Title: Quantum spaces are modular

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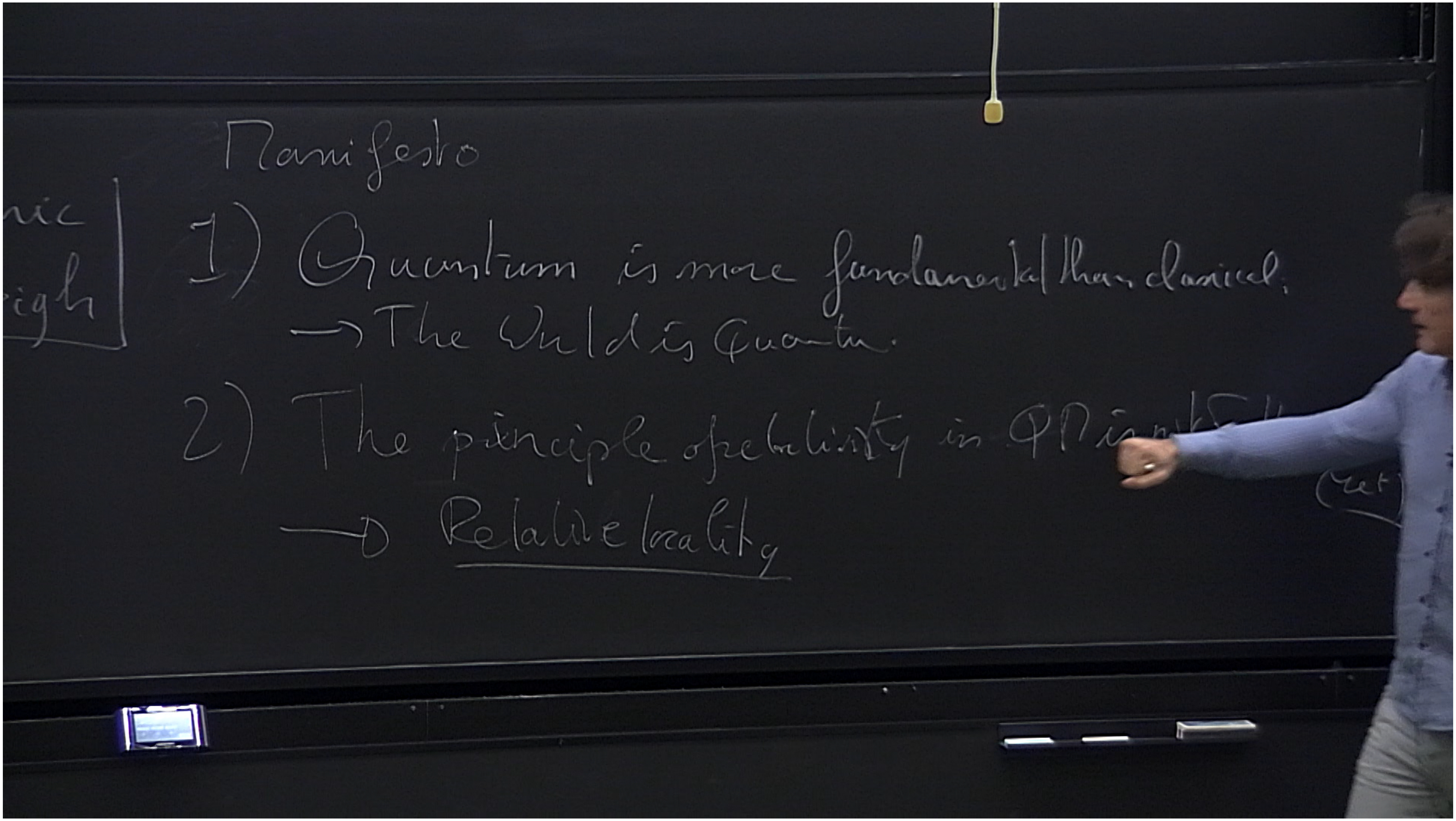
Abstract: <p>In this talk I will review the construction of space starting purely from quantum mechanics and without assuming that the notion of space is attached to a preconceived notion of classical reality. I will show that if one start with the simplest notion of a quantum system encoded into the Heisenberg group algebra one naturally obtain a notion of space that generalizes the usual notion of Euclidean space. Along the way I will try to illustrate how this notion is in a way going back to the roots of the discovery of QM by Heisenberg-Born-Jordan which never made it past the original papers and giving a critical reading of the subsequent interpretation of space and QM that were put forward by Schrodinger and von Neumann. The notion of space that emerge from quantum mechanics is naturally modular in the sense of Aharonov, it also naturally possess a built-in length scale and renders possible to assign a new notion of locality to non-local superpositions. I will illustrate how such space can allow reconciliation of relativity with the presence of a fundamental scale.</p>

<p>I will show how to construct such spaces following the original analysis by Mackay and also show that such modular spaces possess a beautiful geometrical structure that generalizes Riemanian geometry to phase space. A geometry we have named Born geometry. I hope this will open wild speculations on the nature of locality in the presence of quantum mechanics and more broadly the nature of classical reality viewed from a quantum perspective.</p>

What is quantum space?

D. Minic
R. Leigh


Manifesto
1) Quantum is more fundamental than classical;
→ The World is Quantum.
2) The principle of relativity in QM is
→ Relative locality

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Manifesto

1) Quantum is more fundamental than classical;
→ The World is Quantum.

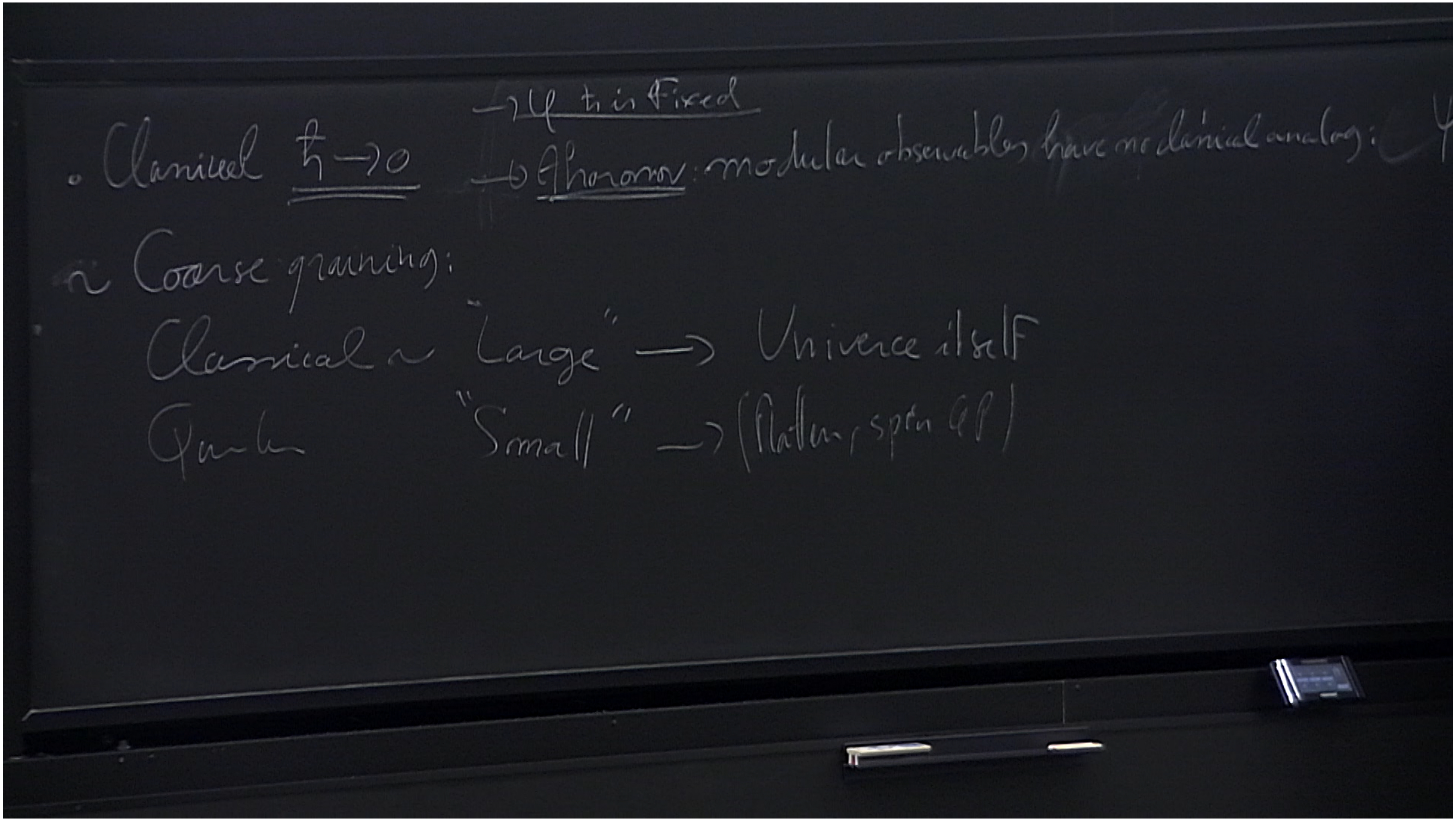
2) The principle of relativity in QFT is not fully understood (yet)
→ Relative locality

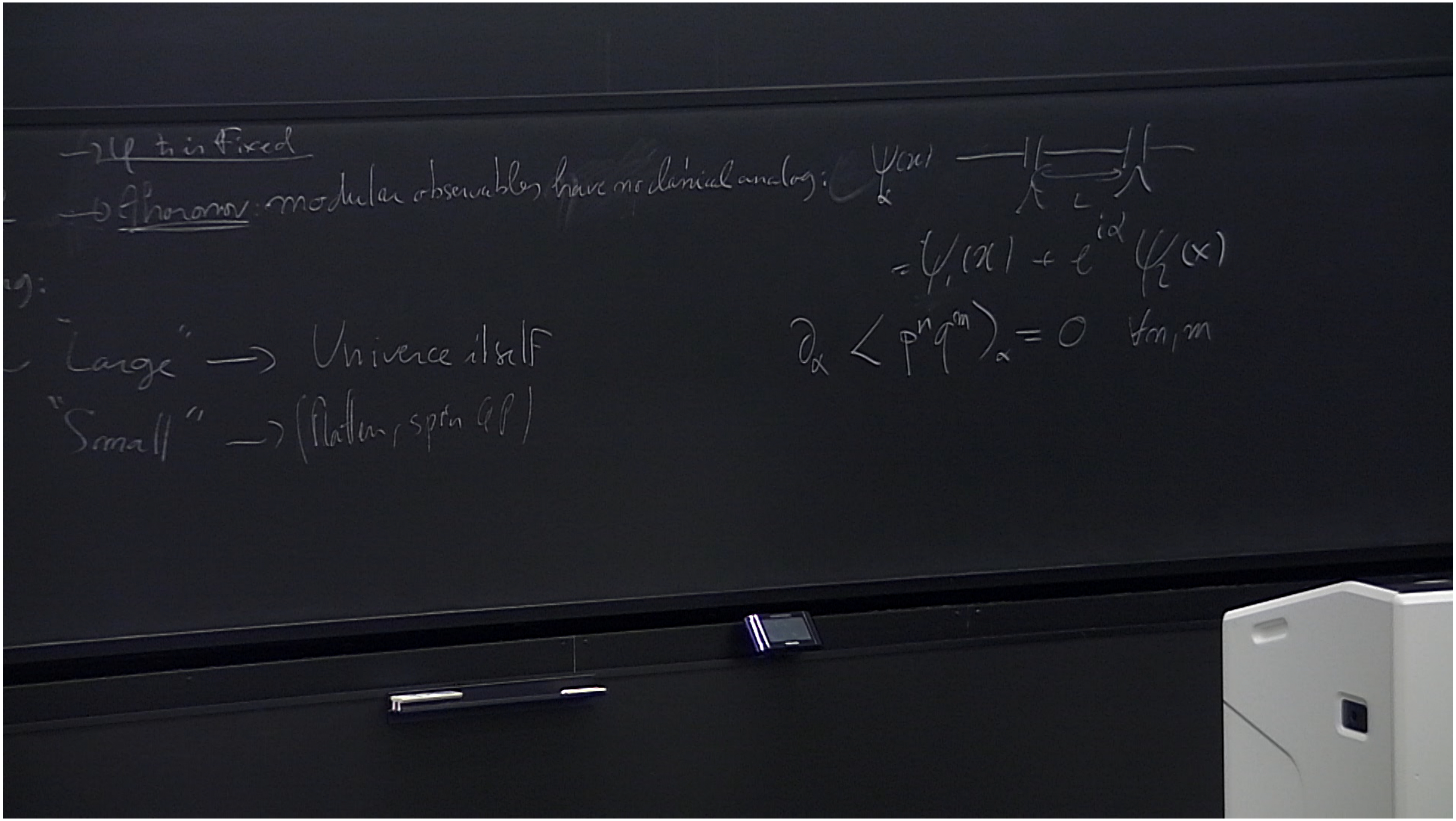
- Classical $\hbar \to 0$

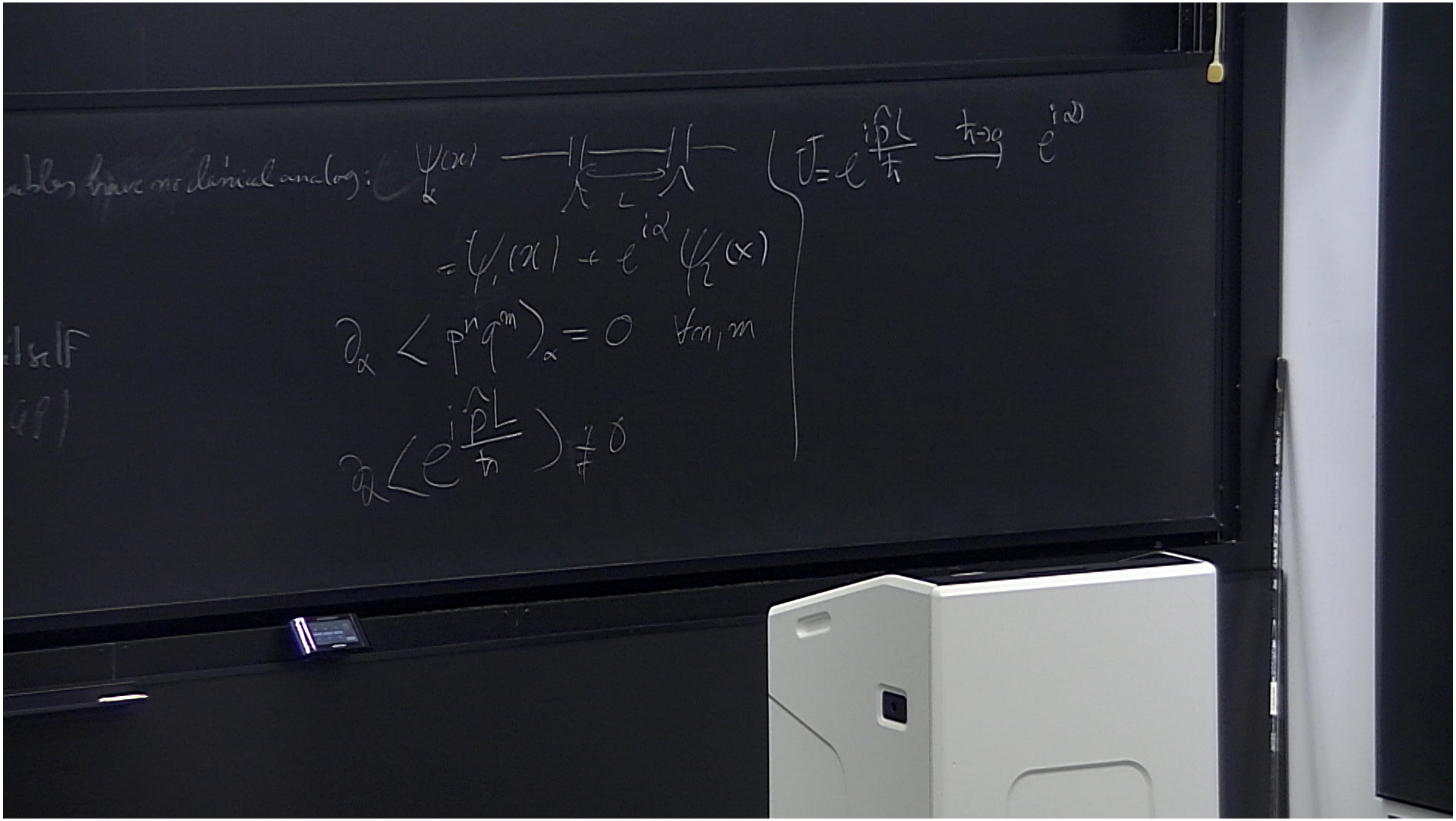
- Coarse graining:

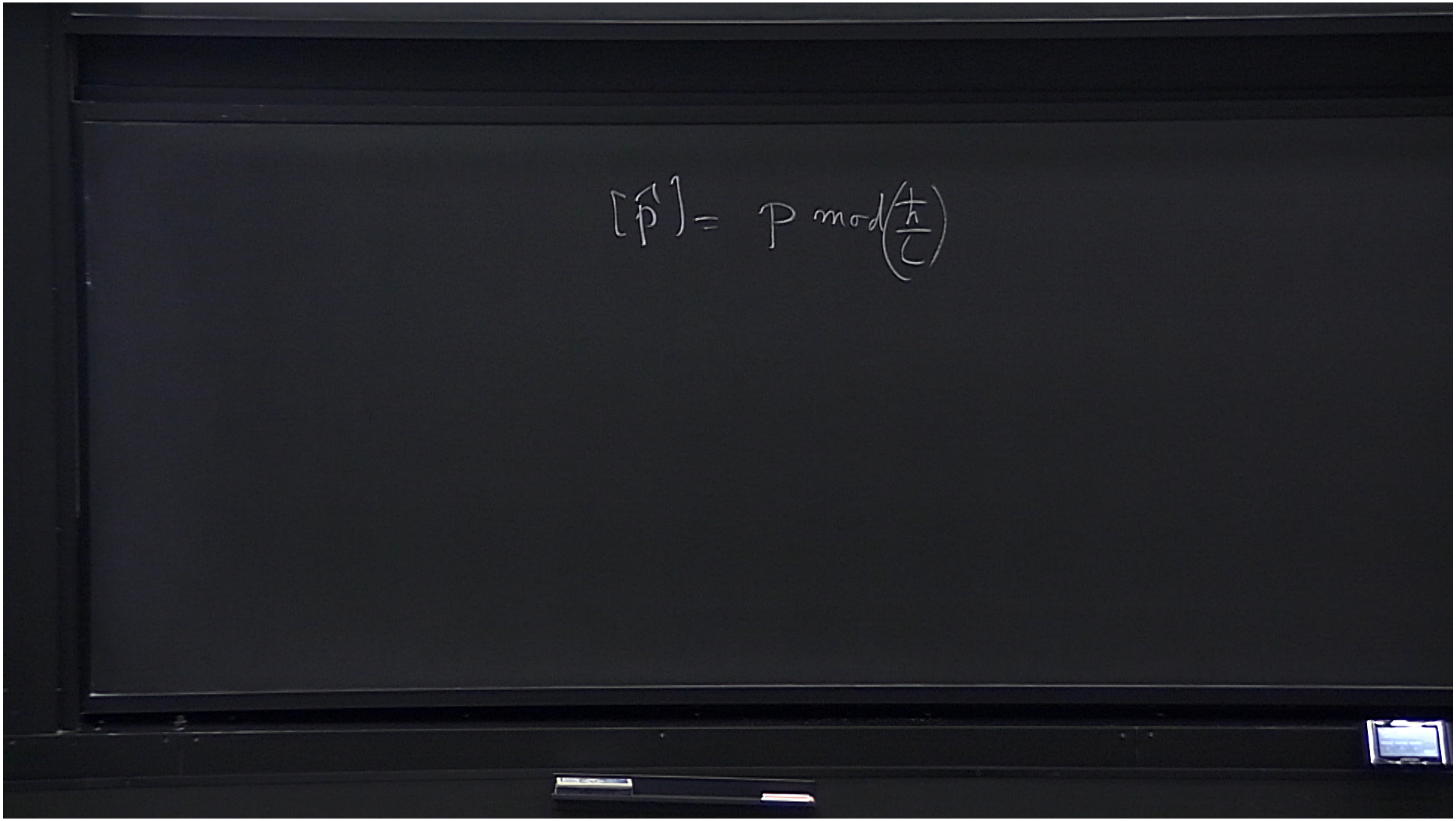
  Classical ~ "Large" $\longrightarrow$ Univerce itself

  Quantum "Small" $\longrightarrow$ (Photon, spin QP)

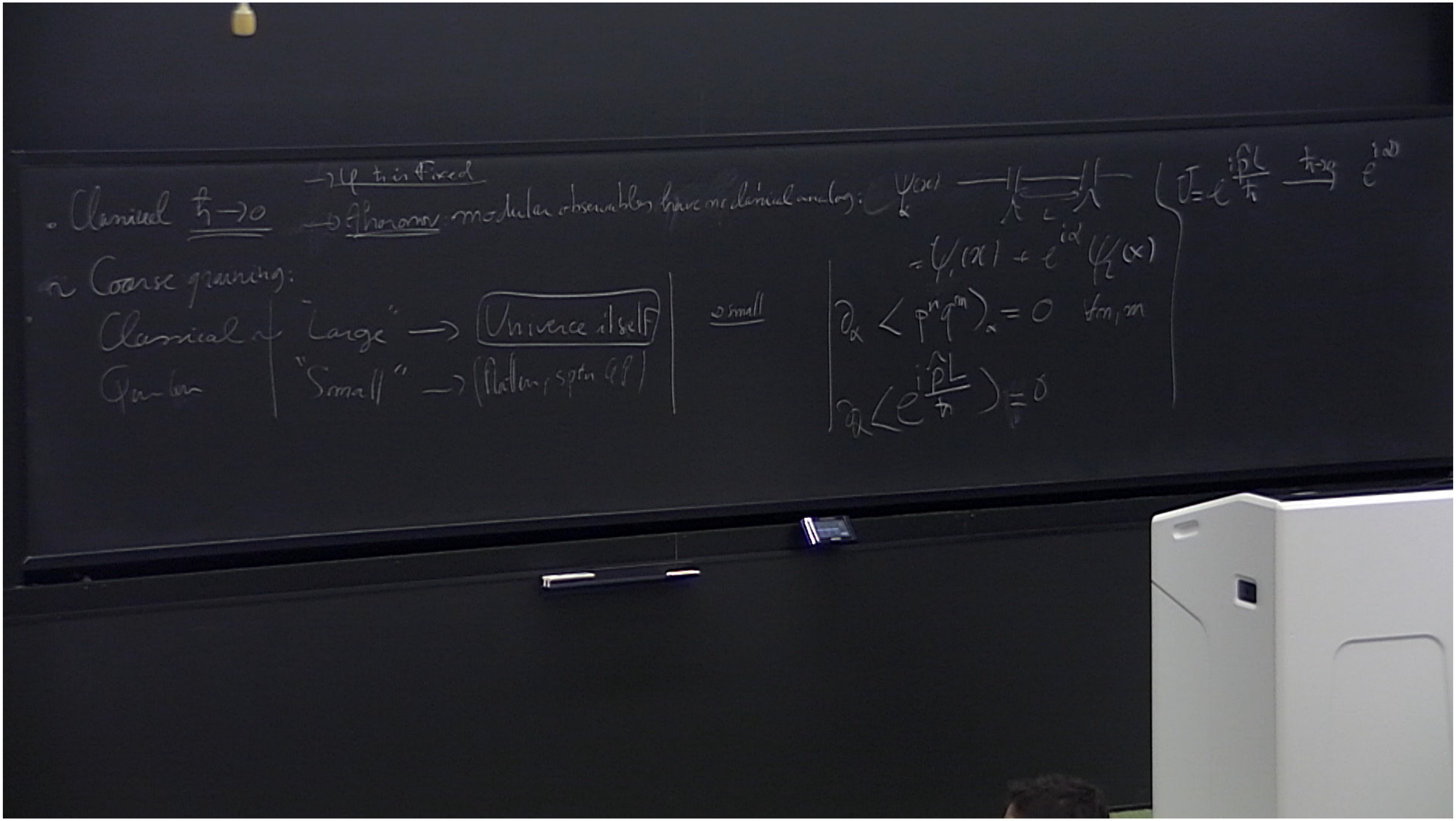

→ If ħ is Fixed
• Classical ħ → 0 → Phenomena: modular observables have no classical analog: ψ
• Coarse graining:
Classical ~ "Large" → Universe itself
Quantum "Small" → (Matter, spin 1/2)


modular observables have no classical analog:
ψ_α(x)
= ψ_1(x) + e^{iα} ψ_2(x)
∂_α ⟨p^n q^m⟩_α = 0 ∀ n, m
"Large" → Universe itself
"Small" →


ables have no classical analog:
$\psi_\alpha(x)$
L
$= \psi_1(x) + e^{i\alpha}\psi_2(x)$
$\partial_\alpha \langle p^n q^m \rangle_\alpha = 0 \quad \forall n, m$
$\partial_\alpha \langle e^{i\frac{\hat{p}L}{\hbar}} \rangle \neq 0$
$U = e^{i\frac{\hat{p}L}{\hbar}} \xrightarrow{\hbar \to 0} e^{i\alpha}$
itself

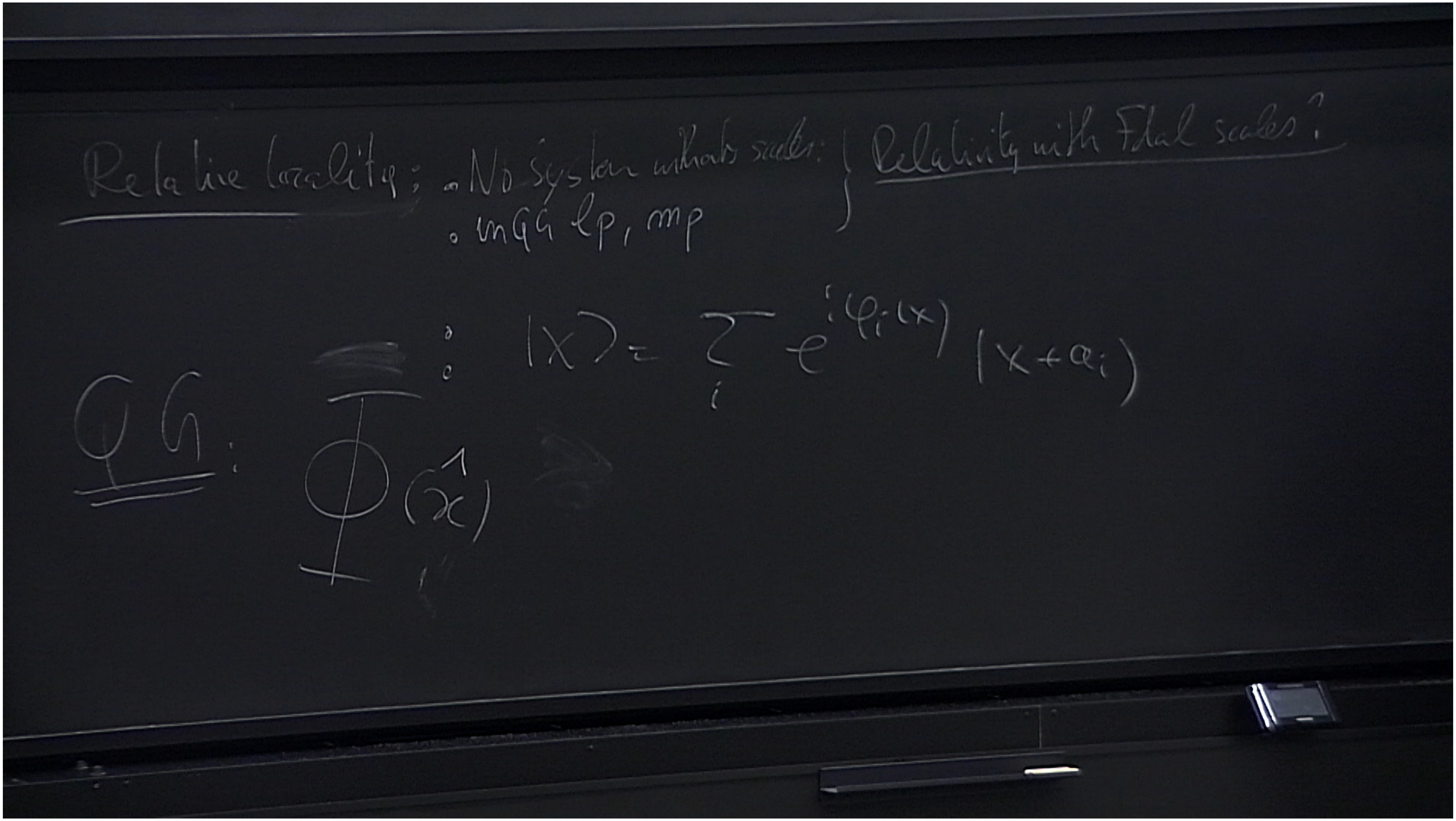


$[\hat{p}'] = p \bmod \left(\frac{\hbar}{L}\right)$


Classical $\hbar \to 0$
$\to \psi$ $\hbar$ is fixed
$\to$ Phenomena: modular observables have no classical analog: $\psi_\alpha(x)$
$= \psi_1(x) + e^{i\alpha}\psi_2(x)$
Coarse graining:
Classical "Large" $\to$ Universe itself
Quantum "Small"
$\partial_\alpha \langle p^n q^m \rangle_\alpha = 0 \quad \forall n, m$
$\partial_\alpha \langle e^{i\hat{p}L/\hbar} \rangle = 0$
$U = e^{i\hat{p}L/\hbar} \xrightarrow{\hbar \to 0} e^{i\alpha}$

- normal → $|0\rangle$ unique. Dec Z-8 IR
- random → ∞ # of vacua: with A.B. ϕhases.
  Y.M., Gravity, QED

$$(\Phi(x), \varphi(y)) = \delta(x-y)$$


Relative locality:
• No system without scales:
• in QG l_p, m_p
Relativity with Fixed scales?
QG:
$|x\rangle = \sum_i e^{i\varphi_i(x)} |x+a_i\rangle$
$\hat{\Phi}(\hat{x})$

...al scales?

$\rightarrow$

$= U_\Lambda |x\rangle$

$$\phi(\Lambda x) = U_\Lambda \phi(x) U_\Lambda^\dagger$$

$$|\Lambda x\rangle = \hat{\Lambda} |x\rangle$$

$\boxed{\text{What is QS?}}$

$\psi(x) \longrightarrow \Phi(x)$

Microcausal Locality

$[\phi(x), \phi(y)] = 0 \quad (x-y)^2 > 0$

$x : \hat{x}; \quad O_n(\hat{x})\, \psi(x) = O_c(x)\, \psi(x)$

Space: Choice of Polarization

" of Commutative Subalgebra

$\boxed{\text{QS?}}$

$\psi(x) \longrightarrow \Phi(x)$

x: $\hat{x}$; $O_n(\hat{x})\,\psi(x) = O_c(x)\,\psi(x)$

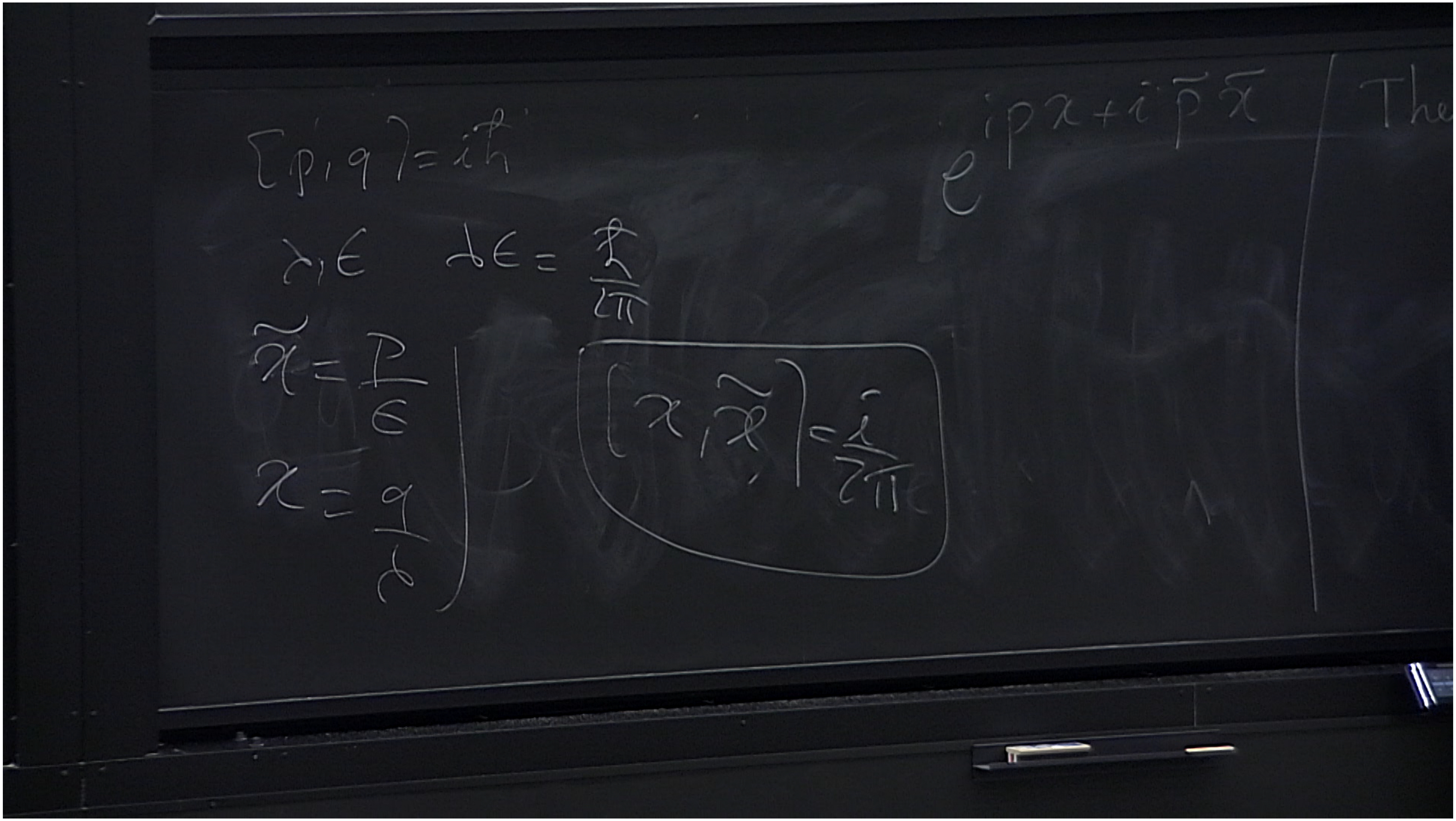
Space: Choice of Polarization

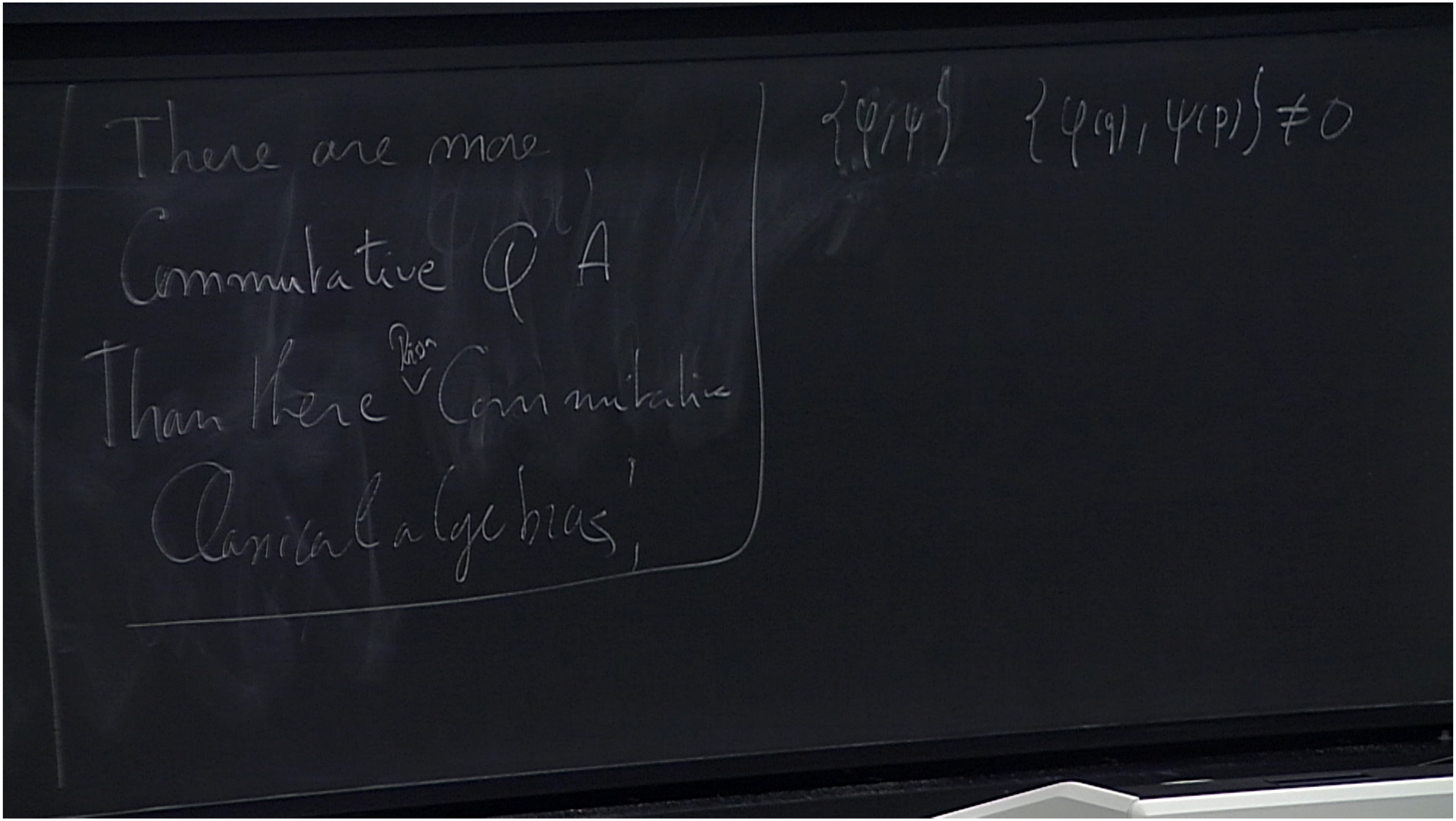
" of Commutative Subalgebra

Microcausal Locality

$[\phi(x), \phi(y)] = 0 \quad (x-y)^2 > 0$

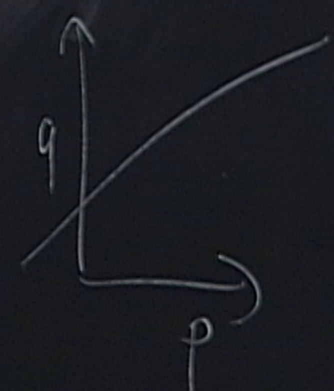
Q. Space is the spectra of a Commutative subalgebra


$[p, q] = i\hbar$
$\lambda, \epsilon \quad \lambda\epsilon = \frac{\hbar}{2\pi}$
$\tilde{x} = \frac{p}{\epsilon}$
$x = \frac{q}{\lambda}$
$[x, \tilde{x}] = \frac{i}{2\pi}$
$e^{ipx + i\tilde{p}\tilde{x}}$
The


There are more
Commutative QA
Than there Poisson Commutative
Classical algebras!
{φ, ψ}   {φ(q), ψ(p)} ≠ 0

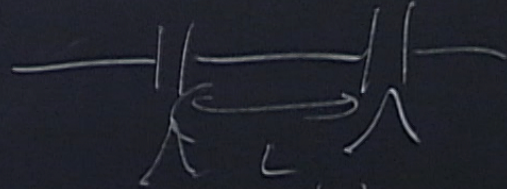
There are more
Commutative QA
Than there Commutative
Classical algebras!

$\{\varphi, \psi\}$ $\{\varphi(q), \psi(p)\} \neq 0$



$R^d \to L^2(\mathbb{R}^d)$

e.g.: $\psi_\alpha(x)$



$$= \psi_1(x) + e^{i\alpha}\psi_2(x)$$

$$\partial_\alpha \langle p^n q^m \rangle_\alpha = 0 \quad \forall n, m$$

$$\partial_\alpha \langle e^{\frac{i\hat{p}L}{\hbar}} \rangle \neq 0$$

$$U(\hat{p}) = e^{\frac{i\hat{p}L}{\hbar}} \xrightarrow{\hbar \to 0} e^{i\alpha} \quad [q]$$

$$V(\hat{q}) = e^{\frac{i2\pi\hat{q}}{L}}$$

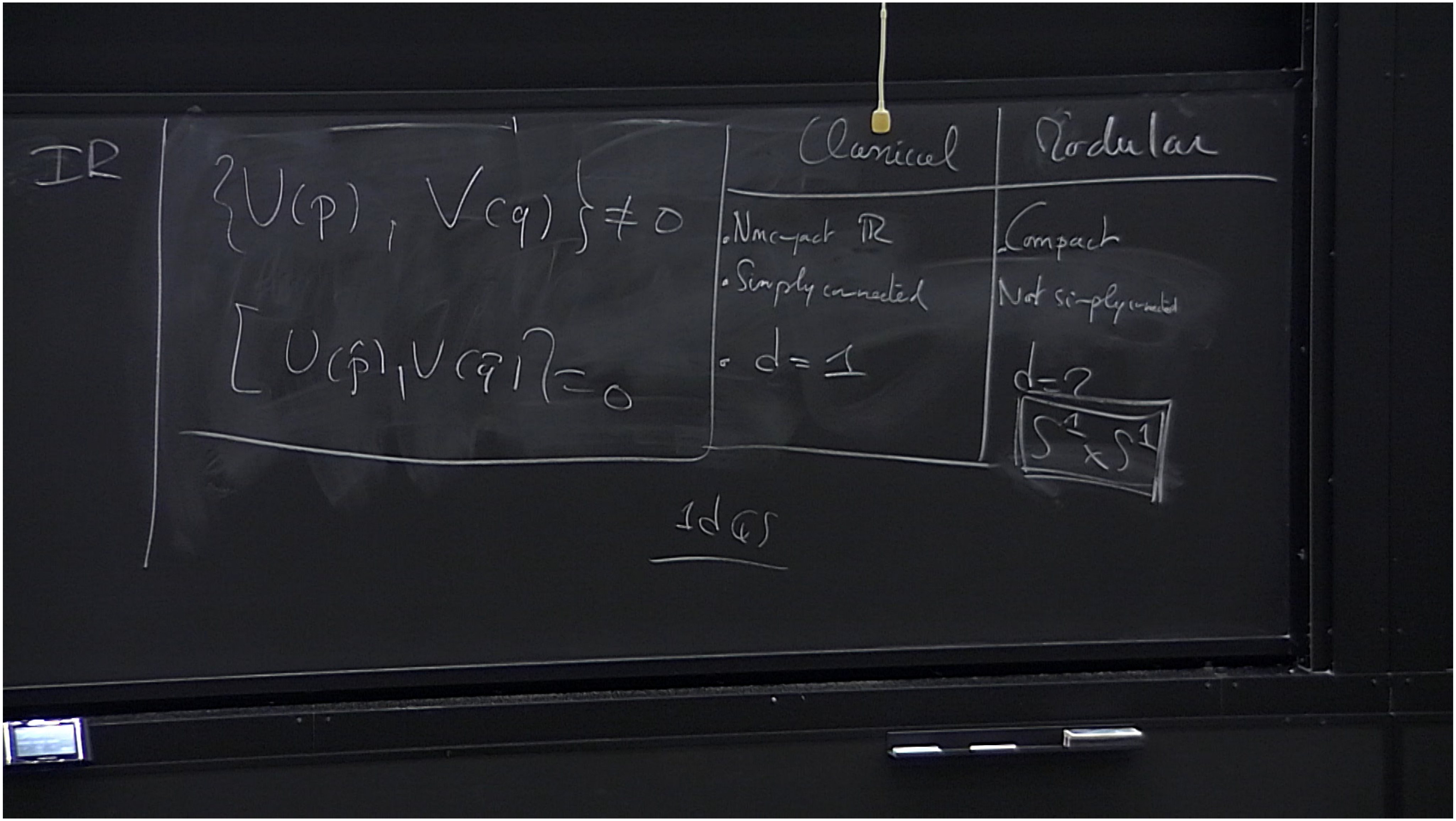
$$U(p) = U\left(p + \frac{\hbar 2\pi}{L}\right)$$

$$V(q) = V(q + L)$$

IR

$$\{U(p), V(q)\} \neq 0$$

$$[U(\hat{p}), V(\hat{q})] = 0$$


IR
$\{U(p), V(q)\} \neq 0$
$[U(\hat{p}), V(\hat{q})] = 0$
Classical
Modular
• Noncompact $\mathbb{R}$
• Simply connected
• $d = 1$
Compact
Not simply connected
$d = 2$
$\boxed{S^1 \times S^1}$
1d QS