

Title: Gravitational Vacuum Decay and Inflation

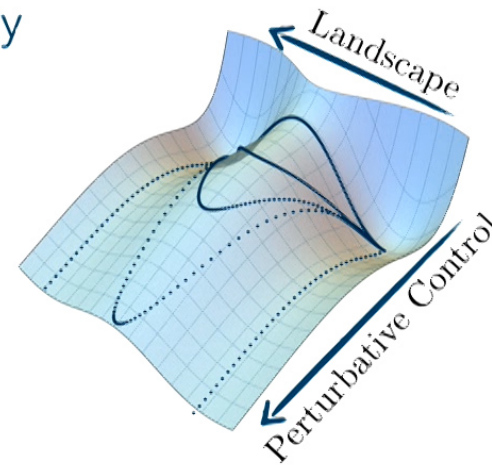
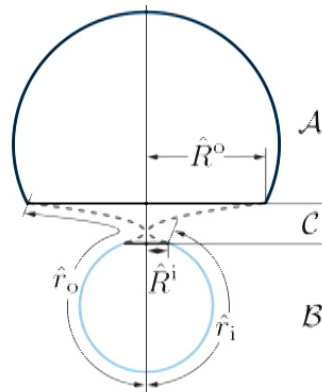
Date: Nov 01, 2016 11:00 AM

URL: <http://pirsa.org/16110047>

Abstract: <p>We argue that moduli stabilization severely constrains the evolution following transitions between weakly coupled de Sitter vacua and can induce a strong selection bias towards inflationary cosmologies. We carefully discuss gravitational vacuum decays and resolve a naive sign ambiguity in the exponential of the decay rate. Equipped with this clear understanding of vacuum decay we then turn towards constraints on the cosmological evolution after transitions in weakly coupled flux compactifications. The energy density of domain walls between vacua typically destabilizes Kahler moduli and triggers a runaway towards large volume. This decompactification phase can collapse the new de Sitter region unless inflation lasts for more than roughly 60 efolds. High scale inflation is vastly favored. Our results illustrate the necessity to understand inflationary initial conditions at least at a basic level, before making predictions in the landscape.</p>

# Gravitational Vacuum Decay and Inflation

Thomas Bachlechner  
Columbia University



T.B., *Inflation Expels Runaways*, hep-th/1608.07576

T.B., F. Denef, K. Eckerle, R. Monten, *to appear*

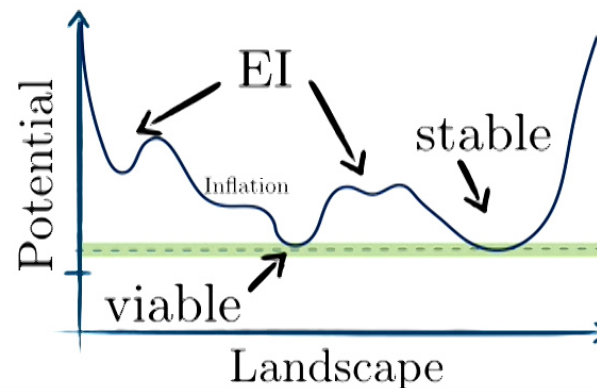
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Cosmological observations are in excellent agreement with inflationary initial state and  $\Lambda$ CDM cosmology.

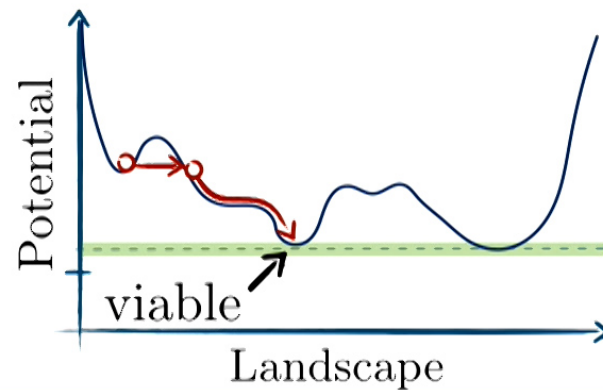
Cartoon of landscape idea:

Vacuum energy : Many vacua + transitions + selection bias

Initial state : Hope for the best?

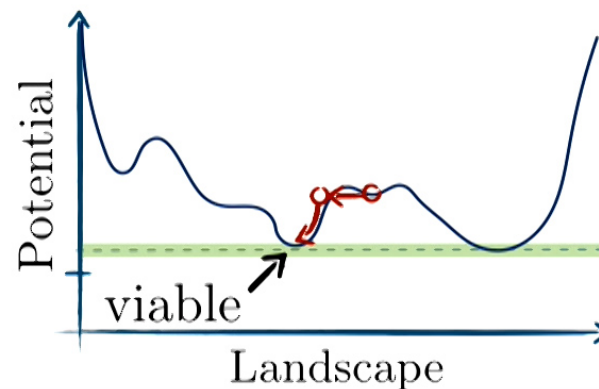


1. Coleman-de Luccia decay from left EI vacuum:  
Transition rate:  $\Gamma \sim e^{-S_E}$ , gives slow roll inflation



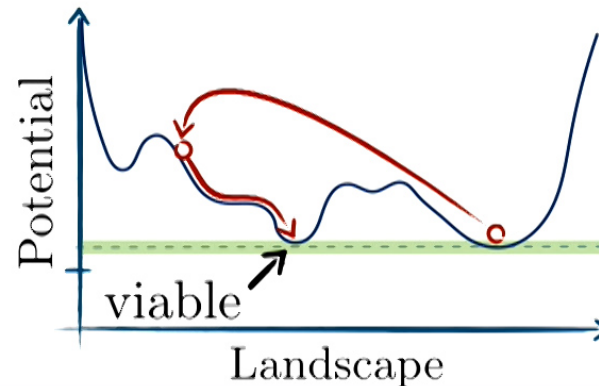


1. Coleman-de Luccia decay from left EI vacuum:  
Transition rate:  $\Gamma \sim e^{-S_E}$ , gives slow roll inflation
2. Coleman-de Luccia decay from right EI vacuum:  
Transition rate:  $\Gamma \sim e^{-S_E}$ , no slow roll inflation



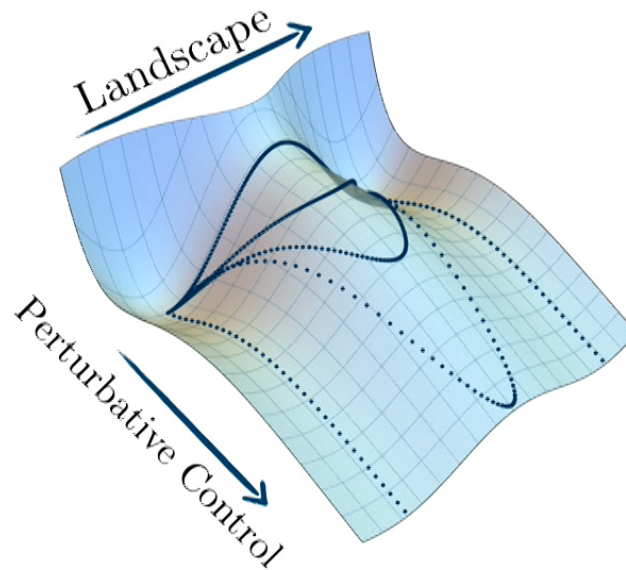


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2. Coleman-de Luccia decay from right EI vacuum:  
Transition rate:  $\Gamma \sim e^{-S_E}$ , no slow roll inflation
3. Farhi-Guth-Gueven transition from stable vacuum:  
Transition rate:  $\Gamma \sim e^{\pm S_E}$ ? Maybe slow roll inflation



Moduli Stabilization is hard.

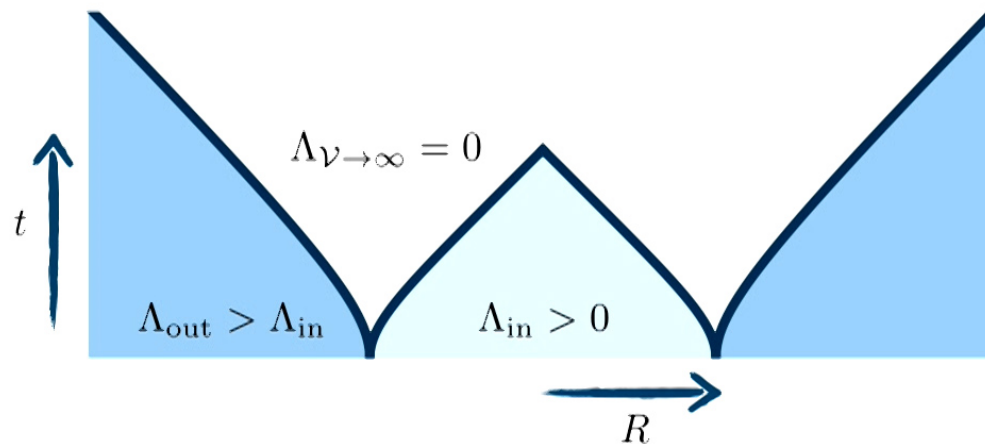
Gravitational couplings destabilize the moduli at the wall.



Moduli Stabilization is hard.

Gravitational couplings destabilize the moduli at the wall.

Try to nucleate bubble, but it collapses!







Let's solve two problems:

1. Carefully compute transition rate in gravitational theory

$$\Gamma_{\mathcal{I} \rightarrow \mathcal{F}} \approx e^{-|\int_{\mathcal{I}}^{\mathcal{F}} dS|}$$

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2. How is the landscape populated if there are no cosmological domain walls?

Bubbles collapse without inflation,

$$\frac{a_{inf}}{a_0} \gtrsim e^{N_{hp}} \sim e^{60}$$

## Gravitational Vacuum Transitions

T.B., F. Denef, K. Eckerle, R. Monten, *to appear*

Spherically symmetric gravity with a thin domain wall

Transition rates from the semi-classical path integral

Gravitational vacuum transition rate

## Inflation Expels Runaways

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Domain walls and moduli stabilization

Viable vacua and inflation in the landscape

## Conclusion

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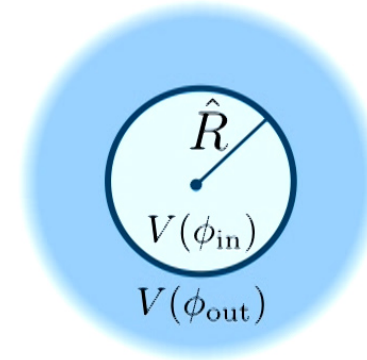
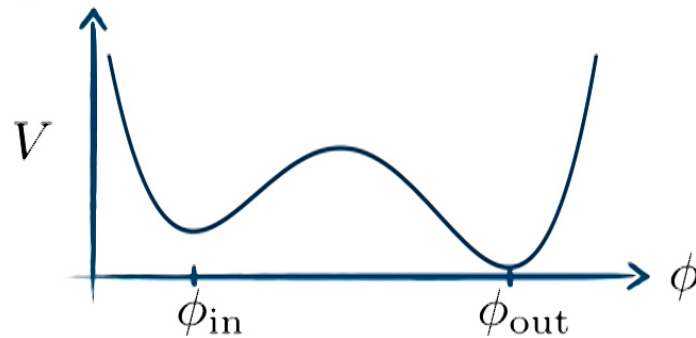
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Consider theory with two de Sitter vacua. Assume a thin domain wall.

Metric: 
$$ds^2 = -AdT^2 + A^{-1}dR^2 + R^2d\Omega_2^2$$

$$A_{\text{in/out}} = 1 - \frac{2GM_{\text{in/out}}}{r} - H_{\text{in/out}}^2 r^2$$

Parameters:  $G$ ,  $\sigma$ ,  $M_{\text{in/out}}$ ,  $H_{\text{in/out}}$

Action:

$$S = \frac{M_{Pl}^2}{2} \int_{\mathcal{M}} d^4x \sqrt{g} \mathcal{R} + \int_{\mathcal{M}} d^4x \sqrt{g} \mathcal{L}_m + \text{b.t.}$$

Matter:

$$T^{\mu\nu} = -g^{\mu\nu} [\sigma \delta(r - \hat{r}) + \rho(r)]$$

Parametrize general, isotropic metric

$$ds^2 = -N^{t^2}(t, r) dt^2 + L^2(t, r) (dr + N^r(t, r) dt)^2 + R^2(t, r) d\Omega_2^2$$

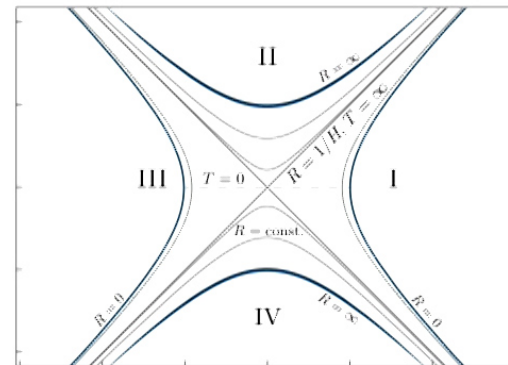
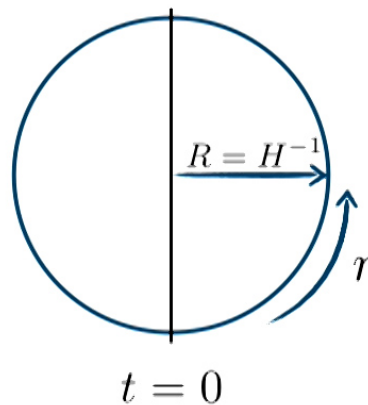
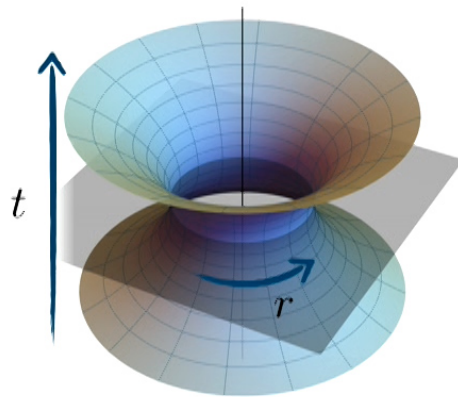


$$ds^2 = -N^{t2}(t, r)dt^2 + L^2(t, r)(dr + N^r(t, r)dt)^2 + R^2(t, r)d\Omega_2^2$$

Simple Example: de Sitter space  $M = 0$ ,  $H > 0$

Pick gauge:  $N^t = 1$ ,  $N^r = 0$

$$L = \cosh(Ht) \quad R = \frac{1}{\chi} \cosh(Ht) \sin(Hr)$$

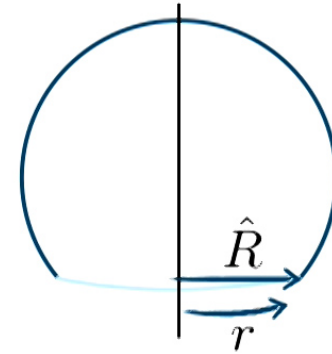




Classical domain wall evolution?

Lapse/shift impose constraints:  $\mathcal{H}_r = \mathcal{H}_t = 0$

Constraints at the domain wall become



$$M_{\text{out}} = \frac{H_{\text{in}}^2 - H_{\text{out}}^2}{2G} \hat{R}_{\text{in}}^3 + 4\pi\sigma \hat{R}_{\text{in}}^2 \text{sign}(\hat{R}'_{\text{in}}) \sqrt{1 - H_{\text{in}}^2 \hat{R}_{\text{in}}^2 - \frac{2GM_{\text{in}}}{\hat{R}_{\text{in}}} + \dot{\hat{R}}_{\text{in}}^2} - 8\pi^2 G \sigma^2 \hat{R}_{\text{in}}^3 + M_{\text{in}}$$

Vacuum energy

Domain wall energy

Surface-Surface interaction



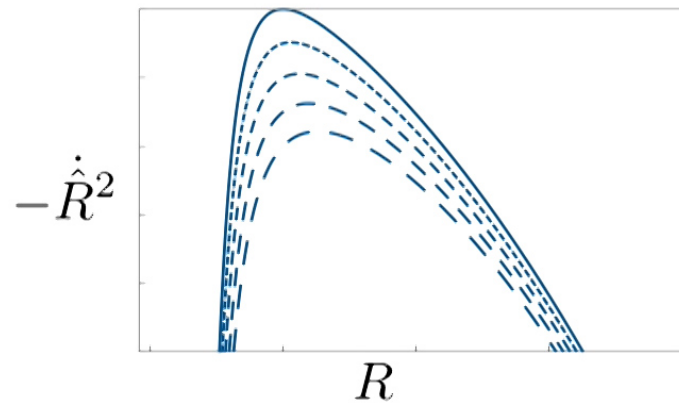
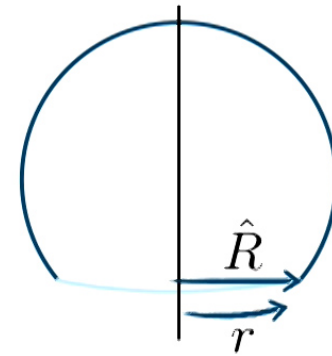


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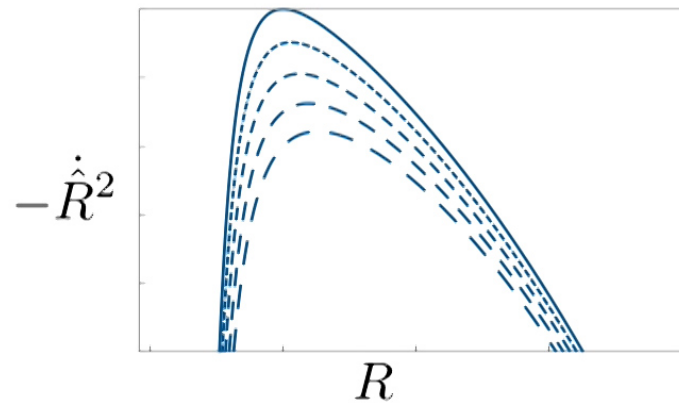
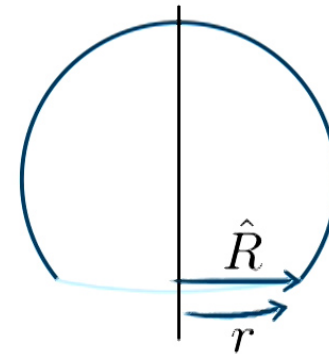


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Tunneling in semi-classical mechanics. Consider a theory, expanded at small  $E$  (c.f.  $f = \pm\sqrt{2[E - V(q)]}$ )

$$\begin{aligned}\tilde{L} &= p\dot{q} + E\dot{\theta} - \Lambda(p - f(q, E)) \\ &\approx p\dot{q} + E\dot{\theta} - \tilde{\Lambda} \left( \underbrace{\frac{1}{f'_{E=0}}(p - f(q, 0)) - E}_{\equiv H} \right)\end{aligned}$$

EoM:  $\dot{\theta} = -\tilde{\Lambda}, \dot{q} = \tilde{\Lambda}/f'_{E=0}, E = H$

In the gauge  $\dot{\theta} = -1$  this is classically equivalent to

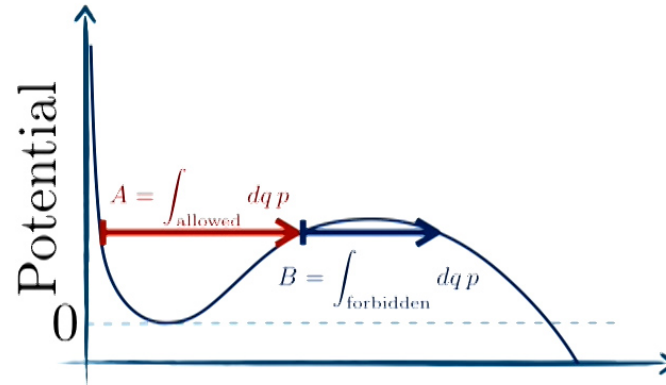
$$L = p\dot{q} - H$$

Canonical quantization,  $E \rightarrow -i\partial_{\theta}$  gives:  $\hat{H}\psi = i\partial_t\psi$

We will only be interested in semiclassical results, disregarding any off-shell contributions.

Compute the poles in the propagator around  $H \approx 0$

$$\begin{aligned}
 \langle q_f | \frac{1}{H - E - i\epsilon} | q_i \rangle &= \int_{t=0}^{\infty} dt e^{i(E+i\epsilon)t} \langle q_f, t | q_i, 0 \rangle \\
 &\sim \int_{t=0}^{\infty} dt e^{i(E+i\epsilon)t} \int \mathcal{D}p \mathcal{D}q \Big|_{q_i}^{q_f} \exp \left( i \int_0^t d\tau (p\dot{q} - H) \right) \\
 &\sim \sum_{\substack{H=E+i\epsilon \\ (q_f, q_i)}} e^{i \int dq p}
 \end{aligned}$$



$$\langle q_c | \frac{1}{H - E - i\epsilon} | q_c \rangle \approx \sum_{n=1}^{\infty} e^{2niA} \left( \sum_{m=0}^{\infty} e^{2miB} \right)^n = \frac{e^{2iA}}{1 - e^{2iA} - e^{2iB}}$$

Expand around  $E = 0$ :  $A \approx (\partial_E A)_{E=0} E \approx E \Delta T$  gives

$$E \approx i \frac{e^{2iB}}{2\Delta T} \quad (\sim e^{\pm \int dq \sqrt{2V}})$$

Remember evolution with time:  $\psi(q, t) \approx e^{-iEt} \psi(q)$

$$\Gamma \sim e^{2iB} = \exp \left( 2i \int_{\text{forbidden}}^{\text{outgoing}} dq p |_{\text{Im}(E) > 0} \right)$$

The sign of the momentum is uniquely determined by the outgoing solution requirement in the allowed region,

$$\dot{q} > 0$$

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Imposing classical constraints away from the shell gives

$$S = \int dt P_{\hat{R}} \dot{\hat{R}} + M_+ \dot{T}(r_{\max}) - \tilde{N} \left( P_{\hat{R}} - f(\hat{R}, M_+) \right)$$

where

$$\pm f(\hat{R}, M_+) = i \frac{\partial_{\hat{R}} \mathbb{A}(\hat{R})}{8G} + \frac{\hat{R}}{G} \log \left( \frac{\hat{R}'_+ - \sqrt{\hat{R}'_+{}^2 - \hat{L}^2 \hat{A}_+}}{\hat{R}'_- - \sqrt{\hat{R}'_-{}^2 - \hat{L}^2 \hat{A}_-}} \sqrt{\frac{\hat{A}_-}{\hat{A}_+}} \right)$$

$\hat{R}$  : Shell radius of curvature

$T$  : Static patch time

$M_+$  : Outside Schwarzschild-de Sitter mass parameter

$$\text{Metric : } ds_{\pm}^2 = -A_{\pm} dT^2 + A_{\pm}^{-1} dR^2 + R^2 d\Omega_2^2$$





Find decay rate  $\Gamma \sim e^{2iB}$

$$2iB = \pm\pi \left( \hat{R}_2 \frac{(H_+^2 - H_+^{-2})^2 + (H_+^2 + H_-^2)\kappa^2}{4\kappa GH_+^2 H_-^2} + \frac{N_{+,I} - N_{+,F} + \frac{\text{sgn}(\dot{\hat{R}}_+)}{2}}{GH_+^2} + \frac{N_{-,I} - N_{-,F} - \frac{1}{2}}{GH_-^2} \right)$$

The velocity of outgoing trajectory:  $\text{sgn}(\dot{\hat{R}}) = \text{sgn}(-2iB) > 0$

$$\Gamma \sim e^{-2|B|}$$

Leading channels for small  $H_+$  :

$H_- < H_+$

$$\Gamma = \Gamma_{CDL} \sim \exp\left(-\frac{27\pi^2\sigma^4}{2\Lambda^3}\right)$$

$H_- > H_+$

$$\Gamma = \Gamma_{FGG} \sim \exp\left(-\frac{\mathbb{A}_{dS}}{4G}\right)$$

### Lessons learned:

Decay rate (& negative modes) crucially depend on an understanding of what clocks are used.

The thin wall vacuum decay rate is given by

$$\Gamma \sim e^{-|S_E|}$$

Vacuum decay is suppressed in the semi-classical limit.

Baby universe nucleation favors low “entropy” states.

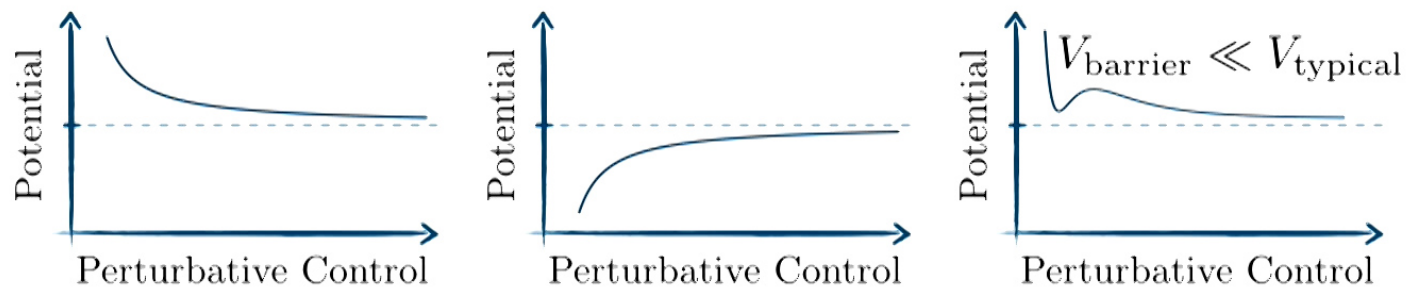
String compactification that gives rise to  $\mathcal{N} = 1$  susy in 4D:

Ricci flat compactification manifolds. Good for control, bad for stability: many fields to not receive a classical potential.

E.g. scale invariance of metric  $\rightarrow$  volume is modulus

Some are lifted by fluxes, others by quantum effects

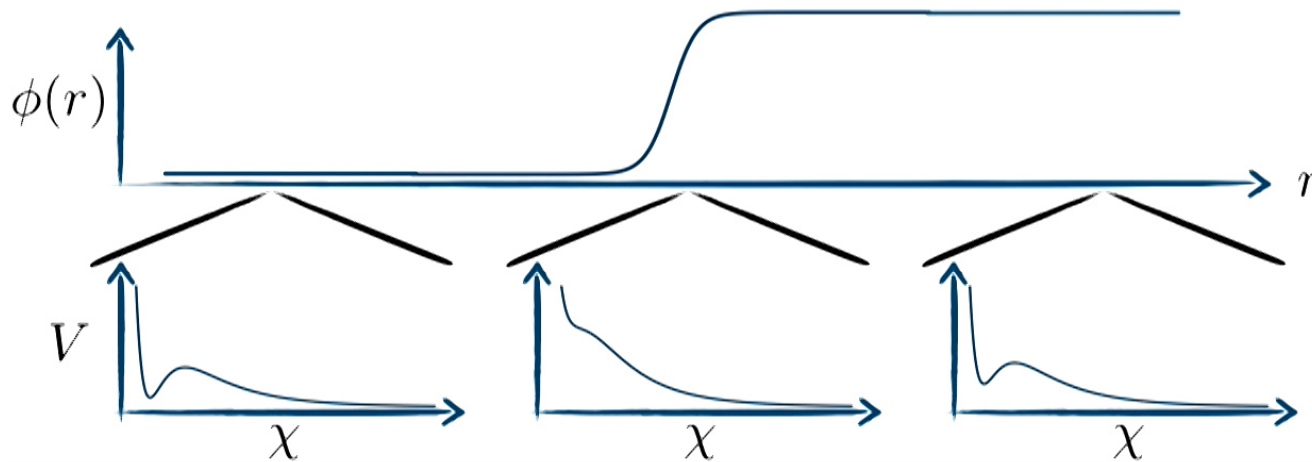
Dine-Seiberg problem: Need at least two non-trivial contributions to potential to stabilize moduli. This is hard!



Let's try to decouple a domain wall in potential  $V(\phi)$  from a modulus  $\chi$ , (e.g. canonical volume modulus)

$$V(\phi, \chi) = V_\phi(\phi) + V_\chi(\chi) + \frac{V_\phi(\phi)\chi}{M_{Pl}}$$

Consider domain wall between two vacua of  $V_\phi$



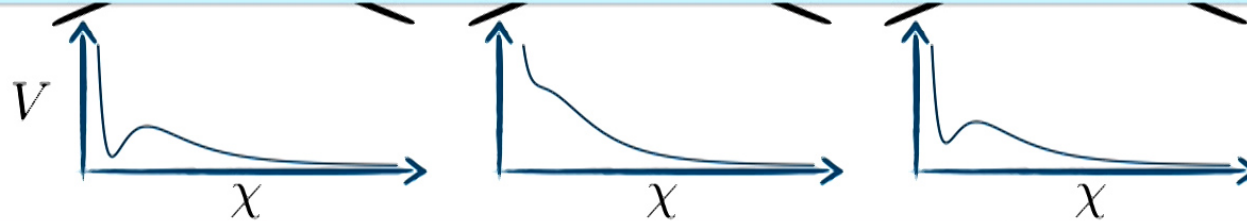
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Modulus classically unstable at domain wall when

$$\frac{\sigma^2}{8M_{Pl}^2} \gtrsim V_{\chi\text{barrier}}$$



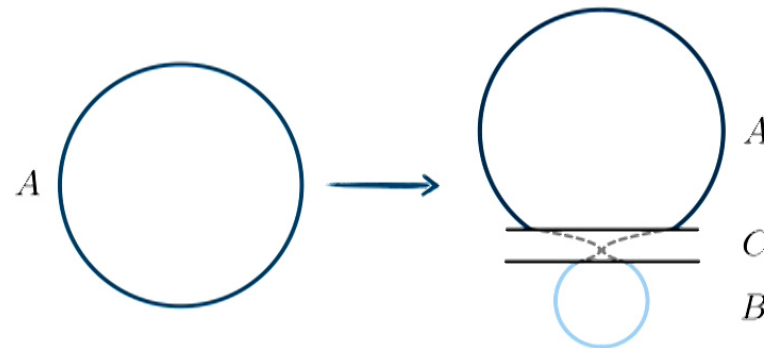
Can gravity stabilize a classically unstable transition?

Reminder: The initial problem can be formulated as

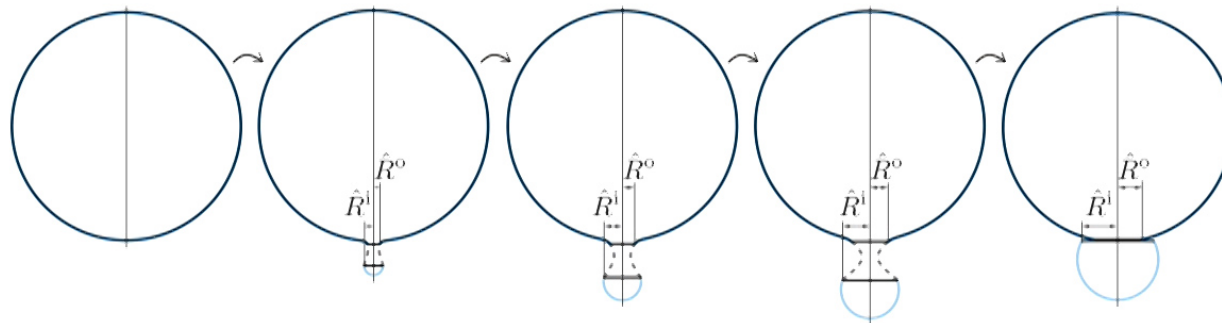
Consider 3 Vacua  $A, B, C$  with  $\Lambda_{A,B} > 0, \Lambda_C = 0$ .  
Only domain walls  $AC$  and  $BC$  are stable.

Does there exist a stable transition  $A$  to  $B$  at late times?

Yes, the double-bubble:



Double-bubble nucleation process



Nucleation rate (  $\kappa$  : domain wall tension,  $H$  : Hubble scale

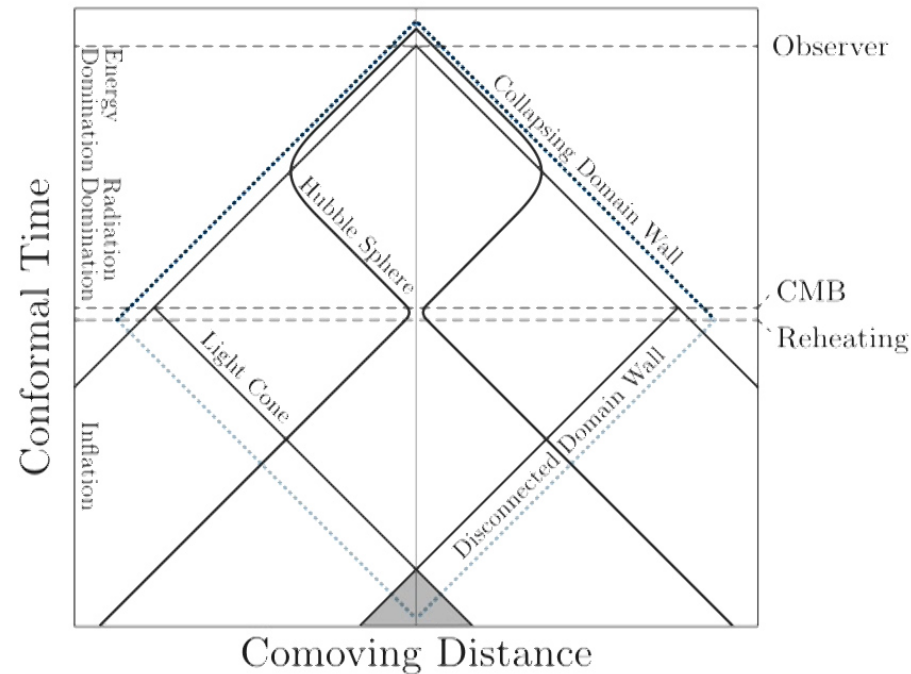
$$\Gamma \sim \exp \left[ -\frac{\pi}{G} \left| \frac{\kappa_{CA}^4}{H_A^2 (H_A^2 + \kappa_{CA}^2)^2} - \frac{H_B^2 + 2\kappa_{BC}^2}{(H_B^2 + \kappa_{BC}^2)^2} \right| \right]$$

High Hubble scales are vastly favored.



But what if the Hubble scale decreases and inflation ends?

At reheating domain wall may start collapsing! Ensure collapsing domain wall never terminates entire bubble.





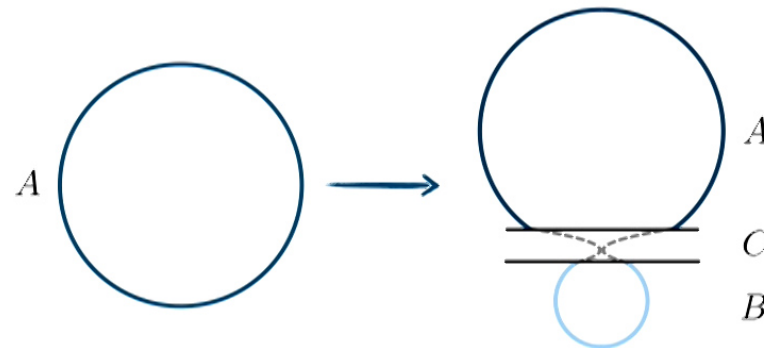
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1. Time is important! Transition rate in gravitational theory

$$\Gamma_{\mathcal{I} \rightarrow \mathcal{F}} \approx e^{-|\int_{\mathcal{I}}^{\mathcal{F}} dS|}$$

2. Moduli stabilization can severely constrain cosmological evolution after vacuum decay.
3. Successful landscape population favors high scale inflation and may require extended period of inflation.

Bubbles collapse without inflation,

$$\frac{a_{inf}}{a_0} \gtrsim e^{N_{hp}} \sim e^{60}$$