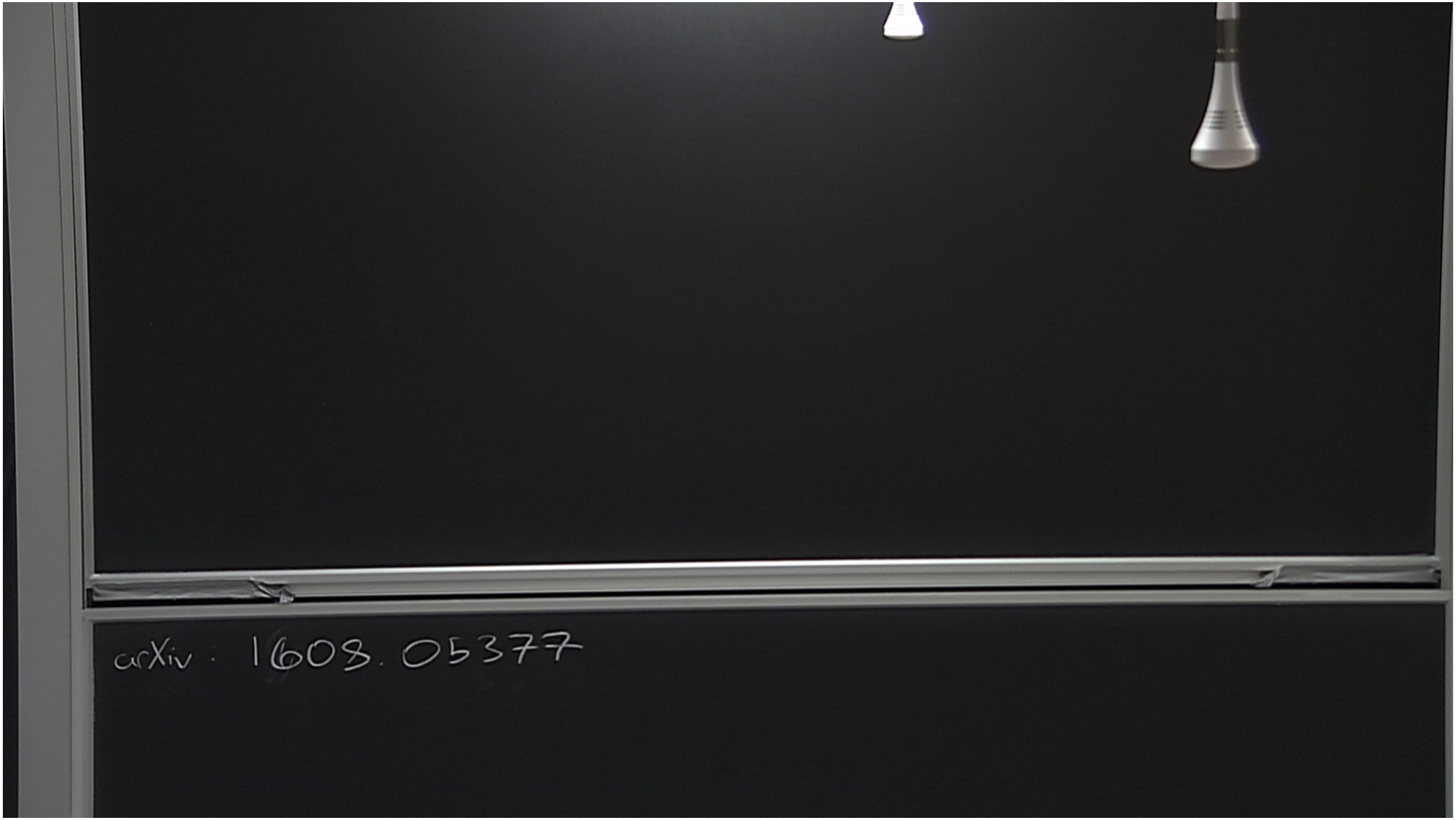
Title: Objective classical branch structure in a many-body wavefunction from spatial redundancy

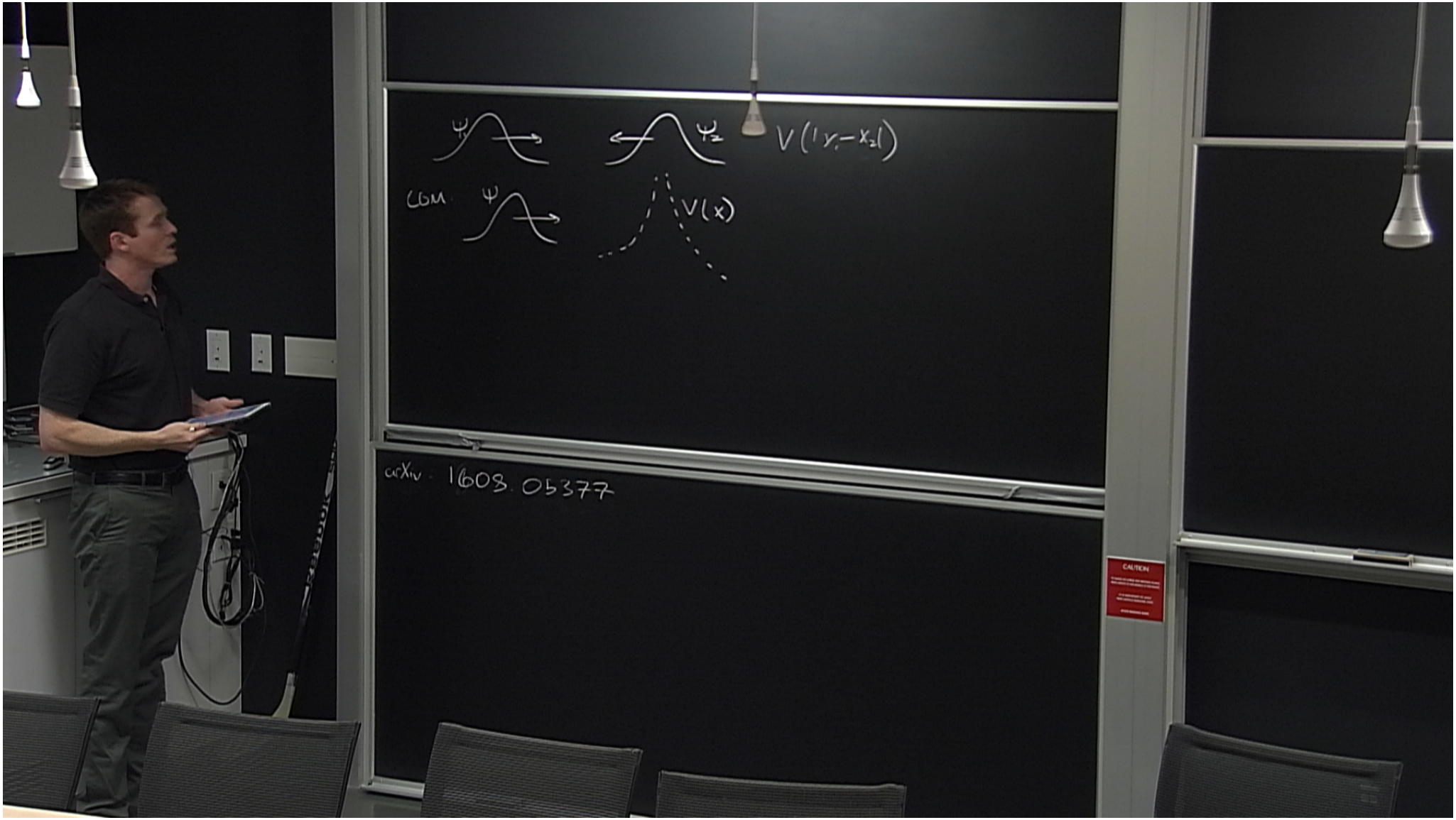
Date: Nov 15, 2016 03:30 PM

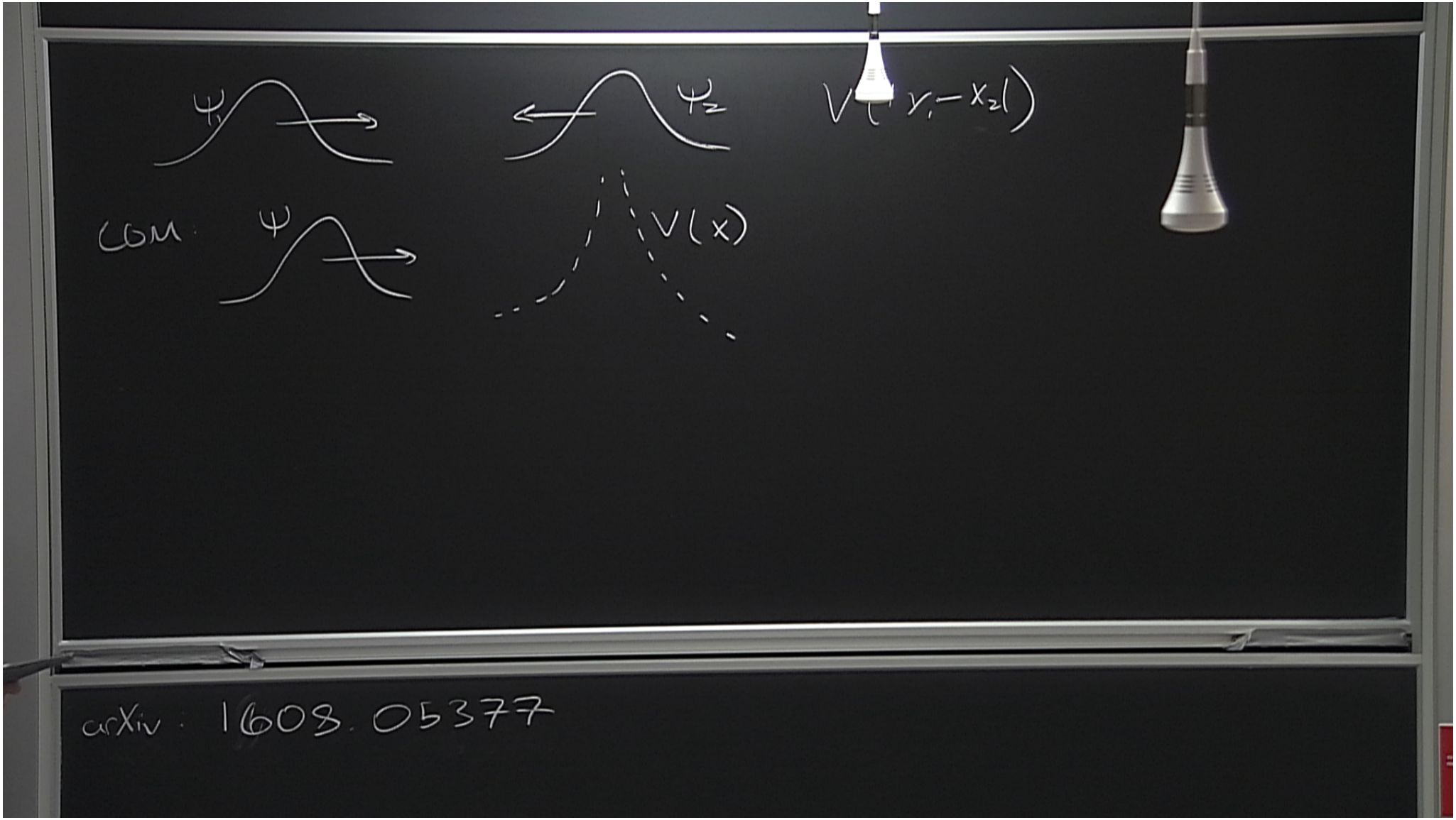
URL: http://pirsa.org/16110079

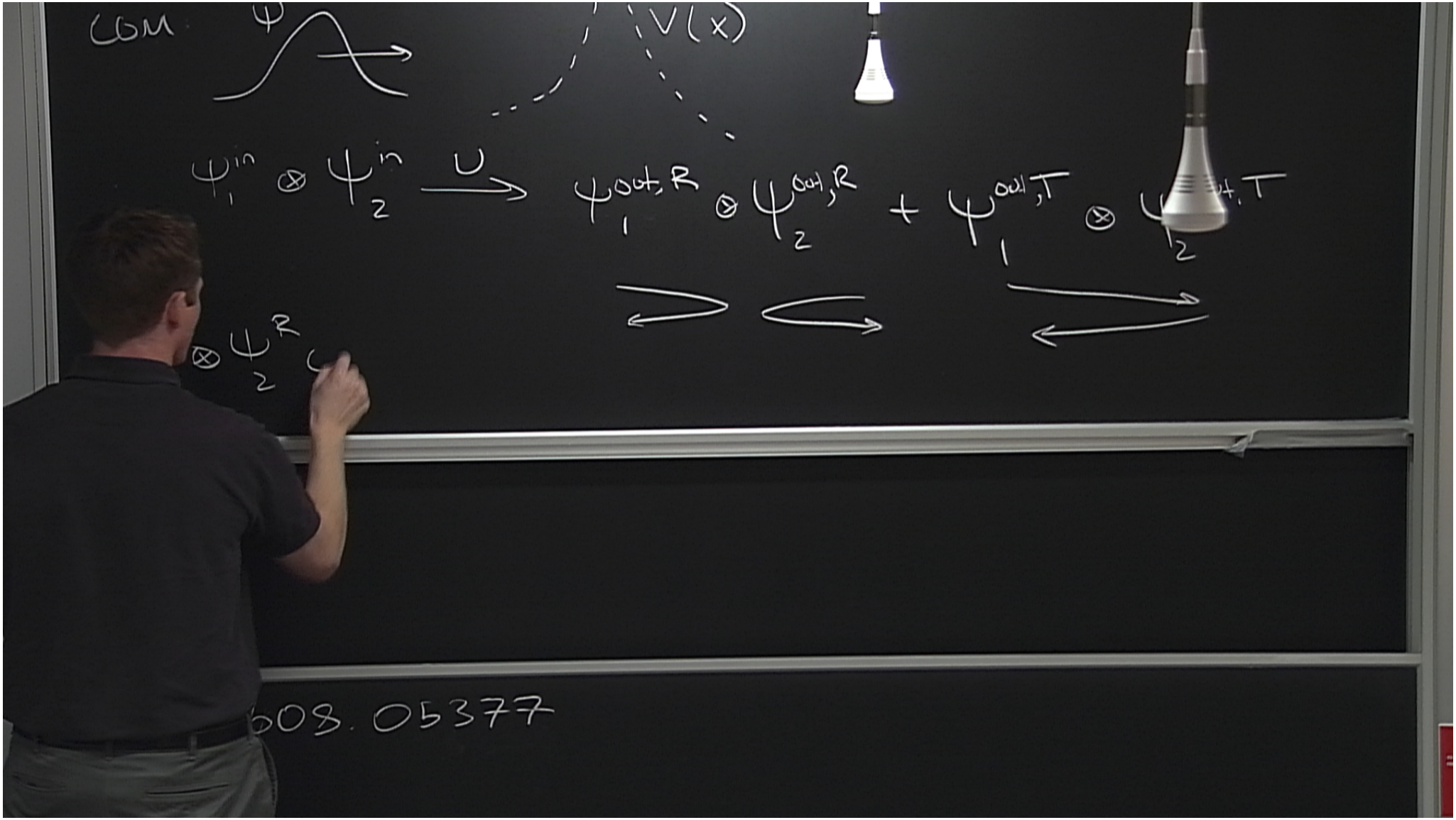
Abstract: When the wavefunction of a macroscopic system unitarily evolves from a low-entropy initial state, there is good circumstantial evidence it develops "branches", i.e., a decomposition into orthogonal components that can't be distinguished from the corresponding incoherent mixture by feasible observations, with each component a simultaneous eigenstate of preferred macroscopic observables. Is this decomposition unique? Can the number of branches decrease in time?

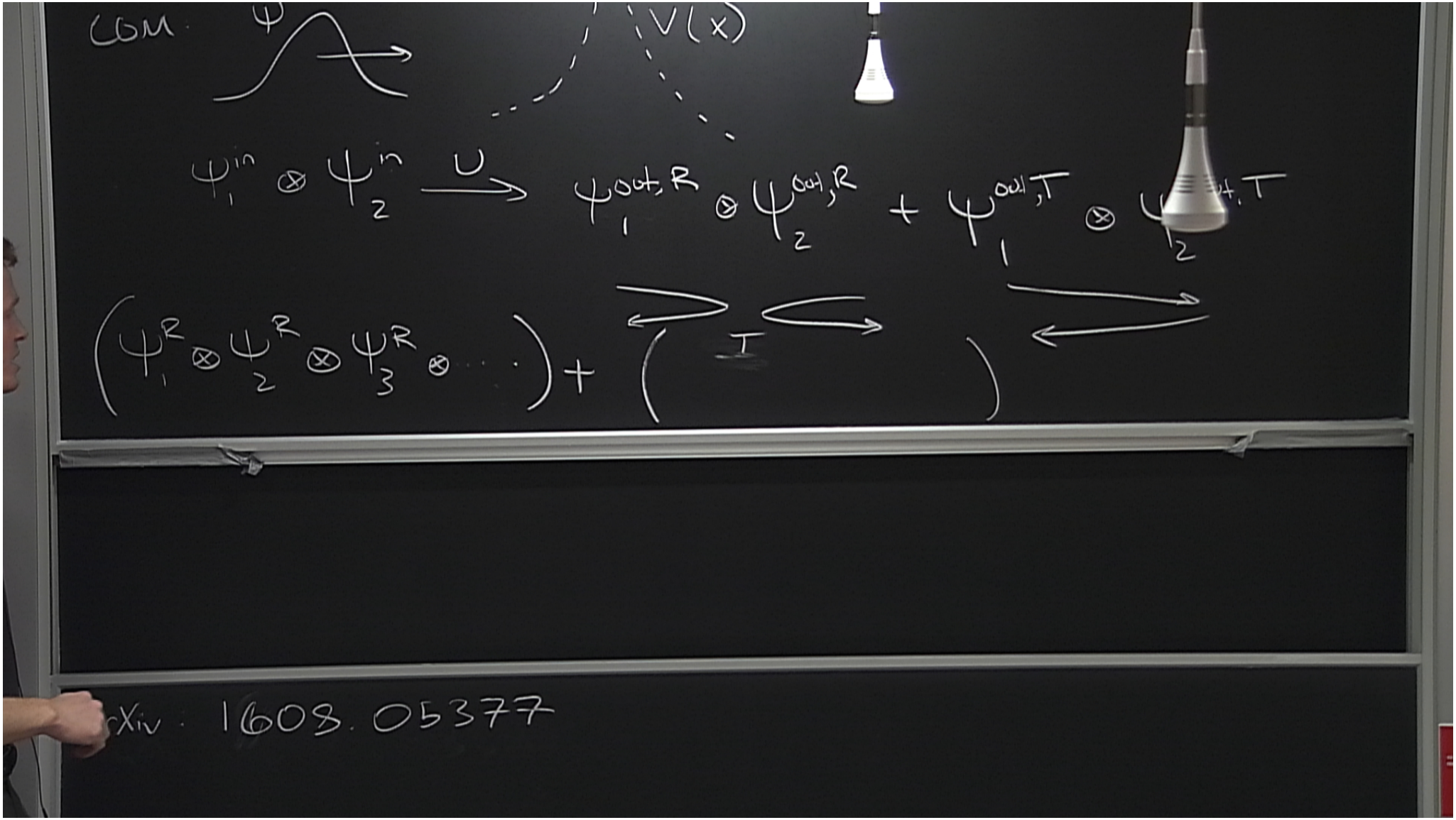
Answering these questions is hard because branches are defined only intuitively, much like early investigations of algorithms prior to the Churchâ€“Turing thesis. A rigorous definition could give a speed up to certain non-stationary matrix-product state numerical simulations, as well as solve Kent's Set Selection problem in the consistent histories formalism, a 20-year-old open challenge in the foundations of quantum mechanics. I introduce some tentative definitions based on the idea of redundant information, and establishing a uniqueness theorems. A key counterexample is provided by the Shor error-correction code, which demonstrates that branch structure with robust, redundant records on macroscopic scales can hide incompatible (noncommuting) structure on microscopic scales.


arXiv: 1608.05377


Ψ_1
Ψ_2
$V(|x_1 - x_2|)$
CoM
Ψ
$V(x)$
arXiv: 1608.05377


ψ_1
ψ_2
$V(|x_1 - x_2|)$
COM
ψ
$V(x)$
arXiv: 1608.05377


COM
V(x)
U
608.05377


COM
V(x)
U
I
1608.05377

$|\vec{\sigma}\rangle = |\sigma_1\rangle \otimes \cdots \otimes |\sigma_N\rangle$ $\qquad \vec{\sigma} = (\sigma_1, \ldots, \sigma_N)$

$$\langle \vec{\sigma} | \Psi \rangle = \sum_{a_1 \cdots a_N} A^{\sigma_1}_{a_1} A^{\sigma_2}_{a_1 a_2} \cdots A^{\sigma_{N-1}}_{a_{N-2} a_{N-1}} A^{\sigma_N}_{a_{N-1}}$$

$=$ 

arXiv: 1608.05377

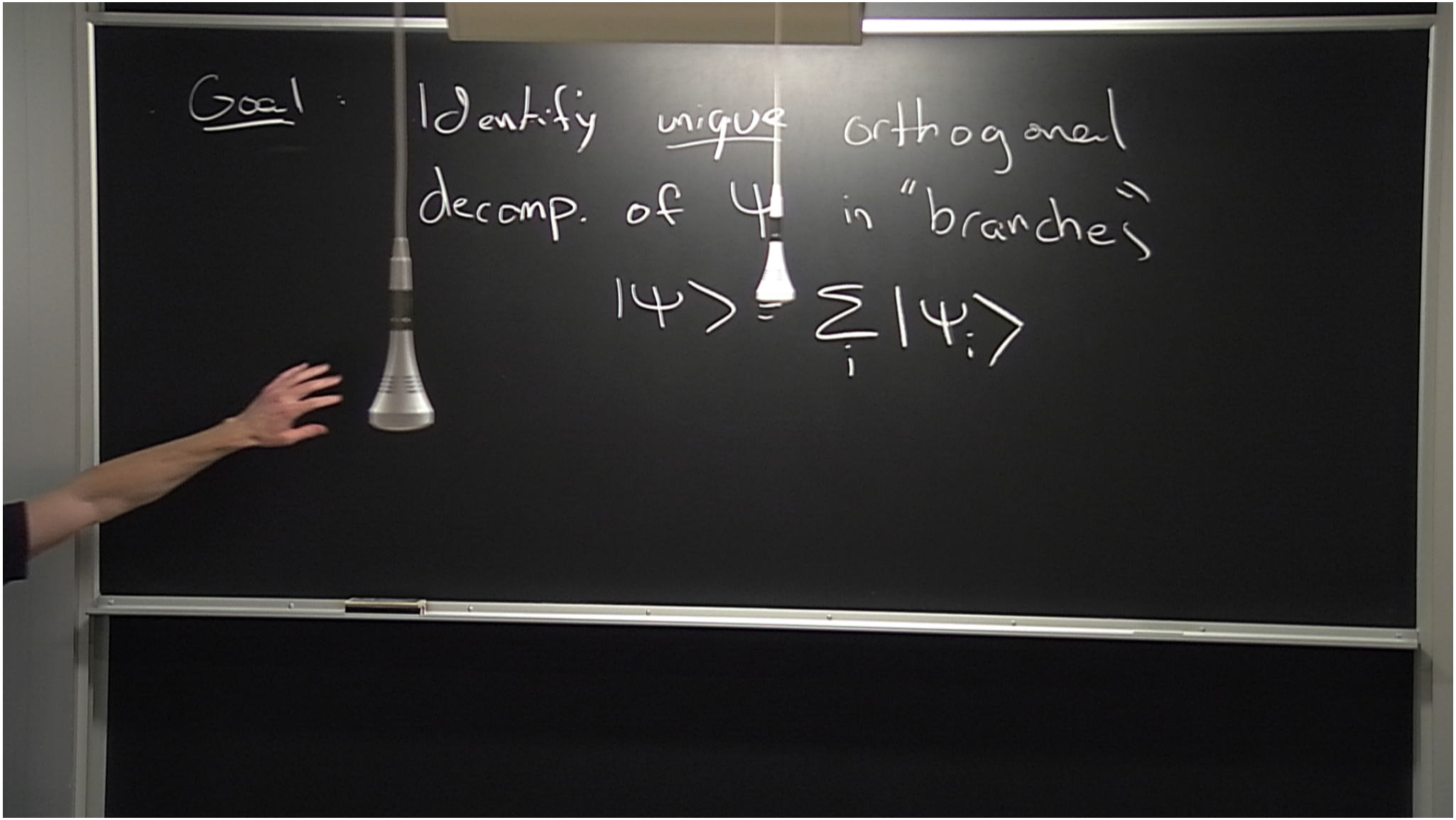
$$\left(\psi_1^R \otimes \psi_2^R \otimes \psi_3^R \otimes \cdots\right) + \left(\quad I \quad \right)$$

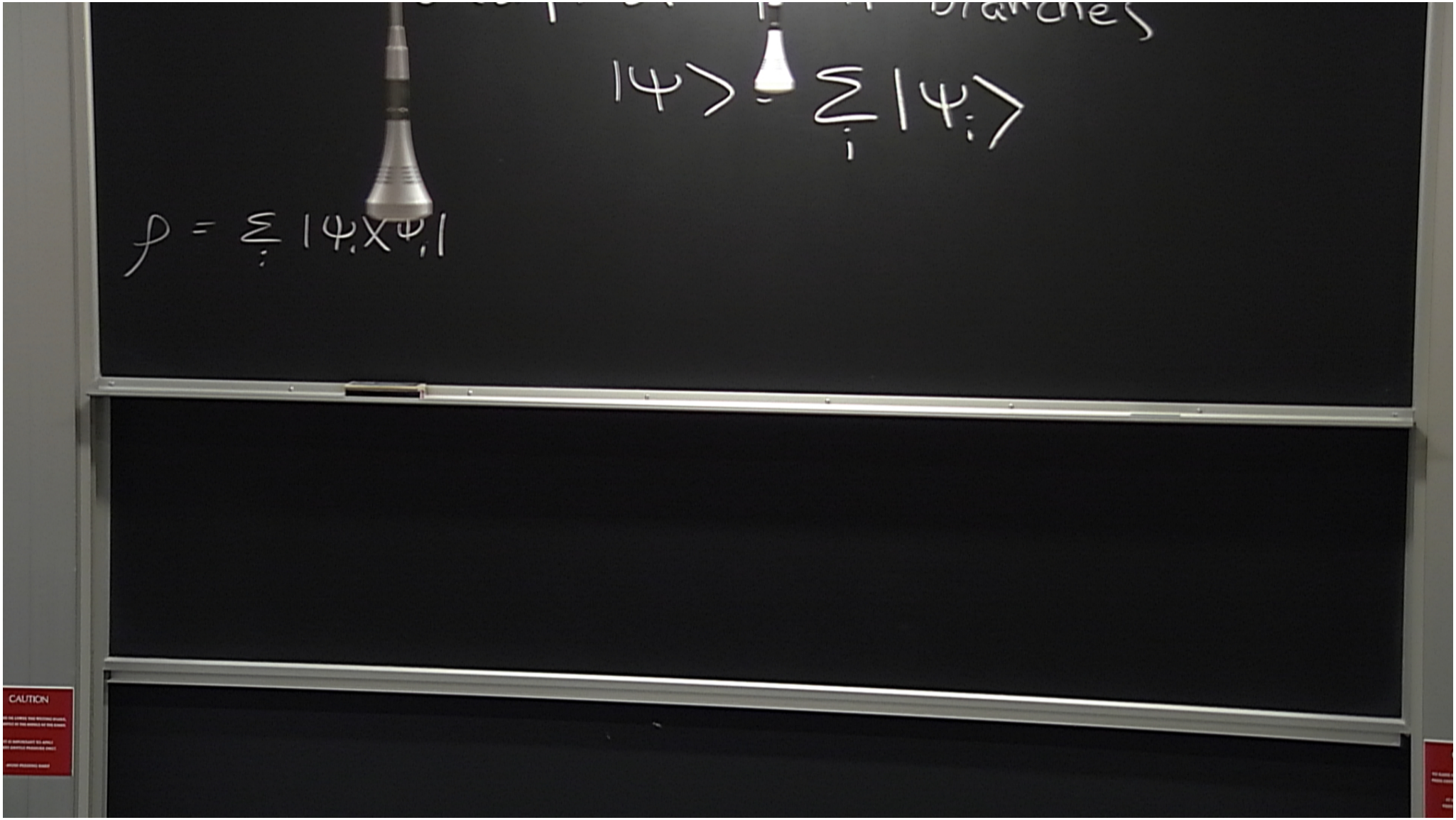
$$|\vec{\sigma}\rangle = |\sigma_1\rangle \otimes \cdots \otimes |\sigma_N\rangle \qquad \vec{\sigma} = (\sigma_1, \ldots, \sigma_N)$$

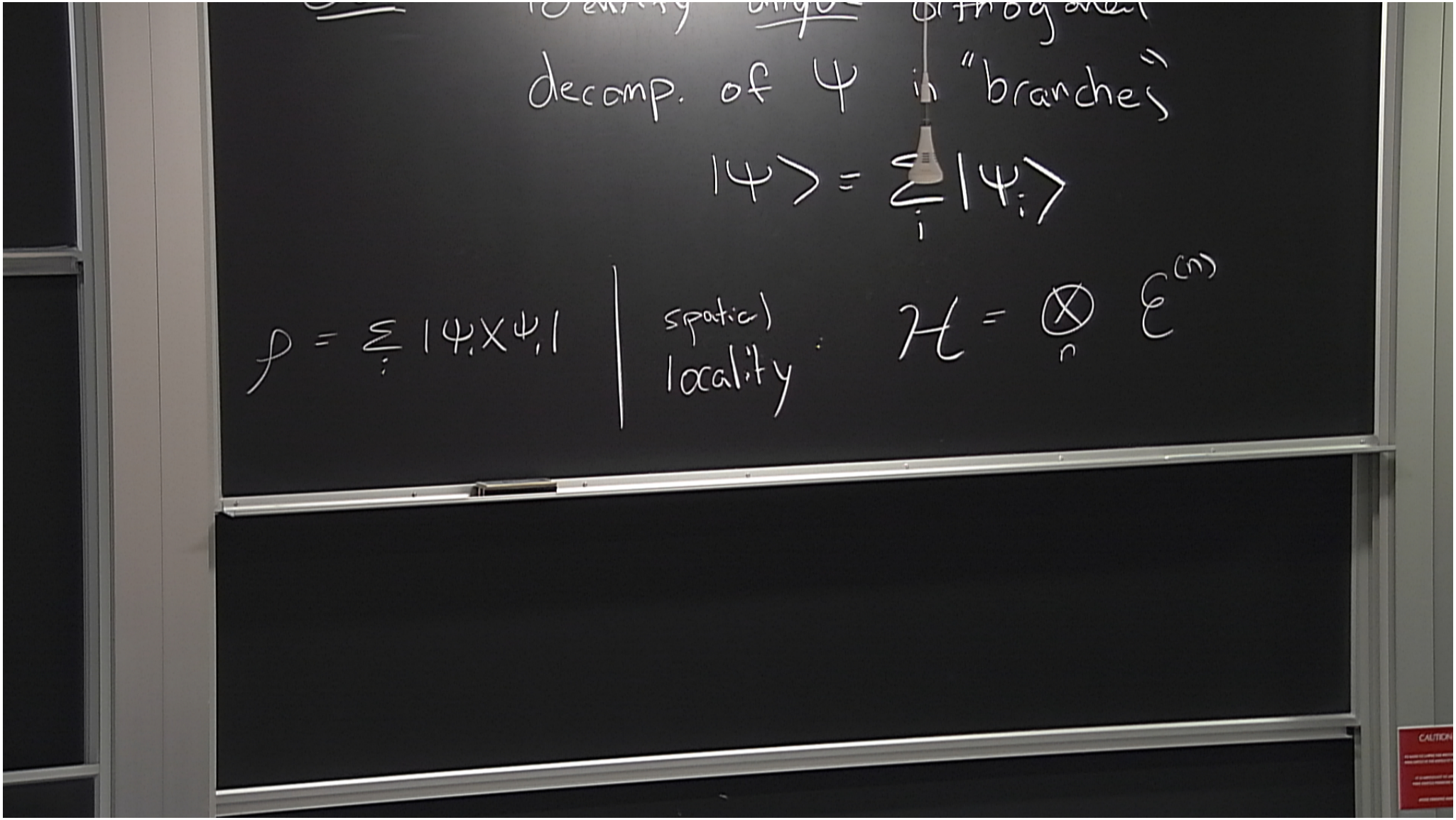
$$\langle \vec{\sigma} | \psi \rangle = \sum_{a_1 \cdots a_N} A^{\sigma_1}_{a_1} A^{\sigma_2}_{a_1 a_2} \cdots A^{\sigma_{N-1}}_{a_{N-2} a_{N-1}} A^{\sigma_N}_{a_{N-1}}$$

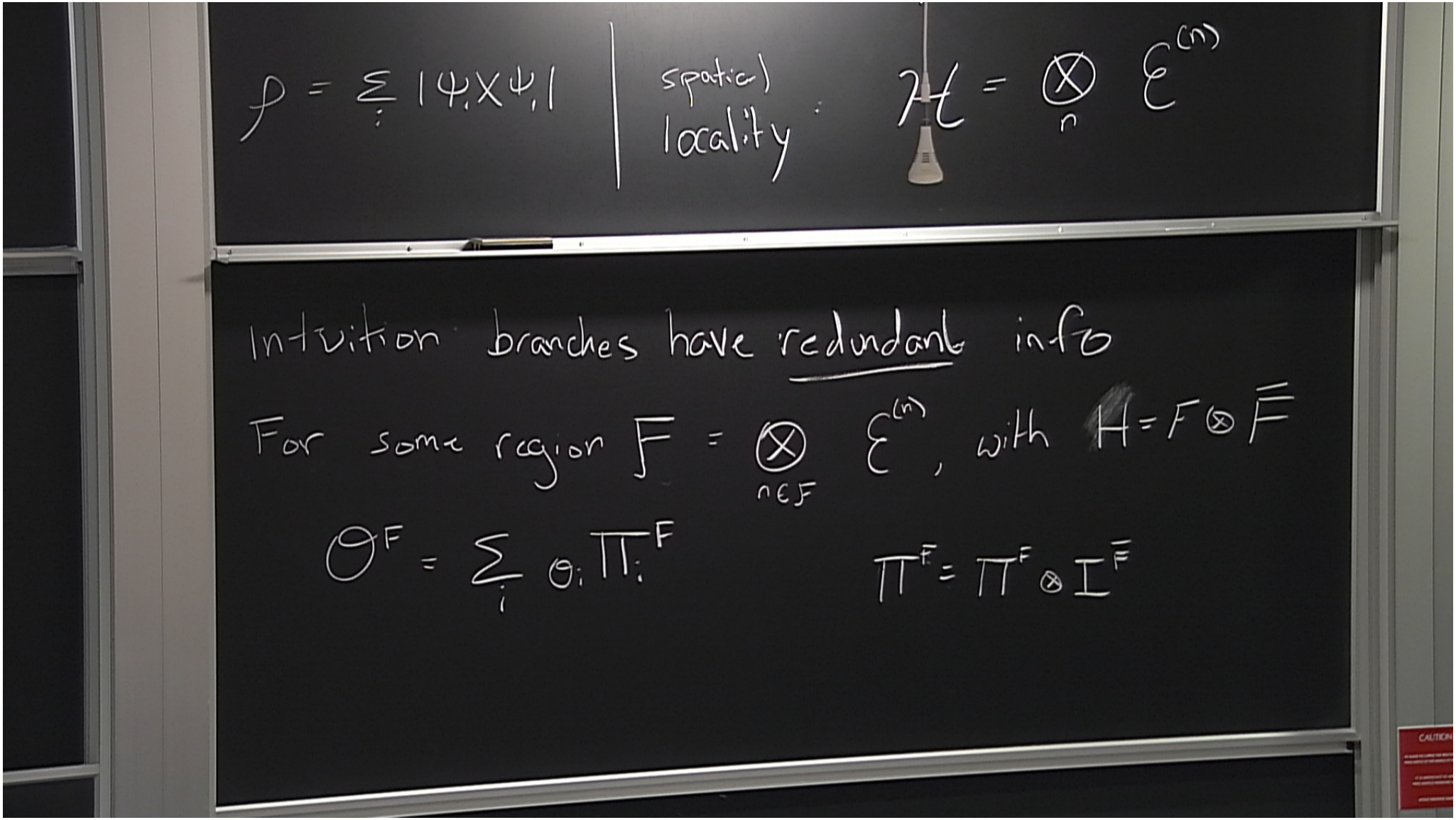
$=$ 

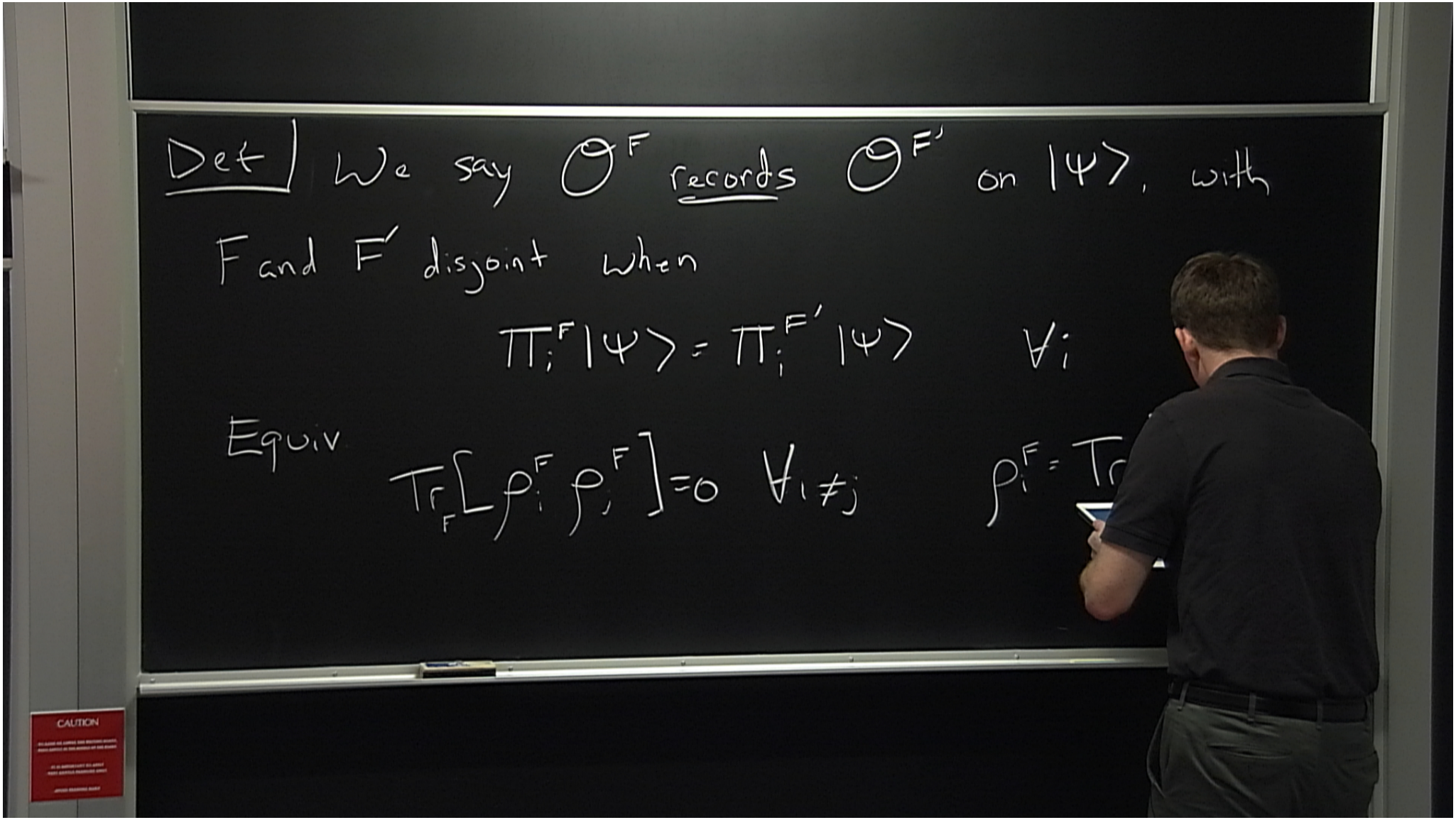
arXiv: 1608.05377


Goal:
Identify unique orthogonal
decomp. of ψ in "branches"
|ψ⟩ = Σ_i |ψ_i⟩


branches
$|\psi\rangle = \sum_i |\psi_i\rangle$
$\rho = \sum_i |\psi_i\rangle\langle\psi_i|$
CAUTION


decomp. of Ψ in "branches"
$|\Psi\rangle = \sum_i |\Psi_i\rangle$
$\rho = \sum_i |\Psi_i\rangle\langle\Psi_i|$
spatial locality
$\mathcal{H} = \bigotimes_n \mathcal{E}^{(n)}$


$\rho = \sum_i |\psi_i\rangle\langle\psi_i|$
spatial locality
$\mathcal{H} = \bigotimes_n \mathcal{E}^{(n)}$
Intuition: branches have redundant info
For some region $F = \bigotimes_{n \in F} \mathcal{E}^{(n)}$, with $H = F \otimes \bar{F}$
$\Theta^F = \sum_i \theta_i \Pi_i^F$
$\Pi^F = \Pi^F \otimes I^{\bar{F}}$


Def/ We say $\mathcal{O}^F$ records $\mathcal{O}^{F'}$ on $|\psi\rangle$, with
F and F' disjoint when
$\Pi_i^F|\psi\rangle = \Pi_i^{F'}|\psi\rangle \quad \forall i$
Equiv
$\mathrm{Tr}_F[\rho_i^F \rho_j^F] = 0 \quad \forall i \neq j$
$\rho_i^F = \mathrm{Tr}$
CAUTION

Def We say $\mathcal{O}^F$ records $\mathcal{O}^{F'}$ on $|\psi\rangle$, with F and F' disjoint when

$$\Pi_i^F |\psi\rangle = \Pi_i^{F'} |\psi\rangle \quad \forall i$$

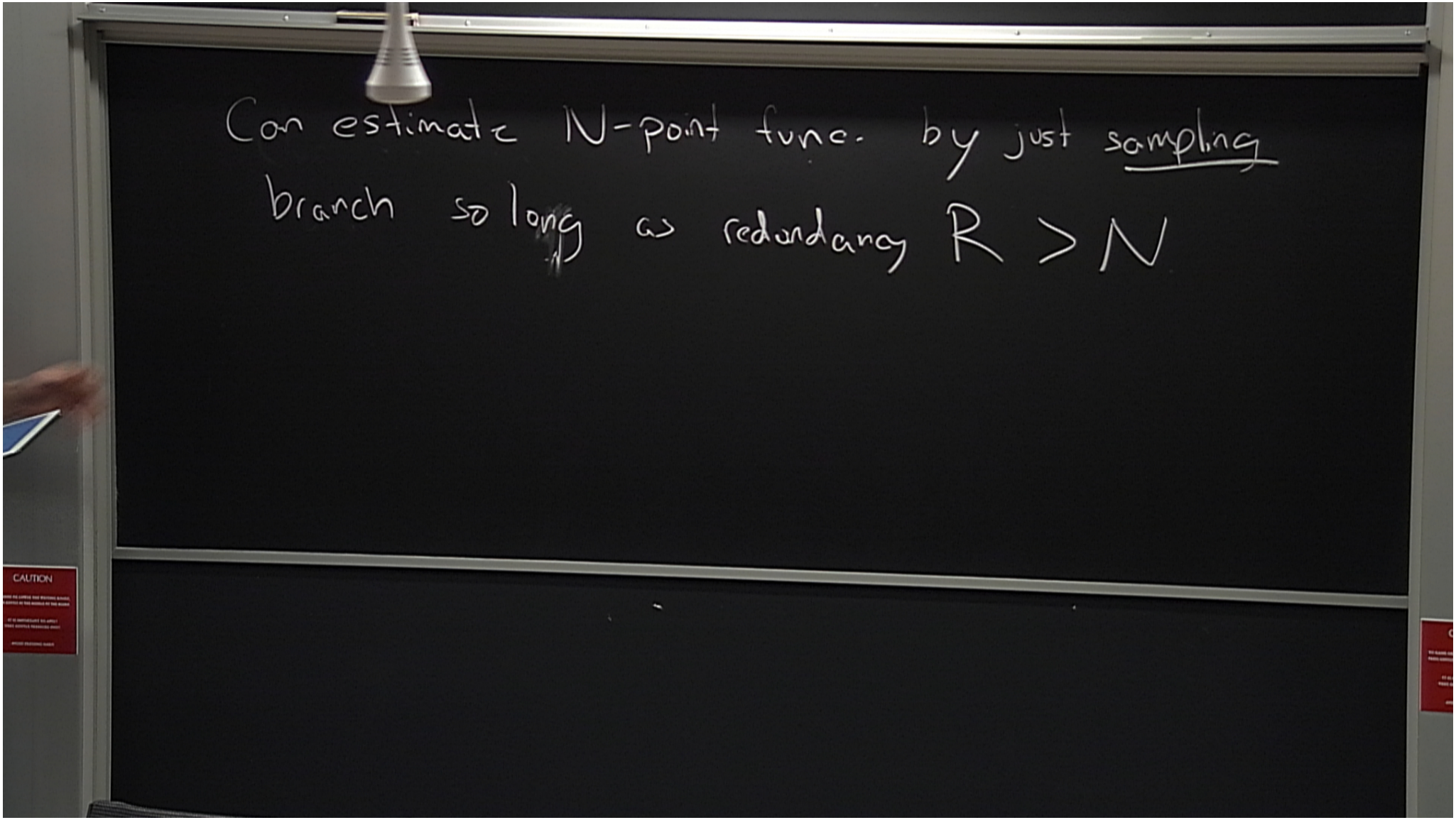
Equiv $\mathrm{Tr}_F[\rho_i^F \rho_j^F] = 0 \quad \forall i \neq j$ $\quad \rho_i^F = \mathrm{Tr}_{\bar{F}}[\Pi_i^{F'} |\psi\rangle\langle\psi| \Pi_i^{F'}]$

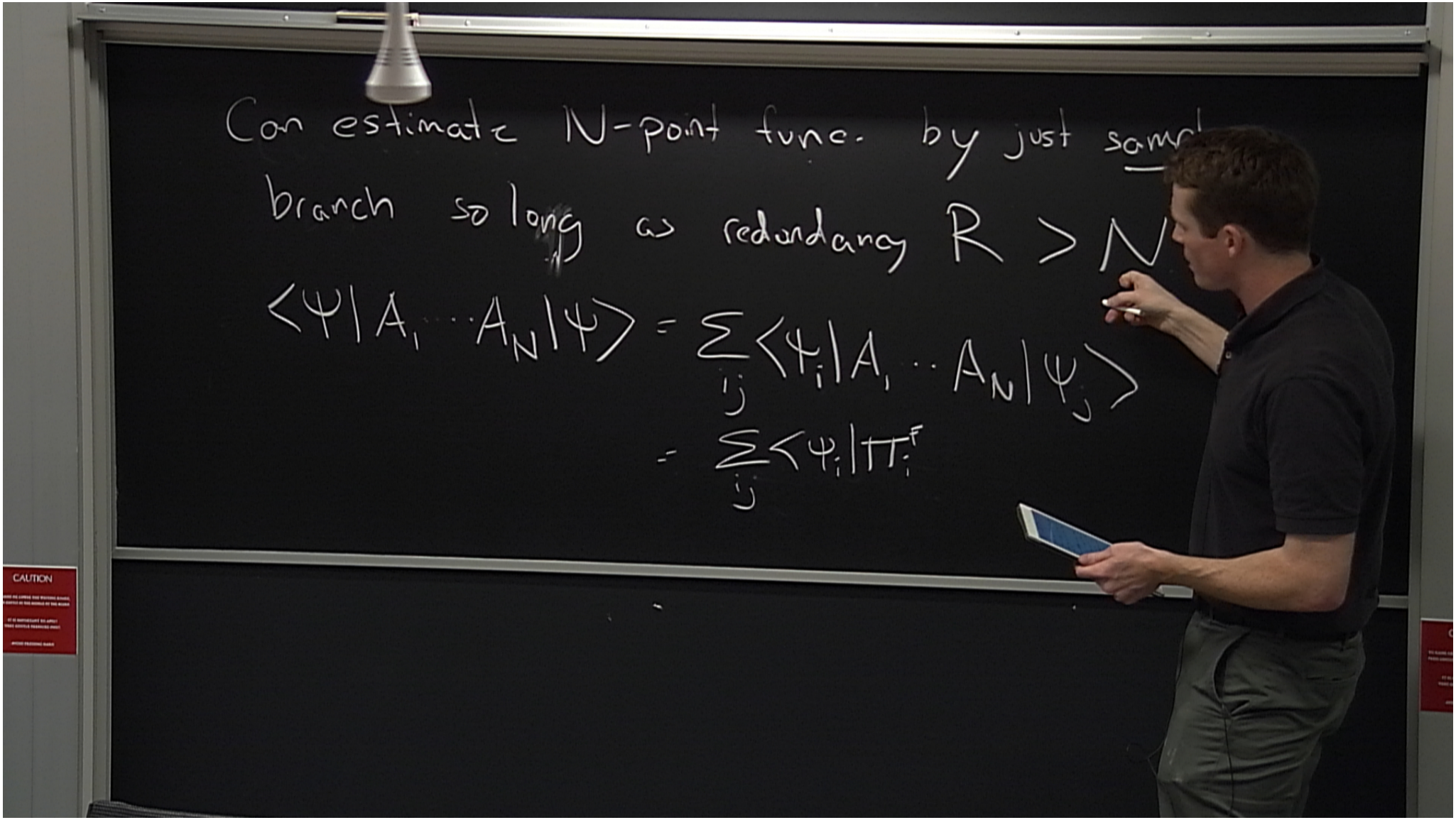
CAUTION

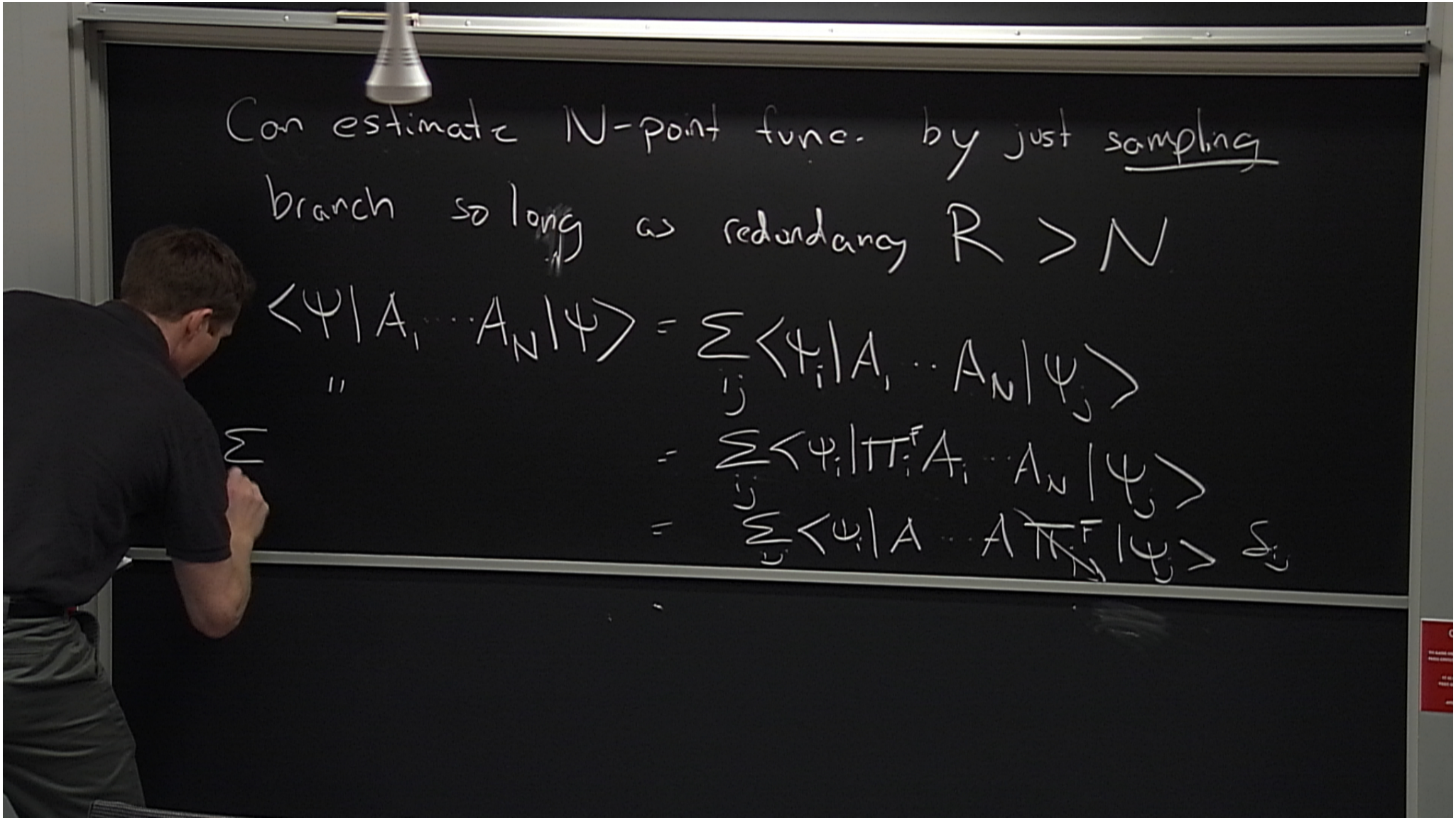
Def] A recorded observ. $\Theta = \{\Theta^F, \Theta^{F'}, \Theta^{F''}, \ldots\}$

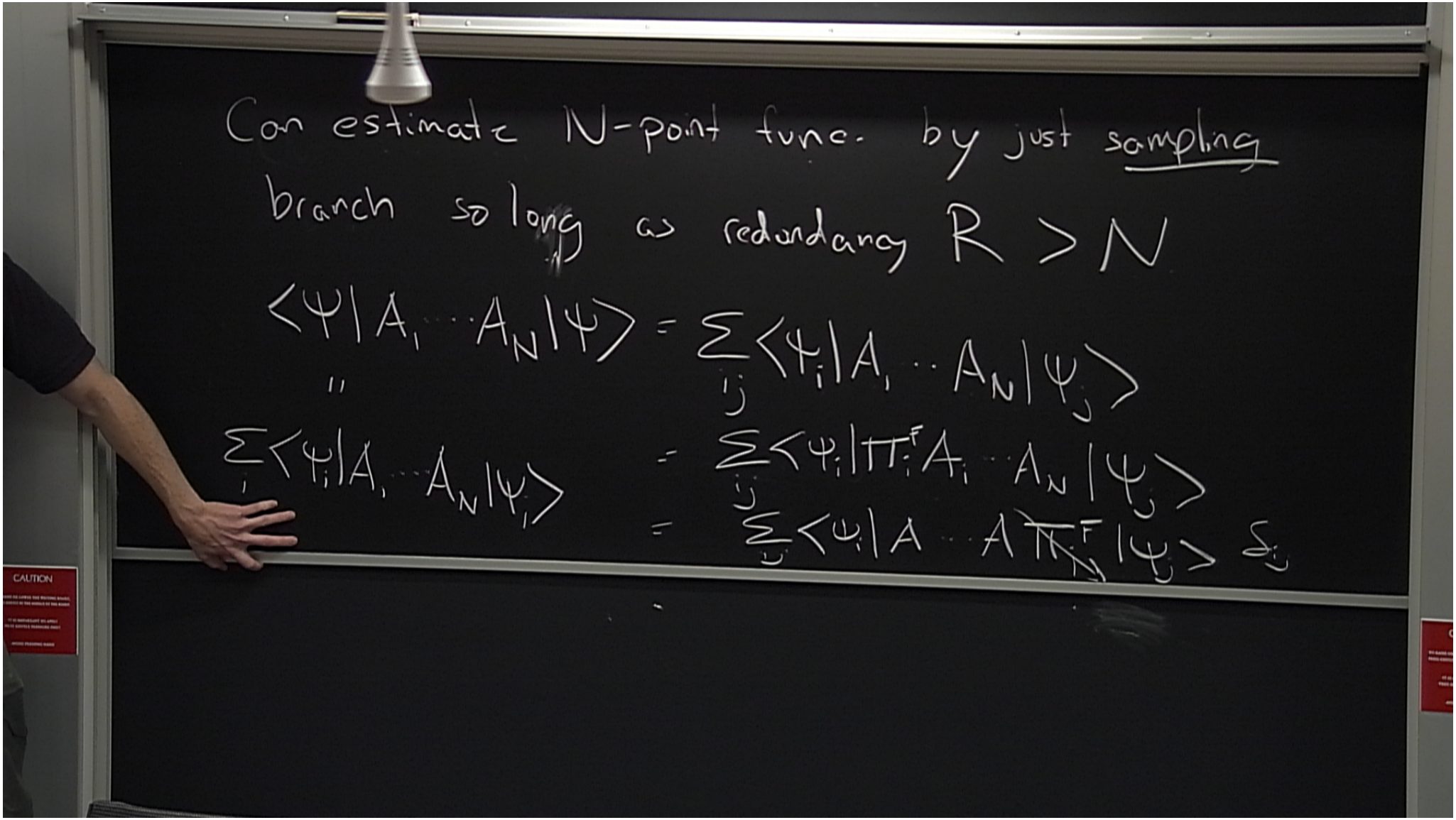
$$|\psi_i\rangle \equiv \Pi_i^F |\psi\rangle = \Pi_i^{F'} |\psi\rangle = \ldots$$

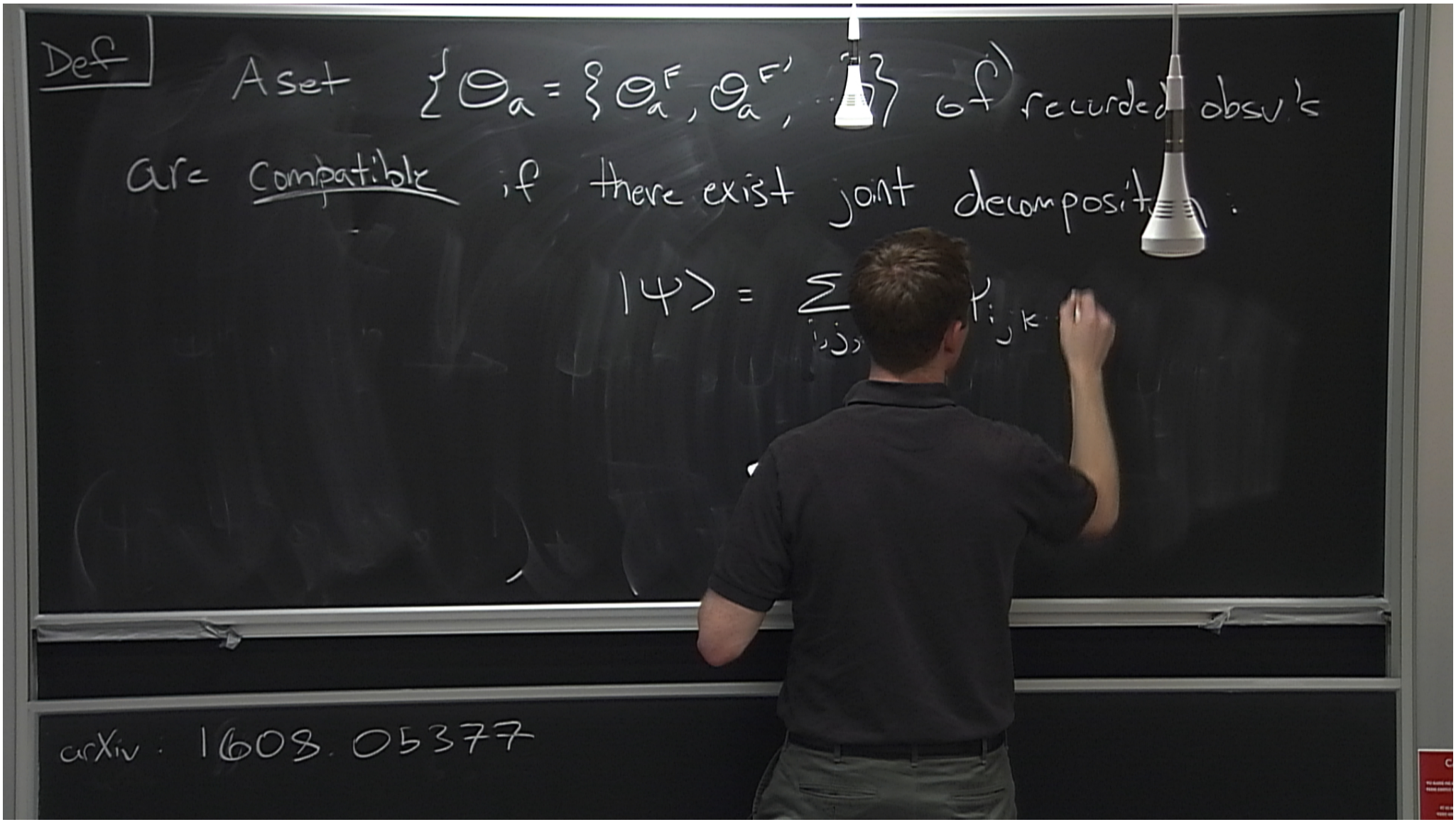
The $|\psi_i\rangle$ are "branches", each Θ^F is "record", $|\Theta| = R$ "redundancy"

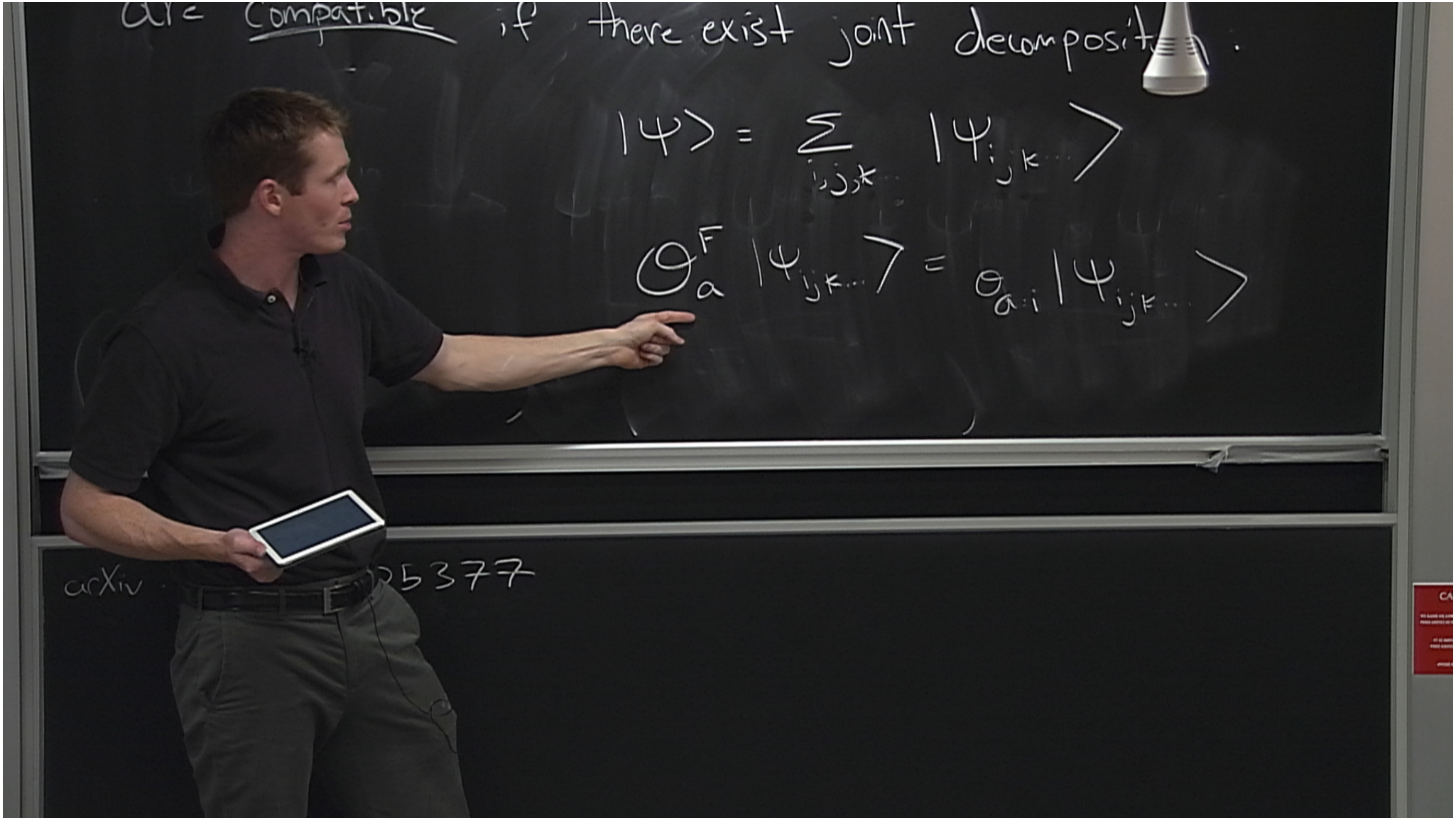

Can estimate N-point func. by just sampling
branch so long as redundancy R > N
CAUTION


Can estimate N-point func. by just sampl
branch so long as redundancy R > N
$\langle\Psi|A_1 \cdots A_N|\Psi\rangle = \sum_j \langle\Psi_i|A_1 \cdots A_N|\Psi_j\rangle$
$- \sum_j \langle\Psi_i|\Pi_i^F$
CAUTION


Can estimate N-point func. by just sampling
branch so long as redundancy R > N
$\langle\Psi|A_1 \cdots A_N|\Psi\rangle = \sum_{ij}\langle\Psi_i|A_1 \cdots A_N|\Psi_j\rangle$
$= \sum_{ij}\langle\Psi_i|\Pi_i^F A_i \cdots A_N|\Psi_j\rangle$
$= \sum_{ij}\langle\Psi_i|A \cdots A\Pi_j^F|\Psi_j\rangle \delta_{ij}$


Can estimate N-point func. by just sampling
branch so long as redundancy R > N
$\langle\psi|A_1 \cdots A_N|\psi\rangle = \sum_{ij}\langle\psi_i|A_1 \cdots A_N|\psi_j\rangle$
$\sum_i\langle\psi_i|A_1 \cdots A_N|\psi_i\rangle = \sum_{ij}\langle\psi_i|\Pi_i^F A_i \cdots A_N|\psi_j\rangle$
$= \sum_{ij}\langle\psi_i|A \cdots A \Pi_j^F|\psi_j\rangle \delta_{ij}$
CAUTION


Def
A set $\{\Theta_a = \{\theta_a^F, \theta_a^{F'}, \ldots\}$ of recorded obsv's
are Compatible if there exist joint decomposition.
$|\psi\rangle = \sum_{i,j,} \gamma_{ijk}$
arXiv: 1608.05377

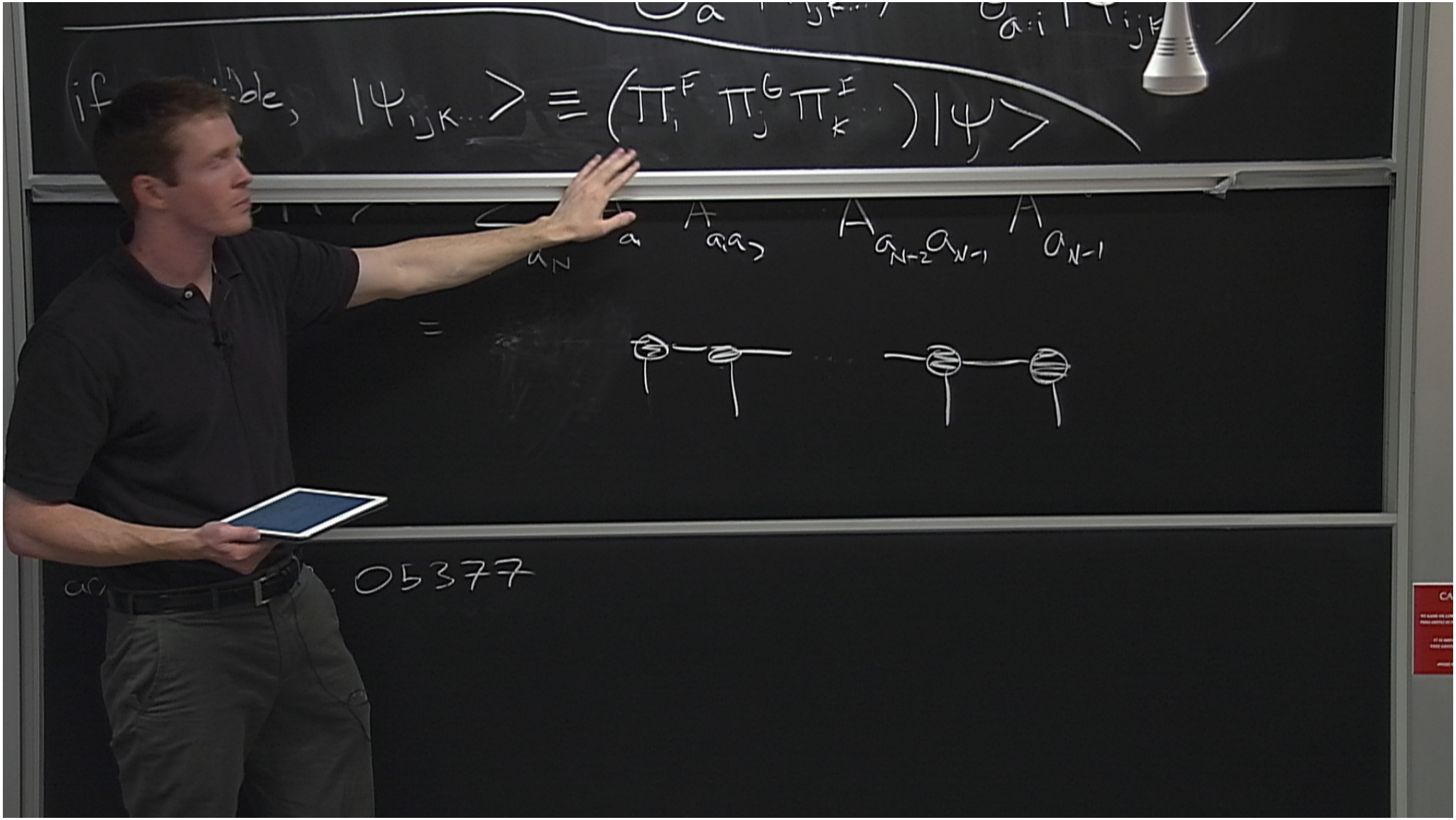

are Compatible if there exist joint decomposit
$|\psi\rangle = \sum_{i,j,k\ldots} |\psi_{ijk\ldots}\rangle$
$\sigma_a^F |\psi_{ijk\ldots}\rangle = \sigma_{a;i} |\psi_{ijk\ldots}\rangle$
arXiv

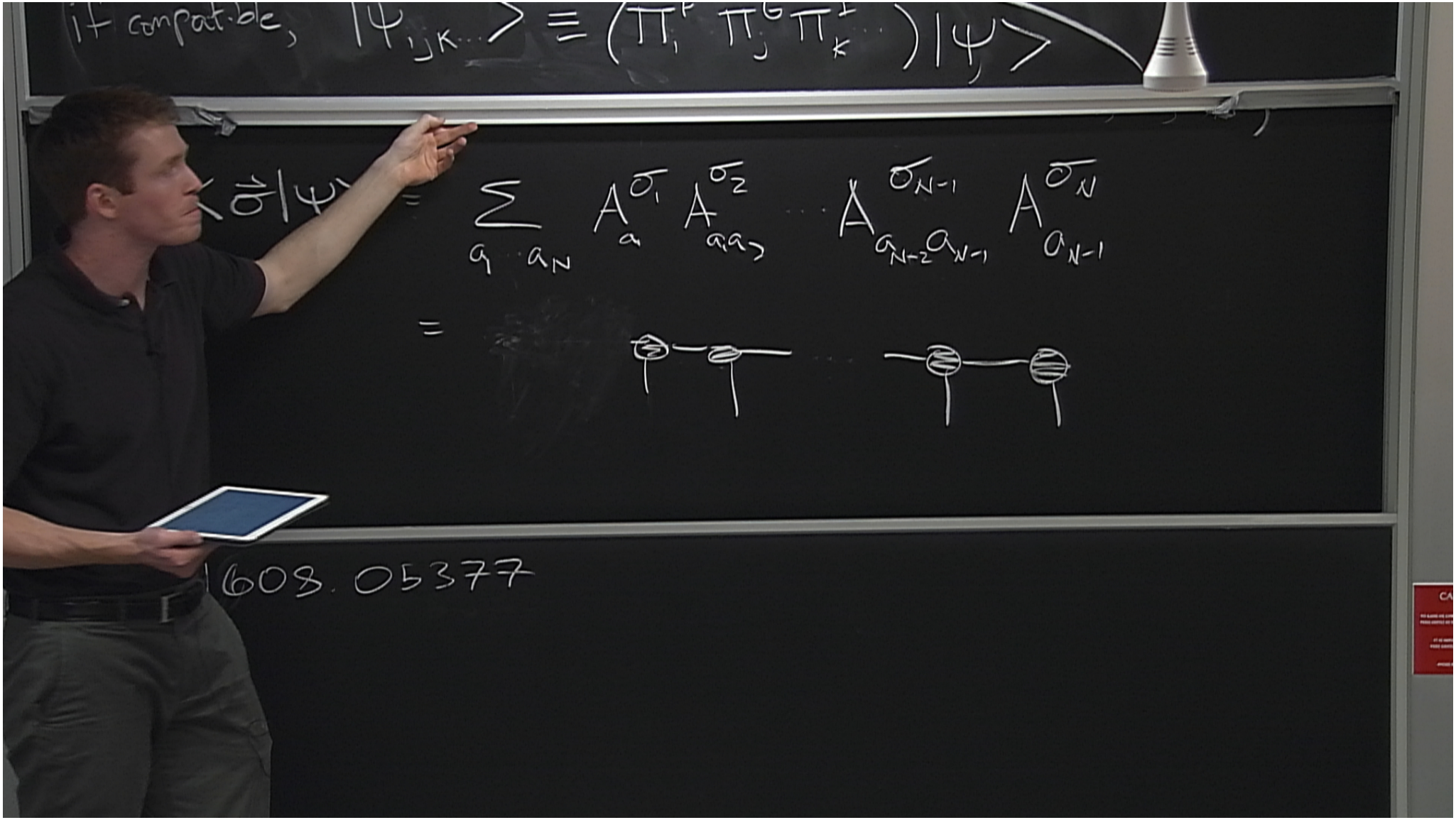
Def A set $\{\Theta_a = \{\Theta_a^F, \Theta_a^{F'}, \ldots\}\}$ of recorded obsv's are compatible if there exist joint decomposition:

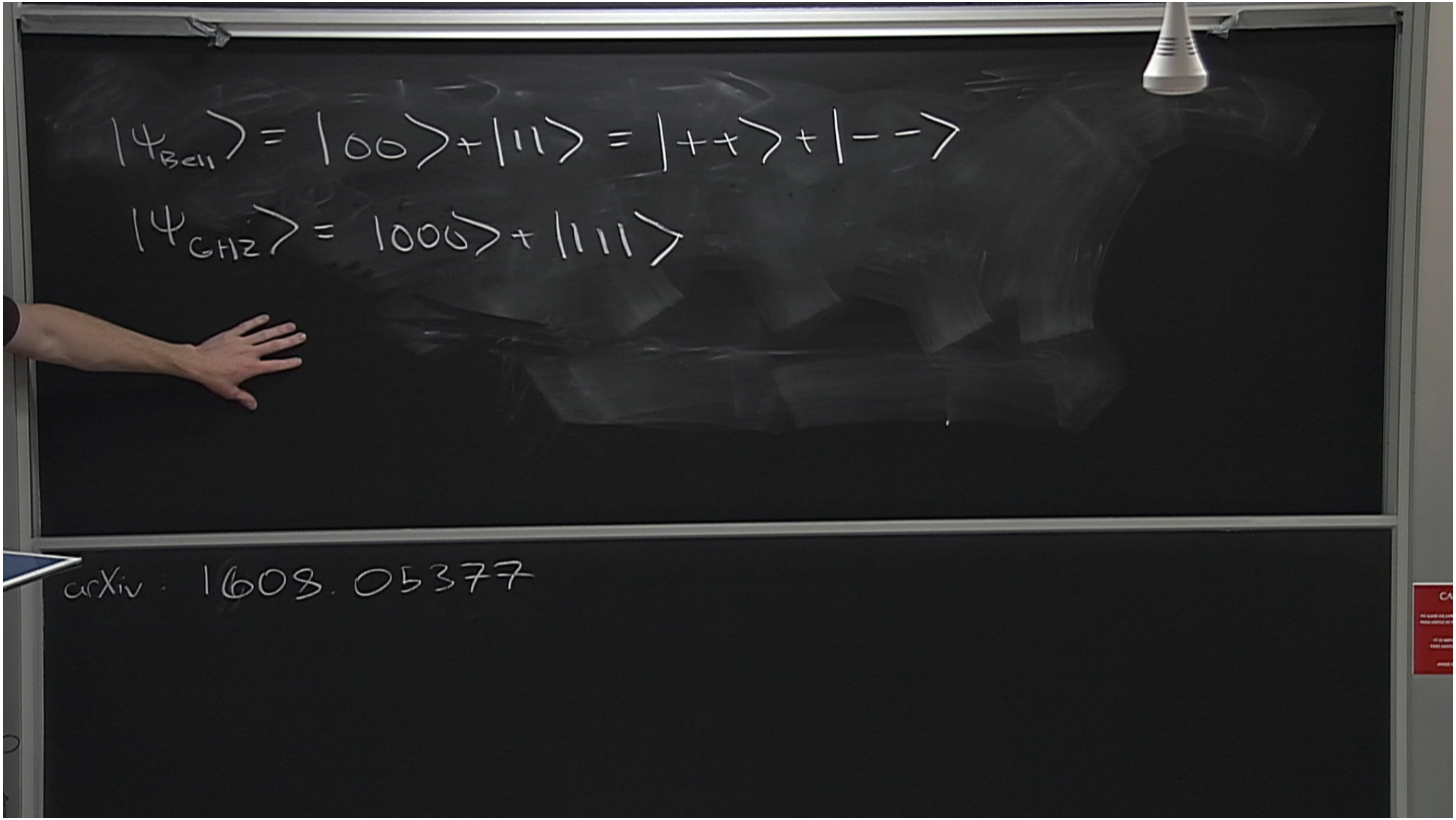
$$|\Psi\rangle = \sum_{ijk\ldots} |\Psi_{ijk\ldots}\rangle$$

$$\Theta_a^F |\Psi_{ijk\ldots}\rangle = \theta_{a:i} |\Psi_{ijk\ldots}\rangle$$

arXiv: 1608.05377

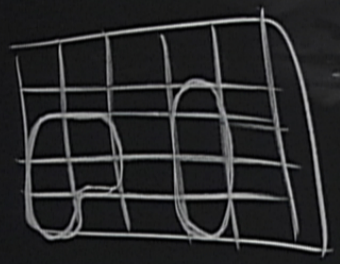

$|\psi_{ijk\ldots}\rangle \equiv (\pi_i^F \pi_j^G \pi_k^F \ldots)|\psi\rangle$
A_{a_N}
$A_{a_1 a_2}$
$A_{a_{N-2} a_{N-1}}$
$A_{a_{N-1}}$
05377


if compatible,
$\sum_{a_1 \cdots a_N} A^{\sigma_1}_{a_1} A^{\sigma_2}_{a_1 a_2} \cdots A^{\sigma_{N-1}}_{a_{N-2} a_{N-1}} A^{\sigma_N}_{a_{N-1}}$
=
1608.05377


$|\Psi_{Bell}\rangle = |00\rangle + |11\rangle = |++\rangle + |--\rangle$
$|\Psi_{GHZ}\rangle = |000\rangle + |111\rangle$
arXiv: 1608.05377

$|\psi_{Bell}\rangle = |00\rangle + |11\rangle = |++\rangle + |--\rangle$

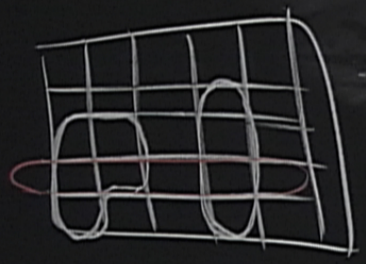
$|\psi_{GHZ}\rangle = |000\rangle + |111\rangle$



1608.05377

$|\psi_{Bell}\rangle = |00\rangle + |11\rangle = |++\rangle + |--\rangle$

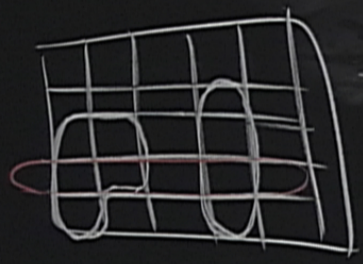
$|\psi_{GHZ}\rangle = |000\rangle + |111\rangle$



arXiv: 1608.05377

$|\psi_{Bell}\rangle = |00\rangle + |11\rangle = |++\rangle + |--\rangle$

$|\psi_{GHZ}\rangle = |000\rangle + |111\rangle$

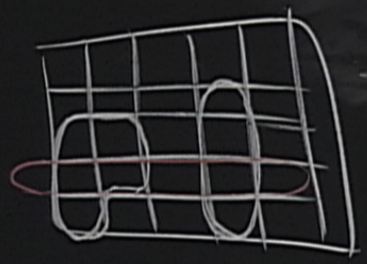




arXiv: 1608.05377

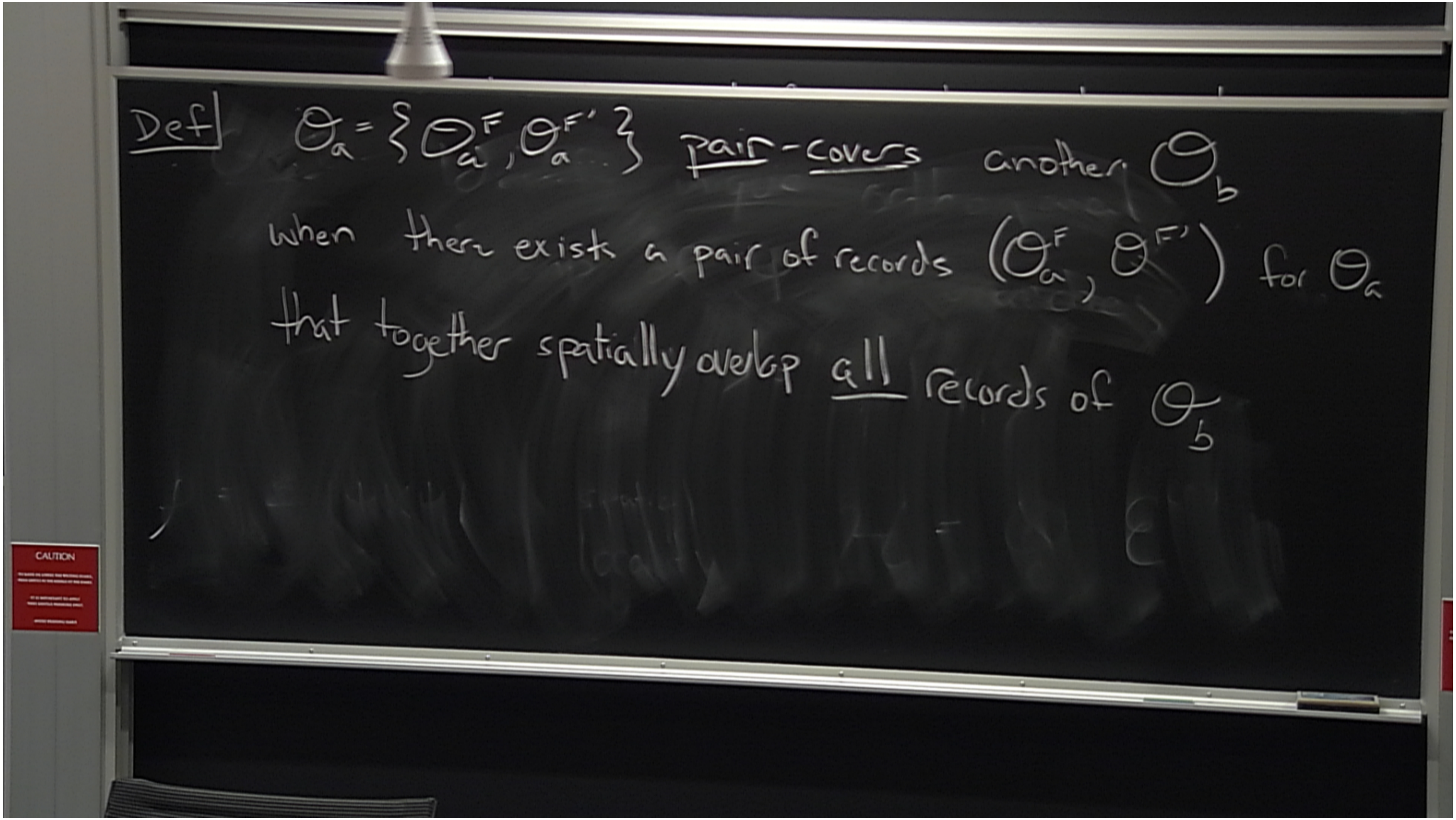
$|\psi_{Bell}\rangle = |00\rangle + |11\rangle = |++\rangle + |--\rangle$

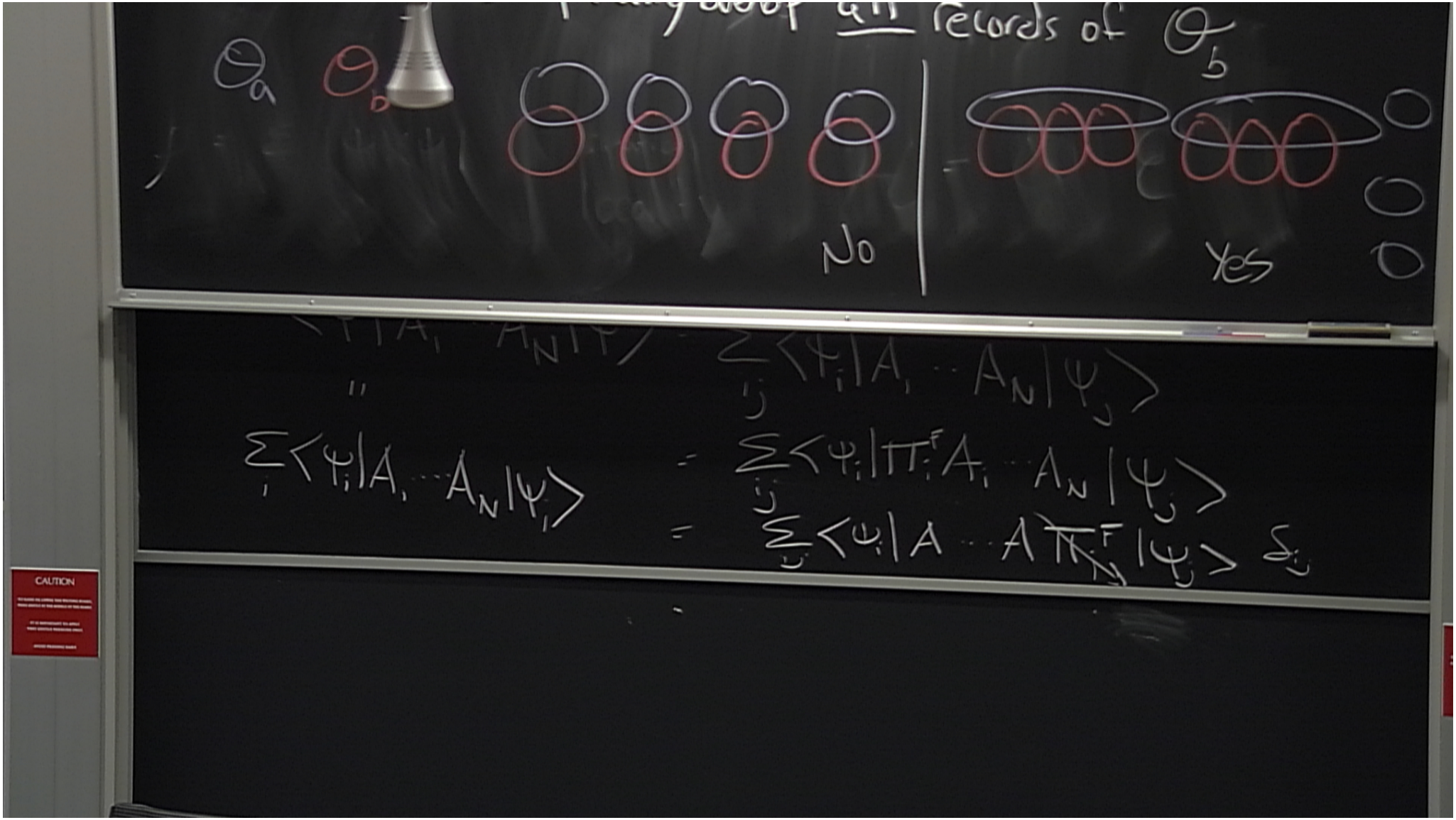
$|\psi_{GHZ}\rangle = |000\rangle + |111\rangle$

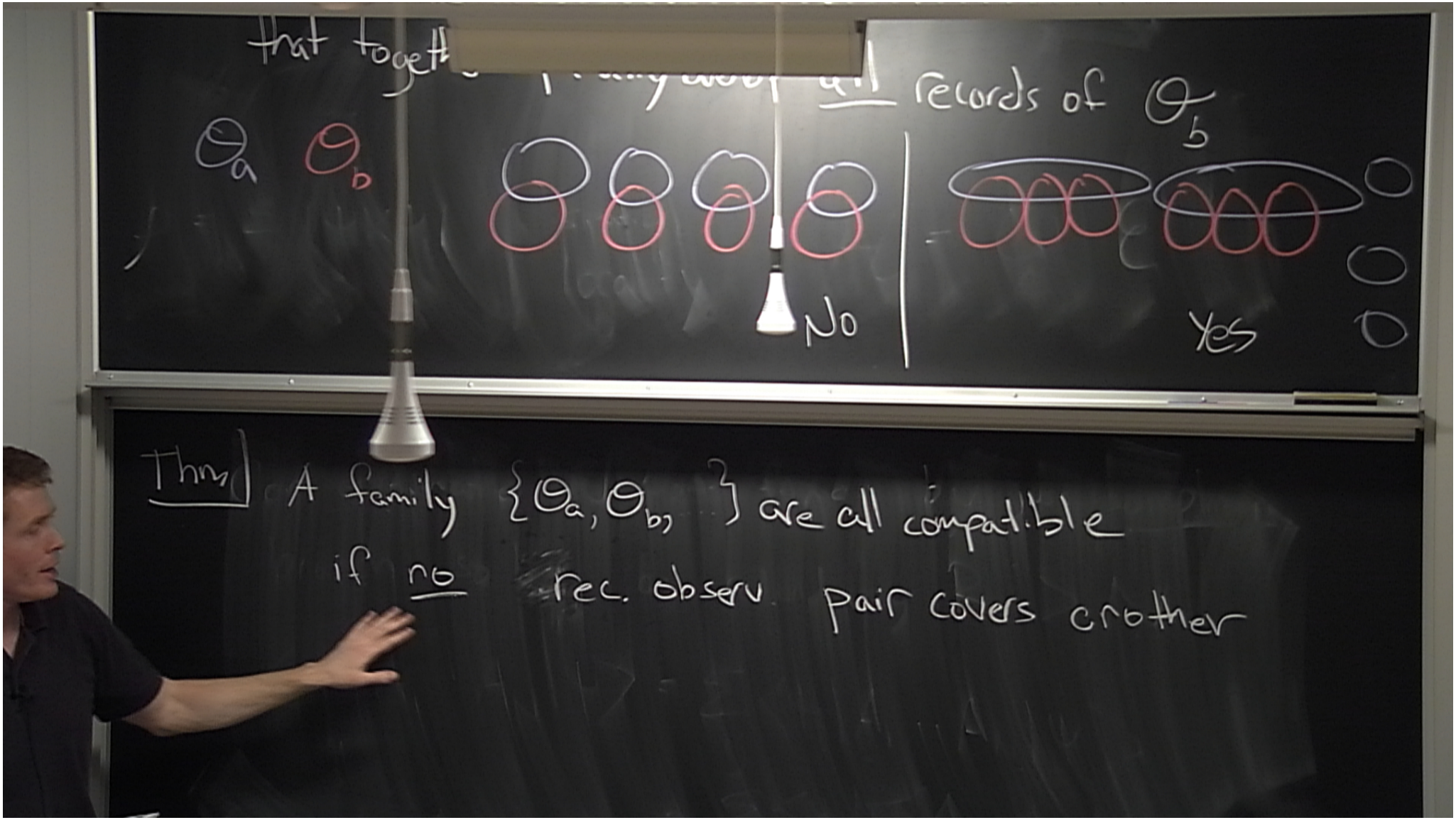


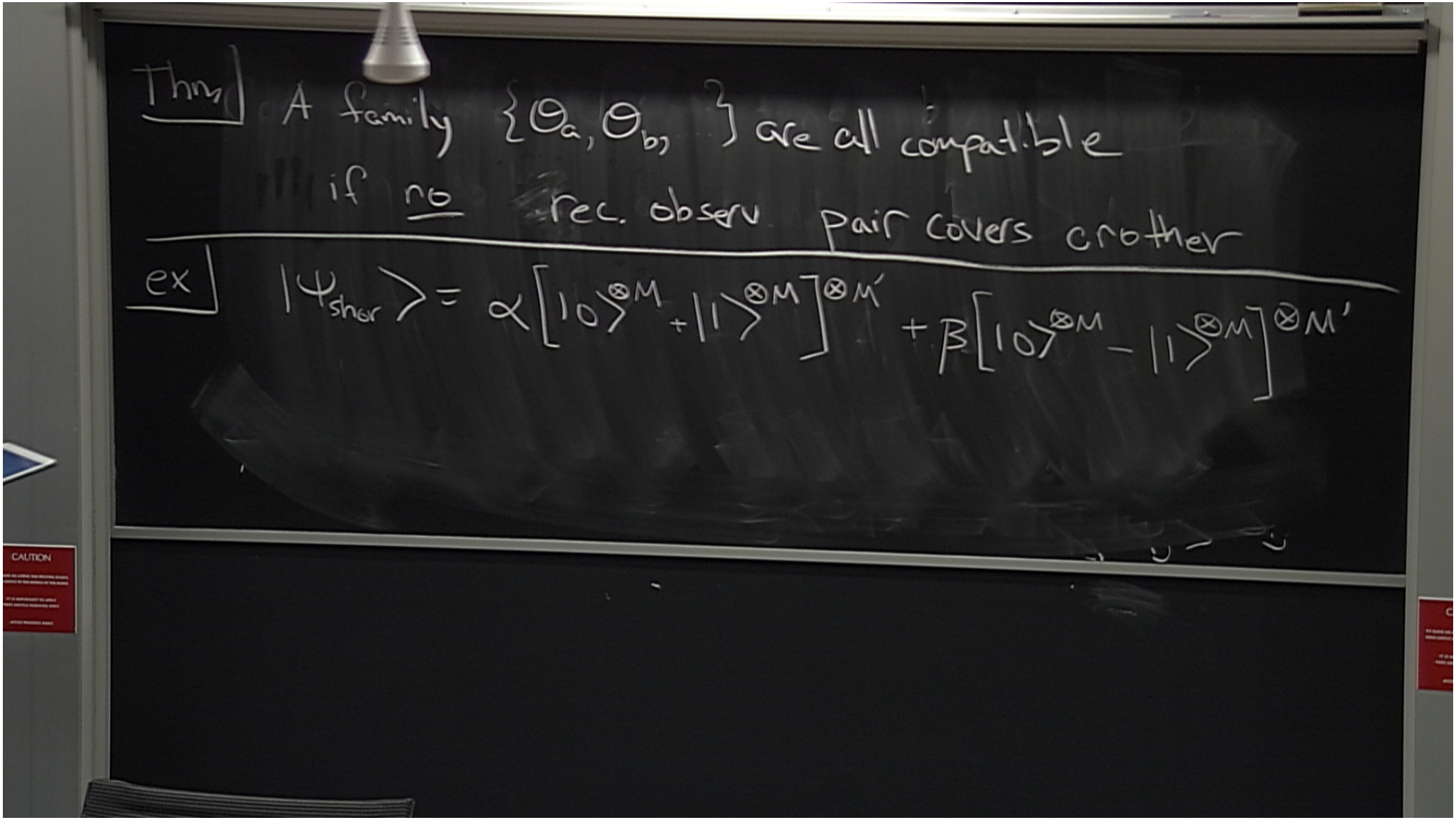


arXiv: 1608.05377


Def
$\Theta_a = \{\Theta_a^F, \Theta_a^{F'}\}$ pair-covers another Θ_b
when there exists a pair of records $(\Theta_a^F, \Theta^{F'})$ for Θ_a
that together spatially overlap all records of Θ_b
CAUTION

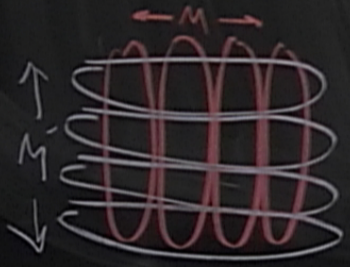

records of
No
Yes
CAUTION

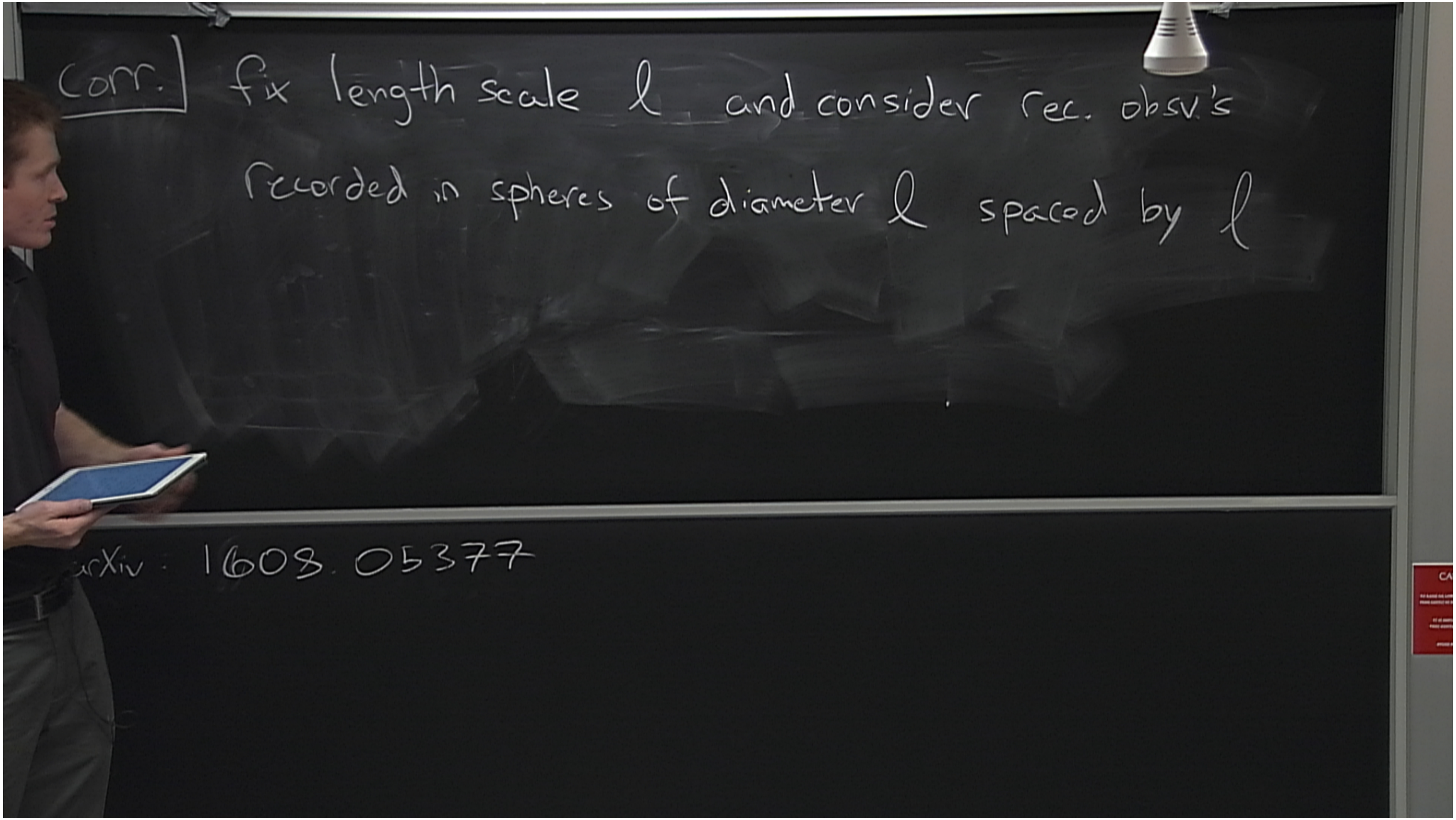

that toget
all records of θ_b
θ_a θ_b
No
Yes
Thm] A family $\{\theta_a, \theta_b, \}$ are all compatible
if no rec. observ. pair covers another

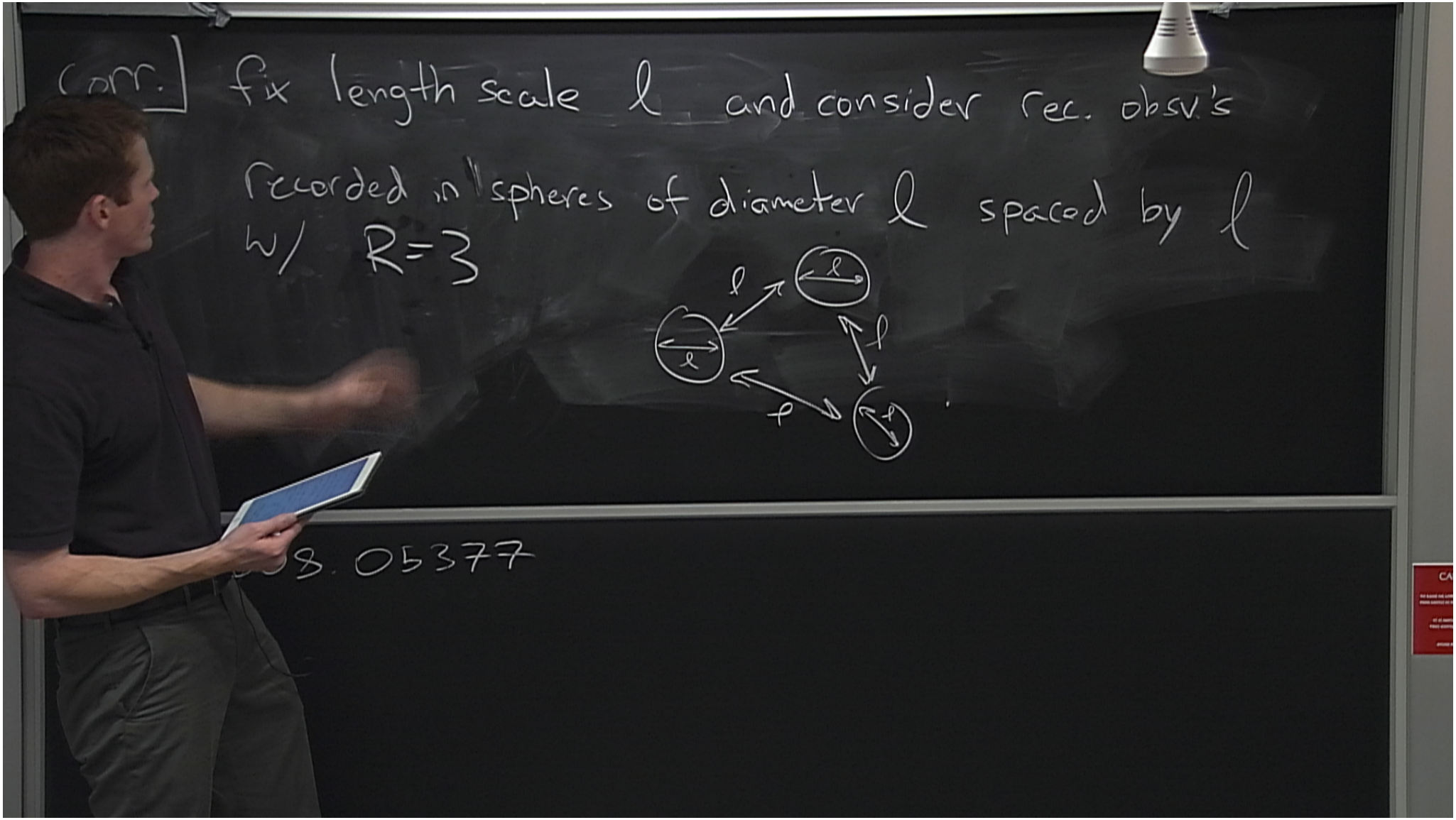

Thm] A family $\{\Theta_a, \Theta_b, \}$ are all compatible
if no rec. observ pair covers another
ex] $|\Psi_{shor}\rangle = \alpha[|0\rangle^{\otimes M} + |1\rangle^{\otimes M}]^{\otimes M'} + \beta[|0\rangle^{\otimes M} - |1\rangle^{\otimes M}]^{\otimes M'}$
CAUTION

Thm A family $\{\Theta_a, \Theta_b, \}$ are all compatible if no rec. observ pair covers another

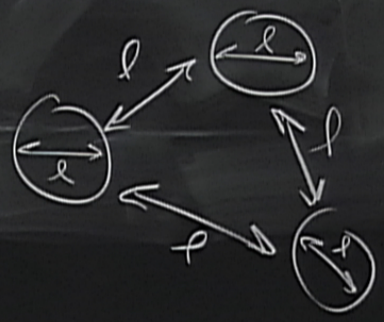
ex $|\Psi_{shor}\rangle = \alpha\left[|0\rangle^{\otimes M} + |1\rangle^{\otimes M}\right]^{\otimes M'} + \beta\left[|0\rangle^{\otimes M} - |1\rangle^{\otimes M}\right]^{\otimes M'}$




Corr. fix length scale ℓ and consider rec. obsv's
recorded in spheres of diameter ℓ spaced by ℓ
arXiv: 1608.05377


[corr.] fix length scale ℓ and consider rec. obsv's
recorded in spheres of diameter ℓ spaced by ℓ
w/ R=3
ℓ
ℓ
ℓ
ℓ
ℓ
ℓ
8.05377

Corr. fix length scale ℓ and consider rec. obsv's recorded in spheres of diameter ℓ spaced by ℓ w/ $R=3$



$\Rightarrow$ compatible.

arXiv: 1608.05377