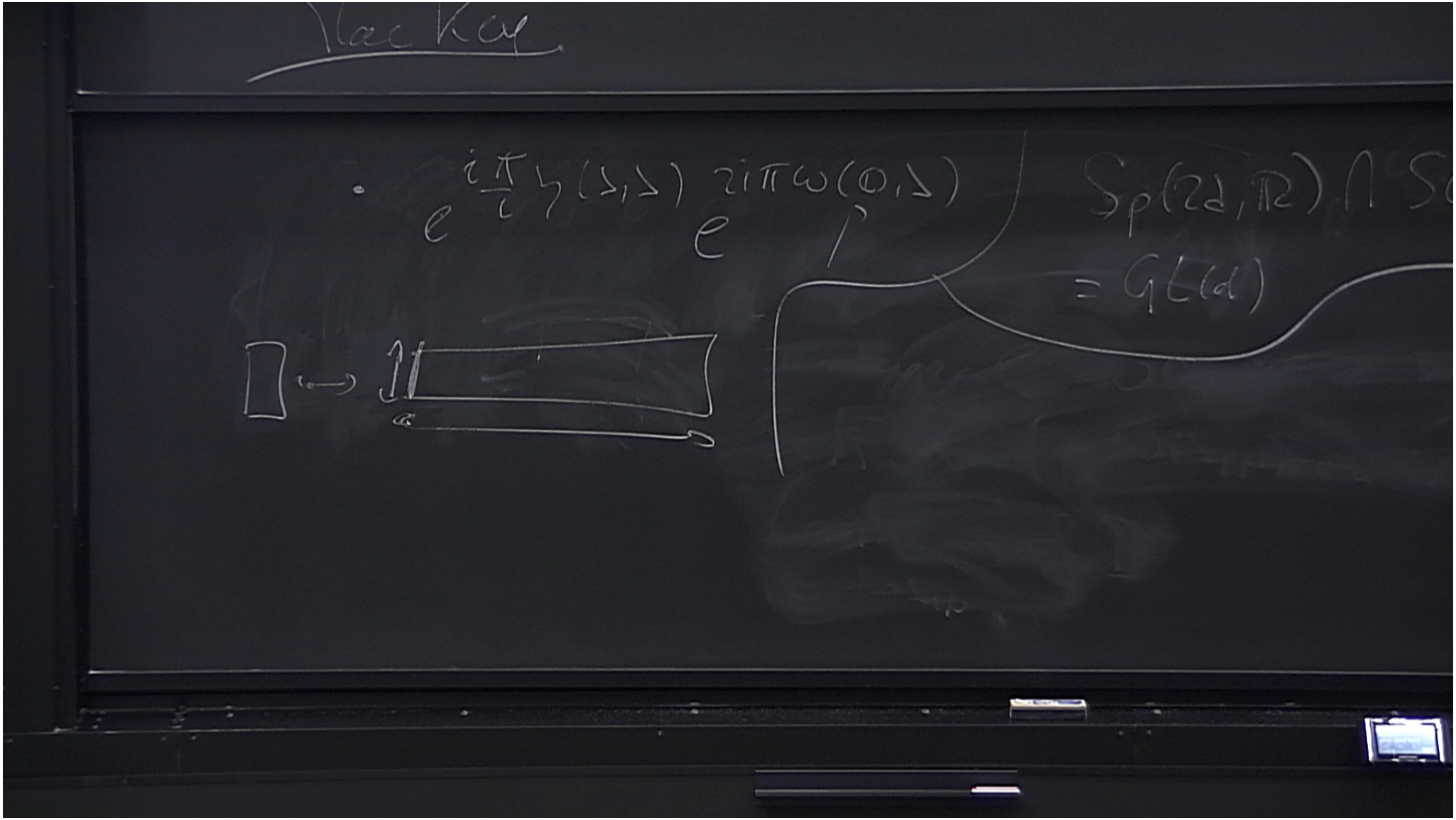

$\eta(P,Q) = \tilde{p}q + \tilde{q}p$
$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $(d,d) \hookrightarrow O(d,d)$ structure.
$\alpha_\eta(\lambda) = e^{\frac{i\pi}{2}\eta(\lambda,\lambda)}$


$P=(p,\tilde{p})$
$Q=(q,\tilde{q})$
$\eta(P,Q)=\tilde{p}q+\tilde{q}p$
$\eta=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
$\alpha_\eta(\lambda)=e^{\frac{i\pi}{2}\eta(\lambda,\lambda$
Bilagrangian structure
$\Rightarrow$ $\eta \rightsquigarrow$ lift
$Sp(2d,\mathbb{R})/Sp(2d,\mathbb{Z})$
$(x,\tilde{x})$
$\rightarrow(\alpha x,\alpha^{-1}\tilde{x})$


Mac Kay
$e^{\frac{i\pi}{k} h(\lambda,\lambda)}$ $e^{2i\pi\omega(0,\lambda)}$


Sp(2d, ℝ) ∩ SO(d, d)


Sp(2d, ℝ) ∩ SO(d, d)
= GL(d)

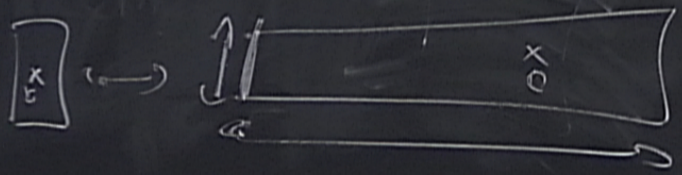

Sp(2d, ℝ) ∩
= GL(d)

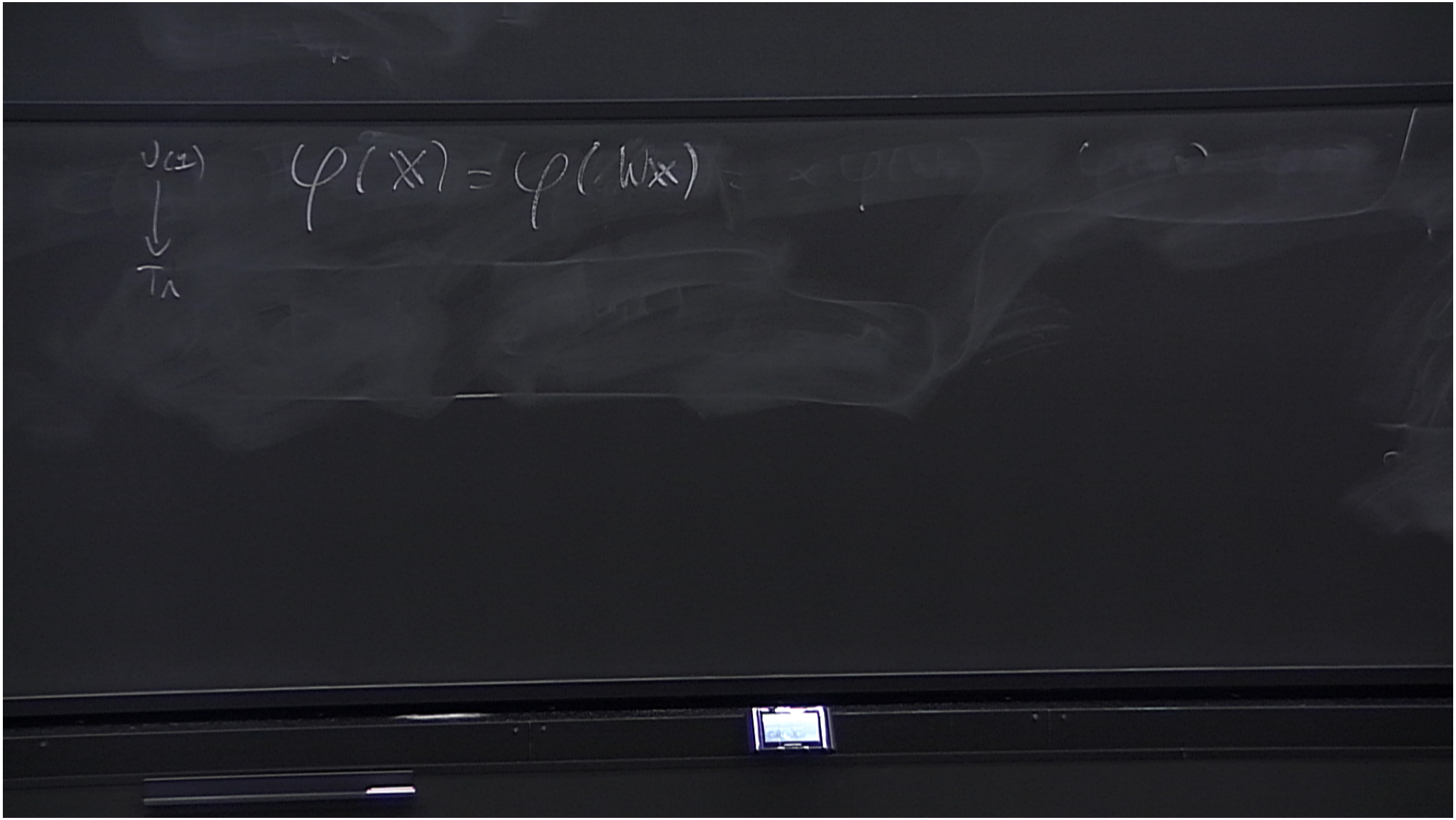
Mackey

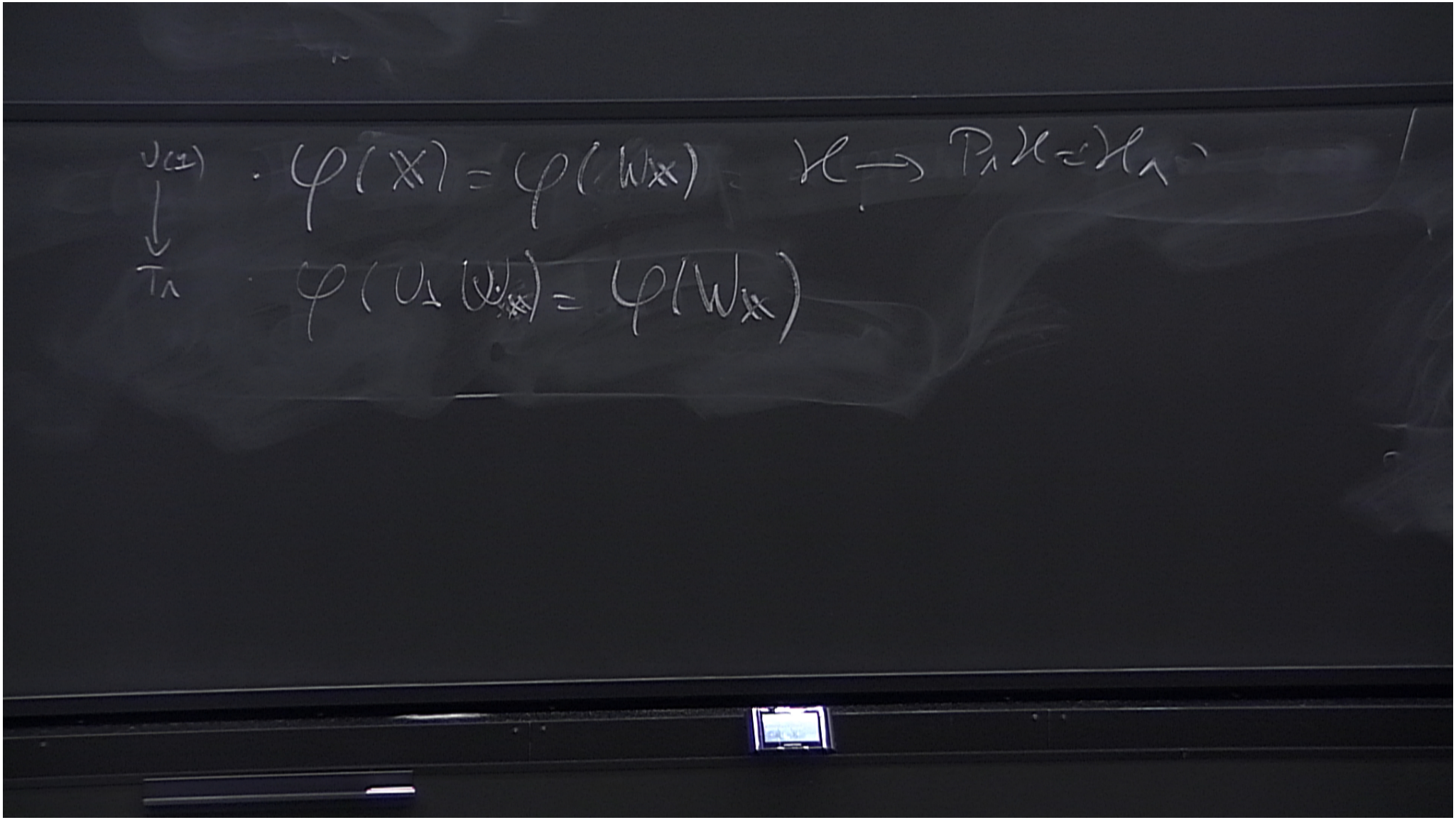
$e^{\frac{i}{\hbar}\frac{\pi}{2}\gamma(\lambda,\lambda)}\, e^{2i\pi\omega(\ell,\lambda)}$

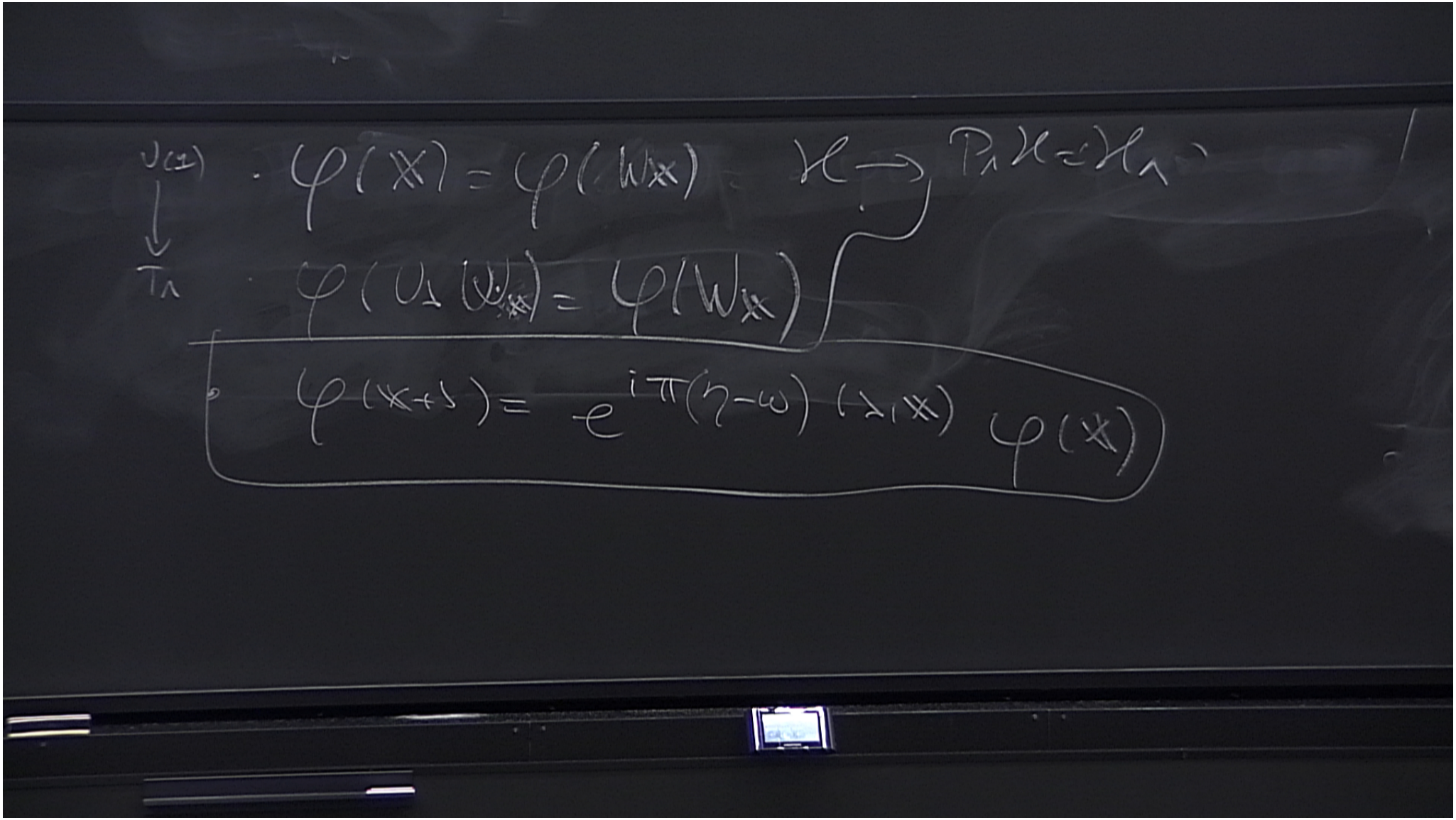
$Sp(2d,\mathbb{R}) \cap$

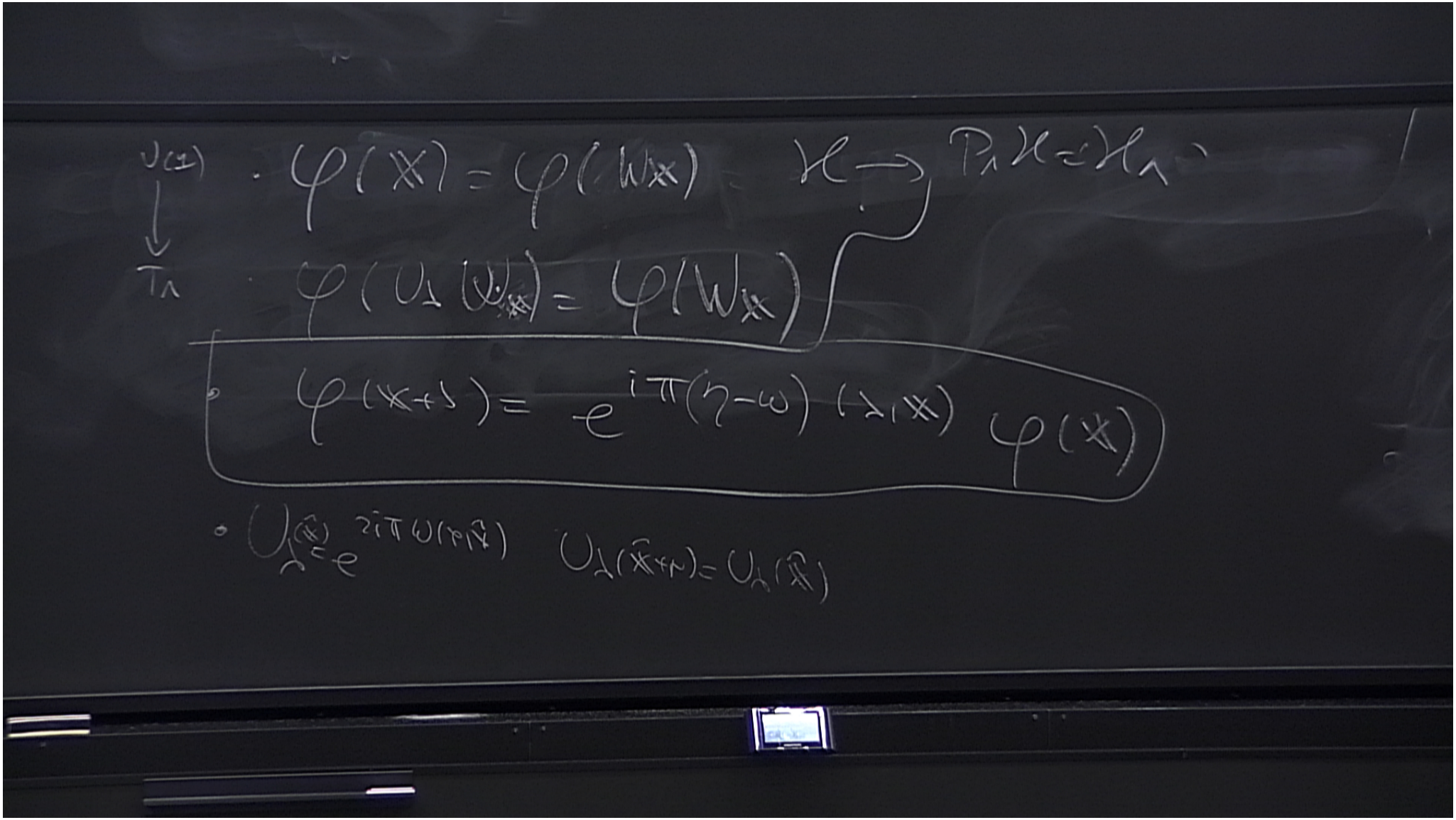
$= GL(d)$

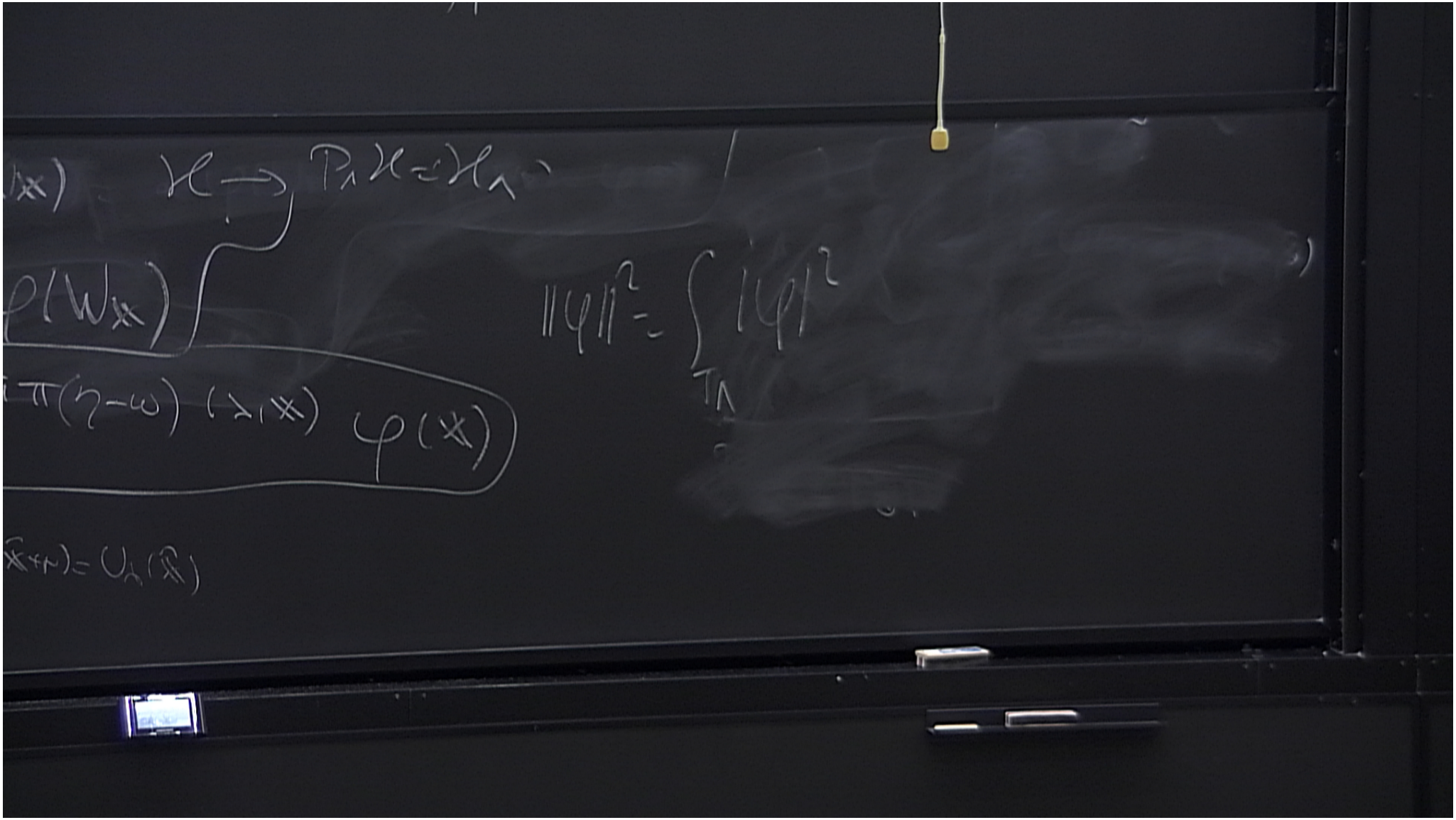


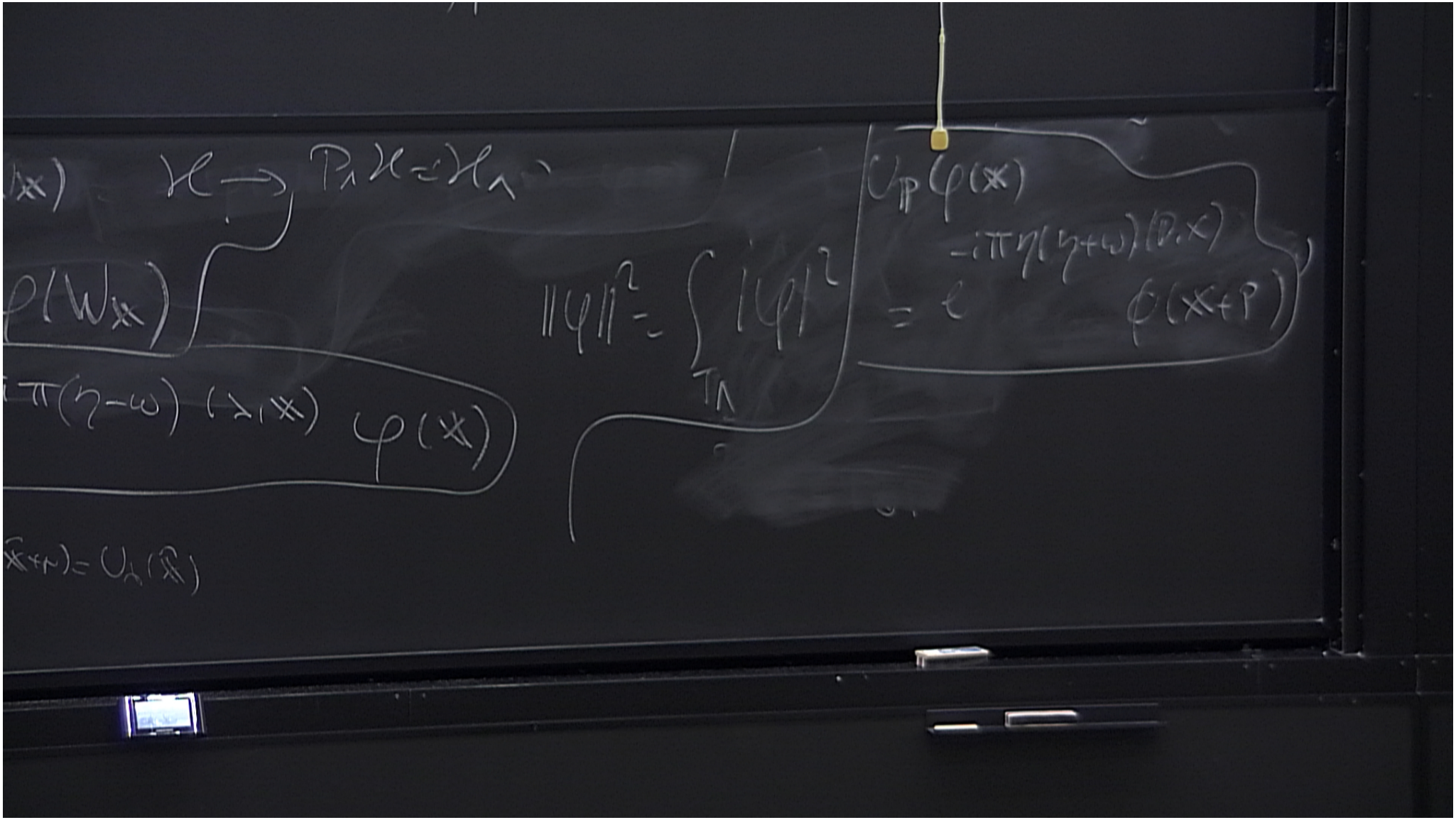

$V(x)$
$\varphi(x) = \varphi(Wx)$
$T\Lambda$


$U(t)$
T_Λ
$\varphi(X) = \varphi(WX)$
$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\varphi(U_t(W_X)) = \varphi(W_X)$


$U(1)$
T_Λ
$\varphi(x) = \varphi(Wx)$
$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\varphi(U_\Lambda(Wx)) = \varphi(Wx)$
$\varphi(x+1) = e^{i\pi(\eta-\omega)(\lambda,x)}\varphi(x)$


$U(1)$
T_Λ
$\varphi(x) = \varphi(Wx)$
$\mathcal{H} \to \mathcal{H}_\Lambda$
$P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\varphi(U_\Lambda(\theta)x) = \varphi(Wx)$
$\varphi(x+1) = e^{i\pi(\eta-\omega)(\lambda,x)}\varphi(x)$
$U_\Lambda(\theta) = e^{2i\pi\theta(\eta\cdot\hat{x})}$
$U_\Lambda(\theta+n) = U_\Lambda(\theta)$




$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\rho(W_x)$
$\pi(\eta-\omega)(\lambda,x)\ \varphi(x)$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$U_p(\varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)}\ \varphi(x+p)$
$x+r) = U_\Lambda(\hat{x})$

$U(1)$ $\varphi(X) = \varphi(WX)$

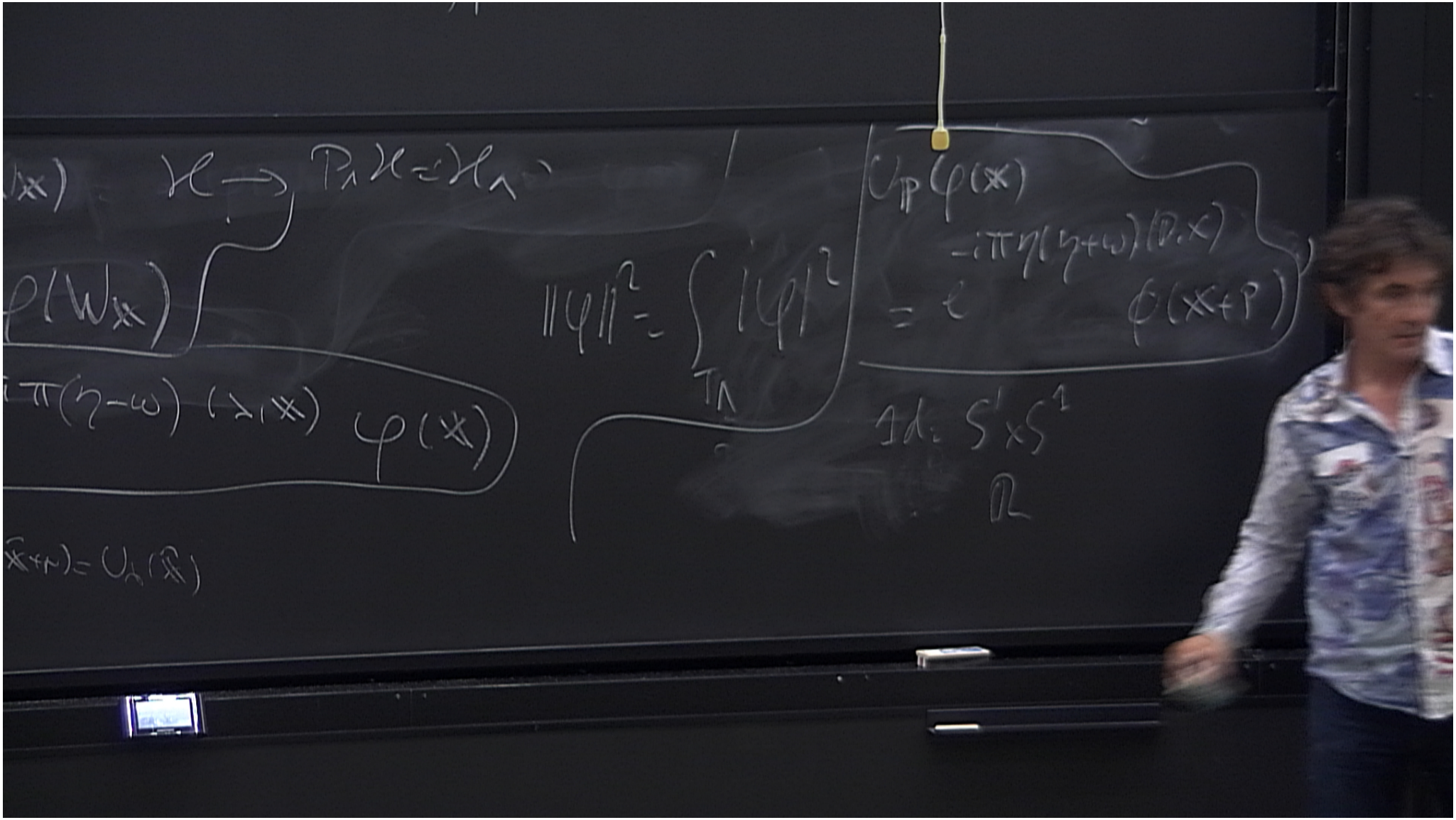
T_Λ $\varphi(U_\Lambda(W)X) = \varphi(WX)$

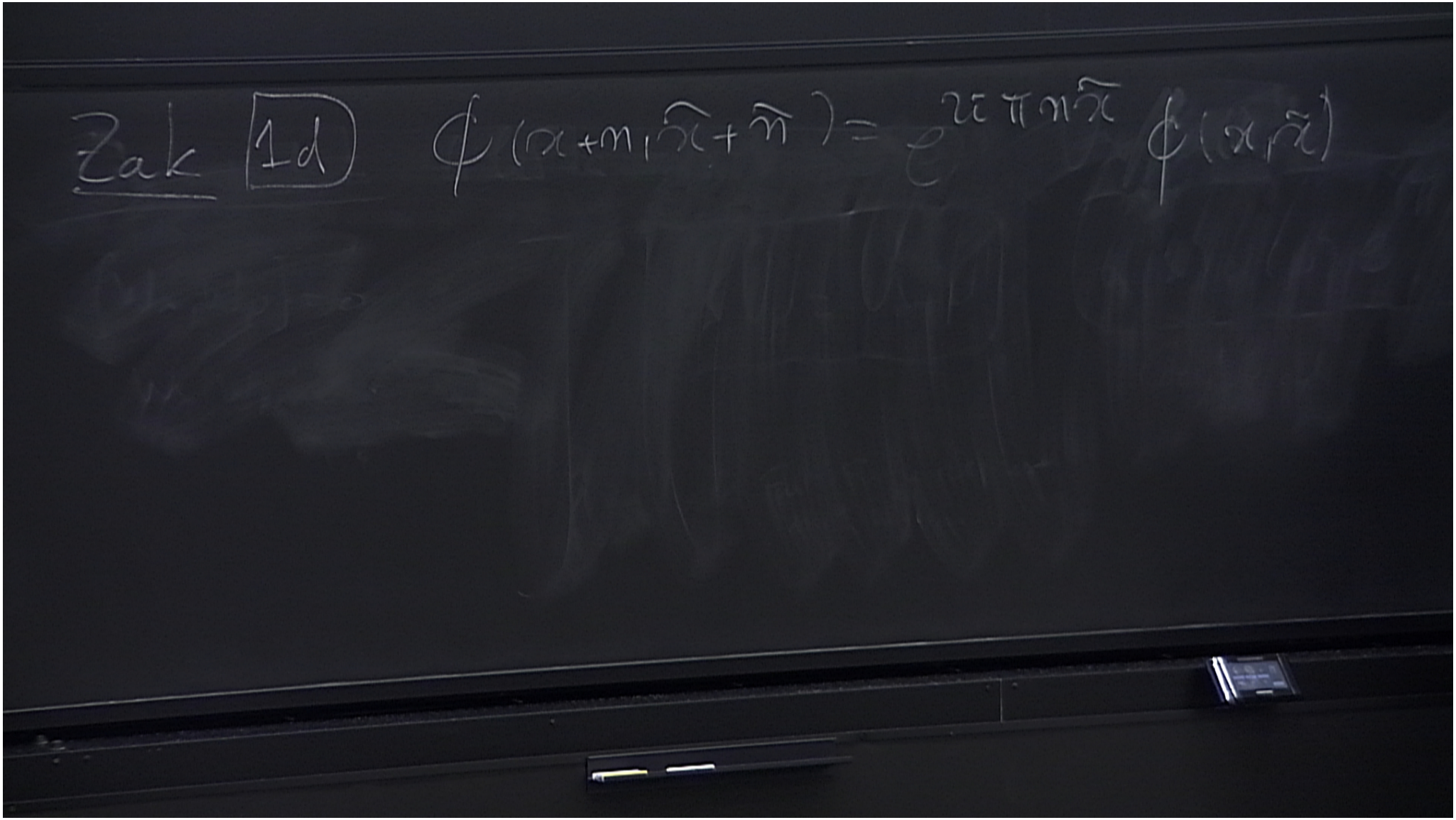
$\varphi(X+\lambda) = e^{i\pi(\eta-\omega)}\varphi(X)$

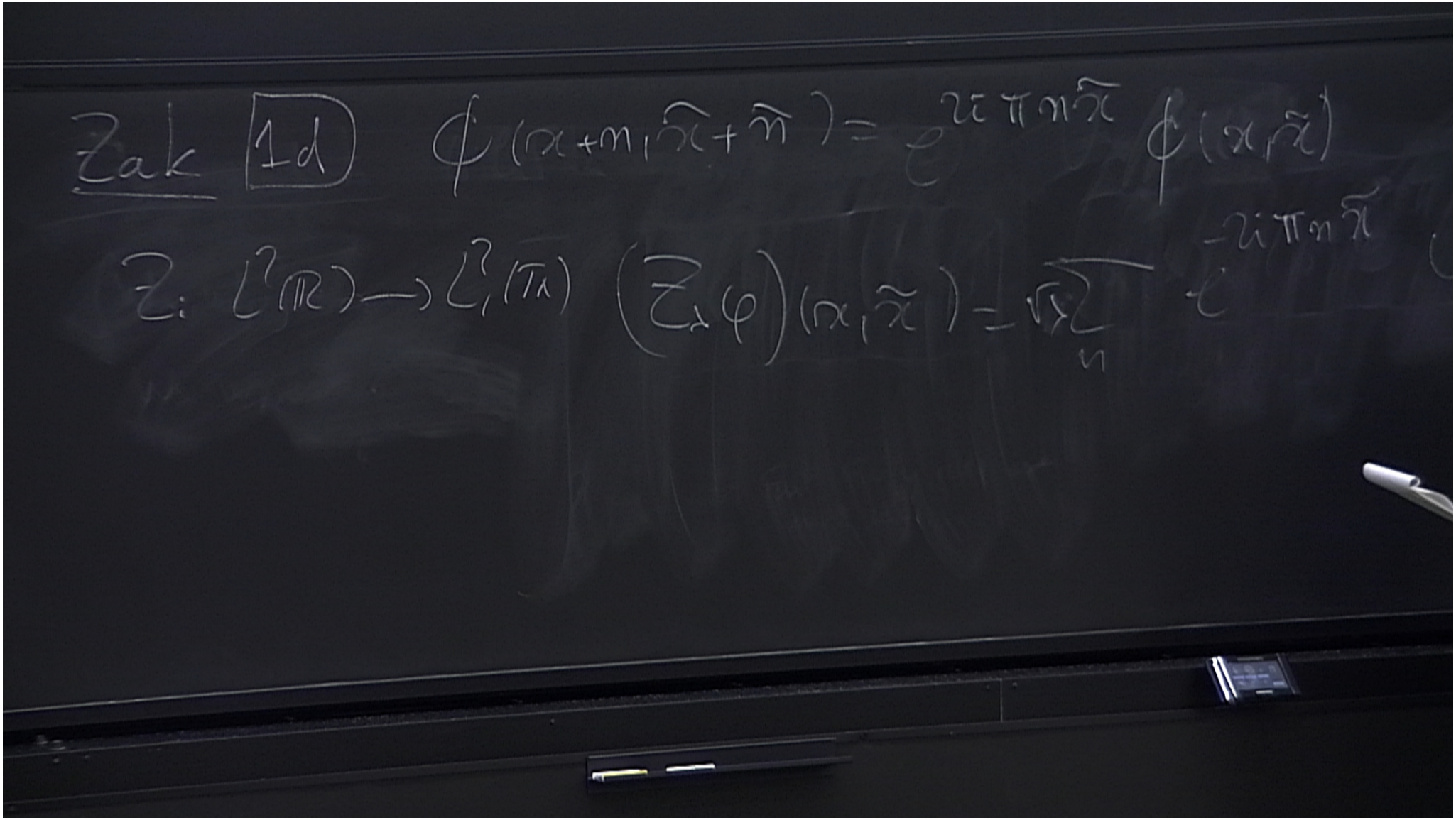


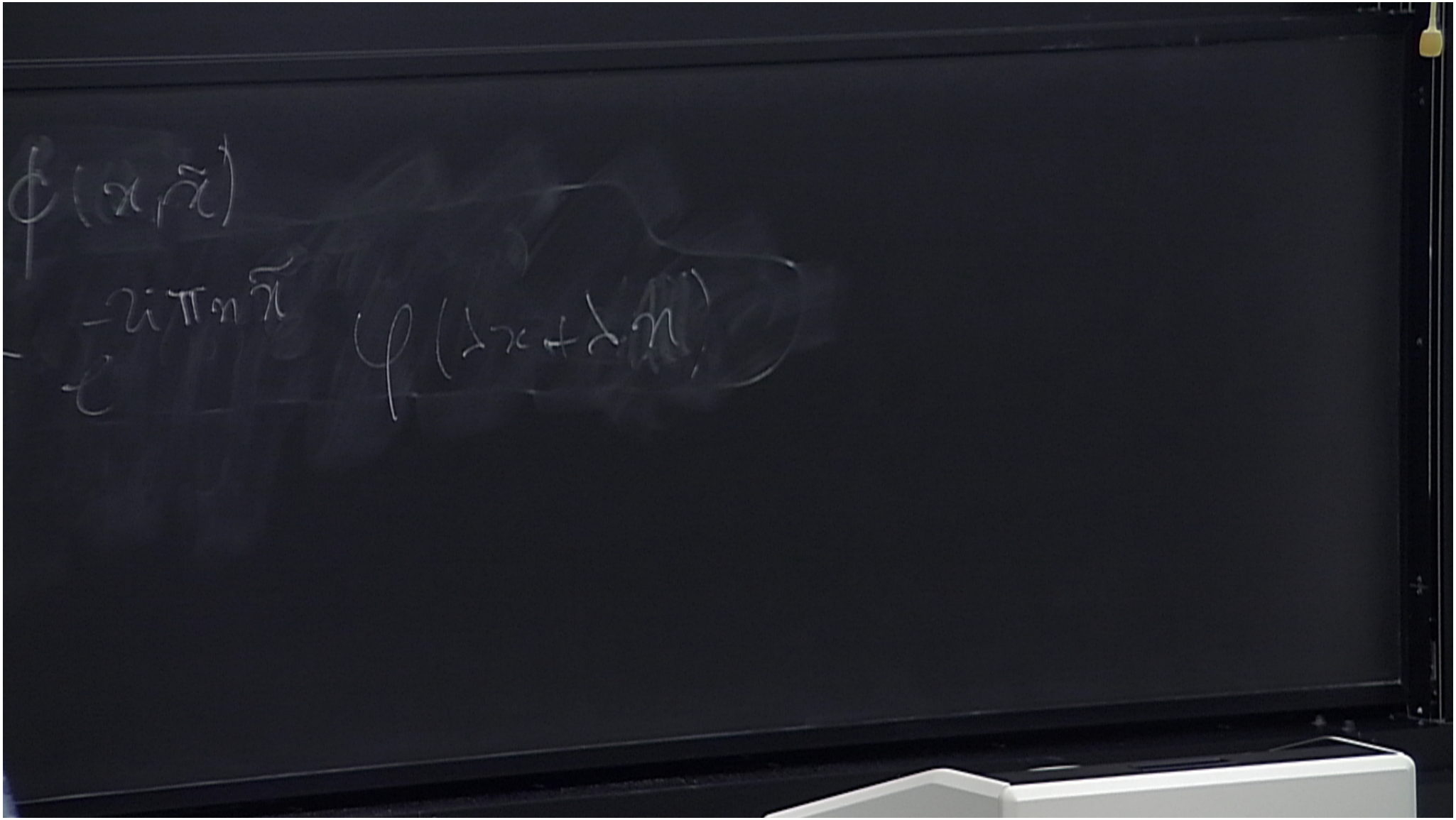
$U_\lambda(X) = e^{2i\pi\omega(p\lambda)}$ $U_\lambda(X+\mu) = U_\lambda(X)$

Backup


$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\|\varphi\|^2 = \int_{T\Lambda} |\varphi|^2$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\mathbb{R}$


Zak 1d
$\phi(x+m, \tilde{x}+\tilde{n}) = e^{2\pi i n\tilde{x}} \phi(x,\tilde{x})$

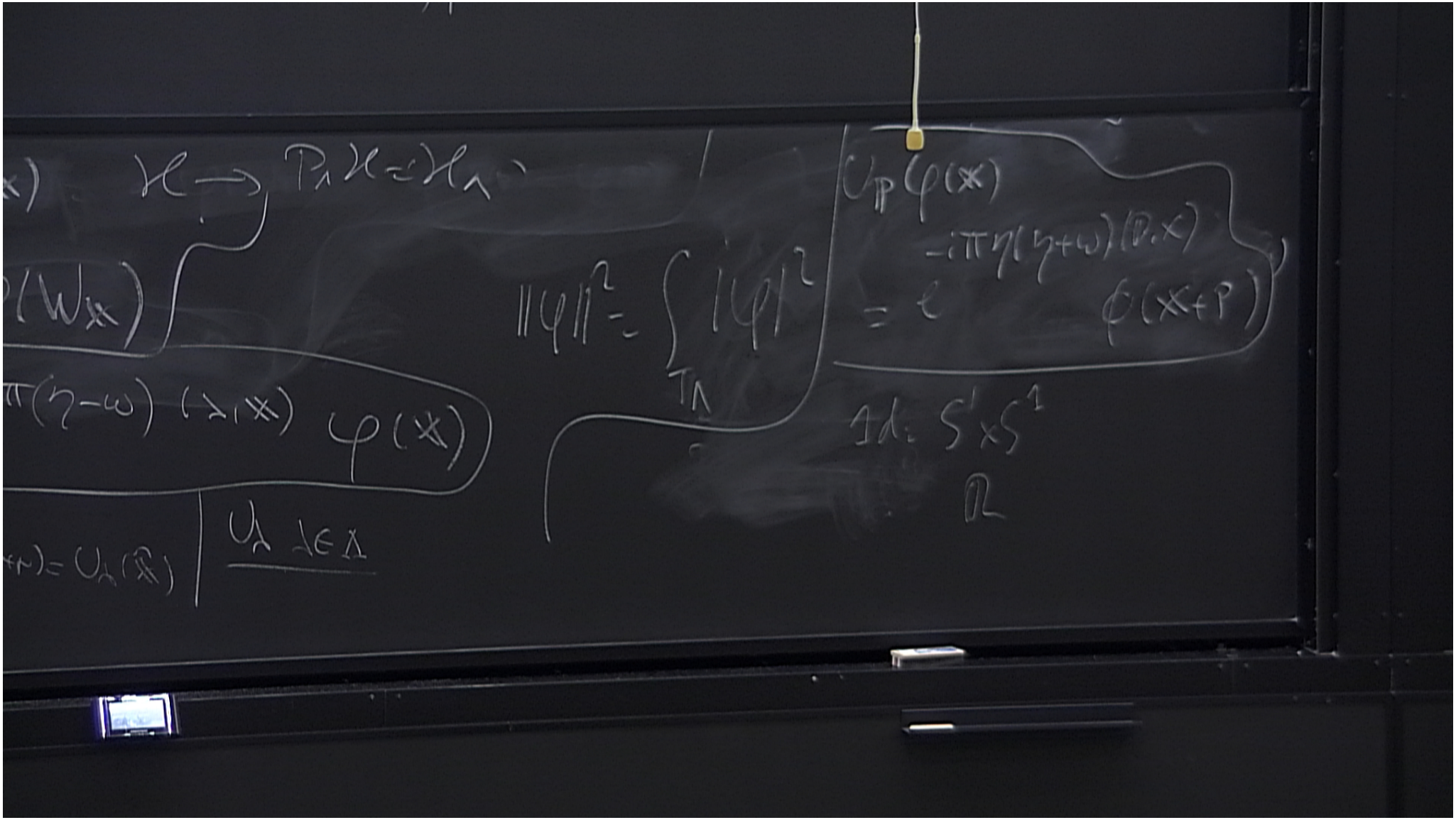

Zak 1d
$\phi(x+m, \tilde{x}+\tilde{n}) = e^{2\pi i n\tilde{x}} \phi(x,\tilde{x})$
$Z: L^2(\mathbb{R}) \to L^2_1(T_\lambda)$
$(Z\varphi)(x,\tilde{x}) = \sum_n$
$e^{-2\pi i n\tilde{x}}$

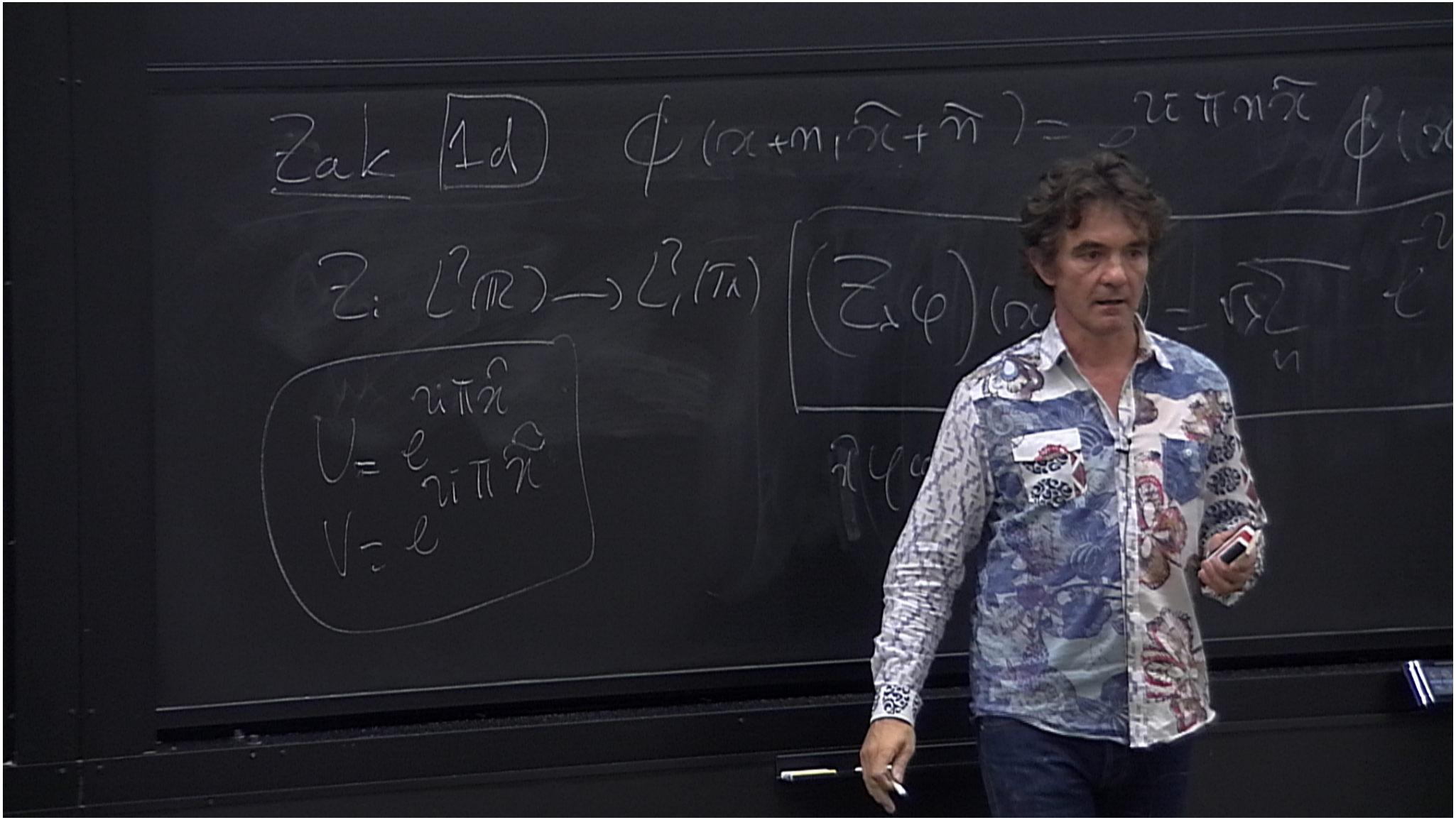

$\phi(x,\tilde{x})$
$-i\pi n\tilde{x}$
$\varphi(\lambda x+\lambda n)$

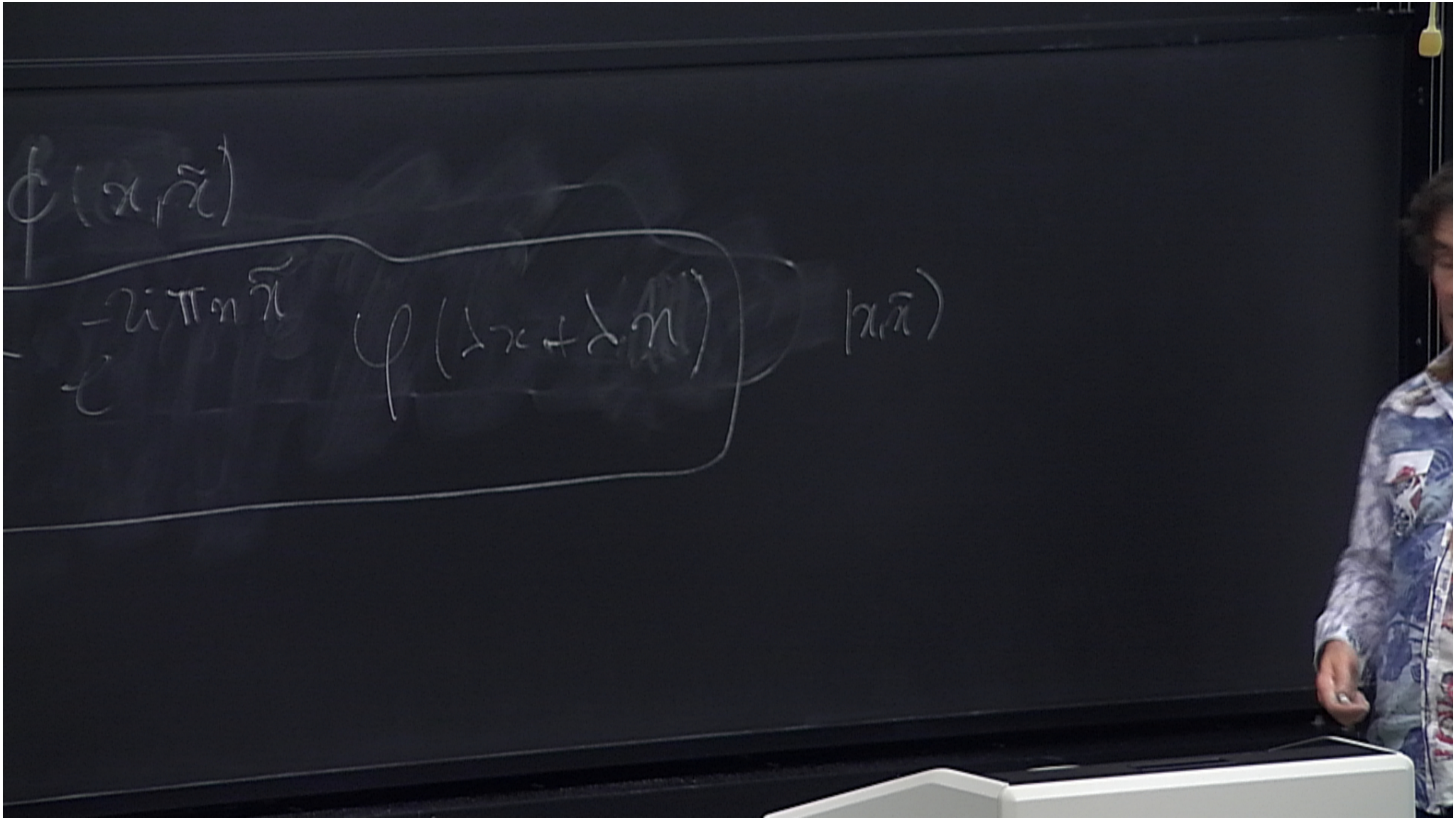
$$\phi(x+m, \tilde{x}+\tilde{n}) = e^{2\pi i n\tilde{x}}\,\phi(x,\tilde{x})$$

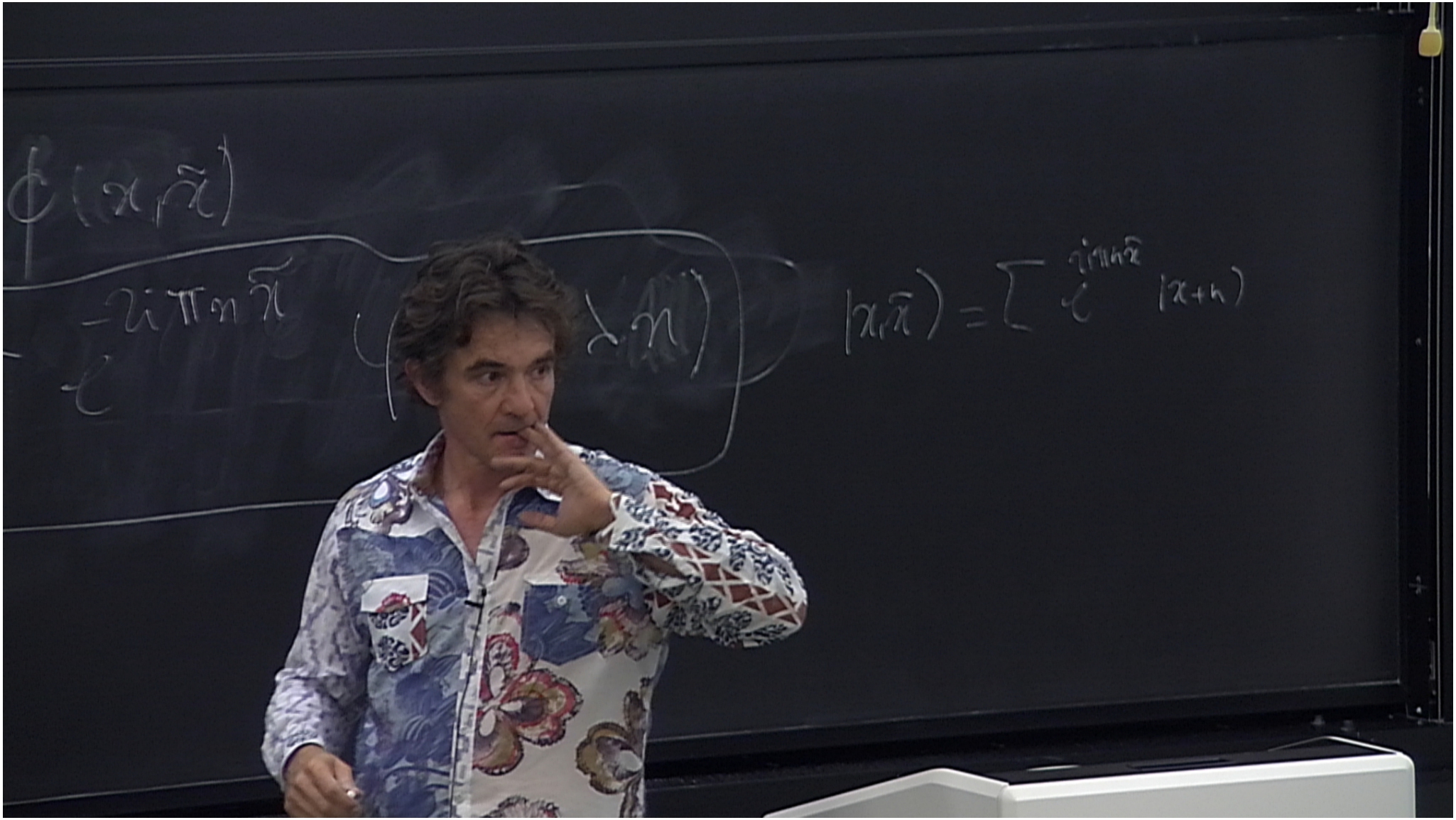
(Th) $$(Z_\lambda \varphi)(x,\tilde{x}) = \sqrt{\lambda}\sum_n e^{-2\pi i n\tilde{x}}\,\varphi(\lambda x + \lambda n)$$

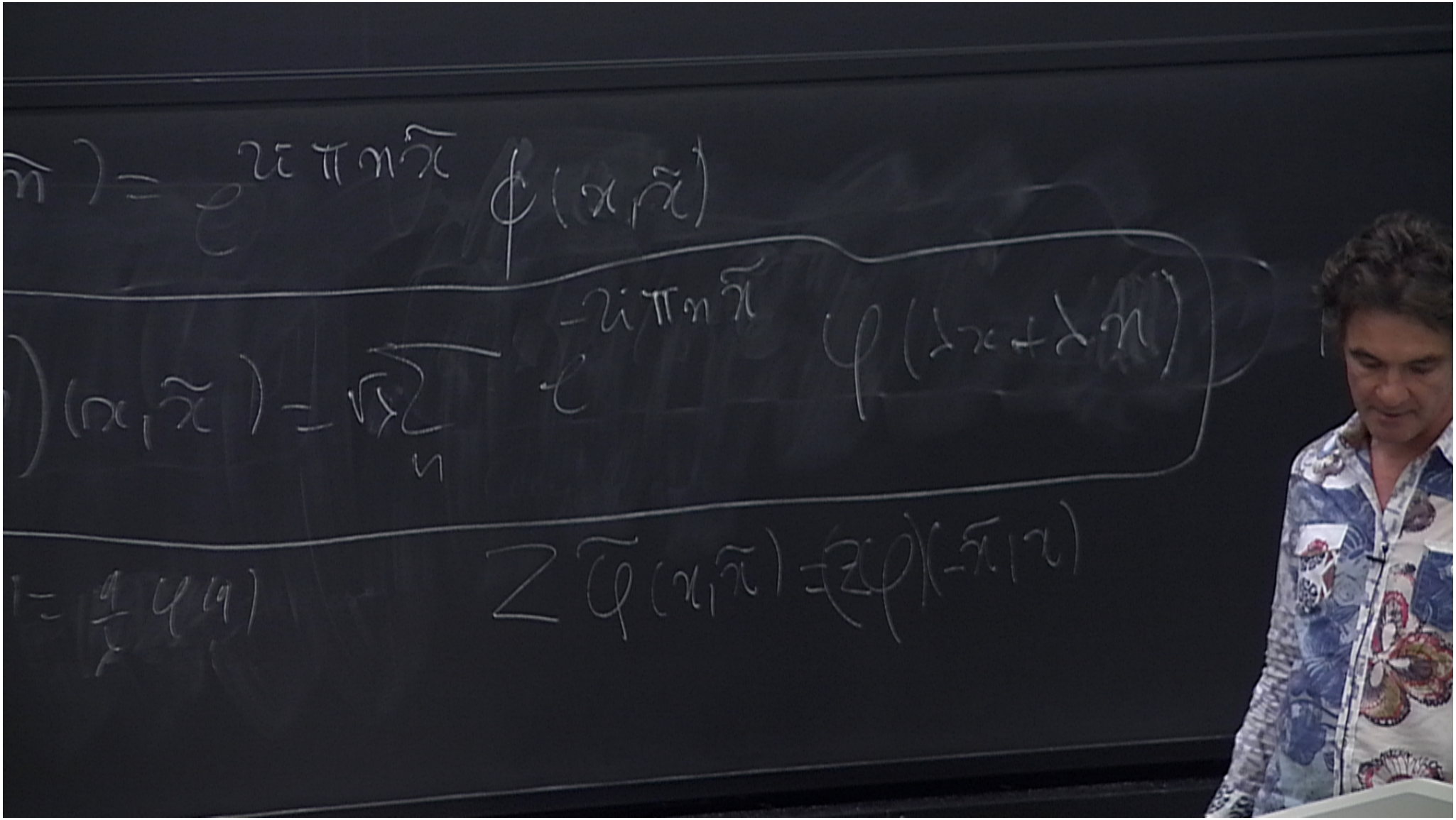
$$\hat{\pi}\,\psi(q) = \frac{q}{\lambda}\,\psi(q)$$

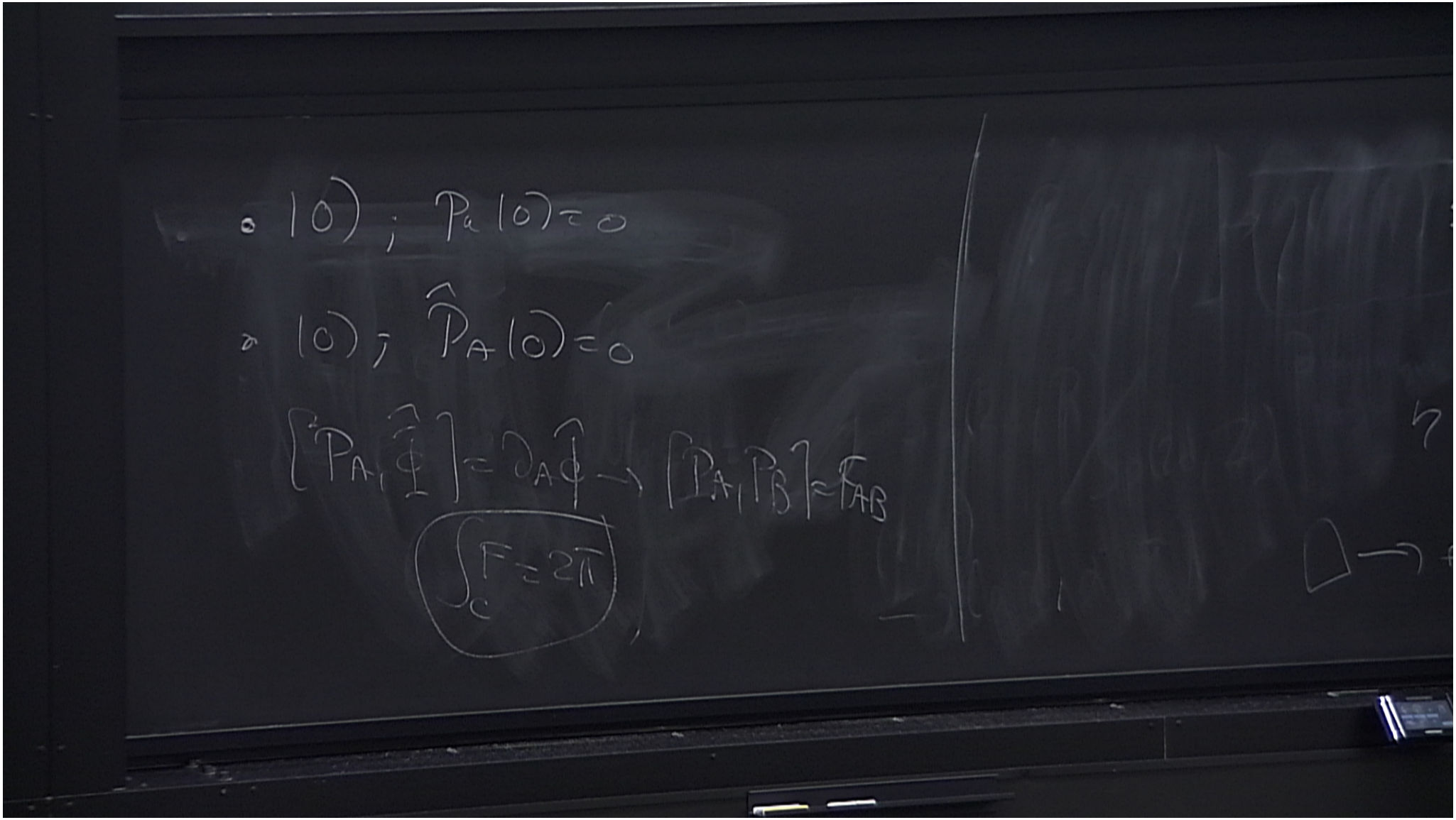

$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\Lambda : S^1 \times S^1$
$\mathbb{R}$
$U_\lambda \; \lambda \in \Lambda$

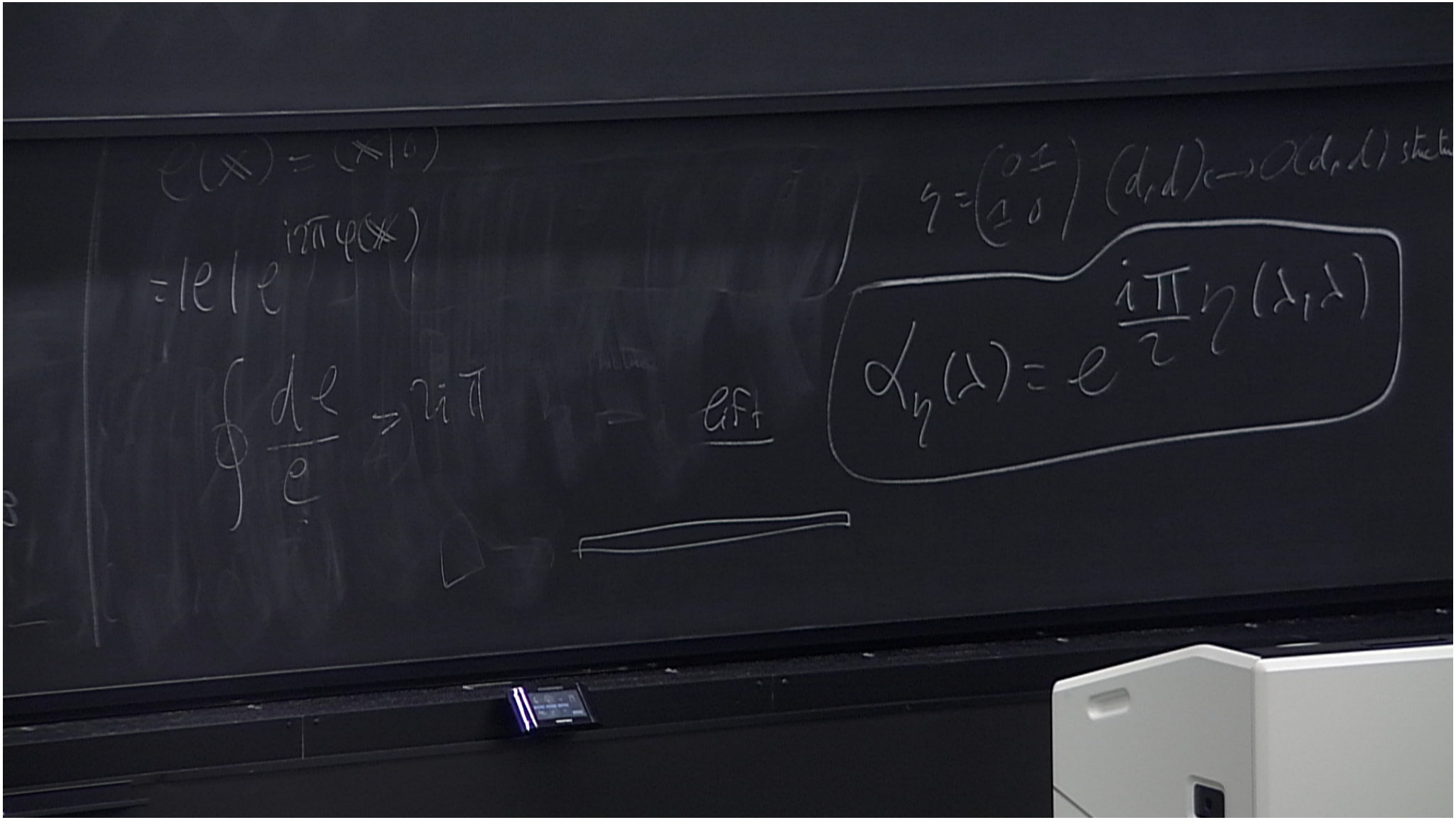

Zak 1d
$\phi(x+m, \tilde{x}+\tilde{n}) = e^{2\pi i n \tilde{x}} \phi(x$
$\mathcal{Z}: L^2(\mathbb{R}) \to L^2(\Lambda)$
$(\mathcal{Z}\phi)(x$
$U = e^{2\pi i \tilde{x}}$
$V = e^{2\pi i \tilde{x}}$

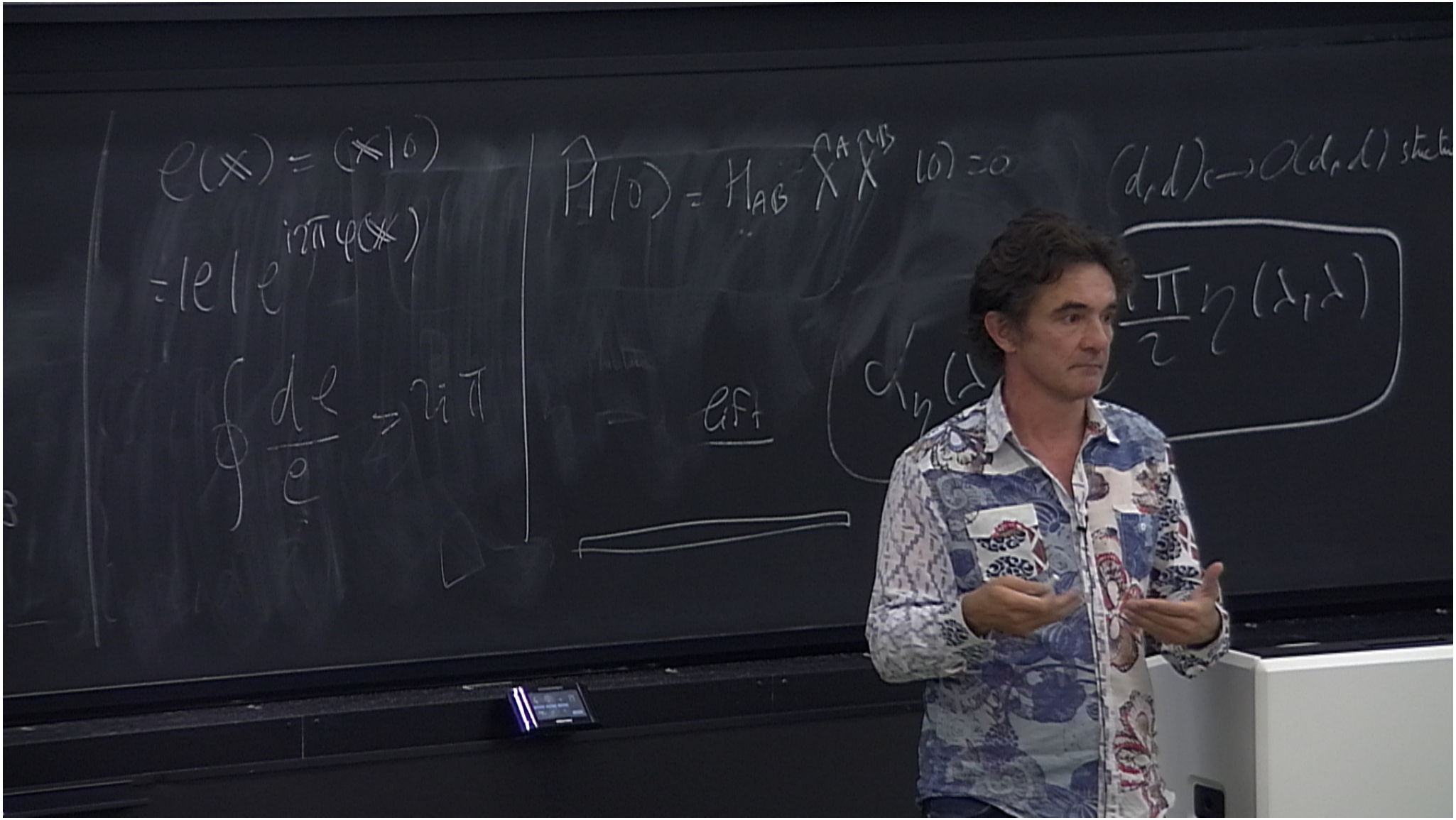

$\phi(x,\tilde{x})$
$-i\pi n\tilde{x}$
$\rho(\lambda x+\lambda n)$
$(x,\tilde{x})$

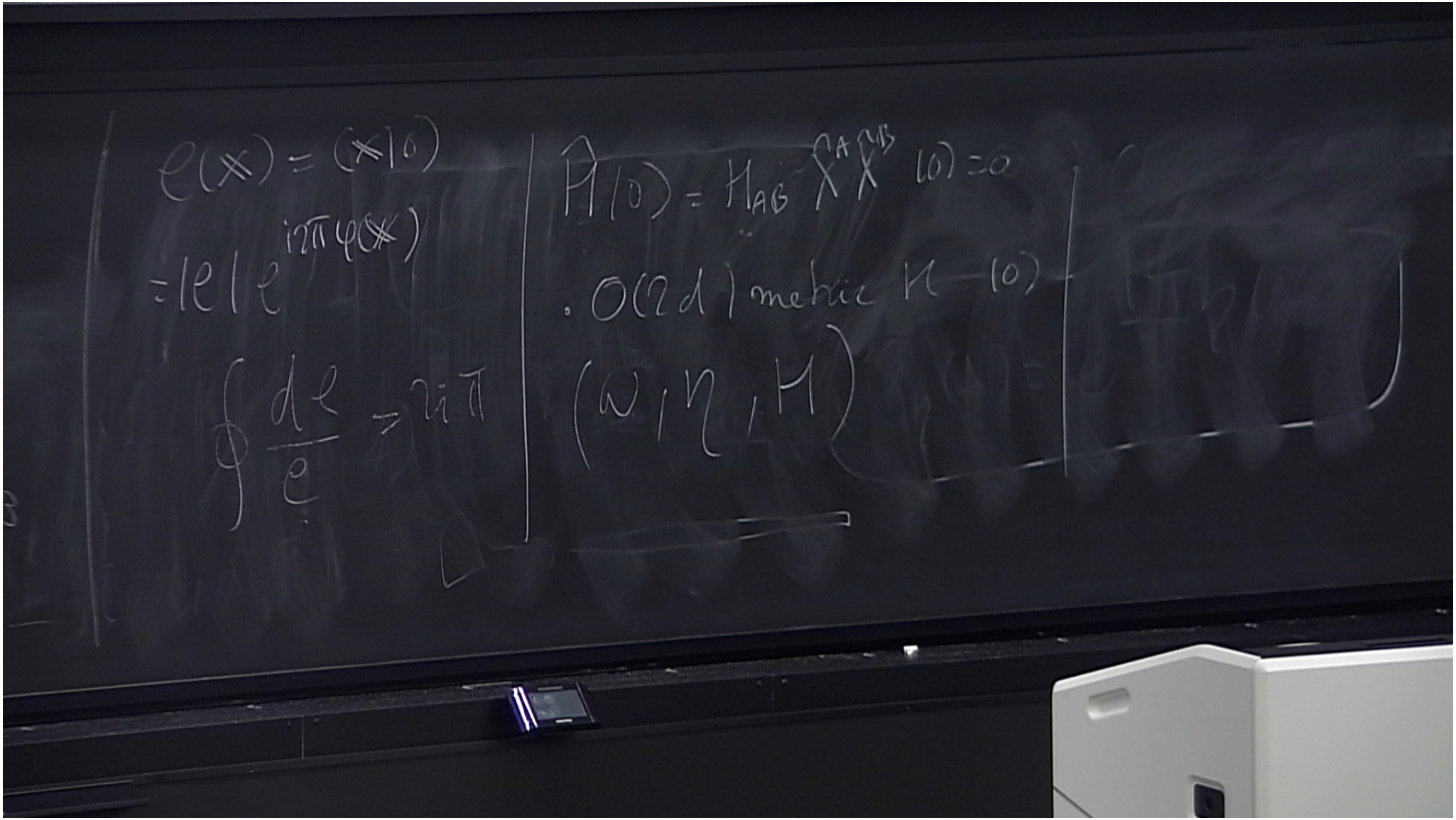


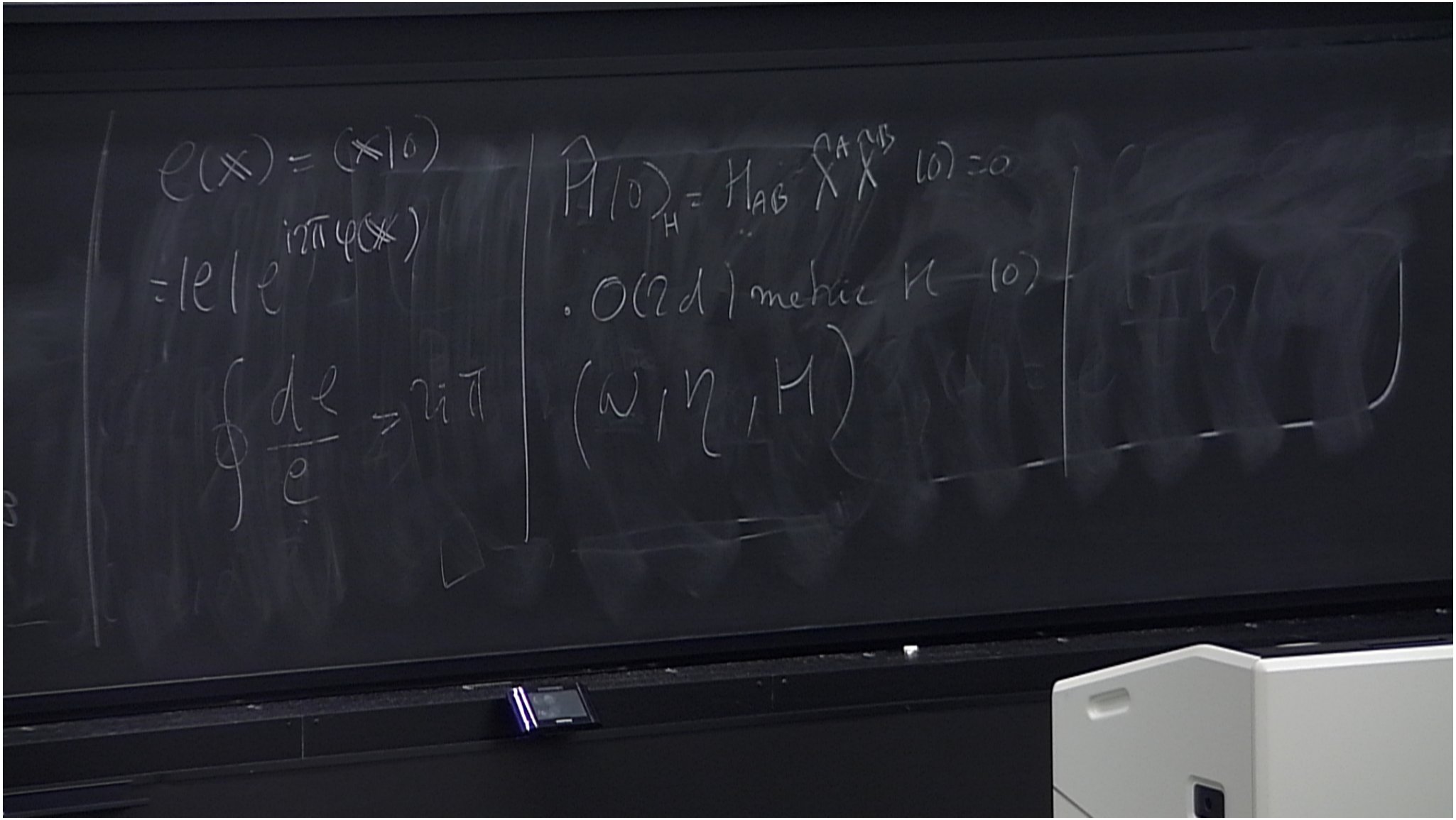

$\phi(x,\tilde{x})$
$\varphi(\lambda x+\lambda n)$
$Z\varphi(x,\tilde{x})=Z\varphi(-x,n)$

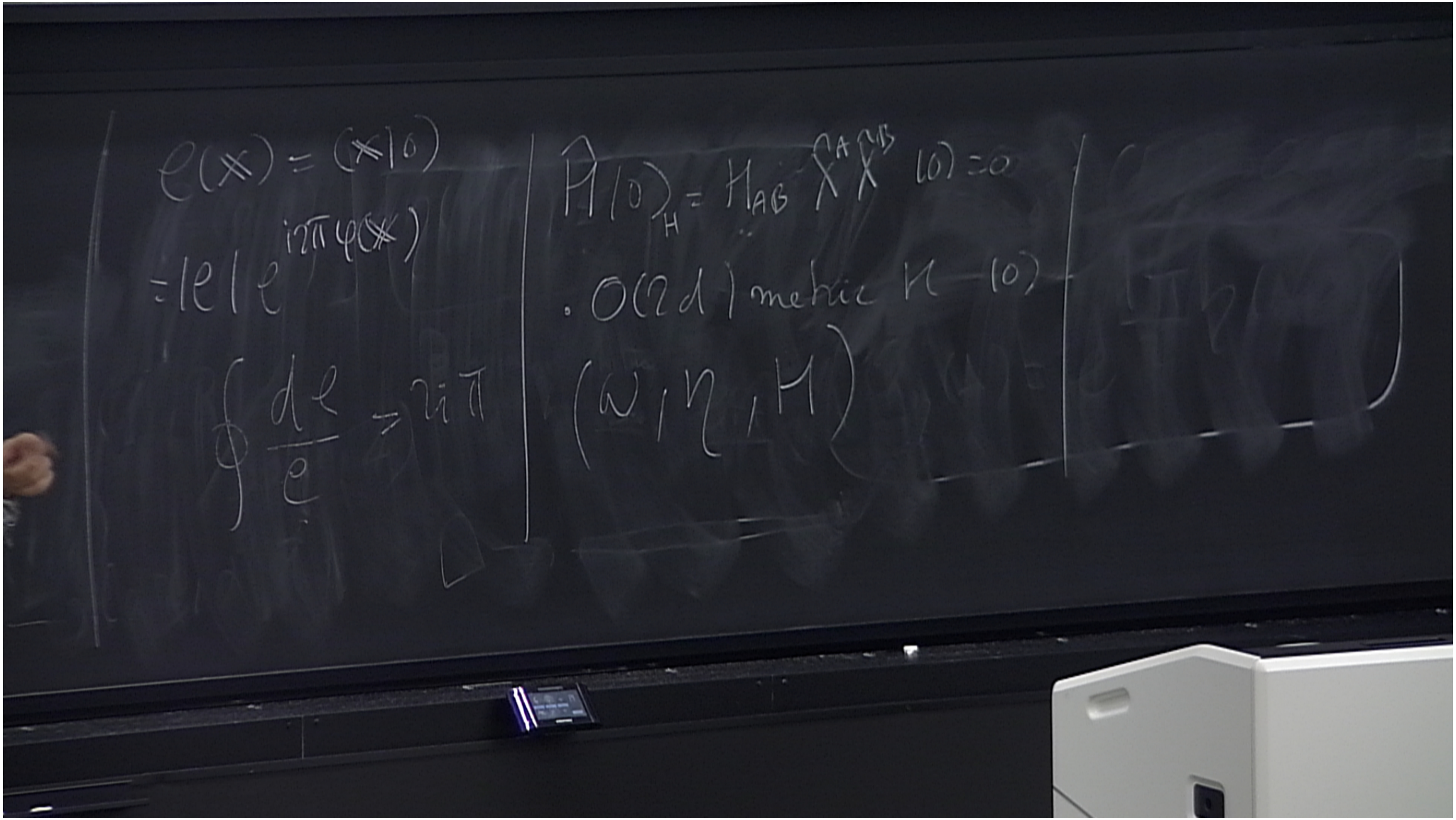

$|0\rangle$; $P_A|0\rangle = 0$
$|0\rangle$; $\hat{P}_A|0\rangle = 0$
$[P_A, \hat{\phi}] = \partial_A \hat{\phi} \rightarrow [P_A, P_B] = F_{AB}$
$\int_C F = 2\pi$

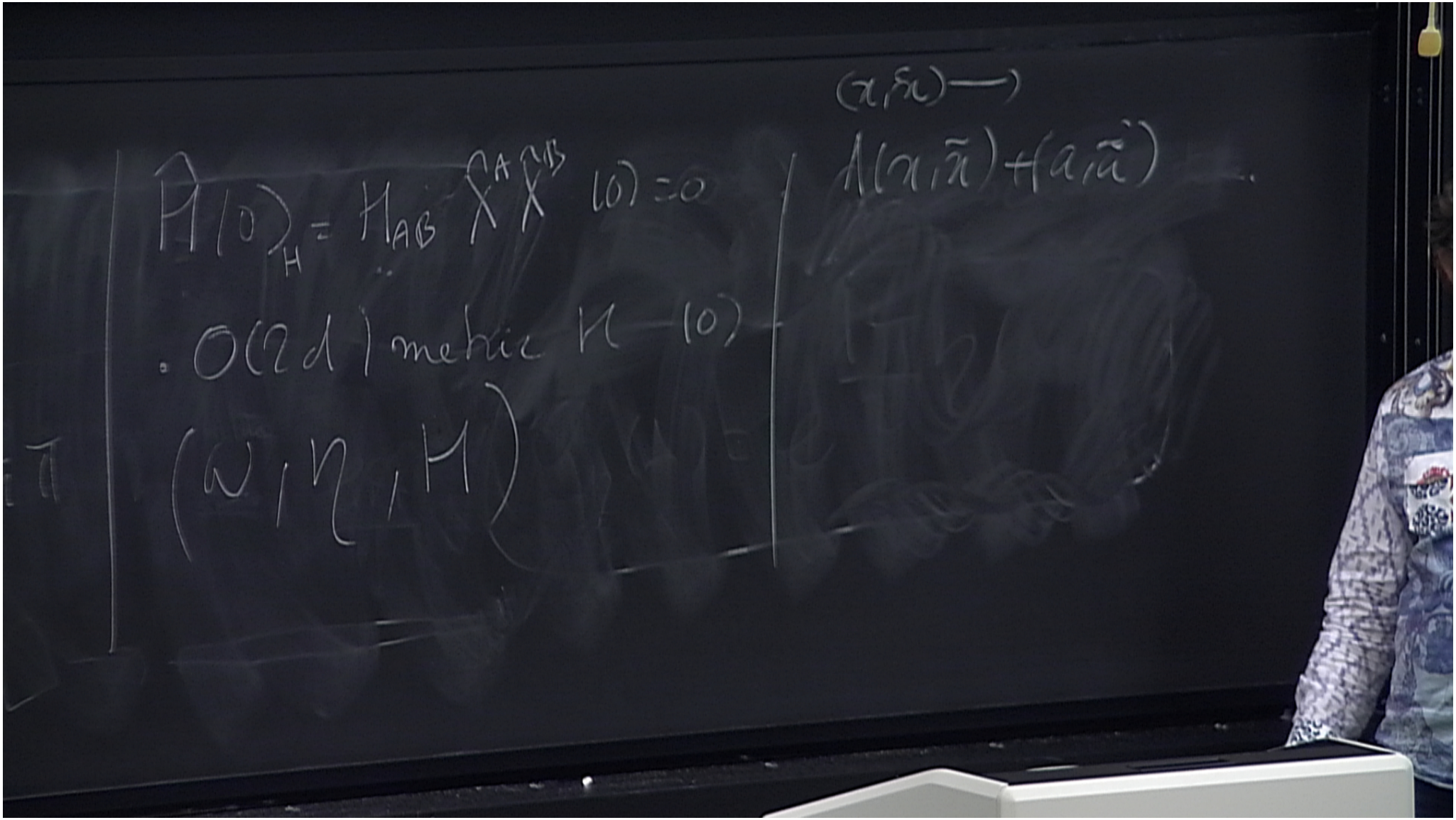

$\ell(\lambda) = (\lambda \mid 0)$
$= |\ell| e^{i2\pi\varphi(\lambda)}$
$\oint \frac{d\ell}{\ell} = 2i\pi$
$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\alpha_\eta(\lambda) = e^{\frac{i\pi}{2}\eta(\lambda,\lambda)}$

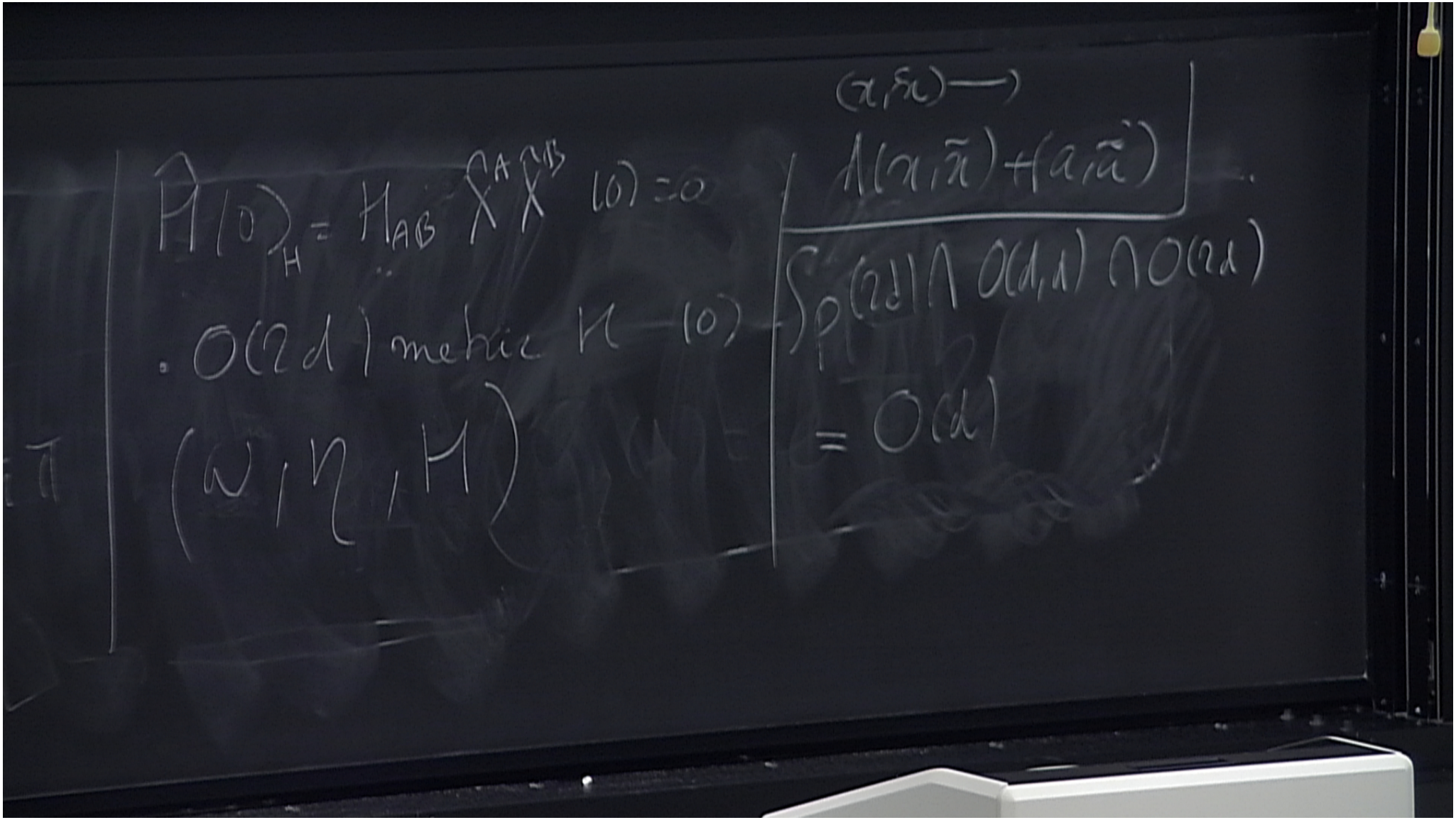

$\rho(x) = (x|0)$
$= |\rho| e^{i2\pi\varphi(x)}$
$\oint \frac{d\rho}{\rho} = 2i\pi$
$\hat{H}(0) = H_{AB} \hat{X}^A \hat{X}^B |0\rangle = 0$
$\frac{i\pi}{2} \eta(\lambda,\lambda)$

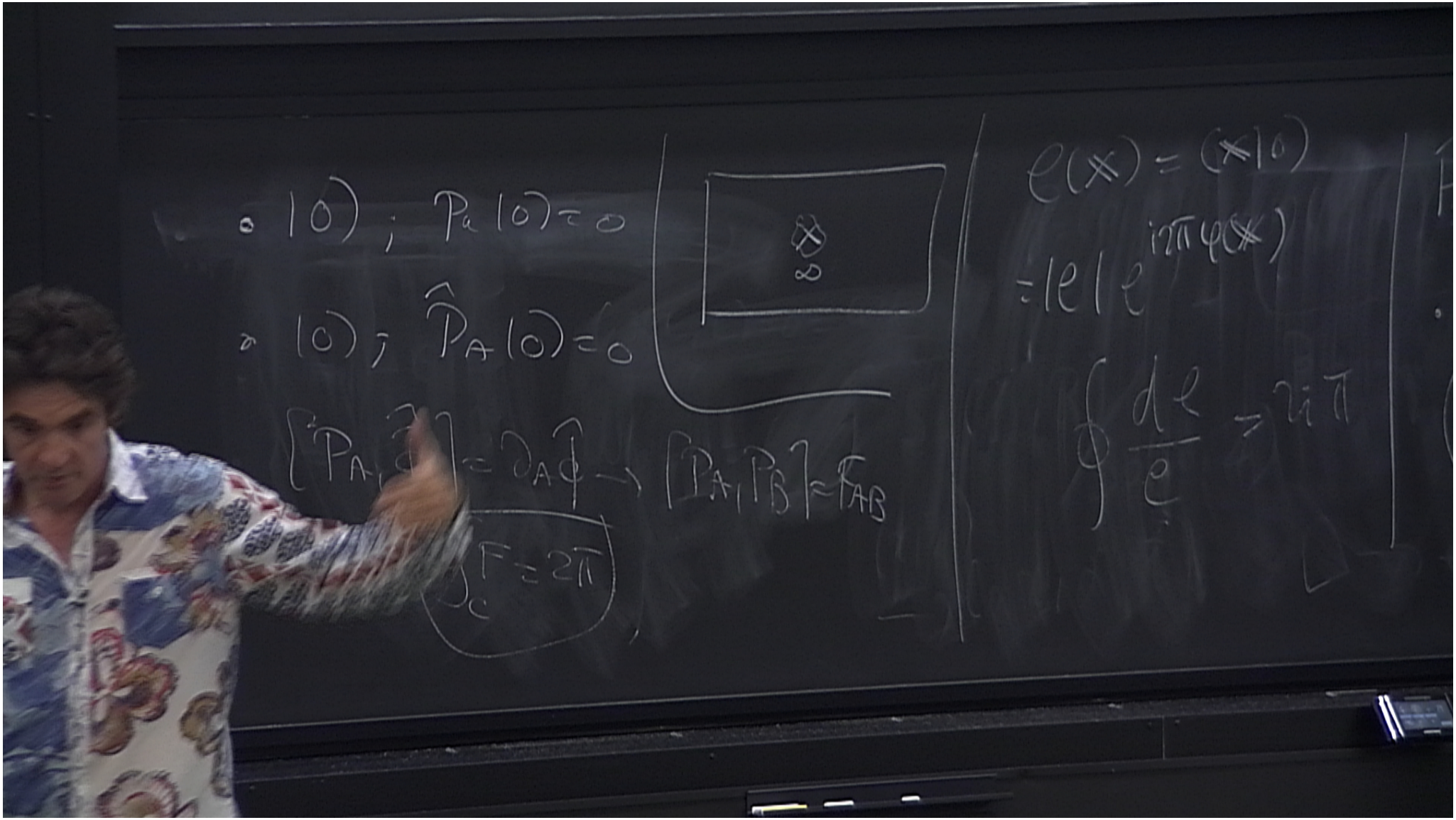

$\rho(x) = (x|0)$
$= |\rho| e^{i2\pi\varphi(x)}$
$\oint \frac{d\rho}{\rho} = 2i\pi$
$\hat{P}(0) = H_{AB} \hat{X}^A \hat{X}^B (0) = 0$
. $O(2d)$ metric $K(0)$
(N, η, H)

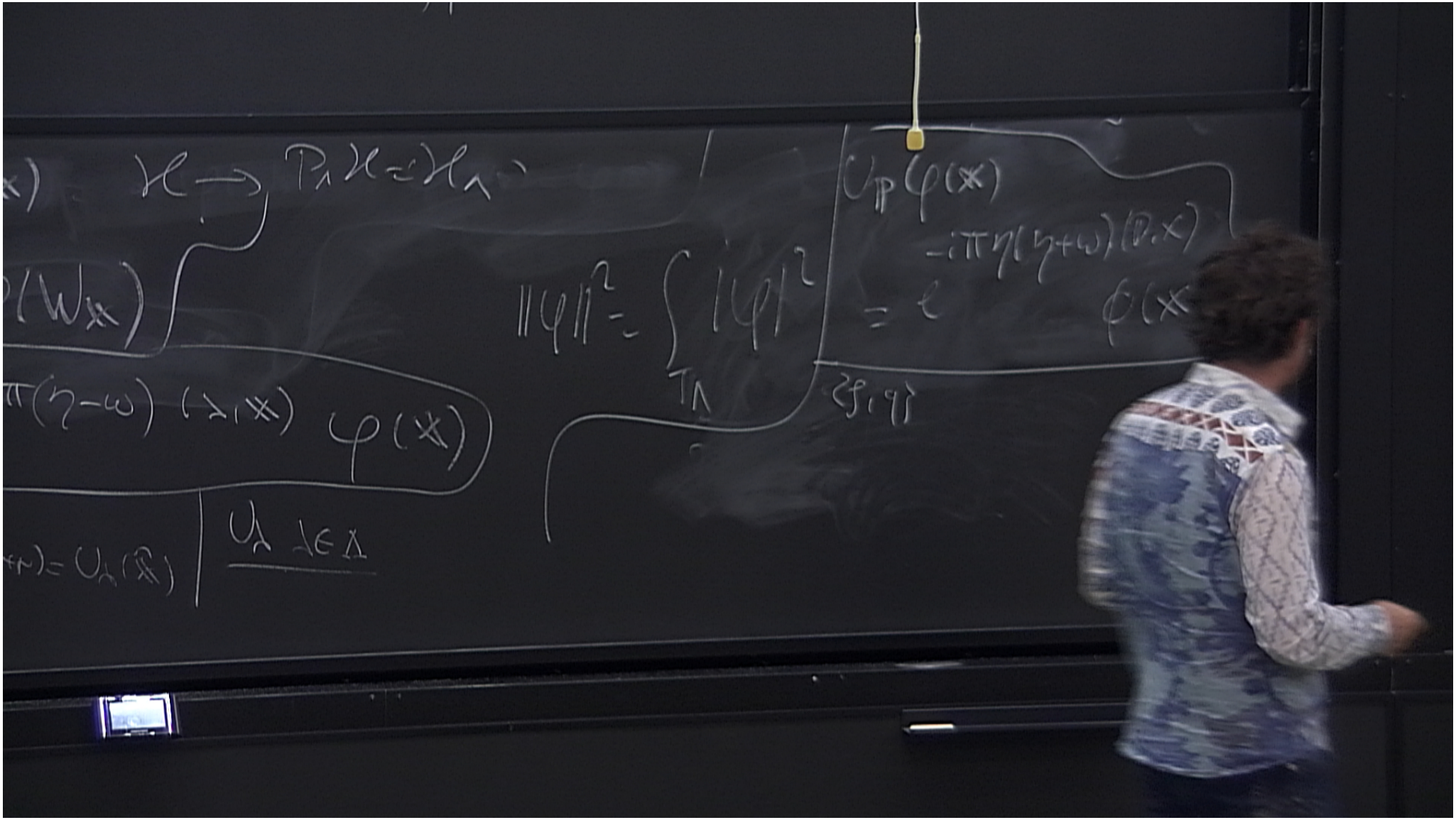

$\ell(x) = (x10)$
$= |\ell| e^{i2\pi\varphi(x)}$
$\oint \frac{d\ell}{\ell} = 2i\pi$
$P(0)_H = H_{AB} X^A X^B (0) = 0$
$O(2d)$ metric $K(0)$
(ω, η, H)

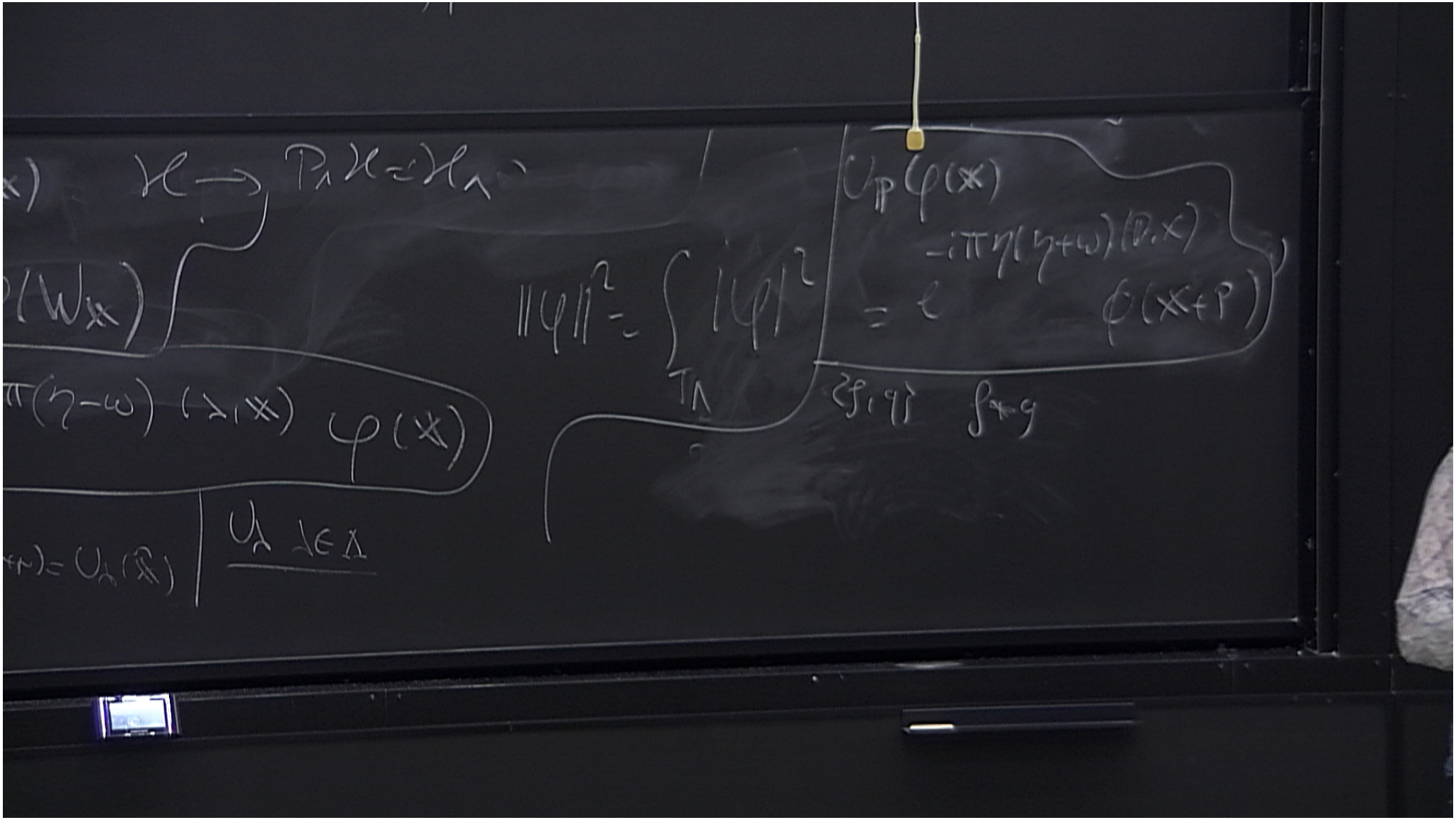

$\rho(x) = (x|0)$
$= |\rho| e^{i2\pi\varphi(x)}$
$\oint \frac{d\rho}{\rho} = 2i\pi$
$\hat{H}(0)_H = H_{AB}\, \hat{x}^A \hat{x}^B (0) = 0$
. $O(2d)$ metric $K(0)$
(ω, η, H)

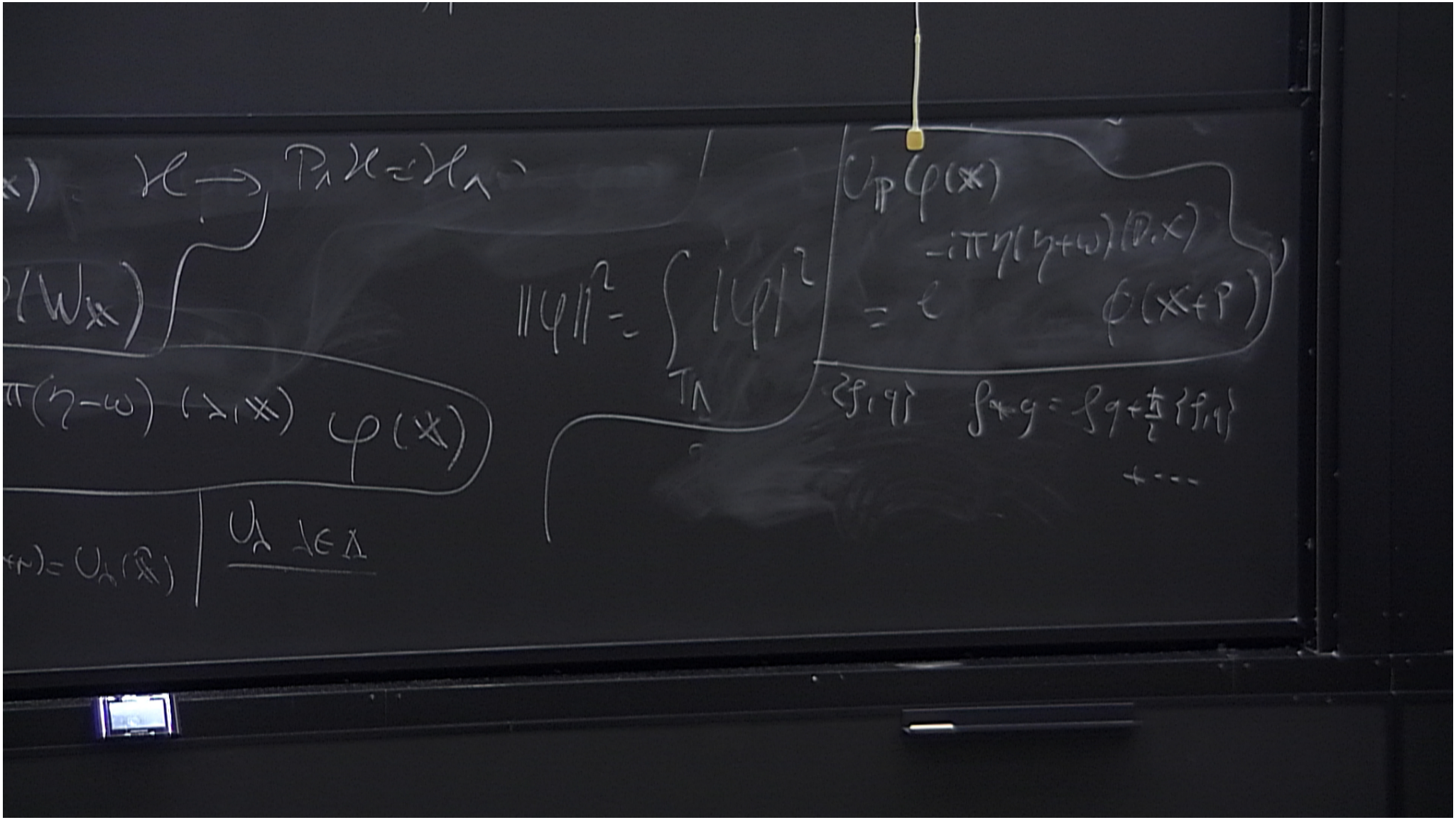

$\hat{H}(0)_H = H_{AB}\,\hat{X}^A\hat{X}^B(0) = 0$
$\Omega(2d)$ metric $K(0)$
(ω, η, H)
$(x, \xi) \longrightarrow$
$\lambda(x, \tilde{x}) + (a, \tilde{a})$

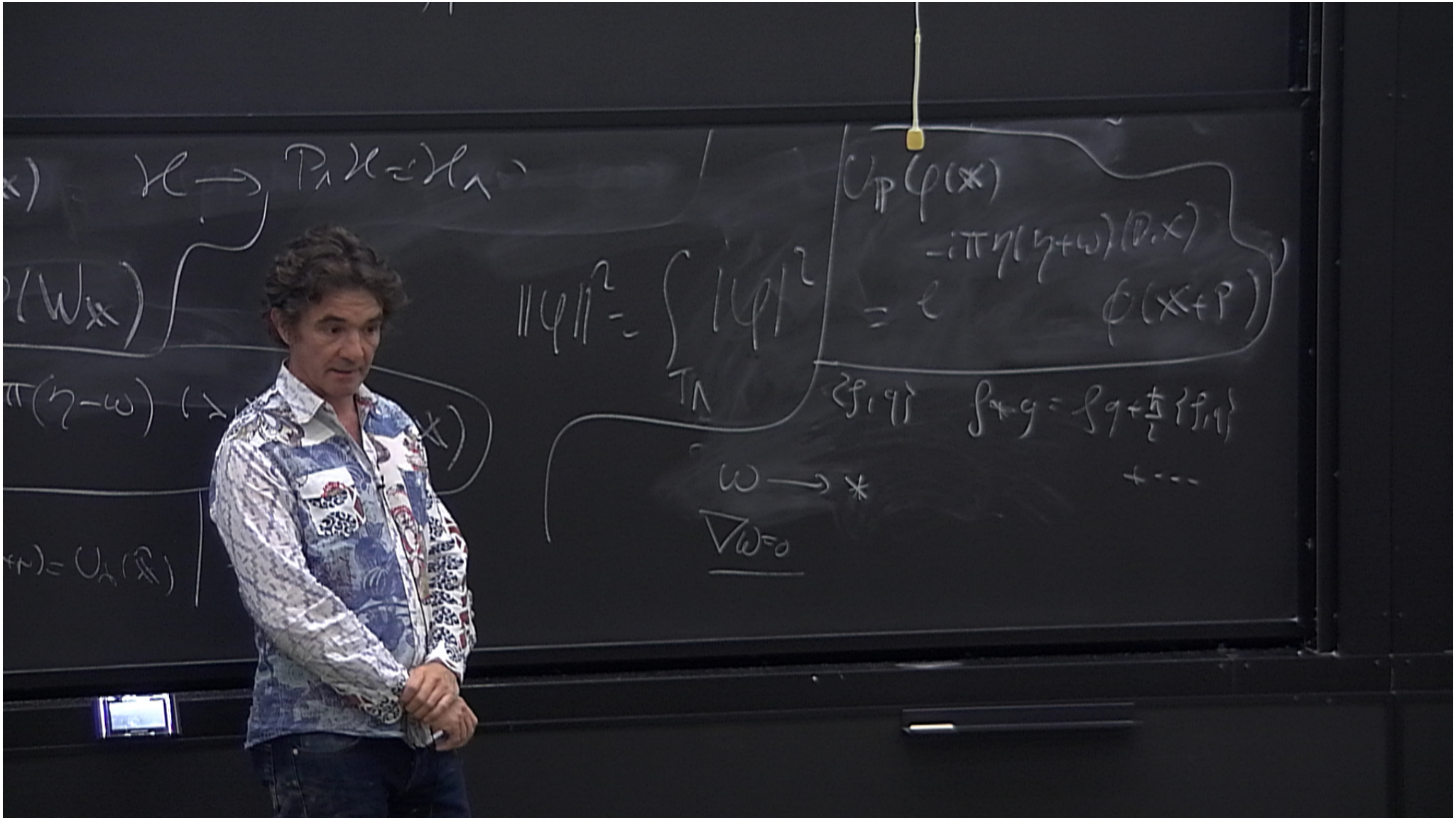

$P(0)_H = H_{AB} X^A X^B (0) = 0$
$O(2d)$ metric $K(0)$
(ω, η, H)
$(x, \xi) \longrightarrow$
$\Lambda(x, \tilde{x}) + (a, \tilde{a})$
$Sp(2d) \cap O(d,d) \cap O(2d)$
$= O(d)$

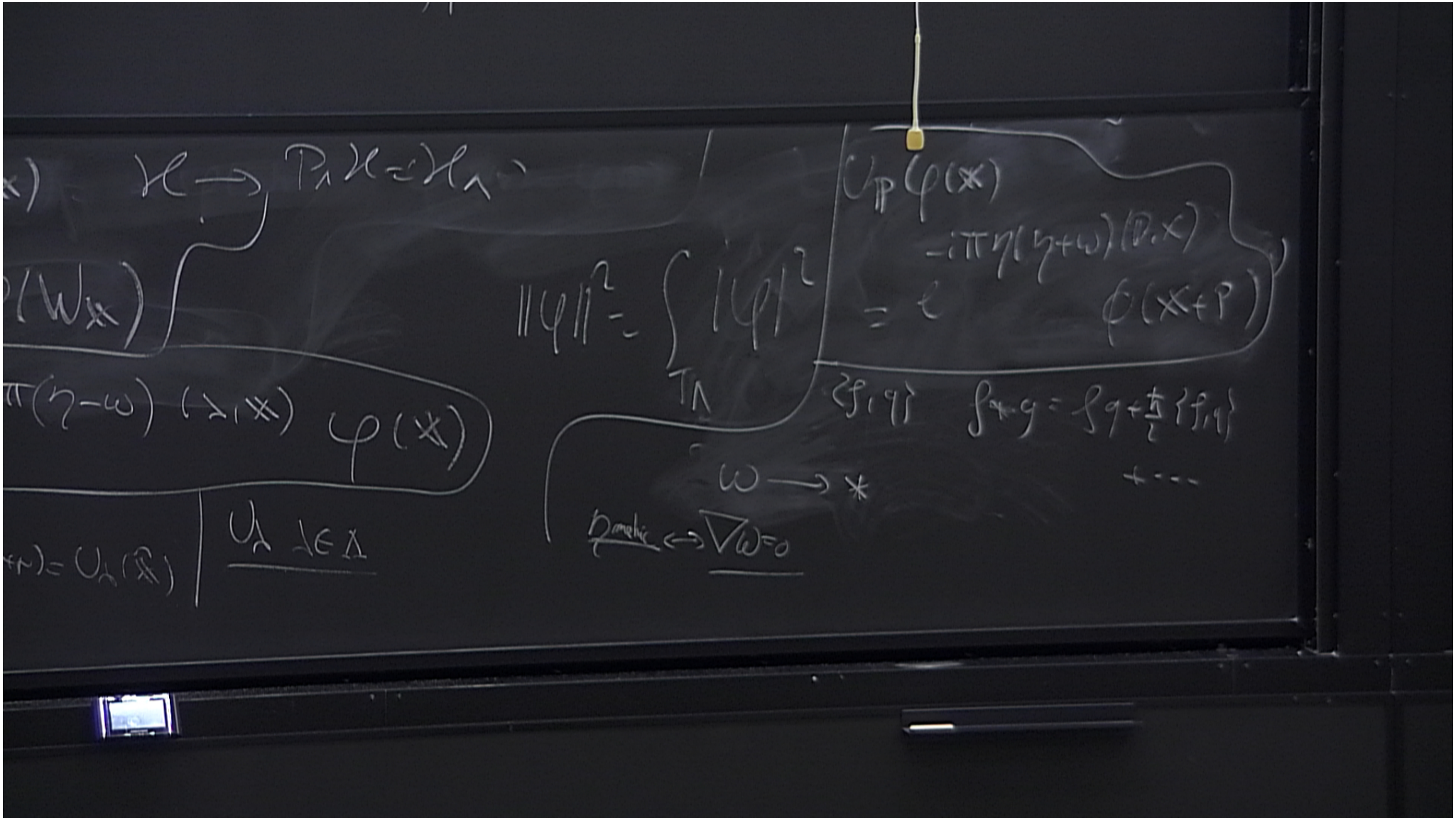

$|0\rangle$; $P_A|0\rangle = 0$
$\hat{P}_A|0\rangle = 0$
$[P_A, \phi] = \partial_A \phi \rightarrow [P_A, P_B] = F_{AB}$
$\oint_C F = 2\pi$
$e(x) = (x|0)$
$= |e| e^{i\pi\varphi(x)}$
$\oint \frac{de}{e} = 2i\pi$

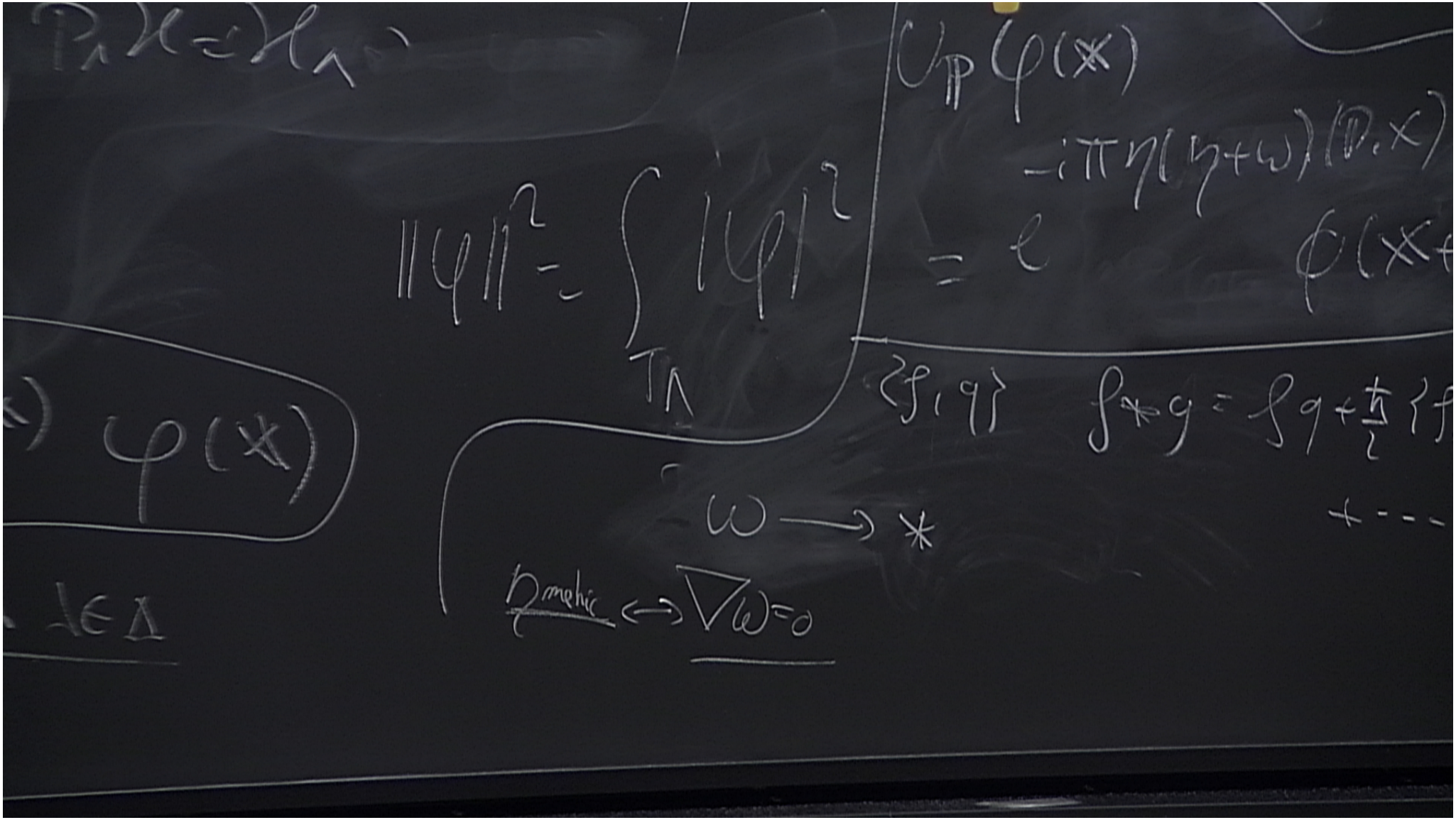



$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\pi(\eta-\omega)(x,x)\ \varphi(x)$
$U_\lambda \ \lambda \in \Lambda$
$\xi, \eta \quad f * g$

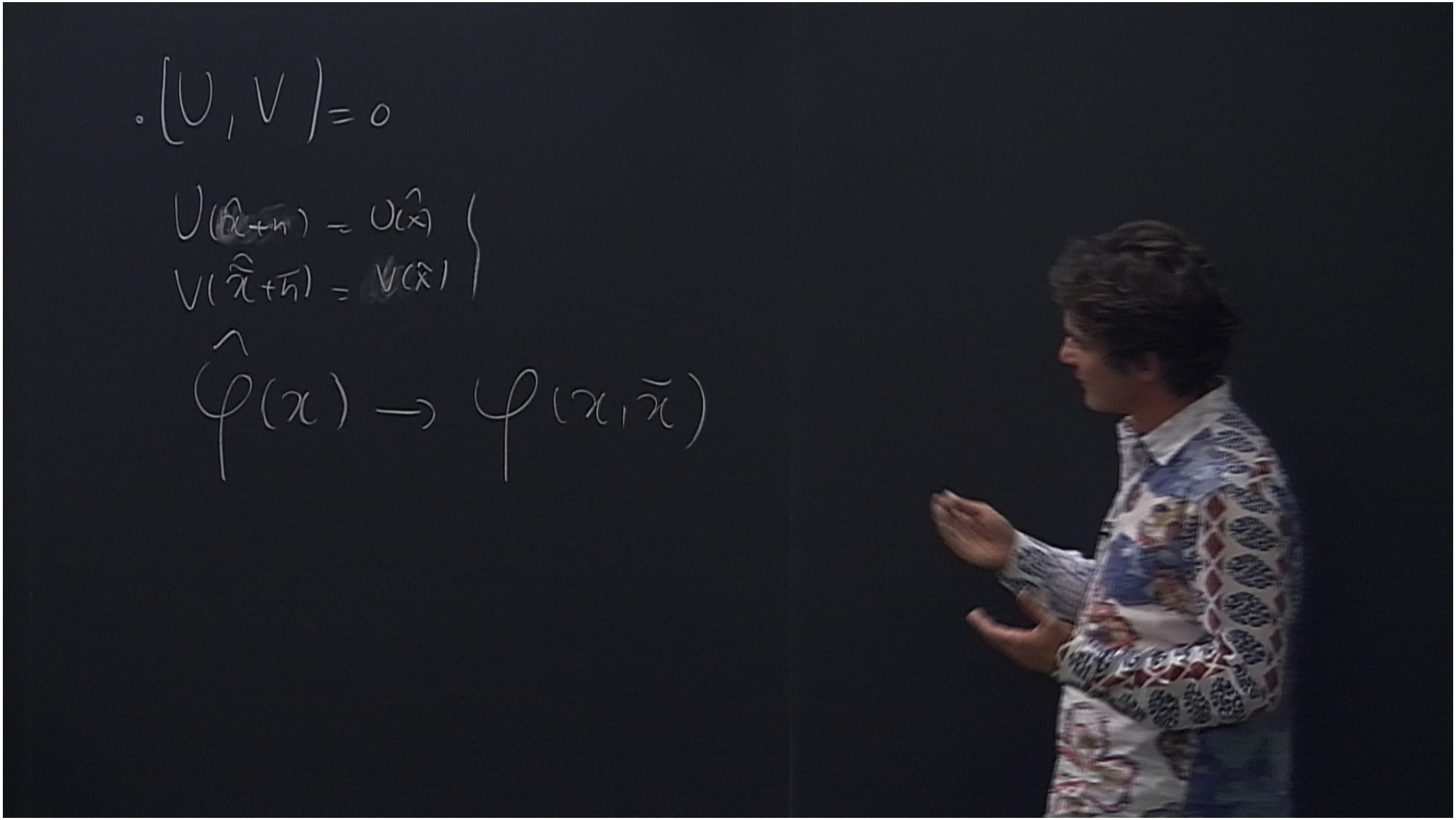

$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$\pi(\eta-\omega)(\lambda,x)\,\varphi(x)$
$\{\xi,\eta\}$
$f * g = fg + \frac{1}{2}\{f,g\} + \cdots$
$U_\lambda \; \lambda \in \Lambda$


$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\omega \to *$
$\nabla\omega = 0$


$\mathcal{H} \to P_\Lambda \mathcal{H} = \mathcal{H}_\Lambda$
$\|\varphi\|^2 = \int_{T_\Lambda} |\varphi|^2$
$U_p \varphi(x) = e^{-i\pi\eta(\eta+\omega)(p,x)} \varphi(x+p)$
$\pi(\eta-\omega)(\lambda,x)\,\varphi(x)$
$\omega \to *$
$U_\lambda \; \lambda \in \Lambda$
$\xi, \eta \quad f * g = fg + \frac{1}{2}\{f,g\} + \cdots$
Dynamic $\leftrightarrow \nabla\omega = 0$


$\|\varphi\|^2 = \int_{T\Lambda} |\varphi|^2$
$U_p \varphi(x)$
$\omega \longrightarrow *$
$\nabla\omega = 0$
$\varphi(x)$
$\in \Lambda$


$\cdot [U, V] = 0$
$U(\hat{x}+n) = U(\hat{x})$
$V(\hat{x}+n) = V(\hat{x})$

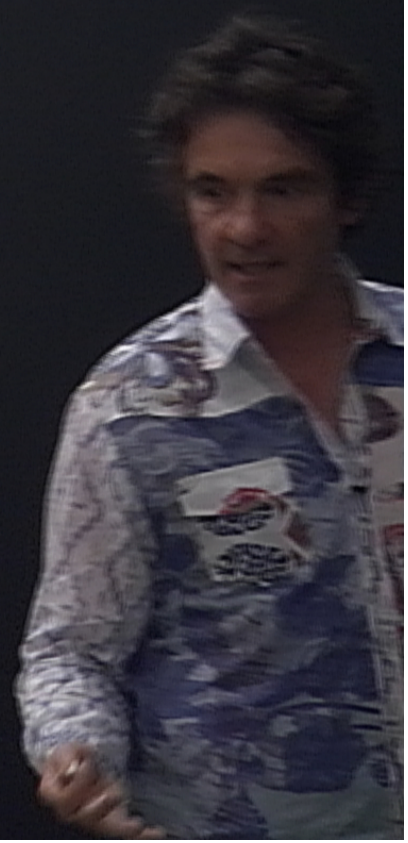

$[U, V] = 0$
$U(\hat{x}+\eta) = U(\hat{x})$
$V(\hat{x}+\eta) = V(\hat{x})$
$\hat{\varphi}(x) \rightarrow \varphi(x, \tilde{x})$

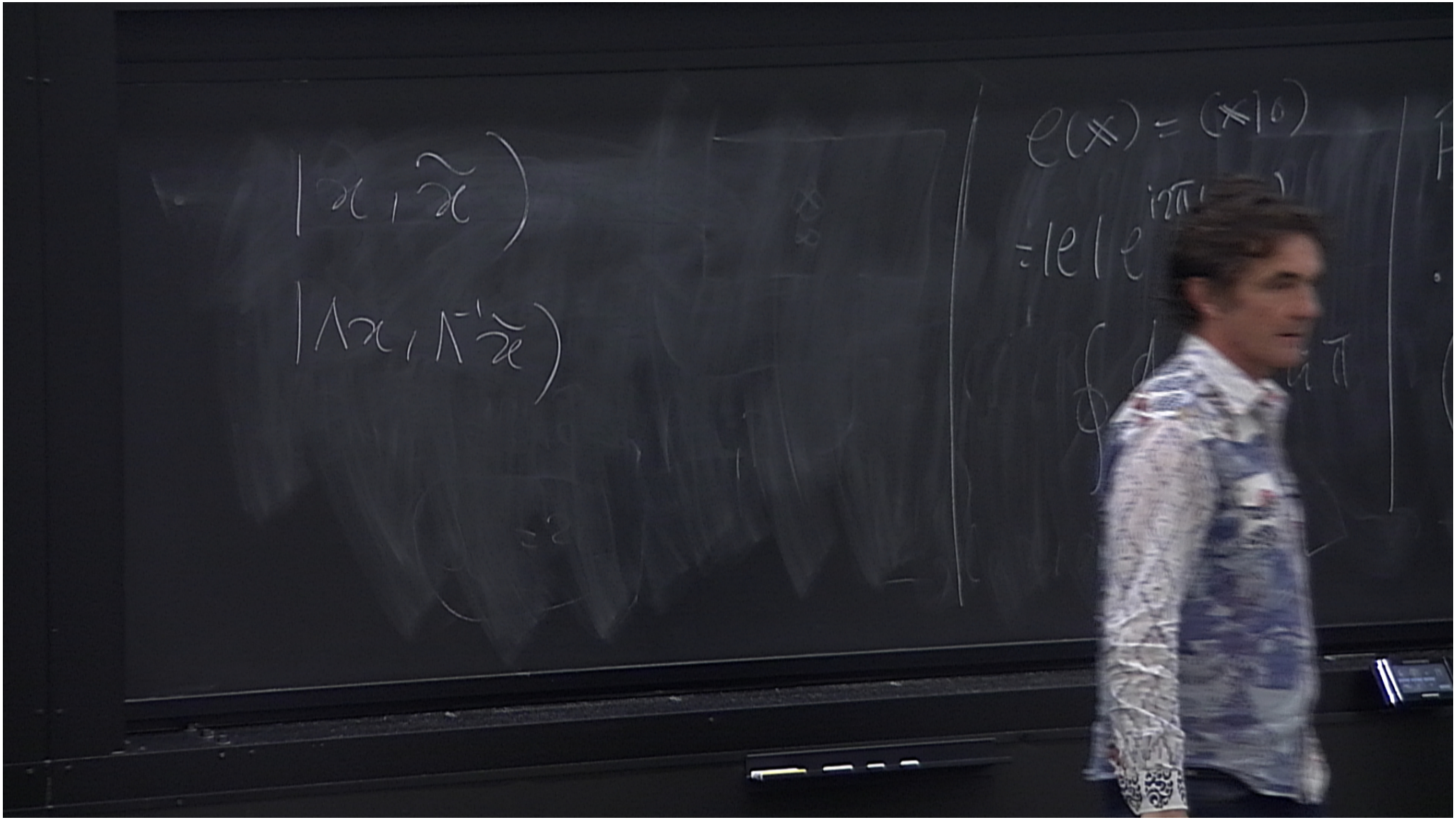
$$[U, V] = 0$$

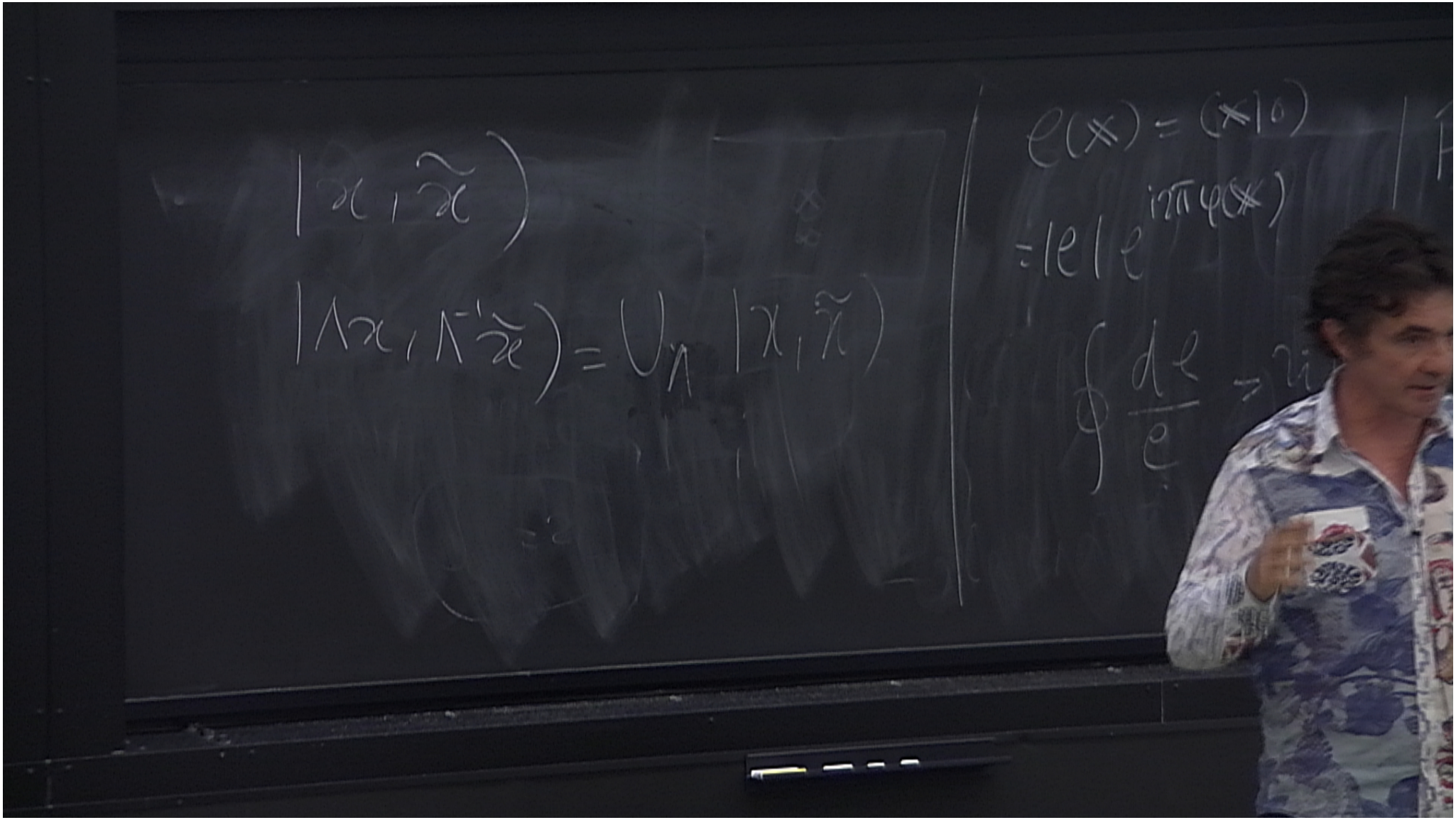
$$\left.\begin{aligned} U(\hat{x}+h) &= U(\hat{x}) \\ V(\hat{x}+h) &= V(\hat{x}) \end{aligned}\right\}$$

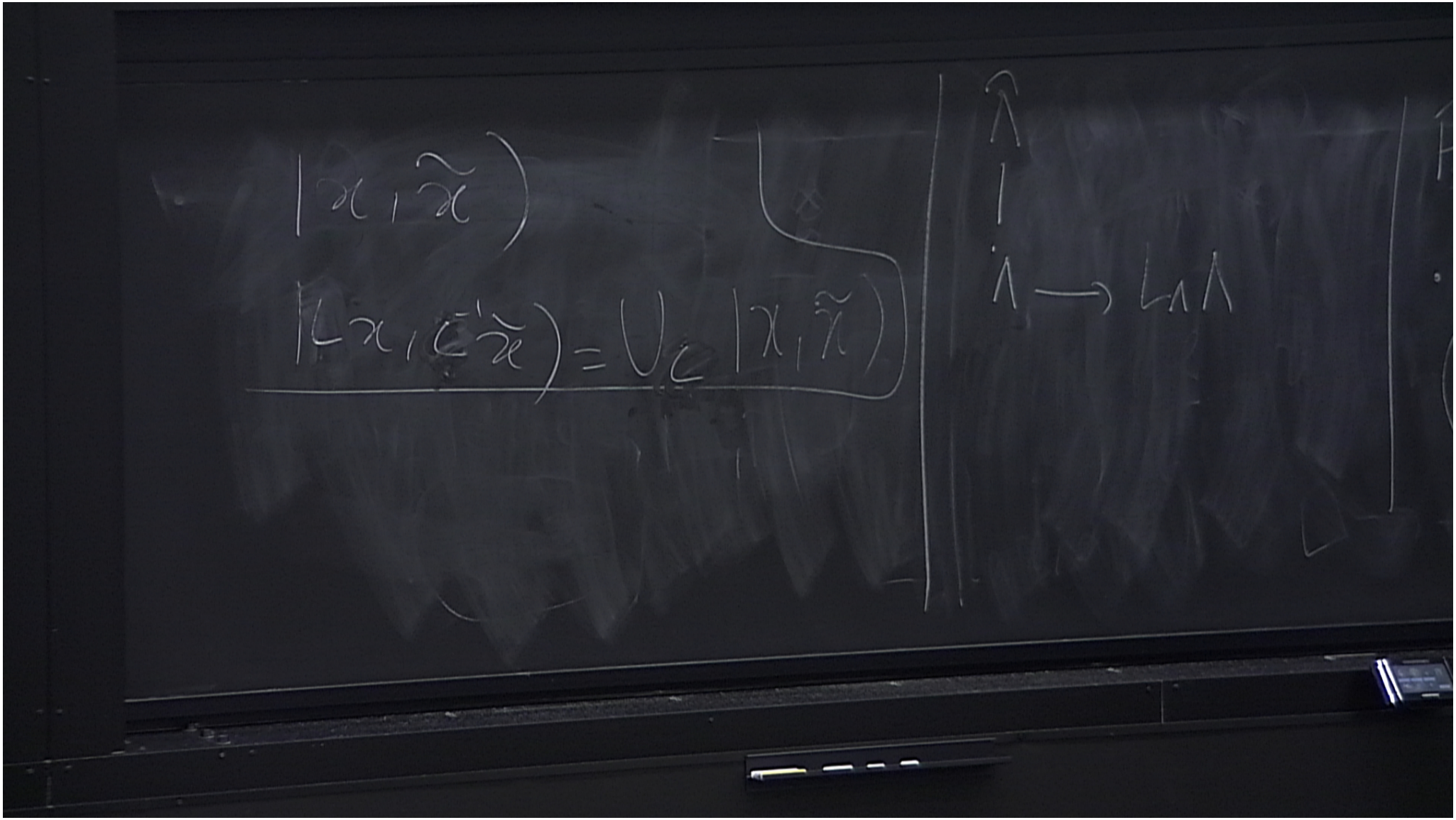
$$\hat{\varphi}(x) \rightarrow \varphi(x, \tilde{x})$$

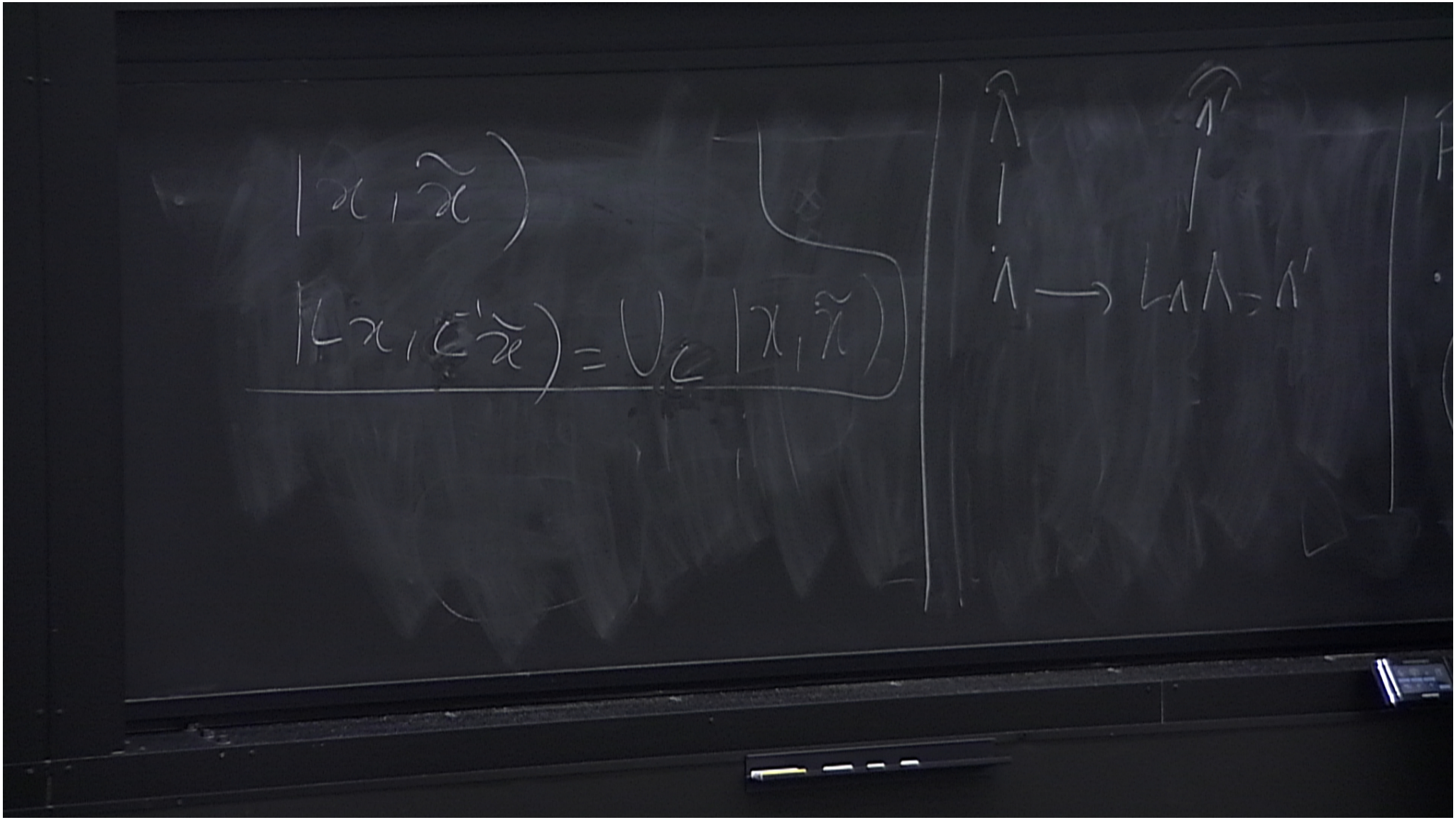
$$= \sum_h e^{ih\tilde{x}} \varphi(x+h)$$

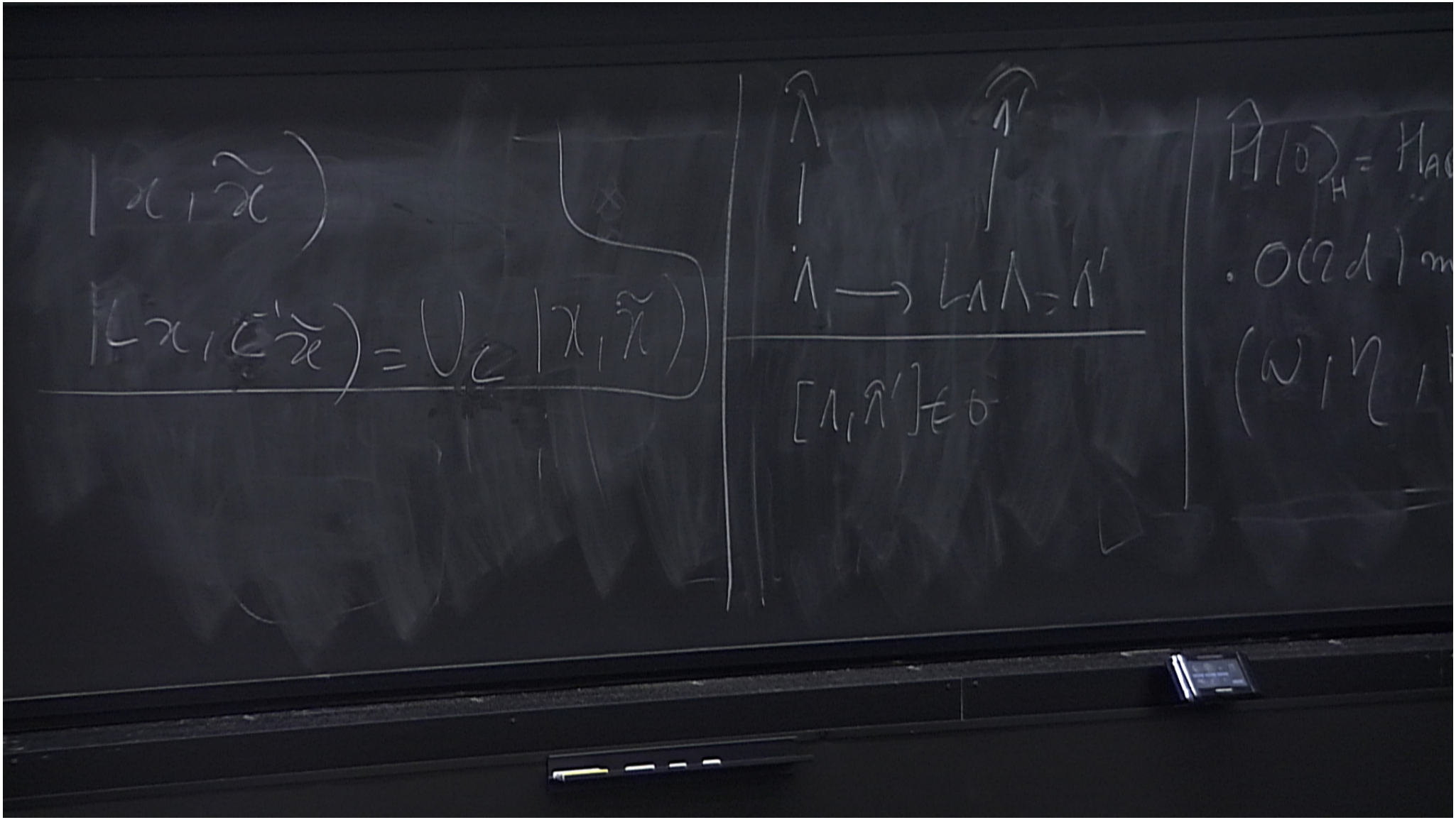


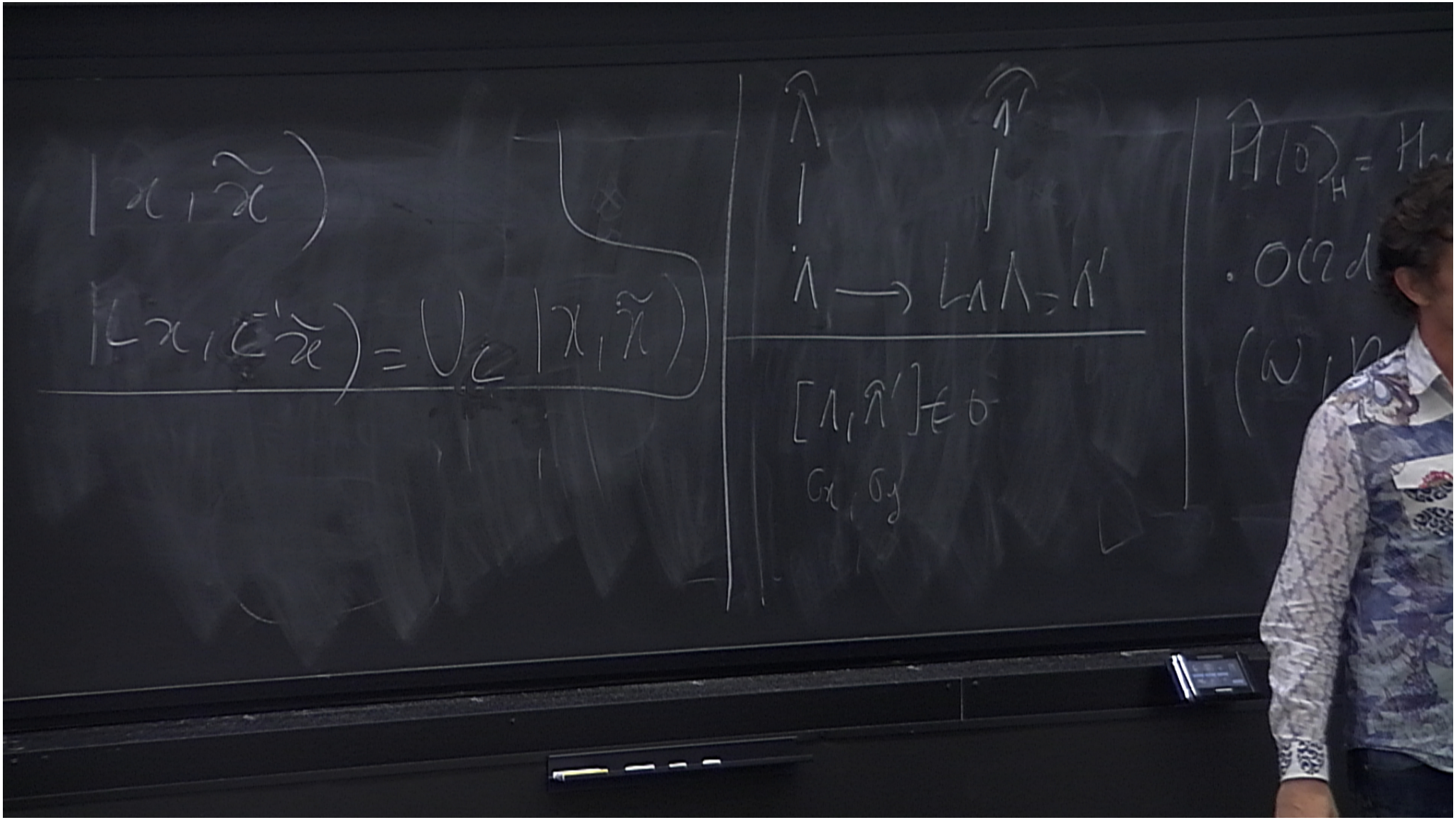

$|x, \tilde{x})$
$|\Lambda x, \Lambda^{-1}\tilde{x})$
$e(x) = (x|0)$

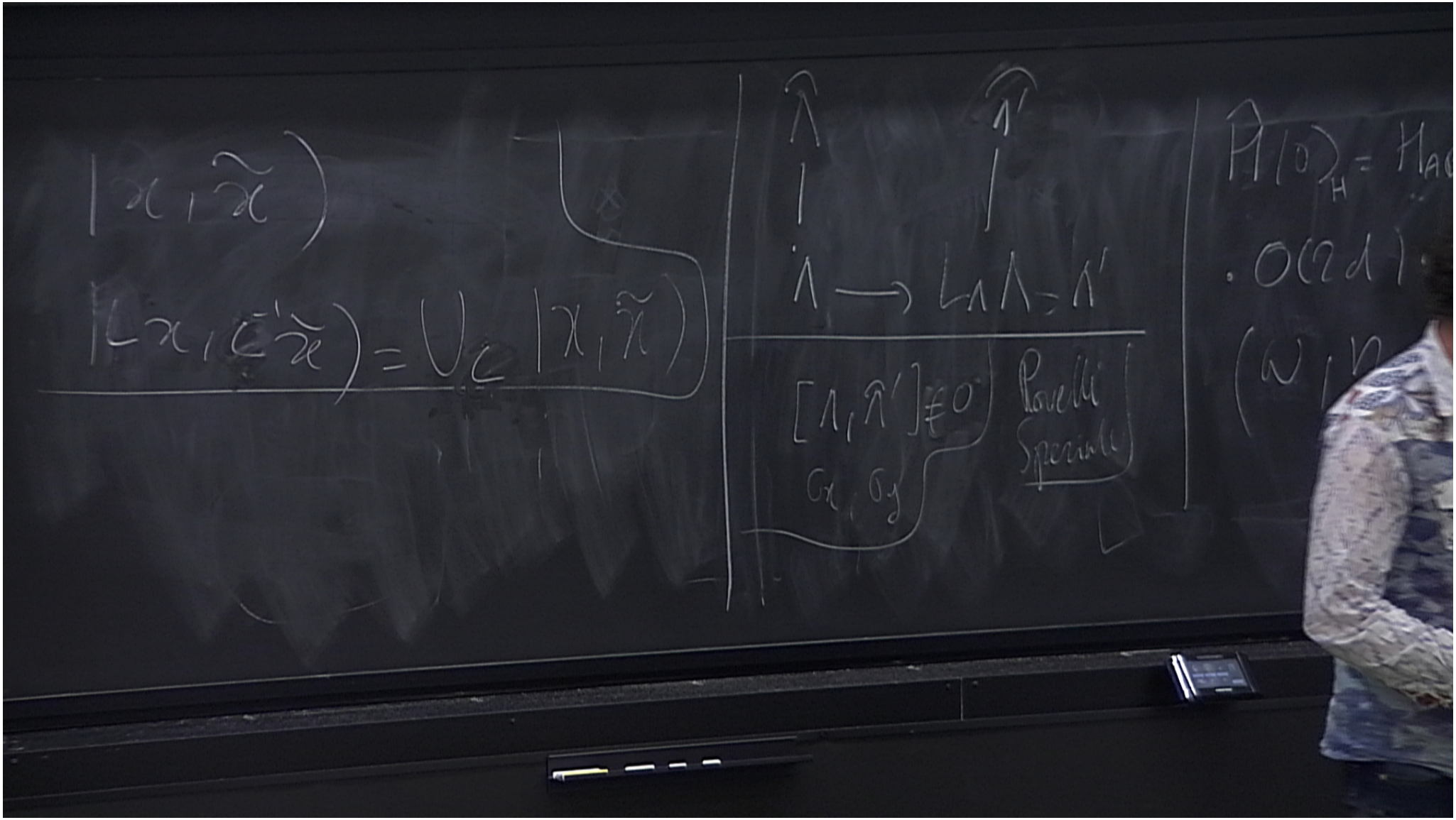

$|x, \tilde{x}\rangle$
$|\Lambda x, \Lambda^{-1}\tilde{x}\rangle = U_\Lambda |x, \tilde{x}\rangle$
$e(*) = (*10)$
$= |e| e^{i\pi\varphi(*)}$
$\oint \frac{de}{e} = 2i$

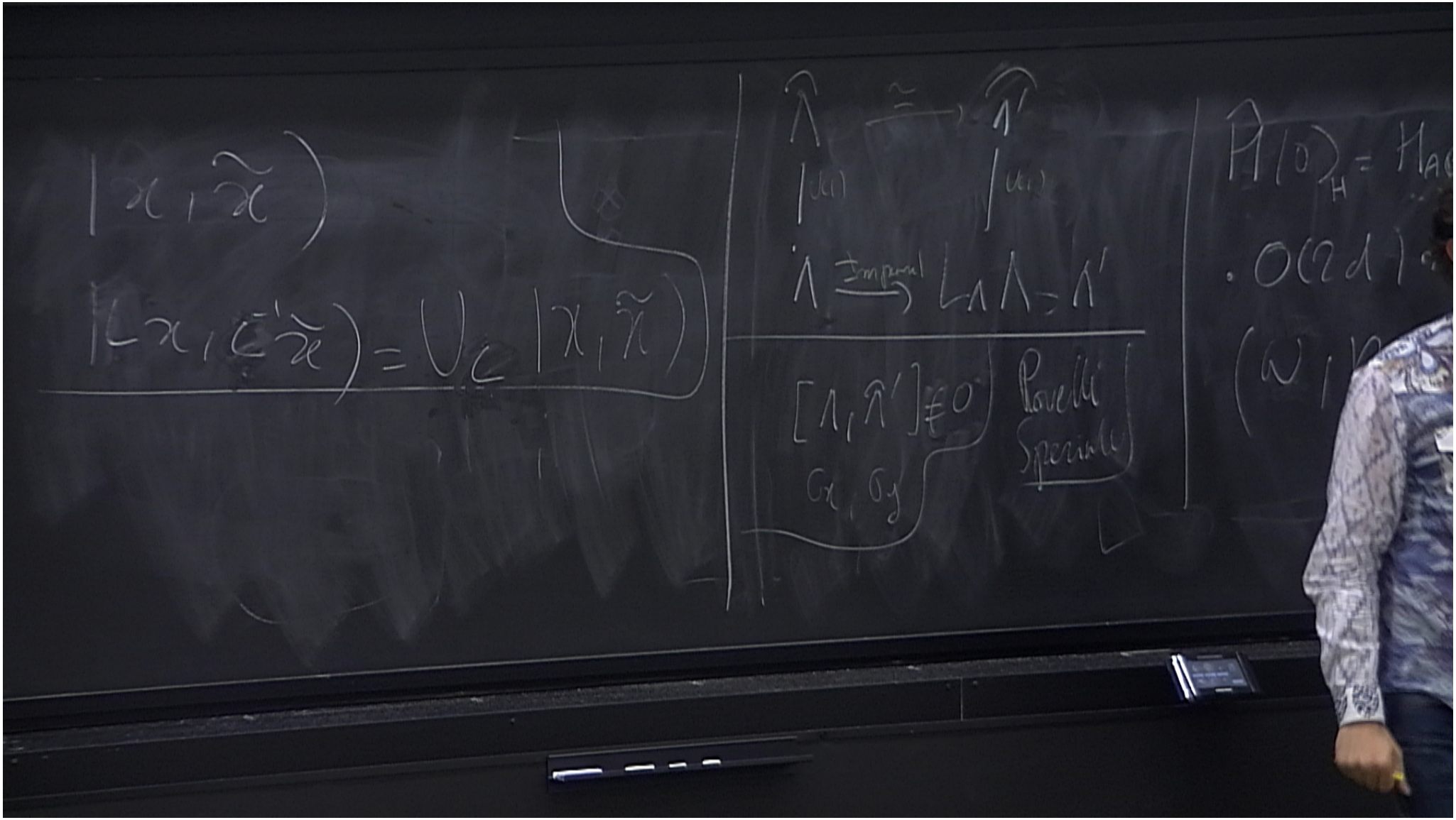

$|x, \tilde{x})$
$|\Lambda x, (\Lambda^{-1})^T \tilde{x}) = U_\Lambda |x, \tilde{x})$
$\Lambda \longrightarrow L_\Lambda \Lambda$

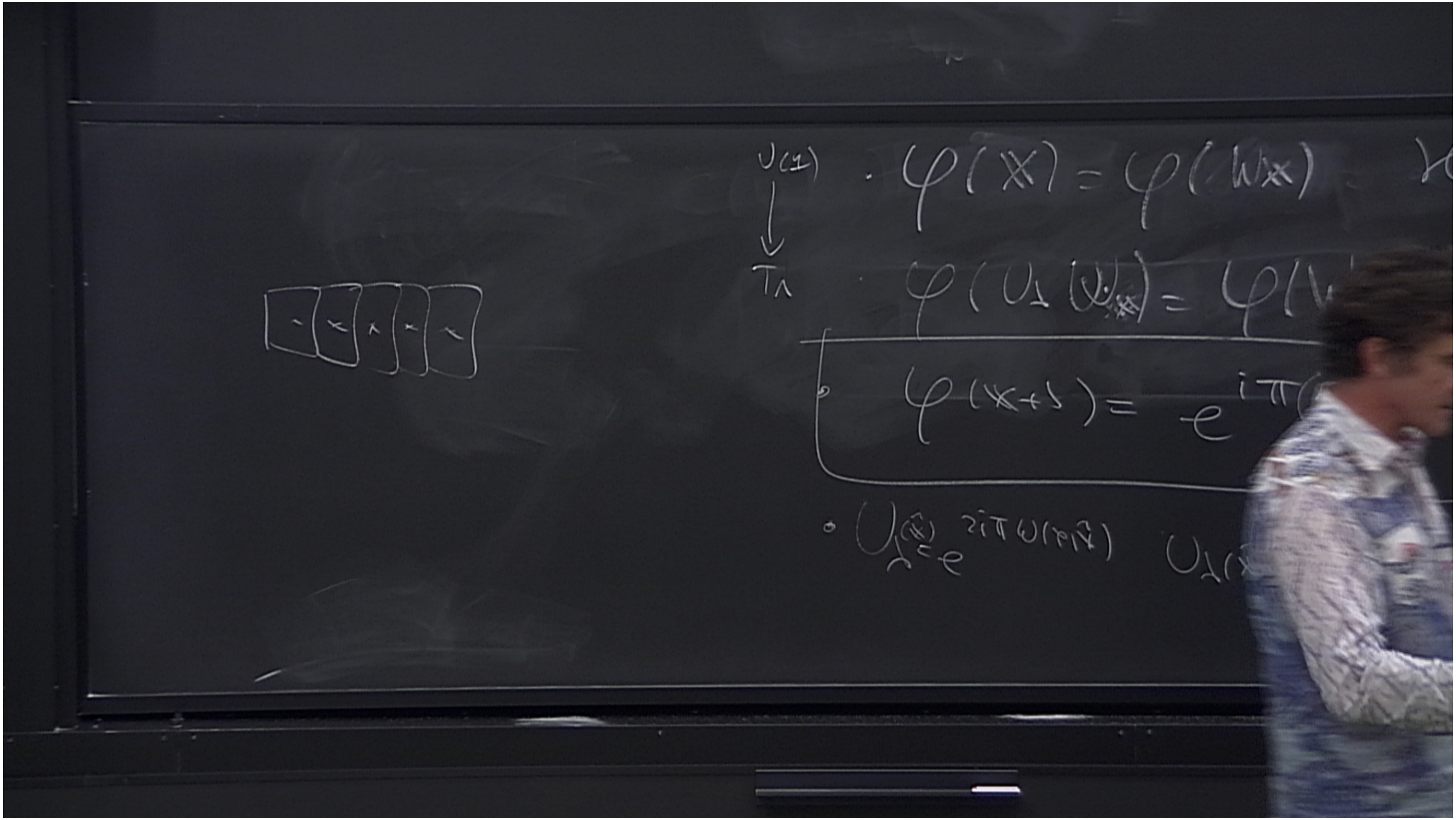

$|x, \tilde{x})$
$|L x, L^{-1} \tilde{x}) = U_L |x, \tilde{x})$
$\Lambda \longrightarrow L_\Lambda \Lambda L_{\Lambda'}^{-1}$

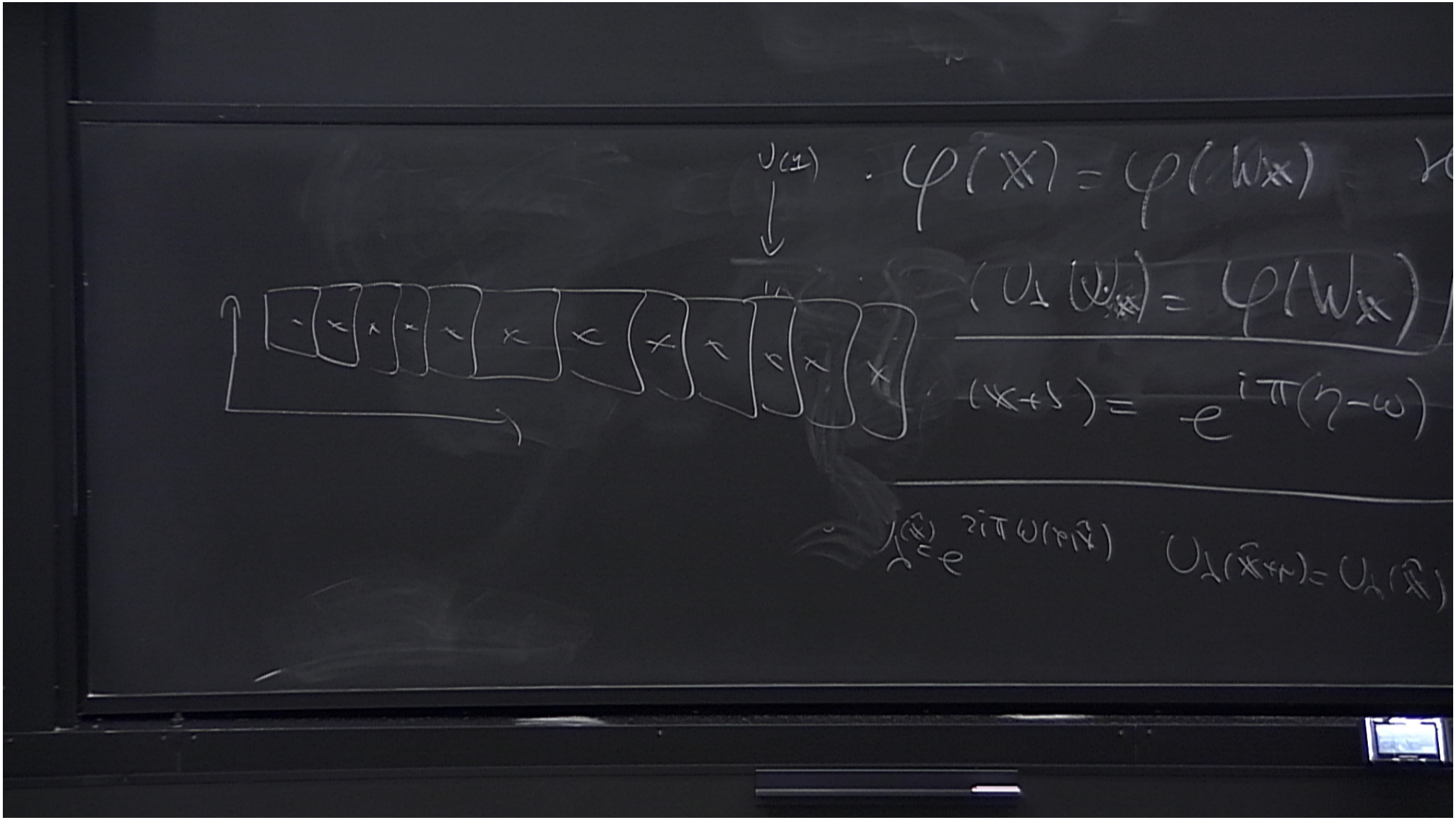

$|x, \tilde{x})$
$|\Lambda x, \Lambda \tilde{x}) = U_\Lambda |x, \tilde{x})$
$\Lambda \longrightarrow L_\Lambda \Lambda = \Lambda'$
$[\Lambda, \Lambda'] \neq 0$
$P(0)_H = H_{AC}$
$O(2d)$

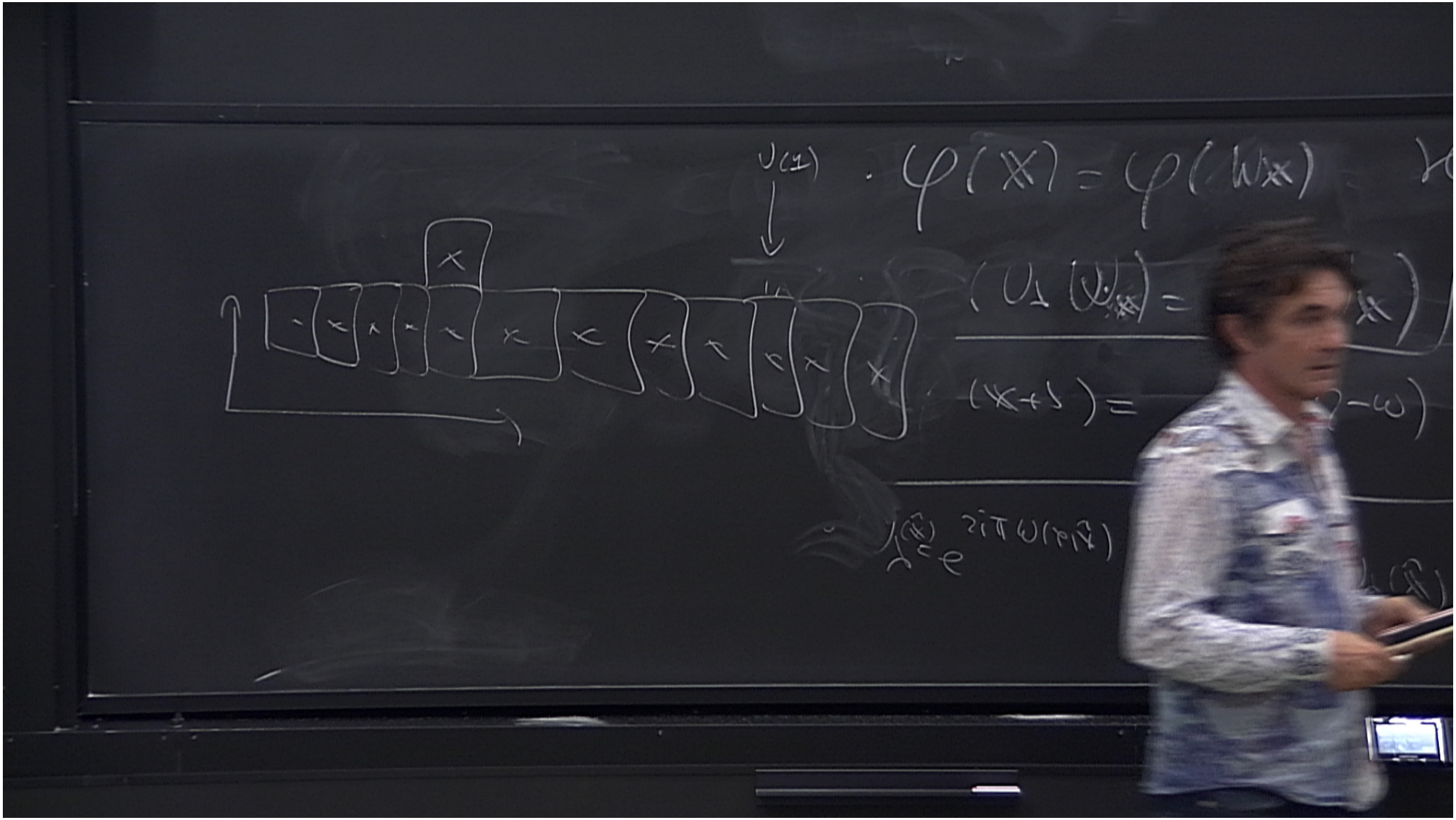

$|x, \tilde{x}\rangle$
$|L x, L \tilde{x}\rangle = U_L |x, \tilde{x}\rangle$
$\Lambda \longrightarrow L_\Lambda \Lambda = \Lambda'$
$[\Lambda, \Lambda'] \neq 0$
G_x, G_y


$|x, \tilde{x})$
$|Lx, \tilde{L}\tilde{x}) = U_L |x, \tilde{x})$
$\Lambda \longrightarrow L_\Lambda \Lambda = \Lambda'$
$[\Lambda, \Lambda'] \neq 0$
Pauli
Speciale
G_x, G_y
$P(0)_H = H_{AC}$
$O(2,1)$
$(W, P$


$|x, \tilde{x}\rangle$
$|Lx, L\tilde{x}\rangle = U_L |x, \tilde{x}\rangle$
$\Lambda \xrightarrow{\text{Impuls}} L_\Lambda \Lambda = \Lambda'$
$[\Lambda, \Lambda'] \neq 0$
Pauli
Spezial


U(1)
T_Λ
φ(X) = φ(WX)
φ(U_Λ(W)X) = φ(W
φ(X+1) = e^{iπ
U_Λ^{(1)} = e^{2iπW(P_Λ^T)}


$U(1)$
$\varphi(x) = \varphi(Wx)$
$U_1(W_{xx}) = \varphi(W_x)$
$(x+1) = e^{i\pi(\gamma-\omega)}$
$U_1(x+n) = U_1(x)$


U(t)
φ(X) = φ(WX)