Title: Majorana zero modes in condensed matter: From topological quantum computing milestones to Majorana-dimer models

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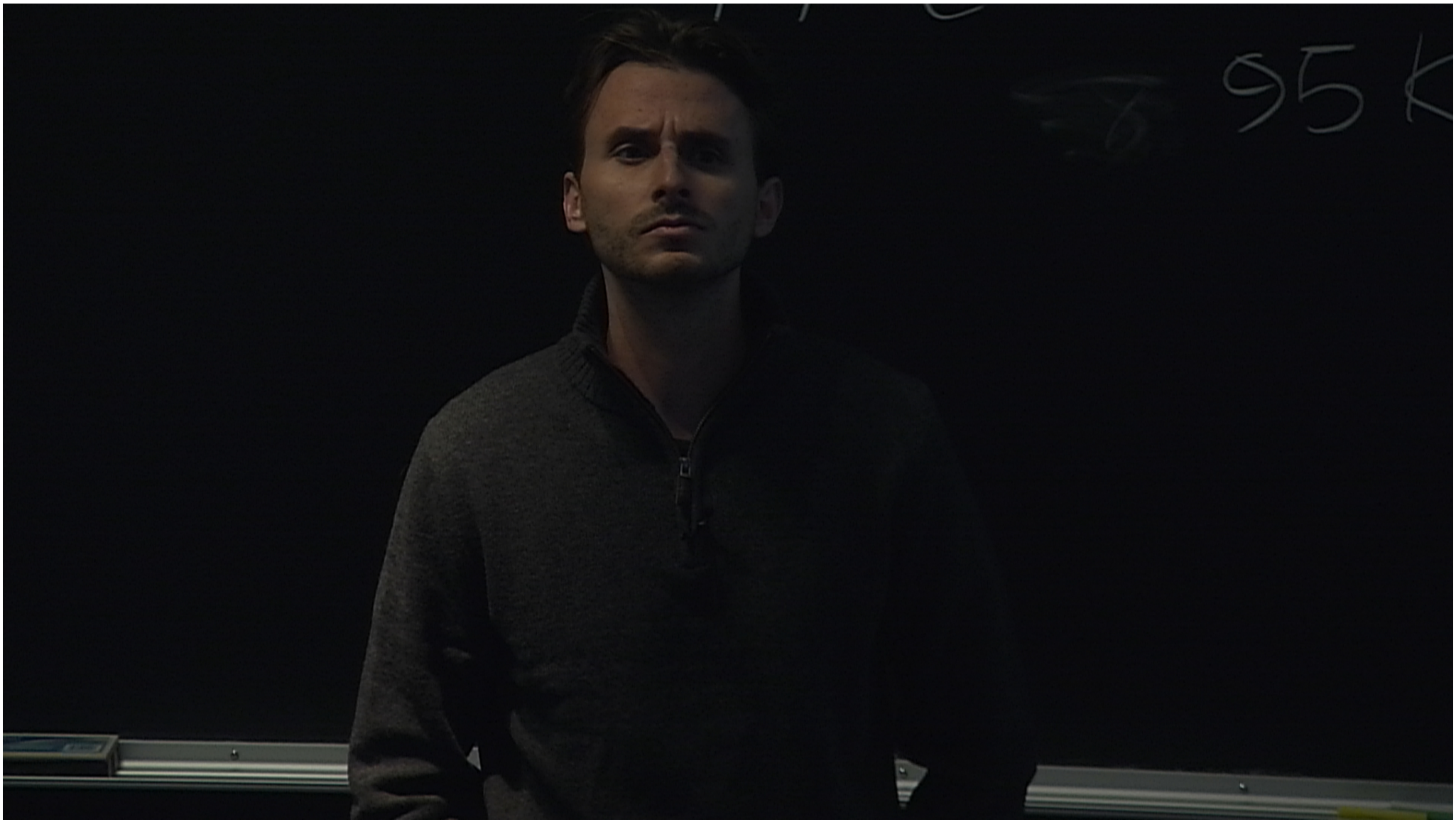
Abstract: Condensed matter realizations of Majorana zero modes constitute potential building blocks of a topological quantum computer and thus have recently been the subject of intense theoretical and experimental investigation. In the first part of this talk, I will introduce a new scheme for preparation, manipulation, and readout of these zero modes in semiconducting wires coated with mesoscopic superconducting islands. This approach synthesizes recent materials growth breakthroughs with tools long successfully deployed in quantum-dot research, notably gate-tunable island couplings, charge-sensing readout, and charge pumping. Guided by these capabilities, we map out numerous milestones that progressively bridge the gap between Majorana zero-mode detection and long-term quantum computing applications. These include (1) detecting non-Abelian anyon â€˜fusion rulesâ€™ in two complementary schemes, one based on charge sensing, the other using a novel Majorana-mediated charge pump, (2) validation of a prototype topological qubit, (3) braiding to demonstrate non-Abelian statistics, and (4) observing the elusive topological phase transition accompanying the onset of Majorana modes. With the exception of braiding, these proposed experiments require only a single wire with as few as two islands, a setup already available in the laboratory. In the second part of the talk, I will introduce a new class of 2D microscopic models---termed â€˜Majorana-dimer modelsâ€™---which generalize well-known quantum dimer models by dressing the bosonic dimers with pairs of Majorana modes. These models host a novel interacting topological phase of matter which has the same bulk anyonic content as the chiral Ising theory, albeit with a fully gapped edge. These seemingly contradictory statements can be reconciled by noting that our phase is inherently fermionic: it can be understood as the product of an Ising phase with a topological p-ip superconductor. Potential physical realizations of this exotic state via a lattice of strongly interacting Majorana modes will be discussed.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Majorana zero modes in condensed matter: From topological quantum computing milestones to Majorana-dimer models

Ryan V. Mishmash

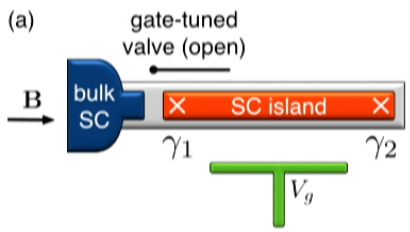
Caltech

References:

- Dave Aasen*, Michael Hell*, RVM*, Andrew Higginbotham*, Jeroen Danon, Martin Leijnse, Thomas Jespersen, Joshua Folk, Charlie Marcus, Karsten Flensberg, and Jason Alicea, PRX **6**, 031016 (2016)
- RVM, Dave Aasen, Andrew Higginbotham, and Jason Alicea, PRB **93**, 245404 (2016)
- Brayden Ware, Jun Ho Son, Meng Cheng, RVM, Jason Alicea, and Bela Bauer, PRB **94**, 115127 (2016)

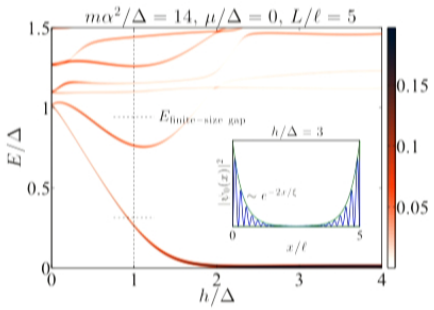
Outline: Two Majorana-based stories

- Milestones toward Majorana-based topological quantum computing
  - Aasen *et al.*, PRX **6**, 031016 (2016)
  - RVM, Aasen, Higginbotham, and Alicea, PRB **93**, 245404 (2016)

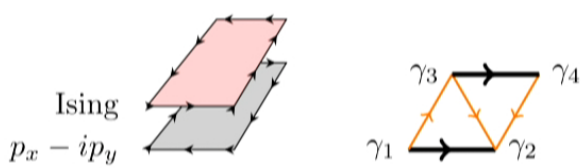


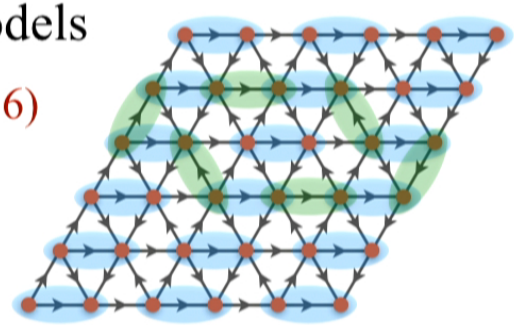
$E_J \gg E_C$

$|\text{even}\rangle$ $|\text{odd}\rangle$



- Ising anyons in Majorana-dimer models
  - Ware, Son *et al.*, PRB **94**, 115127 (2016)





Majorana nanowires: From Kitaev to experiment

- Toy model for <u>1D spinless *p*-wave superconductor</u>: Kitaev (2001)

$$H_{\text{Kitaev}} = -\mu \sum_{x=1}^{L} c_x^\dagger c_x - \frac{1}{2} \sum_{x=1}^{L-1} \left(t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + \text{H.c.} \right)$$

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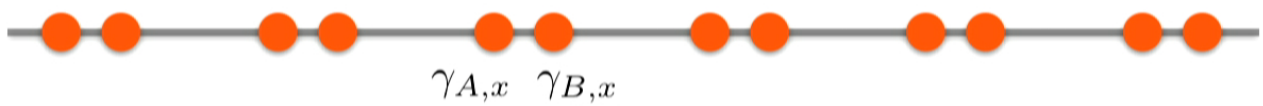

$\mu \neq 0, t = \Delta = 0$: Trivial

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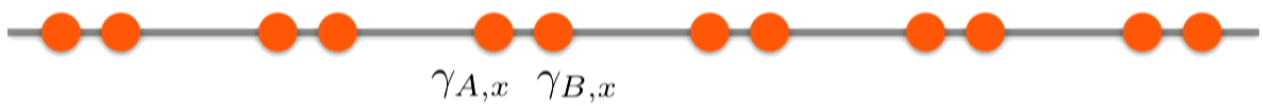
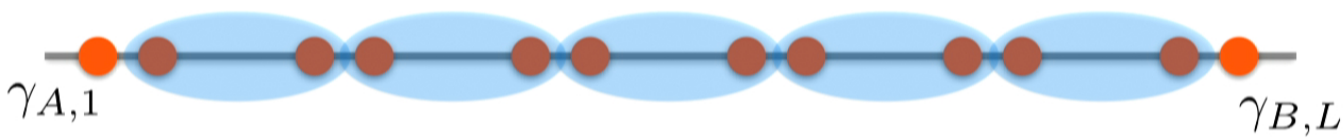


$\mu \neq 0, t = \Delta = 0$: Trivial

$\mu = 0, t = \Delta$: Topological, $f = (\gamma_{A,1} + i\gamma_{B,L})/2 =$ nonlocal!



Majorana nanowires: From Kitaev to experiment

- Recipe for experimental realization of p-wave SC wires
  - Lutchyn, Sau, Das Sarma (2010); Oreg, Refael, von Oppen (2010)
- (Relatively) easy-to-handle ingredients
  - 1D quantum wire w/ spin-orbit coupling

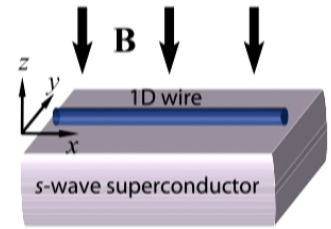
$$H_{\rm wire} = \int dx\, \psi^\dagger \left(-\frac{\partial_x^2}{2m} - \mu - i\alpha\sigma^y \partial_x \right) \psi$$

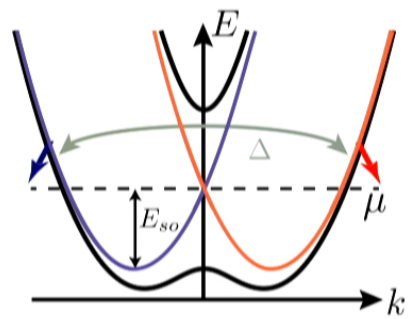
  - External Zeeman field

$$H_{\rm Zeeman} = h \int dx\, \psi^\dagger \sigma^z \psi$$

  - Proximity coupling to s-wave SC

$$H_\Delta = \int dx\, \Delta \left(\psi_\uparrow \psi_\downarrow + {\rm H.c.} \right)$$



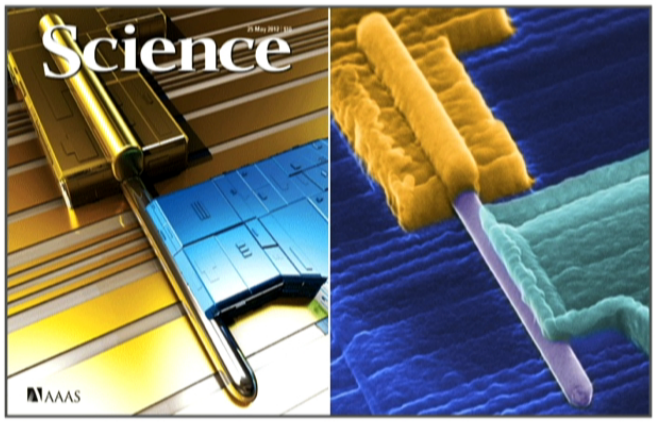


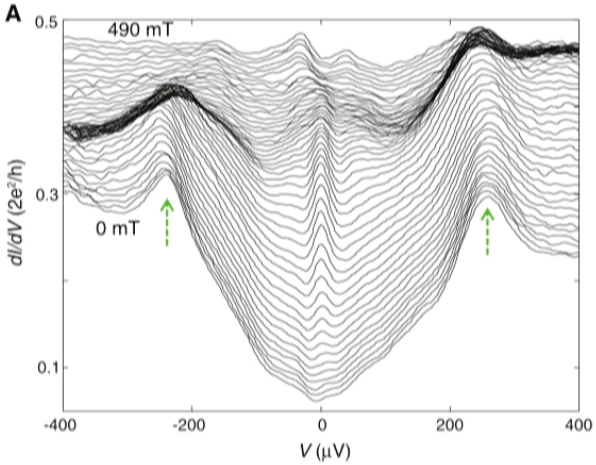
$$\boxed{H = H_{\rm wire} + H_{\rm Zeeman} + H_\Delta}$$

topological criterion : $h > \sqrt{\Delta^2 + \mu^2}$

Majorana nanowires: From Kitaev to experiment

- First (putative) experimental realization: Mourik *et al.* (2012)
  - NS tunneling spectroscopy: Majorana $\rightarrow 2e^2/h$ zero-bias cond. peak

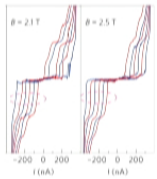




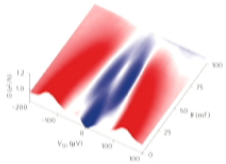
  - Tantalizing evidence for Majorana zero modes, but many lingering issues …
    - Soft gap, no evidence for topological phase transition, etc.

Where does (nanowire-based) Majorana stand today?

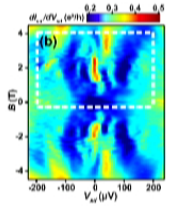
- Signatures of Majoranas found in several experiments/platforms



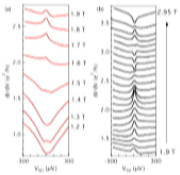
Purdue/
Notre Dame (2012)



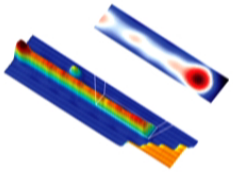
Weizmann (2012)



Lund/Beijing (2012)

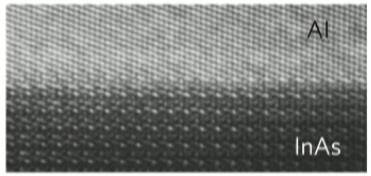


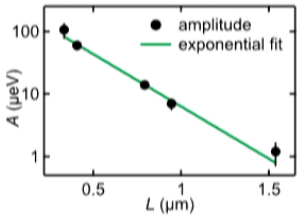
UIUC (2013)

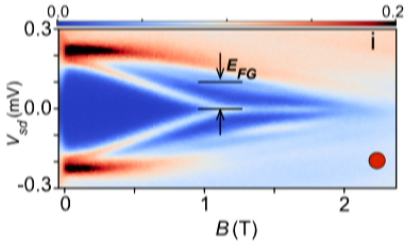


Princeton (2014)

- Quickly improving technology and device quality





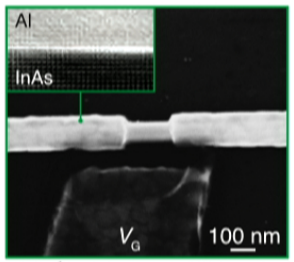


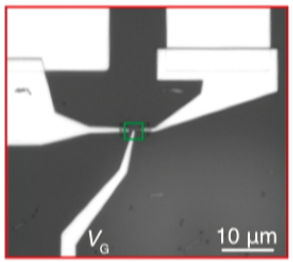
Copenhagen (2015-2016)

- Field is poised to enter a new *control-focused* experimental era!

Two main goals of our work

- Introduce new all-electrical manipulation approach to the Majorana problem





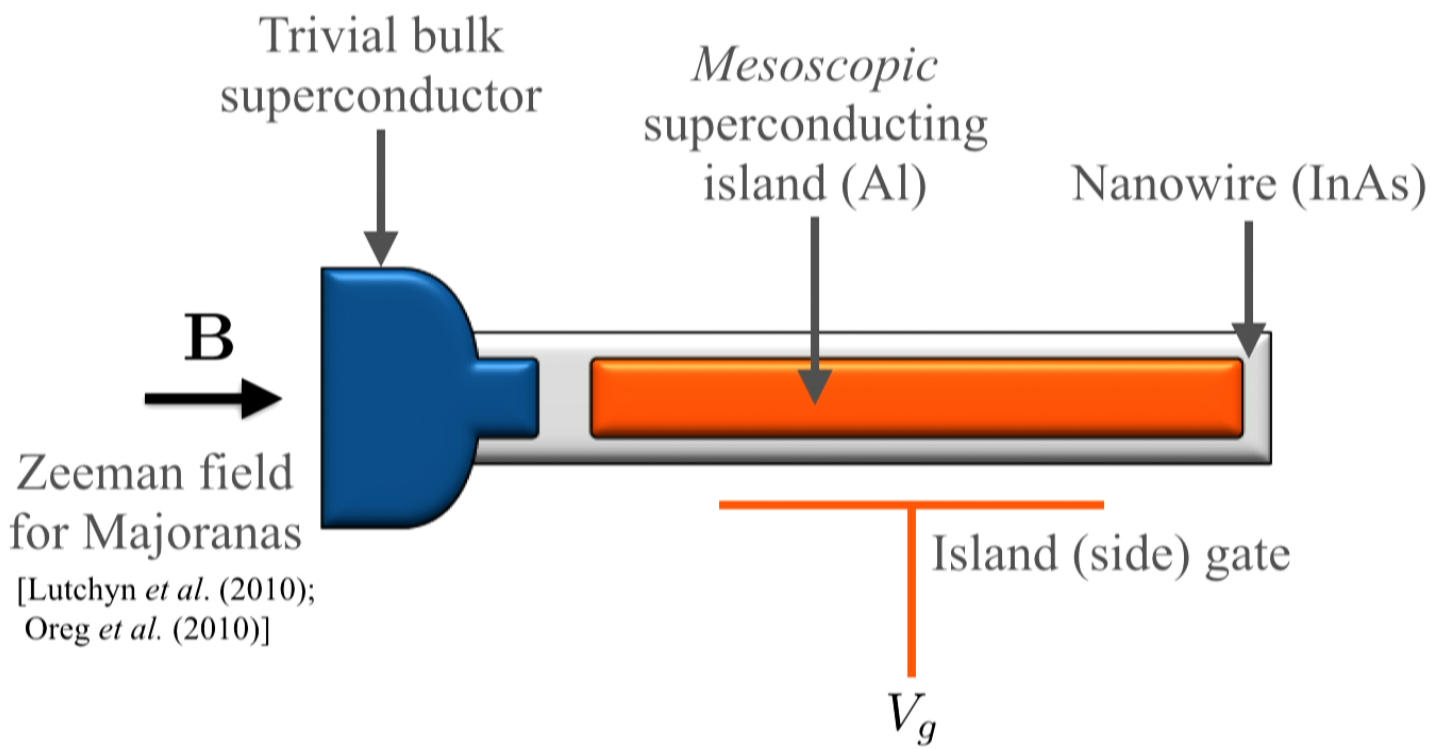
  - Related non-topological qubits already available in the laboratory
    - Larsen *et al.* (2015); de Lange *et al.* (2015)

- Devise series of (relatively) near-term milestone experiments
  - Aim: Advance the field closer to real quantum computing applications
  - Anyon fusion-rule detection, topological qubit validation, braiding …
  - Protocols should be adaptable to other Majorana platforms as well

Outline: Majorana milestones

- All-electrical Majorana control scheme
  - Philosophy: Harness tools from quantum dot / spin qubit community for feasible Majorana experiments
  - cf. Majorana-transmon marriage in, e.g.,
    - Hassler *et al*. (2011), van Heck *et al*. (2012), Hyart *et al*. (2013)
- Majorana milestone experiments
  - Ising anyon fusion-rule detection
  - Topological qubit validation
  - Braiding
  - Observing elusive topological phase transition

Rudiments of Majorana control scheme



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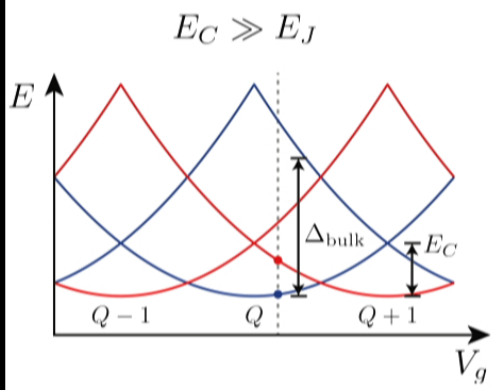
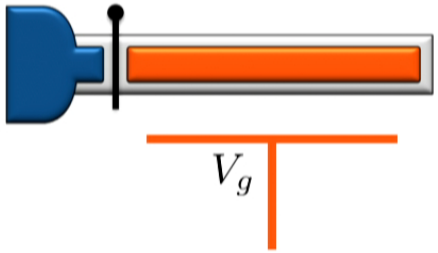
Parity-to-charge conversion (and vice versa)

$$H = H_J[\hat{\varphi}] + E_C(\hat{n} - n_0)^2$$





Note lack of shift between even and odd parabolas due to topological SC:
Liang Fu, PRL (2010); Albrecht *et al.*, Nature (2016)

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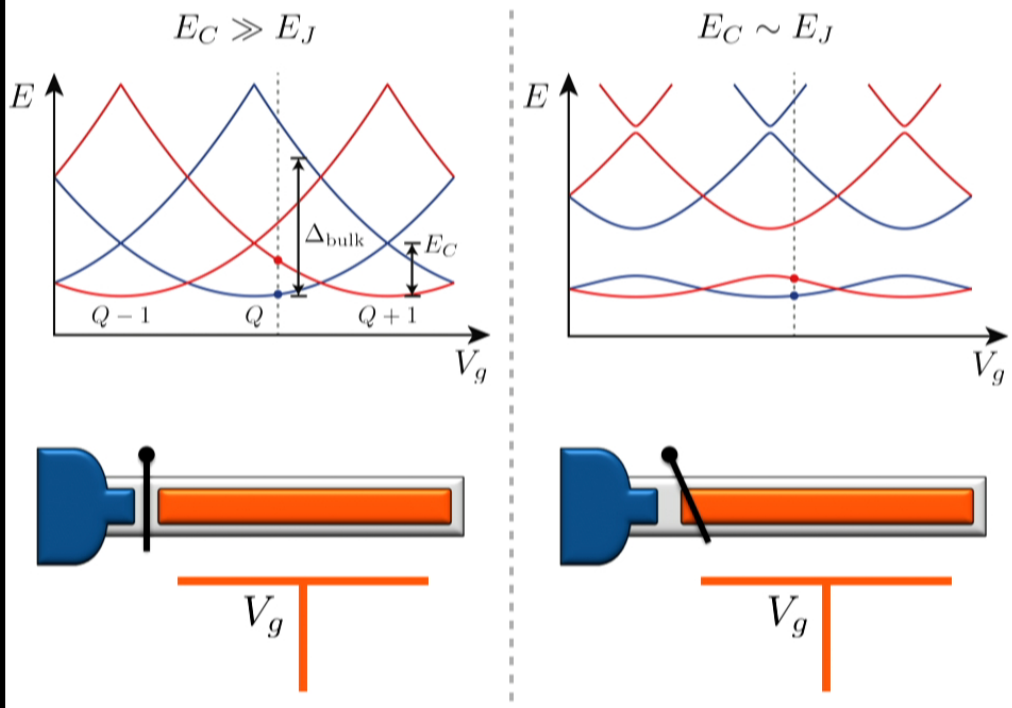

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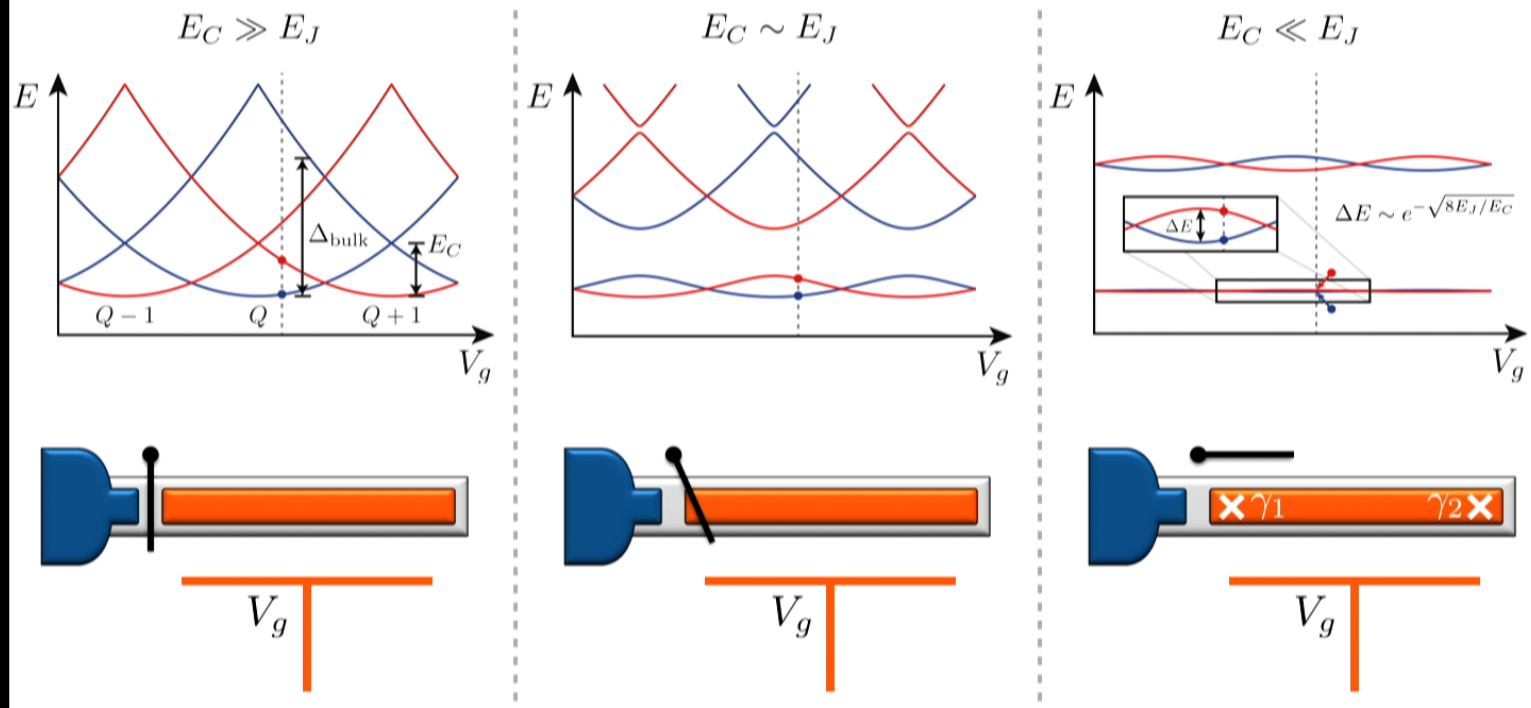
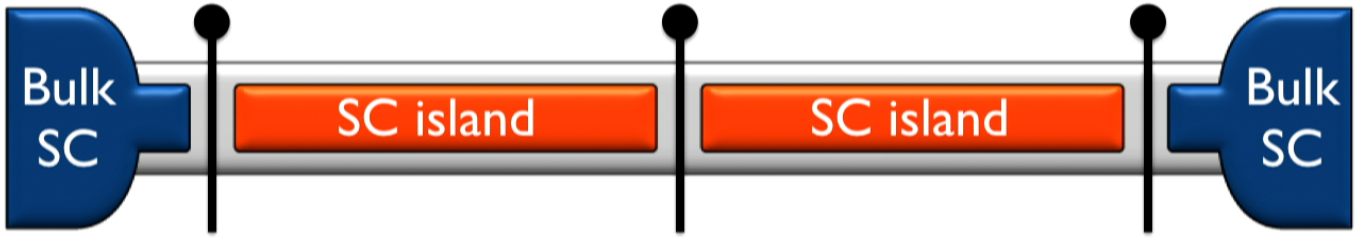


Extend to two islands / two bulk SCs



- Several interesting configurations possible
  - Double quantum dot
  - Single quantum dot
  - Single topological superconductor
  - Double topological superconductor

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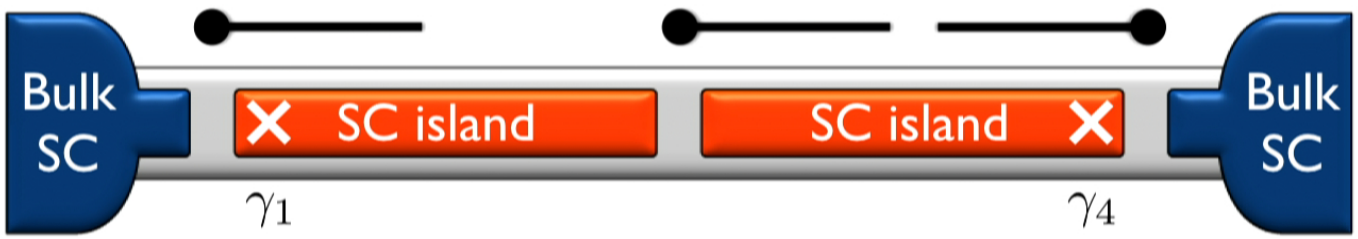

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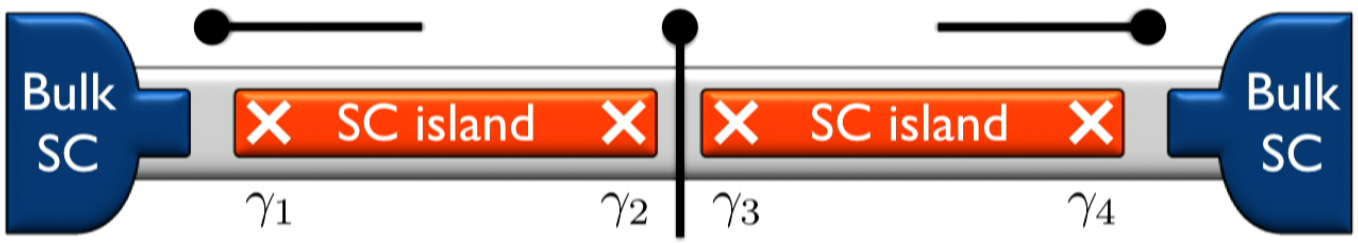

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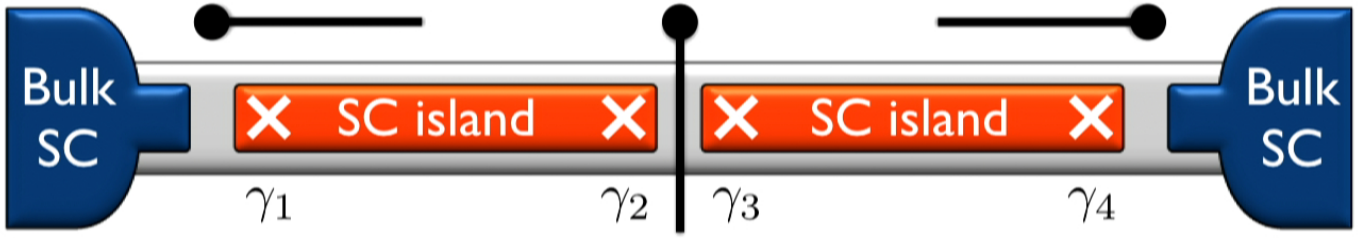

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  - ✓ Double quantum dot
  - ✓ Single quantum dot
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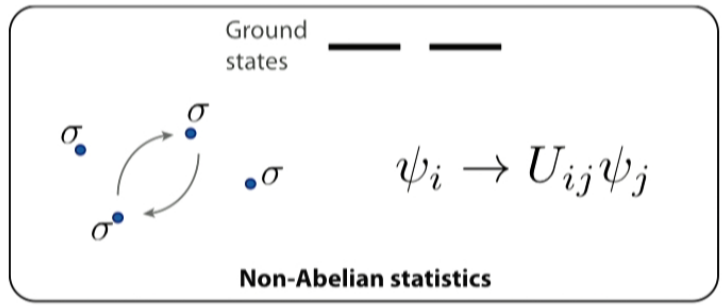
4 Majoranas = prototype

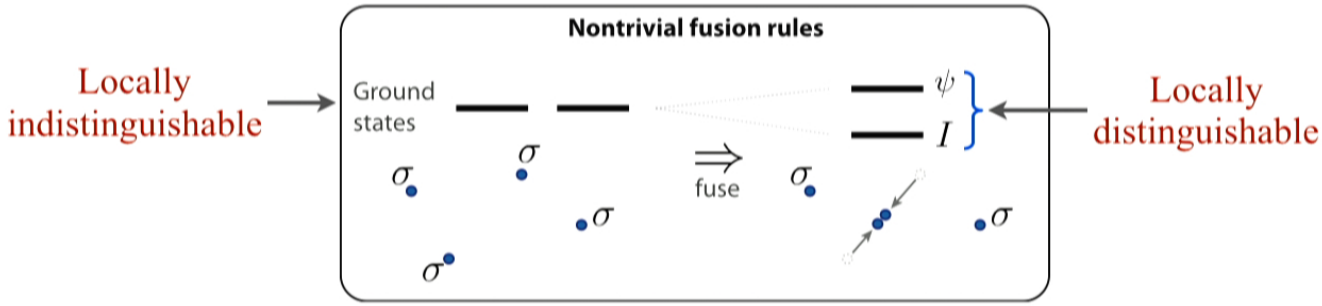
- This is all we need for fusion rules and topo qubit validation!

Outline: Majorana milestones

- All-electrical Majorana control scheme
  - Philosophy: Harness tools from quantum dot / spin qubit community for feasible Majorana experiments
- Majorana milestone experiments
  - Ising anyon fusion-rule detection
    - See also: Bishara, Bonderson, Nayak, Shtengel, and Slingerland, PRB (2009); Alicea *et al.*, Nature Phys. (2011); Ruhman, Berg, and Altman, PRL (2015)
  - Topological qubit validation
  - Braiding
  - Observing elusive topological phase transition

Nontrivial fusion rules vs. non-Abelian statistics

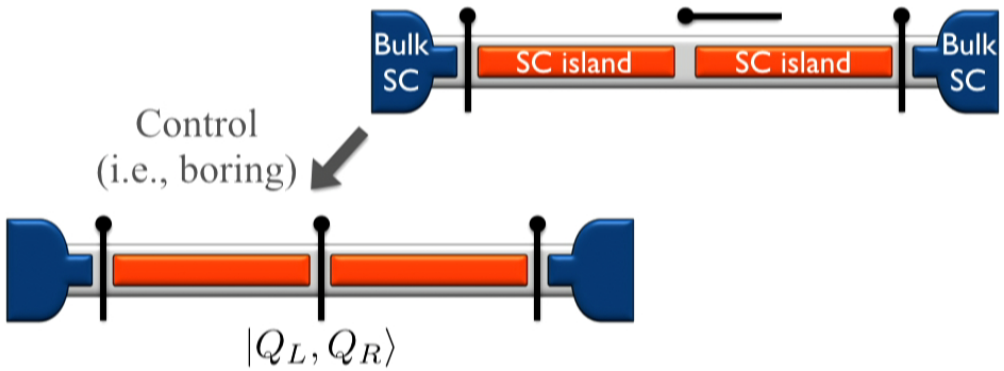




$$\boxed{\sigma \times \sigma = I + \psi}$$

Rigorous proofs: Barkeshli, Bonderson, Cheng, and Wang, arXiv:1410.4540; Rowell and Wang, arXiv:1508.04793

Fusion-rule protocol: Verify $\sigma \times \sigma = I + \psi$



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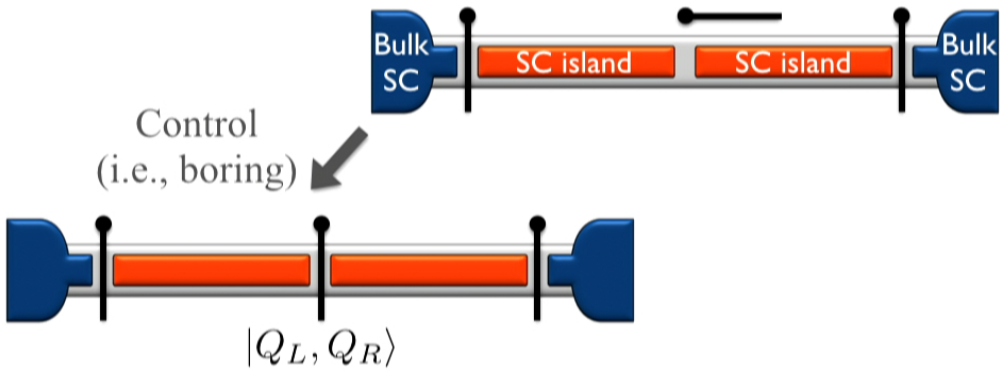

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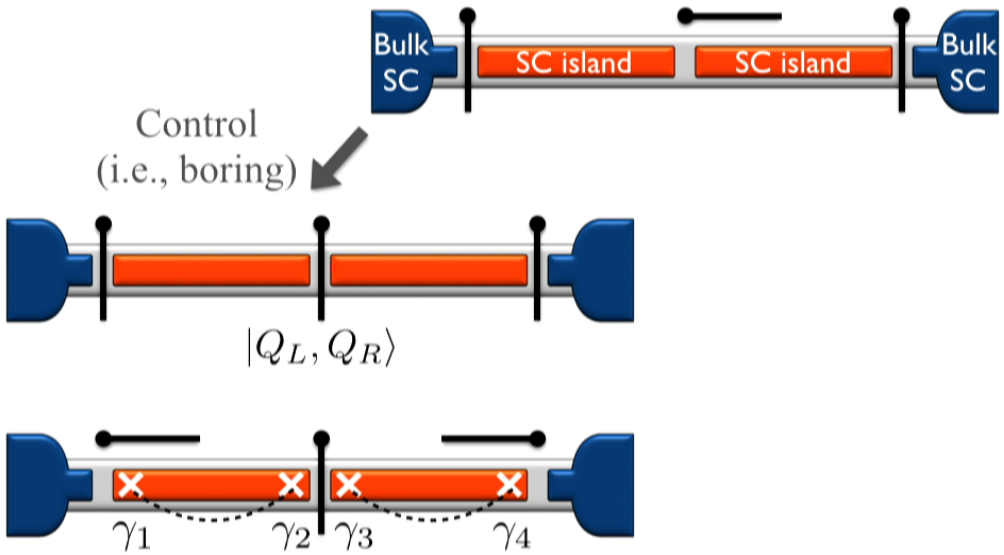

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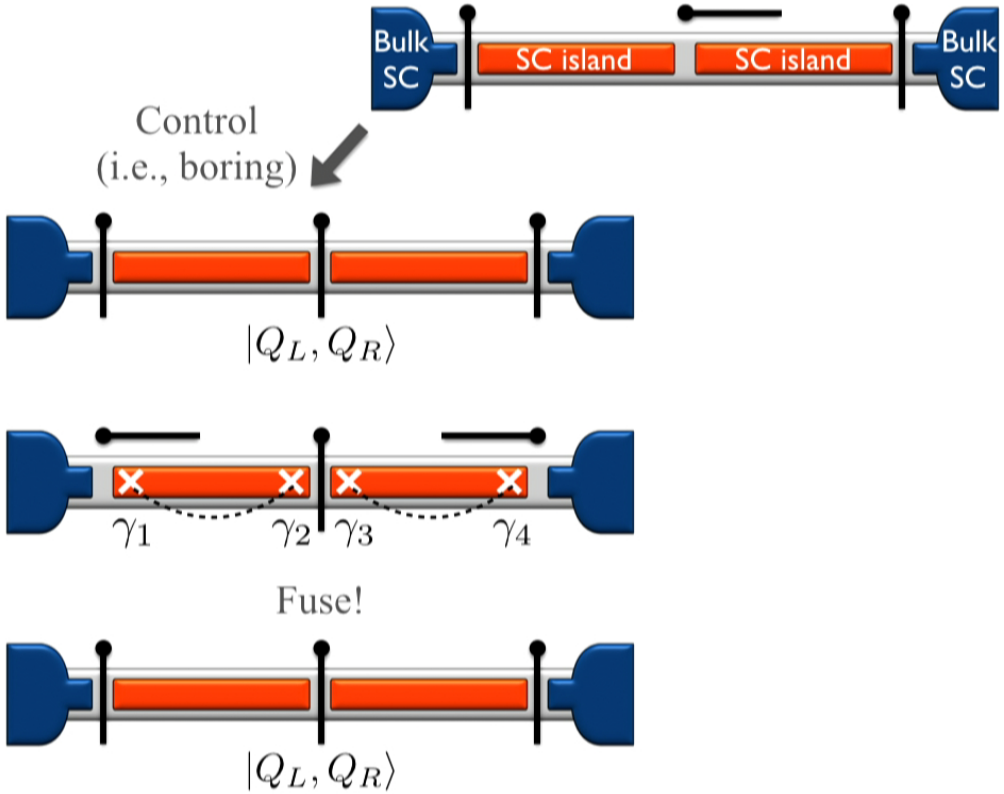
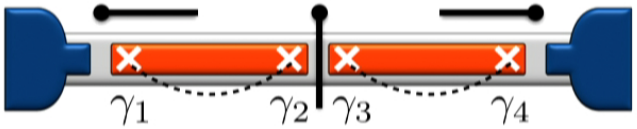


Fusion-rule protocol: Verify $\sigma \times \sigma = I + \psi$

Control
(i.e., boring)

$|Q_L, Q_R\rangle$

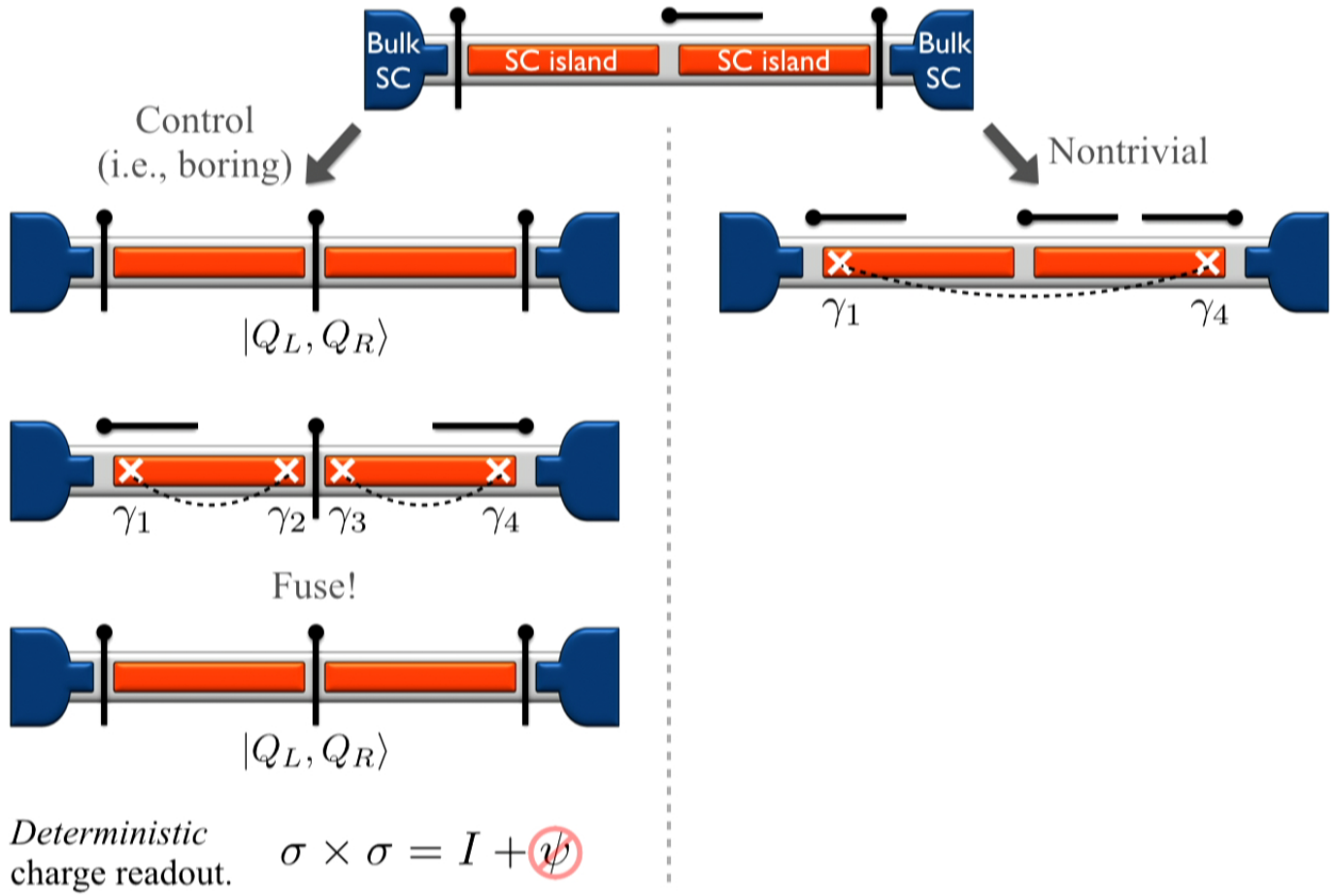


Fuse!

$|Q_L, Q_R\rangle$

Deterministic charge readout. $\sigma \times \sigma = I + \psi$

Fusion-rule protocol: Verify $\sigma \times \sigma = I + \psi$



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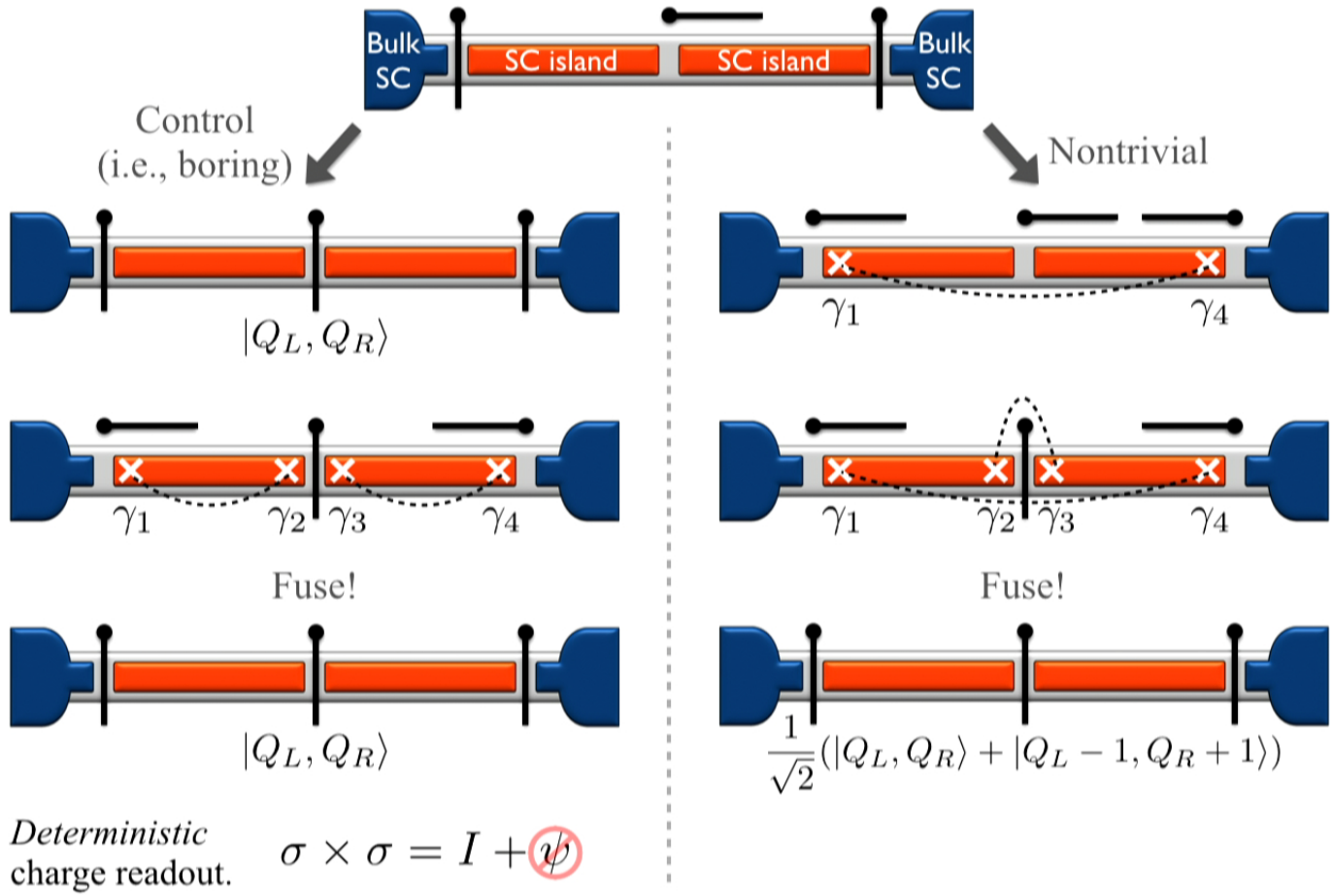
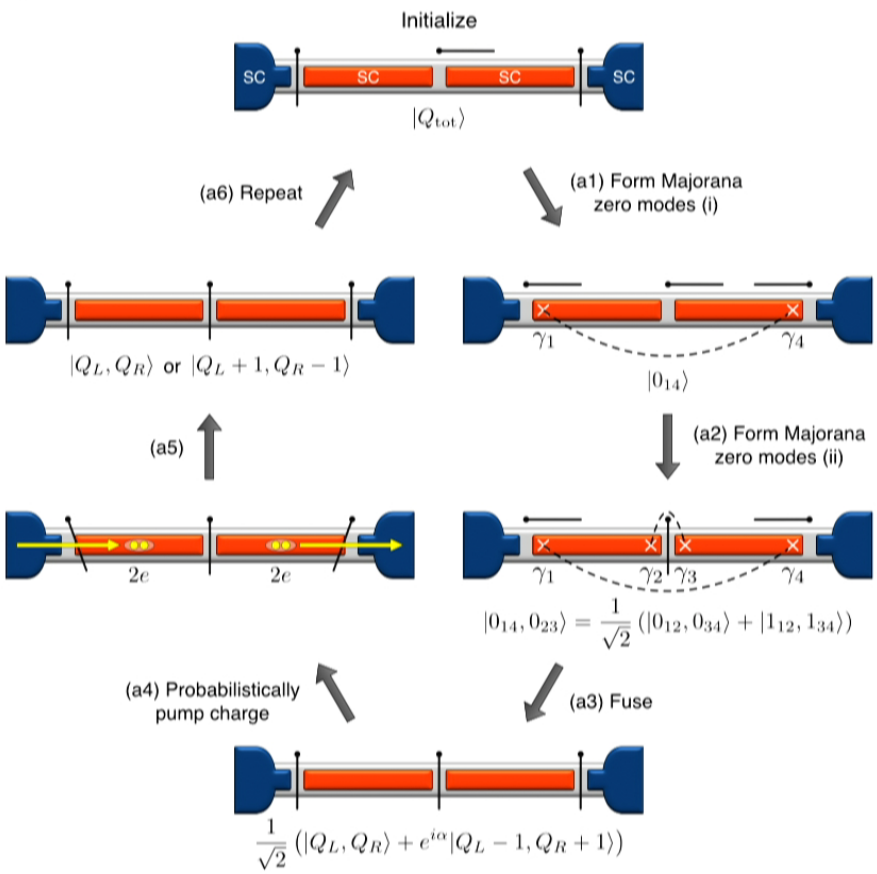


Deterministic charge readout. $\sigma \times \sigma = I + \psi$

Alternative readout scheme: Majorana-mediated charge pump!



- Nontrivial pumping cycle gives quantized dc current!

$$I_{\rm pump} = \frac{1}{2}(2e)f = ef$$

probability of fusing into ψ channel

- Control cycle gives zero current:

$$I_{\rm control} = 0$$

- No need for extra charge sensors in setup
  - Easier experiment …

Several important remarks

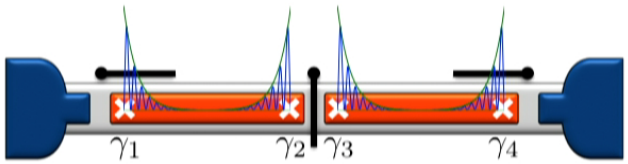
- Control expt. should exclude noise as source of probabilistic readout
- Experiments have interesting non-Abelian character
  - Different outcomes due to different order of manipulations possible only because of (hopefully topological!) ground-state degeneracy
- Protocol rates need to lie within "speed limits"
  - Slow enough to avoid corruption from excited states
  - Fast enough to avoid poisoning / be ignorant of residual degeneracy splitting
  - See also: Hell, Danon, Flensberg, and Leijnse, PRB **94**, 035424 (2016)
  - Thorough experiment should *intentionally violate* these speed limits
    - Glean valuable device information (gaps, residual splitting, poisoning time)

Outline: Majorana milestones

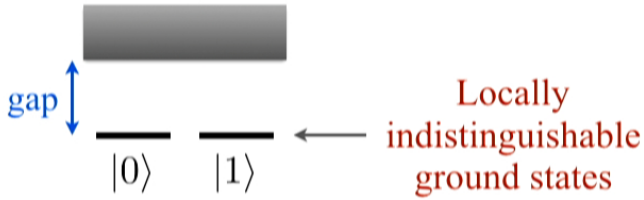
- All-electrical Majorana control scheme
  - Philosophy: Harness tools from quantum dot / spin qubit community for feasible Majorana experiments
- Majorana milestone experiments
  - Ising anyon fusion-rule detection
  - Topological qubit validation
  - Braiding
  - Observing elusive topological phase transition

Topological qubit validation: Preliminaries

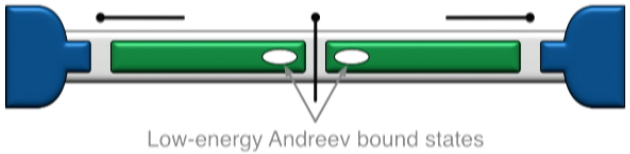
- The two-island device constitutes a *prototype topological qubit*



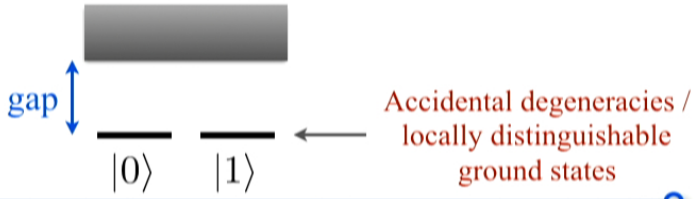
$|0\rangle = |0_{12}, 0_{34}\rangle, \quad |1\rangle = |1_{12}, 1_{34}\rangle$



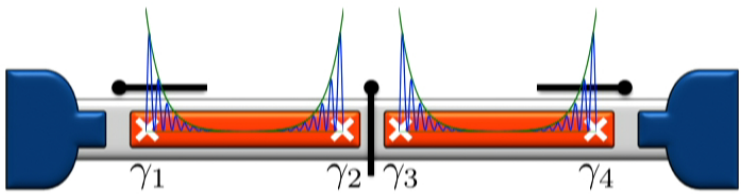
- Important: *From now on, assume usual topological criteria*
  - Work on time scales << poisoning time
  - System confined to ground-state manifold with high probability
- Question: How to *validate* topological protection in this device and distinguish it from a conventional / trivial "Andreev qubit"?

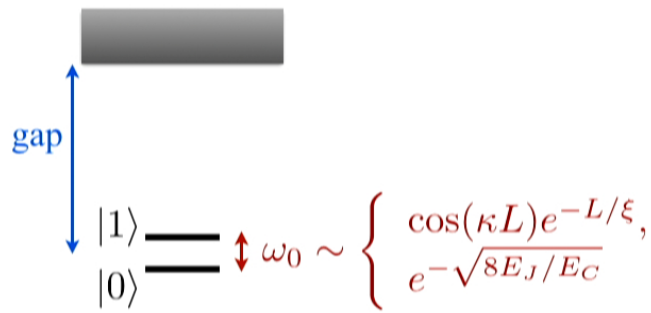


$|0\rangle = |0_L, 0_R\rangle, \quad |1\rangle = |1_L, 1_R\rangle$



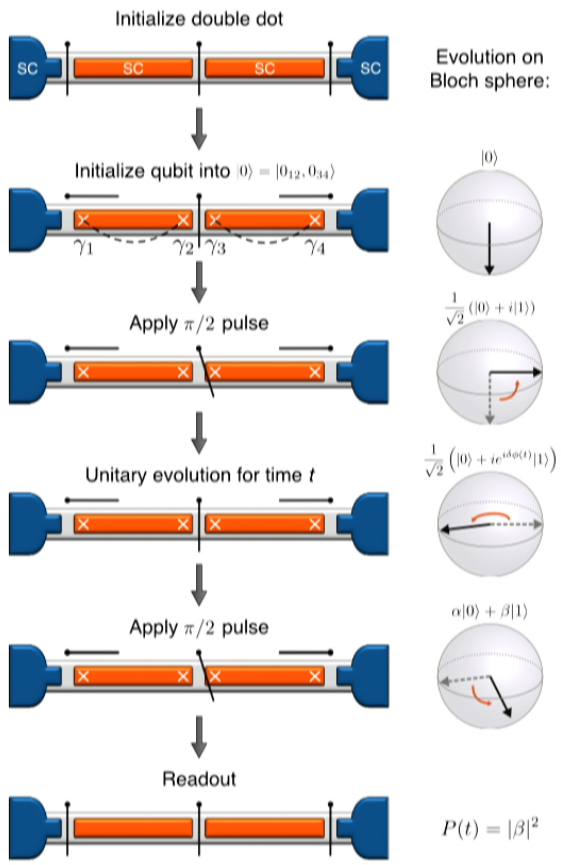
Zooming in on the topological degeneracy





- Low-frequency environmental noise alters ω_0 through, for example, $\xi = \xi(B, \mu, \ldots)$, $\kappa = \kappa(B, \mu, \ldots)$
- Our approach: Can we validate the topological nature of the qubit by examining its splitting ω_0 *and* its coherence times, e.g., T_2?
  - Hint: Topological qubits should have "long" coherence times, but how can we be more precise?
  - What about experimental protocols?

Protocol to measure ω_0 and T_2



- If stochastic (low-frequency) environmental fluctuations have a typical amplitude $\Delta E^z_{\rm typ}$, we find after noise averaging ($1/f$ noise):

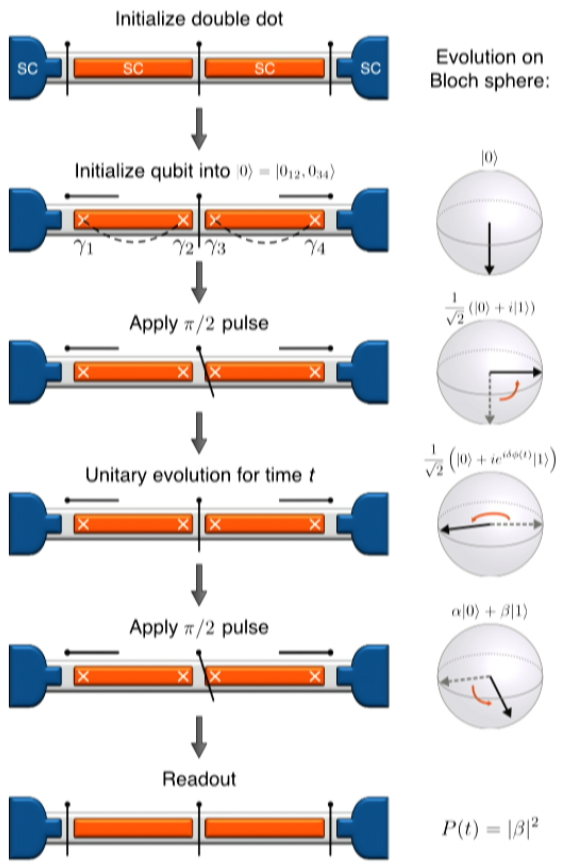
$$\langle P(t)\rangle \approx \frac{1}{2}\left[1 + \cos(\omega_0 t) e^{-(\Delta E^z_{\rm typ} t/\hbar)^2 f(t)}\right]$$

  - Gives direct, time-domain measurement of qubit splitting ω_0!
  - Dephasing time given by

$$T_2 \sim \frac{\hbar}{\Delta E^z_{\rm typ}}$$

- This provides a recipe for measuring the qubit's ω_0 and T_2 (qubit characterization), but how to *validate* its topological nature?

Protocol to measure ω_0 and T_2



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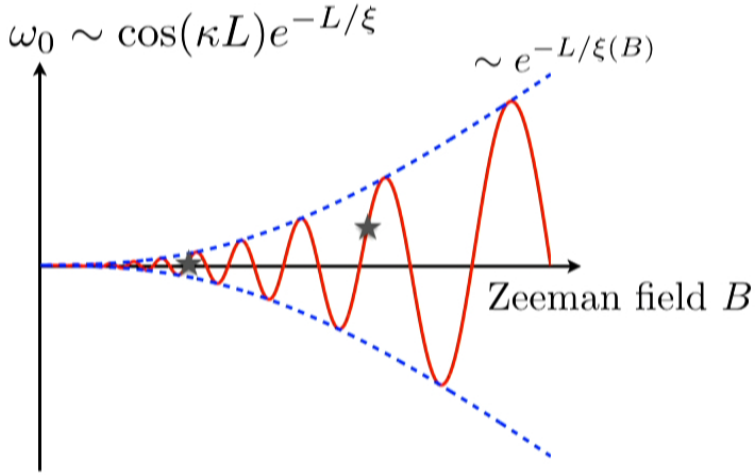
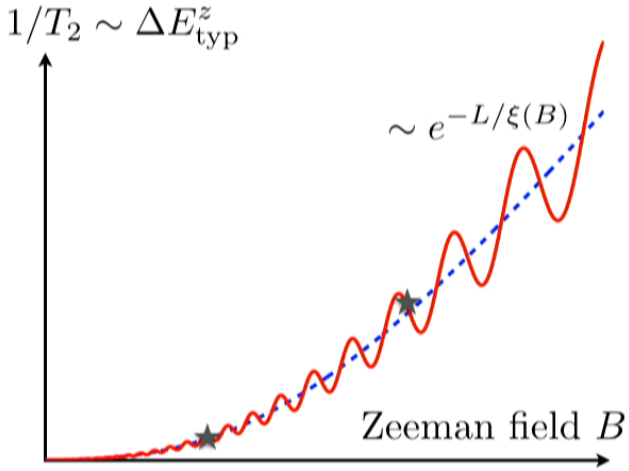
- This provides a recipe for measuring the qubit's ω_0 and T_2 (qubit characterization), but how to *validate* its topological nature?

Unusual noise sensitivity of the topological qubit

- Key point: Since the splitting is exponentially suppressed in L/ξ, *even in the presence of all local noise sources*, so too is its typical noise-induced variation $\Delta E^z_{\rm typ}$





$$\therefore \quad \omega_0 \sim \frac{1}{T_2} \sim \begin{cases} e^{-L/\xi}, & \text{Majorana overlap dominant} \\ e^{-\sqrt{8E_J/E_C}}, & \text{charging energy dominant.} \end{cases}$$

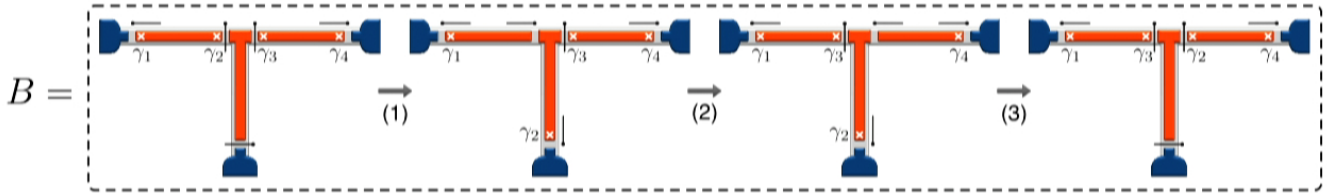
Several important remarks

- For the trivial "Andreev qubit" with accidental degeneracies at zero energy, $\cancel{\omega_0 \sim 1/T_2}$
  - Typical environmental fluctuations need bear no relation to the time-averaged splitting
- For the Majorana-overlap-dominant regime, expect out-of-phase oscillations between ω_0 and $1/T_2$
  - Probing very detailed aspect of the theory
  - cf. Das Sarma, Sau, and Stanescu, PRB **86**, 220506(R) (2012)
- Relaxation time T_1 also has characteristic dependence on ω_0 :

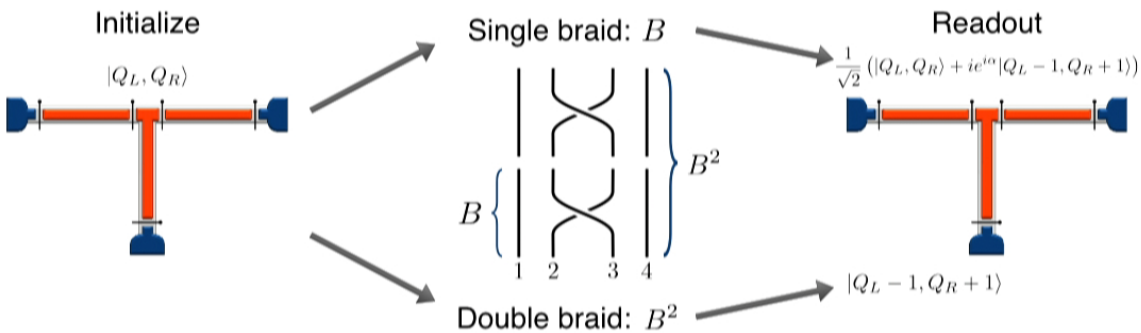
$$\boxed{T_1 \sim \frac{\hbar^2 \omega_0}{(\Delta E^x_{\rm typ})^2} \quad \Rightarrow \quad T_1(\omega_0) \propto \frac{1}{T_2(\omega_0)}} \quad (\text{assumes } \omega_0 \gg \Delta E^x_{\rm typ})$$

All-electrical braiding protocol

(a) Basic braiding operation

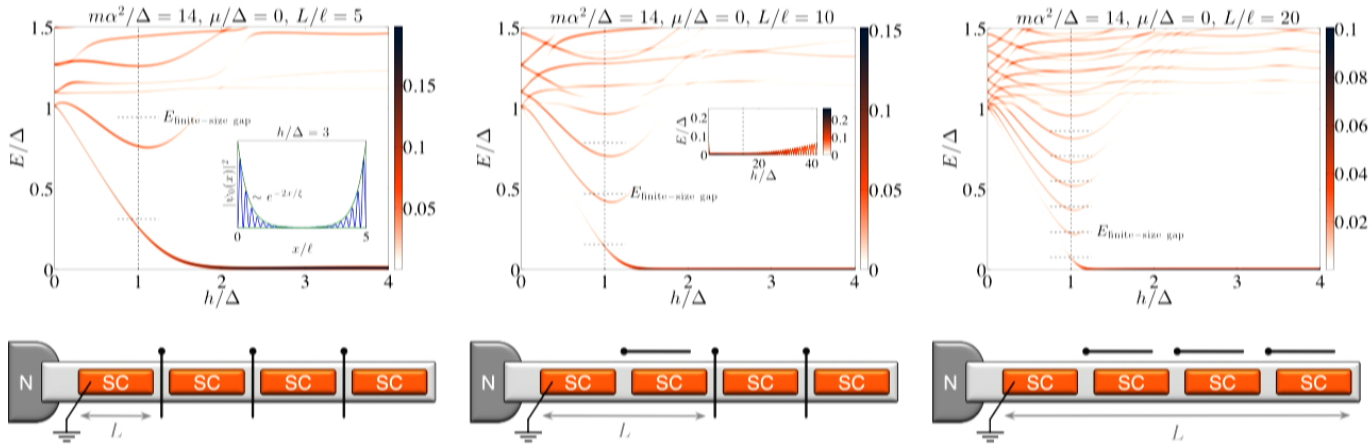


(b) Full protocols: Single and double braid



- Single braid gives probabilistic readout, cf. the fusion-rule protocol
- Double braid gives deterministic readout $\Longrightarrow$ rigidity of qubit rotation!

Approaching a topological phase transition



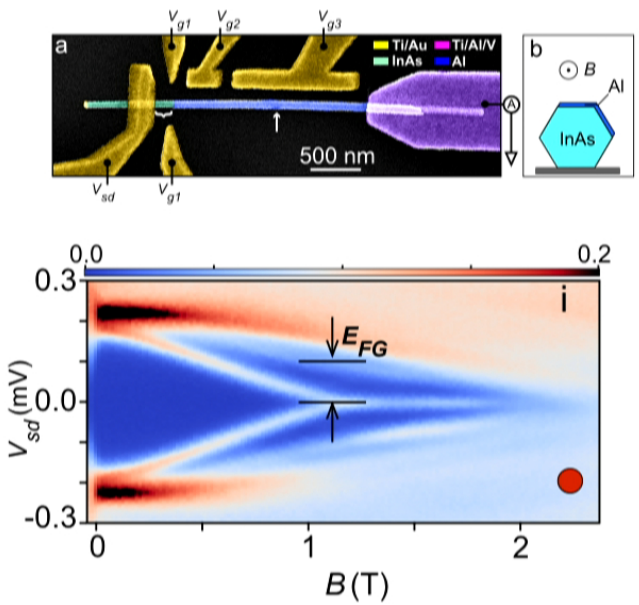
- Finite-size scaling and level tracking of *excited states* at the transition expose eventual, bona fide topological phase transition
- *With strong spin-orbit*, good Majoranas can easily form even if the topological phase transition is severely obliterated into a crossover at finite L
- Allows extraction of spin-orbit coupling in full proximitized hybrid device:

$$\alpha \geq \frac{2LE_{\text{finite-size gap}}}{3\pi\hbar}$$

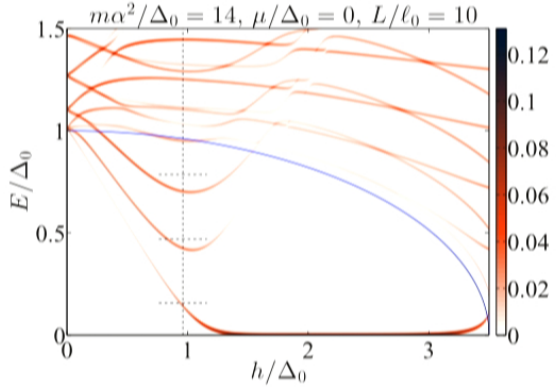
RVM, Aasen, Higginbotham, and Alicea, PRB **94**, 115127 (2016)

Relevance to latest experiments?

- M. T. Deng, E. B. Hansen, J. Danon, M. Leijnse, K. Flensberg, J. Nygard, P. Krogstrup, and C. M. Marcus (unpublished).

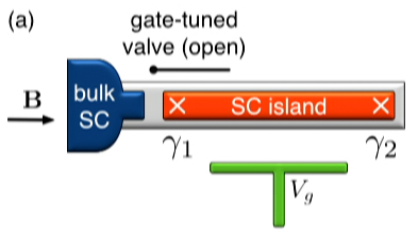


$$\Delta(h) = \Delta_0 \sqrt{1 - \left(\frac{h}{h_{SC}}\right)^2}$$



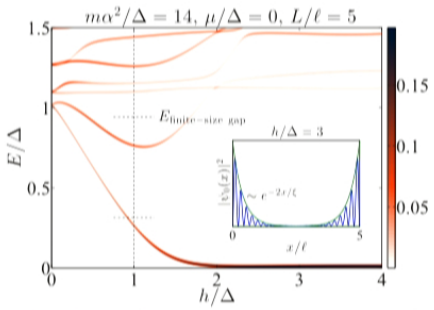
Outline: Two Majorana-based stories

- Milestones toward Majorana-based topological quantum computing
  - Aasen *et al.*, PRX **6**, 031016 (2016)
  - RVM, Aasen, Higginbotham, and Alicea, PRB **93**, 245404 (2016)

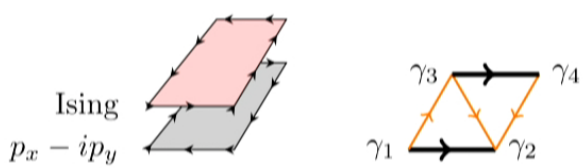


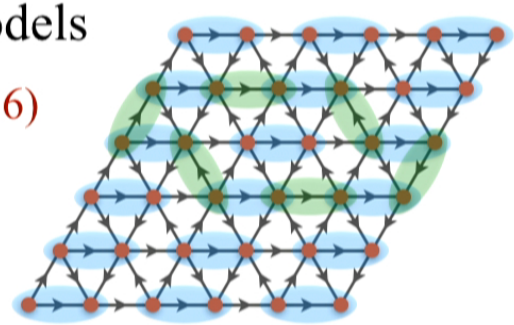
$E_J \gg E_C$

$|\text{even}\rangle$ $|\text{odd}\rangle$



- Ising anyons in Majorana-dimer models
  - Ware, Son *et al.*, PRB **94**, 115127 (2016)





Review: Resonating valence bond (RVB) wfs

- Anderson (1973) + Anderson (1988)

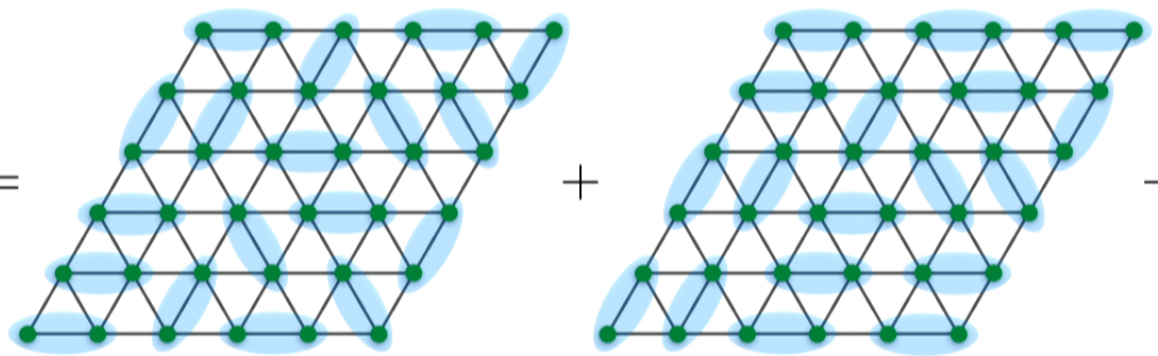
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson

The Resonating Valence Bond State in La_2CuO_4 and Superconductivity

P. W. Anderson

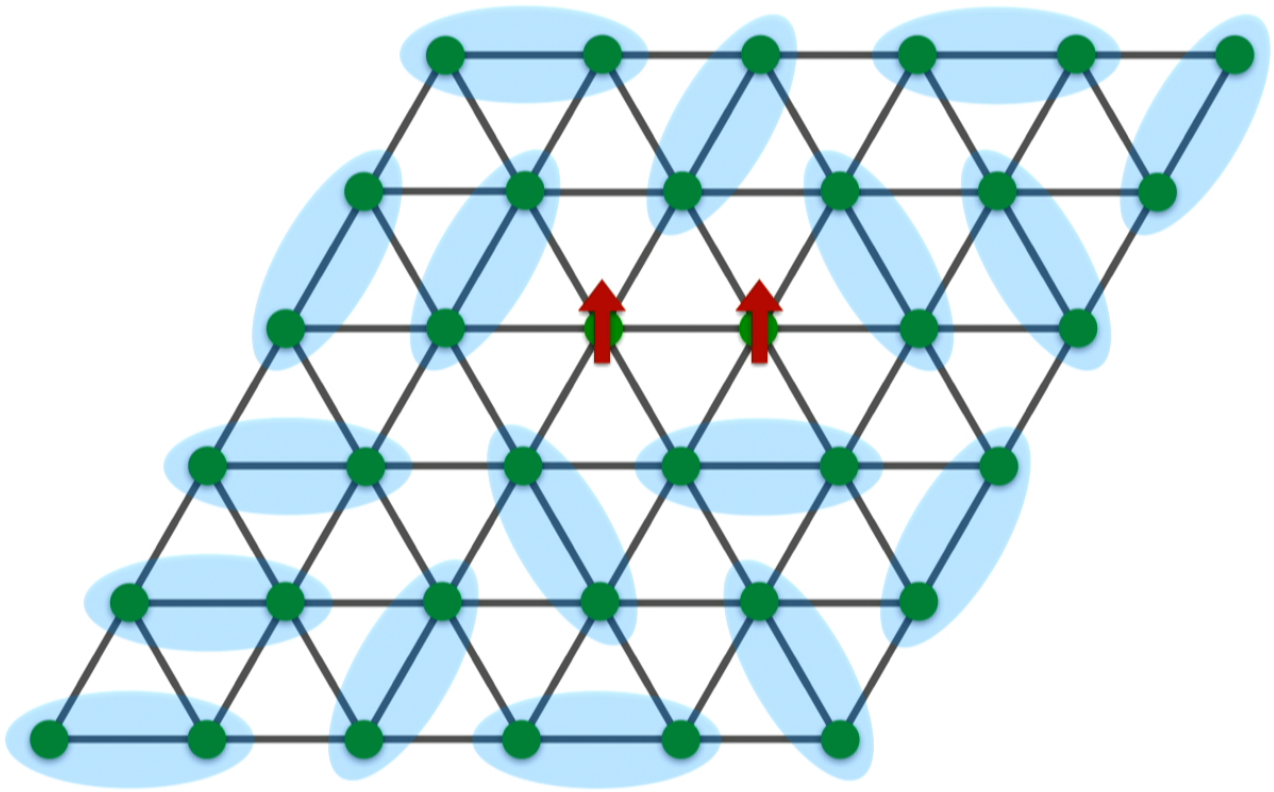
$|\mathrm{RVB}\rangle =$



$+ \cdots$

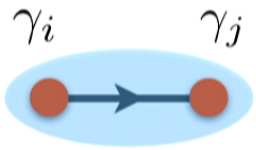
$$= \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$$

Gapped, deconfined spinon excitations



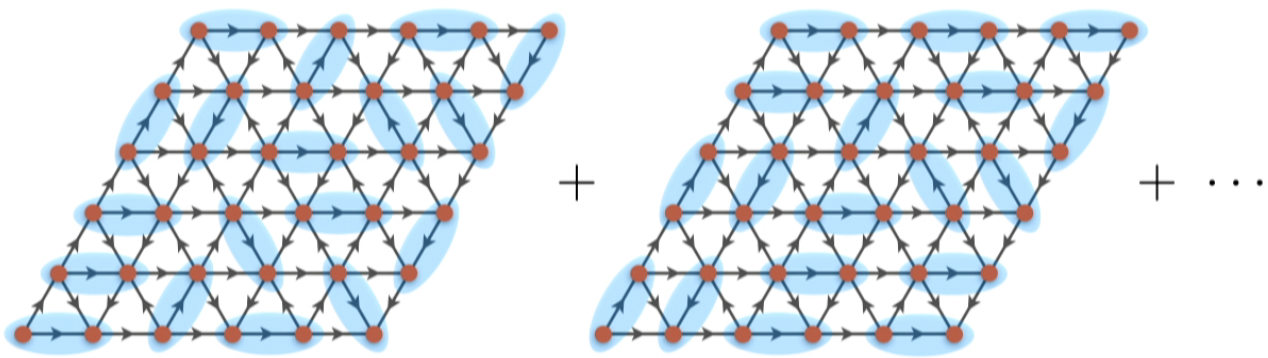
Our goal: Create an RVB out of Majoranas

- ● = spin-1/2 $\longrightarrow$ ● = Majorana mode γ_i
- Pairing spins into singlets $\longrightarrow$ *Fusing* pairs of Majoranas

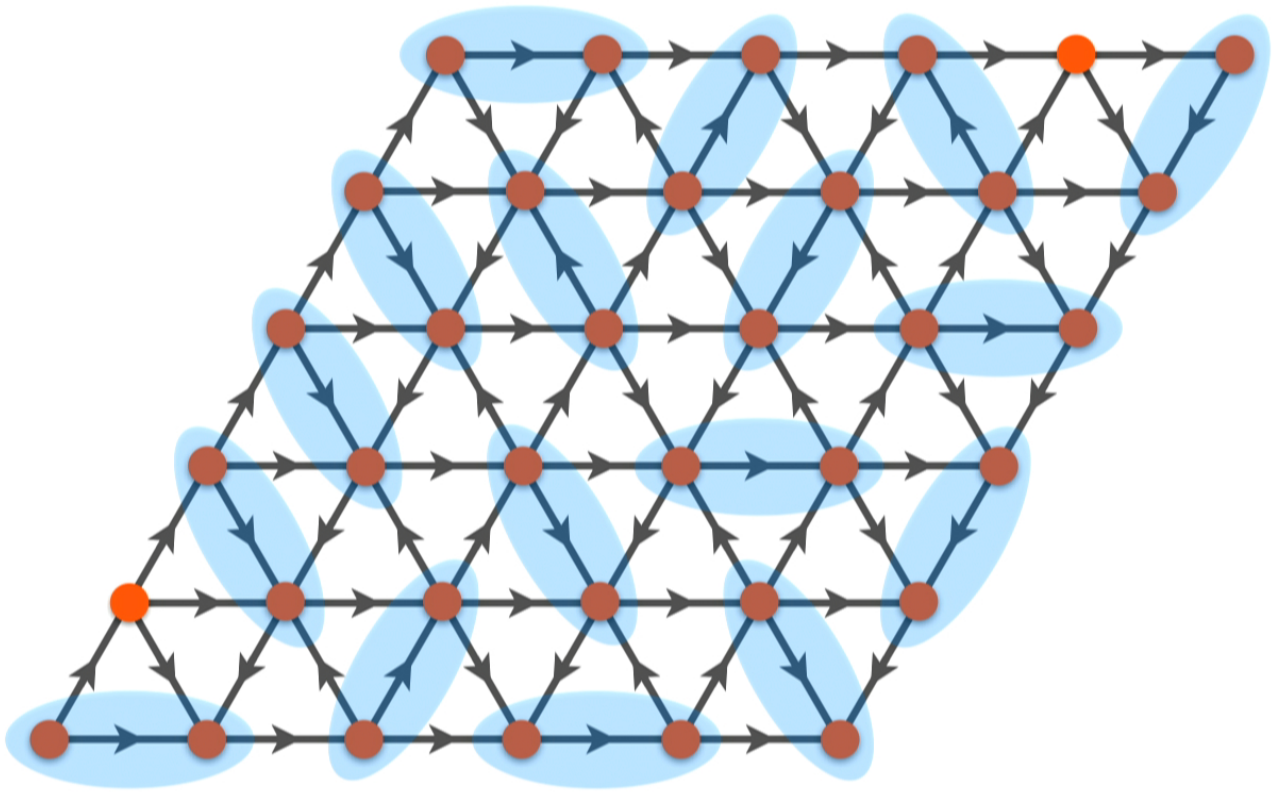


$(s_{ij} = +1)$: $f = \dfrac{\gamma_i - i s_{ij}\gamma_j}{2}$, $\boxed{f^\dagger f = \dfrac{1 - i s_{ij}\gamma_i\gamma_j}{2} = 0 \leftrightarrow i s_{ij}\gamma_i\gamma_j = 1}$

$|\text{Majorana-RVB}\rangle =$

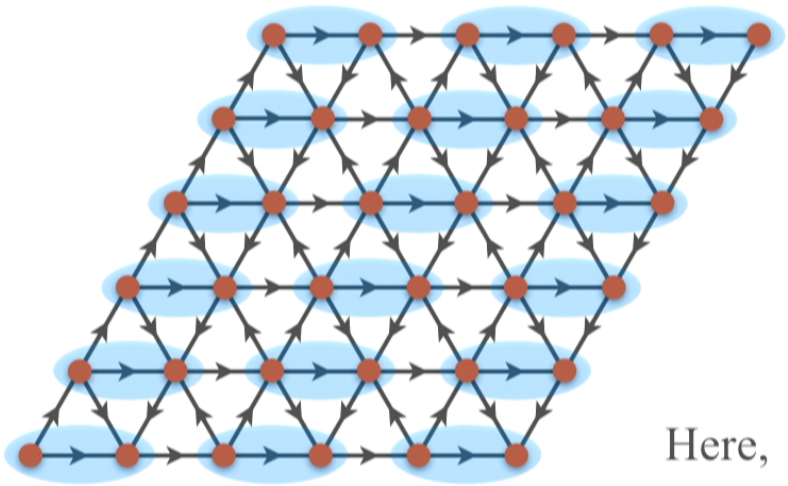


Gapped, deconfined Majorana (Ising) excitations?



"Majorana-dimer" basics

- Hilbert space
  - Bosonic dimers σ_e^z on edges $\otimes$ Majorana fermions γ_i on vertices
  - Different bosonic dimer configurations are orthogonal: $\langle D|D'\rangle = \delta_{D,D'}$
- Kasteleyn orientations and fermion parity
  - *Clockwise-odd rule* guarantees parity conservation on planar graphs



Here,  are *reference dimers*.

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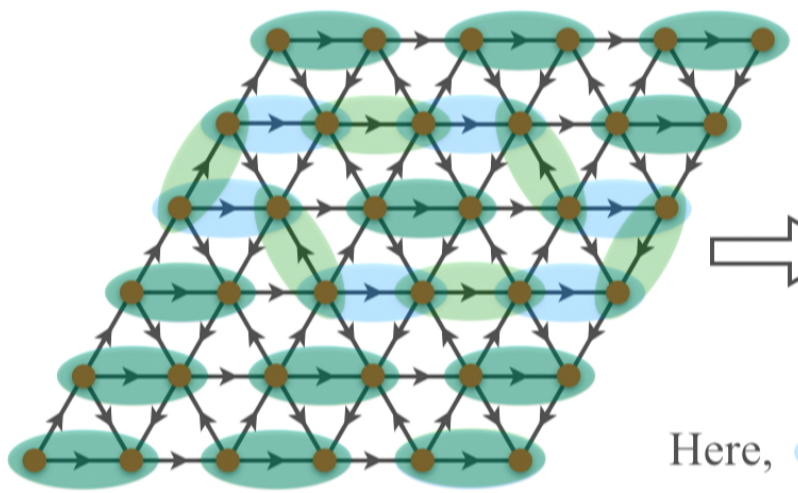


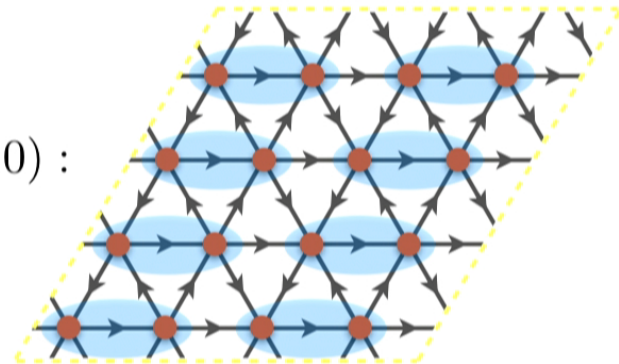
Loops = *topological* Kitaev chains in zero correlation length limit! $(\mu = 0, t = \Delta)$

Here, are *reference dimers*.

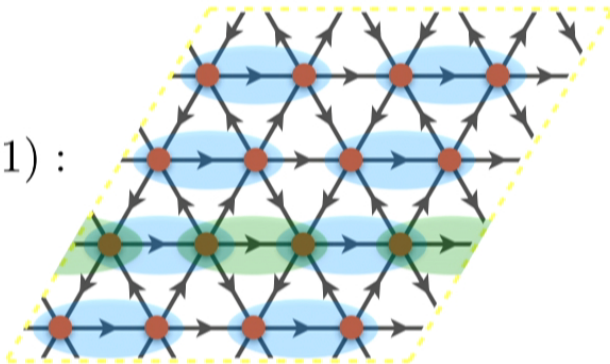
Majorana-dimers on closed manifolds

- Interesting interplay between topological sectors, boundary conditions, and fermion parity

$(0,0)$:



$(0,1)$:



	Boundary cond.			
Dimer sector	PP	PA	AP	AA
(0,0)	+1	+1	+1	+1
(1,0)	−1	+1	−1	+1
(0,1)	−1	−1	+1	+1
(1,1)	−1	+1	+1	−1

Majorana-dimer models

- Goal: Find a parent Hamiltonian for the Majorana-RVB state

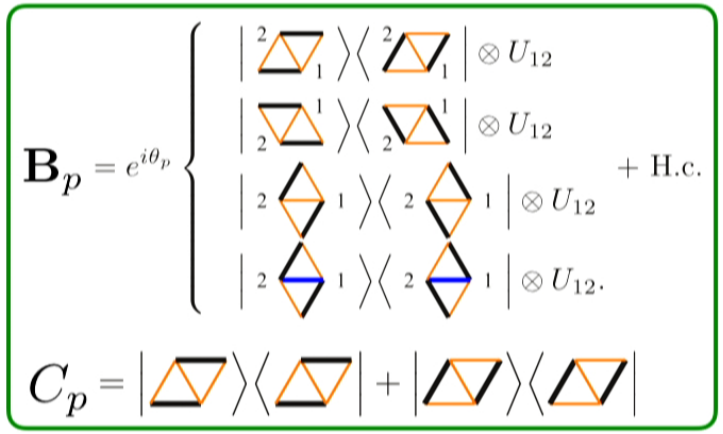
$$|\psi\rangle = \sum_D |F(D)\rangle|D\rangle, \ |F(D)\rangle : \text{ground state of } H_F(D) = -\sum_{e\in D} i s_{ij}\gamma_i\gamma_j$$

- Plan: Dress quantum dimer models with Majorana modes

$$H = \boxed{J_v \sum_v A_v + J_e \sum_e \mathbf{A}_e} \boxed{- \sum_p (t\mathbf{B}_p - vC_p)}$$

$$A_v = \left(\sum_{e\in v} \sigma_e^z + 4\right)^2$$

$$\mathbf{A}_e = \frac{1+\sigma_e^z}{2}\,\frac{1 - i s_{ij}\gamma_i\gamma_j}{2}$$



Line of analysis: Try to map to bosonic QDM

- Work in the restricted dimer subspace by taking $J_e, J_v \to \infty$:

$$\mathcal{H}_r = \{|F(D)\rangle|D\rangle\}$$

- Consider Hamiltonian matrix elements in this subspace:

$$h_{DD'} = \langle F(D')|\langle D'|H|F(D)\rangle|D\rangle = \begin{cases} v\, n_{\text{flippable}}(D), & D' = D \\ -t\, e^{i\varphi_{p,D}}, & D' = D_p \end{cases}$$

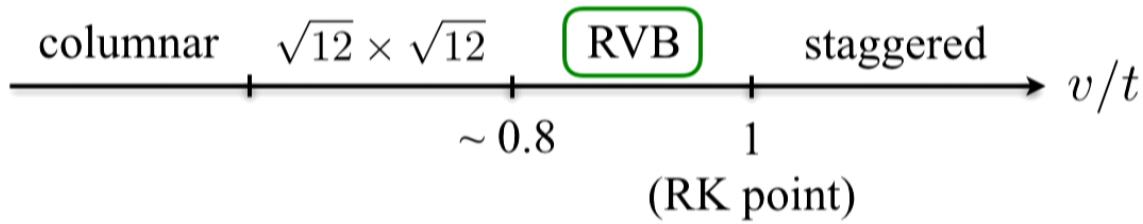
from $e^{\pm i\theta_p}\left(\frac{1 \pm \gamma_1\gamma_2}{\sqrt{2}}\right)$ in H

- Question: Can we choose phases for the fermionic states via $|F(D)\rangle \to e^{i\phi_D}|F(D)\rangle$ such that $e^{i\varphi_{p,D}} = 1$ always?

- If yes, the Majorana-dimer model is *unfrustrated* and will have the same spectrum as the traditional bosonic QDM

Phase diagram on the triangular lattice

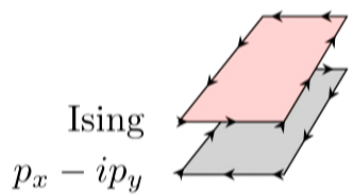
- For a system with OBC, e.g., on a disk, we can analytically prove (nontrivial..) that H is indeed unfrustrated!
- Import known results from Moessner and Sondhi (2001):



- We can thus conclude the following (for OBC):
  - At the RK point, $v = t$, our Majorana-RVB state is the *exact* ground state
  - The Majorana-RVB phase is gapped with a unique ground state
  - The bosonic Z_2 RVB spin liquid contains no gapless edge modes, so our Majorana-RVB phase has a fully gapped edge

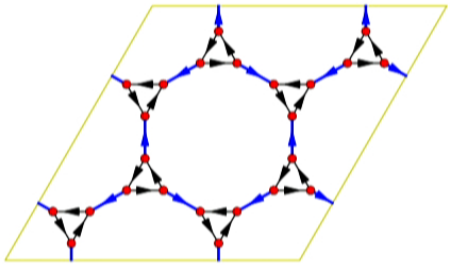
Identifying the likely topological order

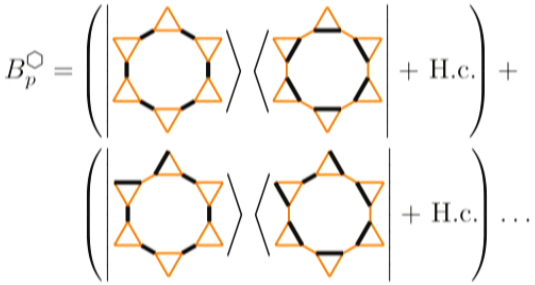
- Evidence for 3-fold ground-state degeneracy on the torus
  - For PBC, we find (numerically on small clusters) that only the nontrivial topological sectors—(1, 0), (0, 1), (1, 1)—remain unfrustrated
  - Exact diagonalization results:
    - RVB states in the (1, 0), (1, 0), and (1, 1) sectors are exact zero-energy ground states at the RK point (with odd fermion parity)
    - Staggered states lift off to excited states in the RVB phase
- Likely extended 3-fold degenerate Majorana-RVB phase!
- Natural candidate for the topological order: $\boxed{\text{Ising} \times (p_x - ip_y)}$
  - ✓ Deconfined, non-Abelian Ising anyon σ
  - ✓ Fully gapped edge: $c_- = \frac{1}{2} - \frac{1}{2} = 0$
  - ✓ 3-fold GS degeneracy on the torus with PBC



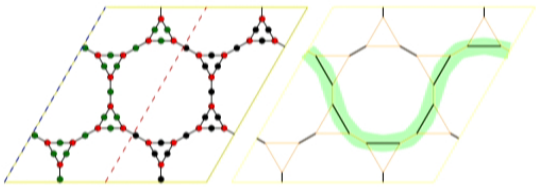
Pinning down the topological order on the Fisher lattice

- Can construct an analogous exactly solvable, *commuting-projector* model on the Fisher lattice





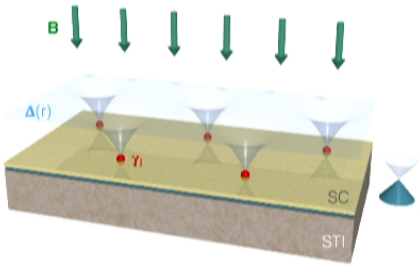
- Compute S and T matrices via minimally entangled states



$$\text{MES} \Rightarrow \begin{cases} S = S_{\text{Ising}} \otimes S_{p_x - ip_y} & \checkmark \\ T = T_{\text{Ising}} \otimes T_{p_x - ip_y} & \checkmark \end{cases}$$

Conclusions: Majorana dimers

- Realized Majorana-RVB state as ground state of (frustration-free) Majorana-dimer models on two different lattices
- Identified the topological order as Ising $\times$ $(p_x - ip_y)$
- Future/ongoing work
  - Replace Majoranas with parafermions
  - Forgo bosonic dimers and realize state in Majorana-only Hilbert space
- Potential physical platforms
  - Arrays of Majorana nanowires
  - Abrikosov vortex lattice of Majoranas
    - $p + ip$ / Fu-Kane superconductor



Chiu, Pikulin, and Franz (2015)