Title: Singularity resolution in loop quantum gravity: Emergence of non-Riemannian spacetimes

Date: Nov 10, 2016 02:30 PM

URL: http://pirsa.org/16110049

Abstract: <p>It is a common expectation in quantum gravity that the fundamental nature of space-time would be radically different from the smooth continuum of classical general relativity. In this talk it shall be shown that a quantum modification from loop quantum gravity crucial for singularity resolution is also responsible for deforming the underlying space-time in a manner which cannot be realized using classical geometric structures. With the minimum requirement that the quantum theory satisfies a well-defined notion of covariance explicit midisuperspace models manifesting such non-Riemannian space-times shall be shown to exhibit the physical phenomenon of non-singular signature change. Robustness and implications of this effect shall also be briefly discussed.</p>

Singularity resolution in loop quantum gravity: Emergence of non-Riemannian spacetimes

Suddhasattwa Brahma

Center for Field Theory and Particle Physics
Fudan University

S.B., 1411.3661
M. Bojowald, S.B., U. Büyükçam & F. D'Ambrosio, 1610.08355
M. Bojowald, S.B., & J. Reyes, 1507.00329
M. Bojowald & S.B., 1407.4444, 1507.00679, 1610.08850, 1610.08850
J. Ben Achour, S.B., J. Grain & A. Marcianò, 1608.07314, 1610.07467

November 10, 2016

Big Picture

→ We work with midisuperspace models in LQG:

- Symmetry reduced model (spherically symmetric gravity models or Gowdy systems) simple enough playground to test ideas of LQG yet shares some features of full $(3+1)$-d gravity such as inhomogeneity and/or anisotropy.
- Quantization of such models exhibit subtle features of the full theory such as singularity resolution.
- Has interesting implications for physical (early-universe) cosmology and black hole models.

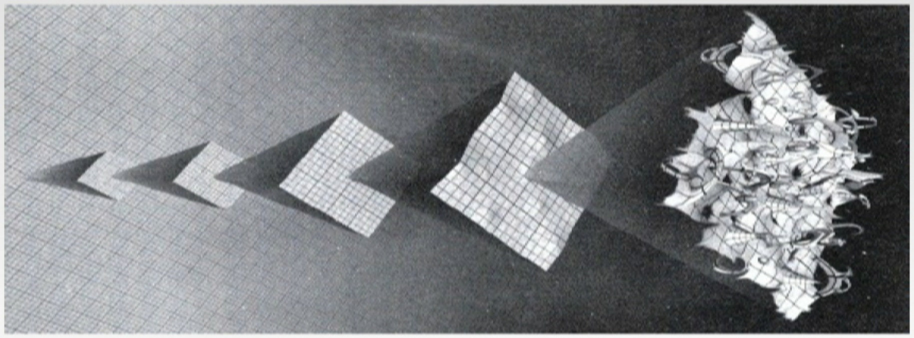
However, important to check consistency conditions of such quantizations ⇒ An important criterion for anomaly-freedom is that the canonical quantization respects some notion of general covariance.

Big Picture

→ Use covariance as a guiding principle to incorporate nontrivial quantum modifications from LQG ⇒ Leads to some deformed notion of underlying symmetries in background independent quantizations.

→ Emergence of non-Riemannian space time structures ⇒ Consequences of such non-classical backgounds.

→ However, several obstructions when quantizing models with local physical degrees of freedom due to such deformations ⇒ A challenge is to avoid them in a consistent fashion.

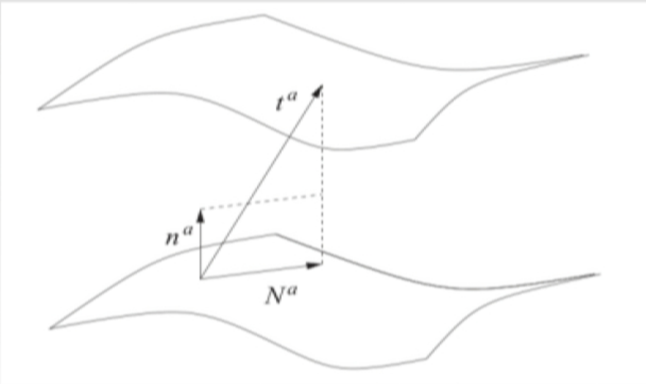


Picture from the blog 'Faithful to Science'.

Covariance in canonical gravity

→ Why is general covariance an important consistency condition in the canonical formulation?

- In the Lagrangian formulation, space and time are treated equally and on the same footing.
- In the Hamiltonian formulation, split space and time by using an arbitrary (time) function to foliate globally hyperbolic spacetime.
- À priori, covariance not manifest.



Is covariance lost due to splitting?

NO!

Hamiltonian for GR:

$$H^{\rm tot}_{\rm grav} = \int \mathrm{d}^3x \left(N C_{\rm grav} + N^a C_a^{\rm grav} - \Lambda^i \mathcal{G}_i\right)$$

where $C_{\rm grav}$, $C_a^{\rm grav}$ & $\mathcal{G}_i$ are the 'Hamiltonian', 'Diffeomorphism' and 'Gauss' Constraints respectively.

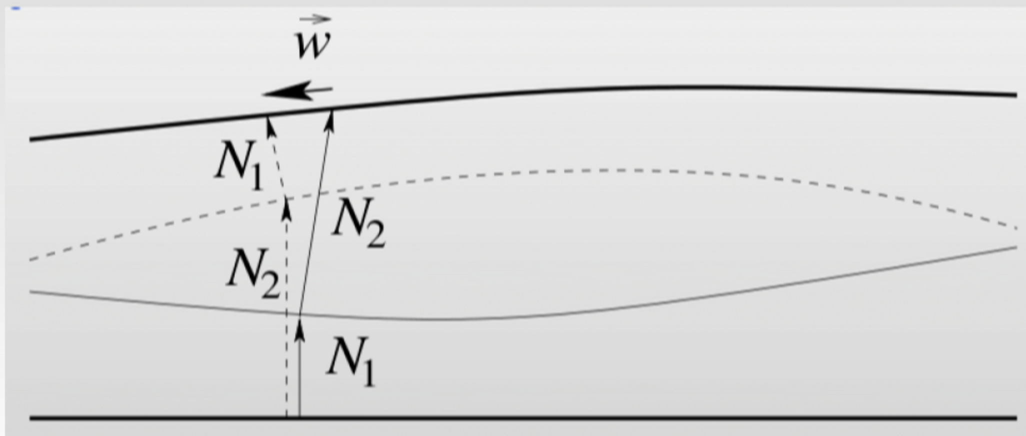
- Constraints are first-class and thus generate gauge transformations, which do not change the physical solutions.
- Hamiltonian Constraint $\rightarrow$ time, Diffeomorphism Constraint $\rightarrow$ spatial co-ordinates, Gauss Constraint $\rightarrow$ rotation of triads (local frame).
- Once these constraints are satisfied, the formalism is space-time covariant, even though we started with slicing of space-time by a time function t.
- The classical constraints satisfy the (Dirac) hypersurface deformation algebra.

Covariance is retained - I

→ Hypersurface deformation algebra (HDA) of classical space-time (generalization of local Poincaré algebra):

$$
\begin{aligned}
\{S(w_1), S(w_1)\} &= S\left(\mathcal{L}_{w_1} w_2\right) \\
\{T(N), S(w)\} &= -T\left(\mathcal{L}_w N\right) \\
\{T(N_1), T(N_2)\} &= S\left(\sharp_q\left(N_1 \mathrm{d} N_2 - N_2 \mathrm{d} N_1\right)\right)
\end{aligned}
$$

with N: function on space, w: vector field and q: metric on spatial slice.



Covariance is retained - II

→ [Dirac, 1951]: Invariance under Hypersurface Deformation Algebra implies general covariance.

- The Hamiltonian and Diffeomorphism Constraints (as derived from GR) satisfy the hypersurface deformation algebra.
- Gauge transformations represent coordinate freedom: space-time Lie derivative of a function given by $\{f, H[\epsilon] + D[\xi^a]\} = \mathcal{L}_{(\epsilon/N, \xi^a + \epsilon N^a/N)} f$ if constraints are satisfied (time direction $t^a = Nn^a + N^a$).

→ [Hojman, Kukař & Teitelboim, 1974-76]: Second-order field equations invariant under hypersurface deformation algebra must equal Einstein's equation ⇒ Slicing Independence.

→ Off-shell property ⇒ not only true for solutions of GR ⇔ any metric can be used to contract indices in the action, not necessarily only solutions of Einstein's equations.

Quantum modifications to spacetime structure

→ Both the Lagrangian density as well as the measure may be subject to quantum corrections

$$S[g] = \frac{1}{2\kappa} \int d^4x \sqrt{|\det g|} \; (R[g] + \cdots)$$

→ Canonical quantization indicates spacetime structures which may not have an underlying (pseudo)-Riemannian geometry any longer ⇒ no metric structure may exist in higher curvature regimes.

→ (Loop) quantum gravity only an 'effective', 'coarse-grained' picture of some exotic microscopic structure (noncommutative geometry, nonassociatve geometry, fractal spacetimes ...?)

What is a generally covariant quantization?

→ Two conditions necessary [M. Bojowald, S.B. and J. Reyes, 2015]:

- The classical constraints must be replaced by generators which still have closed brackets, computed either as Poisson brackets in an effective theory, or as commutators of operators in a quantization.
- Brackets of the new generators of gauge transformations must have a classical limit identical with the classical Dirac algebra.

→ Stronger conditions than anomaly free reformulated systems ⇒ Not sufficient to have a closed quantum (or effective) algebra, but also a well-defined classical limit. Examples:

- $C = H + D; \{C, C\} = 0$ [R. Gambini and J. Pullin, 2013].
- $\{H, H\} = \{D', D'\}$ [Tomlin and Varadarajan, 2012].

What is a generally covariant quantization?

→ Two conditions necessary [M. Bojowald, S.B. and J. Reyes, 2015]:

- The classical constraints must be replaced by generators which still have closed brackets, computed either as Poisson brackets in an effective theory, or as commutators of operators in a quantization.
- Brackets of the new generators of gauge transformations must have a classical limit identical with the classical Dirac algebra.

→ Stronger conditions than anomaly free reformulated systems ⇒ Not sufficient to have a closed quantum (or effective) algebra, but also a well-defined classical limit. Examples:

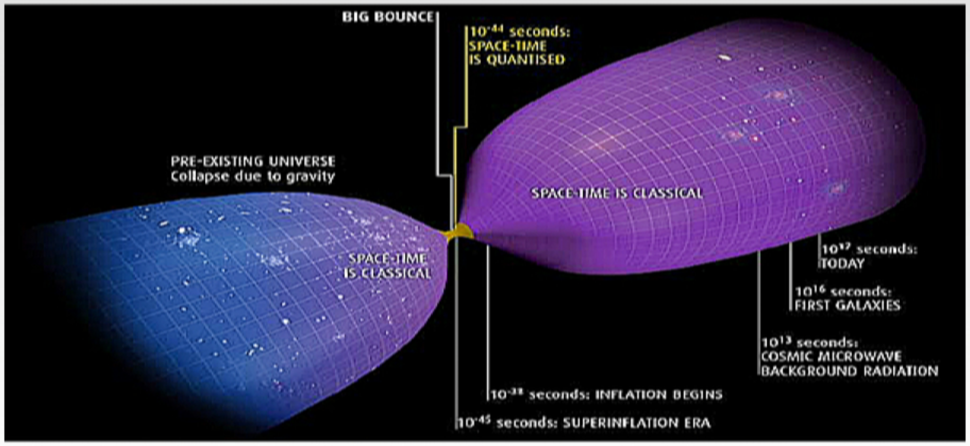
- $C = H + D; \{C, C\} = 0$ [R. Gambini and J. Pullin, 2013].
- $\{H, H\} = \{D', D'\}$ [Tomlin and Varadarajan, 2012].

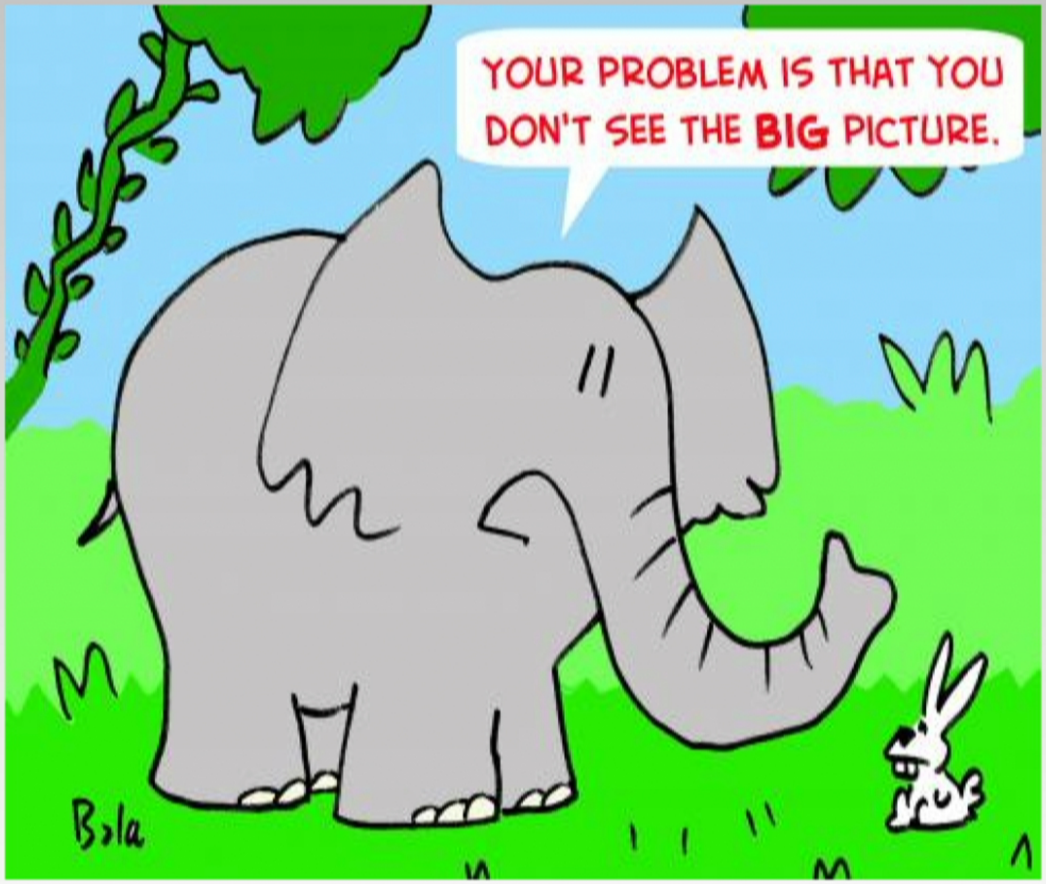
Singularity resolution in LQG

→ Crucial feature of singularity resolution in models of LQG ⇒ Polymerization of connection components to parameterize holonomy modifications $K \to f(K)$ (Usual choice of $\delta^{-1}\sin(\delta K)$ for curvature regularization in lowest spin representation).

→ Examples:

- LQC [M. Bojowald, 2001; A. Ashtekar, T. Pawloski and P. Singh, 2006; ...]
- Black hole models [R. Gambini and J. Pullin, 2013; A. Corichi, J. Olmedo and S. Rastgoo].




YOUR PROBLEM IS THAT YOU DON'T SEE THE BIG PICTURE.
Bala

Recap: Anomaly problem in quantum gravity

→ A first class system of constraints (f_{ij}^k are phase-space functions for gravity):

$$\{C_i, C_j\} = f_{ij}^k C_k$$

→ The constraints are turned into quantum operators (or *effective constraints*) by including quantum corrections:

$$C_i \xrightarrow{\text{quantum corrections}} \hat{C}_i$$

In particular, consider holonomy corrections from LQG – at the heart of singularity resolution.
→ Anomaly freedom implies that the quantum version obeys

$$[\hat{C}_i, \hat{C}_j] \sim \hat{g}_{ij}^k \hat{C}_k$$

→ Classical limit of the quantum constraint algebra must reproduce hypersurface deformation algebra.

Deformed Covariance - I

→ Vacuum spherically symmetric model: [S.B., 2014; M. Bojowald & S.B., 2015; J. Reyes, 2009]

- Two canonical pairs (K_ϕ, E^ϕ) and (K_x, E^x), with $\mathrm{d}s^2 = \frac{(E^\phi)^2}{|E^x|}\mathrm{d}x^2 + |E^x|\mathrm{d}\Omega^2$.
- Implement local holonomy corrections through polymerization: $K_\phi \to f(K_\phi)$ (Keep function arbitrary for our purposes to allow for quantization ambiguities).
- The constraint algebra is closed, but deformed (for the structure function).

$$[H[N_1], H[N_2]] = D\left[\beta q^{ab}\left(N_1 \nabla_b N_2 - N_2 \nabla_b N_1\right)\right]$$

- Deformation function second derivative of holonomy correction function → "Signature Change" around maxima of the bounded function.

Deformed covariance - II

→ Consistent deformation ⇒ no gauge conditions broken.

→ Once β changes sign, HDA has the same sign as for Euclidean gravity.

→ No effective line element available on standard manifold ⇒ $\mathrm{d}x^a$ in $\mathrm{d}s^2 = \tilde{q}_{ab}\mathrm{d}x^a\mathrm{d}x^b$ does not transform under the same deformed gauge transformations as $\tilde{q}_{ab}$.
Field redefinition can absorb β to give standard HDA brackets *as long as* β does not change sign.

→ 'Signature change' resolves classical singularity ⇒ New model of quantum spacetime with no Riemannian structure.

→ Treat Hamiltonian formulation as the fundamental theory and can evaluate all canonical observables of the deformed gauge theory.

HDA as a Lie algebroid

→ Lie algebroid: $(A, [.,.]_A, \rho)$ with $\rho : \Gamma(A) \to \Gamma(TB)$, such that ρ satisfies a homomorphism of Lie algebras and a Leibnitz identity.

→ Hypersurface deformation brackets form a Lie algebroid → Phase space (q_{ab}, K^{ab}) forms base manifold → Lagrangian multipliers (N, N^a) forms $(4 \times \infty)$−dimensional fibers.

→ Lie algebroid morphisms can change the deformation function $\beta(q_{ab}, K^{ab})$:[M. Bojowald, S.B., U. Büyükçam & F. D'Ambrosio, 2016]

- $q_{ab} \mapsto |\beta|^{-1} q_{ab}$ generated by base transformations.
- $N \mapsto \sqrt{|\beta|^{-1}} N$ generated by fiber maps (same as a non-standard normal for β spatially constant).

→ No algebroid morphisms can remove $\text{sgn}(\beta)$ ⇒ No Riemannian structure when β changes sign.

Ubiquitousness of signature change in LQG

→ Similar conclusions for loop quantization of cosmological perturbations with holonomy modifications. [A. Barrau, T. Cailleteau, L. Linsefors & J. Grain, 2012; M. Bojowald & Mielczarek, 2015]

→ Loop quantization of the $(1+1)-$dimensional CGHS black hole [M. Bojowald & S.B., 2016]

- Classical singularity resolved.
- Structure functions get deformed ⇒ nontrivial to prove that the deformation function changes sign but has been shown to be true.

→ Generalized midisuperspace model considered [M. Bojowald & S.B., 2016]

- One inhomogeneous direction assumed for the model (special cases: Schwarzschild black hole, Gowdy models, CGHS black hole, 2-dimensional dilaton gravity etc.)
- Triad terms not fixed according to specific models but any term with proper density weight and up to second order derivatives considered.
- Holonomy modifications necessarily leads to signature change.

(Partial) No-Go Theorems!

[M. Bojowald, S.B. & J. Reyes, 2015; M. Bojowald & S.B., 2015]

→ Theories with local degrees of freedom such as spherical symmetry with matter OR Gowdy models → Implement holonomy corrections → Difficult to attain closure of the algebra.

- Very general form of modification functions assumed.
- Poisson structure after polymerization assumed to be of the form $\{f(K_\phi)(x), E^\phi(y)\} = G\dot{f}\delta(x,y)$.

→ For instance, in the spherical symmetry case, $[H_{\rm grav}, H_{\rm grav}] = \beta D_{\rm grav}$ while $[H_{\rm matter}, H_{\rm matter}] = D_{\rm matter}$. Thus brackets with full constraint $H_{\rm total} = H_{\rm grav} + H_{\rm matter}$ doesn't seem to close into $D_{\rm total}$.

→ Is this unique to our treatment? How big is the problem?

(Partial) No-Go Theorems!

[M. Bojowald, S.B. & J. Reyes, 2015; M. Bojowald & S.B., 2015]

→ Theories with local degrees of freedom such as spherical symmetry with matter OR Gowdy models → Implement holonomy corrections → Difficult to attain closure of the algebra.

- Very general form of modification functions assumed.
- Poisson structure after polymerization assumed to be of the form $\{f(K_\phi)(x), E^\phi(y)\} = G\dot{f}\delta(x,y)$.

→ For instance, in the spherical symmetry case, $[H_{\rm grav}, H_{\rm grav}] = \beta D_{\rm grav}$ while $[H_{\rm matter}, H_{\rm matter}] = D_{\rm matter}$. Thus brackets with full constraint $H_{\rm total} = H_{\rm grav} + H_{\rm matter}$ doesn't seem to close into $D_{\rm total}$.

→ Is this unique to our treatment? How big is the problem?

Comparison with other work

→ Alternative approach to loop quantization of spherically symmetric model [R. Gambini, J. Pullin, J. Olmedo & M. Campiglia, 2013 - 2016].

- Abelianize the difficult part of the constraint algebra by linear redefinition of constraints (with phase-space dependent coefficients).
- This new system of constraints is loop quantized through exactly same polymerization.
- Require covariant quantizations (closed commutators AND classical limit).
- Remarkably, exact same restrictions imposed on the modification functions as in the algebra closure procedure, for vacuum model.
- For spherical symmetry with matter, possible to 'Abelianize' the classical constraints but obstructions for the holonomy-corrected constraints ⇒ Equivalent to the No-Go theorem.

→ Similar conclusions when compared to loop quantization of the CGHS black hole model [A. Corichi, J. Olmedo & S. Rastgoo, 2016; M. Bojowald & S.B., 2016].

Perturbative higher derivative corrections

→ Can fluctuations and higher moments of the quantum state, generic to any QG theory, *undo* the deformations seen in effective constraint analysis of LQG?

NO! [M. Bojowald & S.B., 2014]

- Quantum back-reaction by moments of a state on expectation values does not lead to deformations of structure functions of the classical algebra, even though the constraints themselves receive perturbative quantum corrections (via moments).
- These conclusions from effective constraints are applicable even in deep quantum regimes, even where the semi-classical approximation breaks down.
- In an effective framework, moment terms ⇒ higher derivative corrections. Undeformed algebra for Effective Constraints including moments ⇒ higher curvature effective actions, constructed from curvature invariants, can be derived from such algebra. [Deruelle, Sasaki, Sendouda, & Yamauchi, 2009]

Possible way(s) out

→ Change the polymerization scheme (additional: $E^{\phi} \to \cos(\rho K_{\phi})^{-1} E^{\phi}$) such that the Poisson bracket remains the same after polymerization.[M. Campiglia, R. Gambini, J. Olmedo & J. Pullin, 2016]

→ Background treatment (e.g: FLRW or Schwarzschild spacetimes):

- Loop quantize the background ⇒ Cosmological perturbations on the 'quantum'-corrected spacetime with *dressed* metric, OR Spherically symmetric matter on a 'quantized' lattice.
- Non-matching version of covariance for gravity and matter ⇒ Difficult to check covariance of the full system with gauge fixings.
- In the cosmological case, quantum correction functions introduced only in the background Hamiltonian and not in the perturbation Hamiltonian ⇒ Issues of consistency?

→ Go back to self dual Ashtekar variables?

Possible way(s) out

→ Change the polymerization scheme (additional: $E^{\phi} \to \cos(\rho K_{\phi})^{-1} E^{\phi}$) such that the Poisson bracket remains the same after polymerization.[M. Campiglia, R. Gambini, J. Olmedo & J. Pullin, 2016]

→ Background treatment (e.g: FLRW or Schwarzschild spacetimes):

- Loop quantize the background ⇒ Cosmological perturbations on the 'quantum'-corrected spacetime with *dressed* metric, OR Spherically symmetric matter on a 'quantized' lattice.
- Non-matching version of covariance for gravity and matter ⇒ Difficult to check covariance of the full system with gauge fixings.
- In the cosmological case, quantum correction functions introduced only in the background Hamiltonian and not in the perturbation Hamiltonian ⇒ Issues of consistency?

→ Go back to self dual Ashtekar variables?

Self-dual Ashtekar variables for LQC perturbations

[J. Ben Achour, S.B., J. Grain & A. Marcianò, 2016]

- Scalar cosmological perturbations ⇒ Gravity plus a minimally coupled scalar around a flat, FLRW background.
- Consider holonomy correction functions for the background connection variable (same as for real LQC) ($c \to f(c)$).
- Modifications in not only the background part but also the perturbations part

$$H_{\rm tot}[N] = \frac{1}{2\kappa}\int \mathrm{d}^3x \left(\bar{N}\left[\mathcal{H}^{(0)}(f) + \mathcal{H}^{(2)}(h_1, h_2, h_3, h_4)\right] + \delta N \mathcal{H}^{(1)}(g_1, g_2)\right).$$

- Closure of algebra gives nontrivial consistency conditions on the various modification functions ⇒ Correct classical limit.

→ The structure functions remain unchanged ⇒ Constraint algebra remains undeformed.

→ (Spatial) diffeomorphism constraint necessarily picks up quantum correction ⇒ Indications of Quantum spacetime?

Summary

→ What have we learnt?

- Requiring covariance is a restrictive consistency condition for constraining (canonical) quantum theories of gravity.
- For interesting effects arising from LQG, can give rise to truly quantum spacetime with no classical structures available anymore.
- However, closure of quantum algebra difficult to obtain in some models of LQG.
- Use self-dual variables in midisuperspace models where the no-go theorem was valid and loop quantize models with local degrees of freedom.

Looking ahead

→ Deformed covariance can produce interesting phenomenology:

- Flat limit of deformed covariance models lead to $\kappa-$Minkowski noncommutative spacetime [G. Amelino-Camelia, M. Da Silva, M. Roncho, L. Cesarini & O. Lecian, 2016].

  Study the modified dispersion relation for such deformations [G. Amelino-Camelia, S.B., A. Marcianò & M. Roncho, 2016].

- Study algebra of constraints for noncommutative and nonassociative geometries as well as fractal spacetimes. Suitable coarse graining strategies lead to classical Riemannian geometries.[M. Bojowald, S.B. & U. Büyükçam, *in preparation*]

→ Loop quantization of (symmetry-reduced) models with self-dual Ashtekar variables:

- Understand the gauge transformations generated by modified (spatial) diffeomorphism constraint ⇒ Consequences for Kukař, Teitelbohm, Hoffman uniqueness theorem.
- Derive a polymerization function for $SL(2, C)$ variables for holonomy corrections. Analytic continuation in some models indicate possibility to do so and still get bounded functions. [J. Ben Achour, K. Noui & A. Perez, 2016; J. Ben Achour, J. Grain & K. Noui, 2014].