

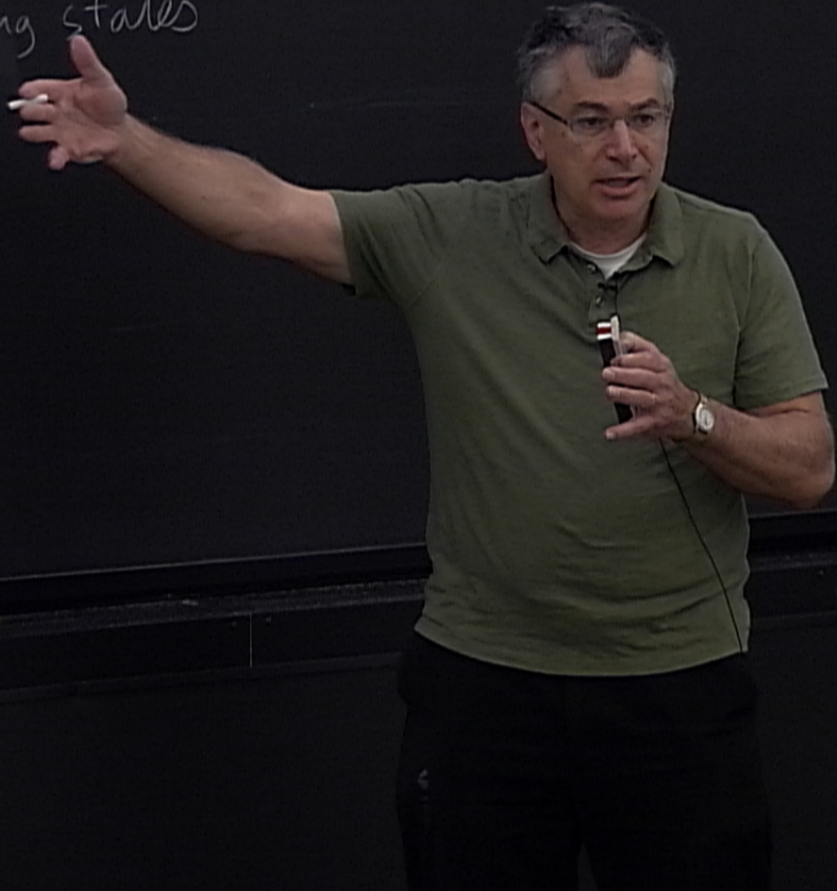
Title: PSI 2016/2017 Condensed Matter - Lecture 4

Date: Nov 10, 2016 10:45 AM

URL: <http://pirsa.org/16110055>

Abstract:

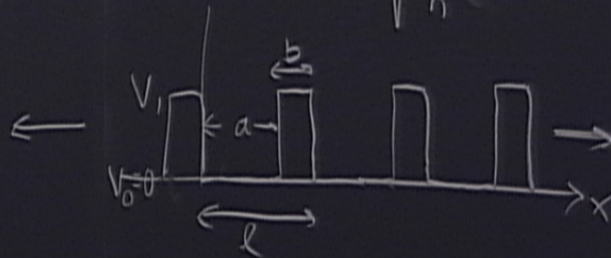
Rydberg states



$$\cos kl = \cosh K_1 b \cos k_0 a + \frac{K_1^2 - k_0^2}{2K_1 k_0} \sinh K_1 b \sin k_0 a \quad E < V_1$$

$$K_1 = \sqrt{\frac{2m}{\hbar^2} (V_1 - E)}$$

$$k_0 = \sqrt{\frac{2m}{\hbar^2} E} \quad (V_0 = 0)$$



Tight-binding

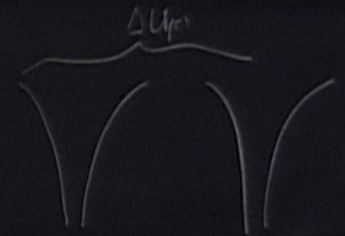
$$\psi_n(\vec{r})$$

Rydberg states

$$\hat{H} = H_{at} + \Delta U(\vec{r})$$

$$H_{at} \psi_n(\vec{r}) = E_n \psi_n(\vec{r})$$

$$\psi_k(\vec{r}) = \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \psi_n(\vec{r} - \vec{R})$$



Other extreme: tight-binding

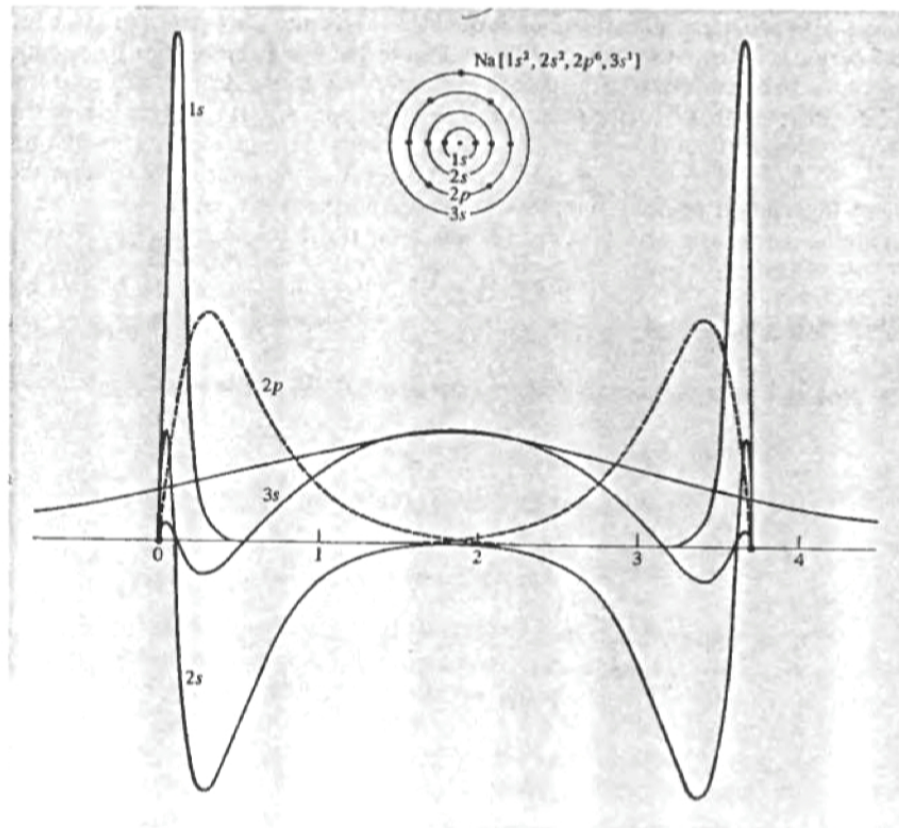


Figure 10.1

Calculated electron wave functions for the levels of atomic sodium, plotted about two nuclei separated by the nearest-neighbor distance in metallic sodium, 3.7 Å. The solid curves are $r\psi(r)$ for the 1s, 2s, and 3s levels. The dashed curve is r times the radial wave function for the 2p levels. Note how the 3s curves overlap extensively, the 2s and 2p curves overlap only a little, and the 1s curves have essentially no overlap. The curves are taken from calculations by D. R. Hartree and W. Hartree, *Proc. Roy. Soc. A*193, 299 (1948). The scale on the r -axis is in angstroms.

Fig. 10.1 from Ashcroft and Mermin

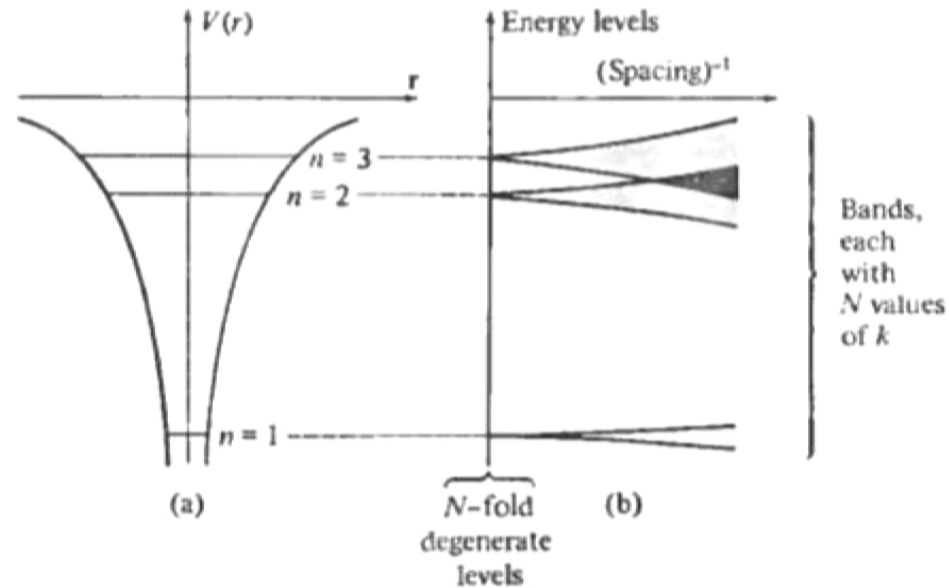


Figure 10.4

(a) Schematic representation of nondegenerate electronic levels in an atomic potential. (b) The energy levels for N such atoms in a periodic array, plotted as a function of mean inverse interatomic spacing. When the atoms are far apart (small overlap integrals) the levels are nearly degenerate, but when the atoms are closer together (larger overlap integrals), the levels broaden into bands.

Fig. 10.4 from Ashcroft and Mermin

The importance of basis states: an example using the hydrogen basis

Lindsay Forestell and Frank Marsiglio

Can. J. Phys. 93: 1009–1014 (2015) [dx.doi.org/10.1139/cjp-2014-0726](https://doi.org/10.1139/cjp-2014-0726)

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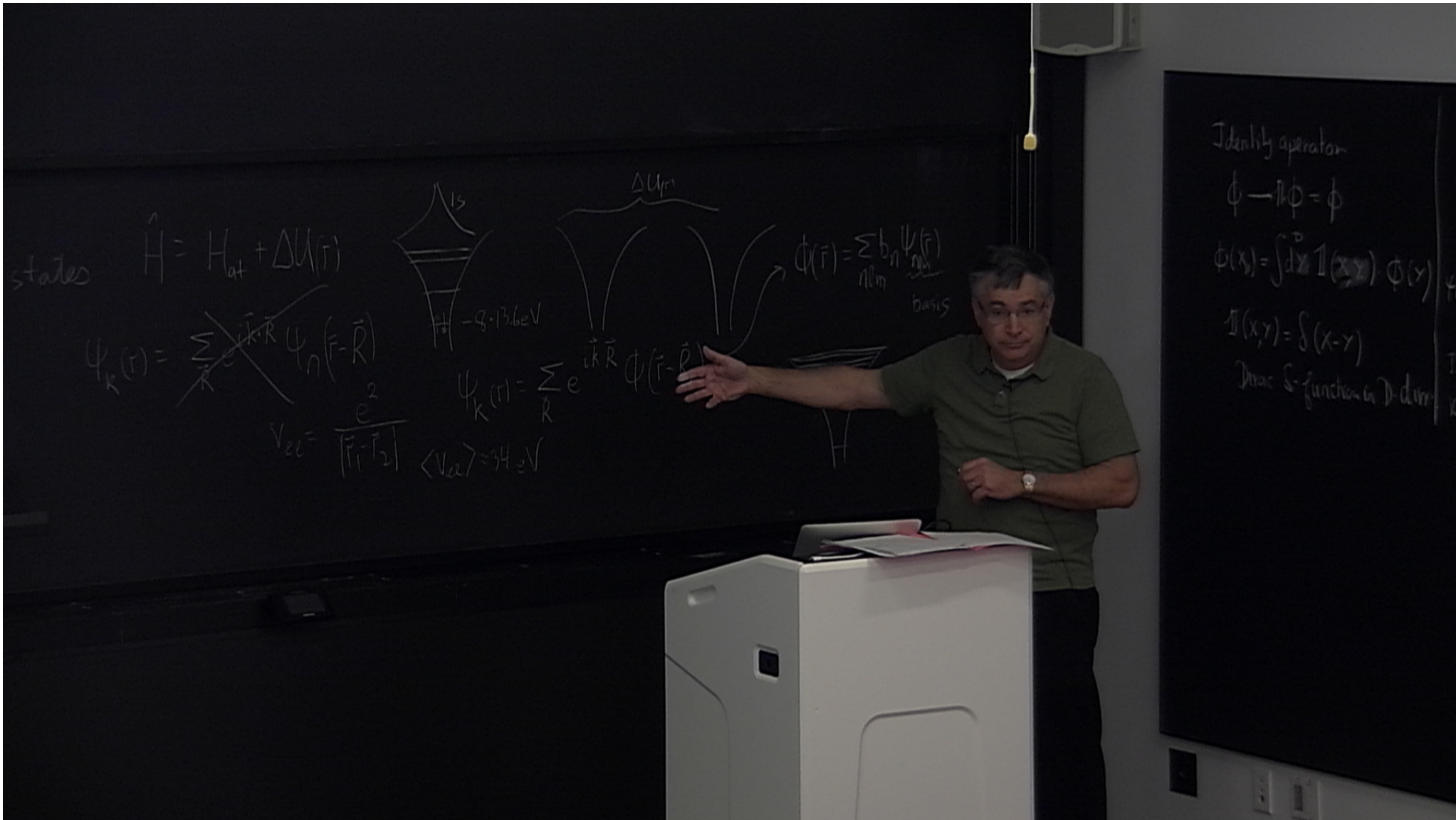
Eur. J. Phys. 34 (2013) 111–128

EUROPEAN JOURNAL OF PHYSICS

[doi:10.1088/0143-0807/34/1/111](https://doi.org/10.1088/0143-0807/34/1/111)

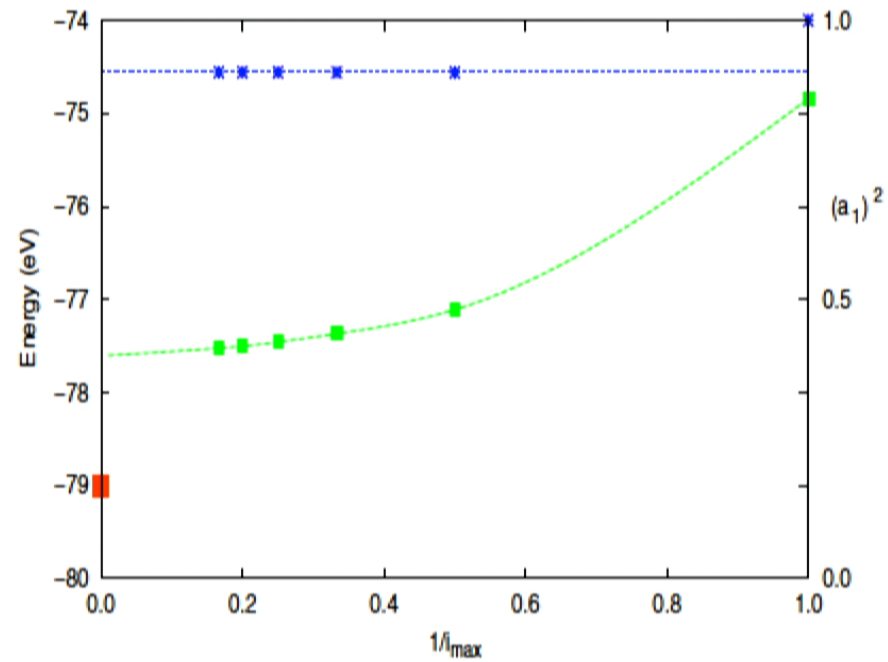
The spectral decomposition of the helium atom two-electron configuration in terms of hydrogenic orbitals

Joel Hutchinson¹, Marc Baker¹ and Frank Marsiglio^{1,2,3}



The spectral decomposition of the helium atom two-electron configuration in terms of hydrogenic orbitals

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$$\begin{aligned}
 H \Psi_k &= \sum_k \Psi_k \\
 H_{at} + \Delta U(r) & \int \phi_m^*(r) d^3r \left[H_{at} + \Delta U(r) \right] \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \phi_n(\vec{r}-\vec{R}) = \epsilon_k \Psi_k \\
 (E_m - \epsilon_k) \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \phi_n(\vec{r}-\vec{R}) + \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(\vec{r}-\vec{R}) &= 0 \\
 (E_m - \epsilon_k) b_m + (E_m - \epsilon_k) \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \phi_n(\vec{r}-\vec{R}) &= - \sum_n b_n \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(r) \\
 & \quad - \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(\vec{r}-\vec{R})
 \end{aligned}$$

$$H \psi_k = \epsilon_k \psi_k \quad (E_m - \epsilon_k) \sum_{\vec{R}}$$

$$\uparrow$$

$$H_{at} + \Delta U(r) \quad [H_{at} + \Delta U(r)] \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \phi_n(\vec{r} - \vec{R}) = \epsilon_k \psi_k \quad (E_m$$

$$\int \phi_m^*(r) d^3r$$

$$(E_m - E_k) \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \Phi_m^*(r) \Phi_n(\vec{r} - \vec{R}) + \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_n(\vec{r} - \vec{R}) = 0$$

$$\Phi_n(\vec{r} - \vec{R}) = \sum_k \psi_k$$

$$(E_m - E_k) b_m + (E_m - E_k) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \Phi_m^*(r) \Phi_n(\vec{r} - \vec{R}) = - \sum_n b_n \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_n(r) \quad \equiv -\beta_{mn}$$

$$(E_k - E_m) b_m = (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_{mn}(\vec{R}) b_n - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_n(\vec{r} - \vec{R}) \quad \equiv -\gamma_{mn}(\vec{R})$$

$$(E_m - E_k) \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Phi_{\eta}(\vec{r} - \vec{R}) + \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r} - \vec{R}) = 0$$

$$\Phi_n(\vec{r} - \vec{R}) = \sum_k \psi_k$$

$$(E_m - E_k) b_m + (E_m - E_k) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Phi_{\eta}(\vec{r} - \vec{R}) = - \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r})$$

$$(E_k - E_m) b_m = (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_{m\eta}(\vec{R}) b_{\eta} - \sum_{\eta} S_{m\eta} b_{\eta} - \sum_{\vec{R} \neq 0} \sum_{\eta} e^{i\vec{k} \cdot \vec{R}} \gamma_{m\eta}(\vec{R}) b_{\eta} - \gamma_{m\eta}(\vec{R})$$

$$(E_m - E_k) \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Phi_{\eta}(\vec{r} - \vec{R}) + \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r} - \vec{R}) = 0$$

$$\Phi_n(\vec{r} - \vec{R}) = \sum_{\eta} b_{\eta} \Phi_{\eta}(\vec{r} - \vec{R})$$

$$(E_m - E_k) b_m + (E_m - E_k) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Phi_{\eta}(\vec{r} - \vec{R}) = - \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r})$$

$$(E_k - E_m) b_m = - (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Phi_{\eta}(\vec{r} - \vec{R}) - \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r})$$

$$(E_k - E_m) b_m = - (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_{m\eta}(\vec{R}) b_{\eta} - \sum_{\eta} b_{\eta} \int d^3r \Phi_m^*(r) \Delta U(r) \Phi_{\eta}(\vec{r})$$

$$H \Psi_k = \epsilon_k \Psi_k$$

$$H_{at} + \Delta U(r) \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \phi_n(\vec{r}-\vec{R}) = \epsilon_k \Psi_k$$

$$\int \phi_m^*(r) d^3r$$

$$(E_m - \epsilon_k) \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \phi_n(\vec{r}-\vec{R}) + \frac{1}{\Omega} \sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(\vec{r}-\vec{R}) = 0$$

$$(E_m - \epsilon_k) b_m + (E_m - \epsilon_k) \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(r) \phi_n(\vec{r}-\vec{R}) = - \sum_n b_n \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(r)$$

$$(E_k - E_m) b_m = - (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} S_{mn}(\vec{R}) b_n - \sum_n S_{mn} b_n - \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} \int d^3r \phi_m^*(r) \Delta U(r) \phi_n(\vec{r}-\vec{R}) b_n = -Y(\vec{R})$$

$$\Psi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi}} \left(\frac{z}{a_0}\right)^2 e^{-2z/a_0} \quad \text{just consider the } 1s \text{ states}$$

but also in \sum_n , consider only 1s

$$S(\vec{R}) = \int d^3r \Psi_{100}^*(\vec{r}) \Psi_{100}(\vec{r}-\vec{R})$$

$$(E_k - E_{1s}) b_{1s} = - (E_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k}\cdot\vec{R}} b_{1s} - B b_{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} Y(\vec{R}) b_{1s}$$

$$E_k = E_{1s} - \frac{B + \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} Y(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k}\cdot\vec{R}}}$$

$$H_{at} + \Delta U(\vec{r}) \left[\sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \phi_n(\vec{r}-\vec{R}) \right] = \epsilon_k \psi_k$$

$$\int \phi_m^*(\vec{r}) d^3r$$

$$(E_m - \epsilon_k) b_m + (E_m - \epsilon_k) \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} \sum_n b_n \int d^3r \phi_m^*(\vec{r}) \phi_n(\vec{r}-\vec{R}) = - \sum_n b_n \int d^3r \phi_m^*(\vec{r}) \Delta U(\vec{r}) \phi_n$$

$$(E_k - E_m) b_m = - (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} S_{m\vec{R}}(\vec{R}) b_n - \sum_n S_{mn} b_n - \sum_{\vec{R} \neq 0} \sum_n e^{i\vec{k}\cdot\vec{R}} \int d^3r \phi_m^*(\vec{r}) \Delta U(\vec{r}) \phi_n$$

$$\psi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$

just consider the 1s states
 $m = 1s$
 but also in \sum_n , consider only 1s

$$(E_k - E_{1s}) b_{1s} = - (E_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k}\cdot\vec{R}} b_{1s} - B b_{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} Y(\vec{R}) b_{1s}$$

$$E_k = E_{1s} - \frac{B + \sum_{\vec{R} \neq 0} e^{i\vec{k}\cdot\vec{R}} Y(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k}\cdot\vec{R}}}$$

include only nearest neighbours

$$S(\vec{R}) = \int d^3r \psi_{100}^*(\vec{r}) \psi_{100}(\vec{r}-\vec{R})$$

$$S(R_{nn}) = e^{-ZR/a_0} \left(1 + \frac{ZR}{a_0} + \frac{1}{3} \left(\frac{ZR}{a_0} \right)^2 \right) \sim 10^{-30}$$

$Z = 11$
 $R = 3.7 \text{ \AA}$
 $a_0 = 0.529 \text{ \AA}$

$$\Psi_n(\vec{r}-\vec{R}) = \sum_n \psi_n(\vec{r}-\vec{R})$$

$$(E_m - E_k) b_m + (E_m - E_k) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \psi_m^*(\vec{r}) \psi_n(\vec{r}-\vec{R}) = - \sum_n b_n \int d^3r \psi_m^*(\vec{r}) \Delta U(\vec{r}) \psi_n(\vec{r})$$

$$= - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_n b_n \int d^3r \psi_m^*(\vec{r}) \Delta U(\vec{r}) \psi_n(\vec{r}-\vec{R})$$

$$(E_k - E_m) b_m = - (E_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_{mn}(\vec{R}) b_n - \sum_n \beta_{mn} b_n - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \gamma_{mn}(\vec{R}) b_n - \gamma_{mn}(\vec{R})$$

states

$$(E_k - E_{1s}) b_{1s} = - (E_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}} \gamma_{1s} - \beta b_{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \gamma(\vec{R}) b_{1s}$$

only 1s

$$\psi_{100}(\vec{r}-\vec{R})$$

$$1 + \frac{ZR}{a_0} + \frac{1}{3} \left(\frac{ZR}{a_0} \right)^2 \sim 10^{-30}$$

$$E_k = E_{1s} - \frac{\beta + \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \gamma(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}}}$$

include only nearest neighbours

$$E_k = E_{1s} + \sum_{nn} e^{i\vec{k} \cdot \vec{R}} \gamma_{nn}(\vec{R})$$



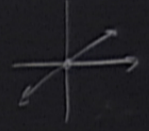
Right

$$(\epsilon_k - E_m) b_m = -(\epsilon_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_m(\vec{R}) b_m - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\vec{R}'} b_{\vec{R}'} \left[\int \psi_m^*(\vec{r}) \Delta U(\vec{r}) \psi_{\vec{R}'}(\vec{r}-\vec{R}) \right]$$

$$= -\sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\vec{R}'} b_{\vec{R}'} \left[\int \psi_m^*(\vec{r}) \Delta U(\vec{r}) \psi_{\vec{R}'}(\vec{r}-\vec{R}) \right] - \gamma_{m\vec{R}}(\vec{k}) b_{\vec{R}}$$

atoms

$$(\epsilon_k - E_{1s}) b_{1s} = -(\epsilon_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}} \frac{b_{1s}}{1s} - B \frac{b_{1s}}{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R}) \frac{b_{1s}}{1s}$$



include only nearest neighbors

$$\epsilon_k = \frac{E_{1s} - B + \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}}}$$

$$\epsilon_k = E_{1s} - \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} Y_{m\vec{R}}(\vec{R})$$

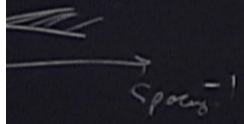
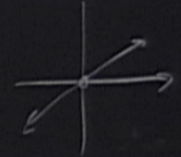
$\left. \begin{matrix} E_{1s} \\ -B \end{matrix} \right\}$



$\sim 10^{-50}$

$$(\epsilon_k - E_m) b_m = -(\epsilon_k - E_m) \sum_{\vec{R} \neq 0} \sum_n e^{i\vec{k} \cdot \vec{R}} S_{mn}(\vec{R}) b_n - \sum_n B_{mn} b_n - \sum_{\vec{R} \neq 0} \sum_n e^{i\vec{k} \cdot \vec{R}} Y_{mn}(\vec{R}) b_n - Y_{mn}(\vec{R})$$

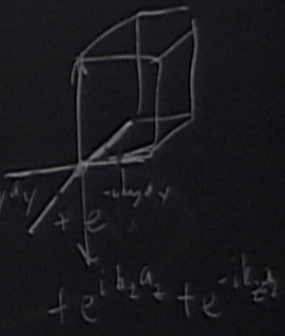
$$(\epsilon_k - E_{1s}) b_{1s} = -(\epsilon_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}} b_{1s} - B b_{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R}) b_{1s}$$



$$\epsilon_k = \frac{E_{1s} - B + \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}}}$$

include only nearest neighbors

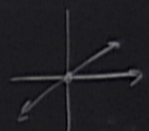
$$\epsilon_k = \frac{E_0 - \sum_{mn} e^{i\vec{k} \cdot \vec{R}} Y_{mn}(\vec{R})}{E_{1s} - B}$$



$\sim 10^{-30}$

$$(\epsilon_k - E_m) b_m = -(\epsilon_k - E_m) \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} S_m(\vec{R}) b_m - \sum_{\vec{R} \neq 0} \sum_{\vec{R}' \neq 0} e^{i\vec{k} \cdot \vec{R}} \sum_{\vec{R}'' \neq 0} b_{\vec{R}''} \left(\sum_{\vec{R}'} \Delta U(\vec{R}') \Delta U(\vec{R}' - \vec{R}) \right) - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} \gamma_m(\vec{R}) b_m - \gamma_m(\vec{R})$$

$$(\epsilon_k - E_{1s}) b_{1s} = -(\epsilon_k - E_{1s}) \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}} \gamma_{1s} - B b_{1s} - \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R}) b_{1s}$$



→ $S_{\text{spins}} = 1$

$$\epsilon_k = \frac{E_{1s} - B + \sum_{\vec{R} \neq 0} e^{i\vec{k} \cdot \vec{R}} Y(\vec{R})}{1 + \sum_{\vec{R} \neq 0} S(\vec{R}) e^{i\vec{k} \cdot \vec{R}}}$$

include only nearest neighbors

$$\epsilon_k = \epsilon_0 - \sum_{\vec{R}} e^{i\vec{k} \cdot \vec{R}} \gamma_m(\vec{R})$$

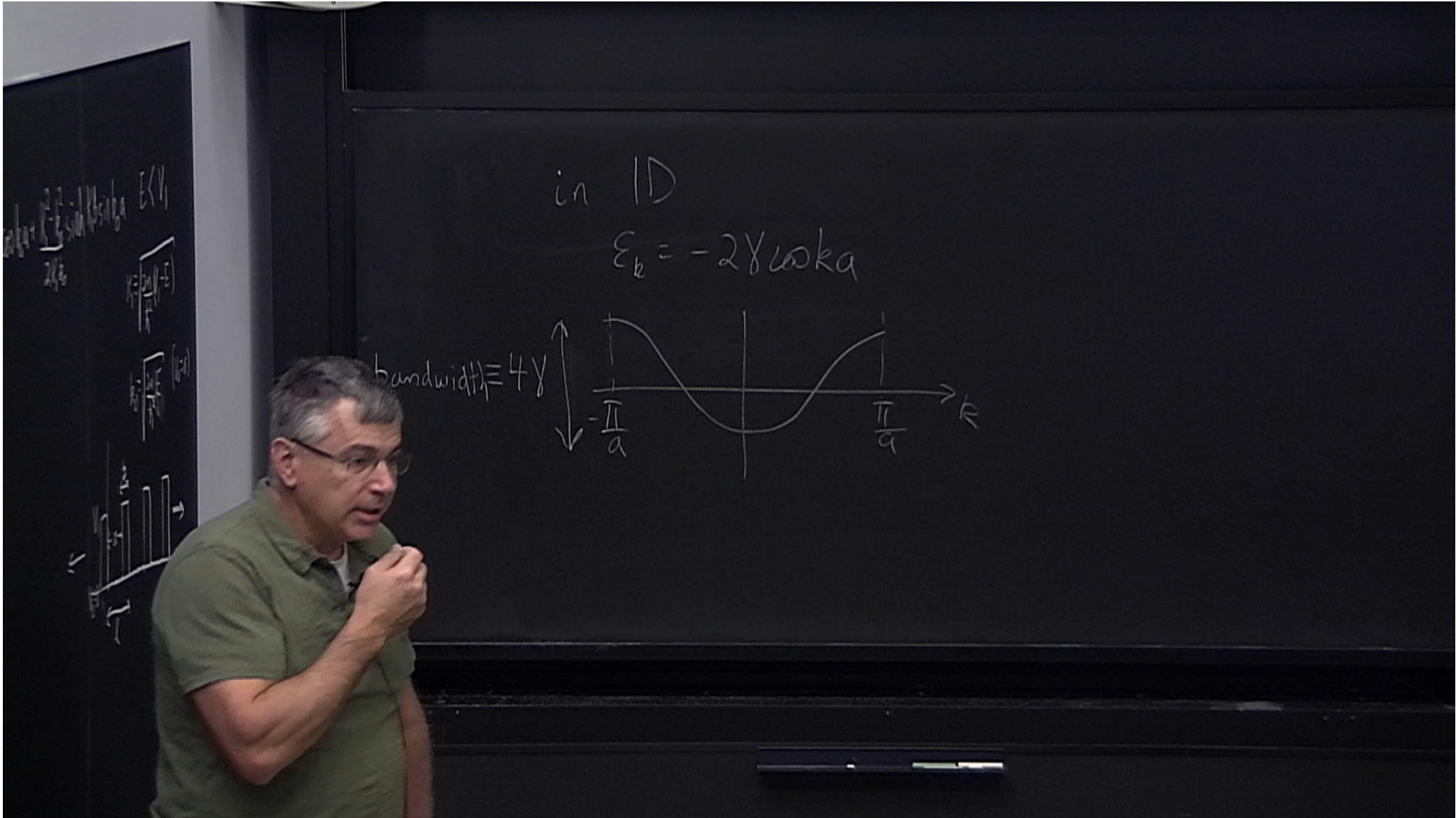
$$E_{1s} = B$$



$$\epsilon_k = \epsilon_0 - 2\gamma (\cos k_x a + \cos k_y a + \cos k_z a) + e^{i b_2 a} + e^{-i b_2 a}$$

$\sim 10^{-30}$

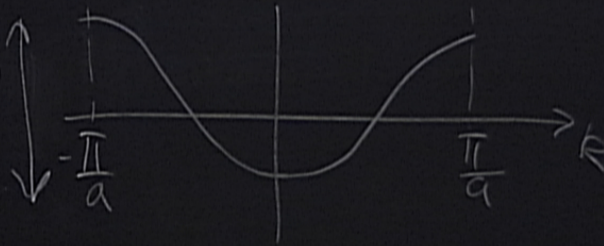
Tight

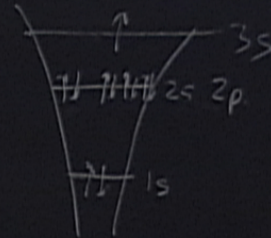
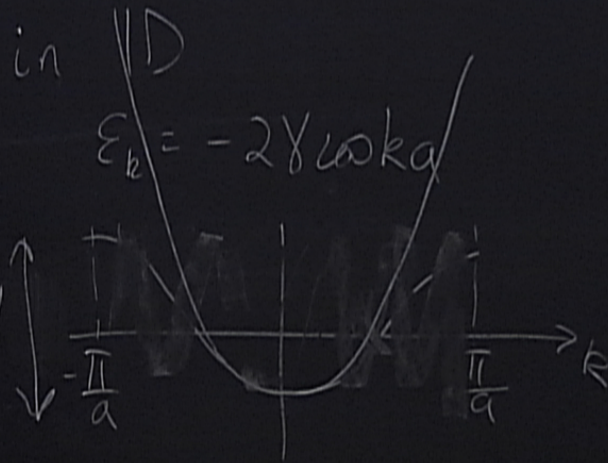


in 1D

$$E_b = -2\gamma u_0 k a$$

bandwidth = 4γ

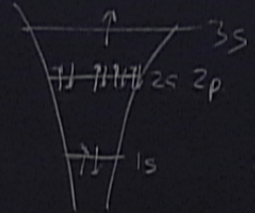
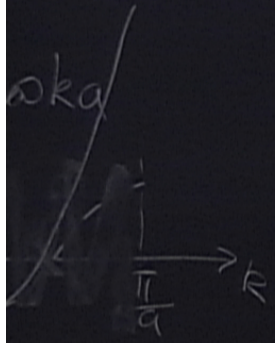




$$E_k = -2\gamma \cos ka \sim -2\gamma \left(1 - \frac{1}{2}(ka)^2 + \dots\right)$$

$$= -2\gamma + \gamma a^2 k^2$$

$$= \epsilon_0 + \frac{\hbar^2 k^2}{2m_{\text{band}}}$$



$$m_{\text{band}}^{(k=0)} = \frac{\hbar^2}{2\gamma a^2}$$

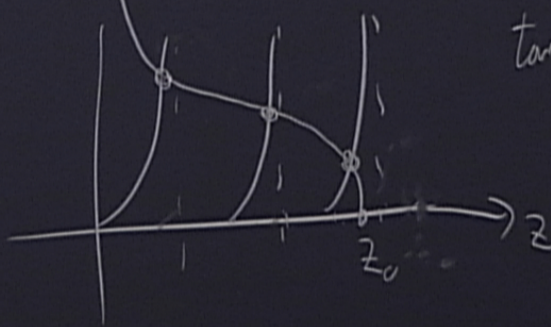
$$\begin{aligned} \epsilon_n &= -2\gamma \cos ka \sim -2\gamma \left(1 - \frac{1}{2}(ka)^2 + \dots\right) \\ &= -2\gamma + \gamma a^2 k^2 \\ &= \epsilon_0 + \frac{\hbar^2 k^2}{2m_{\text{band}}^{(k=0)}} \end{aligned}$$

$$\cos kl = \underbrace{\cosh K_1 b}_{\frac{e^{+K_1 b}}{2}} \cos k_0 a + \frac{K_1^2 - k_0^2}{2K_1 k_0} \sinh K_1 b \sin k_0 a \quad E < V_1$$

$$K_1 = \sqrt{\frac{2m}{\hbar^2}(V_1 - E)}$$

$$k_0 = \sqrt{\frac{2m}{\hbar^2}E} \quad (V_0 = 0)$$

$$2e^{-K_1 b} \cos kl = \left(\cos \frac{k_0 a}{2} - \frac{k_0}{K_1} \sin \frac{k_0 a}{2} \right) \left(\cos \frac{k_0 a}{2} + \frac{K_1}{k_0} \sin \frac{k_0 a}{2} \right)$$

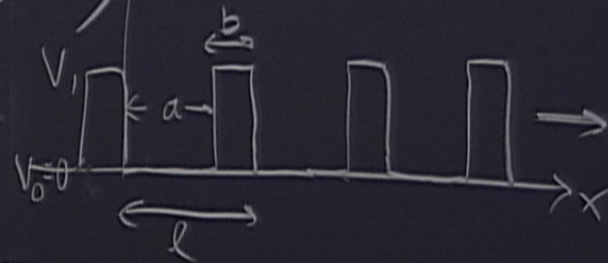


$$\tan \frac{k_0 a}{2} = \frac{K_1}{k_0}$$

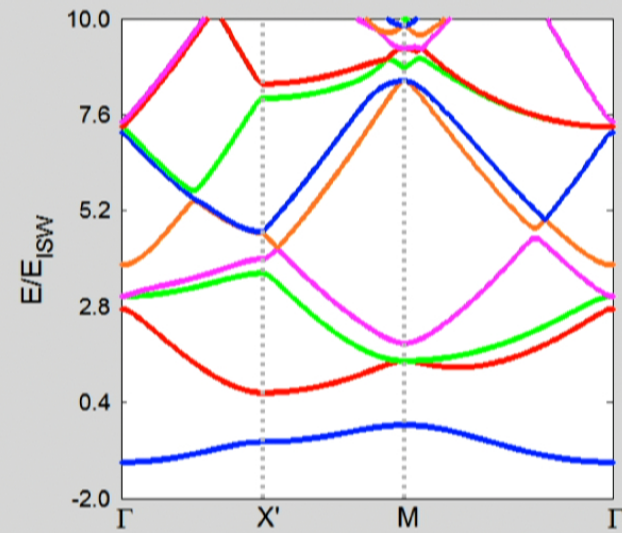
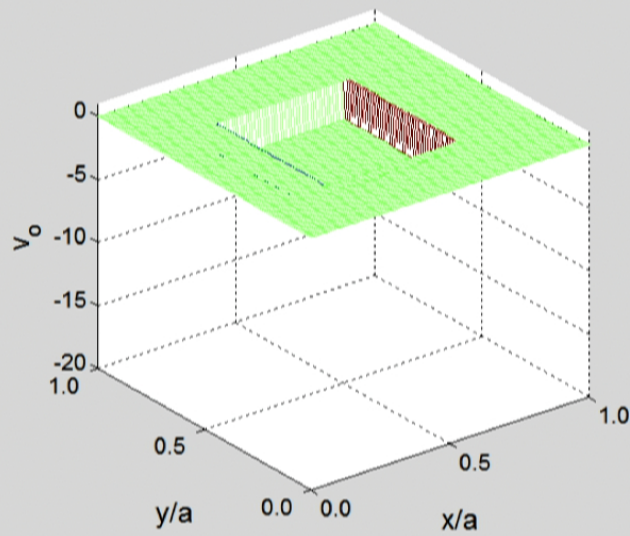
$$z = \frac{b_0 a}{2}$$

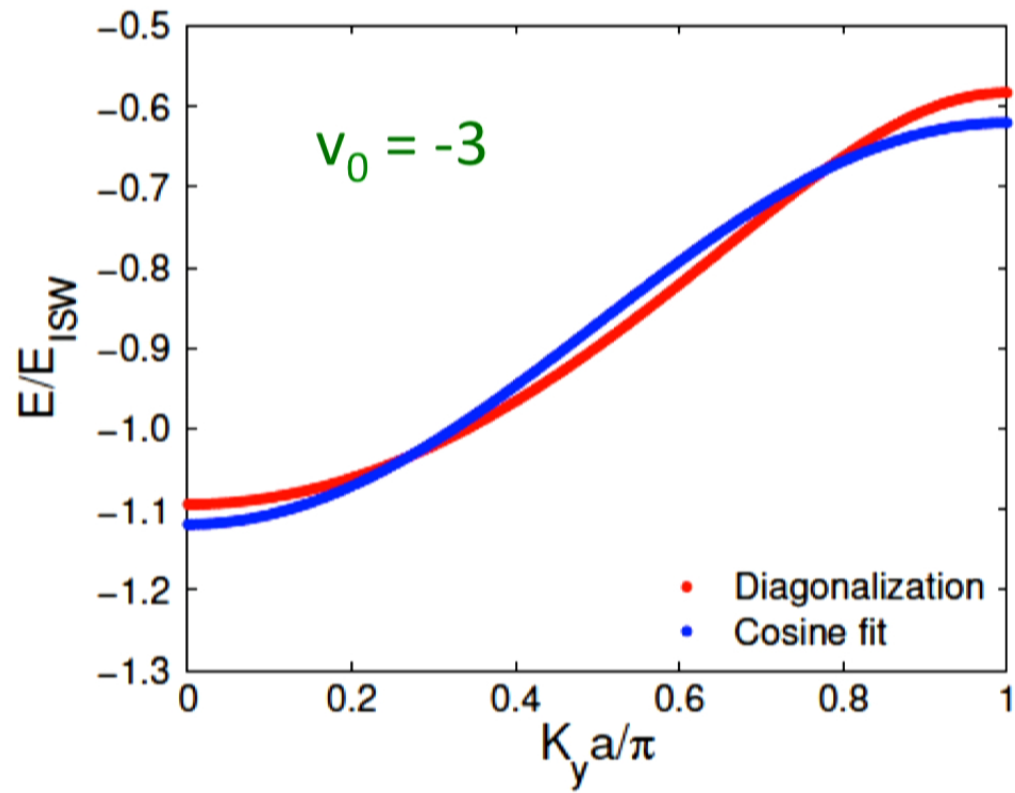
$$\tan z_1 = \sqrt{\frac{z_0}{z_1} - 1}$$

$$z = z_1 + \delta$$



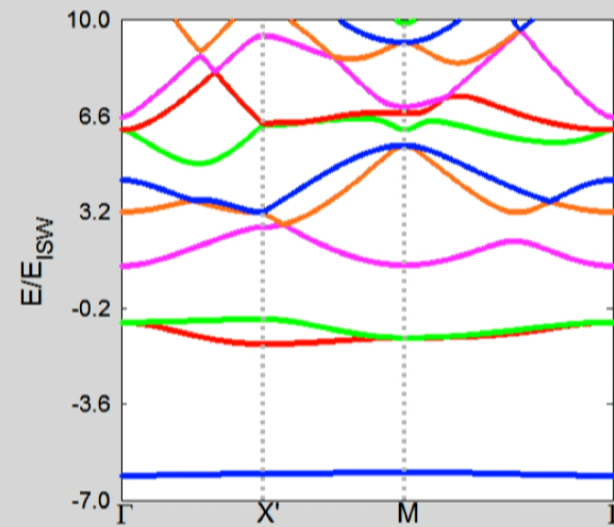
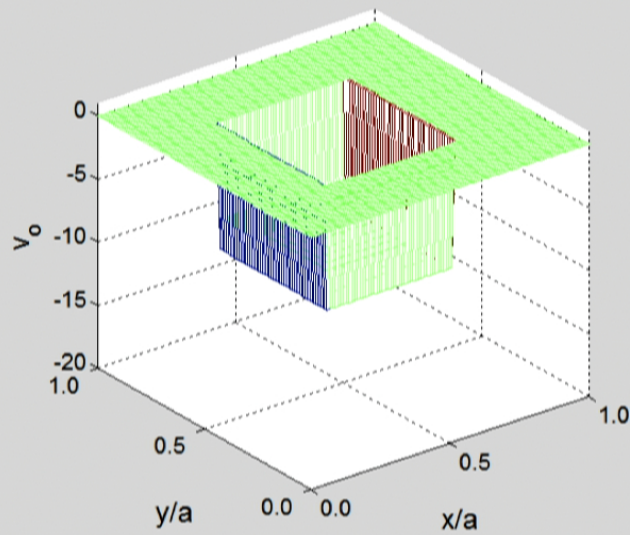
Tight-binding





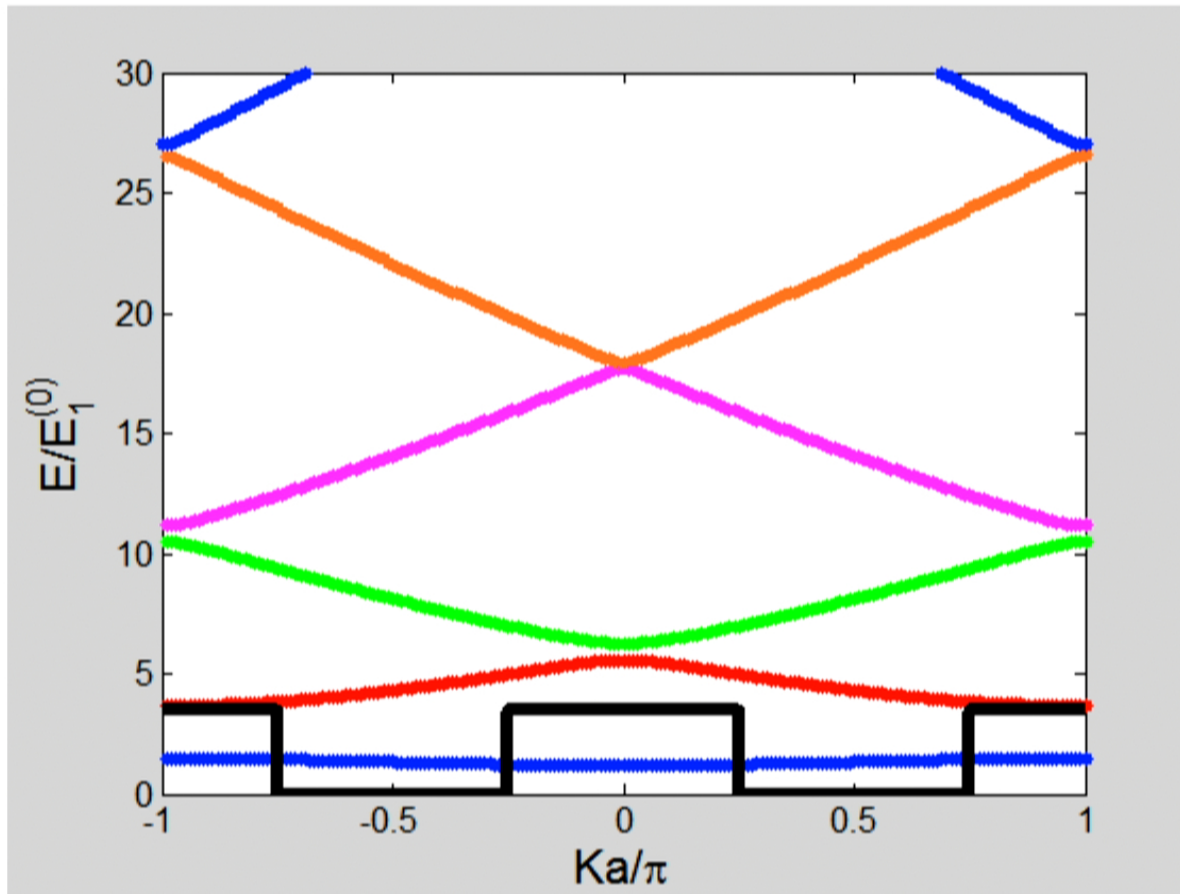
(b) $v_0 = -3, R^2 = 0.9854$

Tight-binding



Stick to 1D

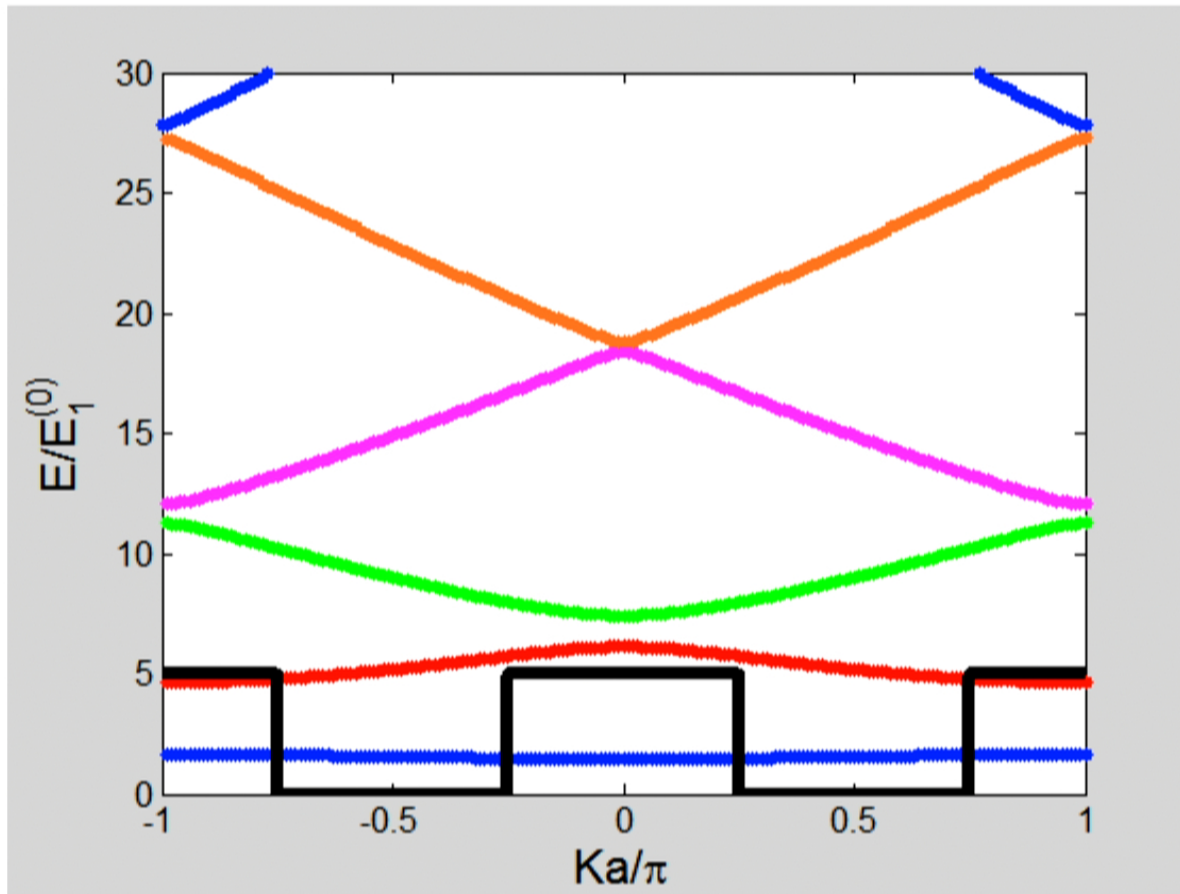
Change the barrier height



Pavelich MSc Thesis

Stick to 1D

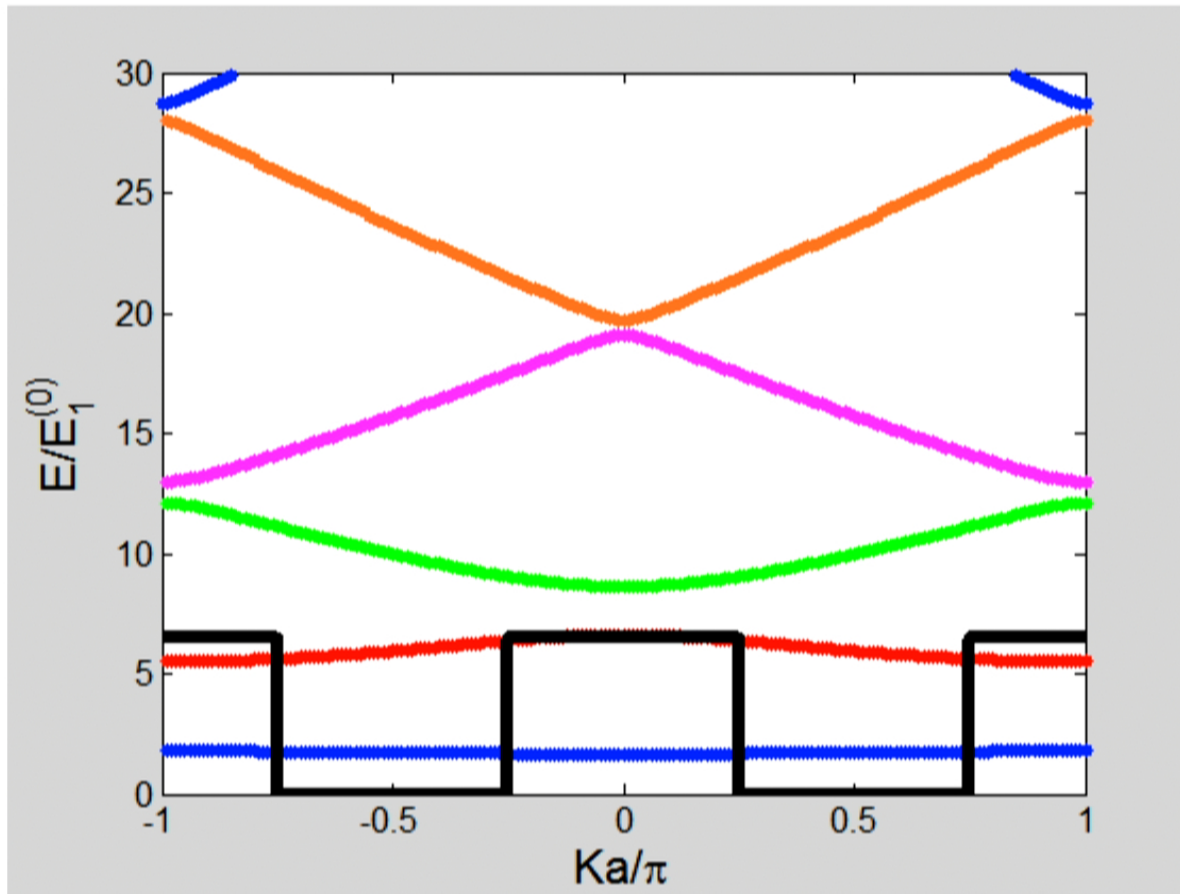
Change the barrier height



Pavelich MSc Thesis

Stick to 1D

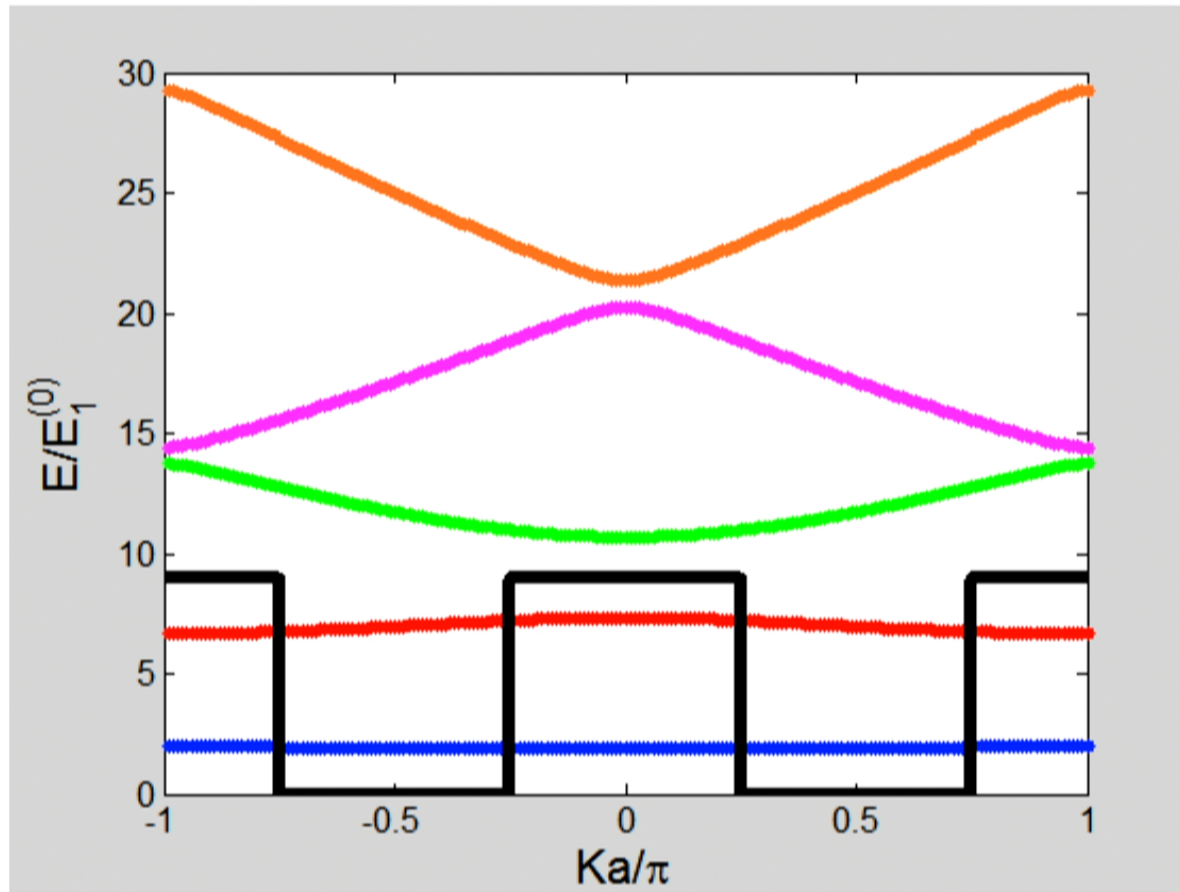
Change the barrier height



Pavelich MSc Thesis

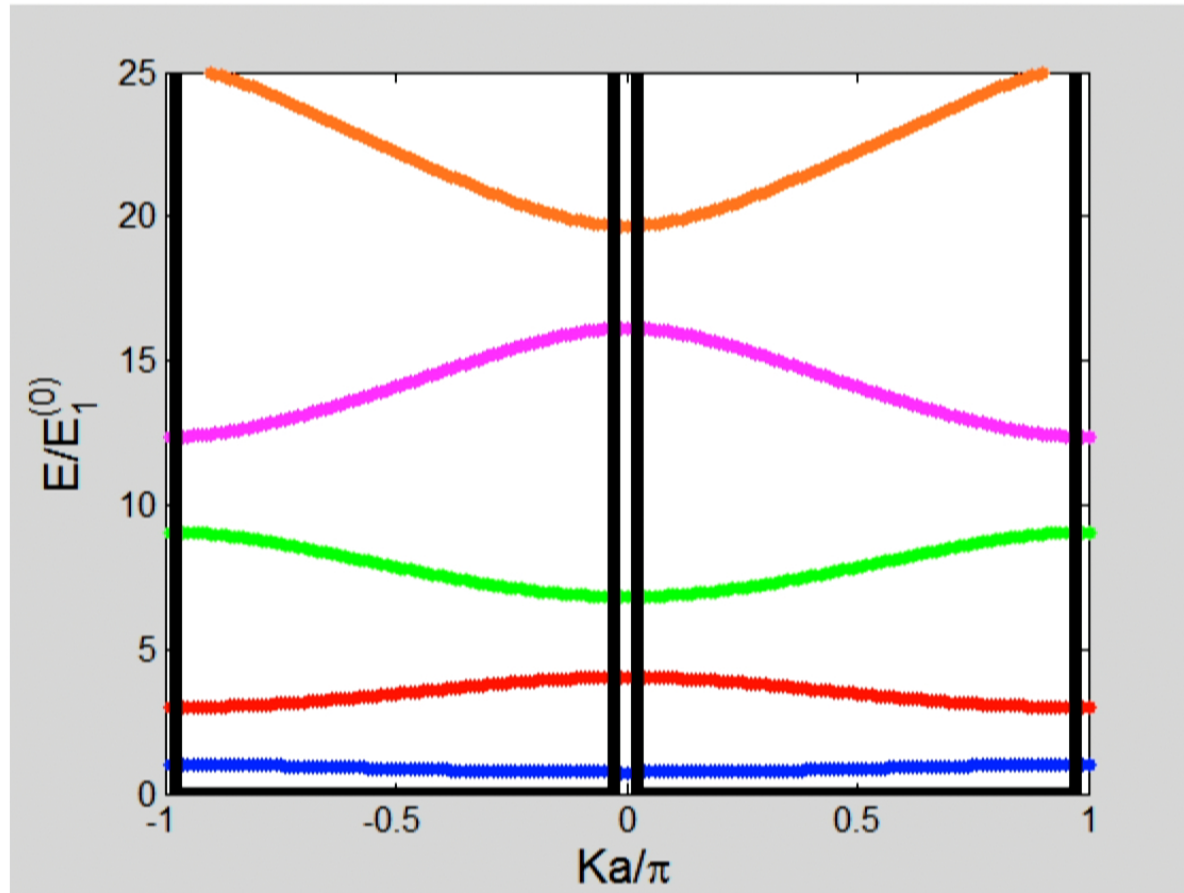
Stick to 1D

Change the barrier height



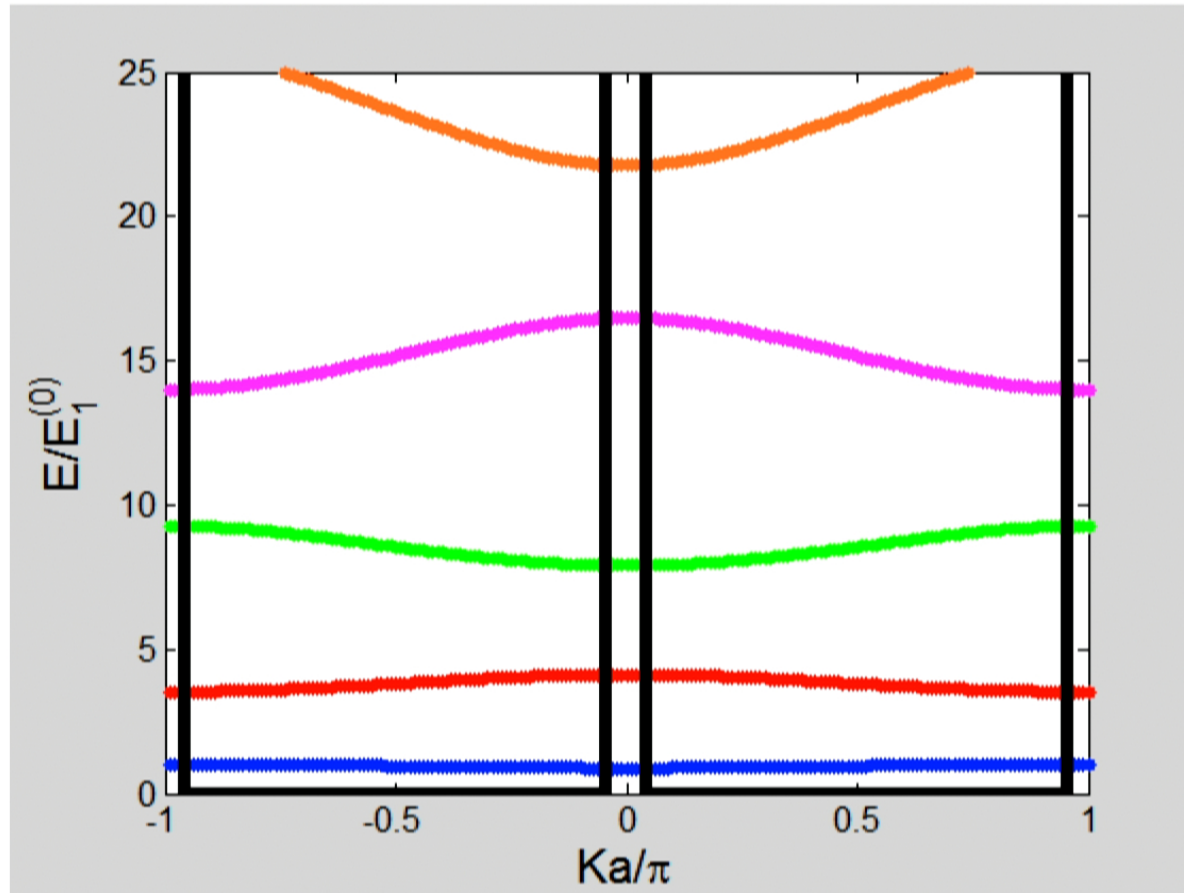
Pavelich MSc Thesis

Stick to 1D Change the barrier width



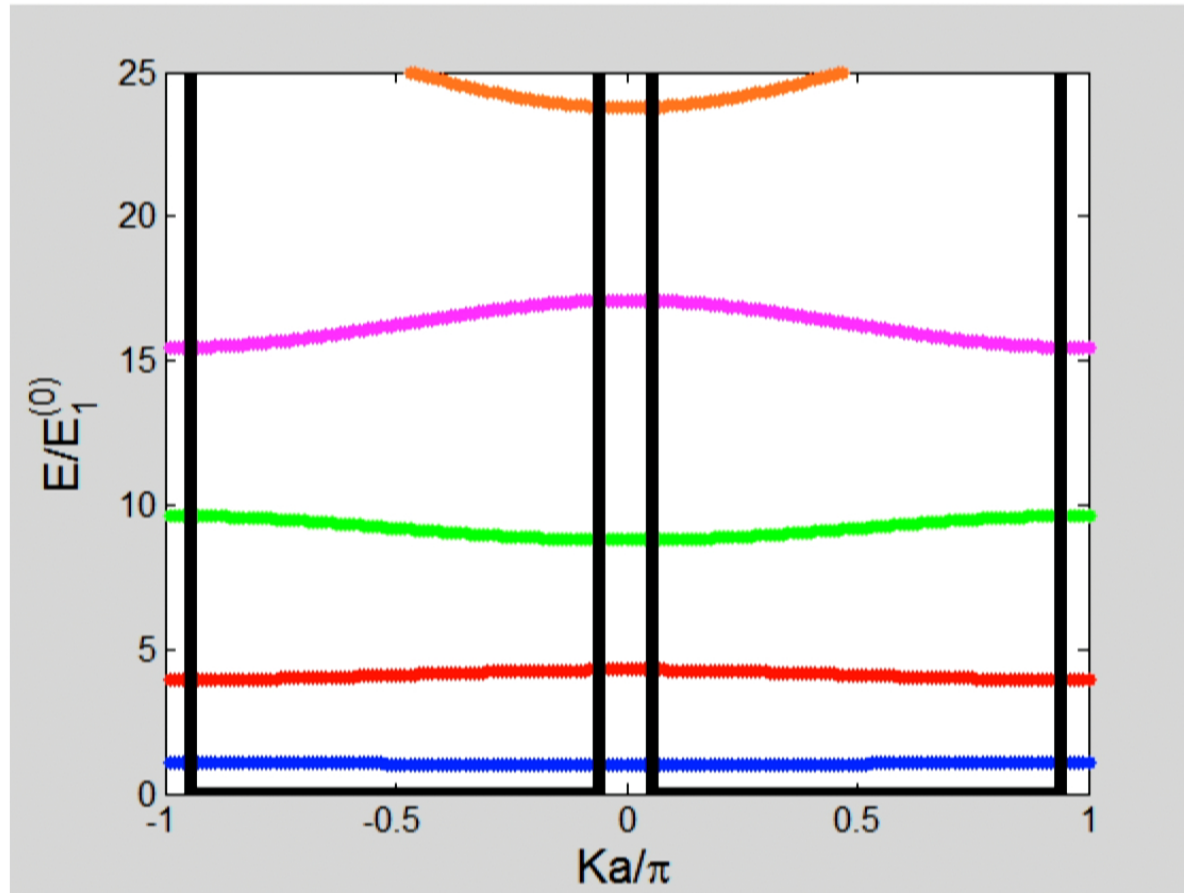
$$V_0 = 40 E_1^{(0)}$$

Stick to 1D Change the barrier width



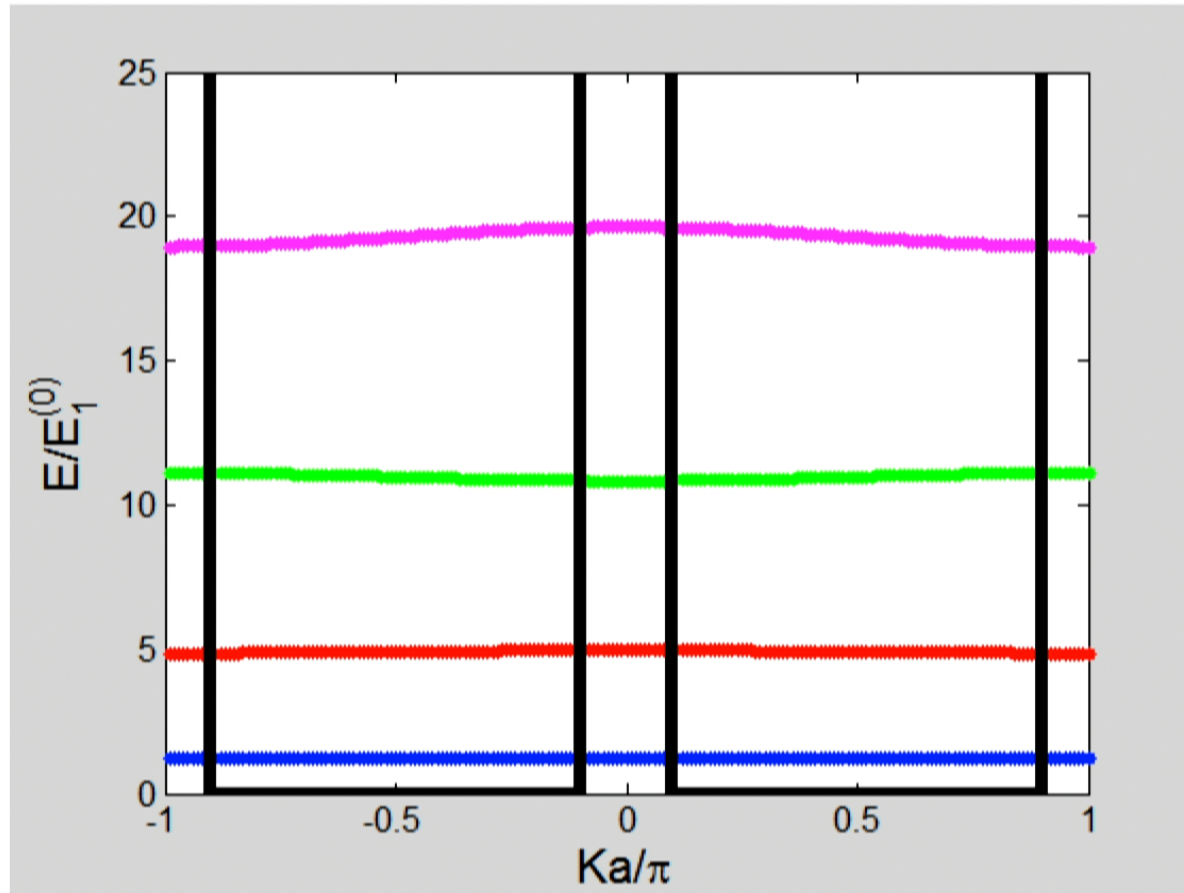
$$V_0 = 40 E_1^{(0)}$$

Stick to 1D Change the barrier width



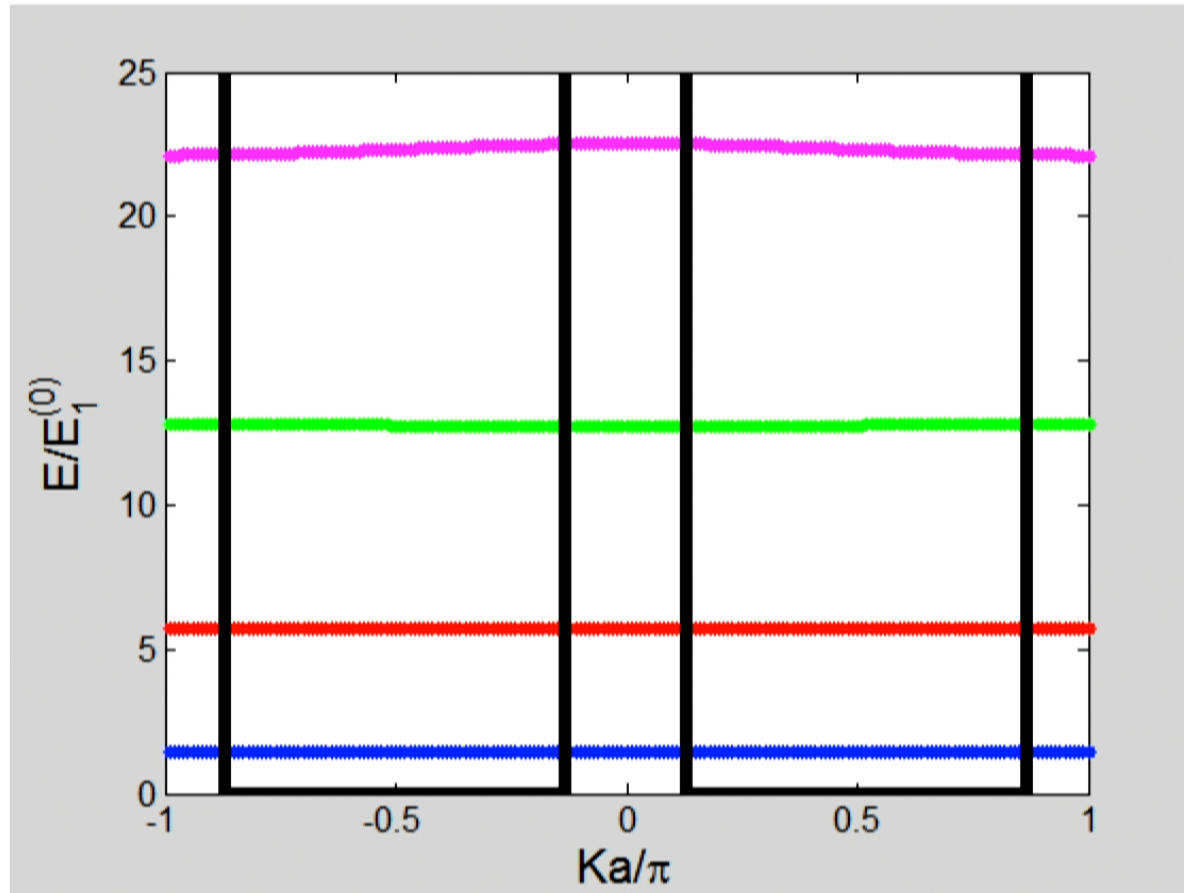
$$V_0 = 40 E_1^{(0)}$$

Stick to 1D Change the barrier width



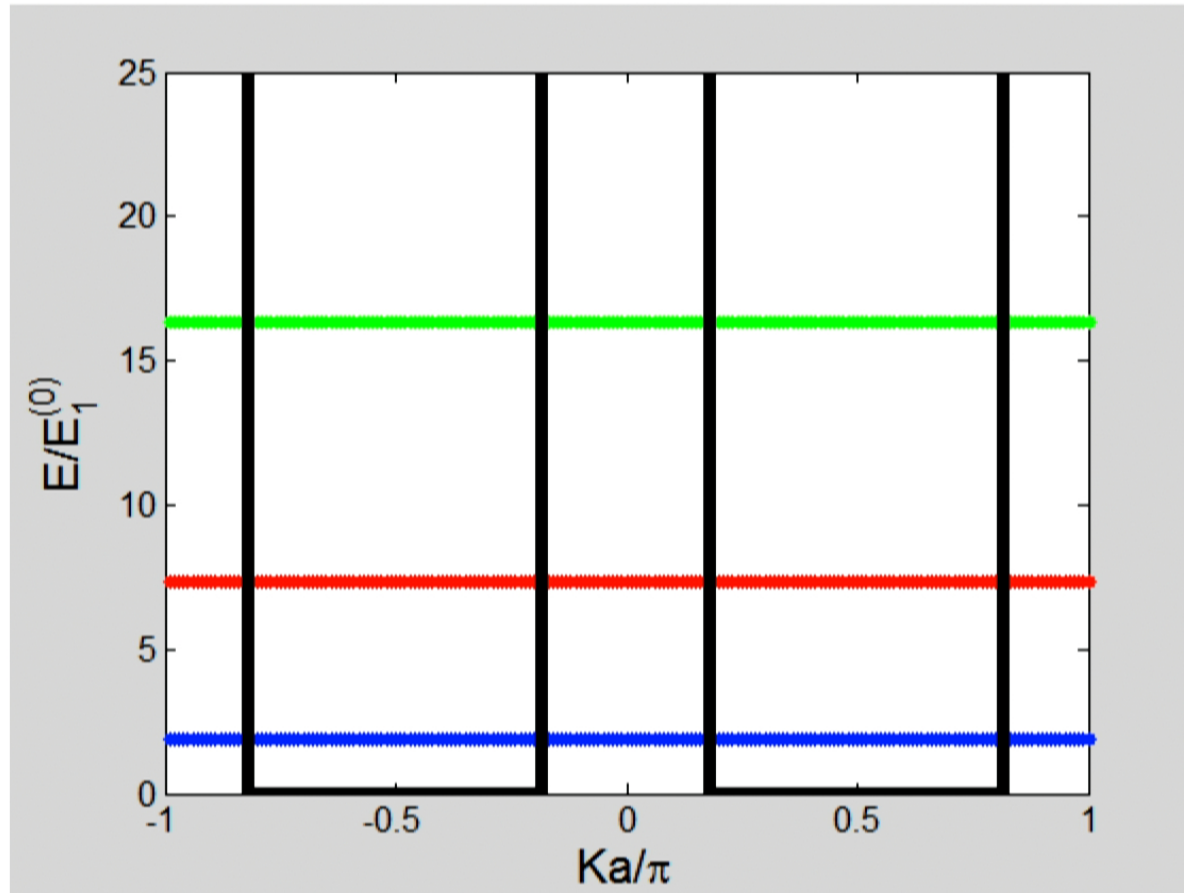
$$V_0 = 40 E_1^{(0)}$$

Stick to 1D Change the barrier width



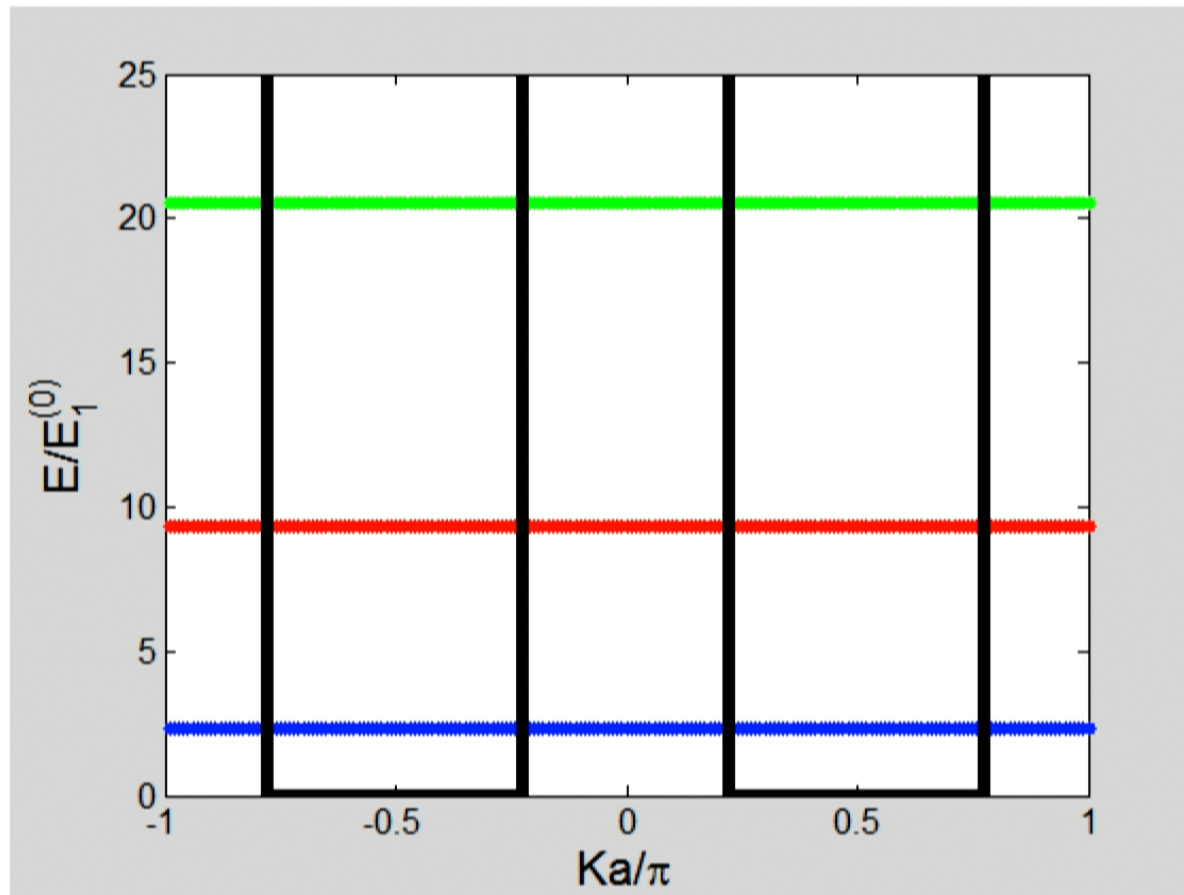
$$V_0 = 40 E_1^{(0)}$$

Stick to 1D Change the barrier width



$$V_0 = 40 E_1^{(0)}$$

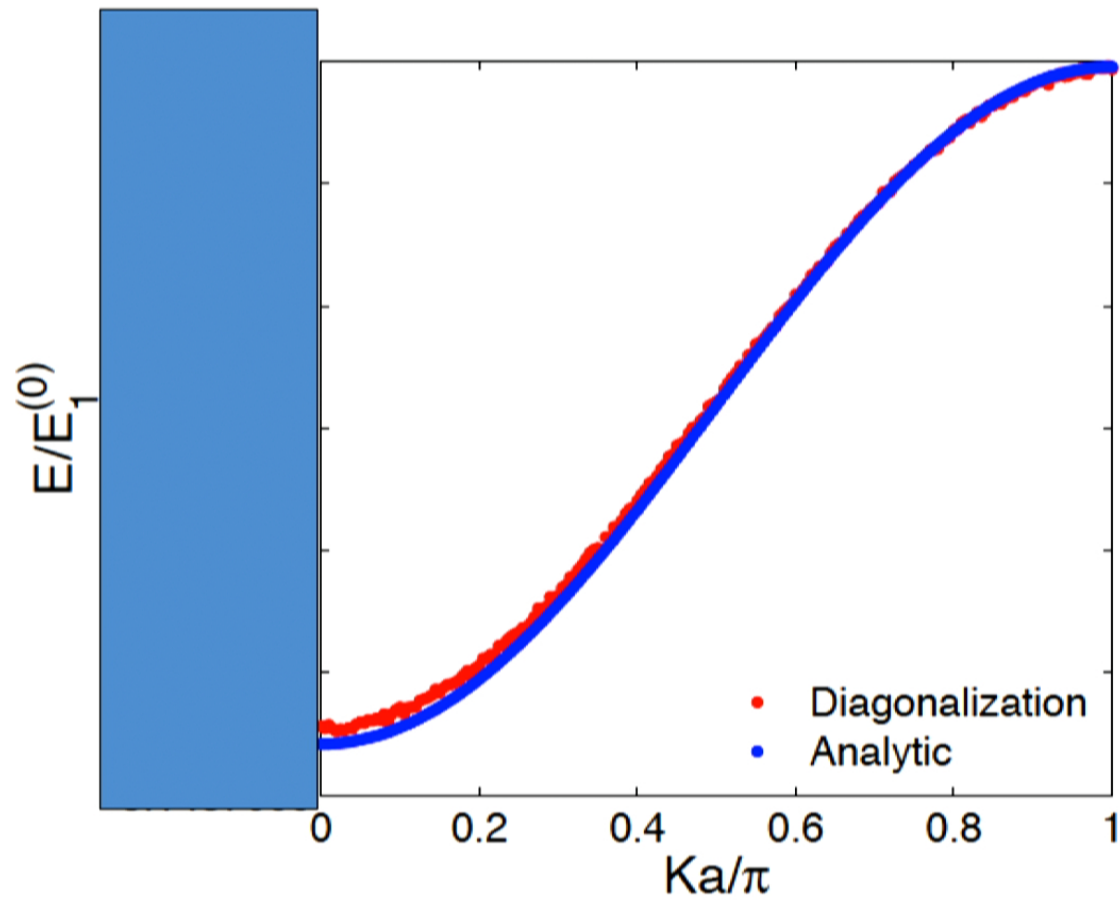
Stick to 1D Change the barrier width



$$V_0 = 40 E_1^{(0)}$$

1D

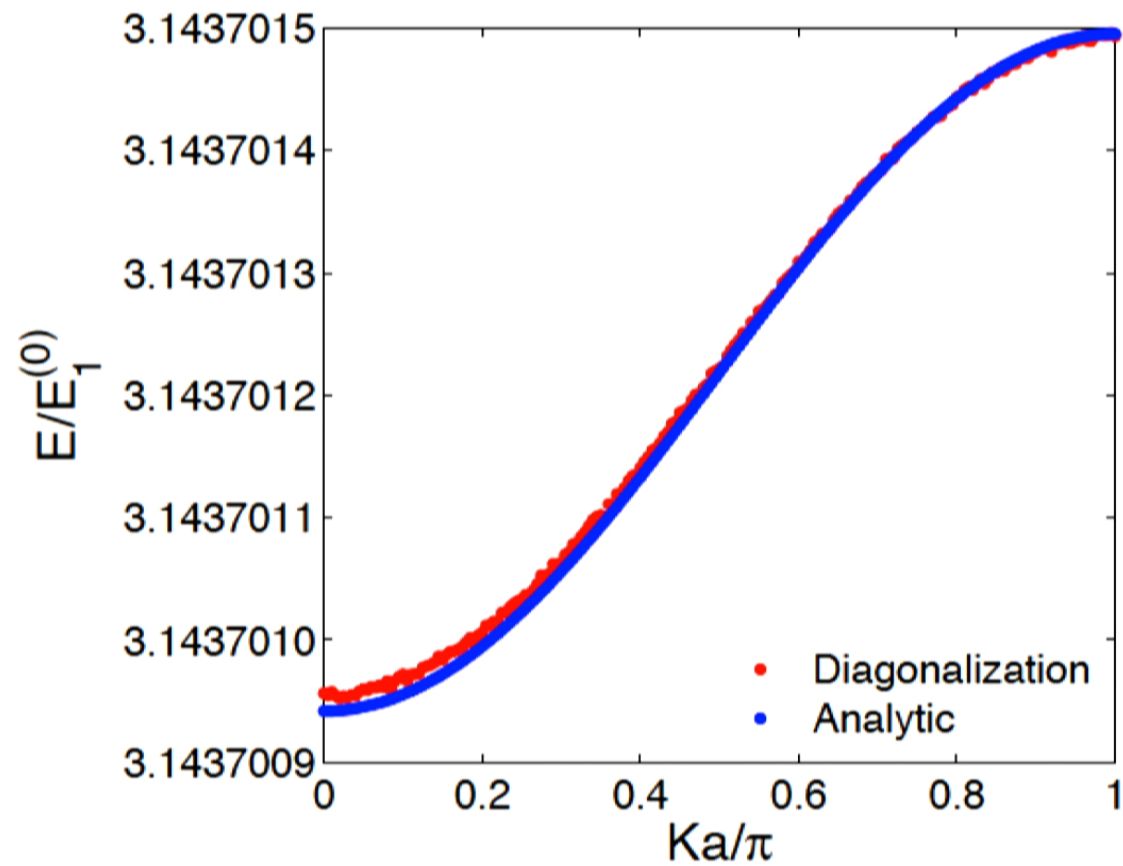
Pavelich thesis (unpublished)



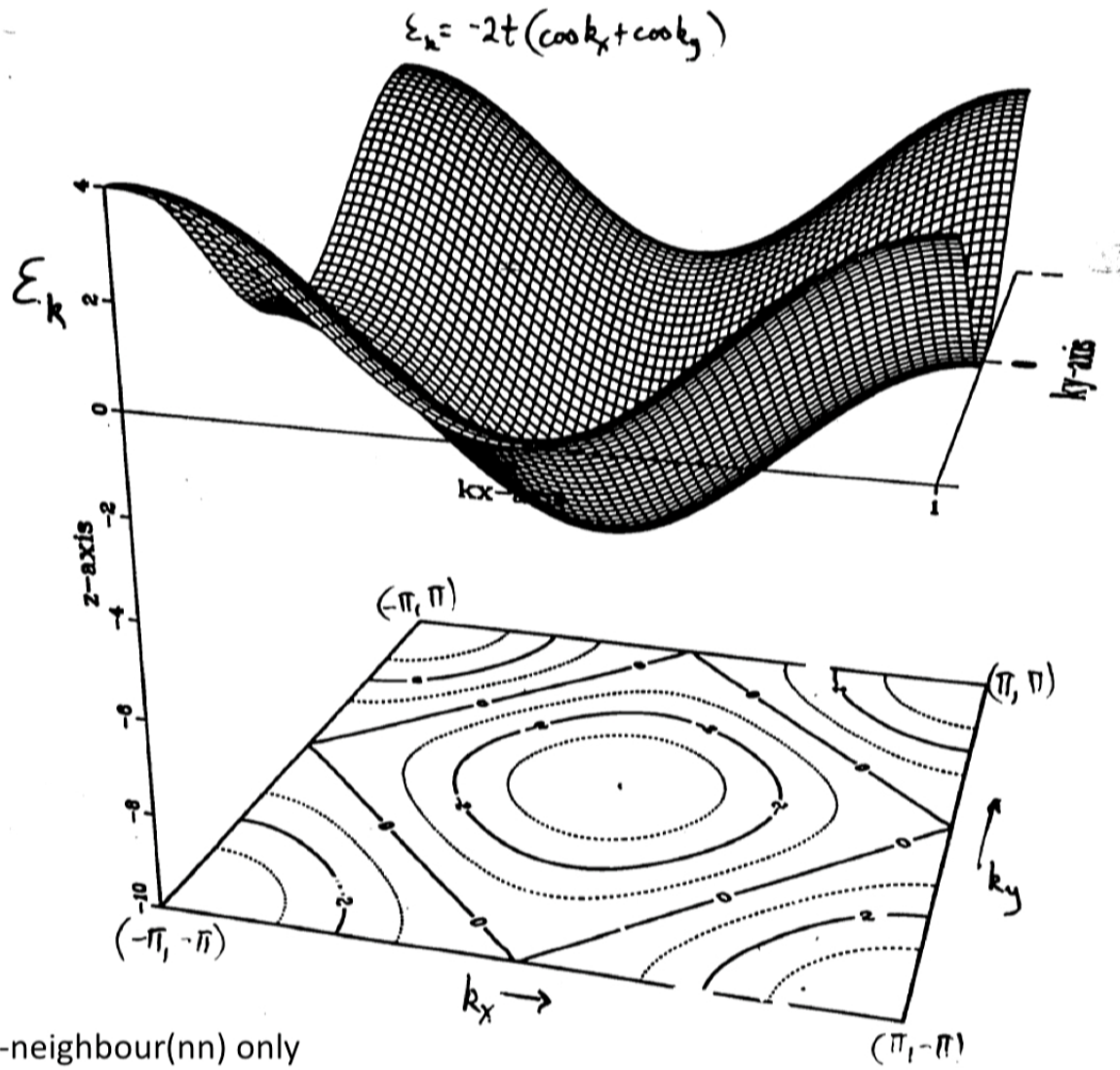
(f) $v_0 = 100, \alpha = 0.887$

1D

Pavelich thesis (unpublished)



(f) $v_0 = 100, \alpha = 0.887$



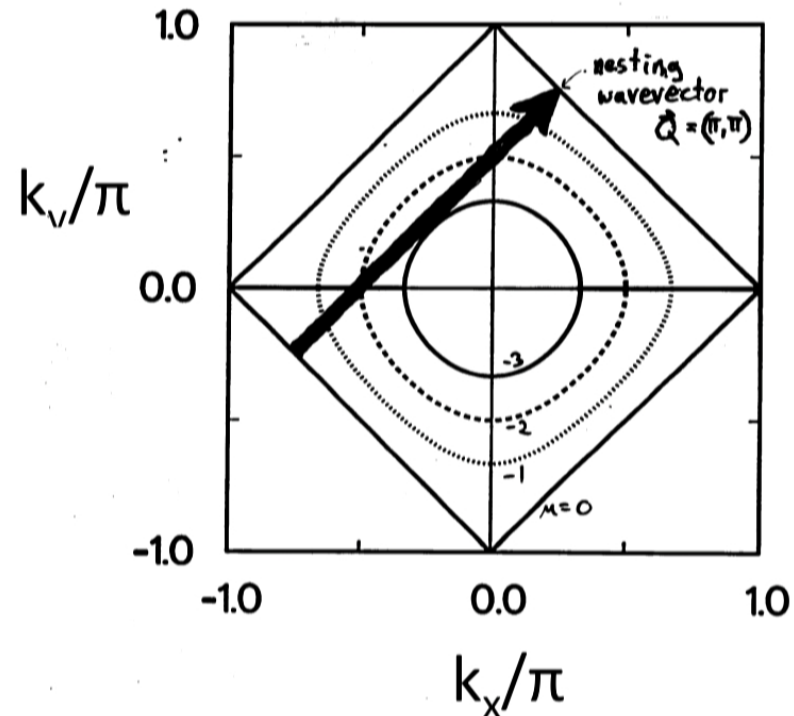
2D nearest-neighbour(nn) only

$$t \equiv 1$$

Nesting

ONE wavevector maps a large piece of the Fermi surface onto another large piece of the Fermi surface.

$$\mu = 0.0, -1.0, -2.0, -3.0$$

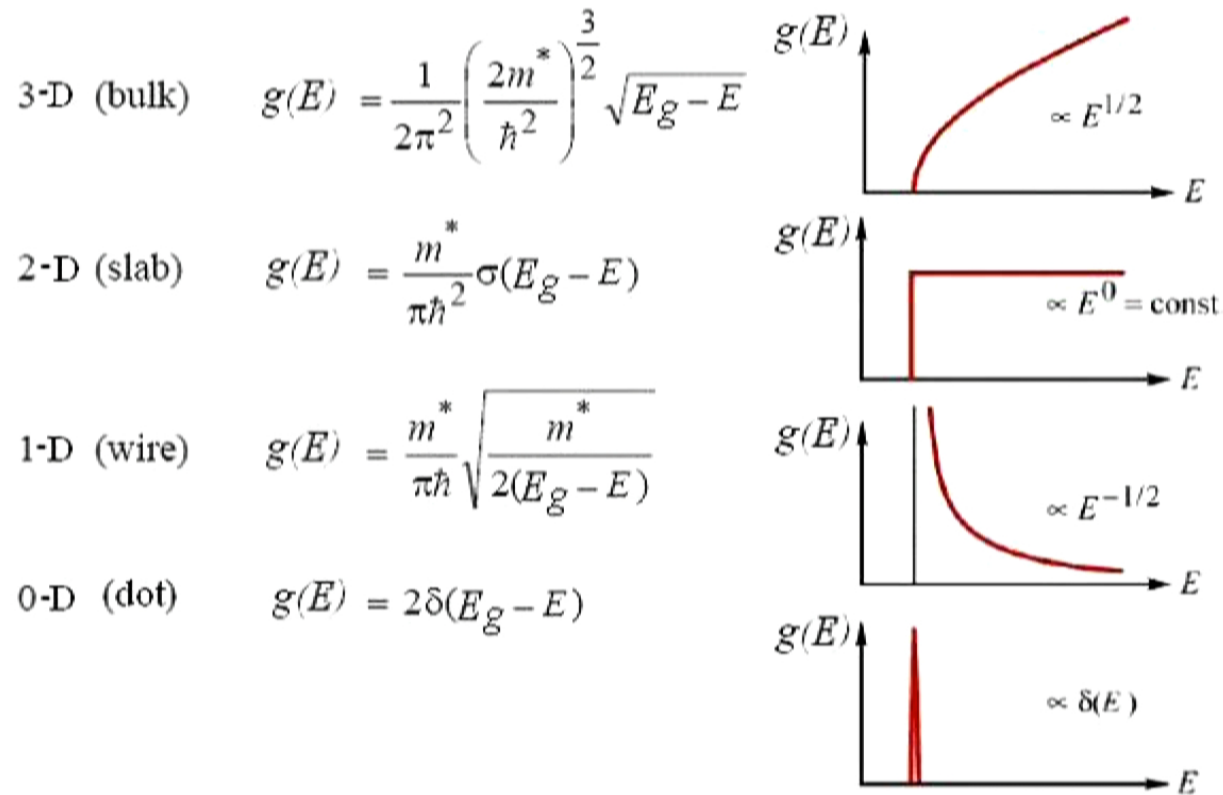


- actually not good for superconductivity
- good for charge density wave (CDW)
or spin density wave (SDW)

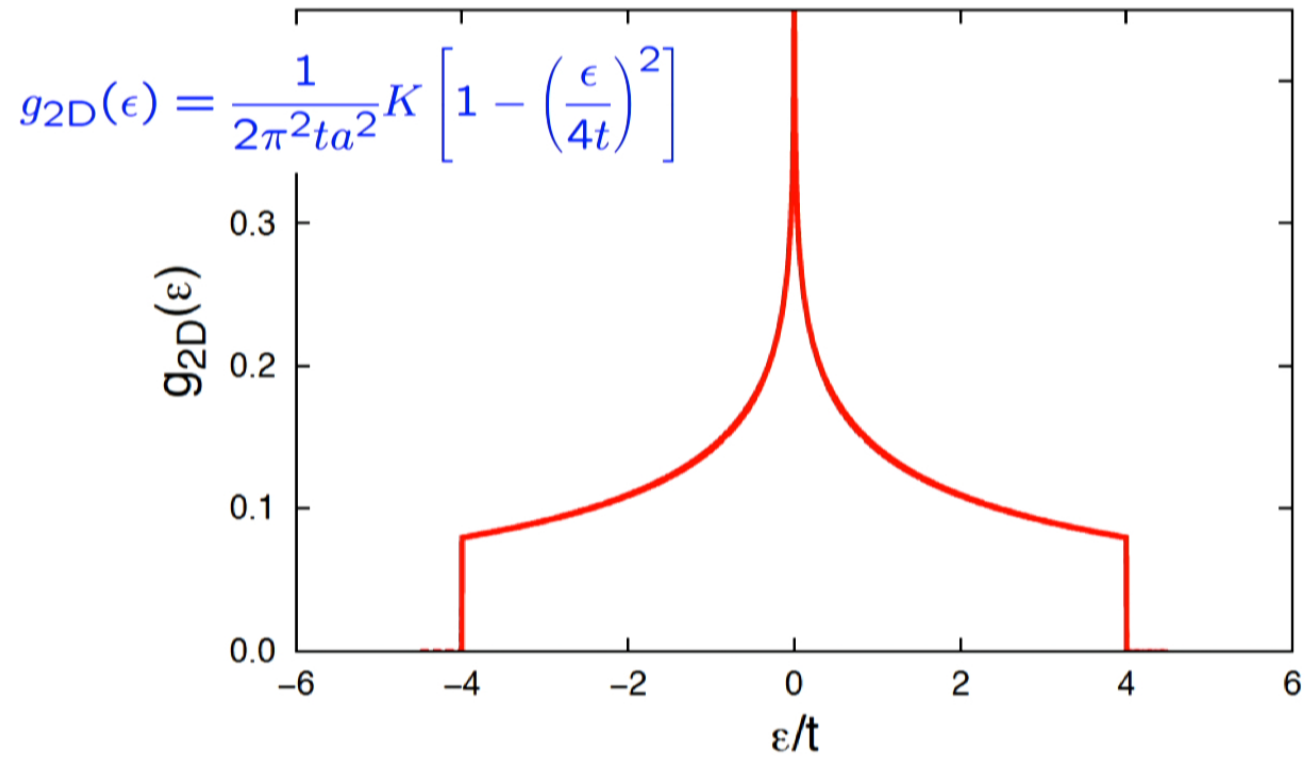
$$g(E) = \int_{-\infty}^{+\infty} \left(\frac{dk}{2\pi} \right)^D \delta(E - E_k)$$

$$E_k = E_g + \frac{\hbar^2 k^2}{2m^*}$$

$$g(E) = \int_{-\infty}^{+\infty} \left(\frac{dk}{2\pi} \right)^D \delta(E - E_k)$$



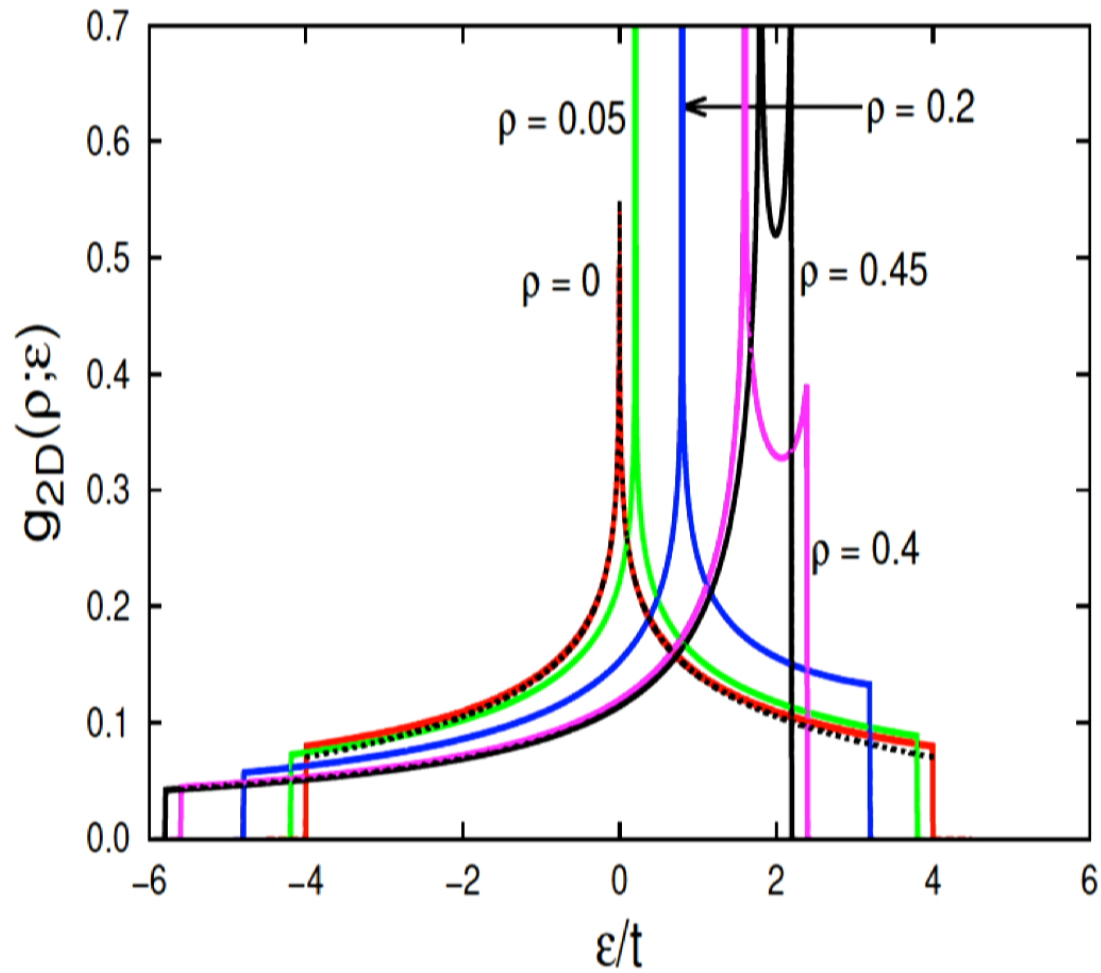
2D Density of States, with NN hopping ($t_2 = 0$)



2D Density of States, with , NNN hopping t_2

$$\rho \equiv t_2/t$$

$$\epsilon_k = -2t [\cos(k_x a) + \cos(k_y a)] - 4t_2 \cos(k_x a) \cos(k_y a)$$



Three Dimensions (NN)

$$\epsilon_k = -2t_s [\cos(k_x a) + \cos(k_y a) + \cos(k_z a)] \quad [\text{sc}]$$

SC

$$\epsilon_k = -8t_b \left[\cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_y a}{2}\right) \cos\left(\frac{k_z a}{2}\right) \right] \quad [\text{bcc}]$$

BCC

$$\epsilon_k = -4t_f \left[\cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_y a}{2}\right) + \cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_z a}{2}\right) + \cos\left(\frac{k_y a}{2}\right) \cos\left(\frac{k_z a}{2}\right) \right] \quad \text{FCC}$$

Three Dimensions

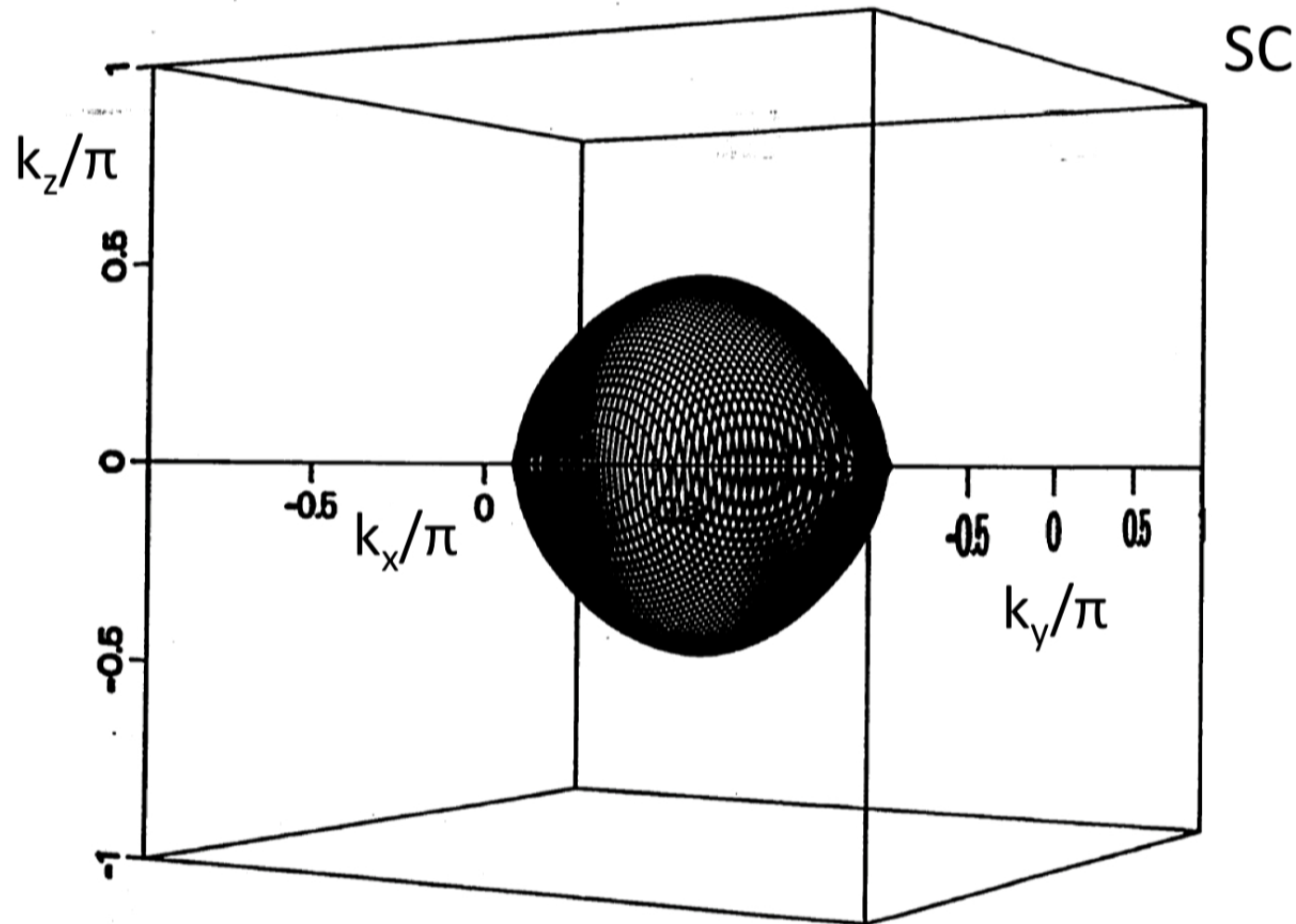
$$\epsilon_k = -2t_s [\cos(k_x a) + \cos(k_y a) + \cos(k_z a)] - 4t_{s2} \left[\cos(k_x a)\cos(k_y a) + \cos(k_x a)\cos(k_z a) + \cos(k_y a)\cos(k_z a) \right] \quad \text{SC} \quad [\text{sc NNN}] \quad (\text{A4})$$

$$\epsilon_k = -8t_b \left[\cos\left(\frac{k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right)\cos\left(\frac{k_z a}{2}\right) \right] - 2t_{b2} [\cos(k_x a) + \cos(k_y a) + \cos(k_z a)] \quad \text{BCC} \quad [\text{bcc NNN}] \quad (\text{A5})$$

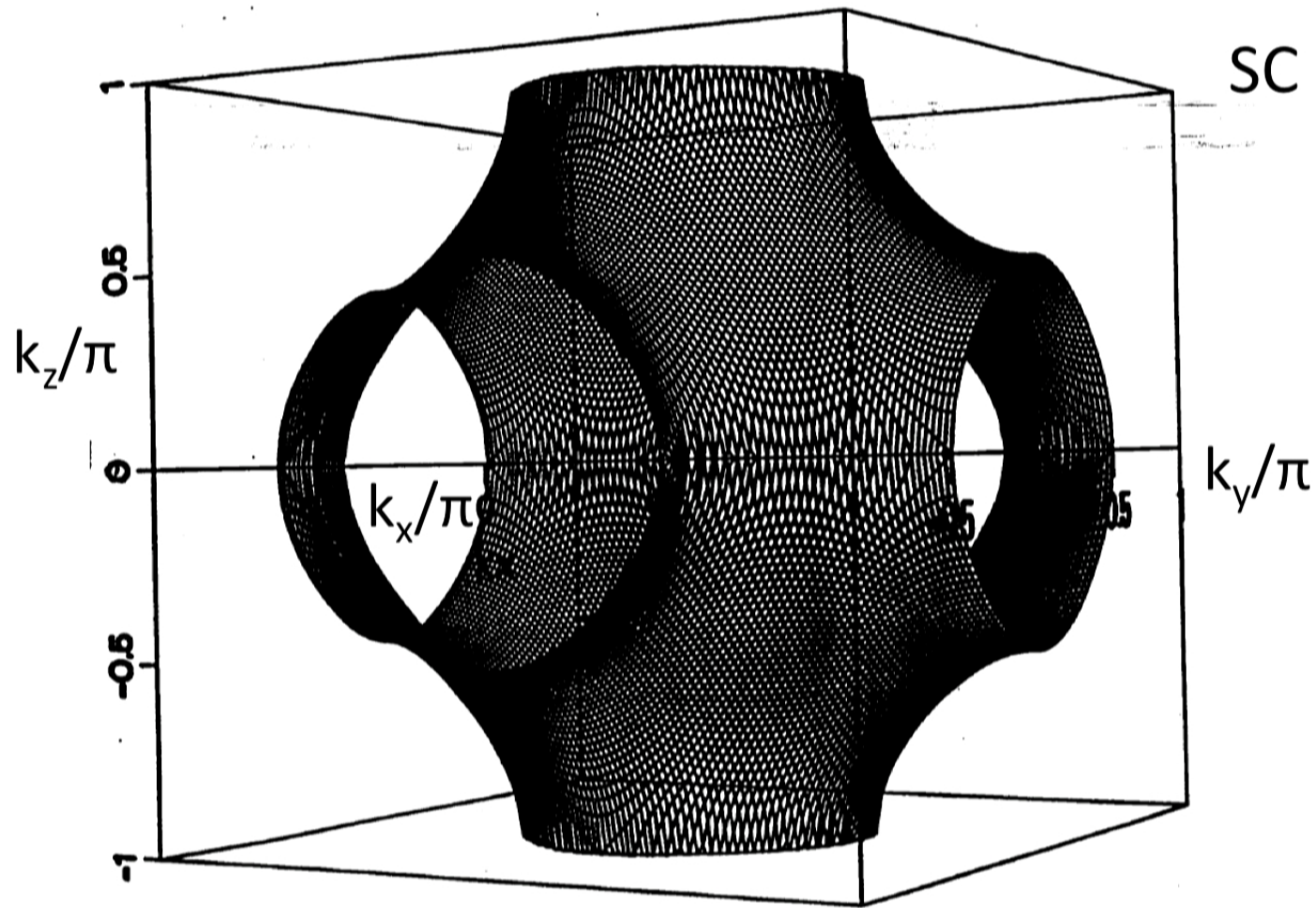
$$\epsilon_k = -4t_f \left[\cos\left(\frac{k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right) + \cos\left(\frac{k_x a}{2}\right)\cos\left(\frac{k_z a}{2}\right) + \cos\left(\frac{k_y a}{2}\right)\cos\left(\frac{k_z a}{2}\right) \right] - 2t_{f2} [\cos(k_x a) + \cos(k_y a) + \cos(k_z a)]. \quad \text{FCC} \quad [\text{fcc NNN}] \quad (\text{A6})$$

Dispersions?

$$-2(\cos k_x + \cos k_y + \cos k_z) = -4.0 = \text{amu} \quad n_x = 512 \quad -60 \quad 0 \quad 40$$



$$-2(\cos k_x + \cos k_y + \cos k_z) = 4.0 = \text{amu} \quad n_x = 512 \quad -60 \quad 0 \quad 40$$



What do the densities of states look like?

J. Phys. Chem. Solids Pergamon Press 1969. Vol. 30, pp. 609–626. Printed in Great Britain.

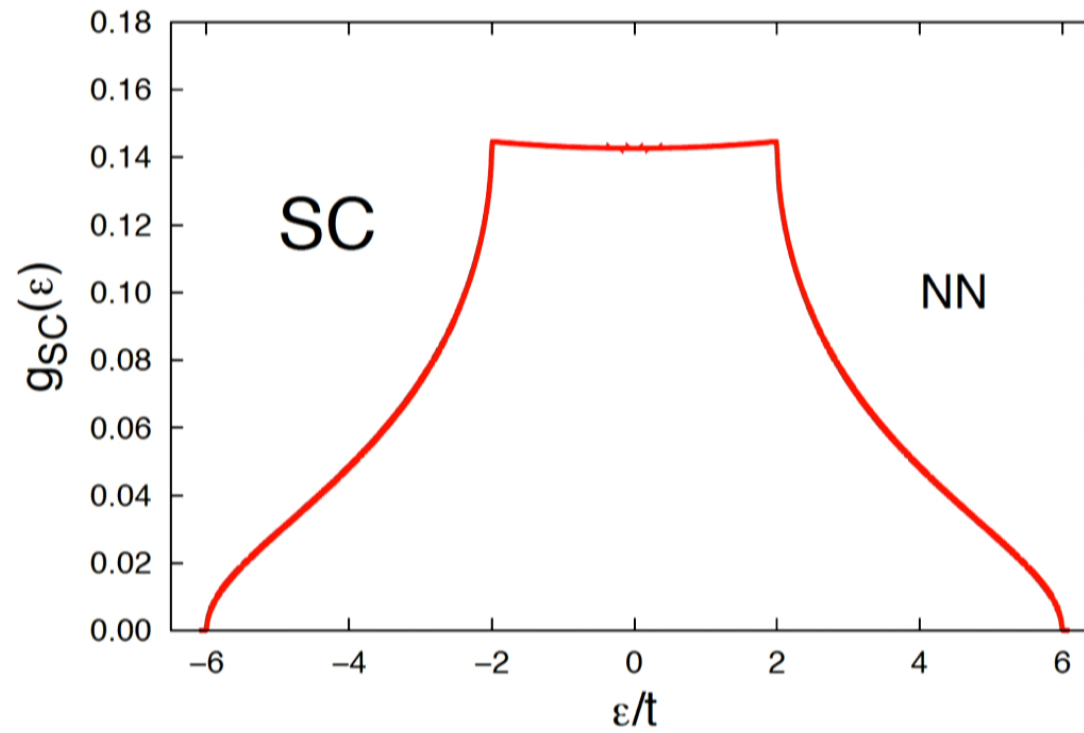
**THE DENSITY OF STATES OF SOME
SIMPLE EXCITATIONS IN SOLIDS**

RAINER J. JELITTO

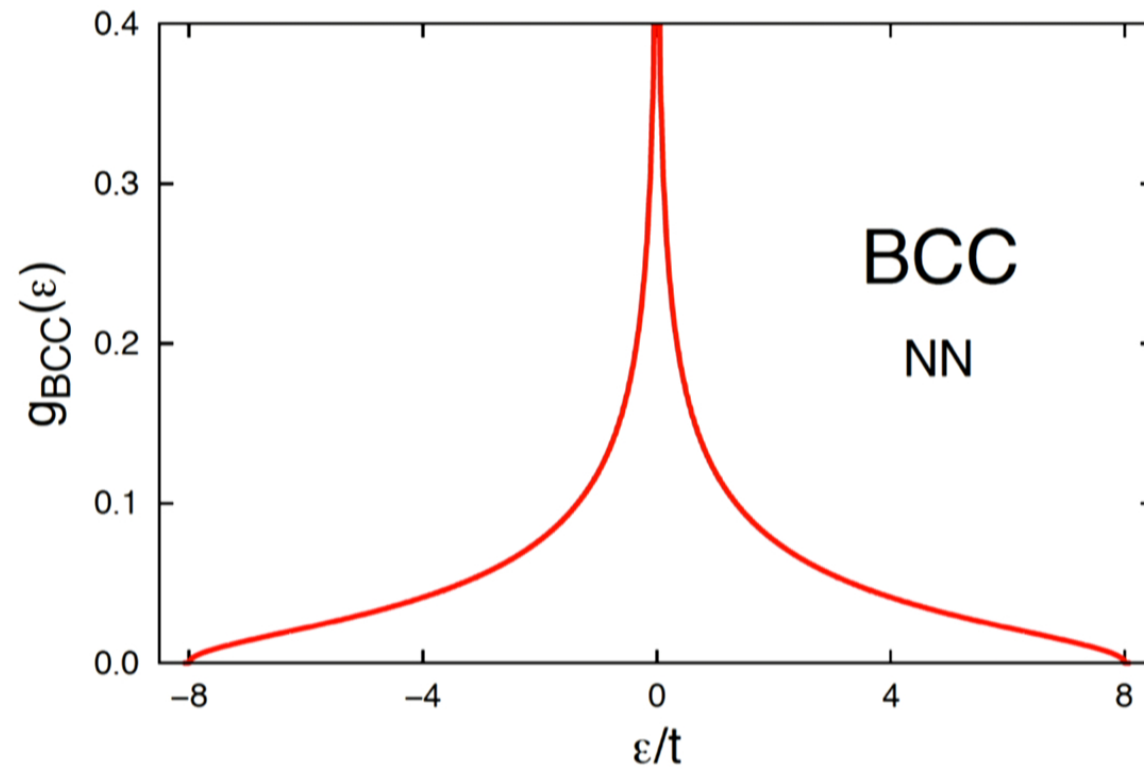
Institut für Theoretische Physik der Universität Kiel, Germany

(Received 4 March 1966)

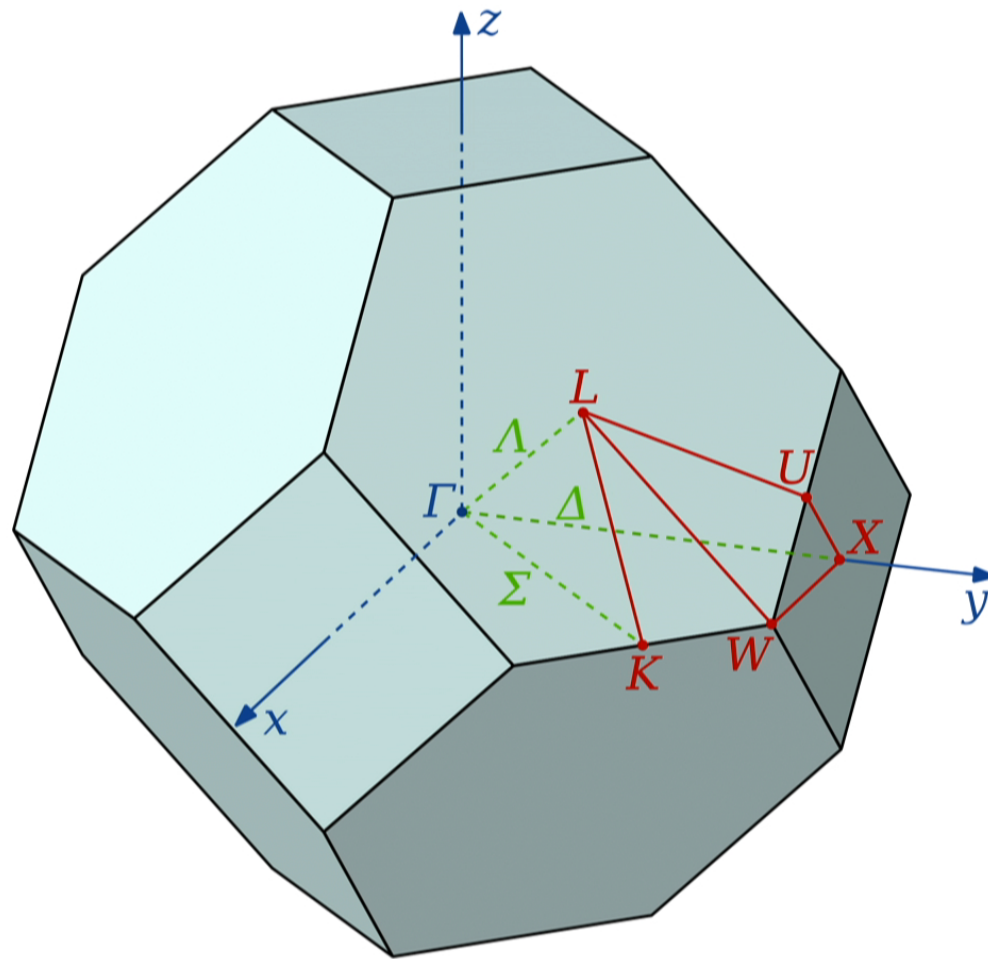
The nearest-neighbour (NN) cases:



The nearest-neighbour (NN) cases:



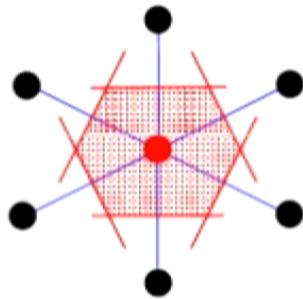
First Brillouin Zone for FCC



FIRST BRILLOUIN ZONES

The Wigner-Seitz cell of the reciprocal lattice is called the first Brillouin zone (FBZ).

Wigner-Seitz cell: primitive cell with lattice point at its center



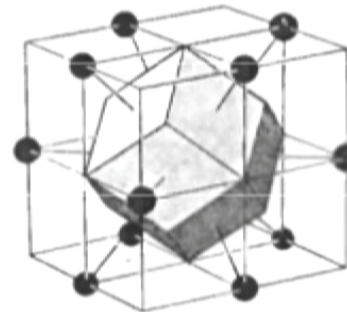
enclosed region is W-S cell for 2D hexagonal lattice

d.l. FCC
r.l. BCC
1st Brillouin zone:



truncated octahedron

d.l. BCC
r.l. FCC
1st Brillouin zone:

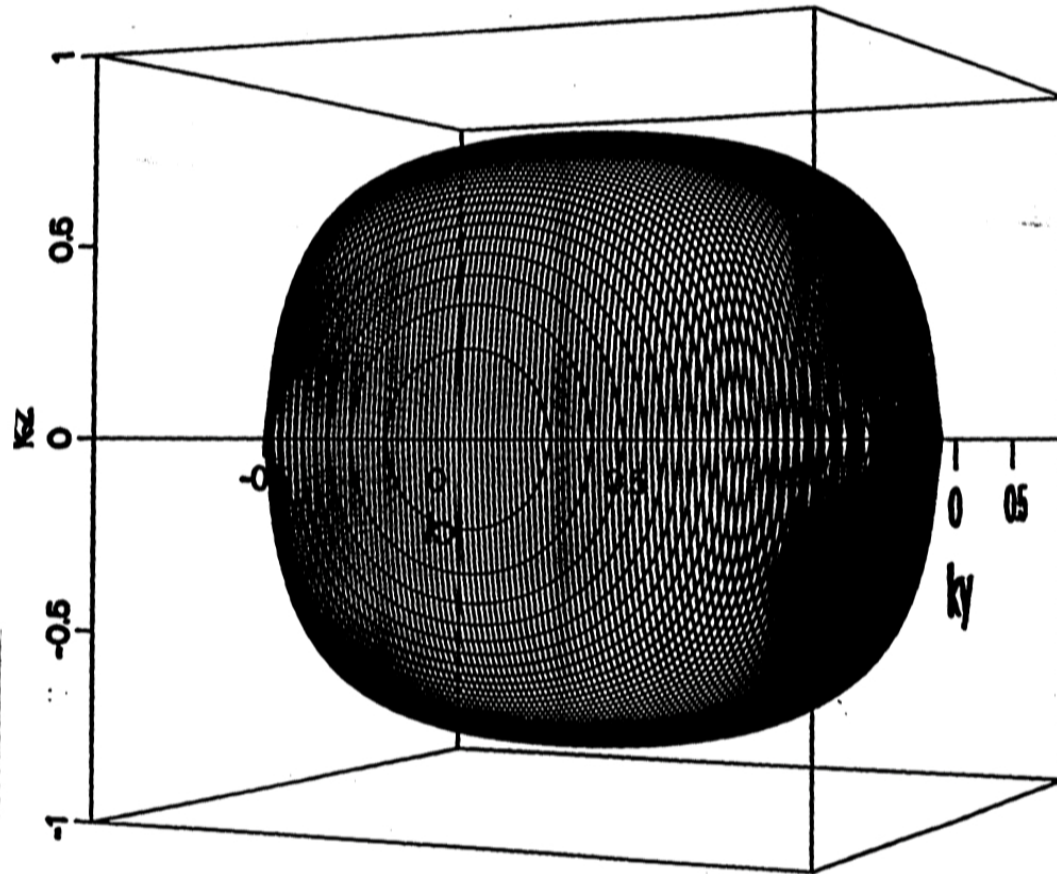


rhombic dodecahedron

268

Tight-binding 3D BCC

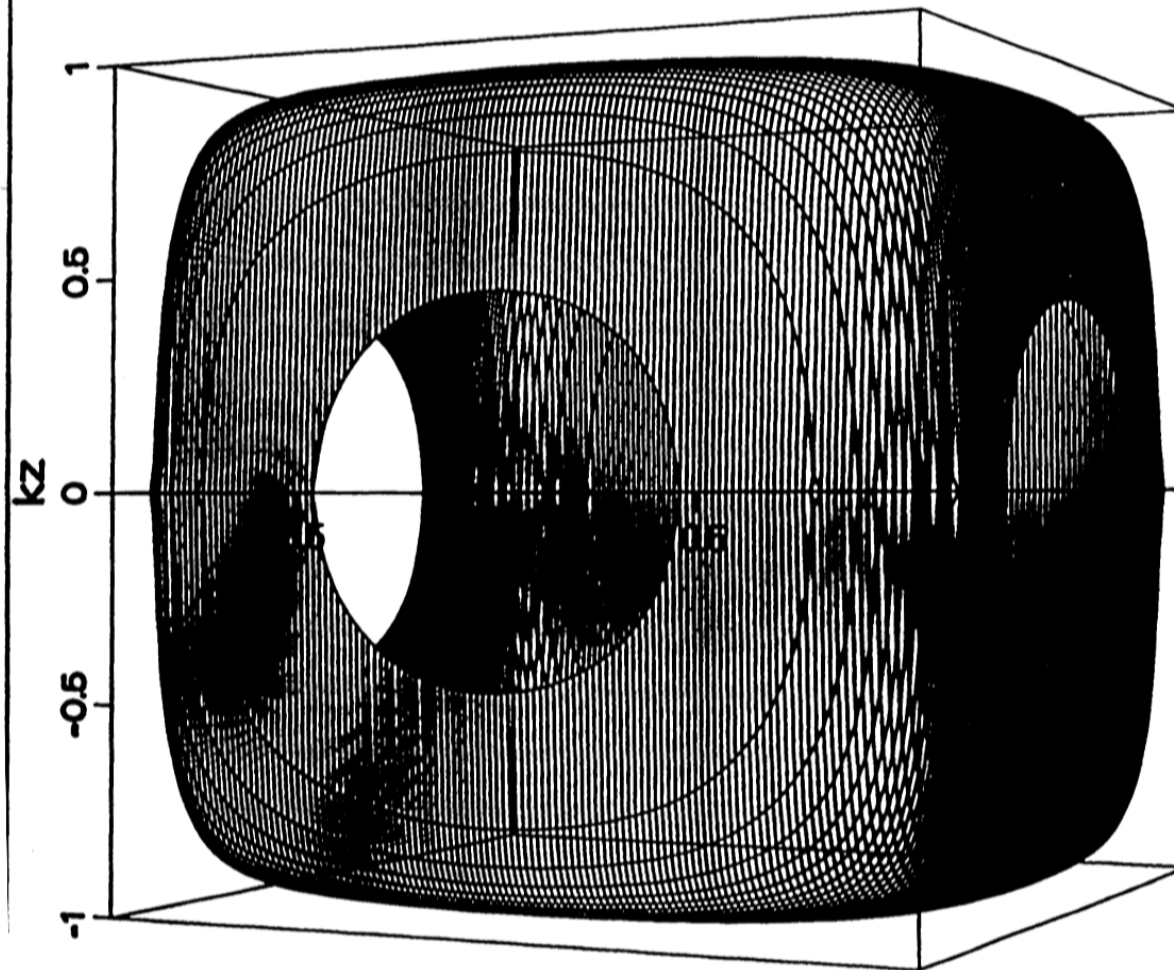
$t_t=0.0$ $t_p=1.0$ $\text{gap}=0.001$ $\text{amu}=-2.0$ -65 0 40



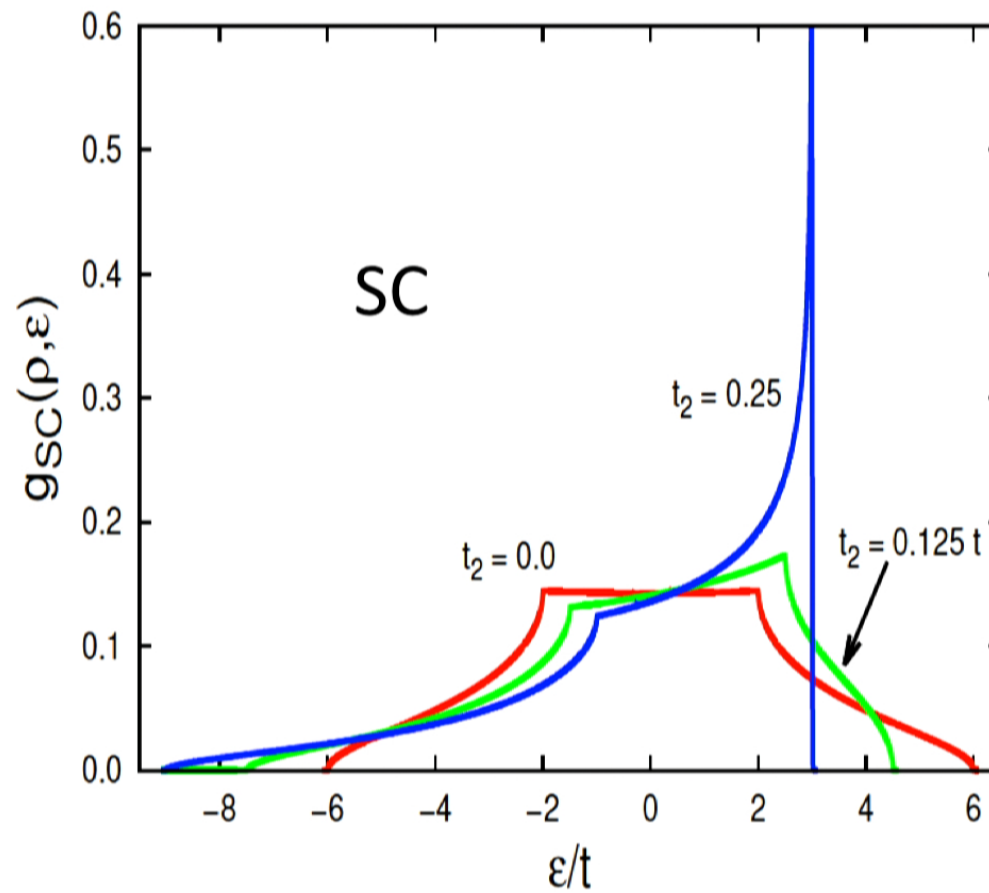
BCC

tt=0.0 tp=1.0 gap=0.001 amu=-0.15 -65 0 40

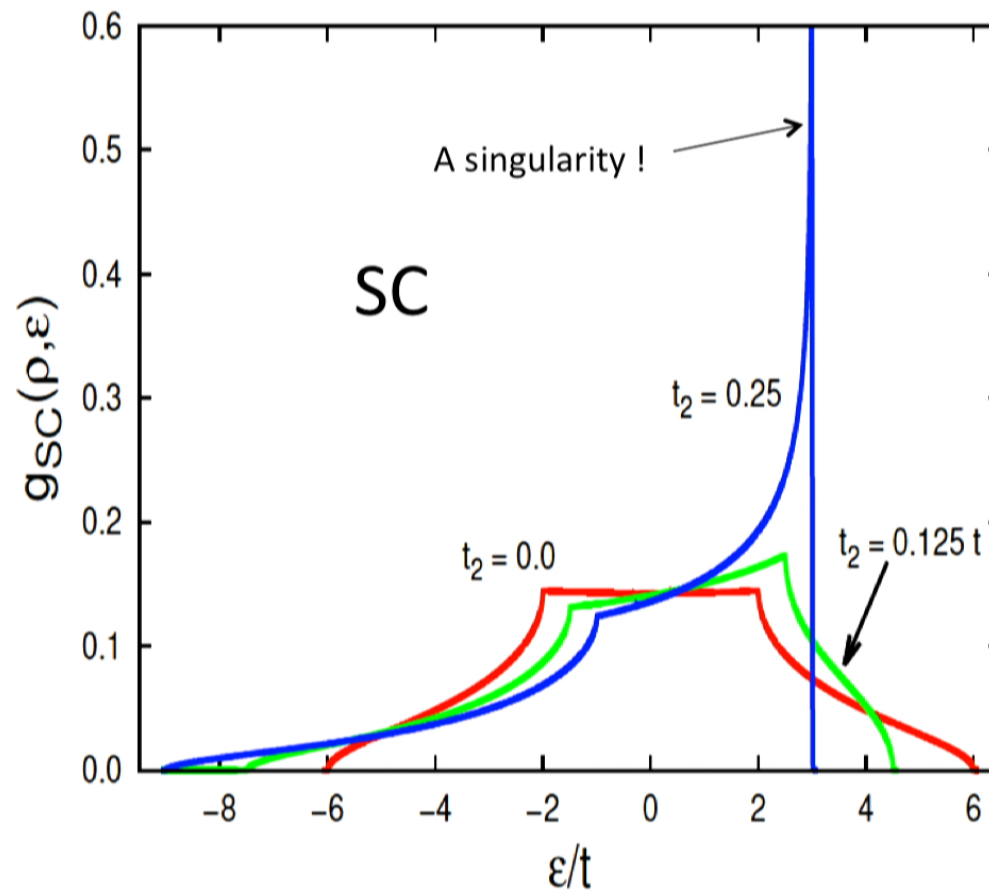
BCC



What does next-nearest neighbour (NNN) hopping do to the densities of states?



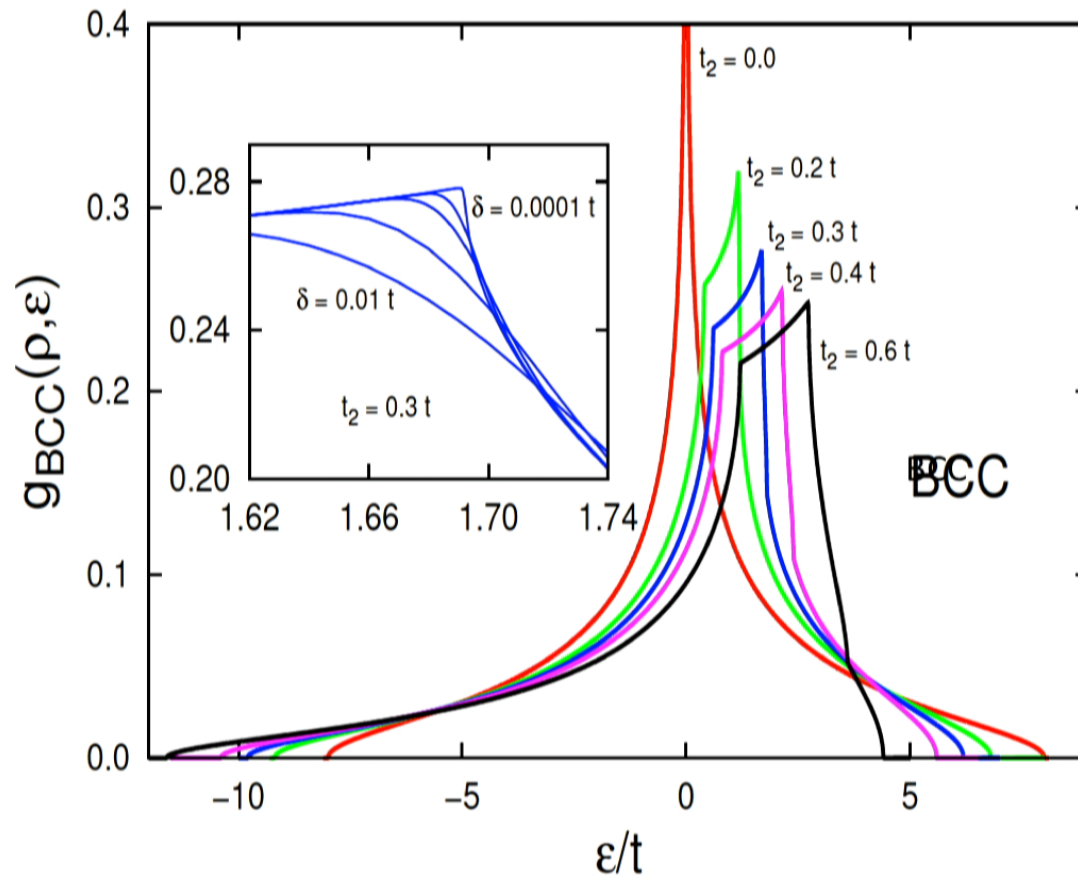
What does next-nearest neighbour (NNN) hopping do to the densities of states?



What does next-nearest neighbour (NNN) hopping do to the densities of states?

BCC

What does next-nearest neighbour (NNN) hopping do to the densities of states?

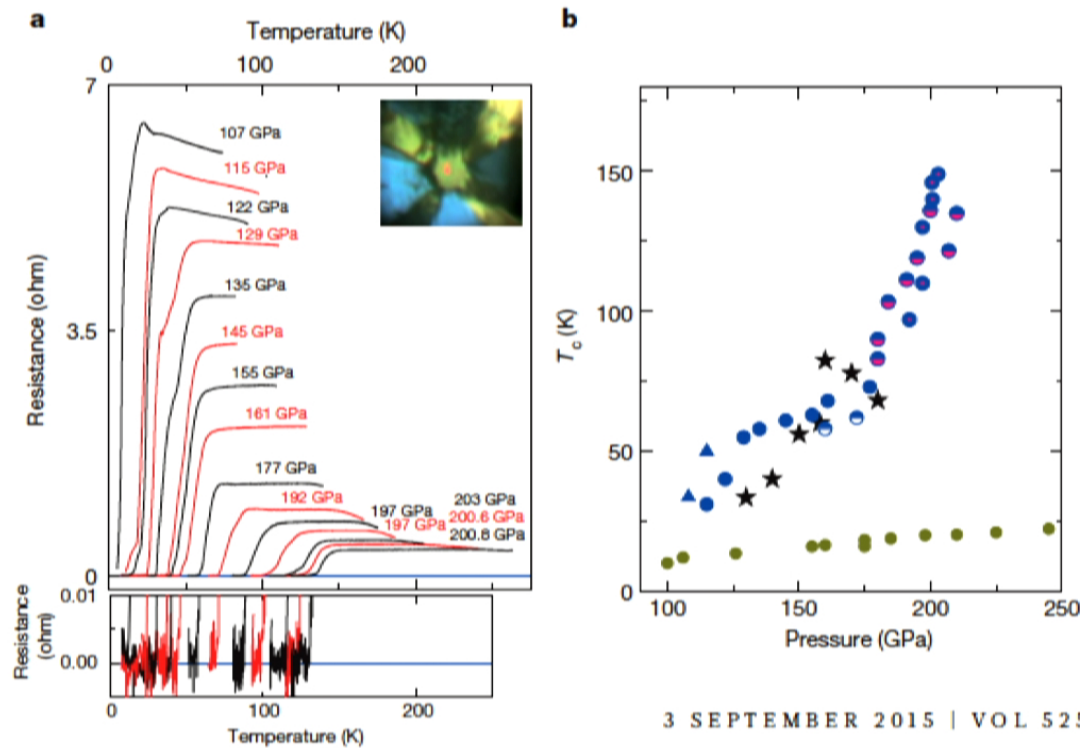


What does next-nearest neighbour (NNN) hopping do to the densities of states?

FCC

Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system

A. P. Drozdov^{1*}, M. I. Eremets^{1*}, I. A. Troyan¹, V. Ksenofontov² & S. I. Shylin²



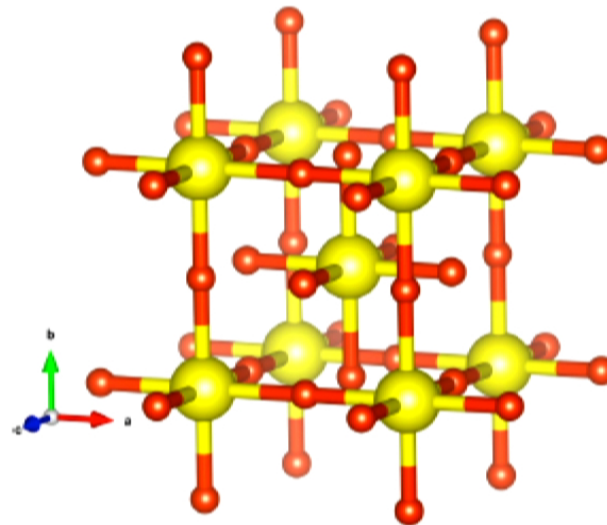
3 SEPTEMBER 2015 | VOL 525 | NATURE | 73

Van Hove singularities and spectral smearing in high-temperature superconducting H₃S

Yundi Quan and Warren E. Pickett*

Department of Physics, University of California Davis, Davis, California 95616, USA

(Received 3 September 2015; published 25 March 2016)



A. Importance of sulfur

FIG. 1. Crystal structure of $Im\bar{3}m$ H₃S. Nearest neighbor S-H bonds are shown. At this nearest neighbor level, the structure consists of two interleaved ReO₃ sublattices, displaced relative to one another by the body-centering vector $(1,1,1)a/2$.

See also Sano et al. Phys.Rev. B93, 094525-1-16 (2016)

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singularities at energies $\epsilon_{lo} < \bar{\epsilon}_{hi}$

$$N(E) = \begin{cases} a_1\sqrt{|b_1 - \epsilon|} + c_1\epsilon + d_1 & \epsilon < \epsilon_{lo} \\ a_2\epsilon + b_2 & \\ a_3\sqrt{|\epsilon + b_3|} + c_3\epsilon + d_3 & \end{cases}$$

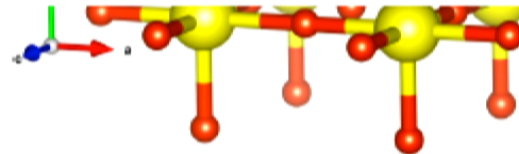


FIG. 1. Crystal structure of $Im\bar{3}m$ H₃S. Nearest bonds are shown. At this nearest neighbor level, the str of two interleaved ReO₃ sublattices, displaced relative by the body-centering vector $(1,1,1)a/2$.

See also Sano et al. Phys.Rev. B93, 094525-1-16 (2016)

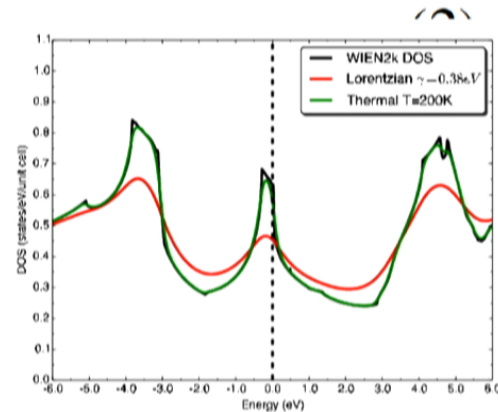


FIG. 8. H₃S density of states without broadening (black line, Wien2k DOS), with thermal broadening at 200 K (green line, which is hard to distinguish from the unbroadened one), and the virtual phonon broadened effective DOS $N(E)$ using a Lorentzian halfwidth of 0.38 eV (red line). Note the large drop in the value at the Fermi energy (dashed vertical line).