Title: PSI 2016/2017 Condensed Matter - Lecture 10

Date: Nov 18, 2016 10:45 AM

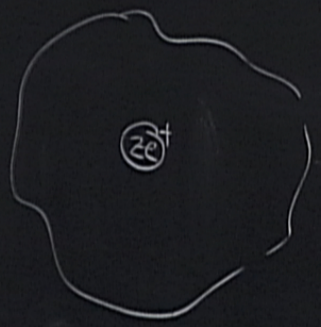
URL: http://pirsa.org/16110061

Abstract:

Screening: 1) fixing ourselves as one of the electrons $e\left[V_{ext}(\vec{r}) + V_{ee}(\vec{r})\right]$

2) electron sees: (a) V_{ext} (test charge)

(b) V_{ee} (the other electrons)



3) this is true for all the electrons

4) repeat

of the elections

(test charge)

(the other elections)

the elections

$$e\left[V_{ext}(\vec{r}) + V_{ee}(\vec{r})\right]$$

$$\varphi_k^0(r) \sim e^{i\vec{k}\cdot\vec{r}}$$

$$\psi_k(\vec{r}) = \left[e^{i\vec{k}\cdot\vec{r}} + \varphi_k^{(1)}(\vec{r})\right]$$

$$n(r) = \sum_k |\psi_k(r)|^2 \approx \sum_k \left[1 + e^{i\vec{k}\cdot\vec{r}}\,\varphi_k^{(1)*}(r) + e^{-i\vec{k}\cdot\vec{r}}\,\varphi_k^{(1)}(r)\right]$$

$e\left[V_{ext}(\vec{r}) + V_{ee}(\vec{r})\right]$

$\varphi_k^0(r) \sim e^{i\vec{k}\cdot\vec{r}}$

ons)

$\psi_k(\vec{r}) = \left[e^{ik\cdot r} + \varphi_k^{(1)}(\vec{r})\right]$

$n(r) = \sum_k |\psi_k(r)|^2 f_k \approx \sum_k \left[1 + e^{i\vec{k}\cdot\vec{r}}\,\varphi_n^{(1)*}(r) + e^{-ik\cdot r}\,\varphi_n^{(1)}(r)\right] f_k$

Fermi fn.

$$\psi_k^{(1)}(r) = \sum_{k'} \frac{|k'\rangle\langle k'|e(V_{ext} + V_{ee})|k\rangle}{\varepsilon_k - \varepsilon_{k'}} \quad \leftarrow e^{ik'r}$$

$$\psi_n^{(1)\sigma}(r) + e^{-ik\cdot r}\psi_n^{(1)}(r)\Big] f_p$$

Screening: 1) fixing ourselves as one of the electrons

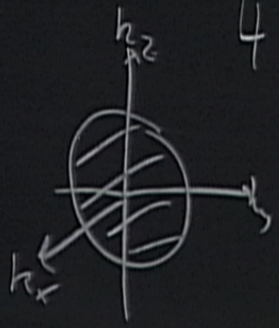
2) electron sees: (a) V_{ext} (test charge)

(b) V_{ee} (the other electrons)

3) this is true for all the electrons

4) repeat





$$\psi_k^{(1)}(r) = \sum_{\substack{k' \\ \xi_{k'} > \xi_F}} \frac{|k'\rangle\langle k'|e(V_{ext} + V_{ee})|k\rangle}{\xi_k - \xi_{k'}} \left(1 - f_{k'}\right) \quad \uparrow e^{ik'r}$$

$$e^{-ik\cdot r}\psi_n^{(1)*}(r) + e^{-ik\cdot r}\psi_n^{(1)}(r)\Big) f_k$$

$$\psi_k^{(1)}(r) = \sum_{k'} \frac{|k'\rangle\langle k'|e(V_{ext} + V_{ee})|k\rangle}{\xi_k - \xi_{k'}} \left(1 - f_{k'}\right) \quad \text{(} \to e^{ik'r} \text{)}$$

$$= \sum_{k'} e^{ik'\cdot r}\left[e\left(V_{ext}(k-k') + V_{ee}(k-k')\right)\right]\left(1 - f_{k'}\right)$$

$$+ e^{-ikr}\psi_k^{(1)}(r)\Big] f_k$$

$$f_{k'} = \frac{1}{e^{\beta(\xi_{k'} - \mu)} + 1}$$

$V_{ext}(\vec{r}) + V_{ee}(\vec{r})\Big]$

$\varphi^0_k(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}}$

$\psi_k(\vec{r}) = \left[e^{ik\cdot r} + \varphi^{(1)}_k(\vec{r})\right]$

$n(\vec{r}) = 2\sum_k |\psi_k(\vec{r})|^2 f_k \approx 2\sum_k \left[1 + e^{ik\cdot r}\varphi^{(1)*}_k(r) + e^{-ikr}\varphi^{(1)}_k(r)\right] f_k$

spin ↑ Fermi fn. ↑

$= n + \Delta n(\vec{r})$ where $\Delta n(\vec{r}) = 2\sum_{kk'} f_k(1-f_{k'})\, e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}$

$q = (k-k')$

$\varphi^{(1)}_k(\vec{r}) = \sum_{k'} \frac{e^{ik'\cdot r}\,|k'\rangle\langle k'|\,e(V_{ext}+V_{ee})\,|k\rangle}{\varepsilon_k - \varepsilon_{k'}}(1-f_{k'})$

$= \sum_{k'} e^{ik'\cdot\vec{r}}\left[e\left(V_{ext}(k-k') + V_{ee}(k-k')\right)\right](1-f_{k'})$

$k \to -k$, $k' \to -k'$

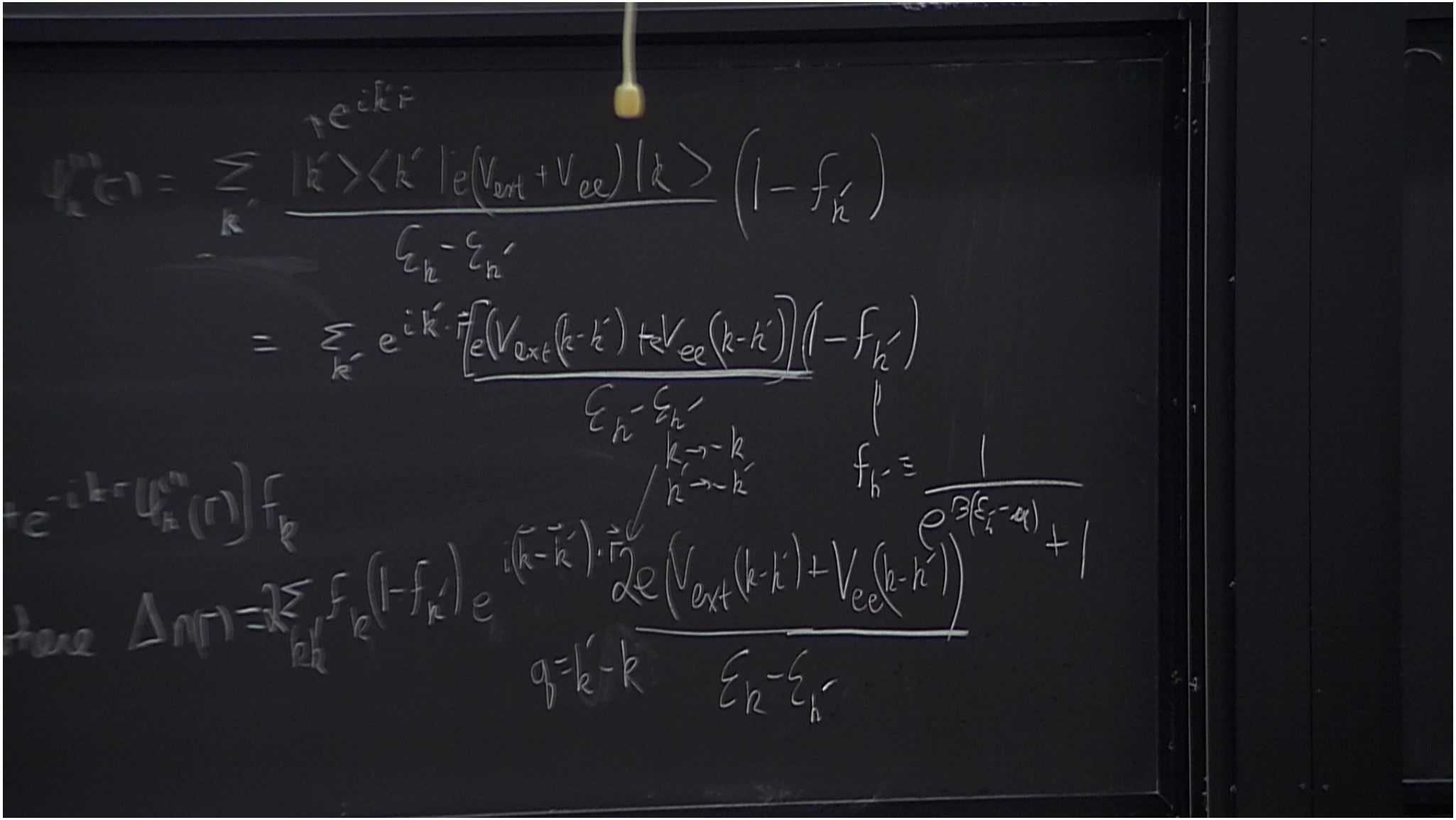
$f_{k'} \equiv \frac{1}{e^{\beta(\varepsilon_{k'}-\mu)}+}$

$2e\left(V_{ext}(k-k') + V_{ee}(k-k')\right)$

$$\Delta n(r) = 4 \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \sum_{k} f_k (1 - f_{k+q})$$

$$\Delta n(r) = 4 \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \sum_{k} f_k(1 - f_{k+q}) e\left[V_{ext}(q) + V_{ee}(q)\right]$$

$$= \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \Delta n(\vec{q}) \Rightarrow \Delta n(\vec{q}) = \frac{4}{N}$$


$\psi_k^{(1)}(r) = \sum_{k'} \frac{|k'\rangle\langle k'|e(V_{ext}+V_{ee})|k\rangle}{\varepsilon_k - \varepsilon_{k'}} (1-f_{k'})$
$e^{ik'\cdot r}$
$= \sum_{k'} e^{ik'\cdot r} \frac{[e(V_{ext}(k-k')+V_{ee}(k-k'))](1-f_{k'})}{\varepsilon_k - \varepsilon_{k'}}$
$k \to -k$
$k' \to -k'$
$f_{k'} = \frac{1}{e^{\beta(\varepsilon_{k'}-\mu)}+1}$
$e^{-ik\cdot r}\psi_k^{(1)}(r)\, f_k$
$\Delta n(r) = 2\sum_{kk'} f_k(1-f_{k'}) e^{i(k-k')\cdot r} \frac{2e(V_{ext}(k-k')+V_{ee}(k-k'))}{\varepsilon_k - \varepsilon_{k'}}$
$q = k'-k$

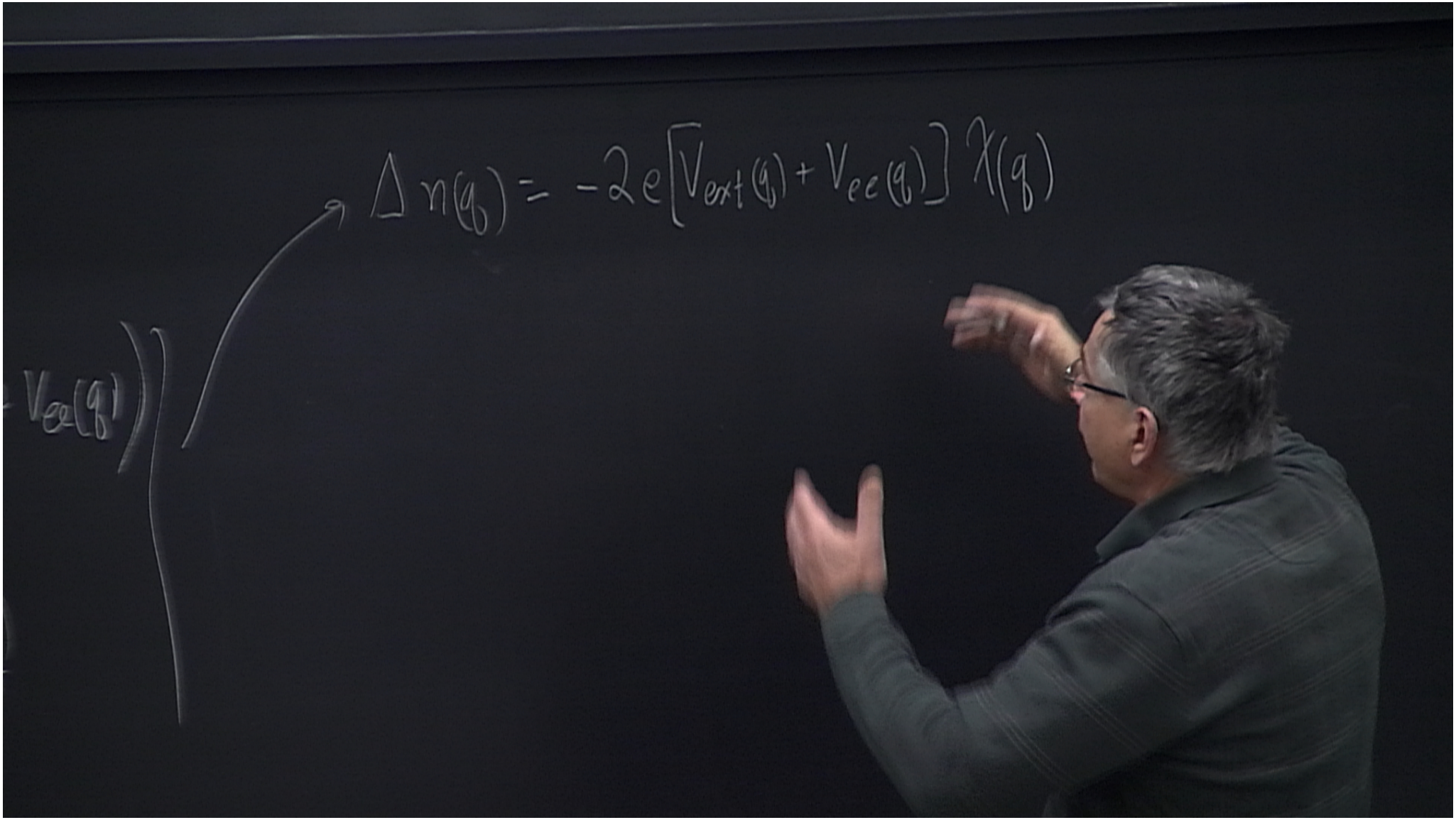
$$\Delta n(\vec{r}) = 4 \sum_{q} e^{i\vec{q}\cdot\vec{r}} \sum_{k} \frac{f_k(1-f_{k+q})\, e\left[V_{ext}(q) + V_{ee}(q)\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

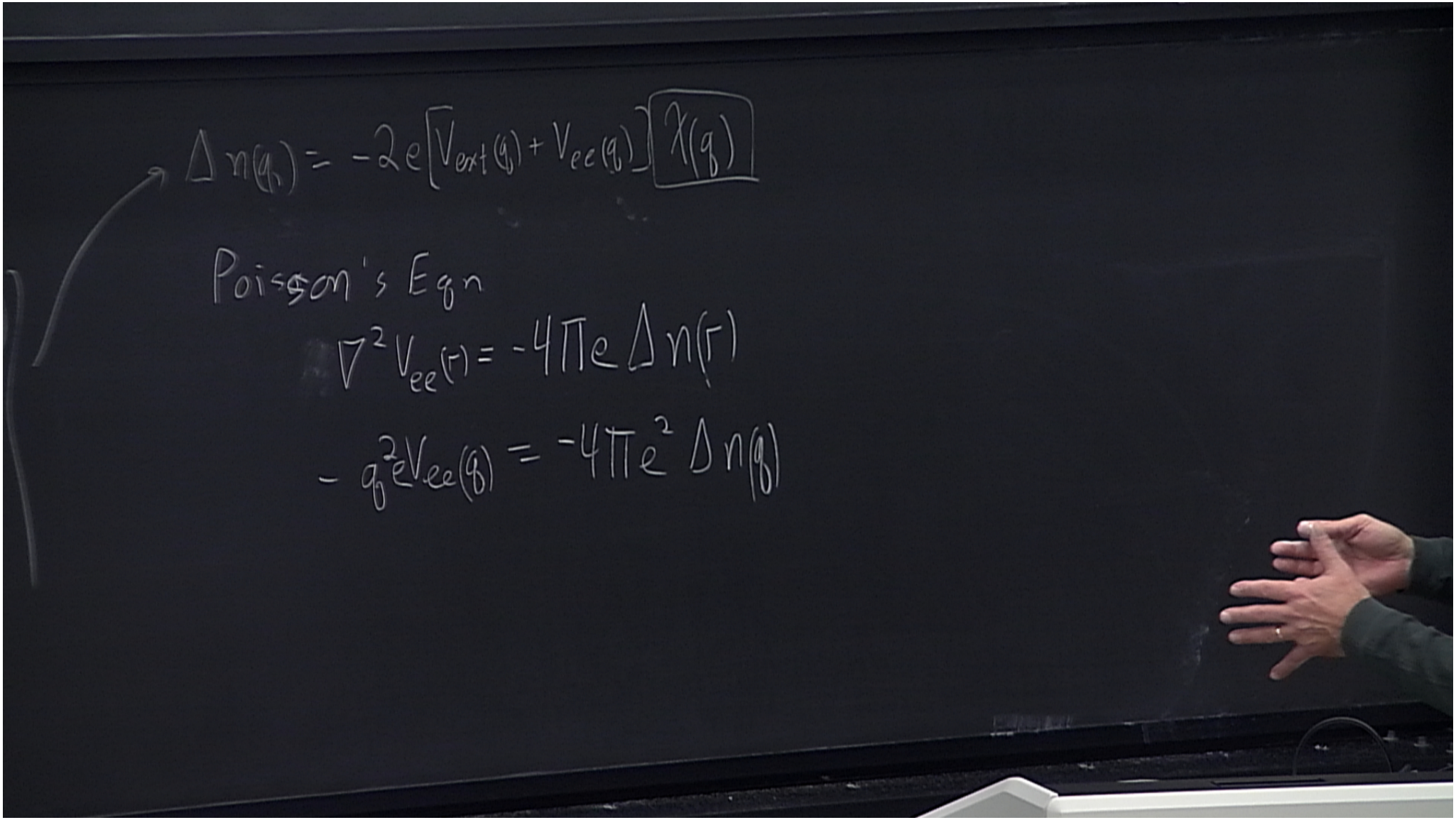
$$= \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \Delta n(\vec{q}) \Rightarrow \Delta n(\vec{q}) = \frac{4}{N} \frac{f_k(1-f_{k+q})}{\varepsilon_k - \varepsilon_{k+q}} e\left(V_{ext}(q) + V_{ee}(q)\right)$$

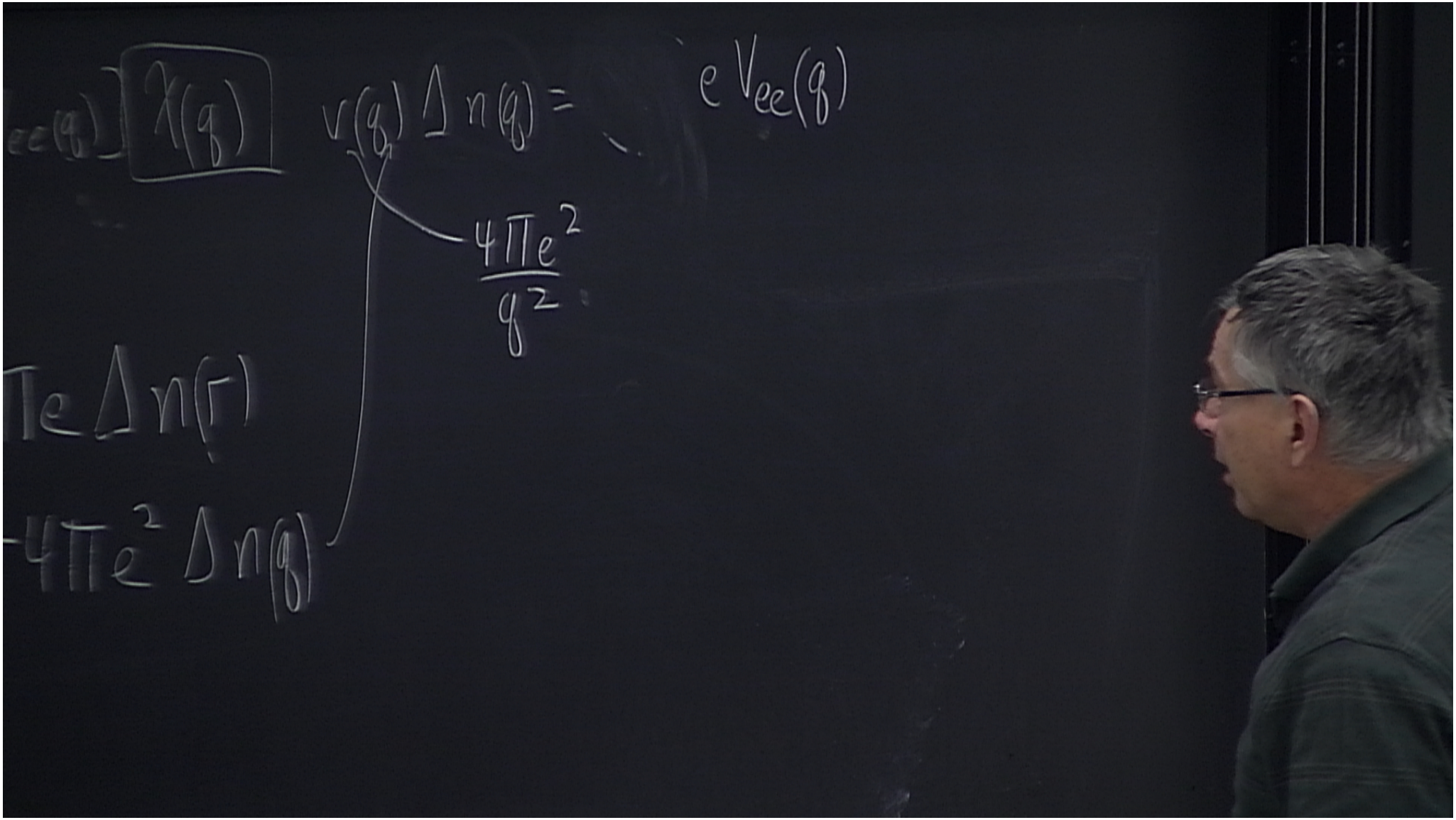
$$\Delta n(\vec{r}) = 4 \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \sum_{k} \frac{f_k(1-f_{k+q})\, e\left[V_{ext}(q) + V_{ee}(q)\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

$$= \sum_{q} e^{-i\vec{q}\cdot\vec{r}} \Delta n(q) \Rightarrow \Delta n(\vec{q}) = -\frac{4}{N} \sum_{k} \frac{f_k(1-f_{k+q})}{\varepsilon_k - \varepsilon_{k+q}} e\left(V_{ext}(q) + V_{ee}(q)\right)$$

define the susceptibility, $\chi(q) = -\frac{2}{N} \sum_{k} \frac{f_k(1-f_{k+q})}{\varepsilon_k - \varepsilon_{k+q}}$


$\Delta n(q) = -2e\left[V_{ext}(q) + V_{ec}(q)\right]\chi(q)$
$V_{ec}(q)$


$\Delta n(q) = -2e[V_{ext}(q) + V_{ee}(q)]\chi(q)$
Poisson's Eqn
$\nabla^2 V_{ee}(r) = -4\pi e \Delta n(r)$
$-q^2 V_{ee}(q) = -4\pi e^2 \Delta n(q)$


χ(q)
v(q) Δn(q) = − eVee(q)
4πe²/q²
πeΔn(r)
−4πe²Δn(q)

$V_{ee}(q) = \chi(q)$ (1)

$\Pi e \Delta n(r)$

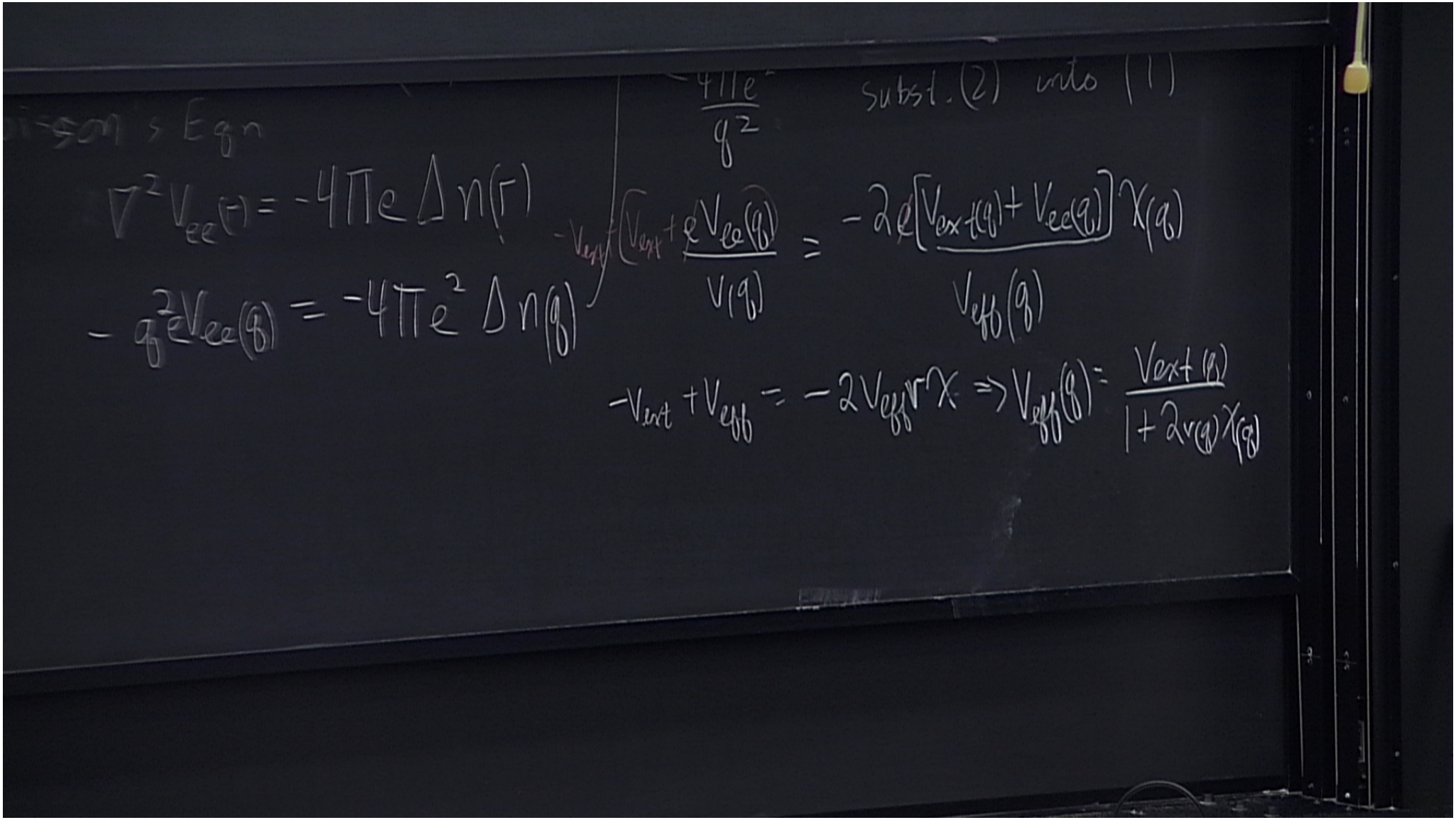
$-4\Pi e^2 \Delta n(q)$

$v(q)\,\Delta n(q) = -eV_{ee}(q)$ (2)

$v(q) = \frac{4\Pi e^2}{q^2}$

subst. (2) into (1)

$\frac{eV_{ee}(q)}{v(q)} = -2e\underbrace{[V_{ext}(q) + V_{ee}(q)]}_{V_{eff}(q)}\chi(q)$


Poisson's Eqn
$\nabla^2 V_{ee}(r) = -4\pi e \Delta n(r)$
$-q^2 V_{ee}(q) = -4\pi e^2 \Delta n(q)$
$\frac{4\pi e^2}{q^2}$
Subst. (2) into (1)
$-V_{ext} + \left[V_{ext} + \frac{e V_{ee}(q)}{V(q)}\right] = -2e\underbrace{\left[V_{ext}(q) + V_{ee}(q)\right]}_{V_{eff}(q)} \chi(q)$
$-V_{ext} + V_{eff} = -2 V_{eff} r \chi \Rightarrow V_{eff}(q) = \frac{V_{ext}(q)}{1 + 2v(q)\chi(q)}$

$$V_{eff}(q) = \frac{V_{ext}(q)}{\varepsilon(q)}$$

static dielectric function

$(\varepsilon(q,\omega))$

$$V_{eff}(q) = \frac{V_{ext}(q)}{\varepsilon(q)}$$

$$\varepsilon(q) = 1 + 2v(q)\chi(q)$$

static dielectric function

$$(\varepsilon(q,\omega))$$

$$\varepsilon(q) = 1 + 2v(q)\chi(q)$$

$$= 1 + \frac{8\pi e^2}{q^2}\chi(q)$$

$$= \frac{V_{ext}(q)}{\varepsilon(q)}$$

dielectric function

$(\varepsilon(q,\omega))$

$$\varepsilon(q) = 1 + 2v(q)\chi(q) \qquad \chi(q) = -\frac{2}{N}\sum_k \frac{f(\varepsilon_k)\left[1 - f(\varepsilon_{k+q})\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

$$= 1 + \frac{8\pi e^2}{q^2}\chi(q) \qquad = -\frac{1}{N}\sum_k \frac{f(\varepsilon_k)\left[1 - f(\varepsilon_{k+q})\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

tion

$$\chi(q) = -\frac{2}{N}\sum_k \frac{f(\varepsilon_k)\left[1 - f(\varepsilon_{k+q})\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

$\chi(q)$

$k' = -(k+q)$

$k = -(k'+q)$

$$= -\frac{1}{N}\sum_k \left[\frac{f(\varepsilon_k)\left(1 - f(\varepsilon_{k+q})\right)}{\varepsilon_k - \varepsilon_{k+q}} - \frac{f_{k+q}\left(1 - f(\varepsilon_k)\right)}{\varepsilon_k - \varepsilon_{k+q}}\right]$$

$$= -\frac{1}{N}\sum_k \frac{f(\varepsilon_k) - f(\varepsilon_{k+q})}{\varepsilon_k - \varepsilon_{k+q}}$$

$$V_{eff}(q) = \frac{V_{ext}(q)}{\varepsilon(q)}$$

$$\varepsilon(q) = 1 + 2v(q)\chi(q) = 1 + \frac{8\pi e^2}{q^2}\chi(q)$$

$$\chi(q) = -\frac{2}{N} = -\frac{1}{N}$$

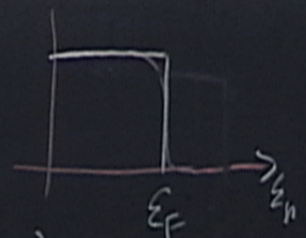
Static dielectric function

$$(\varepsilon(q,\omega))$$

$$\chi(q,\omega+i\delta) = -\frac{1}{N}\sum_{k} \frac{f(\varepsilon_k) - f(\varepsilon_{k+q})}{\varepsilon_k - \varepsilon_{k+q} + \omega + i\delta}$$

$$= -\frac{1}{N}$$

$$\chi(q) = -\frac{2}{N}\sum_k \frac{f(\varepsilon_k)\left[1 - f(\varepsilon_{k+q})\right]}{\varepsilon_k - \varepsilon_{k+q}}$$

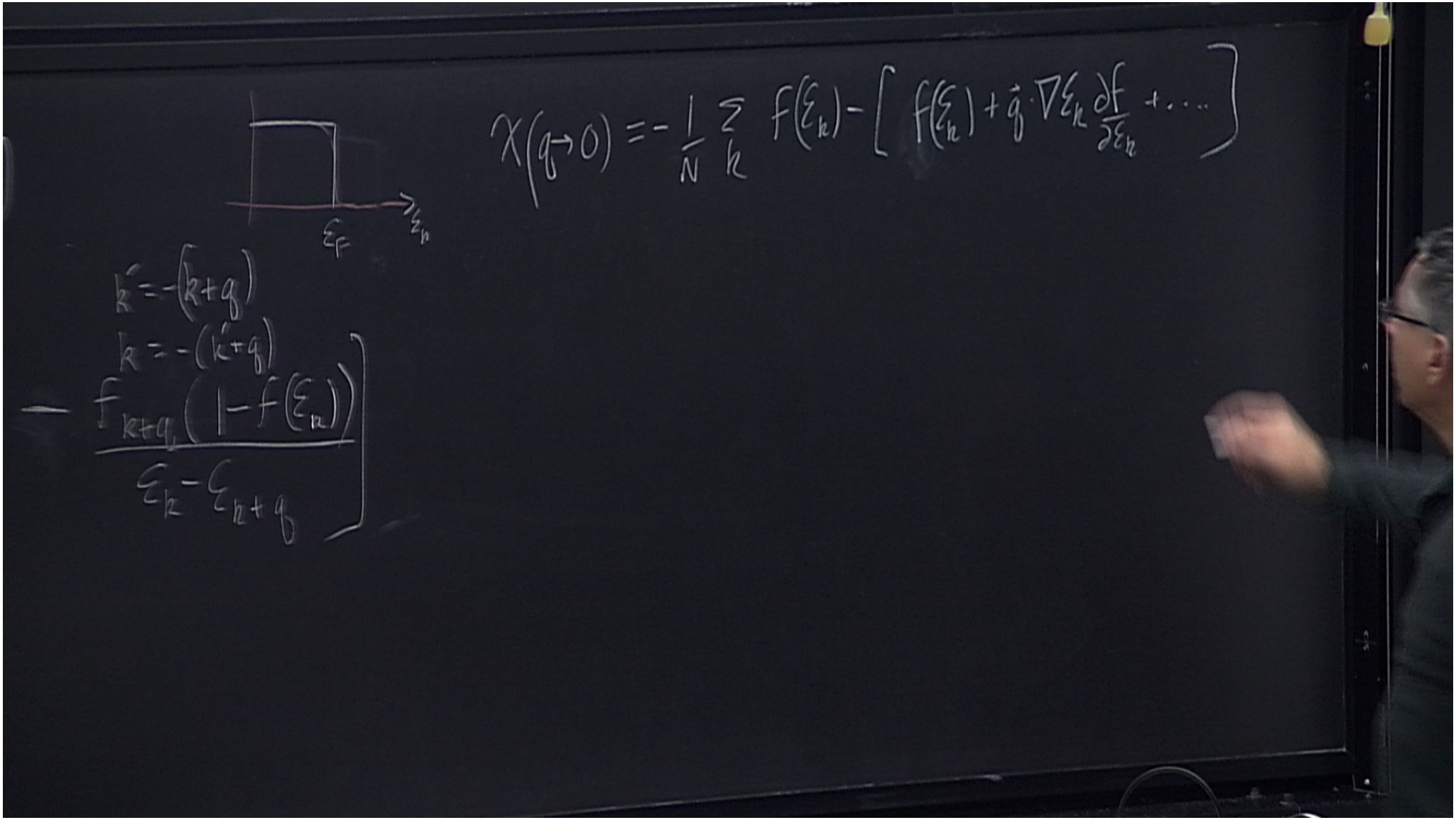


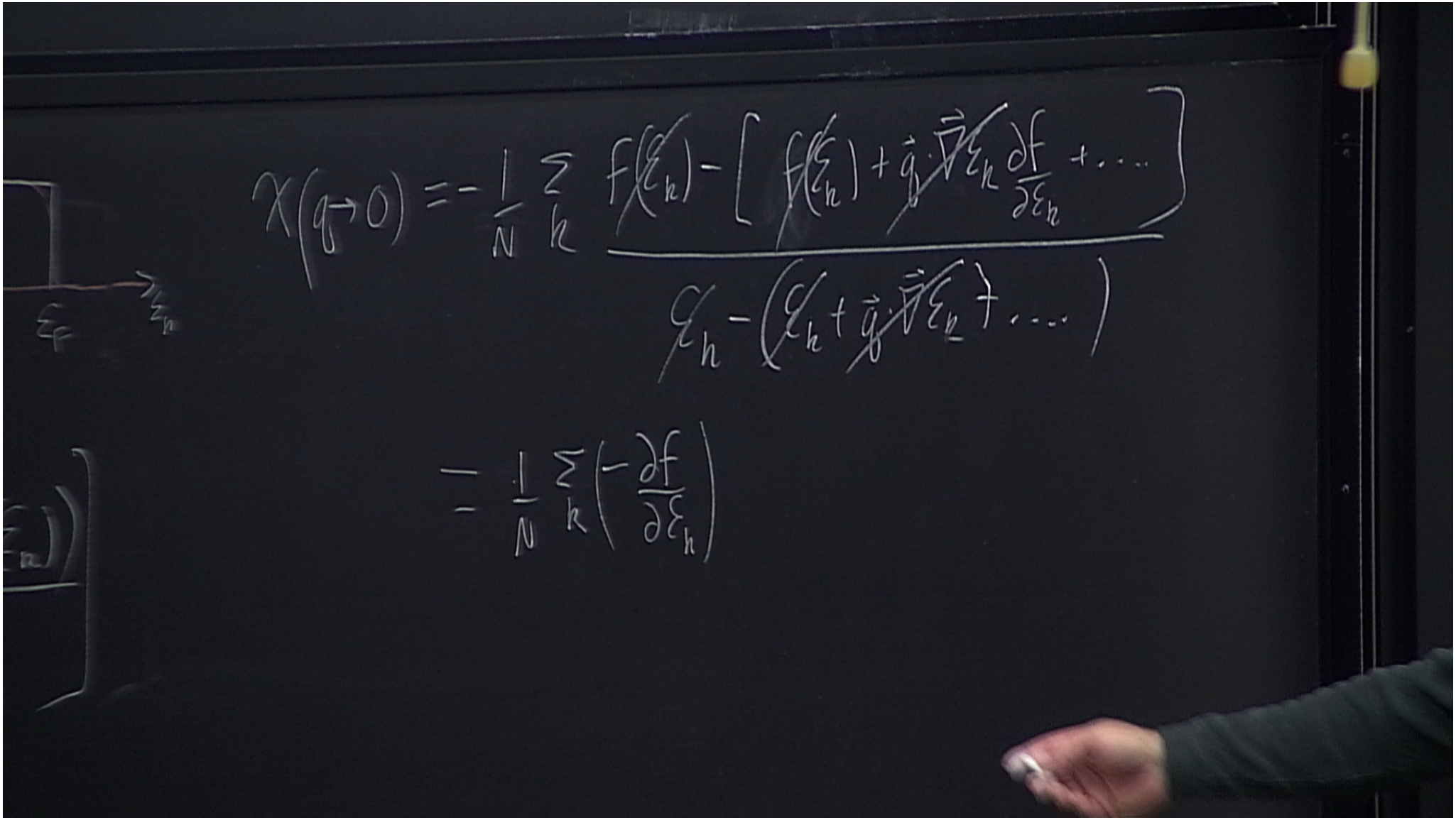
$$k' = -(k+q)$$

$$k = -(k'+q)$$

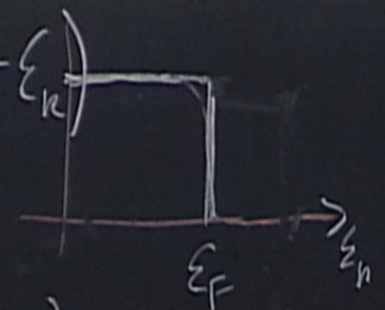
$$= -\frac{1}{N}\sum_k \left[\frac{f(\varepsilon_k)\left(1 - f(\varepsilon_{k+q})\right)}{\varepsilon_k - \varepsilon_{k+q}} - \frac{f_{k+q}\left(1 - f(\varepsilon_k)\right)}{\varepsilon_k - \varepsilon_{k+q}}\right]$$

$$= -\frac{1}{N}\sum_k \frac{f(\varepsilon_k) - f(\varepsilon_{k+q})}{\varepsilon_k - \varepsilon_{k+q}}$$


$\chi(q\to 0) = -\frac{1}{N}\sum_k f(\varepsilon_k) - \left[f(\varepsilon_k) + \vec{q}\cdot\nabla\varepsilon_k \frac{\partial f}{\partial \varepsilon_k} + \dots \right]$
ε_F
ε_k
$k' = -(k+q)$
$k = -(k'+q)$
$- f_{k+q}\left(1 - f(\varepsilon_k)\right)$
$\varepsilon_k - \varepsilon_{k+q}$


$\chi(q \to 0) = -\frac{1}{N}\sum_k \frac{f(\varepsilon_k) - [f(\varepsilon_k) + \vec{q}\cdot\vec{\nabla}\varepsilon_k \frac{\partial f}{\partial \varepsilon_k} + \dots]}{\varepsilon_k - (\varepsilon_k + \vec{q}\cdot\vec{\nabla}\varepsilon_k + \dots)}$
$= -\frac{1}{N}\sum_k \left(-\frac{\partial f}{\partial \varepsilon_k}\right)$
ε_F
ε_k

$T \to 0$ $f(\varepsilon_k) = \Theta(\varepsilon_F - \varepsilon_k)$



$$\chi(q \to 0) = -\frac{1}{N} \sum_k \frac{f(\varepsilon_k)(1 - f(\varepsilon_{k+q}))}{\varepsilon_k - \varepsilon_{k+q}}$$

$k' = -(k+q)$

$k = -(k'+q)$

$$= \frac{1}{N} \sum_k \left[\frac{f(\varepsilon_k)(1 - f(\varepsilon_{k+q}))}{\varepsilon_k - \varepsilon_{k+q}} - \frac{f_{k+q}(1 - f(\varepsilon_k))}{\varepsilon_k - \varepsilon_{k+q}} \right]$$

$$= \frac{1}{N} \sum_k \left(\right)$$

what about $T \to 0$? =

$$\chi(q \to 0) = \frac{1}{N} \sum_k \frac{f(\varepsilon_k) - f(\varepsilon_k + \vec{q}\cdot\vec{\nabla}\varepsilon_k + \dots)}{\varepsilon_k - (\varepsilon_k + \vec{q}\cdot\vec{\nabla}\varepsilon_k + \dots)}$$

$$= \frac{1}{N} \sum_k \left(-\frac{\partial f}{\partial \varepsilon_k} \right)$$

what about $T \to 0$?

$$= \frac{1}{N} \sum_k \delta(\varepsilon - \varepsilon_F) = g(\varepsilon_F)$$

$$= \frac{1}{N} \sum_k \left(-\frac{\partial f}{\partial \varepsilon_k} \right)$$

what about $T \to 0$?

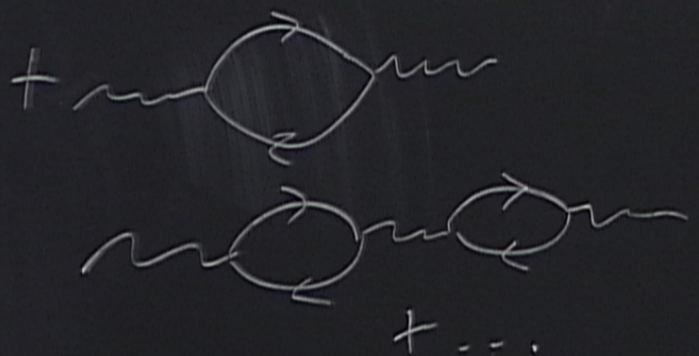
$$= \frac{1}{N} \sum_k \delta(\varepsilon - \varepsilon_k) = g(\varepsilon_F)$$

approximate: $\chi(q) \approx \chi(q=0) = g(\varepsilon_F)$

$$V_{eff}(q) = \frac{V_{ext}(q)}{1 + 2v(q)\chi(q)}$$

assume $V_{ext}(q) = \frac{4\pi Z e^2}{q^2}$

$\approx = \sim$

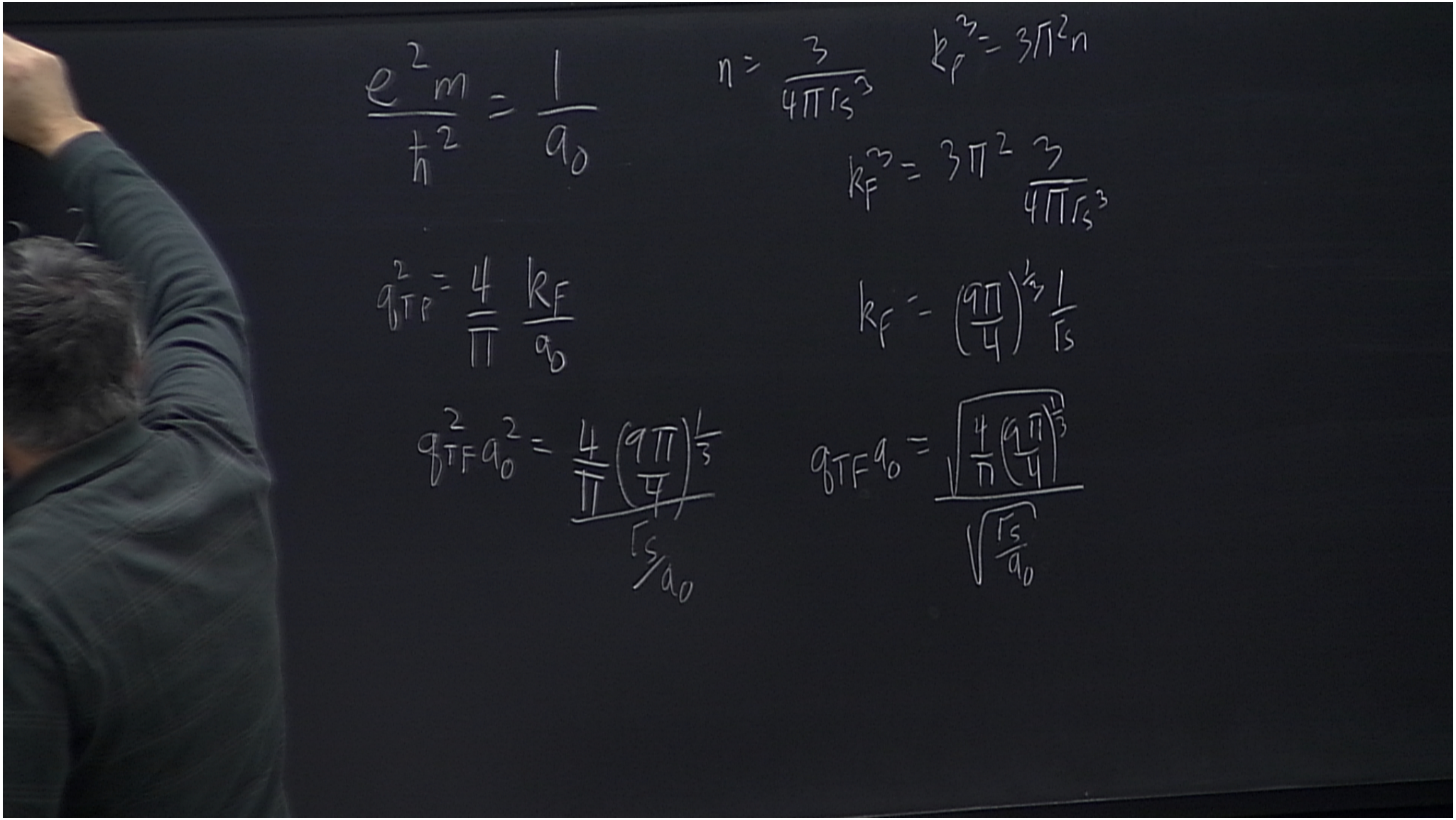


assume $\chi(q) \approx g(\varepsilon_F)$

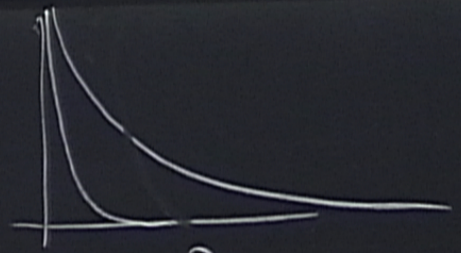
$$V_{eff}(q) = \frac{\frac{4\pi z e^2}{q^2}}{1 + \frac{8\pi e^2}{q^2} g(\varepsilon_F)} = \frac{4\pi z e^2}{q^2 + q_{TF}^2}$$

$$q_{TF}^2 \equiv 8\pi e^2 g(\varepsilon_F)$$

(3D) $$g(\varepsilon_F) = \frac{m}{2\pi\hbar^2} k_F$$


$\frac{e^2 m}{\hbar^2} = \frac{1}{a_0}$
$n = \frac{3}{4\pi r_s^3}$
$k_F^3 = 3\pi^2 n$
$k_F^3 = 3\pi^2 \frac{3}{4\pi r_s^3}$
$q_{TF}^2 = \frac{4}{\pi} k_F a_0$
$k_F = \left(\frac{9\pi}{4}\right)^{1/3} \frac{1}{r_s}$
$q_{TF}^2 a_0^2 = \frac{\frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{1/3}}{r_s/a_0}$
$q_{TF} a_0 = \frac{\sqrt{\frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{1/3}}}{\sqrt{\frac{r_s}{a_0}}}$

$\chi(q) \approx g(\varepsilon_F)$



$$\varphi(q) = \frac{\frac{4\pi z e^2}{q^2}}{1 + \frac{8\pi e^2}{q^2} g(\varepsilon_F)} = \frac{4\pi z e^2}{q^2 + q_{TF}^2}$$

$$q_{TF}^2 \equiv 8\pi e^2 g(\varepsilon_F)$$

(3D) $$g(\varepsilon_F) = \frac{m}{2\pi^2\hbar^2} k_F$$

$$\frac{e^2 m}{\hbar^2} = \frac{1}{a_0}$$

$$n = \frac{3}{4\pi}$$

$$q_{TF}^2 = \frac{4}{\pi} \frac{k_F}{a_0}$$

$$q_{TF}^2 a_0^2 = \frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{\frac{1}{3}}$$

$[s/a_0]$

$n = \frac{3}{4\pi r_s^3}$ $\quad k_F^3 = 3\pi^2 n$

$k_F^3 = 3\pi^2 \frac{3}{4\pi r_s^3}$

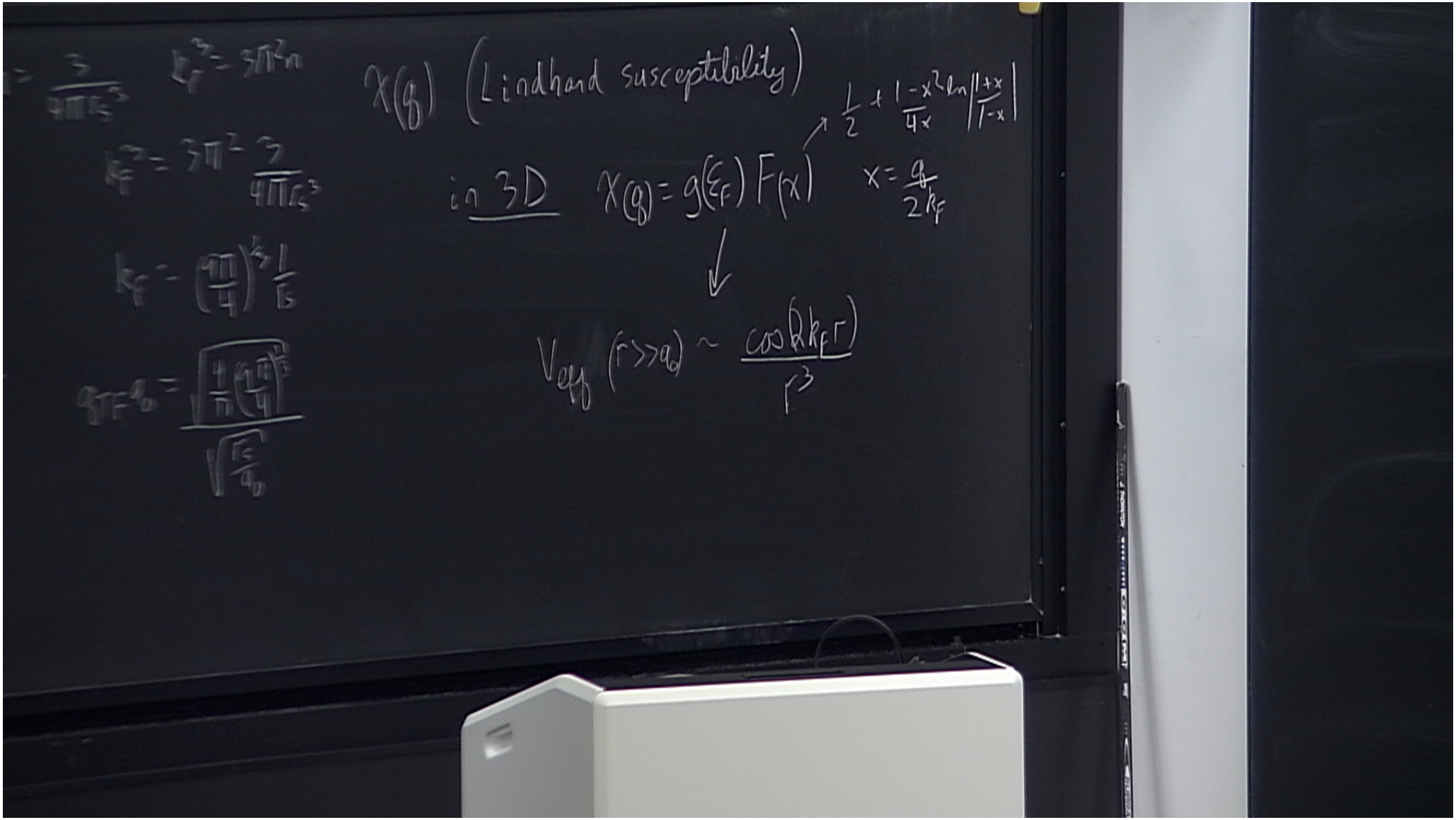
$k_F = \left(\frac{9\pi}{4}\right)^{1/3} \frac{1}{r_s}$

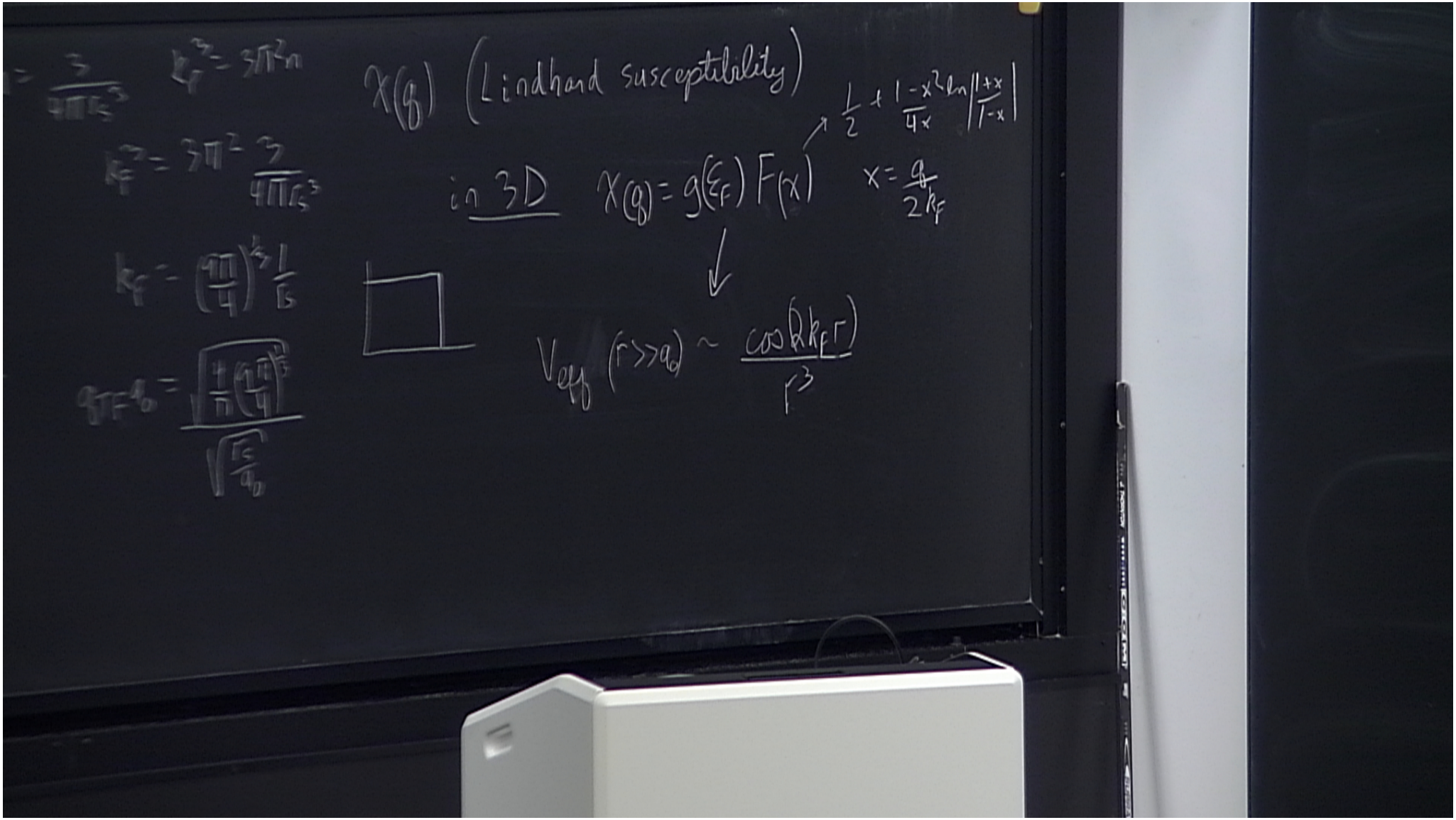
$q_{TF} = q_0 = \sqrt{\frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{\frac{1}{3}}} \Big/ \sqrt{\frac{r_s}{a_0}}$

$\chi(q)$ (Lindhard susceptibility)

in 3D $\quad \chi(q) = g(\varepsilon_F) F(x)$ $\quad x = \frac{q}{2k_F}$

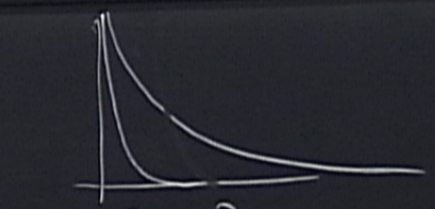
$F(x) \to \frac{1}{2} + \frac{1-x^2}{4x} \ln\left|\frac{1+x}{1-x}\right|$

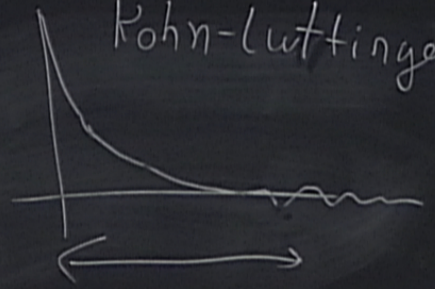

$\chi(q)$ (Lindhard susceptibility)
in 3D $\chi(q) = g(\xi_F) F(x)$
$\frac{1}{2} + \frac{1-x^2}{4x} \ln\left|\frac{1+x}{1-x}\right|$
$x = \frac{q}{2k_F}$
$V_{eff}(r \gg a_0) \sim \frac{\cos(2k_F r)}{r^3}$


χ(q) (Lindhard susceptibility)
in 3D
Veff (r>>a0) ~

Assume $\chi(q) \approx g(\varepsilon_F)$

$$V_{eff}(q) = \frac{\frac{4\pi z e^2}{q^2}}{1 + \frac{4\pi e^2}{q^2} g(\varepsilon_F)} = \frac{4\pi z e^2}{q^2 + q_{TF}^2}$$





$$\frac{e^2 m}{\hbar^2} = \frac{1}{a_0}$$

$$n = \frac{3}{4\pi r_s^3}$$

$$k_F^3 = 3\pi^2 n$$

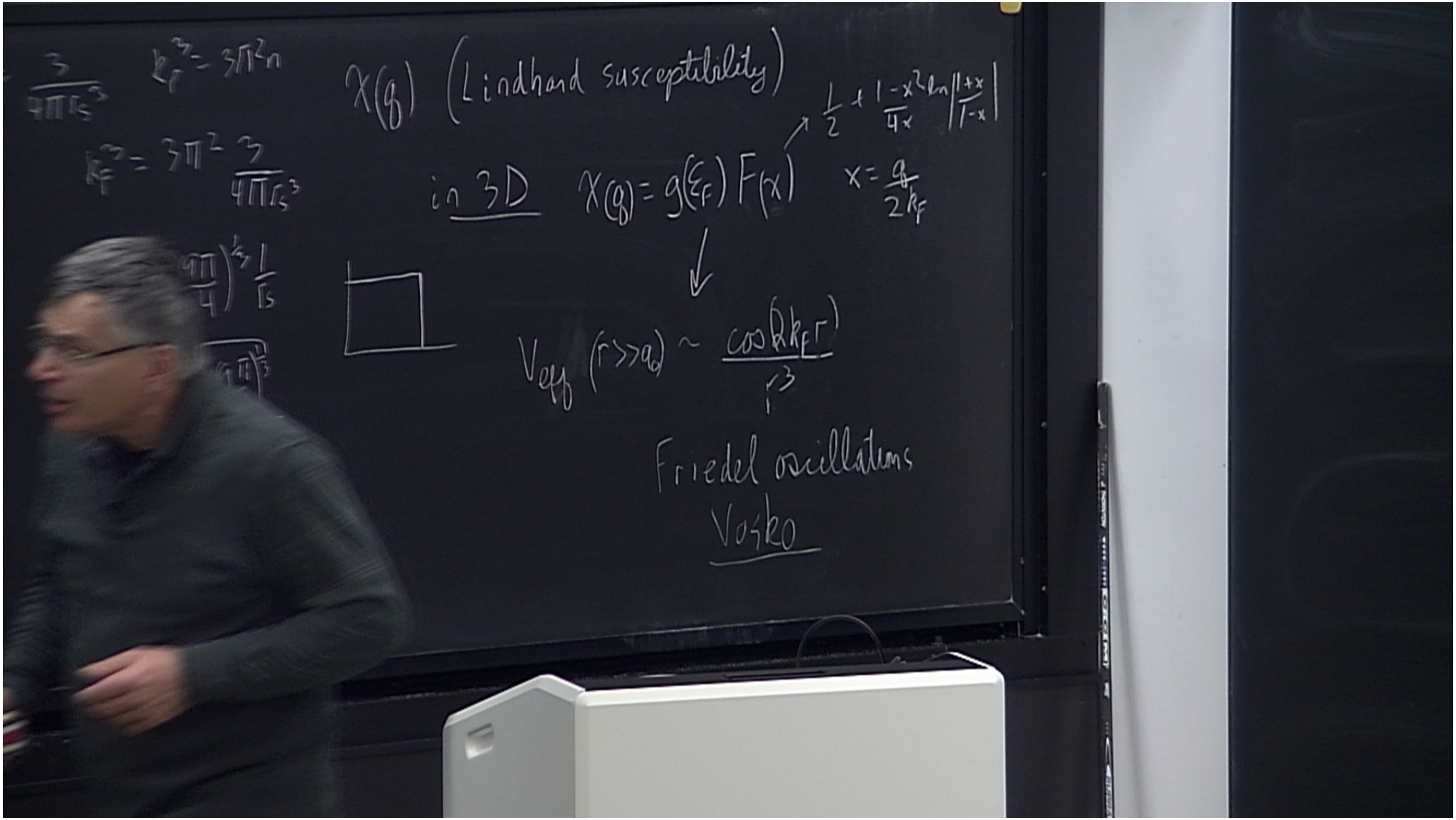
$$k_F^3 = 3\pi^2 \frac{3}{4\pi r_s^3}$$

$$q_{TF}^2 = \frac{4}{\pi} \frac{k_F}{a_0}$$

$$k_F = \left(\frac{9\pi}{4}\right)^{1/3} \frac{1}{r_s}$$

$$q_{TF}^2 a_0^2 = \frac{\frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{1/3}}{r_s/a_0}$$

$$q_{TF} a_0 = \frac{\sqrt{\frac{4}{\pi}\left(\frac{9\pi}{4}\right)^{1/3}}}{\sqrt{\frac{r_s}{a_0}}}$$


χ(q) (Lindhard susceptibility)
in 3D
Friedel oscillations
Vosko

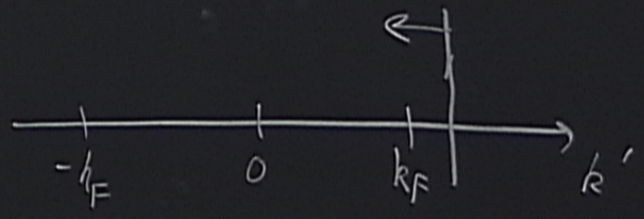
$k' = k + \frac{q}{2}$ $\quad k \to k' - \frac{q}{2}$

In 1D $\quad \chi(q) = -\frac{1}{N}\sum_{k'} \frac{f_{k'-\frac{q}{2}}\left(1 - f_{k'+\frac{q}{2}}\right)}{\frac{\hbar^2}{2m}\left[\left(k'-\frac{q}{2}\right)^2 - \left(k'+\frac{q}{2}\right)^2\right]}$ $\quad k+q \to k'+\frac{q}{2}$

$\left(k'-\frac{q}{2}\right)^2 < k_F^2$

$k' < k_F + \frac{q}{2}$

$= \frac{-2m}{-\hbar^2 2q}\, 2\pi \int_{-\pi}^{\pi} \frac{dk'}{k'}$



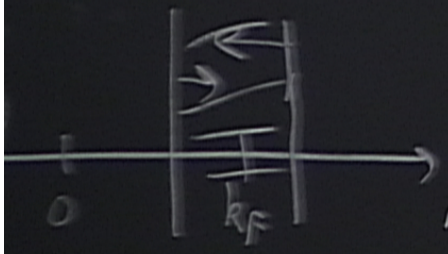
$k \to k' - \frac{q}{2}$

$k + q \to k' + \frac{q}{2}$

$$\left(k' - \frac{q}{2}\right)^2 < k_F^2 \qquad \left(k' + \frac{q}{2}\right)^2 > k_F^2$$

$$k' < k_F + \frac{q}{2} \qquad k' > k_F - \frac{q}{2}$$

$$k' > \frac{q}{2} - k_F \qquad k' < -k_F - \frac{q}{2}$$



In 1D

$$\chi(q) = -\frac{2}{N}\sum_{k'} \frac{f_{k'-\frac{q}{2}}\left(1 - f_{k'+\frac{q}{2}}\right)}{\frac{\hbar^2}{2m}\left[\left(k'-\frac{q}{2}\right)^2 - \left(k'+\frac{q}{2}\right)^2\right]}$$

$k' = k + \frac{q}{2}$ $\quad k \to k' - \frac{q}{2}$

$k + q \to k' + \frac{q}{2}$

$q > 0$

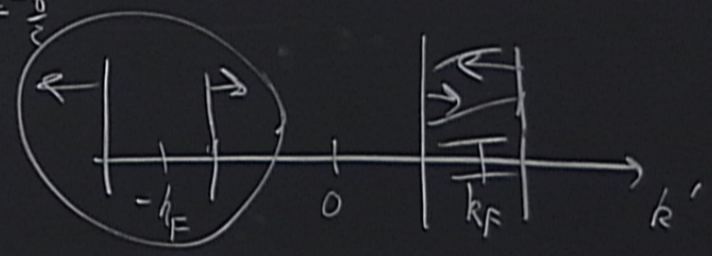
$$= -\frac{2m}{-\hbar^2 2q}\,\frac{2}{2\pi}\int_{k_F - \frac{q}{2}}^{k_F + \frac{q}{2}} \frac{dk'}{k'}$$

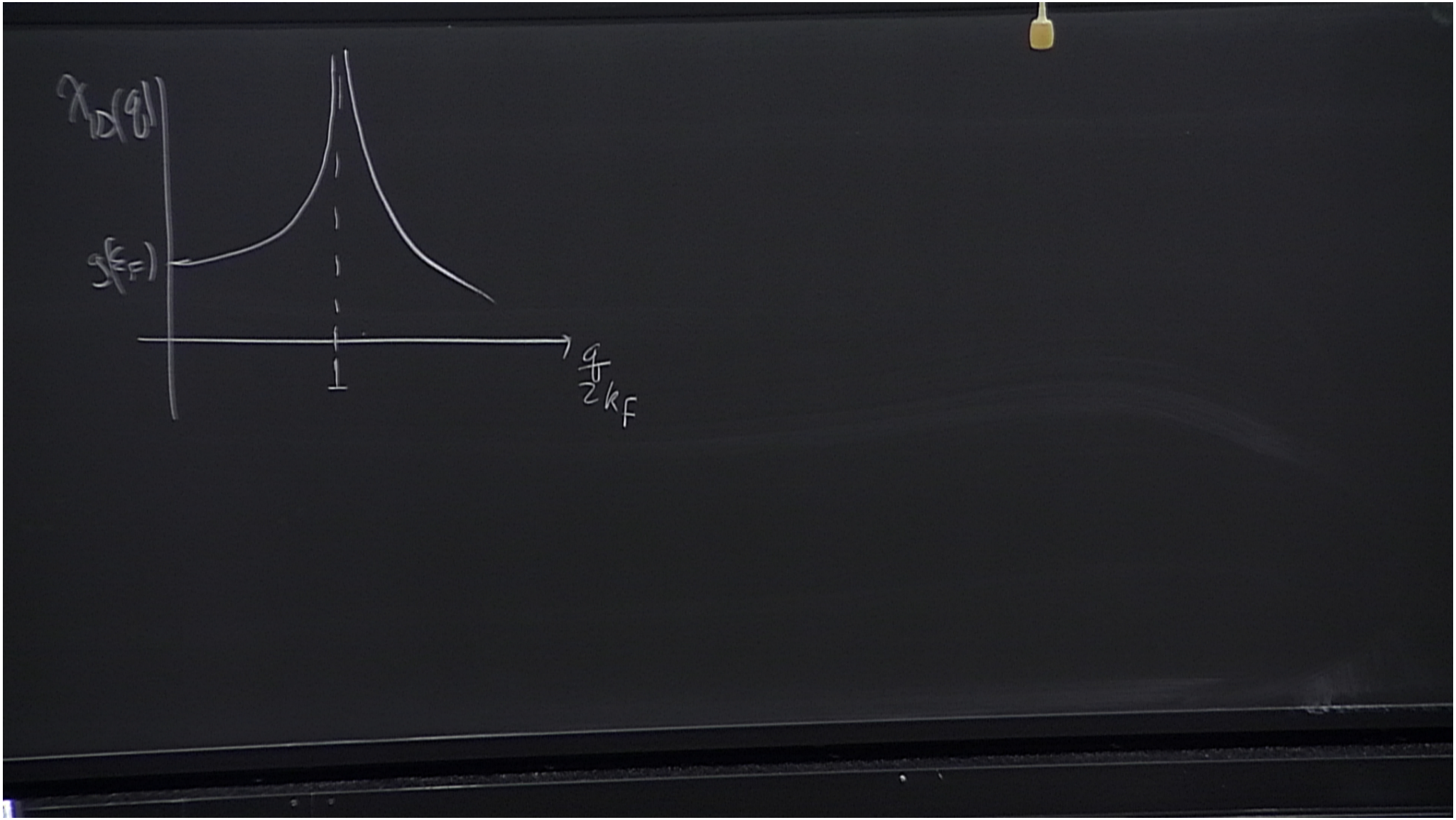
$$= \frac{m}{2\pi\hbar^2}\,\frac{1}{\frac{q}{2k_F}}\,\ln\left|\frac{1 + \frac{q}{2k_F}}{1 - \frac{q}{2k_F}}\right|$$

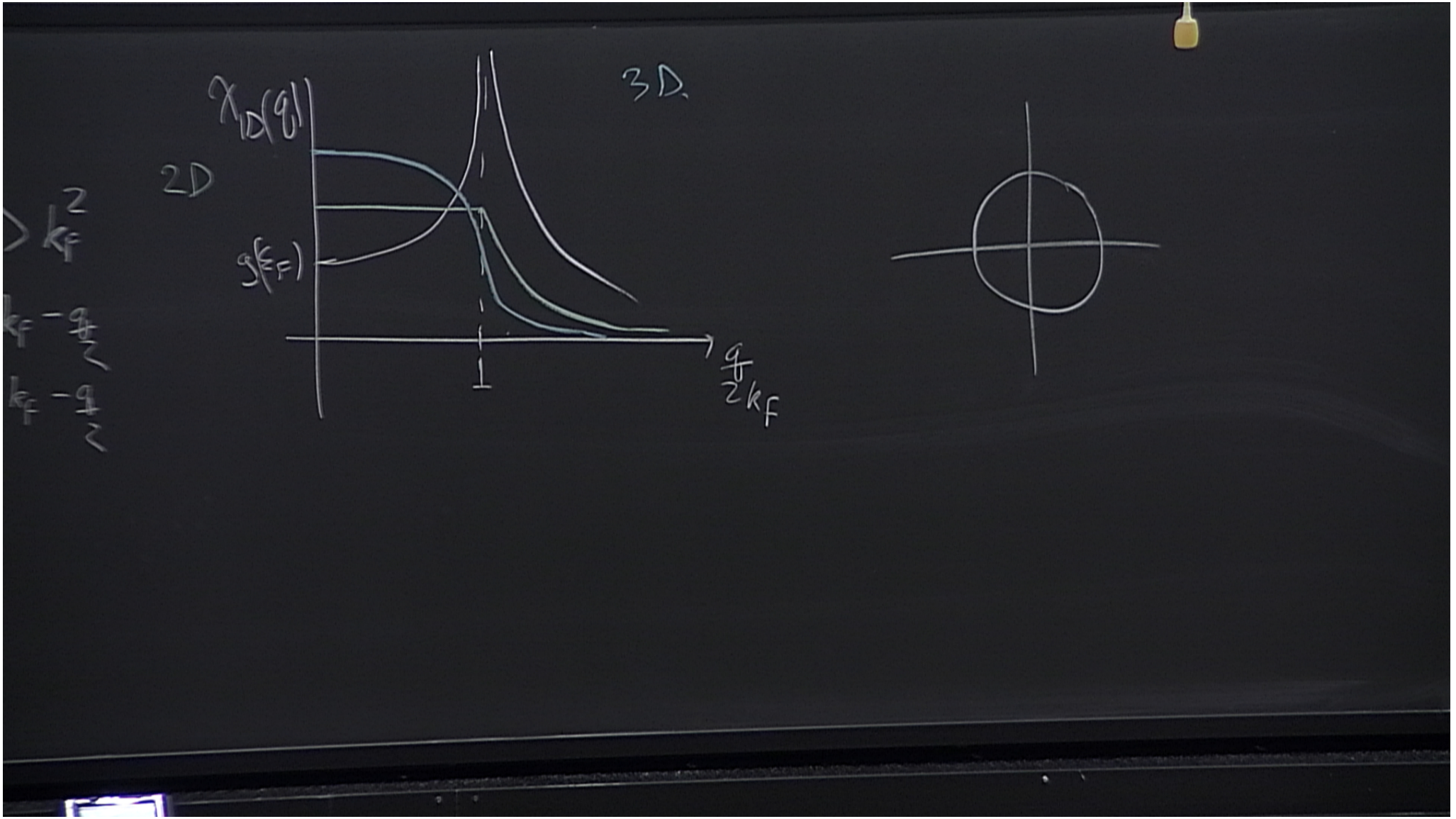
$\left(k' - \frac{q}{2}\right)^2 < k_F^2$

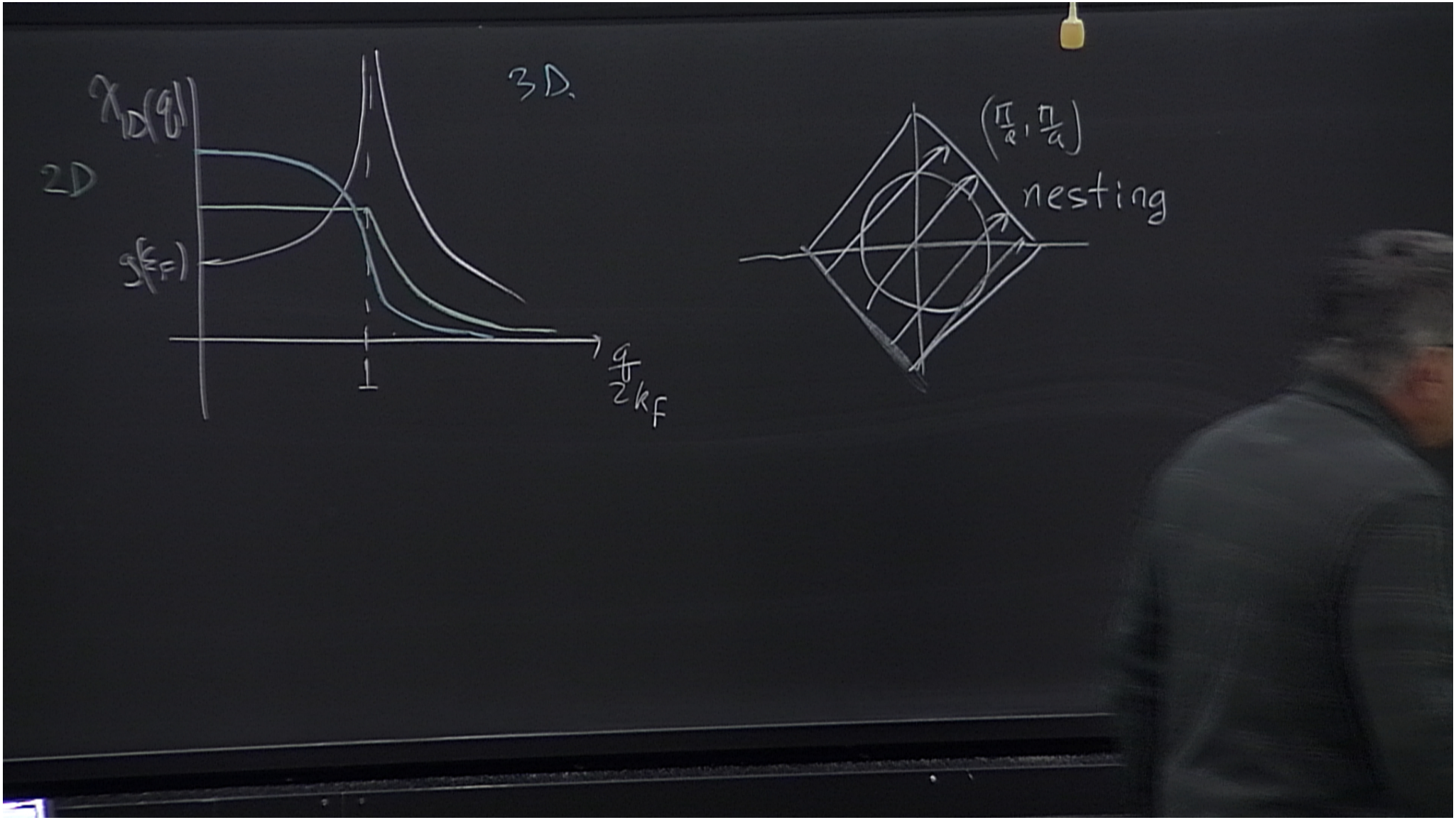
$k' < k_F + \frac{q}{2}$

$k' > \frac{q}{2} - k_F$




$\chi_{LD}(q)$
$g(\epsilon_F)$
1
$\frac{q}{2k_F}$


$\chi_{1D}(q)$
2D
3D.
$g(\epsilon_F)$
1
$\frac{q}{2k_F}$
k_F^2
$k_F - \frac{q}{2}$
$k_F - \frac{q}{2}$


$\chi_{1D}(q)$
2D
$g(\epsilon_F)$
1
$\frac{q}{2k_F}$
3D.
$(\frac{\pi}{a}, \frac{\pi}{a})$
nesting

$$M(\vec{H}) \equiv -\frac{1}{V}\frac{\partial E_0(H)}{\partial H} \quad (T=0)$$

$\vec{m} = -g\mu_B \vec{S}$

negative M is "bad" diamagnetism

positive M is "good" paramagnetism

moments ↓ ↑ $\vec{H}$ ↑

spin ↑ higher energy

↓ low energy soln

$(T=0)$ $\chi_{mag} = \frac{\partial M}{\partial H}$

Curie paramagnetism $\chi \sim \frac{1}{T}$

Pauli " $\chi \sim \frac{1}{T_F}$

is "bad" diamagnetism

is "good" paramagnetism

Magnetic interactions are fake!

dipole-dipole: $U = \frac{1}{r^3}\left(\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})\right)$

Magnetic interactions are fake!

dipole-dipole: $U = \frac{1}{r^3}\left(\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})\right)$

Real!

anti alignment $\uparrow\downarrow$ or $\downarrow\uparrow$ $-\frac{m^2}{r^3}$

ferro alignment $\rightarrow\rightarrow$ or $\leftarrow\leftarrow$ $-\frac{2m^2}{r^3}$

$U \sim 1K$ $T_c \sim 1000K$