Title: PSI 2016/2017 Condensed Matter - Lecture 15

Date: Nov 25, 2016 10:45 AM

URL: http://pirsa.org/16110066

Abstract:

$$|G\rangle = \prod_k \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

minimize $\langle \hat{H} - \mu \hat{N} \rangle \Rightarrow |u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k - \mu}{E_k}\right)$

$$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k - \mu}{E_k}\right)$$

and $u_k v_k = \frac{\Delta_k}{2E_k}$ ← introduced through

$$\Delta_k \equiv -\frac{1}{N}\sum_{k'} V_{kk'} u_{k'} v_{k'}$$

$$\left.\begin{aligned} \Delta_k &= -\frac{1}{N}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \\ n &= \frac{1}{N}\sum_{k'}\left(1 - \frac{\xi_{k'} - \mu}{E_{k'}}\right) \\ E_k &\equiv \sqrt{(\xi_k - \mu)^2 + \Delta_k^2} \end{aligned}\right\} T = 0$$

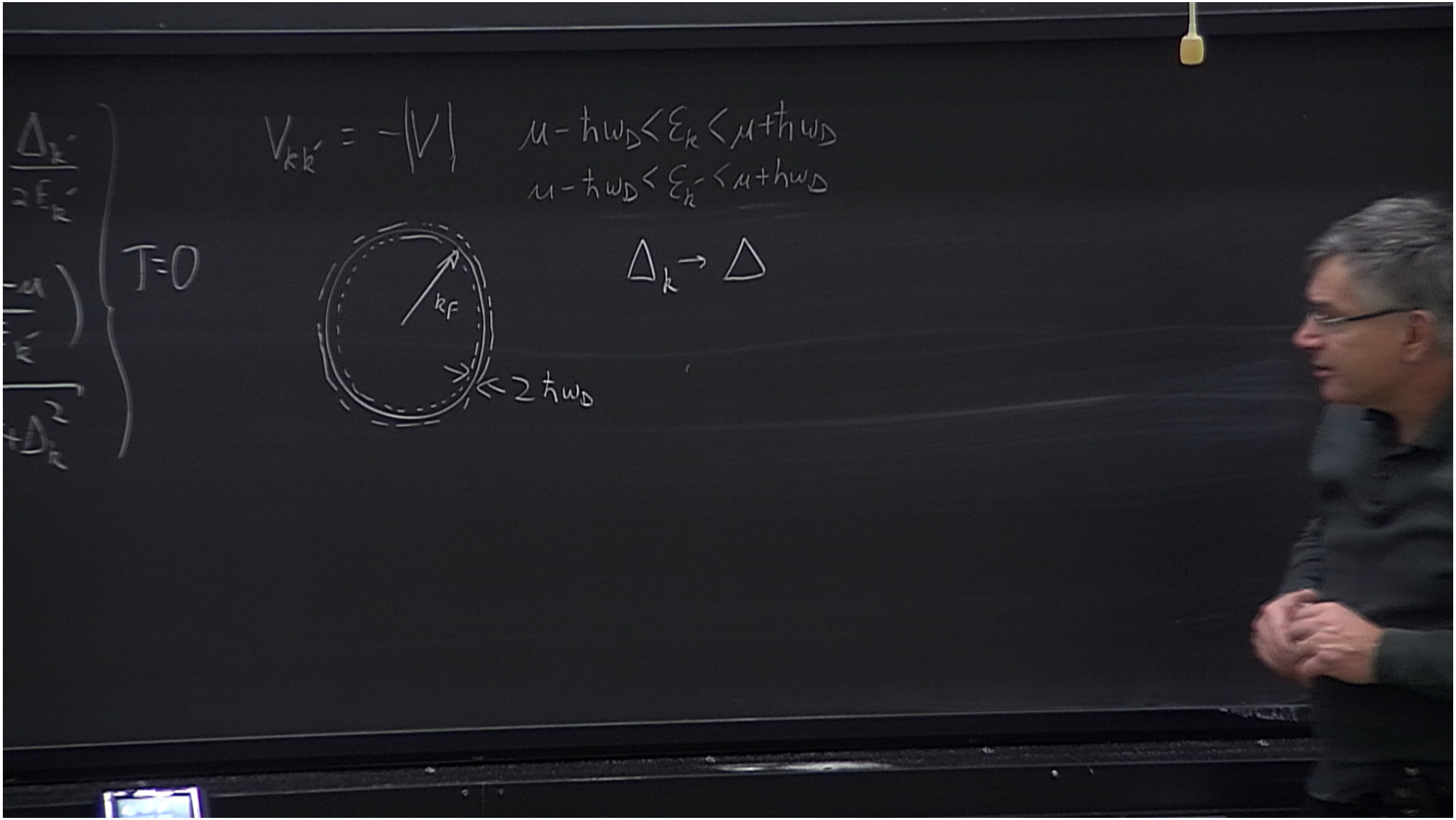
$$V_{kk'} = -|V| \qquad \mu - \hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$$
$$\mu - \hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$$

$\frac{\Delta_{k'}}{2E_{k'}}$

$T=0$



$\sqrt{\quad + \Delta_k^2}$


$V_{kk'} = -|V|$
$\mu - \hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$
$\mu - \hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$
$\Delta_k \to \Delta$
T=0
k_F
$2\hbar\omega_D$

$$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'}\frac{\Delta_{k'}}{2E_{k'}}$$

$$n = \frac{1}{N}\sum_{k'}\left(1-\frac{\xi_{k'}-\mu}{E_{k'}}\right)$$

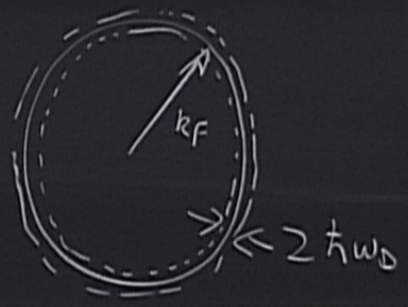
$$E_k = \sqrt{(\xi_k-\mu)^2+\Delta_k^2}$$

$T=0$

$$V_{kk'} = -|V|$$

$$\mu-\hbar\omega_D < \xi_k < \mu+\hbar\omega_D$$
$$\mu-\hbar\omega_D < \xi_{k'} < \mu+\hbar\omega_D$$

$$\Delta_k \to \Delta$$



$$1 = |V|\frac{1}{N}\sum_{k'}\frac{1}{2\sqrt{(\xi_{k'}-\mu)^2+\Delta^2}}$$

$V_{kk'} = -|V|$

$\mu - \hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$

$\mu - \hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$



$\Delta_k \to \Delta$

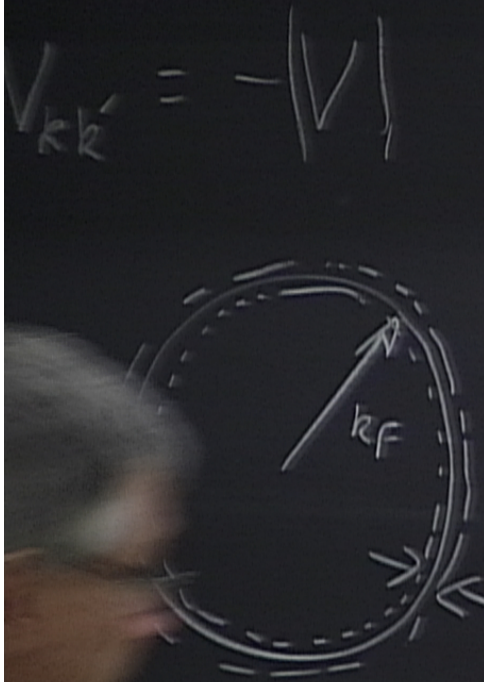
$$\frac{1}{N} = |V| \frac{1}{N} \sum_{k'} \frac{1}{2\sqrt{(\varepsilon_{k'} - \mu)^2 + \Delta^2}}$$

$$= |V| \int_{+\mu - \hbar\omega_D}^{\mu + \hbar\omega_D} d\xi \, \frac{g(\varepsilon)}{2\sqrt{(\varepsilon - \mu)^2 + \Delta^2}}$$

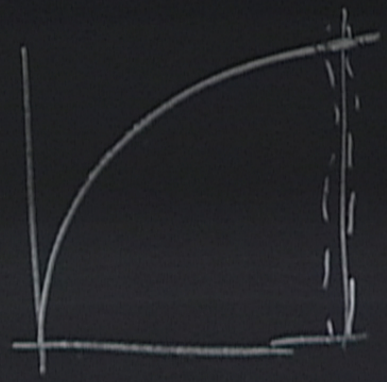
$V_{kk'} = -|V|$

$\mu - \hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$

$\mu - \hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$



$\Delta_k \to \Delta$



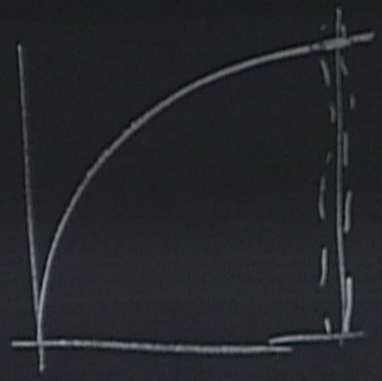
$$\frac{1}{N} = |V| \frac{1}{N} \sum_{k'}' \frac{1}{2\sqrt{(\varepsilon_{k'} - \mu)^2 + \Delta^2}}$$

$$= |V| \int_{+\mu - \hbar\omega_D}^{\mu + \hbar\omega_D} d\xi \, \frac{g(\varepsilon)}{2\sqrt{(\varepsilon - \mu)^2 + \Delta^2}}$$

$\hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$

$\hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$

$\Delta_k \to \Delta$



$g(\varepsilon) \approx g(\varepsilon_F)$

$$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \frac{1}{\sqrt{\varepsilon^2 + \Delta^2}}$$

$$= \ln \frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + \Delta^2}}{\Delta}$$

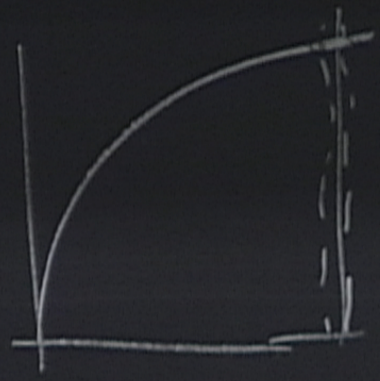
$$\frac{1}{|V|} = \frac{1}{N} \sum_{k}' \frac{1}{2\sqrt{(\varepsilon_{k'} - \mu)^2 + \Delta^2}}$$

$$= |V| \int_{+\mu - \hbar\omega_D}^{\mu + \hbar\omega_D} d\varepsilon \frac{g(\varepsilon)}{2\sqrt{(\varepsilon - \mu)^2 + \Delta^2}}$$

$\hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$

$\hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$

$\Delta_k \to \Delta$



$g(\varepsilon) \approx g(\varepsilon_F)$

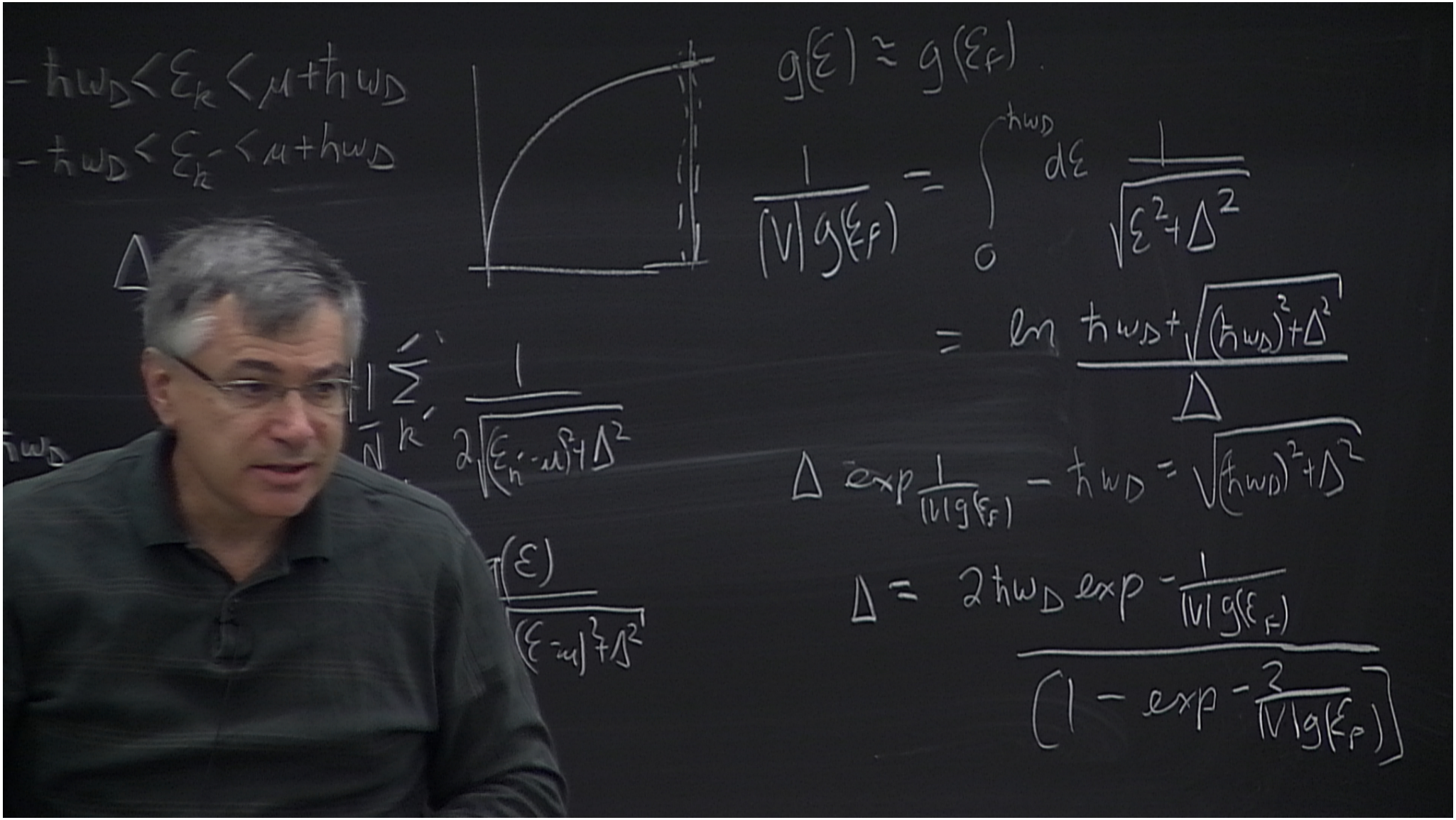
$$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \frac{1}{\sqrt{\varepsilon^2 + \Delta^2}}$$

$$= \ln \frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + \Delta^2}}{\Delta}$$

$$\Delta \exp \frac{1}{|V| g(\varepsilon_F)} - \hbar\omega_D = \sqrt{(\hbar\omega_D)^2 + \Delta^2}$$

$$\frac{1}{|V|} = \frac{1}{N} \sum_{k}' \frac{1}{2\sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}}$$

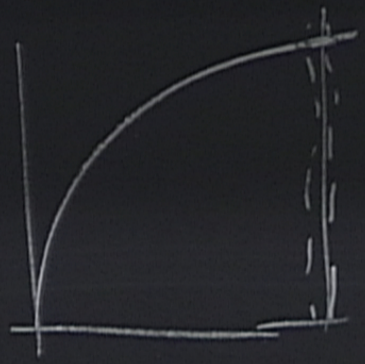
$$= \int_{\mu - \hbar\omega_D}^{\mu + \hbar\omega_D} d\varepsilon \frac{g(\varepsilon)}{2\sqrt{(\varepsilon - \mu)^2 + \Delta^2}}$$


$-\hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$
$-\hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$
$g(\varepsilon) \simeq g(\varepsilon_F)$
$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \frac{1}{\sqrt{\varepsilon^2 + \Delta^2}}$
$= \ln \frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + \Delta^2}}{\Delta}$
$\Delta \exp \frac{1}{|V| g(\varepsilon_F)} - \hbar\omega_D = \sqrt{(\hbar\omega_D)^2 + \Delta^2}$
$\Delta = 2\hbar\omega_D \exp -\frac{1}{|V| g(\varepsilon_F)}$
$\left[1 - \exp -\frac{2}{|V| g(\varepsilon_F)}\right]$

$-\hbar\omega_D < \varepsilon_k < \mu + \hbar\omega_D$

$-\hbar\omega_D < \varepsilon_{k'} < \mu + \hbar\omega_D$

$\Delta_k \to \Delta$



$g(\varepsilon) \simeq g(\varepsilon_F)$

$$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \frac{1}{\sqrt{\varepsilon^2 + \Delta^2}}$$

$$= \ln \frac{\hbar\omega_D + \sqrt{(\hbar\omega_D)^2 + \Delta^2}}{\Delta}$$

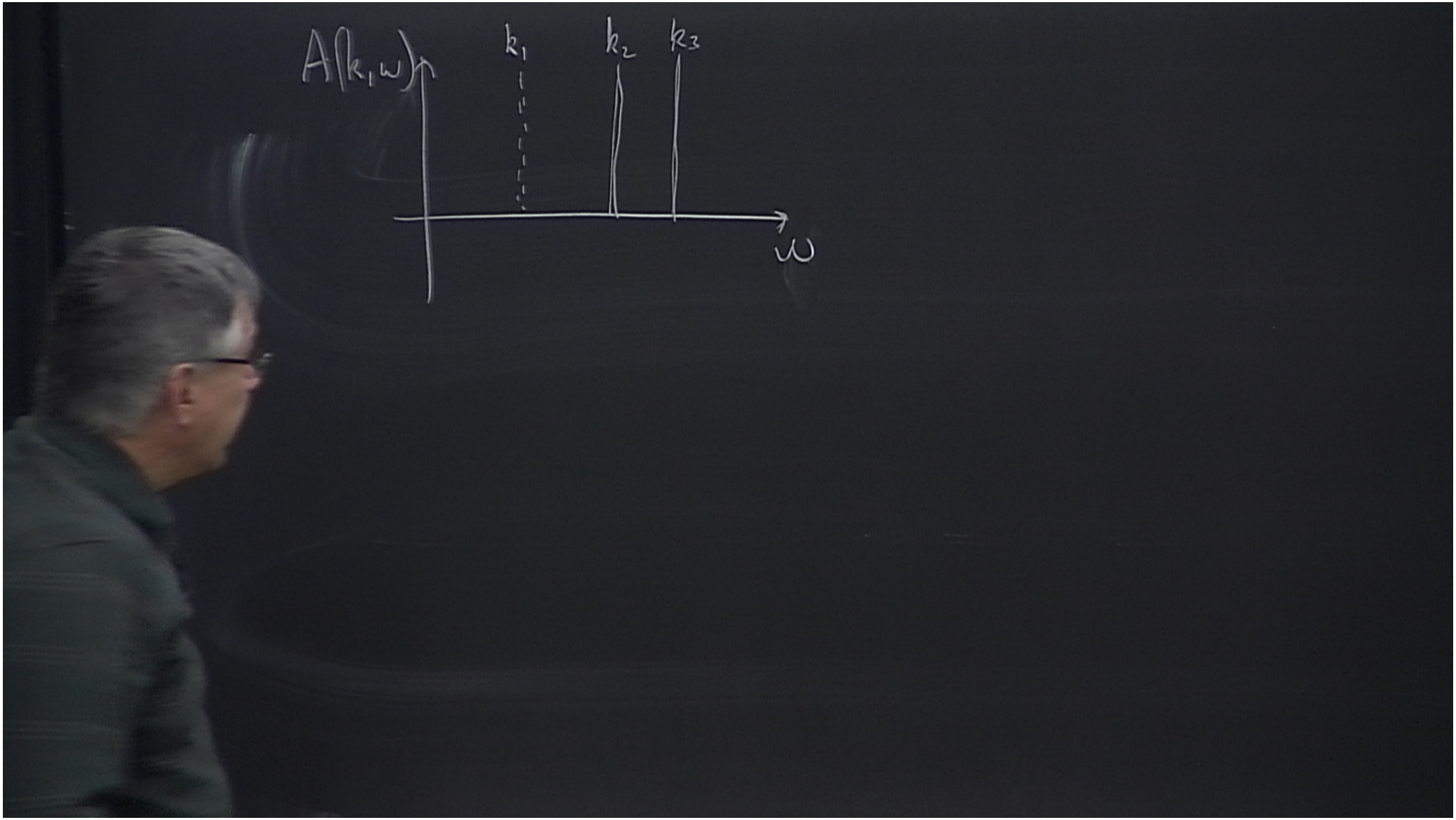
$$\Delta \exp \frac{1}{|V| g(\varepsilon_F)} - \hbar\omega_D = \sqrt{(\hbar\omega_D)^2 + \Delta^2}$$

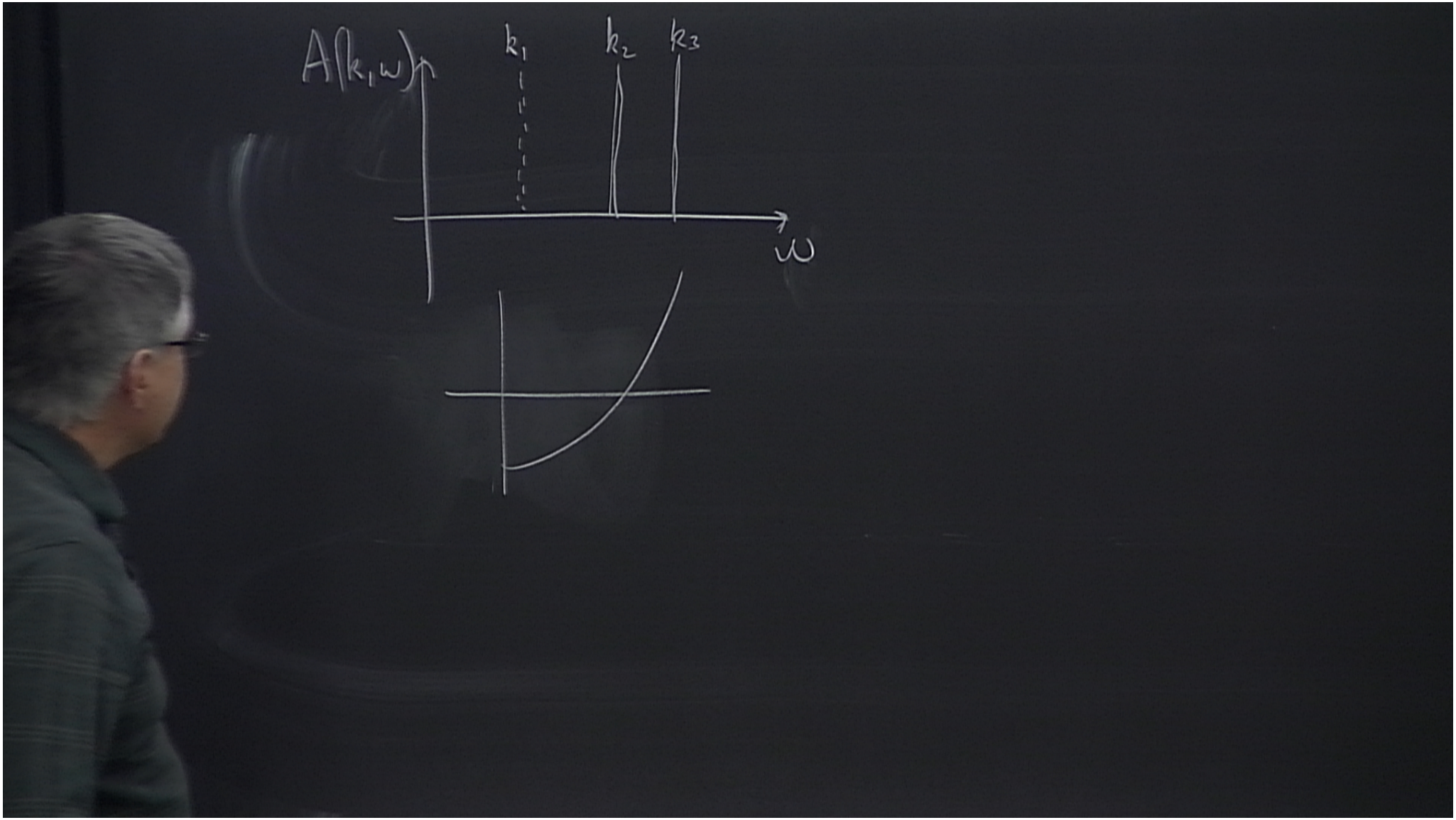
$$\Delta = 2\hbar\omega_D \exp -\frac{1}{|V| g(\varepsilon_F)} \Big/ \left[1 - \exp -\frac{2}{|V| g(\varepsilon_F)}\right]$$

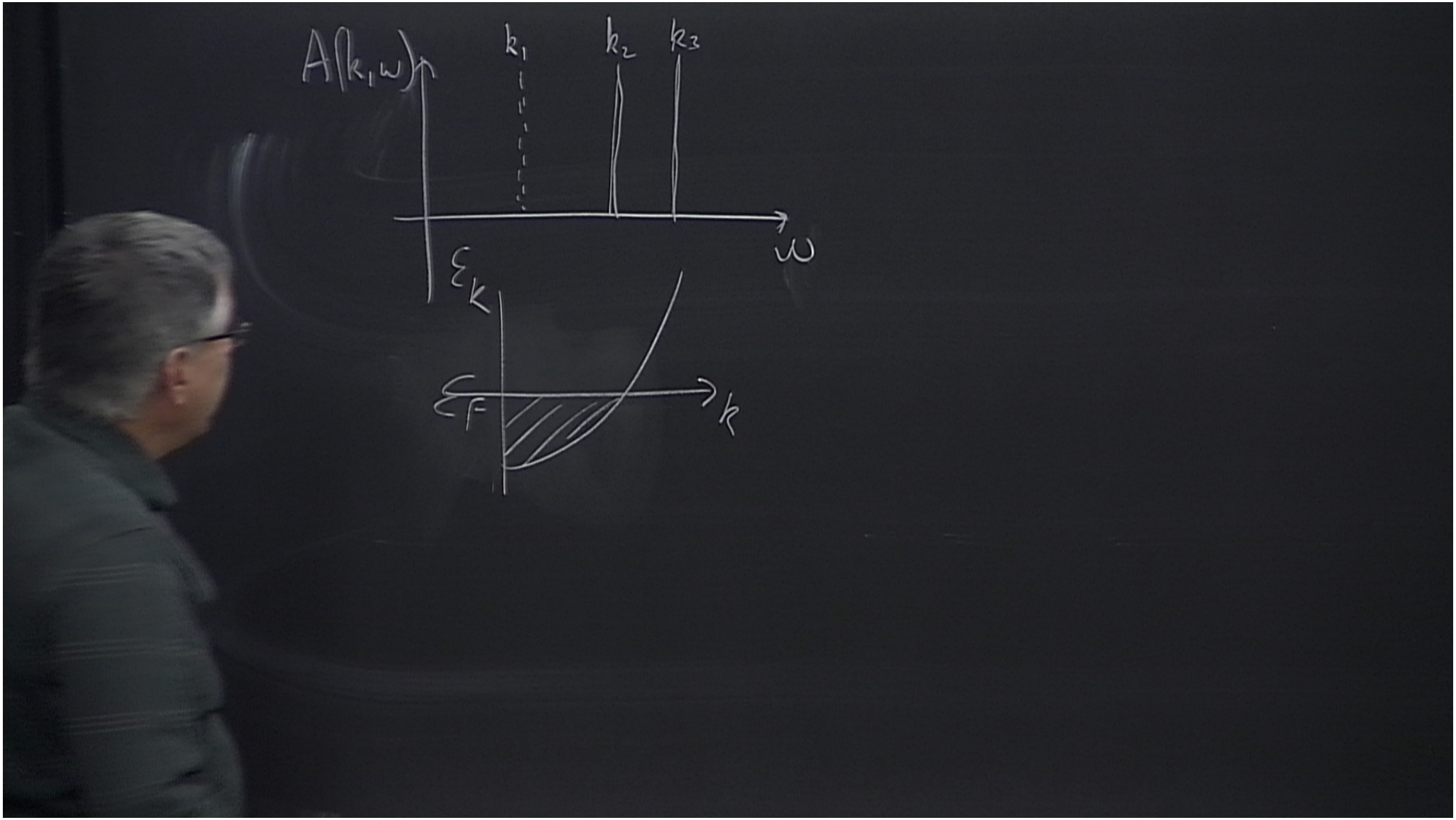
$\hbar\omega_D$

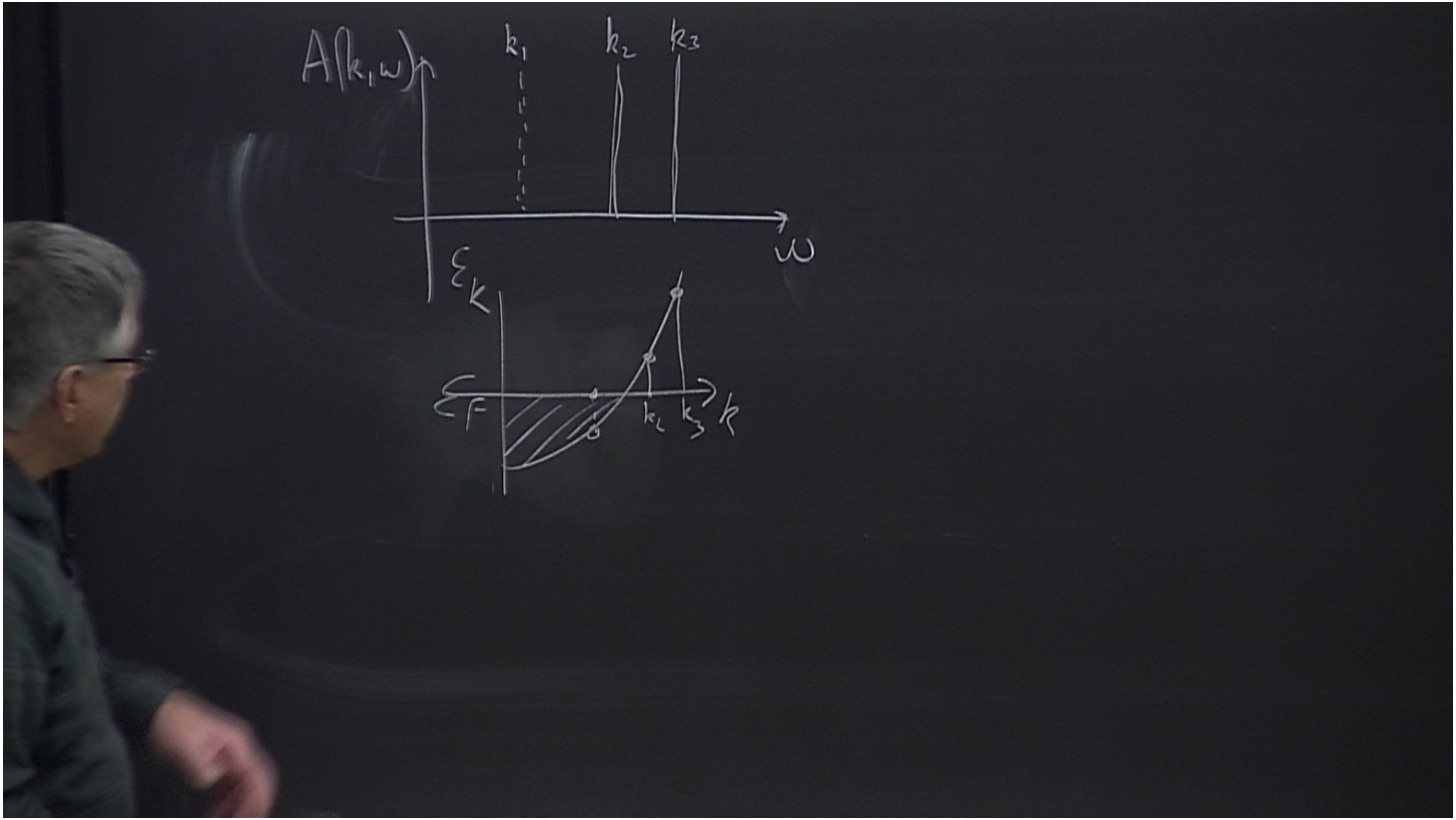
$$1 = |V| \frac{1}{N} \sum_{k'} \frac{1}{2\sqrt{(\varepsilon_{k'} - \mu)^2 + \Delta^2}}$$

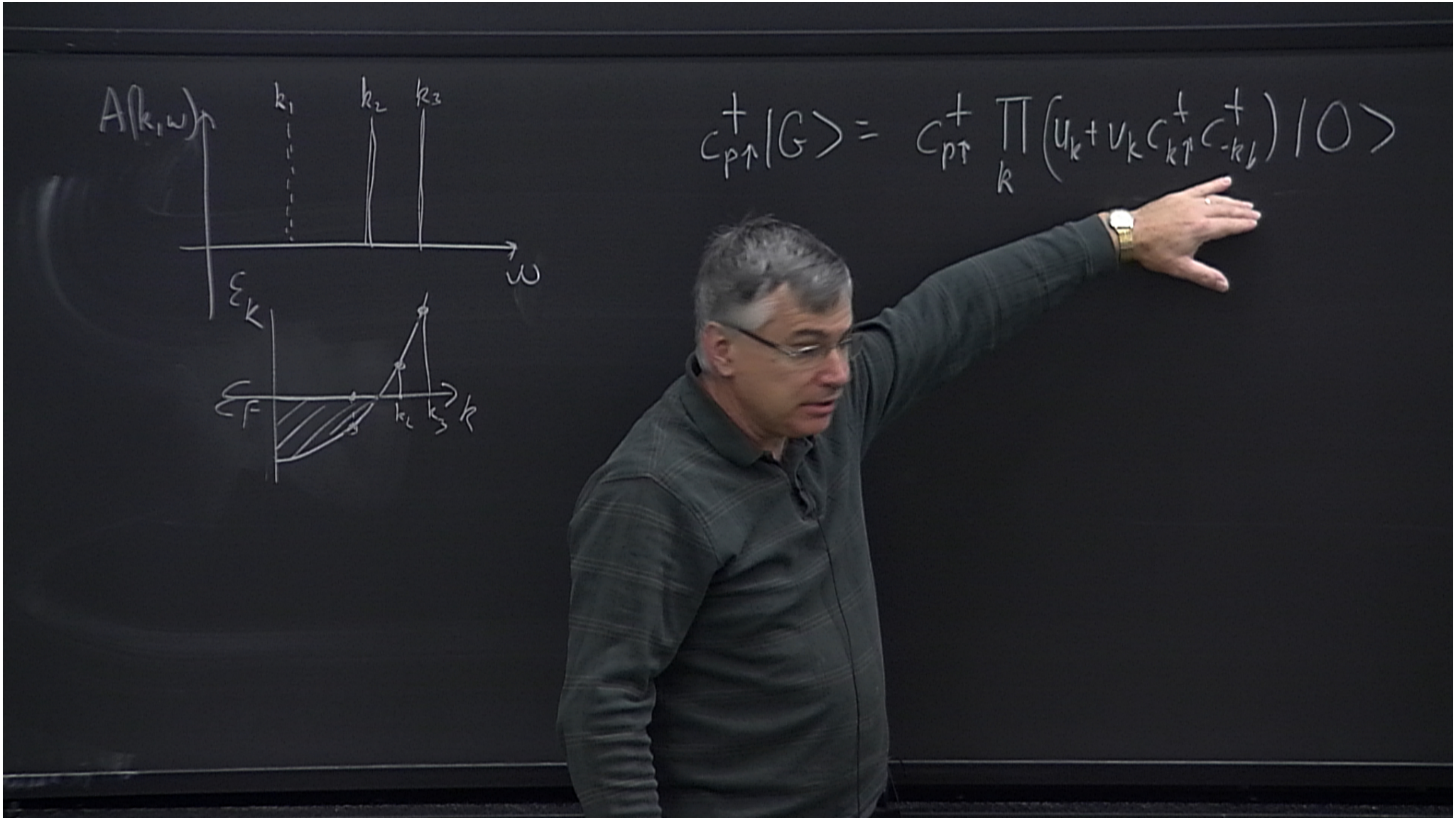
$$= |V| \int_{\mu - \hbar\omega_D}^{\mu + \hbar\omega_D} d\varepsilon \frac{g(\varepsilon)}{2\sqrt{(\varepsilon - \mu)^2 + \Delta^2}}$$

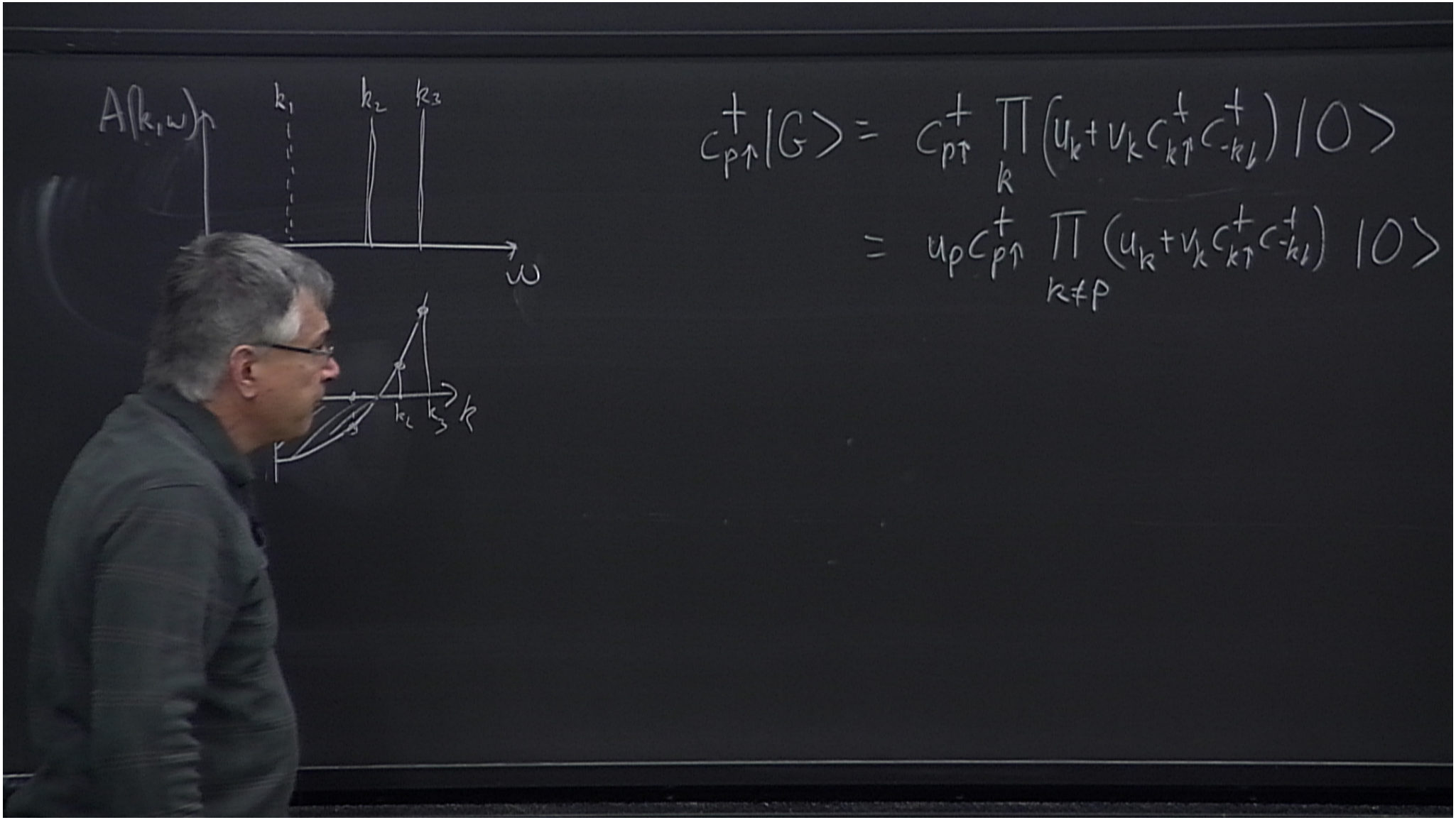

A(k,ω)
k1
k2
k3
ω


A(k,ω)
k1
k2
k3
ω


A(k,ω)
k1
k2
k3
ω
εk
εF
k

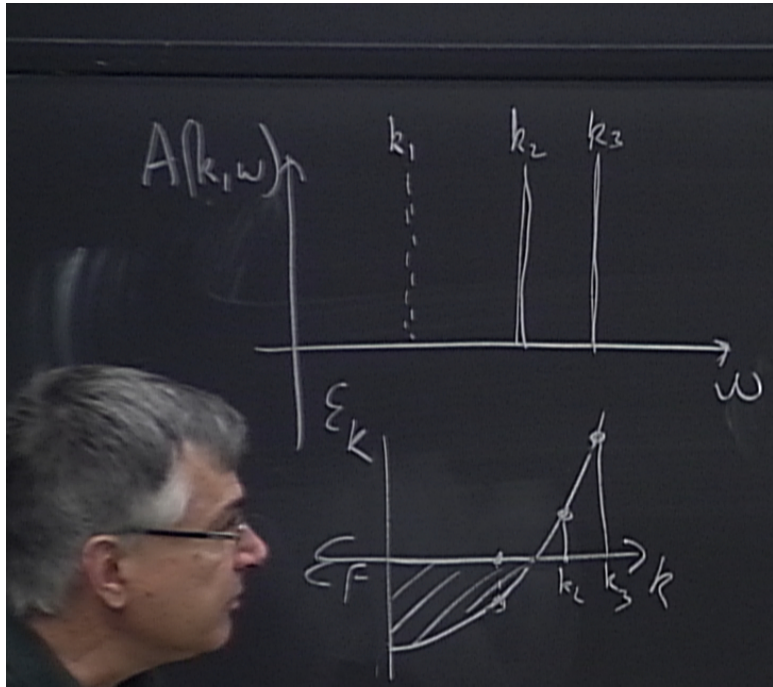

A(k,ω)
k1
k2
k3
ω
εk
εF
kL
k3
k


A(k,ω)
k1
k2
k3
ω
εk
εF
kF
k1
k2
k3
k
c†pσ|G⟩ = c†pσ ∏k (uk + vk c†kσ c†−kσ)|0⟩



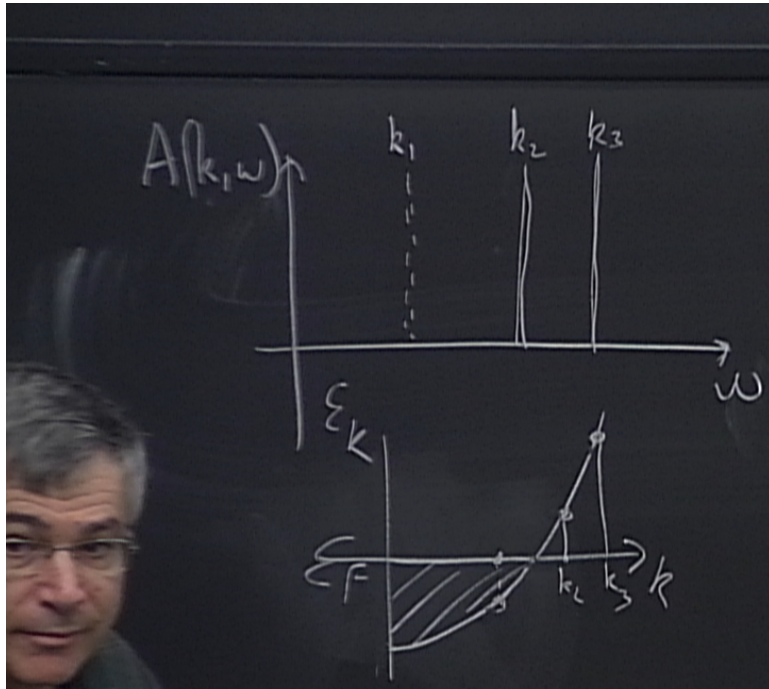
$$c^{+}_{p\uparrow}|G\rangle = c^{+}_{p\uparrow}\prod_{k}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$

$$= u_p c^{+}_{p\uparrow}\prod_{k\neq p}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$



$$c^{+}_{p\uparrow}|G\rangle = c^{+}_{p\uparrow}\prod_{k}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$

$$= u_p c^{+}_{p\uparrow} \underbrace{\prod_{k\neq p}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)}_{|4_p\rangle}|0\rangle$$

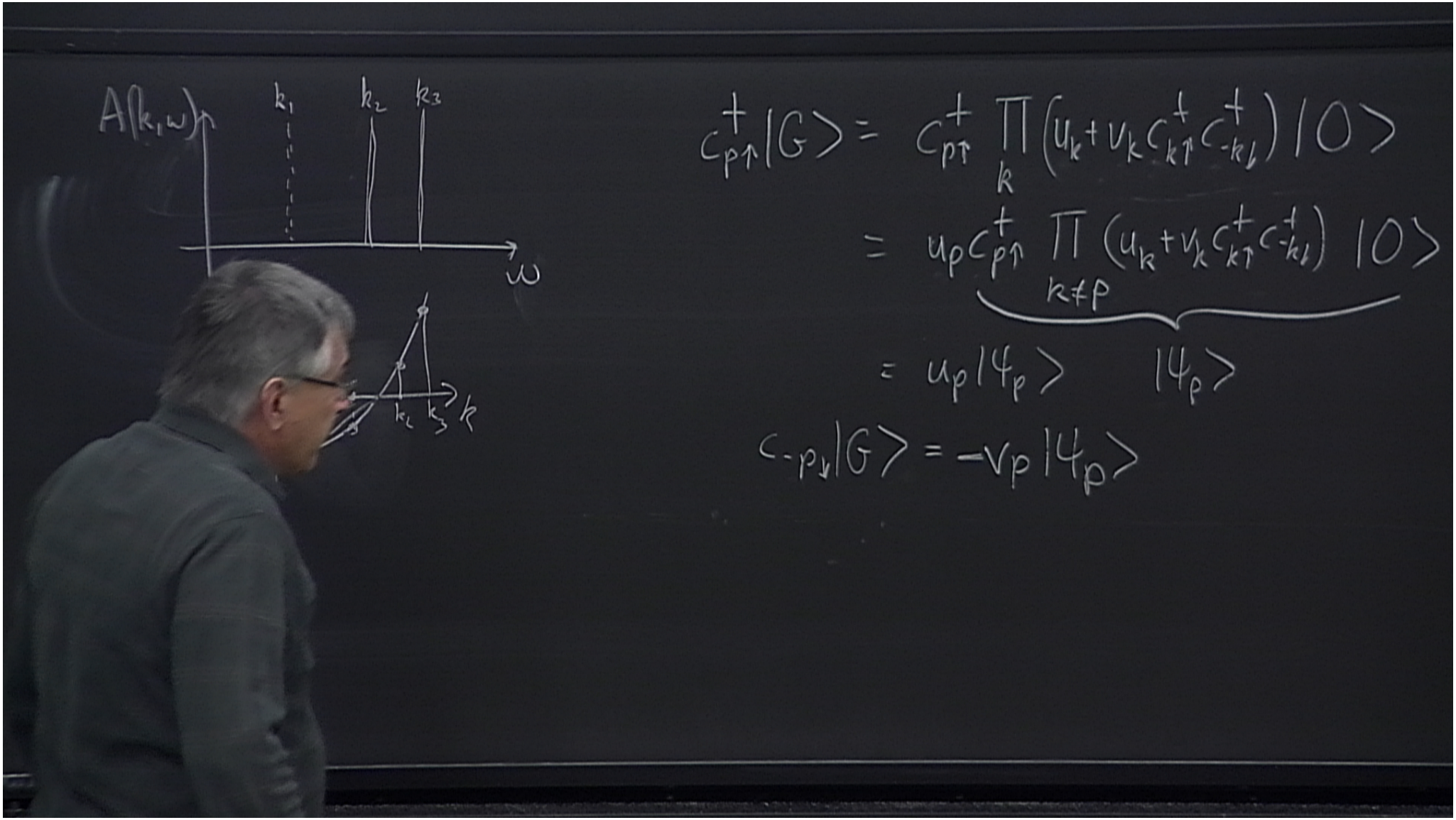


$$c^{+}_{p\uparrow}|G\rangle = c^{+}_{p\uparrow}\prod_{k}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$

$$= u_p c^{+}_{p\uparrow}\underbrace{\prod_{k\neq p}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)}_{}|0\rangle$$

$$= u_p|1_p\rangle \qquad |1_p\rangle$$

$$c_{-p\downarrow}|G\rangle = v_p|1_p\rangle$$

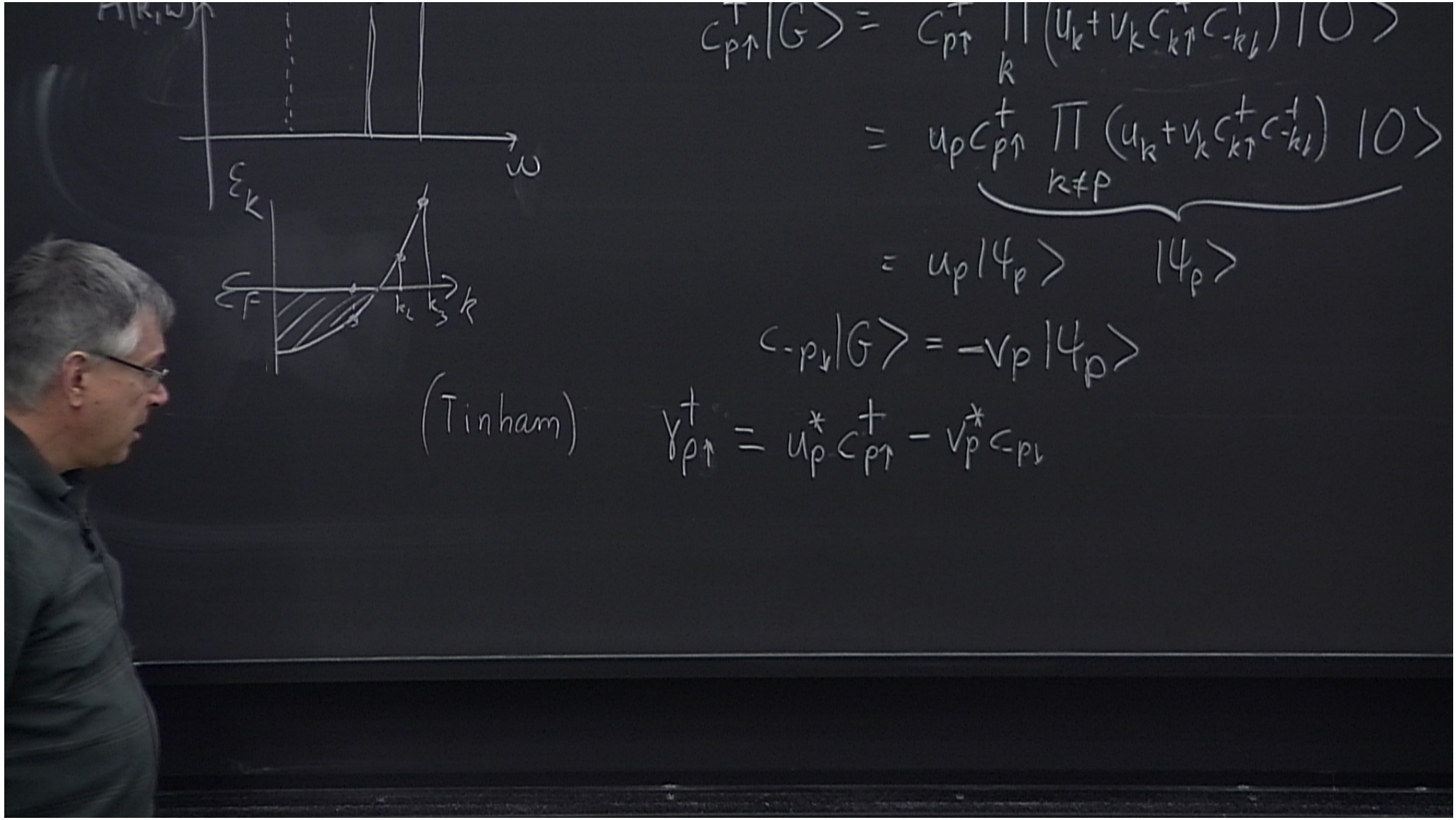


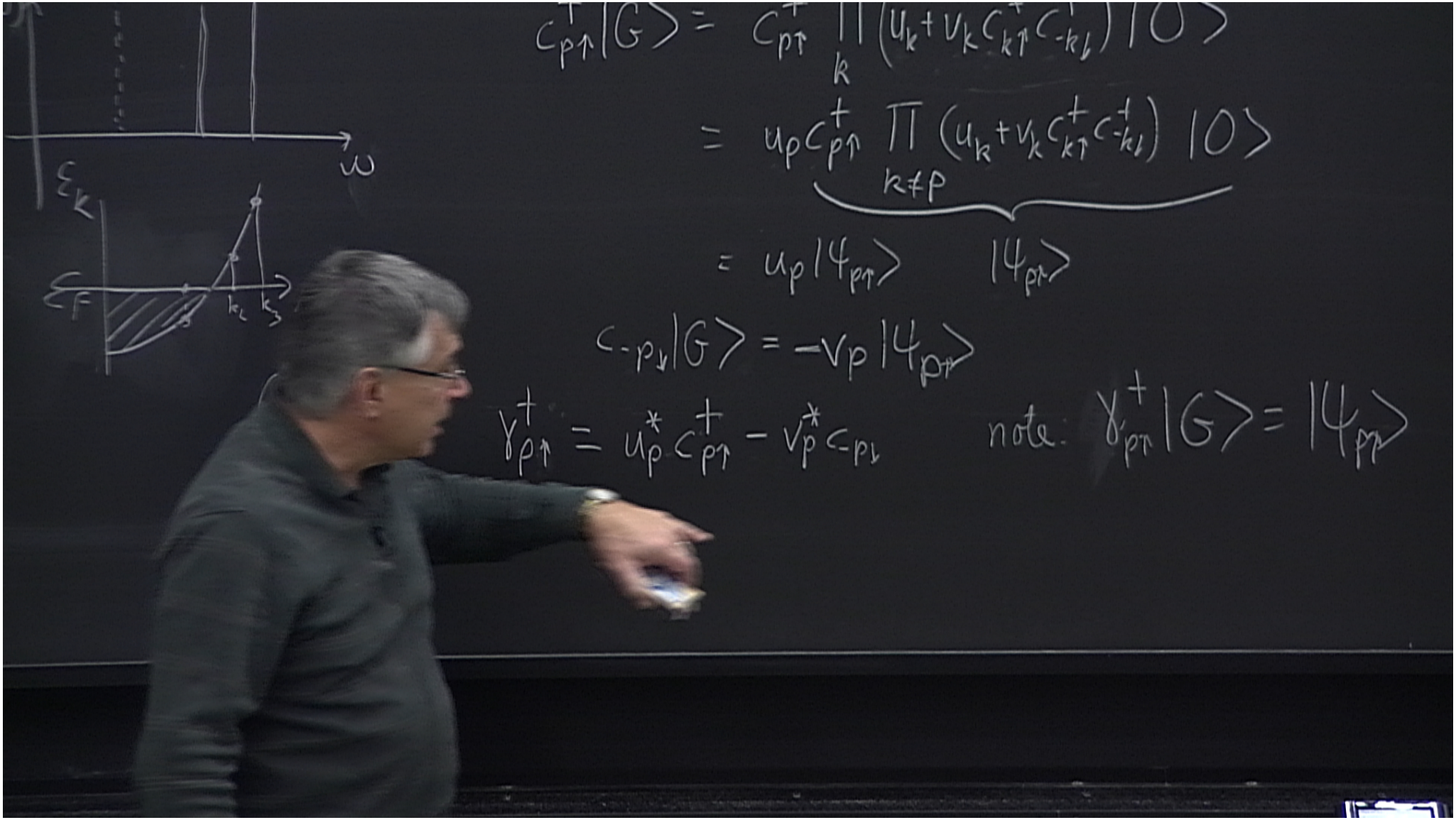
$$c^{+}_{p\uparrow}|G\rangle = c^{+}_{p\uparrow}\prod_{k}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$

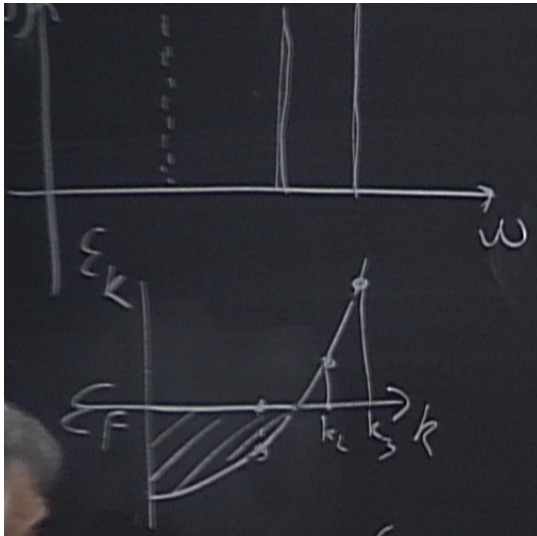
$$= u_p c^{+}_{p\uparrow}\underbrace{\prod_{k\neq p}\left(u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)}_{}|0\rangle$$

$$= u_p |1_p\rangle \qquad |1_p\rangle$$

$$c_{-p\downarrow}|G\rangle = -v_p |1_p\rangle$$


$A(k,\omega)$
ω
ε_k
ε_F
k_1
k_2
k
$c^\dagger_{p\uparrow}|G\rangle = c^\dagger_{p\uparrow} \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$
$= u_p c^\dagger_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})}\ |0\rangle$
$= u_p |1_p\rangle \quad |1_p\rangle$
$c_{-p\downarrow}|G\rangle = -v_p |1_p\rangle$
(Tinham) $\gamma^\dagger_{p\uparrow} = u^*_p c^\dagger_{p\uparrow} - v^*_p c_{-p\downarrow}$


$c^\dagger_{p\uparrow}|G\rangle = c^\dagger_{p\uparrow} \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$
$= u_p c^\dagger_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})}_{} |0\rangle$
$= u_p |1_{p\uparrow}\rangle \quad |1_{p\uparrow}\rangle$
$c_{-p\downarrow}|G\rangle = -v_p |1_{p\uparrow}\rangle$
$\gamma^\dagger_{p\uparrow} = u^*_p c^\dagger_{p\uparrow} - v^*_p c_{-p\downarrow}$
note: $\gamma^\dagger_{p\uparrow}|G\rangle = |1_{p\uparrow}\rangle$
ω
ε_k
ε_F
k_1
k_2



$$c^{+}_{p\uparrow}|G\rangle = c^{+}_{p\uparrow}\prod_k (u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow})|0\rangle$$

$$= u_p c^{+}_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow})}\ |0\rangle$$

$$= u_p |\psi_{p\uparrow}\rangle \quad |\psi_{p\uparrow}\rangle$$

$$c_{-p\downarrow}|G\rangle = -v_p |\psi_{p\uparrow}\rangle$$

quasiparticle.

(Tinham)

$$\hat{\gamma}^{+}_{p\uparrow} = u^{*}_p \hat{c}^{+}_{p\uparrow} - v^{*}_p \hat{c}_{-p\downarrow}$$

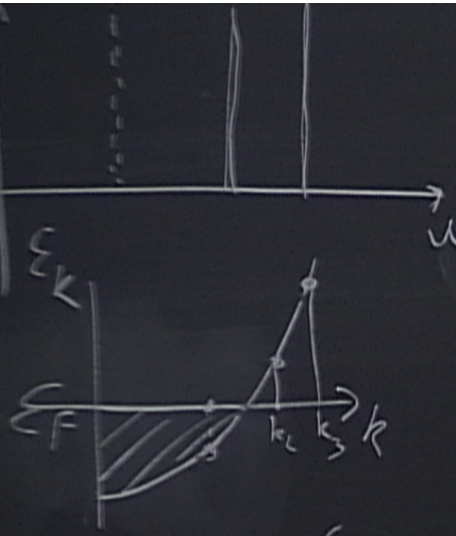
$$\hat{\gamma}_{-p\downarrow} = u_p \hat{c}_{-p\downarrow} + v_p \hat{c}^{+}_{p\uparrow}$$

note: $\gamma^{+}_{p\uparrow}|G\rangle = |\psi_{p\uparrow}\rangle$

$\hat{\gamma}_{-p\downarrow}|G\rangle = 0$

$$c^{\dagger}_{p\uparrow}|G\rangle = c^{\dagger}_{p\uparrow} \prod_k (u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})|0\rangle$$

$$= u_p c^{\dagger}_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})}_{|\psi_{p\uparrow}\rangle} |0\rangle$$



Bogoliubov quasiparticles $u_p|\psi_{p\uparrow}\rangle$ $|\psi_{p\uparrow}\rangle$

$$c_{-p\downarrow}|G\rangle = -v_p|\psi_{p\uparrow}\rangle$$

(Tinham)

$$\hat{\gamma}^{\dagger}_{p\uparrow} = u^*_p \hat{c}^{\dagger}_{p\uparrow} - v^*_p \hat{c}_{-p\downarrow}$$

$$\hat{\gamma}_{-p\downarrow} = u_p \hat{c}_{-p\downarrow} + v_p \hat{c}^{\dagger}_{p\uparrow}$$

quasiparticle.

note: $\hat{\gamma}^{\dagger}_{p\uparrow}|G\rangle = |\psi_{p\uparrow}\rangle$

$\hat{\gamma}_{-p\downarrow}|G\rangle = 0$

$$c^\dagger_{p\uparrow}|G\rangle = c^\dagger_{p\uparrow}\prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$$

$$= u_p c^\dagger_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})}\ |0\rangle$$

$|\psi_{p\uparrow}\rangle$

Bogoliubov quasiparticle $u_p|\psi_{p\uparrow}\rangle$

$$c_{-p\downarrow}|G\rangle = -v_p|\psi_{p\uparrow}\rangle$$

$$\hat{\gamma}^\dagger_{p\uparrow} = u_p^* \hat{c}^\dagger_{p\uparrow} - v_p^* \hat{c}_{-p\downarrow}$$

$$\hat{\gamma}_{-p\downarrow} = u_p \hat{c}_{-p\downarrow} + v_p \hat{c}^\dagger_{p\uparrow}$$

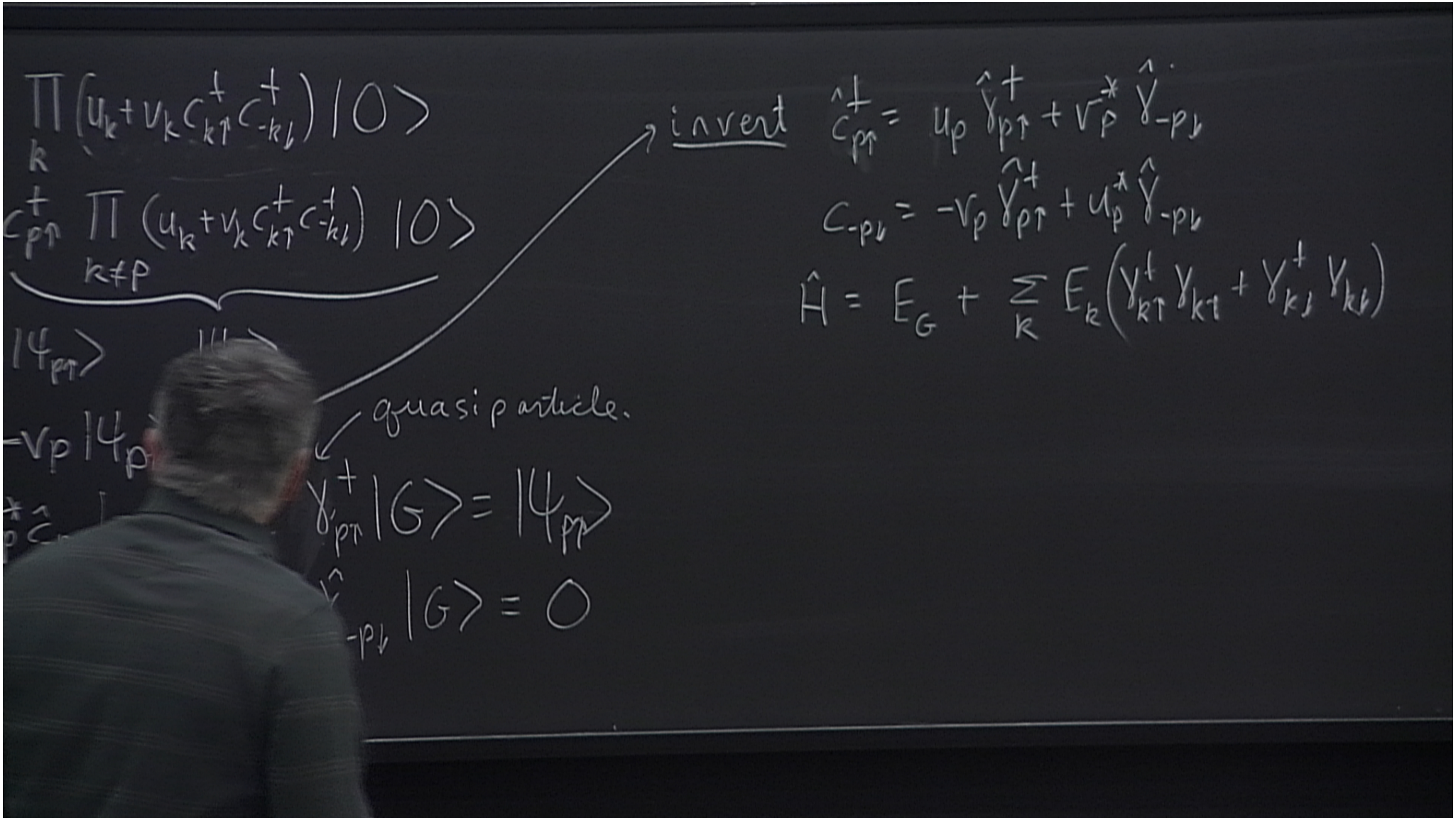
note: $\hat{\gamma}^\dagger_{p\uparrow}|G\rangle = |\psi_{p\uparrow}\rangle$ quasiparticle.

$$\hat{\gamma}_{-p\downarrow}|G\rangle = 0$$

invert

$$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$$

$$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$$


$\prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$
$c^\dagger_{p\uparrow} \prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$
$|\psi_{p\uparrow}\rangle$
$-v_p |\psi_p$
invert
$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$
$\hat{H} = E_G + \sum_k E_k (\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow})$
quasiparticle.
$\gamma^\dagger_{p\uparrow} |G\rangle = |\psi_{p\uparrow}\rangle$
$\hat{\gamma}_{-p\downarrow} |G\rangle = 0$

$$\prod_k \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

$$c^\dagger_{p\uparrow} \prod_{k\neq p} \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

$|\psi_{p\uparrow}\rangle \qquad |\psi_{p\uparrow}\rangle$

$-v_p|\psi_{p\uparrow}\rangle$

$u_p^* \hat{c}_{p\downarrow}$

$v_p \hat{c}^\dagger_{p\uparrow}$

$\varepsilon_{-k} = \varepsilon_k$

invert

$$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$$

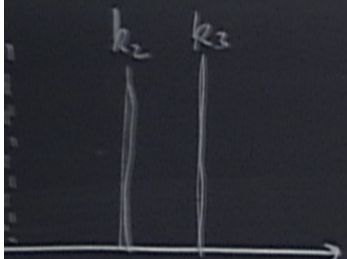
$$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$$

$$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow}\gamma_{k\downarrow}\right)$$

quasiparticle.

note: $\gamma^\dagger_{p\uparrow}|G\rangle = |\psi_{p\uparrow}\rangle$

$\hat{\gamma}_{-p\downarrow}|G\rangle = 0$

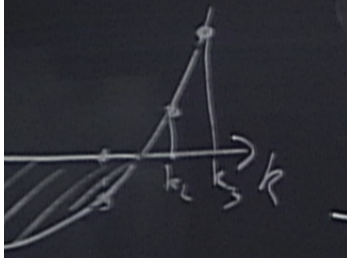


$$c^\dagger_{p\uparrow}|G\rangle = c^\dagger_{p\uparrow}\prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$$

$$= u_p c^\dagger_{p\uparrow} \underbrace{\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})}\ |0\rangle$$

$|4_{p\uparrow}\rangle$

Bogoliubov quasiparticles $u_p|4_{p\uparrow}\rangle$



-Valatin

$$c_{-p\downarrow}|G\rangle = -v_p|4_{p\uparrow}\rangle$$

(Tinham)

$$\hat{\gamma}^\dagger_{p\uparrow} = u^*_p \hat{c}^\dagger_{p\uparrow} - v^*_p \hat{c}_{p\downarrow}$$

$$\hat{\gamma}_{-p\downarrow} = u_p \hat{c}_{-p\downarrow} + v_p \hat{c}^\dagger_{p\uparrow}$$

inve

$\varepsilon_{-k} = \varepsilon_k$

quasiparticle.

note: $\gamma^\dagger_{p\uparrow}|G\rangle = |4_{p\uparrow}\rangle$

$\hat{\gamma}_{-p\downarrow}|G\rangle = 0$

$\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$

$\prod_{k\neq p} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$

$|\psi_{p\uparrow}\rangle$ $|\psi_{p\uparrow}\rangle$

$c_{p\uparrow} |\psi_{p\uparrow}\rangle$

$c_{-p\downarrow}$

$c^\dagger_{p\uparrow}$

$\varepsilon_{-k} = \varepsilon_k$

invert

$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$

$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$

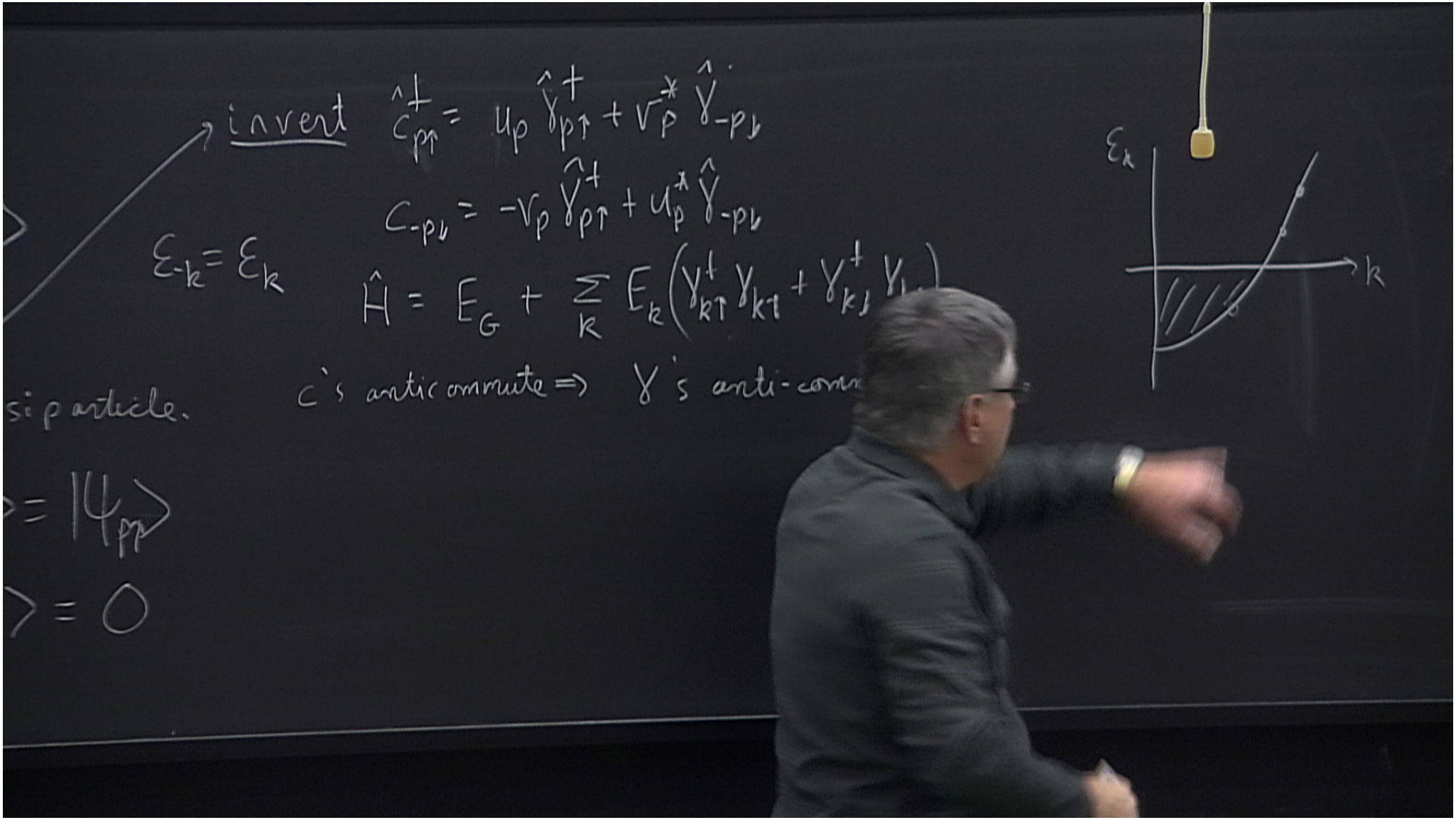
$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$

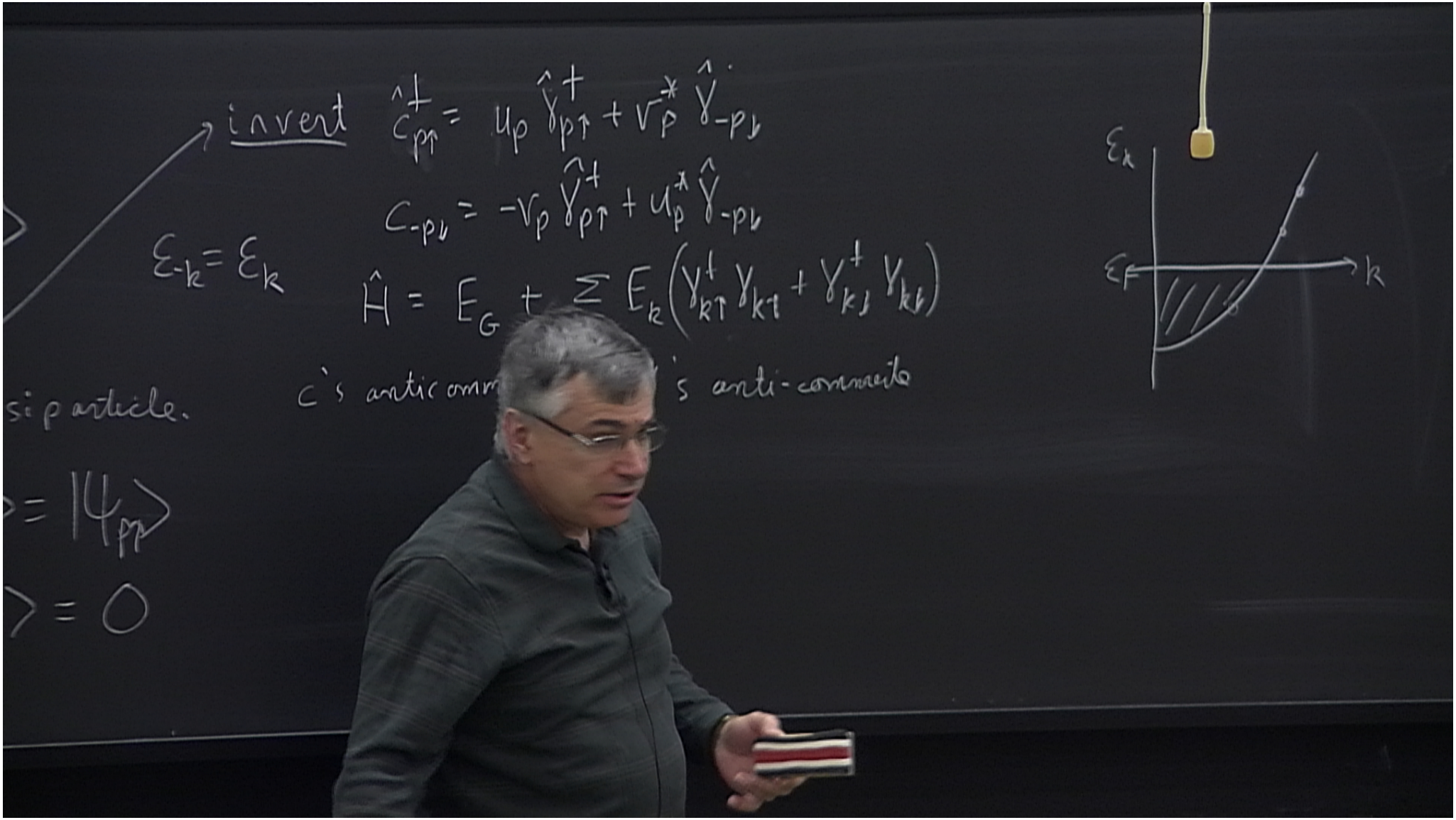
c's anticommute $\Rightarrow$ γ's anti-commute

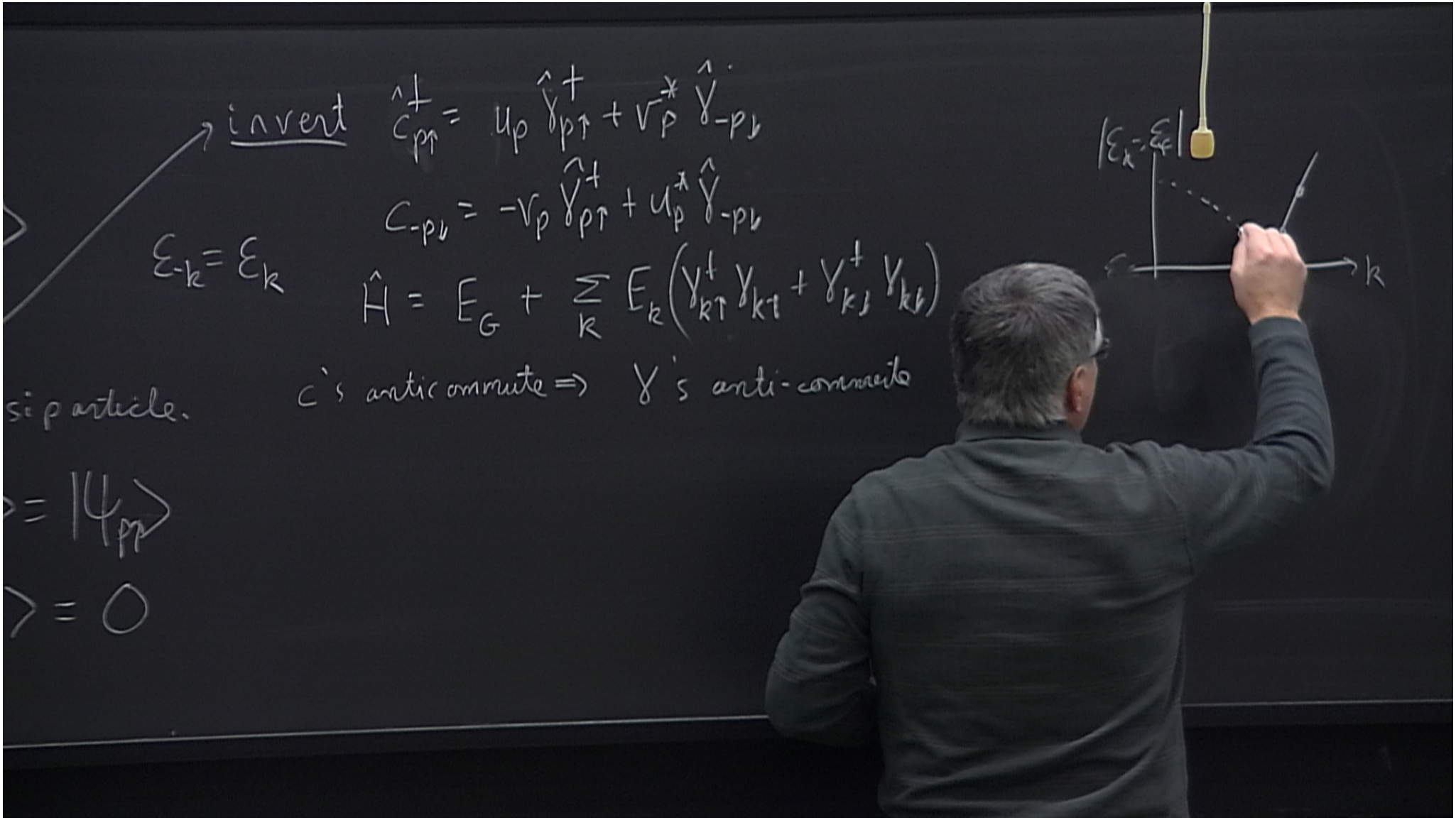
quasiparticle.

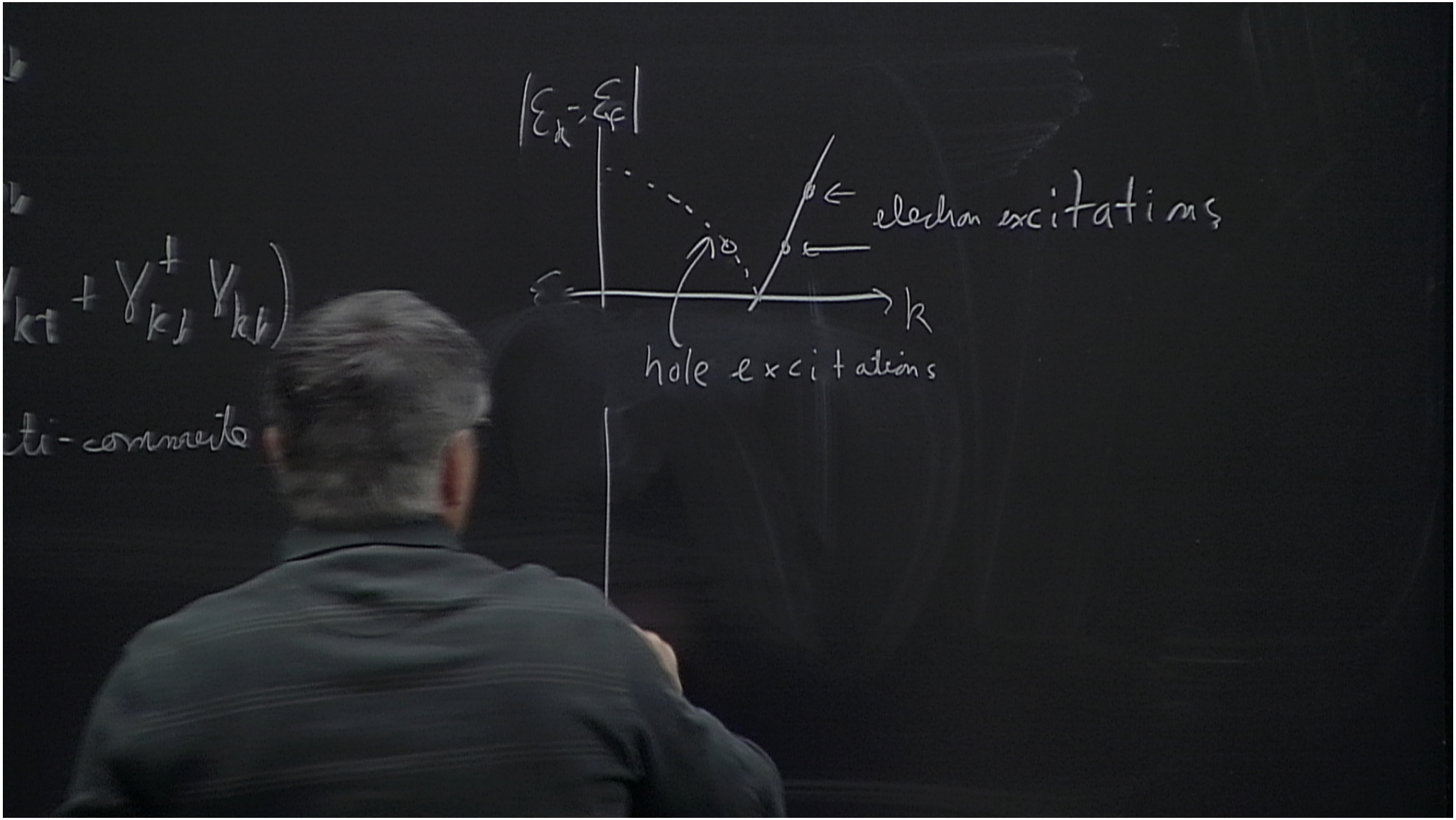
note: $\gamma^\dagger_{p\uparrow} |G\rangle = |\psi_{p\uparrow}\rangle$

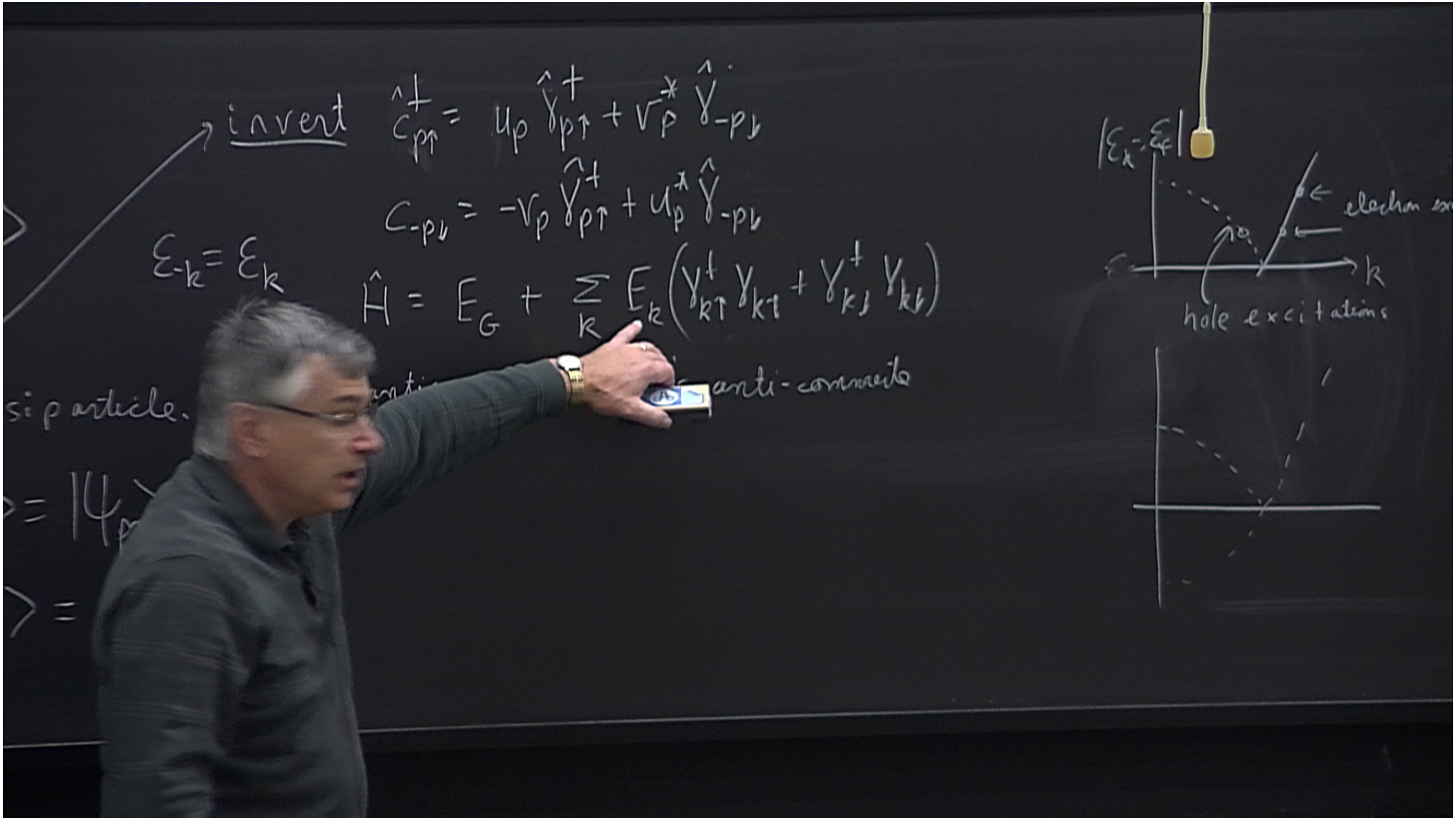
$\hat{\gamma}_{-p\downarrow} |G\rangle = 0$

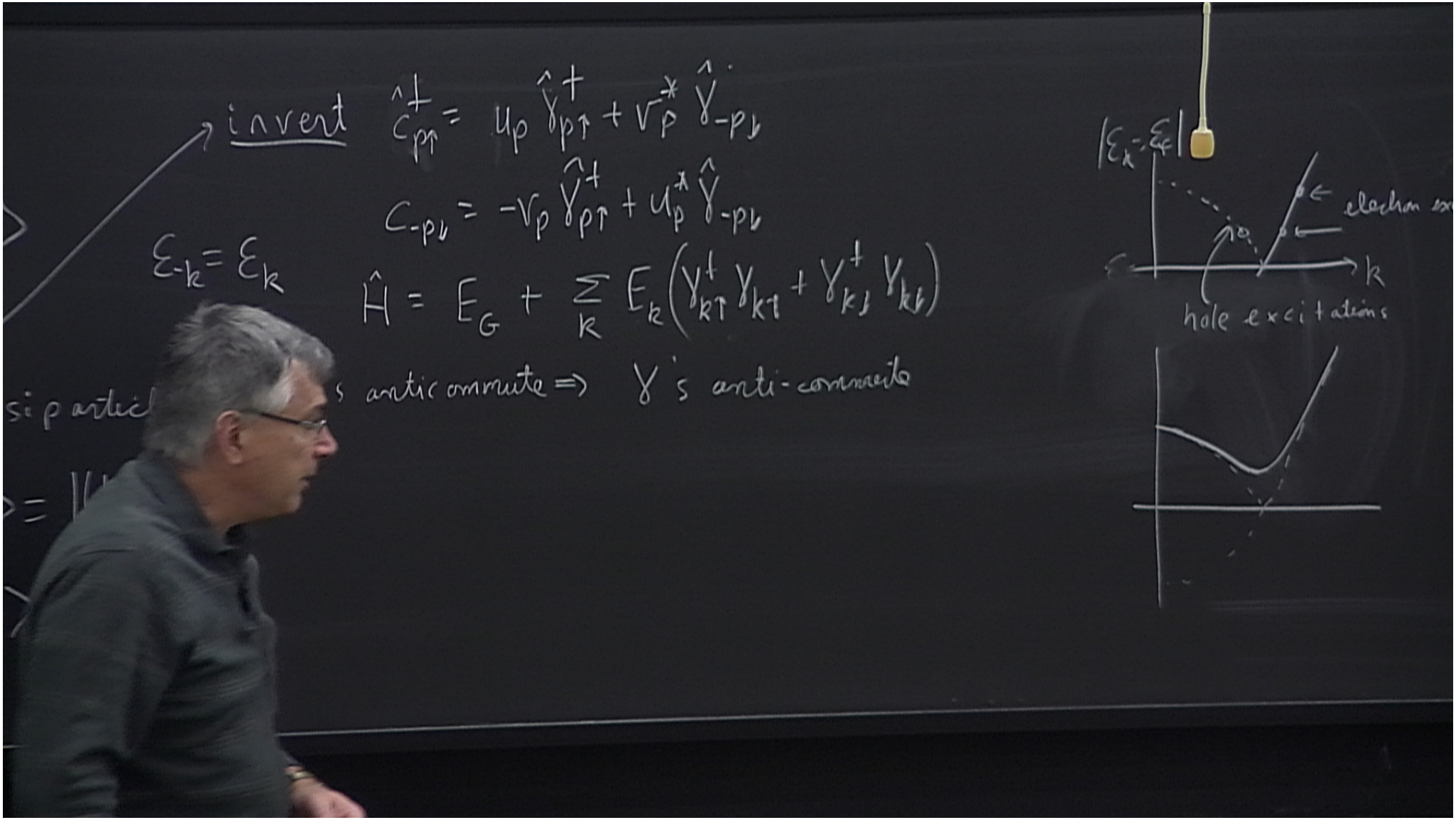

invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$
$\varepsilon_{-k} = \varepsilon_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$
c's anticommute ⇒ γ's anti-comm
si particle.
$= |\psi_{pp}\rangle$
$\rangle = 0$
ε_k
k


invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
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ε_k
ε_F
k


invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
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c's anticommute ⇒ γ's anti-commute
si particle.
$= |\psi_{pp}\rangle$
$\rangle = 0$
$|\varepsilon_k - \varepsilon_F|$
ε_F
k


$|\varepsilon_a - \varepsilon_f|$
electron excitations
ε
k
hole excitations


invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^{*} \hat{\gamma}_{-p\downarrow}$
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$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$
anti-commute
particle.
$|\varepsilon_k - \varepsilon_F|$
k
hole excitations


invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^{*} \hat{\gamma}_{-p\downarrow}$
$\varepsilon_{-k} = \varepsilon_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$
s anticommute ⇒ γ's anti-commute
si partic
$|\varepsilon_k - \varepsilon_F|$
k
hole excitations

invert $\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$

$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^{*} \hat{\gamma}_{-p\downarrow}$

$\varepsilon_{-k} = \varepsilon_k$

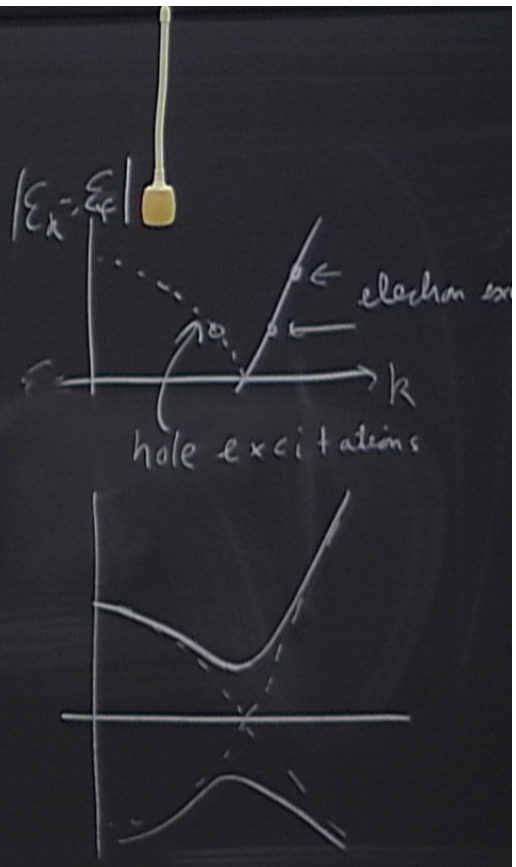
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$

c ... mute $\Rightarrow$ γ's anti-commute

... particle.

$= |\psi_{pp}\rangle$

$\rangle = 0$



invert $\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$

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$\varepsilon_{-k} = \varepsilon_k$

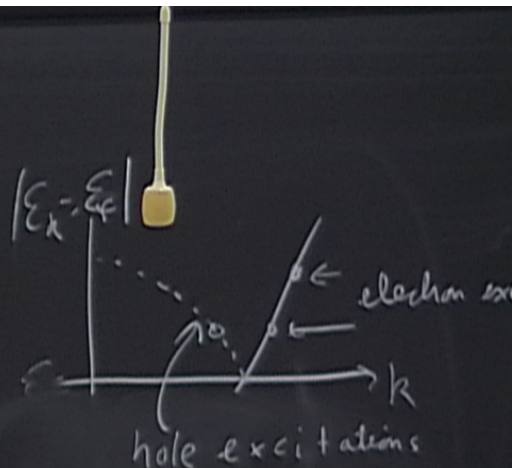
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$

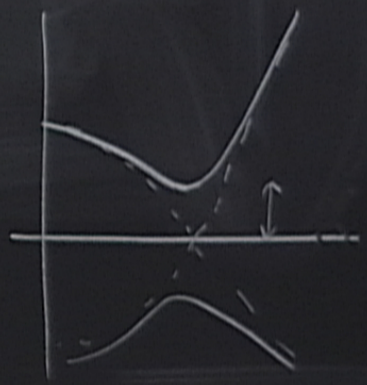
c's anticommute $\Rightarrow$ γ's anti-commute

si particle.

$= |\psi_{pp}\rangle$

$\rangle = 0$





invert $\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$

$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$

$\varepsilon_{-k} = \varepsilon_k$

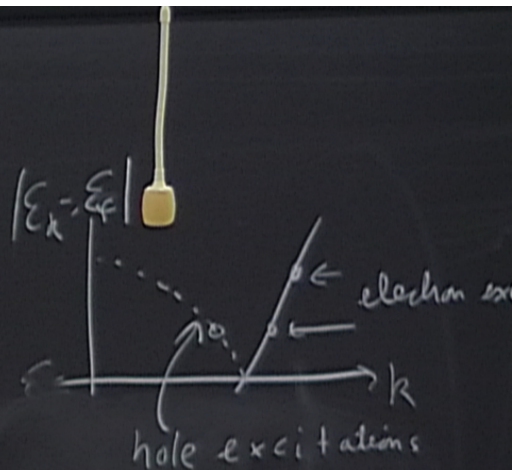
$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$

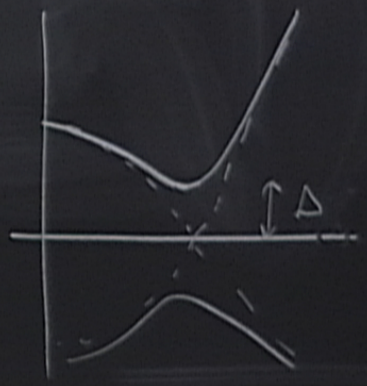
c's anticommute $\Rightarrow$ γ's anti-commute

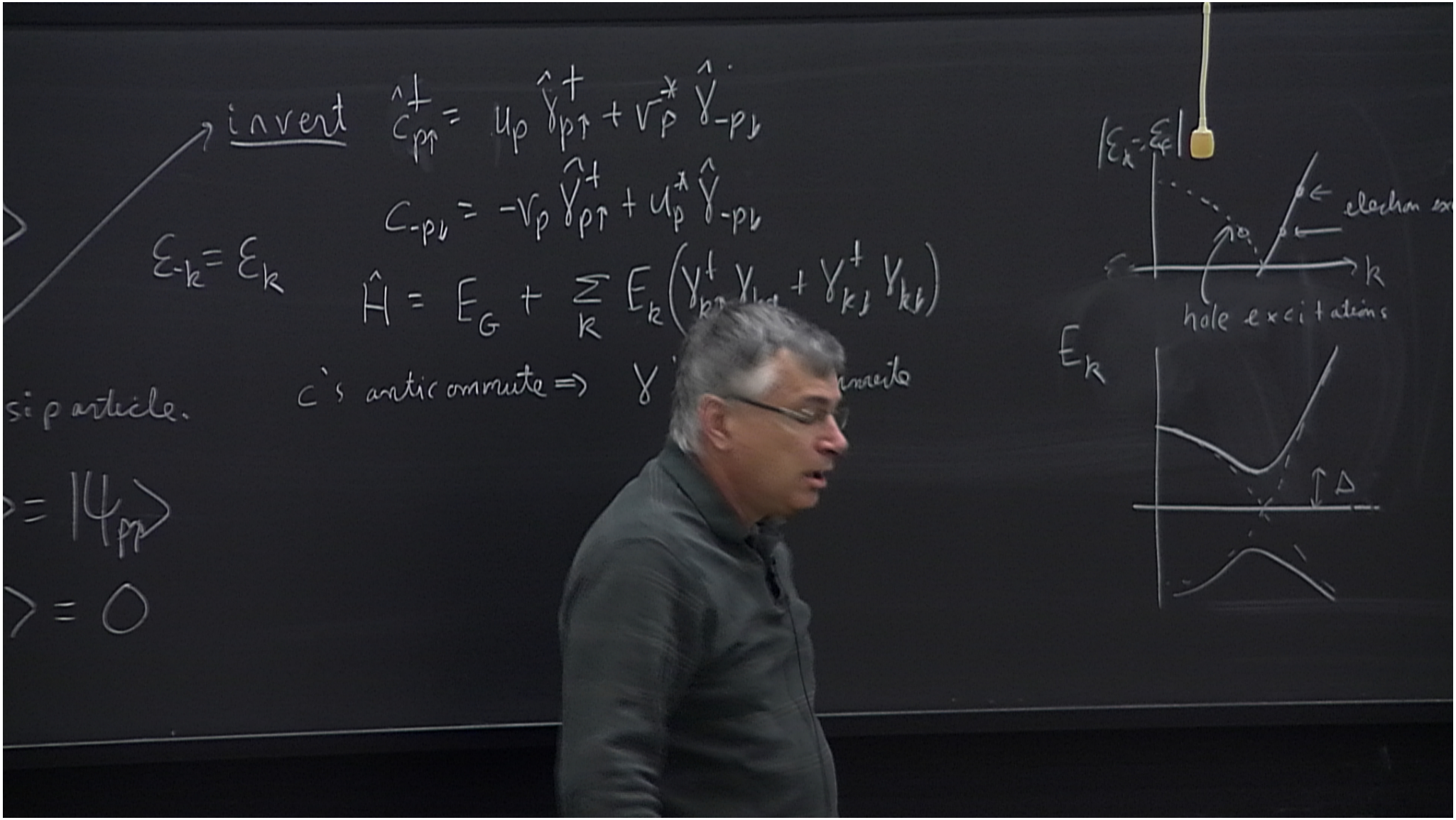
si particle.

$= |\psi_{pp}\rangle$

$\rangle = 0$






invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^{*} \hat{\gamma}_{-p\downarrow}$
$\varepsilon_{-k} = \varepsilon_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$
c's anticommute ⇒
$|\varepsilon_k - \varepsilon_F|$
electron ex
k
hole excitations
E_k
Δ

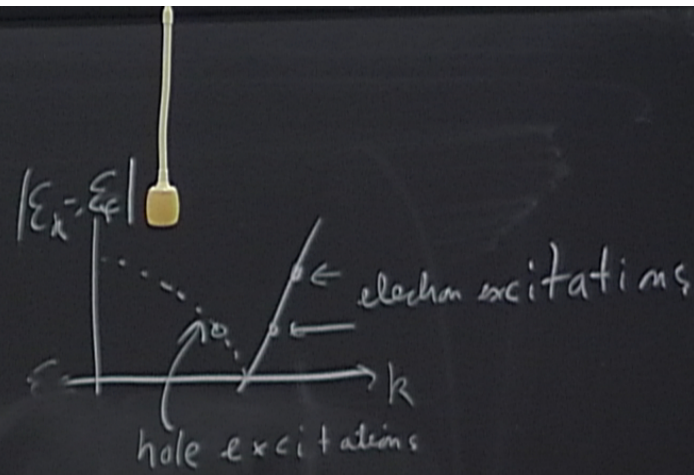
invert $\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$

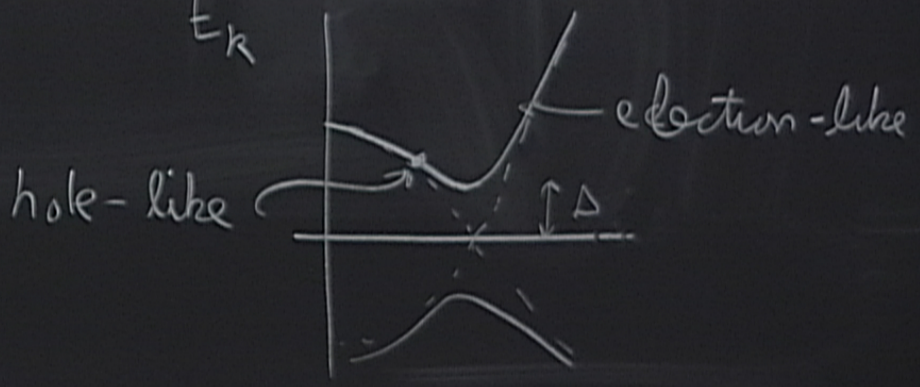
$c_{-p\downarrow} = -v_p \hat{\gamma}^{\dagger}_{p\uparrow} + u_p^{*} \hat{\gamma}_{-p\downarrow}$

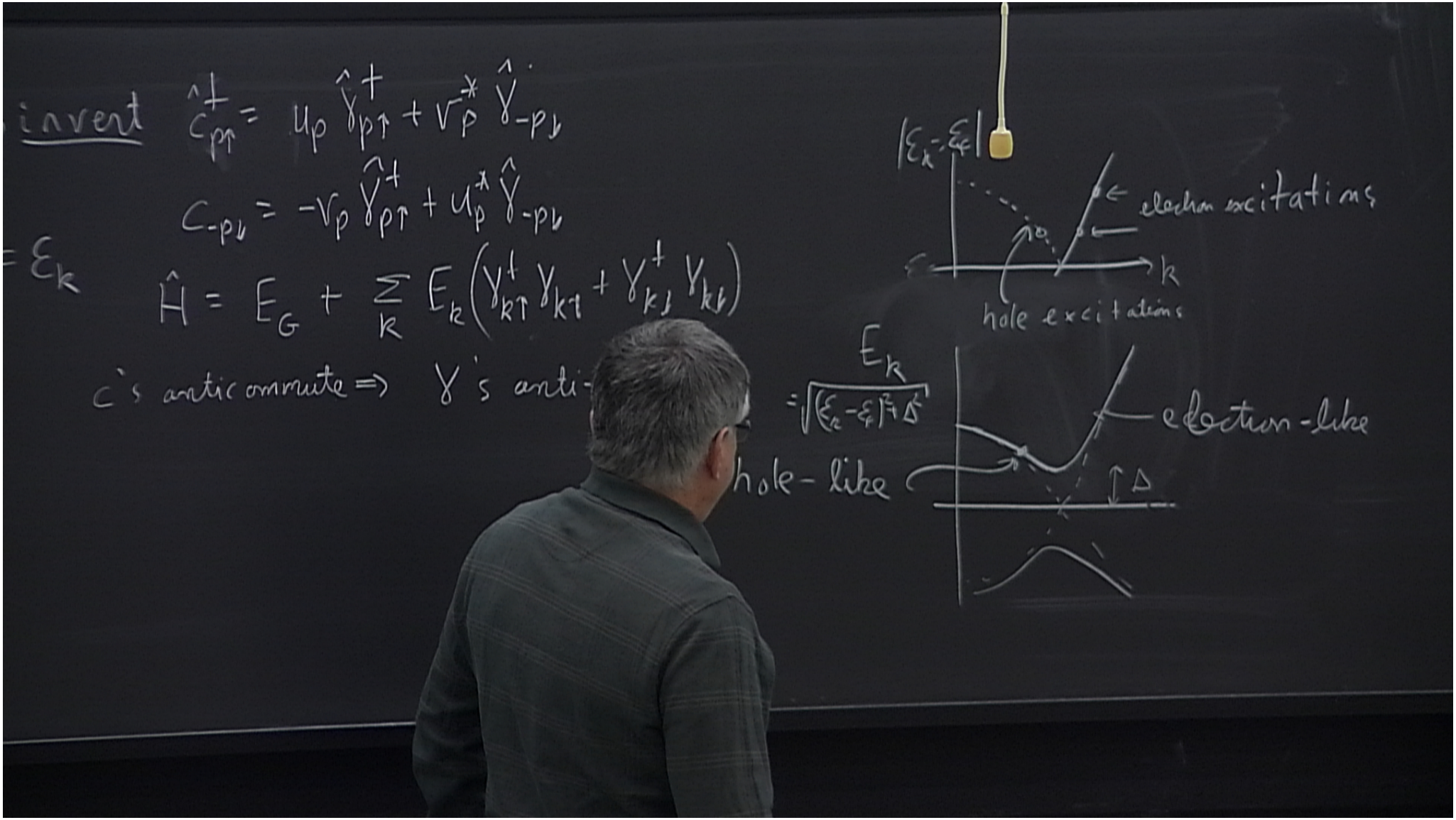
$= \varepsilon_k$

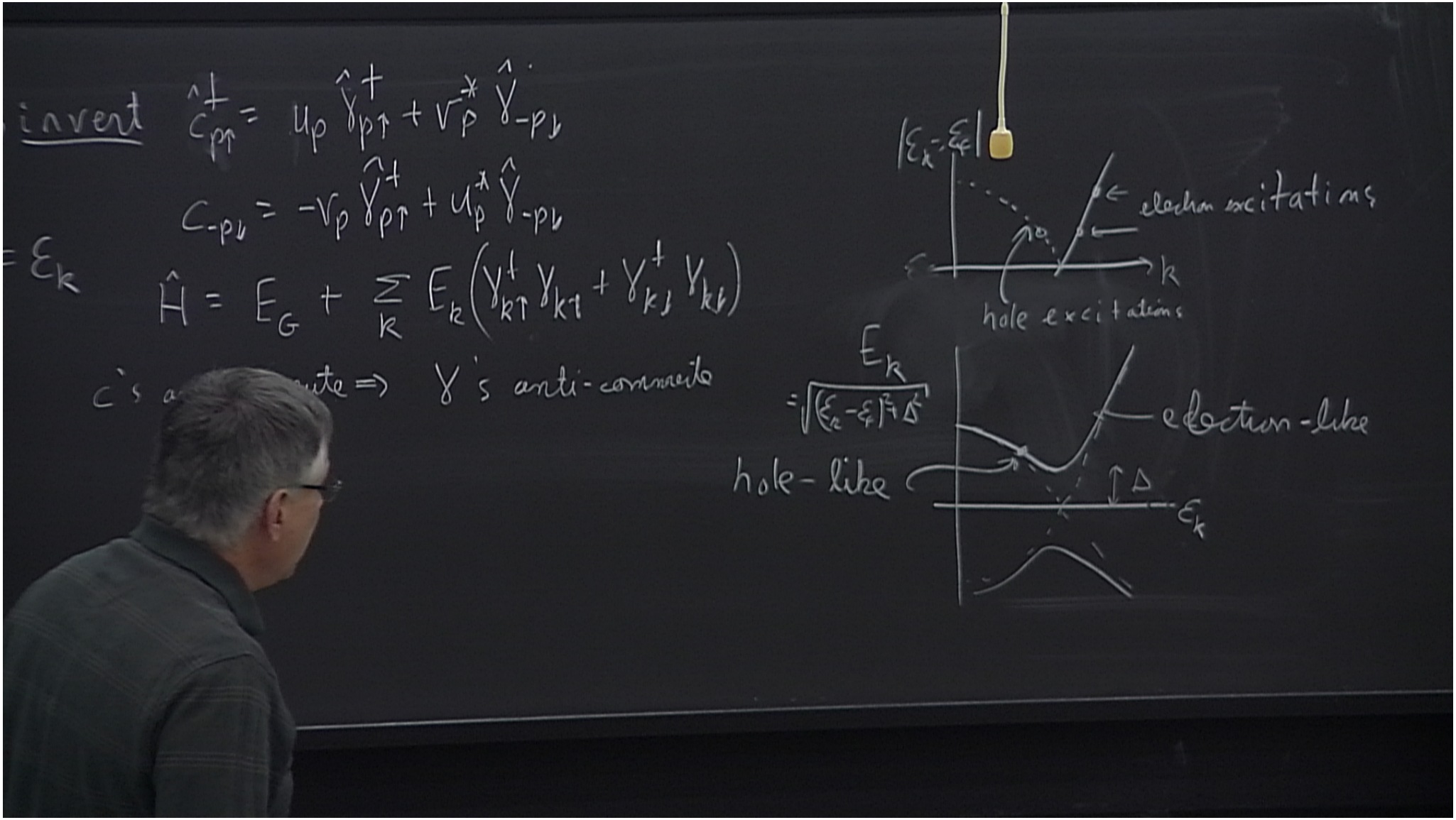
$\hat{H} = E_G + \sum_k E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$

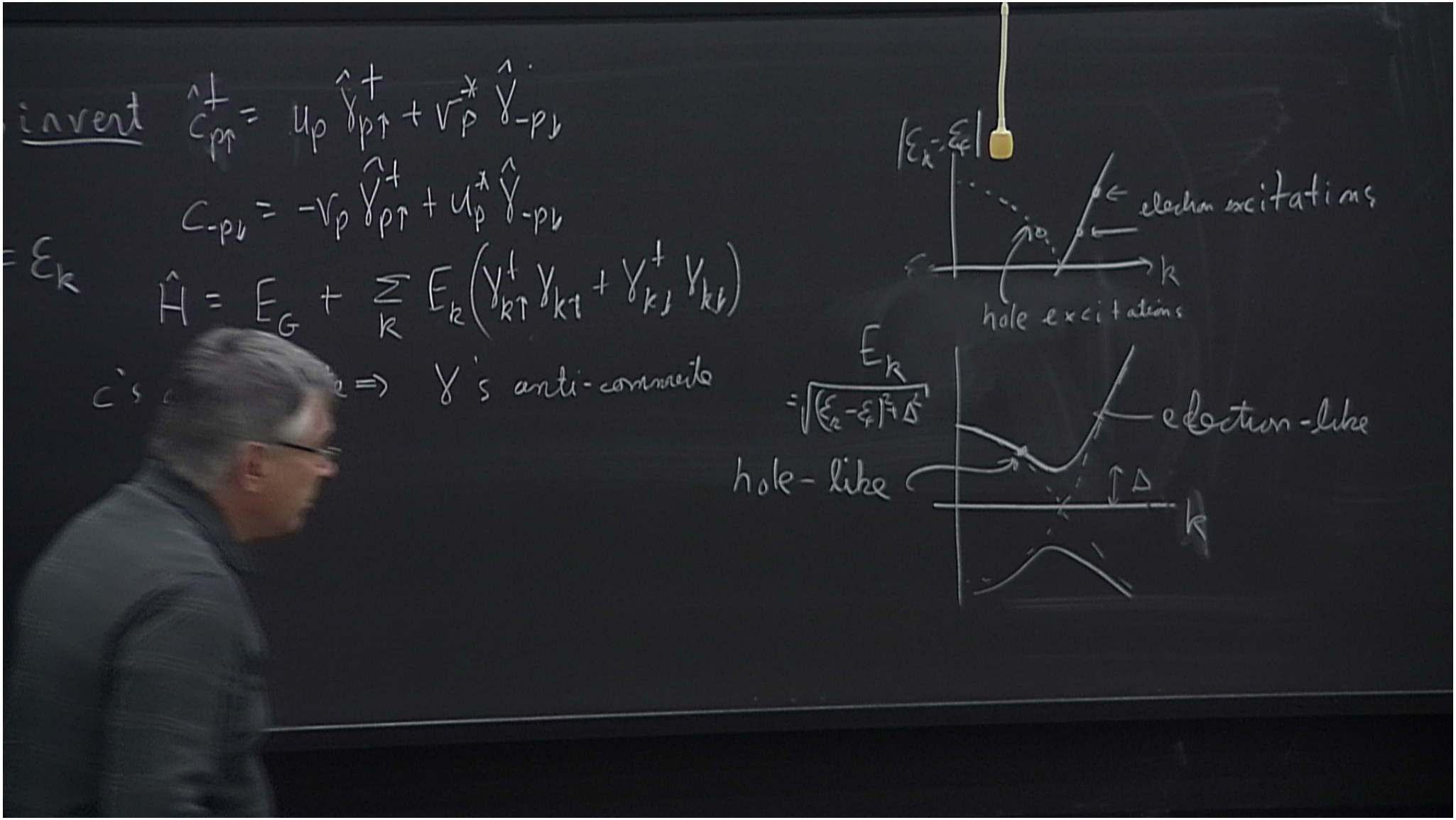
c's anticommute $\Rightarrow$ γ's anti-commute

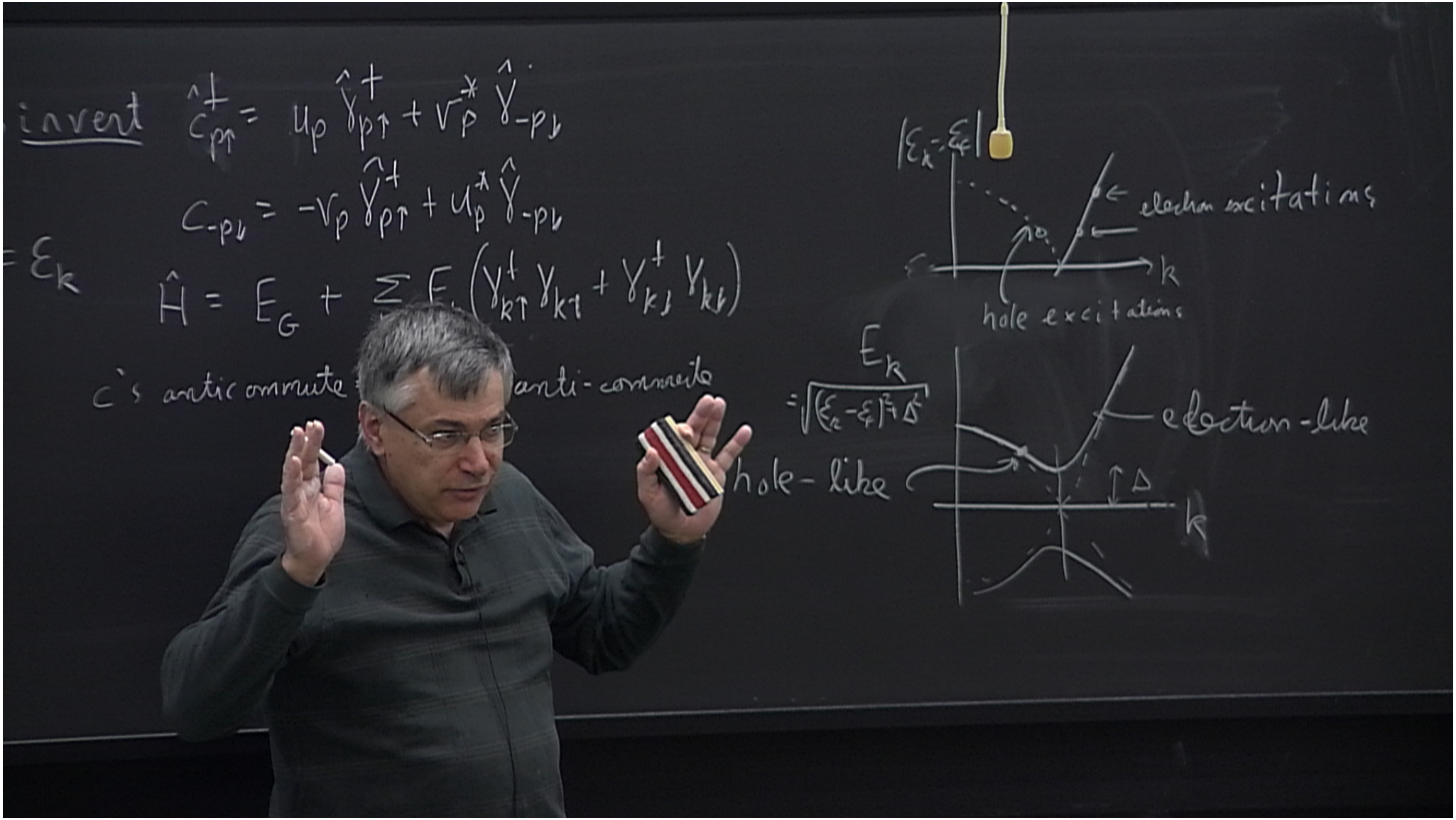


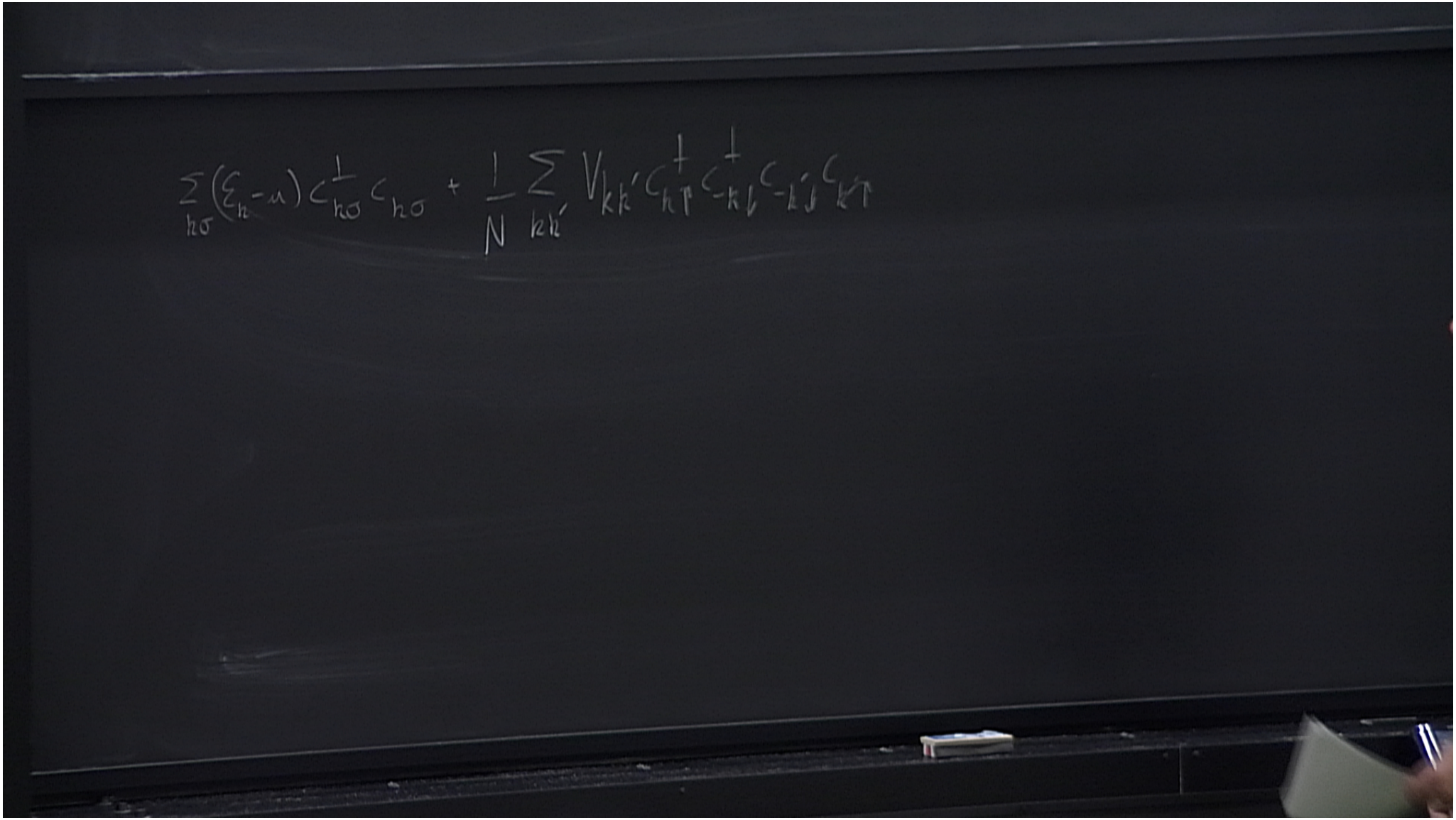


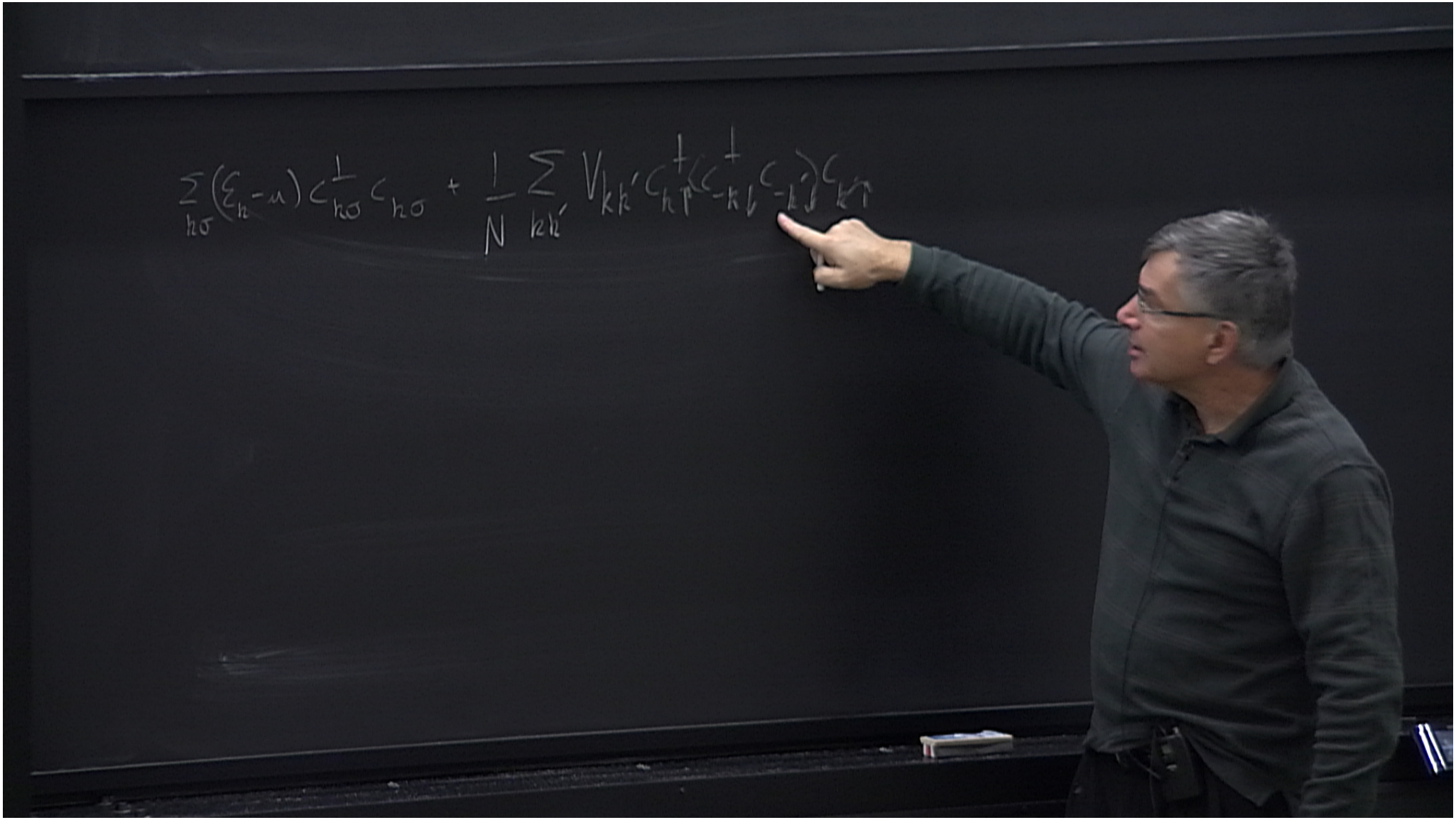

invert
$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$
$= \xi_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$
c's anticommute ⇒ γ's anti-
$E_k = \sqrt{(\xi_k - \xi_f)^2 + \Delta^2}$
hole-like
$|\xi_k - \xi_f|$
electron excitations
k
hole excitations
electron-like
Δ


invert
$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$
$= \varepsilon_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$
c's a ute $\Rightarrow$ γ's anti-commute
$|\varepsilon_k - \varepsilon_f|$
electron excitations
k
hole excitations
$E_k = \sqrt{(\varepsilon_k - \varepsilon_f)^2 + \Delta^2}$
electron-like
hole-like
Δ
ε_k


invert
$\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$
$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$
$= \varepsilon_k$
$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$
$\Rightarrow$ γ's anti-commute
$|\varepsilon_k - \varepsilon_f|$
electron excitations
hole excitations
k
$E_k = \sqrt{(\varepsilon_k - \varepsilon_f)^2 + \Delta^2}$
electron-like
hole-like
Δ
k

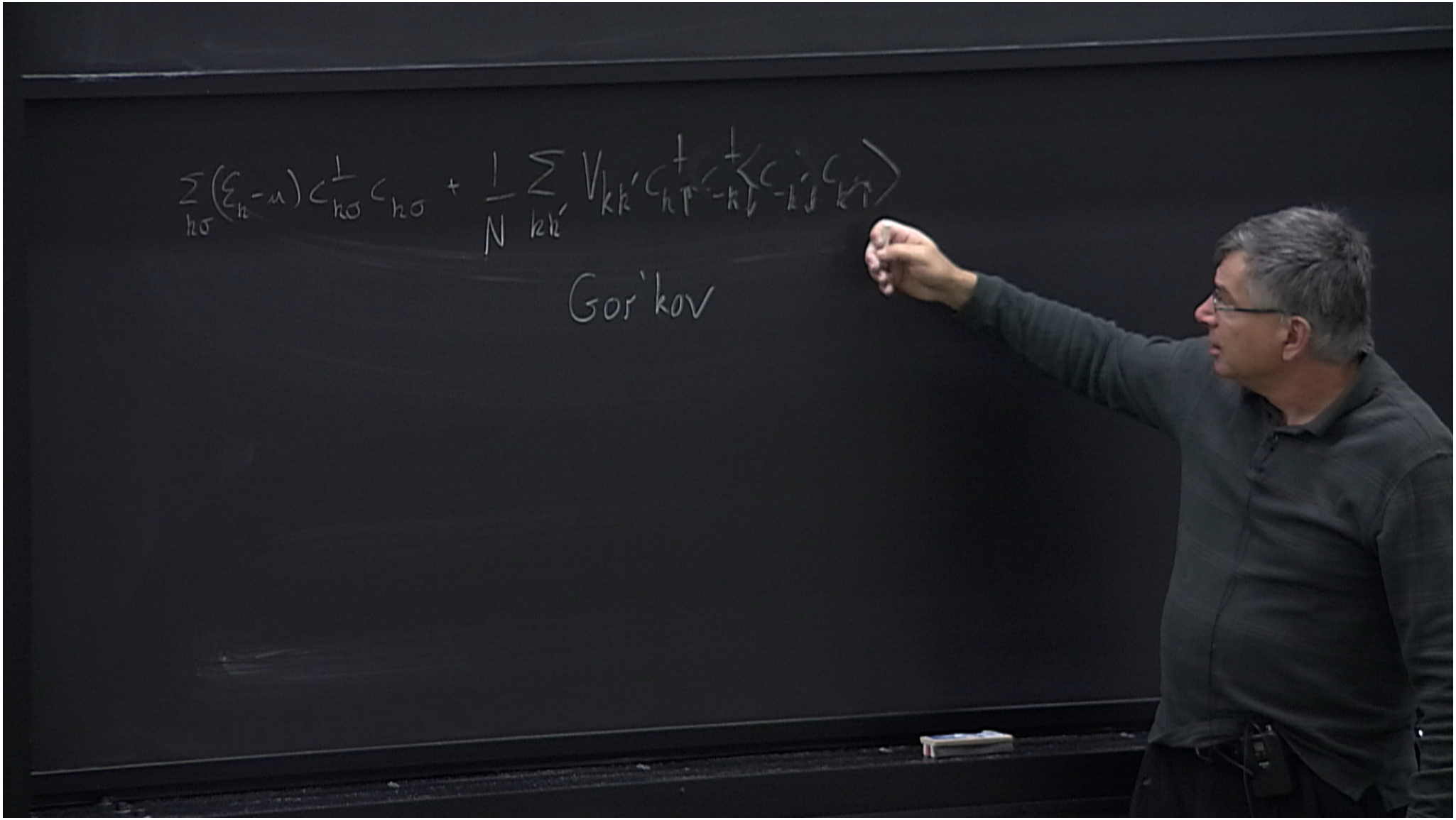

invert
$\hat{c}^{\dagger}_{p\uparrow} = u_p \hat{\gamma}^{\dagger}_{p\uparrow} + v_p^{*} \hat{\gamma}_{-p\downarrow}$
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$\hat{H} = E_G + \sum E_k \left(\gamma^{\dagger}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{\dagger}_{k\downarrow} \gamma_{k\downarrow} \right)$
c's anticommute
anti-commute
$E_k = \sqrt{(\xi_k - \xi_f)^2 + \Delta^2}$
hole-like
$|\xi_k - \xi_f|$
electron excitations
hole excitations
k
electron-like
Δ
k


$\sum_{k\sigma} (\varepsilon_k - \mu) c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{N} \sum_{kk'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$


$\sum_{k\sigma}(\varepsilon_k - \mu) c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$

$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

Gor'kov


$\sum_{k\sigma}(\varepsilon_k - \mu) c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$
Gor'kov

$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'}\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

Gor'kov

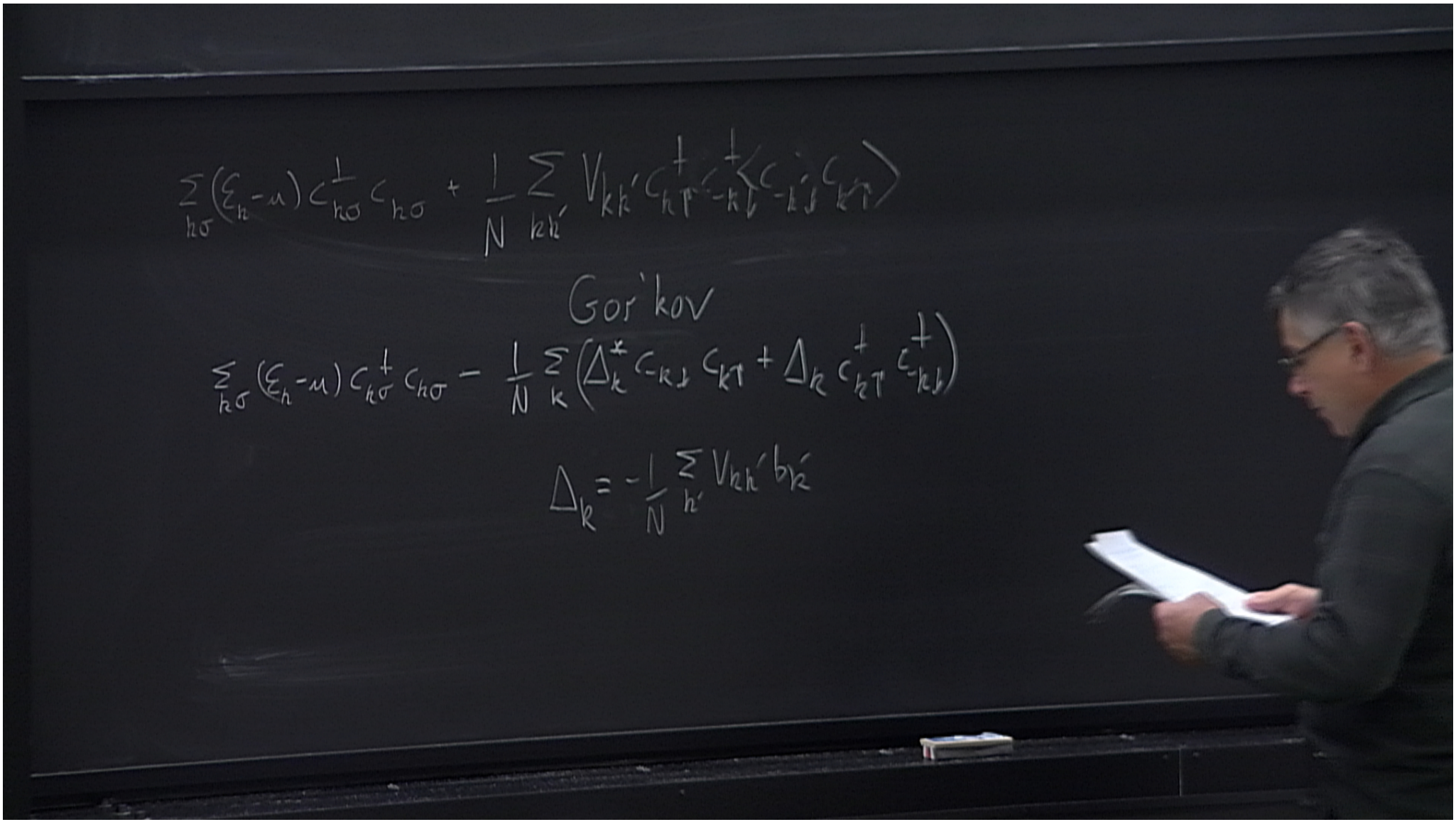
$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} - \frac{1}{N}\sum_{k}\left(\Delta^{*}_{k}\, c_{-k\downarrow} c_{k\uparrow} + \Delta_k\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)$$

$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

Gor'kov

$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} - \frac{1}{N}\sum_{k}\left(\Delta^{*}_{k} c_{-k\downarrow} c_{k\uparrow} + \Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)$$

$$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} b_{k'}$$



$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'}\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \langle c_{-k\downarrow} c_{k'\uparrow} \rangle$$

Gor'kov

$$\sum_{k\sigma}(\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} - \frac{1}{N}\sum_{k}\left(\Delta^{*}_{k}\, c_{-k\downarrow} c_{k\uparrow} + \Delta_k\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)$$

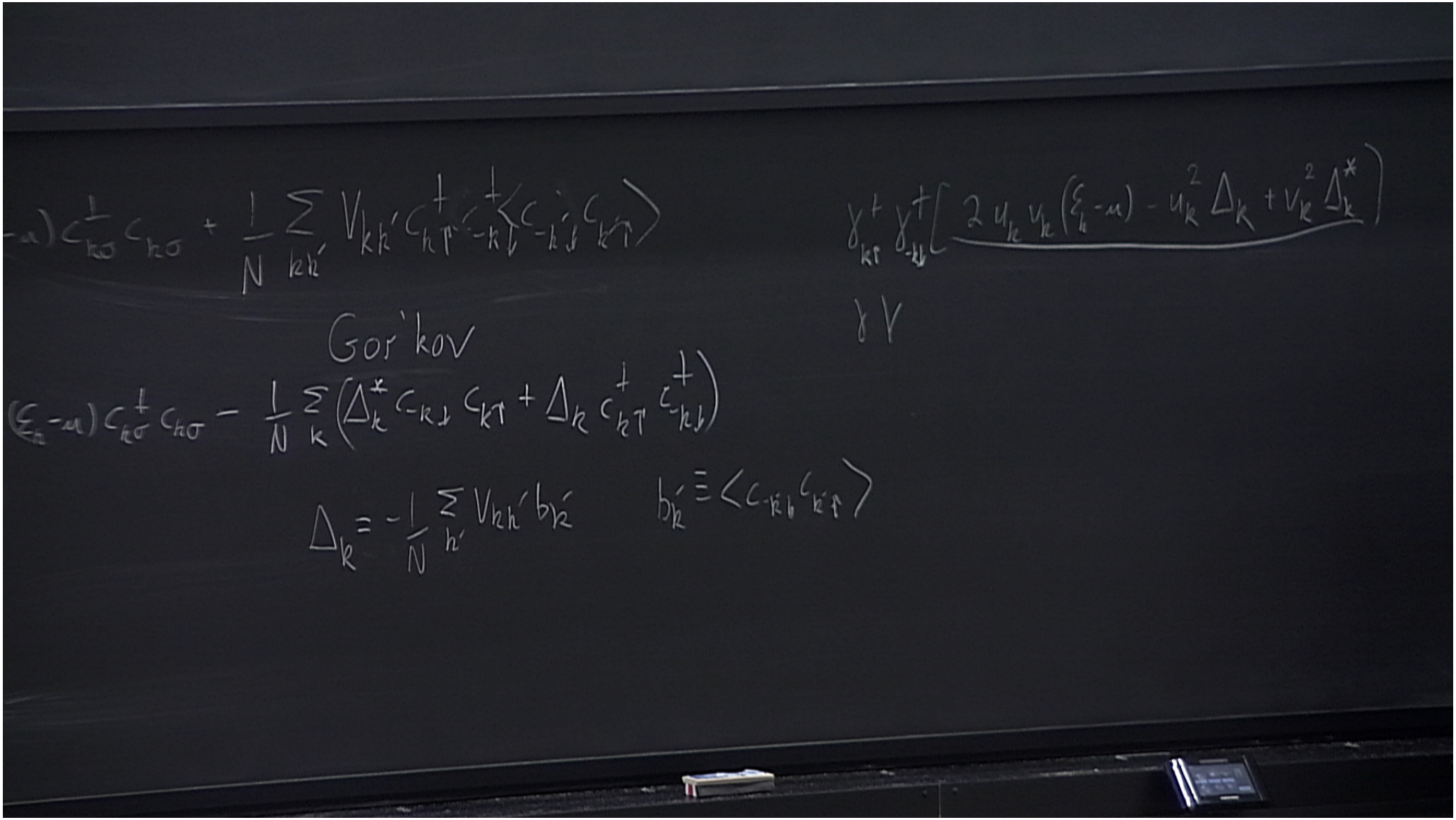
$$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} b_{k'} \qquad b_{k'} \equiv \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

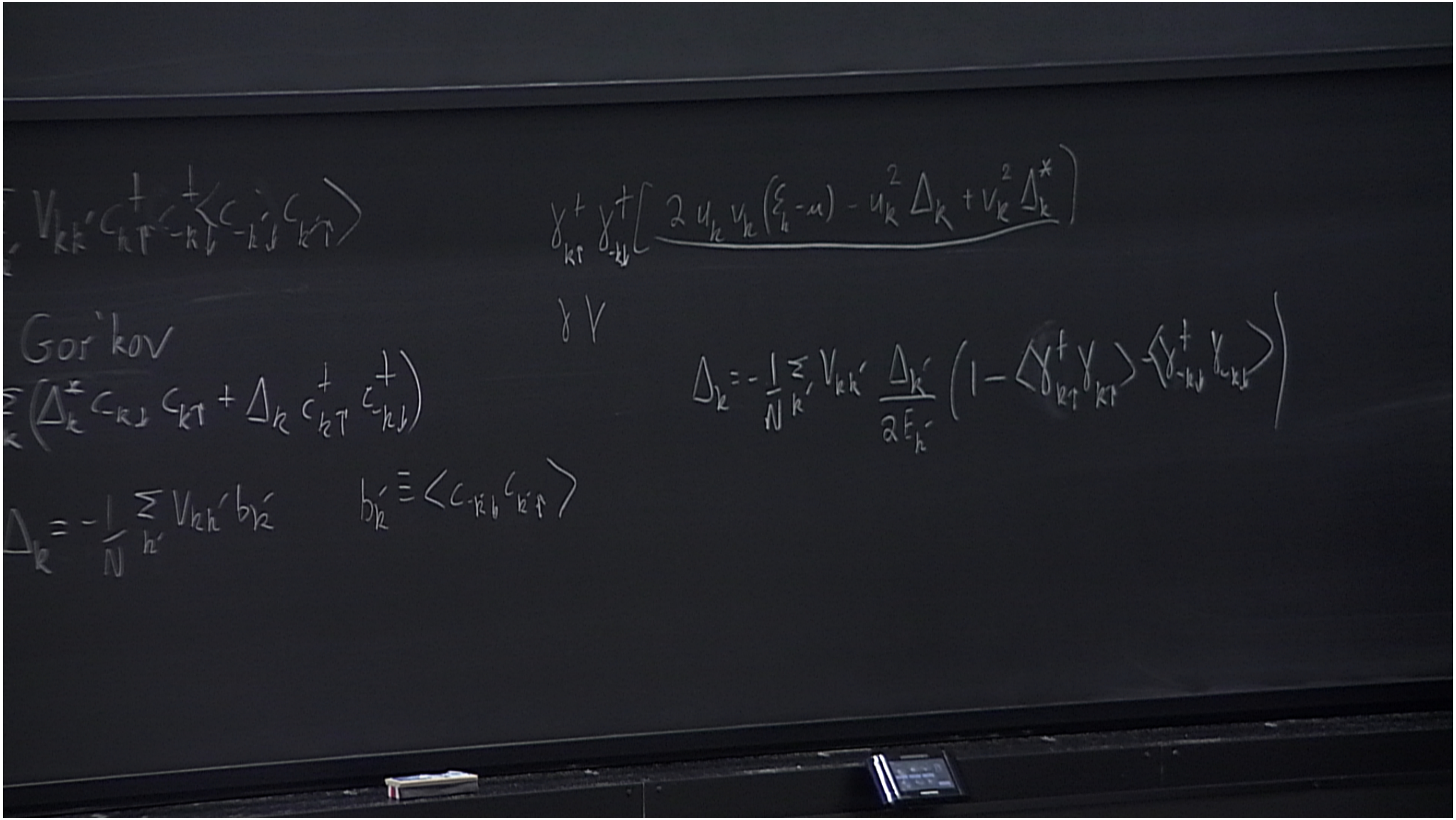
$$\sum_{k\sigma}(\varepsilon_k-\mu)c^\dagger_{k\sigma}c_{k\sigma}+\frac{1}{N}\sum_{kk'}V_{kk'}c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\langle c_{-k\downarrow}c_{k'\uparrow}\rangle \qquad \gamma^+\gamma^+$$

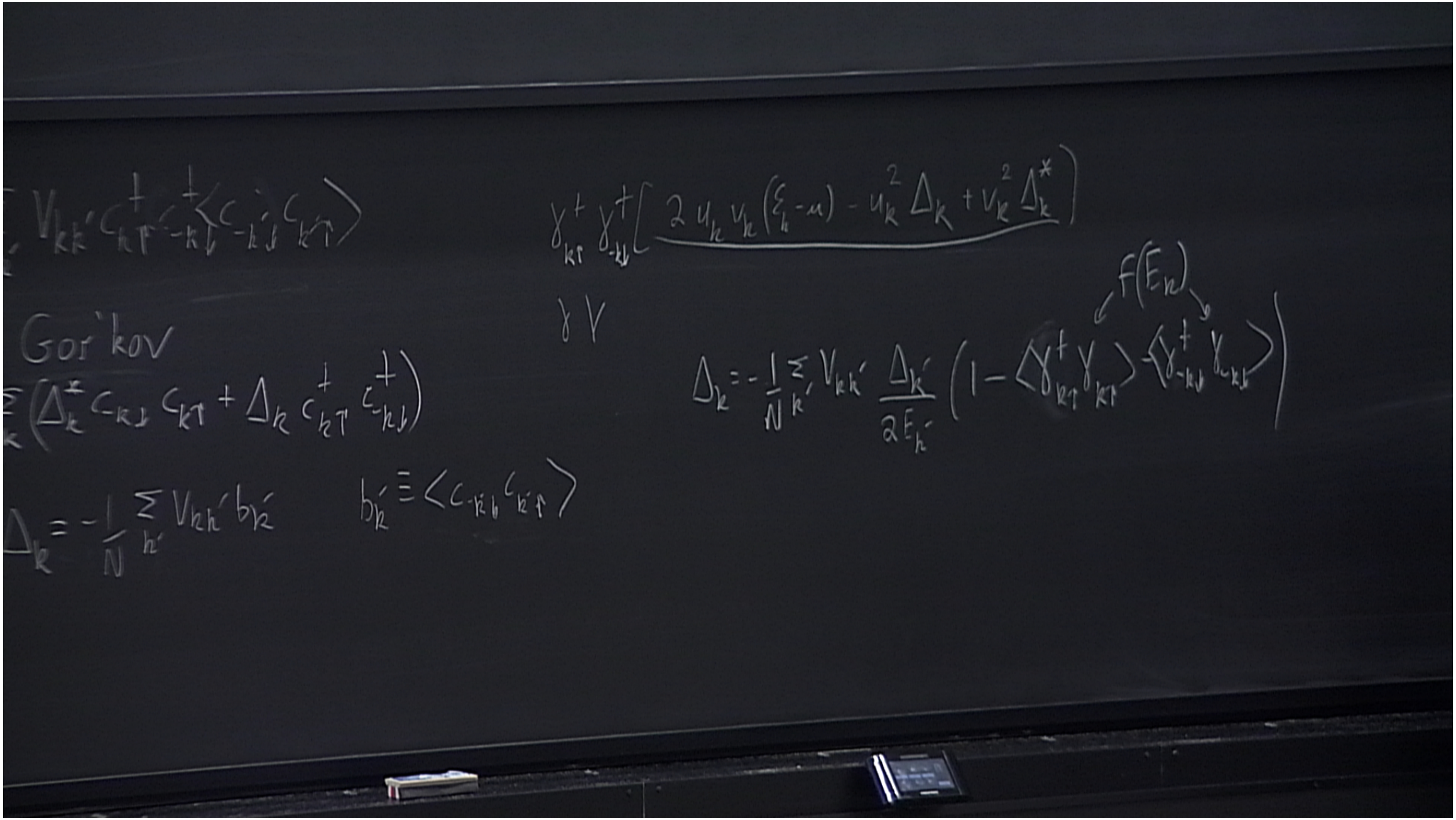
Gor'kov

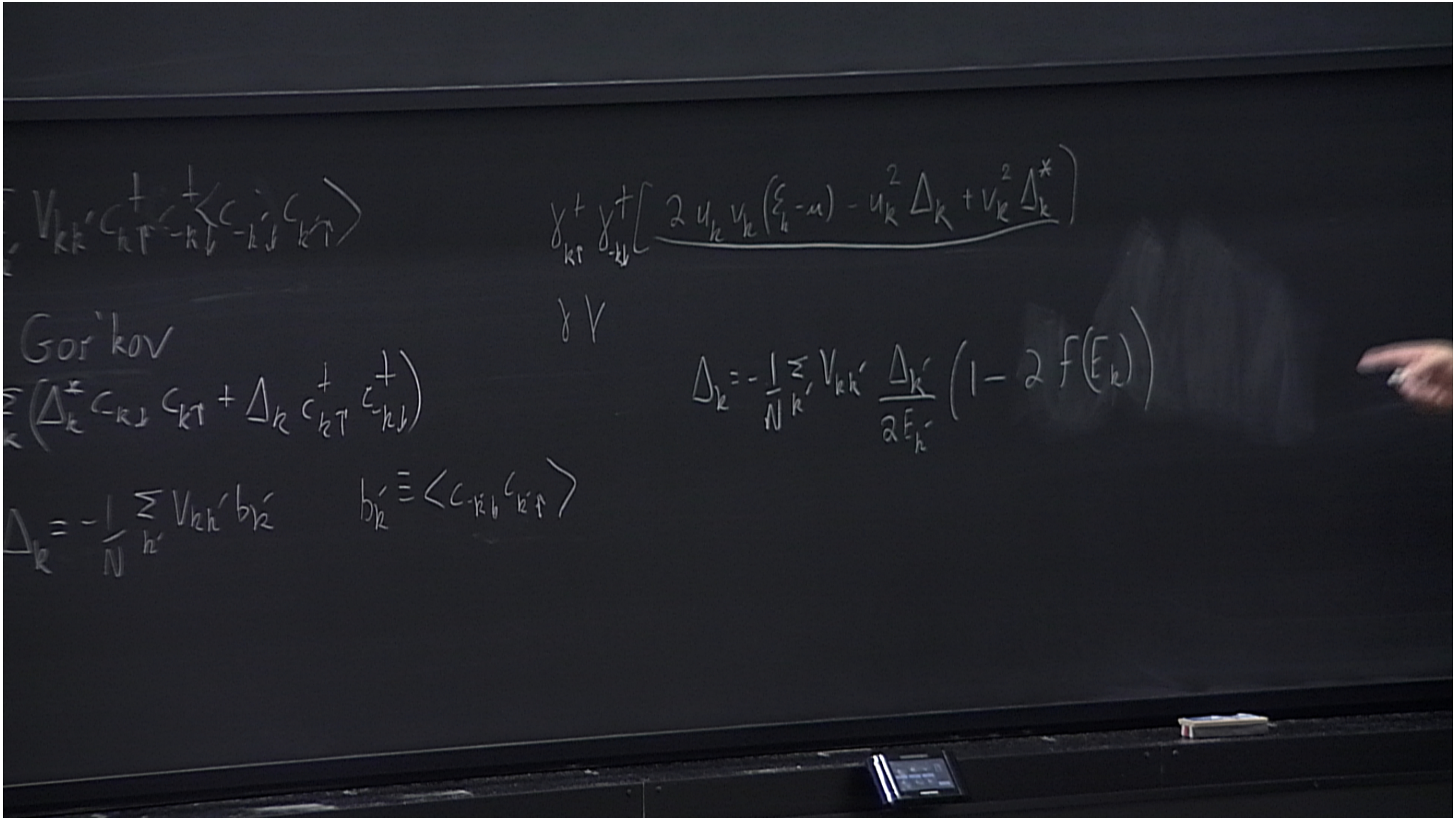
$$\sum_{k\sigma}(\varepsilon_k-\mu)c^\dagger_{k\sigma}c_{k\sigma}-\frac{1}{N}\sum_k\left(\Delta^*_k c_{-k\downarrow}c_{k\uparrow}+\Delta_k c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\right)$$

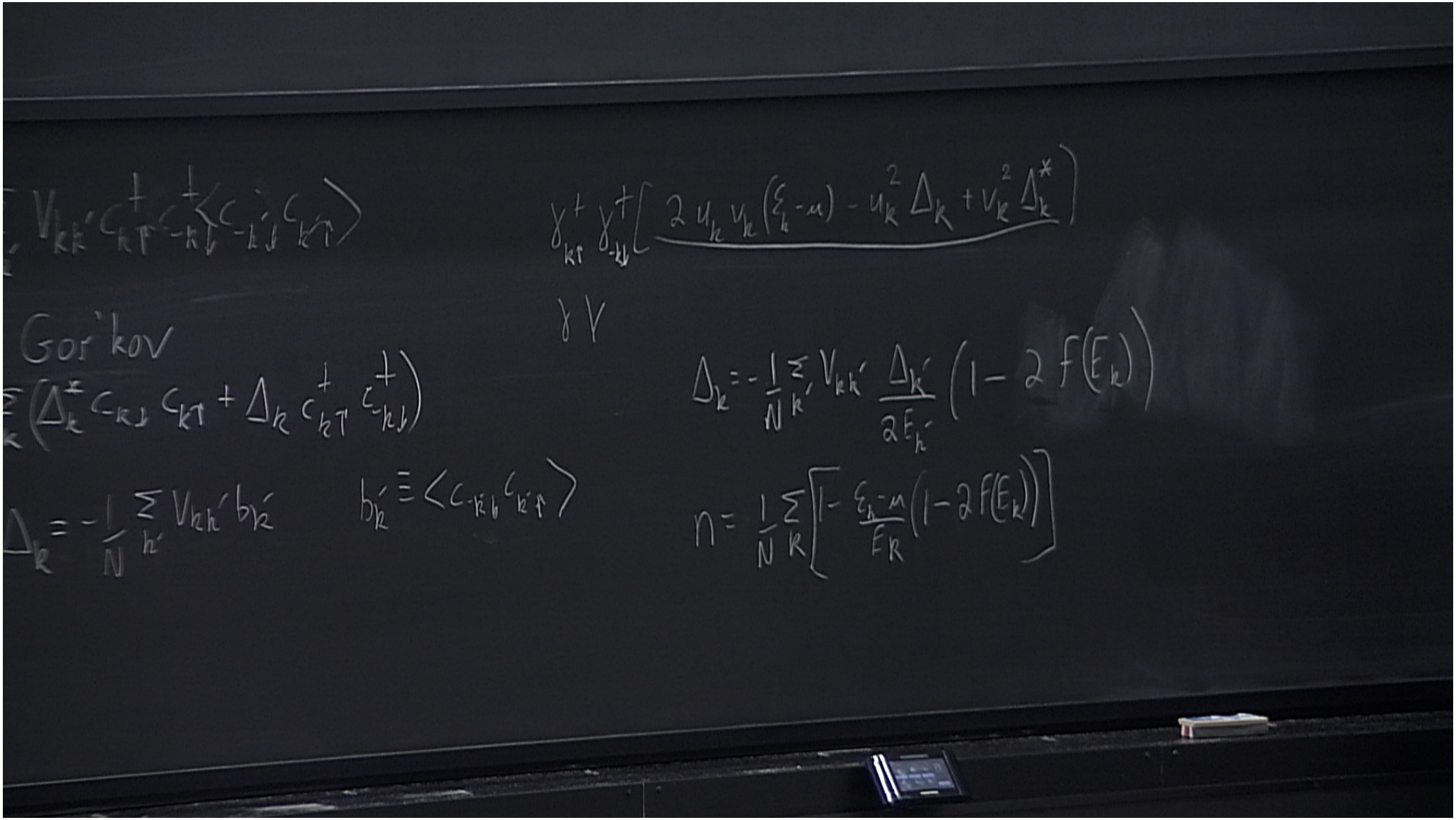
$$\Delta_k=-\frac{1}{N}\sum_{k'}V_{kk'}b_{k'} \qquad b_{k'}\equiv\langle c_{-k\downarrow}c_{k'\uparrow}\rangle$$

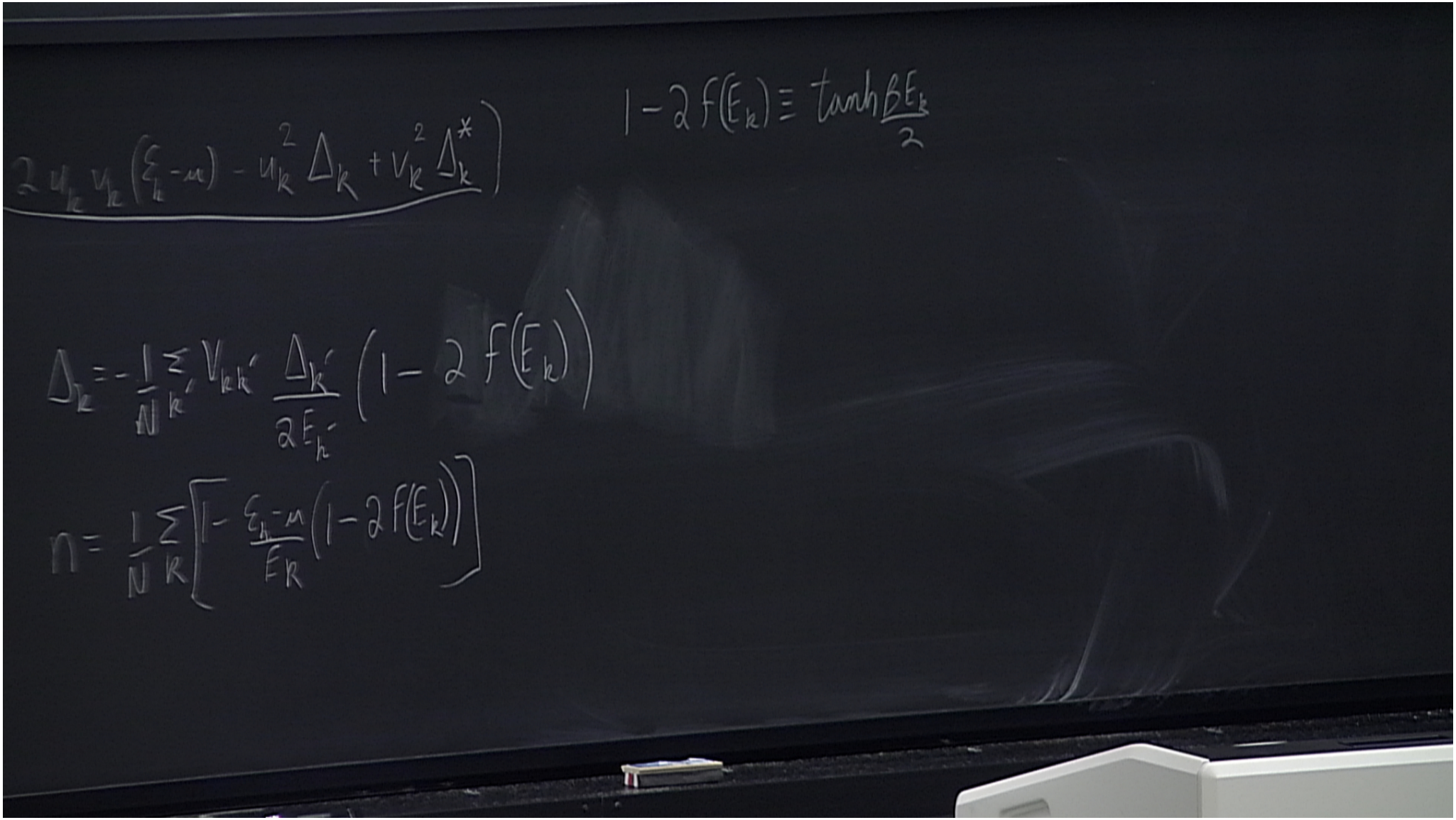

Gor'kov

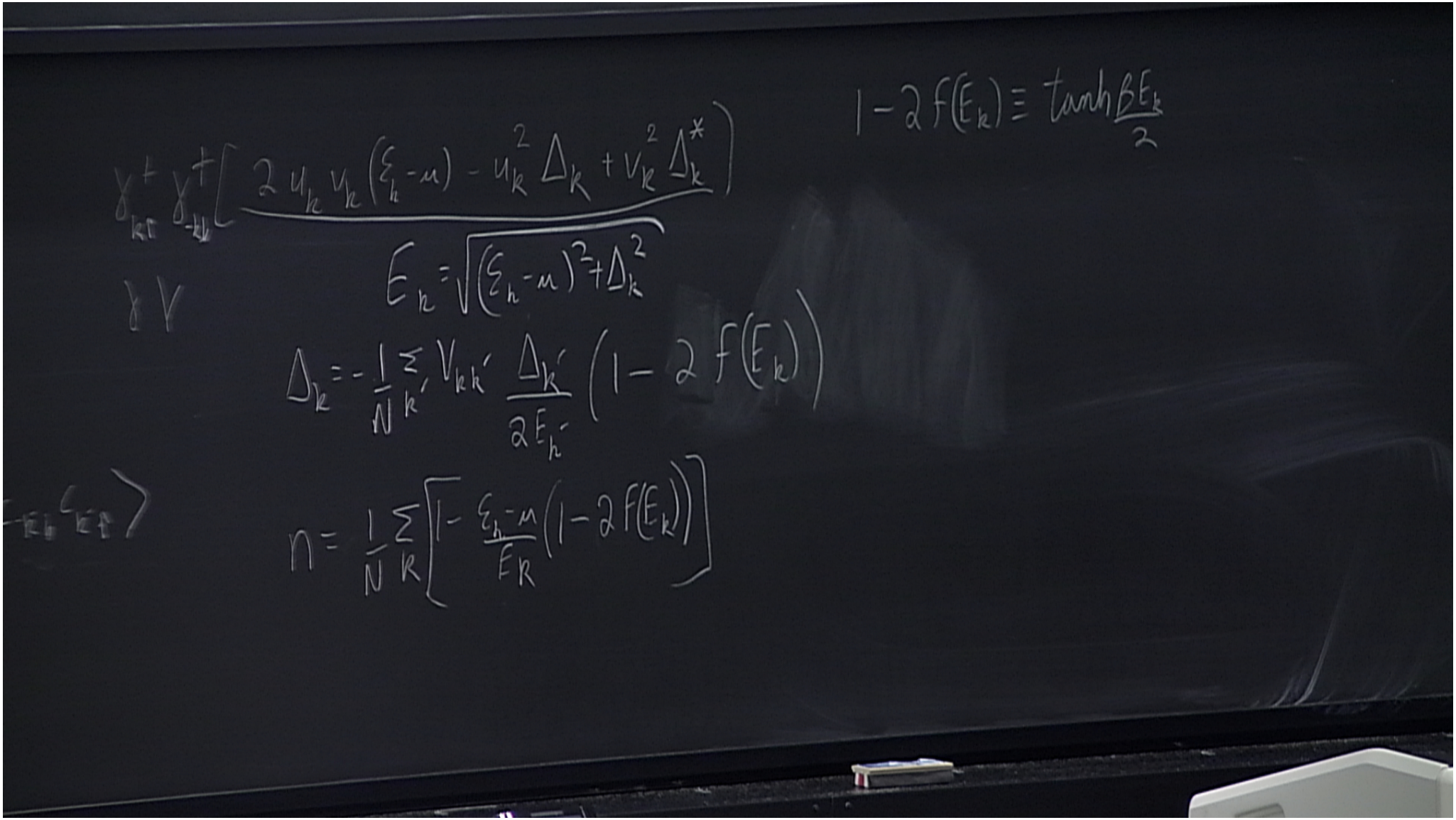

Gor'kov

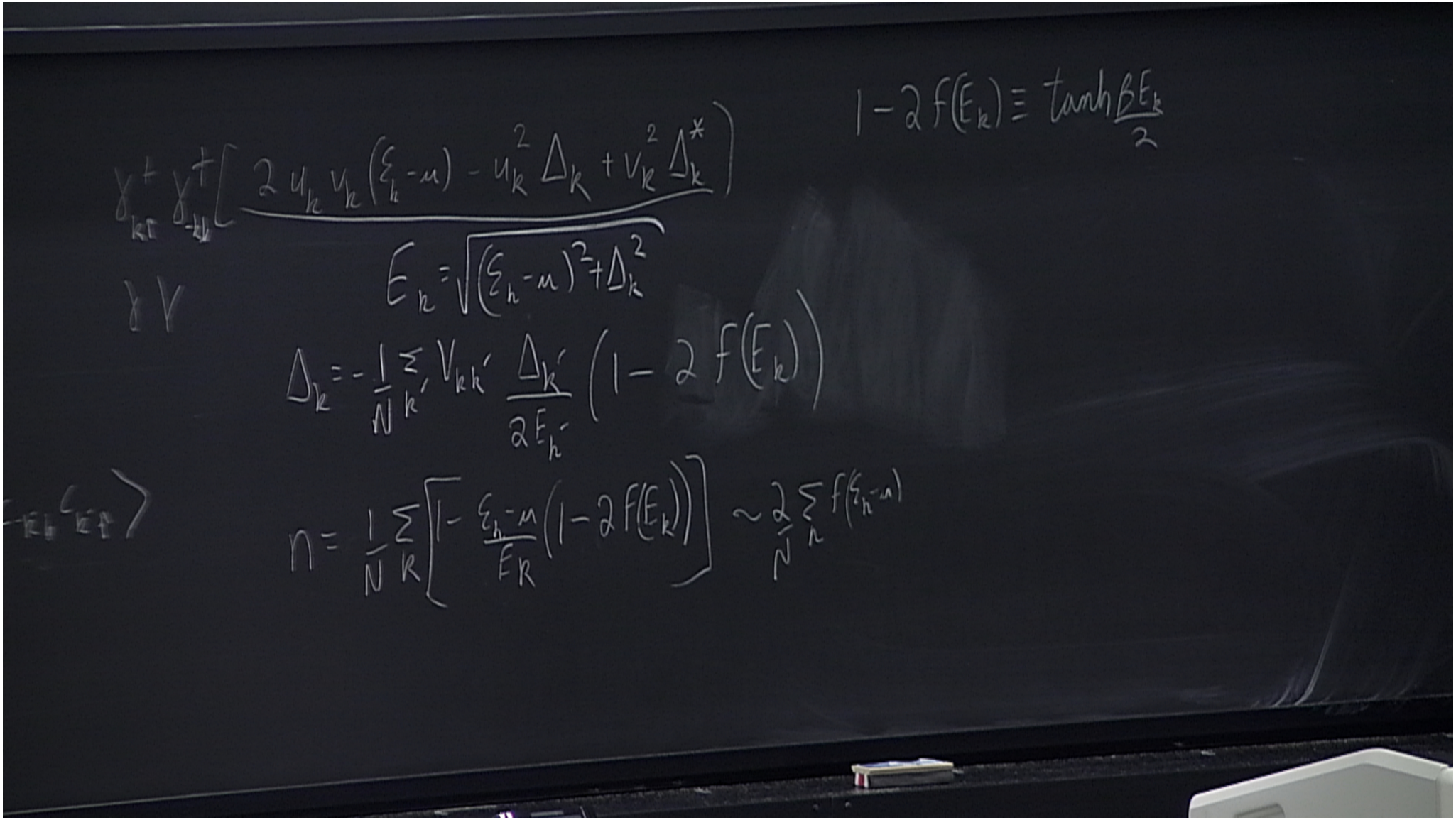

Gor'kov
$\gamma^\dagger_{k\uparrow}\,\gamma^\dagger_{-k\downarrow}\left[\underbrace{2u_k v_k(\xi_k-\mu) - u_k^2\Delta_k + v_k^2\Delta_k^*}\right)$
γV
$f(E_{-k})$
$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'}\frac{\Delta_{k'}}{2E_{k'}}\left(1-\langle\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow}\rangle-\langle\gamma^\dagger_{-k\downarrow}\gamma_{-k\downarrow}\rangle\right)$
$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} b_{k'}$
$b_{k'} \equiv \langle c_{-k\downarrow} c_{k'\uparrow}\rangle$


Gor'kov


Gor'kov
$\gamma_{k\uparrow}^{+}\gamma_{-k\downarrow}^{+}\left[2u_k v_k(\xi_k-\mu)-u_k^2\Delta_k+v_k^2\Delta_k^*\right)$
γV
$\Delta_k=-\frac{1}{N}\sum_{k'}V_{kk'}\frac{\Delta_{k'}}{2E_{k'}}\left(1-2f(E_k)\right)$
$b_{k'}\equiv\langle c_{-k\downarrow}c_{k'\uparrow}\rangle$
$n=\frac{1}{N}\sum_k\left[1-\frac{\xi_k-\mu}{E_k}\left(1-2f(E_k)\right)\right]$


$2u_k v_k(\xi_k - \mu) - u_k^2 \Delta_k + v_k^2 \Delta_k^*$
$1 - 2f(E_k) \equiv \tanh \frac{\beta E_k}{2}$
$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} (1 - 2f(E_k))$
$n = \frac{1}{N}\sum_k \left[1 - \frac{\xi_k - \mu}{E_k}(1 - 2f(E_k))\right]$


$\gamma_{k\uparrow}^{+}\,\gamma_{-k\downarrow}^{+}\left[\dfrac{2u_k v_k(\xi_k-\mu)-u_k^2\Delta_k+v_k^2\Delta_k^*}{E_k=\sqrt{(\xi_k-\mu)^2+\Delta_k^2}}\right)$
$1-2f(E_k)\equiv\tanh\frac{\beta E_k}{2}$
γV
$\Delta_k=-\frac{1}{N}\sum_{k'}V_{kk'}\frac{\Delta_{k'}}{2E_{k'}}\left(1-2f(E_k)\right)$
$n=\frac{1}{N}\sum_k\left[1-\frac{\xi_k-\mu}{E_k}\left(1-2f(E_k)\right)\right]$


$\gamma^+_{k\uparrow}\,\gamma^+_{-k\downarrow}\left[\dfrac{2u_k v_k(\xi_k-\mu) - u_k^2\Delta_k + v_k^2\Delta_k^*}{E_k=\sqrt{(\xi_k-\mu)^2+\Delta_k^2}}\right)$
γV
$1-2f(E_k) \equiv \tanh\frac{\beta E_k}{2}$
$\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'}\frac{\Delta_{k'}}{2E_{k'}}\left(1-2f(E_k)\right)$
$n = \frac{1}{N}\sum_k\left[1-\frac{\xi_k-\mu}{E_k}\left(1-2f(E_k)\right)\right] \sim \frac{2}{N}\sum_k f(\xi_k-\mu)$

invert $\hat{c}^\dagger_{p\uparrow} = u_p \hat{\gamma}^\dagger_{p\uparrow} + v_p^* \hat{\gamma}_{-p\downarrow}$

$c_{-p\downarrow} = -v_p \hat{\gamma}^\dagger_{p\uparrow} + u_p^* \hat{\gamma}_{-p\downarrow}$

$\varepsilon_{-k} = \varepsilon_k$

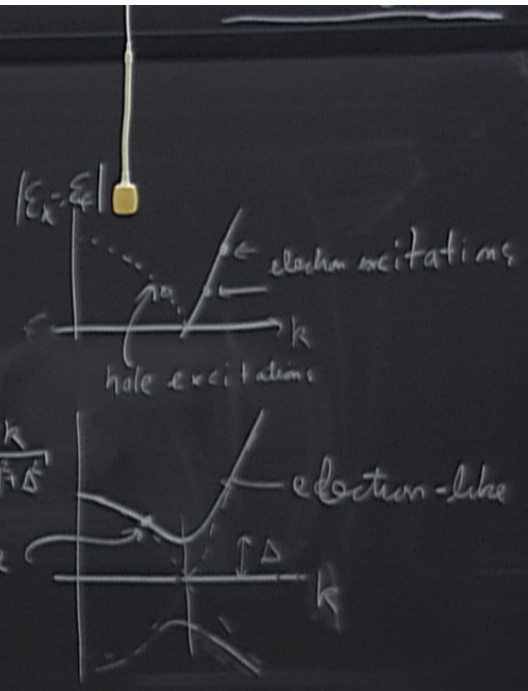
$$\hat{H} = E_G + \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow} \gamma_{k\downarrow} \right)$$

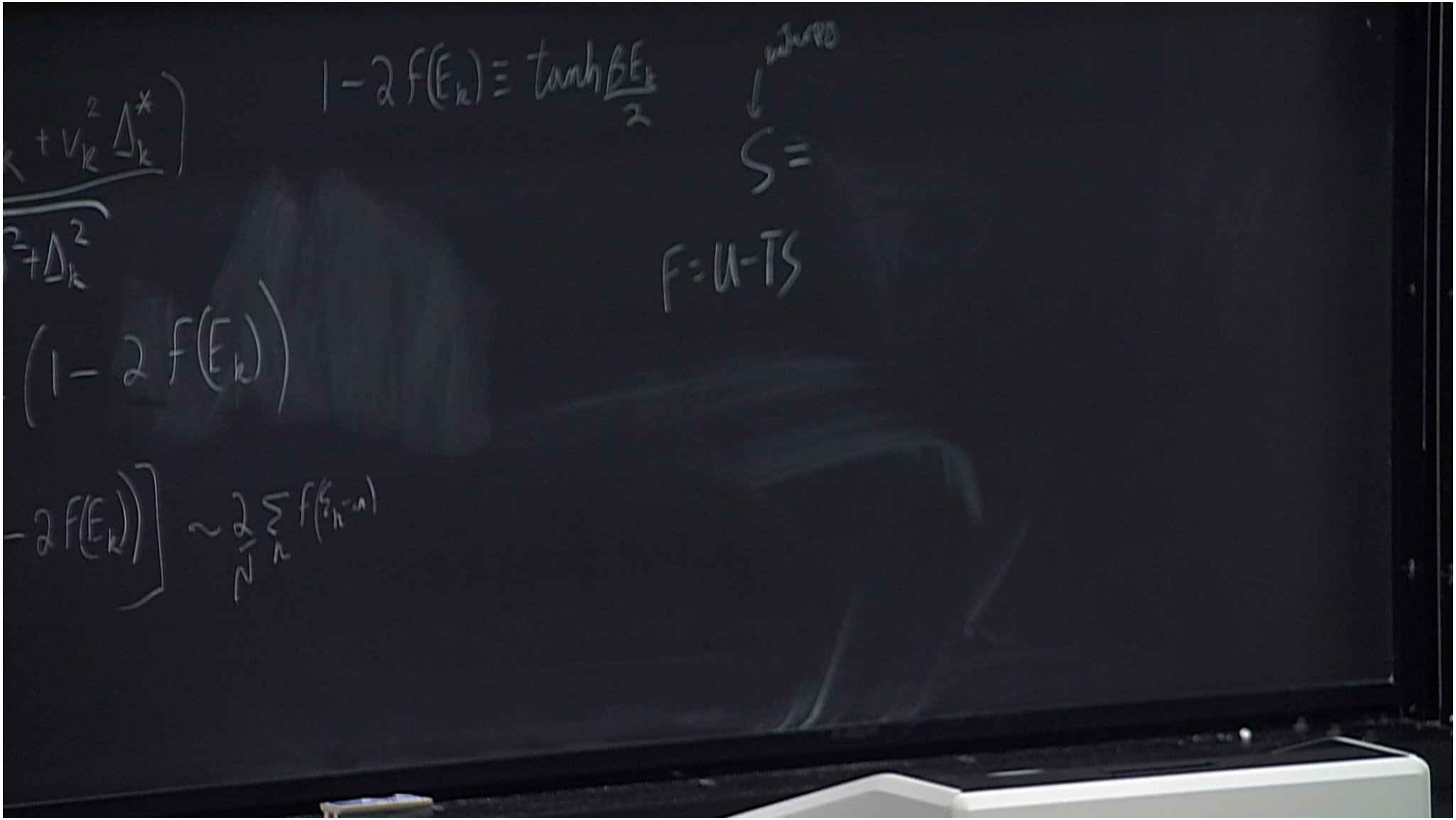
c's anticommute $\Rightarrow$ γ's anti-commute

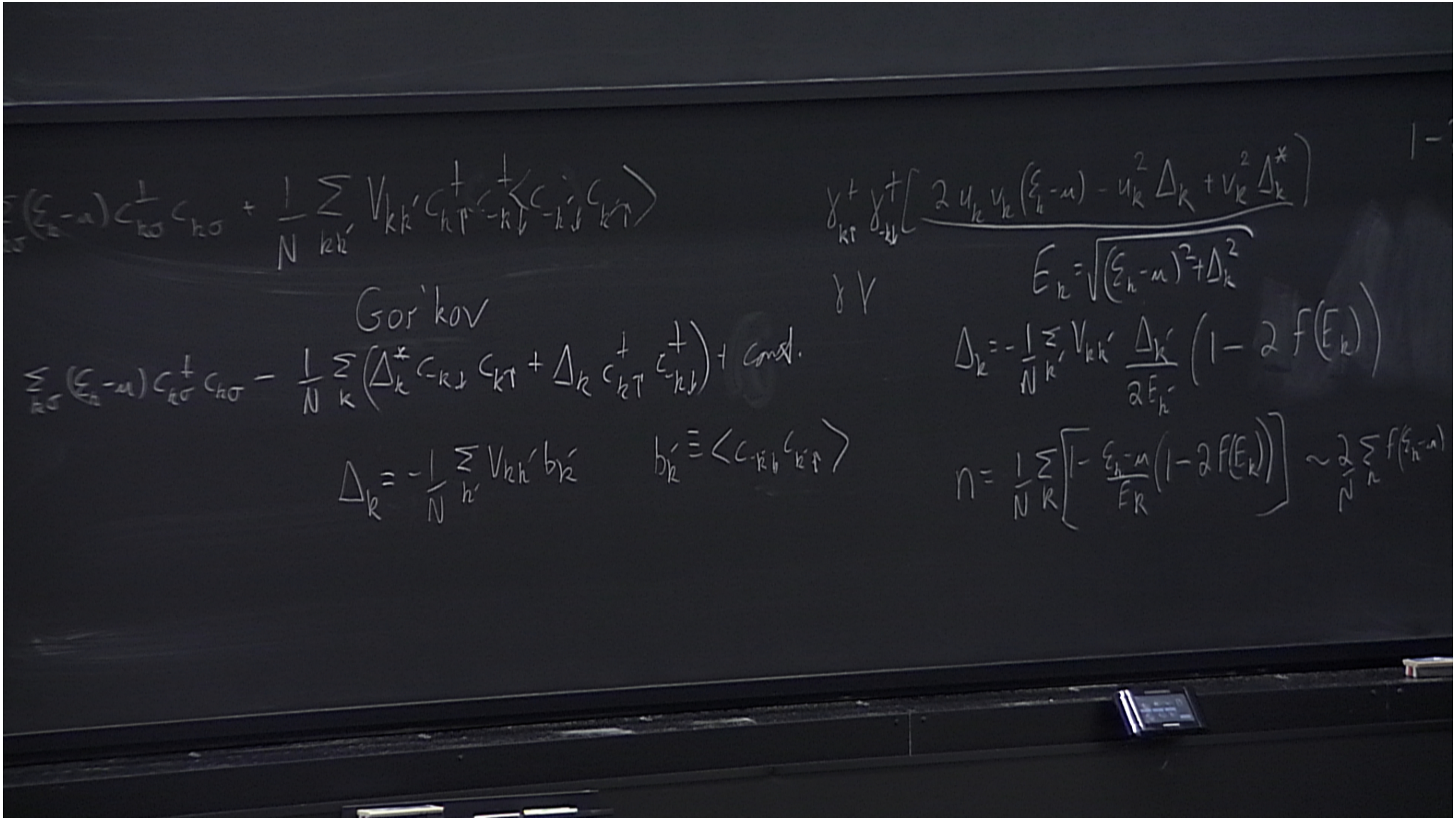
asi particle.

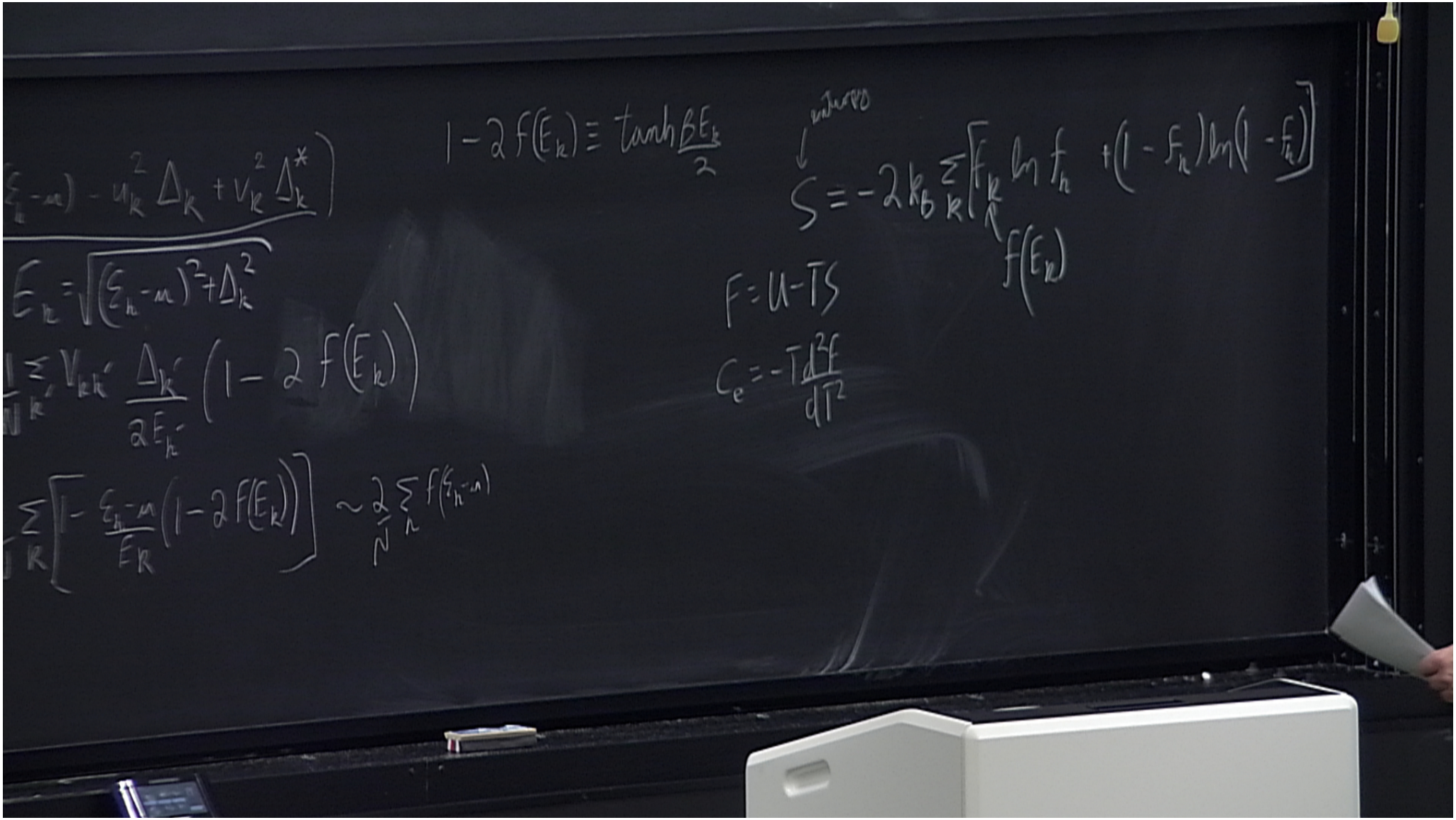
$\rangle = |\psi_{p\uparrow}\rangle$

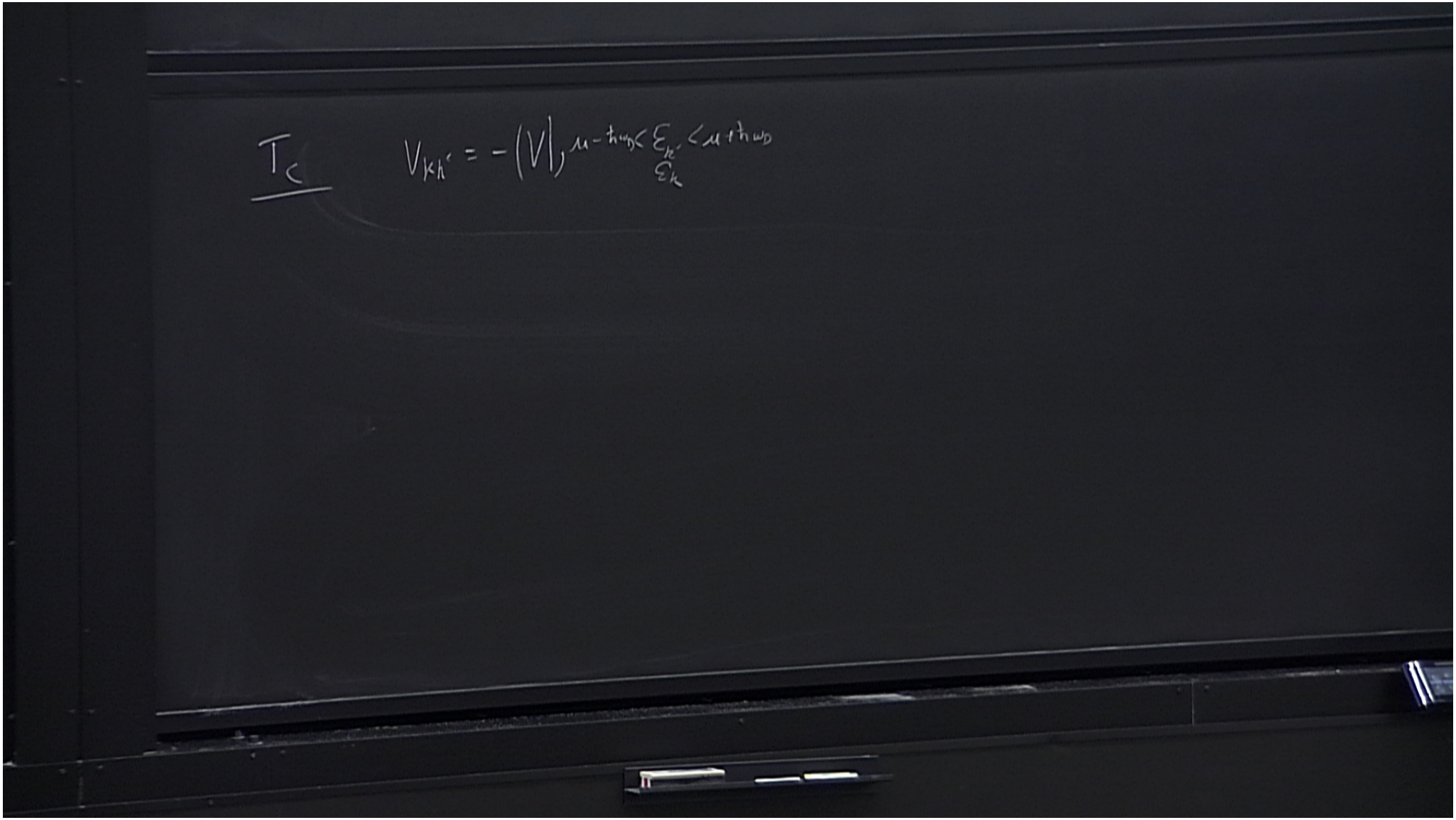
$G\rangle = 0$




$1-2f(E_k) \equiv \tanh\frac{\beta E_k}{2}$
$S =$
$F = U - TS$
$\left(1-2f(E_k)\right)$


Gor'kov


$1-2f(E_k) \equiv \tanh\frac{\beta E_k}{2}$
$E_k = \sqrt{(\xi_k-\mu)^2+\Delta_k^2}$
$F = U - TS$
$C_e = -T\frac{d^2f}{dT^2}$
$S = -2k_B \sum_k \left[f_k \ln f_k + (1-f_k)\ln(1-f_k) \right]$
$f(E_k)$


T_c
$V_{kk'} = -|V|, \quad \mu - \hbar\omega_D < \varepsilon_{k'}, \varepsilon_k < \mu + \hbar\omega_D$

T_c

$$V_{kk'} = -|V|, \quad \mu - \hbar\omega_D < \begin{matrix} \varepsilon_{k'} \\ \varepsilon_k \end{matrix} < \mu + \hbar\omega_D$$

weak coupling $\Rightarrow$ μ stays fixed

T_c

$$V_{kk'} = -|V|, \quad \mu - \hbar\omega_D < \begin{matrix}\varepsilon_{k'} \\ \varepsilon_k\end{matrix} < \mu + \hbar\omega_D$$

weak coupling $\Rightarrow$ μ stays fixed

$$\Delta = |V| g(E_F) \int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\varepsilon \, \frac{\Delta}{2\sqrt{(\varepsilon-\mu)^2+\Delta^2}} \tanh\left(\frac{\sqrt{(\varepsilon-\mu)^2+\Delta^2}}{2k_B T_c}\right)$$

weak coupling $\Rightarrow \mu$ stays fixed

$$\Delta = |V| g(E_F) \int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\xi \, \frac{\Delta}{2\sqrt{(\xi-\mu)^2+\Delta^2}} \tanh\left(\frac{\sqrt{(\xi-\mu)^2+\Delta^2}}{2k_B T_c}\right)$$

T_c

$V_{kk'} = -|V|, \quad \mu - \hbar\omega_D < \begin{matrix} \varepsilon_{k'} \\ \varepsilon_k \end{matrix} < \mu + \hbar\omega_D$

weak coupling $\Rightarrow$ μ stays fixed

$$\Delta = |V| g(\varepsilon_F) \int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\varepsilon \, \frac{\Delta}{2\sqrt{(\varepsilon-\mu)^2+\Delta^2}} \tanh\left(\frac{\sqrt{(\varepsilon-\mu)^2+\Delta^2}}{2k_B T_c}\right)$$

$$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\varepsilon}{2k_B T_c}\right)}{\varepsilon} \qquad x = \frac{\varepsilon}{2k_B T_c}$$

$$= \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \, \frac{\tanh x}{x}$$

$$= |V| g(E_F) \int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\varepsilon \, \frac{\Delta}{2\sqrt{(\varepsilon-\mu)^2+\Delta^2}} \tanh\left(\frac{\sqrt{(\varepsilon-\mu)^2+\Delta^2}}{2k_B T_c}\right)$$

$$\frac{}{g(E_F)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\varepsilon}{2k_B T_c}\right)}{\varepsilon} \qquad x = \frac{\varepsilon}{2k_B T_c}$$

$$= \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \, \frac{\tanh x}{x}$$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

$$\tanh \frac{\hbar\omega_D}{2k_B T_c} \ln \frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$$

$$\frac{\tanh x}{x}$$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

$$\frac{1}{g(\varepsilon_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$$

$\frac{\tanh x}{x}$

$k_B T_c \sim 1.5\ meV$

$\hbar\omega_D \sim 10\text{-}25\ meV$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

$$\frac{1}{g(E_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$$

$\frac{\tanh x}{x}$

$u = \tanh x \qquad dv = \frac{dx}{x}$

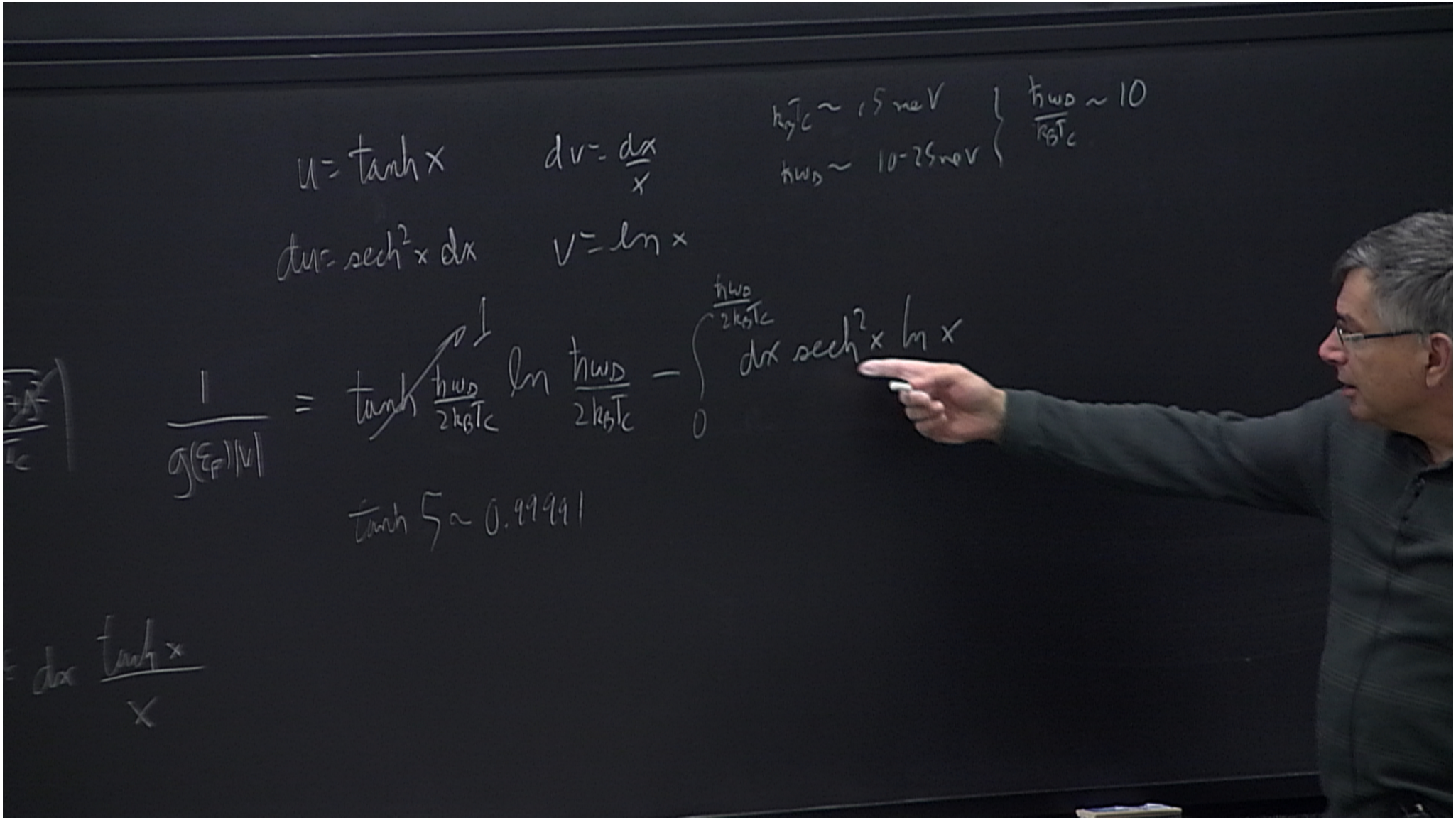
$du = \operatorname{sech}^2 x \, dx \qquad v = \ln x$

$k_B T_c \sim .5 \text{ meV}$

$\hbar\omega_D \sim 10\text{-}25 \text{ meV}$

$\left.\begin{array}{l} k_B T_c \sim .5 \text{ meV} \\ \hbar\omega_D \sim 10\text{-}25 \text{ meV} \end{array}\right\} \frac{\hbar\omega_D}{k_B T_c} \sim 10$

$$= \tanh \frac{\hbar\omega_D}{2k_B T_c} \ln \frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$$


$u = \tanh x$
$dv = \frac{dx}{x}$
$du = \operatorname{sech}^2 x\, dx$
$v = \ln x$
$k_B T_c \sim 1.5\ meV$
$\hbar\omega_D \sim 10\text{-}25\ meV$
$\frac{\hbar\omega_D}{k_B T_c} \sim 10$
$\frac{1}{g(E_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$
$\tanh 5 \sim 0.9991$
$\int dx \frac{\tanh x}{x}$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$k_B T_c \sim 1.5\ meV$

$\hbar\omega_D \sim 10\text{-}25\ meV$ $\Big\}$ $\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

$$\frac{1}{g(E_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx \operatorname{sech}^2 x \ln x$$

$\tanh\frac{\hbar\omega_D}{2k_B T_c} \to 1$

$\tanh 5 \sim 0.9991$

~ 0.0002

$\int dx \frac{\tanh x}{x}$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$k_B T_c \sim 1.5\ meV$, $\hbar\omega_D \sim 10\text{-}25\ meV$ $\Rightarrow \frac{\hbar\omega_D}{k_B T_c} \sim 10$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

$$\frac{1}{g(\varepsilon_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\infty} dx \operatorname{sech}^2 x \ln x$$

$\tanh 5 \sim 0.9991$

$\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$\int dx \frac{\tanh x}{x}$

$u = \tanh x \qquad dv = \frac{dx}{x}$

$du = \operatorname{sech}^2 x\, dx \qquad v = \ln x$

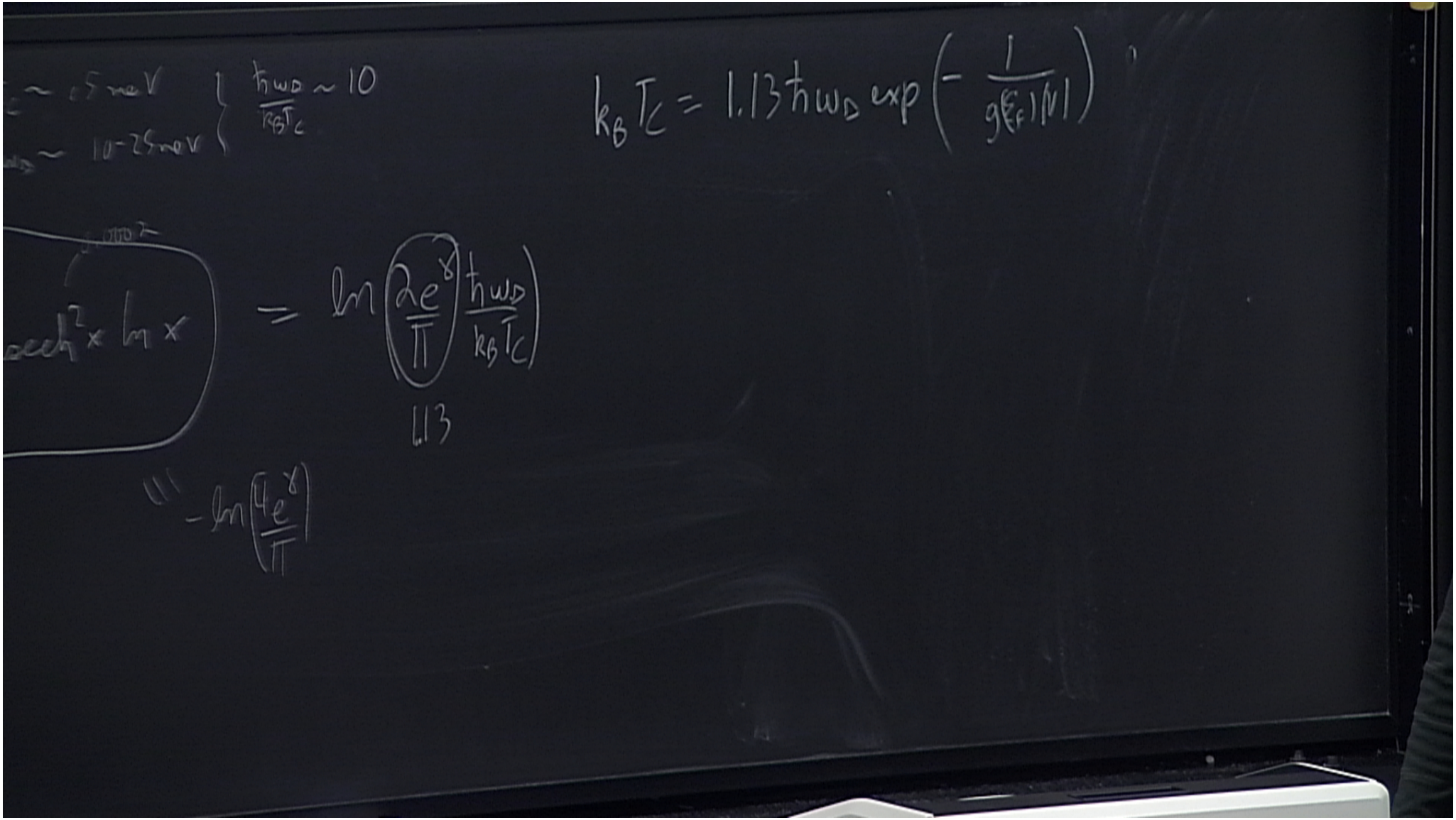
$k_B T_c \sim 1.5\ meV$, $\hbar\omega_D \sim 10\text{-}25\ meV$ $\Rightarrow \frac{\hbar\omega_D}{k_B T_c} \sim 10$

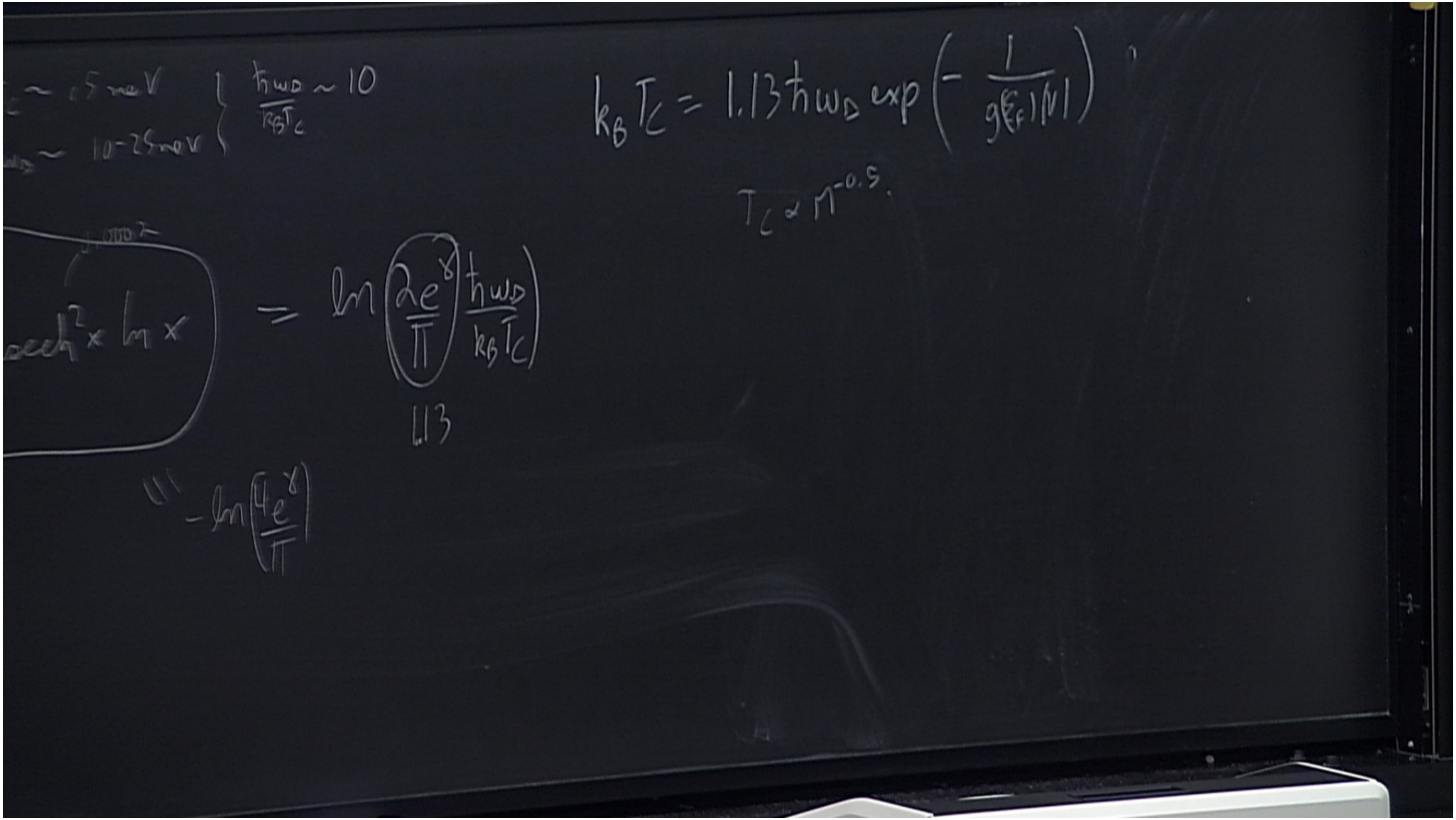
$$\frac{1}{g(E_F)|V|} = \tanh\frac{\hbar\omega_D}{2k_B T_c} \ln\frac{\hbar\omega_D}{2k_B T_c} - \int_0^{\frac{\hbar\omega_D}{2k_B T_c}} dx\, \operatorname{sech}^2 x \ln x = \ln\left(\frac{2e^{\gamma}}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$$

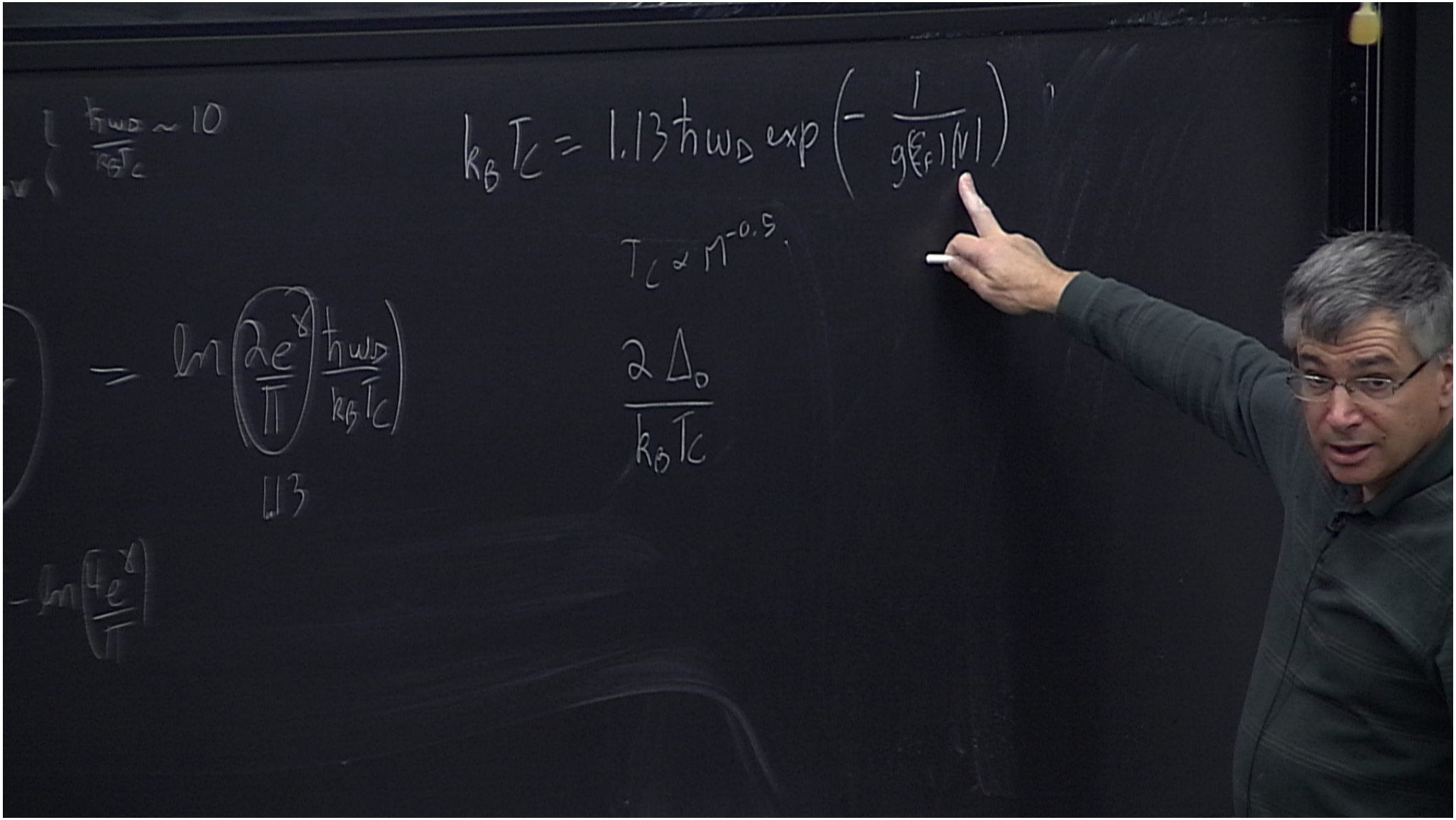
$\tanh 5 \sim 0.9991$

$= -\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$= \int dx \frac{\tanh x}{x}$


~ .5 meV
~ 10-25 meV
$\frac{\hbar\omega_D}{k_B T_c} \sim 10$
$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$
$\operatorname{sech}^2 x \ln x$
$= \ln\left(\frac{2e^\gamma}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$
1.13
$-\ln\left(\frac{4e^\gamma}{\pi}\right)$


$\sim$ 5 meV
$\sim$ 10-25 meV
$\frac{\hbar\omega_D}{k_B T_c} \sim 10$
$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$
$T_c \propto M^{-0.5}$
$\operatorname{sech}^2 x \ln x$
$= \ln\left(\frac{2e^\gamma}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$
1.13
$-\ln\left(\frac{4e^\gamma}{\pi}\right)$


$\frac{\hbar\omega_D}{k_B T_c} \sim 10$
$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$
$T_c \propto M^{-0.5}$
$= \ln\left(\frac{2e^{\gamma}}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$
1.13
$\frac{2\Delta_0}{k_B T_c}$
$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$

$T_c \propto M^{-0.5}$.

$= \ln\left(\frac{2e^{\gamma}}{\pi} \frac{\hbar\omega_D}{k_B T_c}\right)$

1.13

$\frac{2\Delta_0}{k_B T_c} = 3.53$ universal!

$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$$

$T_c \propto M^{-0.5}$.

$$= \ln\left(\frac{2e^{\gamma}}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$$

1.13

$$\frac{2\Delta_0}{k_B T_c} = 3.53$$

universal!

$$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$$

$\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$

$T_c \propto M^{-0.5}$.

$= \ln\left(\frac{2e^{\gamma}}{\pi} \frac{\hbar\omega_D}{k_B T_c}\right)$

1.13

$\frac{2\Delta_0}{k_B T_c} = 3.53$

universal ΔC. Cen

C_{es}

C_{en}

T/T_c

$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$$

$T_c \propto M^{-0.5}$

$$= \ln\left(\frac{2e^{\gamma}}{\pi}\frac{\hbar\omega_D}{k_B T_c}\right)$$

1.13

$$\frac{2\Delta_0}{k_B T_c} = 3.53$$

universal

ΔC

C_{en}

C_{es}

C_{en}

γT_c

T/T_c

$$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$$

$\frac{\hbar\omega_D}{k_B T_c} \sim 10$

$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$

$\frac{\Delta C}{\gamma T_c} = 1.43$

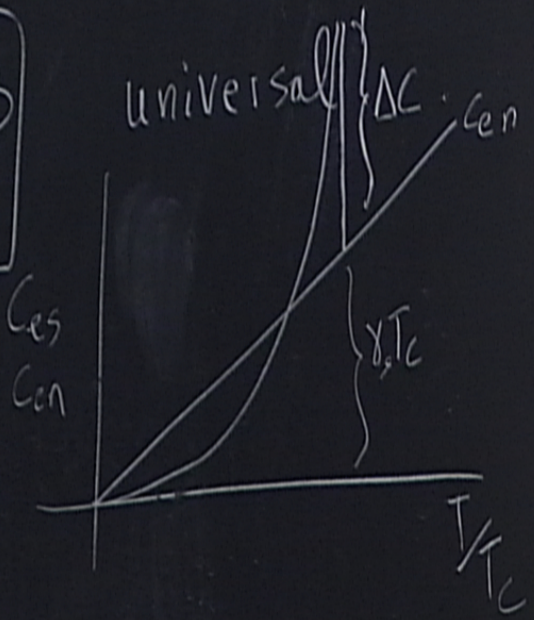
$T_c \propto M^{-0.5}$.

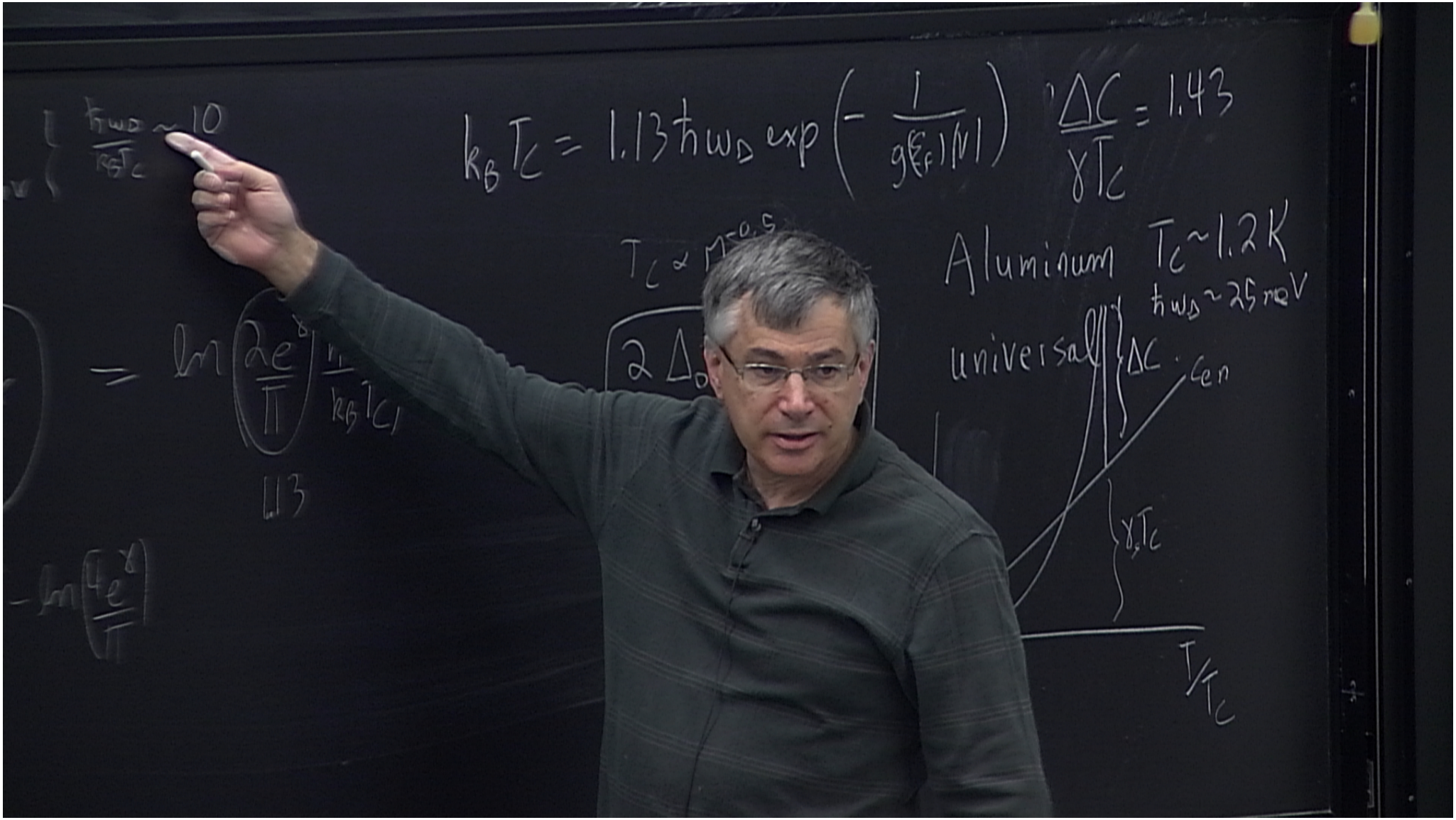
$= \ln\left(\frac{2e^{\gamma}}{\pi} \frac{\hbar\omega_D}{k_B T_c}\right)$

1.13

$\frac{2\Delta_0}{k_B T_c} = 3.53$

$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$




$\frac{\hbar\omega_D}{k_B T_c} \sim 10$
$k_B T_c = 1.13 \hbar\omega_D \exp\left(-\frac{1}{g(E_F)|V|}\right)$
$\frac{\Delta C}{\gamma T_c} = 1.43$
$T_c \propto M^{-0.5}$
Aluminum $T_c \sim 1.2$ K
$\hbar\omega_D \sim 25$ meV
universal
ΔC
γT_c
T/T_c
$= \ln\left(\frac{2e^{\gamma}}{\pi}\right) \frac{1}{k_B T_c}$
1.13
$-\ln\left(\frac{4e^{\gamma}}{\pi}\right)$

$T_c \propto M^{-0.5}$

γ_c

Aluminum $T_c \sim 1.2\,K$

$\hbar\omega_D \sim 25\,meV$

$$\frac{2\Delta_0}{k_B T_c} = 3.53$$

universal

ΔC

C_{en}

C_{es}

C_{en}

$\gamma_c T_c$

T_c

$$\frac{1}{|V|\, g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T)}}{2 k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}}$$

$$\frac{1}{|V|\, g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}}$$

$$\frac{\Delta(T)}{\Delta(0)} = h\left(\quad\right)$$

$$\frac{1}{|V|\, g(\varepsilon_f)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}}$$

$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}\right)$$

$$\frac{1}{|V|\, g(\varepsilon_F)} = \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}}$$

$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, |V| g(\varepsilon_F)\right)$$

$$\frac{1}{(V/g(E_F))} = \int_0^{\hbar\omega_D} d\varepsilon \frac{\tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}}$$

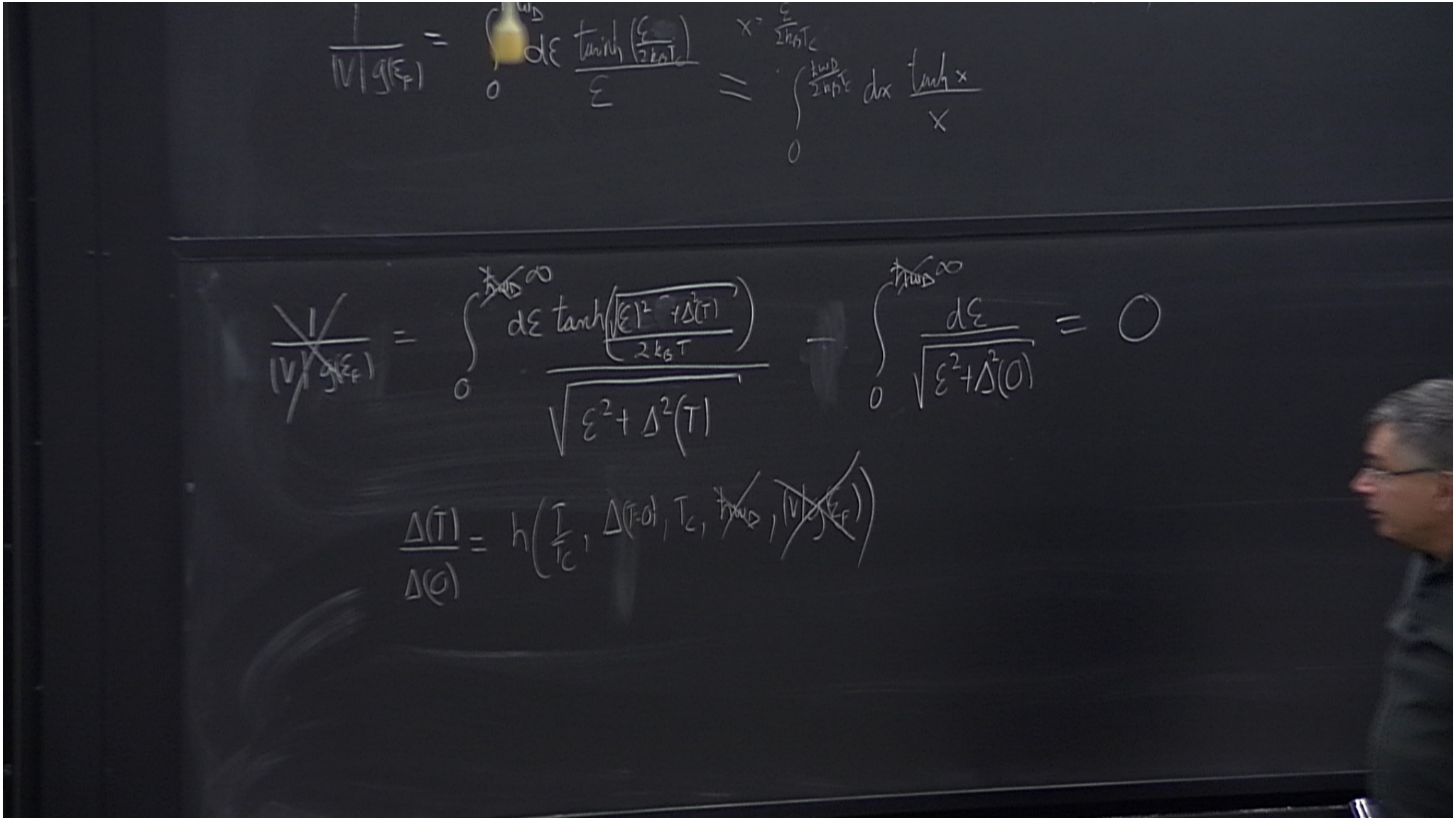
$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, (V/g(E_F))\right)$$

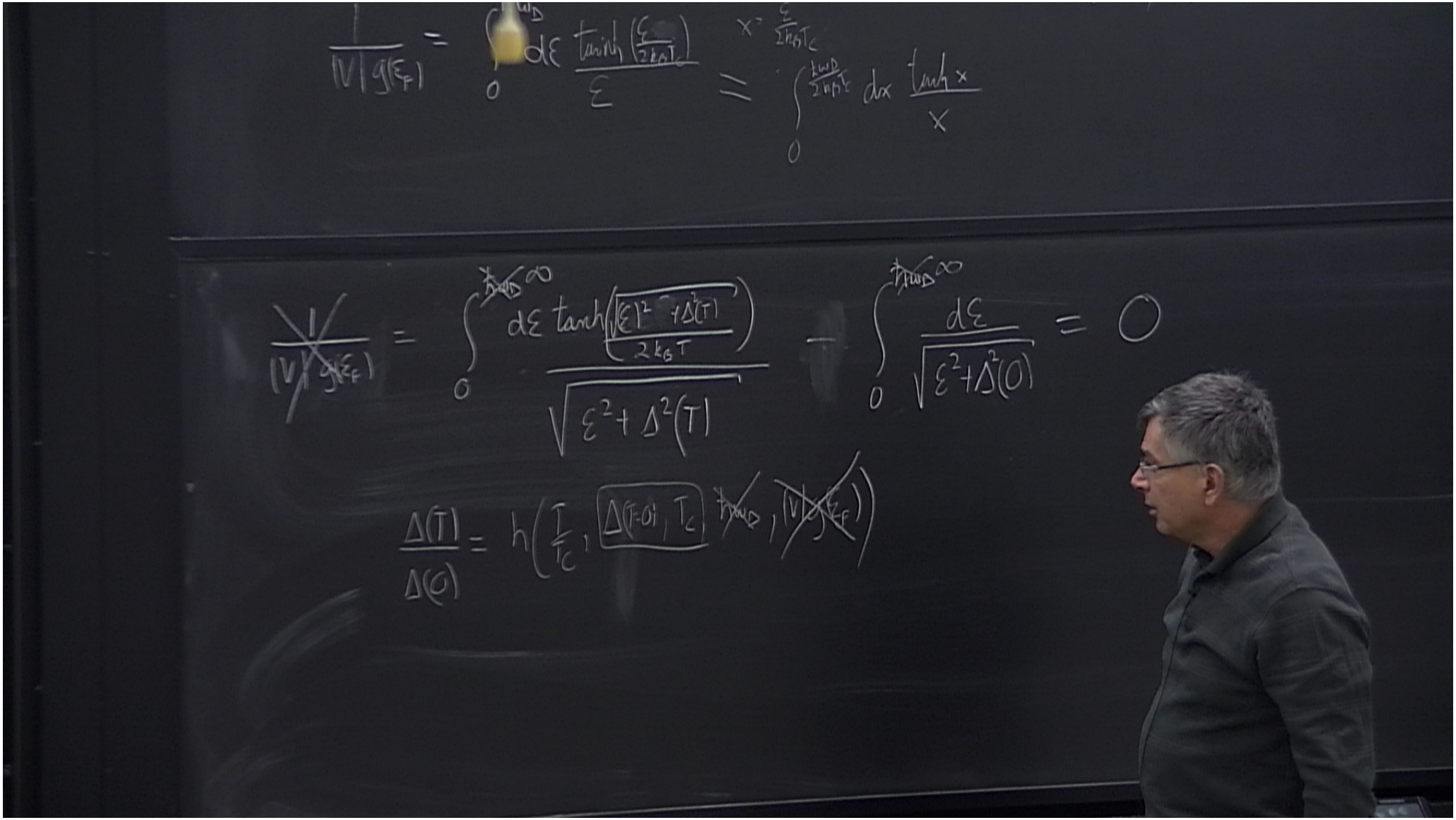
$$\frac{1}{(V)\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D} \frac{d\varepsilon \tanh\left(\frac{\sqrt{(\varepsilon)^2 + \Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2(0)}}$$

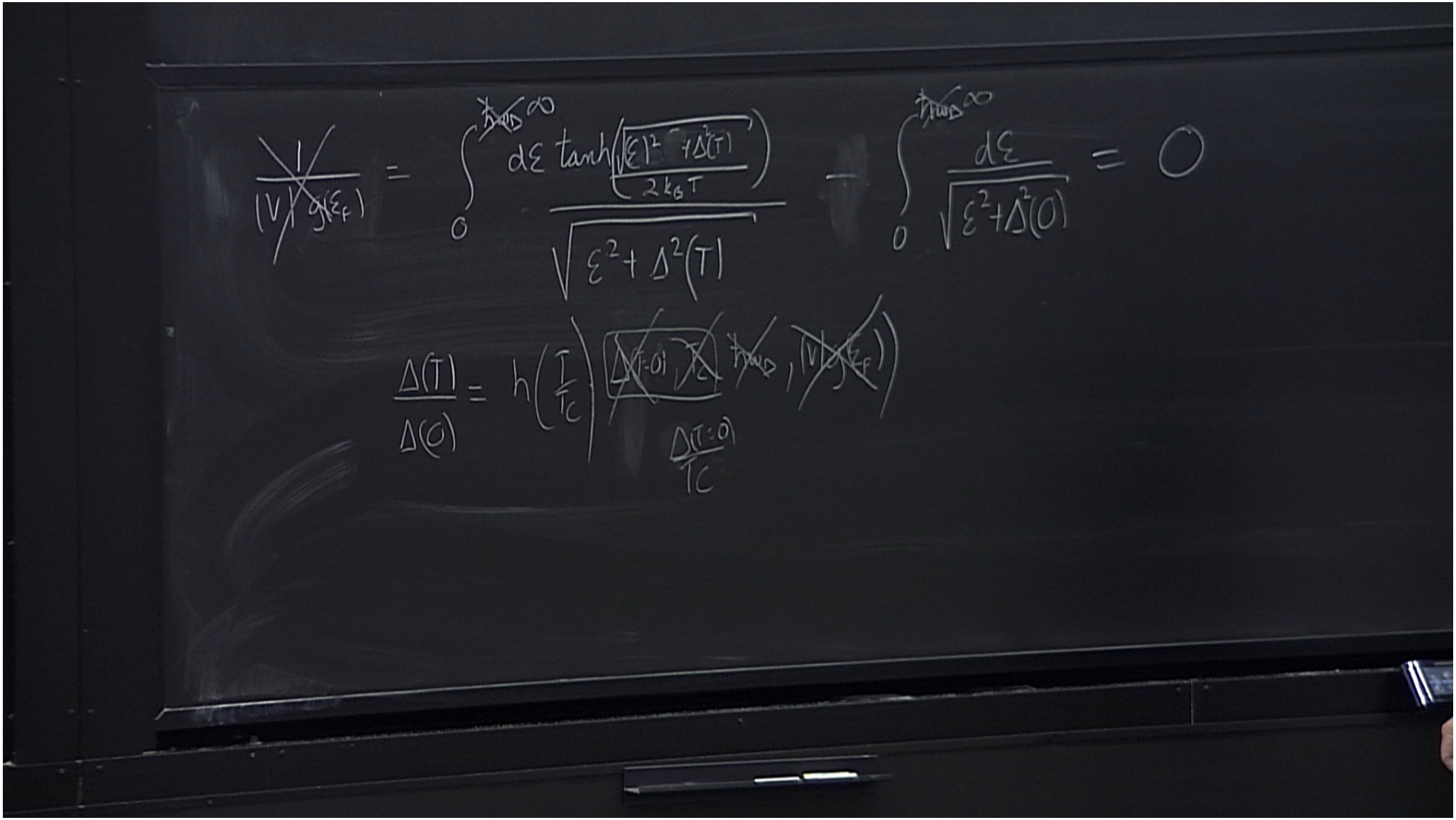
$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, (V)\,g(\varepsilon_F)\right)$$

$$\frac{1}{(V)\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D} \frac{d\varepsilon \tanh\left(\frac{\sqrt{(\varepsilon)^2 + \Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2 + \Delta^2(T)}} - \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2(0)}} = 0$$

$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, (V)\,g(\varepsilon_F)\right)$$

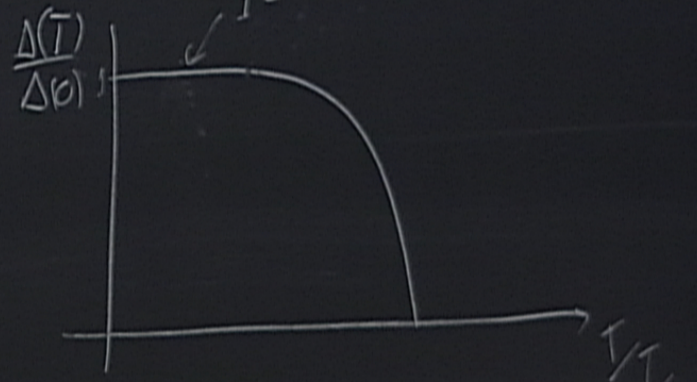

$\frac{1}{|V|\,g(\varepsilon_F)} = \int_0^{\omega_D} d\varepsilon \frac{\tanh\left(\frac{\varepsilon}{2k_BT_c}\right)}{\varepsilon} = \int_0^{\frac{\hbar\omega_D}{2\pi k_BT_c}} dx \frac{\tanh x}{x}$
$x = \frac{\varepsilon}{2\pi k_BT_c}$
$\frac{1}{|V|\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D\to\infty} \frac{d\varepsilon \tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D\to\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$
$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, |V|\,g(\varepsilon_F)\right)$


$\frac{1}{|V|\,g(E_F)} = \int_0^{\omega_D} d\varepsilon \frac{\tanh\left(\frac{\varepsilon}{2k_BT}\right)}{\varepsilon} \quad x = \frac{\varepsilon}{2k_BT_c}$
$= \int_0^{\frac{\hbar\omega_D}{2k_BT}} dx \frac{\tanh x}{x}$
$\frac{1}{|V|\,g(E_F)} = \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon \tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$
$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \Delta(T=0), T_c, \hbar\omega_D, |V|\,g(E_F)\right)$


$\frac{1}{(V)\, g(\varepsilon_F)} = \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon \tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_B T}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$
$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_C}\right)$
$\Delta(T=0), \varepsilon, \hbar\omega_D, (V)\, g(\varepsilon_F)$
$\frac{\Delta(T=0)}{T_C}$

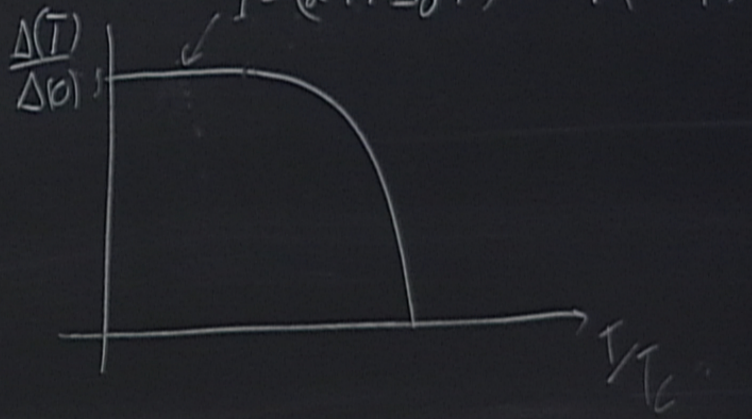
$$\frac{1}{(V)\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D \to \infty} d\varepsilon \frac{\tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$$

$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}\right) \quad \left(\frac{\Delta(T=0)}{T_c}, \hbar\omega_D, (V)\,g(\varepsilon_F)\right)$$



$$\frac{1}{(V)\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D \to \infty} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$$

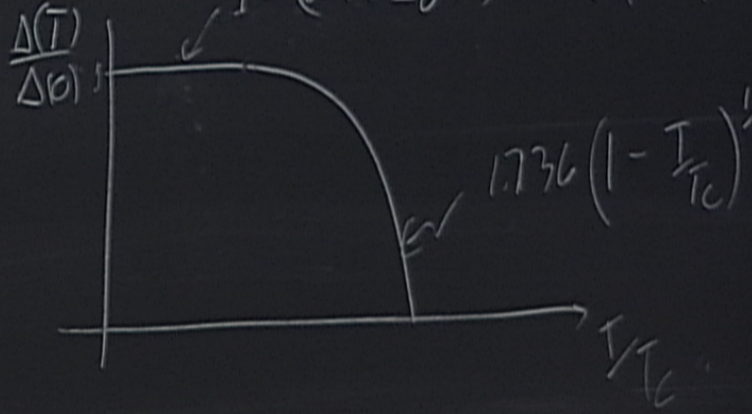
$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}, \frac{\Delta(T=0)}{T_c}, \hbar\omega_D, (V)\,g(\varepsilon_F)\right)$$



$$\frac{1}{(V)\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D \to \infty} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$$

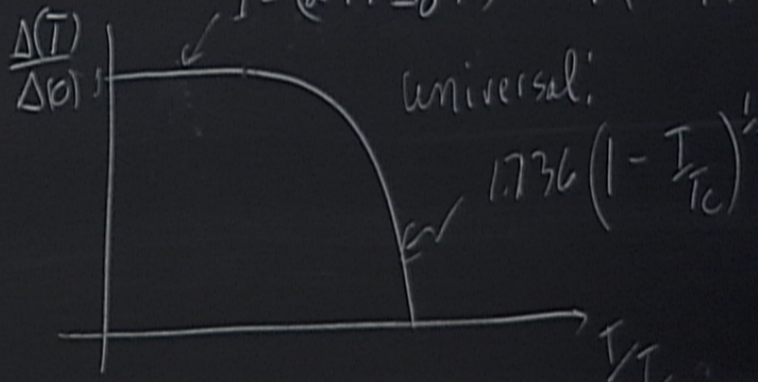
$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_C}, \frac{\Delta(T=0)}{T_C}, \hbar\omega_D, (V)\,g(\varepsilon_F)\right)$$

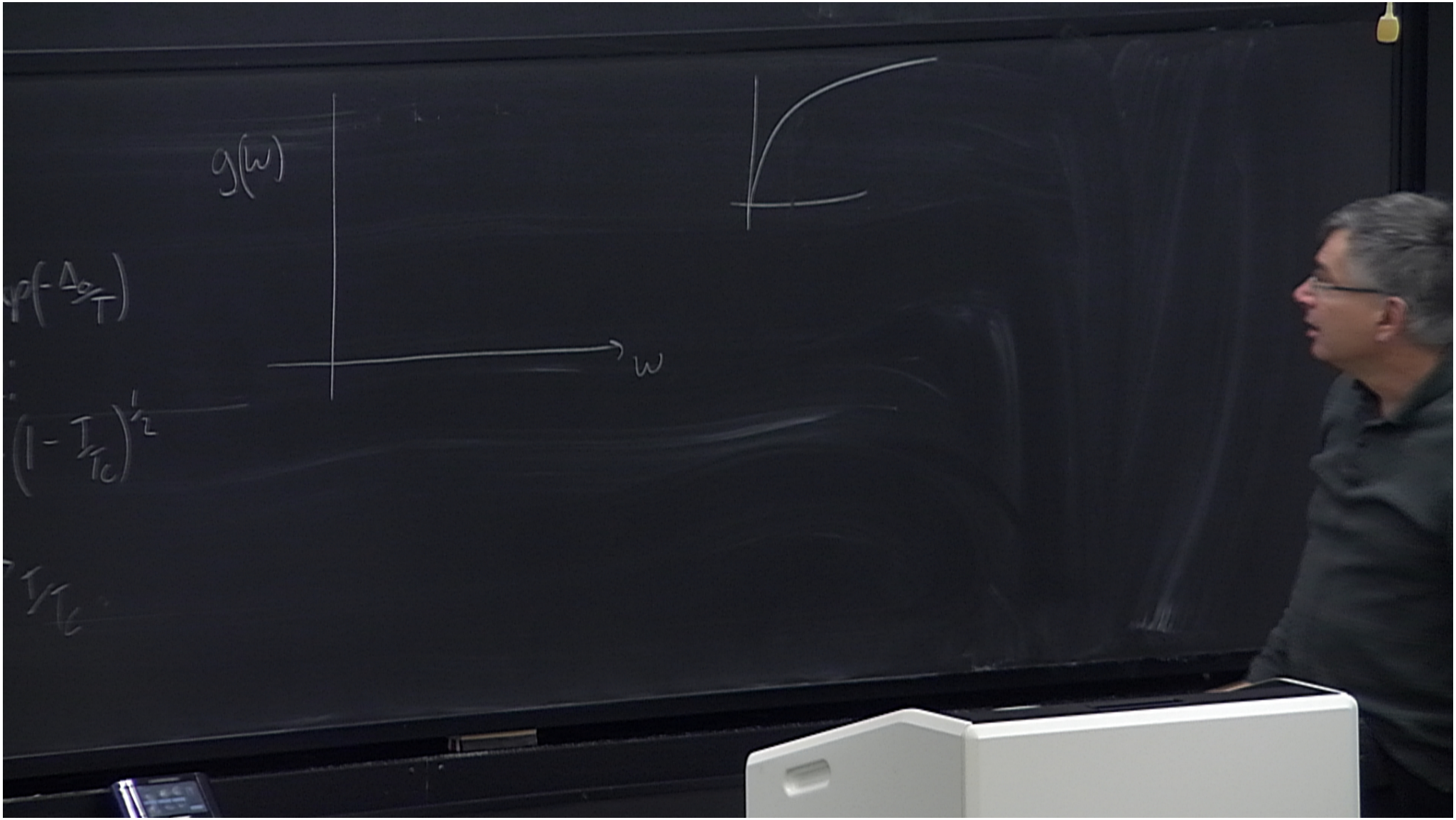
$$1 - (2\pi\Delta_0 T)^{\frac{1}{2}} \exp\left(-\frac{\Delta_0}{T}\right)$$

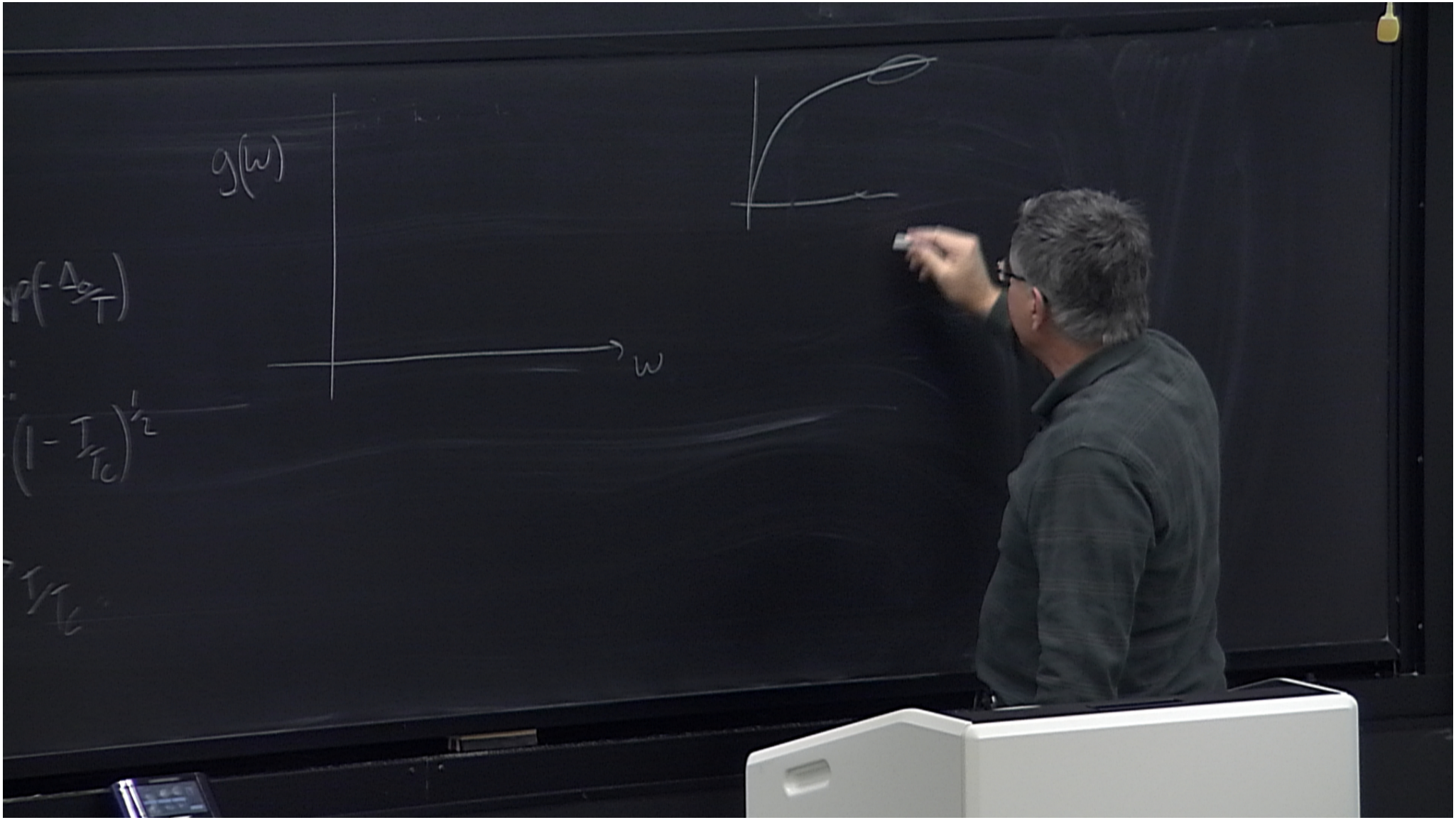


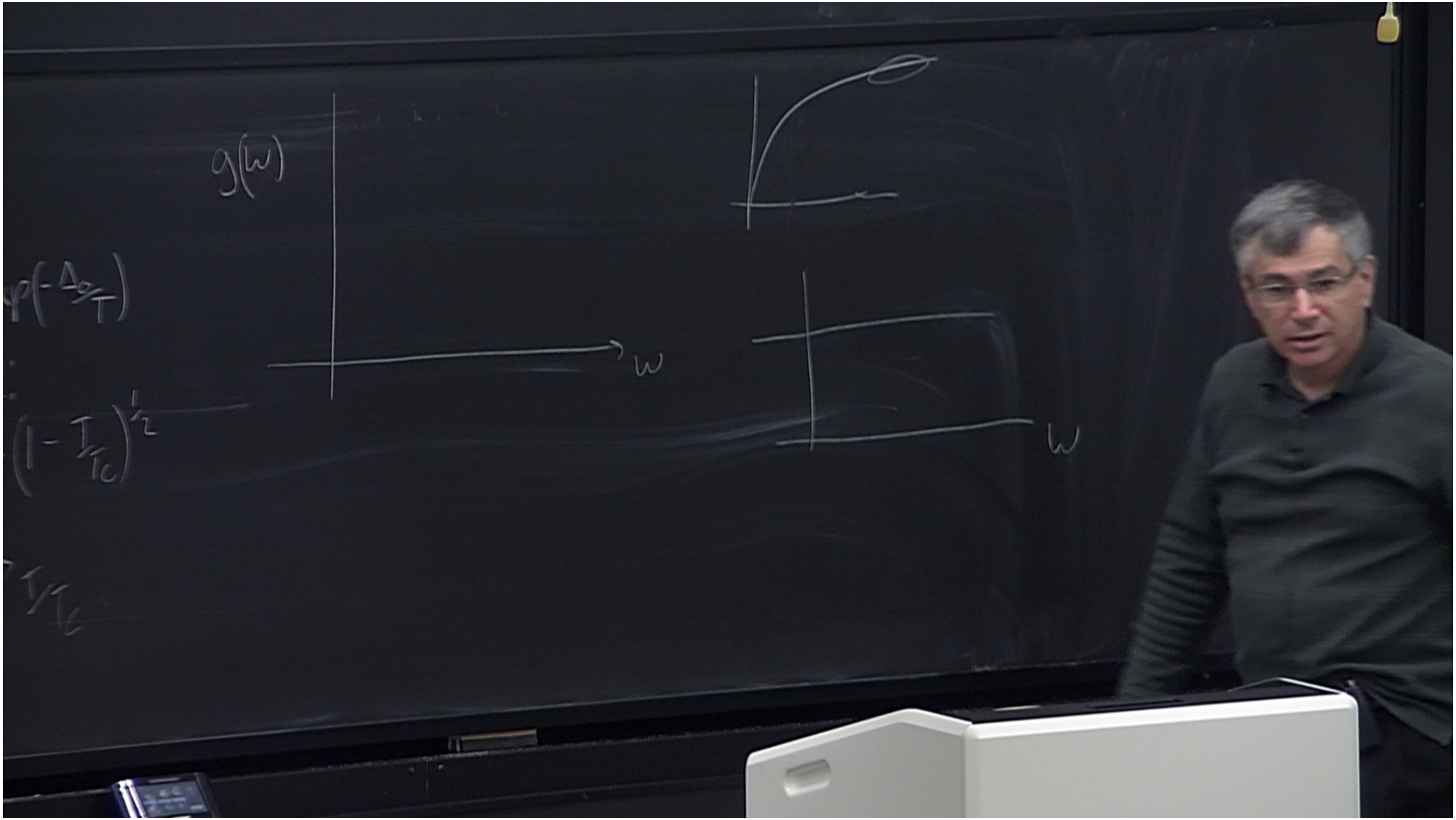
$$\frac{1}{|V|\,g(\varepsilon_F)} = \int_0^{\hbar\omega_D \to \infty} d\varepsilon \, \frac{\tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)}}{2k_BT}\right)}{\sqrt{\varepsilon^2+\Delta^2(T)}} - \int_0^{\hbar\omega_D \to \infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2(0)}} = 0$$

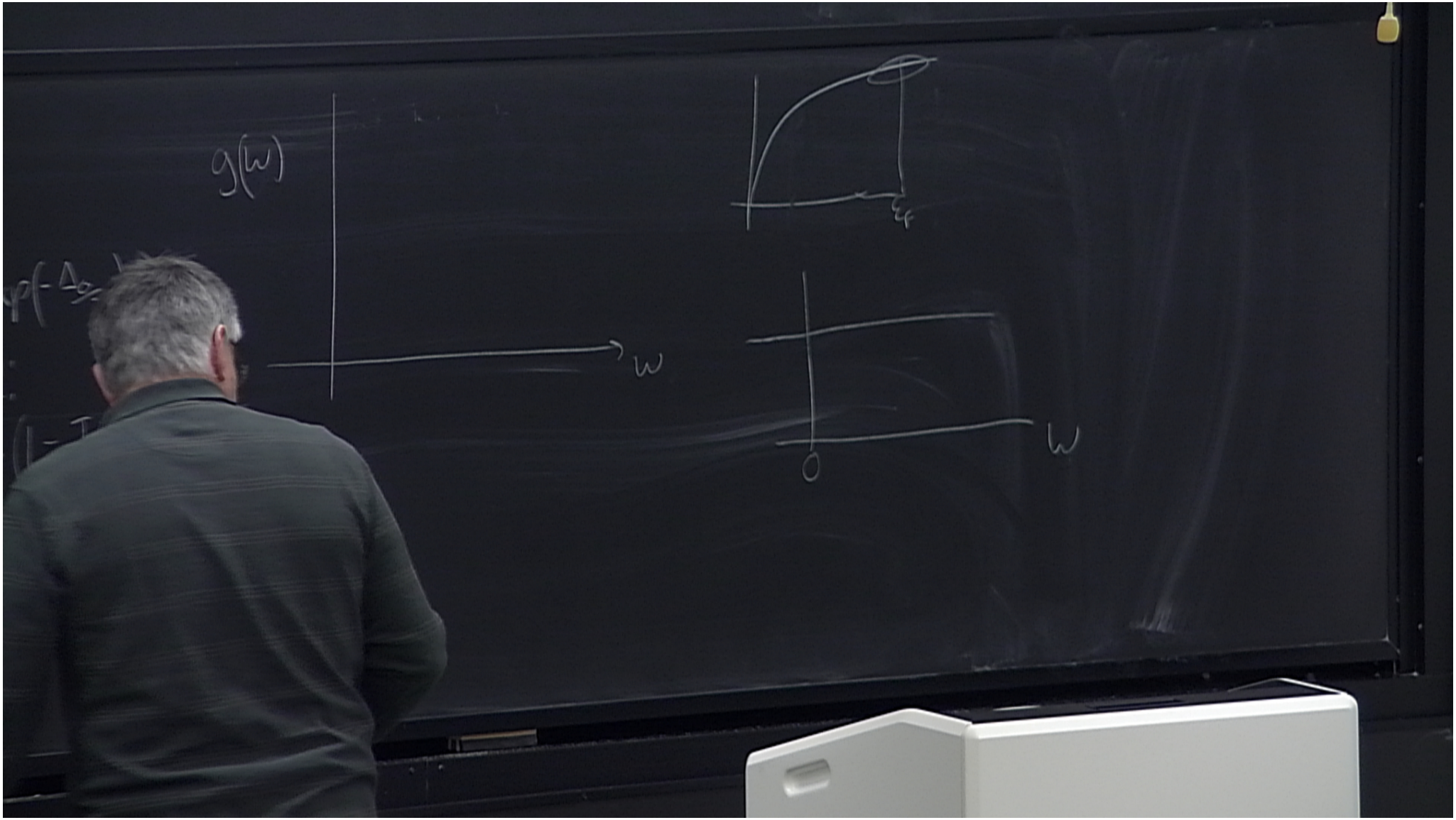
$$\frac{\Delta(T)}{\Delta(0)} = h\left(\frac{T}{T_c}\right) \quad \frac{\Delta(T=0)}{T_c}$$

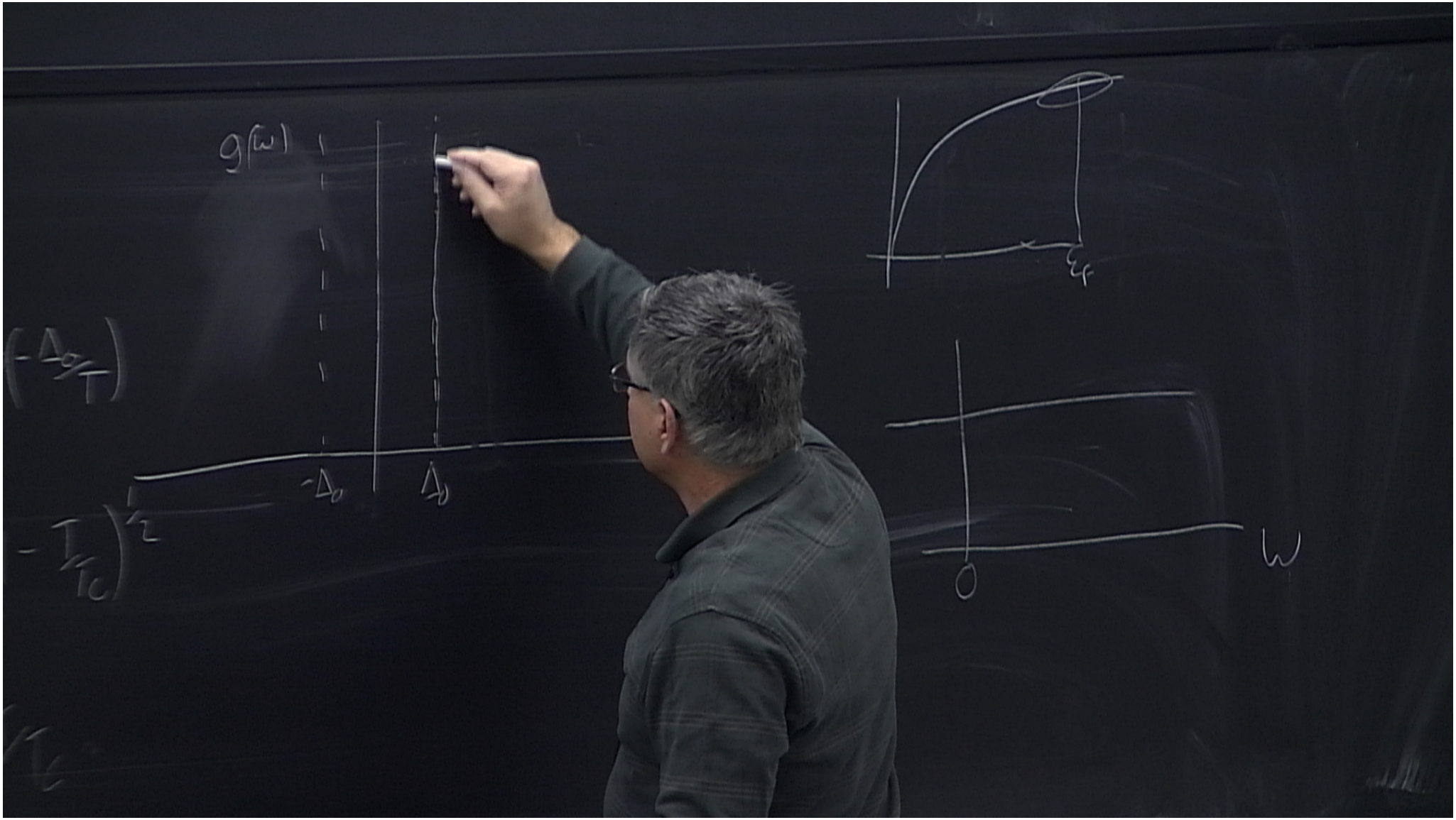


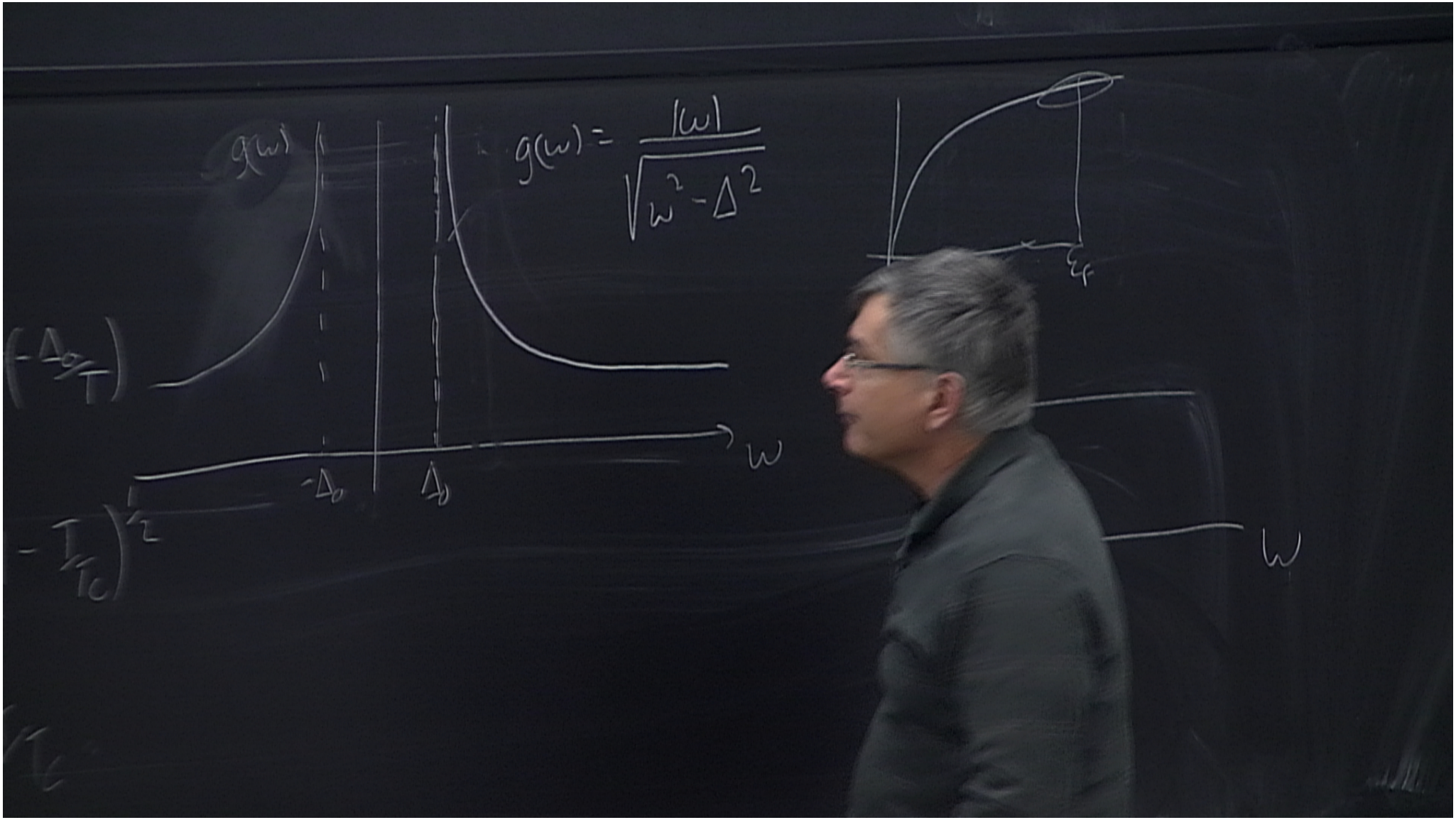

g(w)
w

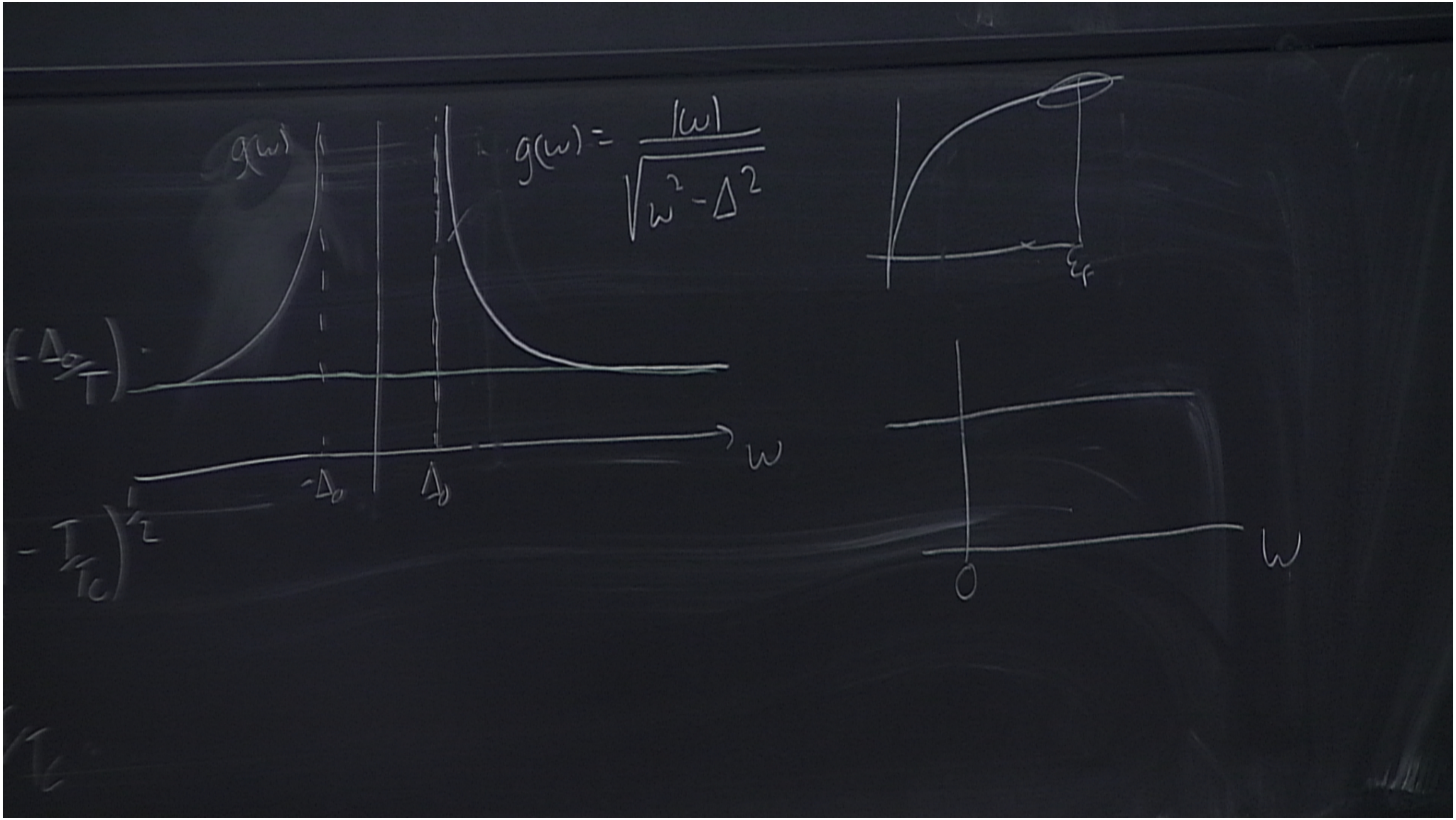

g(w)
w

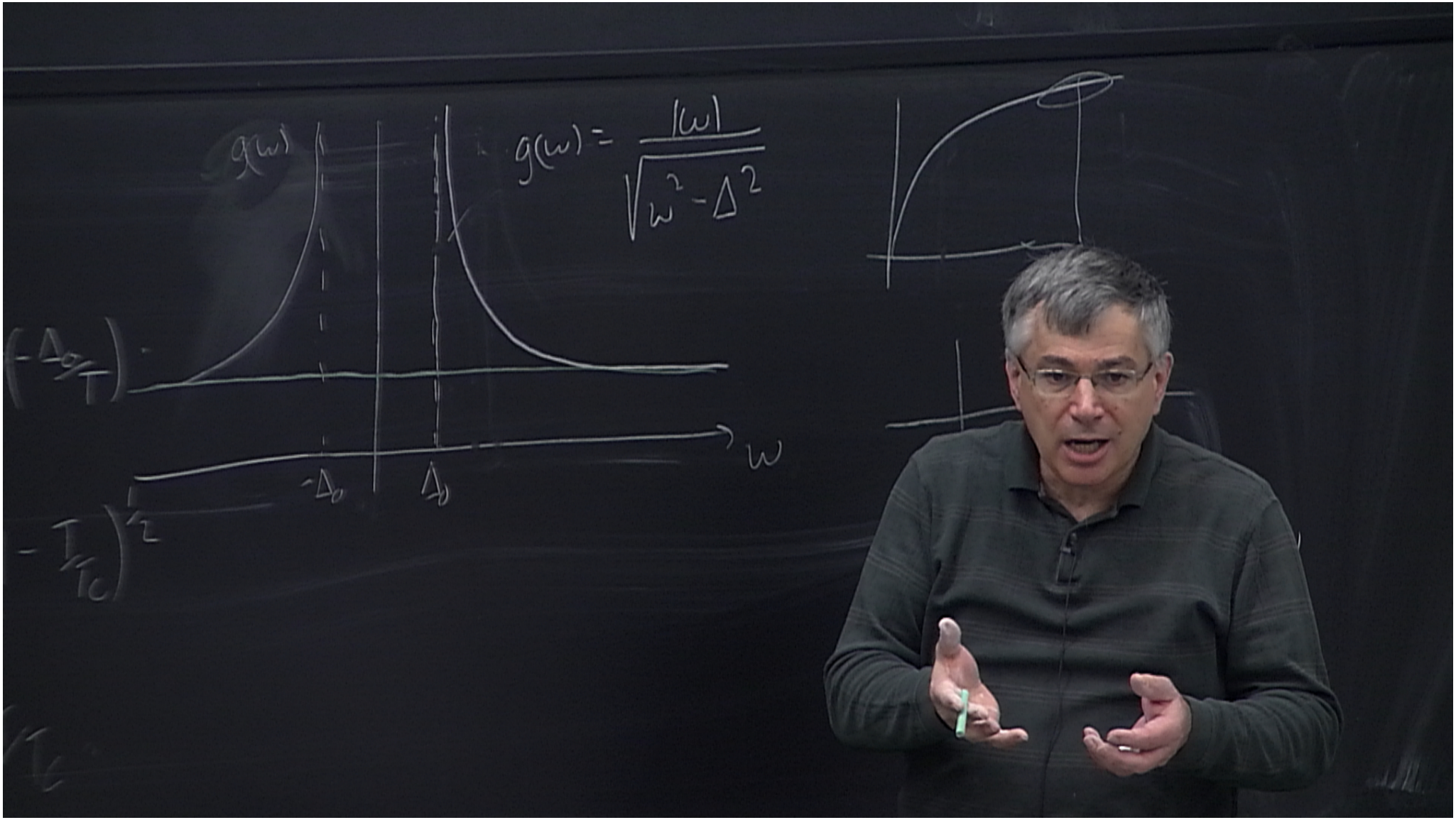

g(w)
w
w

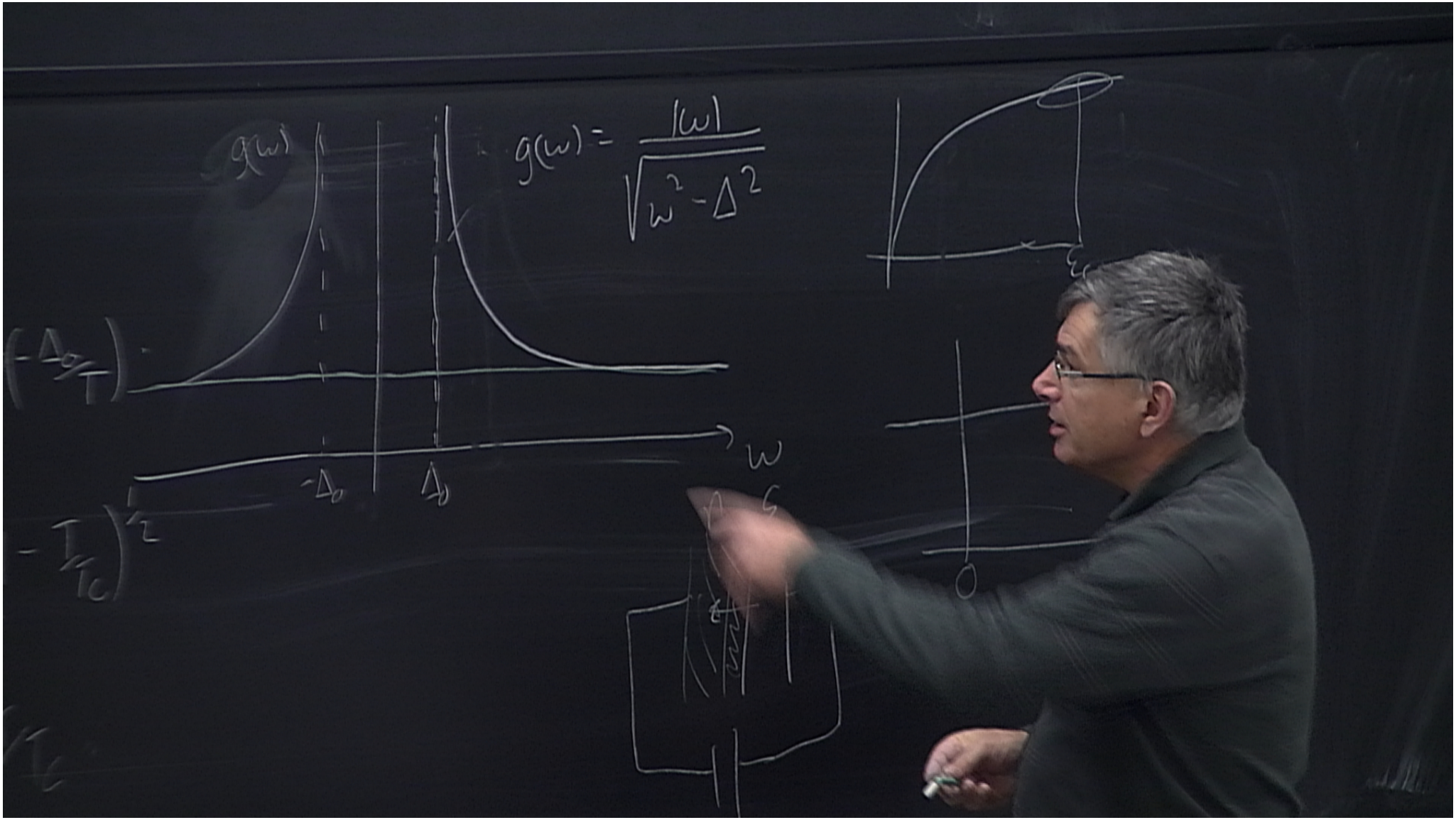

g(w)
w
w

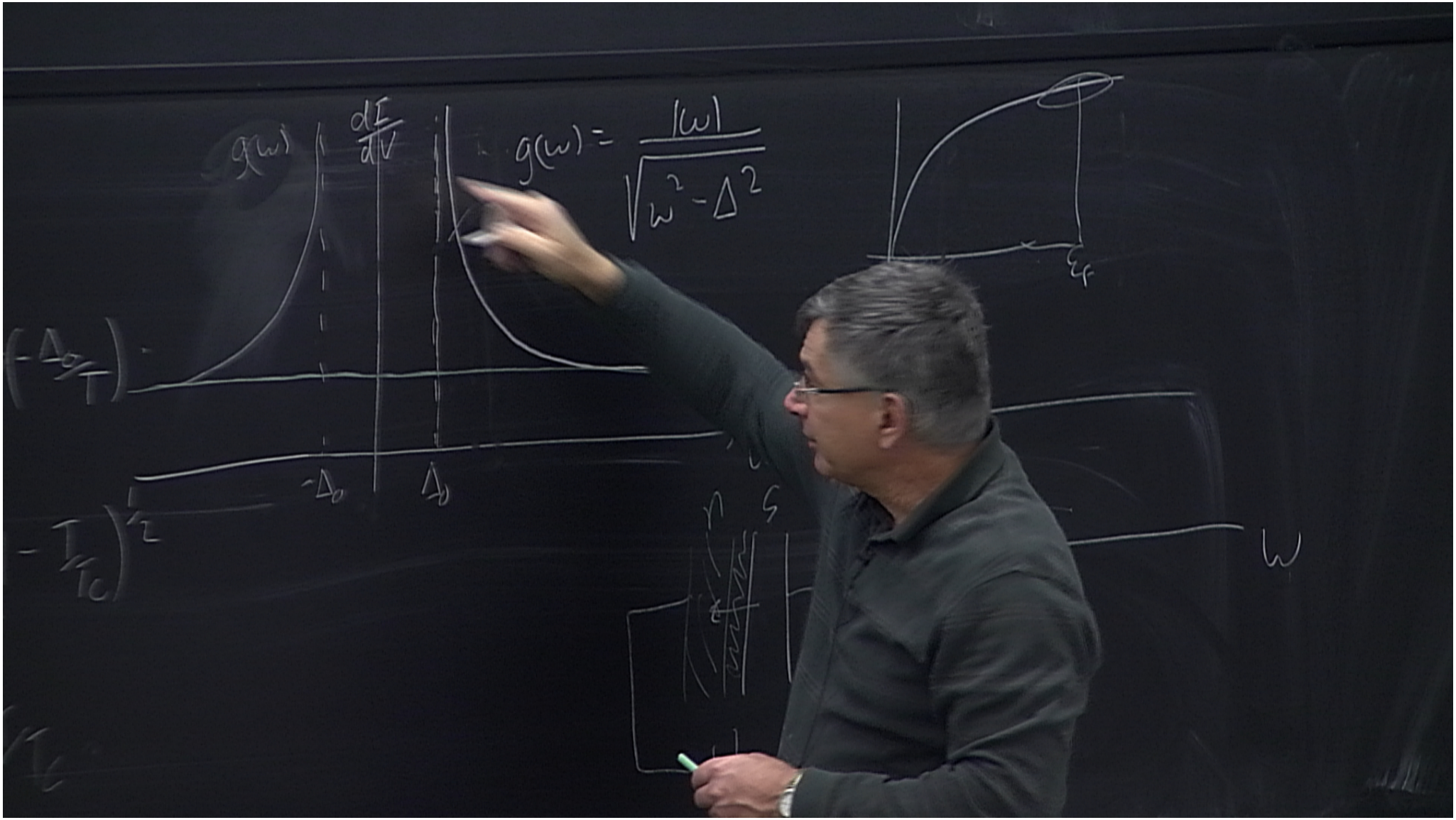

g(ω)
$-\Delta_0$
Δ_0
E_F
ω
0

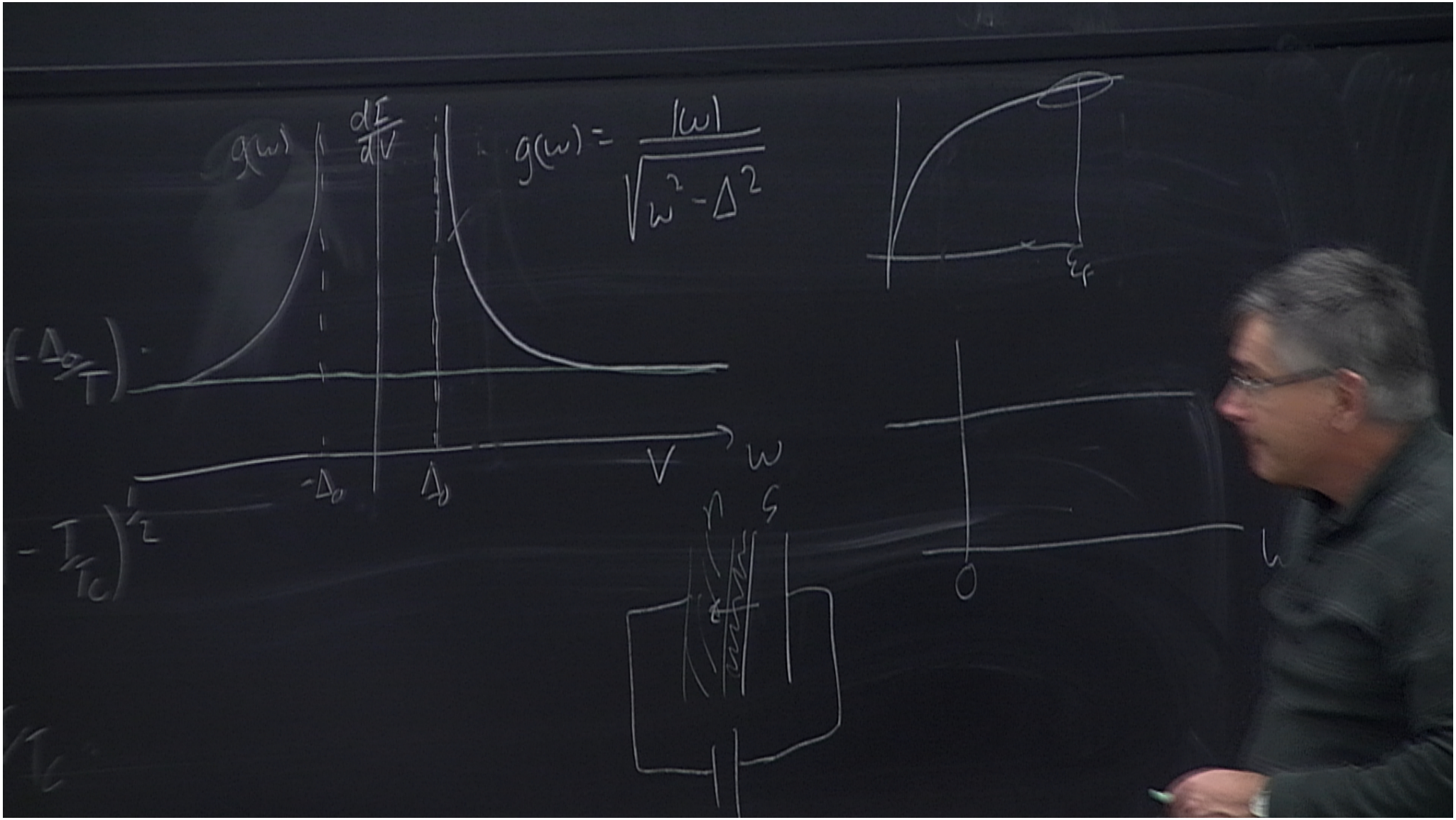

g(ω)
g(ω) = |ω| / √(ω² − Δ²)
E_F
(−Δ₀/T)
−Δ₀
Δ₀
ω
ω

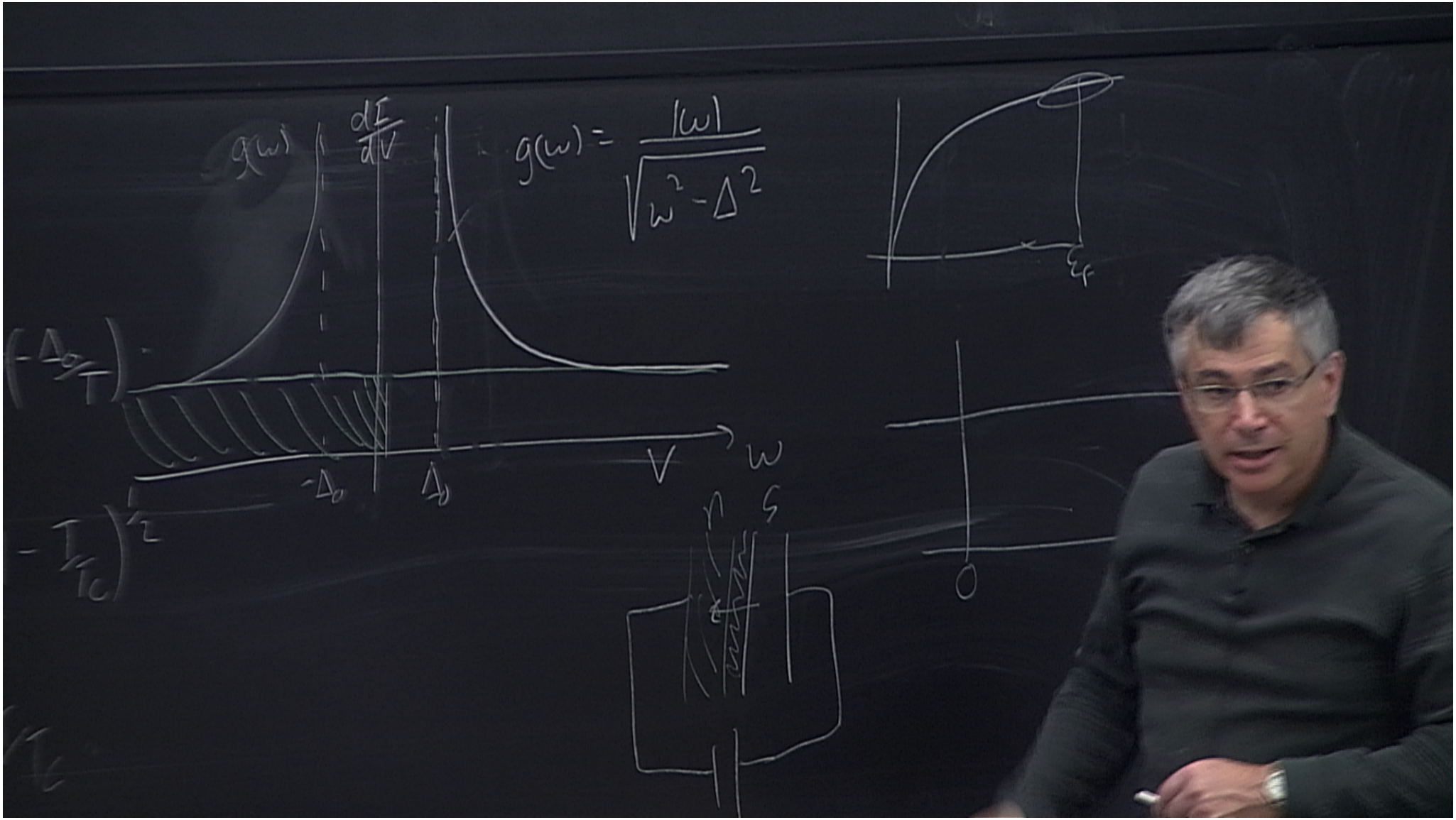

g(ω)
g(ω) = |ω| / √(ω² − Δ²)
ω
−Δ₀
Δ₀
ω

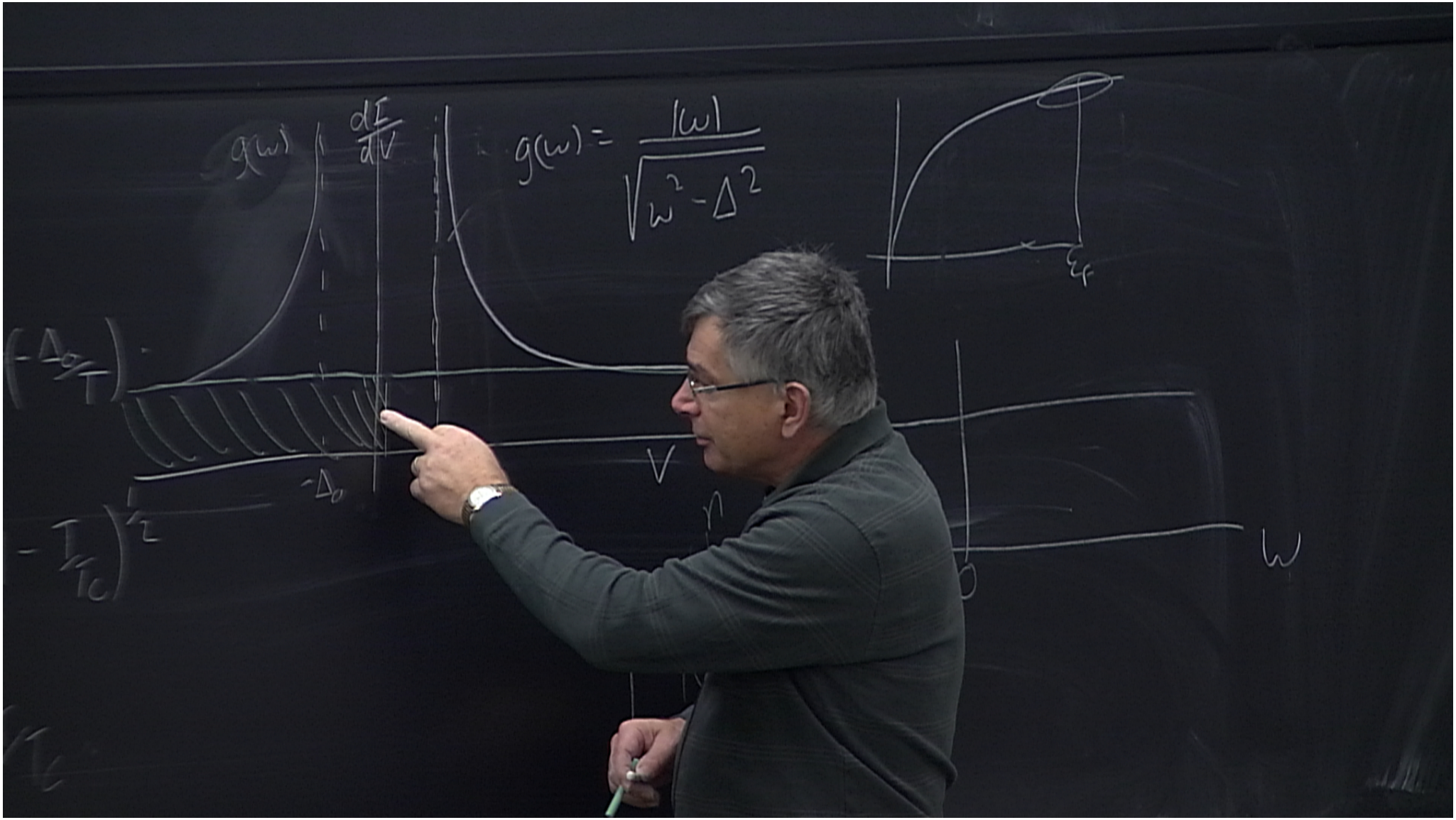

g(ω)
g(ω) = |ω| / √(ω² − Δ²)
−Δ₀
Δ₀
ω

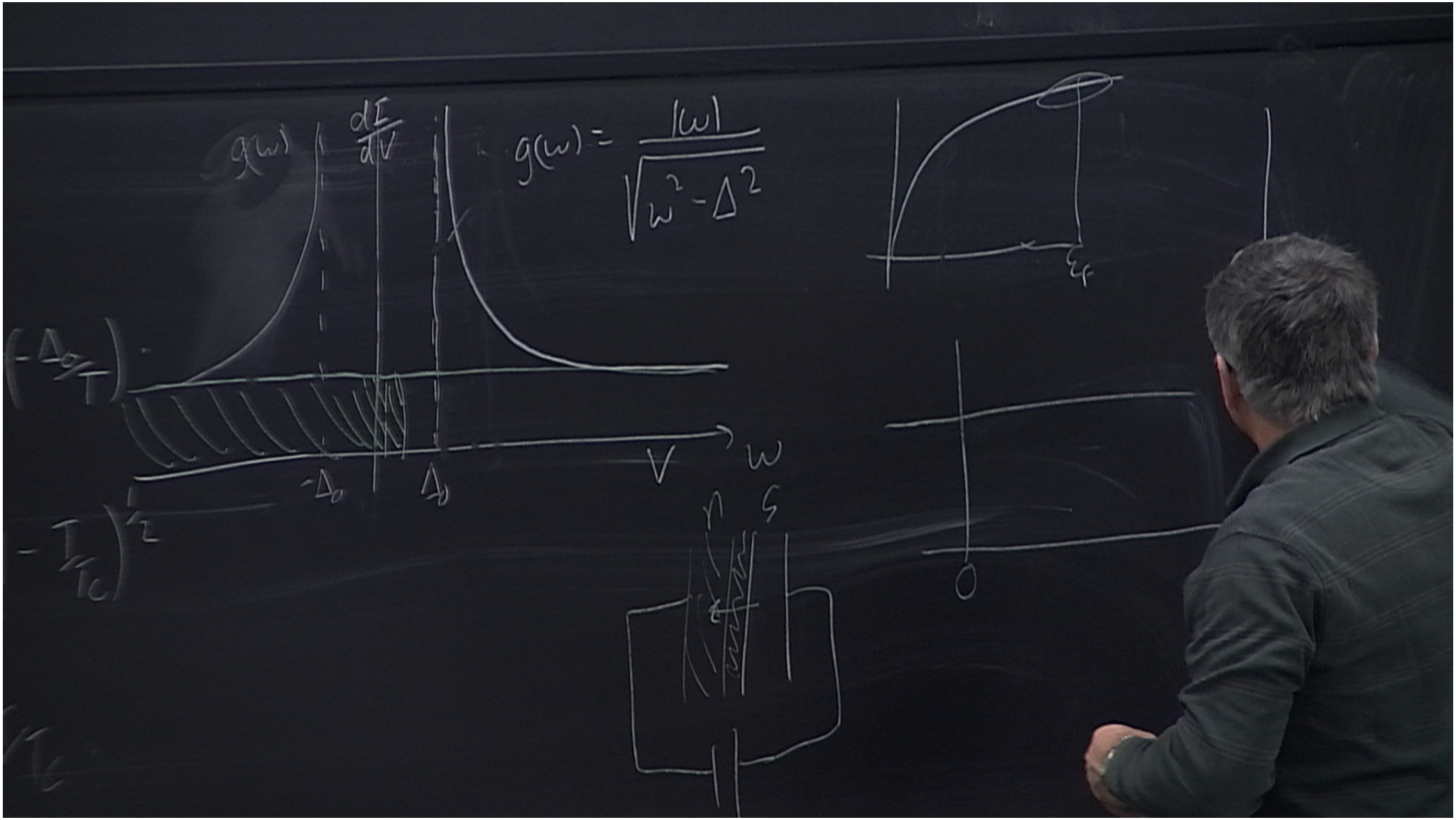

g(ω)
g(ω) = |ω| / √(ω² − Δ²)
−Δ₀
Δ₀
ω
0

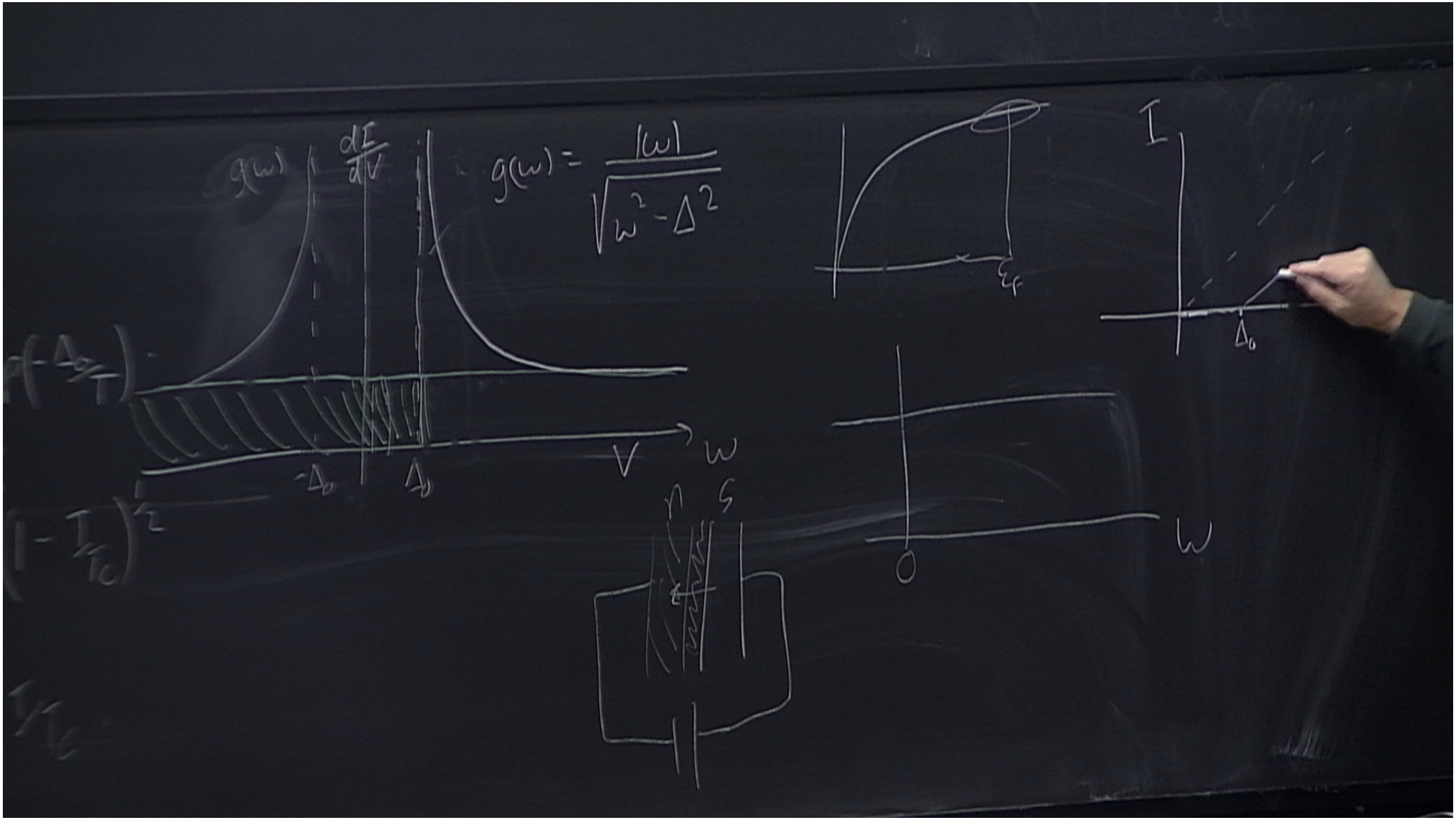

g(ω)
dF/dV
g(ω) = |ω| / √(ω² − Δ²)
E_F
−Δ_0
Δ_0
ω

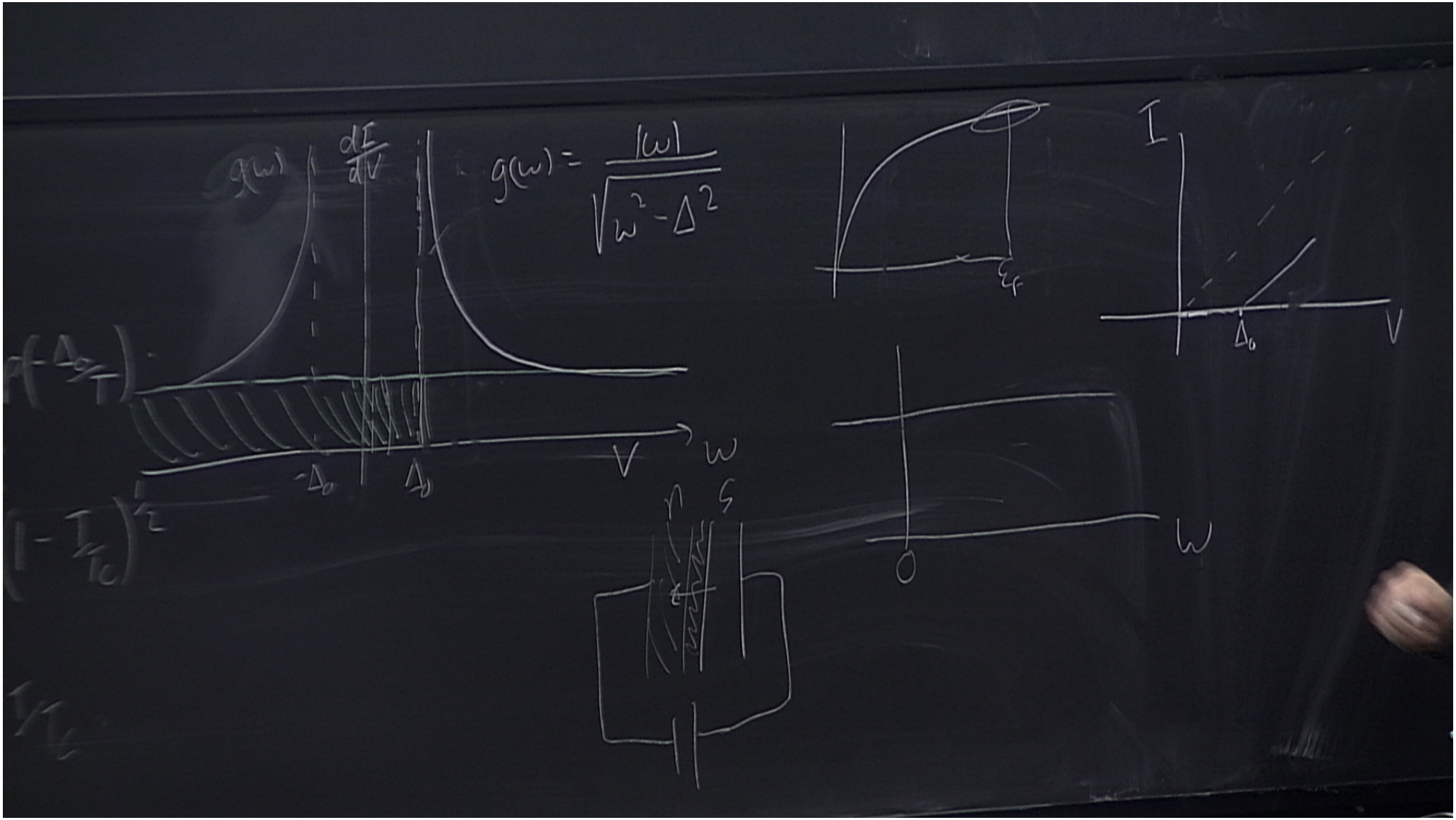

g(ω)
dF/dV
g(ω) = |ω| / √(ω² − Δ²)
−Δ₀
Δ₀
V
ω
E_F
0

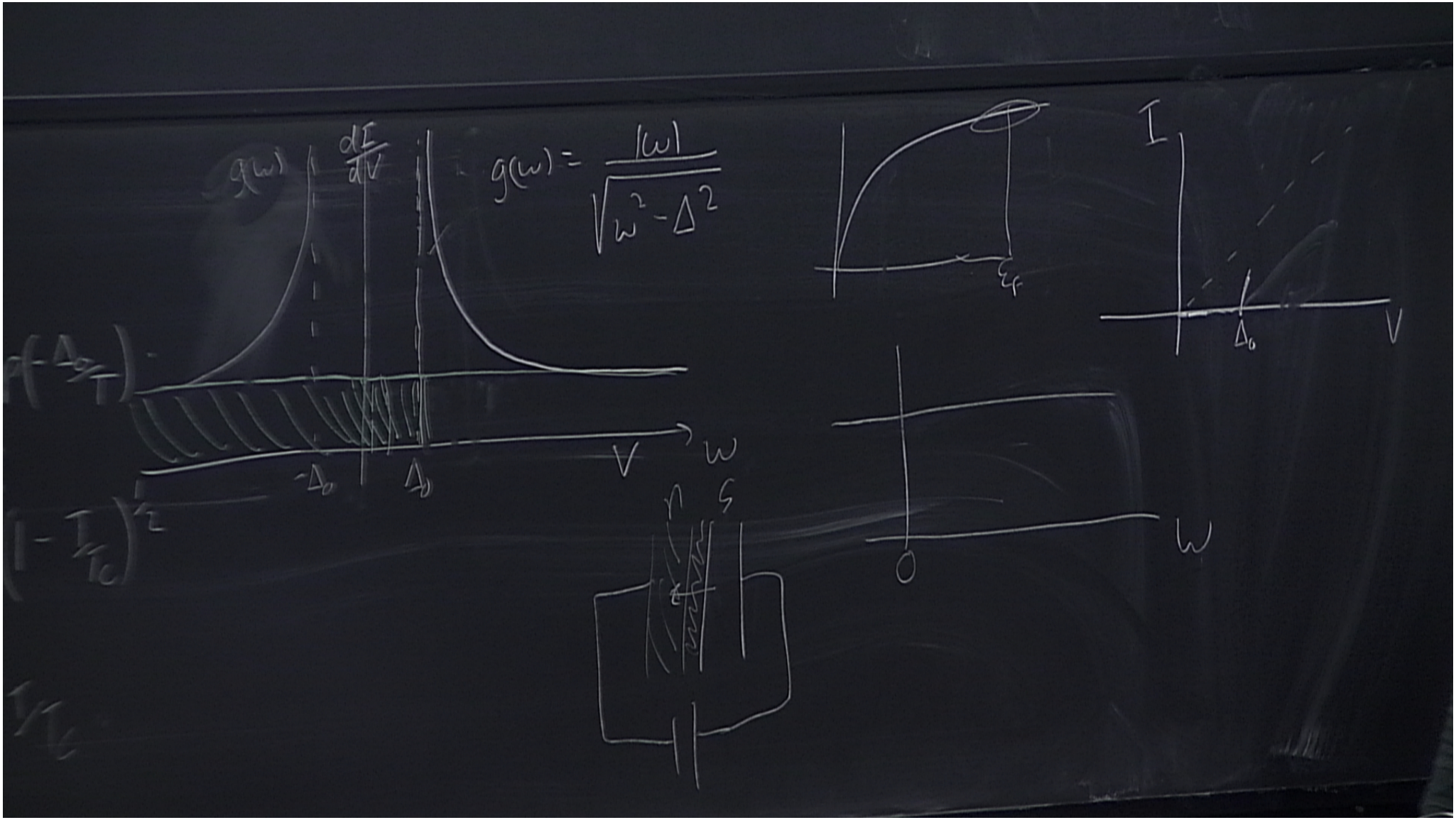

g(ω)
dE/dV
g(ω) = |ω| / √(ω² − Δ²)
E_F
−Δ_0
V
ω
−Δ_0
Δ_0
0

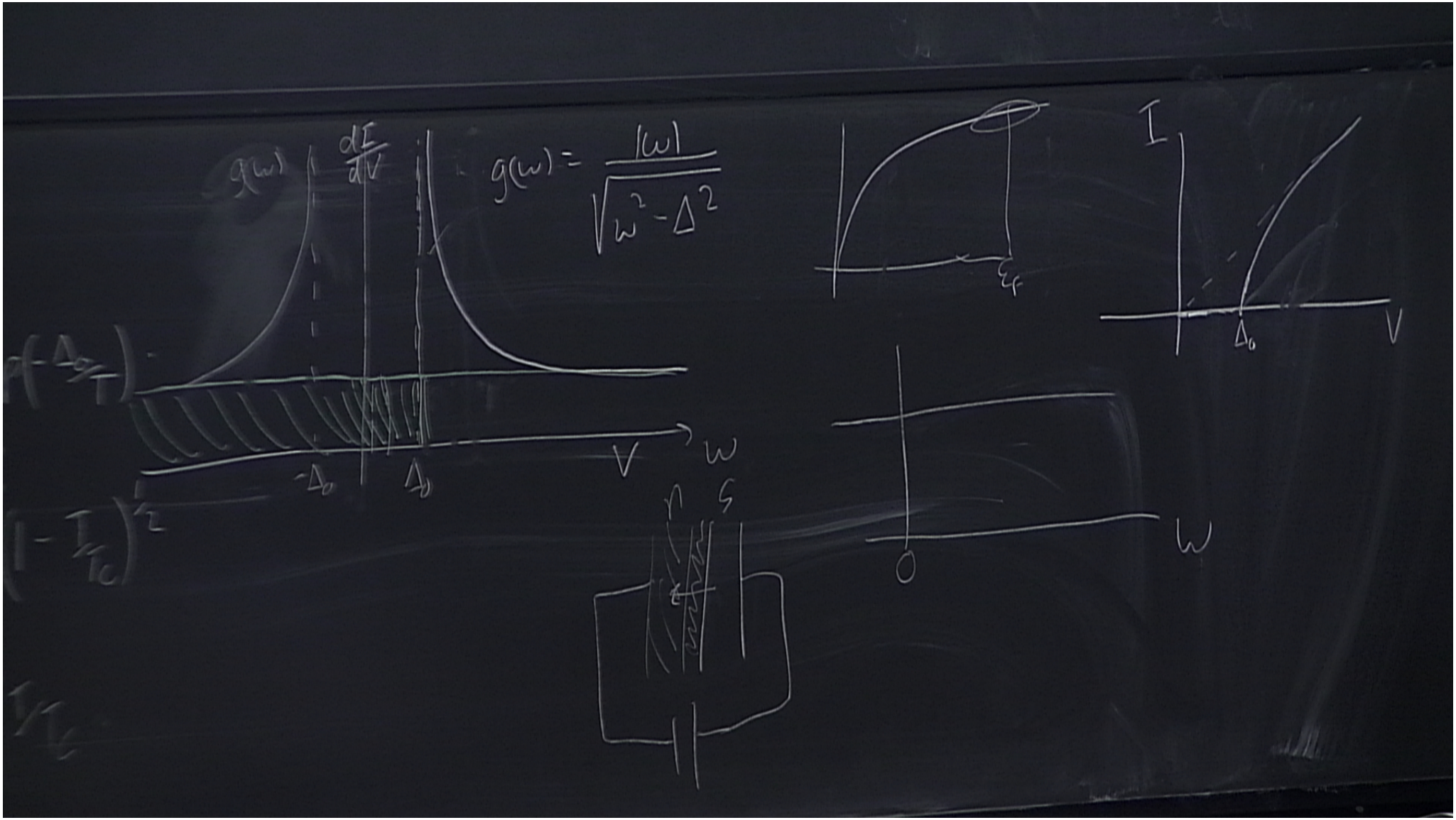

g(ω)
dE/dV
g(ω) = |ω| / √(ω² − Δ²)
E_F
−Δ_0
V
n
ω
0

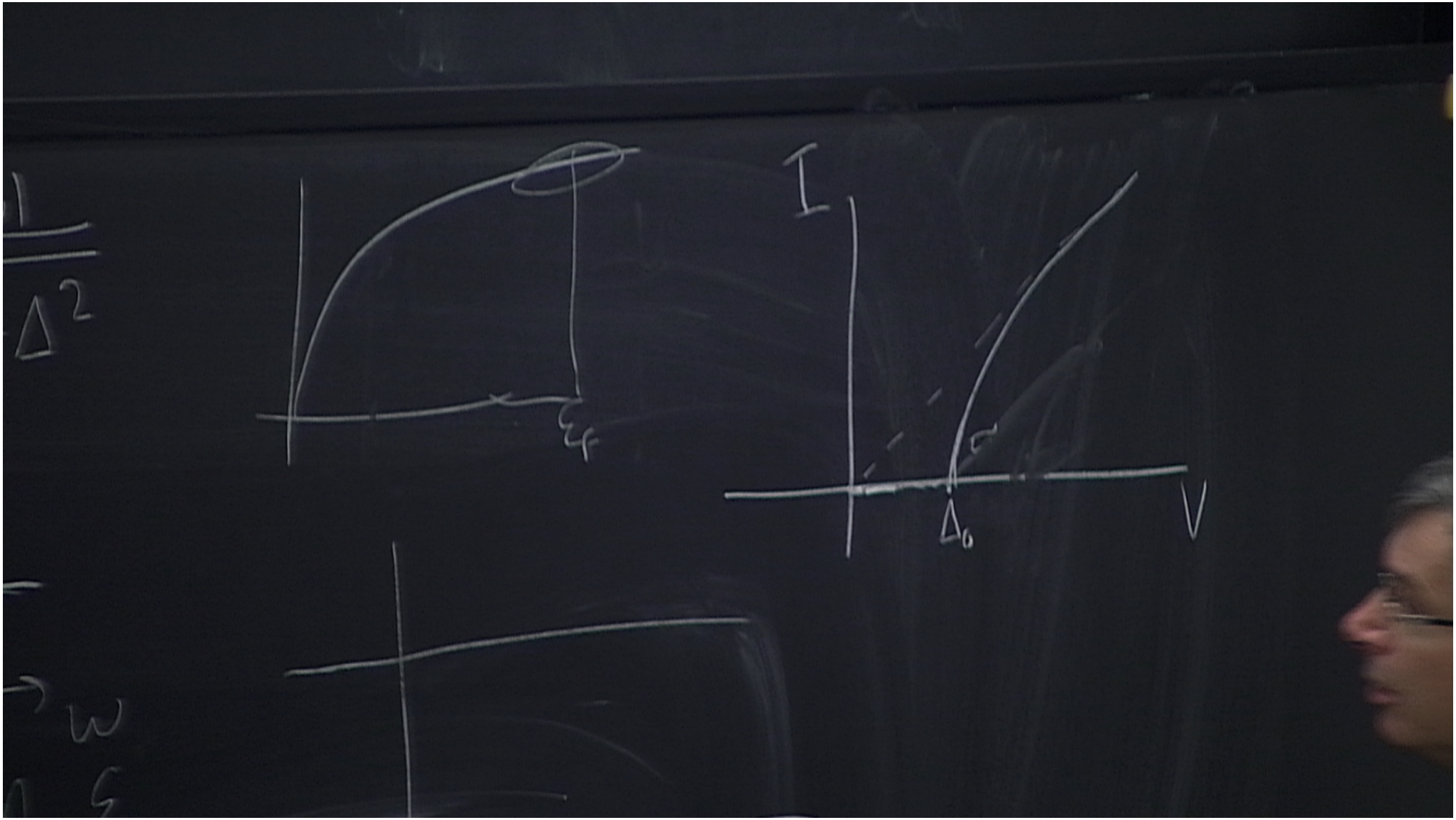

g(ω)
dF/dV
g(ω) = |ω| / √(ω² − Δ²)
−Δ
Δ
V
ω
ε_F

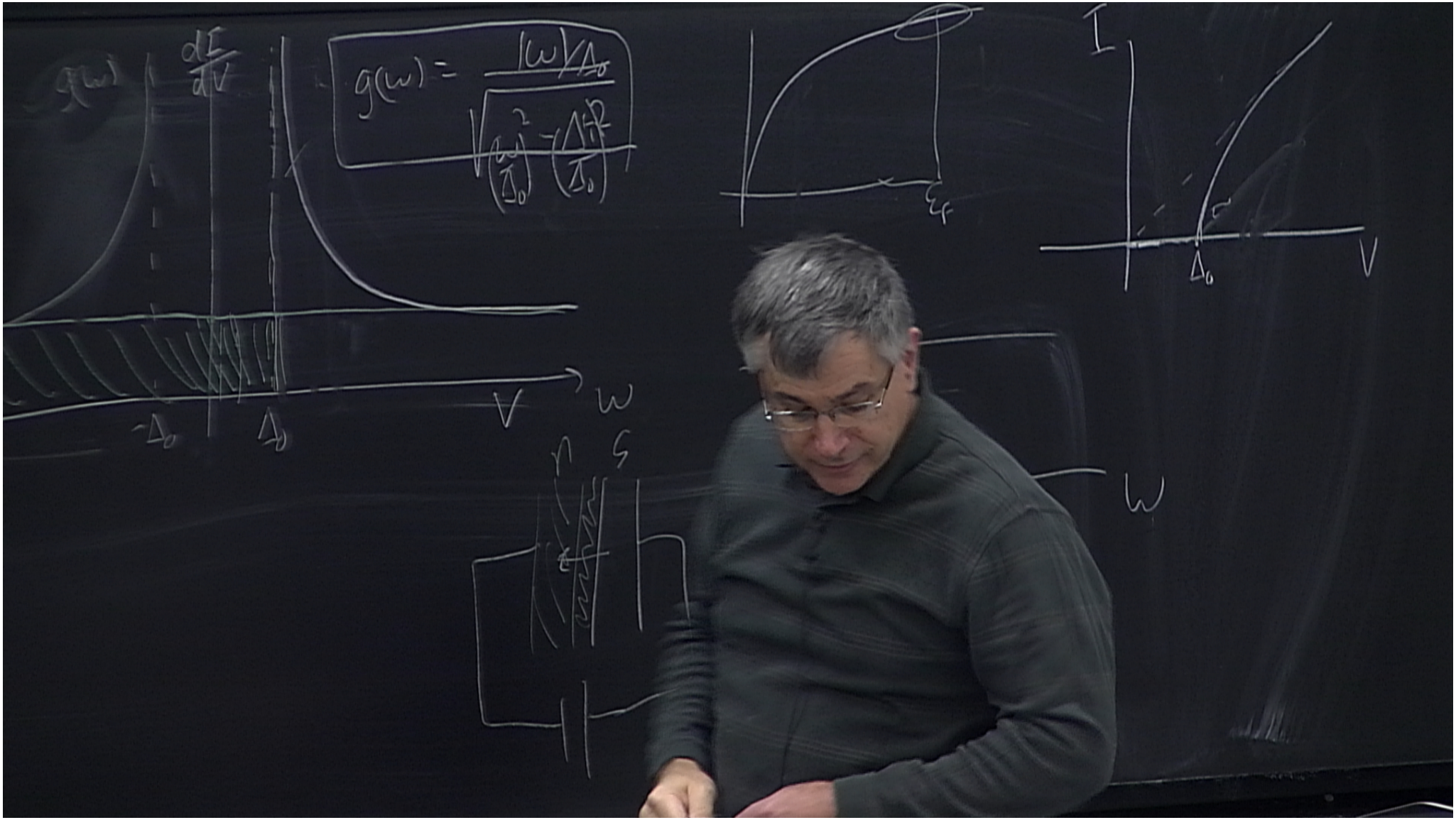

g(ω)
dI/dV
g(ω) = |ω| / √(ω² − Δ²)
I
V
ω
−Δ₀
Δ₀
0
ω

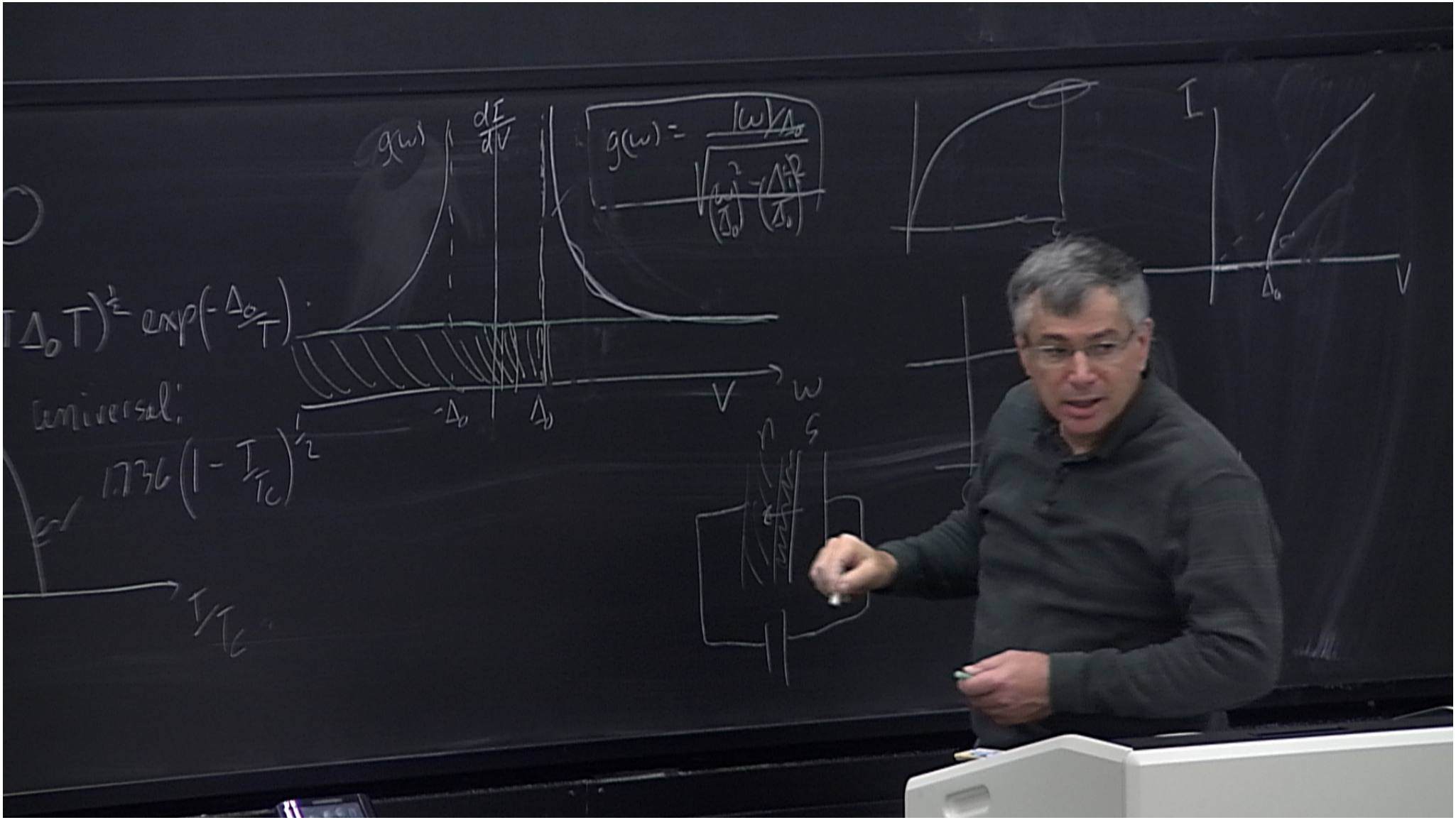

g(w)
dI/dV
g(w) = |w| / √(w² − Δ²)
I
V
w
−Δ₀
Δ₀
0

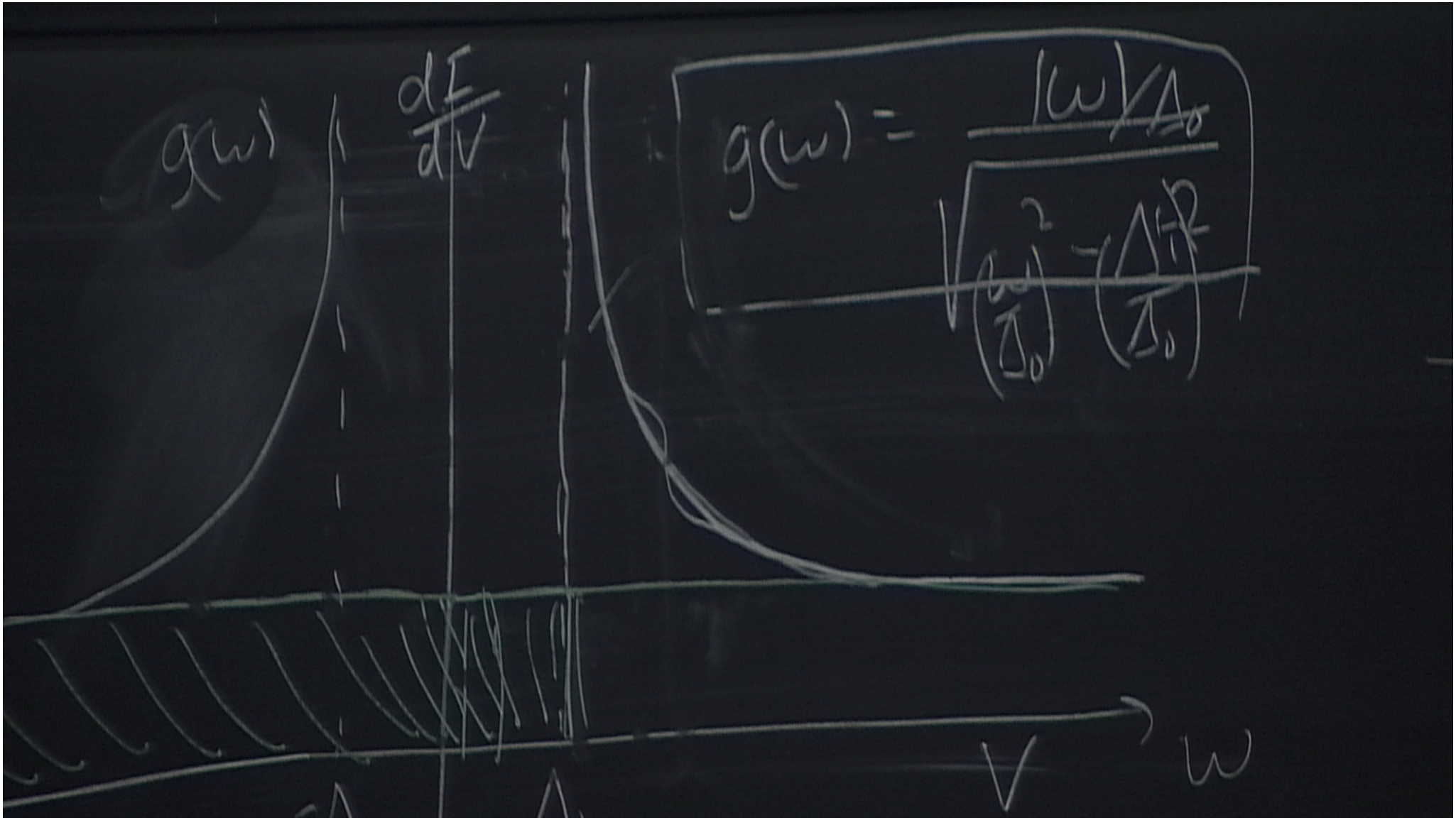

g(ω)
dI/dV
g(ω) = |ω| / √(ω² − Δ²)
I
V
Δ₀
−Δ₀
ω
0
ξ
η


g(ω)
dI/dV
g(ω) = |ω| / √(ω² − Δ²)
I
V
ω
−Δ₀
Δ₀
0
ξ
η


I
V
Δ_0
Δ^2


g(ω)
dI/dV
g(ω) = |ω|/Δ0 / √((ω/Δ0)^2 − (Δ1/Δ0)^2)
V
ω
−Δ0
Δ0
ξ
ω
I
V
Δ0


g(ω)
dI/dV
g(ω) = |ω|/Δ₀
√((ω/Δ₀)² − (Δ/Δ₀)²)
I
V
Δ₀
−Δ₀
Δ₀
V
universal:
1.736 (1 − I/Ic)^½
I/Ic


g(ω)
dF/dV
g(ω) = |ω|/Δ₀ / √((ω/Δ₀)² − (Δ/Δ₀)²)
V
ω
Δ