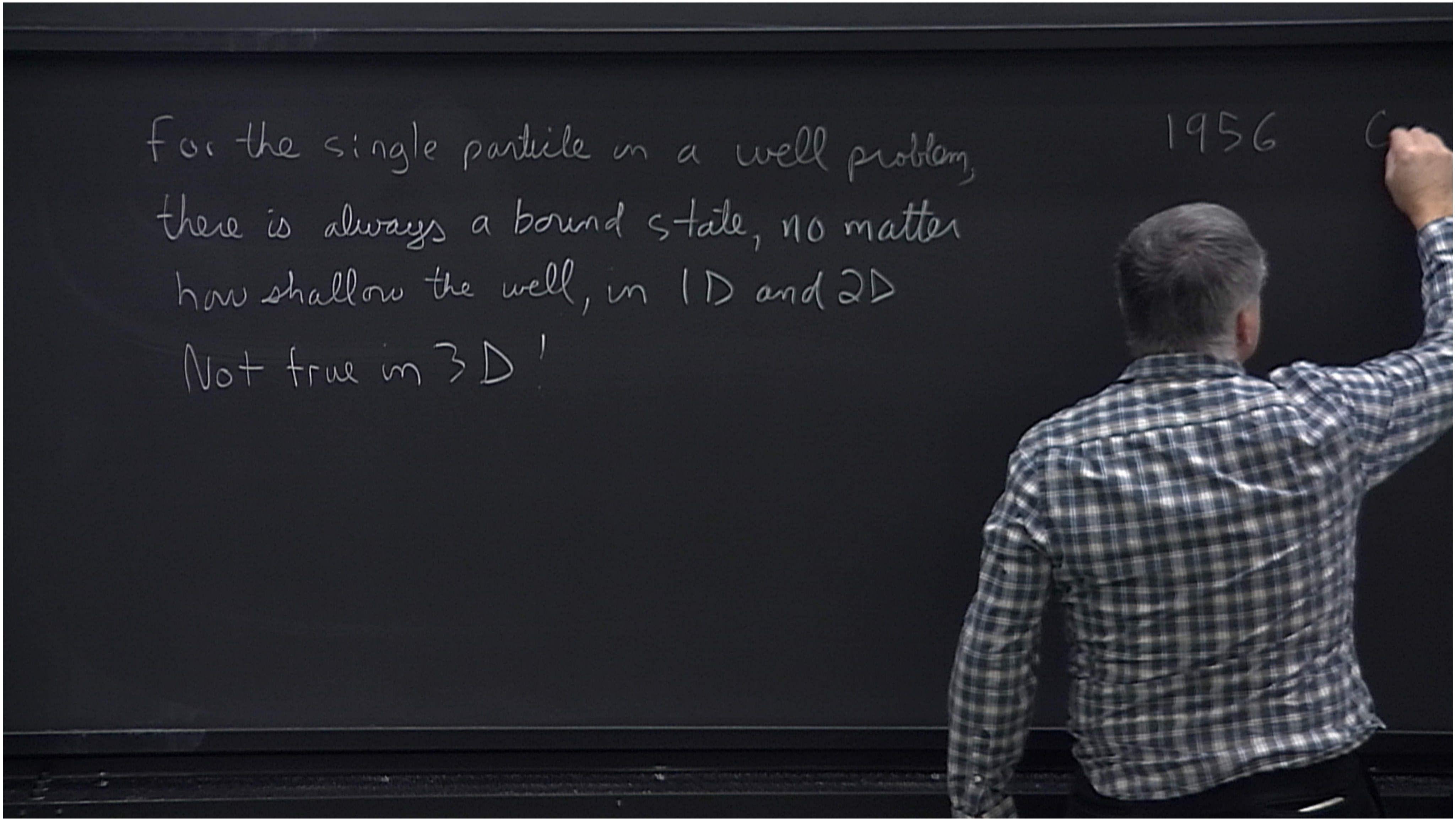
Title: PSI 2016/2017 Condensed Matter - Lecture 14

Date: Nov 24, 2016 10:45 AM

URL: http://pirsa.org/16110065

Abstract:


For the single particle in a well problem,
there is always a bound state, no matter
how shallow the well, in 1D and 2D
Not true in 3D!
1956

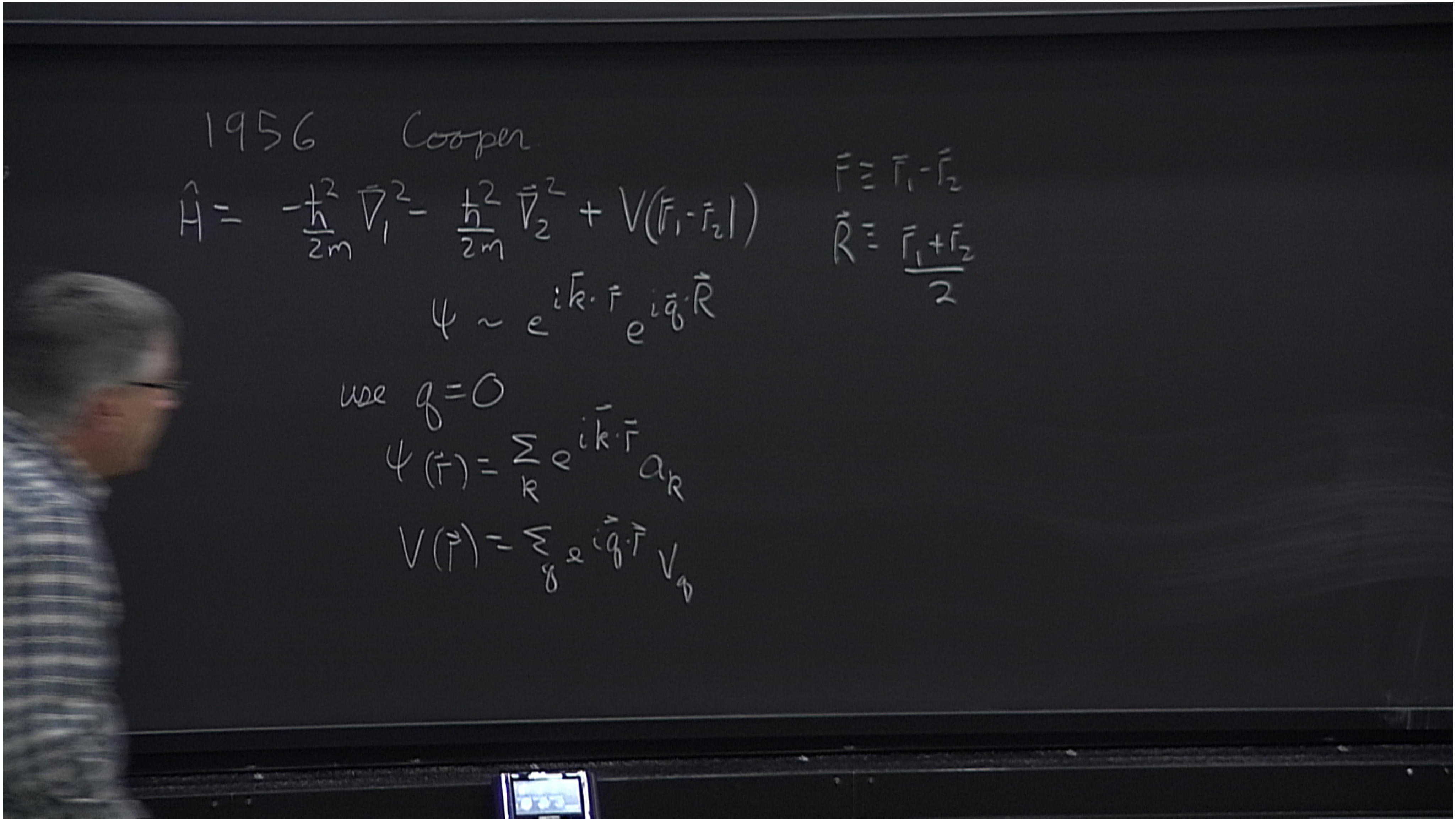
1956 Cooper

$$\hat{H} = -\frac{\hbar^2}{2m}\vec{\nabla}_1^2 - \frac{\hbar^2}{2m}\vec{\nabla}_2^2 + V(|\vec{r}_1 - \vec{r}_2|)$$

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$$

$$\vec{R} \equiv \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$\psi \sim e^{i\vec{k}\cdot\vec{r}}\, e^{i\vec{q}\cdot\vec{R}}$$


1956 Cooper
$\hat{H} = -\frac{\hbar^2}{2m}\vec{\nabla}_1^2 - \frac{\hbar^2}{2m}\vec{\nabla}_2^2 + V(|\vec{r}_1 - \vec{r}_2|)$
$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$
$\vec{R} \equiv \frac{\vec{r}_1 + \vec{r}_2}{2}$
$\psi \sim e^{i\vec{k}\cdot\vec{r}} e^{i\vec{q}\cdot\vec{R}}$
use $q=0$
$\psi(\vec{r}) = \sum_k e^{i\vec{k}\cdot\vec{r}} a_k$
$V(\vec{r}) = \sum_q e^{i\vec{q}\cdot\vec{r}} V_q$

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$$

$$\vec{R} \equiv \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$H\psi = E\psi$$

$$2\sum_k \frac{\hbar^2 k^2}{2m} a_k e^{i\vec{k}\cdot\vec{r}} + \sum_{kq} e^{i\vec{k}\cdot\vec{r}} e^{i\vec{q}\cdot\vec{r}} a_k V_q = E \sum_k e^{i\vec{k}\cdot\vec{r}} a_k$$

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$$

$$\vec{R} \equiv \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$H\psi = E\psi$$

$$2\sum_k \frac{\hbar^2 k^2}{2m} a_k e^{i\vec{k}\cdot\vec{r}} + \sum_{k'q} \underbrace{e^{i\vec{k}'\cdot\vec{r}} e^{i\vec{q}\cdot\vec{r}}}_{\substack{k = k' + q \\ q = k - k'}} a_{k'} v_q = E \sum_k e^{i\vec{k}\cdot\vec{r}} a_k$$

$$\left[E - 2\varepsilon_k\right] a_k = \sum_{k'} v_{k-k'} a_{k'}$$

$$H\psi = E\psi$$

$$\vec{r}_1 - \vec{r}_2$$

$$\frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$2\sum_k \frac{\hbar^2 k^2}{2m} a_k e^{i\vec{k}\cdot\vec{r}} + \sum_{k'q} \underbrace{e^{i\vec{k}'\cdot\vec{r}}\, e^{i\vec{q}\cdot\vec{r}}}_{} a_{k'} V_q = E \sum_k e^{i\vec{k}\cdot\vec{r}} a_k$$

$$k = k' + q$$

$$q = k - k'$$

$$\left[E - 2\varepsilon_k\right] a_k = \sum_{k'} V_{k-k'} a_{k'}$$

$$= -|V| \sum_{k'}{}' a_{k'}$$

$$V_{k-k'} = -|V| \text{ for } \varepsilon_{k'} < \hbar\omega_D, \quad \varepsilon_k < \hbar\omega_D$$

$$H\psi = E\psi$$

$$\bar{r}_1 - \bar{r}_2$$

$$\frac{\bar{r}_1 + \bar{r}_2}{2}$$

$$2\sum_k \frac{\hbar^2 k^2}{2m} a_k e^{i\bar{k}\cdot\bar{r}} + \sum_{k'q} \underbrace{e^{i\bar{k}'\cdot\bar{r}}\, e^{i\bar{q}\cdot\bar{r}}} a_{k'} V_q = E \sum_k e^{i\bar{k}\cdot\bar{r}} a_k$$

$$k = k' + q$$

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$$V_{k-k'} = -|V| \text{ for } \varepsilon_{k'} < \hbar\omega_D, \quad \varepsilon_k < \hbar\omega_D$$

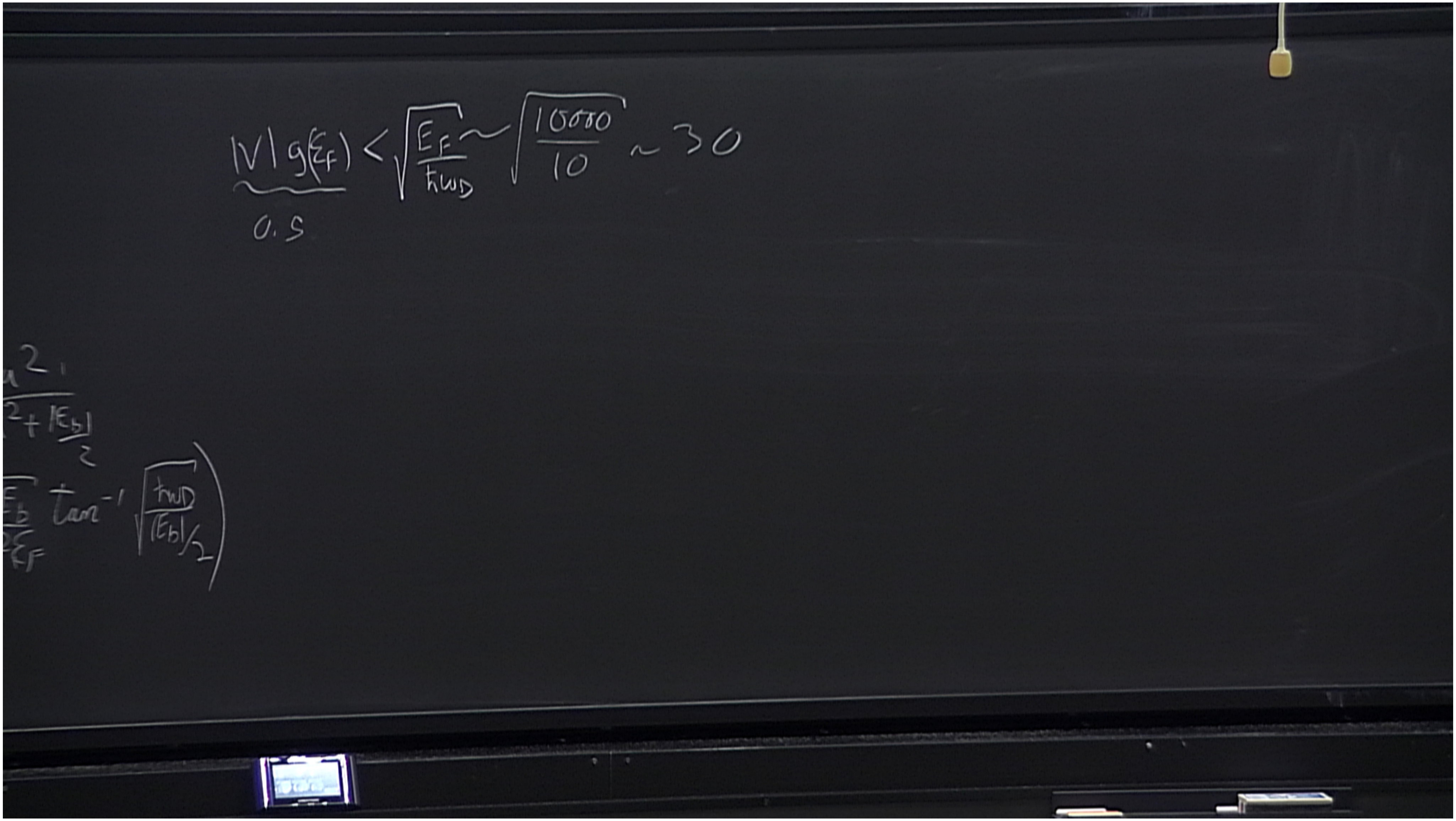
$$= -|V| \sum_{k'}{}' a_{k'}$$

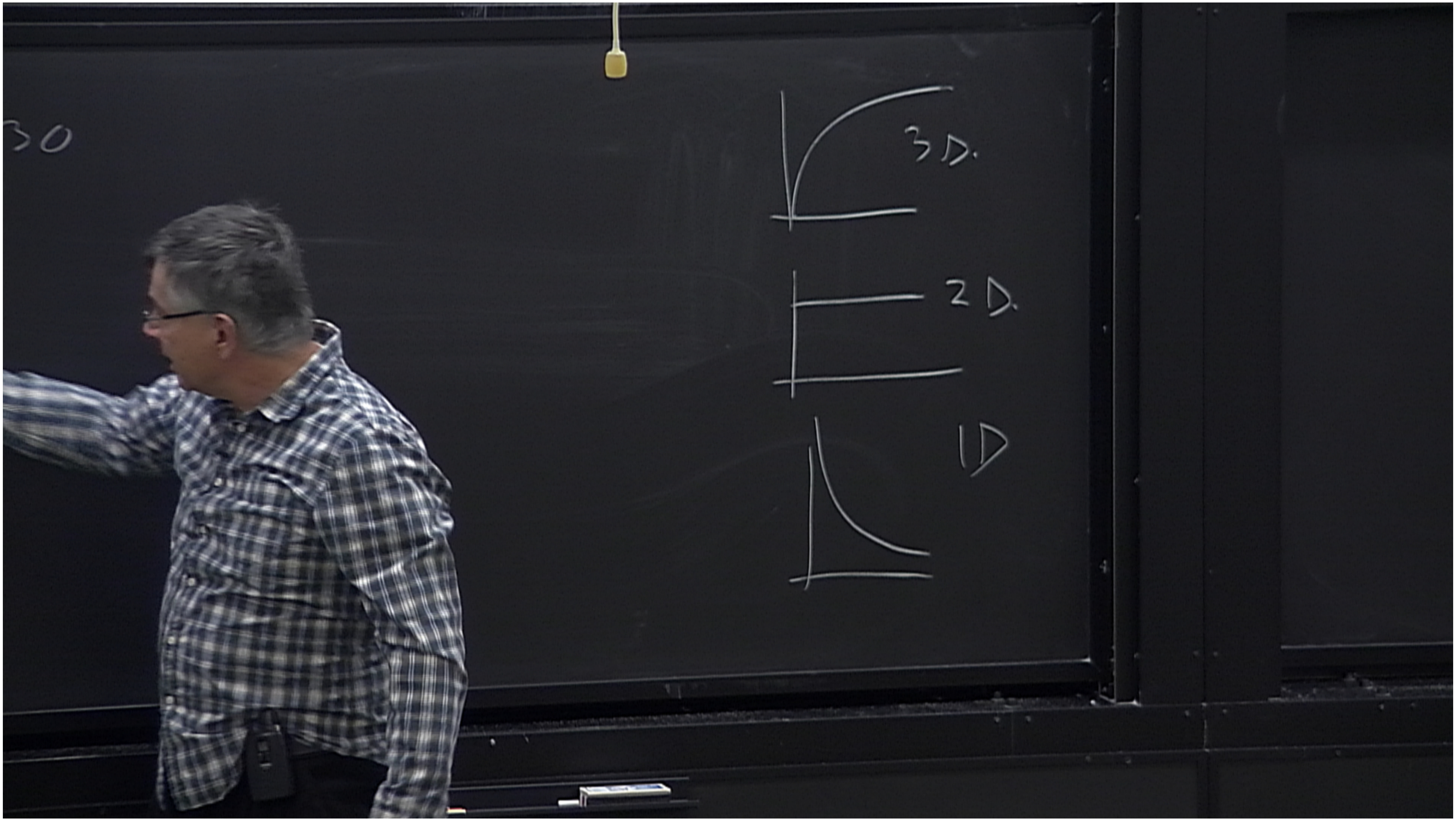
$$\sum_k{}' a_k = C$$

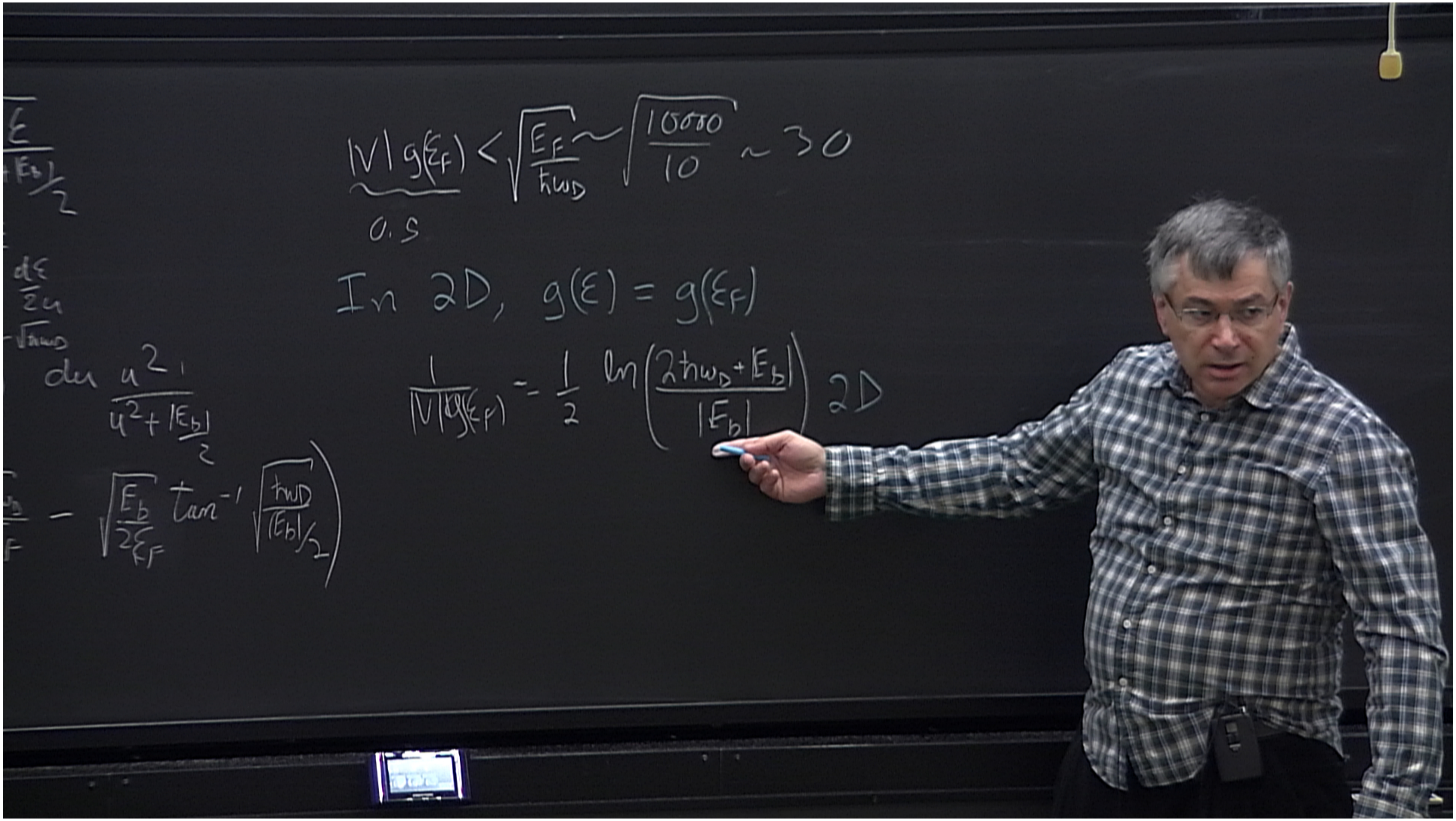
$$C = -|V| \sum_k{}' \frac{1}{E - 2\varepsilon_k}$$

for binding, $E = -|E_b|$

$$1 = +|V| \sum_k{}' \frac{1}{|E_b| + 2\varepsilon_k}$$


$\underbrace{|V|\, g(\varepsilon_F)}_{0.5} < \sqrt{\frac{E_F}{\hbar\omega_D}} \sim \sqrt{\frac{10000}{10}} \sim 30$


30
3D.
2D.
1D


|V| g(ε_F) < √(E_F / ħω_D) ~ √(10000/10) ~ 30
0.5
In 2D, g(ε) = g(ε_F)
1/(|V| g(ε_F)) = 1/2 ln((2ħω_D + |E_b|)/|E_b|) 2D

$$\underbrace{|V|\,g(\varepsilon_F)}_{0.5} < \sqrt{\frac{E_F}{\hbar\omega_D}} \sim \sqrt{\frac{10000}{10}} \sim 30$$

In 2D, $g(\varepsilon) = g(\varepsilon_F)$

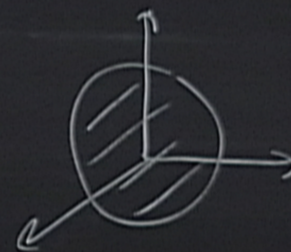
$$\frac{1}{|V|\,g(\varepsilon_F)} = \frac{1}{2}\ln\left(\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right) \quad 2D$$

$$du\,\frac{u^2}{u^2 + \frac{|E_b|}{2}}$$

$$- \sqrt{\frac{E_b}{2\varepsilon_F}}\,\tan^{-1}\sqrt{\frac{\hbar\omega_D}{|E_b|/2}}$$

$$\underbrace{|V|\, g(\epsilon_F)}_{0.5} < \sqrt{\frac{E_F}{\hbar\omega_D}} \sim \sqrt{\frac{10000}{10}} \sim 30$$

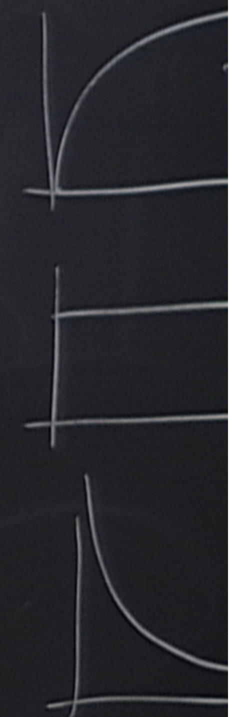
Cooper said that there is an inert Fermi sea



In 2D, $g(\epsilon) = g(\epsilon_F)$

$$\frac{1}{|V|\, g(\epsilon_F)} = \frac{1}{2} \ln\left(\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right) \quad 2D$$

$$\sqrt{\frac{\hbar\omega_D}{|E_b|/2}}\Bigg)$$



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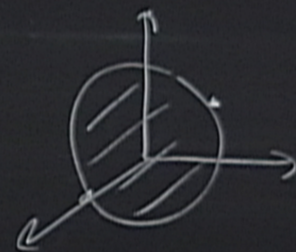

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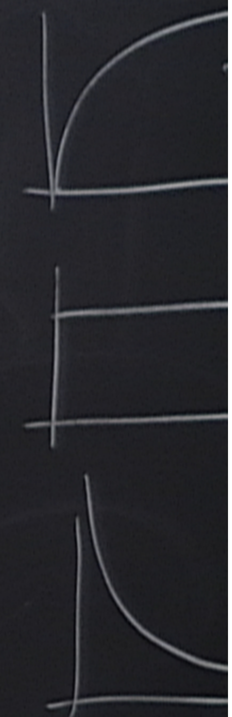


$\boxed{\text{for binding}}$, $E = -|E_b|$

$E = 2\varepsilon_F - |E_b|$

$\varepsilon_F < \varepsilon_k < \varepsilon_F + \hbar\omega_D$

$$1 = +|V| \sum_k \frac{1}{|E_b| + 2\varepsilon_k}$$

$$= \frac{|V|}{2} \int_0^{\hbar\omega_D} d\varepsilon \, \frac{g(\varepsilon)}{\varepsilon + |E_b|/2}$$

$g(\varepsilon) = a\sqrt{\varepsilon}$ in 3D

$$= g(\varepsilon_F)\sqrt{\frac{\varepsilon}{\varepsilon_F}}$$

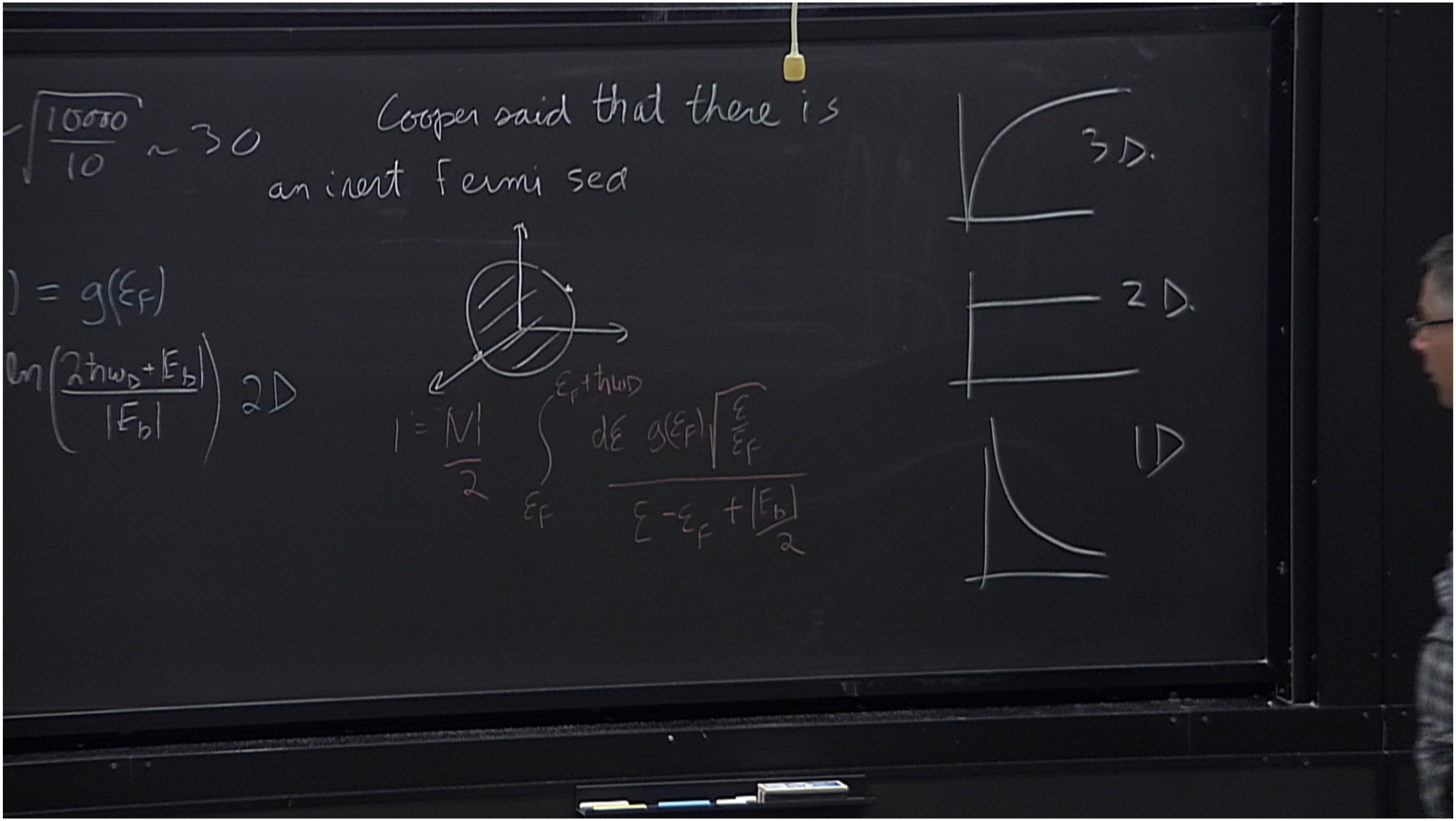
$$1 = \frac{|V| a}{2} \int_0^{\hbar\omega_D} d\varepsilon \, \frac{\sqrt{\varepsilon}}{\varepsilon + |E_b|/2}$$

$u = \sqrt{\varepsilon}$

$du = \frac{d\varepsilon}{2u}$

$$1 = \frac{|V| g(\varepsilon_F)}{2\sqrt{\varepsilon_F}} \, 2 \int_0^{\sqrt{\hbar\omega_D}} du \, \frac{u^2}{u^2 + |E_b|/2}$$

$$\frac{1}{|V| g(\varepsilon_F)} = \left(\sqrt{\frac{\hbar\omega_D}{\varepsilon_F}} - \sqrt{\frac{E_b}{2\varepsilon_F}} \tan^{-1} \sqrt{\frac{\hbar\omega_D}{|E_b|/2}} \right)$$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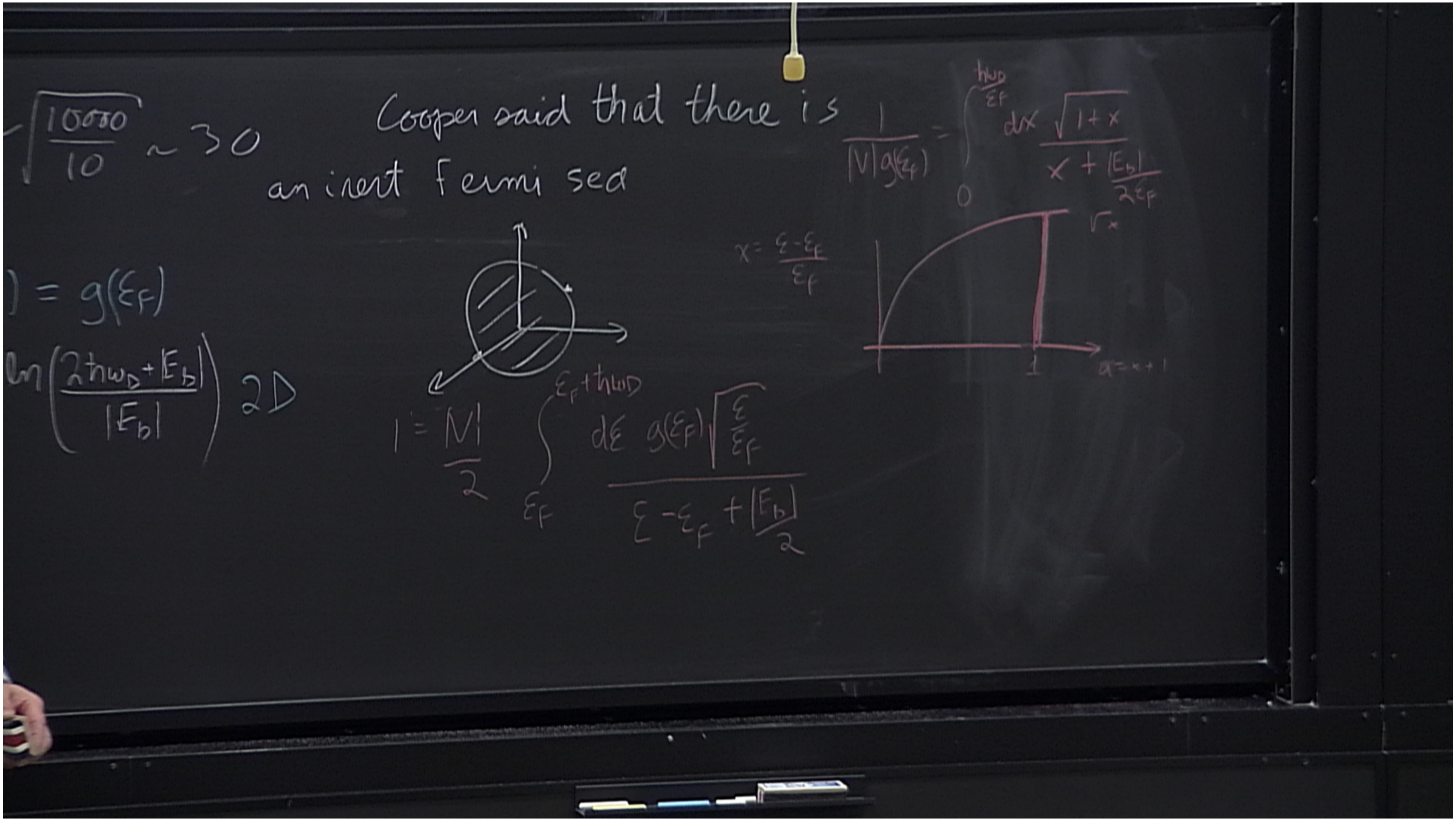
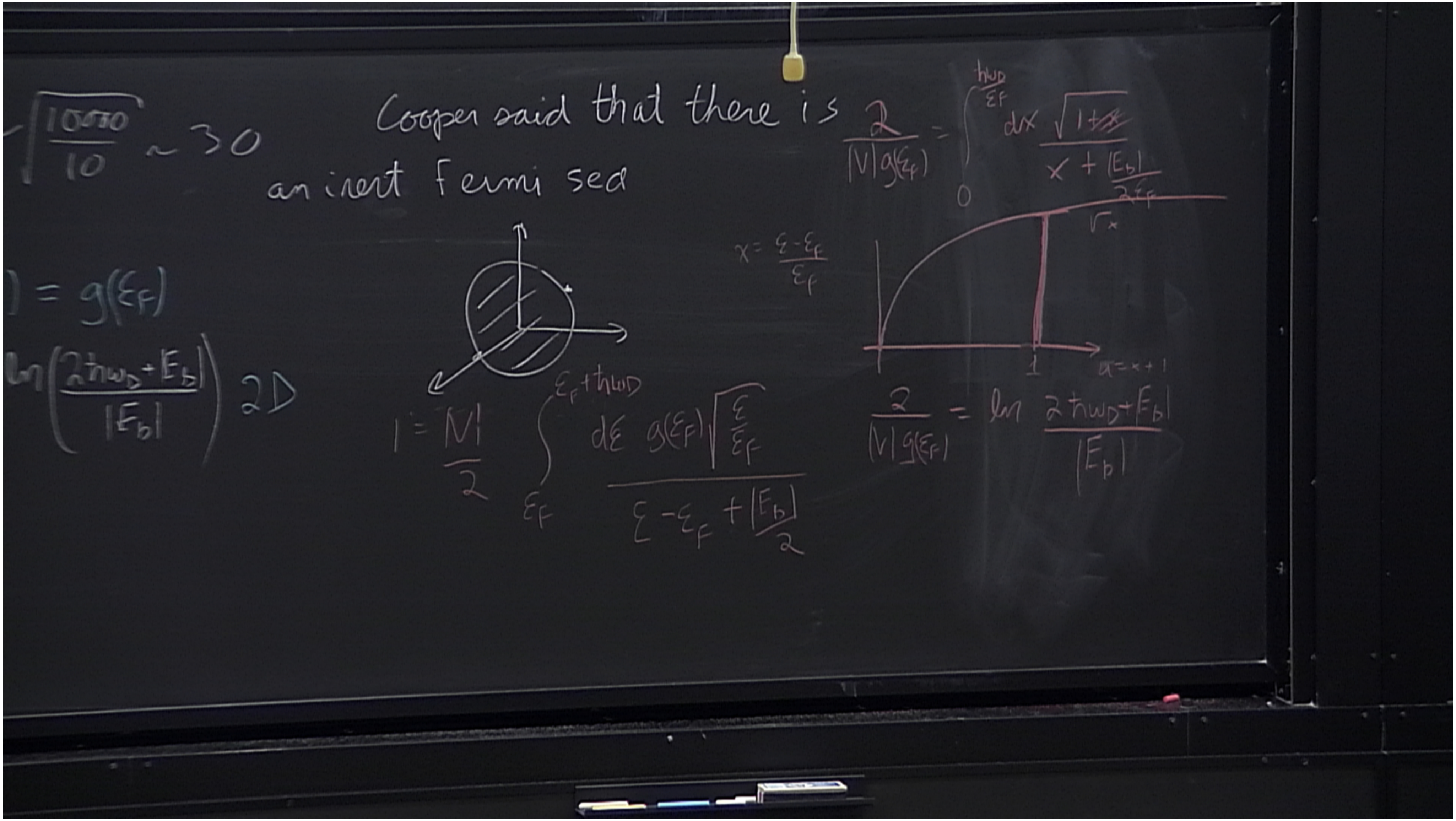
an inert Fermi sea
~ 30
= g(ε_F)
2D
ε_F + ħω_D
dε g(ε_F)
ε_F
3D.
2D.
1D


Cooper said that there is
an inert Fermi sea
$\sqrt{\frac{10000}{10}} \sim 30$
$= g(\varepsilon_F)$
$\ln\left(\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right)$ 2D
$1 = \frac{|V|}{2} \int_{\varepsilon_F}^{\varepsilon_F+\hbar\omega_D} \frac{d\varepsilon\, g(\varepsilon_F)\sqrt{\frac{\varepsilon}{\varepsilon_F}}}{\varepsilon - \varepsilon_F + \frac{|E_b|}{2}}$
$x = \frac{\varepsilon - \varepsilon_F}{\varepsilon_F}$
$\frac{1}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D/\varepsilon_F} dx \frac{\sqrt{1+x}}{x + \frac{|E_b|}{2\varepsilon_F}}$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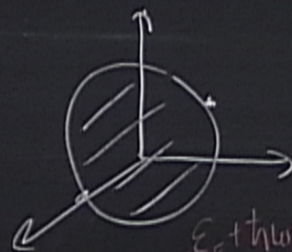
$x = \frac{\varepsilon - \varepsilon_F}{\varepsilon_F}$
$\frac{2}{|V| g(\varepsilon_F)} = \int_0^{\hbar\omega_D/\varepsilon_F} dx \frac{\sqrt{1+x}}{x + \frac{|E_b|}{2\varepsilon_F}}$
$\sqrt{x}$
1
$\alpha = x + 1$
$\frac{2}{|V| g(\varepsilon_F)} = \ln \frac{2\hbar\omega_D + |E_b|}{|E_b|}$

$\sqrt{\frac{10000}{10}} \sim 30$

Cooper said that there is an inert Fermi sea

$) = g(\varepsilon_F)$

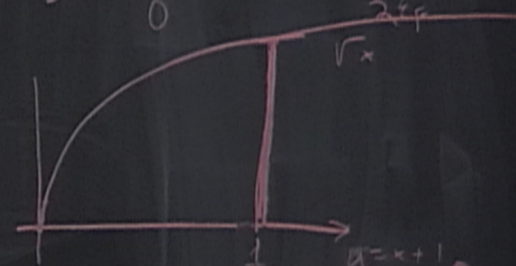
$\ln\left(\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right)$ 2D



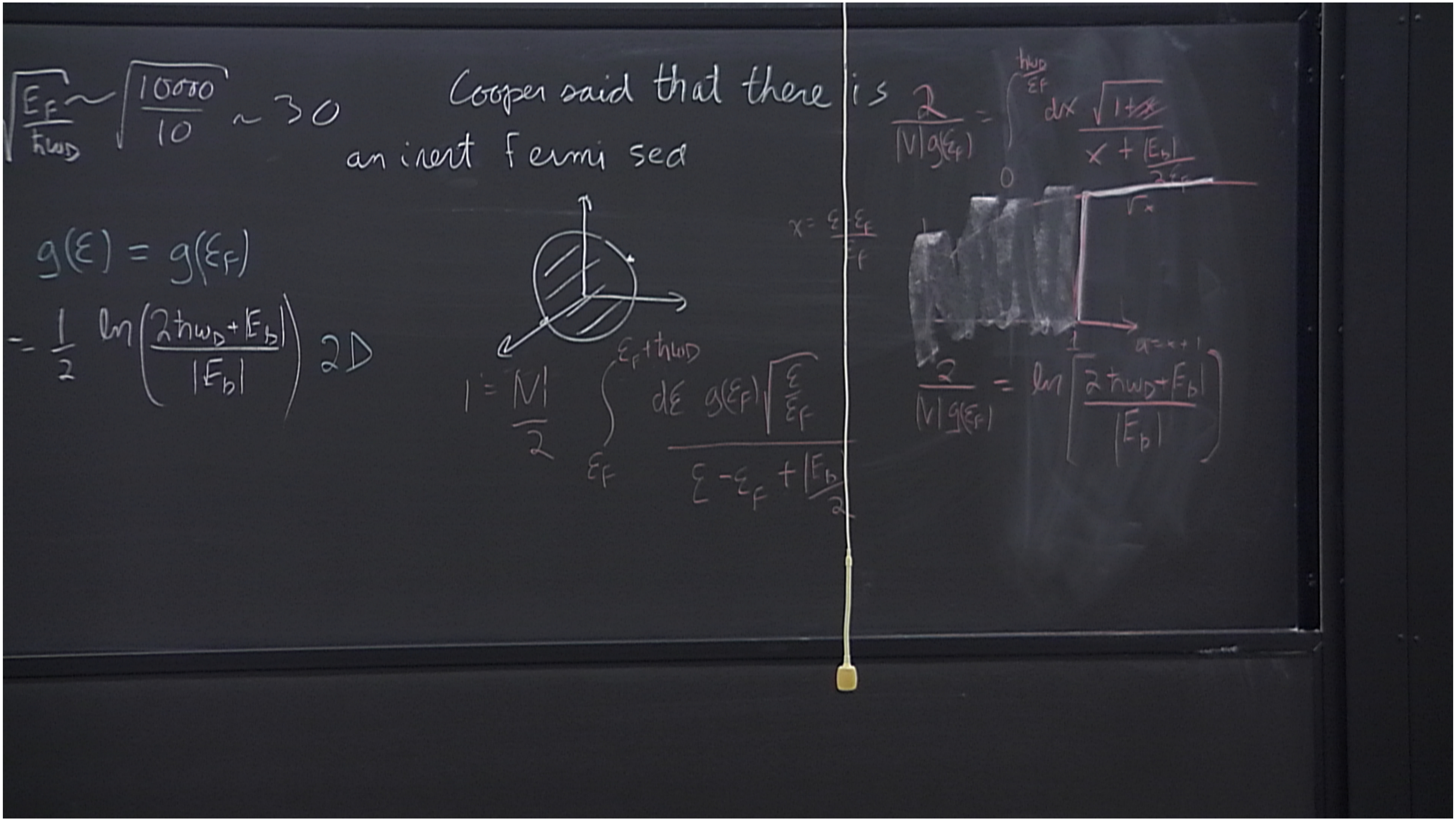
$$1 = \frac{|V|}{2} \int_{\varepsilon_F}^{\varepsilon_F + \hbar\omega_D} \frac{d\varepsilon \; g(\varepsilon_F)\sqrt{\frac{\varepsilon}{\varepsilon_F}}}{\varepsilon - \varepsilon_F + \frac{|E_b|}{2}}$$

$x = \frac{\varepsilon - \varepsilon_F}{\varepsilon_F}$

$$\frac{2}{|V| g(\varepsilon_F)} = \int_0^{\frac{\hbar\omega_D}{\varepsilon_F}} dx \frac{\sqrt{1 + x}}{x + \frac{|E_b|}{2\varepsilon_F}}$$



$$\frac{2}{|V| g(\varepsilon_F)} = \ln\left[\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right]$$


$\sqrt{\frac{E_F}{\hbar\omega_D}} \sim \sqrt{\frac{10000}{10}} \sim 30$
Cooper said that there is an inert Fermi sea
$g(\varepsilon) = g(\varepsilon_F)$
$= \frac{1}{2} \ln\left(\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right)$ 2D
$1 = \frac{|V|}{2} \int_{\varepsilon_F}^{\varepsilon_F + \hbar\omega_D} d\varepsilon \, \frac{g(\varepsilon_F)\sqrt{\frac{\varepsilon}{\varepsilon_F}}}{\varepsilon - \varepsilon_F + \frac{|E_b|}{2}}$
$x = \frac{\varepsilon - \varepsilon_F}{\varepsilon_F}$
$\frac{2}{|V| g(\varepsilon_F)} = \int_0^{\frac{\hbar\omega_D}{\varepsilon_F}} dx \, \frac{\sqrt{1+\cancel{x}}}{x + \frac{|E_b|}{2\varepsilon_F}}$
$\sqrt{x}$
$a = x + 1$
$\frac{2}{|V| g(\varepsilon_F)} = \ln\left[\frac{2\hbar\omega_D + |E_b|}{|E_b|}\right]$

$$|E_b| = \frac{2\hbar\omega_D \exp\left(-\frac{2}{g(\xi_F)|V|}\right)}{1-\exp\left(-\frac{2}{g(\xi_F)|V|}\right)}$$

1) binding in 3D no matter how small $|V|$ is.

3D "looks" like 2D

2) result is non-analytic in $|V|$

3) BCS is the same except for the 2 in the exponent

4) isotope effect

Kamerlingh-Onnes + Tuyn

4) isotope effect.

Kamerlingh-Onnes + Tuyn (1923) no ~~isotope~~ effect.

1950 Reynolds et al & Maxwell et al. } Hg $T_c \propto M^{-\alpha}$

(V) is.

(V)

the 2 in the exponent

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Kamerlingh-Onnes + Tuyn (1923) no ~~iso~~tope effect.

1950 Reynolds et al & Maxwell et al } Hg $\quad T_c \propto M^{-\alpha}$

$\alpha \sim 0.46$

$\omega_D \propto \sqrt{\frac{K}{M}}$ (Cooper: $\underline{\alpha = 0.5}$

(V)

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|V| is.

|V|

the 2 in the exponent

5) Weak coupling : $|E_b| = 2\hbar\omega_D \exp\left(-\frac{2}{g(\epsilon_F)|V|}\right)$

=

...ll $|V|$ is.

...n $|V|$

fa the 2 in the exponent

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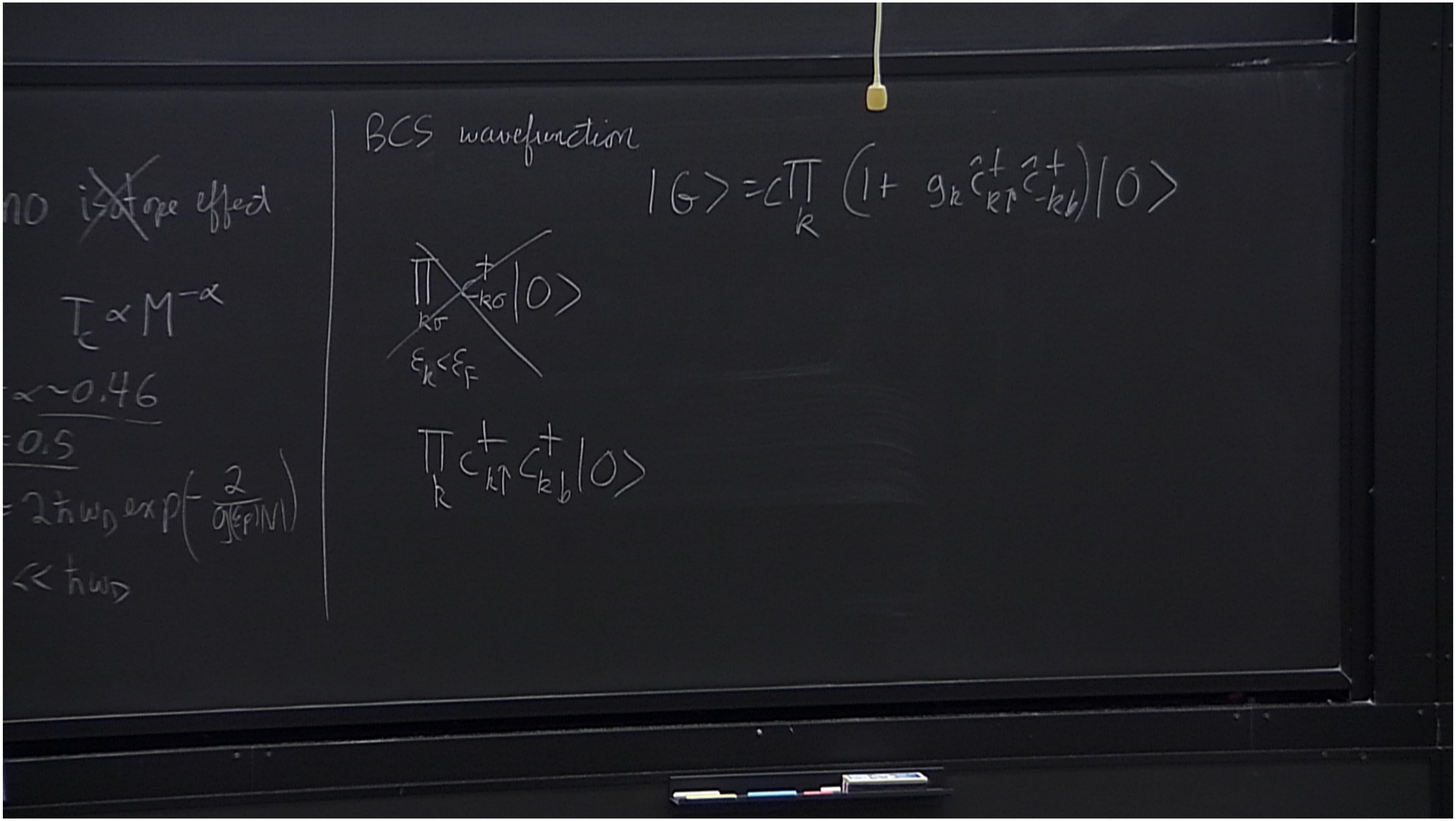
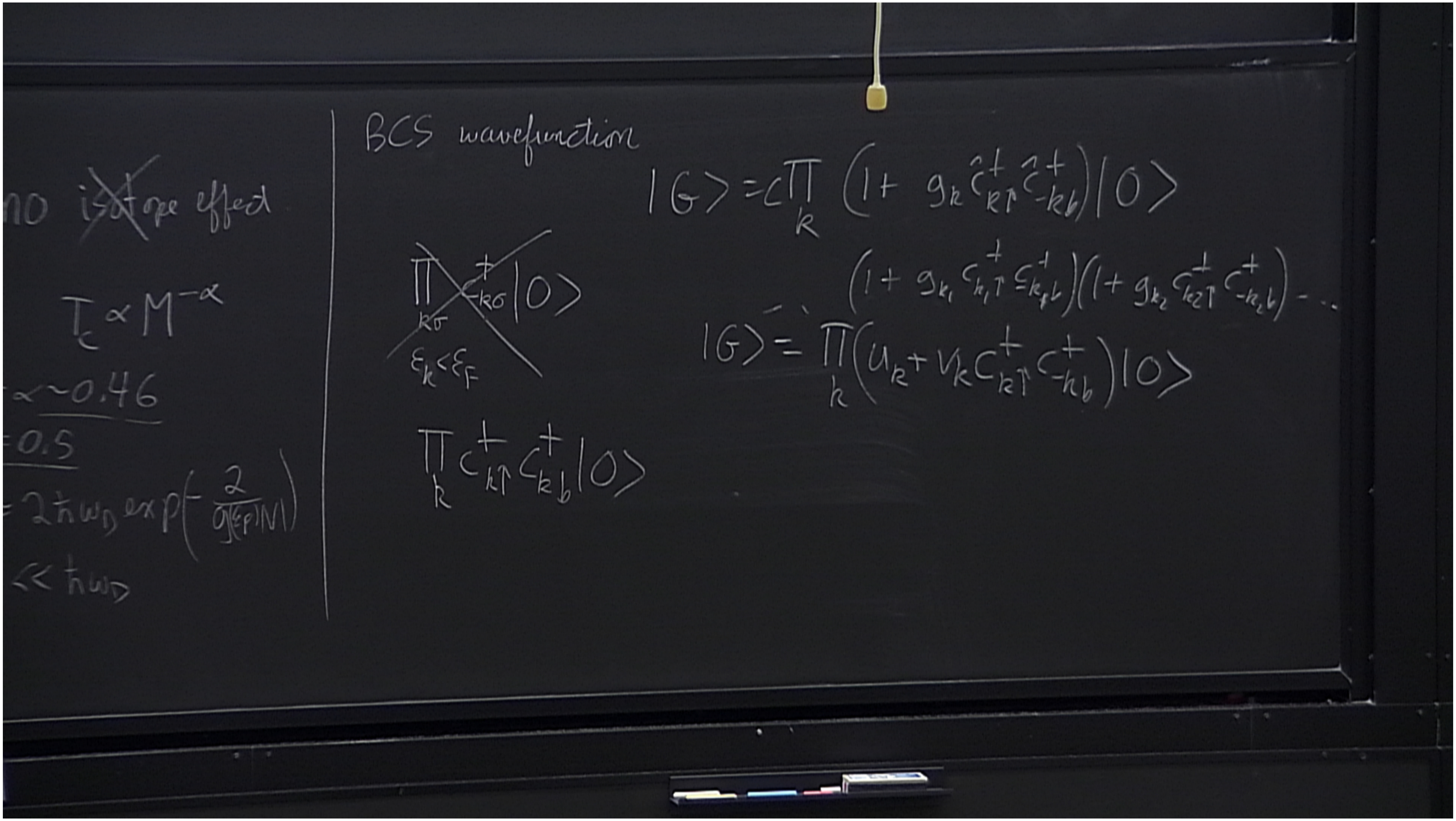
$\alpha \sim 0.46$

$\omega_D \propto \sqrt{\frac{K}{M}}$ (Cooper: $\alpha = 0.5$

5) Weak coupling : $|E_b| = 2\hbar\omega_D \exp\left(-\frac{2}{g\rho(\epsilon_F)|V|}\right)$

$\Rightarrow |E_b| \ll \hbar\omega_D$

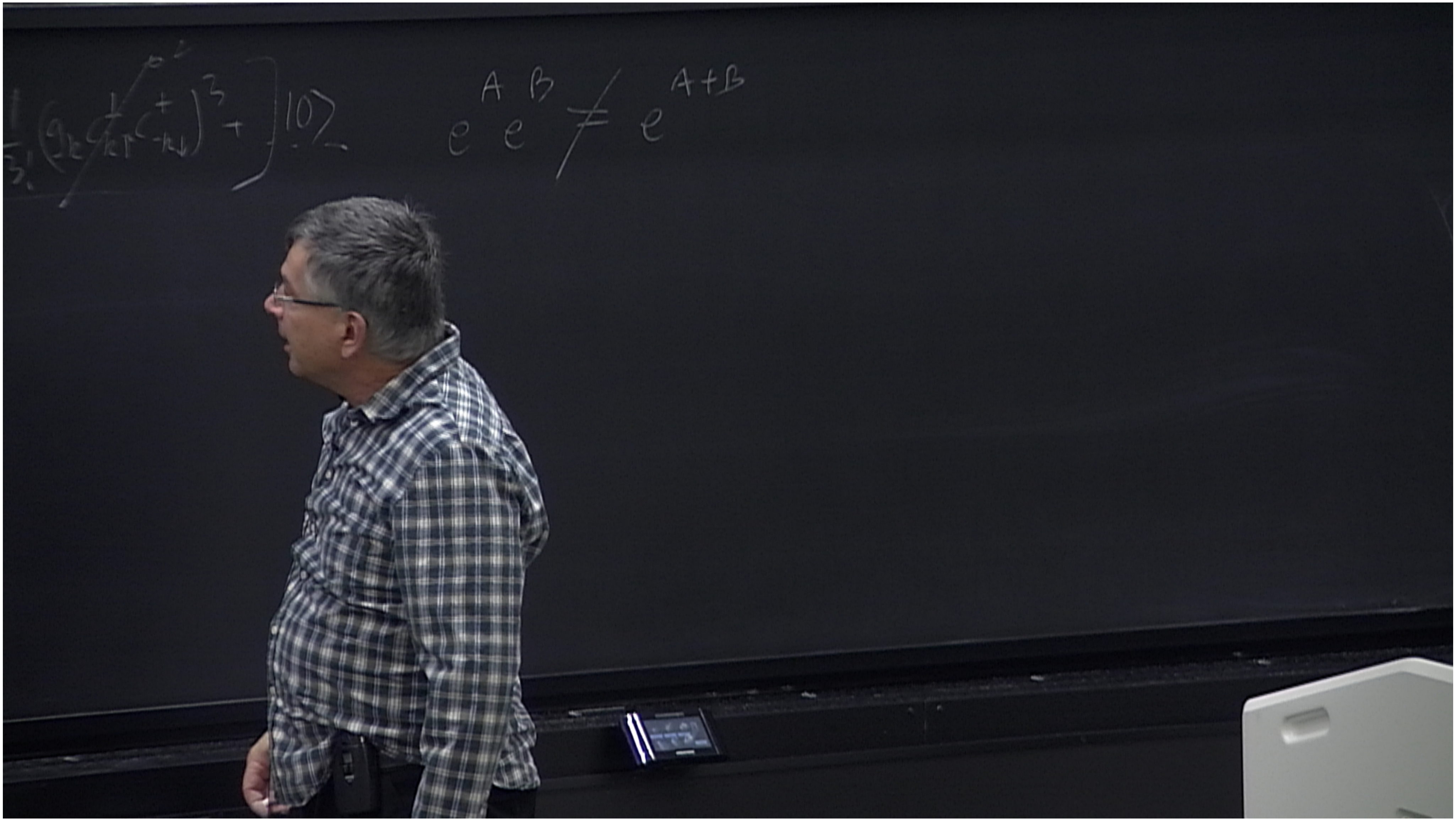

no ~~isotope~~ effect
$T_c \propto M^{-\alpha}$
$\alpha \sim 0.46$
$= 0.5$
$= 2\hbar\omega_D \exp\left(-\frac{2}{g(\epsilon_F)|V|}\right)$
$\ll \hbar\omega_D$
BCS wavefunction
$\cancel{\prod_{k\sigma} c^\dagger_{k\sigma}|0\rangle \quad \epsilon_k < \epsilon_F}$
$\prod_k c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle$
$|G\rangle = c\prod_k \left(1 + g_k \hat{c}^\dagger_{k\uparrow}\hat{c}^\dagger_{-k\downarrow}\right)|0\rangle$


no ~~isotope~~ effect
$T_c \propto M^{-\alpha}$
$\alpha \sim 0.46$
$= 0.5$
$= 2\hbar\omega_D \exp\left(-\frac{2}{g(\epsilon_F)|N|}\right)$
$\ll \hbar\omega_D$
BCS wavefunction
$\cancel{\prod_{k\sigma} c^\dagger_{k\sigma}|0\rangle \quad \epsilon_k < \epsilon_F}$
$\prod_k c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle$
$|G\rangle = c\prod_k \left(1 + g_k \hat{c}^\dagger_{k\uparrow}\hat{c}^\dagger_{-k\downarrow}\right)|0\rangle$
$\left(1 + g_{k_1} c^\dagger_{k_1\uparrow} c^\dagger_{-k_1\downarrow}\right)\left(1 + g_{k_2} c^\dagger_{k_2\uparrow} c^\dagger_{-k_2\downarrow}\right)\cdots$
$|G\rangle = \prod_k \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$

$$|G\rangle = \prod_k \left[1 + g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \frac{1}{2}\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^2 + \frac{1}{3!}\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^3 + \ldots \right] |0\rangle$$

$$|G\rangle = \prod_k \left[1 + g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \cancel{\frac{1}{2}\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^2} + \cancel{\frac{1}{3!}\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^3} + \cdots \right] |0\rangle \qquad e^A$$

$$\prod_k \exp\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)$$


$e^A e^B \neq e^{A+B}$

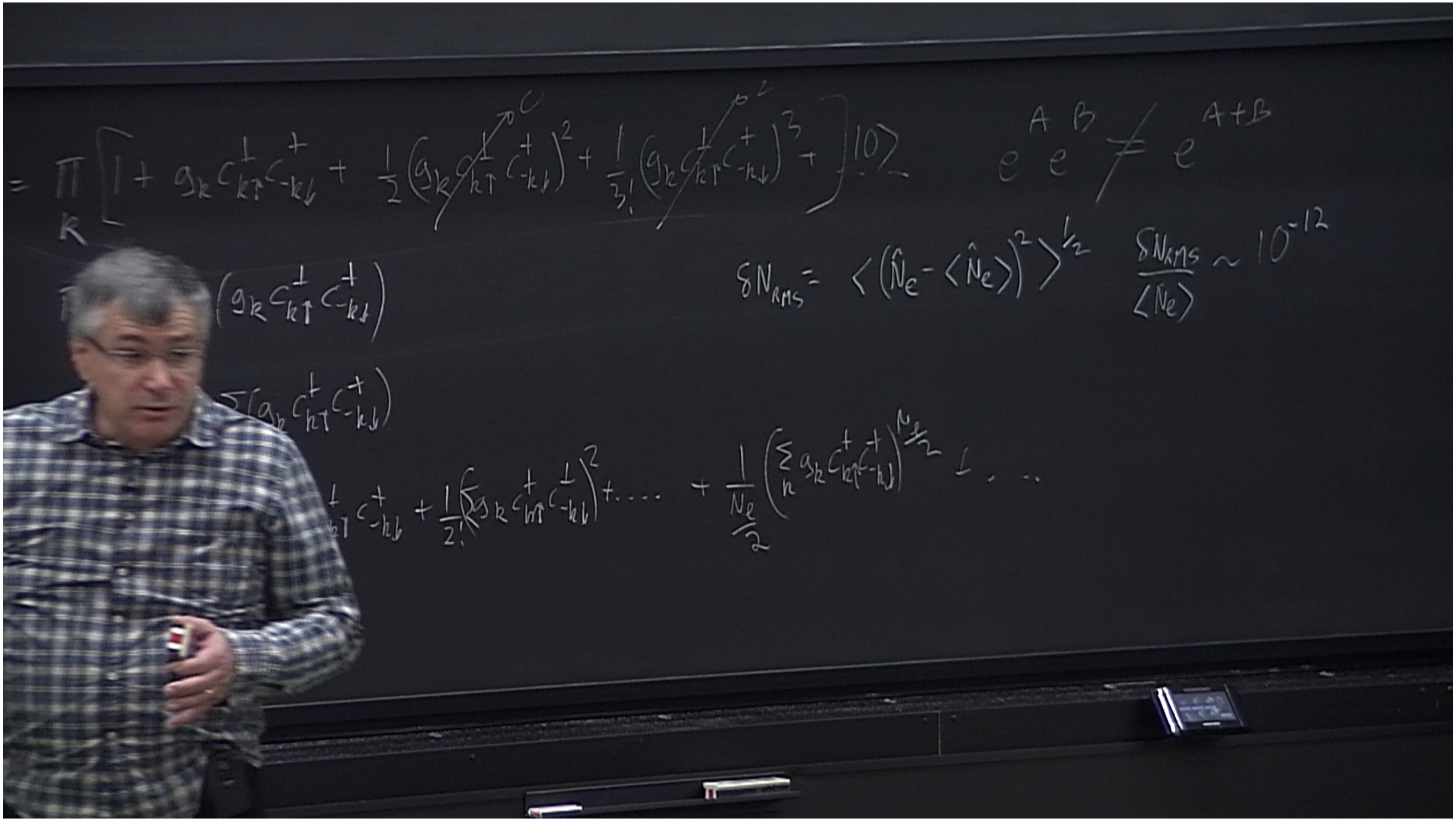
$$|G\rangle = \prod_k \left[1 + g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \frac{1}{2} \cancelto{0}{\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^2} + \frac{1}{3!} \cancelto{0}{\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^3} + \right] |0\rangle$$

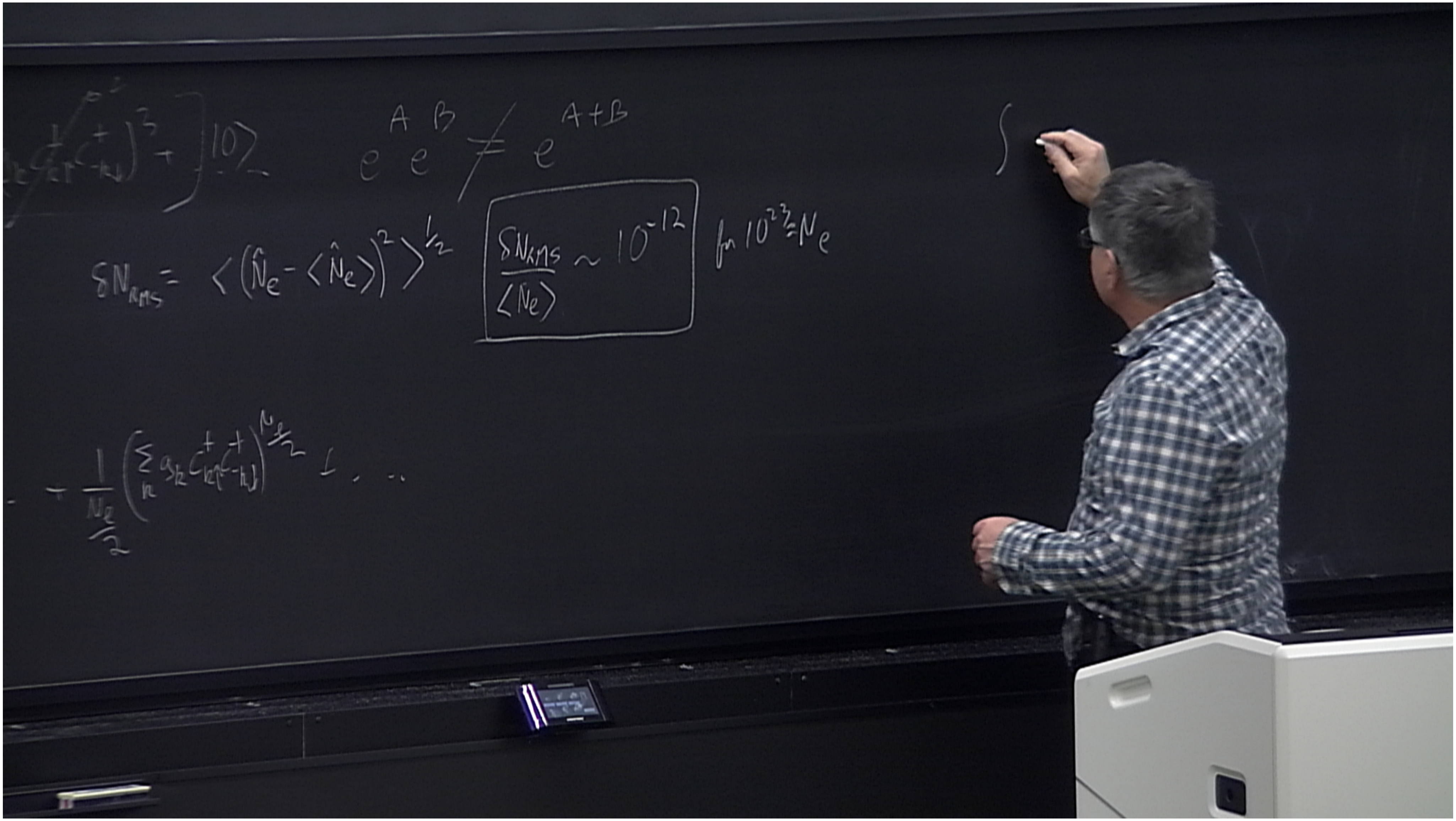
$$e^A e^B \neq e^{A+B}$$

$$\prod_k \exp\left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)$$

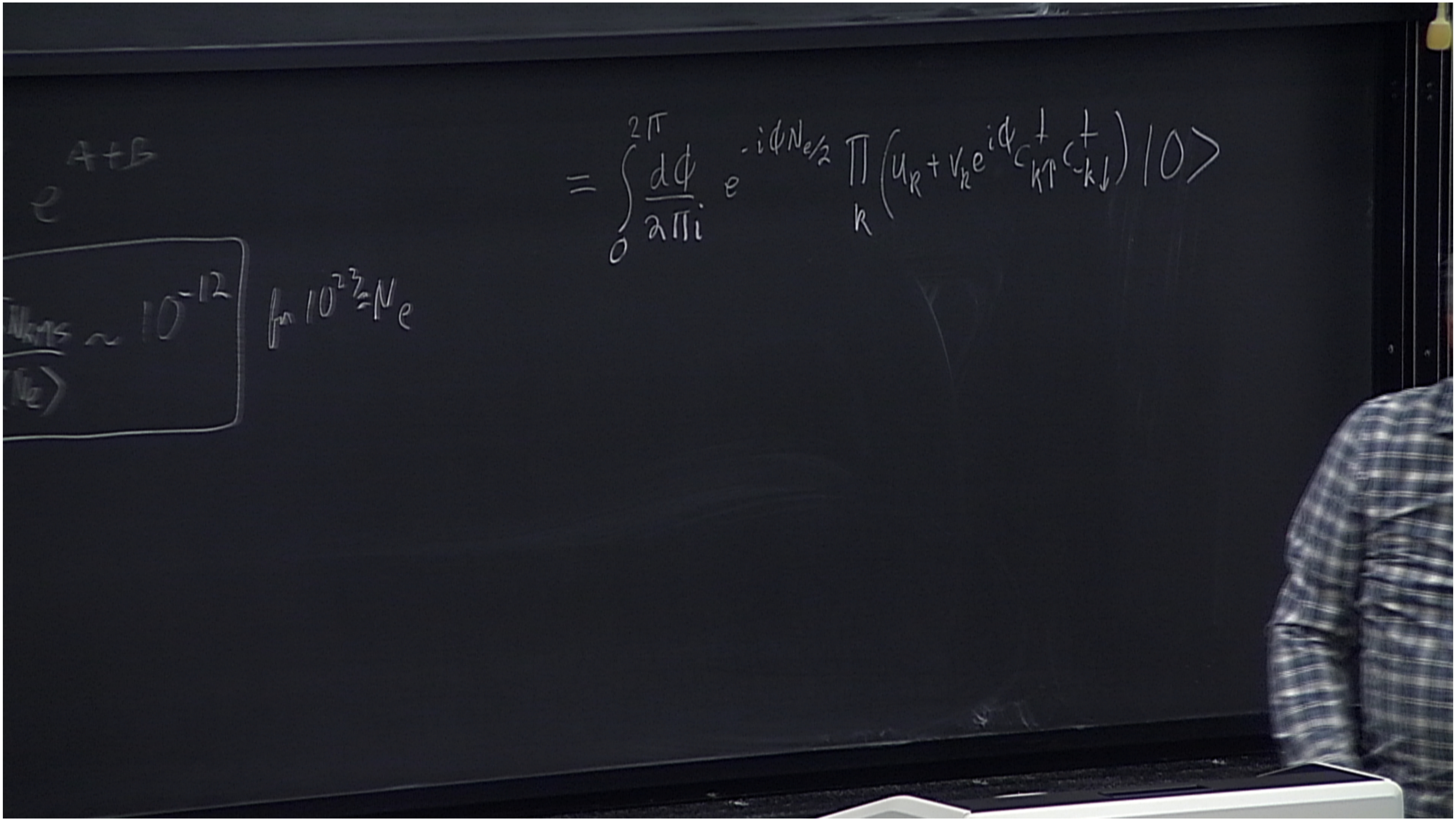
$$= \exp \sum_k \left(g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)$$

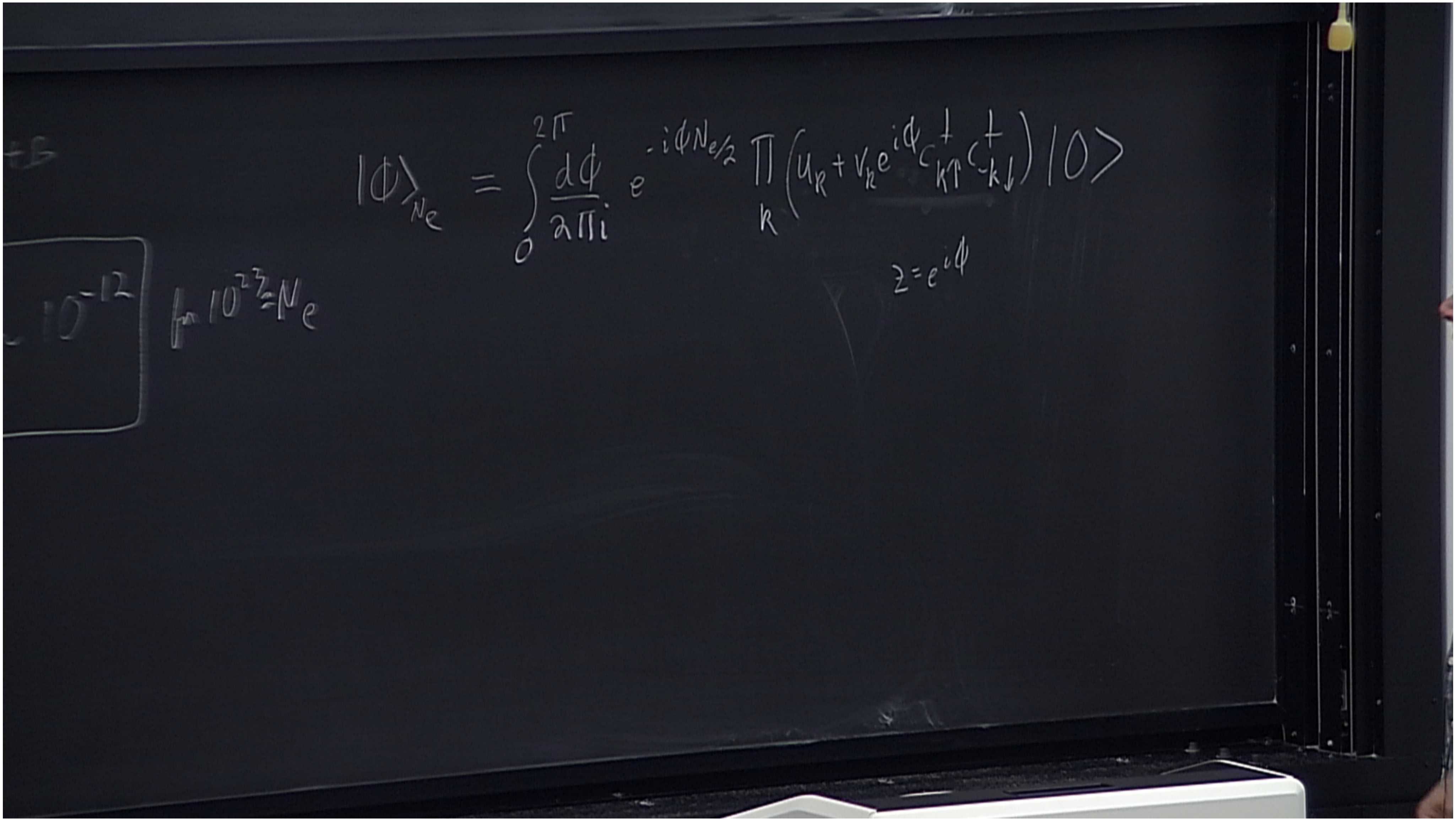
$$= 1 + \sum_k g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \frac{1}{2!}\left(\sum g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^2 + \ldots + \frac{1}{\frac{N_e}{2}} \left(\sum_k g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^{\frac{N_e}{2}} + \ldots$$


$e^A e^B \neq e^{A+B}$
$\delta N_{RMS} = \langle (\hat{N}_e - \langle \hat{N}_e \rangle)^2 \rangle^{1/2}$
$\frac{\delta N_{RMS}}{\langle N_e \rangle} \sim 10^{-12}$



$e^A e^B \neq e^{A+B}$
$\delta N_{RMS} = \langle (\hat{N}_e - \langle \hat{N}_e \rangle)^2 \rangle^{1/2}$
$\frac{\delta N_{RMS}}{\langle N_e \rangle} \sim 10^{-12}$
for $10^{23} \simeq N_e$


e^{A+B}
$= \int_0^{2\pi} \frac{d\phi}{2\pi i} e^{-i\phi N_e/2} \prod_k \left(u_k + v_k e^{i\phi} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right) |0\rangle$
$\sim 10^{-12}$
for $10^{22} \simeq N_e$


$|\phi\rangle_{N_e} = \int_0^{2\pi} \frac{d\phi}{2\pi i} e^{-i\phi N_e/2} \prod_k \left(u_k + v_k e^{i\phi} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right) |0\rangle$
$z = e^{i\phi}$
10^{-12}
for $10^{23} \simeq N_e$

$$\langle G|G\rangle = \langle 0|\prod_k \left(u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}\right)\left(u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)|0\rangle$$

$$= \prod_k \left[|u_k|^2 + |v_k|^2\right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$$

$$\langle G|G\rangle = \langle 0 \prod_k \left(u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}\right) \left(u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}\right)|0\rangle$$

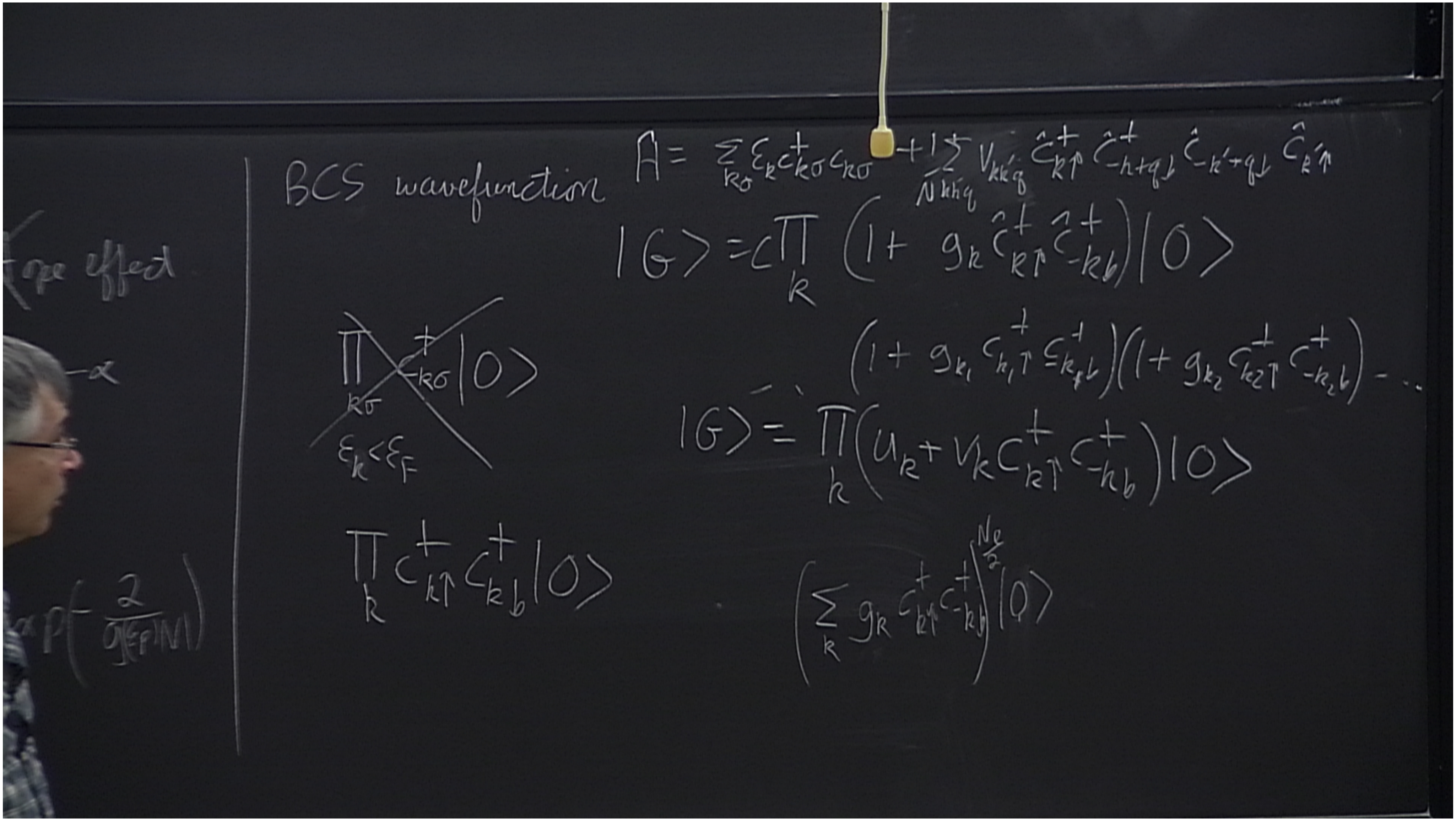
$$= \prod_k \left[|u_k|^2 + |v_k|^2\right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$$

$$\langle G|\hat{H} - \mu \hat{N}|G\rangle$$

$$\langle G | G \rangle = \langle 0 | \prod_k \left(u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow} \right) \left(u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right) | 0 \rangle$$

$$= \prod_k \left[|u_k|^2 + |v_k|^2 \right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$$

$$\frac{\langle G | \hat{H} - \mu \hat{N} | G \rangle}{\langle G | G \rangle} = 2 \sum_k (\varepsilon_k - \mu) v_k^2 + \frac{1}{N} \sum_k$$


BCS wavefunction
$\hat{H} = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'q} V_{kk'q} \hat{c}^\dagger_{k\uparrow} \hat{c}^\dagger_{-k+q\downarrow} \hat{c}_{-k'+q\downarrow} \hat{c}_{k'\uparrow}$
$|G\rangle = c\prod_k \left(1 + g_k \hat{c}^\dagger_{k\uparrow}\hat{c}^\dagger_{-k\downarrow}\right)|0\rangle$
$\left(1+g_{k_1} c^\dagger_{k_1\uparrow} c^\dagger_{-k_1\downarrow}\right)\left(1+g_{k_2} c^\dagger_{k_2\uparrow} c^\dagger_{-k_2\downarrow}\right)\cdots$
$\cancel{\prod_{k\sigma} c^\dagger_{k\sigma}|0\rangle}$
$\varepsilon_k<\varepsilon_F$
$|G\rangle = \prod_k \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$
$\prod_k c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle$
$\left(\sum_k g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^{N_e/2}|0\rangle$
ope effect
$\exp\left(-\frac{2}{g(\varepsilon_F)|V|}\right)$

...e effect

$\eta^{-\alpha}$

$p\left(-\frac{2}{g N(\varepsilon_F)|V|}\right)$

BCS wavefunction

$$\hat{H} = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{N}\sum_{kk'q} V_{kk'q}\, \hat{c}^\dagger_{k\uparrow} \hat{c}^\dagger_{-k+q\downarrow} \hat{c}_{-k'+q\downarrow} \hat{c}_{k'\uparrow}$$

BCS said let's use the "reduced" Hamiltonian

$$\boxed{q=0}$$

$$|G\rangle = C\prod_k \left(1 + g_k \hat{c}^\dagger_{k\uparrow}\hat{c}^\dagger_{-k\downarrow}\right)|0\rangle$$

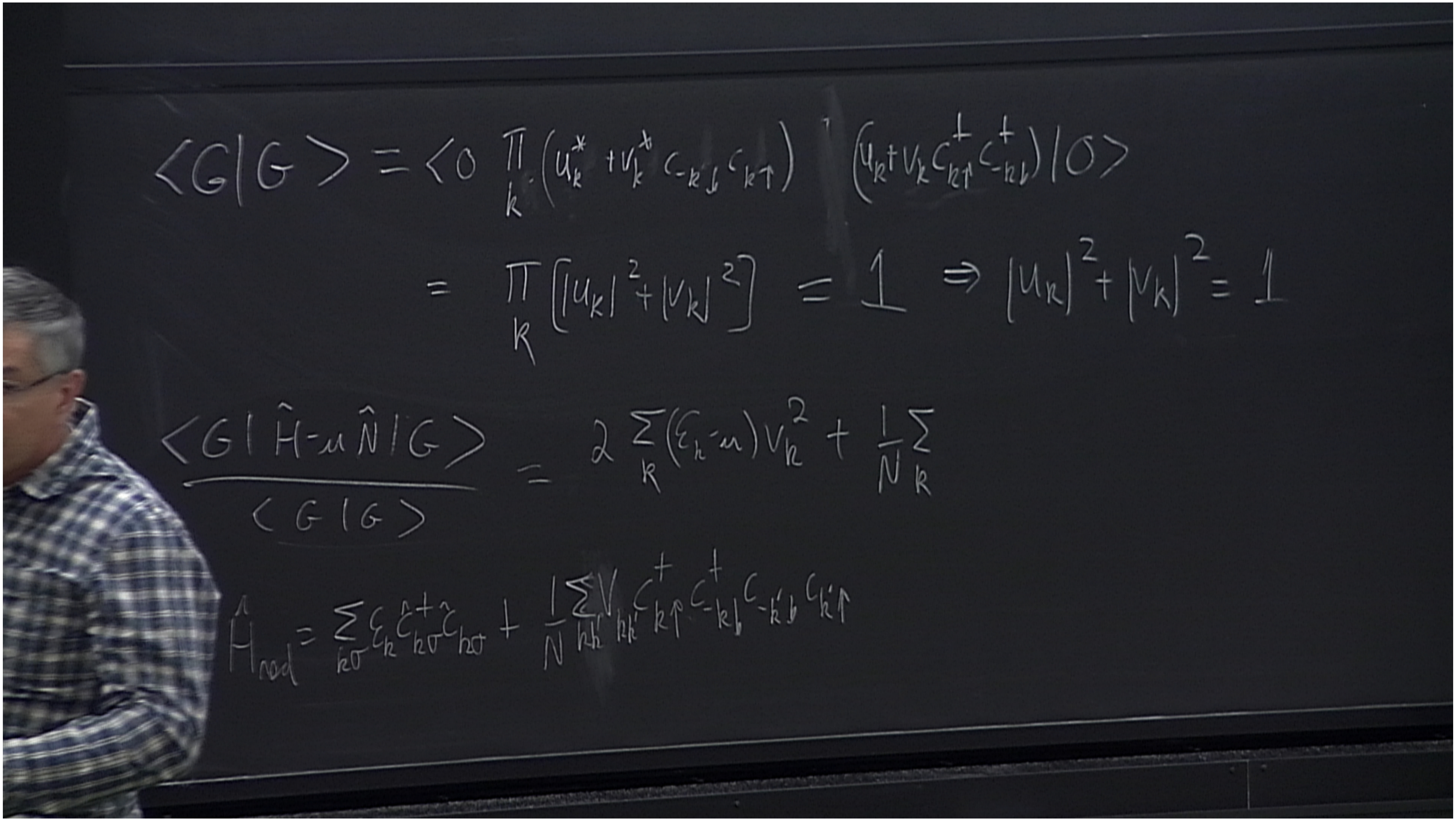
$$\left(1+g_{k_1} c^\dagger_{k_1\uparrow} c^\dagger_{-k_1\downarrow}\right)\left(1+g_{k_2} c^\dagger_{k_2\uparrow} c^\dagger_{-k_2\downarrow}\right)\cdots$$

$$\cancel{\prod_{k\sigma} c^\dagger_{k\sigma}|0\rangle \quad \varepsilon_k<\varepsilon_F}$$

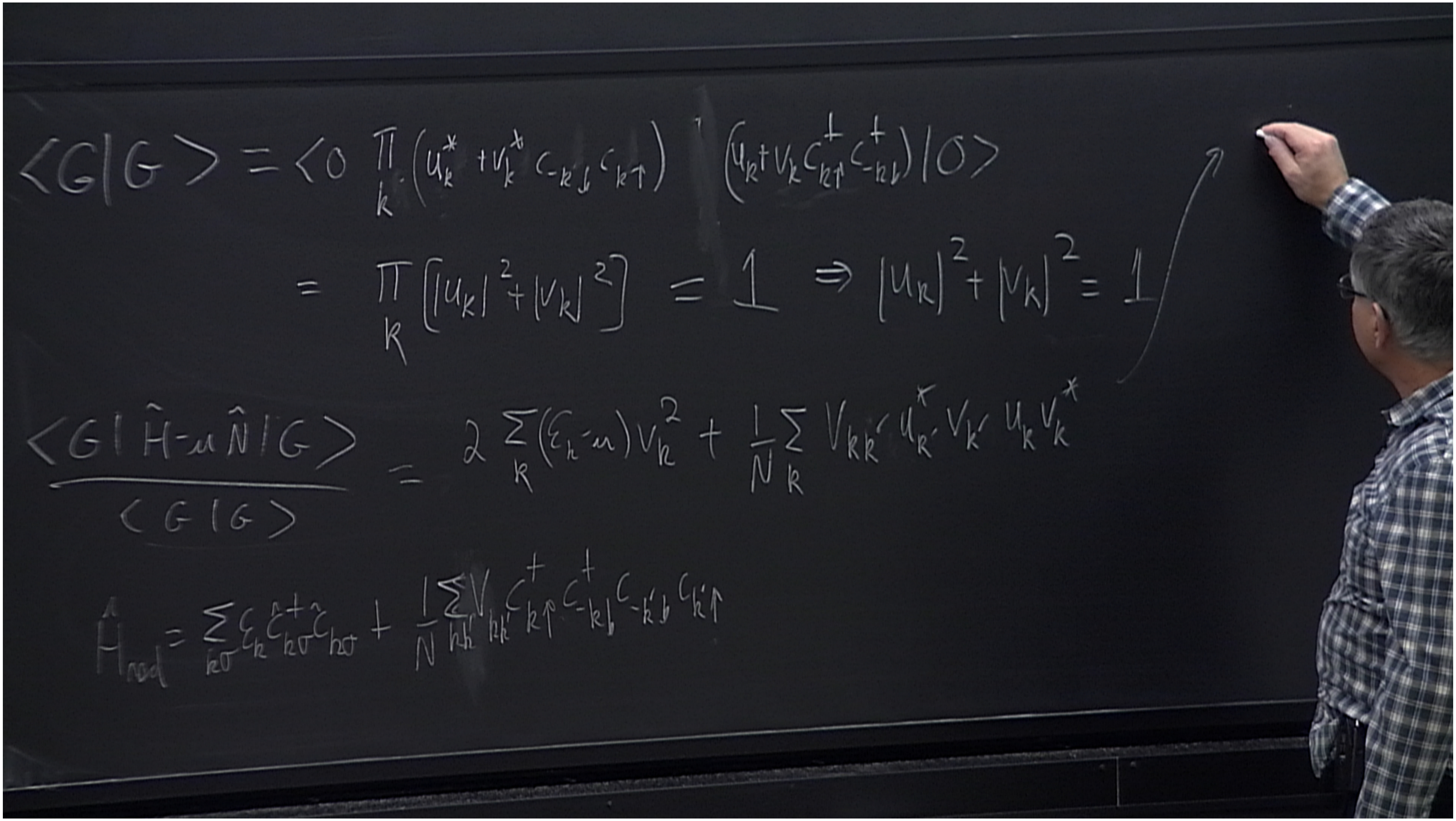
$$|G\rangle = \prod_k \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

$$\prod_k c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle$$

$$\left(\sum_k g_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)^{\frac{N_e}{2}}|0\rangle$$



$\langle G|G\rangle = \langle 0| \prod_k (u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}) (u_k + v_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow})|0\rangle$
$= \prod_k [|u_k|^2 + |v_k|^2] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$
$\frac{\langle G|\hat{H} - \mu \hat{N}|G\rangle}{\langle G|G\rangle} = 2\sum_k (\varepsilon_k - \mu) v_k^2 + \frac{1}{N}\sum_k$
$\hat{H}_{red} = \sum_{k\sigma} \varepsilon_k \hat{c}^{+}_{k\sigma} \hat{c}_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^{+}_{k\uparrow} c^{+}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$


$\langle G|G\rangle = \langle 0| \prod_k (u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}) (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$
$= \prod_k \left[|u_k|^2 + |v_k|^2\right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$
$\frac{\langle G|\hat{H} - \mu \hat{N}|G\rangle}{\langle G|G\rangle} = 2\sum_k (\varepsilon_k - \mu) v_k^2 + \frac{1}{N}\sum_k V_{kk'} u_{k'}^* v_{k'} u_k v_k^*$
$\hat{H}_{red} = \sum_{k\sigma} \varepsilon_k \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$

$$\langle 0| \prod_k \left(u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}\right) \left(u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger\right)|0\rangle$$

$$\prod_k \left[|u_k|^2 + |v_k|^2\right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$$

assume u_k, v_k are real

$$\langle \, \rangle = 2\sum_k (\varepsilon_k - \mu) v_k^2 + \frac{1}{N}\sum_k V_{kk'} u_{k'}^* v_{k'} u_k v_k^*$$

$$\ldots_{\mathrm{ho}} + \frac{1}{N}\sum_{kk'} V_{kk'} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-k'\downarrow} c_{k'\uparrow}$$

$$\langle G|G\rangle = \langle 0|\prod_k \left(u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}\right) \left(u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger\right)|0\rangle$$

$$= \prod_k \left[|u_k|^2 + |v_k|^2\right] = 1 \Rightarrow |u_k|^2 + |v_k|^2 = 1$$

assum

2

$$K \equiv \frac{\langle G|\hat{H} - \mu\hat{N}|G\rangle}{\langle G|G\rangle} = 2\sum_k (\varepsilon_k - \mu) v_k^2 + \frac{1}{N}\sum_k V_{kk'} u_{k'}^* v_{k'} u_k v_k^*$$

$$\hat{H}_{red} = \sum_{k\sigma} \varepsilon_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \frac{1}{N}\sum_{kk'} V_{kk'} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-k'\downarrow} c_{k'\uparrow}$$

$u_p^2 + v_p^2 = 1$ $\quad \frac{\partial v_k}{\partial v_p} = \delta_{kp}$ $\quad \frac{\partial u_k}{\partial v_p} = -\frac{v_p}{u_p}\delta_{k,p}$

assume u_k, v_k are real

$$\frac{\partial K}{\partial v_p} = 0 \qquad 4(\epsilon_p - \mu) v_p + \frac{1}{N}\sum_{kk'} V_{kk'} \left(\delta_{k'p} \left[u_p - \frac{v_p^2}{u_p}\right] u_k v_k + u_{k'} v_{k'} \delta_{k,p} \left(u_p - \frac{v_p^2}{u_p}\right)\right)$$

$u_p^2 + v_p^2 = 1$ $\quad \frac{\partial v_k}{\partial v_p} = \delta_{kp}$ $\quad \frac{\partial u_k}{\partial v_p} = -\frac{v_p}{u_p}\delta_{k,p}$ $\quad V^*_{kk'} = V_{k'k}$

assume u_k, v_k are real

$$\frac{\partial K}{\partial v_p} = 0 \qquad 4(\varepsilon_p - \mu) v_p + \frac{1}{N}\sum_{kk'} V_{kk'}\left(\delta_{k'p}\left[u_p - \frac{v_p^2}{u_p}\right] u_k v_k + \left(u_{k'} v_{k'} \delta_{k,p}\left(u_p - \frac{v_p^2}{u_p}\right)\right)\right)$$

$$\Delta_k \equiv -\frac{1}{N}\sum_{k'} V_{kk'} u_{k'} v_{k'}$$

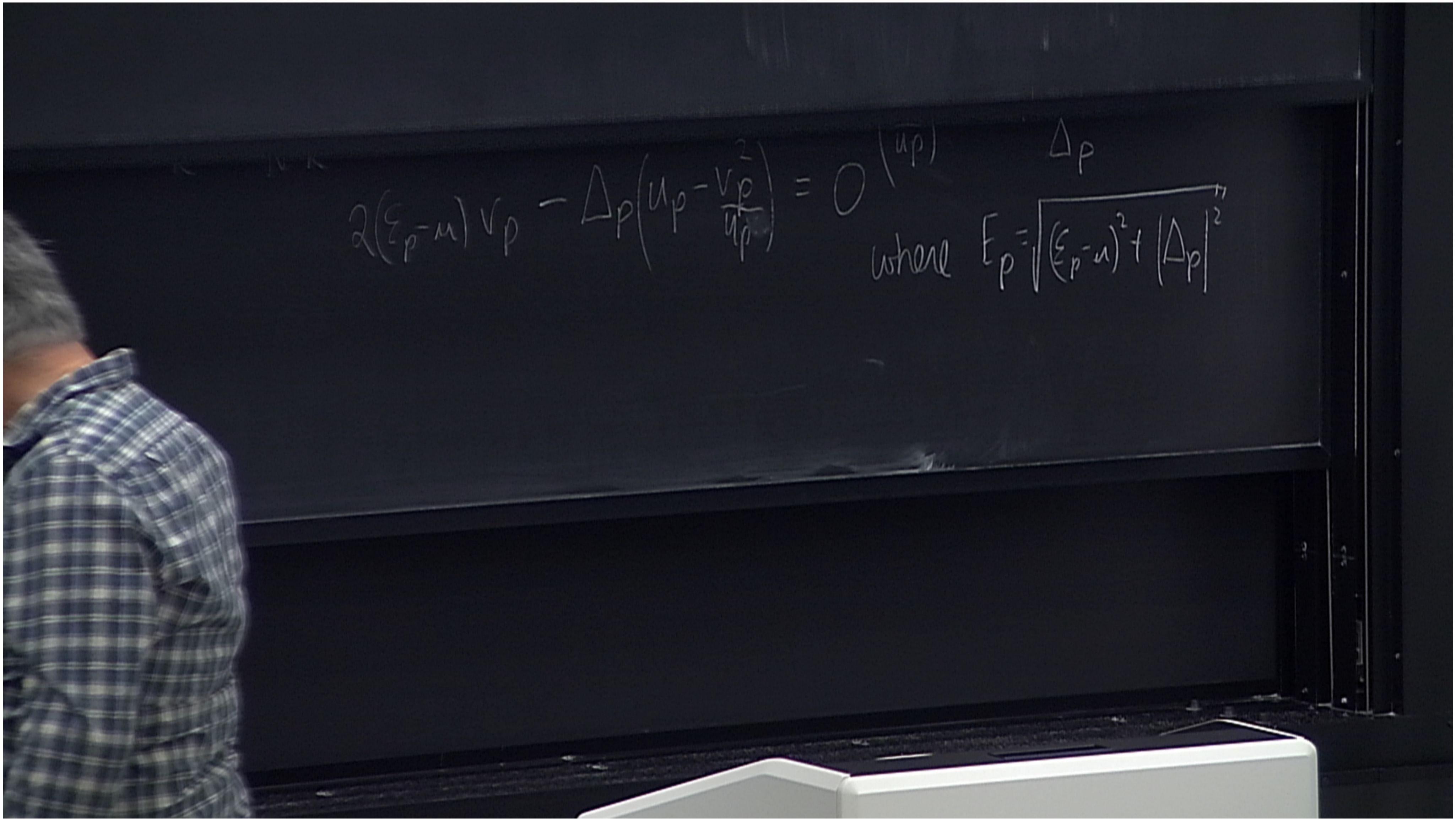
$$u_p^2 + v_p^2 = 1 \qquad \frac{\partial v_k}{\partial v_p} = \delta_{kp} \qquad \frac{\partial u_k}{\partial v_p} = -\frac{v_p}{u_p}\delta_{k,p} \qquad V^*_{kk'} = V_{k'k}$$

assume u_k, v_k are real

$$\frac{\partial K}{\partial v_p} = 0 \qquad 4(\varepsilon_p - \mu)v_p + \frac{1}{N}\sum_{kk'} V_{kk'}\left(\delta_{k'p}\left[u_p - \frac{v_p^2}{u_p}\right]u_k v_k + \left(u_{k'}v_{k'}\delta_{k,p}\left(u_p - \frac{v_p^2}{u_p}\right)\right)\right)$$

$$\Delta_k \equiv -\frac{1}{N}\sum_{k'} V_{kk'} u_{k'} v_{k'}$$

$$2(\varepsilon_p - \mu)v_p - \Delta_p\left(u_p - \frac{v_p^2}{u_p^2}\right) = 0$$


$2(\varepsilon_p - \mu) v_p - \Delta_p\left(u_p - \frac{v_p^2}{u_p}\right) = 0$
where $E_p = \sqrt{(\varepsilon_p - \mu)^2 + |\Delta_p|^2}$

$\frac{\partial \ldots}{\partial v_p} = \delta_{kp}$ $\ldots_p$ real $\quad \frac{\partial u_k}{\partial v_p} = -\frac{v_p}{u_p}\delta_{k,p}$

$$V^*_{kk'} = V_{k'k}$$

$$\ldots(\varepsilon - \mu) v_p + \frac{1}{N}\sum_{kk'} V_{kk'} \left(\delta_{k'p}\left[u_p - \frac{v_p^2}{u_p}\right] u_k v_k + u_{k'} v_{k'} \delta_{k,p}\left(u_p - \frac{v_p^2}{u_p}\right)\right)$$

$$\Delta_k \equiv -\frac{1}{N}\sum_{k'} V_{kk'} u_{k'} v_{k'}$$

$$2(\varepsilon_p-\mu)v_p - \Delta_p\left(u_p - \frac{v_p^2}{u_p}\right) = 0$$

$$\left(\frac{v_p}{u_p}\right) = \frac{E_p - (\varepsilon_p-\mu)}{\Delta_p}$$

where $E_p = \sqrt{(\varepsilon_p-\mu)^2 + |\Delta_p|^2}$

$$u_p^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_p - \mu}{E_p}\right)$$

$$v_p^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \mu}{E_p}\right)$$

$$u_p v_p = \frac{\Delta_p}{2E_p}$$

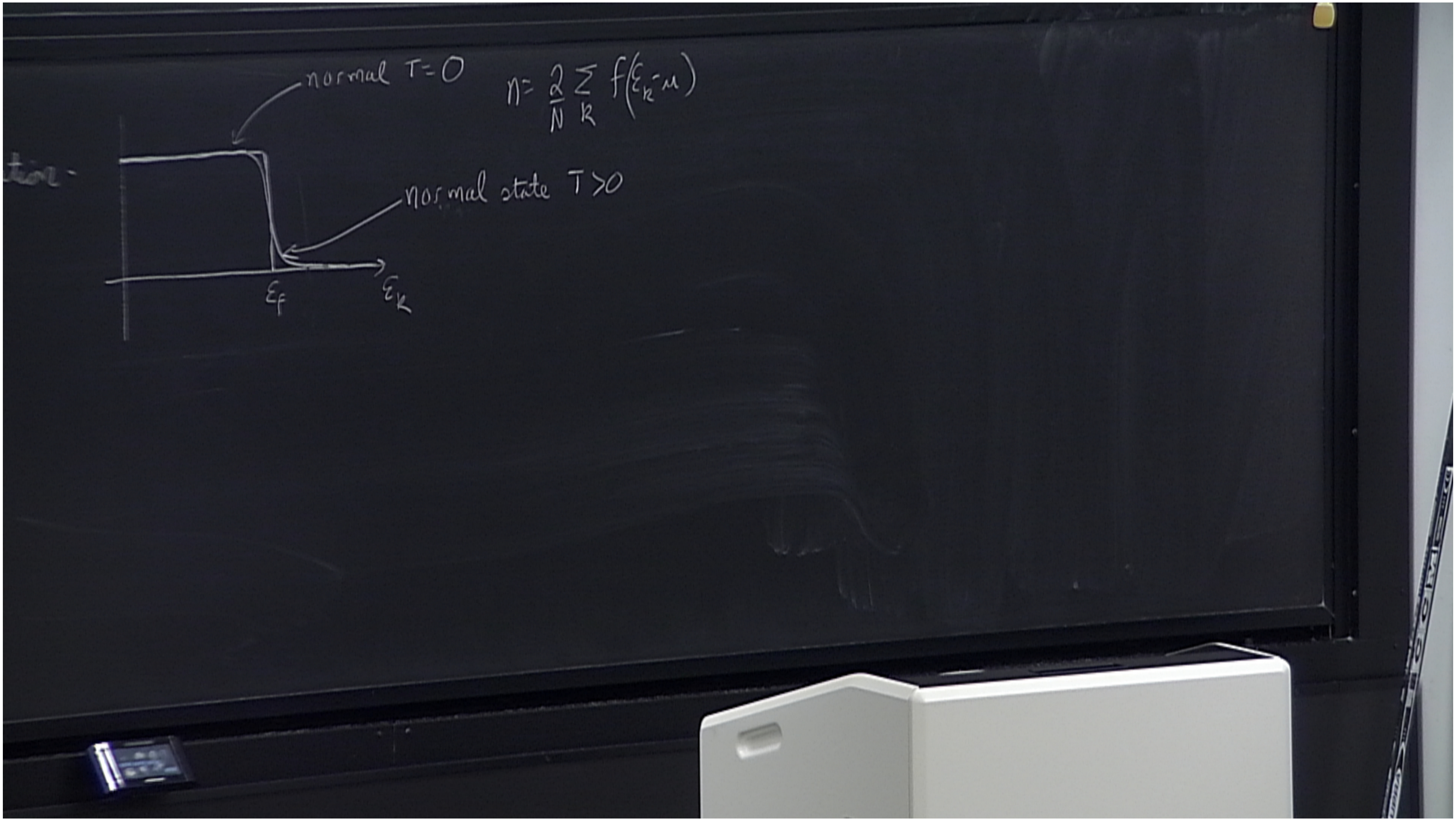
$$\boxed{\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}}$$

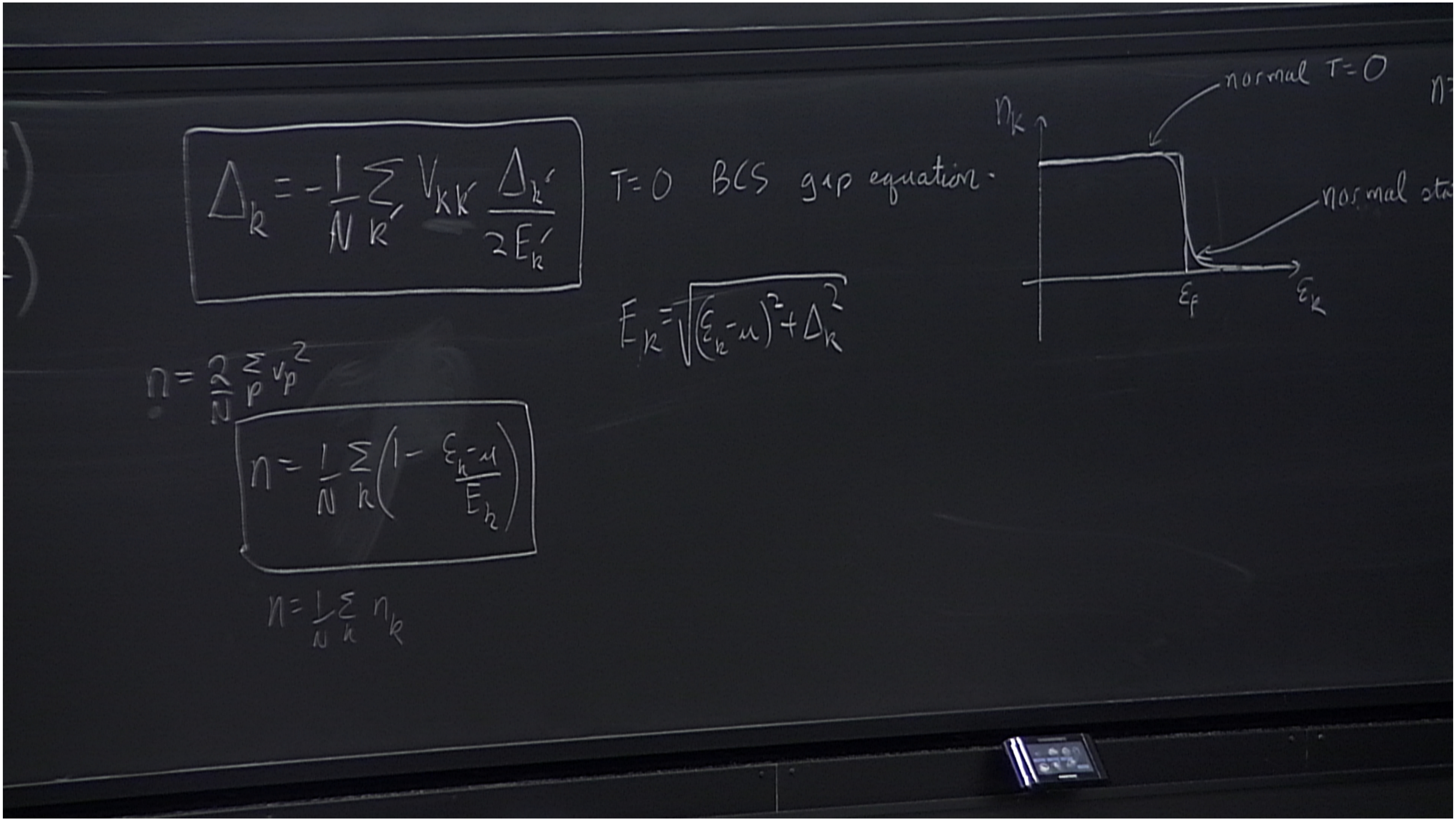
$T=0$ BCS gap equation.

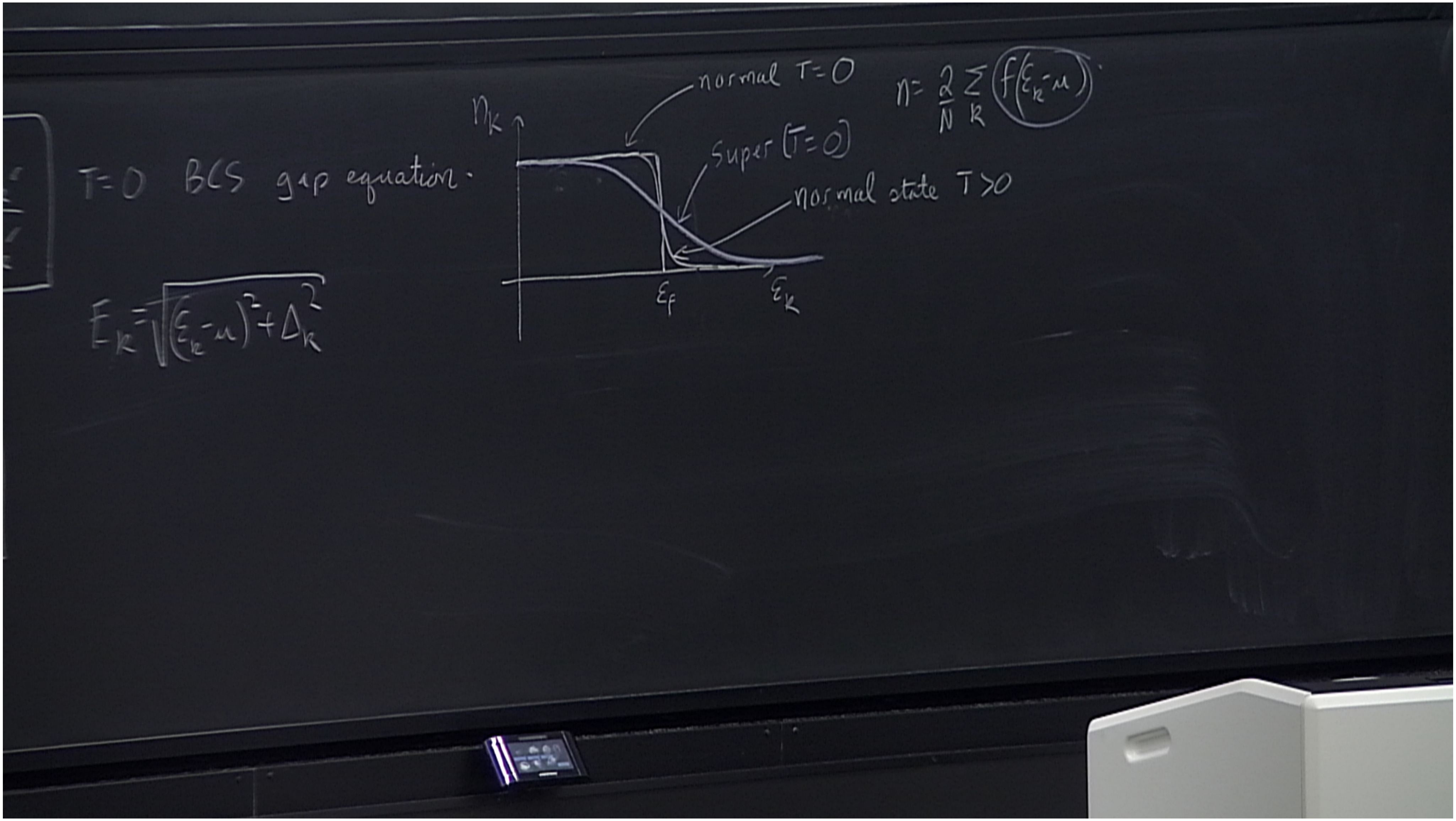
$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

$$n = \frac{2}{N}\sum_p v_p^2$$

$$\boxed{n = \frac{1}{N}\sum_k \left(1 - \frac{\varepsilon_k - \mu}{E_k}\right)}$$


normal T=0
n= 2/N Σ_k f(ε_k - μ)
normal state T>0
ε_F
ε_k


$\boxed{\Delta_k = -\frac{1}{N}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}}$
T=0 BCS gap equation.
$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$
$n = \frac{2}{N}\sum_p v_p^2$
$\boxed{n = \frac{1}{N}\sum_k \left(1 - \frac{\epsilon_k - \mu}{E_k}\right)}$
$n = \frac{1}{N}\sum_k n_k$
n_k
normal T=0
normal sta
ϵ_F
ϵ_k


T=0 BCS gap equation.
$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$
n_k
normal T=0
Super (T=0)
normal state T>0
ε_F
ε_k
$n = \frac{2}{N} \sum_k f(\varepsilon_k - \mu)$