Title: Transport in Chern-Simons-Matter Theories

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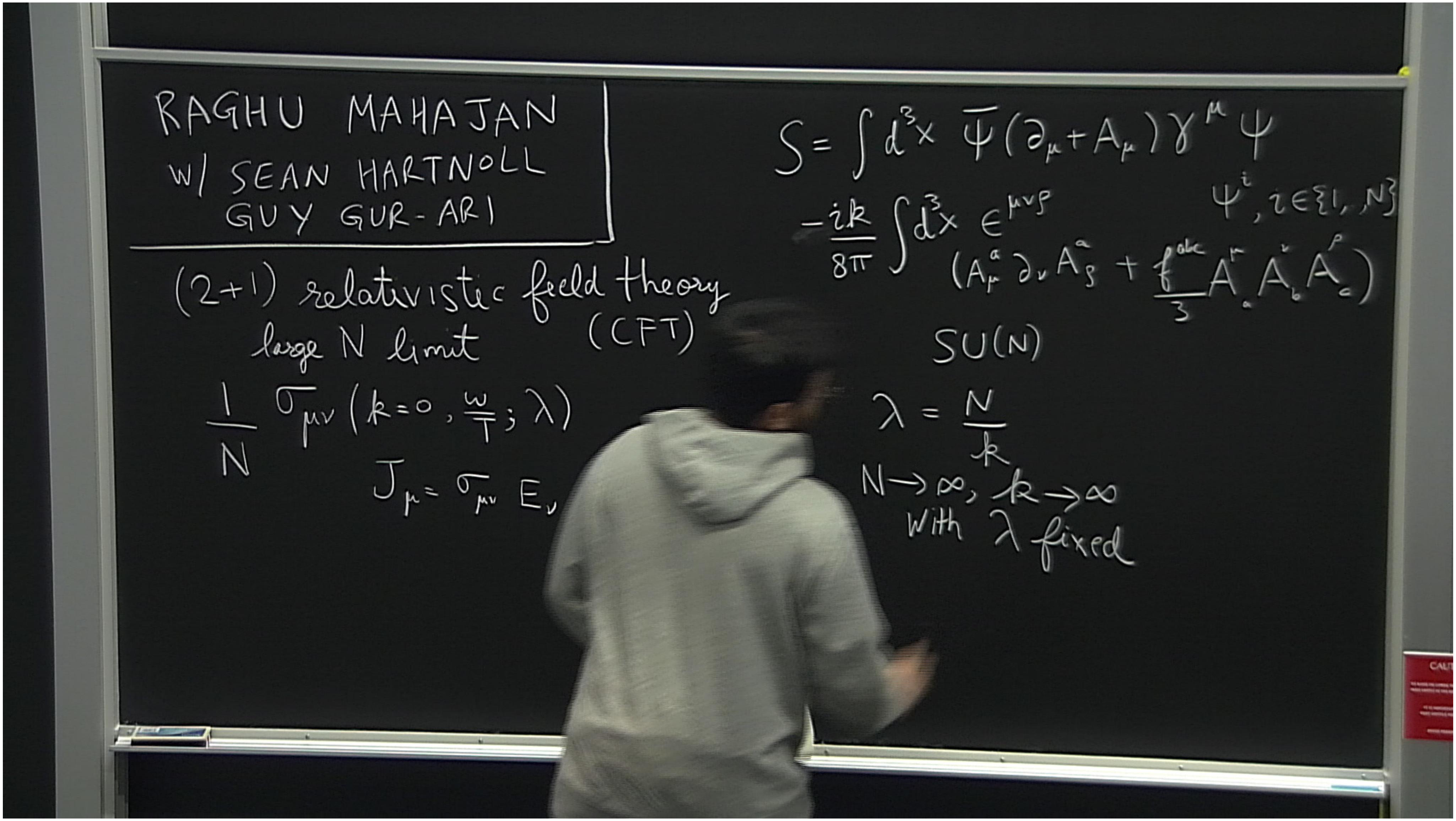
Abstract: <p>The frequency-dependent longitudinal and Hall conductivities â€” Ïƒ_xx and Ïƒ_xy â€” are dimensionless functions of Ï‰/T in 2+1 dimensional CFTs at nonzero temperature. These functions characterize the spectrum of charged excitations of the theory and are basic experimental observables. We compute these conductivities for large N Chern-Simons theory with fermion matter. The computation is exact in the â€™t Hooft coupling Î» at N = âˆž. We describe various physical features of the conductivity, including an explicit relation between the weight of the delta function at Ï‰ = 0 in Ïƒ_xx and the existence of infinitely many higher spin conserved currents in the theory. We also compute the conductivities perturbatively in Chern-Simons theory with scalar matter and show that the resulting functions of Ï‰/T agree with the strong coupling fermionic result. This provides a new test of the conjectured 3d bosonization duality. In matching the Hall conductivities we resolve an outstanding puzzle by carefully treating an extra anomaly that arises in the regularization scheme used. </p>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(2+1) relativistic field theory
large N limit (CFT)
$\frac{1}{N}\sigma_{\mu\nu}(k=0, \frac{\omega}{T}; \lambda)$
$J_\mu = \sigma_{\mu\nu} E_\nu$
$S = \int d^3x\, \bar{\Psi}(\partial_\mu + A_\mu)\gamma^\mu \psi$
$-\frac{ik}{8\pi}\int d^3x\, \epsilon^{\mu\nu\rho} (A^a_\mu \partial_\nu A^a_\rho + \frac{f^{abc}}{3} A^\mu_a A^\nu_b A^\rho_c)$
$\psi^i, i \in \{1, \ldots, N\}$
SU(N)
$\lambda = \frac{N}{k}$
$N \to \infty, k \to \infty$
with λ fixed

$$J^\mu = i\,\bar{\Psi}\gamma^\mu\Psi$$

$$\sigma_{\mu\nu} = \lim_{z\to\omega+i\varepsilon} \frac{G^R_{J_\mu J_\nu}(z) - X_{\mu\nu}}{iz}$$

1) Fermion propagator

2) Exact vertex

3)

$$A_+ := A_1 - iA_2 = 0$$

(Gauge)

$$\frac{1}{N}\sigma_{xx}\left(\frac{\omega}{T}\right) = \frac{D}{T}\delta\left(\frac{\omega}{T}\right) + \cdots$$

$$\frac{D}{\pi} = \chi_{xx} - G^R_{xx}\Big|_{\omega\to 0}$$

$$f(\tau) = \langle J(\tau)J(0)\rangle_\beta$$

$$= \frac{1}{Z}\sum_{n,m} e^{-\beta E_n}|J_{nm}|^2 e^{\tau(E_n-E_m)}$$

$$f(i\omega_k) = \frac{1}{Z}\sum_{n,m}|J_{nm}|^2 \frac{e^{-\beta E_m}-e^{-\beta E_n}}{i\omega_k+E_n-E_m}$$

$$\chi = \frac{1}{Z}\sum_{n,m}|J_{nm}|^2 \times \frac{e^{-\beta E_m}-e^{-\beta E_n}}{E_n-E_m}$$

$$G^R(\omega) = f(\omega+i\varepsilon)$$

$$G^R\Big|_{\omega=0} = \frac{1}{Z}\sum_{\substack{n,m\\ E_n\neq E_m}}|J_{nm}|^2\frac{e^{-\beta E_m}-e^{-\beta E_n}}{E_n-E_m}$$

$$D = \frac{1}{Z}\sum_{\substack{n,m\\E_n=E_m}} |J_{nm}|^2 e^{-\beta E_m}\cdot\beta$$

$$= \beta\cdot\lim_{t_0\to\infty}\frac{1}{t_0}\int_0^{t_0} dt\,\langle J(t)\,J(0)\rangle$$

(By $(\star)$ with $\tau = it$)

$$\frac{DT}{\pi} \geq \sum_{a,b}\langle J_x Q_a\rangle C_{ab}\langle Q_b^\dagger J_x\rangle$$

where $C_{ab} \geq 0$ (positive def. matrix)

and Q_a's are any set of conserved charges

$$D = \frac{1}{Z} \sum_{\substack{n,m \\ E_n = E_m}} |J_{nm}|^2 e^{-\beta E_m} \cdot \beta$$

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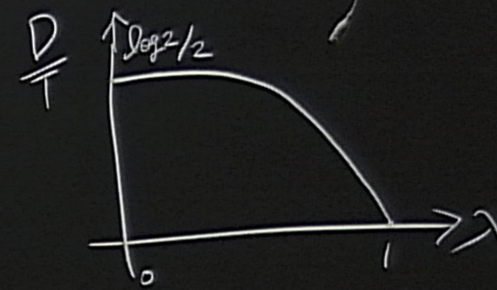
(By $(*)$ with $\tau = it$)



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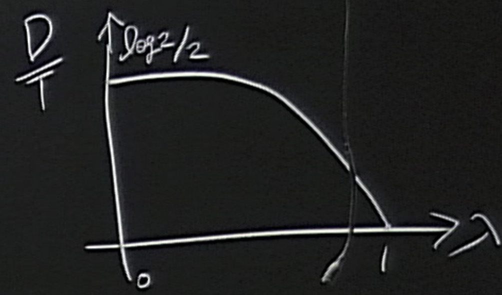


$$= \beta \cdot \lim_{t_0 \to \infty} \frac{1}{t_0} \int_0^{t_0} dt \, \langle J(t) J(0) \rangle$$

(By (*) with $\tau = it$)

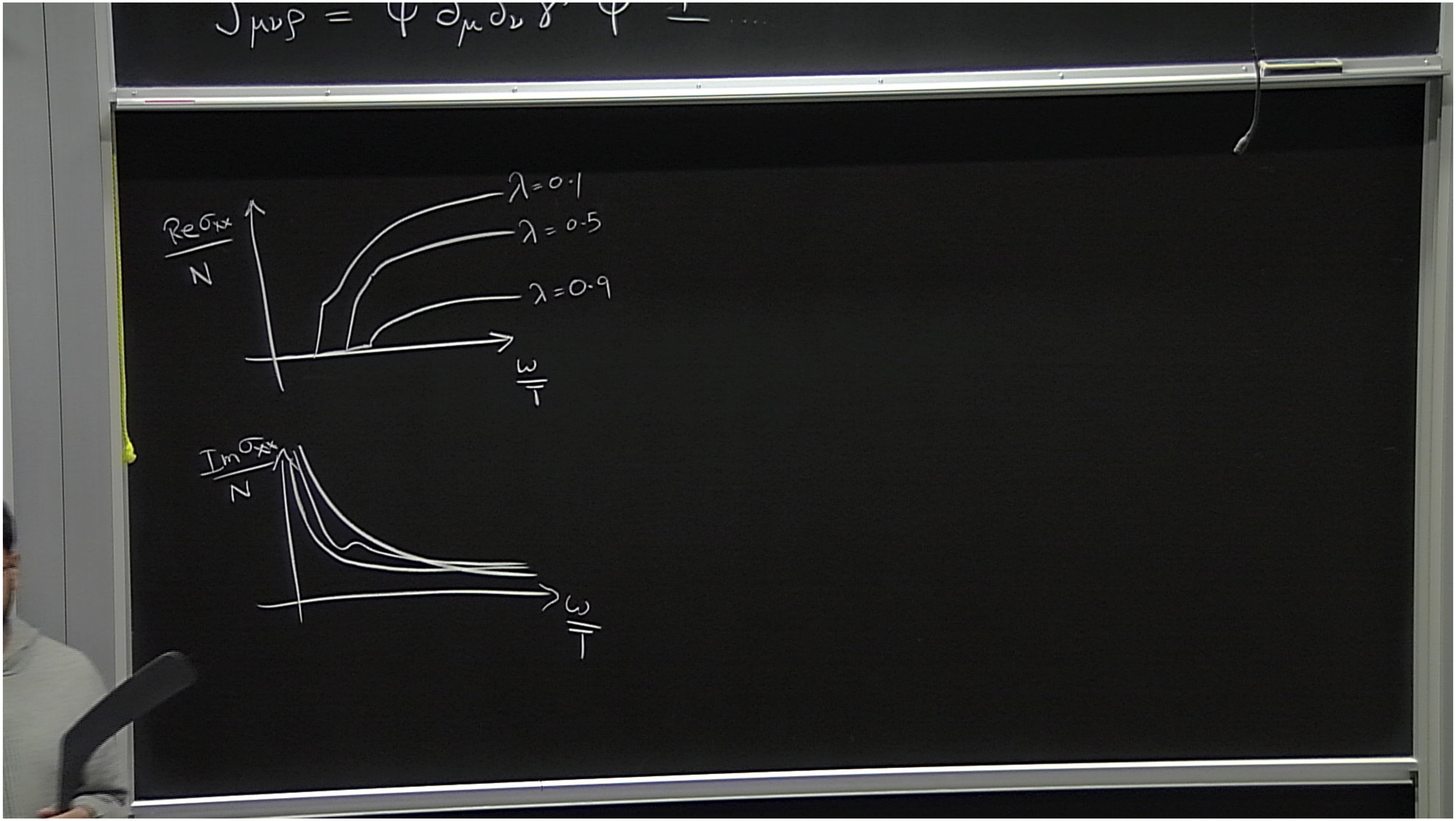
$$J_x = c_a Q_a + \tilde{J}$$

$$\frac{DT}{\pi} \geq \sum_{a,b} \langle J_x Q_a \rangle C_{ab} \langle Q_b^{\dagger} J_x \rangle$$

where $C_{ab} \geq 0$ (positive def. matrix)

and Q_a's are any set of conserved charges

$$J_{\mu\nu\rho} = \bar{\psi} \partial_\mu \partial_\nu \gamma^\rho \psi \pm \ldots$$


Re σxx / N
λ=0.1
λ=0.5
λ=0.9
ω / T
Im σxx / N
ω / T

σ_{xy}

$T=0$

$$J^\mu J^\nu = \tau \cdot \frac{p^\mu p^\nu - \delta^{\mu\nu} p^2}{|p|} + \frac{\kappa}{2\pi} \epsilon^{\mu\nu\rho} p^\rho$$

$\kappa =$ Hall conductivity

$$\kappa_f = \frac{N_f}{4\lambda_f} \sin^2\left(\frac{\pi \lambda_f}{2}\right)$$

$$\kappa_b = \frac{N_b}{4\lambda_b} \sin^2\left(\frac{\pi \lambda_b}{2}\right)$$

$$\kappa_f - \kappa_b = \frac{N_f/\lambda_f}{4} \in \frac{\mathbb{Z}}{4} \; (!)$$

$\lambda_f > 0$

CS + ψ $\equiv$ CS + boson

$$\lambda_f - 1 = \lambda_b$$

$$-\frac{N_f}{\lambda_f} = \frac{N_b}{\lambda_b}$$

σ_{xy}

$T=0$

$$J^\mu J^\nu = \tau \cdot \frac{p^\mu p^\nu - \delta^{\mu\nu} p^2}{|p|} + \frac{\kappa}{2\pi} \epsilon^{\mu\nu\rho} p^\rho$$

$\kappa =$ Hall conductivity

$$\kappa_f = \frac{N_f}{4\lambda_f} \sin^2\left(\frac{\pi \lambda_f}{2}\right) + \frac{N_f}{2} + \frac{N_f \lambda_f}{4}$$

$$\kappa_b = \frac{N_b}{4\lambda_b} \sin^2\left(\frac{\pi \lambda_b}{2}\right) - \frac{N_b \lambda_b}{4}$$

$$\kappa_f - \kappa_b = \frac{N_f/\lambda_f}{4} \in \frac{\mathbb{Z}}{4} \; (!)$$

$\lambda_f > 0$

$CS + \psi \equiv CS +$ boson

$$\lambda_f - 1 = \lambda_b$$

$$-\frac{N_f}{\lambda_f} = \frac{N_b}{\lambda_b}$$

$$W[B^\mu] = \int d\psi \, dA \, \exp(-S[\psi, A] - J_\mu B^\mu)$$

path integral

$$\text{corr } J \sim \frac{\partial}{\partial B} \text{ on } \log W$$

$$\ln W \sim \frac{K}{4\pi} \epsilon^{\mu\nu\rho} B_\mu \partial_\nu B_\rho$$

$K \in$ integer

$$W[B^\mu] = \int_{\text{path integral}} d\psi \, dA \; \exp\left(-S[\psi, A] - J_\mu B^\mu\right)$$

$\frac{\partial}{\partial B}$ on $\log W$

corr $J \sim \frac{\partial}{\partial B}$

$$\ln W \sim \frac{k}{4\pi} \epsilon^{\mu\nu\rho} B_\mu \partial_\nu B_\rho$$

$k \in$ integer

free massive fermion

$$\int \frac{d^3q}{(2\pi)^3} \mathrm{Tr}\left\{ \gamma^\mu \frac{q\cdot\gamma + m}{q^2+m^2} \gamma^\nu \frac{(q-p)\cdot\gamma + m}{(q-p)^2+m^2} \right\}$$

$$= \epsilon^{\mu\nu\rho} \frac{1}{2\pi} \frac{m}{|p|} \arcsin \frac{|p|}{\sqrt{4m^2+p^2}}$$

$$\begin{cases} UV \\ IR \qquad \kappa = 1/2 \end{cases}$$

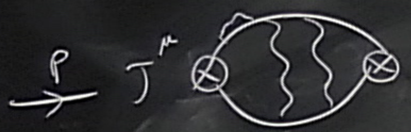
$$\chi = \frac{1}{Z} \sum_{n,m} |J_{nm}|^2 \times \frac{e^{-\beta E_m} - e^{-\beta E_n}}{E_n - E_m}$$

$$G^R(\omega) = f(\omega + i\varepsilon)$$

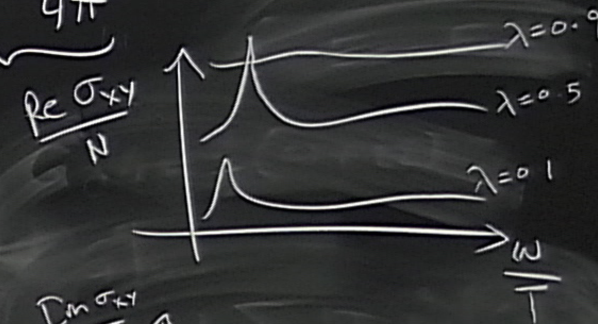
$$G^R\big|_{\omega=0} = \frac{1}{Z} \sum_{\substack{n,m \\ E_n \neq E_m}} |J_{nm}|^2 \frac{e^{-\beta E_m} - e^{-\beta E_n}}{E_n - E_m}$$

free massive fermion

$$\mathcal{L} = \bar{\Psi}\,\partial_\mu \gamma^\mu \psi + \underbrace{B\wedge dB\,\frac{(-1/2)}{4\pi}} + J\cdot B$$

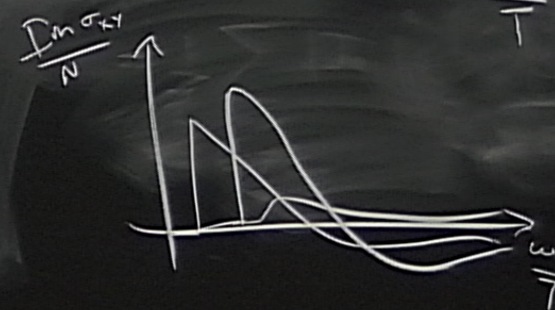


$[\Psi,$



$$\int \frac{d^3q}{(2\pi)^3}\,\mathrm{Tr}\left\{\gamma^\mu\,\frac{q\cdot\gamma+m}{q^2+m^2}\,\gamma^\nu\,\frac{(q-p)\cdot\gamma+m}{(q-p)^2+m^2}\right\}$$

$$= \epsilon^{\mu\nu\rho}\,\frac{1}{2\pi}\,\frac{m}{|p|}\,\arcsin\frac{|p|}{\sqrt{4m^2+p^2}}$$



$$\begin{cases} \text{UV} & k = 0 - 1/2 = -1/2 \\ \text{IR} & k = 1/2 - 1/2 = 0 \end{cases}$$