

Title: PSI 2016/2017 Condensed Matter - Lecture 12

Date: Nov 22, 2016 10:45 AM

URL: <http://pirsa.org/16110063>

Abstract:

convention

$$H_{\text{Heis}} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

take  $\vec{B} = B\hat{z}$

$$H_{\text{Heis}} = -g\mu_B \sum_i \vec{S}_i \cdot \vec{B}_{\text{eff}}$$

$$\vec{B}_{\text{eff}} = B_{\text{eff}} \hat{z}$$

Ferromagnet:  $J = -|J_H|$

$$H_{\text{Heis}} = -|J_H| \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

↑ AM  
nearest neighbors only

$$\boxed{M = g\mu_B \vec{S}}$$

$$\vec{S} \rightarrow \vec{J} = \vec{L} + \vec{S}$$

$$S_i \rightarrow \langle S \rangle \quad H_{\text{Heis}} \rightarrow -|J_H| S^2 N \frac{z}{z} - g\mu_B N S B \quad g(\text{JLS})$$

$$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle \quad \uparrow \uparrow \uparrow \uparrow$$

$$g \rightarrow 2$$

$$B_{\text{eff}} = B + \frac{|J_H| z}{g\mu_B} \frac{z}{z} S = B + \frac{|J_H| z M}{(g\mu_B)^2}$$

atomic "spin" in a magnetic field

$$\hat{H} = -g\mu_B J_z B$$

$$M = -\frac{\partial F}{\partial B}$$

$$F = -k_B T \ln Z$$

$$M = k_B T \frac{\partial \ln Z}{\partial B}$$

$$H_{\text{Heis}} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$
 take  $\vec{B} = B\hat{z}$ 

$$H_{\text{Heis}} = -g\mu_B \sum_i \vec{S}_i \cdot \vec{B}_{\text{eff}}$$

$$B_{\text{eff}} = B_{\text{eff}} \hat{z}$$

Ferromagnet  $J = |J_{ij}|$ 

$$H_{\text{Heis}} = -|J_{ij}| \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

$$M = g\mu_B \sum_i \vec{S}_i$$

$$B_{\text{eff}} = B + |J_{ij}| z \frac{M}{g\mu_B} = B + \frac{|J_{ij}| z M}{(g\mu_B)^2}$$

atomic spin magnetic field  $g\mu_B$ 

$$\vec{A} = -g\mu_B \frac{\vec{J}}{z} B$$

$$M = \frac{-\partial F}{\partial B}$$

$$F = -k_B T \ln Z$$

$$M = k_B T \frac{\partial \ln Z}{\partial B}$$

$$Z = \sum_n e^{-\beta E_n}$$

$$E_n = -g\mu_B I_z B$$
 where  $I_z = J, J-1, \dots, -J+1, J$

$$Z = \sum_{I_z} e^{+\beta g\mu_B I_z B} = \frac{e^{+\beta g\mu_B B (J+1/2)} - e^{-\beta g\mu_B B (J+1/2)}}{e^{+\beta g\mu_B B} - e^{-\beta g\mu_B B}}$$

$$\vec{S} \rightarrow \vec{J} = \vec{L} + \vec{S}$$

$$g \rightarrow 2$$

$$S_i \rightarrow \langle S \rangle$$

$$H_{\text{Heis}} \rightarrow -|J_{ij}| S^2 N \frac{z}{2} - g\mu_B N S B$$

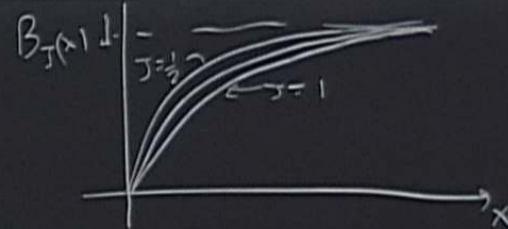
$$S_i = \langle S_i \rangle + S_i - \langle S_i \rangle \uparrow \uparrow \uparrow$$

$$M = g \mu_B J \left[ \coth\left(x \frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right) \right]$$

where  $x \equiv \frac{g \mu_B J B}{k_B T}$   $B_J(x) \equiv$  Brillouin function

$$J = S = \frac{1}{2}$$

$$M = \mu_B \tanh x \quad x = \frac{\mu_B B}{k_B T}$$



what  
We

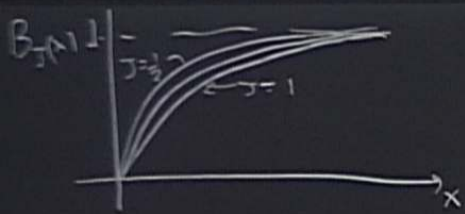
$$\frac{M}{\mu_B} =$$

$$y = \frac{J}{4}$$

$$B=0$$

$$\left( \frac{1}{z} \frac{\partial}{\partial B} \right) \quad J_2 = J$$

$$e^{2\alpha M B z} - e^{-2\alpha M B z}$$



what do we have?

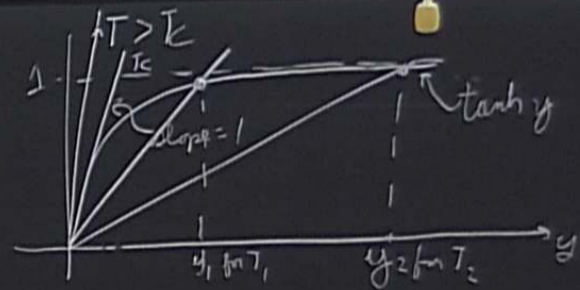
We have  $B_{eff}$

$$\frac{M}{M_B} = \tanh \frac{M_B}{k_B T} \left( B + \frac{|J_H| z M}{4 M_B^2} \right)$$

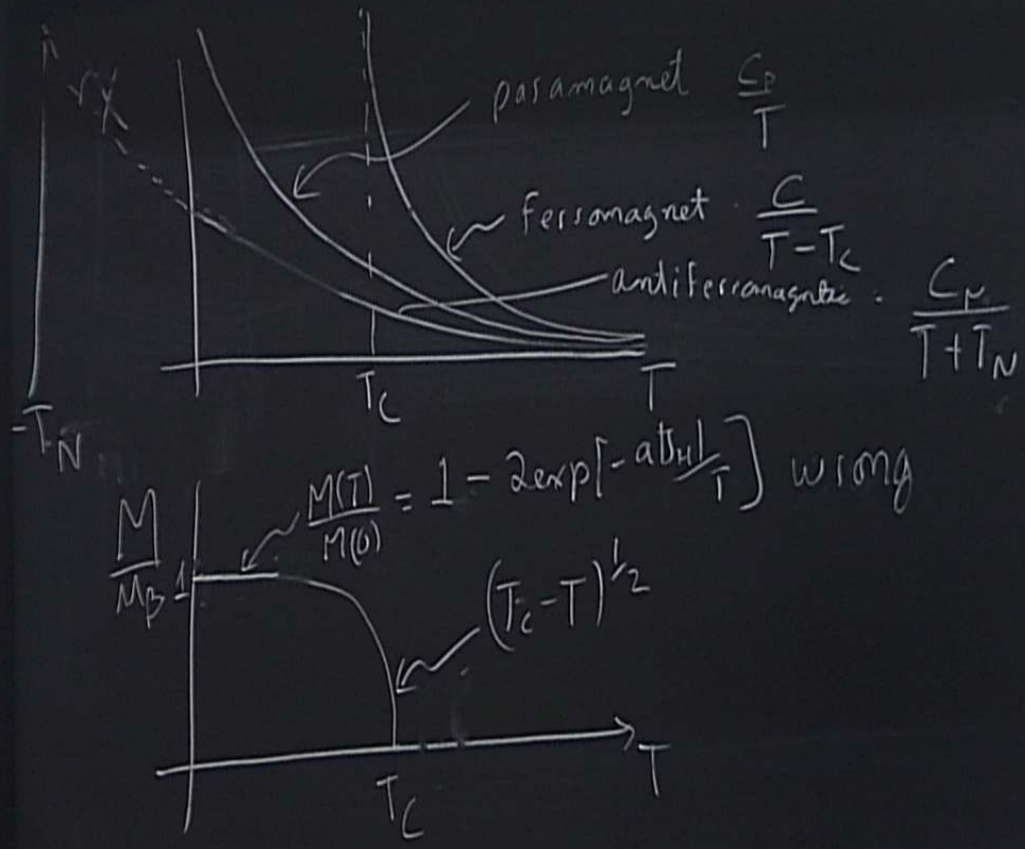
$B=0$

$$y = \frac{|J_H| z}{4 k_B T} \frac{M}{M_B}$$

$$\rightarrow \frac{4 k_B T}{|J_H| z} y = \tanh y$$



$$k_B T_c = \frac{|J_H| z}{4}$$



$$S_i = \langle S_i \rangle + S_i - \langle S \rangle \quad \uparrow \uparrow \uparrow$$

$$M_{\text{total}} \rightarrow -\frac{1}{2} S^2 N \frac{z}{z} - g \mu_B N > B \quad g(JLS)$$

$$g \rightarrow 2$$

$$M = -\frac{\partial F}{\partial B}$$

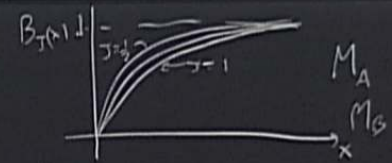
$$F = -k_B T \ln Z$$

$$M = k_B T \frac{1}{z} \frac{\partial z}{\partial B}$$

$$z = \sum_{\lambda} e^{\lambda}$$

$$z = \sum_{S_i} e^{\lambda}$$

$$M = g \mu_B J \left[ \coth\left(x \frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right) \right]$$

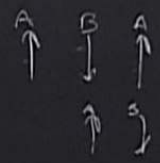


what do we have?  
We have Beff

where  $x \equiv \frac{g \mu_B J B}{k_B T}$   $B_J(x) \equiv$  Brillouin function

$$J = S = \frac{1}{2}$$

$$M = \mu_B \tanh x \quad x = \frac{\mu_B B}{k_B T}$$

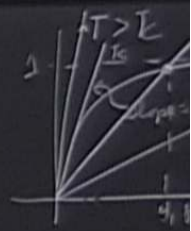


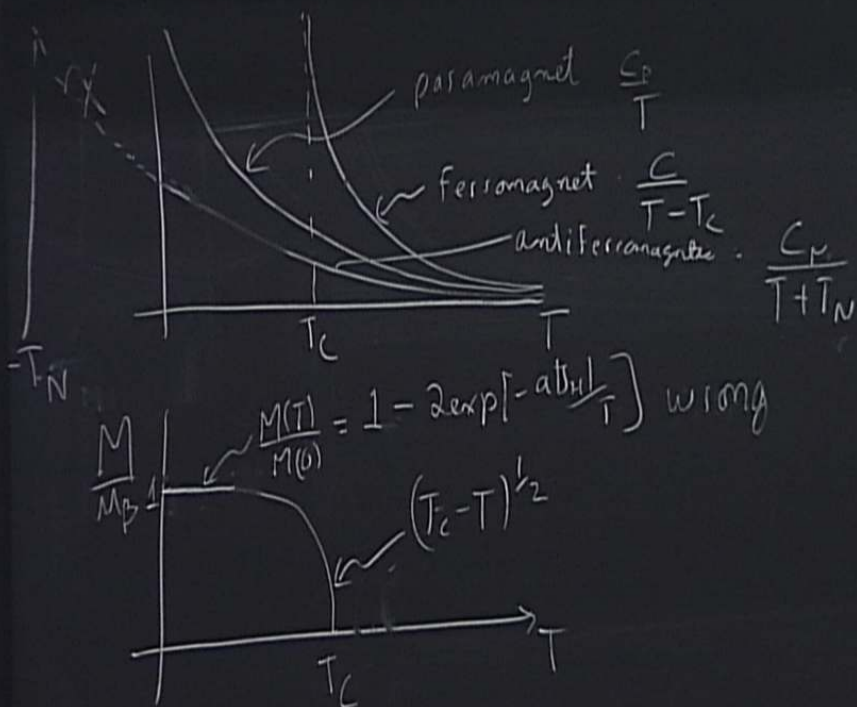
$$B=0$$

$$\frac{M}{M_B} = \tanh \frac{M_B}{k_B T} \left( B + \frac{|J_H| z M}{4 M_B^2} \right)$$

$$y = \frac{|J_H| z M}{4 k_B T} \frac{M}{\mu_B}$$

$$\frac{4 k_B T}{|J_H| z} y = \tanh y$$





Spin waves (magnons)

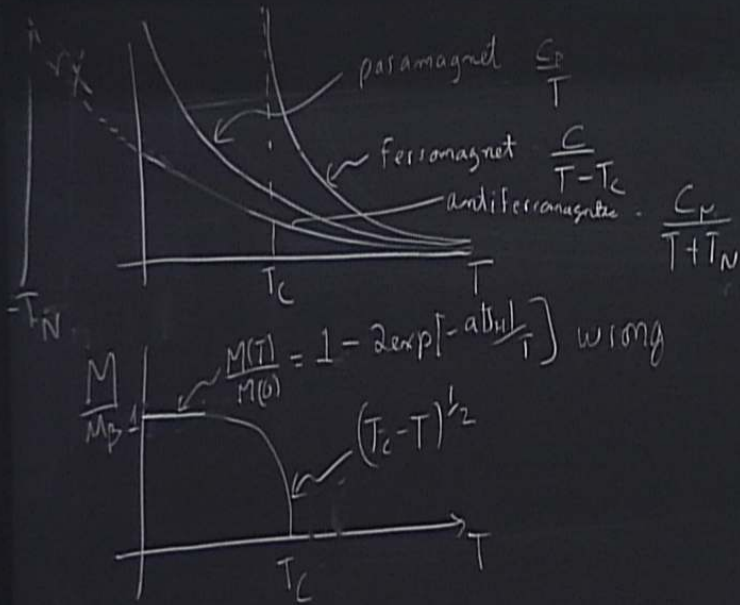
Holstein-Primakoff (1940)

Schwinger (1965)

$$\hat{S}_{iz} |S_1, S_{iz}\rangle = S_{iz} |S_1, S_{iz}\rangle$$

$$\hat{S}_{z+} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}+1)} |S_1, S_{iz}+1\rangle$$

$$\hat{S}_{z-} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}-1)} |S_1, S_{iz}-1\rangle$$



Spin waves (magnons)

Holstein-Primakoff (1940)

Schwinger (1965)  $(\hat{n}_i)$  boson

$$\hat{S}_{iz} |S_1, S_{iz}\rangle = S_{iz} |S_1, S_{iz}\rangle$$

$$\hat{S}_{i+} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}+1)} |S_1, S_{iz}+1\rangle$$

$$\hat{S}_{i-} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}-1)} |S_1, S_{iz}-1\rangle$$

$$\hat{n}_i = S - \hat{S}_{iz}$$

$$b_i^\dagger b_i$$

$$[b_i, b_i^\dagger] = 1 \quad S = \frac{3}{2}$$



$$[S_{+1}, S_-] = -2S_2$$

$$[S_{2+}, S_+] = -S_4$$

$$[S_{2-}, S_-] = -S_4$$

Ferromagnet  $J = -|J_{ij}|$

$$H_{\text{eff}} = -|J_{ij}| \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

$$H_{\text{eff}} \rightarrow -|J_{ij}| S^2 N \frac{z}{2} - g\mu_B N S B$$

$$\vec{S}_i = \langle S_i \rangle + S_i - \langle S_i \rangle \quad \uparrow \uparrow \uparrow$$

$$M = g\mu_B \sum_i \vec{S}_i$$

$$\vec{S} \rightarrow \vec{J} = \vec{L} + \vec{S}$$

$$g(JLS)$$

$$g \rightarrow 2$$

$$B_{\text{eff}} = B + |J_{ij}| z \frac{z}{2} S = B + \frac{|J_{ij}| z M}{(g\mu_B)^2}$$

atomic spin in a magnetic field

$$\hat{H} = -g\mu_B \vec{J} \cdot \vec{B}$$

$$M = -\frac{\partial F}{\partial B}$$

$$F = -k_B T \ln Z$$

$$M = k_B T \frac{\partial \ln Z}{\partial B}$$

$$Z = \sum_n e^{-\beta E_n}$$

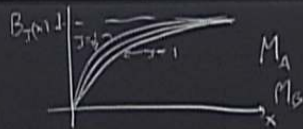
$$E_n = -g\mu_B J_z B$$

where  $J_z = -J, -J+1, \dots, J-1, J$

$$Z = \sum_{J_z} e^{+g\mu_B J_z B}$$

$$= \frac{e^{g\mu_B B(J+1/2)} - e^{-g\mu_B B(J+1/2)}}{e^{g\mu_B B/2} - e^{-g\mu_B B/2}}$$

$$M = g\mu_B J \left[ \coth\left(x \frac{2J+1}{2J}\right) - \coth\left(\frac{x}{2J}\right) \right]$$

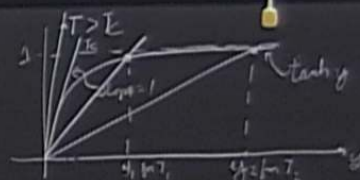


what do we have?  
We have  $B_{\text{eff}}$

$$\frac{M}{M_B} = \tanh \frac{M_B}{k_B T} \left( B + \frac{|J_{ij}| z M}{4\mu_B^2} \right)$$

$$y = \frac{|J_{ij}| z M}{4k_B T} \frac{M}{\mu_B}$$

$$\frac{4k_B T}{|J_{ij}| z} y = \tanh y$$

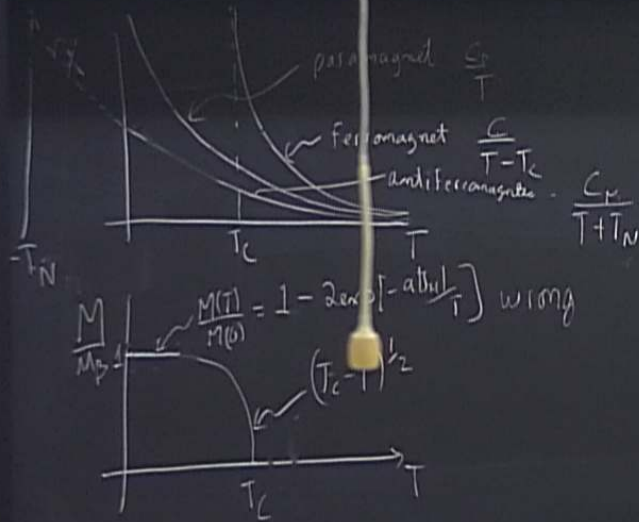


$$\chi = \frac{\partial M}{\partial H}$$

$$\chi = \frac{C}{T - T_c} \quad T > T_c$$

$$k_B T_c = \frac{|J_{ij}| z}{4}$$

$$J = \frac{4t^2}{U}$$



spin waves (magnons)

Holstein-Primakoff (1940)

Schwinger (1965)

$|\hat{n}_i\rangle$  boson

$$\hat{S}_{iz} |S_1, S_{iz}\rangle = S_{iz} |S_1, S_{iz}\rangle$$

$$\hat{S}_{i+} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}+1)} |S_1, S_{iz}+1\rangle$$

$$\hat{S}_{i-} |S_1, S_{iz}\rangle = \sqrt{S(S+1) - S_{iz}(S_{iz}-1)} |S_1, S_{iz}-1\rangle$$

$$\hat{n}_i = S - \hat{S}_{iz}$$

$$\hat{n}_i^\dagger \hat{n}_i$$

$$[\hat{n}_i, \hat{n}_i^\dagger] = 1 \quad S = \frac{3}{2}$$

$$[\hat{S}_{i+}, \hat{S}_z] = -2\hat{S}_z$$

$$[\hat{S}_{i+}, \hat{S}_z] = +\hat{S}_z$$

$$[\hat{S}_{i-}, \hat{S}_z] = -\hat{S}_z$$

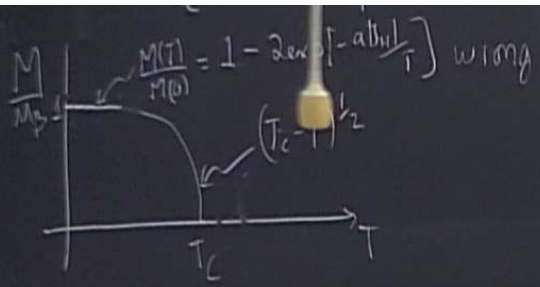
$$\hat{S}_{i-} = \sqrt{2S} \hat{b}_i^\dagger \sqrt{1 - \frac{\hat{n}_i}{2S}}$$

$$\hat{S}_{i+} = \sqrt{2S} \sqrt{1 - \frac{\hat{n}_i}{2S}} \hat{b}_i$$

$$S_{iz} = S - \hat{n}_i$$

$$\text{where } \hat{n}_i \equiv \hat{b}_i^\dagger \hat{b}_i$$

$$\hat{H}_{Hers} = J \sum_{\langle ij \rangle}$$



$$\xi_{2+} |s_1, s_2\rangle = \sqrt{s(s+1) - s_2(s_2+1)} |s_1, s_2+1\rangle$$

$$\xi_{2-} |s_1, s_2\rangle = \sqrt{s(s+1) - s_2(s_2-1)} |s_1, s_2-1\rangle$$

$$[s_2, s_2] = s_2$$

$$[s_2, s_2] = -s_2$$

where  $\hat{n}_i \equiv b_i^\dagger b_i$

$$\hat{H}_{HRS} = J \sum_{iS} \left[ \xi_{2S} s_{i+S} + \frac{1}{2} (s_{i+} s_{i+S-} + s_{i-} s_{i+S+}) \right]$$

$$= (J) \sum_{iS} \left[ (s - n_i)(s - n_{i+S}) + \frac{1}{2} \left( \sqrt{2s} \sqrt{1 - \frac{n_i}{2s}} b_i \sqrt{2s} b_{i+S}^\dagger \sqrt{1 - \frac{n_{i+S}}{2s}} \right. \right.$$

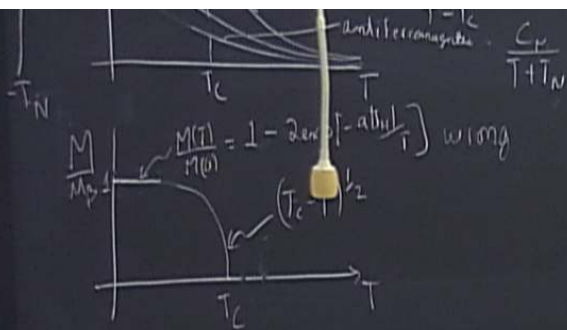
$$\left. \left. + \sqrt{2s} b_i^\dagger \sqrt{1 - \frac{n_i}{2s}} \sqrt{2s} \sqrt{1 - \frac{n_{i+S}}{2s}} b_{i+S} \right) \right]$$

$J = -|J_H|$

$$[n_i, b_i] = -b_i$$

$$[n_i, b_i^\dagger] = b_i^\dagger$$

$$\hat{H}_{HRS} = E_0 + |J_H| S \sum_{iS} b_i^\dagger b_i + b_{i+S}^\dagger b_{i+S} - b_i^\dagger b_{i+S} - b_i b_{i+S} + O(b^4)$$



Schwingen (1965)

$$\hat{S}_{i\pm} |S_1, S_{i\pm}\rangle = S_{i\pm} |S_1, S_{i\pm}\rangle$$

$$\hat{S}_{i+} |S_1, S_{i\pm}\rangle = \sqrt{S(S+1) - S_{i\pm}(S_{i\pm}+1)} |S_1, S_{i\pm} \pm 1\rangle$$

$$\hat{S}_{i-} |S_1, S_{i\pm}\rangle = \sqrt{S(S+1) - S_{i\pm}(S_{i\pm}-1)} |S_1, S_{i\pm} - 1\rangle$$

$$[b_i, b_i^\dagger] = 1$$

$$[S_{i+}, S_{i-}] = 2S_{i-}$$

$$[S_{i+}, S_{i+}] = 0$$

$$[S_{i-}, S_{i-}] = 0$$

$$S_{i\pm} = S - n_i$$

where  $n_i \equiv b_i^\dagger b_i$

$$\hat{H}_{Hers} = J \sum_{iS} \left[ S_{i2} S_{i+2S} + \frac{1}{2} (S_{i+} S_{i+2S} + S_{i-} S_{i+2S}) \right]$$

$$= (J) \sum_{iS} \left[ (S - n_i)(S - n_{i+2S}) + \frac{1}{2} \left( \sqrt{2S} \sqrt{1 - \frac{n_i}{2S}} b_i \sqrt{2S} b_{i+2S}^\dagger \sqrt{1 - \frac{n_{i+2S}}{2S}} \right. \right.$$

$$\left. \left. + \sqrt{2S} b_i^\dagger \sqrt{1 - \frac{n_i}{2S}} \sqrt{2S} \sqrt{1 - \frac{n_{i+2S}}{2S}} b_{i+2S} \right) \right]$$

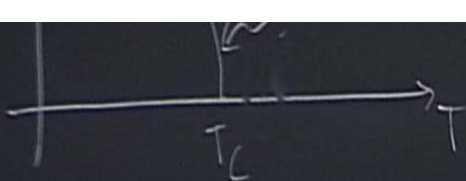
$(n_i, b_i) = -b_i$   
 $(n_i, b_i^\dagger) = b_i^\dagger$   
 $J = -|J_H|$

$$\hat{H}_{Hers} = E_0 + |J_H| S \sum_{iS} (b_i^\dagger b_i + b_{i+2S}^\dagger b_{i+2S} - b_i b_{i+2S} - b_{i+2S} b_i) + O(b_i^2)$$

$$b_i^\dagger = \frac{1}{\sqrt{2S}} \sum_{q, \sigma} e^{i\vec{q} \cdot \vec{R}_i} b_{i, q, \sigma}^\dagger$$

$$= E_0 + |J_H| S \sum_{iS} \sum_{q, \sigma} \sum_{q', \sigma'} (1 + e^{i\vec{q}' \cdot \vec{R}_S} - e^{i\vec{q} \cdot \vec{R}_S} - e^{-i\vec{q} \cdot \vec{R}_S}) b_{i, q, \sigma}^\dagger b_{i, q', \sigma'}$$

$$= E_0 + |J_H| S \sum_{q, \sigma} 4S \sin^2 \frac{qS}{2} b_{i, q, \sigma}^\dagger b_{i, q, \sigma} = E_0 + \sum_{q, \sigma} \omega_{q, \sigma} b_{i, q, \sigma}^\dagger b_{i, q, \sigma} \quad \omega_{q, \sigma} = 4|J_H| S \sum_{\sigma'} \sin^2 \frac{qS}{2}$$



$$\langle v_g \rangle = \frac{1}{e^{\beta \hbar \omega_0} - 1}$$

$$M(T) = M(0) - \frac{g \mu_B \sum}{g} \frac{1}{e^{\beta \hbar \omega_0} - 1}$$

$g \mu_B S N$  ↑

$$= M(0) \left( 1 - \frac{1}{3} \left( \frac{1}{N g} \sum \frac{1}{e^{\beta \hbar \omega_0} - 1} \right) \right)$$

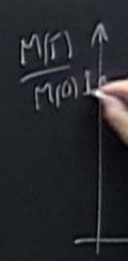
$\int d\omega \delta(\omega - \omega_0)$

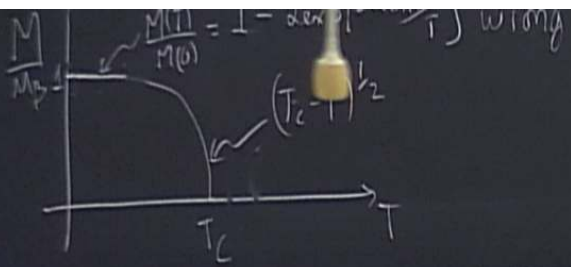
$$\frac{M(T)}{M(0)} = 1 - \frac{1}{3} \int_0^\infty \frac{d\omega g(\omega)}{e^{\beta \hbar \omega} - 1} \xrightarrow{x = \beta \hbar \omega} a \sqrt{x}$$

$$= 1 - \frac{1}{3 B^{3/2}} \int_0^\infty dx \frac{\sqrt{x}}{e^x - 1}$$

$$\frac{M(T)}{M(0)} = 1 - b T^{3/2}$$

Bloch's  $T^{3/2}$  law





$$\xi_{z+} |s_1, s_{12}\rangle = \sqrt{s(s+1) - s_{12}(s_{12}+1)} |s_1, s_{12}+1\rangle$$

$$\xi_{z-} |s_1, s_{12}\rangle = \sqrt{s(s+1) - s_{12}(s_{12}-1)} |s_1, s_{12}-1\rangle$$

$$[s_{z+}, s_{z+}] = 0$$

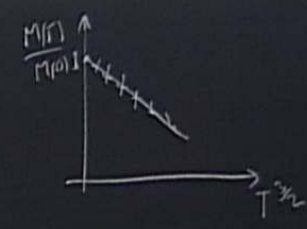
$$[s_{z+}, s_z] = -s_{z+}$$

where  $a_i \equiv b_i$

$$\langle \nu_g \rangle = \frac{1}{e^{\beta \omega_g} - 1}$$

$$\frac{M(T)}{M(0)} = 1 - \frac{1}{S} \int_0^\infty d\omega \frac{g(\omega)}{e^{\beta \omega} - 1} \xrightarrow{x = \beta \omega} a \sqrt{x}$$

$$= 1 - \frac{1}{S \beta^{3/2}} \int_0^\infty dx \frac{\sqrt{x}}{e^x - 1}$$



$$M(T) = M(0) - g_{\mu_B} \sum_g \frac{1}{e^{\beta \omega_g} - 1}$$

$$= M(0) \left( 1 - \frac{1}{S} \left( \frac{1}{N} \sum_g \frac{1}{e^{\beta \omega_g} - 1} \right) \right)$$

$\int d\omega g(\omega)$

$$\frac{M(T)}{M(0)} = 1 - b T^{3/2}$$

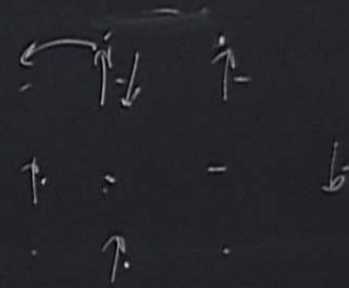
Bloch's  $T^{3/2}$  law



# Hubbard model

$$\hat{H} = -t \sum_{i \rightarrow j} (c_{i\uparrow}^\dagger c_{j\uparrow} + c_{i\downarrow}^\dagger c_{j\downarrow}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Coulomb interaction between electrons

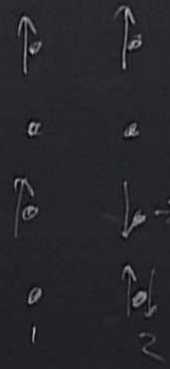
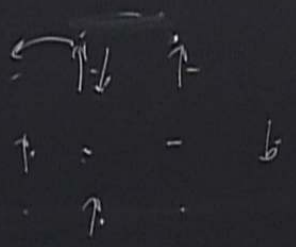


$$M = k_B T \ln \frac{\partial Z}{\partial B}$$

$$Z = \sum_{\{s_i\}} e^{-\beta H(\{s_i\})}$$

$$E_d + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Coulomb interaction between electrons



states:

1) second 2 first

$$c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger |0\rangle$$

2) up first down second - obeys 6 kT rule

$$c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger |0\rangle$$

$$c_{2\downarrow}^\dagger c_{1\uparrow}^\dagger |0\rangle$$

$$p_4) S^N \frac{E}{Z} - g \mu_B N S B \quad g(JLS)$$

$$g \rightarrow 2$$

$$M = -\frac{\partial F}{\partial B}$$

$$F = -k_B T \ln Z$$

$$M = k_B T \frac{1}{Z} \frac{\partial Z}{\partial B}$$

$$Z = \sum_n e^{-\beta \epsilon_n}$$

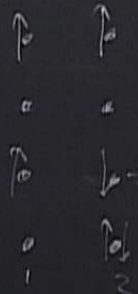
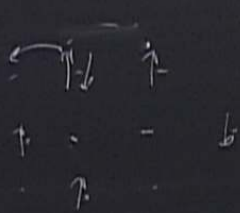
$$E_{m_j} = -g \mu_B J_z B$$

where  $J_z = -J, -J+1, \dots, J-1, J$

$$Z = \sum_{J_z=-J}^J e^{+\beta g \mu_B J_z B} = \frac{e^{\beta g \mu_B B (J+1/2)} - e^{-\beta g \mu_B B (J+1/2)}}{e^{\beta g \mu_B B} - e^{-\beta g \mu_B B}}$$

$$\sum_i n_{i\uparrow} n_{i\downarrow}$$

Coulomb interaction between electrons



states:

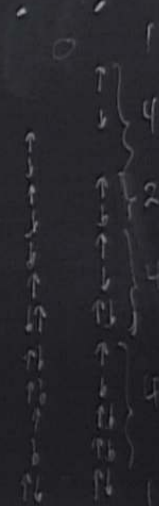
1) second 2 first

$$c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger |0\rangle$$

2) up first down second - obeys 6th rule

$$c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger |0\rangle$$

$$c_{2\downarrow}^\dagger c_{1\uparrow}^\dagger |0\rangle$$



$$|\psi\rangle = \sum_i a_i |\psi_i\rangle$$

$$\sum_i \langle \psi | H \psi \rangle a_i = E a_i$$

$$[A] a$$

Ferromagnet:  $J = -|J_H|$

$$H_{\text{Huis}} = -|J_H| \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

$$H_{\text{Huis}} \rightarrow -|J_H| S^2 N \frac{z}{2} - g\mu_B N S B \quad g(JLS)$$

$$\vec{S}_i = \langle S_i \rangle + S_i - \langle S_i \rangle \quad \uparrow \uparrow \uparrow$$

$$M = g\mu_B \vec{S}$$

$$\vec{S} \rightarrow \vec{J} = \vec{L} + \vec{S}$$

$$g \rightarrow 2$$

$$B_{\text{eff}} = B + \frac{|J_H| z}{2} \frac{z}{2} S = B + \frac{|J_H| z M}{(g\mu_B)^2}$$

atomic spin in a magnetic field

$$\hat{H} = -g\mu_B \vec{J} \cdot \vec{B}$$

$$M = \frac{-\partial F}{\partial B}$$

$$F = -k_B T \ln Z$$

$$M = k_B T \frac{\partial \ln Z}{\partial B}$$

$$Z = \sum_n e^{-\beta E_n}$$

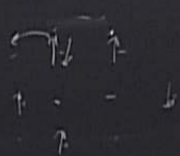
$$E_m = -g\mu_B J_z B$$

$$Z = \sum_{J_z} e^{+\beta g\mu_B J_z B} = \frac{e^{+\beta g\mu_B B(J+1/2)} - e^{-\beta g\mu_B B(J+1/2)}}{e^{+\beta g\mu_B B} - e^{-\beta g\mu_B B}}$$

Hubbard model

$$\hat{H} = -t \sum_{\langle i,j \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Coulomb interaction between electrons



states:

1) lowest state

$$c_{i\uparrow}^\dagger c_{j\uparrow} |0\rangle$$

2) up first down second state is 1st state

$$c_{i\uparrow}^\dagger c_{j\downarrow} |0\rangle$$

$$c_{i\downarrow}^\dagger c_{j\uparrow} |0\rangle$$

$$|1\rangle = \frac{1}{\sqrt{2}} (c_{i\uparrow}^\dagger c_{j\downarrow} + c_{i\downarrow}^\dagger c_{j\uparrow}) |0\rangle$$

$$\frac{1}{\sqrt{2}} \langle 1 | H | 1 \rangle = E_1$$

$$[A] \vec{a} = E \vec{a}$$

