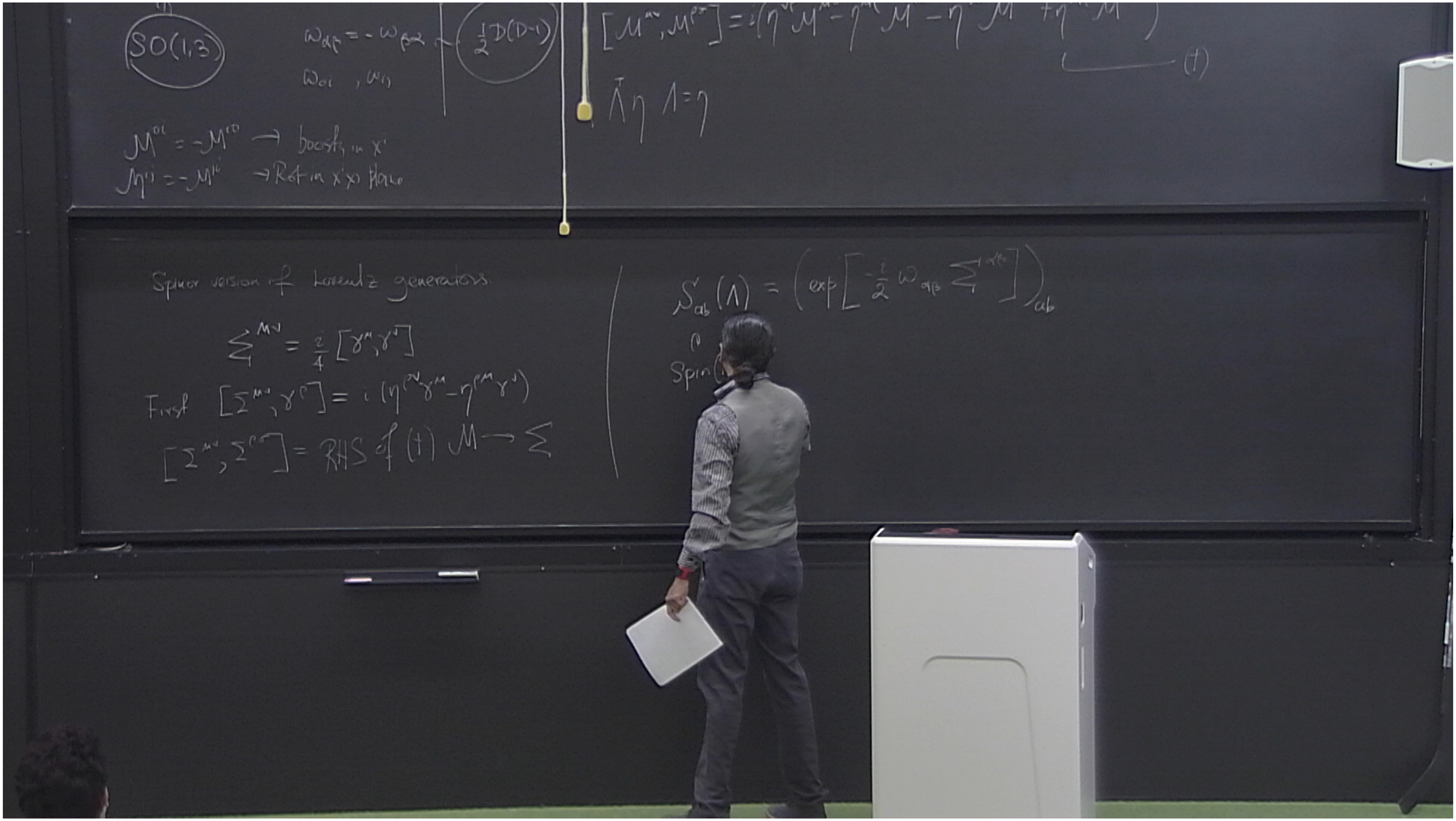
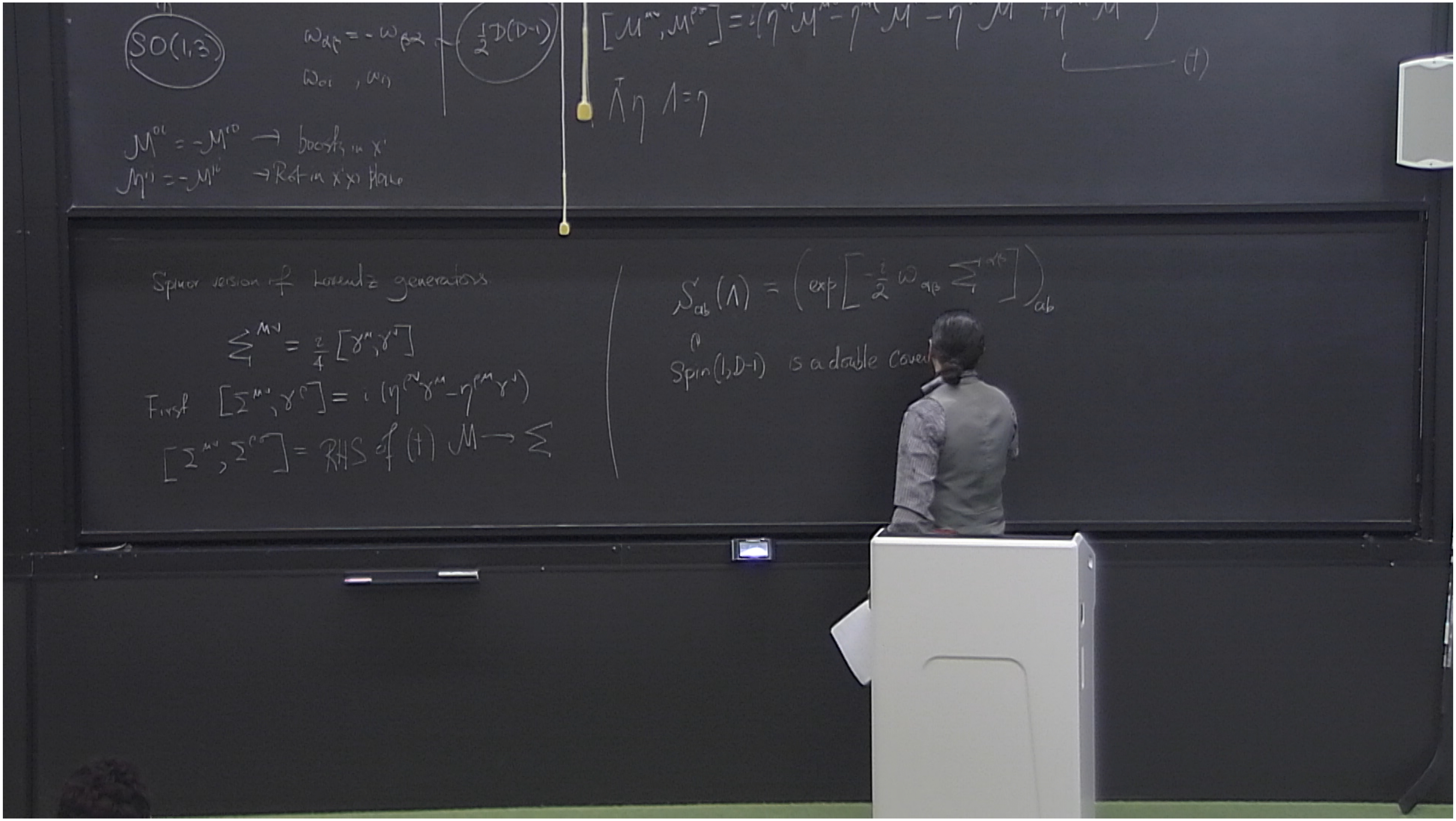
Title: Non-perturbative emergence of non-Fermi liquid behaviour in d=2 quantum critical metals

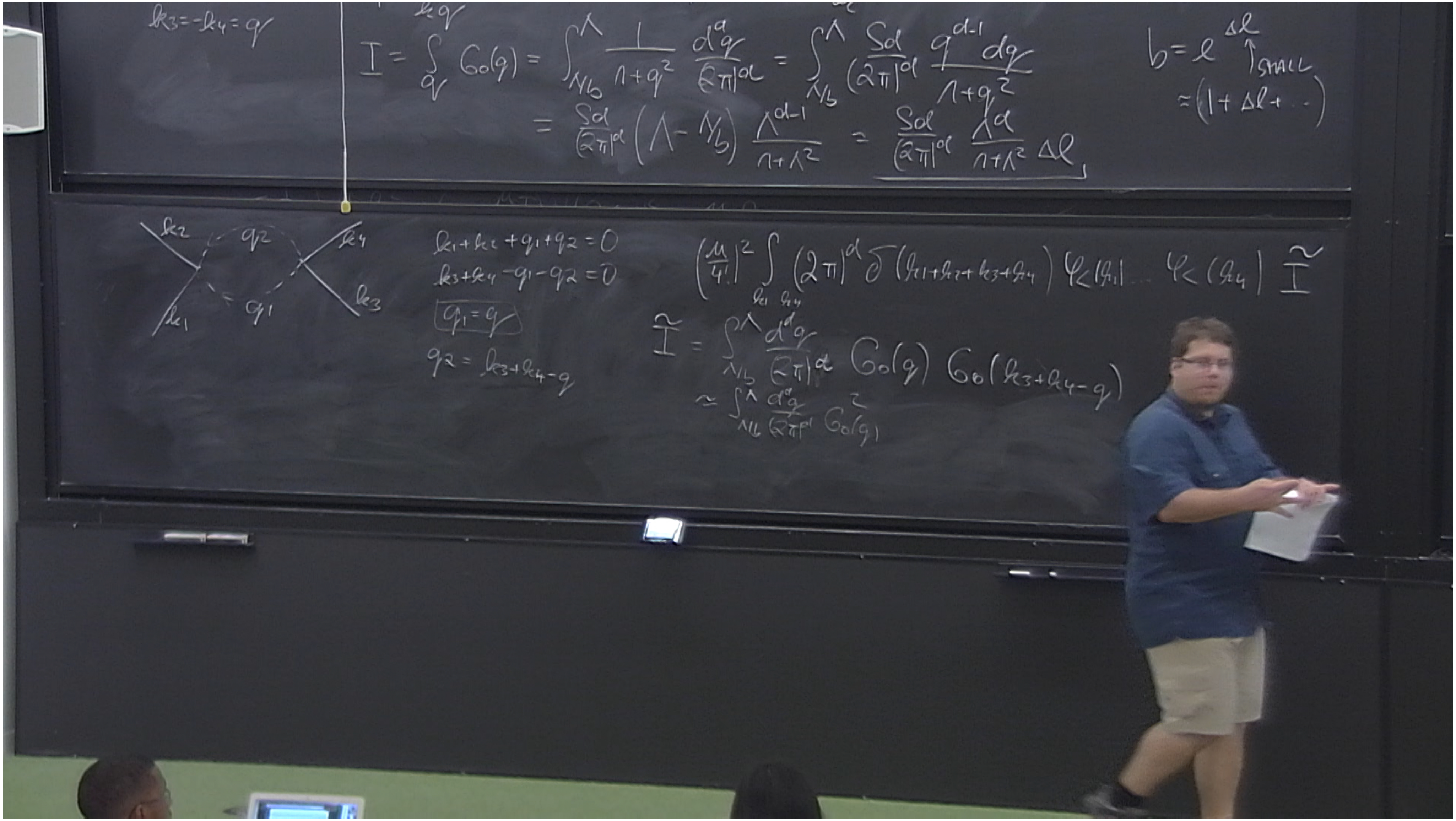
Date: Nov 04, 2016 03:45 PM

URL: http://pirsa.org/16110035

Abstract: <p>We consider d=2 fermions at finite density coupled to a critical boson. In the quenched or Bloch-Nordsieck approximation, where one takes the limit of fermion flavors N_fâ†’0, the fermion spectral function can be determined {exactly}. We show that one can obtain this non-perturbative answer thanks to a specific identity of fermionic two-point functions in the planar local patch approximation. The resulting spectrum is that of a non-Fermi liquid: quasiparticles are not part of the exact fermionic excitation spectrum of the theory. Instead one finds continuous spectral weight with power law scaling excitations. Moreover, at low energies there are three such excitations at three different Fermi surfaces, two with a low energy Green's function Gâˆ¼(Ï‰âˆ’v k)^âˆ’1/2 and one with Gâˆ¼|Ï‰+k|^âˆ’1/3. We proceed to study this model with a finite N_f but still neglecting fermionic loops with 3 or more vertices, motivated by multiloop cancellations at large k_F. We still find a non-Fermi liquid but with a different IR spectrum.</p>


SO(1,3)
Spinor version of Lorentz generators
First
RHS of (†)
exp
Spin


SO(1,3)
Spinor version of Lorentz generators
Spin(1,D-1) is a double cover
RHS of (†)



k3 = -k4 = q
I = ∫_q G0(q) = ∫_{Λ/b}^{Λ} 1/(1+q^2) d^d q/(2π)^d = ∫_{Λ/b}^{Λ} Sd/(2π)^d q^{d-1} dq/(1+q^2)
= Sd/(2π)^d (Λ - Λ/b) Λ^{d-1}/(1+Λ^2) = Sd/(2π)^d Λ^d/(1+Λ^2) Δl
b = e^{Δl} SMALL
≈ (1 + Δl + . .)
k2 q2 k4
k1 q1 k3
k1 + k2 + q1 + q2 = 0
k3 + k4 - q1 - q2 = 0
q1 = q
q2 = k3 + k4 - q
(u/4!)^2 ∫_{k1...k4} (2π)^d δ(k1+k2+k3+k4) φ<(k1) ... φ<(k4) Ĩ
Ĩ = ∫_{Λ/b}^{Λ} d^d q/(2π)^d G0(q) G0(k3+k4-q)
≃ ∫_{Λ/b}^{Λ} d^d q/(2π)^d G0(q)^2


Non-Perturbative Two-Point Functions of a Quantum Critical Metal
Petter Säterskog
Leiden University
November 4, 2016

Collaborators

Koenraad Schalm, my PhD supervisor in Leiden
Balazs Meszena, former PhD student in Leiden
Andrey Bagrov, Postdoc in Nijmegen

Outline

- A quantum critical metal model in 2+1 dimensions
- Holographic, matrix and vector large N
- Quenched $N_f \to 0$
- Ongoing work: Finite N_f
- Conclusion and Next steps

Hertz-Millis model

Fermions at finite density coupled to critical bosons:

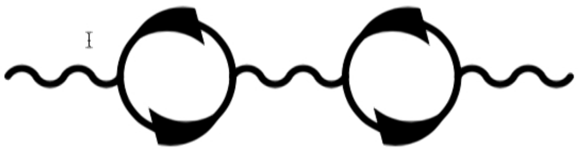
$$S = \int \mathrm{d}\tau \mathrm{d}x^d \left(\bar{\psi}(\mu - \partial_\tau - \frac{\nabla^2}{2m})\psi + \frac{1}{2}\phi(\partial_\tau^2 + \nabla^2)\phi + \lambda\phi\bar{\psi}\psi + g\phi^4 \right)$$

- λ, fermion-boson interaction. Scaling dimension $(d-3)/2$
- g, boson self-interaction. Scaling dimension $d-3$

ϕ is an order parameter field of e.g. Ising-nematic or spin density wave transitions,
Putative model of some heavy fermion systems and high-T_c materials

Previous studies

- $d = 3$ is marginal. Admits perturbative treatment[1] [2] [3]
  Landau damping changes IR of boson $S = \int (k^2 + \gamma \left|\frac{\omega}{k}\right|)|\phi|^2$.



- $d = 2$ is strongly coupled. Landau Fermi liquid theory breakdown[4].
  Fermion sign-problem prohibits Monte-Carlo, except certain cases[5].
  Full description still an open problem.
  This is the topic of our research.

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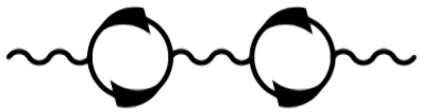

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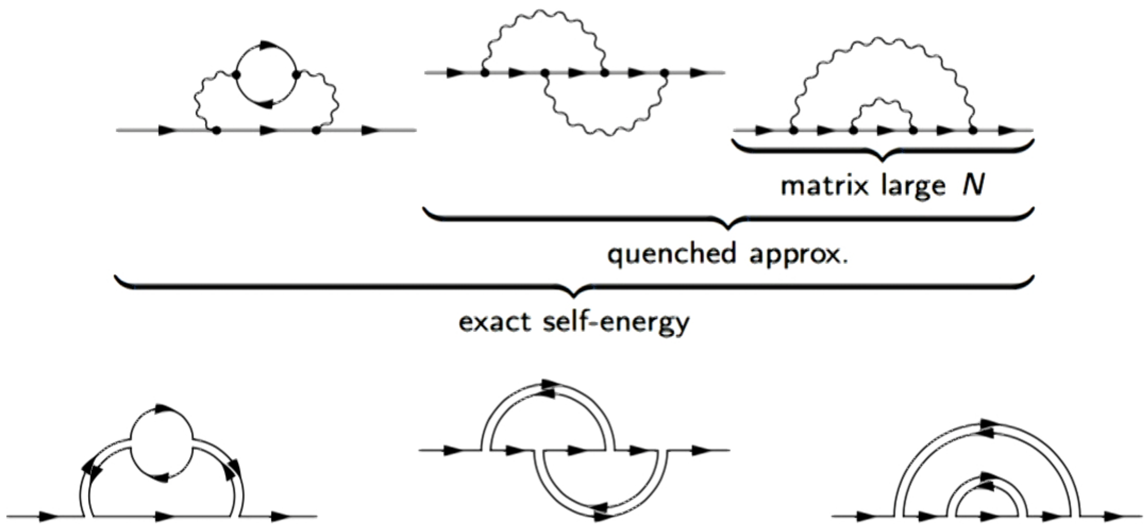
[1] J. Hertz, Phys. Rev. B **14** (1976) 1165
[2] A. J. Millis, Phys. Rev. B **48** (1993) 7183
[3] S. Sachdev, "Quantum Phase Transitions" (2011)
[4] P. A. Lee, Phys. Rev. Lett. **63** (1989) 680
[5] E. Berg, M. A. Metlitski, S. Sachdev, Science **338** (2012) 6114

Previous studies: Matrix large N, $SU(N)$

Interaction $\int \mathrm{d}x^{d+1}\bar{\psi}_i\phi_{ij}\psi_j$ [6] [7]. 1PI diagrams at order λ^4:



They find flow of $v \to 0$ and subsequent destruction of the Fermi surface.

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[6] A. L. Fitzpatrick, S. Kachru, J. Kaplan and S. Raghu, Phys. Rev. B **89** (2014) 165114

[7] G. Torroba and H. Wang Phys. Rev. B **90** (2014) 165144

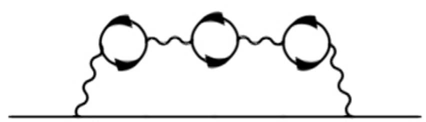
Previous studies: Vector large N, $U(N)$

N_f fermion flavours and coupling constant $\lambda = g/\sqrt{N_f}$

Large N_f gives strong Landau-damping $\implies$ RPA two-point functions

Perhaps too easy?[8] [9]

The limit can be controlled by a dynamical critical exponent $z = 2 + \epsilon$ for the boson[10]. That theory then gives the RPA in the large N_f limit for two-point functions. At each order in λ, the relevant diagram is:



$$\Sigma_{\mathrm{RPA}} = \frac{-i\lambda^{4/3}\mathrm{sgn}(\omega)|\omega|^{2/3}}{(2\pi)^{2/3}\sqrt{3}(N_f k_F)^{1/3}}$$

[8] Sung-Sik Lee, Phys. Rev. B **80**, 165102 (2009)

[9] M. A. Metlitski and S. Sachdev, Phys. Rev. B **82** (2010) 075127

[10] D. F. Mross, J. McGreevy, H. Liu and T. Senthil, Phys. Rev. B **82** (2010) 045121

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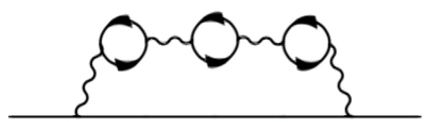

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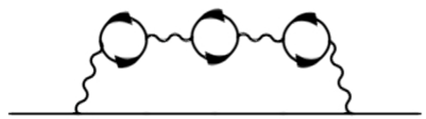

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[10] D. F. Mross, J. McGreevy, H. Liu and T. Senthil, Phys. Rev. B **82** (2010) 045121

Quenched Limit

We "extend" the model to have N_f flavours of fermions, all coupling to the single boson via $\lambda\phi\bar{\psi}_i\psi_i$. We then take the limit $N_f \to 0$.

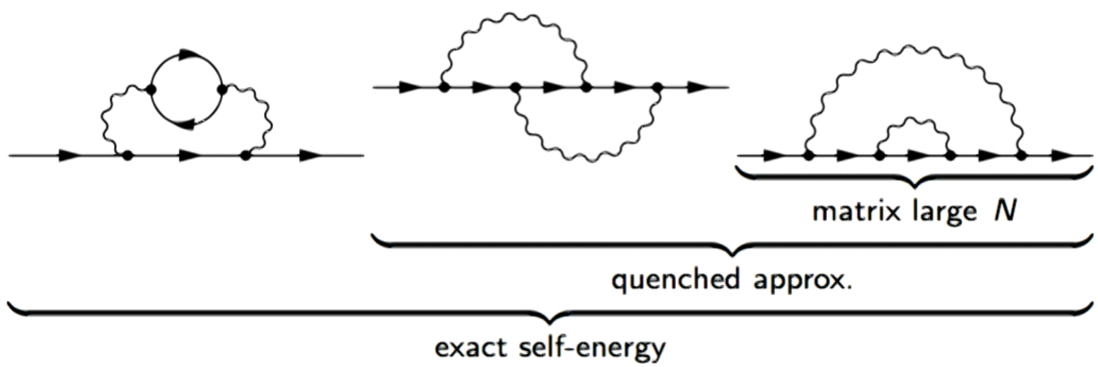
This limit is similar to the probe fermions in holography where they are coupled to a quantum critical system[11] [12].

The fermions live in a field of critical bosons that give the fermionic correlator non-perturbative corrections. However, the fermions do not affect the bosons at all:

$$G_B(K) = G_{B0}(K)$$

[11] H. Liu, J. McGreevy, and D. Vegh Phys. Rev. D **83** (2009) 065029
[12] M. Cubrovic, J. Zaanen and K. Schalm, Science **325** (2009) 5939

1PI diagrams at order λ^4:



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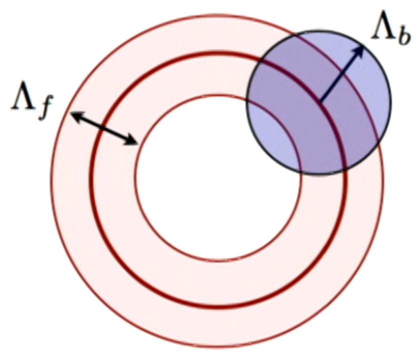
Cut-offs

Introduce cut-offs around low-energy excitations



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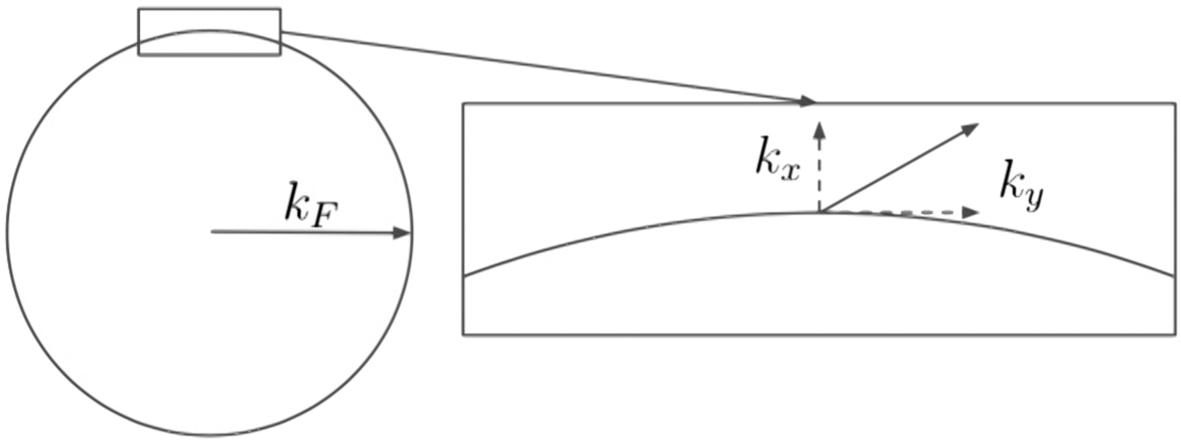
We study limit $\Lambda_b, \Lambda_f \ll k_F = \sqrt{2m\mu}$

[13] Image credit: A. L. Fitzpatrick, G. Torroba, H. Wang, Phys.Rev. B **91** (2015) 195135

Patch theory

In the quenched limit, $N_f = 0$, and for energies far below k_F we can consistently consider just a patch of the "Fermi surface" for calculating the fermion two-point function.

$$G_0(\omega, k) = \frac{1}{i\omega - k^2/2m + \mu} \approx \frac{1}{i\omega - vk_x}$$



$G(x)$ for $N_f \to 0$ fermions coupled to Gaussian scalar

We calculate the fermion two-point function. Integrating out the fermion we have

$$Z[J, J^\dagger] = \int \mathcal{D}\phi \, \mathrm{e}^{\left(-S_b[\phi]-S_{\det}[\phi]-\int d^3z d^3z' J_i^\dagger(z) G^i{}_j[\phi](z;z') J^j(z')\right)}$$

with

$$(-\partial_\tau + iv\partial_x + \lambda\phi(z))\, G[\phi](z,z') = \delta^3(z-z')$$

$$S_{\det} = \int \mathrm{d}x\mathrm{d}y\mathrm{d}\tau \left[-N_f \mathrm{Tr} \ln G^{-1}[\phi]\right]$$

Taking functional derivatives with respect to the sources, the full fermion Green's function is then given by a path integral over only the bosonic field:

$$\langle \psi_j^\dagger(z)\psi^i(0)\rangle_{\text{exact}} = \delta^i_j G(z,z') = \delta^i_j \frac{\int \mathcal{D}\phi\, G[\phi](z,z') e^{-S_b[\phi]-S_{\det}[\phi]}}{\int \mathcal{D}\phi\, e^{-S_b[\phi]-S_{\det}[\phi]}}$$

Two-point function of $N_f \to 0$ fermions coupled to Gaussian scalar

We solve the first order PDE and integrate ϕ. This gives

$$G(\tau, x) = G_0(\tau, x) \exp\left(I(\tau, x)\right)$$

$$I(\tau, x) = \lambda^2 \int \frac{\mathrm{d}k_0 \mathrm{d}k_x \mathrm{d}k_y}{(2\pi)^3} G_0^2\left(k_0, k_x\right) G_B\left(k_0, k_x, k_y\right) \left[\cos\left(ik_0\tau - ik_x x\right) - 1\right]$$

$\implies$ In a large k_F theory with $N_f = 0$ fermions and a Gaussian scalar we have an explicit way of calculating the fermion two-point function in terms of the boson two-point function.

Quenched Limit Results[14]

Analytical continuation + Fourier transform gives retarded two-point function:

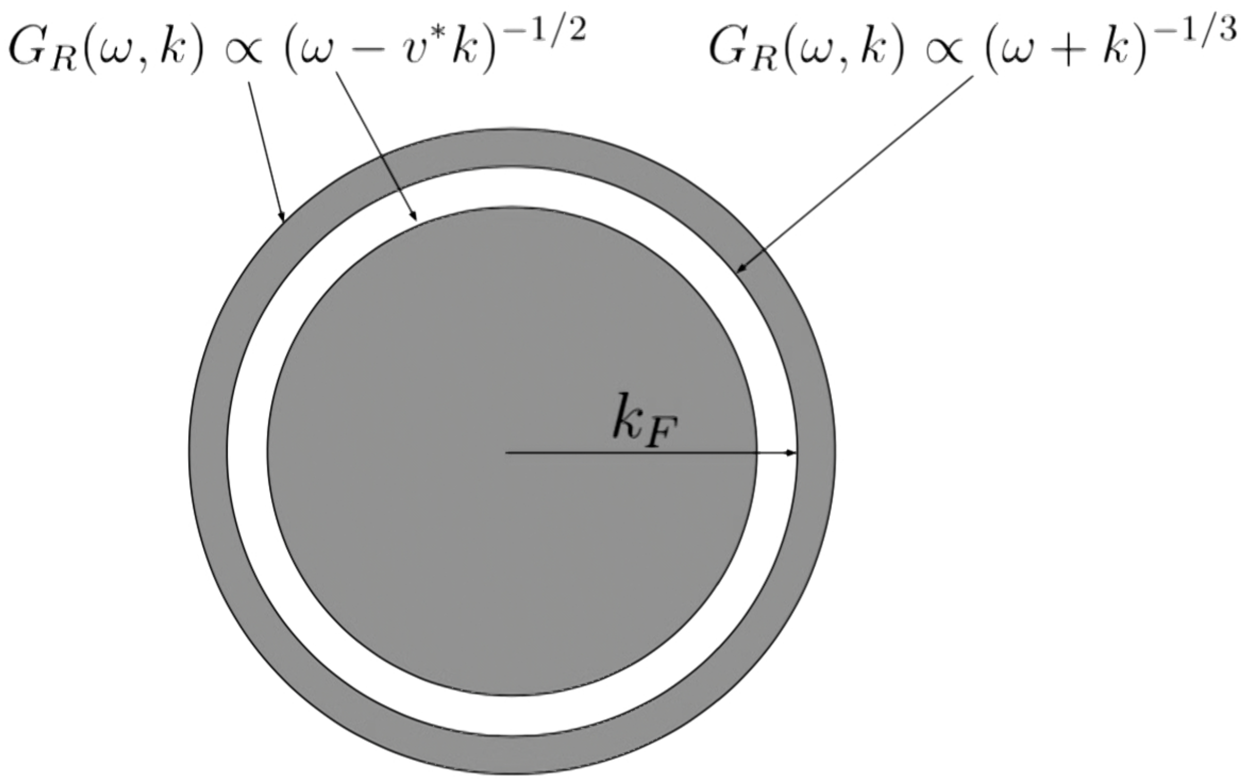
$$G_R(\omega, k_x) = \frac{1}{\omega - k_x v + \frac{\lambda^2}{4\pi\sqrt{1-v^2}}\sigma(\omega, k_x)}$$

where $\sigma(\omega, k_x)$ is the root, within $0 < \mathrm{Im}(\sigma) < i\pi$, of the transcendental function:

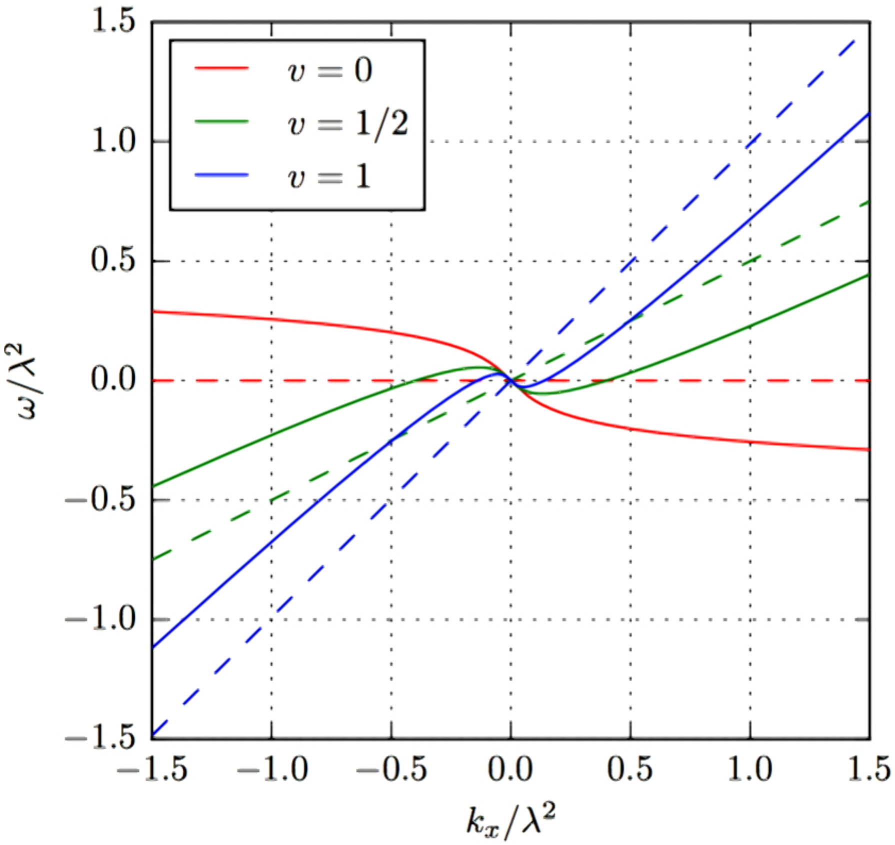
$$\frac{\lambda^2}{4\pi\sqrt{1-v^2}}\left[\sinh(\sigma) - \sigma\cosh(\sigma)\right] + v\omega - k_x - \cosh(\sigma)(\omega - k_x v + i0) = 0$$

[14] B. Meszena, PS, A. Bagrov, K. Schalm, Phys. Rev. B **94** (2016) 115134

Quenched Limit Results - Fermi surface topology



Quenched Limit Results - v-dependence



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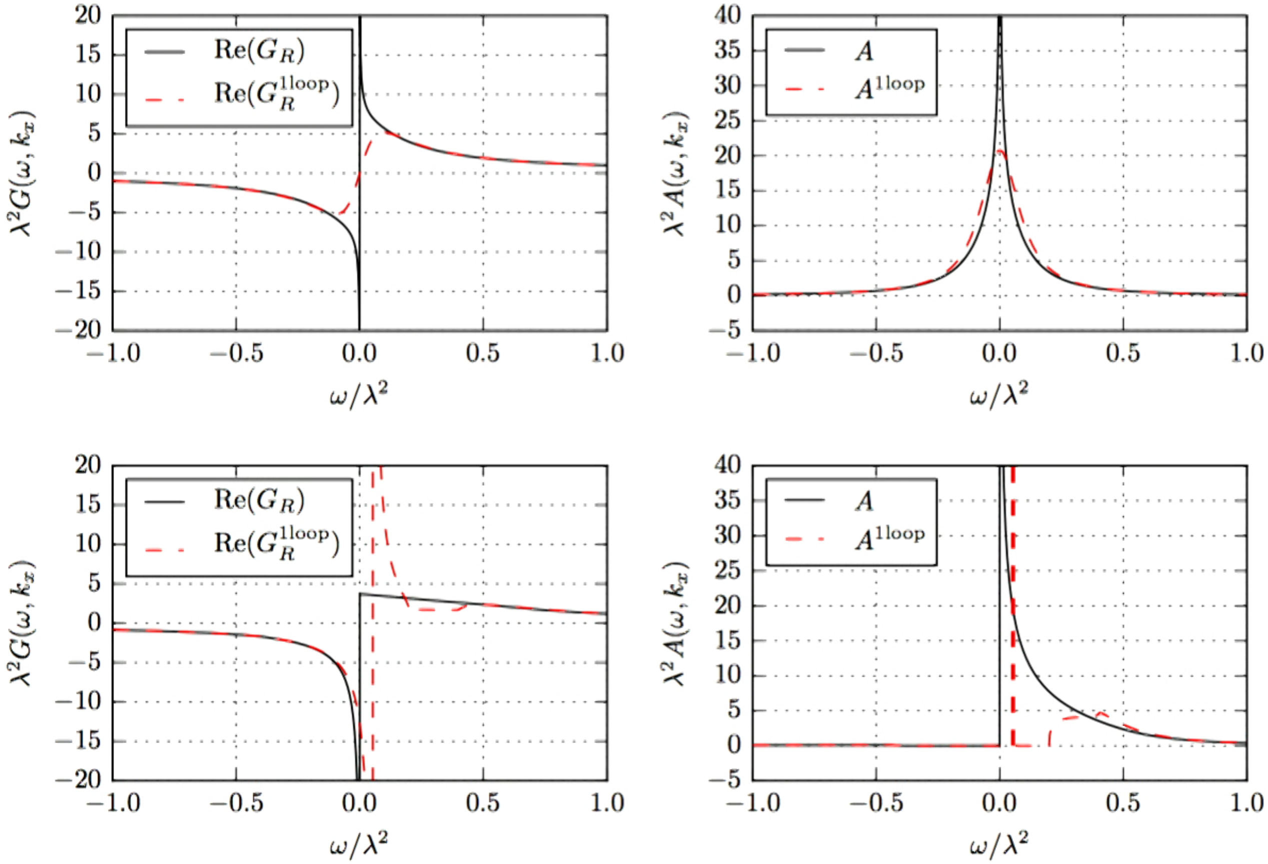
Quenched Limit Results - 1-Loop Comparison



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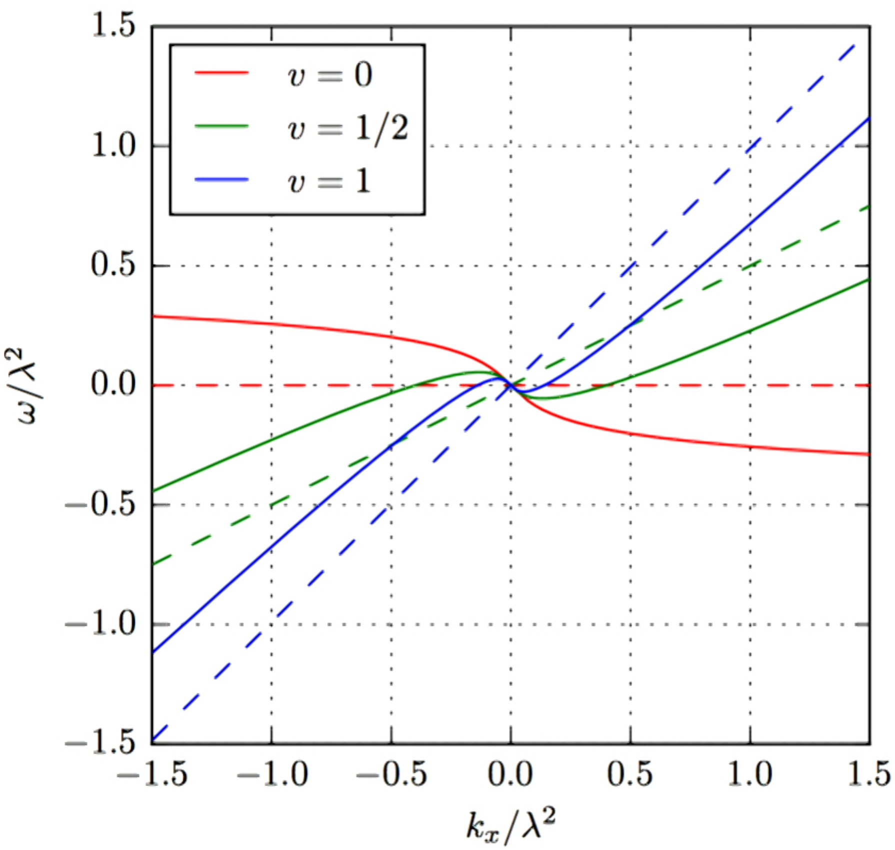

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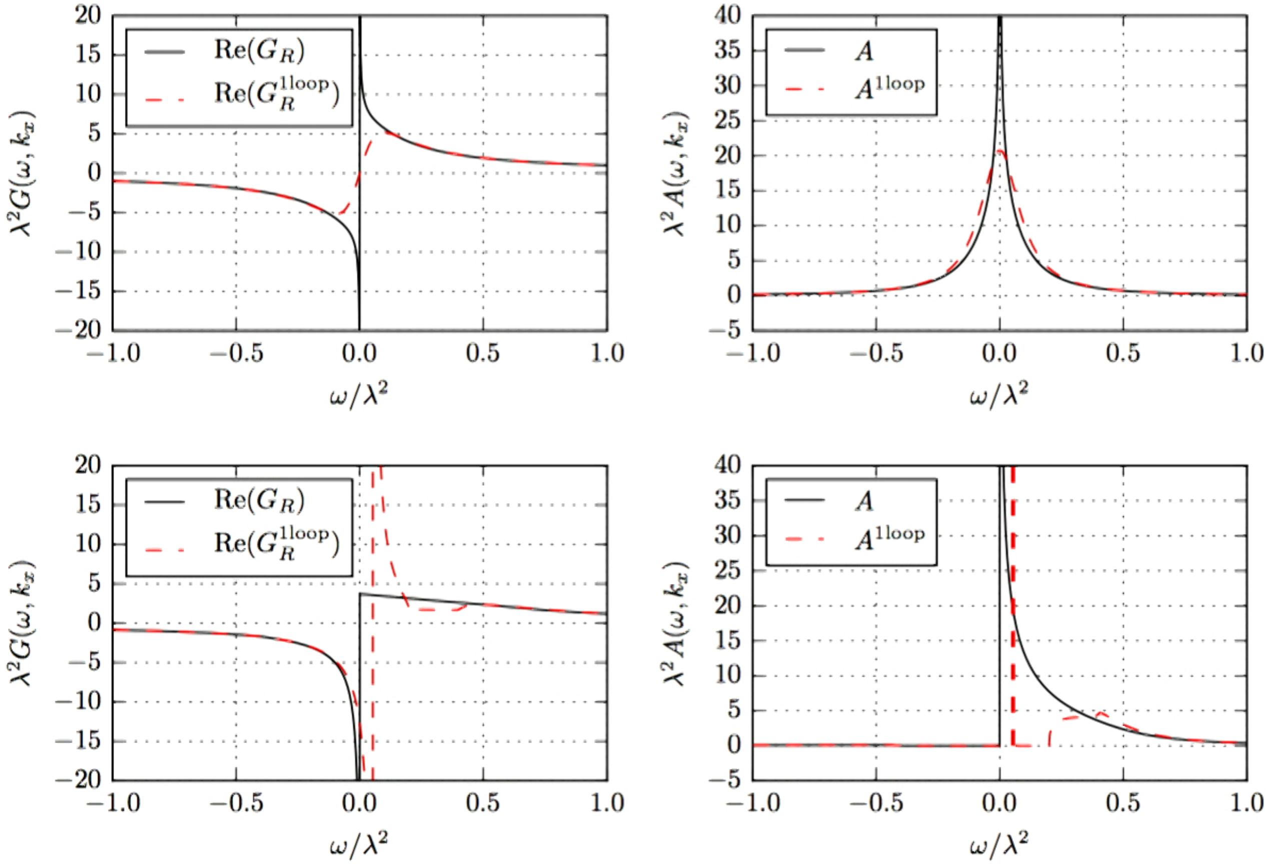

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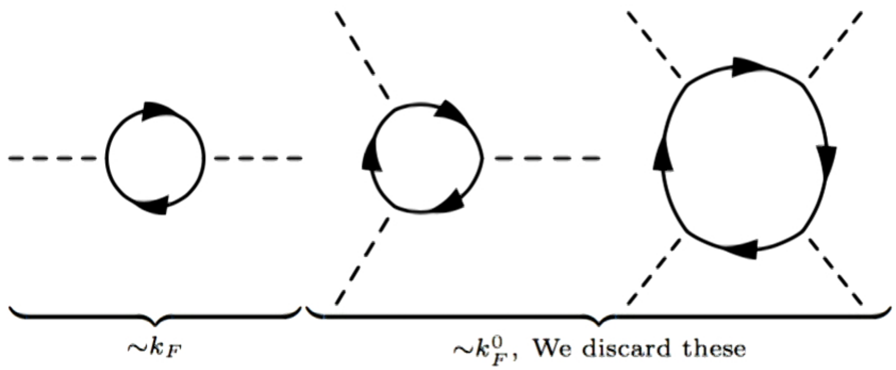
Does this Fermi surface splitting survive N_f corrections?

Multi-loop cancellation at large k_F

Fermion loops with $n = 3$ or more vertices cancel upon symmetrization in Luttinger liquids.

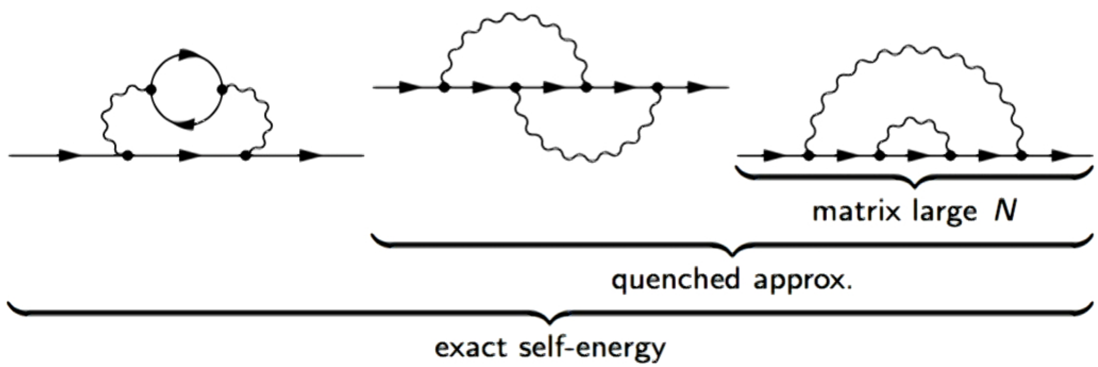
Large cancellations also happen at higher dimensions for $n > 2$ as $\Lambda_b, \Lambda_f \ll k_F$ [15] This has motivated us to study the elementary quantum

critical metal model by discarding all fermion loops with 3 or more insertions.



[15] A. Neumayr, W. Metzner, Phys. Rev. B **58** (1998) 15449

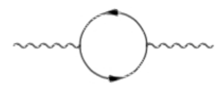
1PI diagrams at order λ^4:



Large k_F effective theory

Integrating out the fermions only keeping 2 vertex fermion loops gives correction to boson kinetic term:

$$S_{b,\text{eff}} = \int \frac{\mathrm{d}\omega \mathrm{d}k^2}{(2\pi)^3} \frac{1}{2} \phi \Big(\omega^2 + k_x^2 + k_y^2 + \underbrace{\frac{\lambda^2 N_f k_F |\omega|}{2\pi v \sqrt{v^2(k_x^2 + k_y^2) + \omega^2}}} \Big) \phi$$



We can study this "n=2 fermion loop" theory by using this scalar effective action coupled to fermions in quenched limit, $N_f \to 0$.
At level of two-point functions we have:

$$\text{“}S_b + \underbrace{S_f + S_{\text{int}}}_{\text{Only } n=2 \text{ loops}} \text{”} = \text{“}S_{b,\text{eff}} + \lim_{N_f \to 0} [S_f + S_{\text{int}}]\text{”}$$

Finite $N_f k_F$

The two-vertex fermion loop corrected boson polarization is given by

$$I(\tau,x) = \lambda^2 \int \frac{\mathrm{d}\omega \mathrm{d}k_x \mathrm{d}k_y}{(2\pi)^3} \frac{\cos(\tau\omega - xk_x) - 1}{(i\omega - k_x v)^2 \left(\omega^2 + k_x^2 + k_y^2 + \frac{\lambda^2 N_f k_F |\omega|}{2\pi v \sqrt{v^2(k_x^2+k_y^2)+\omega^2}}\right)}$$

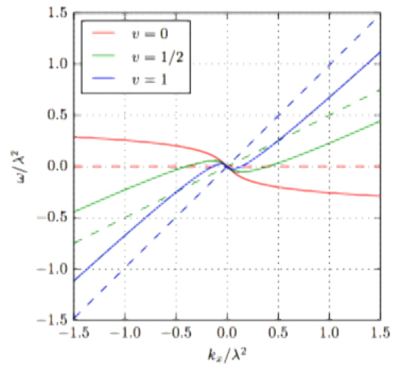
$$G(\tau,x) = G_0(\tau,x)\exp(I(\tau,x))$$

$v = 1$

v is the fermion dispersion at the free fermi momentum, k_F.

$N_f k_F = 0$: All finite v qualitatively the same
$N_f k_F = \infty$: All v same up to rescaling of coordinates

The $v = 1$ case is representative of all $0 < v < 1$ in the $N_f k_F$ extremes
$v = 1$ gives considerable extra symmetry



$\Longrightarrow$ We only study this case

Real-space solution as a series

$$I(r,\theta) = \frac{\lambda f\left(r\lambda\sqrt{N_f k_F/(2\pi)^3},\theta\right)}{\sqrt{N_f k_F/(2\pi)^3}}, \qquad f(\tilde{r},\theta) = \sum_{n=1}^{\infty} f_n \tilde{r}^n$$

$$f_n = \frac{\pi^{n-2} e^{i\theta}(-1)^n \Gamma\left(\frac{n+1}{4}\right) |\sin(\theta)|^{\frac{1+3n}{2}}}{72(3n-1)\Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{1+3n}{4}\right)} \cdot$$

$$\cdot \left({}_2F_1\left(\frac{n+3}{2}, \frac{n+5}{4}; \frac{5}{2}; \cos^2(\theta)\right)(n+1)\cdot \right.$$

$$\cdot \cos^2(\theta)((1-3n)\cos(\theta) - i(n+1)\sin(\theta))$$

$$\left. + \, {}_2F_1\left(\frac{n+1}{4}, \frac{n+1}{2}; \frac{3}{2}; \cos^2(\theta)\right) 6((1-2n)\cos(\theta) - i\sin(\phi)) \right)$$

Intermezzo: Low energy approximation

A small N_f limit keeping Landau damped boson correlator has been used for the same calculation before. [16] [17] [18]

$$I_{\text{``IR''}} = \lambda^2 \int \frac{\mathrm{d}\omega \mathrm{d}k_x \mathrm{d}k_y}{(2\pi)^3} \frac{\cos(\tau\omega - xk_x) - 1}{(i\omega - k_x v)^2 \left(k_y^2 + \frac{4\pi^2\lambda^2 N_f k_F |\omega|}{v^2 |k_y|}\right)}$$

$$G_{\text{``IR''}}(\tau, x) = \frac{1}{2\pi(ix - v\tau)} \exp\left(-\frac{|x|}{l_0^{1/3}(|x| + iv\,\mathrm{sgn}(x)\tau)^{2/3}}\right)$$

where the length scale l_0 is given by

$$l_0^{1/3} = \frac{3\sqrt{3}(2\pi)^{2/3} v^{2/3} (N_f k_F)^{1/3}}{2\Gamma\left(\frac{2}{3}\right)\lambda^{4/3}}$$

They did not present a momentum space correlation.

[16] L. B. Ioffe, D. Lidsky, and B. L. Altshuler, Phys. Rev. Lett. **73** (1994) 472
[17] B. L. Altshuler, L. B. Ioffe, and A. J. Millis, Phys. Rev. B **50** (1994) 14048
[18] W. Metzner, C. Castellani, C. Di Castro, arXiv: cond-mat/9701012 (1997)

Intermezzo: Low energy approximation

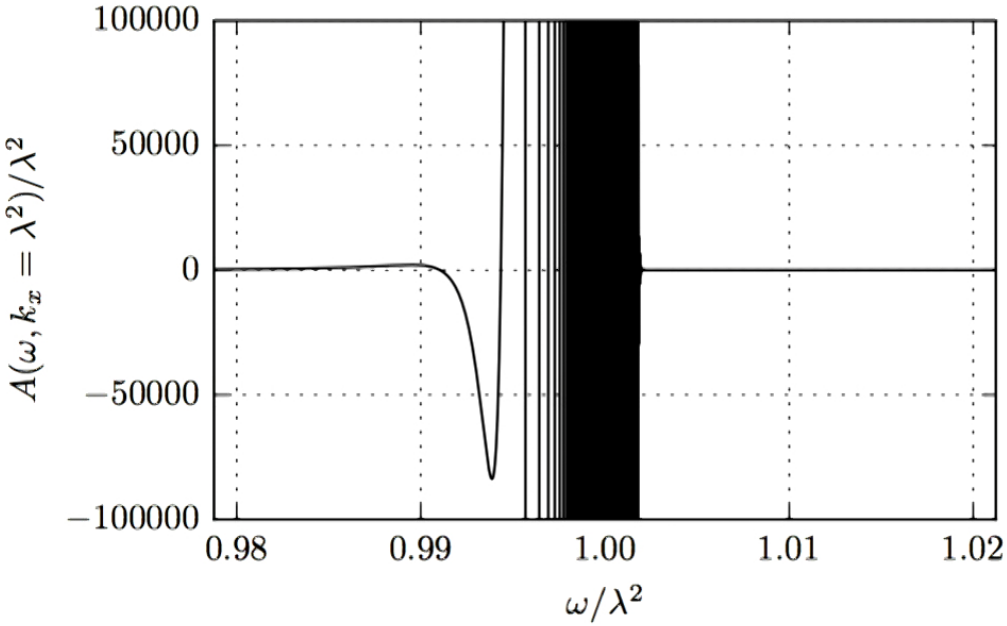
Fourier transforming the real space fermion correlation function gives:

$$\underset{\text{``IR''}}{G}(\omega, k_x) = \frac{1}{i\omega - k_x v} \cos\left(\frac{\omega}{v l_0^{1/2} (\omega/v + i k_x)^{3/2}} \right)$$
$$+ \frac{6\sqrt{3} i \Gamma\left(\frac{1}{3}\right) \omega^{2/3}}{8\pi l_0^{1/3} v^{5/3} (\omega/v + i k_x)^2} {}_1F_2\left(1; \frac{5}{6}, \frac{4}{3}; -\frac{\omega^2}{4 l_0 v^2 (\omega/v + i k_x)^3}\right) +$$
$$+ \frac{3\sqrt{3} i \Gamma\left(-\frac{1}{3}\right) \omega^{4/3}}{8\pi l_0^{2/3} v^{7/3} (\omega/v + i k_x)^3} {}_1F_2\left(1; \frac{7}{6}, \frac{5}{3}; -\frac{\omega^2}{4 l_0 v^2 (\omega/v + i k_x)^3}\right)$$

Manifestly holomorphic in upper half plane, but problematic...

Intermezzo: Low energy approximation

The spectral function is negative close to the singularity



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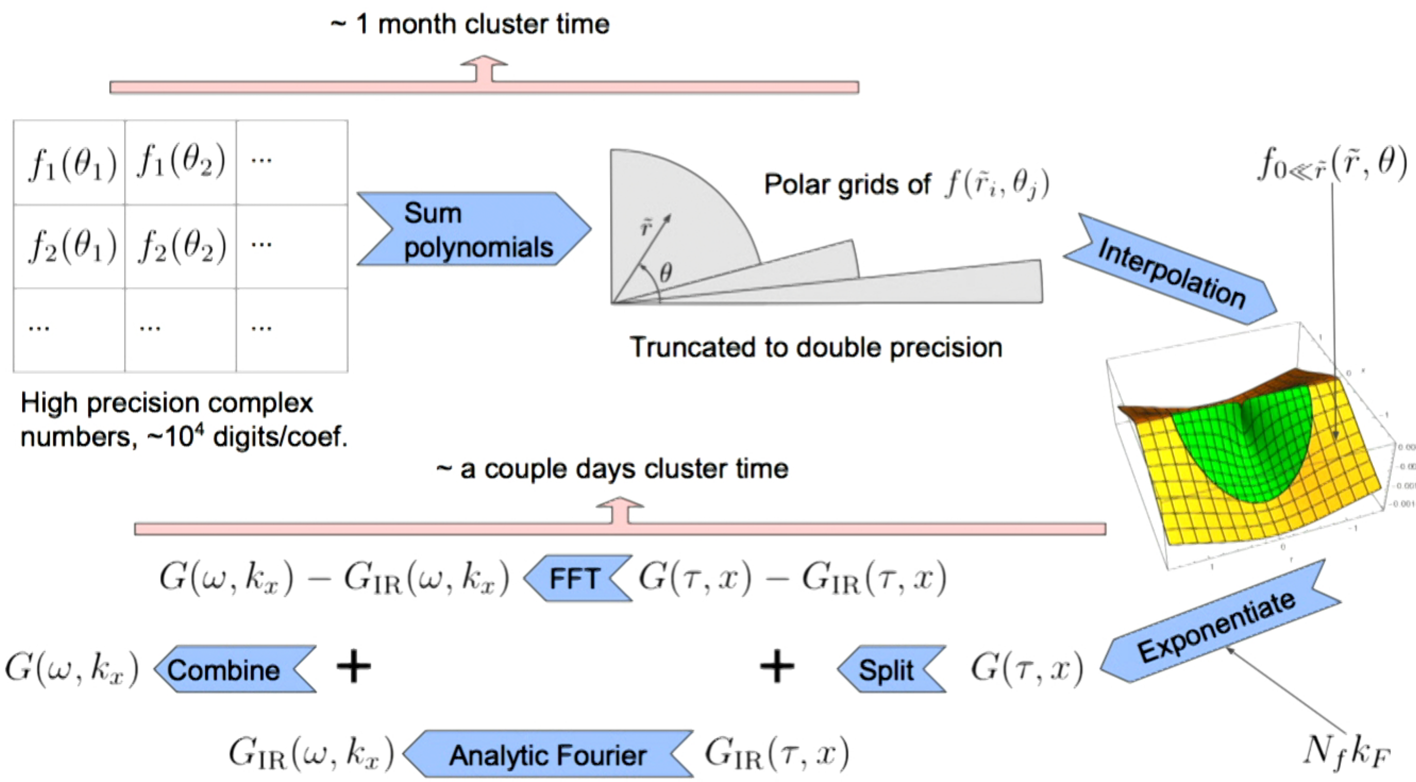
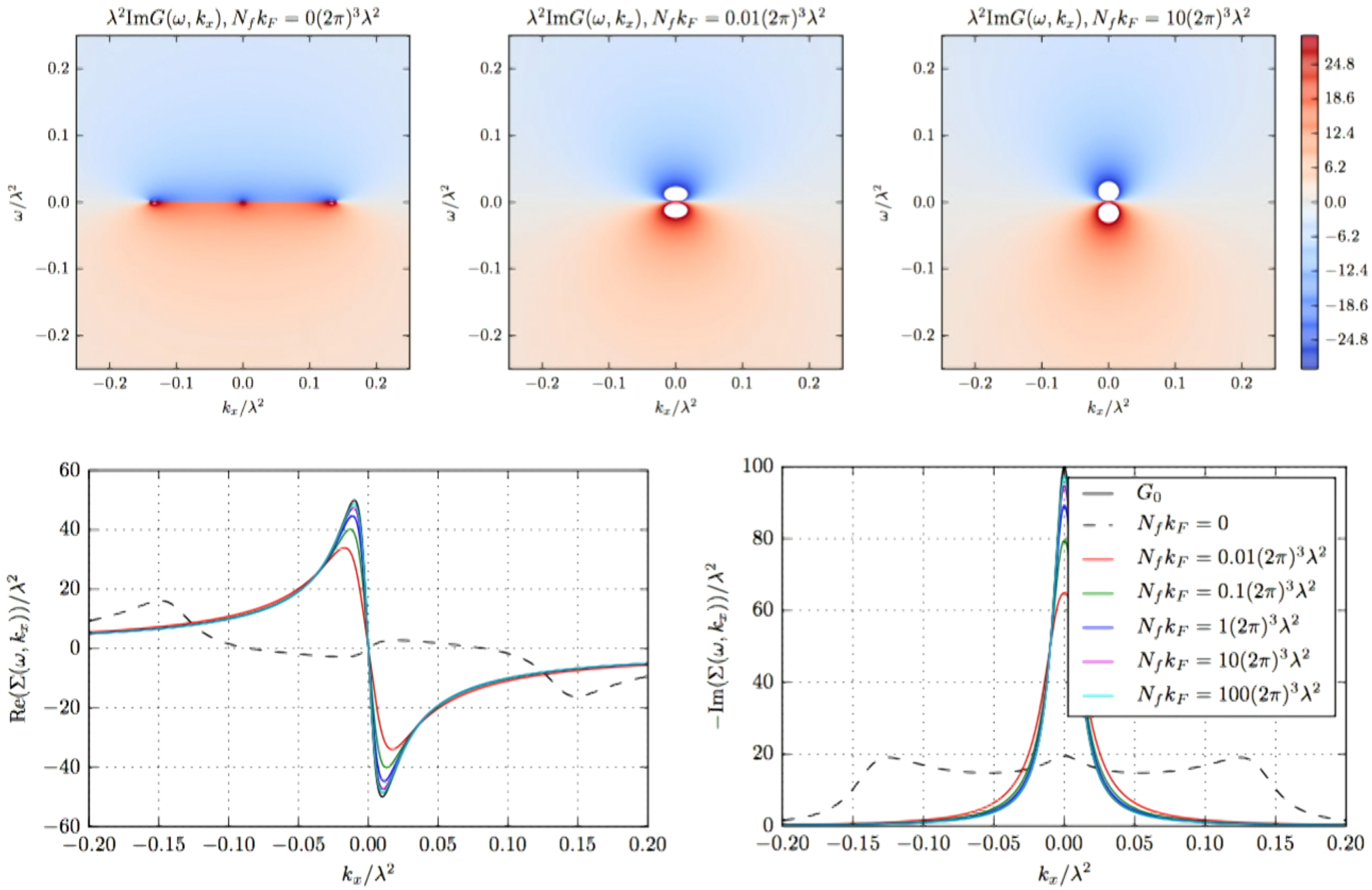
Numeric procedure



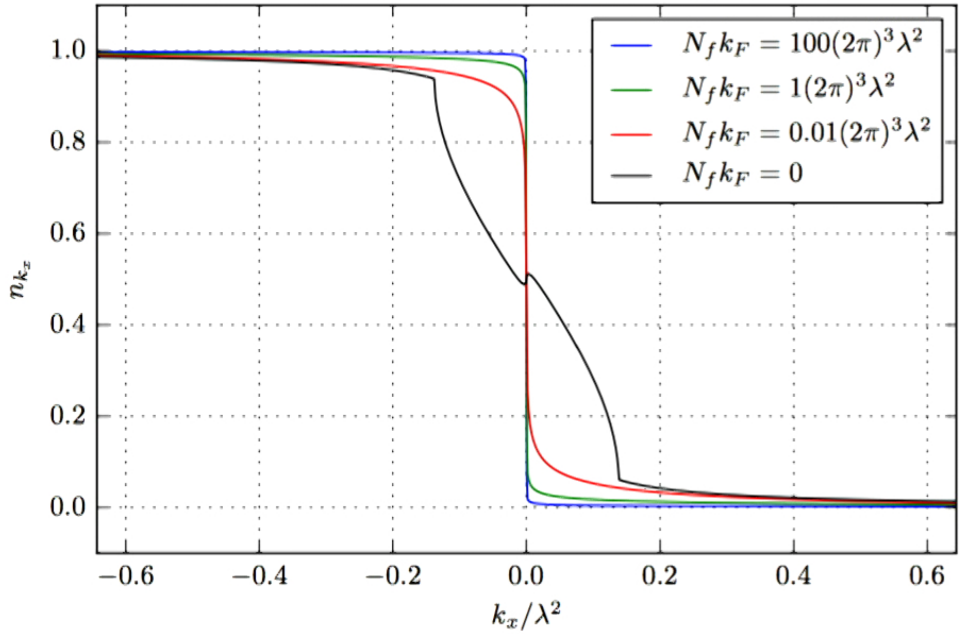
Exact fermion two point functions at large k_F



Momentum distribution function

$$n_{k_x} = \int_{-\infty}^{0} \mathrm{d}\omega A(\omega, k_x)$$

Fermi-liquids have a n_{k_x}-discontinuity at the fermi surface.



Limits

IR expansion:

$$G_{\mathrm{IR}}(\omega, k_x) = \mathrm{e}^{\frac{\lambda}{3\sqrt{2\pi N_f k_F}}} \left[\frac{1}{i\omega - k_x} \cos\left(\frac{\omega}{l_0^{1/2}(\omega + ik_x)^{3/2}} \right) \right.$$

$$+ \frac{6\sqrt{3} i \Gamma\left(\frac{1}{3}\right) \omega^{2/3}}{8\pi l_0^{1/3}(\omega + ik_x)^2} \, {}_1F_2\left(1; \frac{5}{6}, \frac{4}{3}; -\frac{\omega^2}{4 l_0 (\omega + ik_x)^3}\right) +$$

$$\left. + \frac{3\sqrt{3} i \Gamma\left(-\frac{1}{3}\right) \omega^{4/3}}{8\pi l_0^{2/3}(\omega + ik_x)^3} \, {}_1F_2\left(1; \frac{7}{6}, \frac{5}{3}; -\frac{\omega^2}{4 l_0 (\omega + ik_x)^3}\right) \right]$$

$$\approx \mathrm{e}^{\frac{\lambda}{3\sqrt{2\pi N_f k_F}}} \frac{1}{i\omega - k_x - \Sigma_{\mathrm{RPA}}} \quad \text{for small } \omega \text{ and fixed } k_x$$

$$\approx \mathrm{e}^{\frac{\lambda}{3\sqrt{2\pi N_f k_F}}} \frac{1}{i\omega - k_x - \frac{4\pi}{3\sqrt{3}} \Sigma_{\mathrm{RPA}}} \quad \text{for small } \omega \text{ and fixed } k_x/\omega$$

Large $N_f k_F$-limit:

$$\Sigma_{\mathrm{RPA}}(\omega, k_x) = \frac{-i\lambda^{4/3} \mathrm{sgn}(\omega) |\omega|^{2/3}}{(2\pi)^{2/3}\sqrt{3}(N_f k_F)^{1/3}}$$

Summary

- **Two point functions of the elementary quantum critical metal are solvable in the strict $N_f = 0$ limit.**
- **They are also solvable (numerically) when neglecting > 3 vertex fermion loops for general $N_f k_F$.**
- We can express the fermion real space two point function as a (converging) infinite series.
- We find the low energy limit is given by the Fourier transform of a previous result, up to a multiplicative factor.

Next Steps

- Higher-point functions
- Explicit N_f-correction through four point function
- Boson self-interactions through Monte-Carlo methods?
- Lorentzian correlators. This can benefit from a holographic approach[19].

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[19] W. Witczak-Krempa, E. S. Sørensen, S. Sachdev, Nature Physics **10** 2014