Title: PSI 2016/2017 Condensed Matter - Lecture 12

Date: Nov 22, 2016 10:45 AM

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Abstract:

$$H_{Heis} = \bigcirc J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \oplus g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

convention

AM

Ferromagnet $J = -|J_H|$

$$H_{Heis} = -|J_H| \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

$\langle i,j \rangle$ ← nearest neighbours only

$$S_i \to \langle S \rangle \quad H_{Heis} \to -|J_H| S^2 N \frac{z}{2} - g\mu_B N S B$$

$$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle \qquad \uparrow\uparrow\uparrow\uparrow$$

take $\vec{B} = B\hat{z}$

(AM)

$$\boxed{\vec{M} = g\mu_B \vec{S}}$$

$$\vec{S} \to \vec{J} = \vec{L} + \vec{S}$$

$g(JLS)$

$g \to 2$

$$H_{Heis} = -g\mu_B \sum_i \vec{S}_i \cdot \vec{B}_{eff} \qquad \vec{B}_{eff} = B_{eff}\hat{z}$$

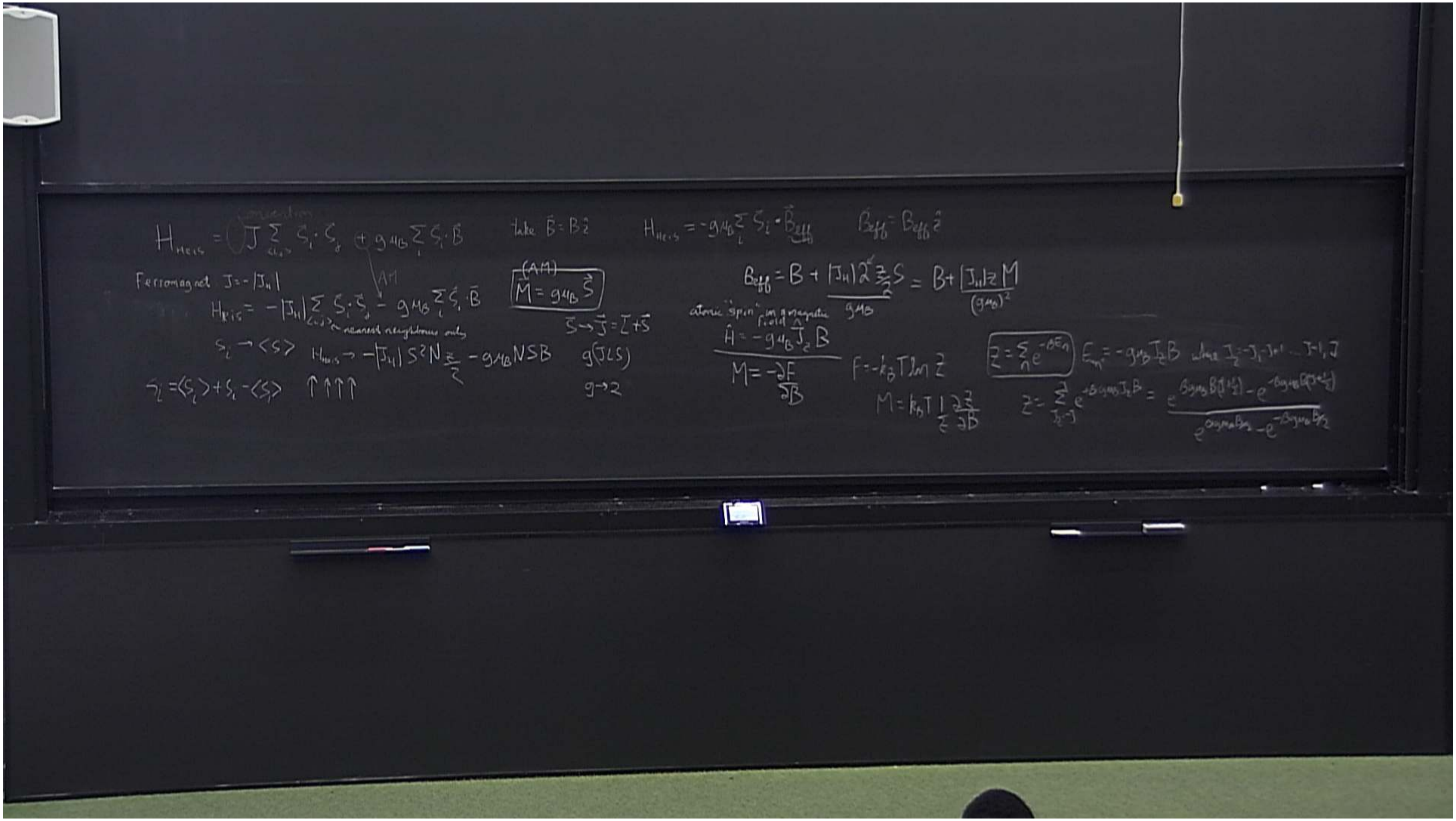
$$B_{eff} = B + \frac{|J_H| 2 \frac{z}{2} S}{g\mu_B} = B + \frac{|J_H| z M}{(g\mu_B)^2}$$

atomic "spin" in a magnetic field

$$\hat{H} = -g\mu_B J_z B$$

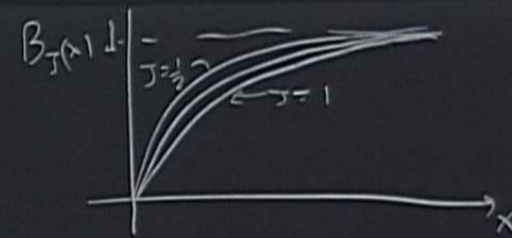
$$M = -\frac{\partial F}{\partial B} \qquad F = -k_B T \ln Z$$

$$M = k_B T \frac{1}{Z} \frac{\partial Z}{\partial B}$$


convention
$H_{Heis} = J \sum_{\langle i,j \rangle} S_i \cdot S_j + g\mu_B \sum_i S_i \cdot \vec{B}$
take $\vec{B} = B\hat{z}$
$H_{Heis} = -g\mu_B \sum_i S_i \cdot \vec{B}_{eff}$
$\vec{B}_{eff} = B_{eff}\hat{z}$
Ferromagnet $J = -|J_H|$
AM
$H_{Heis} = -|J_H| \sum_{\langle i,j \rangle} S_i \cdot \vec{S}_j - g\mu_B \sum_i S_i \cdot \vec{B}$
nearest neighbours only
(AM)
$\boxed{\vec{M} = g\mu_B \vec{S}}$
$\vec{S} \to \vec{J} = \vec{L} + \vec{S}$
$g(JLS)$
$g \to 2$
$S_i \to \langle S \rangle$
$H_{Heis} \to -|J_H| S^2 N \frac{z}{2} - g\mu_B NSB$
$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle$
↑↑↑↑
$B_{eff} = B + \frac{|J_H| 2 \frac{z}{2} S}{g\mu_B} = B + \frac{|J_H| z M}{(g\mu_B)^2}$
atomic "spin" in a magnetic field
$\hat{H} = -g\mu_B \hat{J}_z B$
$M = -\frac{\partial F}{\partial B}$
$F = -k_B T \ln Z$
$M = k_B T \frac{1}{Z} \frac{\partial Z}{\partial B}$
$\boxed{Z = \sum_n e^{-\beta E_n}}$
$E_m = -g\mu_B J_z B$ where $J_z = -J, -J+1, \ldots J-1, J$
$Z = \sum_{J_z=-J}^{J} e^{+\beta g\mu_B J_z B} = \frac{e^{\beta g\mu_B B(J+\frac{1}{2})} - e^{-\beta g\mu_B B(J+\frac{1}{2})}}{e^{\beta g\mu_B B/2} - e^{-\beta g\mu_B B/2}}$

$$M = g\mu_B J \underbrace{\left[\coth\left(x\,\frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right)\right]}$$

where $x \equiv \dfrac{g\mu_B\, JB}{k_B T}$ $\quad B_J(x) \equiv$ Brillouin function



$J = S = \frac{1}{2}$

$$M = \mu_B \tanh x \qquad x = \frac{\mu_B B}{k_B T}$$

$\underline{B=0}$

what

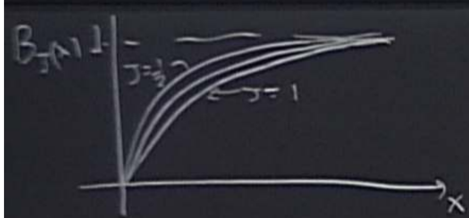
We

$\dfrac{M}{\mu_B} =$

$y = \dfrac{[J_{\ldots}}{4\ldots}$

$J_z = -J$

$e^{\beta g \mu_B B/2} - e^{-\beta g \mu_B B}$



function

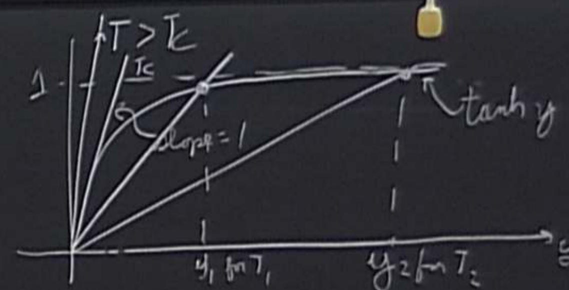
what do we have?

We have B_{eff}

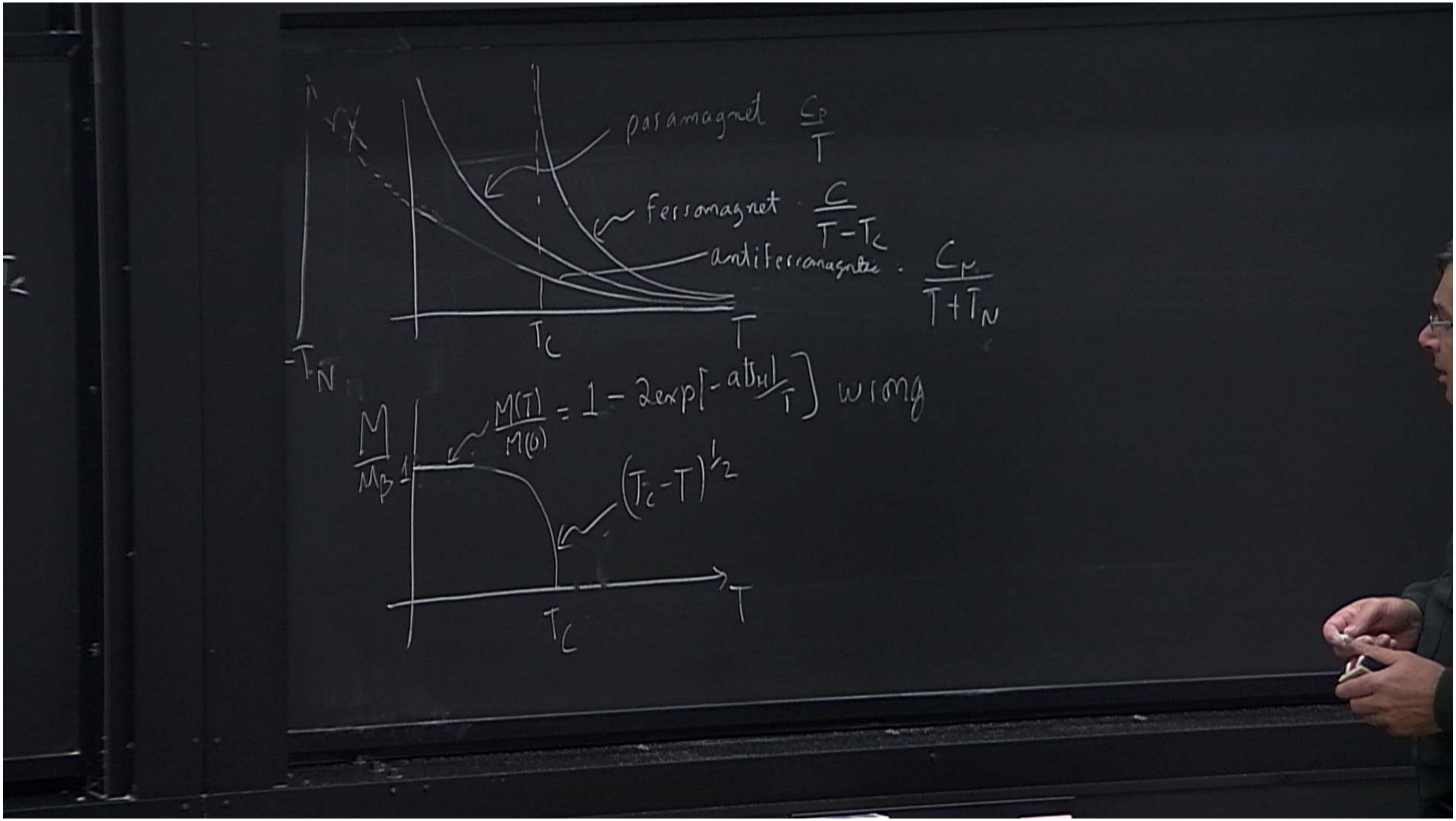
$$\frac{M}{\mu_B} = \tanh \frac{\mu_B}{k_B T}\left(B + \frac{|J_H| z M}{4\mu_B^2}\right)$$

$\underline{B=0}$ $$y = \frac{|J_H| z}{4 k_B T}\frac{M}{\mu_B}$$

$$\frac{4 k_B T}{|J_H| z}\, y = \tanh y$$



$$k_B T_c = \frac{|J_H| z}{4}$$

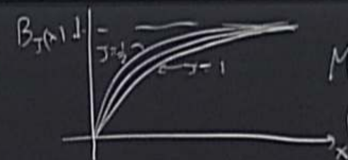

χ
paramagnet C_p / T
ferromagnet C / (T − T_C)
antiferromagnetic C_N / (T + T_N)
T_C
T
−T_N
M(T)/M(0) = 1 − 2exp[−a|J_H|/T] wrong
M/M_B
1
(T_c − T)^{1/2}
T_C
T

$H_{Heis} \to -|J_H| S^2 N \frac{z}{2} - g\mu_B N S B$ $g(JLS)$ $M = -\frac{\partial F}{\partial B}$ $F = -k_B T \ln Z$ $Z = \sum_n e$

$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle$ $\uparrow\uparrow\uparrow\uparrow$ $g \to 2$ $\boxed{M = k_B T \frac{1}{Z}\frac{\partial Z}{\partial B}}$ $Z = \sum_{J_z=-J} e$

$$M = g\mu_B J \underbrace{\left[\coth\left(x\frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right)\right]}$$

where $x \equiv \frac{g\mu_B JB}{k_B T}$ $B_J(x) \equiv$ Brillouin function



$J = S = \frac{1}{2}$ $M = \mu_B \tanh x$ $x = \frac{\mu_B B}{k_B T}$

M_A

M_B

A B A

$B = 0$

$(-1)^i S$

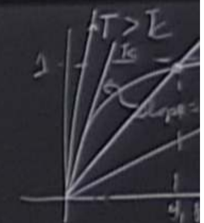
what do we have?

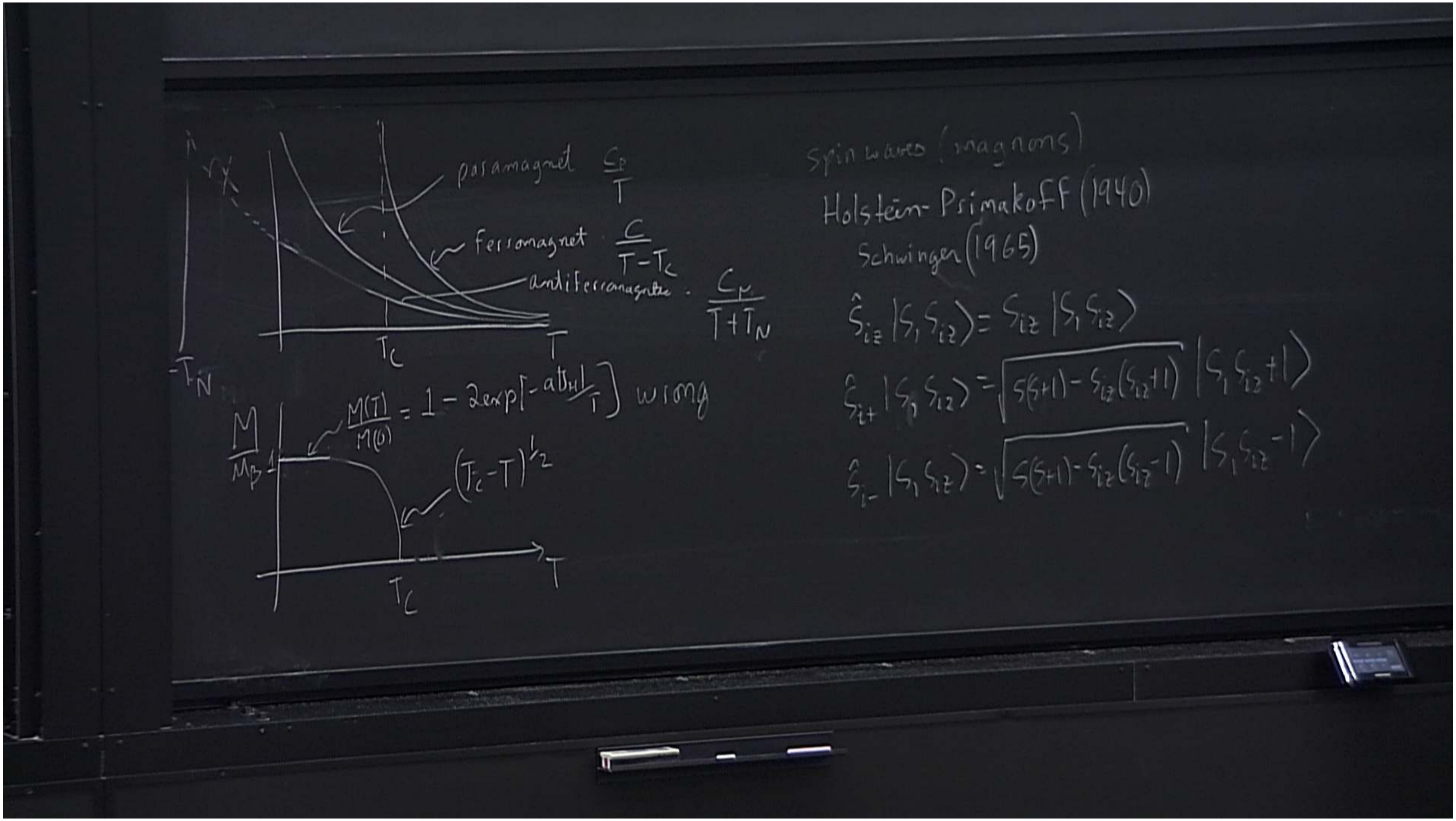
We have B_{eff}

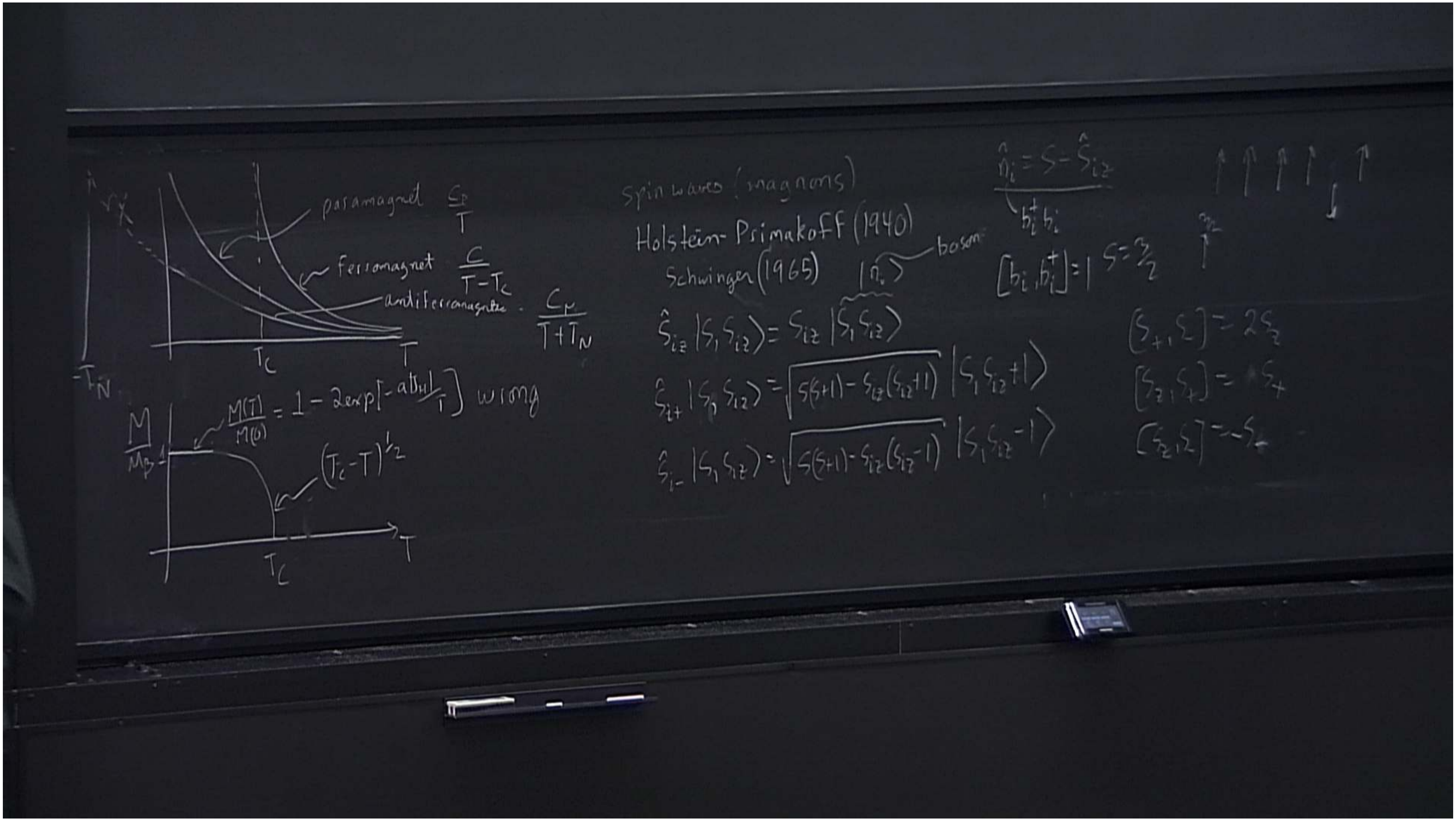
$$\frac{M}{\mu_B} = \tanh \frac{\mu_B}{k_B T}\left(B + \frac{|J_H| z M}{4\mu_B^2}\right)$$

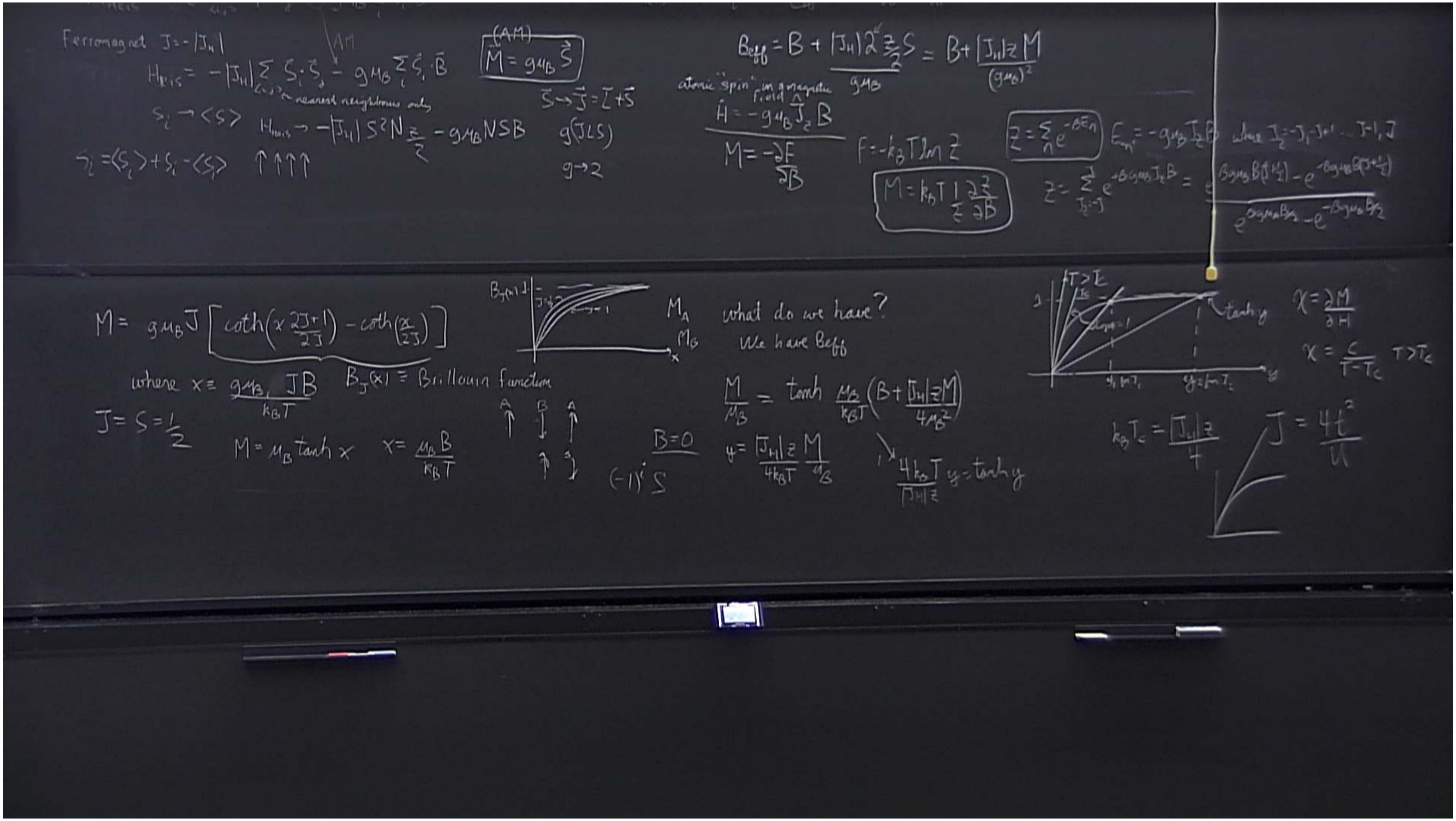
$$y = \frac{|J_H| z}{4 k_B T}\frac{M}{\mu_B}$$

$$\frac{4 k_B T}{|J_H| z} y = \tanh y$$

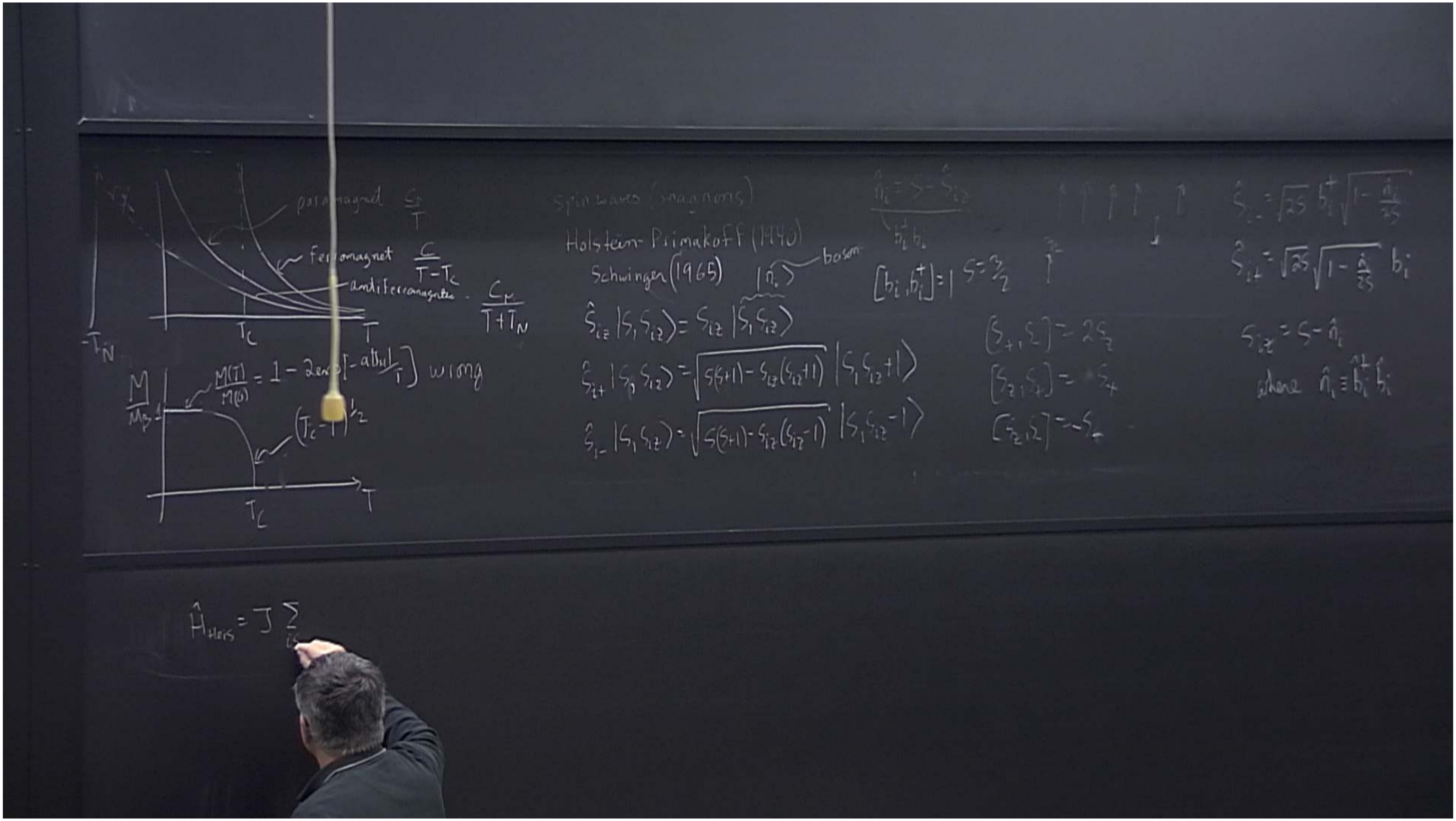


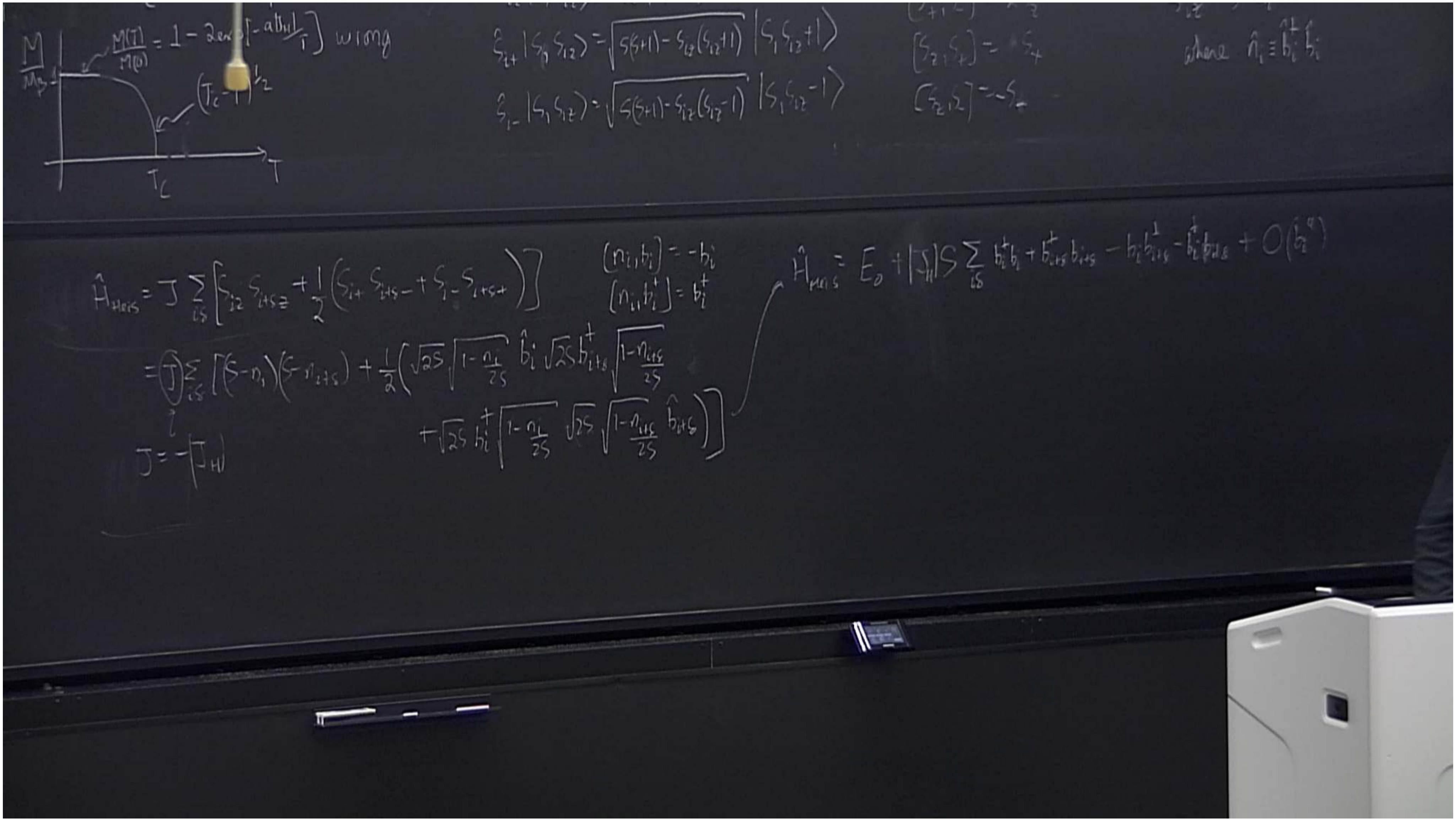

spin waves (magnons)
Holstein-Primakoff (1940)
Schwinger (1965)
paramagnet C/T
ferromagnet C/(T − Tc)
antiferromagnet C_N/(T + T_N)
Tc
T
−T_N
M(T)/M(0) = 1 − 2exp[−aJ.../T] wrong
M/M_B
(Tc − T)^1/2
Ŝ_iz |S, S_iz⟩ = S_iz |S, S_iz⟩
Ŝ_i+ |S, S_iz⟩ = √(S(S+1) − S_iz(S_iz+1)) |S, S_iz+1⟩
Ŝ_i− |S, S_iz⟩ = √(S(S+1) − S_iz(S_iz−1)) |S, S_iz−1⟩


paramagnet
ferromagnet
antiferromagnet
wrong
Spin waves (magnons)
Holstein-Primakoff (1940)
Schwinger (1965)
boson

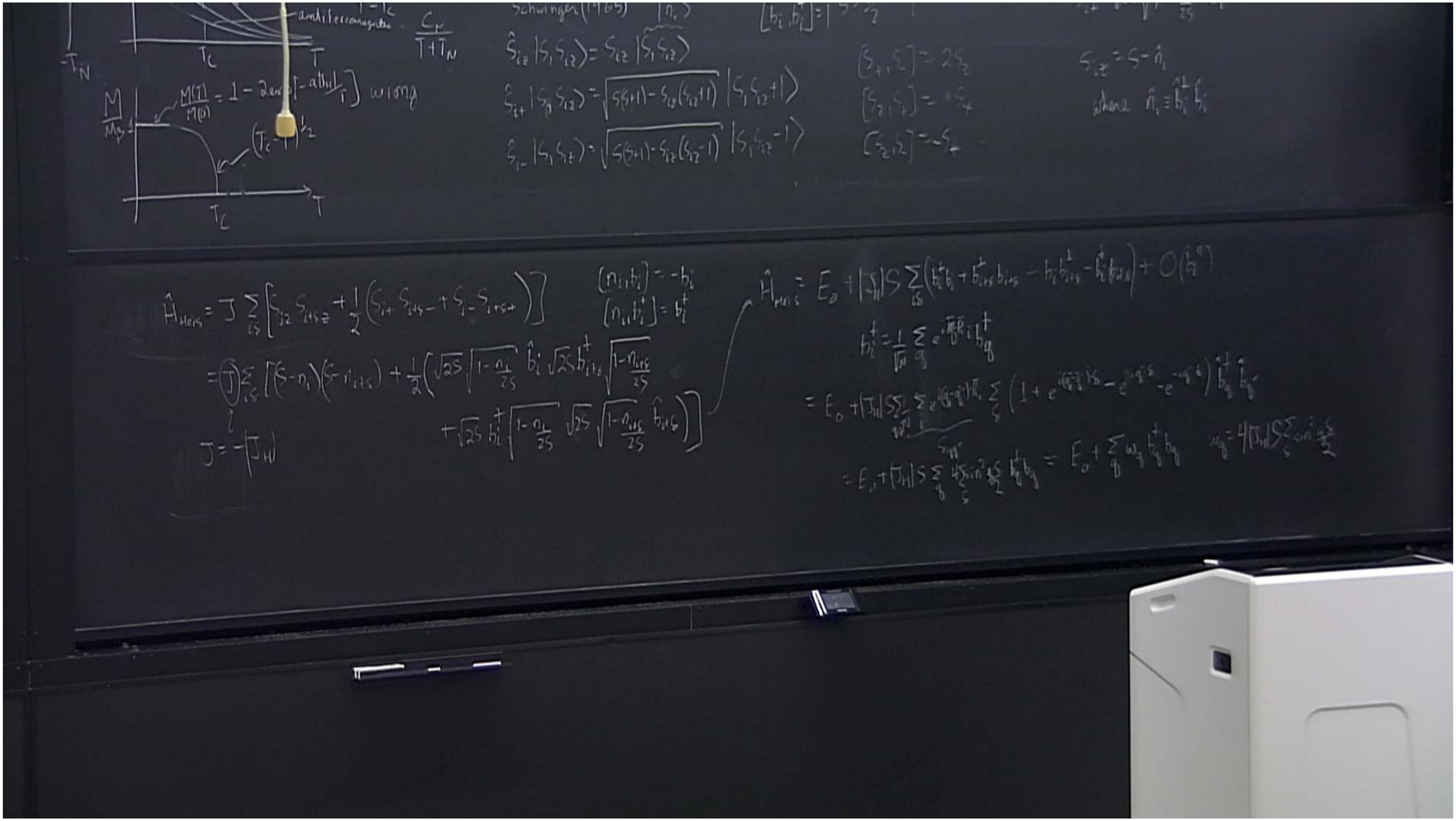


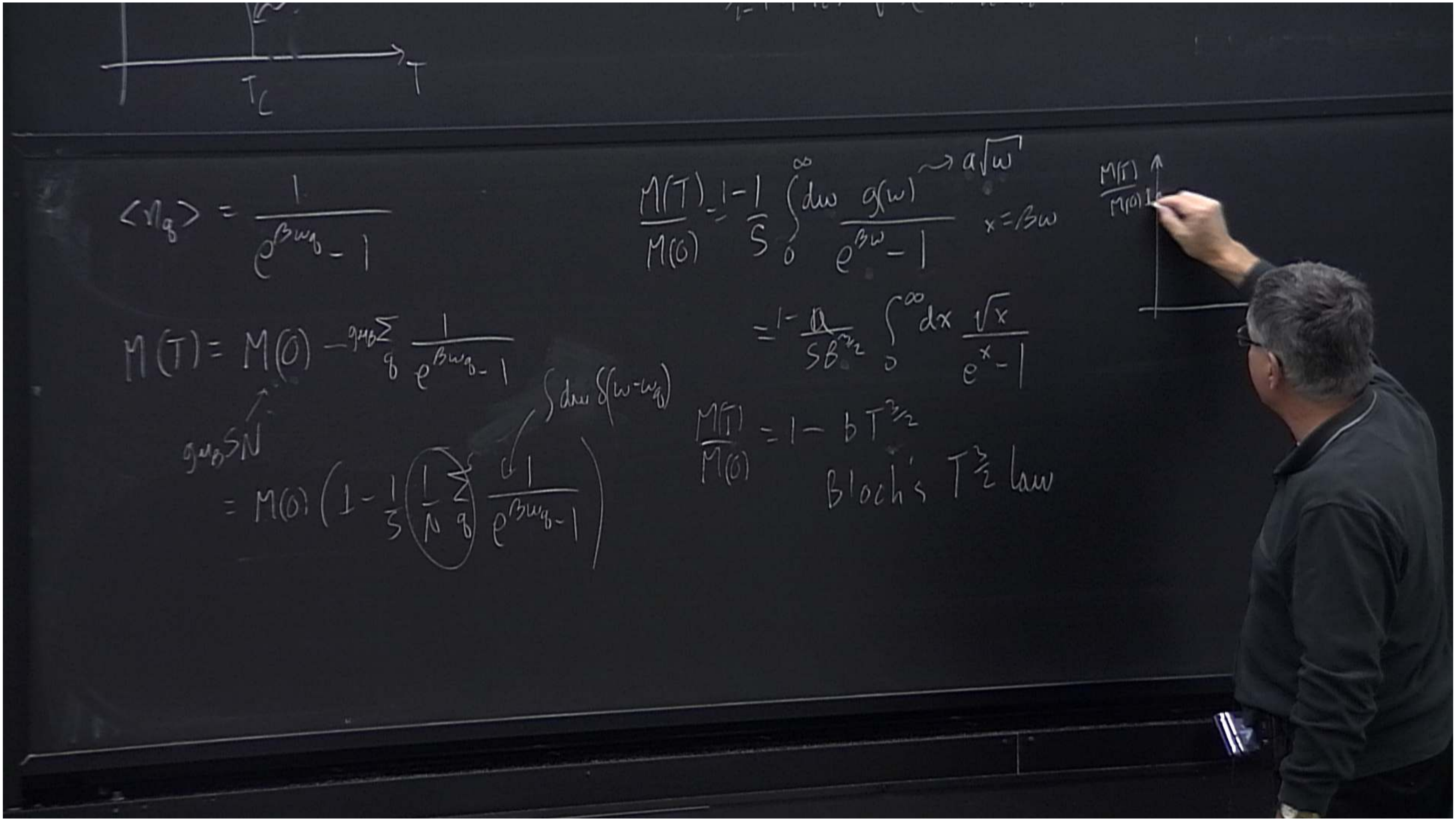
Ferromagnet $J = -|J_H|$
$H_{Heis} = -|J_H| \sum_{\langle i,j \rangle} S_i \cdot S_j - g\mu_B \sum_i S_i \cdot B$
nearest neighbours only
$S_i \to \langle S \rangle$ $H_{Heis} \to -|J_H| S^2 N \frac{z}{2} - g\mu_B NSB$
$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle$ ↑↑↑↑
(AM)
$\boxed{\vec{M} = g\mu_B \vec{S}}$
$\vec{S} \to \vec{J} = \vec{L} + \vec{S}$
$g(JLS)$
$g \to 2$
$B_{eff} = B + |J_H| 2 \frac{z}{2} \frac{S}{g\mu_B} = B + \frac{|J_H| z M}{(g\mu_B)^2}$
atomic "spin" in a magnetic field
$\hat{H} = -g\mu_B \hat{J}_z B$
$M = -\frac{\partial F}{\partial B}$
$F = -k_B T \ln Z$
$\boxed{M = k_B T \frac{1}{Z} \frac{\partial Z}{\partial B}}$
$\boxed{Z = \sum_n e^{-\beta E_n}}$ $E_n = -g\mu_B J_z B$ where $J_z = -J, -J+1, \dots, J-1, J$
$Z = \sum_{J_z=-J}^{J} e^{+\beta g\mu_B J_z B} = \frac{e^{\beta g\mu_B B(J+\frac{1}{2})} - e^{-\beta g\mu_B B(J+\frac{1}{2})}}{e^{\beta g\mu_B B/2} - e^{-\beta g\mu_B B/2}}$
$M = g\mu_B J \left[\coth\left(x \frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right) \right]$
where $x \equiv \frac{g\mu_B JB}{k_B T}$ $B_J(x)$ = Brillouin function
$J = S = \frac{1}{2}$
$M = \mu_B \tanh x$ $x = \frac{\mu_B B}{k_B T}$
$B_J(x)$
M_A
M_B
$B = 0$
$(-1)^i S$
what do we have?
We have B_{eff}
$\frac{M}{\mu_B} = \tanh \frac{\mu_B}{k_B T}\left(B + \frac{|J_H| z M}{4\mu_B^2}\right)$
$y = \frac{|J_H| z}{4 k_B T} \frac{M}{\mu_B}$
$\frac{4 k_B T}{|J_H| z} y = \tanh y$
$T > T_c$
tanh y
$\chi = \frac{\partial M}{\partial H}$
$\chi = \frac{C}{T - T_c}$ $T > T_c$
$k_B T_c = \frac{|J_H| z}{4}$
$J = \frac{4t^2}{U}$

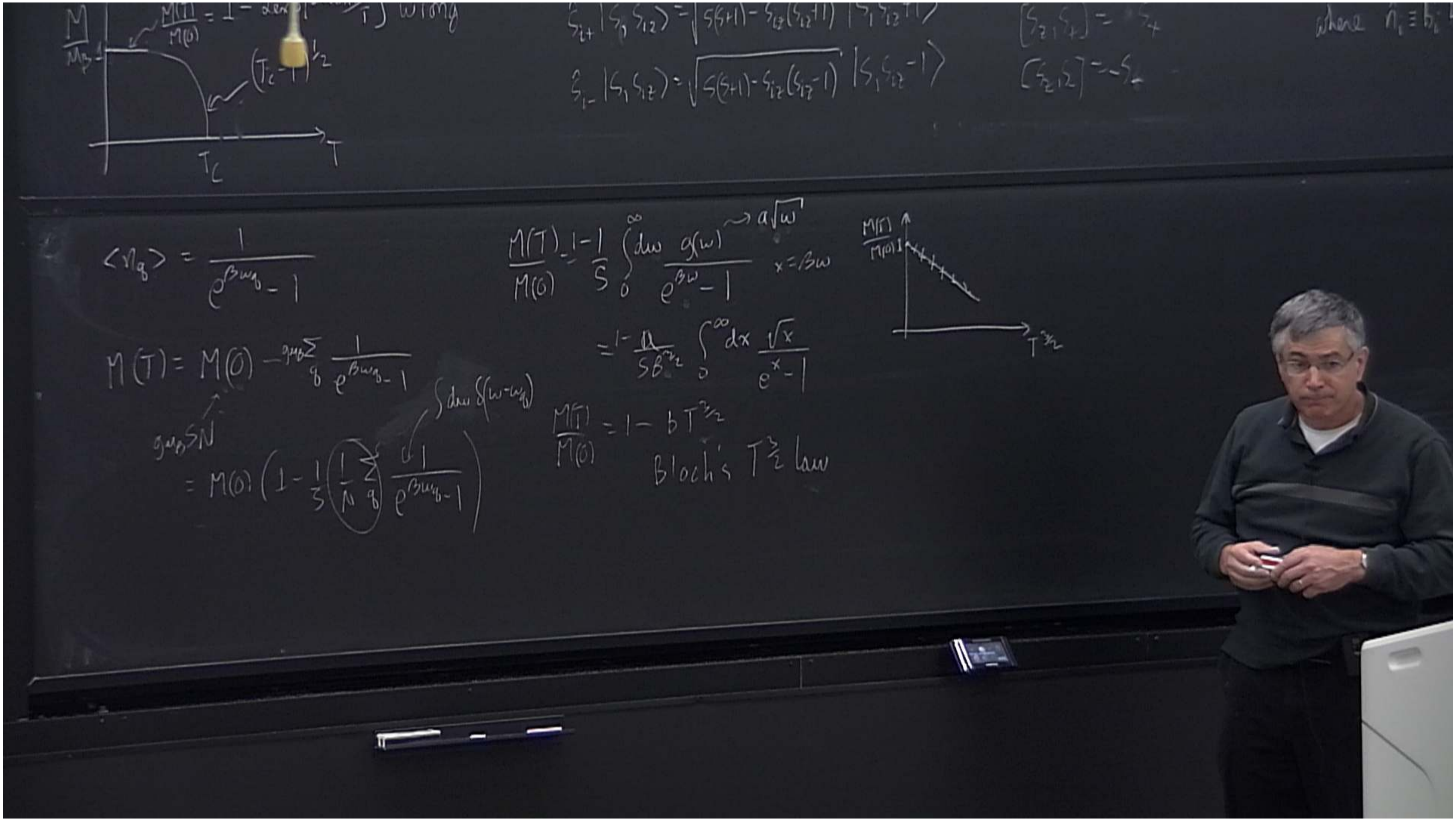

paramagnet
ferromagnet
antiferromagnetic
Holstein-Primakoff (1940)
Schwinger (1965)
boson
where



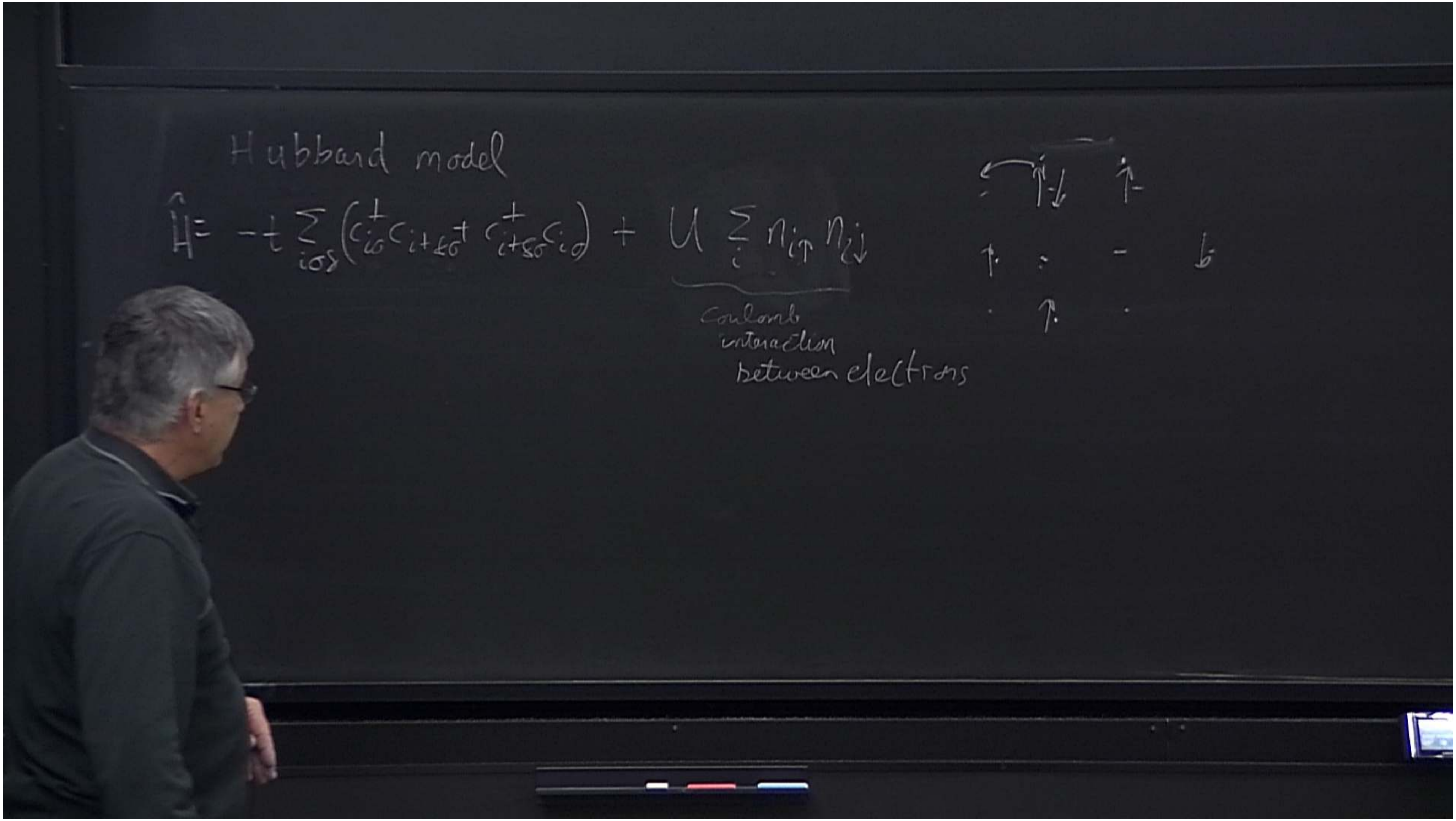
$\frac{M(T)}{M(0)} = 1 - 2\exp[-a|J_H|/T]$ wrong
$(T_c - T)^{1/2}$
$S_{i+}|S, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}+1)}\,|S, S_{iz}+1\rangle$
$S_{i-}|S, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}-1)}\,|S, S_{iz}-1\rangle$
where $\hat{n}_i \equiv \hat{b}_i^\dagger \hat{b}_i$
$\hat{H}_{Heis} = J \sum_{i\delta} \left[S_{iz} S_{i+\delta z} + \frac{1}{2}\left(S_{i+} S_{i+\delta -} + S_{i-} S_{i+\delta +} \right) \right]$
$[n_i, b_i] = -b_i$
$[n_i, b_i^\dagger] = b_i^\dagger$
$J = -|J_H|$
$\hat{H}_{Heis} = E_0 + |J_H| S \sum_{i\delta} b_i^\dagger b_i + b_{i+\delta}^\dagger b_{i+\delta} - b_i b_{i+\delta}^\dagger - b_i^\dagger b_{i+\delta} + O(b_i^4)$

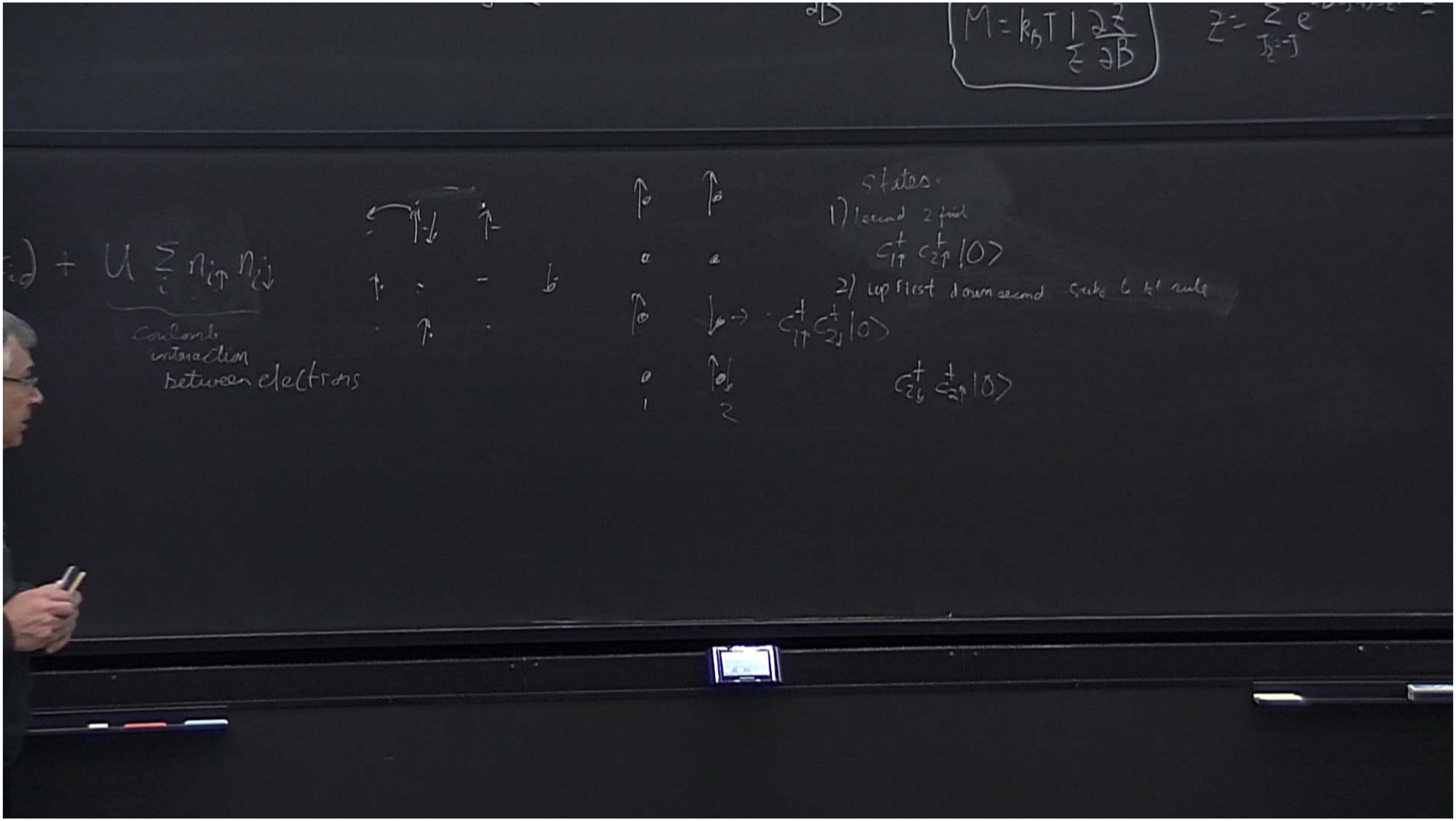

antiferromagnetic
$\frac{C_V}{T+T_N}$
$\frac{M(T)}{M(0)} = 1 - 2\exp[-a|J_H|/T]$ wimg
$\hat{S}_{iz}|S,S_{iz}\rangle = S_{iz}|S,S_{iz}\rangle$
$\hat{S}_{i+}|S,S_{iz}\rangle = \sqrt{S(S+1)-S_{iz}(S_{iz}+1)}\,|S,S_{iz}+1\rangle$
$\hat{S}_{i-}|S,S_{iz}\rangle = \sqrt{S(S+1)-S_{iz}(S_{iz}-1)}\,|S,S_{iz}-1\rangle$
$S_{iz} = S - \hat{n}_i$
where $\hat{n}_i \equiv \hat{b}_i^\dagger \hat{b}_i$
$\hat{H}_{Heis} = J\sum_{i\delta}\left[S_{iz}S_{i+\delta z} + \frac{1}{2}\left(S_{i+}S_{i+\delta -} + S_{i-}S_{i+\delta +}\right)\right]$
$[n_i, b_i] = -b_i$
$[n_i, b_i^\dagger] = b_i^\dagger$
$J = -|J_H|$
$\hat{H}_{Heis} = E_0 + |J_H| S \sum_{i\delta}\left(b_i^\dagger b_i + b_{i+\delta}^\dagger b_{i+\delta} - b_i b_{i+\delta}^\dagger - b_i^\dagger b_{i+\delta}\right) + O(b^4)$
$b_i^\dagger = \frac{1}{\sqrt{N}}\sum_q e^{-i\vec{q}\cdot\vec{R}_i} b_q^\dagger$
$\omega_q = 4|J_H| S \sum_\delta \sin^2\frac{q\cdot\delta}{2}$

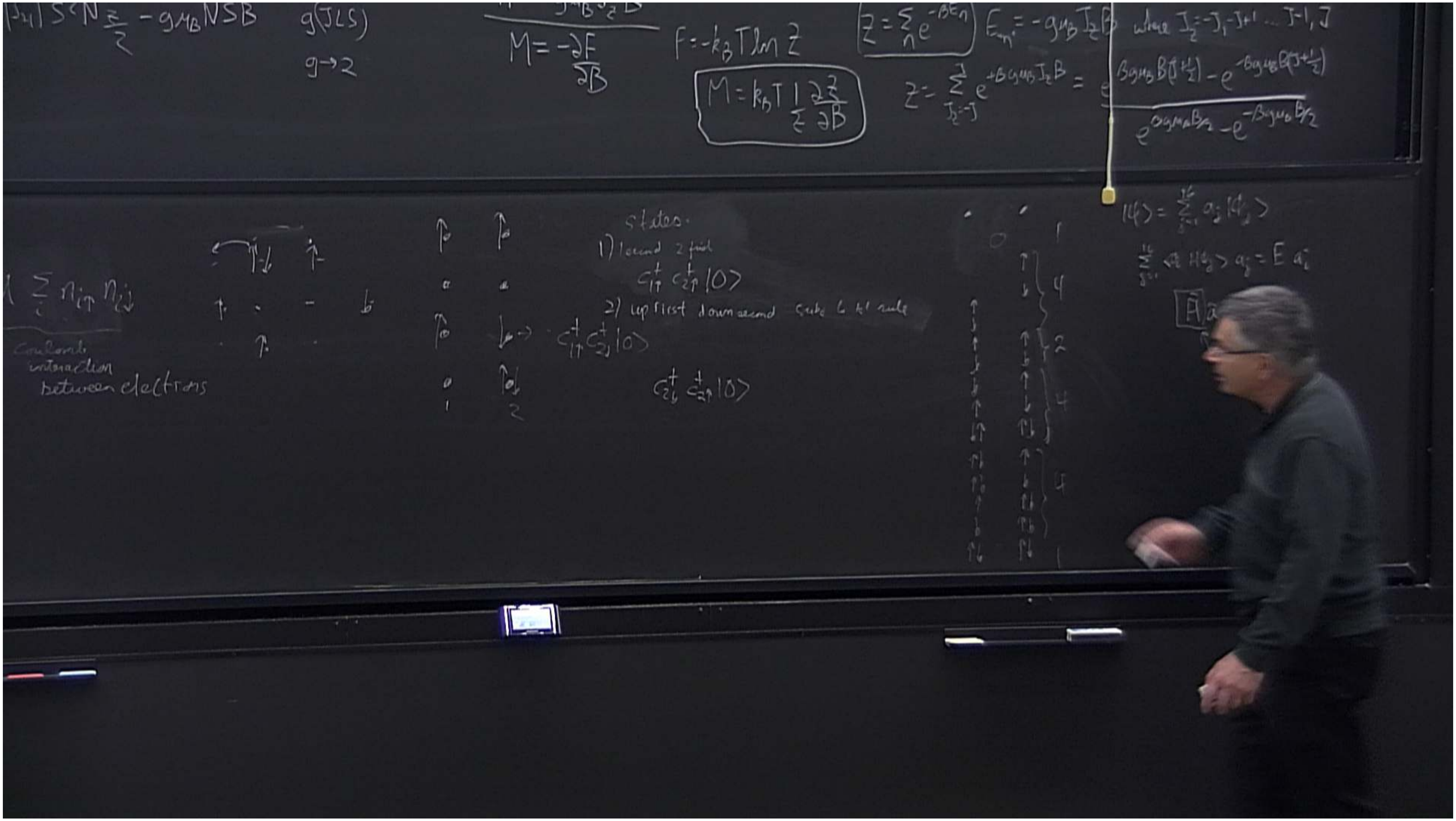

T_C
T
⟨n_q⟩ = 1 / (e^{βω_q} − 1)
M(T) = M(0) − gμ_B Σ_q 1 / (e^{βω_q} − 1)
gμ_B SN
= M(0) (1 − 1/S (1/N Σ_q 1 / (e^{βω_q} − 1)))
∫dω δ(ω − ω_q)
M(T)/M(0) = 1 − 1/S ∫_0^∞ dω g(ω) / (e^{βω} − 1)
a√ω
x = βω
= 1 − a / (Sβ^{3/2}) ∫_0^∞ dx √x / (e^x − 1)
M(T)/M(0) = 1 − bT^{3/2}
Bloch's T^{3/2} law
M(T)/M(0)

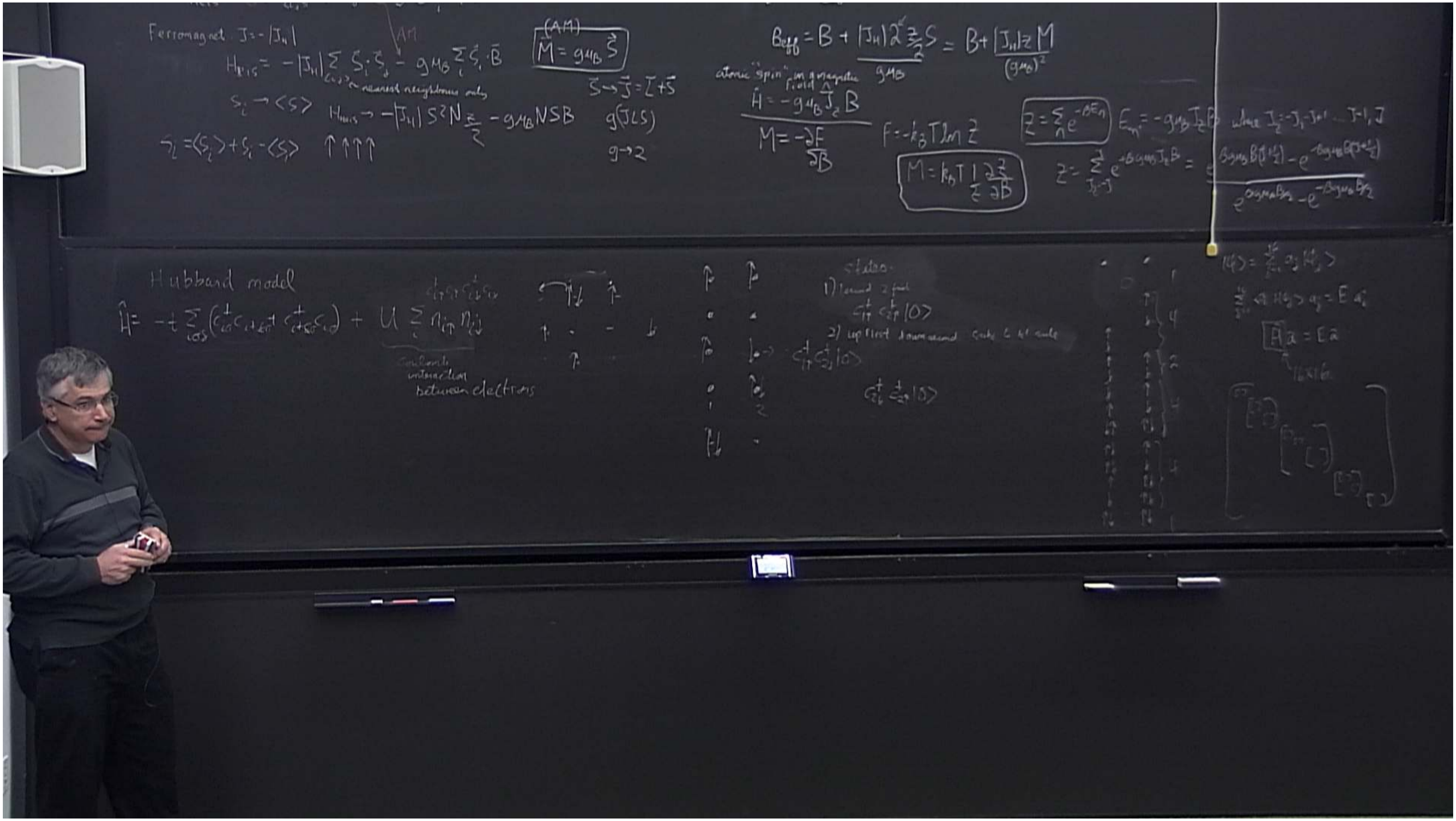


$\langle n_q \rangle = \frac{1}{e^{\beta\omega_q} - 1}$
$M(T) = M(0) - g\mu_B \sum_q \frac{1}{e^{\beta\omega_q} - 1}$
$g\mu_B S N$
$= M(0)\left(1 - \frac{1}{S}\frac{1}{N}\sum_q \frac{1}{e^{\beta\omega_q} - 1}\right)$
$\int d\omega\, \delta(\omega - \omega_q)$
$\frac{M(T)}{M(0)} = 1 - \frac{1}{S}\int_0^\infty d\omega \frac{g(\omega)}{e^{\beta\omega} - 1}$
$a\sqrt{\omega}$
$x = \beta\omega$
$= 1 - \frac{a}{S\beta^{3/2}}\int_0^\infty dx \frac{\sqrt{x}}{e^x - 1}$
$\frac{M(T)}{M(0)} = 1 - bT^{3/2}$
Bloch's $T^{3/2}$ law
$T^{3/2}$


Hubbard model
$\hat{H} = -t \sum_{i\sigma\delta} \left(c_{i\sigma}^{\dagger} c_{i+\delta\sigma} + c_{i+\delta\sigma}^{\dagger} c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$
Coulomb
interaction
between electrons


M = k_B T 1/Z ∂Z/∂B
states.
1) 1 second 2 first
c†1↑ c†2↑ |0>
2) up first down second
c†1↑ c†2↓ |0>
c†2↓ c†2↑ |0>
U Σ n_i↑ n_i↓
Coulomb
interaction
between electrons


g(JLS)
g→2
M = -∂F/∂B
F = -k_B T ln Z
M = k_B T (1/Z) ∂Z/∂B
Z = Σ_n e^{-βE_n}
states.
up first down second
Coulomb
interaction
between electrons


Ferromagnet
nearest neighbours only
atomic "spin" in a magnetic field
Hubbard model
Coulomb interaction between electrons
states