

Title: No-go results and promising directions in the space of causal diamonds

Date: Nov 01, 2016 02:00 PM

URL: <http://pirsa.org/16110039>

Abstract: <p>The space of causal diamonds recently brought to attention by de Boer et al. and Czech et al. provides an organizing principle for the dependence of entanglement entropy in conformal field theories on the spatial subregion considered. I will show that the inclusion relation of causal diamonds does not give rise to a consistent notion of a causal structure and thus does not provide an alternate metric on this space. I will also show that the entanglement entropy of ball shaped regions is not enough to reconstruct the areas of higher dimensional bulk surfaces in a static geometry. Instead, I will provide a new direction for the reconstruction of the areas of bulk surfaces from boundary data in holography, using the shape derivatives of the entanglement entropy of nearly ball shaped regions.</p>

Reconstruction of bulk surface areas from Entanglement

Charles Rabideau
Univ. of Pennsylvania

$$S(A, \rho) = - \text{tr} \rho_A \log \rho_A \quad \rho_A = \text{tr}_{\bar{A}} \rho$$

In holog. : $S(A, M)$

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$$\text{In holog. : } S(A, M) = \min_{\hat{A} \supset \partial A = \partial \hat{A}} \text{area}(\hat{A})$$

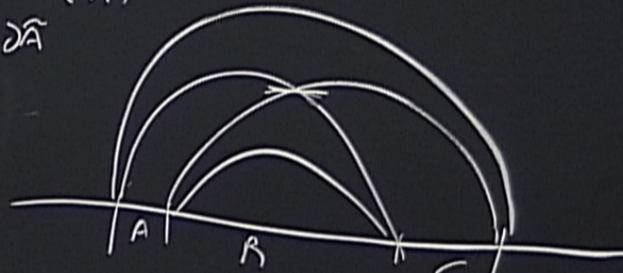
Reconstruction of bulk surface areas from Entanglement

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$$S(A, \rho) = - \text{tr} \rho_A \log \rho_A \quad \rho_A = \text{tr}_{\bar{A}} \rho$$

In holog. : $S(A, M) = \min_{\hat{A} \supset A} \text{area}(\hat{A})$
 $\hat{A} \supset A \Rightarrow \partial A = \partial \hat{A}$

• Entropy Cone



$$S_{AB} + S_{BC} = S_B + S_{ABC}$$

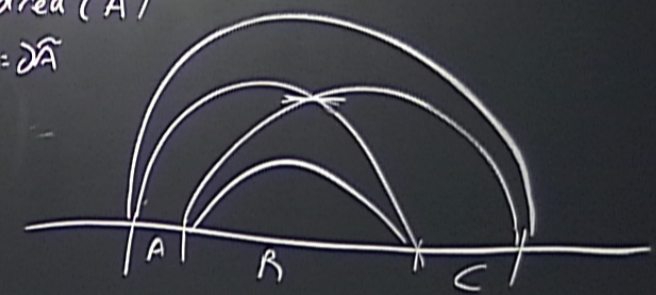
st. area areas from entanglement

Rabideau
Univ. of Pennsylvania

$$S(A, \rho) = - \text{tr} \rho_A \log \rho_A \quad \rho_A = \text{tr}_B \rho$$

In holog. : $S(A, M) = \min_{\tilde{A} \supset A} \text{area}(\tilde{A})$
 $\tilde{A} \supset A \implies \partial \tilde{A} = \partial A$

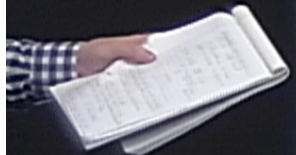
- Entropy cone
- $S(A, \langle \rho_i \rangle)$



$$S_{AB} + S_{BC} \geq S_B + S_{ABC}$$

$R(\partial M) \subset P(\partial M)$. all subsets of ∂M

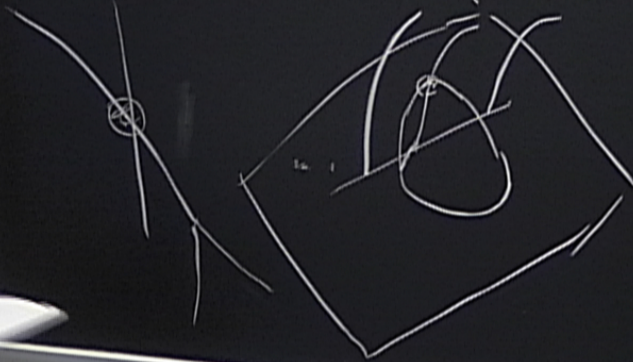
$S: P(\partial M) \rightarrow \mathbb{R}$ $S|_{R(\partial M)}$



$R(\partial M) \subset P(\partial M)$. all subsets of ∂M

$S: P(\partial M) \rightarrow \mathbb{R}$ $S|_{R(\partial M)}$

$\exists S: R(\partial M) = P(\partial M)$

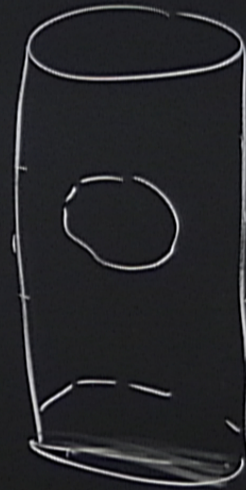
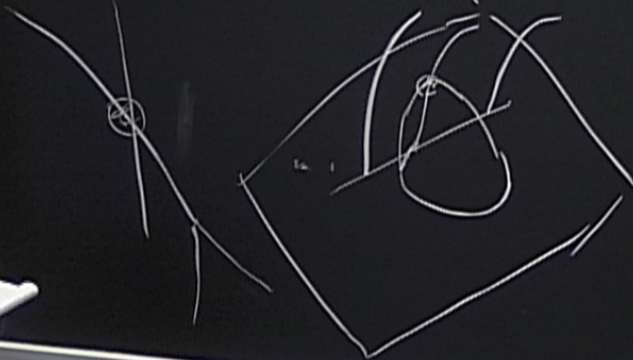


$R(\partial M) \subset P(\partial M)$. all subsets of ∂M

$S: P(\partial M) \rightarrow \mathbb{R}$

$S|_{R(\partial M)}$

$\exists S: R(\partial M) = P(\partial M)$



CAUTION
Warning: Do not touch the surface of the board.
If you touch the surface of the board, you may be injured.

$$\frac{d=2}{}$$

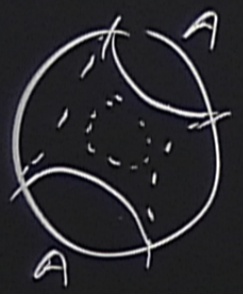
AdS₃



$$\partial H_2 = \mathbb{R}$$

$P(\partial M) = \text{Unions of intervals}$

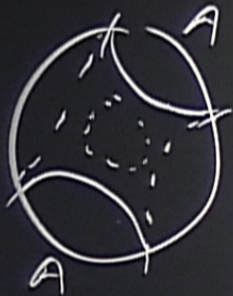
$R(\partial M) = \text{connected intervals}$



CAUTION
DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT
DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT
DO NOT TOUCH THE BOARD

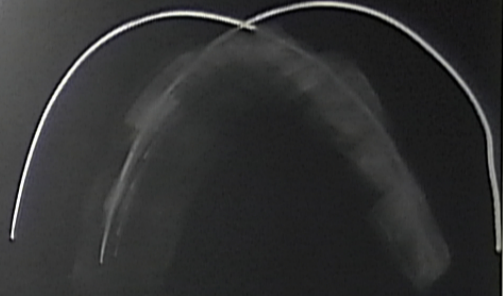
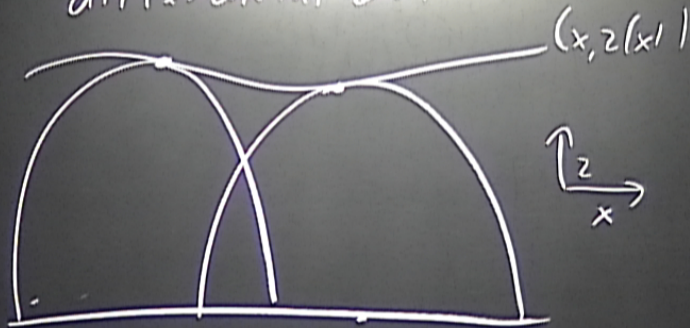
CAUTION
DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT
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AdS_3
 \downarrow
 H^2
 $\mathbb{H}^2 = \mathbb{R}$



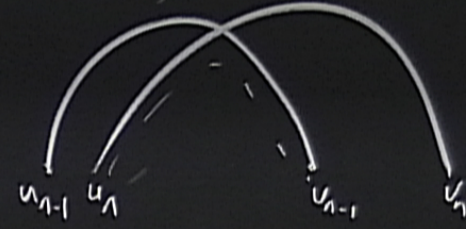
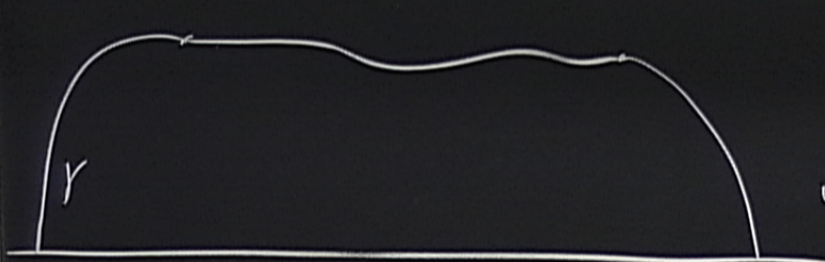
$R(\partial M) = \text{connecco}$

differential ent.:



$x_i(x)$
 $I(x)$
 $R(x)$

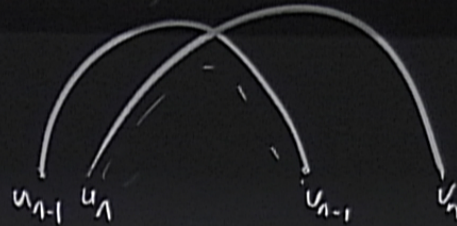
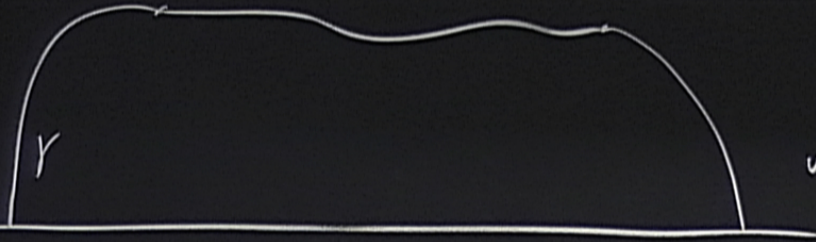
$$\text{area}(y) = S(I_0) + \sum_n [S(I_n) - S(I_n \cap I_{n-1})]$$



$$\text{area}(\gamma) = \text{bdy terms} + \int -\frac{\partial S}{\partial v} dv$$

$$\begin{aligned} S(I_n \cap I_{n-1}) &= S(u_n, v_{n-1}) \\ &= S(I_n) + \frac{\partial S}{\partial v} (v_n - v_{n-1}) \\ &\quad \Delta v \end{aligned}$$





$$S(I_n \cap I_{n-1}) = S(u_n, u_{n-1})$$

$$= S(I_n) + \partial_v S \left(\frac{u_n - u_{n-1}}{\Delta v} \right)$$

$$d \text{area}(I) = \text{bdy terms} + \int_B \frac{\partial S}{\partial v} dv \rightarrow S(u, v)$$

$$\int_B \frac{\partial S}{\partial ?} \rightarrow (d-1) \text{ form}$$

$$B_d \sim dS_d$$

Ad
H_d

CAUTION

AdS_{d+1}
 \downarrow
 H_d

Claim: given a geometry is there a $(d-1)$ form C ,
if given a bulk $^{(d)}$ surface $N \ni N_0 \subset B_d$ it.

$$\int_{N_0} C = \text{area}(N) (\text{+ bdy terms}) ?$$

No!



AdS_{d+1}
H_d

Claim: given a geometry is there a (d-1) form C ,
given a bulk ^(d-1) surface $N \ni N_B \subset B_d$ s.t.

$$\int_{N_B} C = \text{area}(N) (\text{+ bdy terms}) ?$$

No!



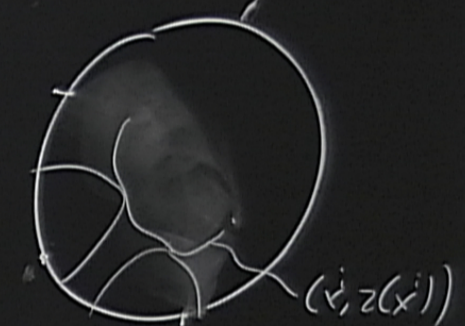
H_d given a bulk $(d-1)$ surface $N \ni N_B \subset B_d \partial$

$$\int_{N_D} \mathcal{L} = \text{area}(N) (\text{+ boundary terms}) ?$$

N_0'

$\partial d - 1 = \dim(E)$ N_x $B_d \times S^{d-1}$ $x \in \partial X$

$$E \sim (x^i, z, \tilde{z}^i) \sim (x_0^i, R, t^i) \sim (x_0^i, x^i, z)$$



$N_{(d-1)} \rightsquigarrow \tilde{N}_{(d-1)} = (x^i, z(x), \partial; z)$

$$\text{area}(N) = \int_N \sqrt{g_{ind}} d\sigma = \int_{\tilde{N}} A$$



$$E \sim (x^i, z, \dot{z}^i) \sim (x_0^i, R, t^i) \sim (x_0^i, x^i, z)$$

$$N_{(d-1)} \rightsquigarrow \tilde{N}_{(d-1)} = (x^i, z(x), \partial_i z) \quad \text{area}(N) = \int_N \sqrt{g_{ind}} d\sigma = \int_{\tilde{N}} A_{d-1}$$



$$A_1 = \frac{dx + z dz}{2\sqrt{1+z^2}}$$

$h = \frac{V_{d-1}}{V_d}$
 $\rightarrow V$

$$E \sim (x', z, z') \sim (x_0, \kappa, \ell) \sim (x_0, x_1, \ell)$$

$$N_{(d-1)} \rightsquigarrow \tilde{N}_{(d-1)} = (x', z(x), \partial; z) \quad \text{area}(N) = \int_N \sqrt{g_{ind}} d\sigma = \int_{\tilde{N}} A_{d-1}$$



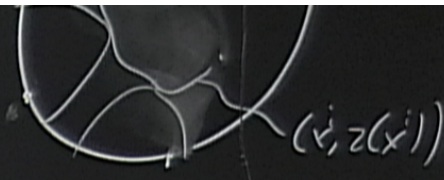
$$A_1 = \frac{dx + i dz}{2\sqrt{1 + z^2}} \quad dz(x) = \frac{\partial z}{\partial x} dx$$

$(h - V_{n-1})$
 $\rightarrow v$



$$E \sim (x^i, z, \dot{z}^i) \sim (x_0^i, R, t^i) \sim (x_0^i, x^i, z)$$

$$N_{(d-1)} \rightsquigarrow \tilde{N}_{(d-1)} = (x^i, z(x), \partial_i z) \quad \text{area}(N) = \int_N \sqrt{g_{ind}} d\sigma = \int_{\tilde{N}} A_{d-1}$$

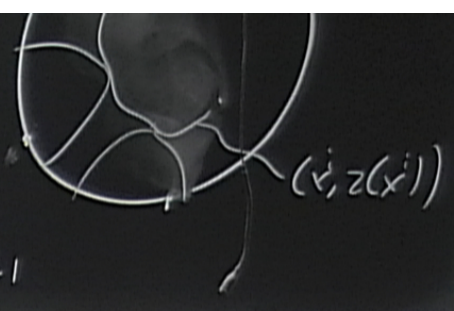


$$A_1 = \frac{dx + z dz}{2\sqrt{1+z^2}} \quad dz(x) = \frac{\partial z}{\partial x} dx$$

$$\int_{N_B} c_{d-1} = \int_{\tilde{N}} A \quad C = A + df$$

$\partial d-1 = \dim(N_x)$ B_{d-1}

$$E \sim (x^i, z, \tilde{z}^i) \sim (x_0^i, R, t^i) \sim (x_0^i, x^i, z)$$



$$N_{(d-1)} \rightsquigarrow \tilde{N}_{(d-1)} = (x^i, z(x^i), \partial; z) \quad \text{area}(N) = \int_N \sqrt{g_{ind}} d\sigma = \int_{\tilde{N}} A_{d-1}$$

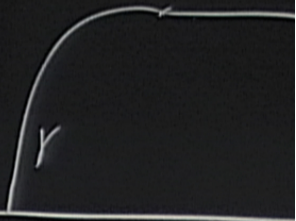
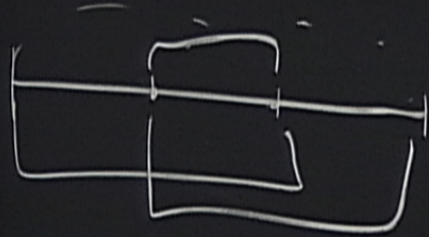
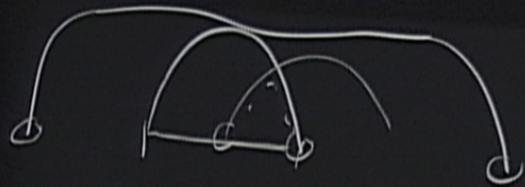
$\langle \rangle | + z$

$$\int_{N_B} c_{d-1} = \int_{\tilde{N}} A \quad C = A + d f$$

$$dC = dA$$



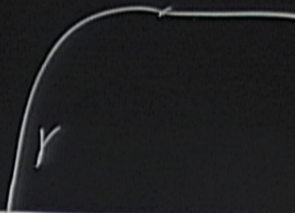
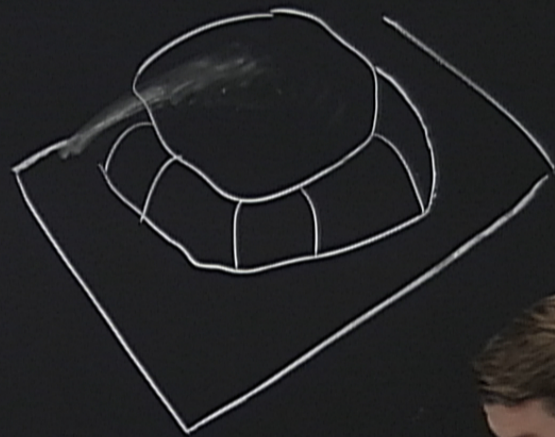
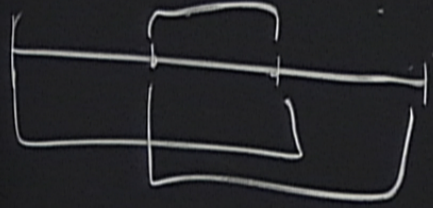
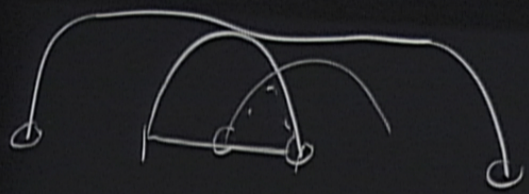
$$S_{AB} + S_{BC} = S_D + S_{ABCD}$$



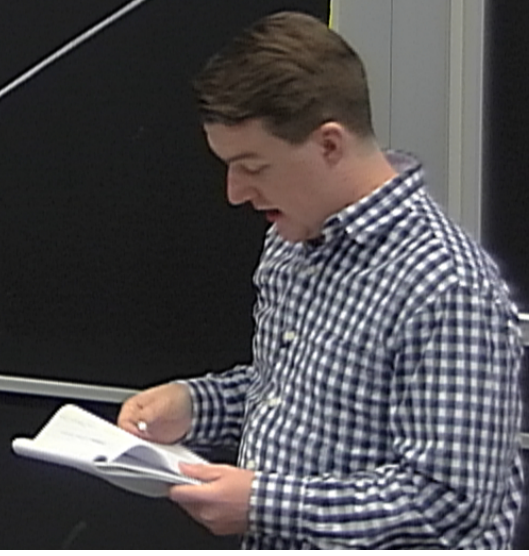
area (1) =

$$S_{AB} + S_{BC} \geq S_B + S_{ABC}$$

$d=3$



area(1) =

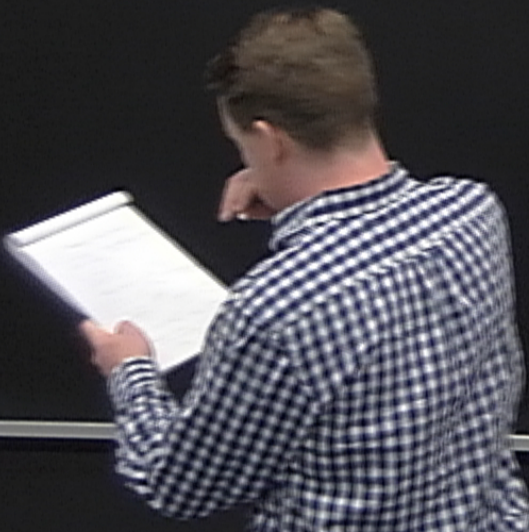
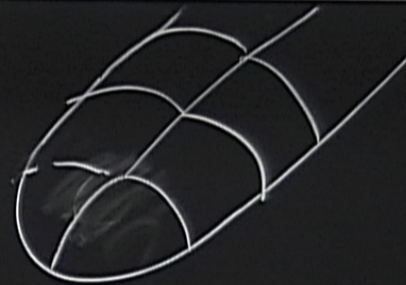


1-parameter family of balls

$$R(\alpha)$$

$$x_0(\alpha)$$

$$y_0(\alpha)$$



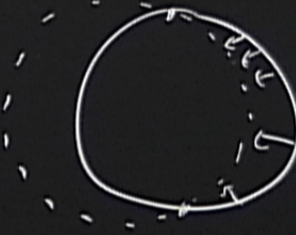
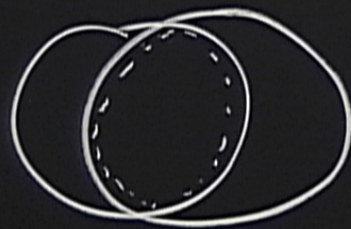
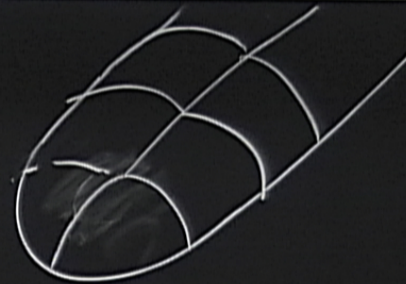
1-parameter family of balls

$R(\alpha)$

$x_0(\alpha)$

$y_0(\alpha)$

y_0 x_0 α



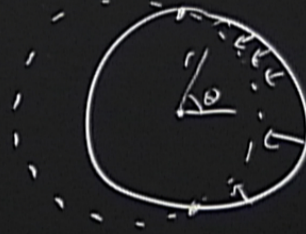
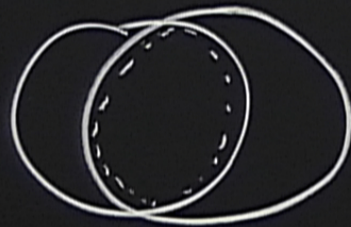
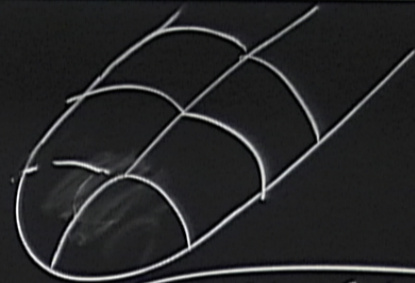
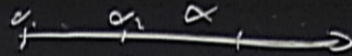
$$\text{Area}(\text{worm}) = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$$



CAUTION
Do not lean against the chalkboard.
Please do not touch the chalkboard.
If a replacement is used,
please do not touch the chalkboard.

1-parameter family of balls

$R(\alpha)$
 $x_0(\alpha)$
 $y_0(\alpha)$



$$S(A) = S(r(\theta))$$

$$B_n = r(\theta, x_0, y_0, R)$$

$$r(\theta, 0, 0, R) = R$$

$$\text{Area(worm)} = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$$

CAUTION
 Do not touch the blackboard when the board is hot.
 It is recommended to wear heat-resistant gloves when touching the board.

(θ)

$y, R)$

R

$$B_n \cap B_{n-1} \sim r(\theta; x_{n-1}, y_{n-1}, R_n) + \mathcal{J}r(\theta)$$

$$\mathcal{J}r = \begin{cases} 0 \end{cases}$$

$$B_n \cap B_{n-1} \sim r(\theta; x_n, y_n, R_n) + \delta r(\theta)$$

$$\delta r = \begin{cases} 0 & \epsilon > \theta_0 \\ -\frac{\partial r(\theta)}{\partial x_0} \frac{\partial x_0}{\partial \alpha} \Delta \alpha + \frac{\partial r}{\partial R} \frac{\partial R}{\partial \alpha} \Delta \alpha \end{cases}$$

$$= \delta_{\alpha} r \Delta \alpha$$

(θ)

$y, R)$

R

$$B_n \cap B_{n-1} \sim r(\theta; x_n, y_n, R_n) + \delta r(\theta)$$

$$\delta r = \begin{cases} 0 & \theta > \theta_0 \\ -\frac{\partial r(\theta)}{\partial x_0} \frac{\partial x_0}{\partial \alpha} \Delta \alpha + \frac{\partial r}{\partial R} \frac{\partial R}{\partial \alpha} \Delta \alpha \end{cases}$$

$$= \delta_\alpha r \Delta \alpha$$

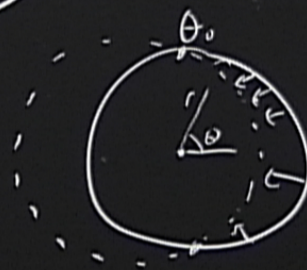
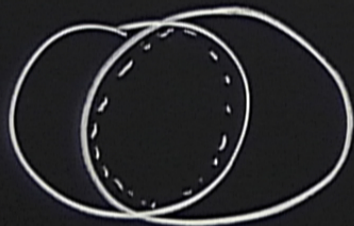
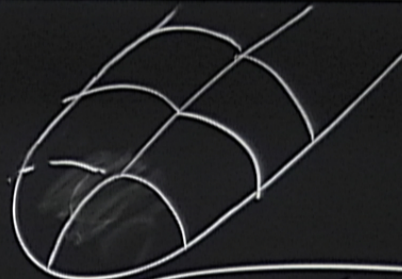
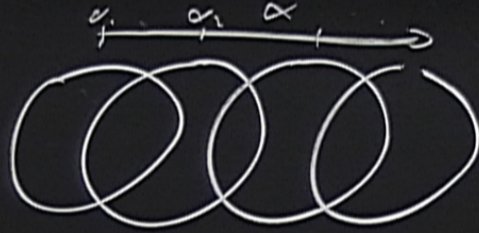
$$S(B_n \cap B_{n-1}) = S(B_n) + \int \frac{\partial S}{\partial r(\theta)} \delta_\alpha r(\theta) \Delta \alpha d\theta$$

1-parameter family of balls

$$R(\alpha)$$

$$x_0(\alpha)$$

$$y_0(\alpha)$$



$$S(A) = S(r(\theta))$$

$$B_n = r(\theta, x_0, y_0, R)$$

$$R_1 = R$$

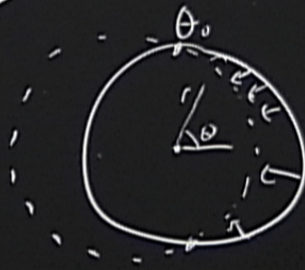
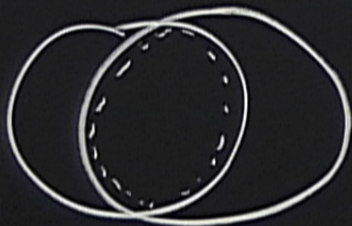
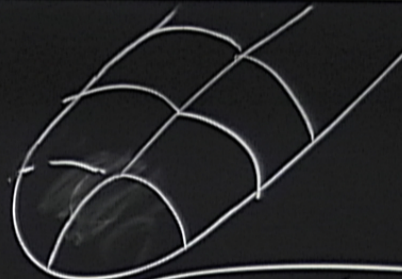
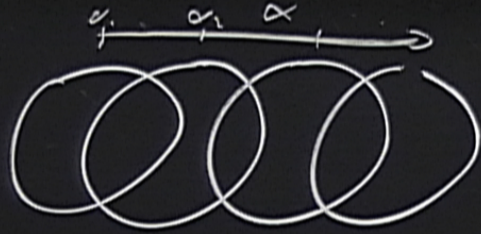
$$\text{Area}(worm) = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$$

$$= bdy + \int \frac{\partial S}{\partial r(\theta)} r(\theta) dk d\theta$$



1-parameter family of balls

$R(\alpha)$
 $x_0(\alpha)$
 $y_0(\alpha)$



$$S(A) = S(r(\theta))$$

$$B_n = r(\theta, x_0, y_0, R)$$

$$r(\theta, 0, 0, R) = R$$

$$\text{Area}(worm) = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$$

$$= bdy + \int \frac{\partial S}{\partial r(\theta)} r(\theta) d\theta$$

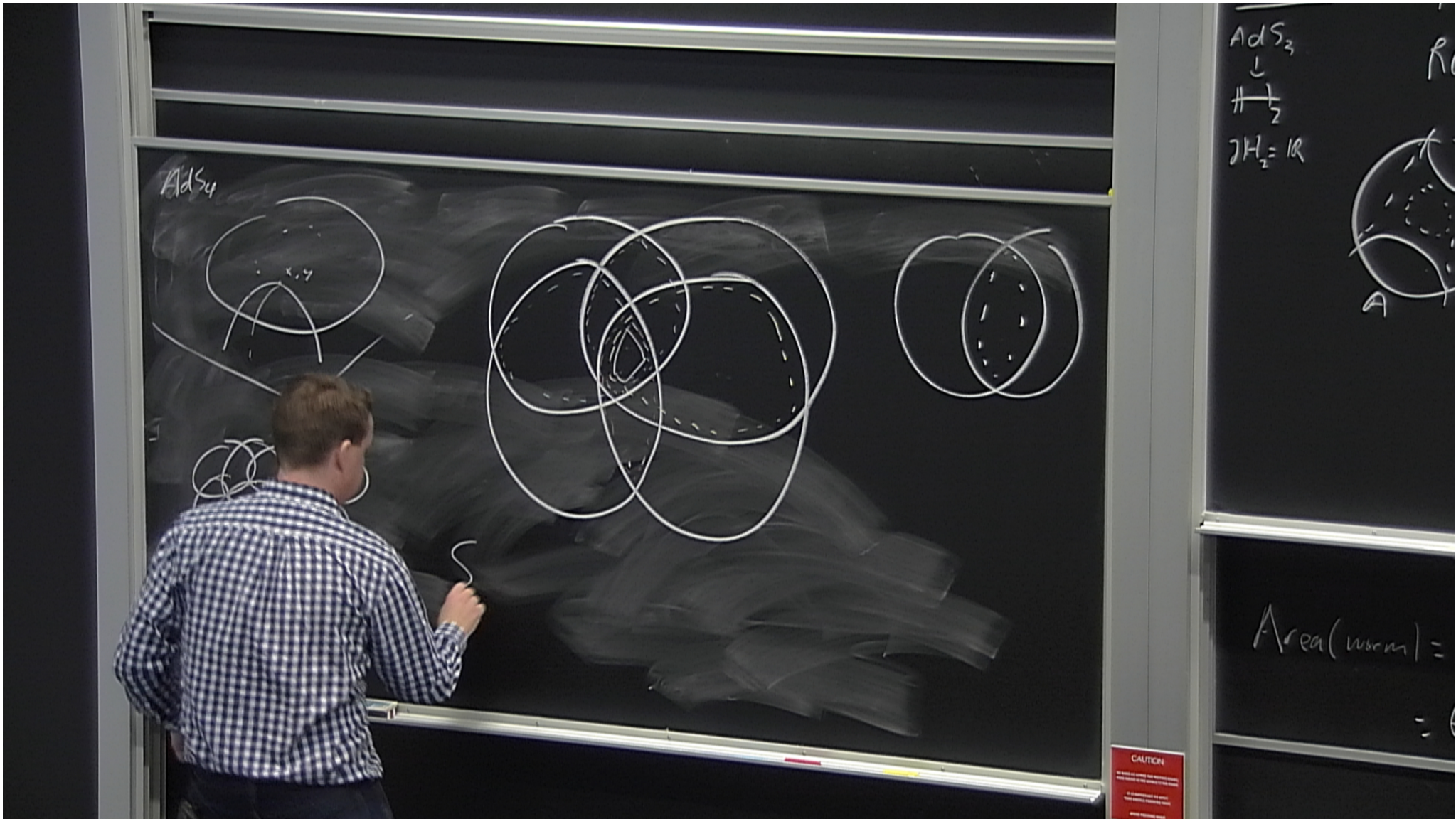


B_n

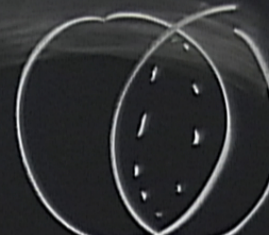
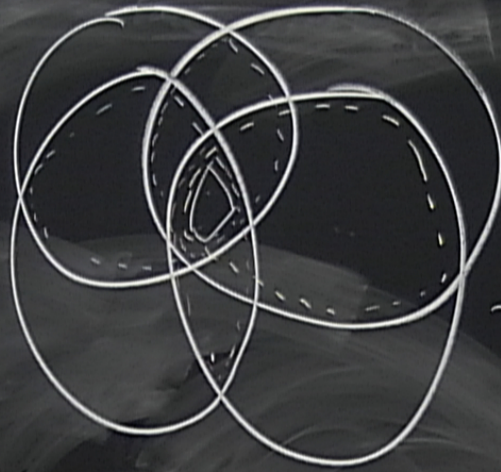
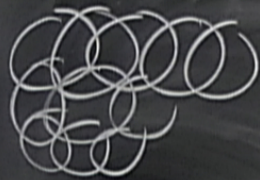
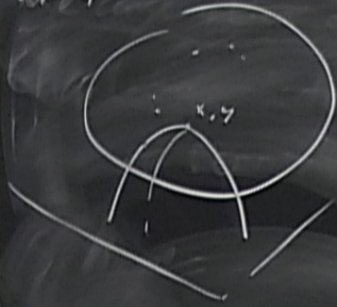
$S(B_n)$



CAUTION
 DO NOT TOUCH THE SURFACE
 OF THE BOARD OR THE BOARD
 AS IT IS DAMAGED BY HAND



AdS₄



$$\sum_{n,m} S(B_{n,m}) - S(B \cap B_{n+1}) + S(B \cap B_{m-1}) + S(B \cap B_{n-1} \cap B_{m-1} \cap B_{n+m-1})$$

AdS₃
 \perp
 \mathbb{H}^2
 $\mathbb{H}^2 = \mathbb{R}$



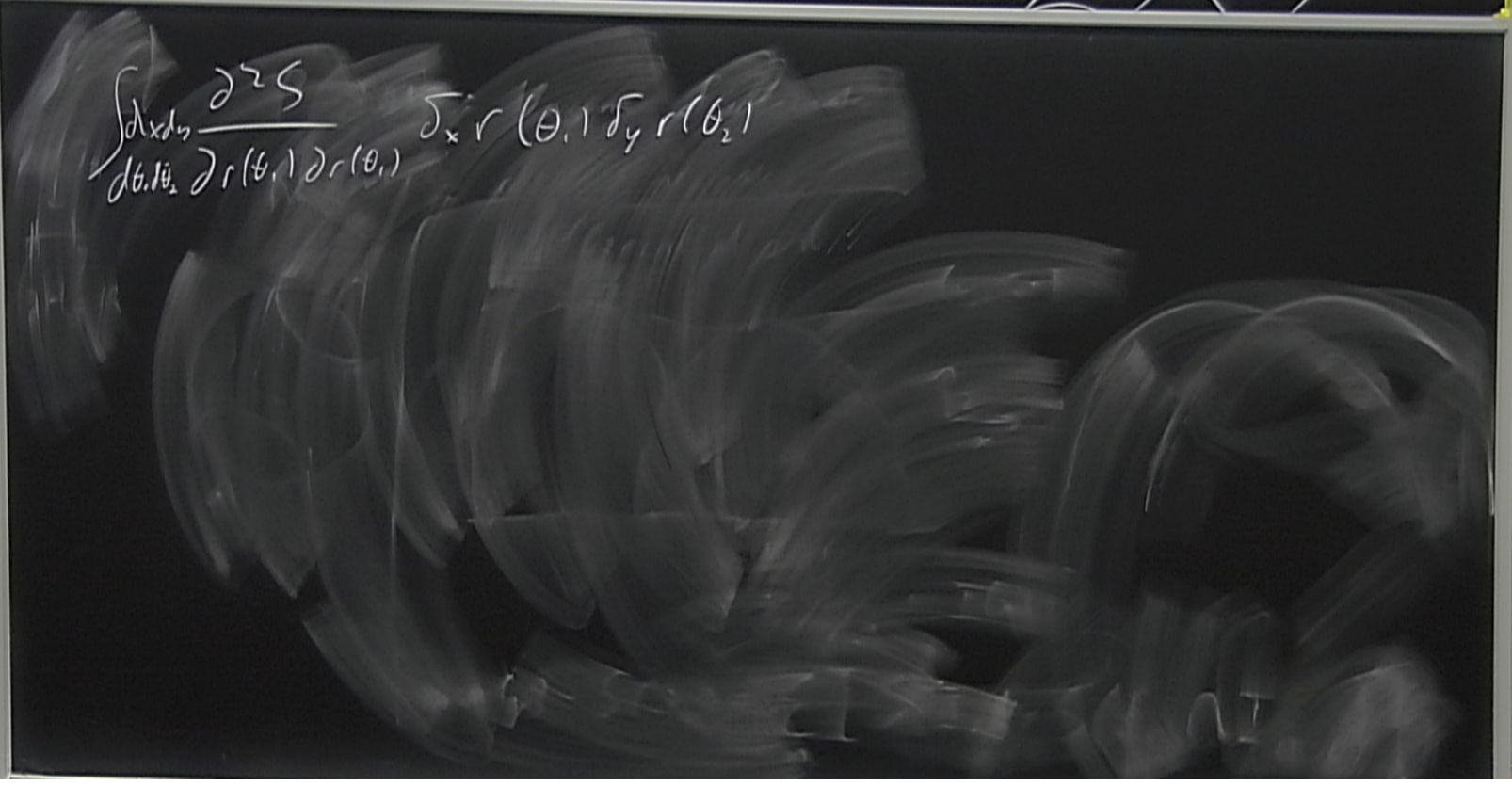
Area(worm) =



$\rightarrow (d-1)$ form

1-parameter family of balls

$$\int_{\mathbb{R}^d} dx_1 \dots dx_d \frac{\partial^2 S}{\partial b_1 \partial b_2} \delta_x r(\theta_1) \delta_y r(\theta_2)$$



CAUTION
Do not sit on the floor or on the bench.
Do not touch the floor or the bench.
Do not touch the floor or the bench.

$\rightarrow (d-1)$ form

parameter families of balls

$$\int_{\partial B} \frac{\partial \Sigma}{\partial r(\theta_1) \partial r(\theta_2)} \delta_x r(\theta_1) \delta_y r(\theta_2)$$

$$F(x_0, 1) - F(x_0 + \Delta x, 1) - F(x_0 + \Delta y) + F(x_0 + \Delta x + \Delta y)$$



CAUTION
Do not sit on the floor
Do not touch the floor
Do not touch the floor

$\rightarrow (d-1)$ form

$$\int_{\partial B} \frac{\partial \psi}{\partial n} \frac{1}{r(\theta_1)} \frac{1}{r(\theta_2)} \delta_x r(\theta_1) \delta_y r(\theta_2)$$

$$\begin{aligned} & F(x_0) - F(x_0 + \Delta x) \\ & - F(x_0 + \Delta y) \\ & + F(x_0 + \Delta x + \Delta y) \\ & = \sum_i \frac{\partial F}{\partial x^i} \Delta x^i \Delta y^i \end{aligned}$$



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$\rightarrow (d-1)$ Form

$$\int_{\partial B} \frac{\partial \zeta}{\partial n} d\sigma = \int_{\partial B} \left(\frac{\partial \zeta}{\partial x_1} \nu_1 + \dots + \frac{\partial \zeta}{\partial x_d} \nu_d \right) d\sigma$$

\downarrow
 Green's
 $R(x, y)$
 x, y

$$F(x_0) - F(x_0 + \Delta x) + F(x_0 + \Delta x + \Delta y) = \sum_i \frac{\partial F}{\partial x_i} \Delta x_i$$



CAUTION
 No sharp or pointed objects allowed.
 No eating or drinking in the lecture hall.
 All equipment to be used must be returned to the front.

$\rightarrow (d-1)$ Form

$$\int_{\partial B} \frac{\partial z}{\partial r} \frac{\partial z}{\partial \theta} \frac{\partial z}{\partial \phi} \dots$$

$$\int_{\partial B} r(\theta, \phi) \frac{\partial z}{\partial r}(\theta, \phi)$$

↓

$$R(x, y)$$

$$x, y$$

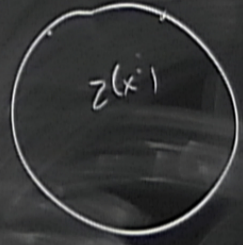
$$F(x_0, y_0) - F(x_0 + \Delta x, y_0)$$

$$- F(x_0, y_0 + \Delta y)$$

$$+ F(x_0 + \Delta x, y_0 + \Delta y)$$

$$= \sum_i \frac{\partial F}{\partial x^i} \Delta x^i + \frac{\partial F}{\partial y^j} \Delta y^j$$

$$\int_{\partial B} z$$



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$\rightarrow (d-1)$ form

$$\int_{\partial \Omega} \frac{\partial \psi}{\partial n} dS$$

\downarrow

Mezrei

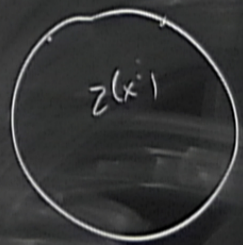
$$\int_{\partial \Omega} \delta_x r(\theta_1) \delta_y r(\theta_2)$$

\downarrow

$$R(x, y)$$

x, y

$$\frac{\int_{\partial \Omega} \delta^{d-1} S}{\delta}$$

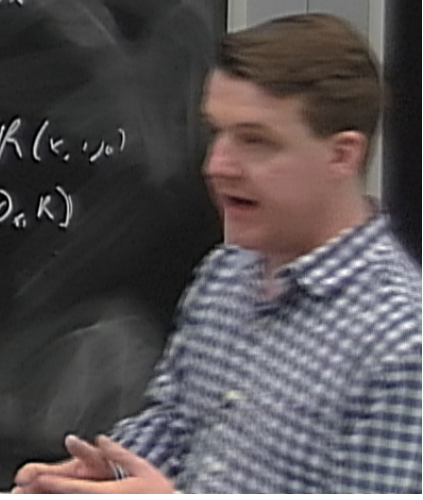


$$F(x_0, y) - F(x_0 + \Delta x, y) - F(x_0 + \Delta y)$$

$$+ F(x_0 + \Delta x + \Delta y)$$

$$= \sum_i \frac{\partial F}{\partial x^i} \Delta x^i \Delta y^i$$

$$\int F(R(x, y)) \partial_n R$$



CAUTION
Do not touch the board or the chalkboard eraser.
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