Title: Theories with indefinite causal structure

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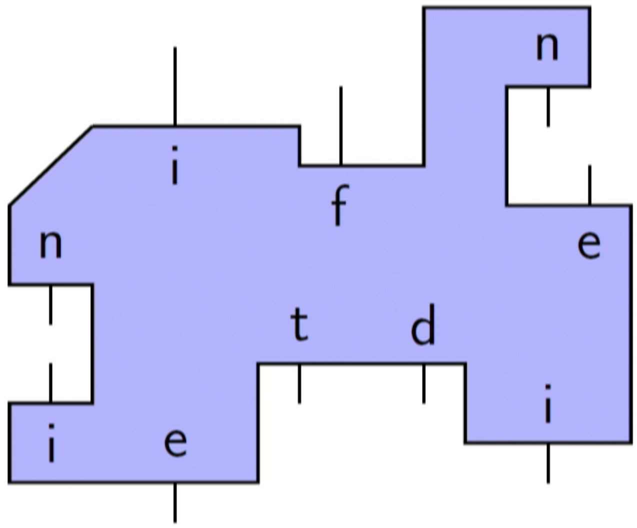
Abstract: <p>To describe observed phenomena in the lab and to apply superposition principle to gravity, quantum theory needs to be generalized to incorporate indefinite causal structure. Practically, indefinite causal structure offers advantage in communication and computation. Fundamentally, superposing causal structure is one approach to quantize gravity (spacetime metric is equivalent to causal structure plus conformal factor, so quantizing causal structure effectively quantizes gravity). </p>

<p>We develop a framework to do Operational Probabilistic Theories (OPT) with indefinite causal structure. For the interest of quantum gravity, this framework gives a general prescription to quantize causal structure, assuming linearity is intact. For the interest of quantum foundations, this framework can support new experimental tests about the validity of quantum theory in complex Hilbert space. It also offers opportunities for constructing new OPT models to substitute ordinary quantum theory. Along this direction, we identify principles that single out the complex Hilbert space theory within the general framework.</p>

Theories with



causal structure

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Motivation: superposing causality/spacetime

- Quantum gravity: apply superposition principle to spacetime and causal structure
- Information theory: describe observed phenomenon; look for advantages in communication and computation; incorporate indefinite causality for causal inference problems

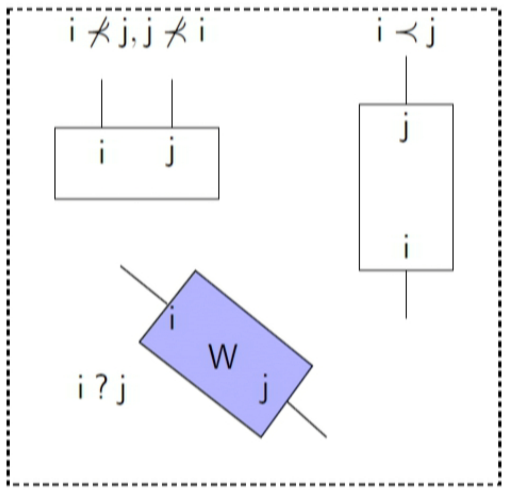
Previous works:

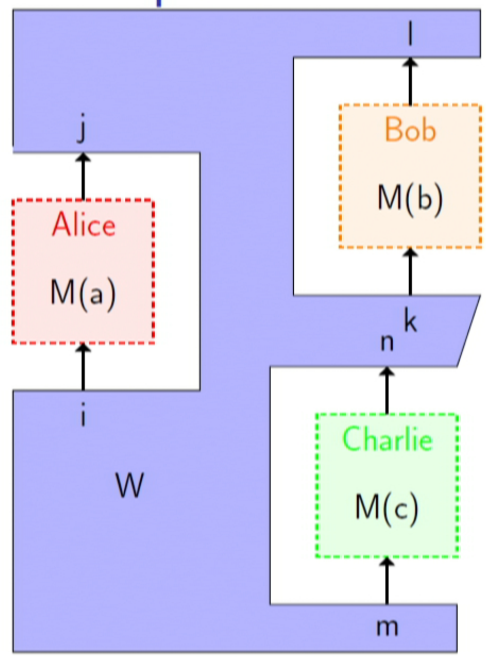
- Hardy 2005 (causaloid framework);
  Hardy 2009 (circuit framework)
- Leifer 2006 (noncommutative probability theory);
  Leifer and Spekkens 2013 (conditional state)
- Chiribella, D'Ariano, Perinotti and Valiron 2013 (quantum switch)
- Oreshkov, Costa and Brukner 2012 (process);
  Oreshkov and Cerf 2015; 2016 (time-symmetric formulation)

*Example: process in complex Hilbert space [1]

Alice's operations: $\{M(a)\}_{a \in \text{Classical Info}}$

$M(a) \in L(H_{in} \otimes H_{out})$ (Choi state)

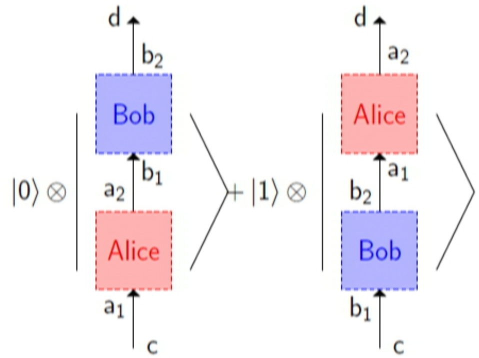




- Operational approach
- Causal order indefinite
- $\mathrm{Tr}[\,\cdot\, W] : \mathrm{M}(a|x) \otimes \mathrm{M}(b|y) \otimes \mathrm{M}(c|z) \to [0, 1]$, where $W \in L(\prod_{\otimes} H_i H_j H_k H_l H_m H_n)$ is self-adjoint.
- Ordinary theory is a special case

[1] Oreshkov, Costa and Brukner 2012

Applications



Superposition of causal order [2]

$$W = |w\rangle\langle w| \in L(\textstyle\prod_{\otimes} H_{ctr} H_c H_{a_1} H_{a_2} H_{b_1} H_{b_2} H_d),$$

$$|w\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle\, |\phi\rangle^{ca_1}\, |\phi\rangle^{a_2 b_1}\, |\phi\rangle^{b_2 d} + |1\rangle\, |\phi\rangle^{cb_1}\, |\phi\rangle^{b_2 a_1}\, |\phi\rangle^{a_2 d} \big),$$

Alice: $M(a) \in L(H_{a_1} \otimes H_{a_2})$, Bob: $M(b) \in L(H_{b_1} \otimes H_{b_2})$

[2] Chiribella, D'Ariano, Perinotti & Valiron 2013; Oreshkov, Costa & Brukner 2012

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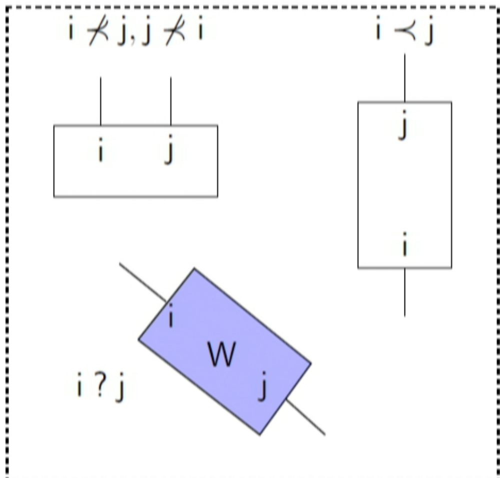

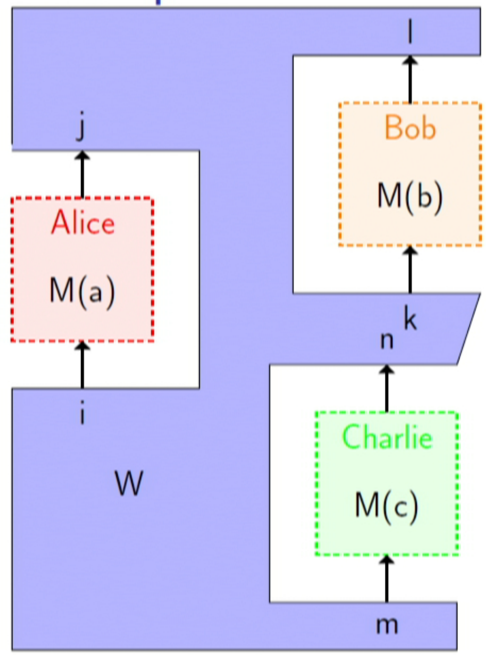

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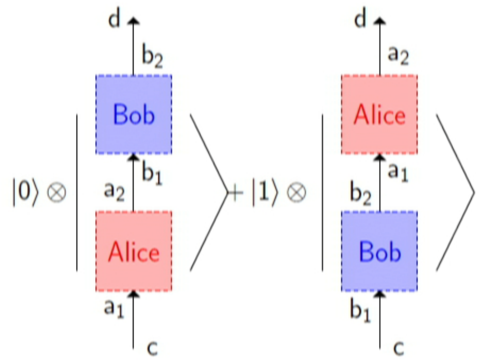

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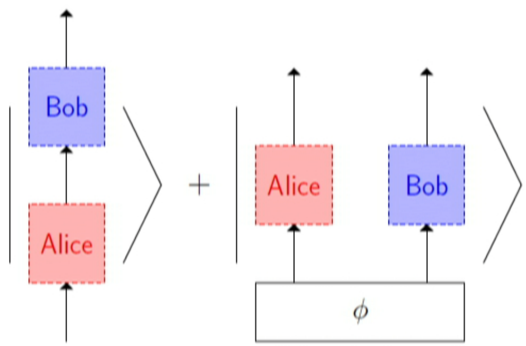


Superposition of direct-cause and common-cause [3]

[2] Chiribella, D'Ariano, Perinotti & Valiron 2013; Oreshkov, Costa & Brukner 2012

[3] MacLean, Ried, Spekkens & Resch 2016; Feix & Brukner 2016

General process framework[4]

Express operational probabilistic theories of indefinite causal structure in general (e.g., classical; real and quaternion etc.)

Motivations:

- Design experiments to test complex Hilbert space theory
- Construct theories beyond complex Hilbert space
- Identify resources of indefinite causal structure

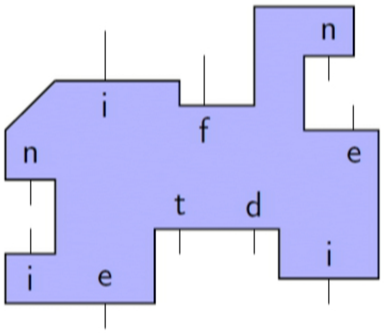
Compare with Hardy's Causaloid framework:

- Make more explicit connection with the graphical circuit framework
- Use process space as the unifying space

[4] D.J. to appear

Process spaces

Causal relation among subsystems indefinite $\Longrightarrow$ unify state, transformation, effect ... spaces: process spaces



- Linearity: vector space
  (motivation: classical mixture obey linearity)
- Probability: positive cone
  (analog: density operators are positive operators)

Expressing operational probabilistic theories (OPT)

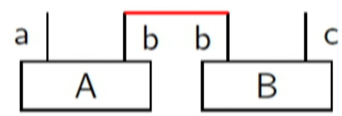
An OPT is defined by:

- Associating a mathematical object to each operation
  –Composition rule obey linearity
- Assigning a probability for each configuration

Example (Complex Hilbert space theory)

- Operations represented by Choi states $A^{ab}, B^{bc}, \cdots$ (self-adjoint operators)
- Composition rule:

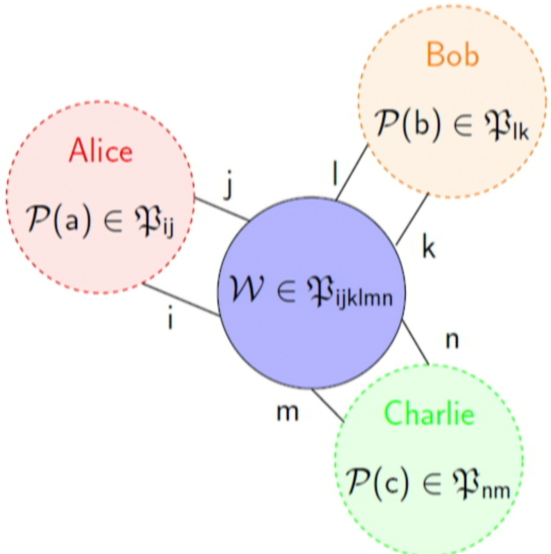
$$A^{ab}, B^{bc} \mapsto C^{ac} = \mathrm{Tr}_b \, A^{ab^T} B^{bc}$$

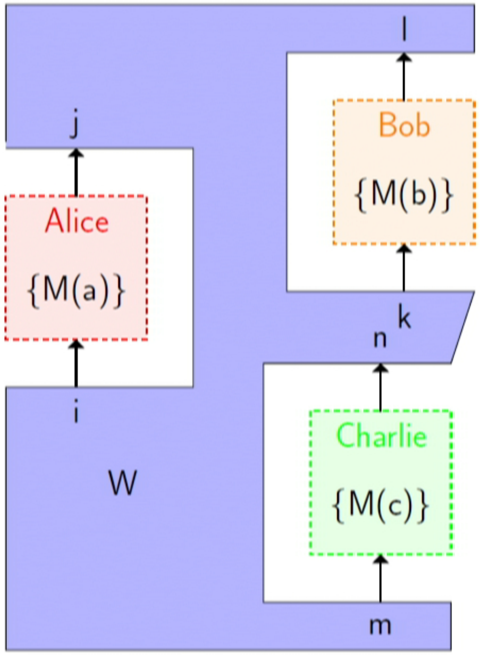


- Probabilities follow from composition rule:

$$A^a, B^a \mapsto \mathrm{Tr}_a \, A^{a^T} B^a$$

*Summary of the framework





To keep the framework general:

- Does not impose normalization
- Does not distinguish inputs and outputs (cf. Oreshkov and Cerf 2015)
- Does not impose no-signalling

Principles for the complex Hilbert space theory

Motivations:

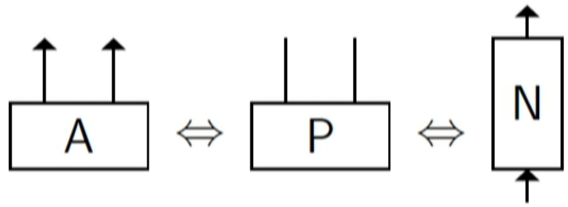
- Why complex numbers?
- Why complex numbers for quantum gravity?
- Suggest systematic ways to go beyond complex Hilbert space
- Identify structures/resources enabling or forbidding operational/informational tasks (e.g., no-cloning)

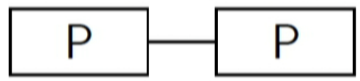
Jordan algebra approach [5]

- Jordan algebra (1932): real vector space with a bilinear product such that $x \cdot y = y \cdot x$ and $(x^2 \cdot y) \cdot x = x^2 \cdot (y \cdot x)$. $(x \cdot y := 1/2(xy + yx))$
- Jordan-von Neumann-Wigner (1934) classification of finite dimensional formally real Jordan algebra:
  –Self-adjoint matrices on real, complex, and quaternionic Hilbert spaces
  –Spin factor (can be embedded in real self-adjoint matrices)
  –Self-adjoint octonionic 3×3 matrices (Albert algebra)
- Zel'manov (1979): infinite dimension no new exceptions
- Koecher (1957)-Vinberg (1961) reconstruction theorem: link to geometric properties of ordered vector spaces

[5] Wilce 2012a, 2012b

*Principles

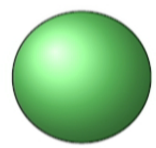
Isomorphism 

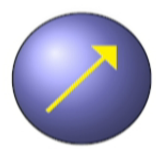
Identification 

> 0, for $P \neq 0$

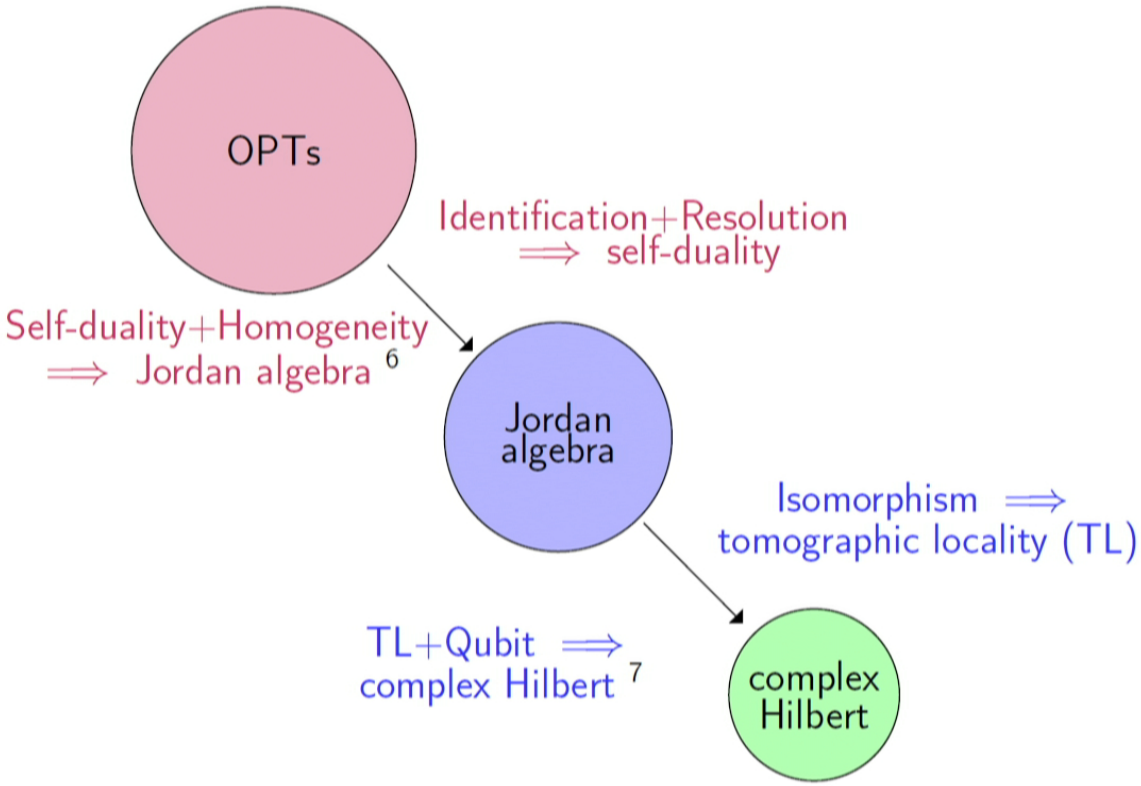
Resolution $\exists\, P, Q \neq 0$, 

$= 0$

Homogeneity 

Qubit $\exists$ 

Deriving complex Hilbert space



[6] Koecher-Vinberg reconstruction theorem
[7] Hanche-Olsen 1985; Barnum & Wilce 2014

Conclusion

Summary:

- Adopted operational approach to indefinite causal structure (independent of manifold)
- Constructed framework to express OPTs assuming linearity and non-negativity
- Identified principles for complex Hilbert space theory

Outlook:

- State-transformation isomorphism
- Information processing tasks from subsets of principles
- Relax principle(s)?
- Generalize to infinite dimensions
- Information-theoretic aspect of spacetime