Title: No-go results and promising directions in the space of causal diamonds

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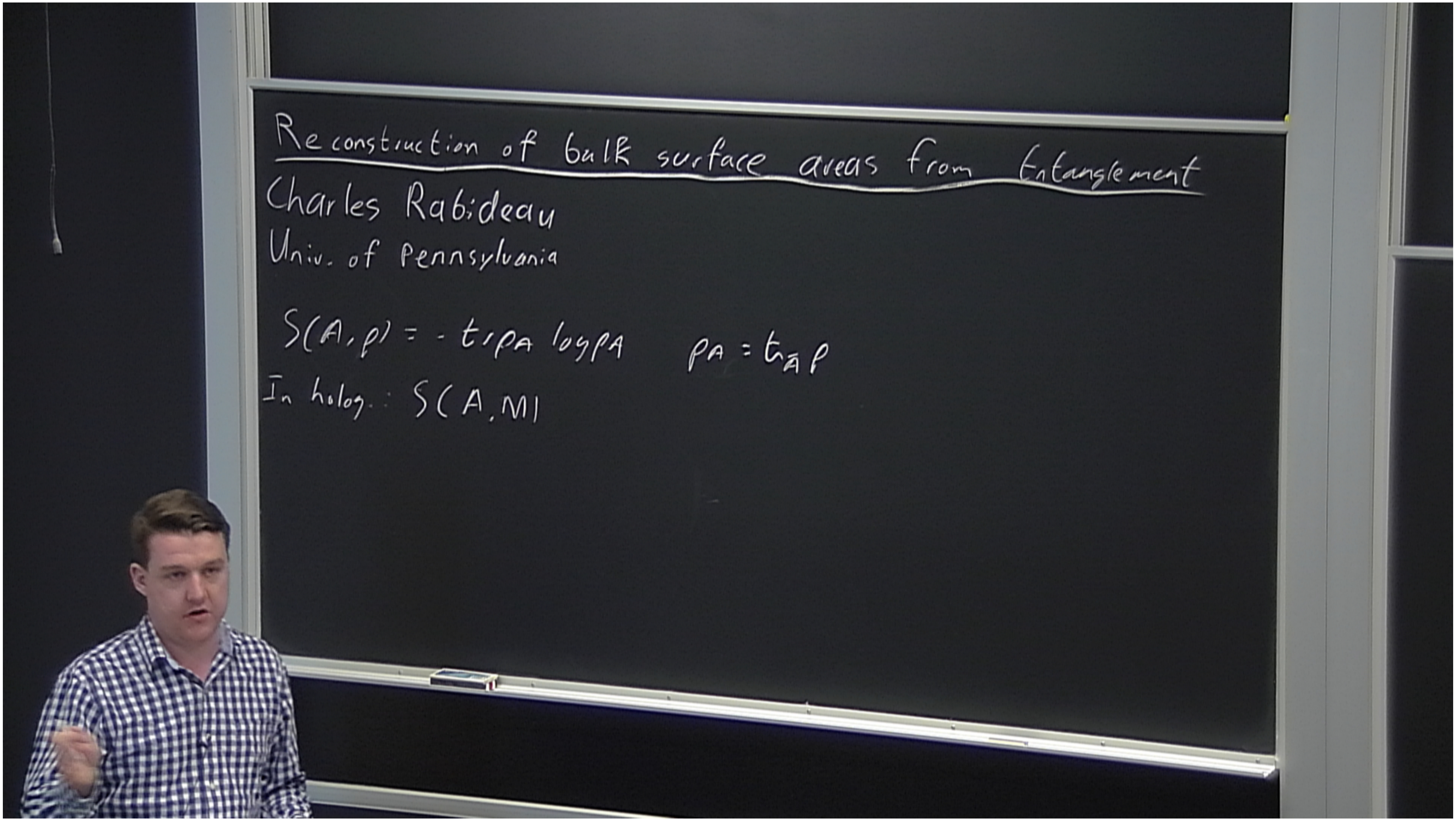
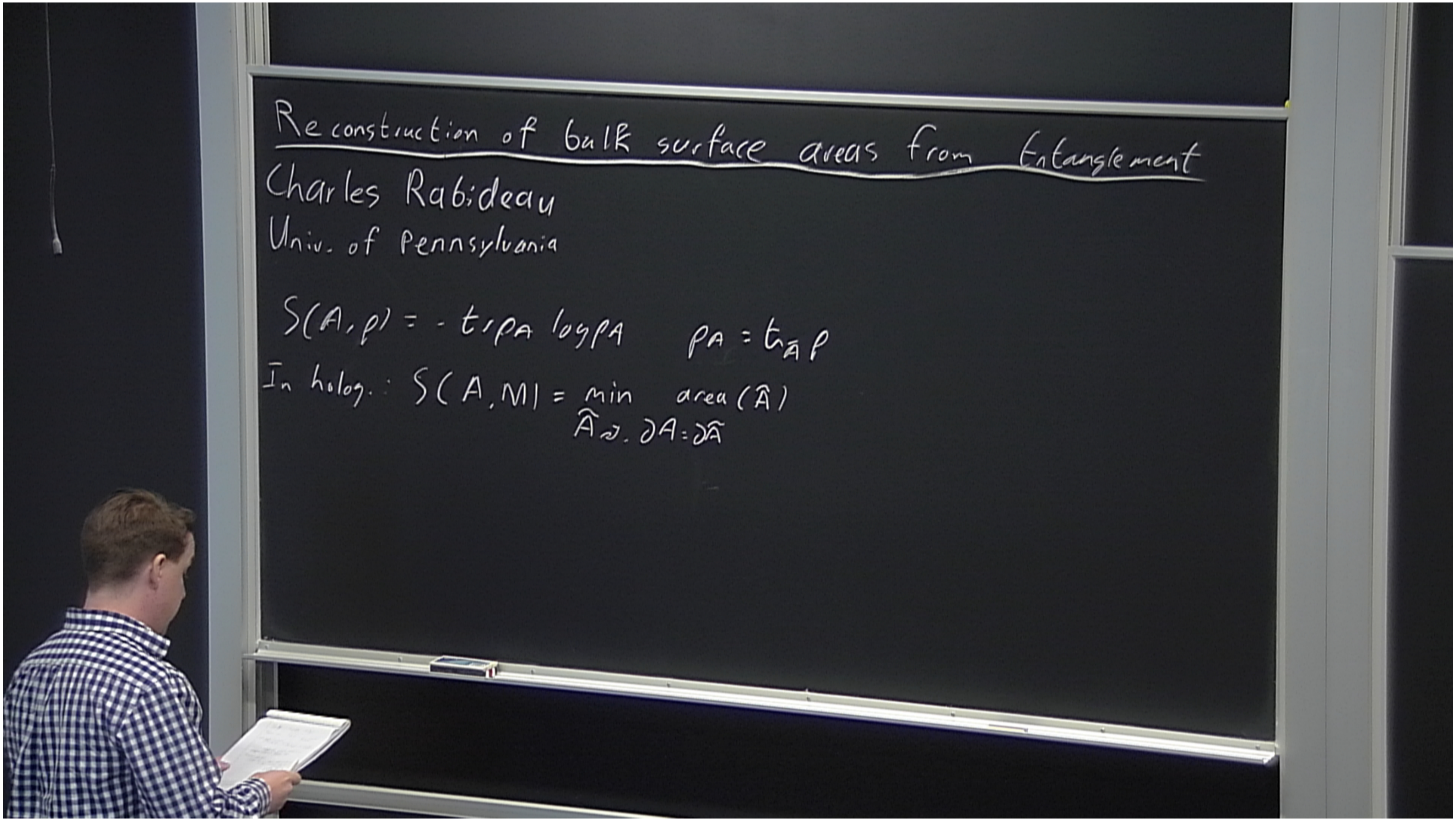
Abstract: <p>The space of causal diamonds recently brought to attention by de Boer et al. and Czech et al. provides an organizing principle for the dependence of entanglement entropy in conformal field theories on the spatial subregion considered. I will show that the inclusion relation of causal diamonds does not give rise to a consistent notion of a causal structure and thus does not provide an alternate metric on this space. I will also show that the entanglement entropy of ball shaped regions is not enough to reconstruct the areas of higher dimensional bulk surfaces in a static geometry. Instead, I will provide a new direction for the reconstruction of the areas of bulk surfaces from boundary data in holography, using the shape derivatives of the entanglement entropy of nearly ball shaped regions.</p>


Reconstruction of bulk surface areas from Entanglement
Charles Rabideau
Univ. of Pennsylvania
S(A, ρ) = - tr ρA log ρA
ρA = trĀ ρ
In holog.: S(A, M)


Reconstruction of bulk surface areas from Entanglement
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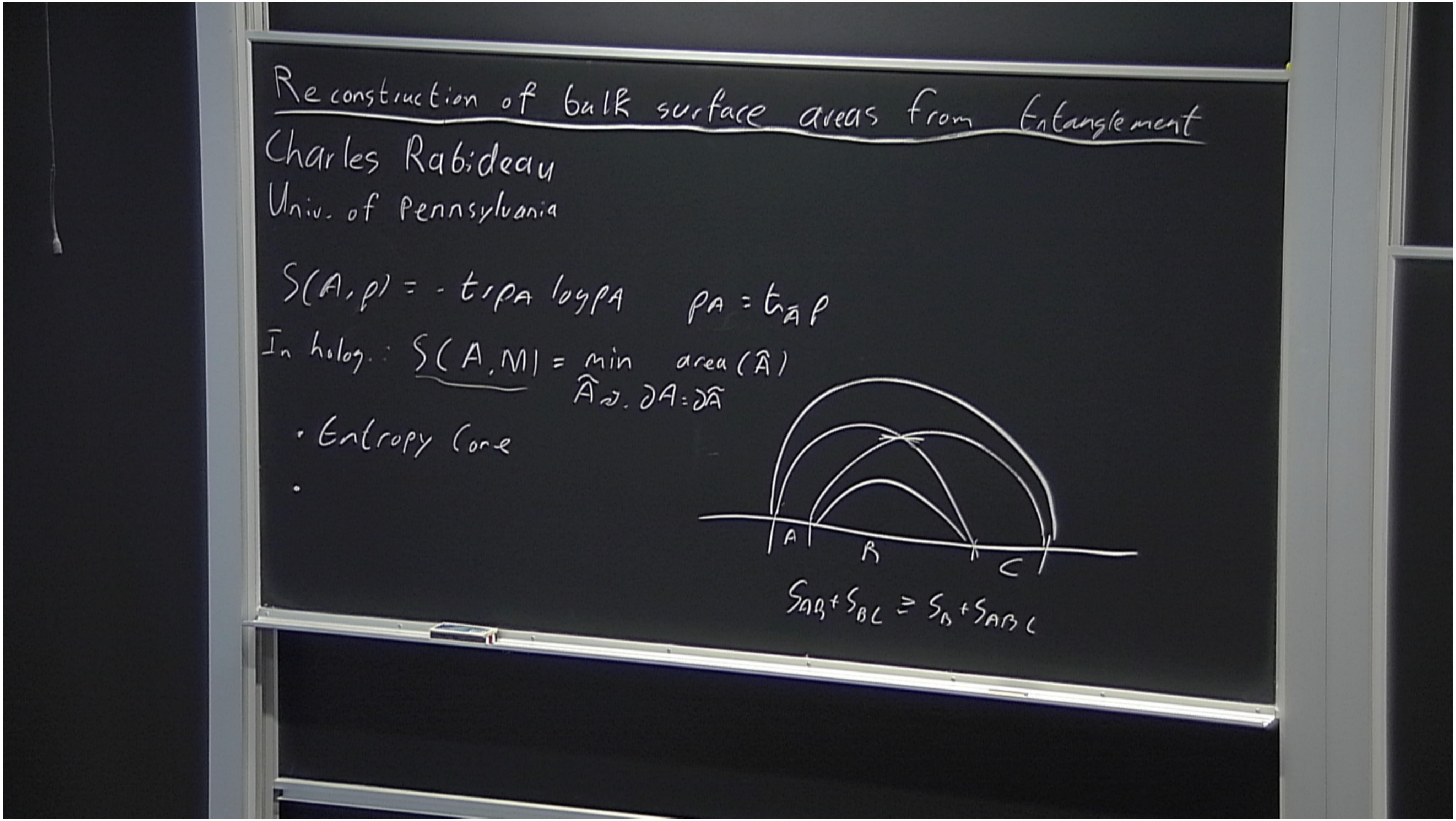
$S(A,\rho) = -\mathrm{tr}\,\rho_A \log \rho_A \quad \rho_A = \mathrm{tr}_{\bar{A}}\,\rho$
In holog.: $S(A, M) = \min_{\hat{A}\ \mathrm{s.t.}\ \partial A = \partial\hat{A}} \mathrm{area}(\hat{A})$


Reconstruction of bulk surface areas from Entanglement
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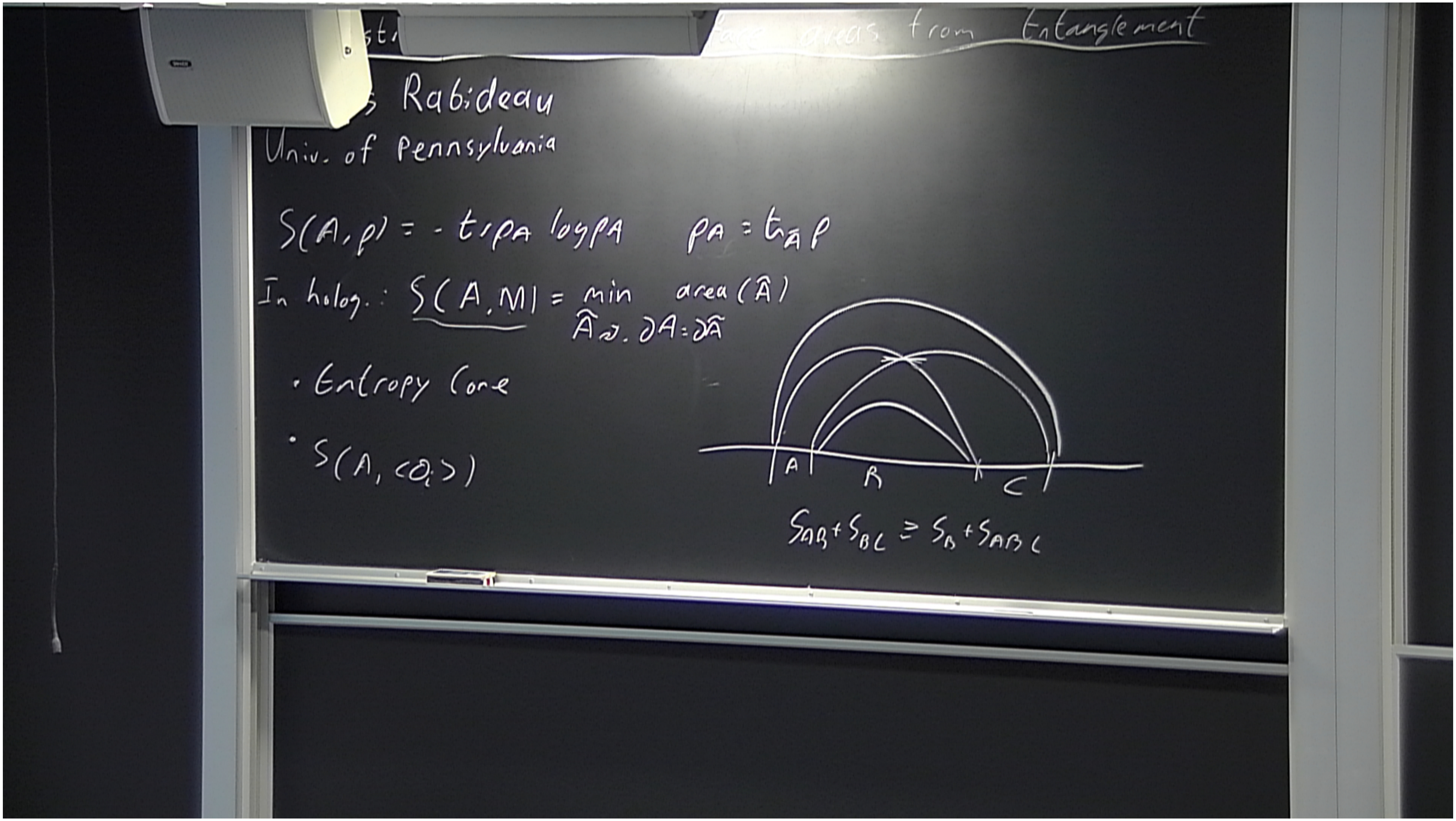
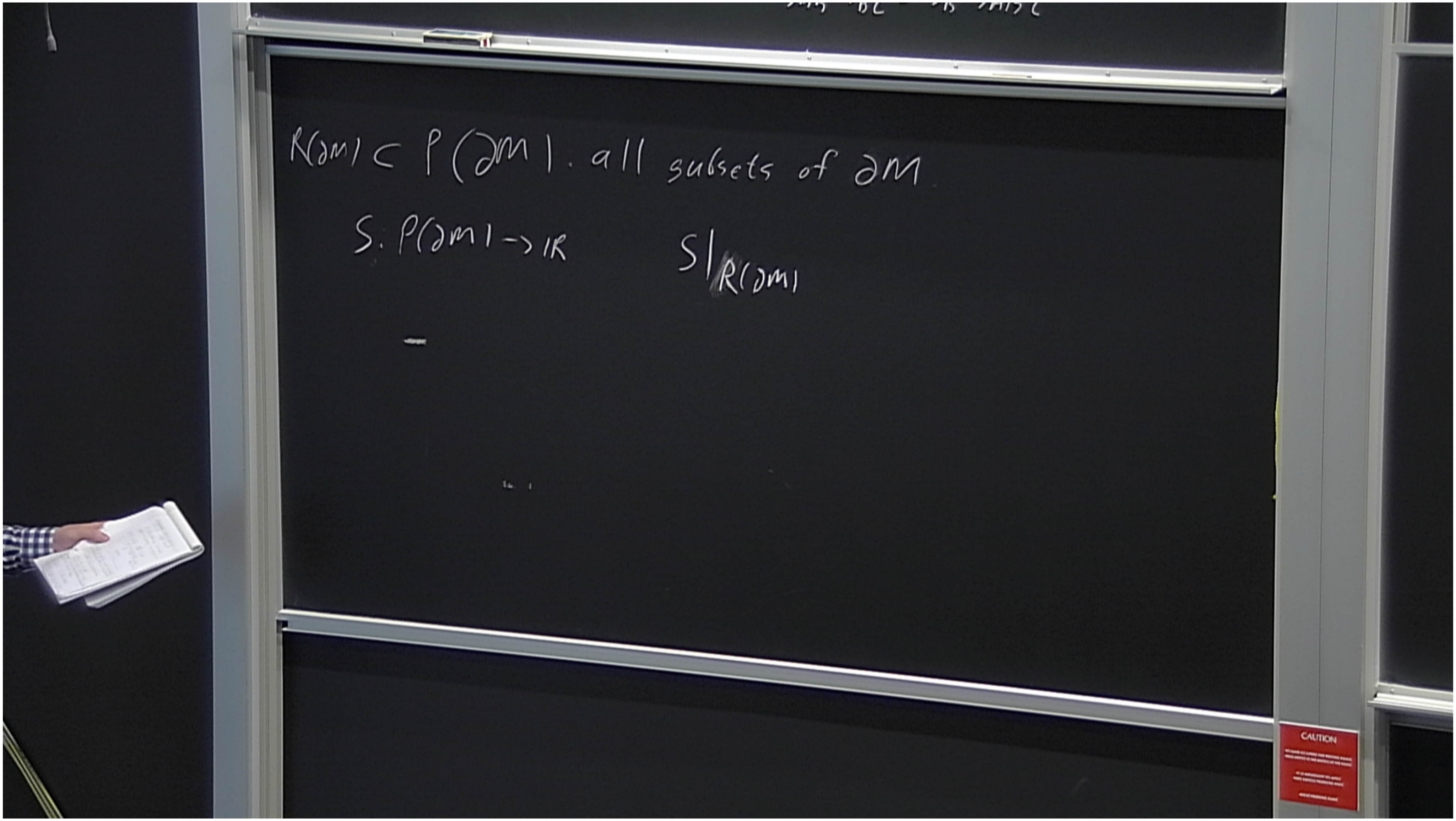
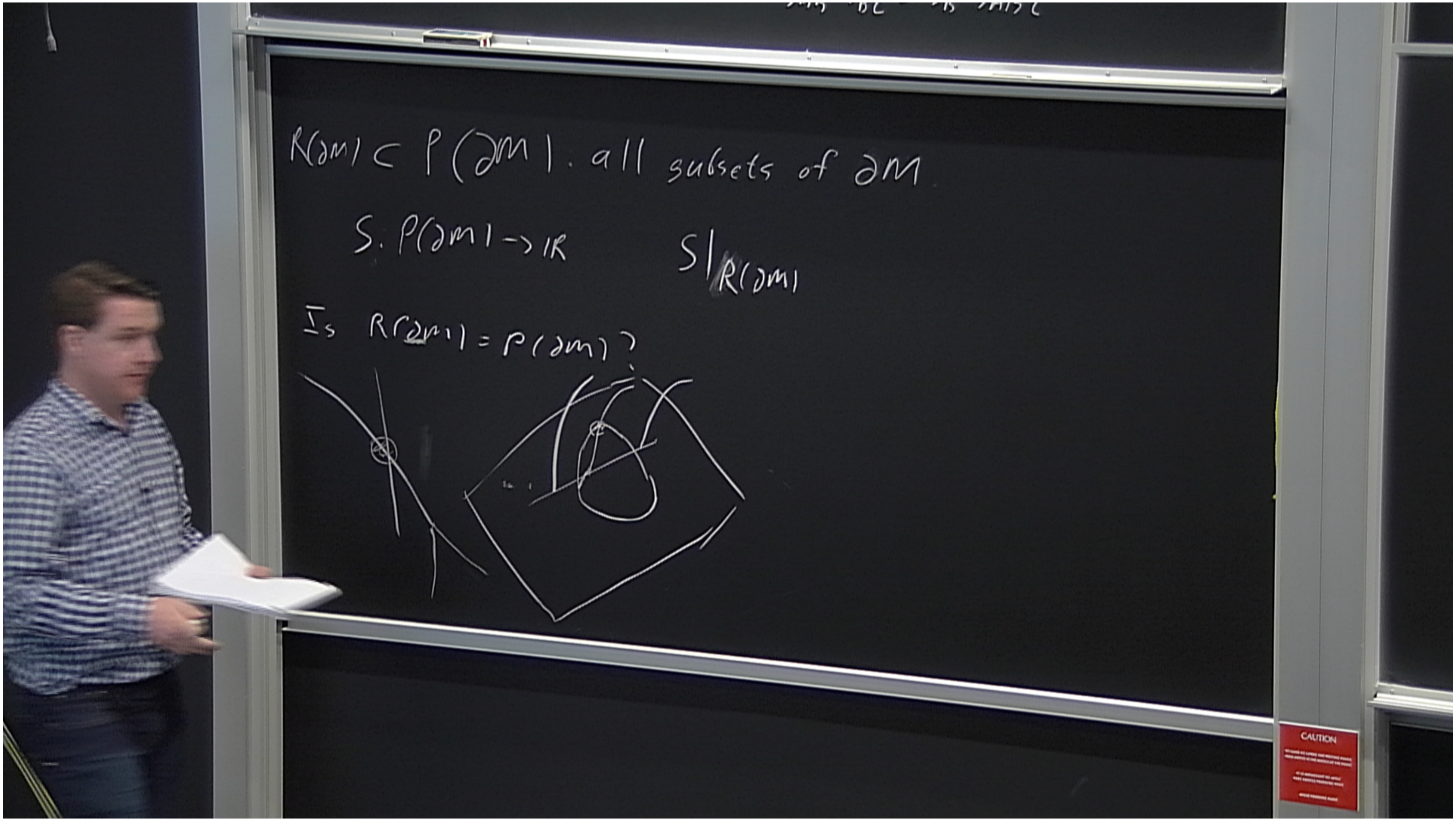
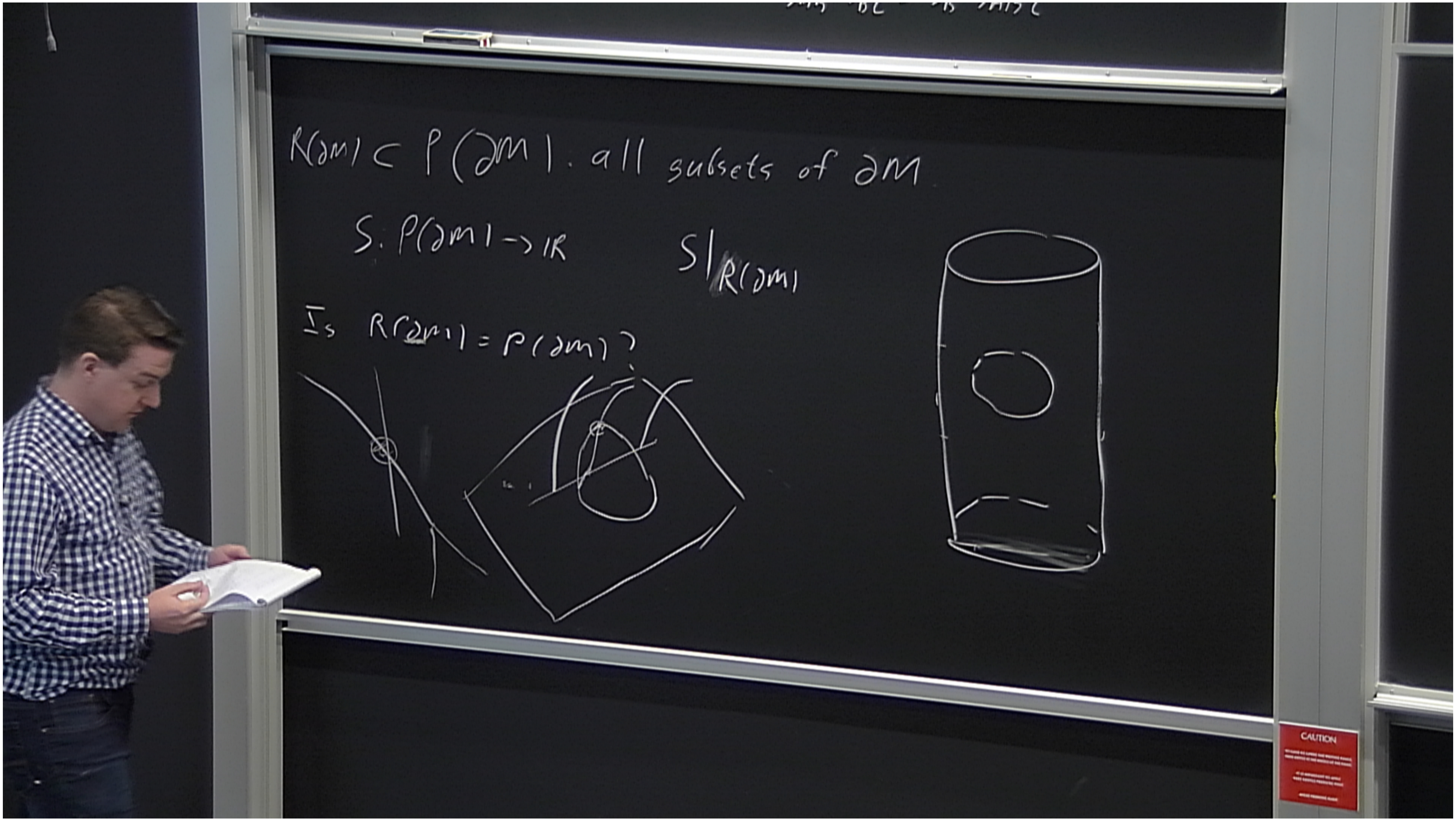
$S(A,\rho) = -\mathrm{tr}\,\rho_A \log \rho_A$ $\rho_A = \mathrm{tr}_{\bar{A}}\,\rho$
In holog.: $S(A,M) = \min_{\hat{A}\ \mathrm{s.t.}\ \partial A = \partial\hat{A}} \mathrm{area}(\hat{A})$
• Entropy Cone
•
A
B
C
$S_{AB} + S_{BC} \geq S_B + S_{ABC}$

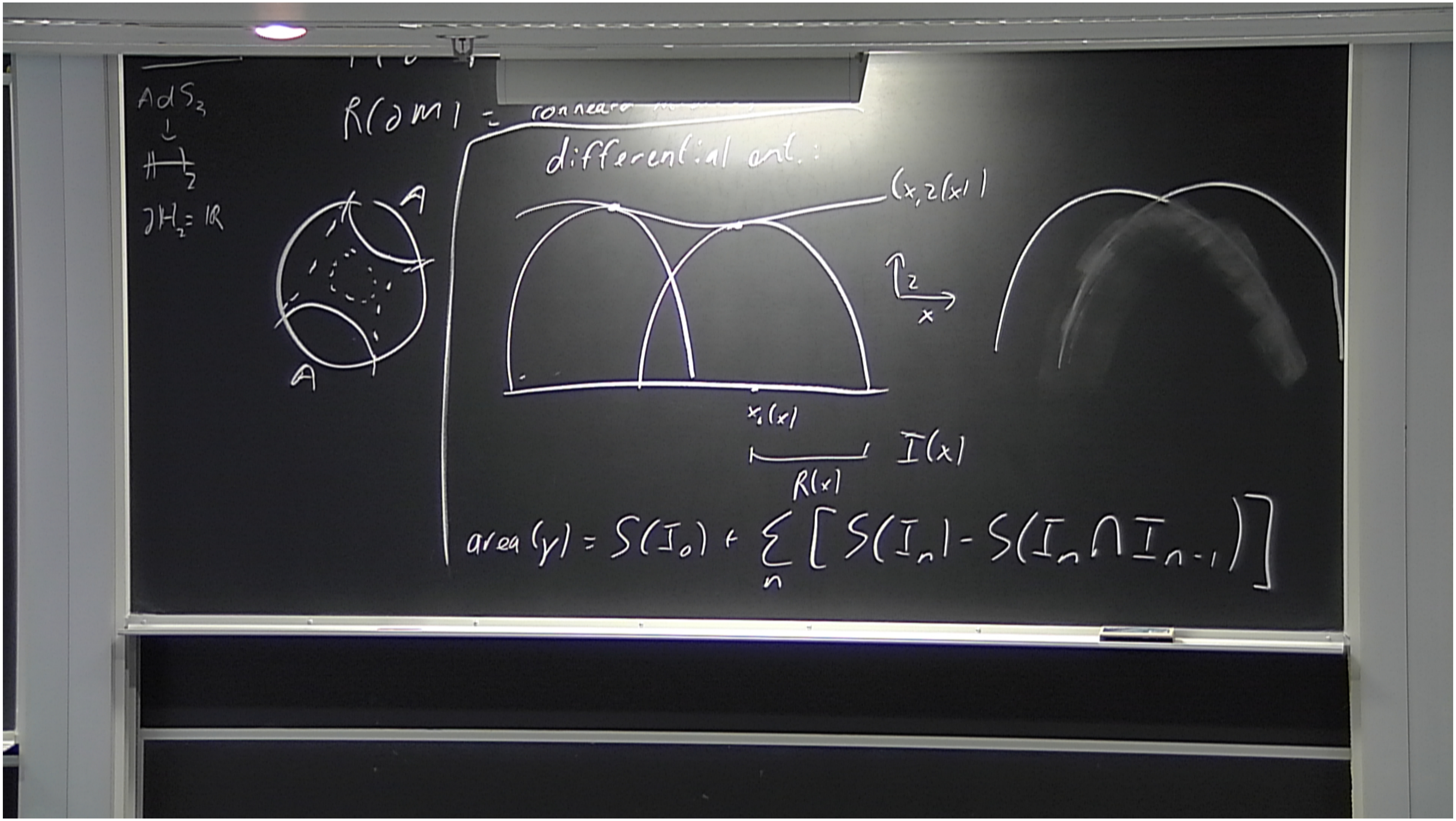

from Entanglement
Rabideau
Univ. of Pennsylvania
S(A,ρ) = - tr ρA log ρA   ρA = trĀ ρ
In holog.: S(A,M) = min area(Â)
• Entropy Cone
• S(A, <Oi>)
A
B
C
SAB + SBC ≥ SB + SABC

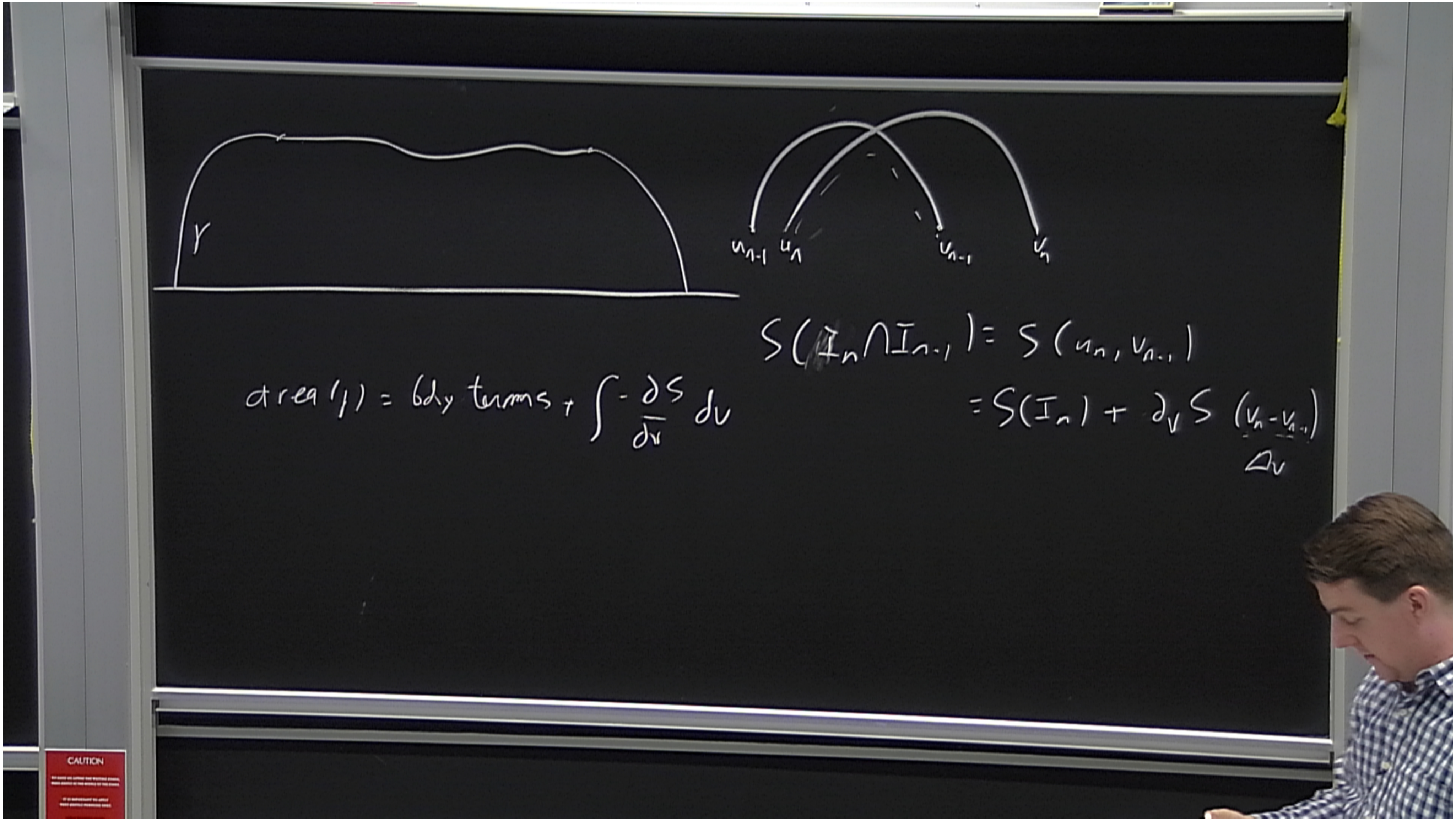

$R(\partial M) \subset P(\partial M)$. all subsets of ∂M
$S: P(\partial M) \to \mathbb{R}$
$S|_{R(\partial M)}$

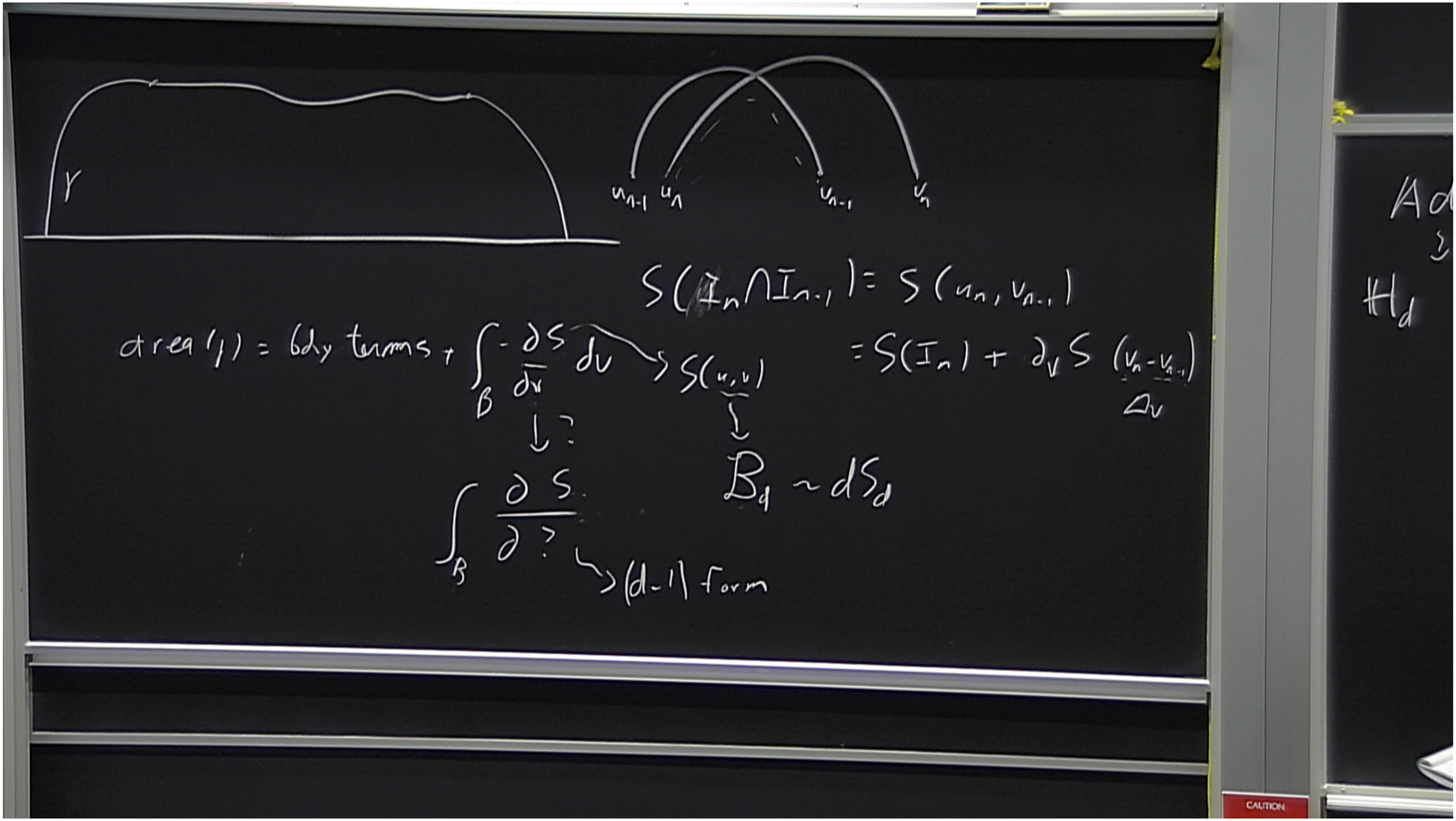

R(∂M) ⊂ P(∂M). all subsets of ∂M
S: P(∂M) → ℝ
S|R(∂M)
Is R(∂M) = P(∂M)?

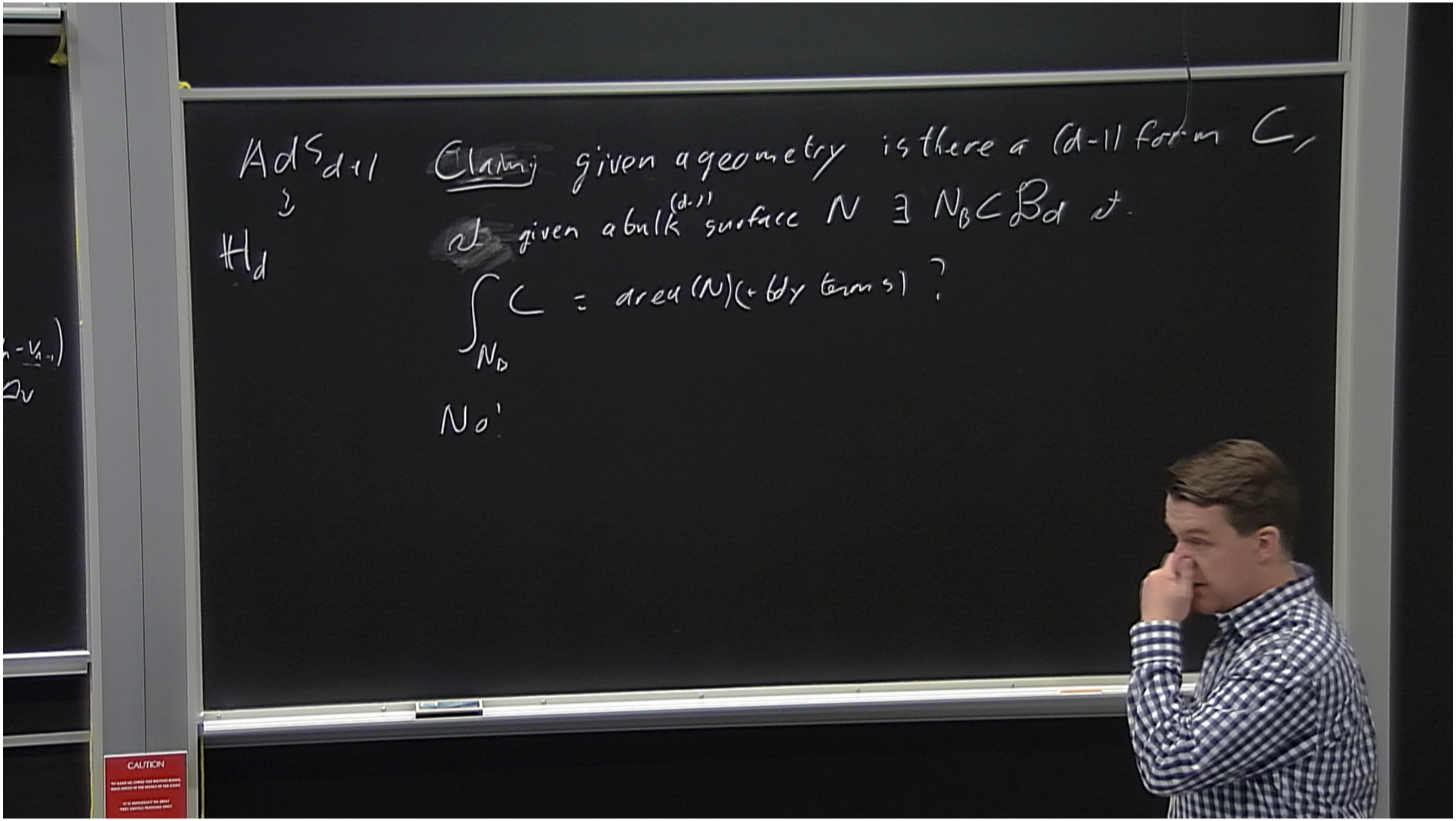

R(∂M) ⊂ P(∂M) . all subsets of ∂M
S: P(∂M) → ℝ
S|R(∂M)
Is R(∂M) = P(∂M)?

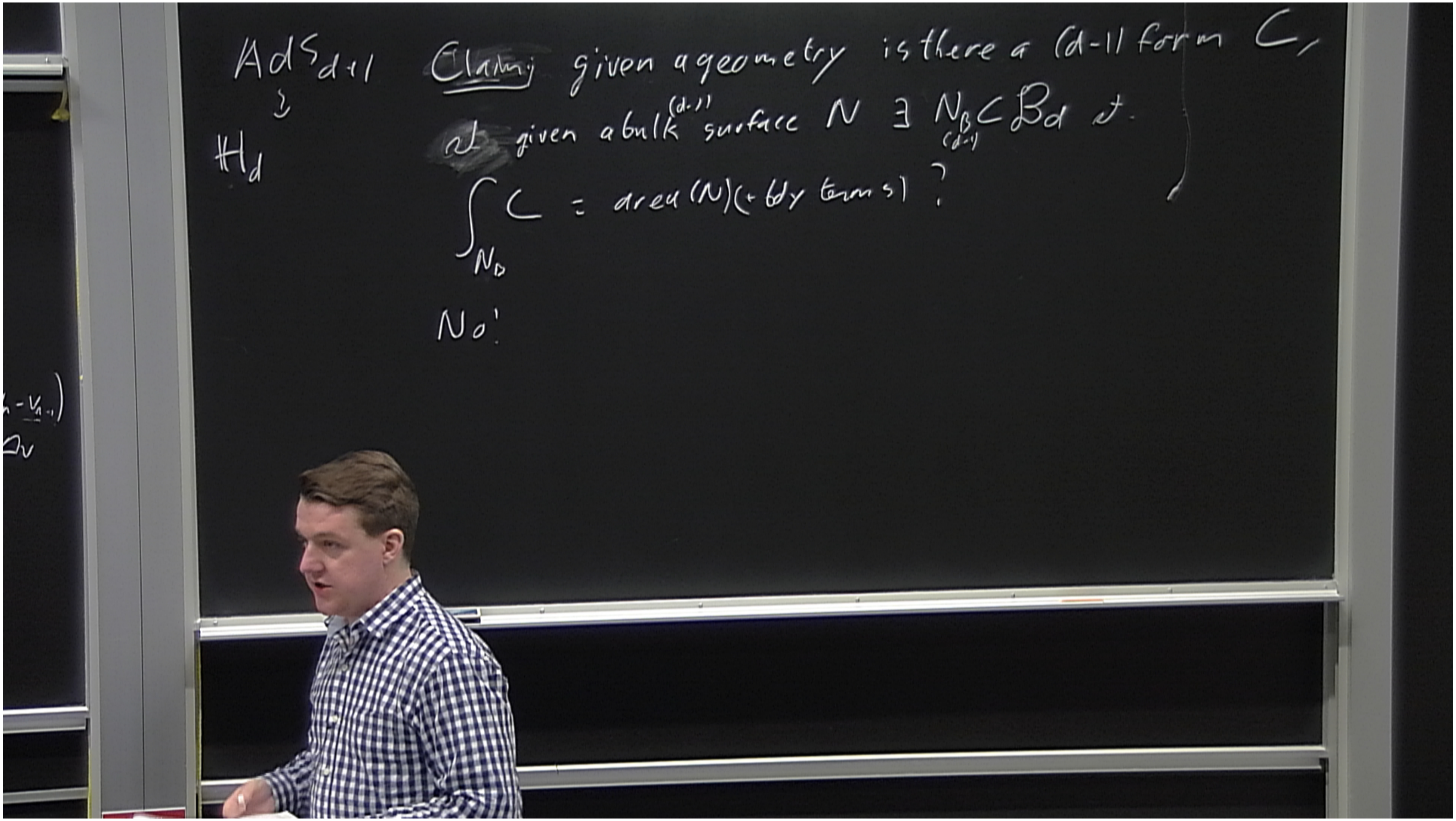

d=2)
AdS_3
H_2
∂H_2 = R
P(∂M) = Unions of intervals
R(∂M) = connected intervals
A
A
CAUTION
CAUTION

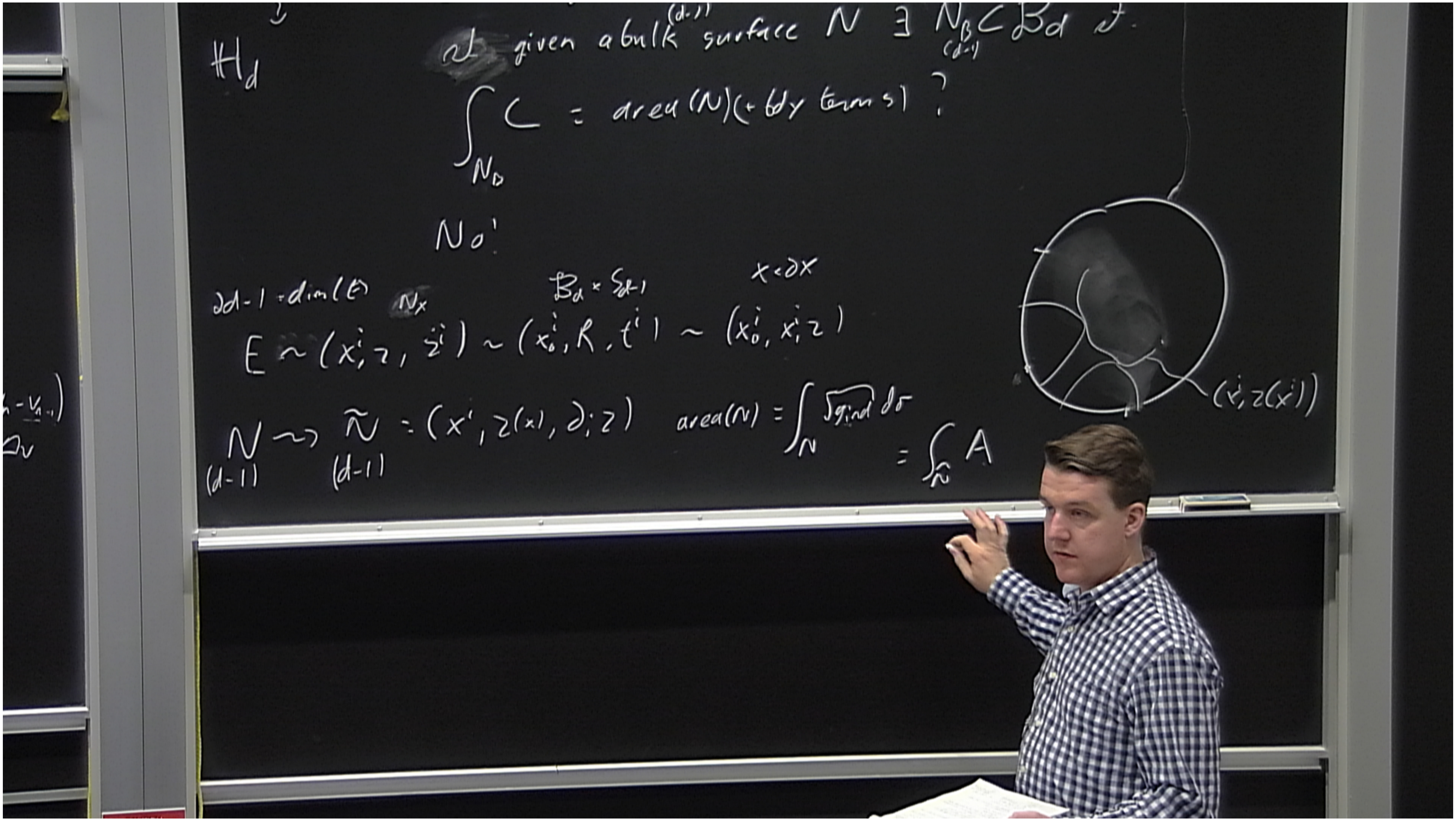

AdS3
∂H2 = R
R(∂M) = connected
A
A
differential ent.:
(x, z(x))
z
x
x0(x)
I(x)
R(x)
area(γ) = S(I0) + Σn [S(In) - S(In ∩ In-1)]

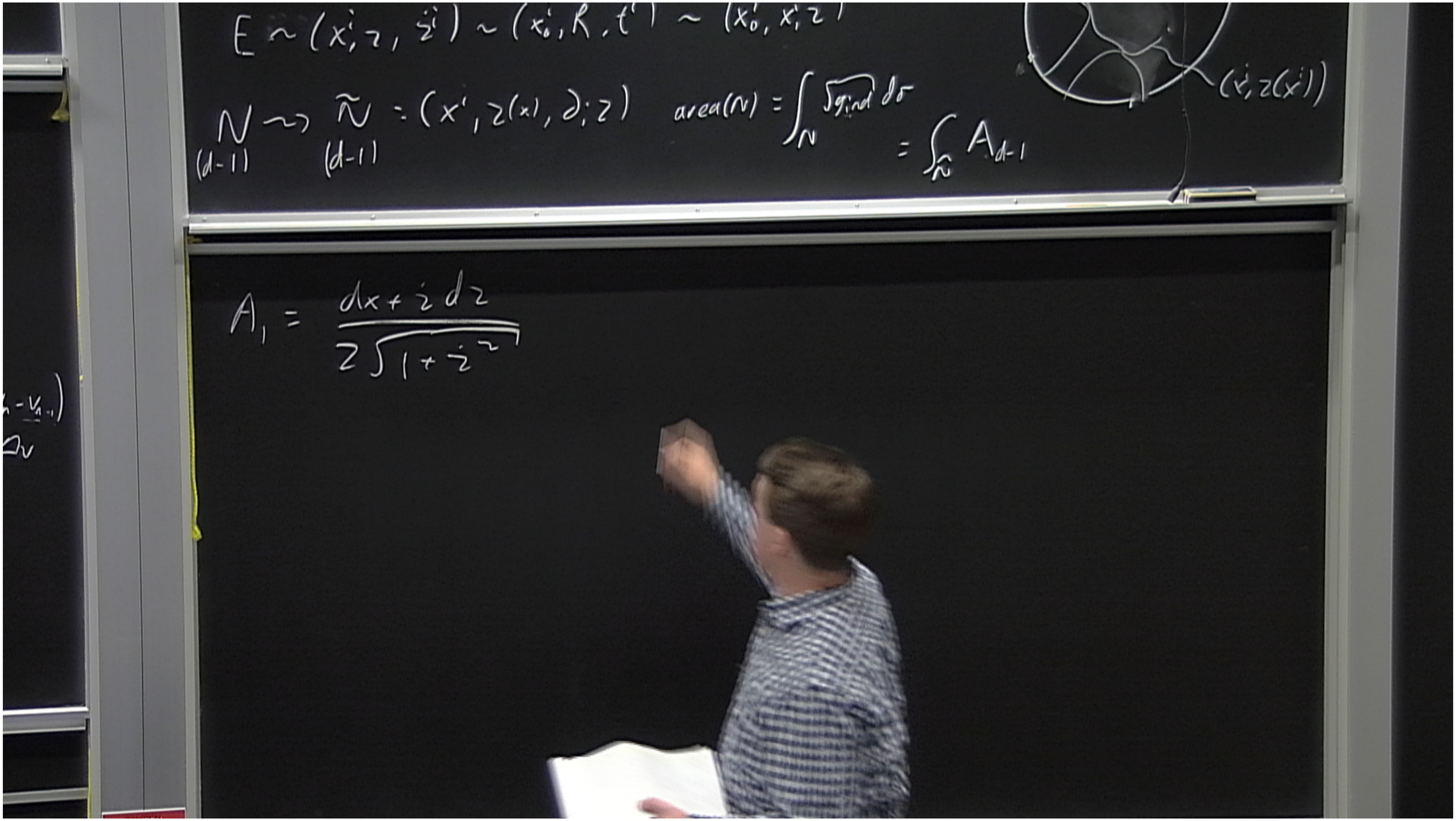

γ
u_{n-1} u_n v_{n-1} v_n
$S(I_n \cap I_{n-1}) = S(u_n, v_{n-1})$
$= S(I_n) + \partial_v S \, (v_n - v_{n-1})$
Δv
$\text{area}(\gamma) = \text{bdy terms} + \int -\frac{\partial S}{\partial v} dv$

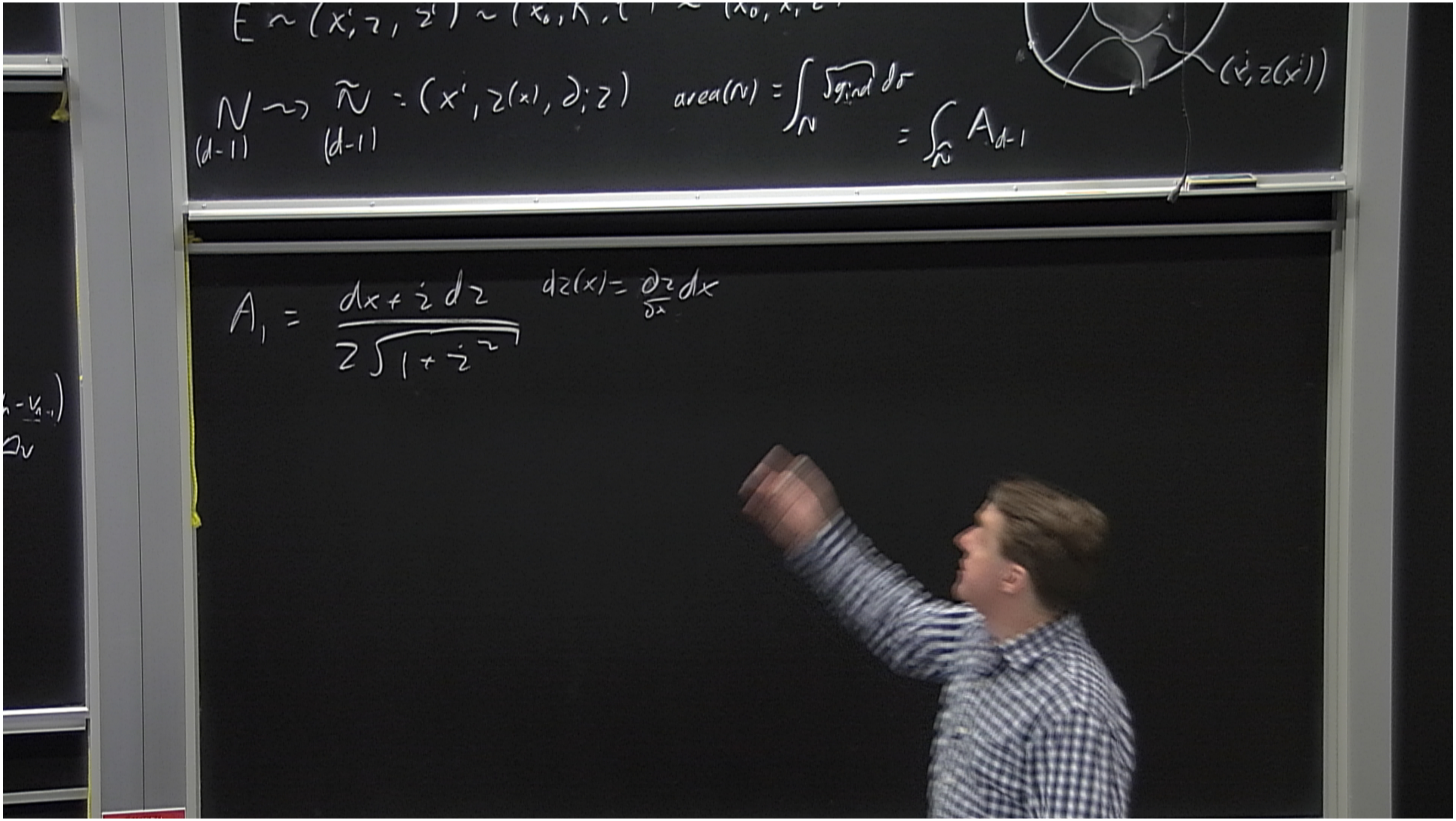

γ
u_{n-1} u_n v_{n-1} v_n
$S(I_n \cap I_{n-1}) = S(u_n, v_{n-1})$
area(γ) = bdy terms + $\int_B -\frac{\partial S}{\partial v} dv$
$S(u,v)$
$= S(I_n) + \partial_v S \, (v_n - v_{n-1})$
Δv
$B_d \sim dS_d$
$\int_B \frac{\partial S}{\partial ?}$
(d-1) form
Ad
H_d


AdS_{d+1}
H_d
Claim: given a geometry is there a (d-1) form C,
st. given a bulk (d-1) surface N ∃ N_B ⊂ B_d st.
∫_{N_B} C = area(N) (+ bdy terms) ?
No:

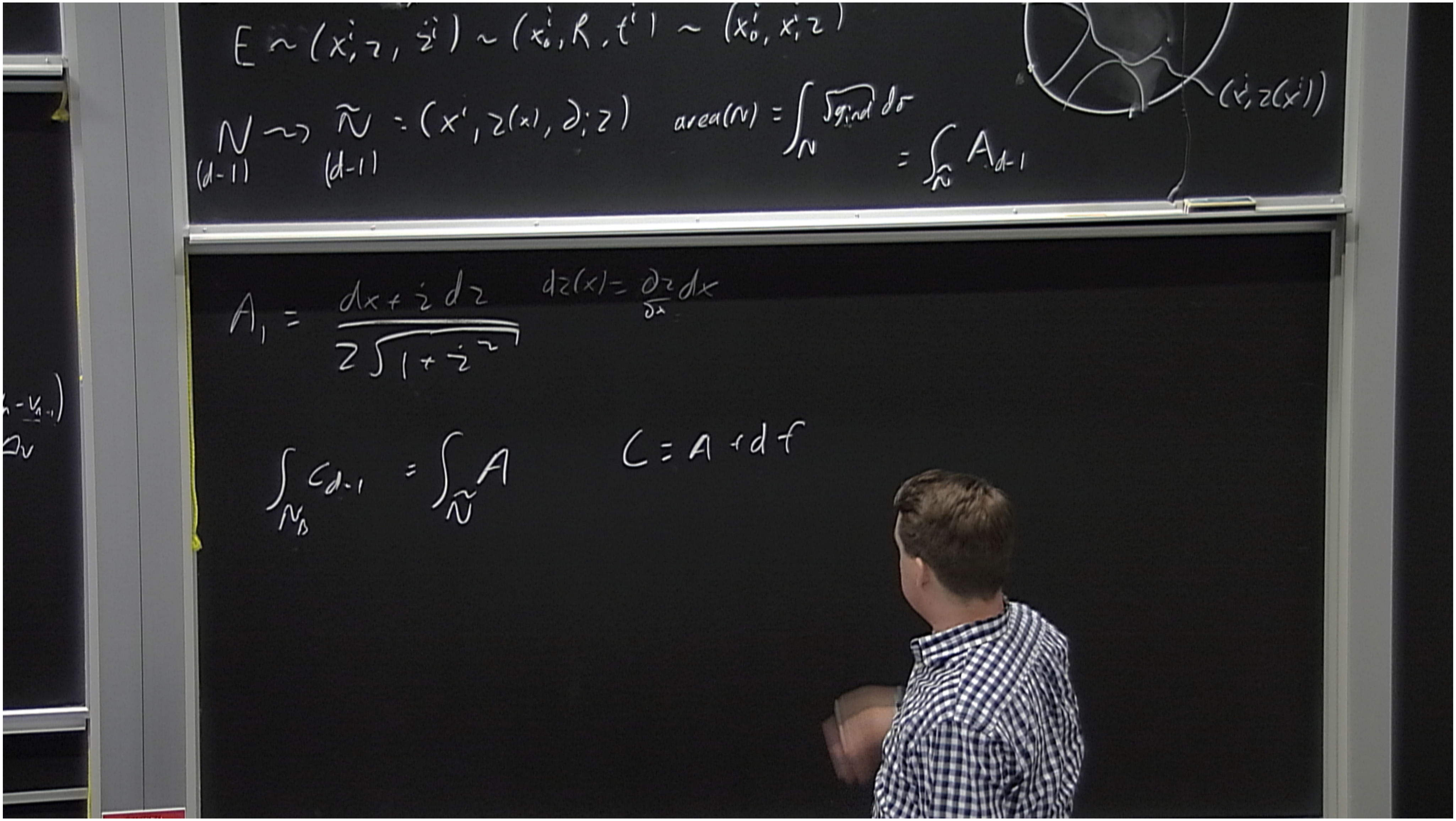

AdS d+1
Hd
Claim: given a geometry is there a (d-1) form C,
st given a bulk (d-1) surface N ∃ NB ⊂ Bd st.
∫NB C = area(N) (+ bdy terms) ?
No!

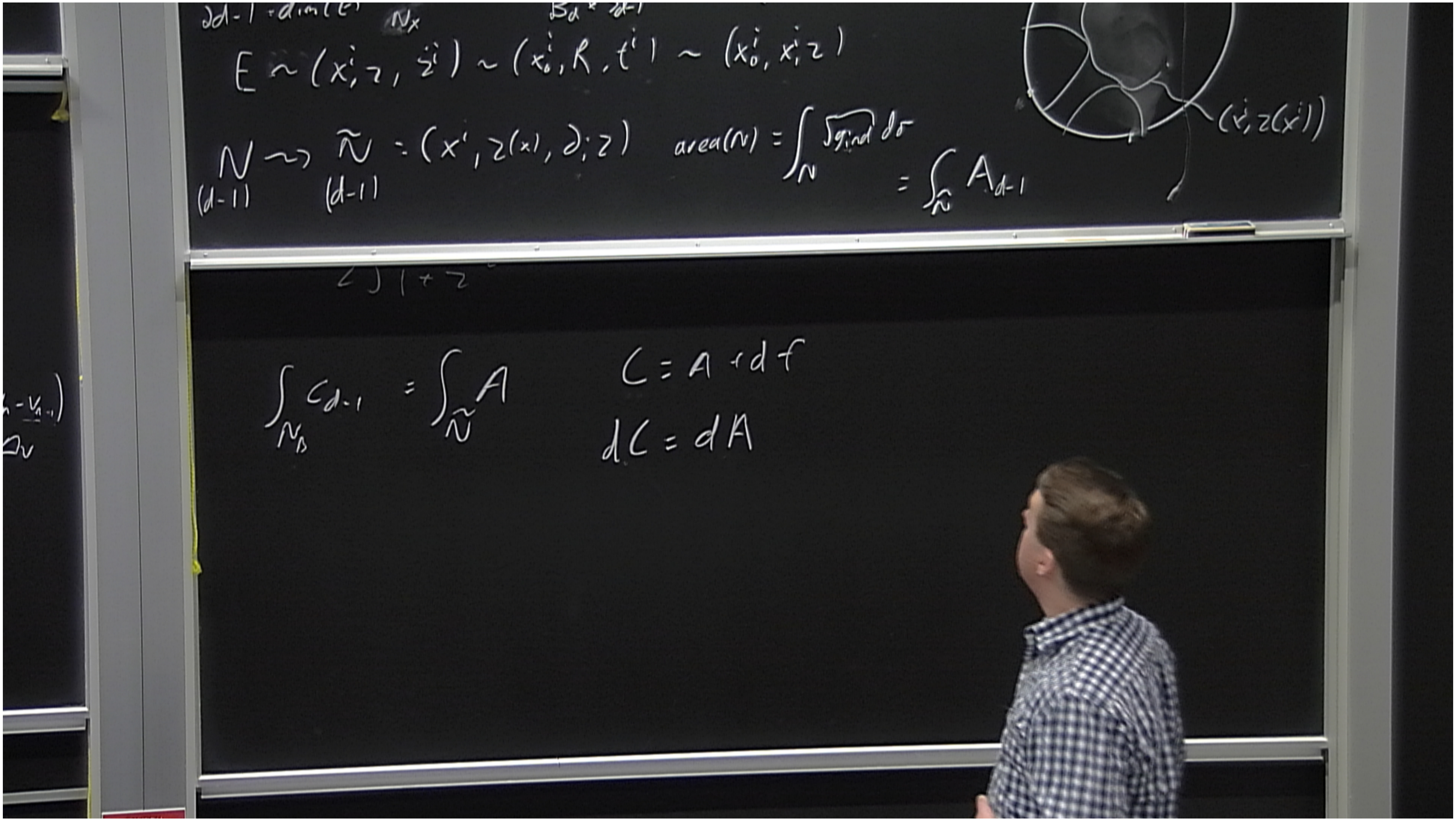

H_d
given a bulk (d-1) surface N ∃ N_B ⊂ B_d s.t.
∫_{N_B} C = area(N) (+ bdy terms) ?
N_B :
∂d-1 = dim(E) N_x B_d × S_{d-1} x∈∂X
E ~ (x^i, z, ż^i) ~ (x_0^i, R, t^i) ~ (x_0^i, x^i, z)
N (d-1) → Ñ (d-1) = (x^i, z(x), ∂_i z) area(N) = ∫_N √g_ind dσ = ∫_Ñ A
(x^i, z(x^i))

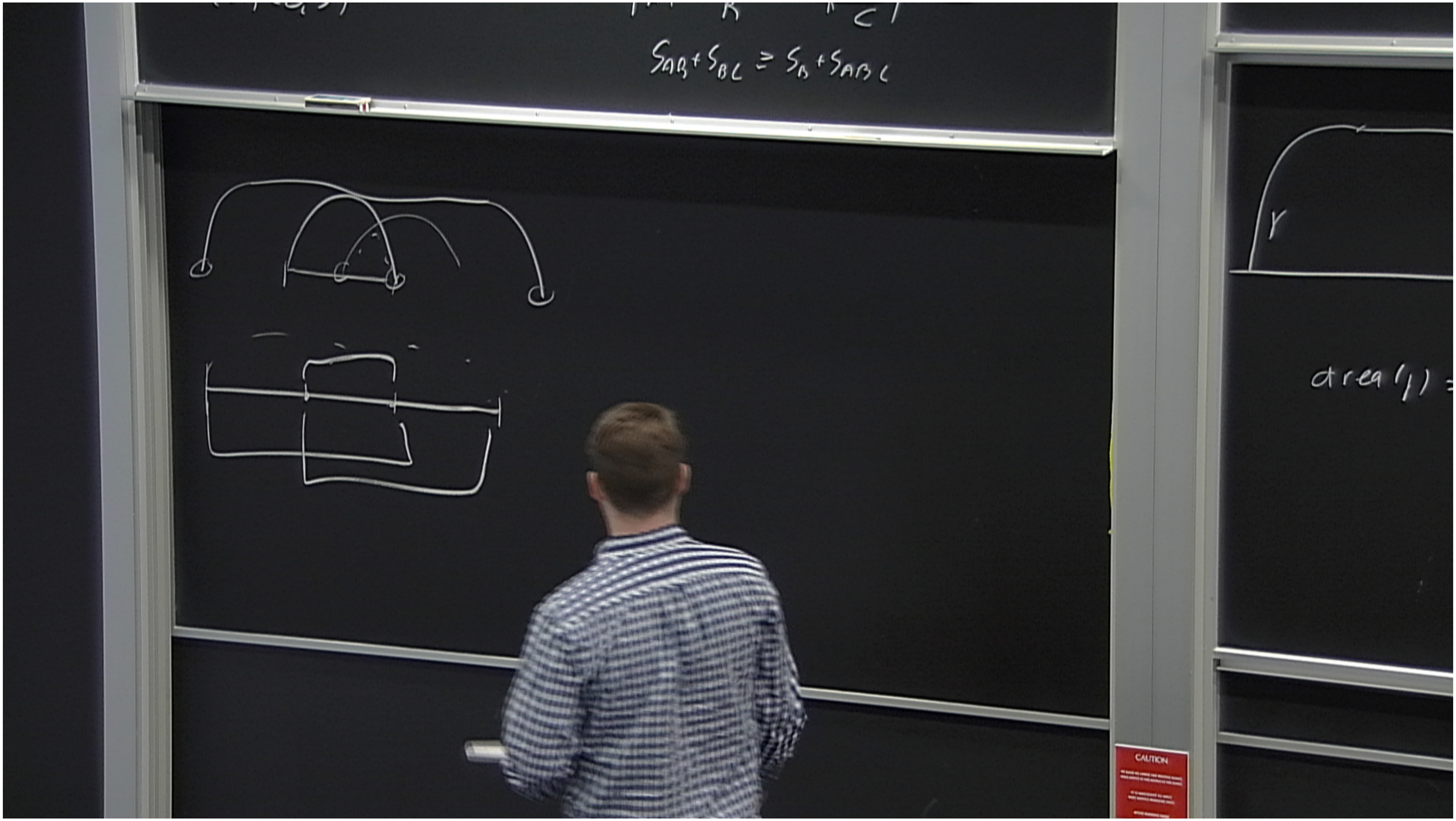

E ~ (x^i, z, ż^i) ~ (x_0^i, R, t^i) ~ (x_0^i, x_i z)
N ⇝ Ñ = (x^i, z(x), ∂_i z)
(d-1) (d-1)
area(N) = ∫_N √g_ind dσ
= ∫_Ñ A_{d-1}
(x^i, z(x^i))
A_1 = (dx + ż dz) / (z √(1 + ż²))

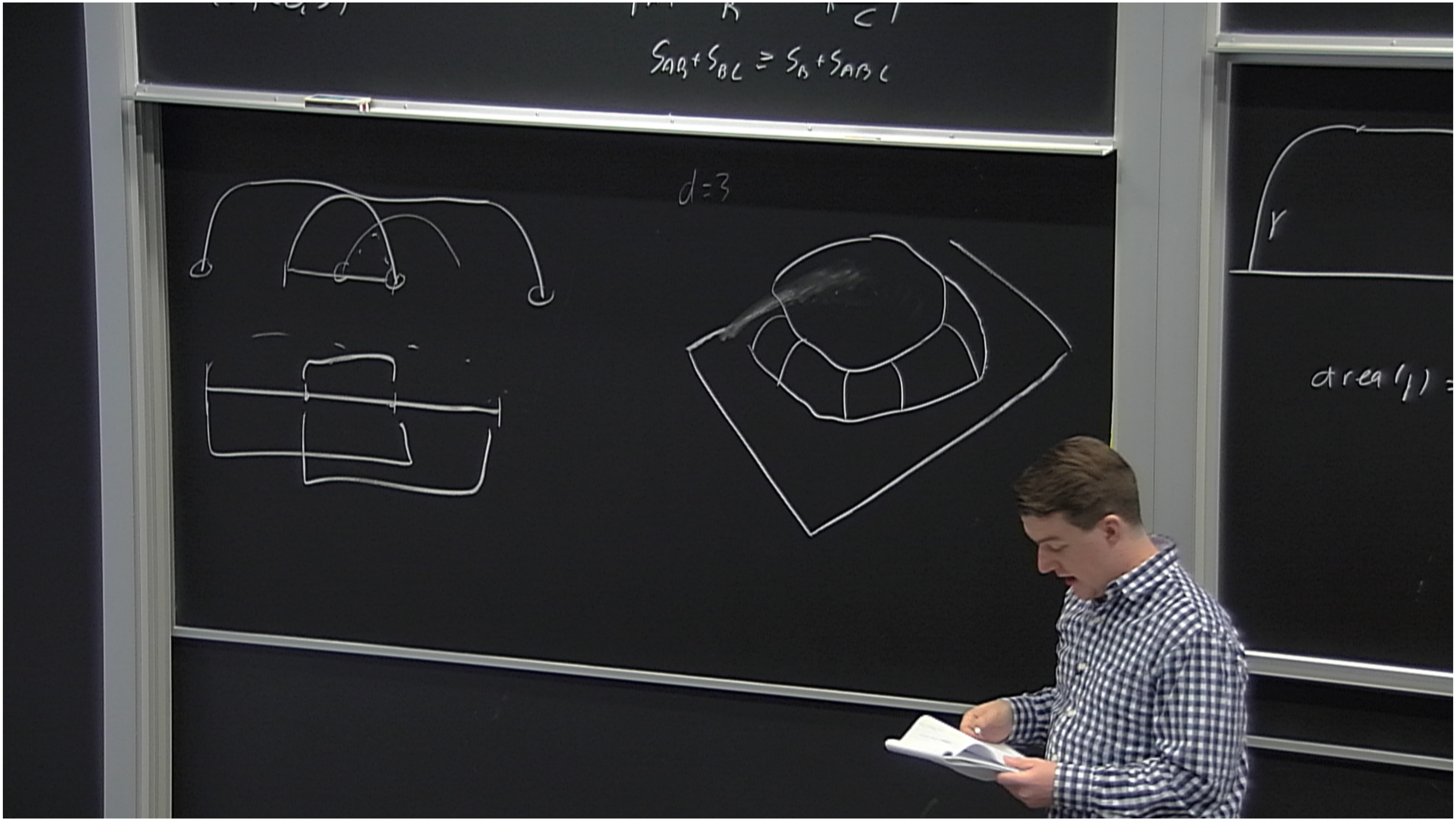


$N \rightsquigarrow \tilde{N} = (x^i, z(x), \partial_i z)$
$(d-1)$ $(d-1)$
$\mathrm{area}(N) = \int_N \sqrt{g_{ind}}\, d\sigma$
$= \int_{\tilde{N}} A_{d-1}$
$(x^i, z(x^i))$
$A_1 = \frac{dx + \dot{z}\, dz}{z\sqrt{1+\dot{z}^2}}$
$dz(x) = \frac{\partial z}{\partial x} dx$

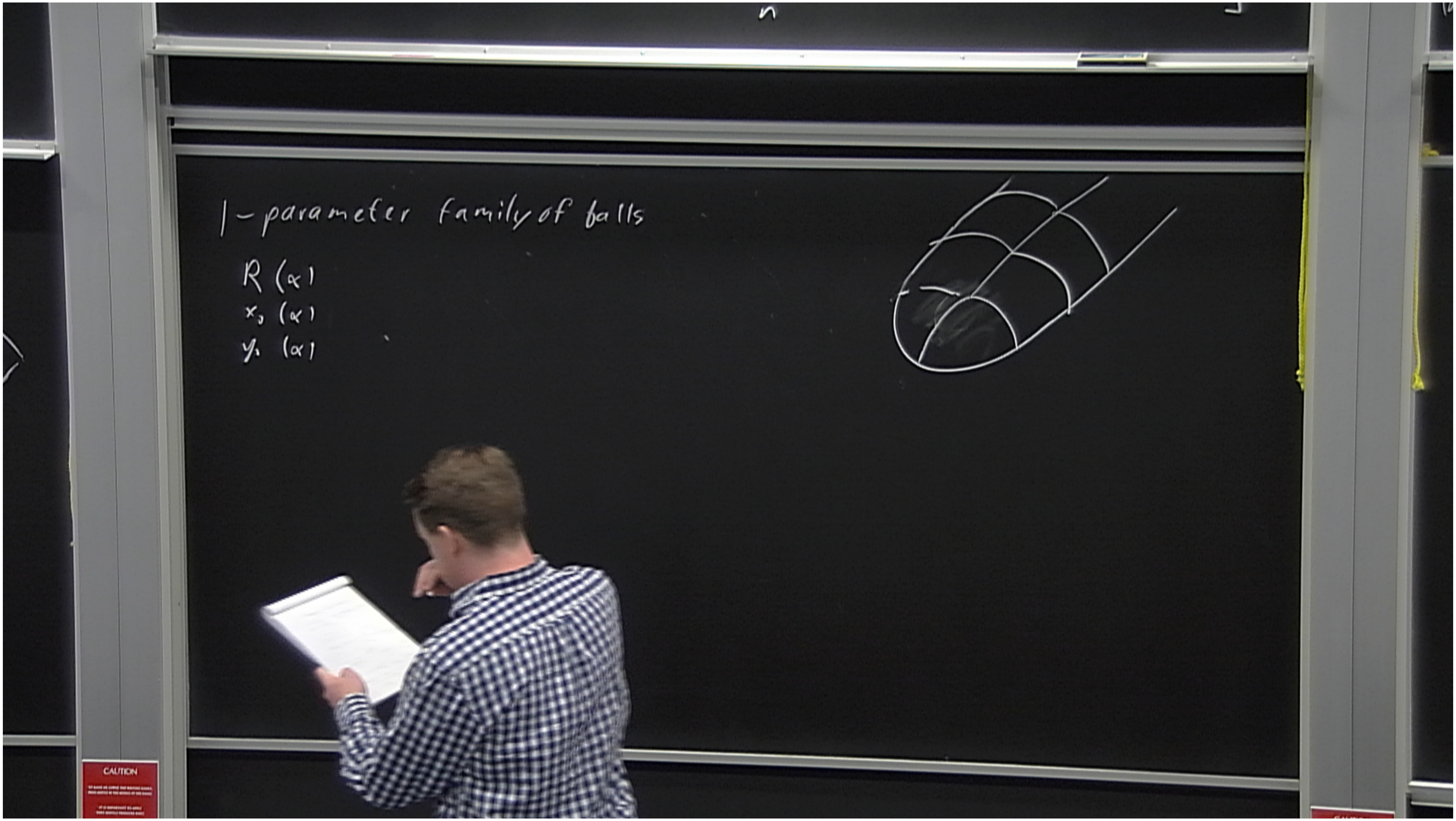

$E \sim (x^i, z, \dot{z}^i) \sim (x_0^i, R, t^i) \sim (x_0^i, x^i, z)$
$\underset{(d-1)}{N} \rightsquigarrow \underset{(d-1)}{\tilde{N}} = (x^i, z(x), \partial_i z)$
$\text{area}(N) = \int_N \sqrt{g_{ind}}\, d\sigma = \int_{\tilde{N}} A_{d-1}$
$(x^i, z(x^i))$
$A_1 = \frac{dx + \dot{z}\, dz}{z\sqrt{1+\dot{z}^2}}$
$dz(x) = \frac{\partial z}{\partial x} dx$
$\int_{N_B} C_{d-1} = \int_{\tilde{N}} A$
$C = A + df$

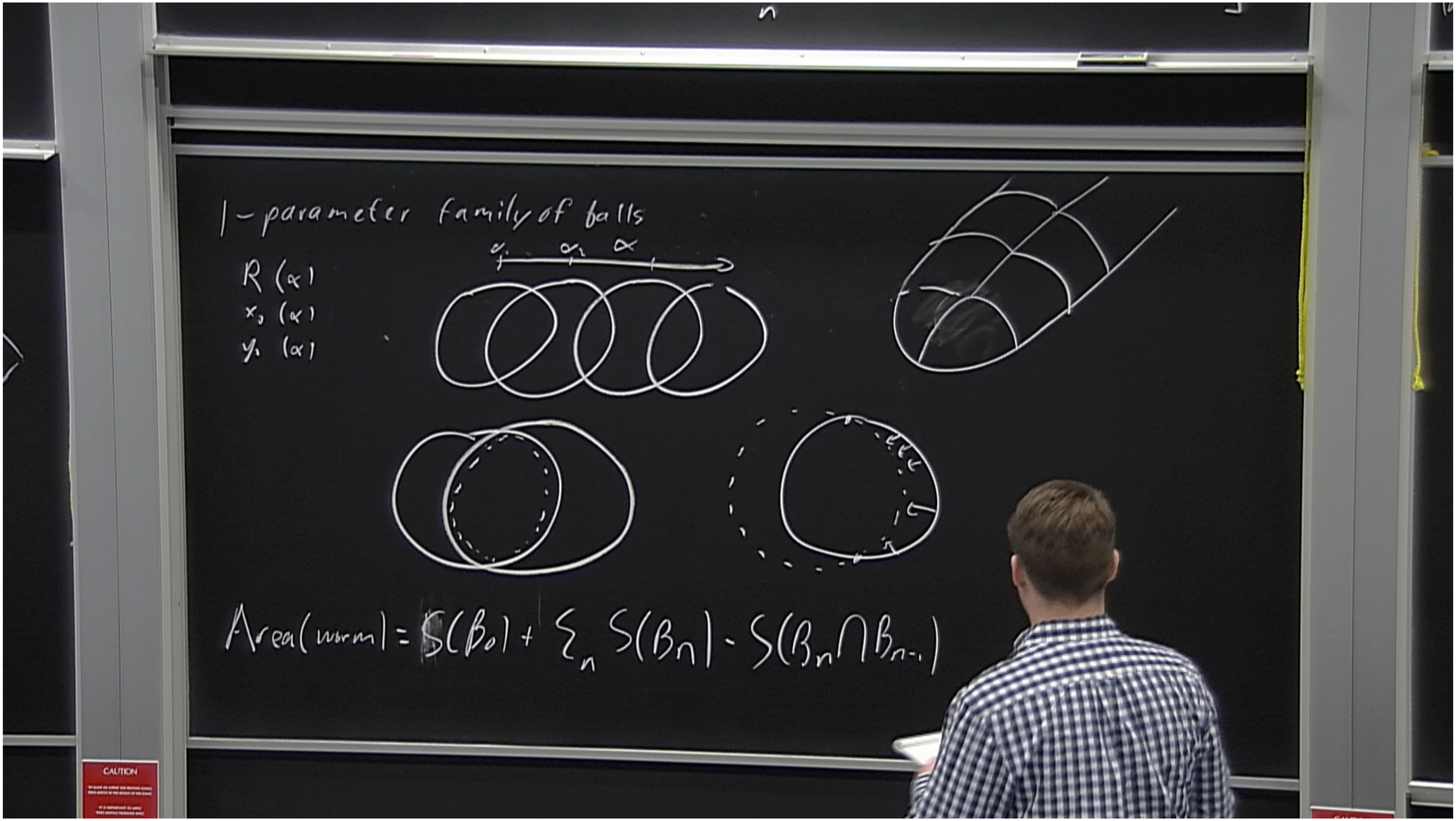

E ~ (x^i; z, z^i) ~ (x_0^i, R, t^i) ~ (x_0^i, x; z)
Ñ ~> Ñ = (x^i, z(x), ∂; z)
(d-1)
(d-1)
area(N) = ∫_N √g_ind dσ
= ∫_Ñ A_{d-1}
(x^i, z(x^i))
∫_{N_B} C_{d-1} = ∫_Ñ A
C = A + df
dC = dA

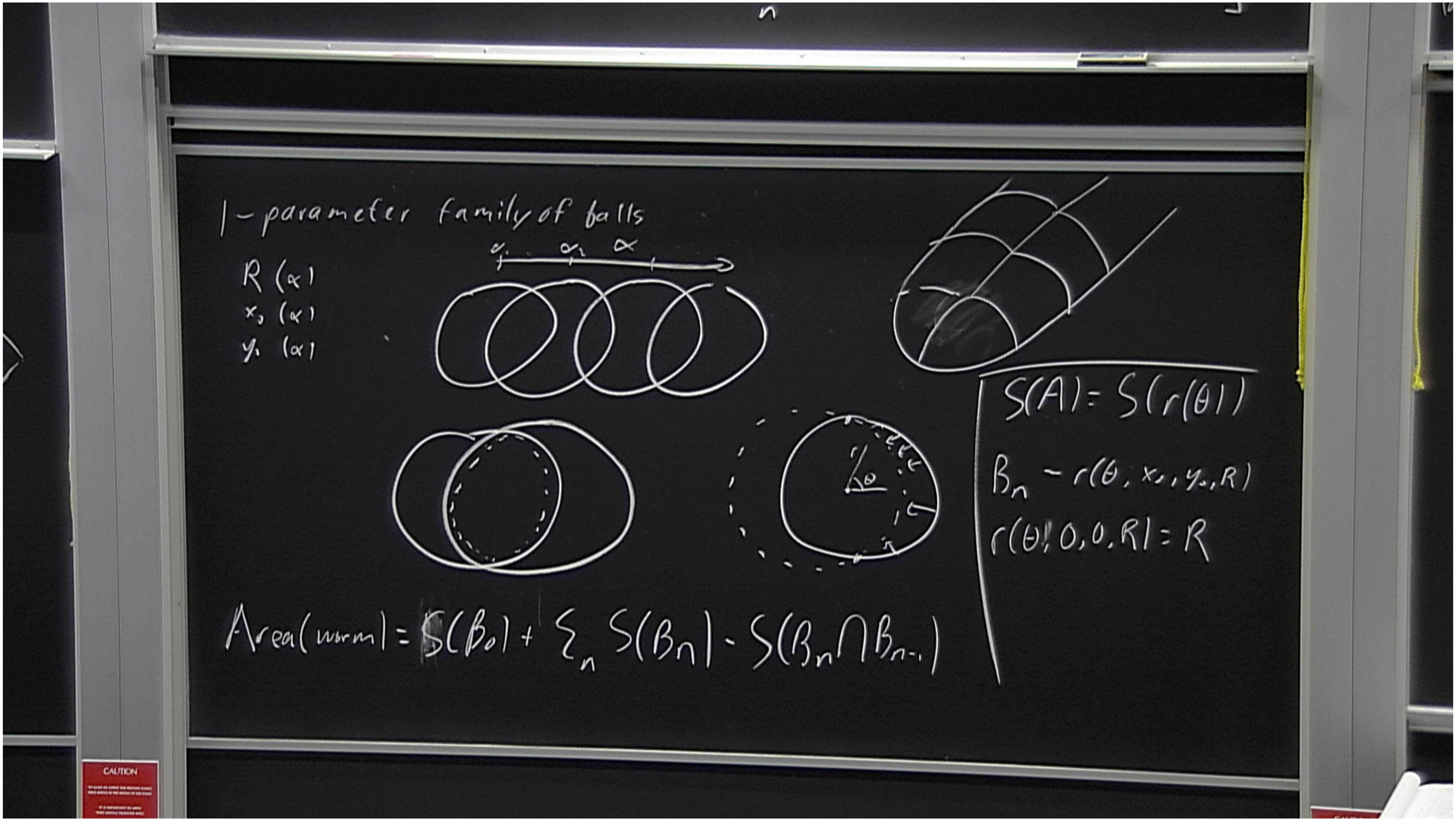

SAB + SBC = SB + SABC
γ
area(γ) =

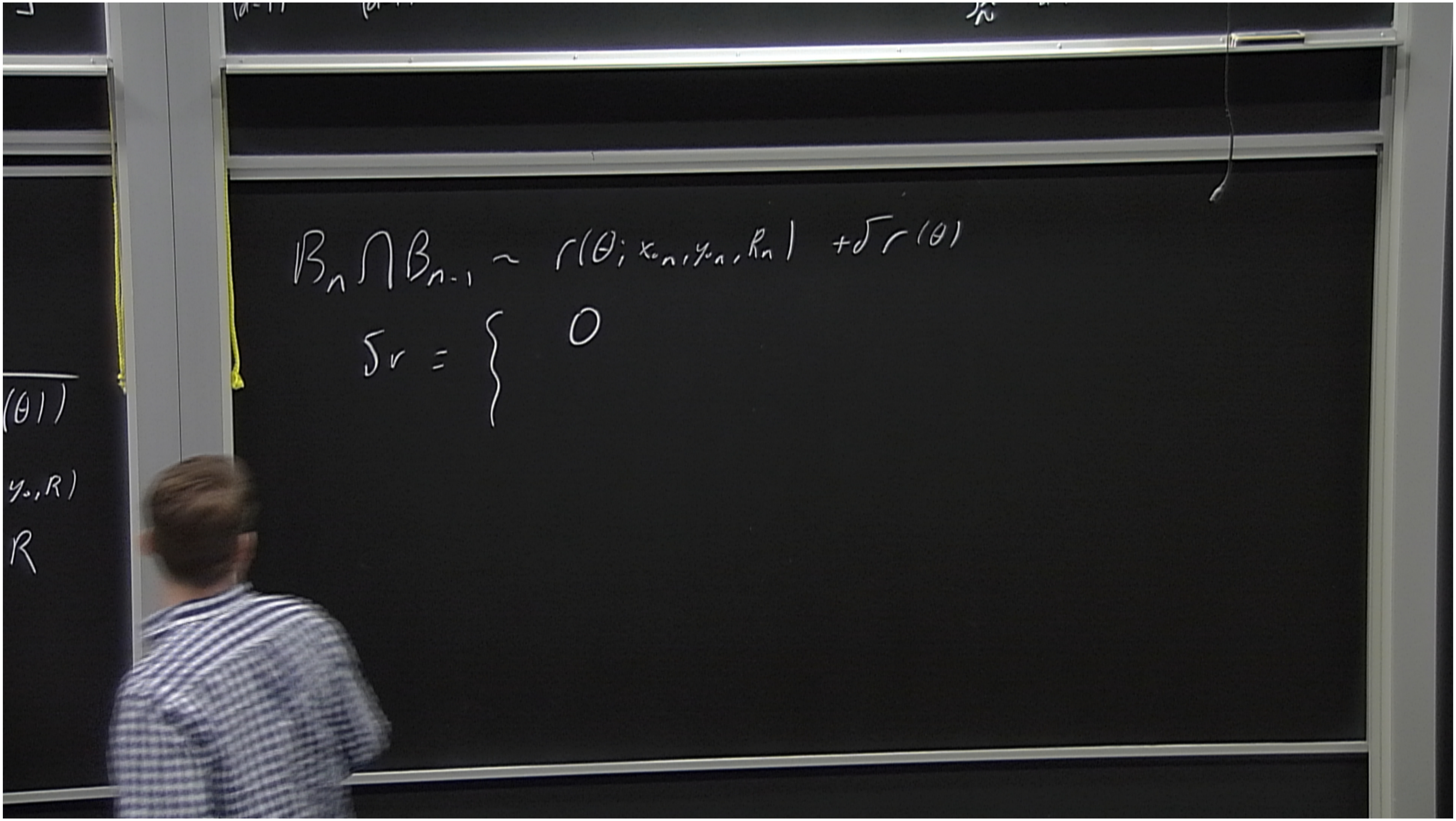


$S_{AB} + S_{BC} \geq S_B + S_{ABC}$
d=3
γ
area(γ) =


1-parameter family of balls
$R(\alpha)$
$x_0(\alpha)$
$y_0(\alpha)$


1-parameter family of balls
$R(\alpha)$
$x_0(\alpha)$
$y_0(\alpha)$
α_1 α_2 α
$\mathrm{Area(worm)} = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$


1-parameter family of balls
α_1 α_2 α
$R(\alpha)$
$x_0(\alpha)$
$y_0(\alpha)$
$S(A) = S(r(\theta))$
$B_n \sim r(\theta, x_0, y_0, R)$
$r(\theta, 0, 0, R) = R$
$\mathrm{Area(worm)} = S(B_0) + \sum_n S(B_n) - S(B_n \cap B_{n-1})$

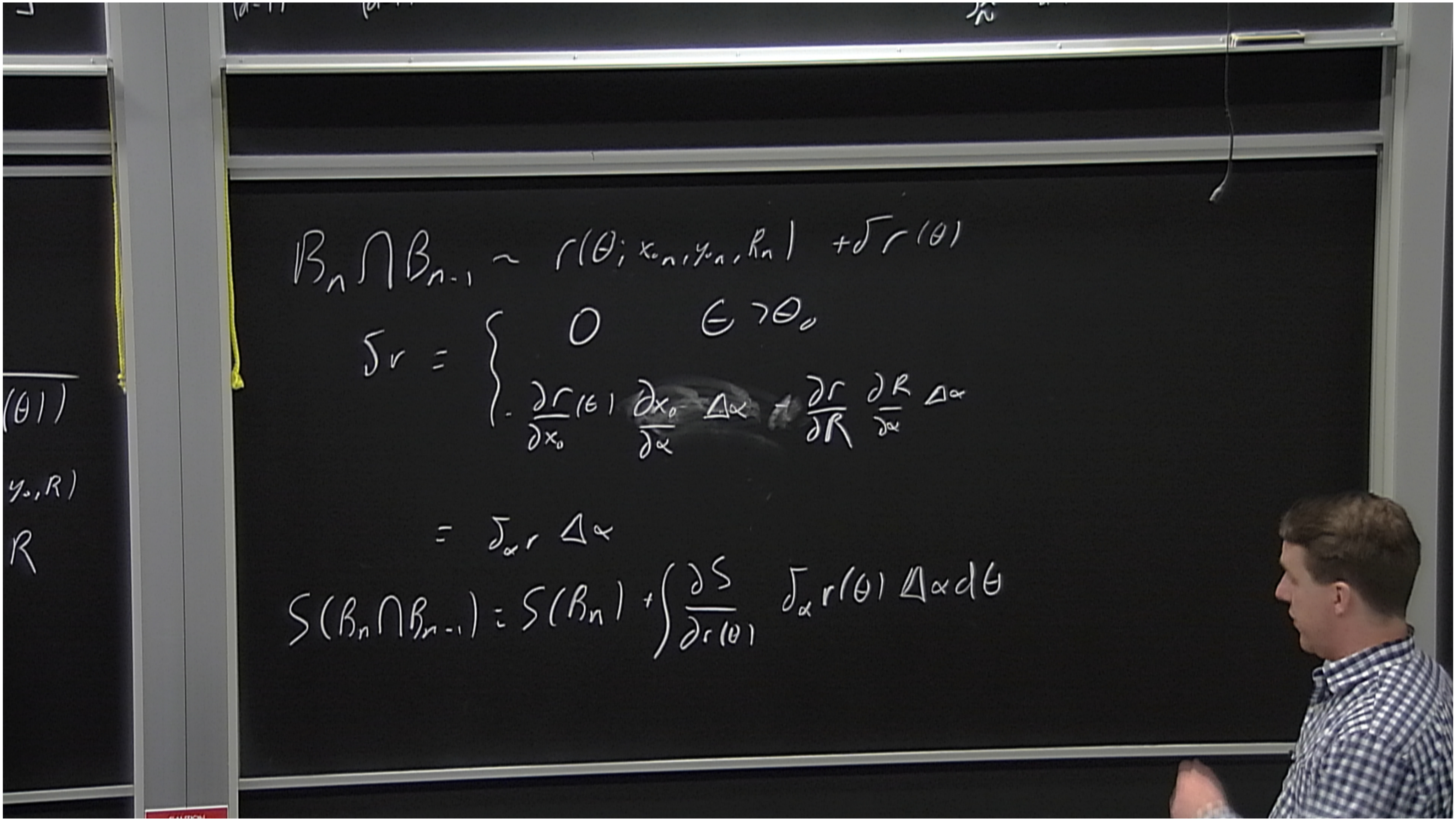


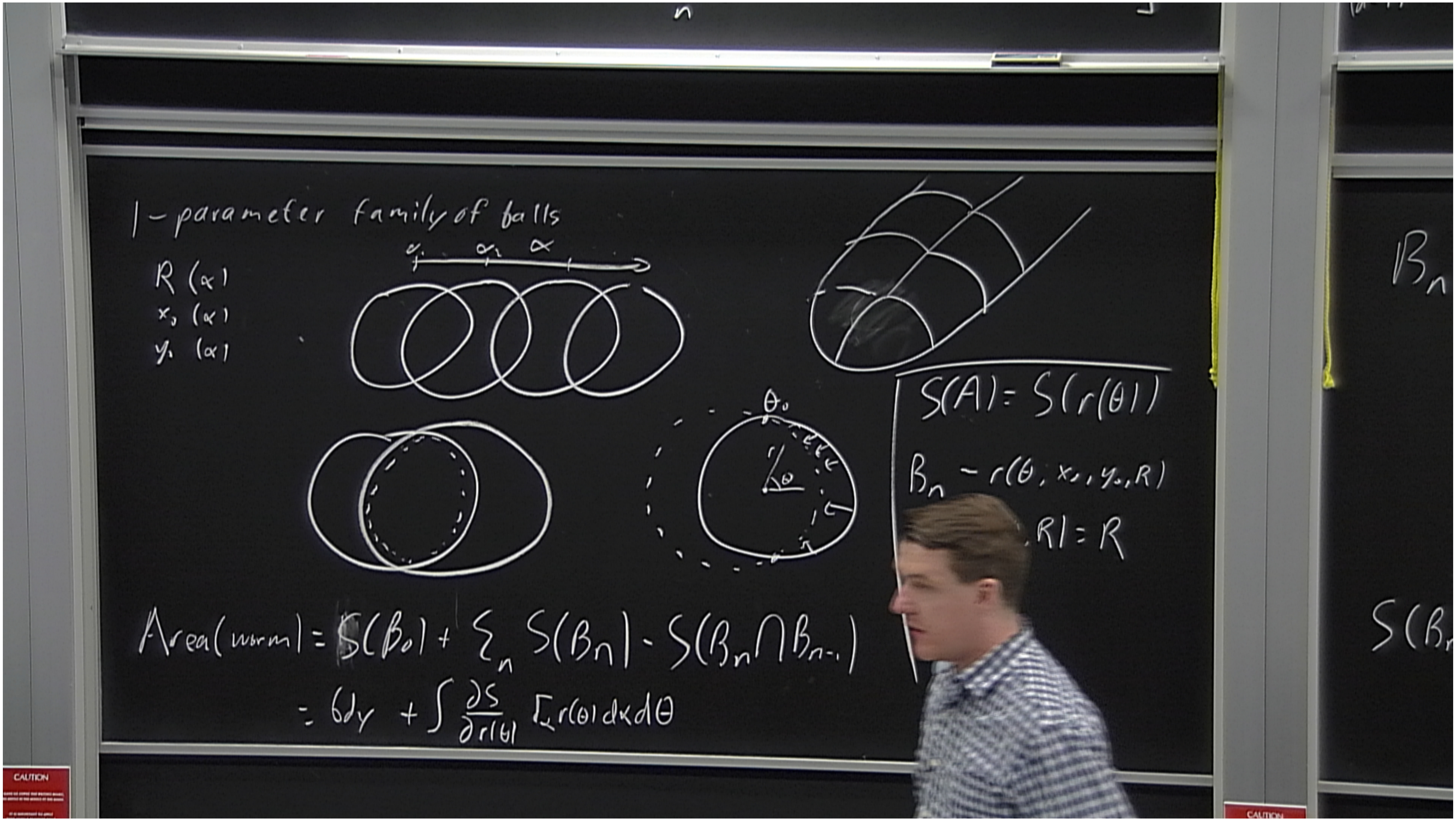
$B_n \cap B_{n-1} \sim r(\theta; x_{0n}, y_{0n}, R_n) + \delta r(\theta)$
$\delta r = \{ \; 0$

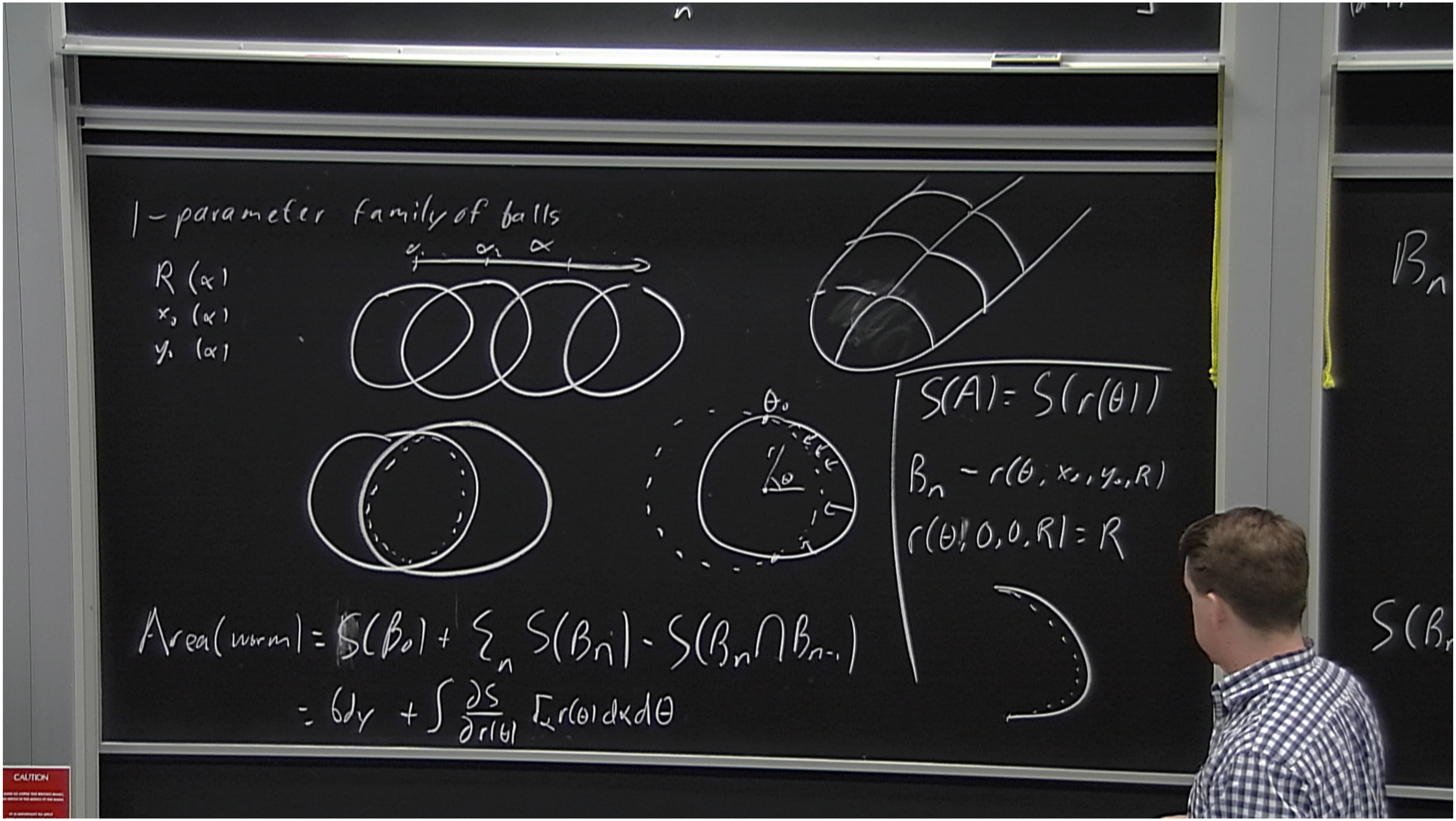
$$B_n \cap B_{n-1} \sim r(\theta; x_{0n}, y_{0n}, R_n) + \delta r(\theta)$$

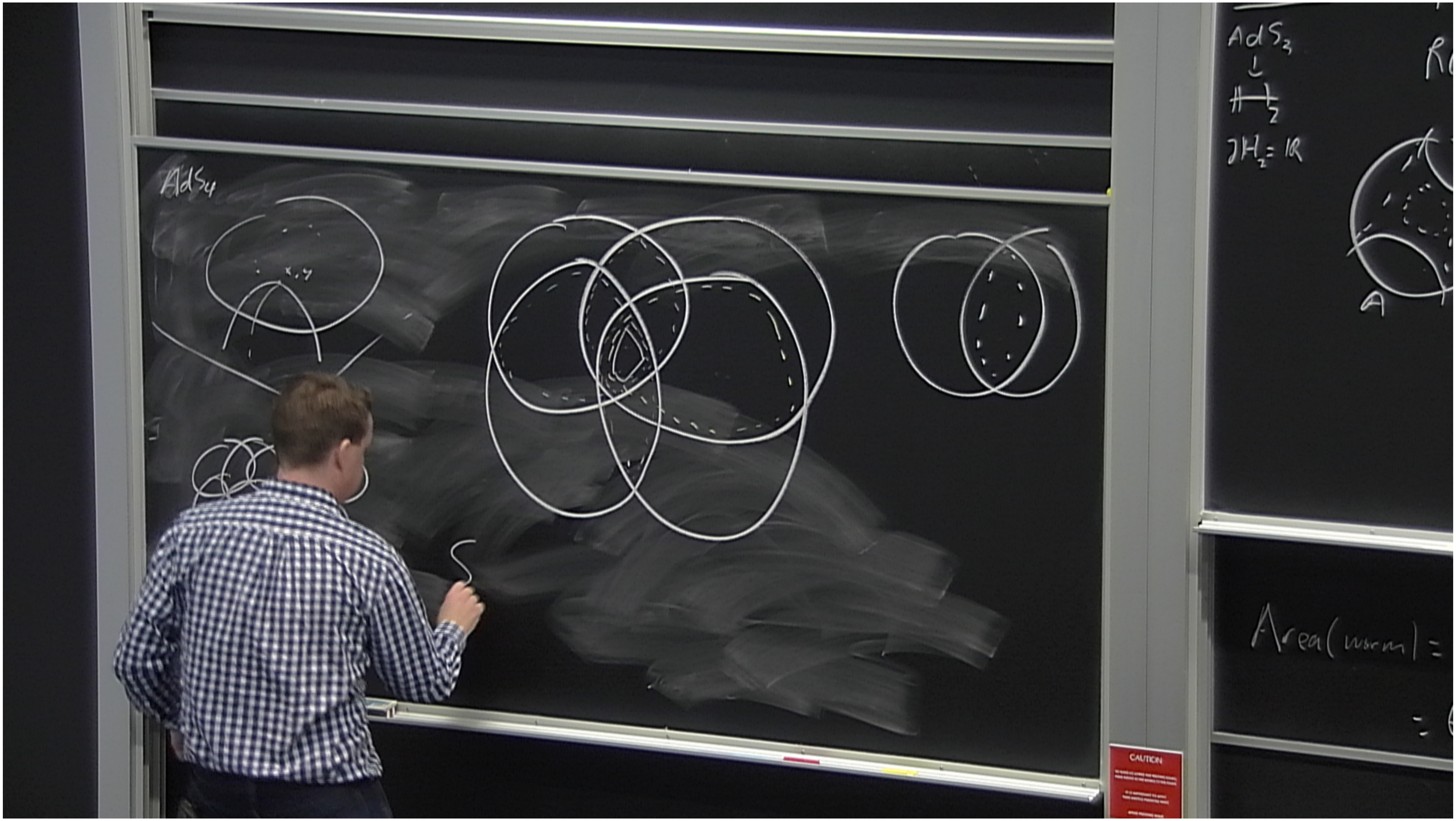
$$\delta r = \begin{cases} 0 & \theta > \theta_0 \\ \frac{\partial r}{\partial x_0}(\theta)\,\frac{\partial x_0}{\partial \alpha}\,\Delta\alpha + \frac{\partial r}{\partial R}\,\frac{\partial R}{\partial \alpha}\,\Delta\alpha \end{cases}$$

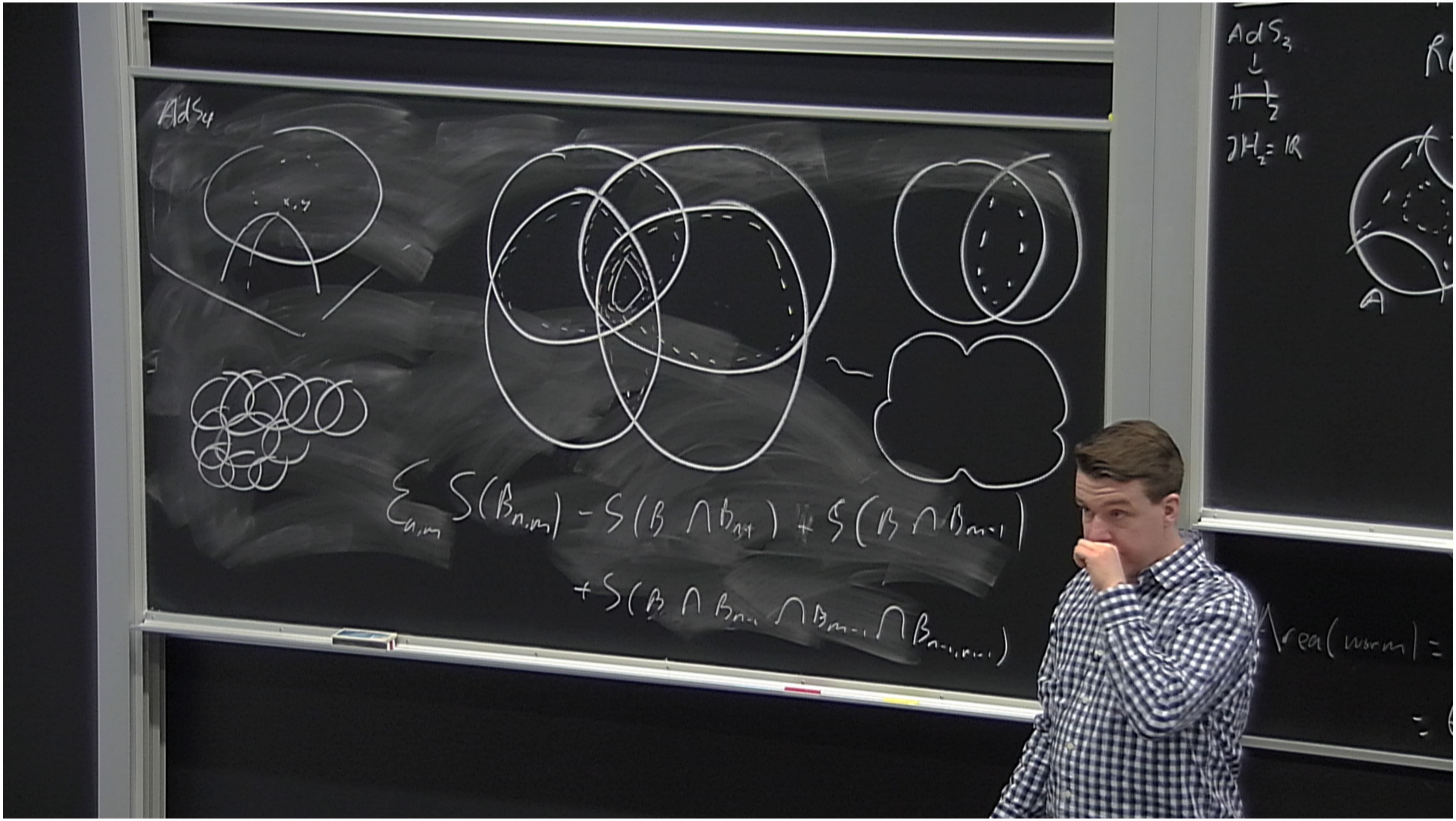
$$= \delta_\alpha r \, \Delta\alpha$$

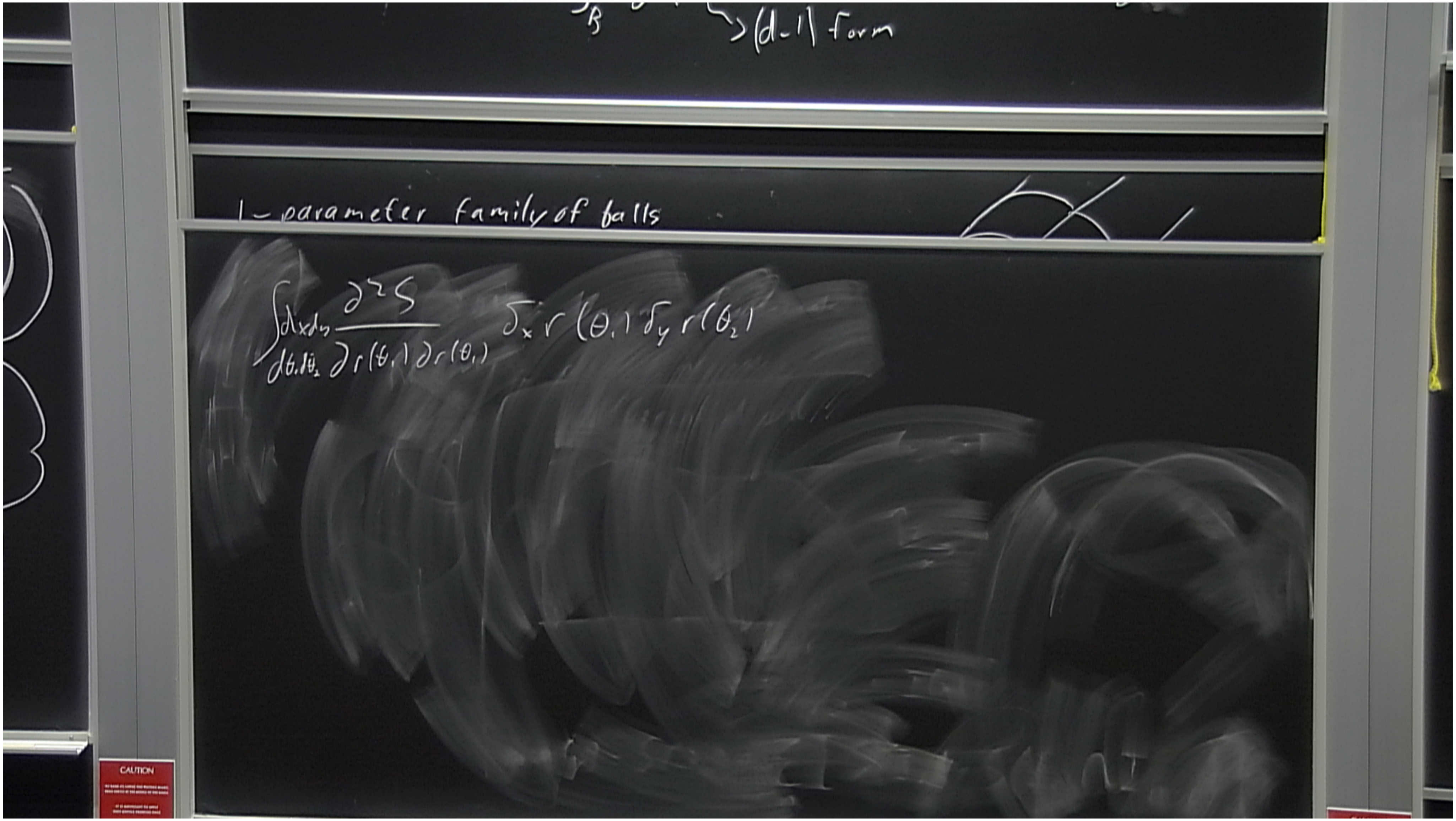

$B_n \cap B_{n-1} \sim r(\theta; x_{0n}, y_{0n}, R_n) + \delta r(\theta)$
$\delta r = \begin{cases} 0 & \theta > \theta_0 \\ \frac{\partial r}{\partial x_0}(\theta) \frac{\partial x_0}{\partial \alpha} \Delta\alpha + \frac{\partial r}{\partial R} \frac{\partial R}{\partial \alpha} \Delta\alpha \end{cases}$
$= \delta_\alpha r \, \Delta\alpha$
$S(B_n \cap B_{n-1}) = S(B_n) + \int \frac{\partial S}{\partial r(\theta)} \delta_\alpha r(\theta) \Delta\alpha \, d\theta$

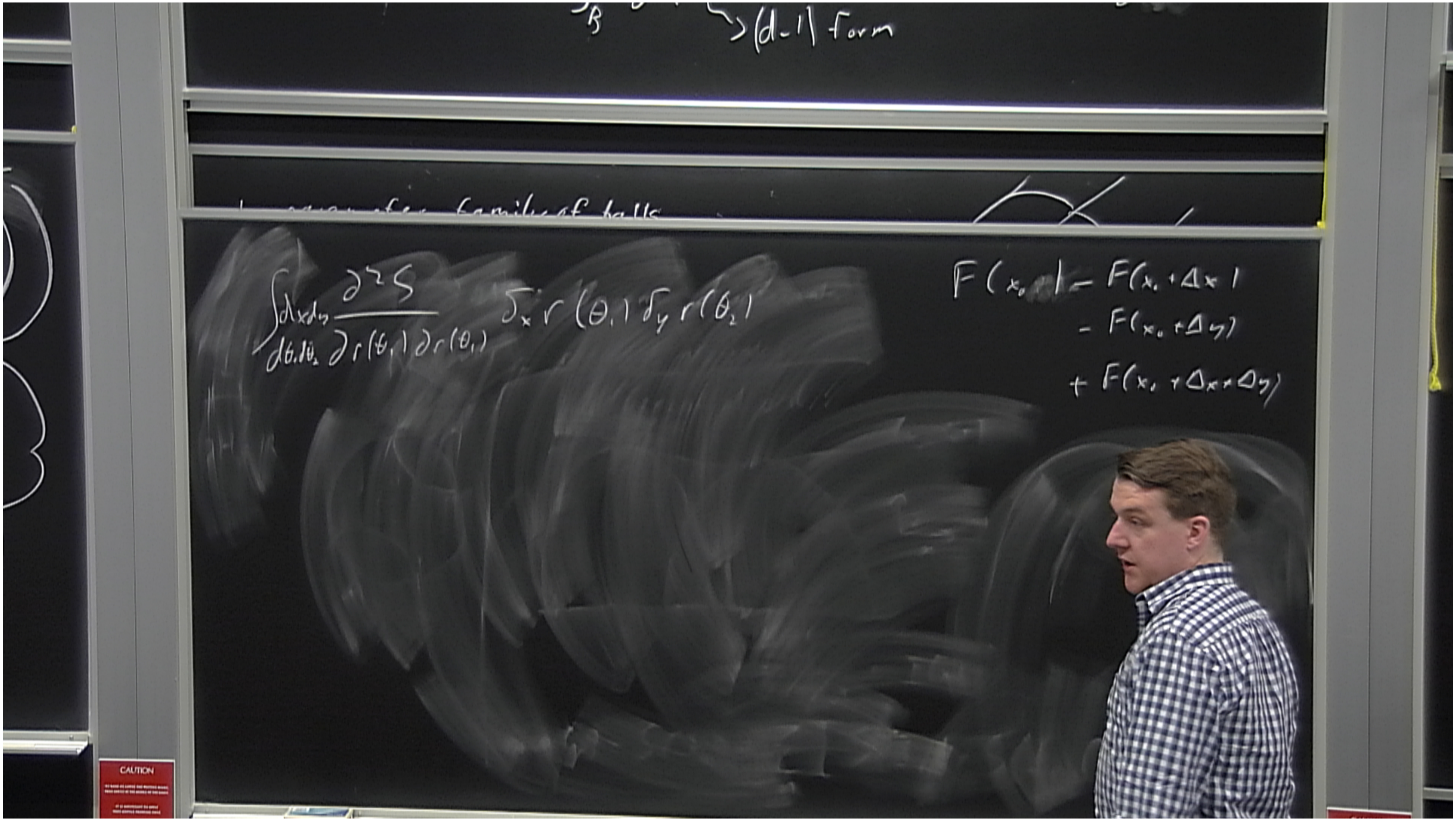

1-parameter family of balls
R (α)
x₀ (α)
y₀ (α)
S(A) = S(r(θ))
Area(worm) = S(B₀) + Σₙ S(Bₙ) - S(Bₙ ∩ Bₙ₋₁)

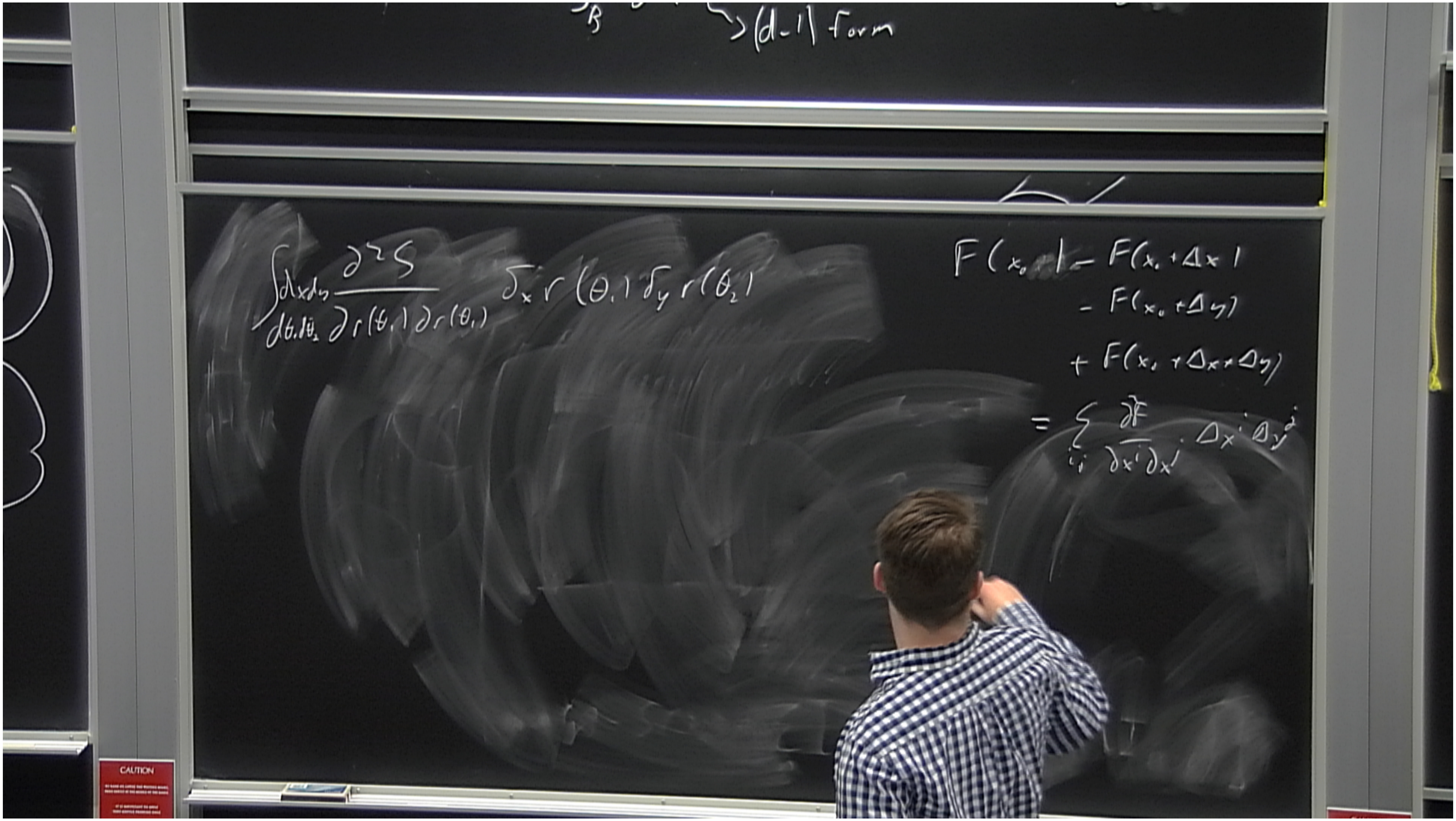

1-parameter family of balls
R (α)
x₀ (α)
y₀ (α)
S(A) = S(r(θ))
Bn ~ r(θ, x₀, y₀, R)
r(θ, 0, 0, R) = R
Area(worm) = S(B₀) + Σn S(Bn) - S(Bn ∩ Bn-1)
= 6dy + ∫ ∂S/∂r(θ) r(θ) dα dθ
CAUTION


AdS$_4$
x, y
AdS$_3$
$\mathbb{H}_2$
$\partial \mathbb{H}_2 = \mathbb{R}$
A
Area(worm) =
CAUTION

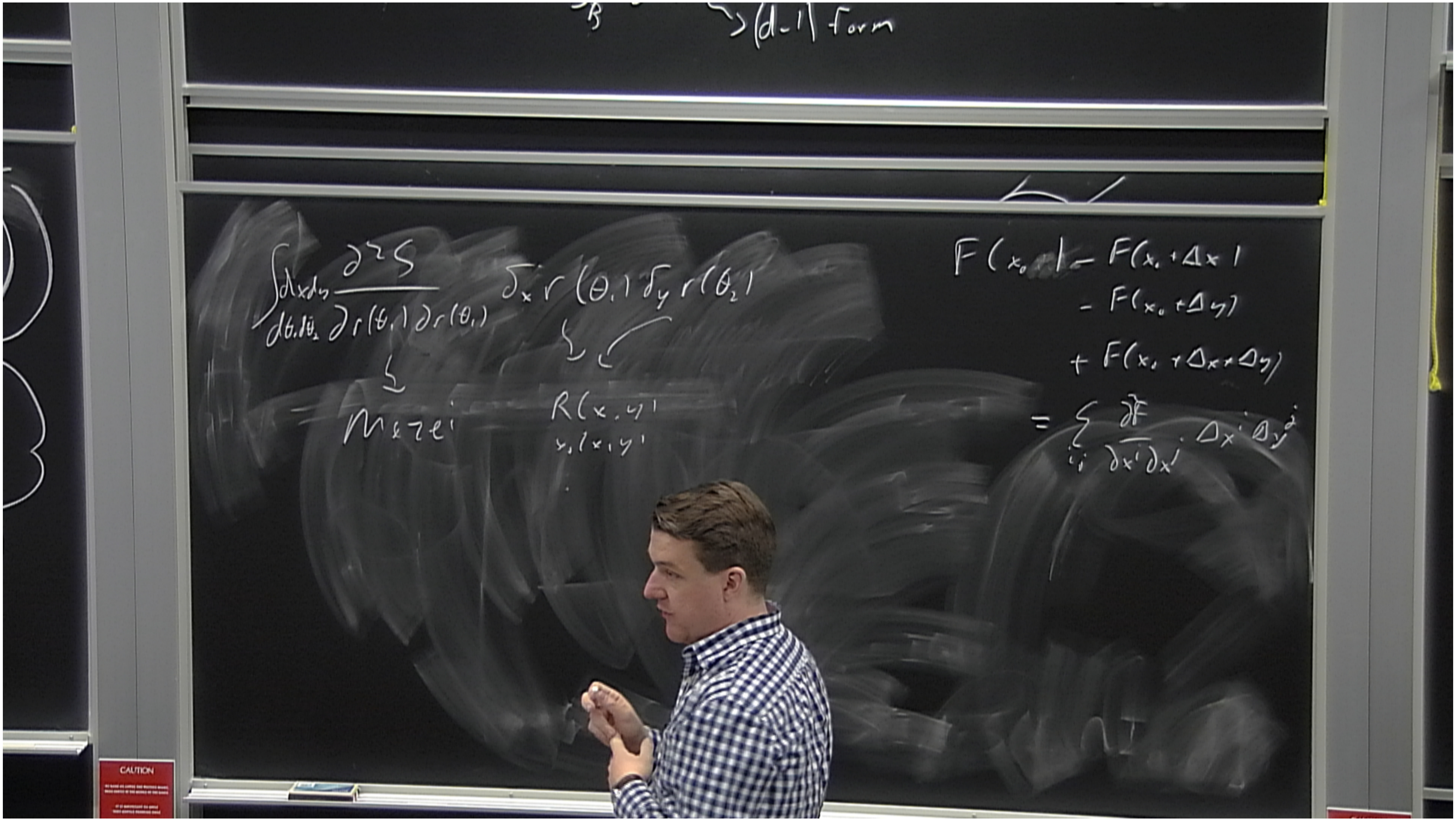

AdS4
AdS3
Area(worm) =

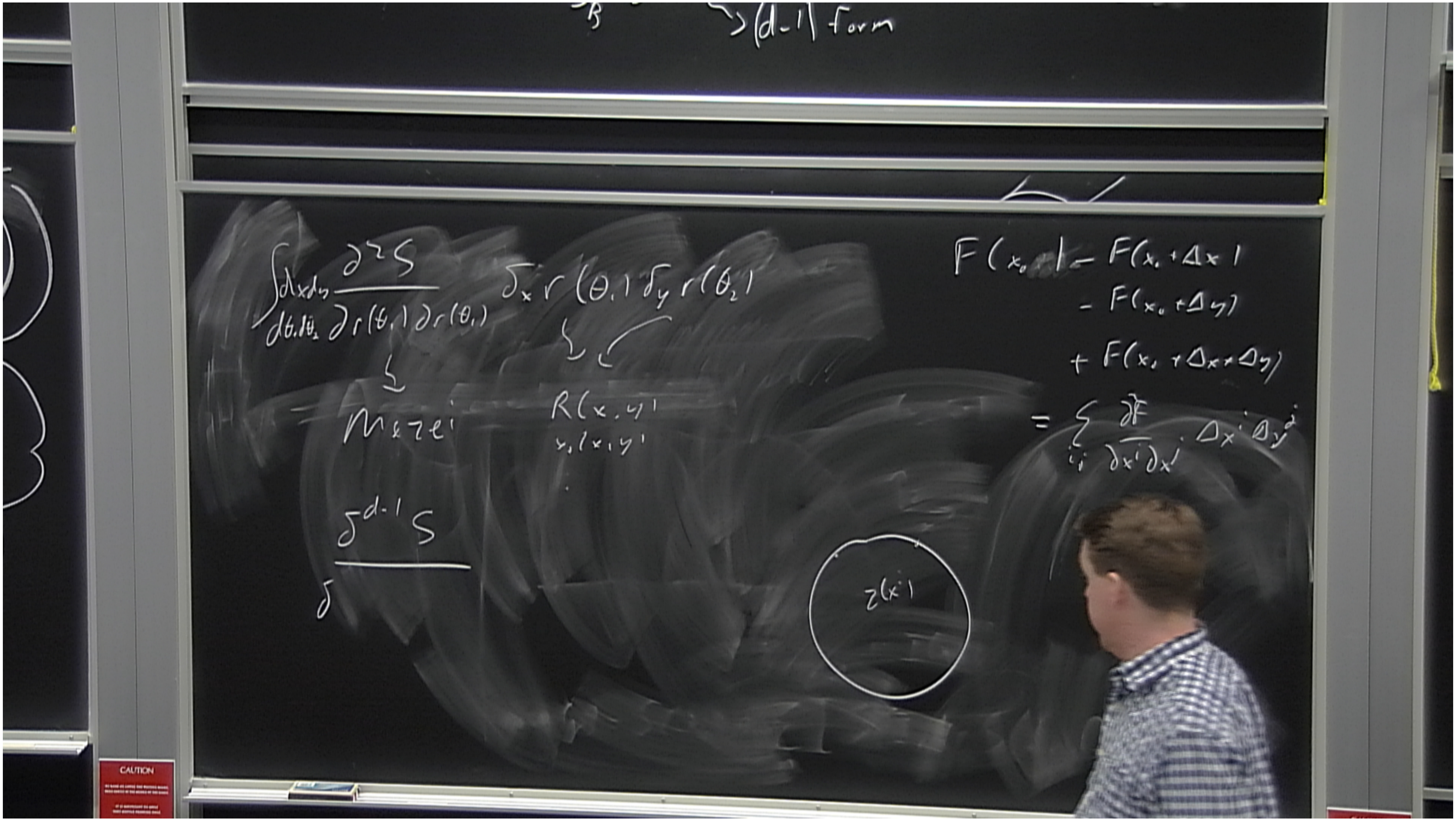

(d-1) form
1-parameter family of balls
$\int dx\,dy\,\frac{\partial^2 S}{\partial r(\theta_1)\,\partial r(\theta_2)}\,\partial_x r(\theta_1)\,\partial_y r(\theta_2)$
$d\theta_1 d\theta_2$

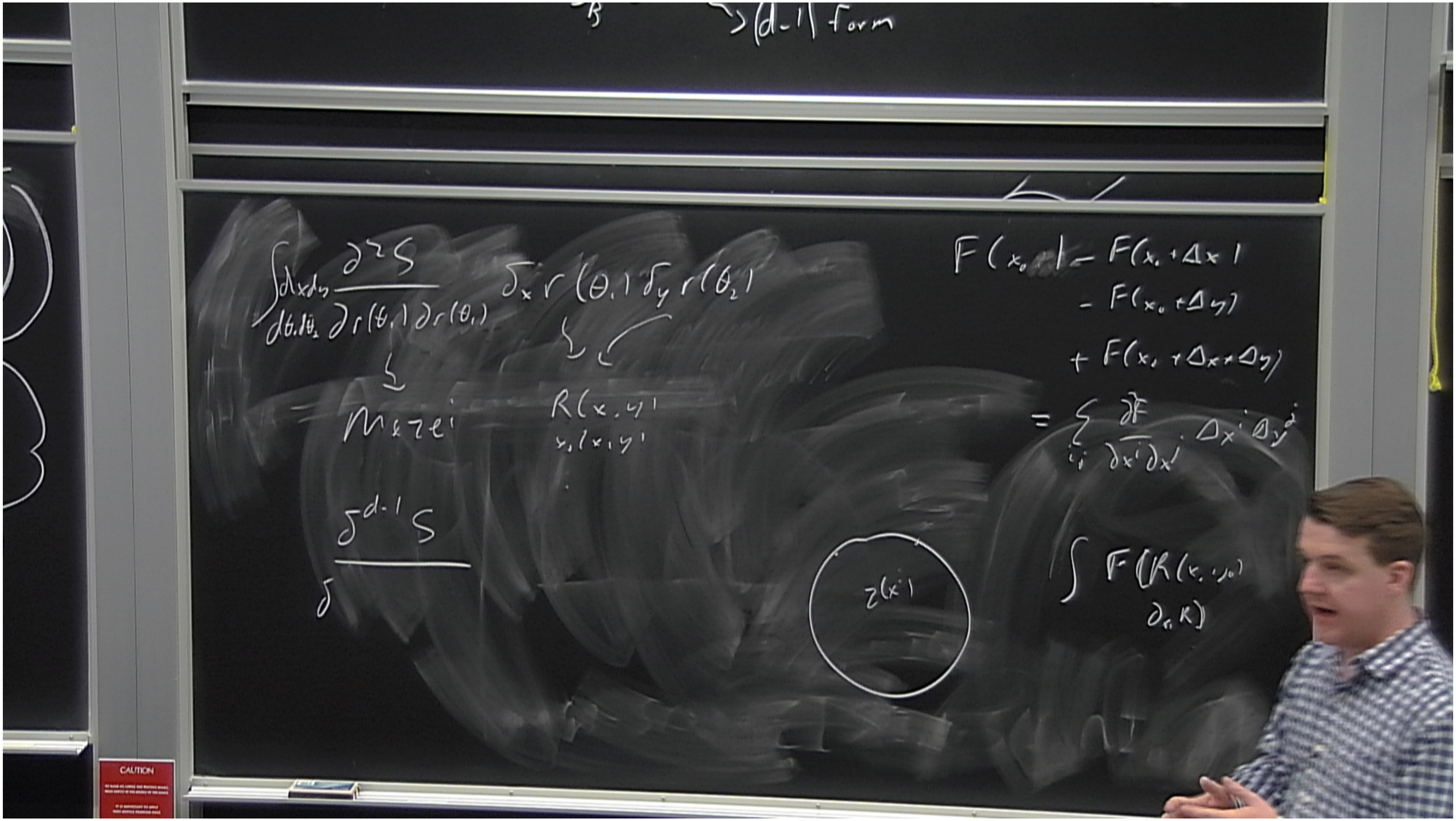

(d-1) form
CAUTION



(d-1) form
∫dx dy / dθ₁dθ₂ ∂²S / ∂r(θ₁)∂r(θ₂) ∂ₓr(θ₁) ∂ᵧr(θ₂)
F(x₀) − F(x₀+Δx)
− F(x₀+Δy)
+ F(x₀+Δx+Δy)
= Σᵢⱼ ∂F / ∂xⁱ∂xʲ Δxⁱ Δyʲ


(d-1) form
R(x, y)
- F(x0 + Δy)
+ F(x0 + Δx + Δy)


(d-1) form
F(x₀) − F(x₀+Δx)
− F(x₀+Δy)
+ F(x₀+Δx+Δy)
R(x,y)
CAUTION


(d-1) form
CAUTION