Title: Michael Cates: Bulletproof Custard: Fluids That Stop Flowing When You Push Them Too Hard

Date: Nov 02, 2016 07:00 PM

URL: http://pirsa.org/16110034

Abstract: <p>When small, hard particles are suspended in a fluid, they make it more resistant to flow. The higher the particle concentration, the higher the viscosity. Add enough particles and fluid stops flowing entirely, becoming a jammed solid - this makes intuitive sense.

Less intuitive and more intriguing are suspensions that flow smoothly if pushed gently, but that suddenly solidify if you push too hard. This behaviour is called Discontinuous Shear Thickening (DST). You can try it yourself by mixing cornstarch with water - in the right proportions, the mixture will flow smoothly when stirred gently, but will refuse to flow at all if stirred too hard.

More than an interesting kitchen trick, DST has important real-world consequences. It can cause catastrophic failure of industrial pumping equipment, but can also have life-saving applications to bulletproof vests.

For many years, the mechanism behind DST was unclear, but we have very recently found a new and stunningly simple explanation based on the idea that the contacts between particles become less lubricated and more frictional as the force between them increases. Although this dependence is typically gradual, when a fluid gets close to the â€œjammingâ€• point, global instabilities can result in the sudden switching from liquid to solid.

Michael Cates (Lucasian Professor of Mathematics, University of Cambridge) will explain this peculiar form of â€œbulletproof custardâ€• with a few equations, plenty of diagrams, and even some hands-on demonstrations.</p>

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BULLET-PROOF CUSTARD:

Fluids that stop flowing when you push them too hard

- Soft matter: what and why?
- Dense suspensions under flow
- Shear-thickening suspensions
    - What we are trying to explain
    - How we explain it
- How is this useful?

BULLET-PROOF CUSTARD:

Fluids that stop flowing when you push them too hard

R. Chacko, S. Fielding (Durham)

B. Guy, M. Hermes, W. Poon (Edinburgh)

R. Mari (Cambridge)

M. Wyart (Lausanne)

Soft Matter: What and Why?

Polymers

Micelles

Suspensions

Emulsions

Foams

Liquid crystals

Soft Matter: What and Why?

Polymers

Micelles

Suspensions

Emulsions

Foams

Liquid crystals

Engine oil

Shampoo

Toothpaste

Mayonnaise

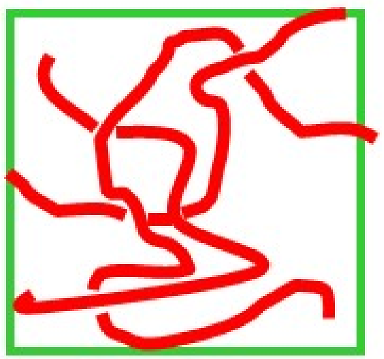
Shaving cream

Wet soap (slime)

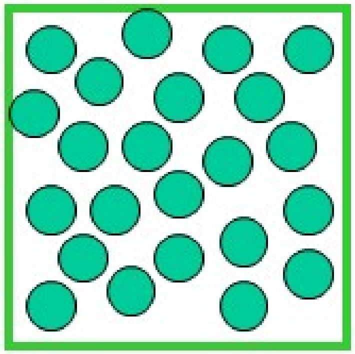
Soft Matter: What and Why?

Polymers	Engine oil	All plastics
Micelles	Shampoo	Pipeline additives
Suspensions	Toothpaste	Advanced Ceramics
Emulsions	Mayonnaise	Pharmaceuticals
Foams	Shaving cream	Environment cleanup
Liquid crystals	Wet soap (slime)	Smartphone displays

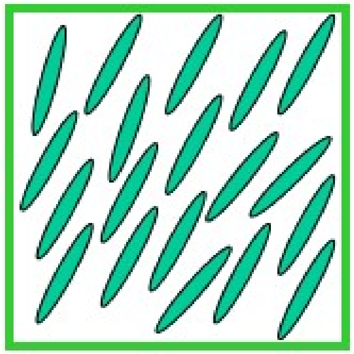
Soft Matter: What and Why?



polymers:
molecular spaghetti



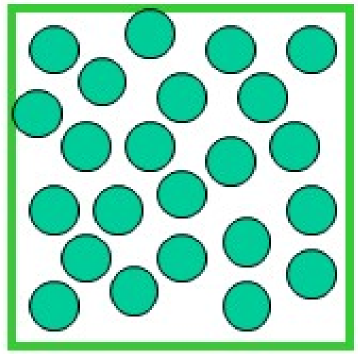
suspensions:
microscopic
ping-pong balls



liquid crystals:
molecular needles

Soft Matter: What and Why?

polymers:
molecular spaghetti



suspensions:
microscopic
ping-pong balls

liquid crystals:
molecular needles

- How does internal structure control macroscopic behaviour?
- More specifically, how do these materials flow?

Jamming and Unjamming Events

Jamming and Unjamming Events

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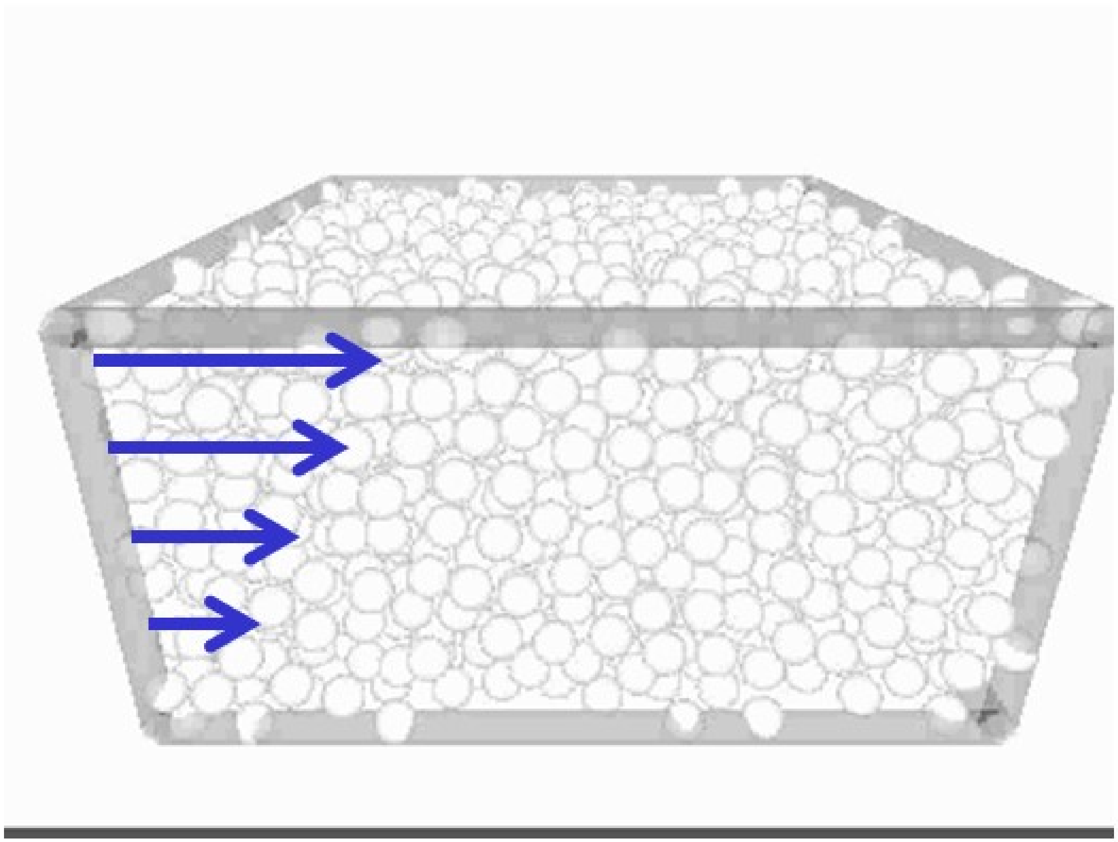
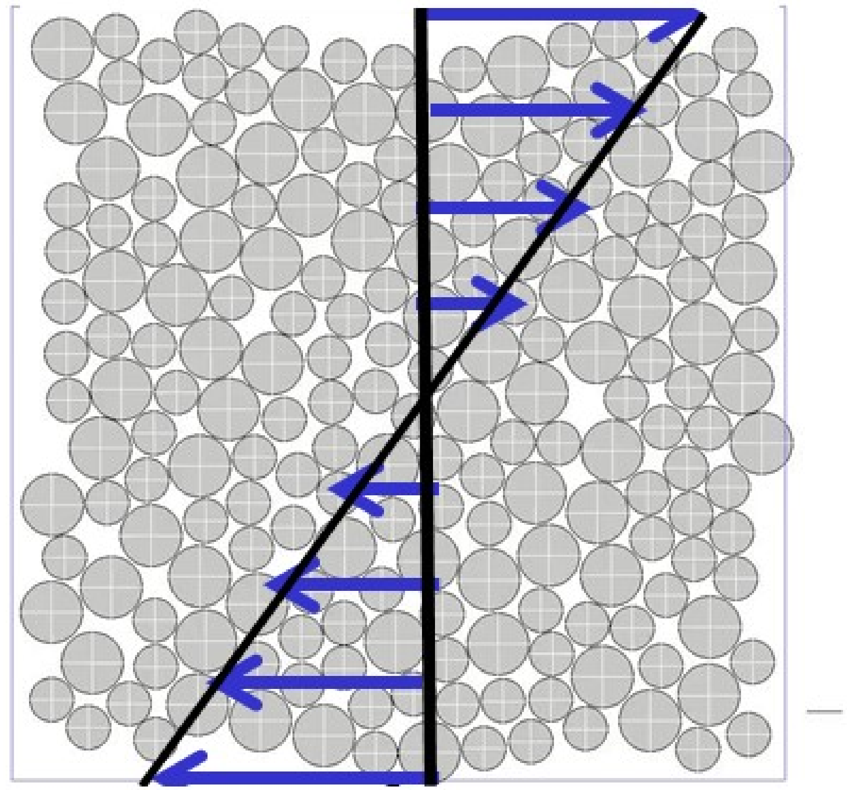
Dense Suspensions under Flow





microscopy experiments
particle size 1 micron

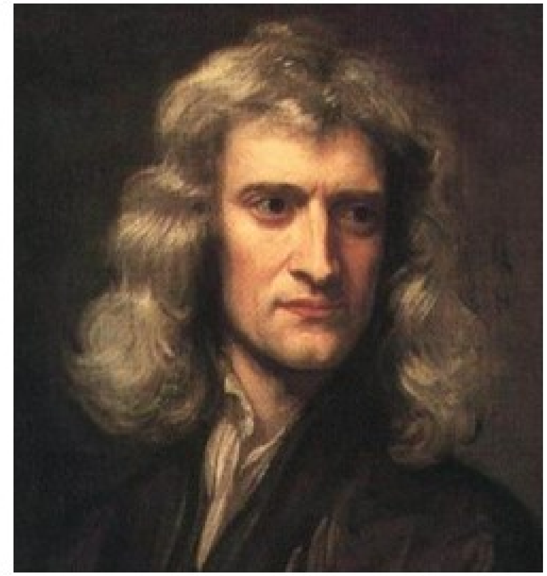
computer simulations

Newton's Law of Viscosity

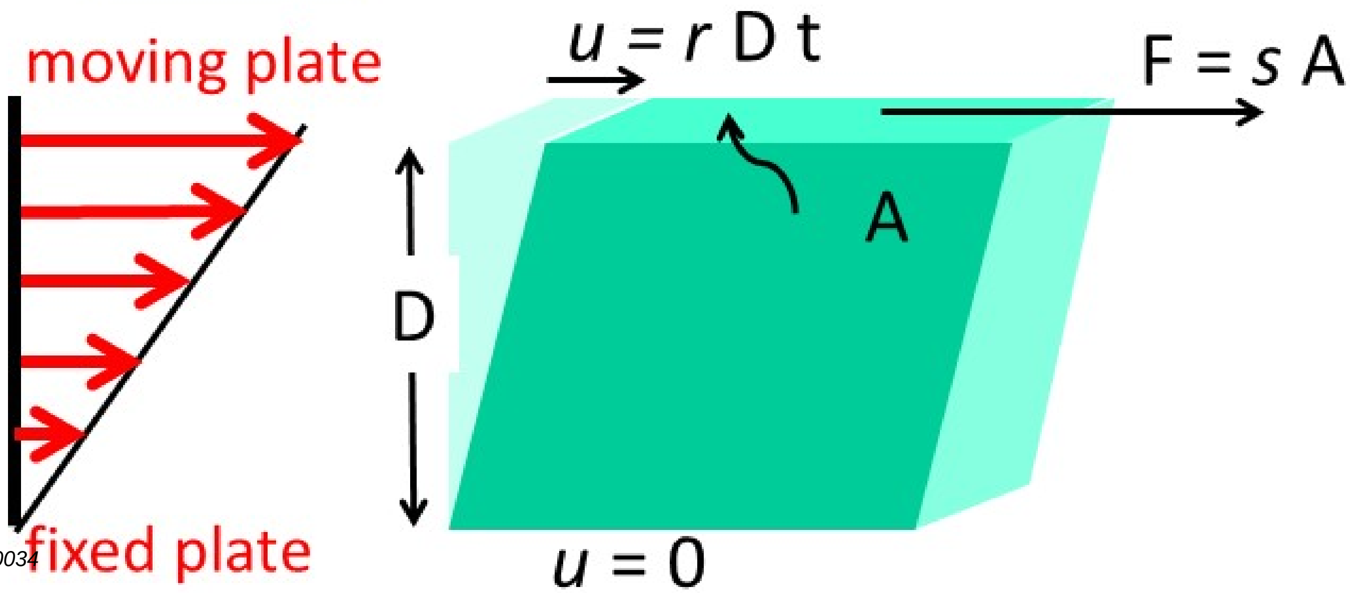
Principia (1687)

stress s = force/area

strain rate r = speed/depth



$$s = \nu r$$



Newton's Law of Viscosity

Principia (1687)

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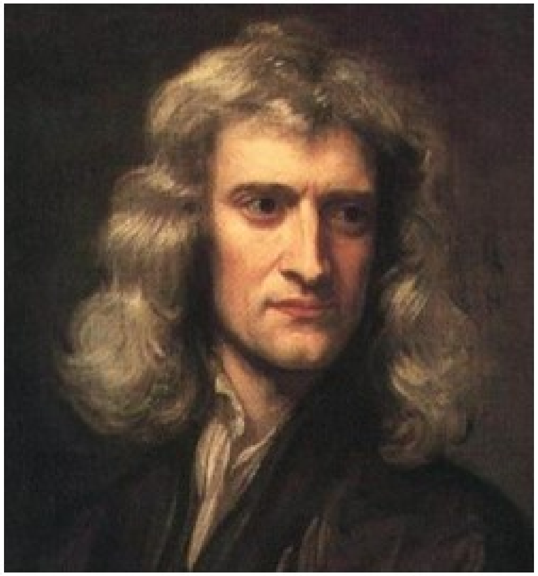
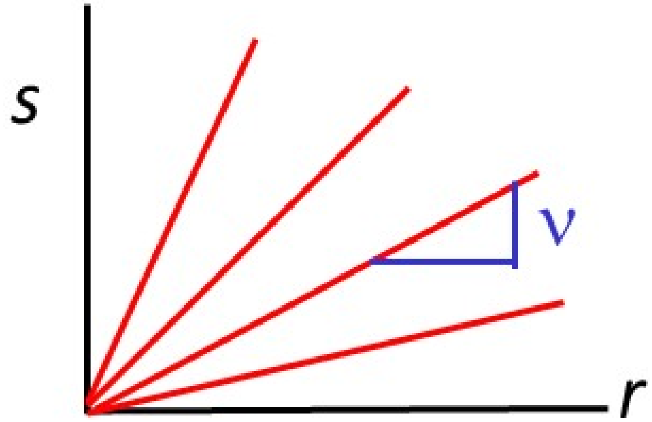


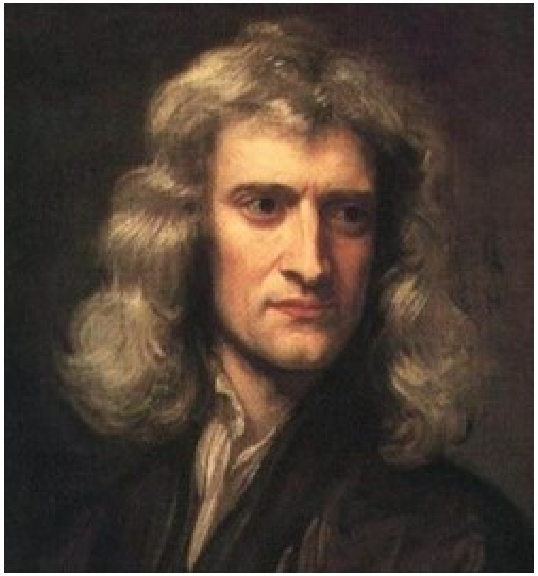
Flow curves *s*(*r*)



Newtonian

Newton's Law of Viscosity

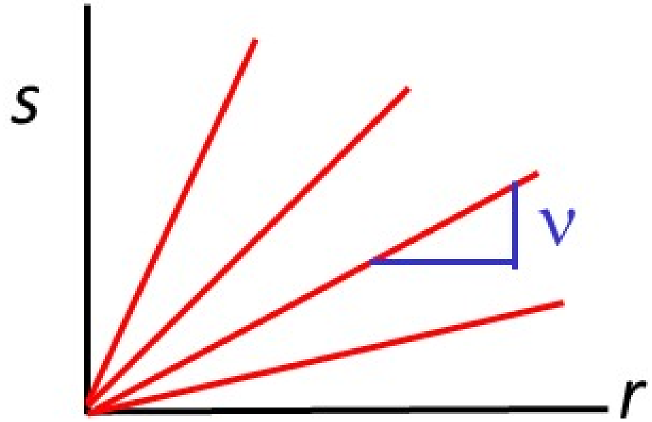
Principia (1687)



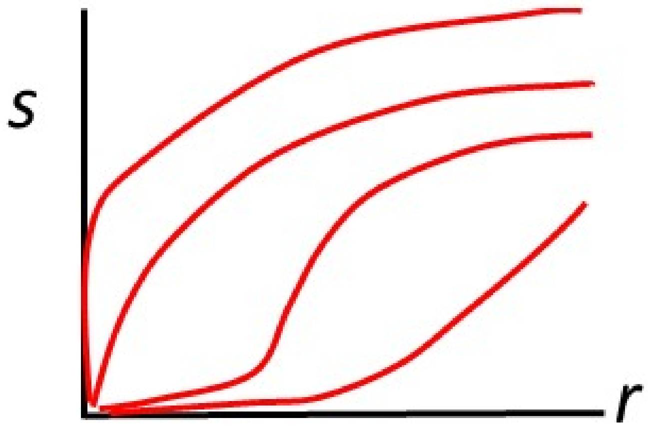
stress s = force/area

strain rate r = speed/depth

Flow curves $s(r)$



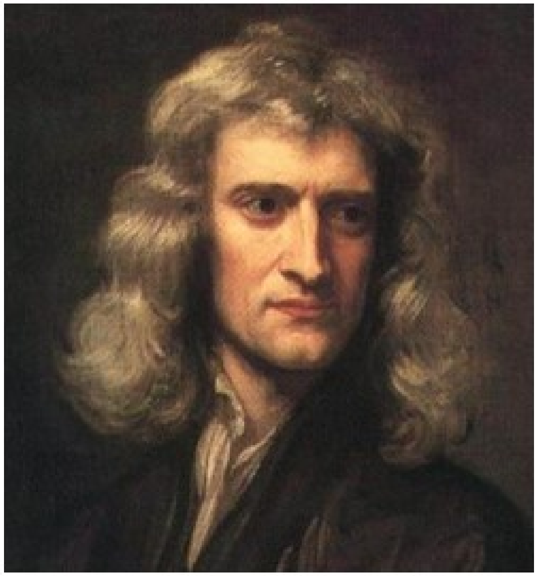
Newtonian



Non-Newtonian

Newton's Law of Viscosity

Principia (1687)



stress s = force/area

strain rate r = speed/depth

Flow curves $s(r)$

Newtonian	Non-Newtonian
water	spit
olive oil	engine oil
honey	putty
molten glass	molten plastic

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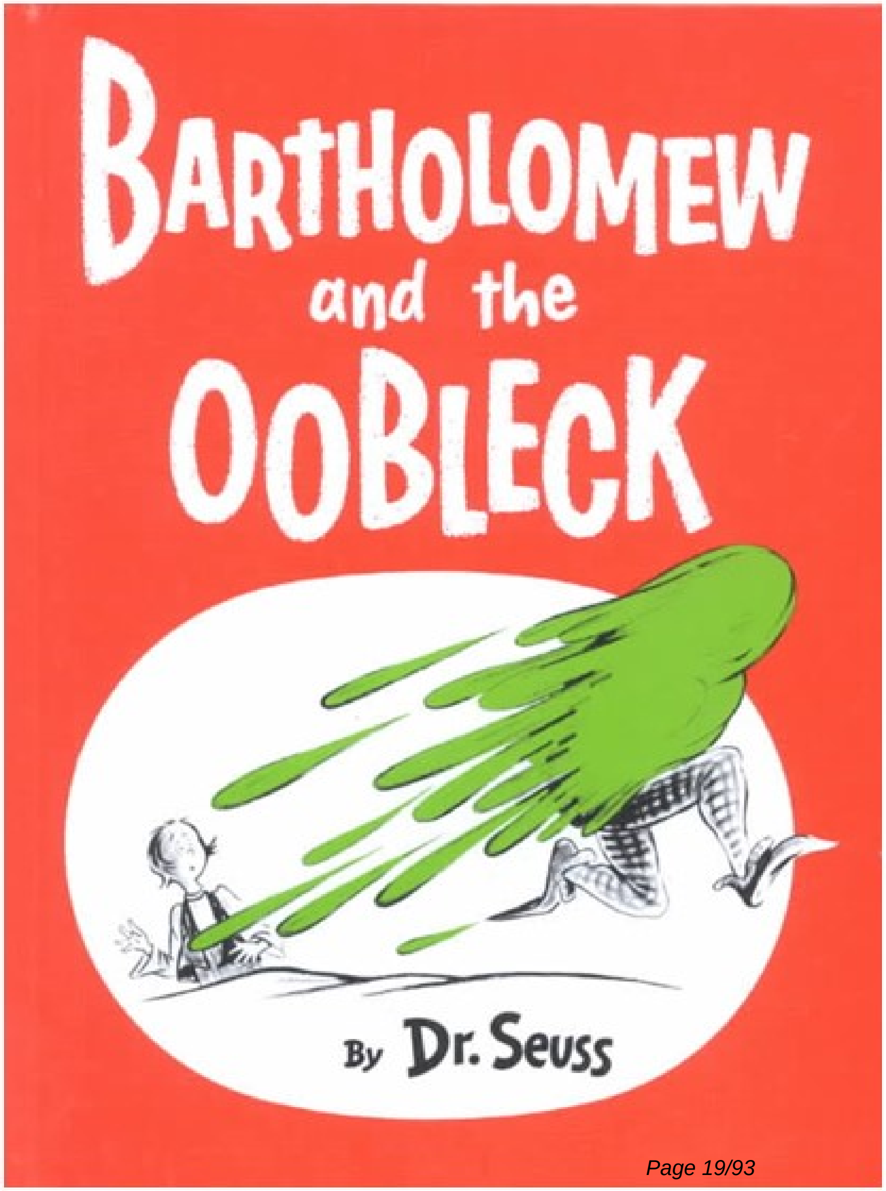
Shear-Thickening Suspensions

- Liquids that solidify
  at large stress

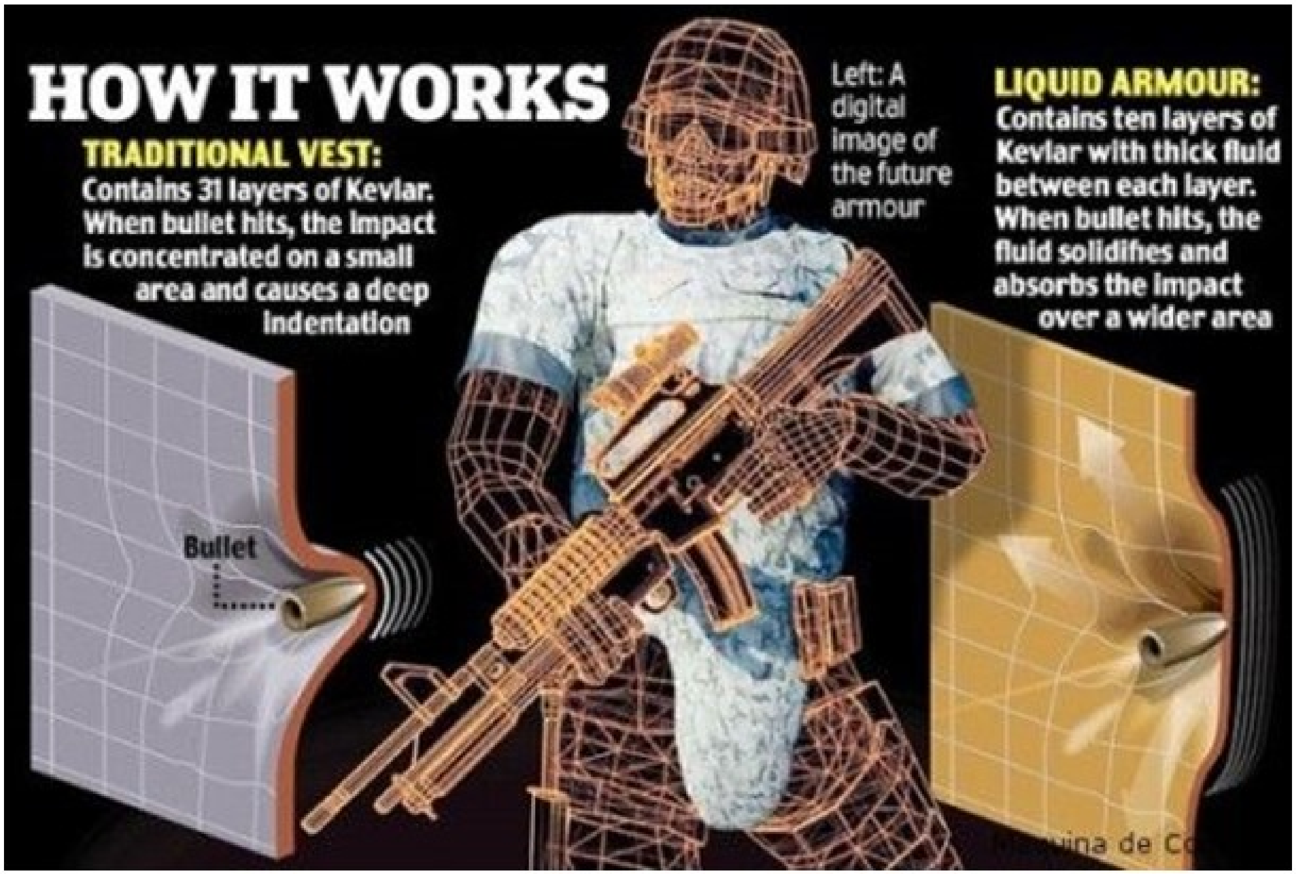
Shear-Thickening Suspensions

- Liquids that solidify at large stress

"Oobleck":
cornstarch or custard powder
+ a little water



Shear-Thickening Suspensions



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Patents:
N. Wagner
U Delaware

The revolutionary liquid armour suit that is made from bullet-proof 'custard'

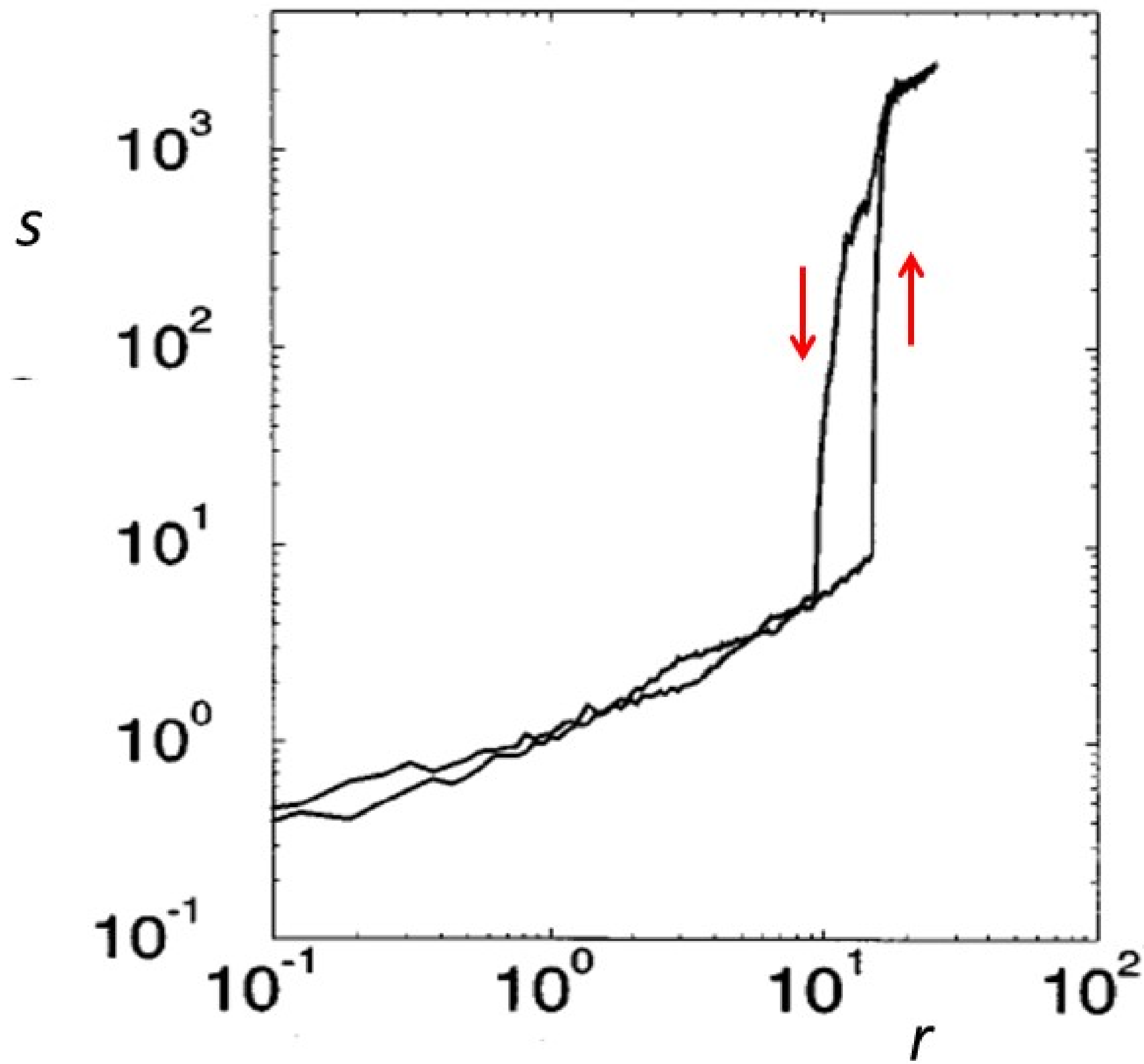

HORM
CUERO
www.youtube.com/watch?v=yHlAcASsf6U


HORMI
CUERO
www.youtube.com/watch?v=yHlAcASsf6U

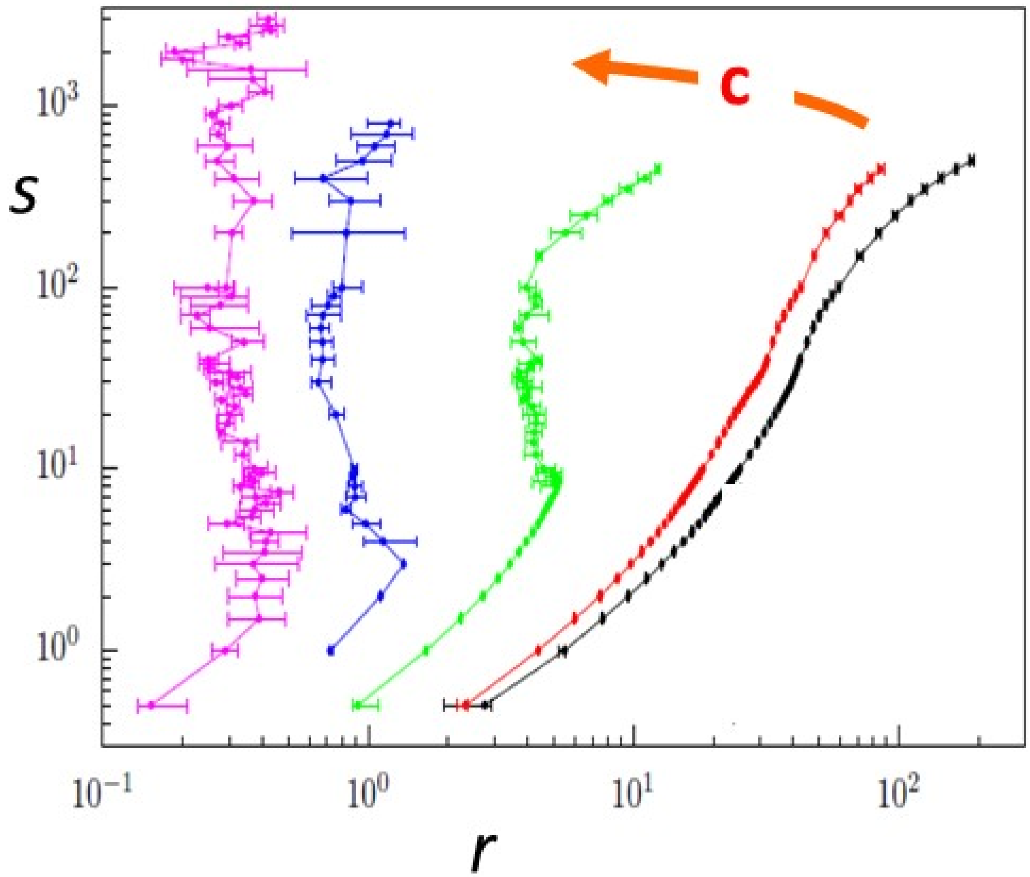
Shear-Thickening Suspensions

Shear-Thickening Suspensions

J. Bender + N. Wagner
Journal of Rheology
40, 899 (1996)



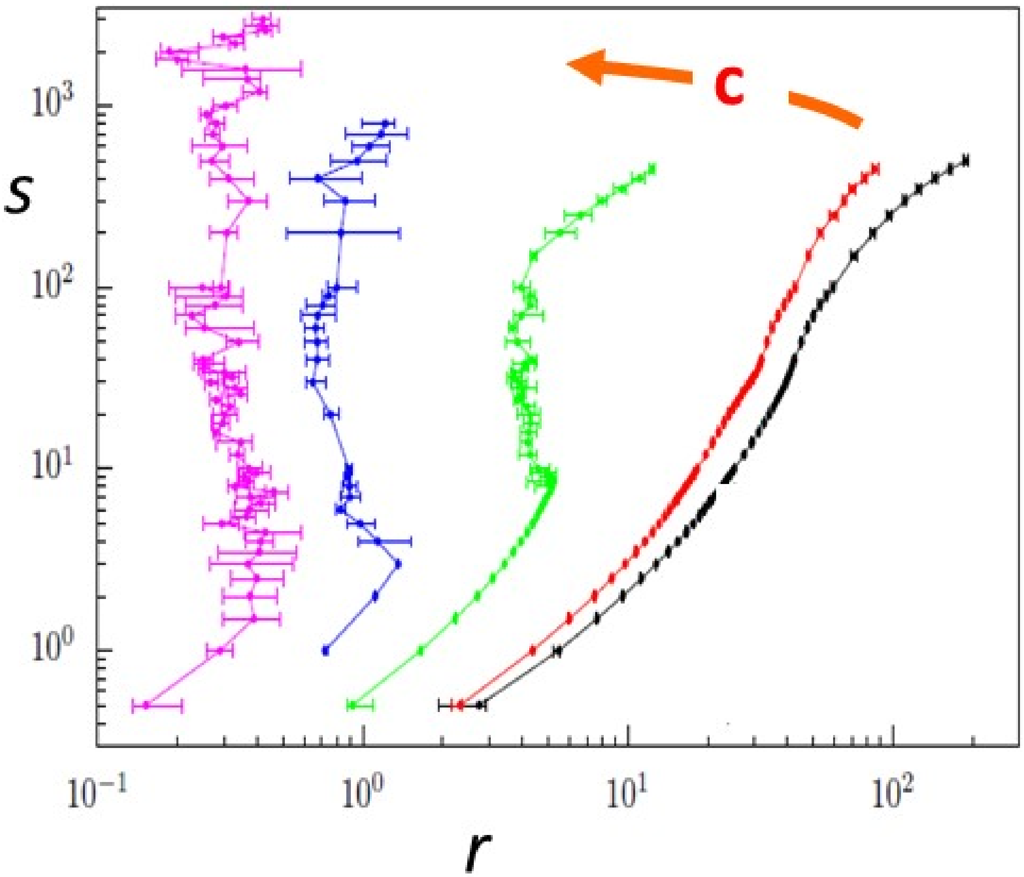
Shear-Thickening Suspensions

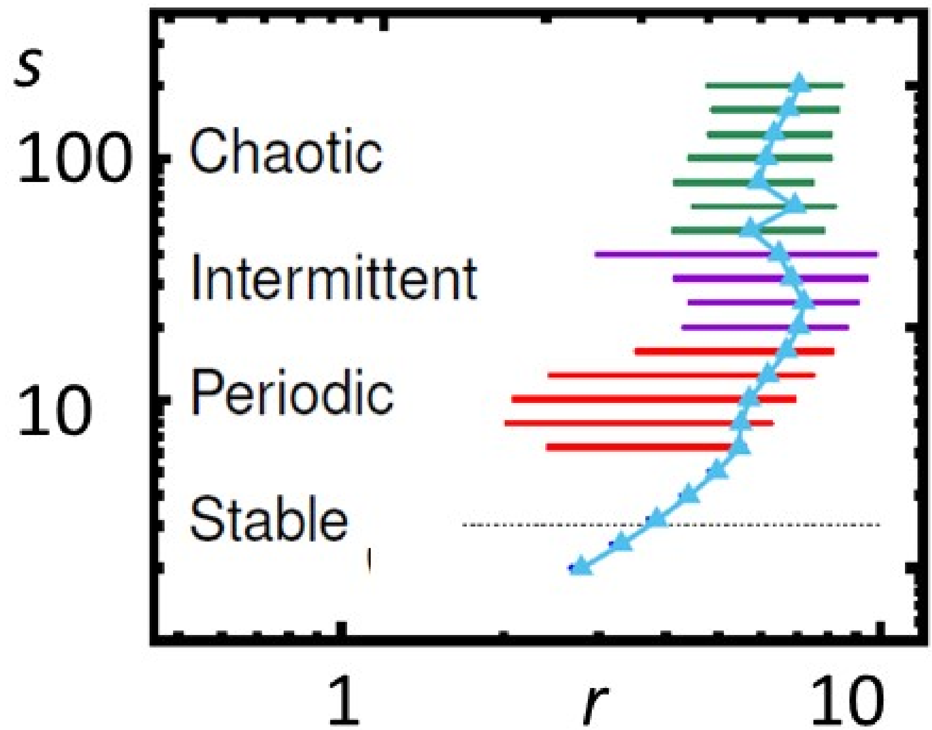


M. Hermes et al,
Journal of Rheology 60, 905 (2016)

c = N $(4\pi/3)R^3/V$
particle concentration
(dimensionless)

Shear-Thickening Suspensions





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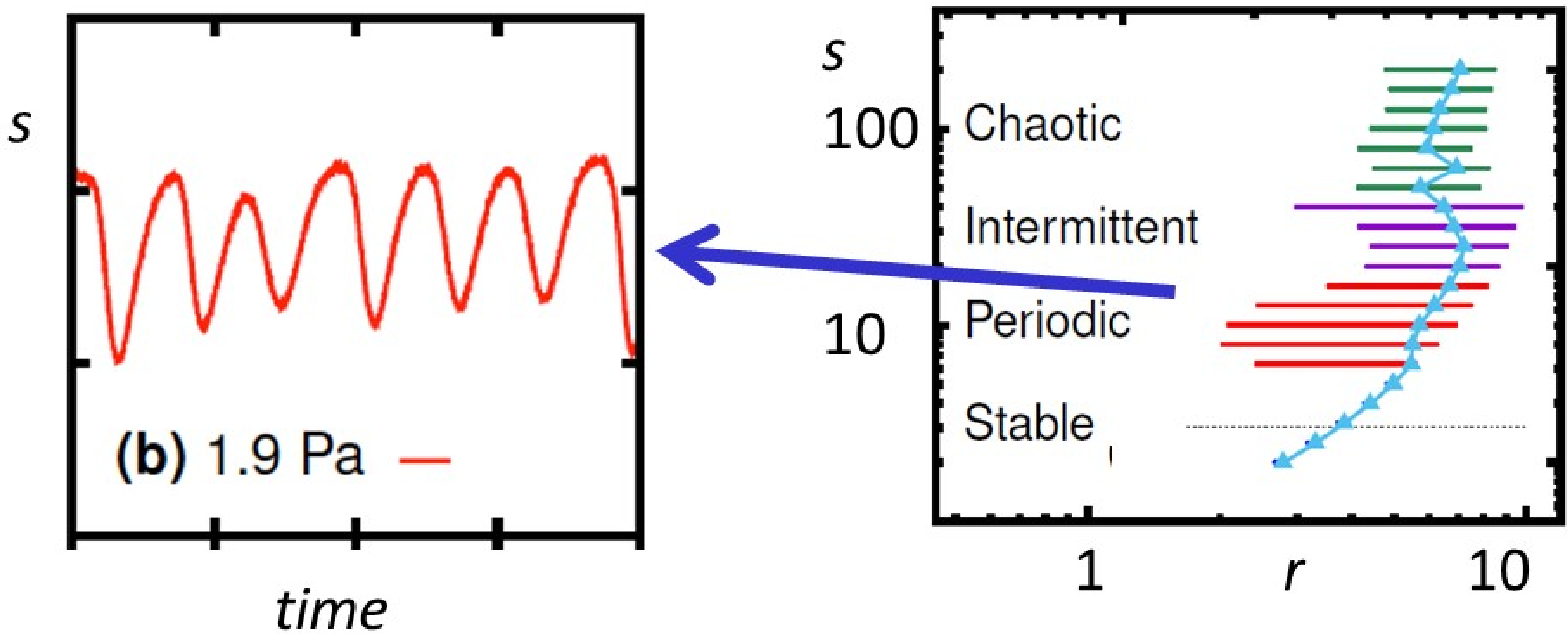

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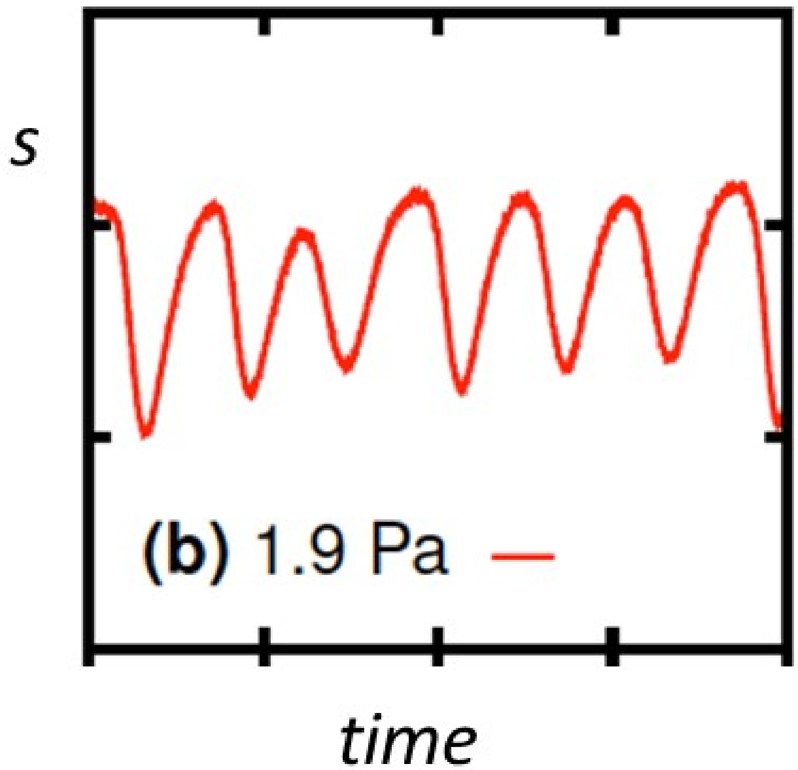

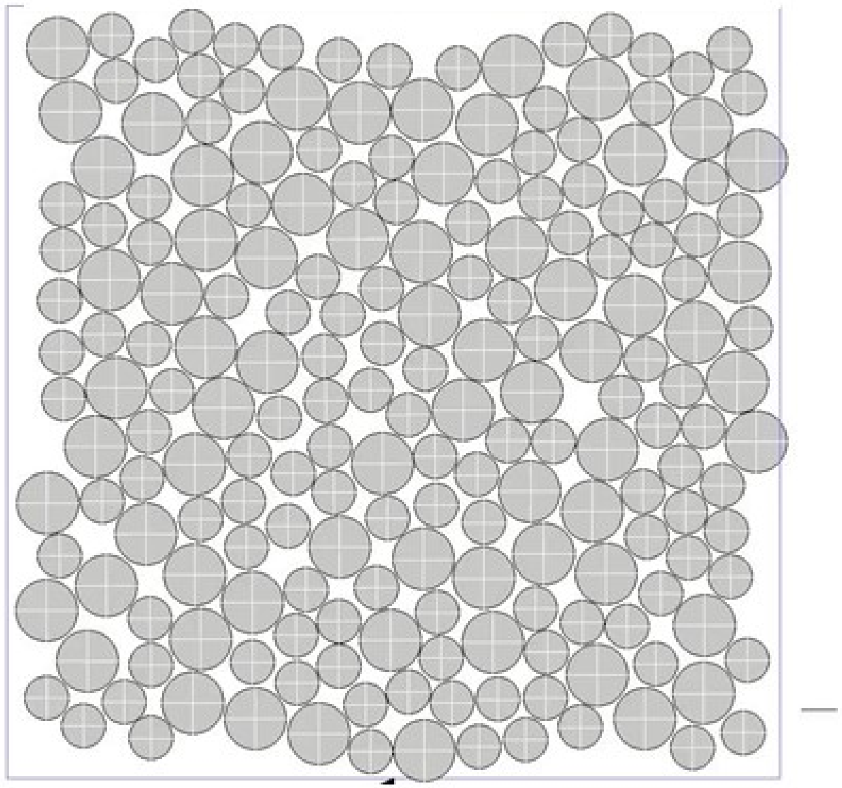

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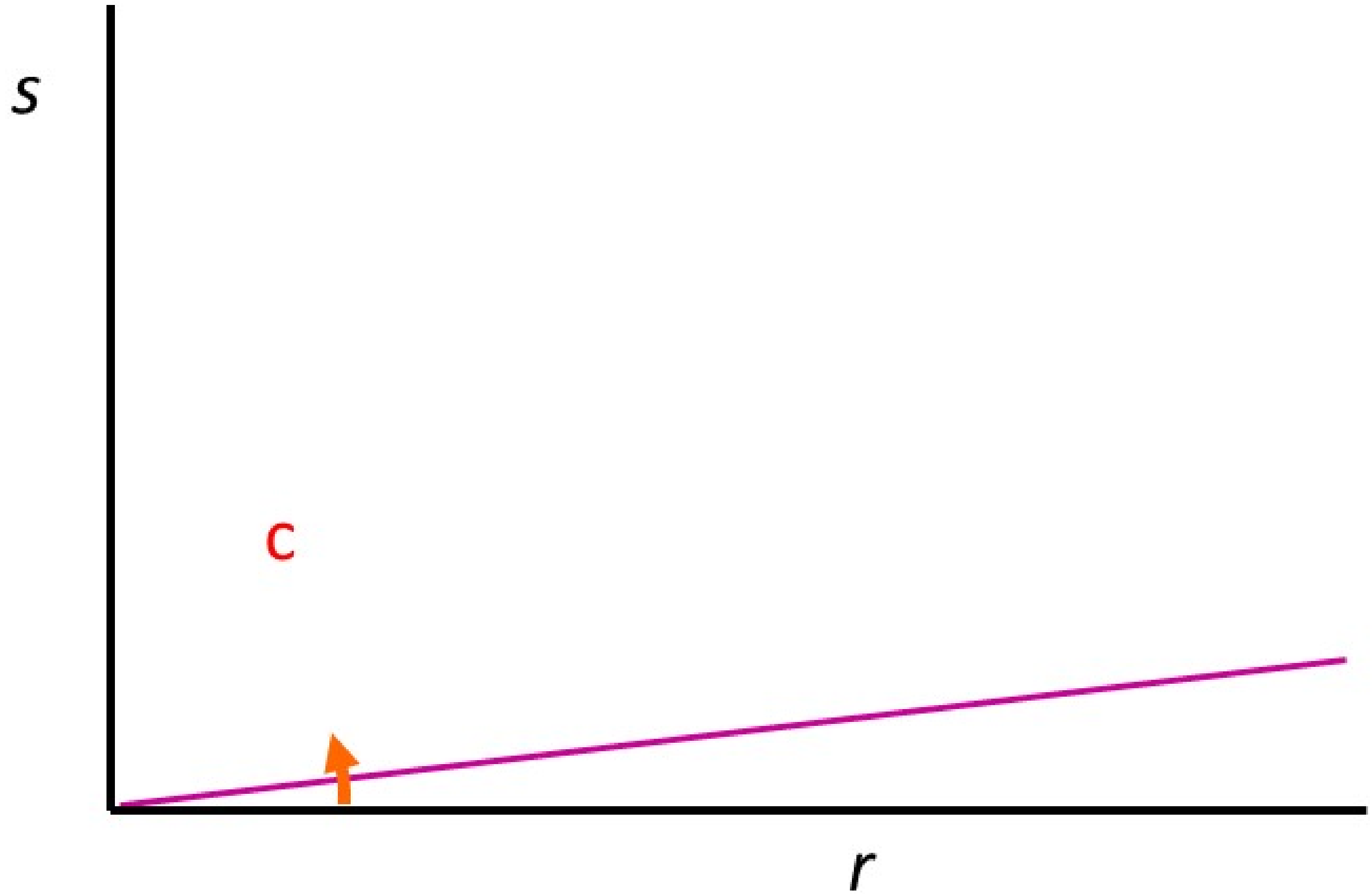
Shear-Thickening Suspensions

Data is explained if:



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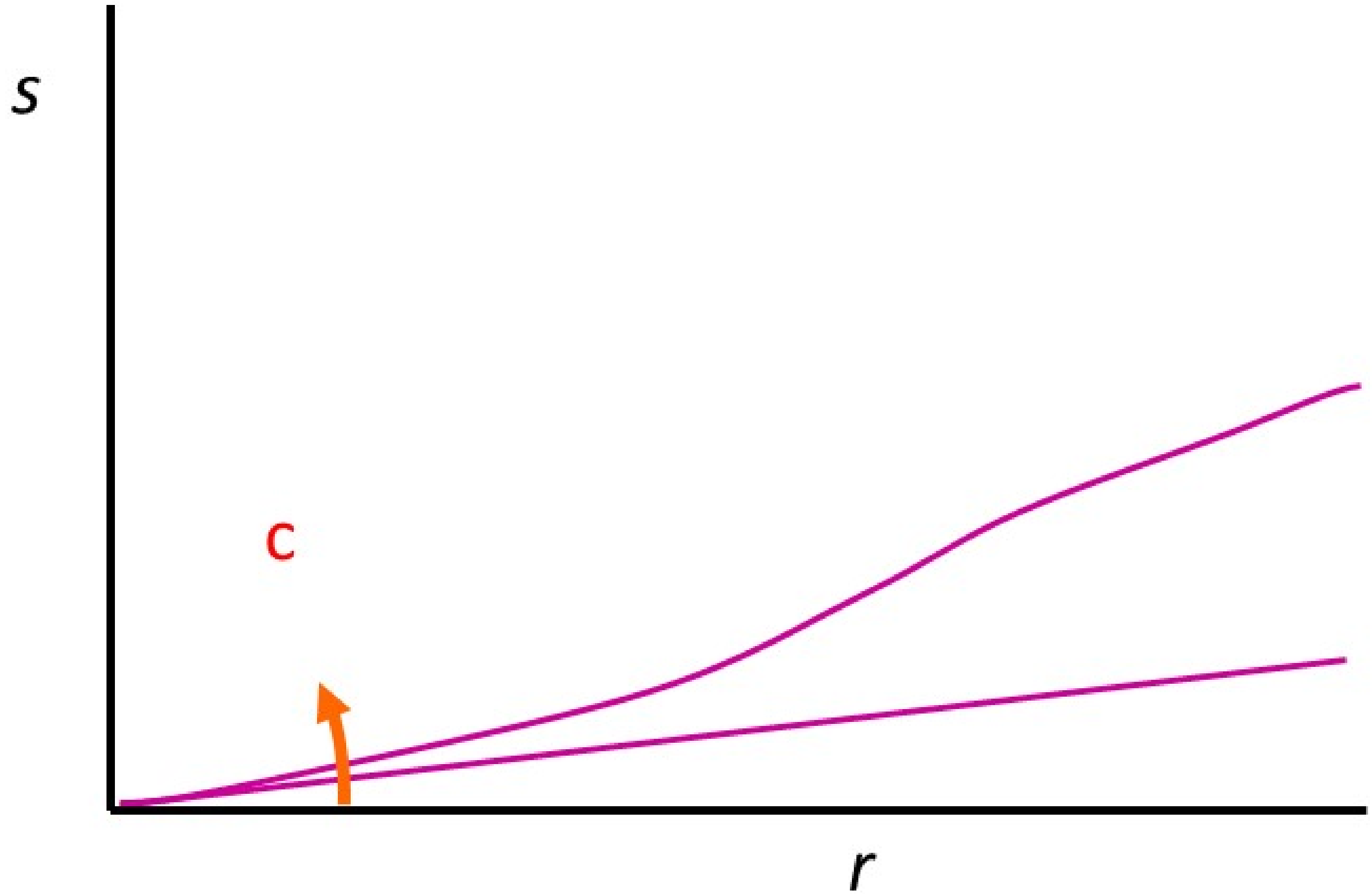

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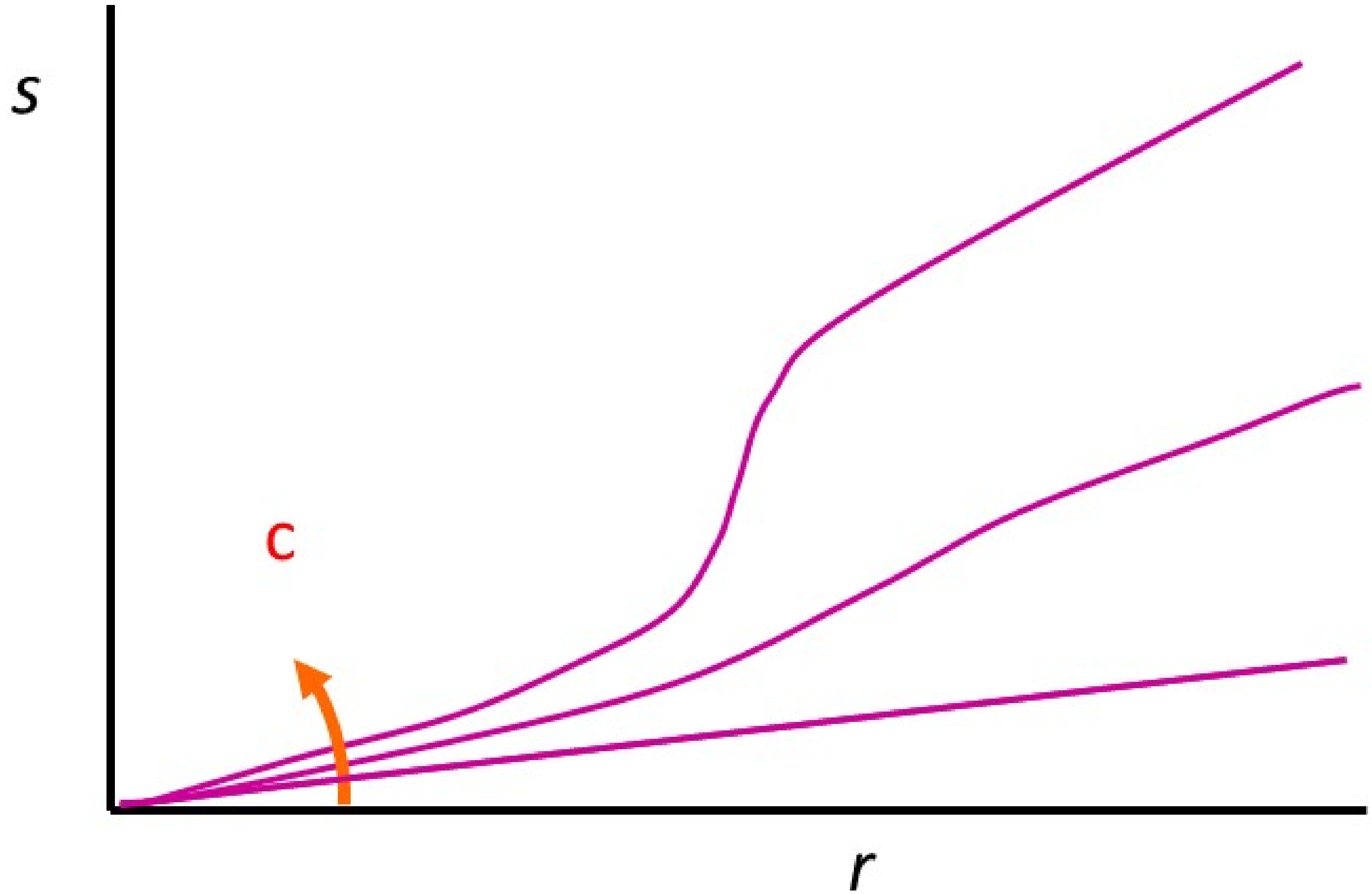

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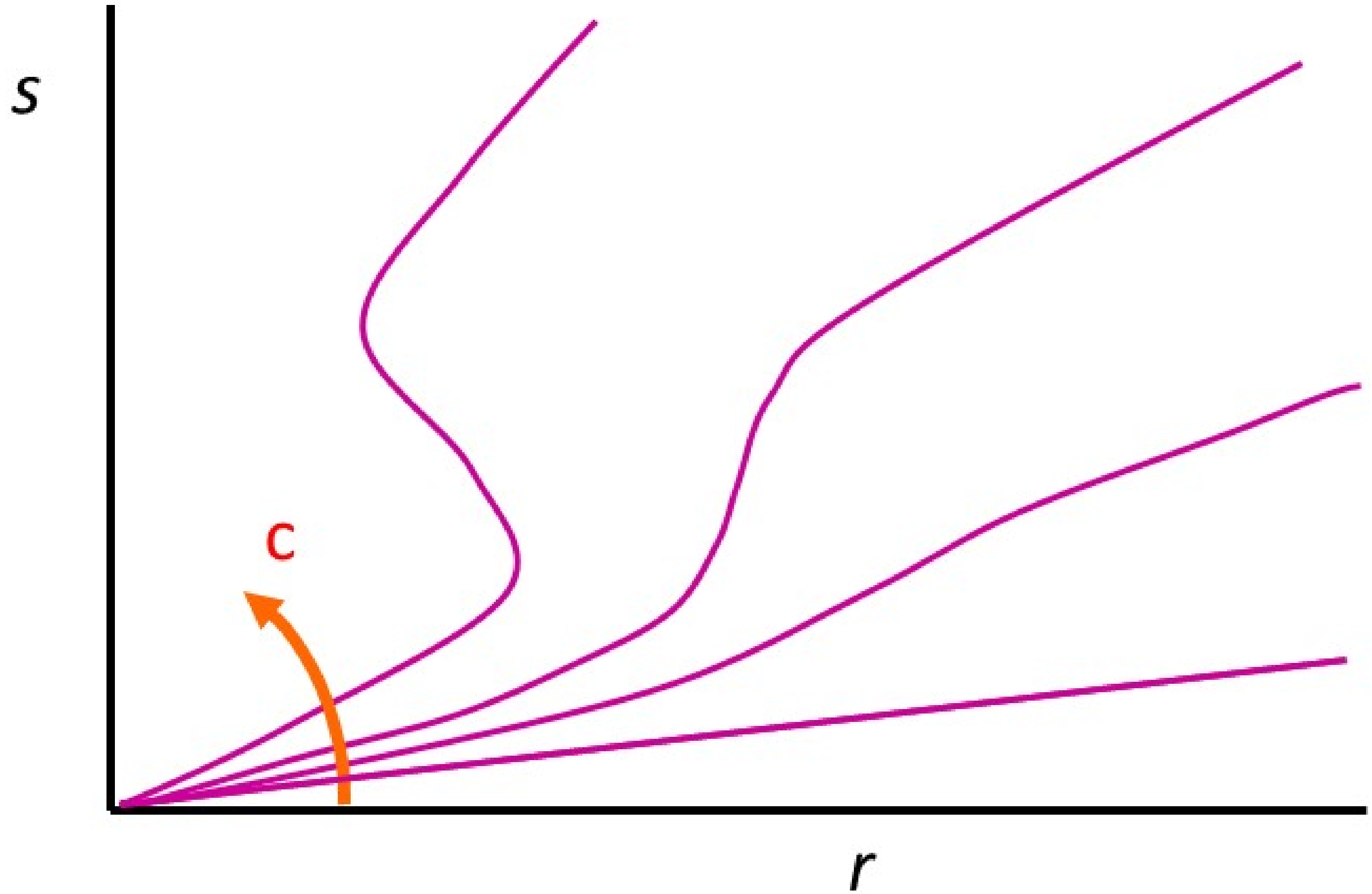

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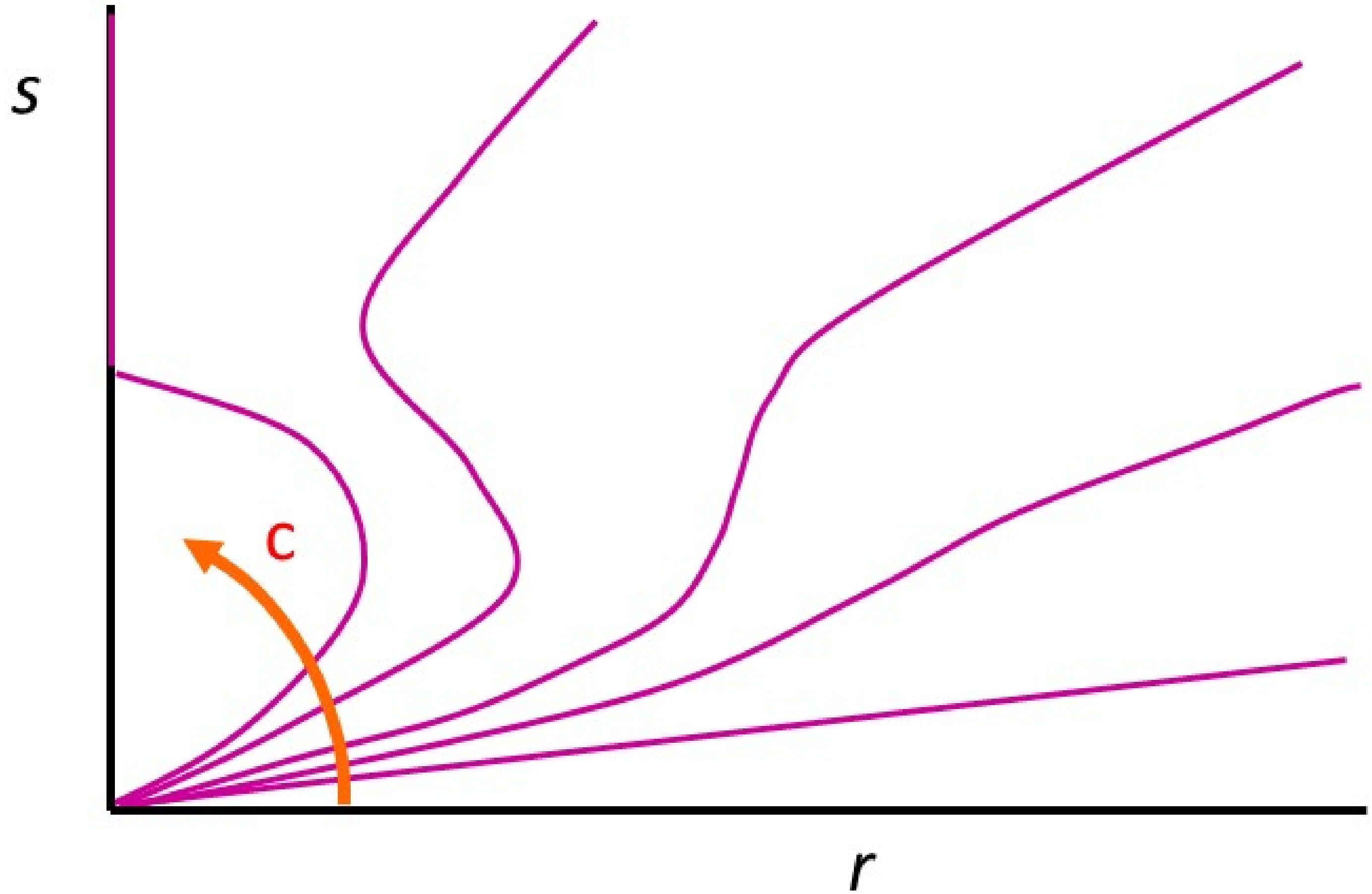
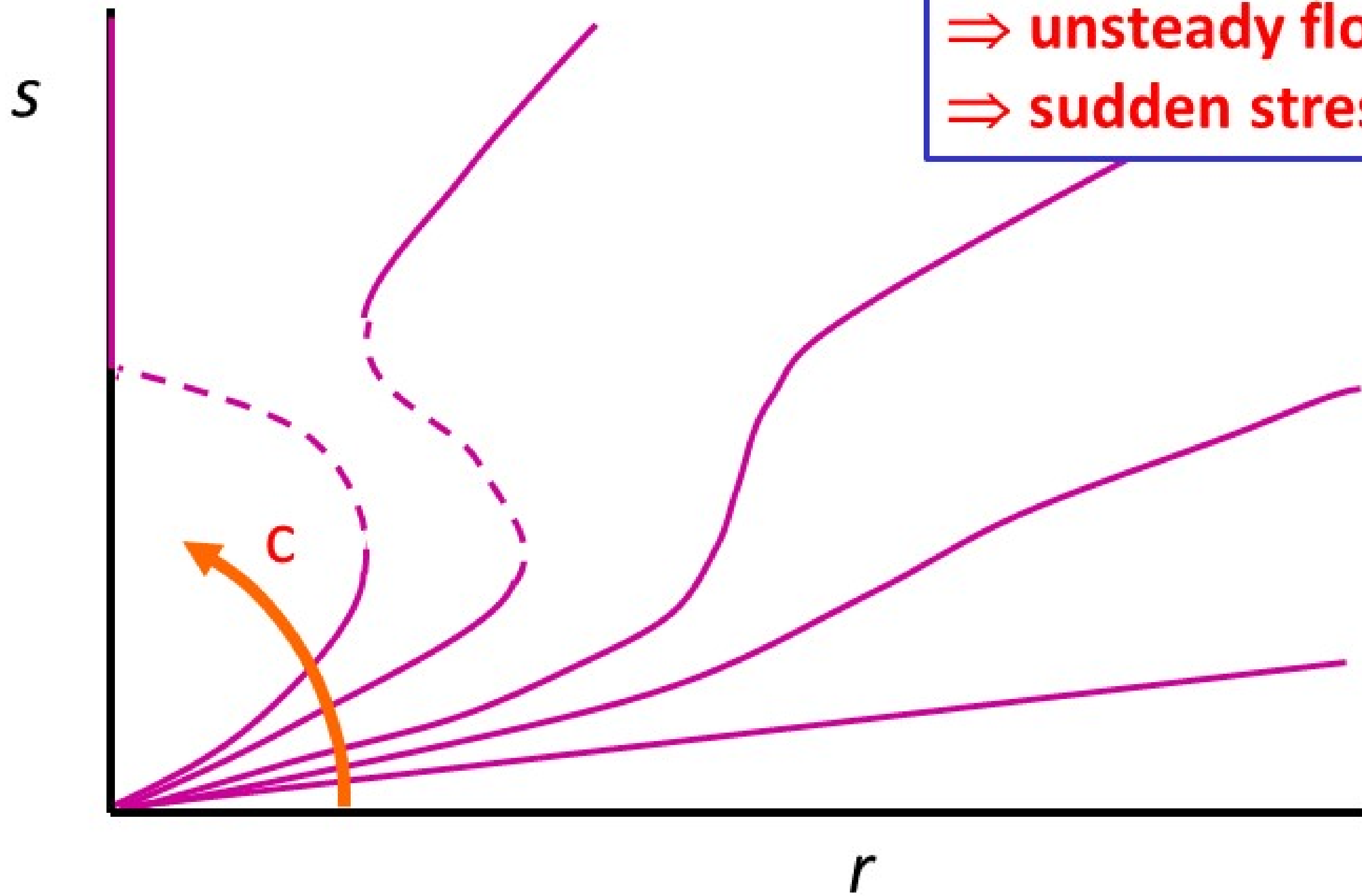


Shear-Thickening Suspensions

Data is explained if:

dotted parts: unstable
⇒ unsteady flow
⇒ sudden stress jumps



Shear-Thickening Suspensions

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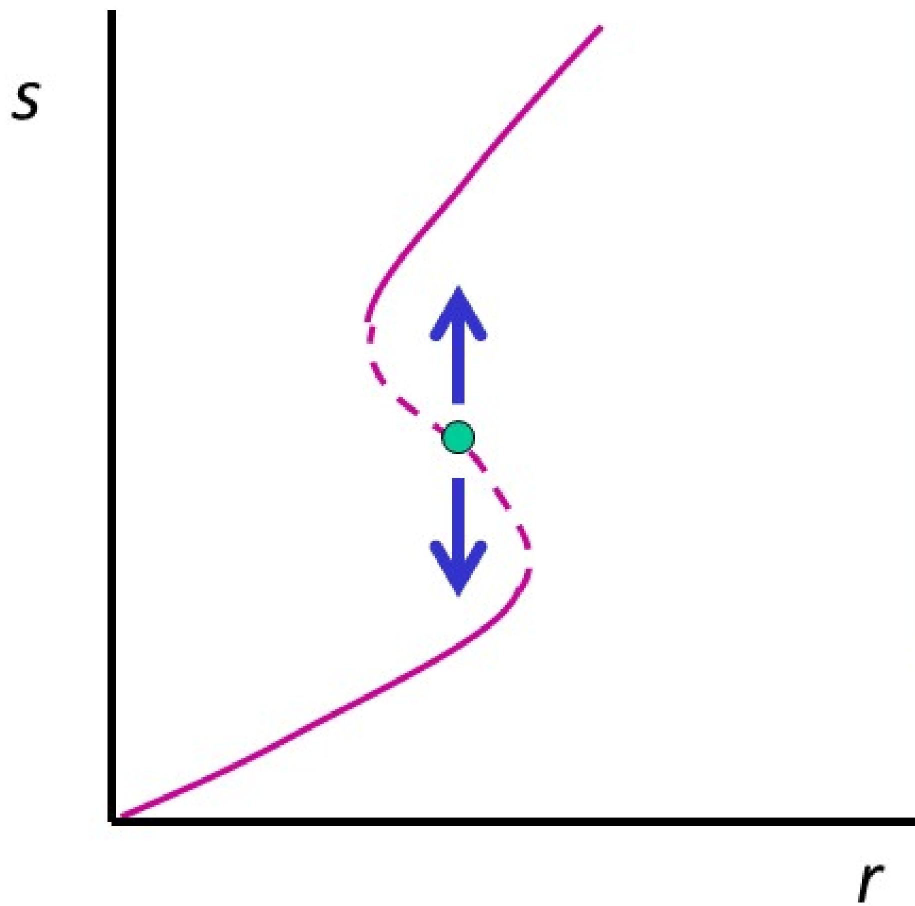
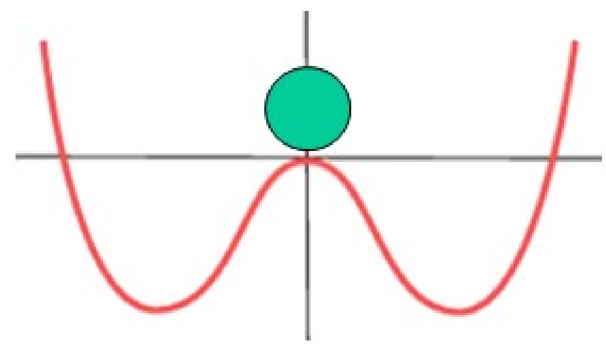


instability:
stress↘ strain rate↗

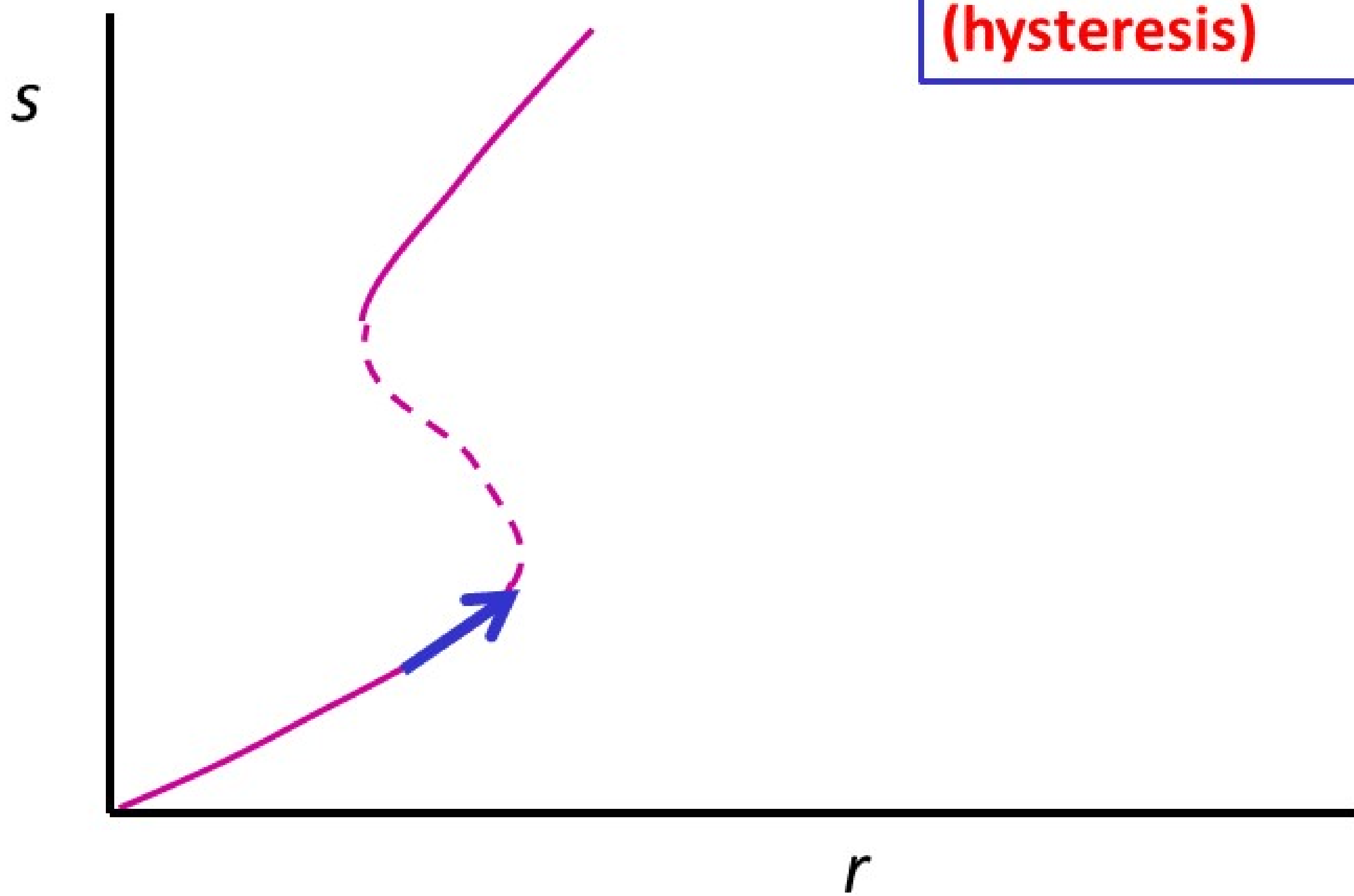
compare particle in V(x)



Shear-Thickening Suspensions

Data is explained if:

sudden stress jumps (hysteresis)



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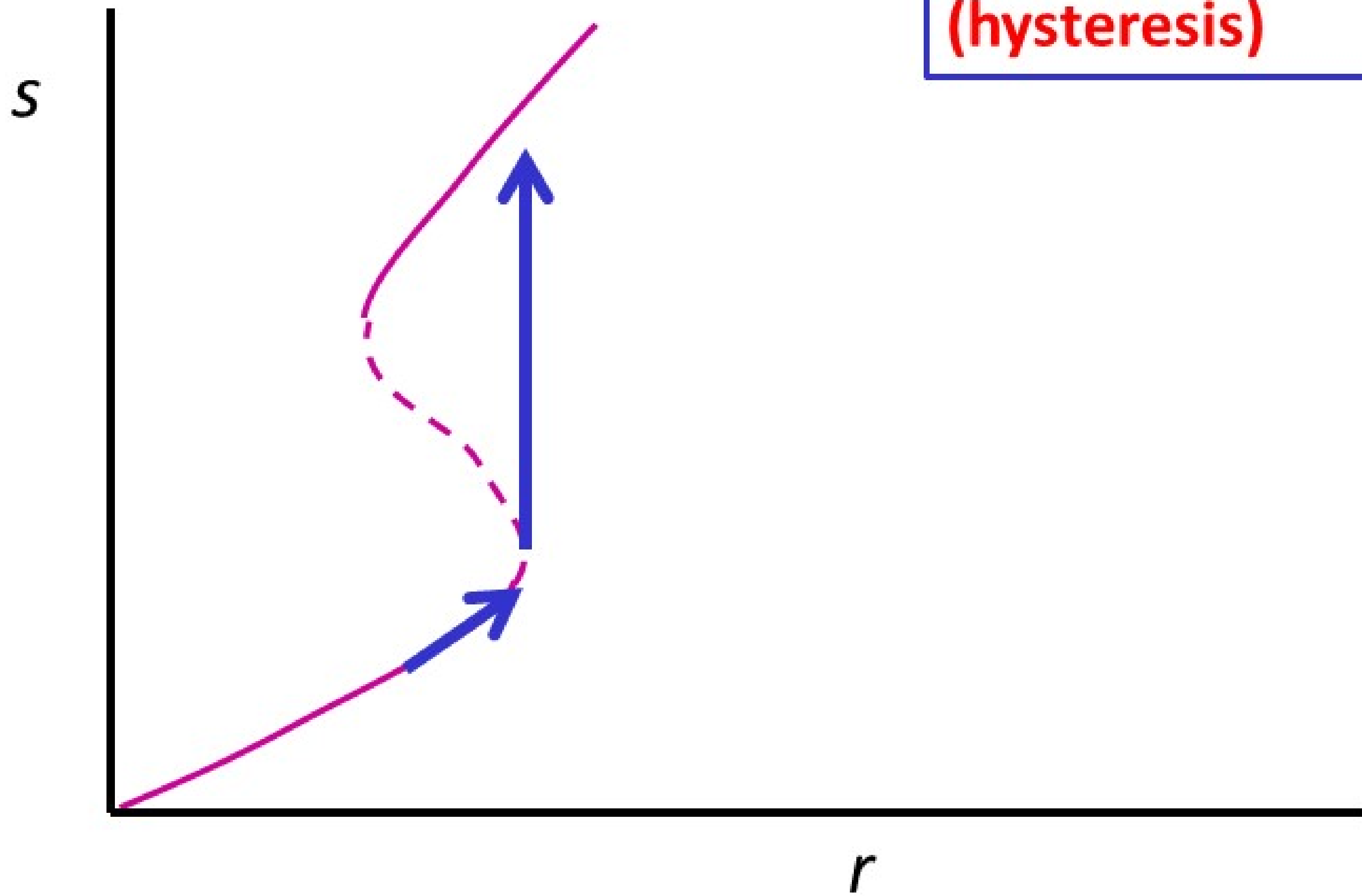

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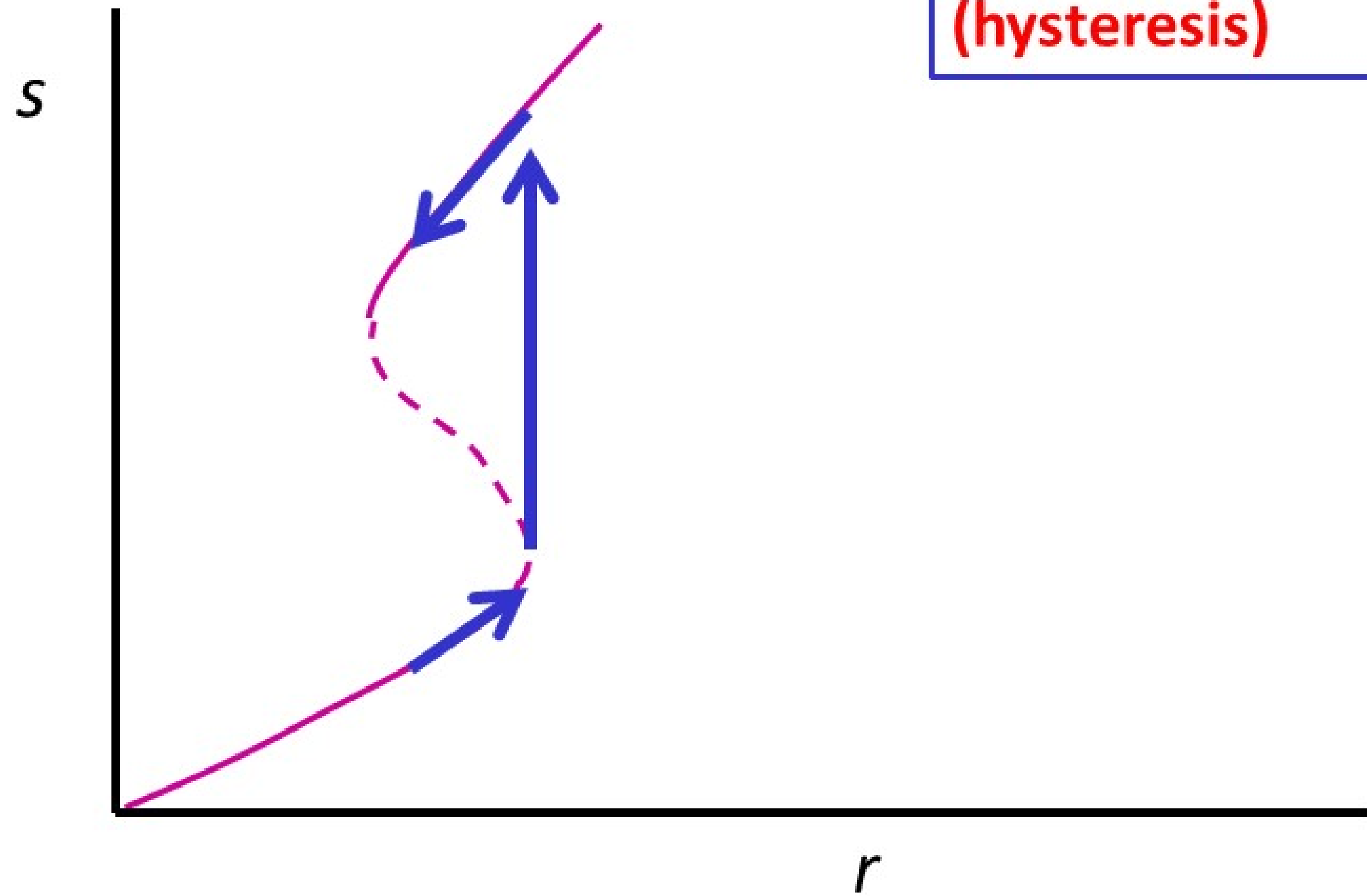

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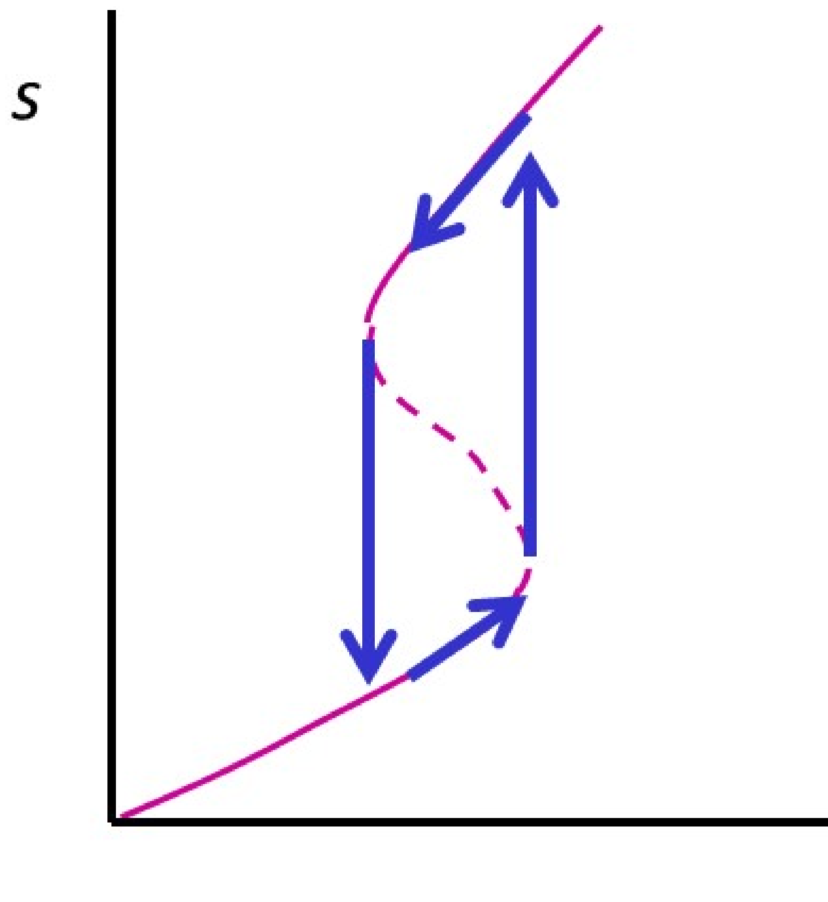

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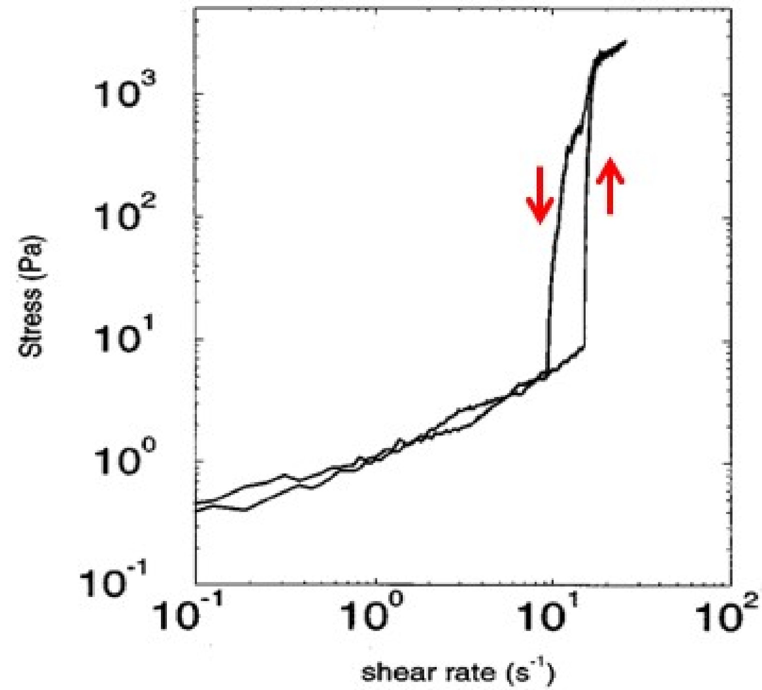

so what's the problem?

Shear-Thickening Suspensions

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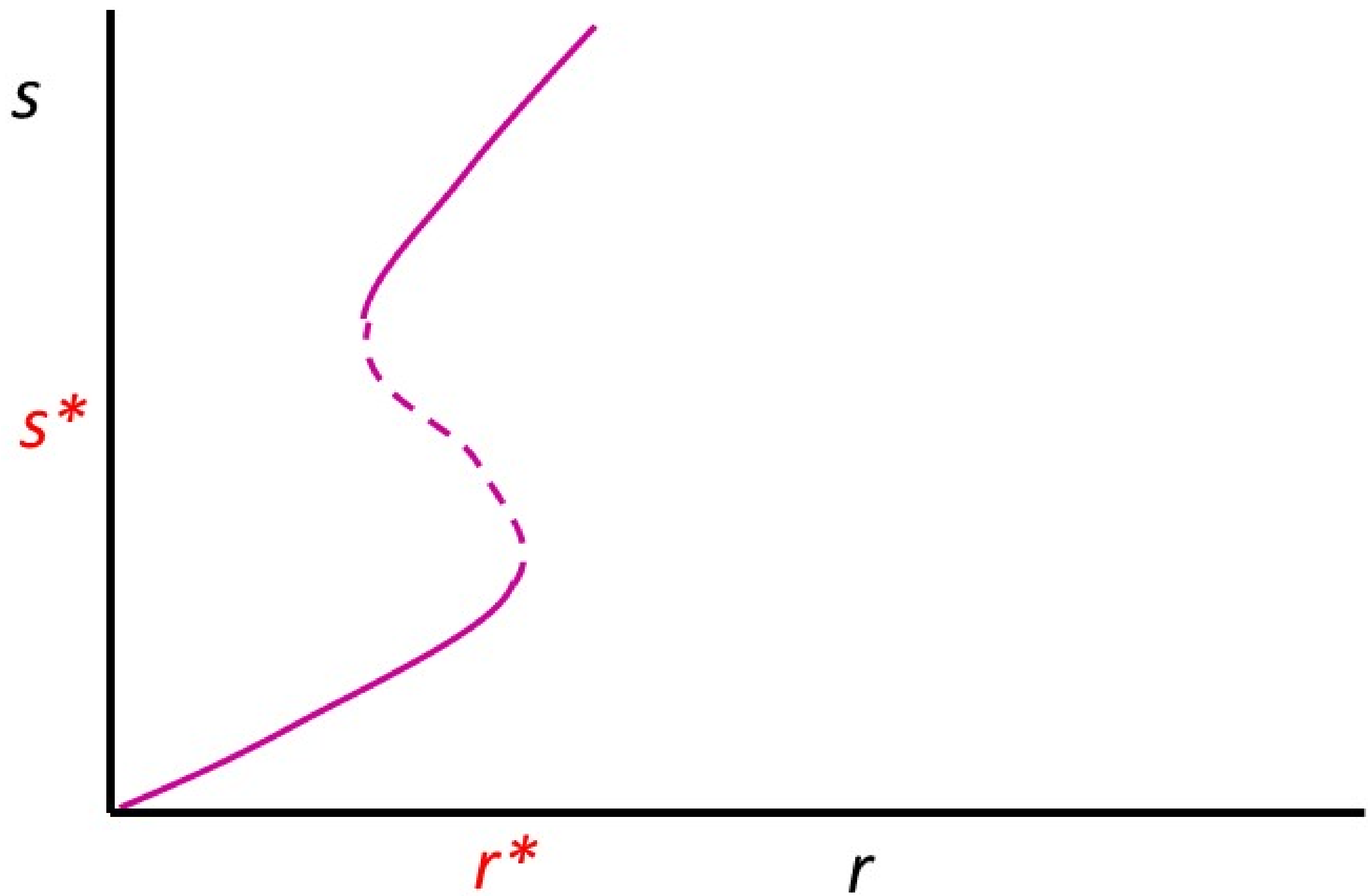

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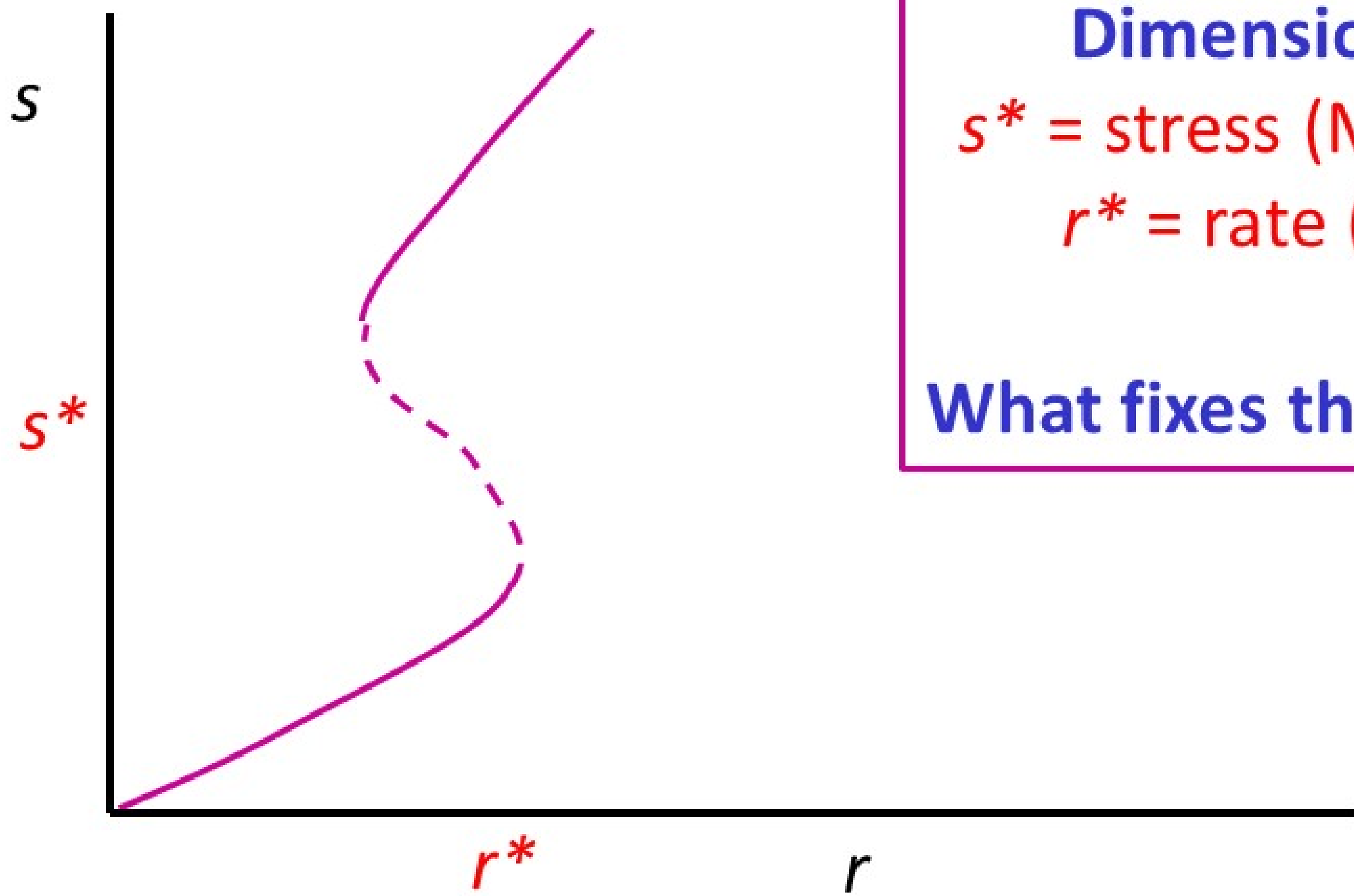


Dimensions

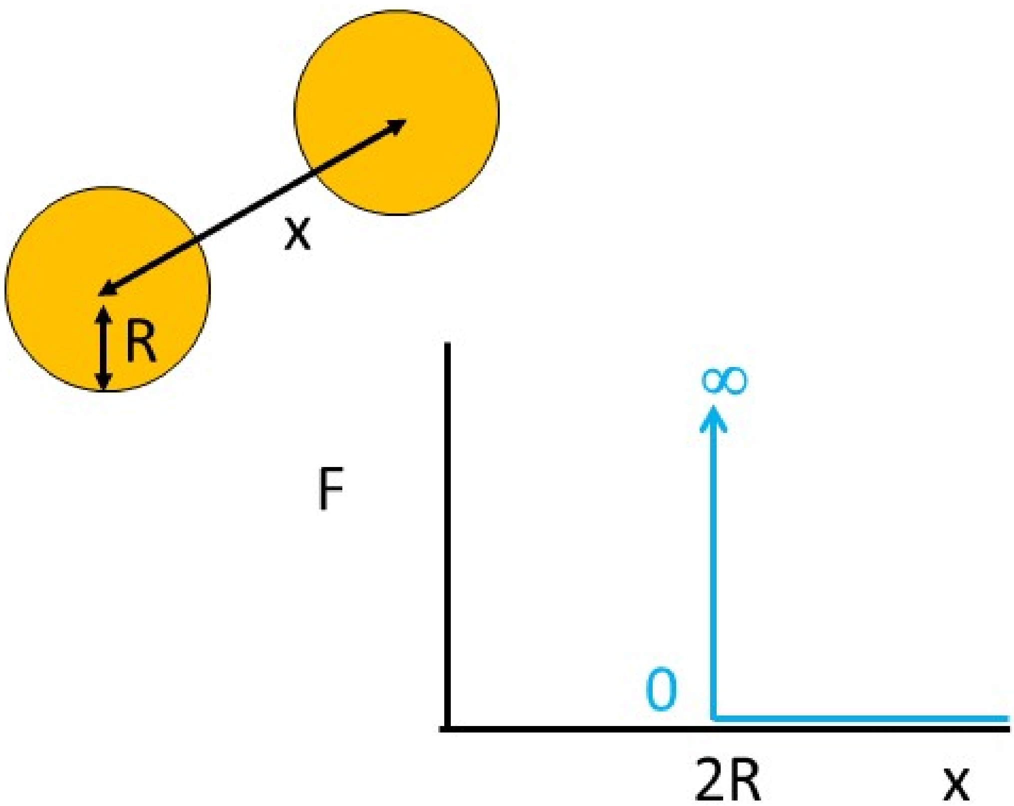
s^* = stress ($ML^{-1}T^{-2}$)

r^* = rate (T^{-1})

What fixes the scale?

Standard Model: Microscopic Ping-Pong Balls

- hard spheres in a viscous solvent ν_s :
- inertia not relevant (overdamped) :



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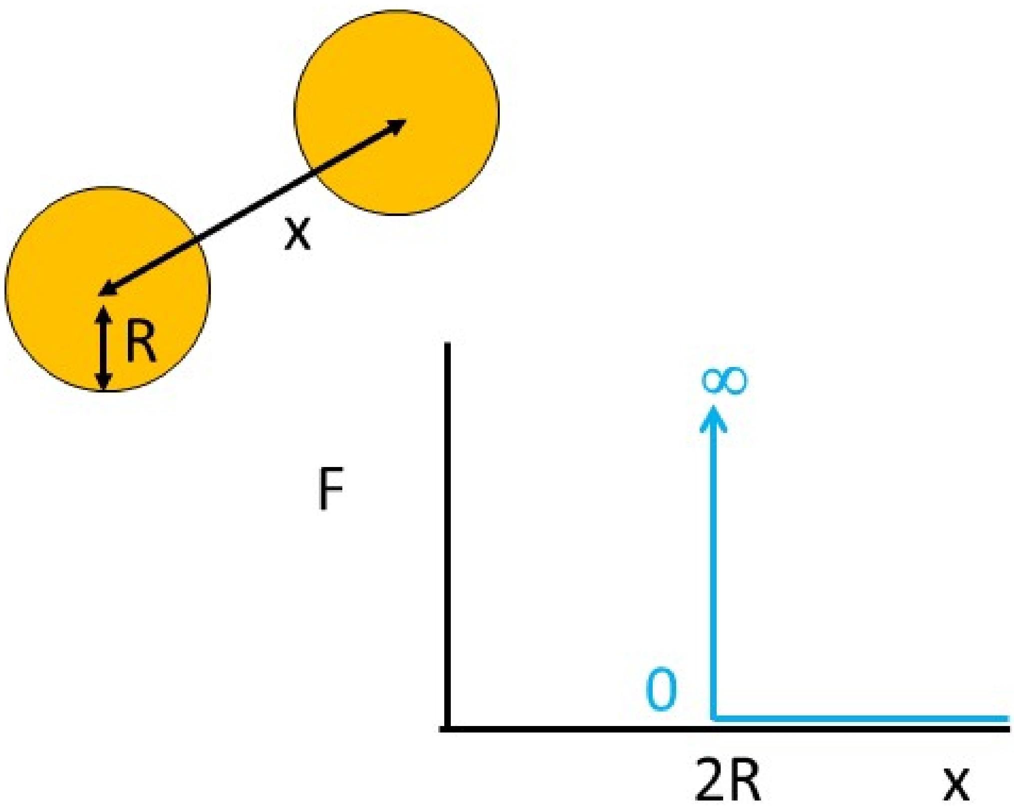

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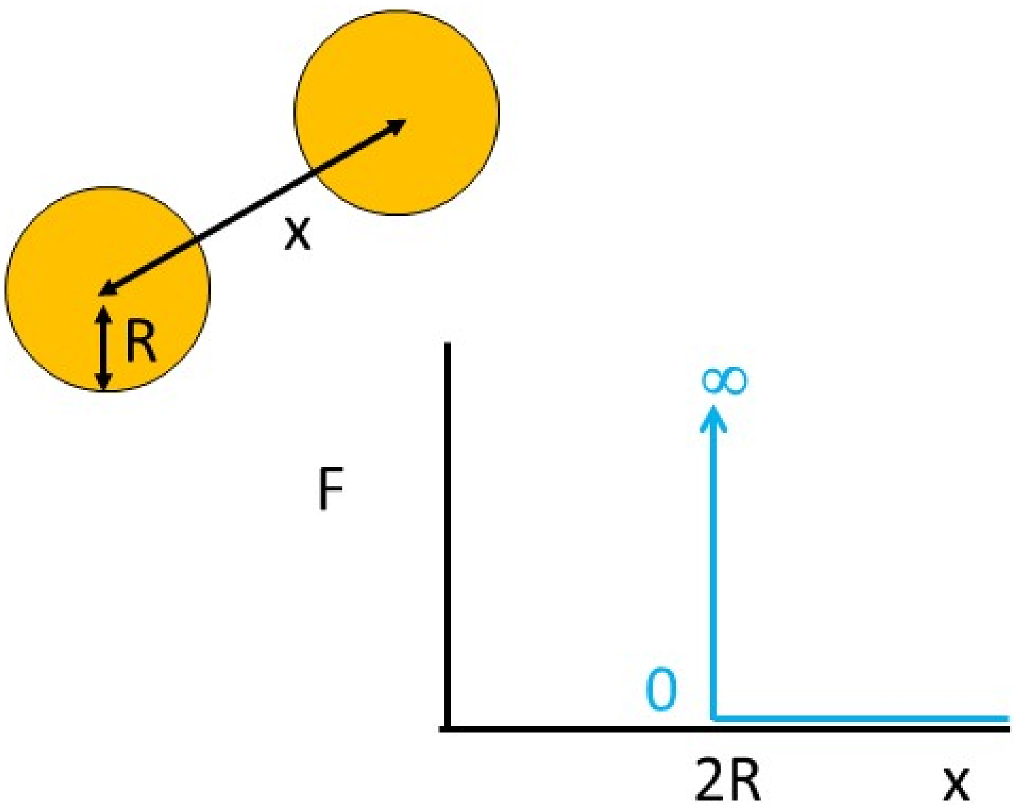

scale-free interaction:

R: L but no T
F: no L or T

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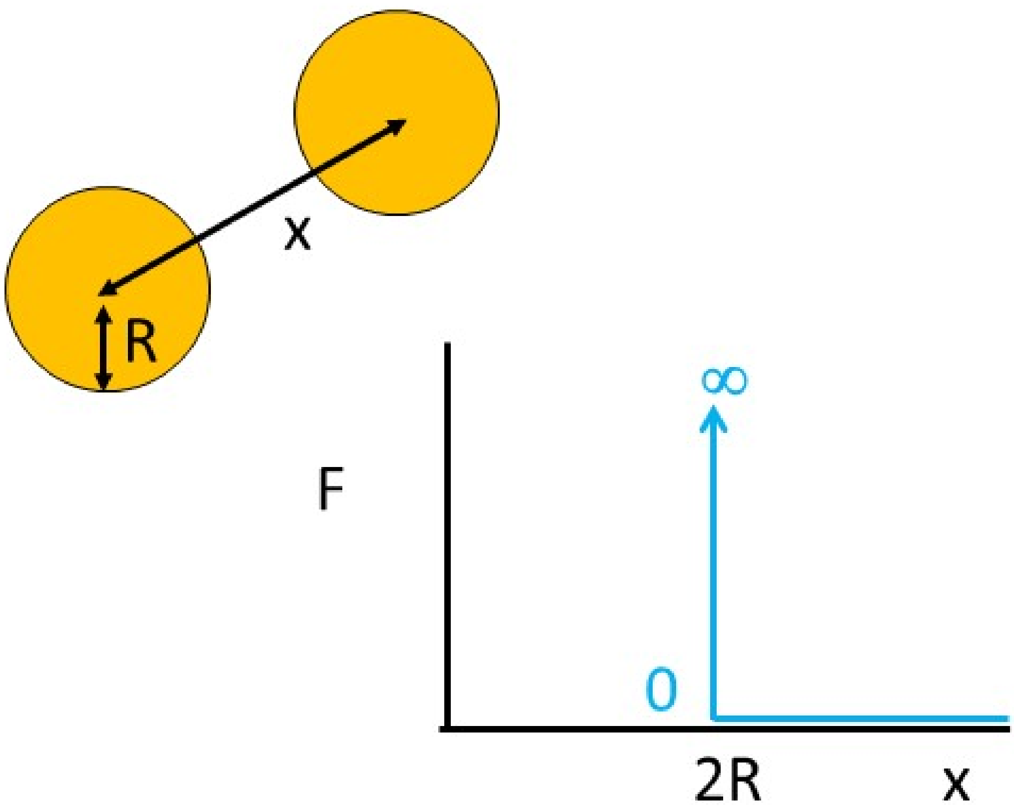


scale-free interaction:

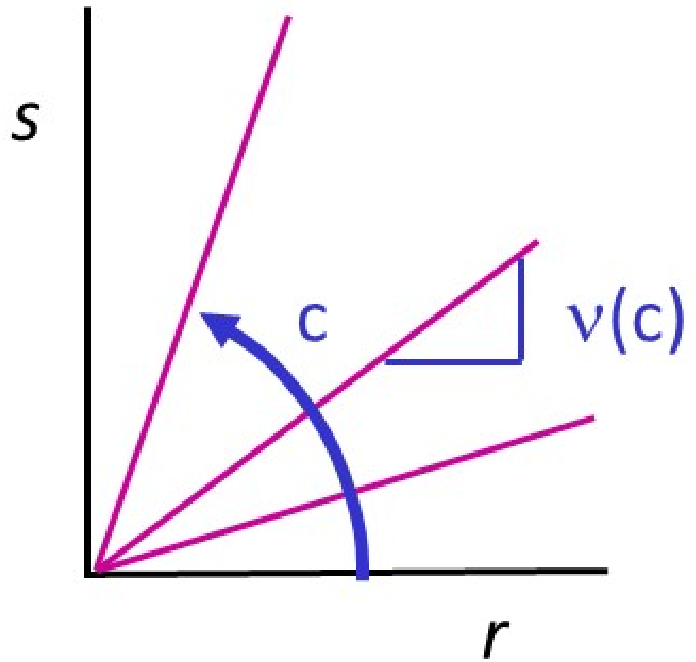
R: L but no T
F: no L or T

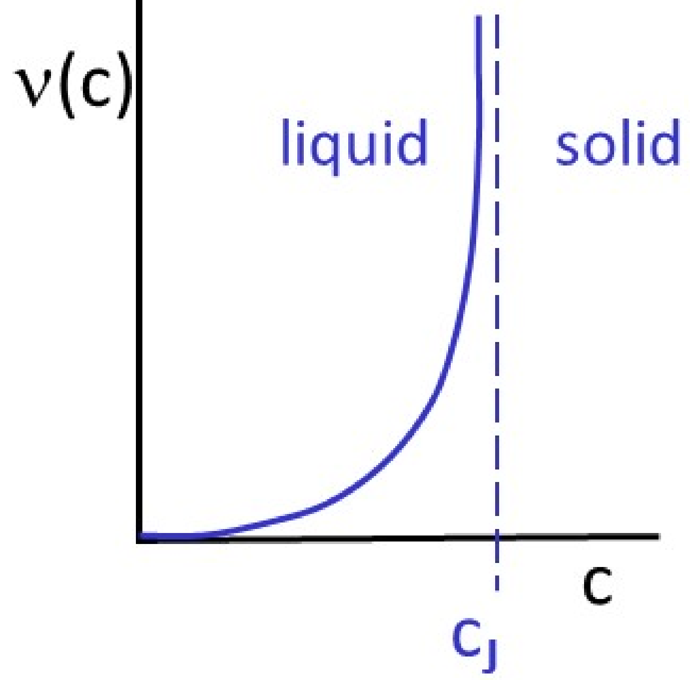
cannot make *s** or *r**
from R, F, ν_s

No-Wiggle Theorem!

Standard Model: Microscopic Ping-Pong Balls

either perfectly Newtonian, or no flow at all





- Key parameter: Jamming point c_J

Standard Model: Microscopic Ping-Pong Balls

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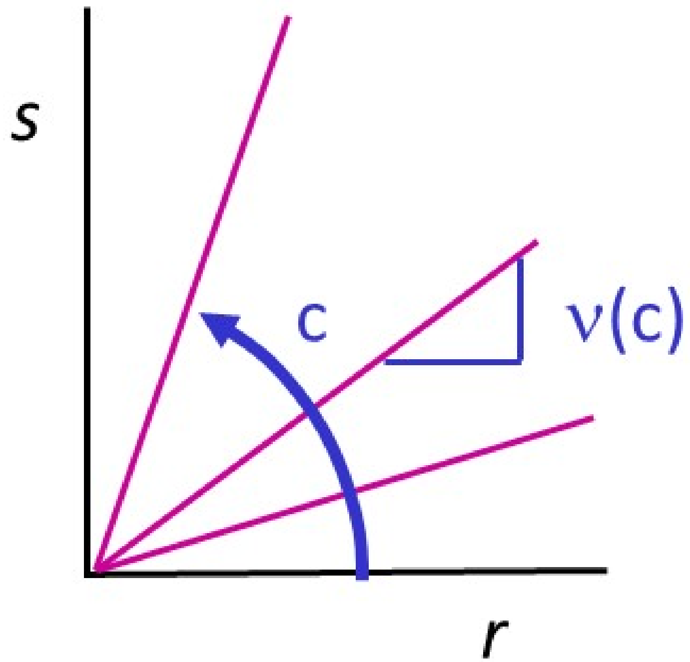
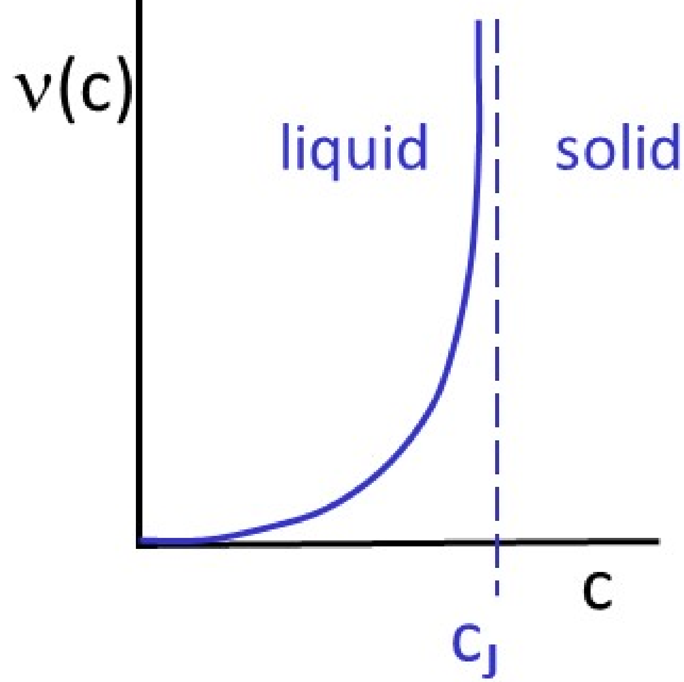
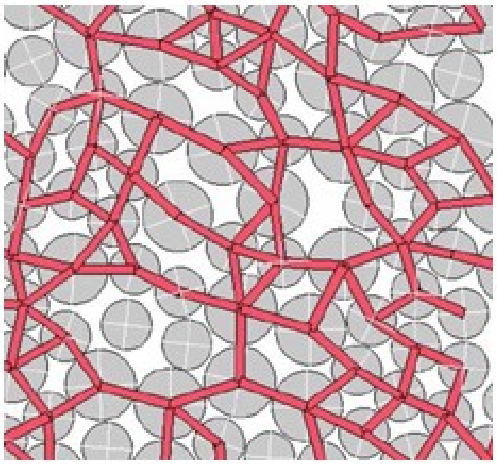




- Key parameter: Jamming point c_J
- Controlled by **contact friction** [realized only in 2012]

Why Friction Matters



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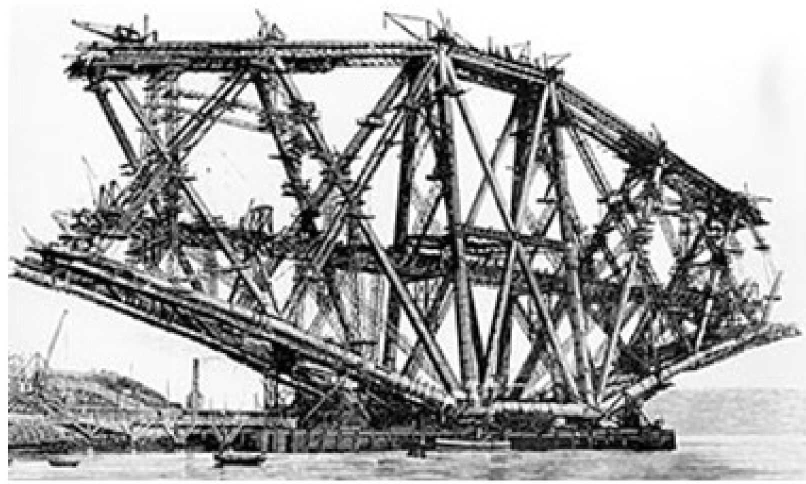
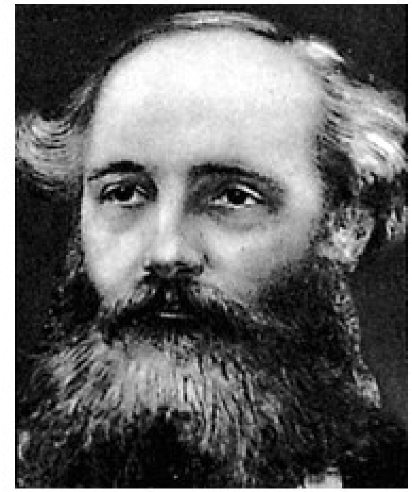


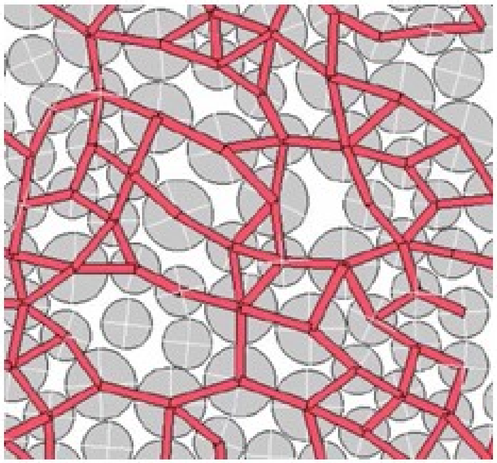


James Clerk Maxwell 1831-79

dense suspension

Forth Rail Bridge

Why Friction Matters



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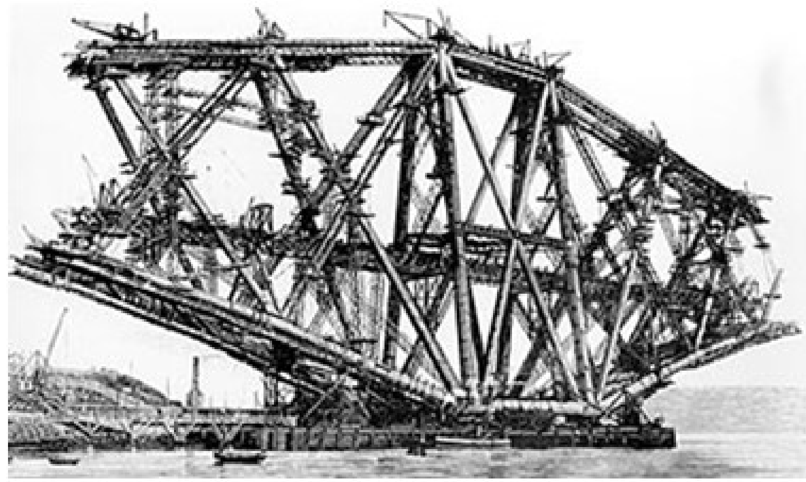
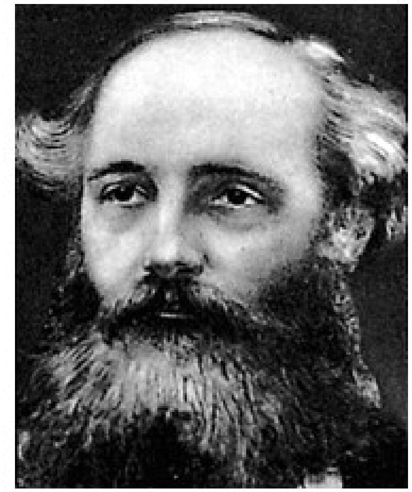




James Clerk Maxwell 1831-79

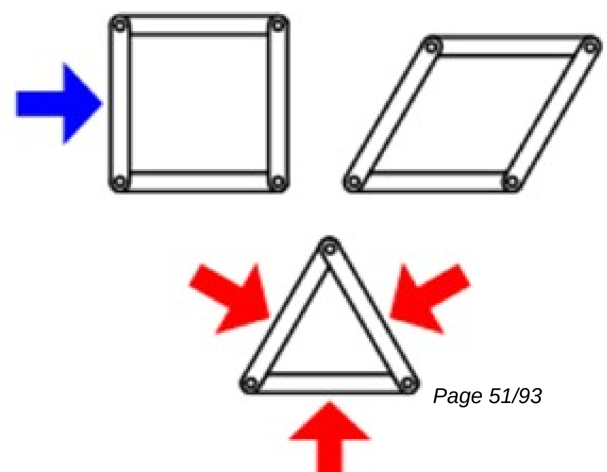
dense suspension

Forth Rail Bridge

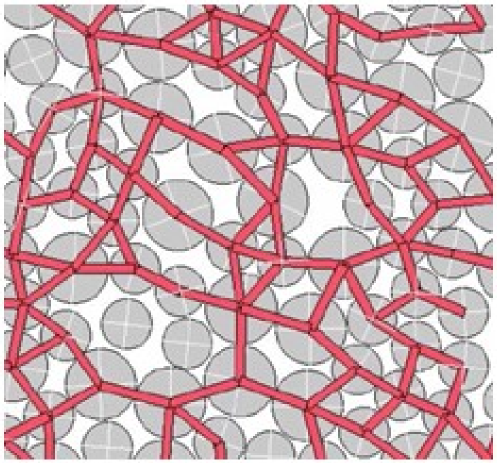
How many connections needed for rigidity?

fixed joints: constrain angles and lengths

hinged joints: constrain lengths only



Why Friction Matters



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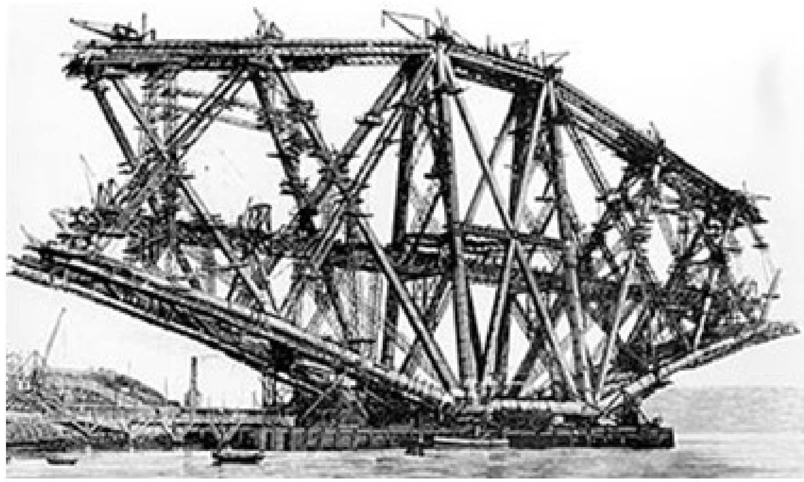
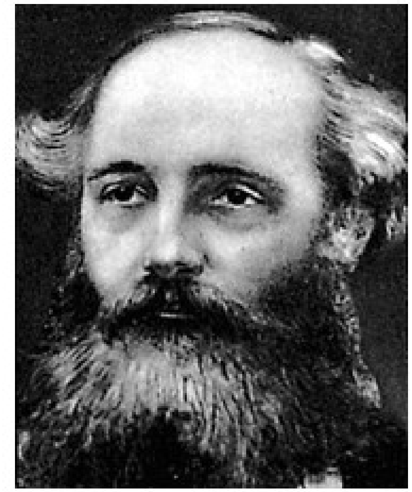


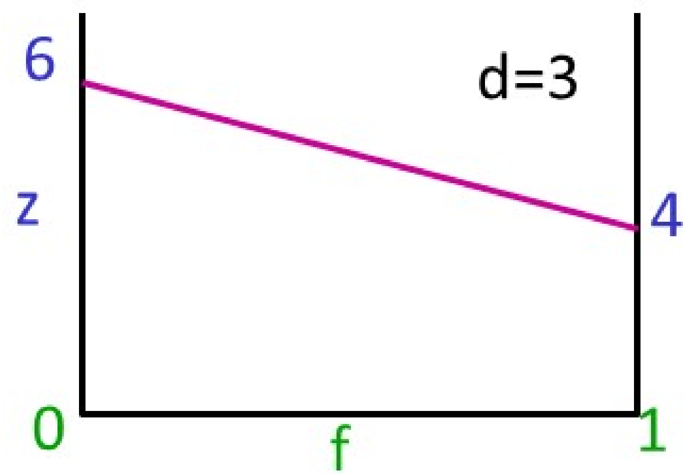


z(c) = contacts per particle

fraction f rolling, (1-f) sliding

rigid when z > 4f + 6(1-f)

or 3f+4(1-f) in 2 dimensions



Counting Equations

Static equilibrium of N spheres (d=3)

all rolling   all sliding

Net forces:

Net torques:

Equations (E):

Contacts:

Variables (V):

Rigid for V > E:

Counting Equations

Static equilibrium of N spheres (d=3)

	all rolling	all sliding
Net forces:	3N	3N
Net torques:	3N	0
Equations (E):	6N	3N
Contacts:		
Variables (V):		
Rigid for V > E:		

Counting Equations

Static equilibrium of N spheres (d=3)

	all rolling	all sliding
Net forces:	3N	3N
Net torques:	3N	0
Equations (E):	6N	3N
Contacts:	zN/2	zN/2
Variables (V):	3zN/2	zN/2
Rigid for V > E:		

Counting Equations

Static equilibrium of N spheres (d=3)

	all rolling	all sliding
Net forces:	3N	3N
Net torques:	3N	0
Equations (E):	6N	3N
Contacts:	zN/2	zN/2
Variables (V):	3zN/2	zN/2
Rigid for V > E:	z > 4	z > 6

rigidity: many static solutions

Weighted sum: $z > 4f + 6(1\text{-}f)$

Counting Equations

Static equilibrium of N discs (d=2) or spheres (d=3)

	all rolling	all sliding
Net forces:	Nd	Nd
Net torques:	Nd(d-1)/2	0
Equations (E):	Nd(d+1)/2	Nd
Contacts:	zN/2	zN/2
Variables (V):	zNd/2	zN/2
Rigid for V > E:	z > d+1	z > 2d

rigidity:
many static
solutions

Weighted sum: $z > (d+1)f \quad + \quad 2d(1-f)$

Final Outcome (d=3)

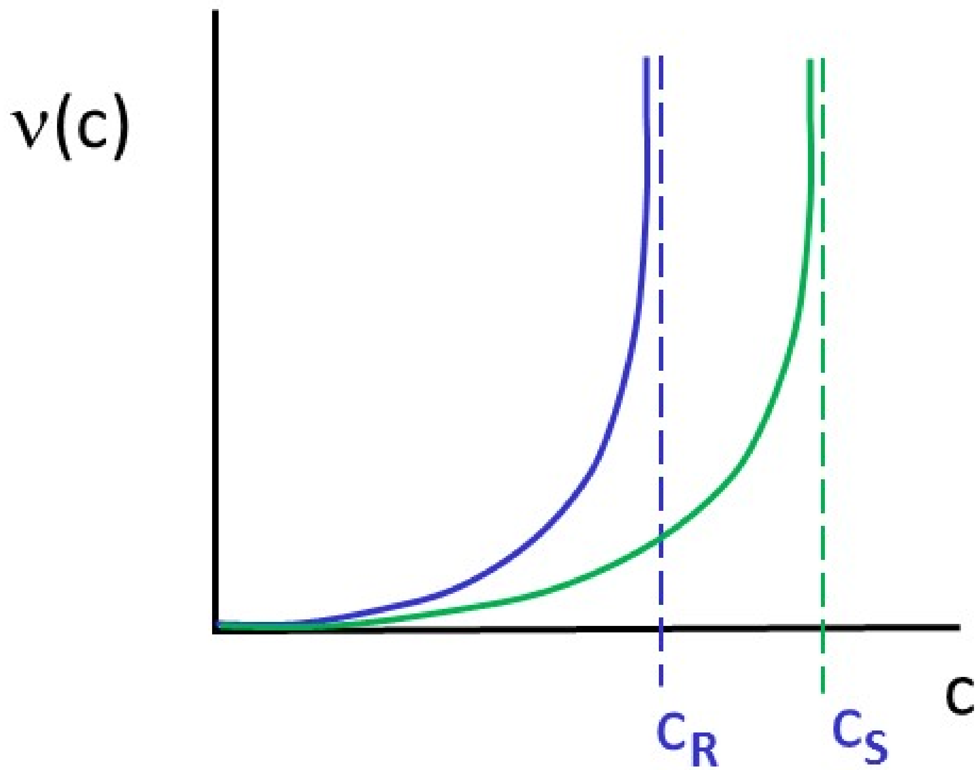
max friction (all rolling):

$c_J = c_R = 0.58$

zero friction (all sliding):

$c_J = c_S = 0.64$

$z(0.58) = 4$	$z(0.64) = 6$



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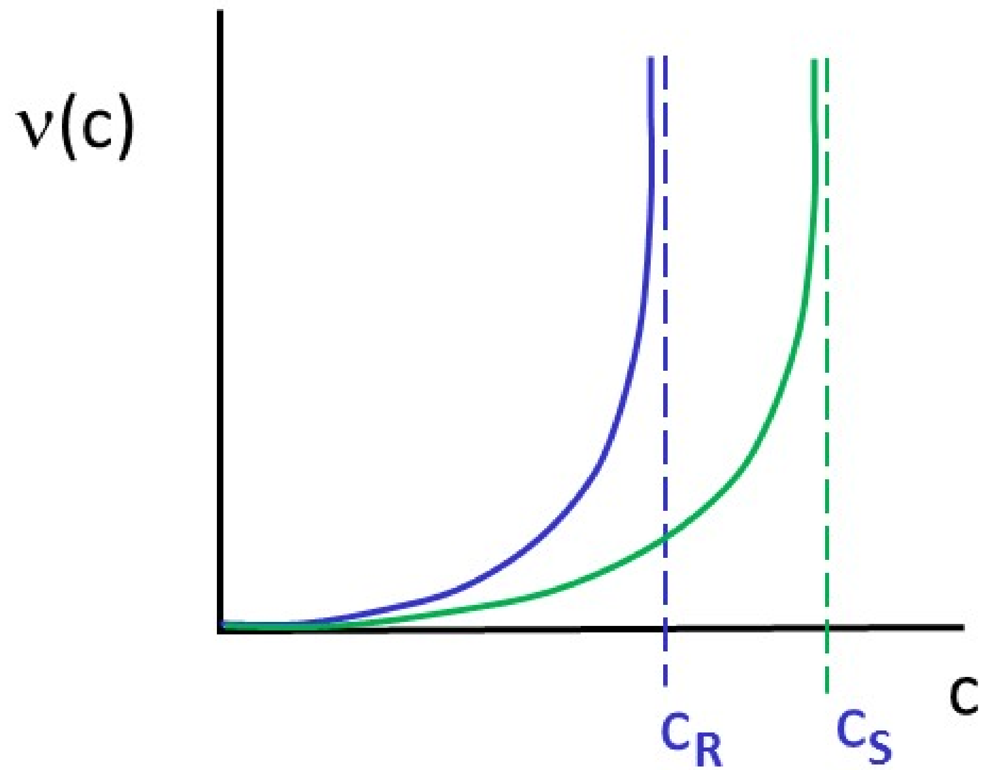
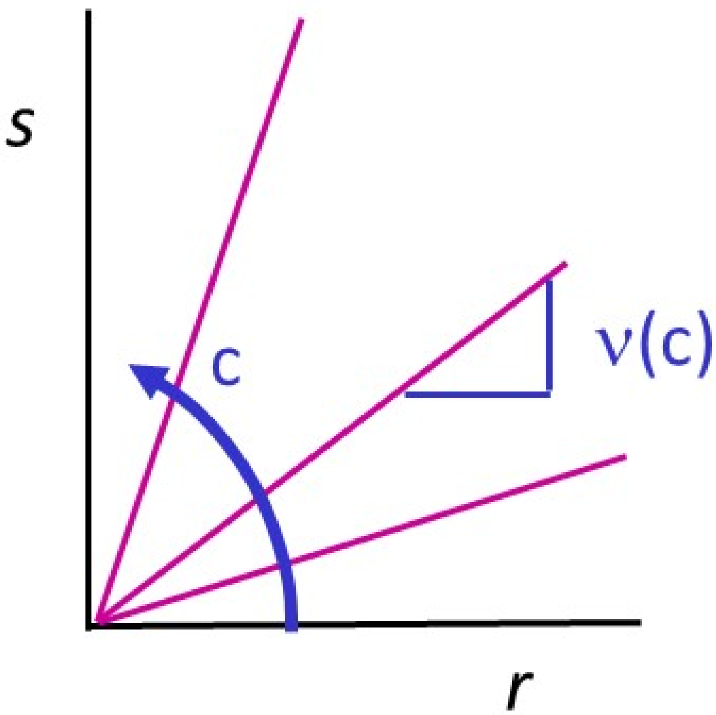


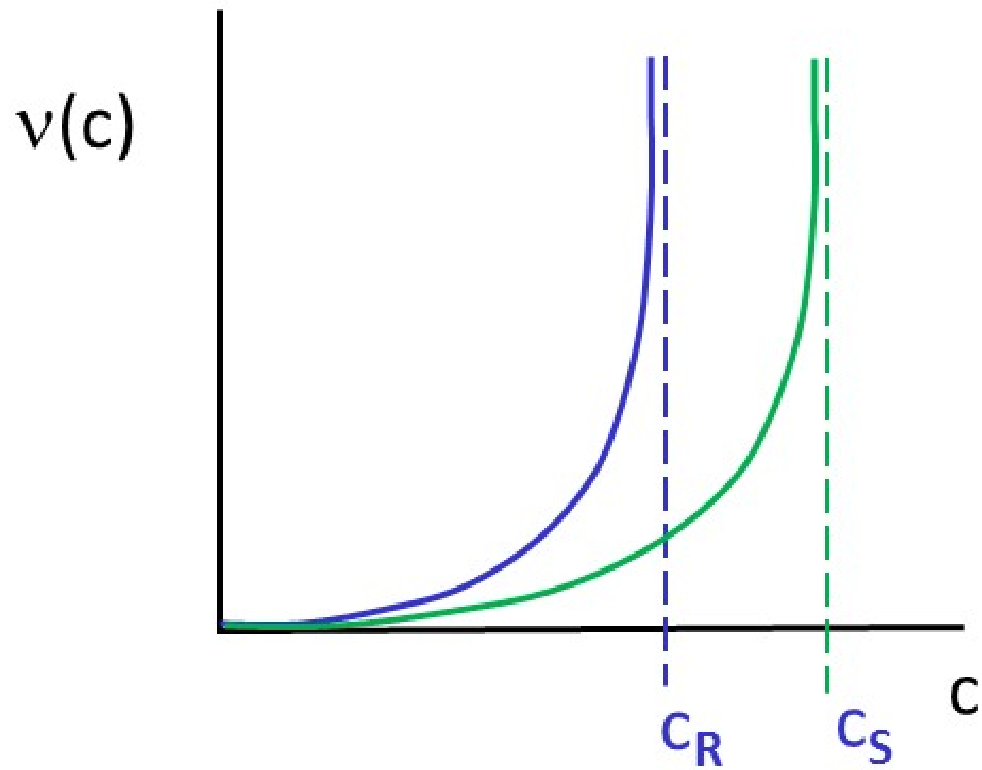
$z(0.58) = 4 \qquad z(0.64) = 6$

Friction coefficient (max tangential/normal force):

Dimensionless: Still no wiggles!

Summary: Standard Model

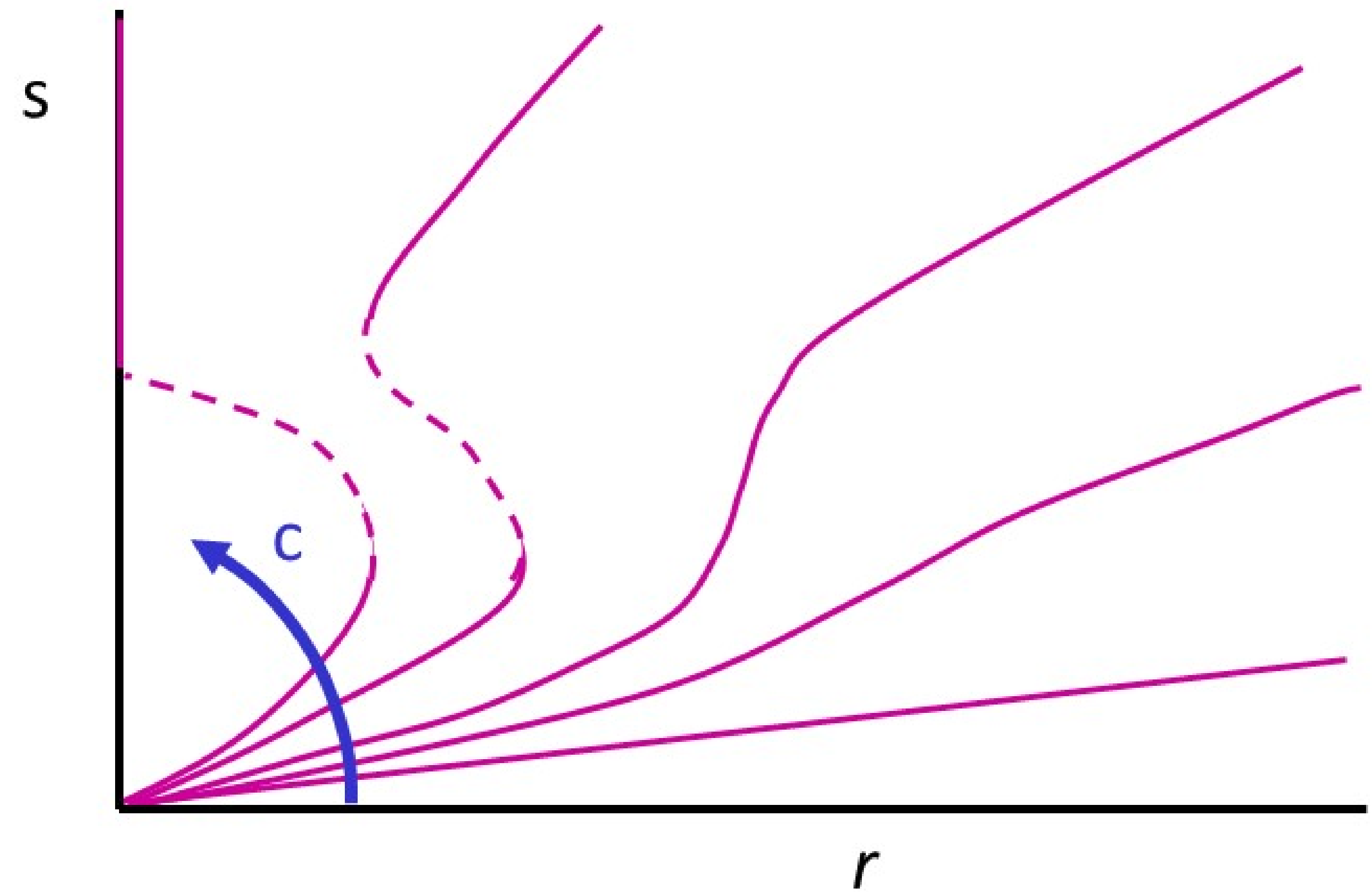




- Either Newtonian fluid, or jammed solid
- Friction-dependent jamming point c_J

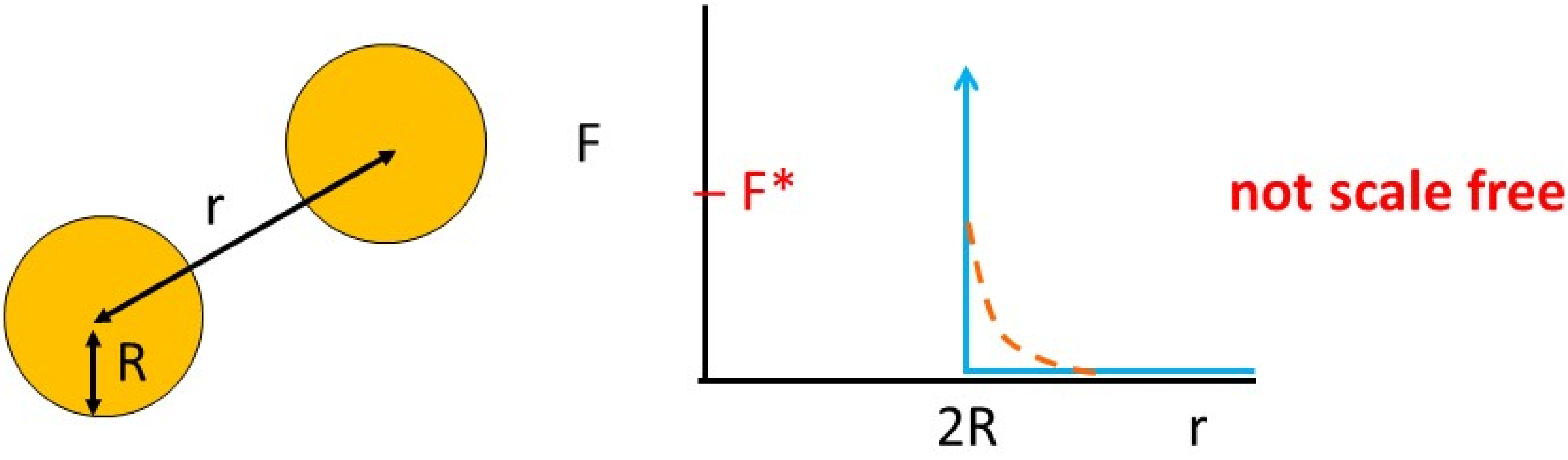
Shear-Thickening Suspensions

We still want this instead:



Stress-Dependent Friction

Hard spheres + additional short range repulsions



Stress-Dependent Friction

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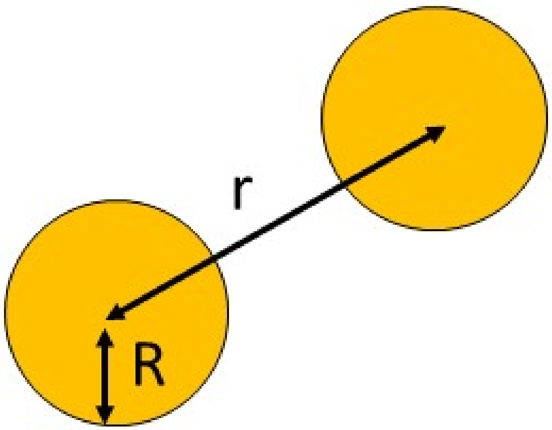
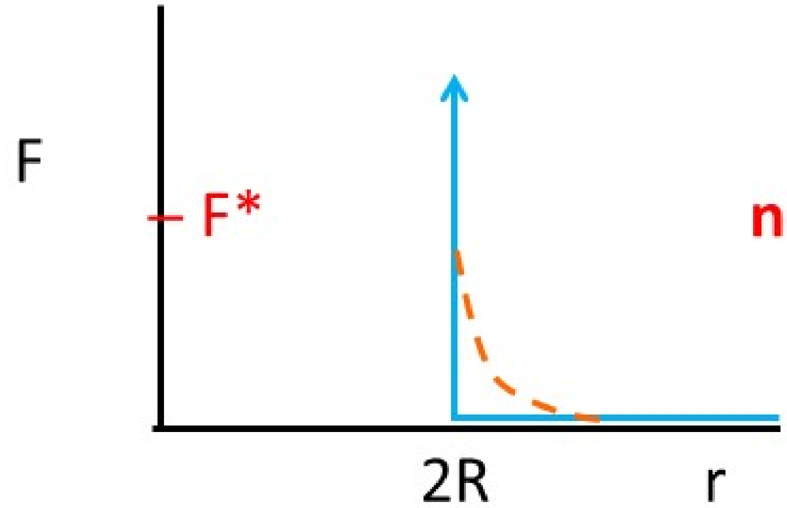




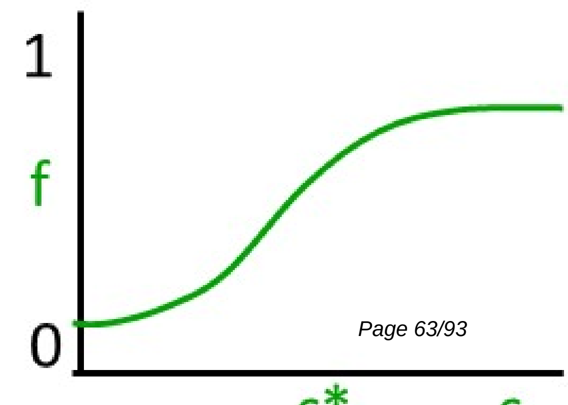
not scale free

Stress scale: $s^* \approx F^*/R^2$

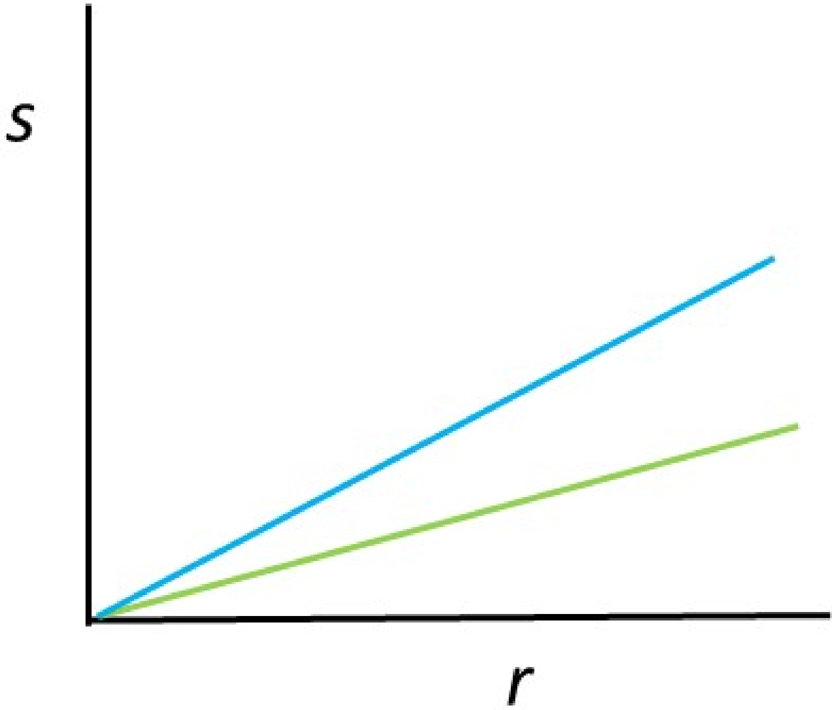
$s << s^*$: mostly frictionless contacts

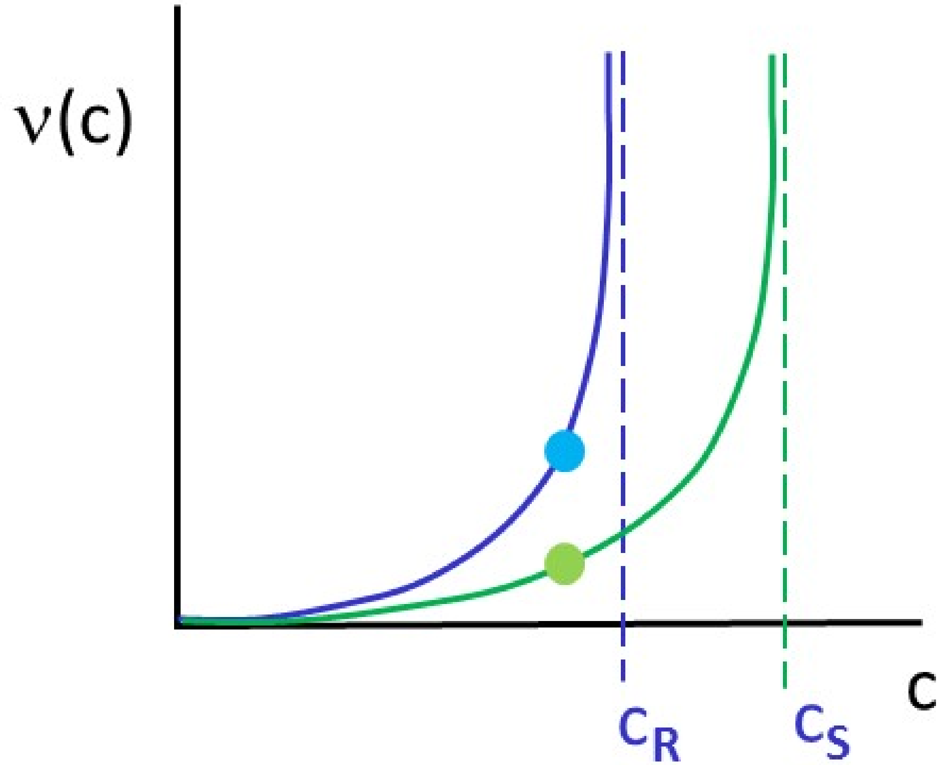
$s >> s^*$: mostly frictional contacts

fraction of rolling contacts $f(s)$

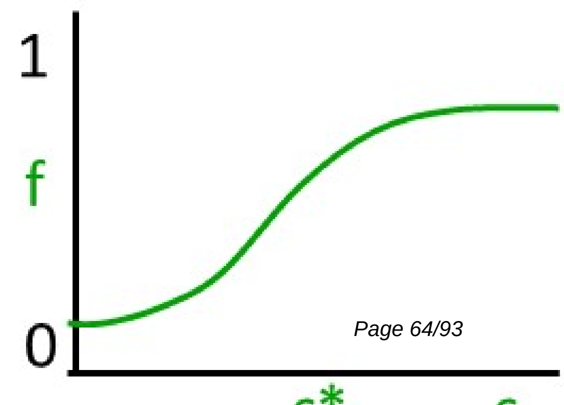


Stress-Dependent Friction

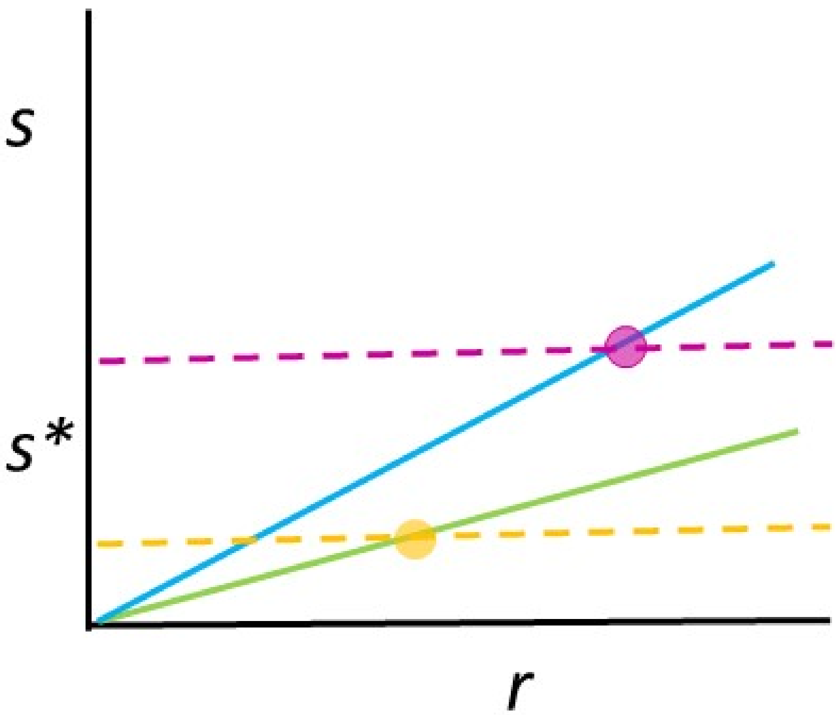


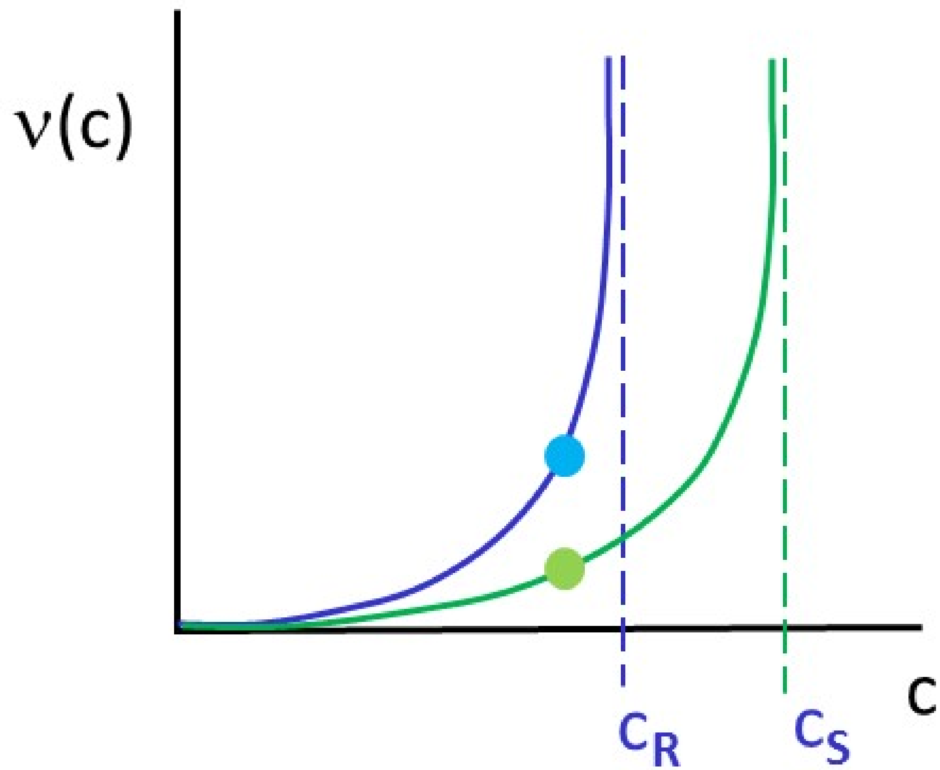


fraction of rolling contacts f(*s*)

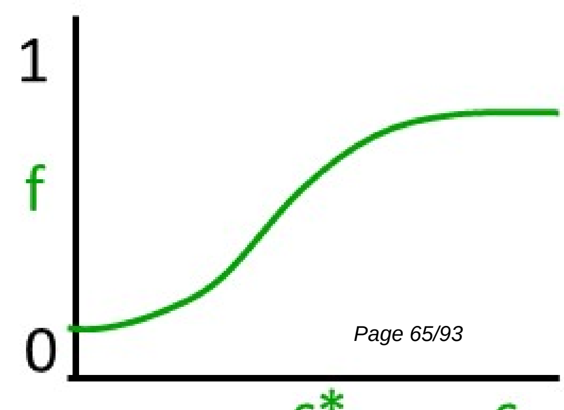


Stress-Dependent Friction

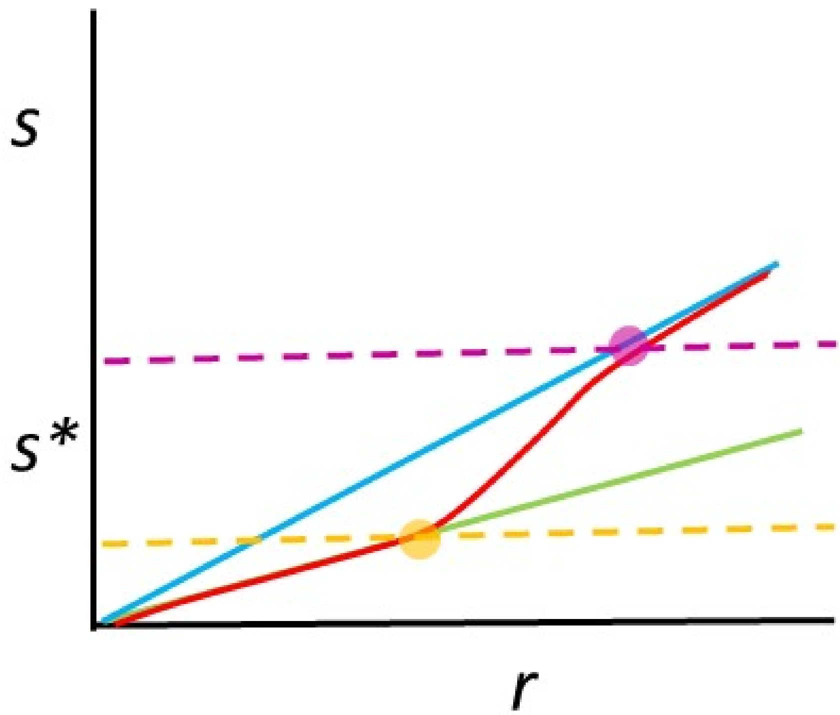


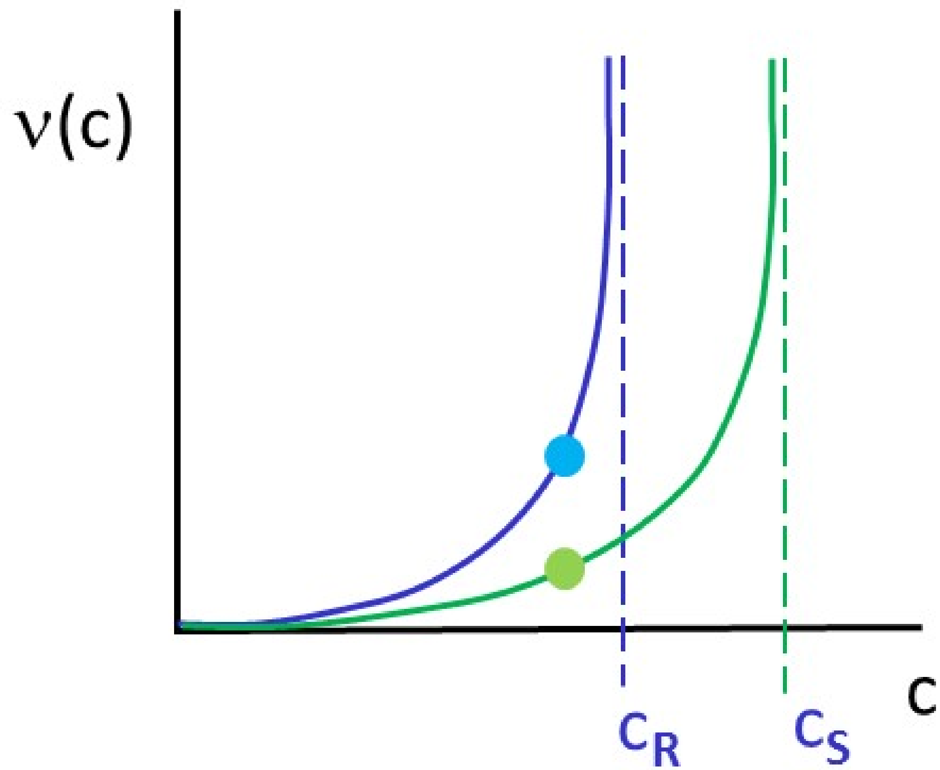


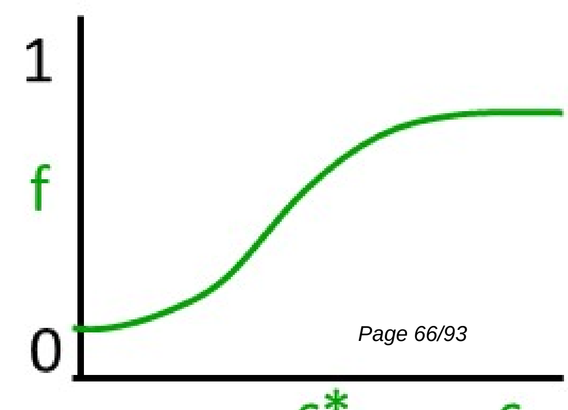
fraction of rolling contacts f(s)



Stress-Dependent Friction

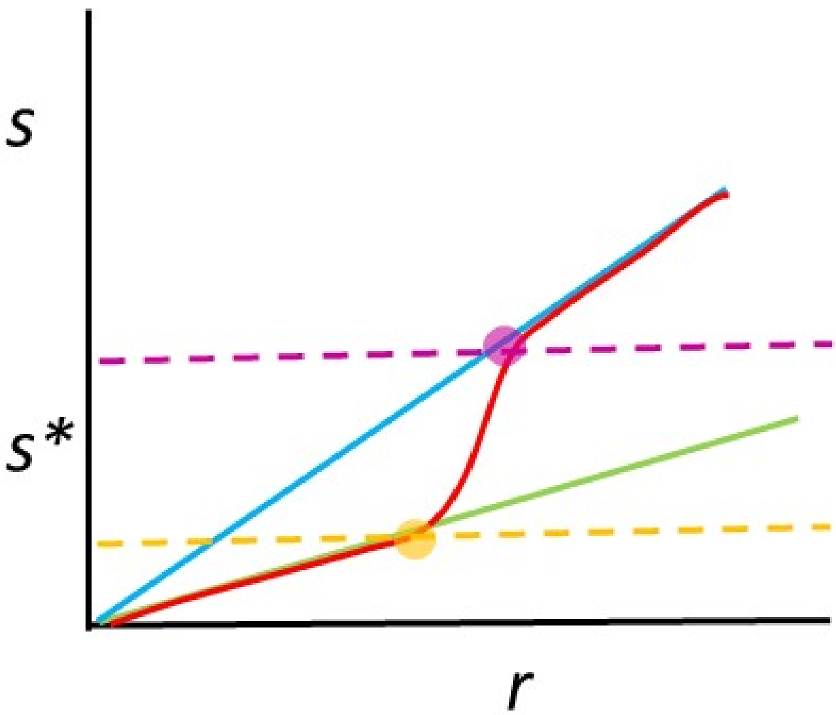


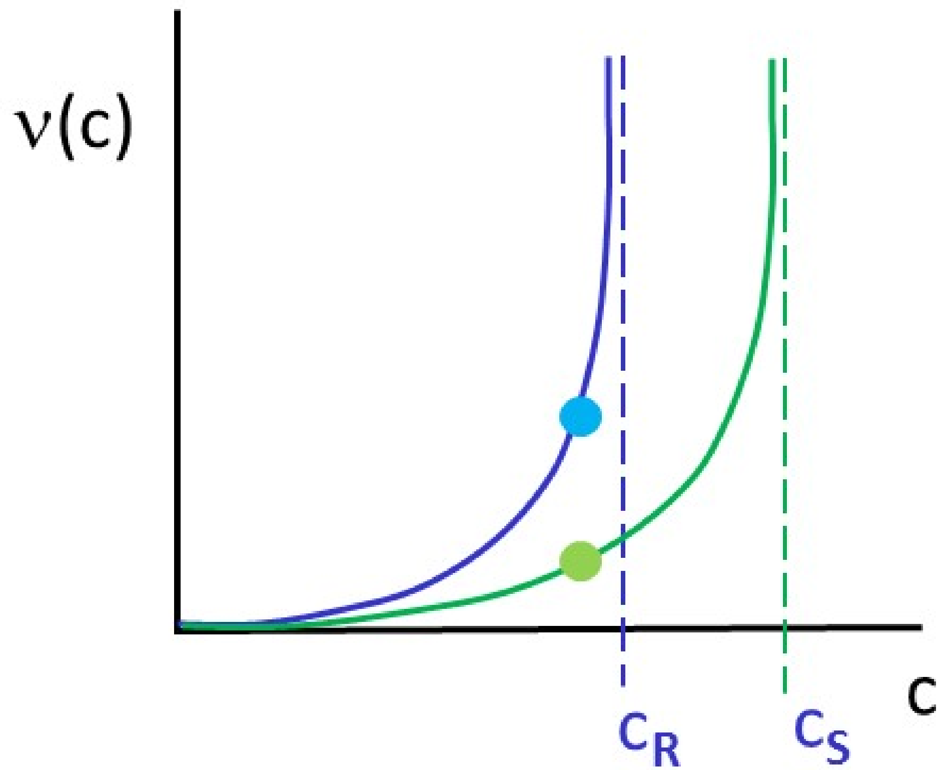




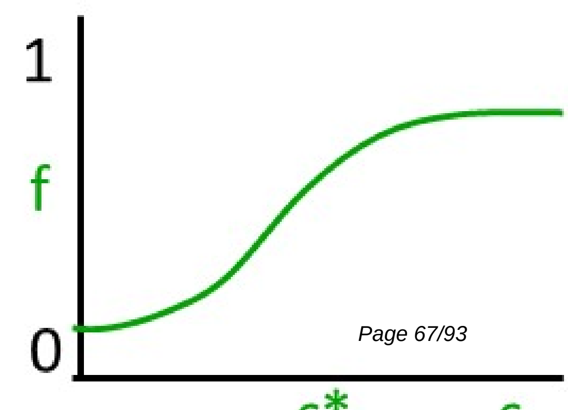
fraction of rolling contacts f(*s*)

Stress-Dependent Friction

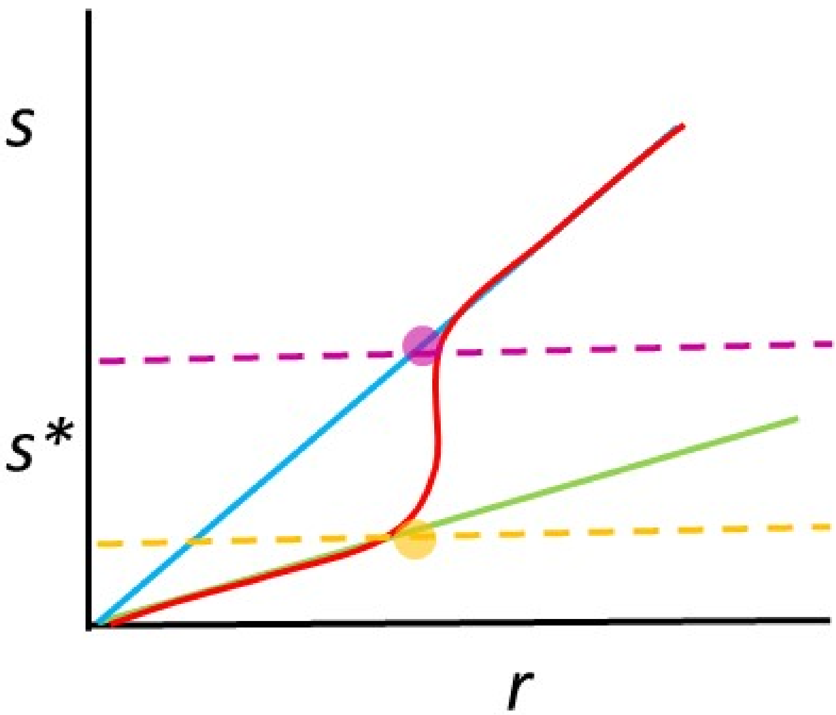


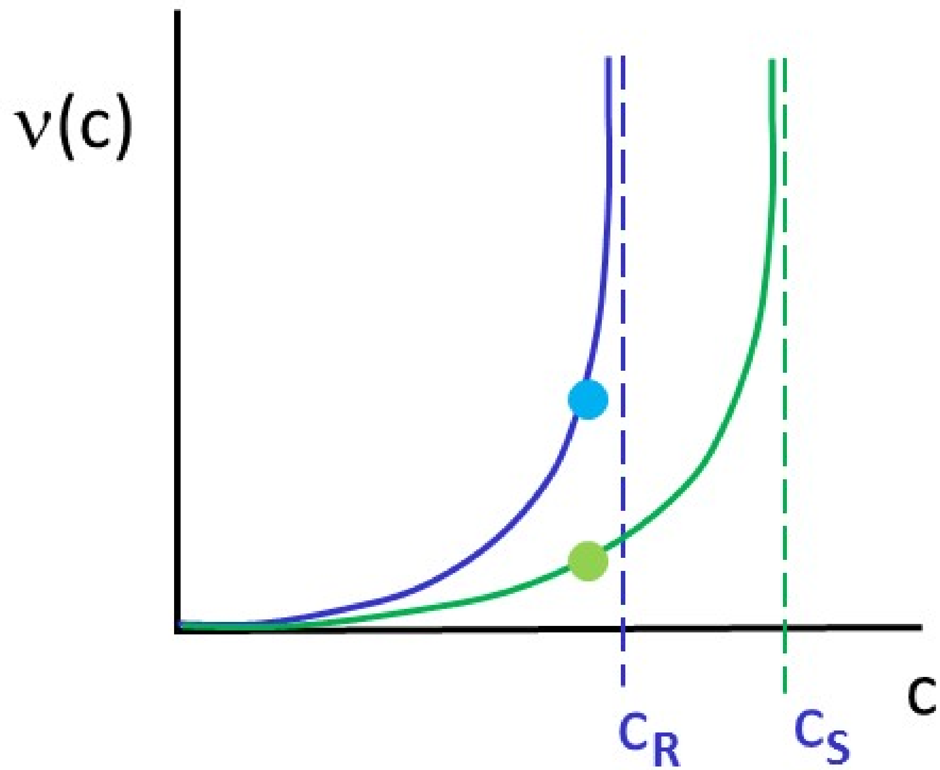


fraction of rolling contacts f(*s*)

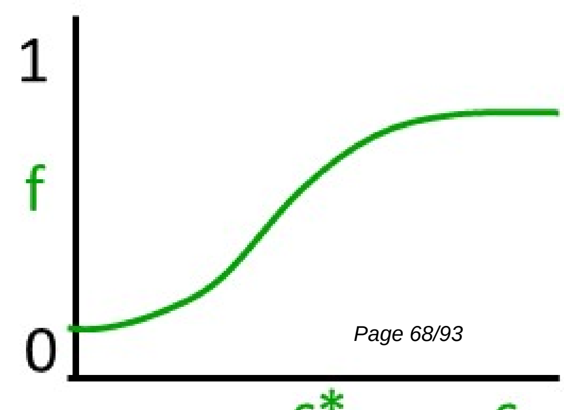


Stress-Dependent Friction

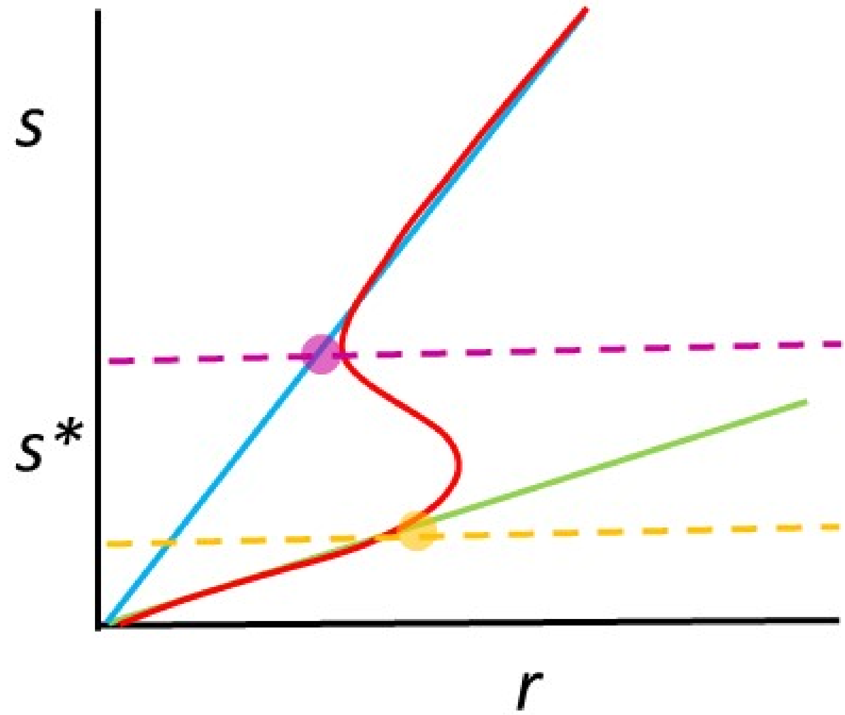


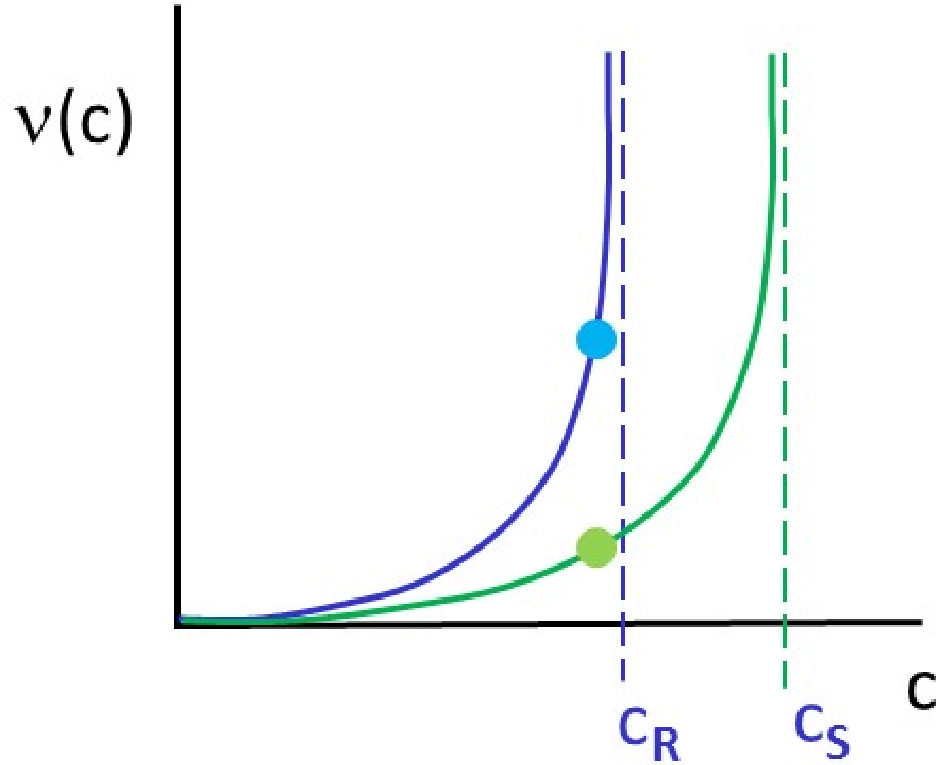


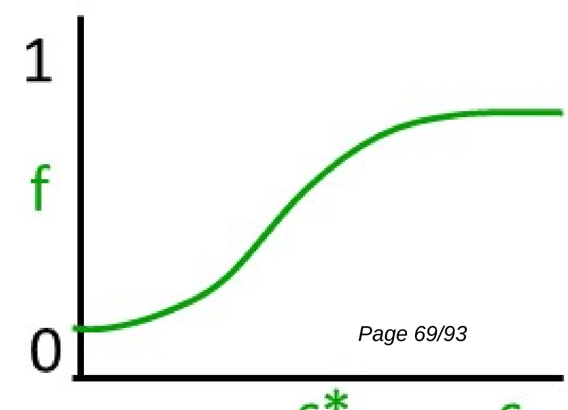
fraction of rolling contacts f(*s*)



Stress-Dependent Friction

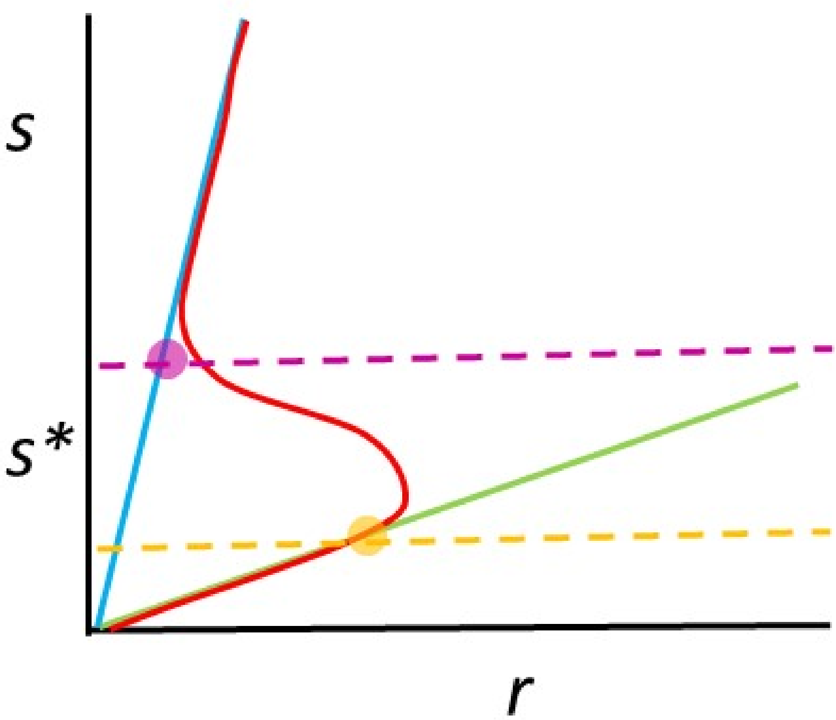


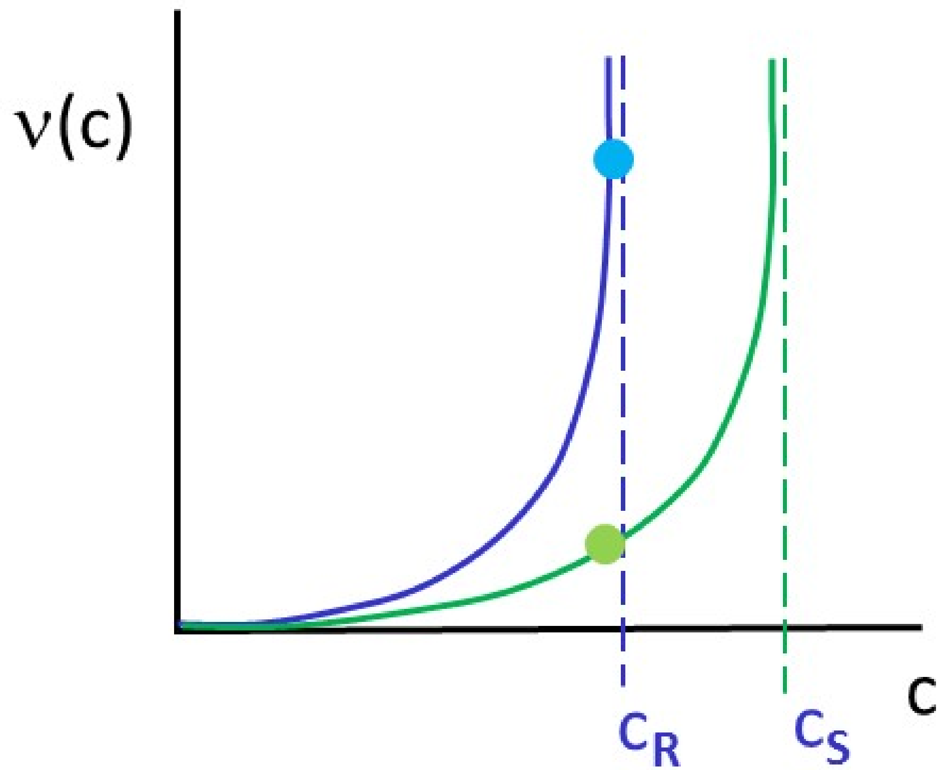




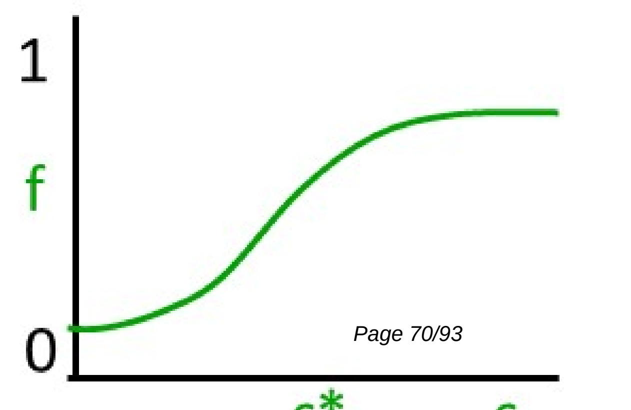
fraction of rolling contacts f(*s*)

Stress-Dependent Friction

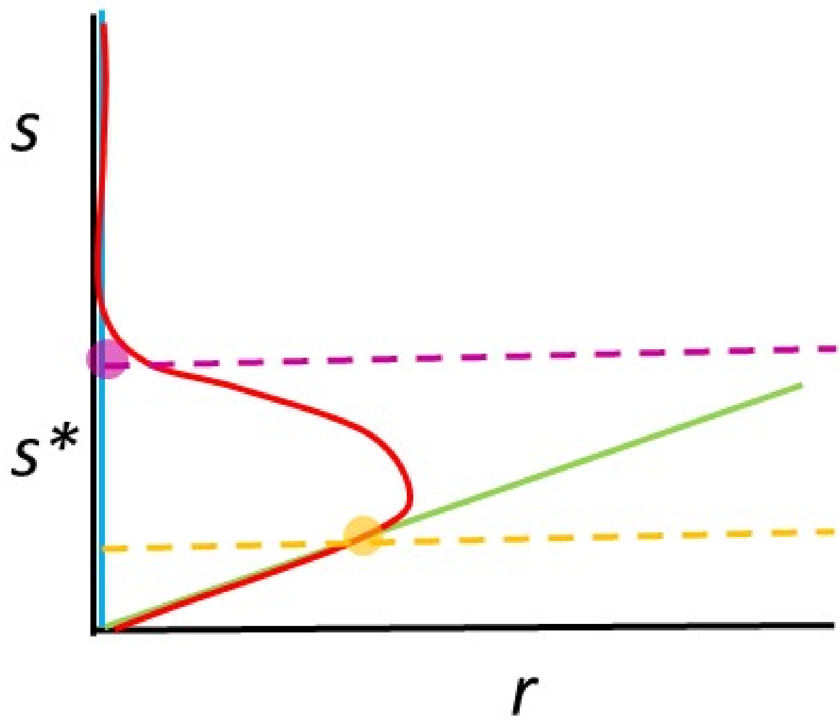


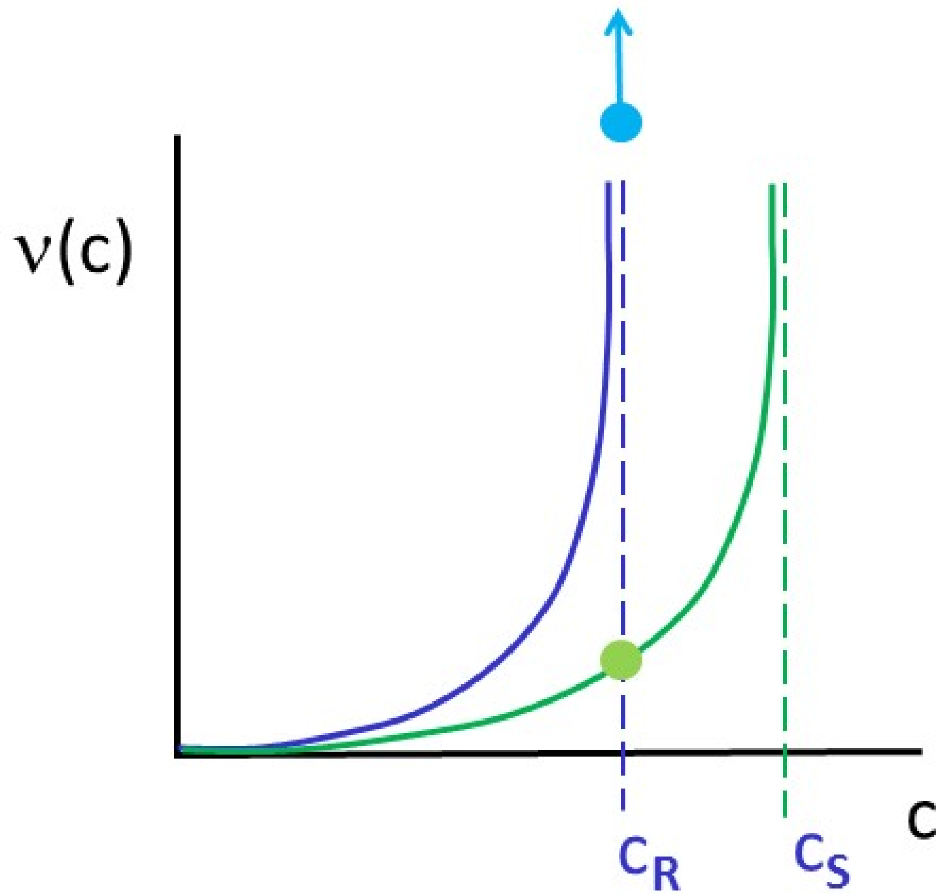


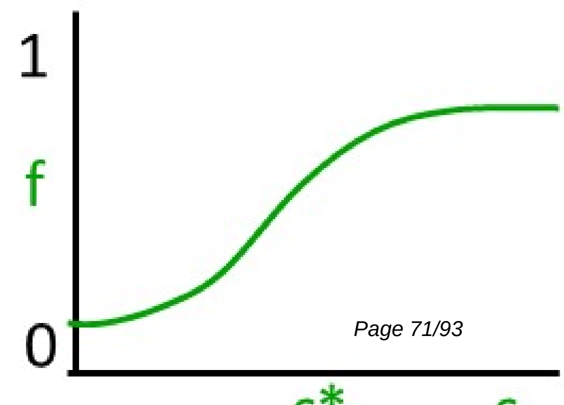
fraction of rolling contacts f(*s*)



Stress-Dependent Friction



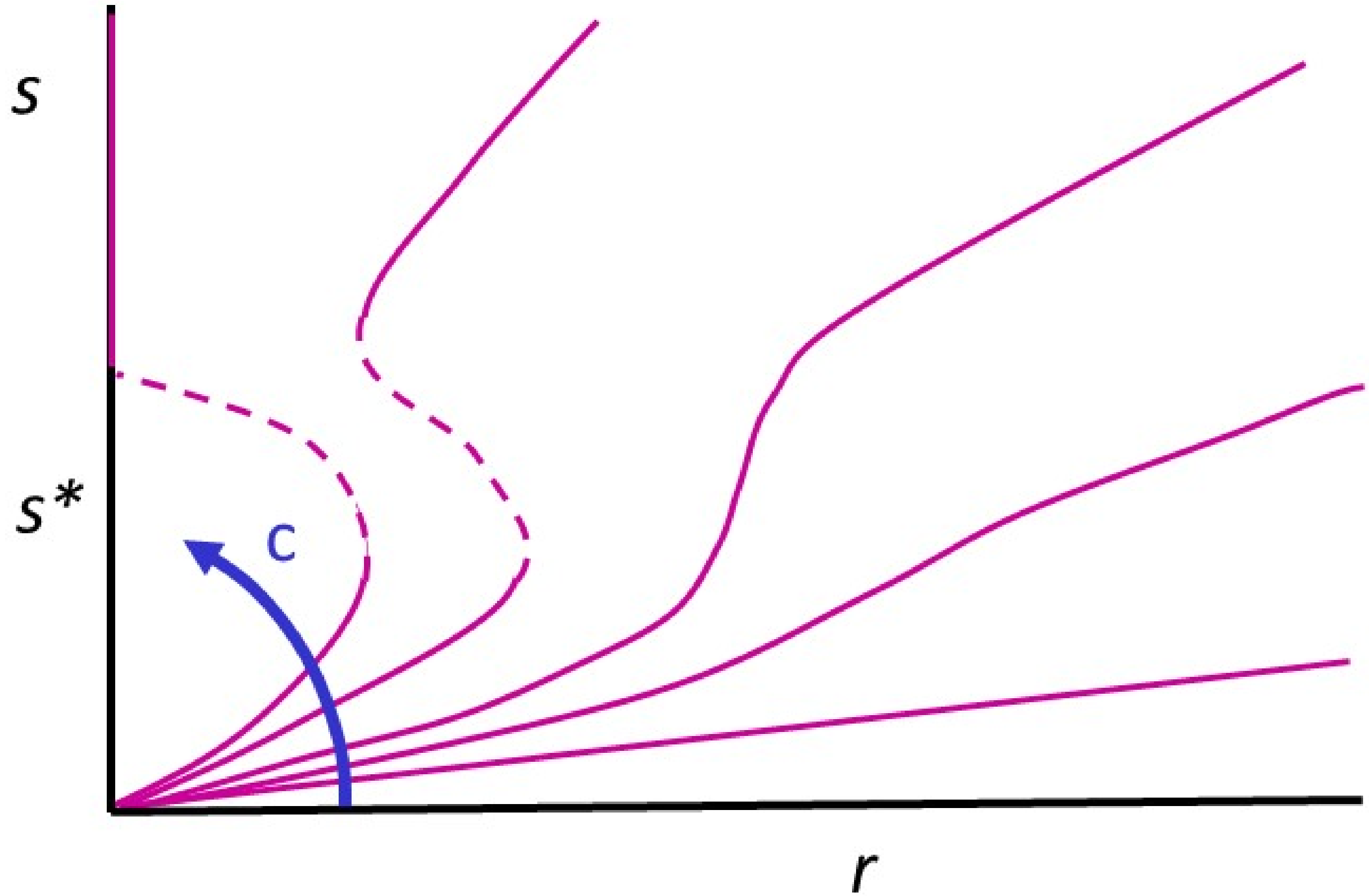




fraction of rolling contacts f(*s*)

Stress-Dependent Friction

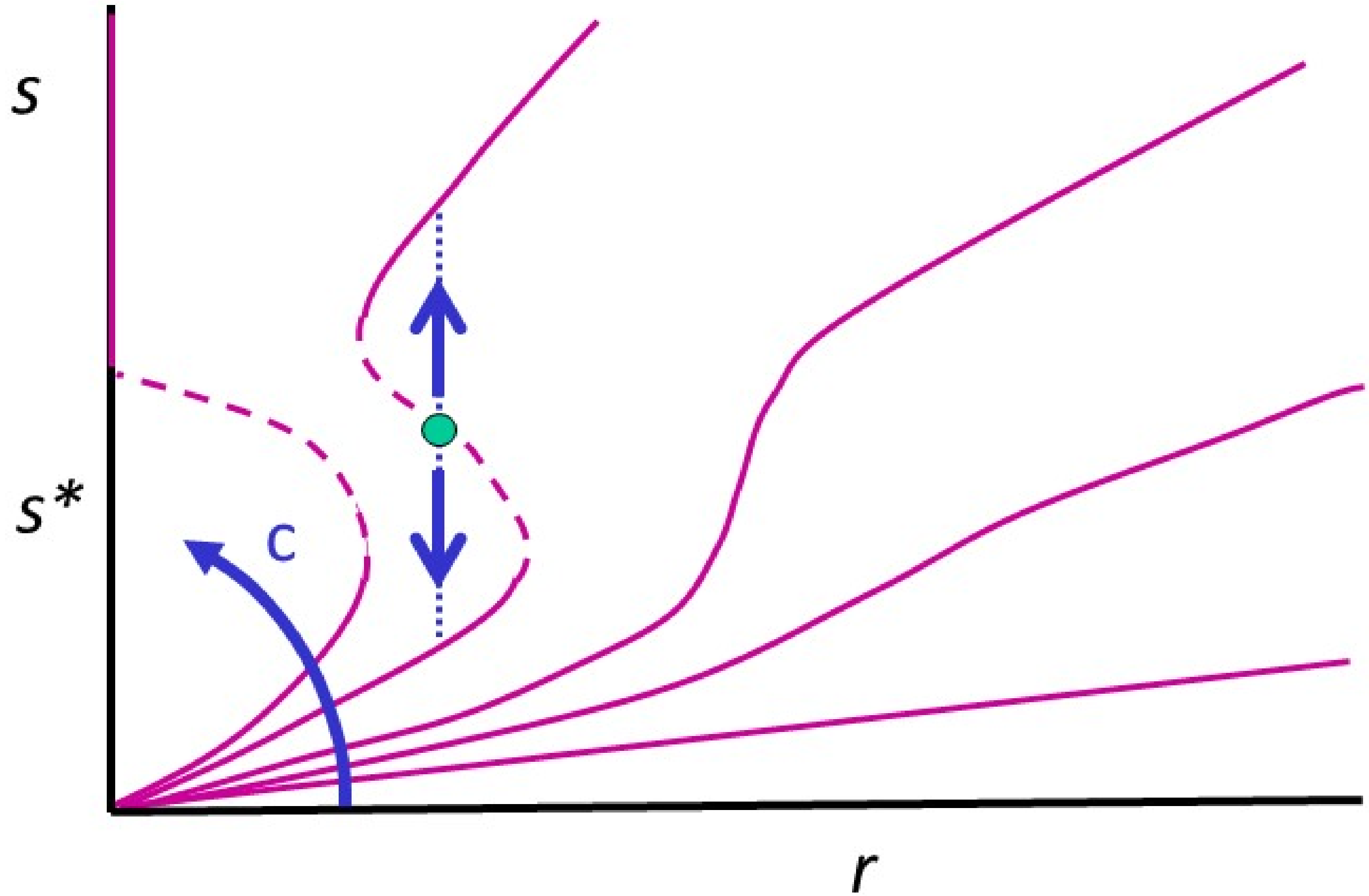
this explains everything!



Cates and Wyart, Phys Rev Lett (2014)

Stress-Dependent Friction

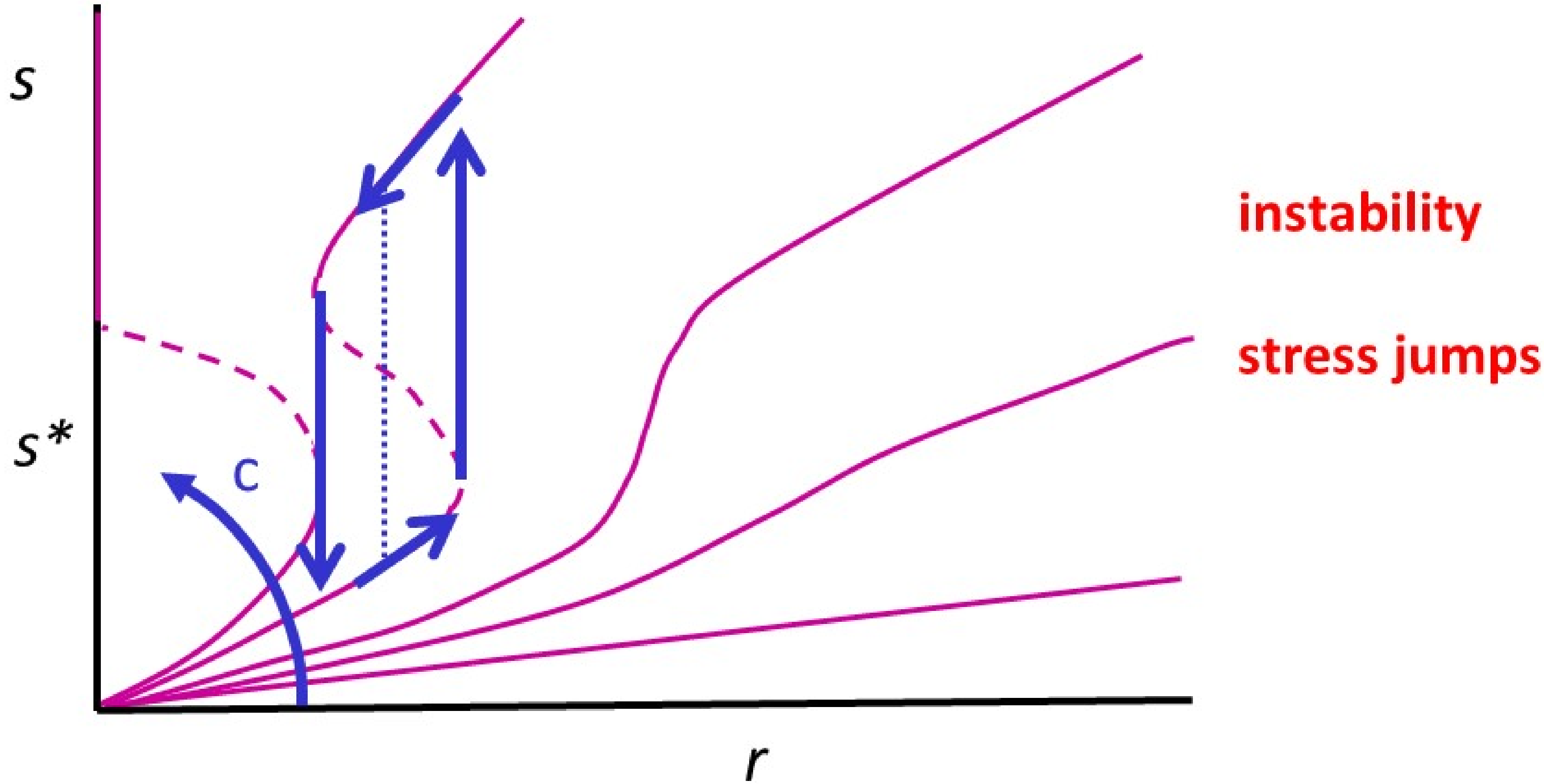
this explains everything!



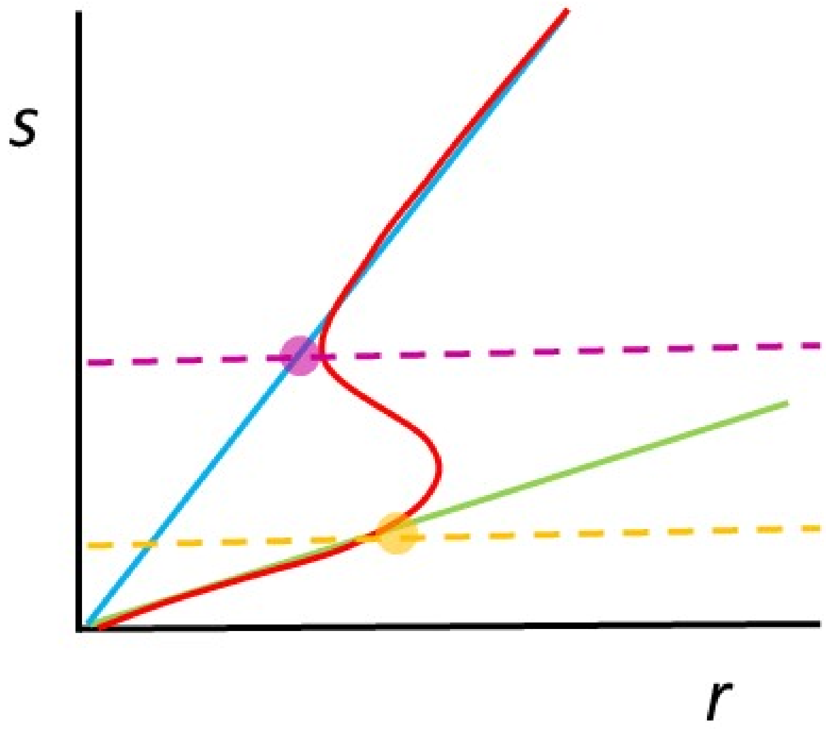
instability

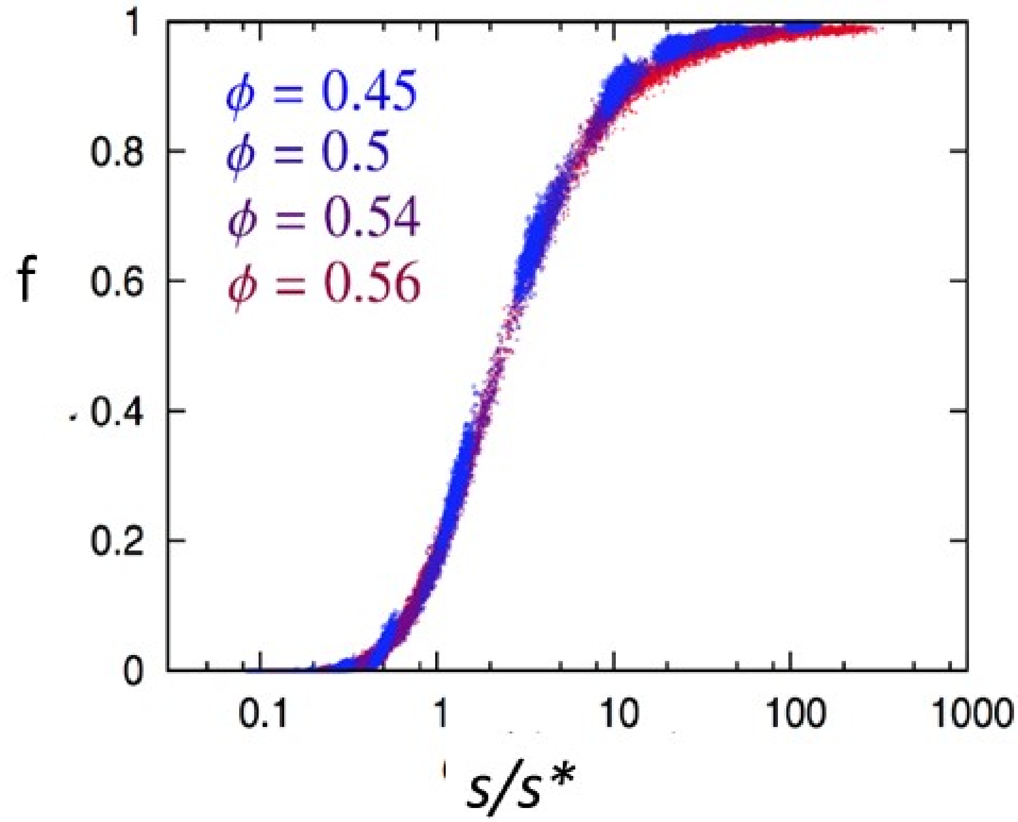
Stress-Dependent Friction

this explains everything!



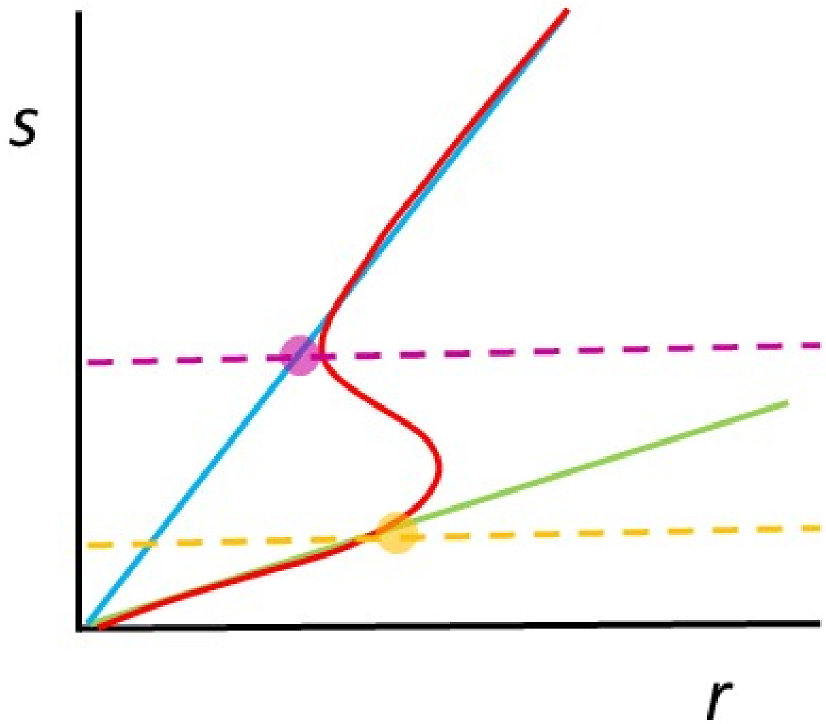
Stress-Dependent Friction

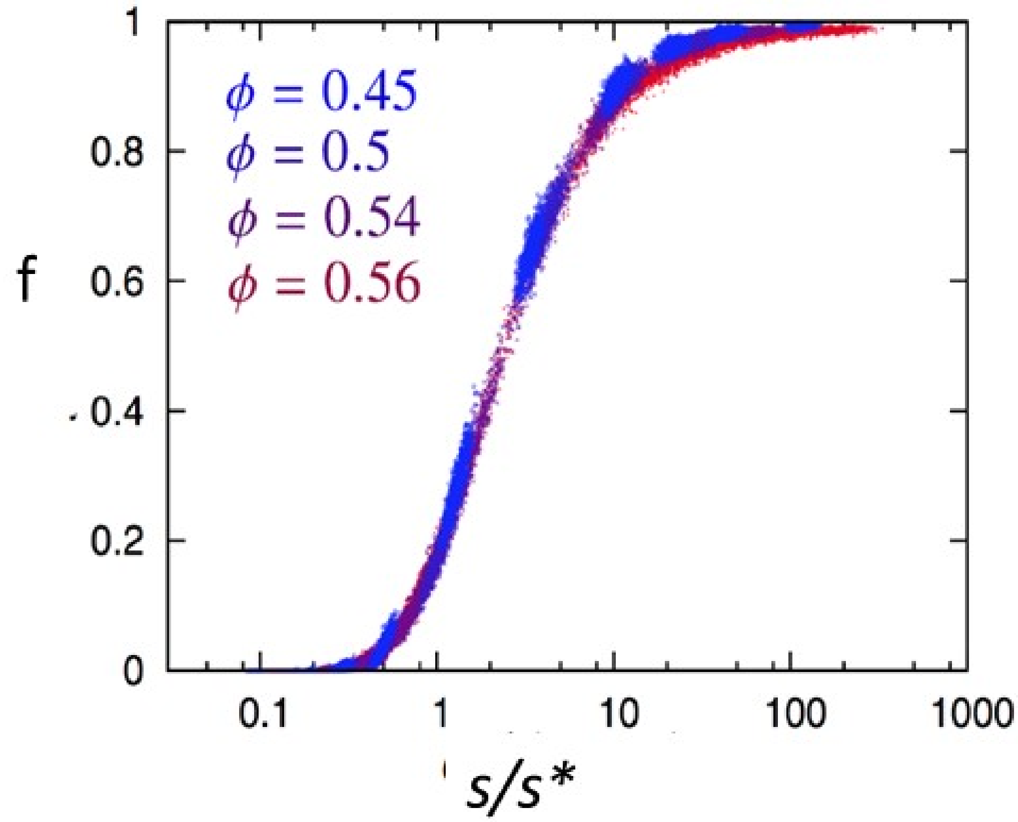




computer simulation data
(R Mari et al, JoR 2014)

Stress-Dependent Friction

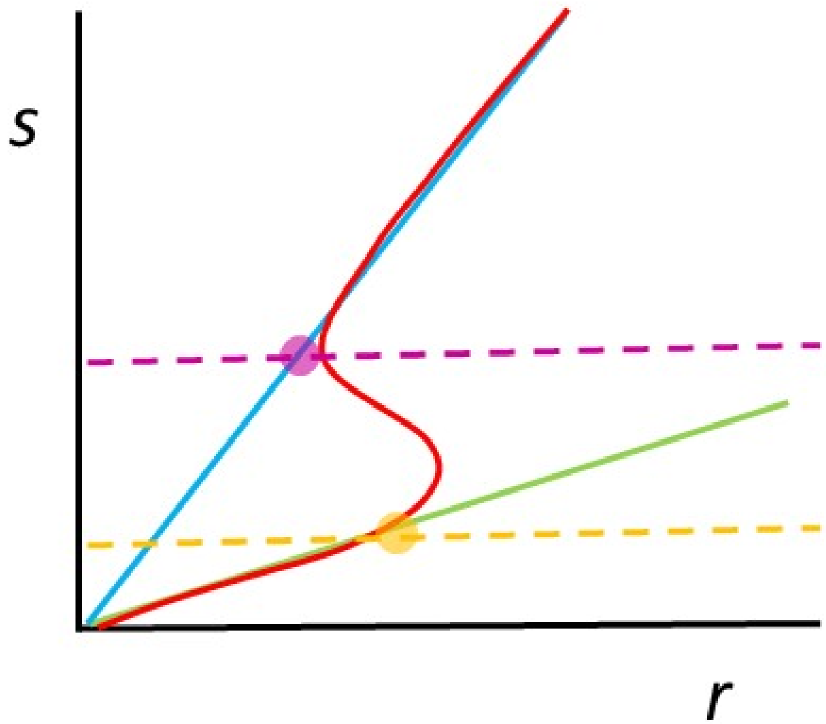


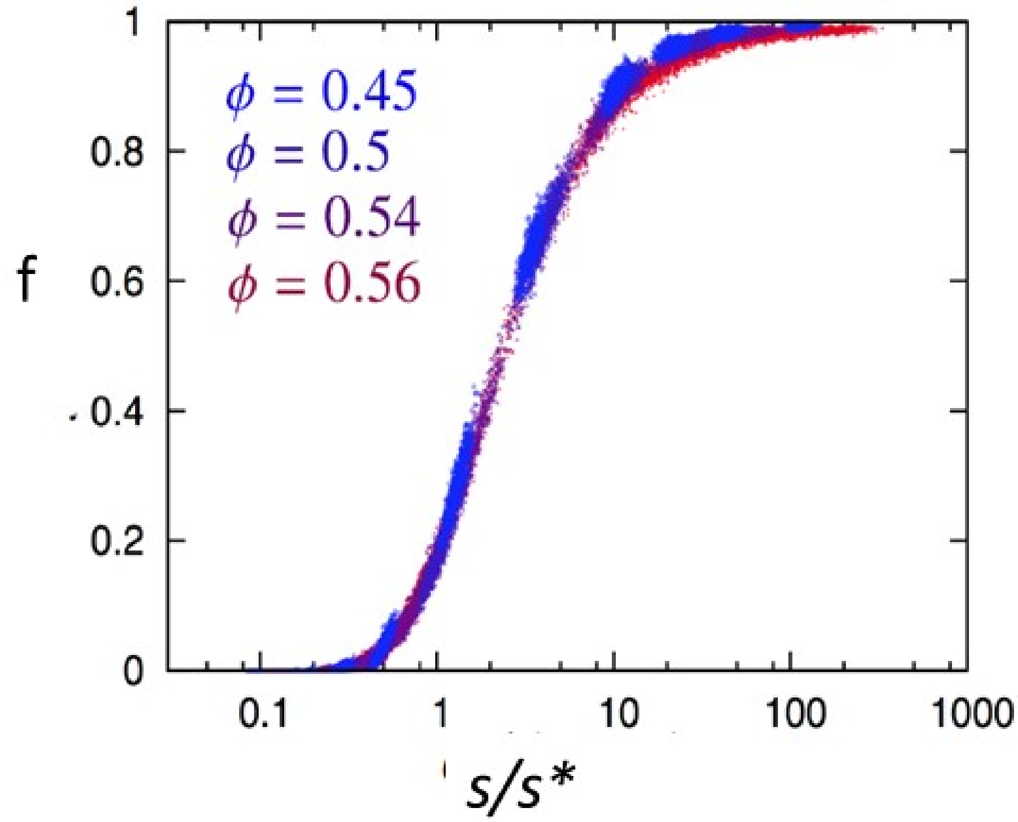


computer simulation data
(R Mari et al, JoR 2014)

this happens for any reasonable function f

Stress-Dependent Friction





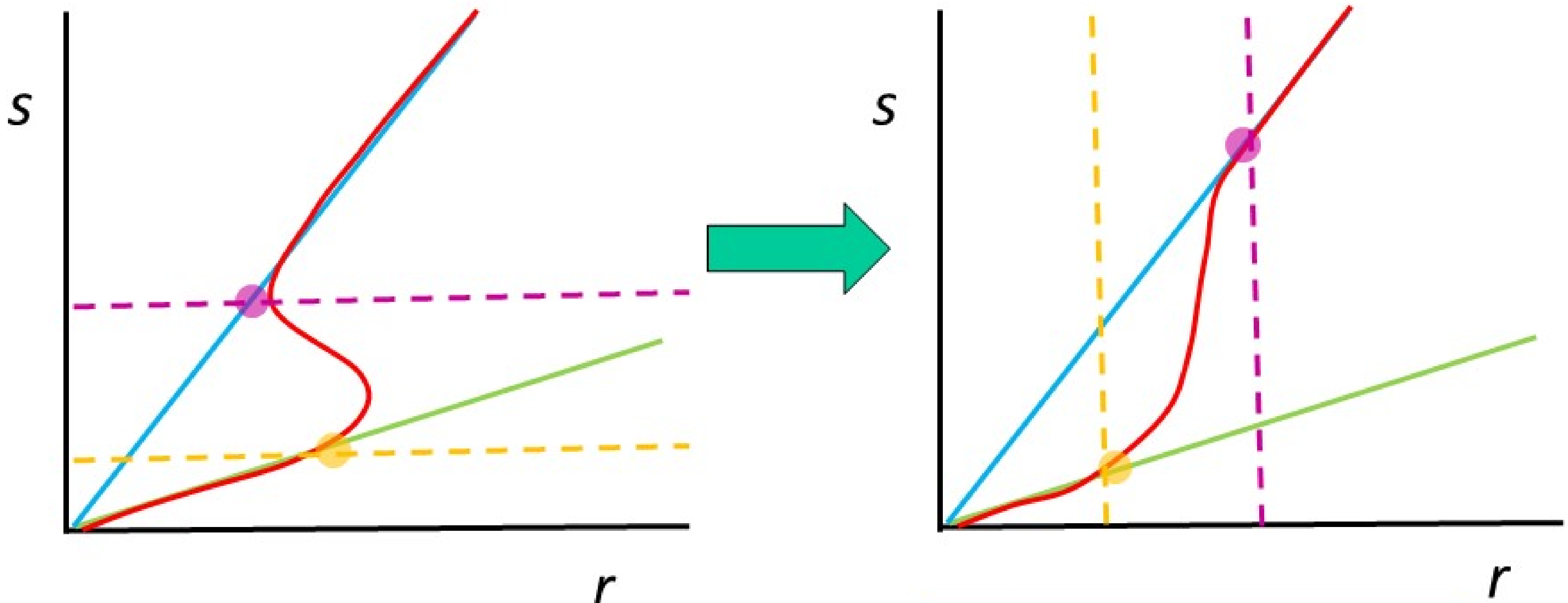
computer simulation data
(R Mari et al, JoR 2014)

this happens for any reasonable function f

- power of 'graphical mathematics'

Stress-Dependent Friction

rate-dependent friction would not suffice:



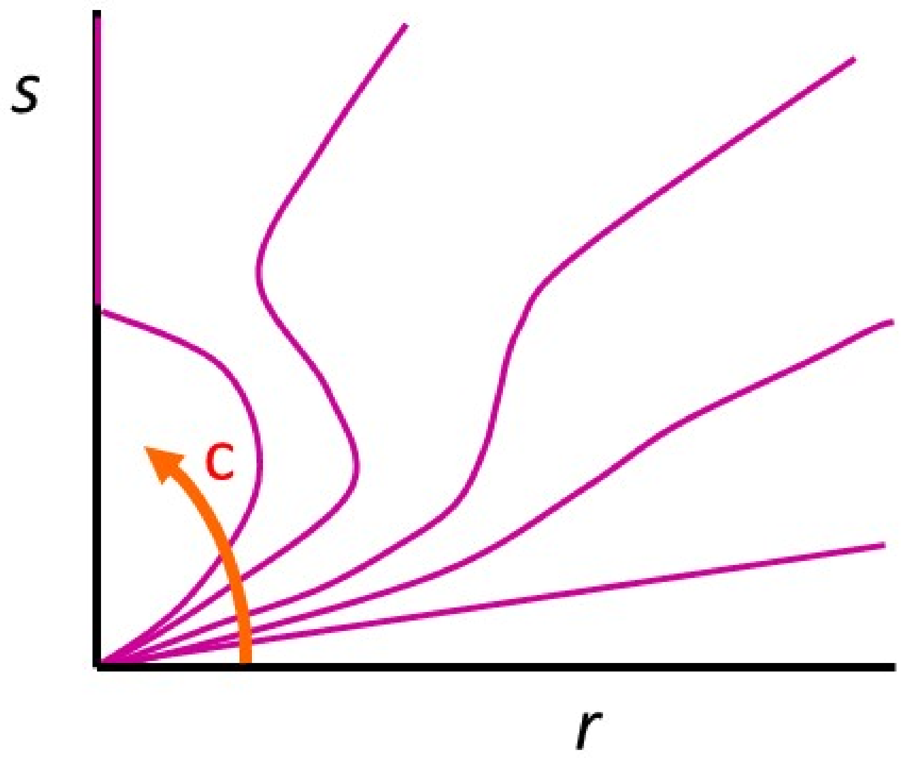
curves get steep...
but never fold back!

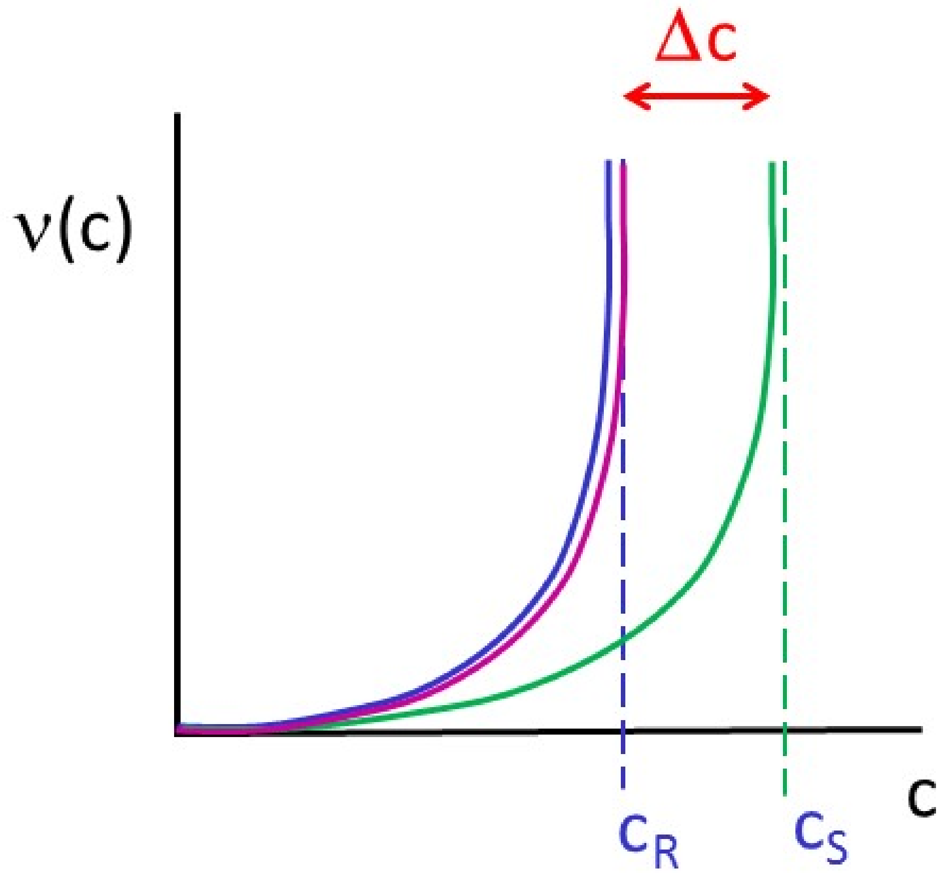
BULLET-PROOF CUSTARD:

Fluids that stop flowing when you push them too hard

- Soft matter: what and why?
- Dense suspensions under flow
- Shear-thickening suspensions
  - What we are trying to explain
  - How we explain it
- How is this useful?

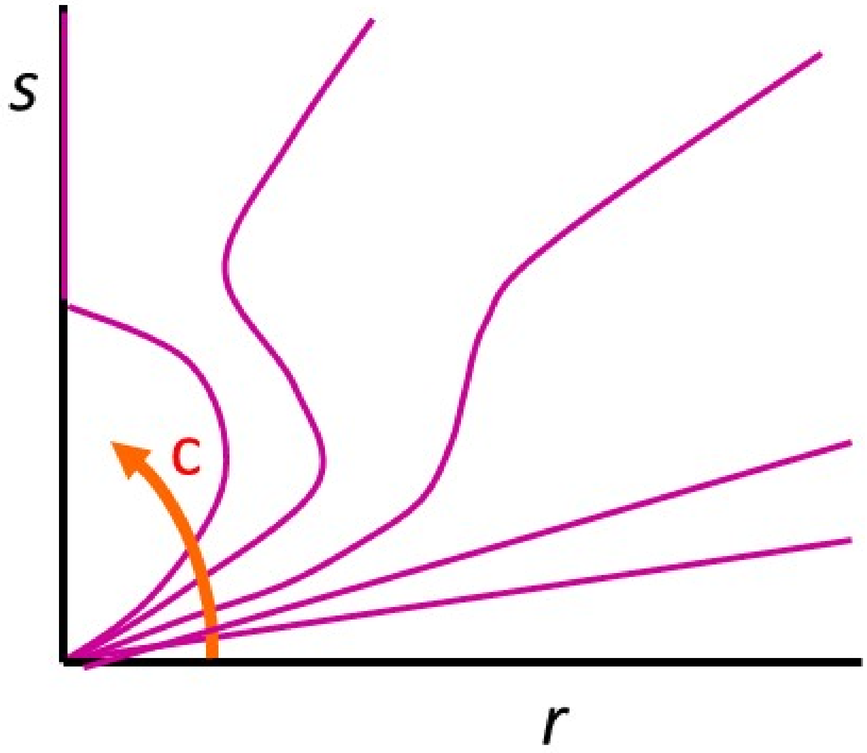
Dense Suspensions: Contact Engineering

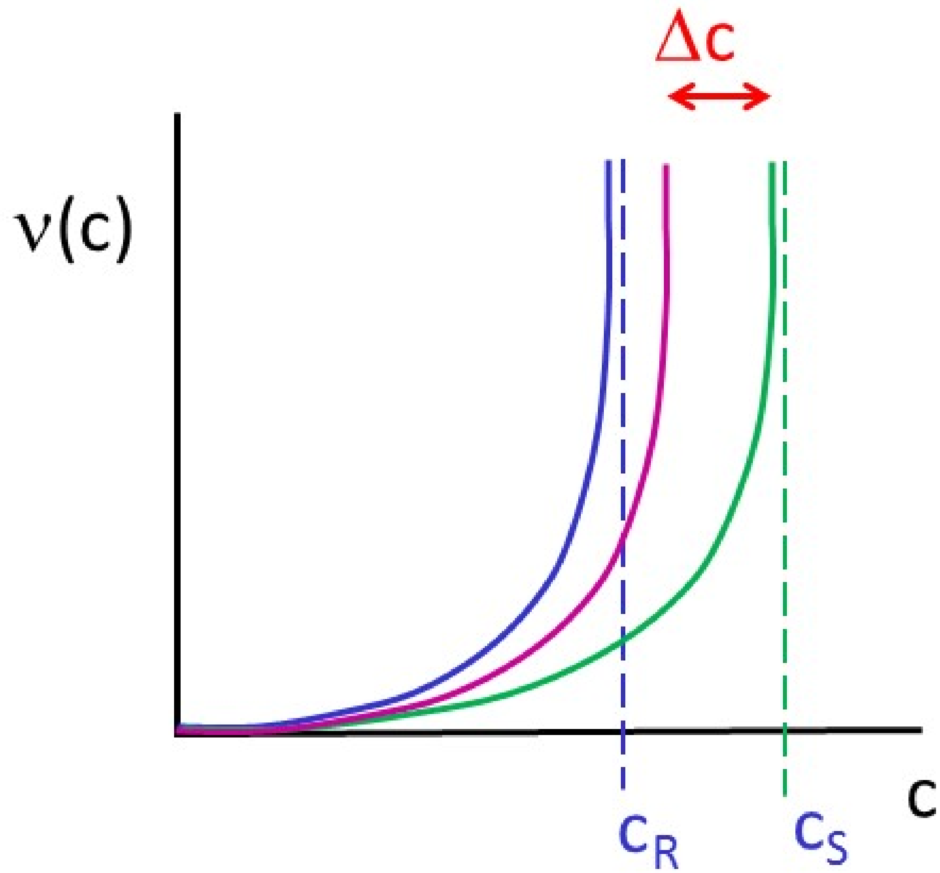




restore control by reducing friction:
absorb slippery molecules

Dense Suspensions: Contact Engineering





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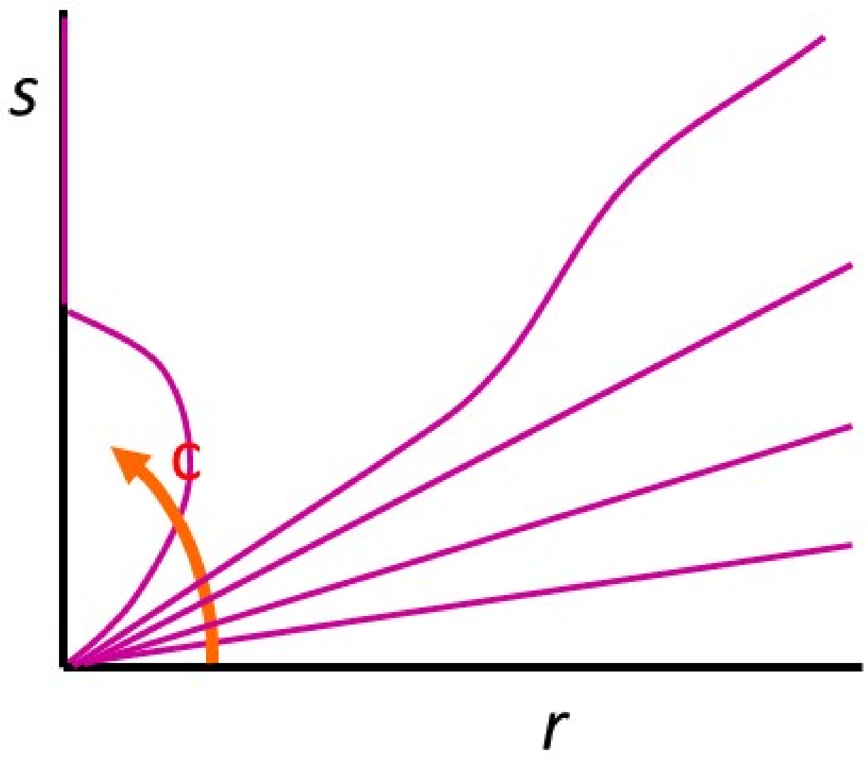
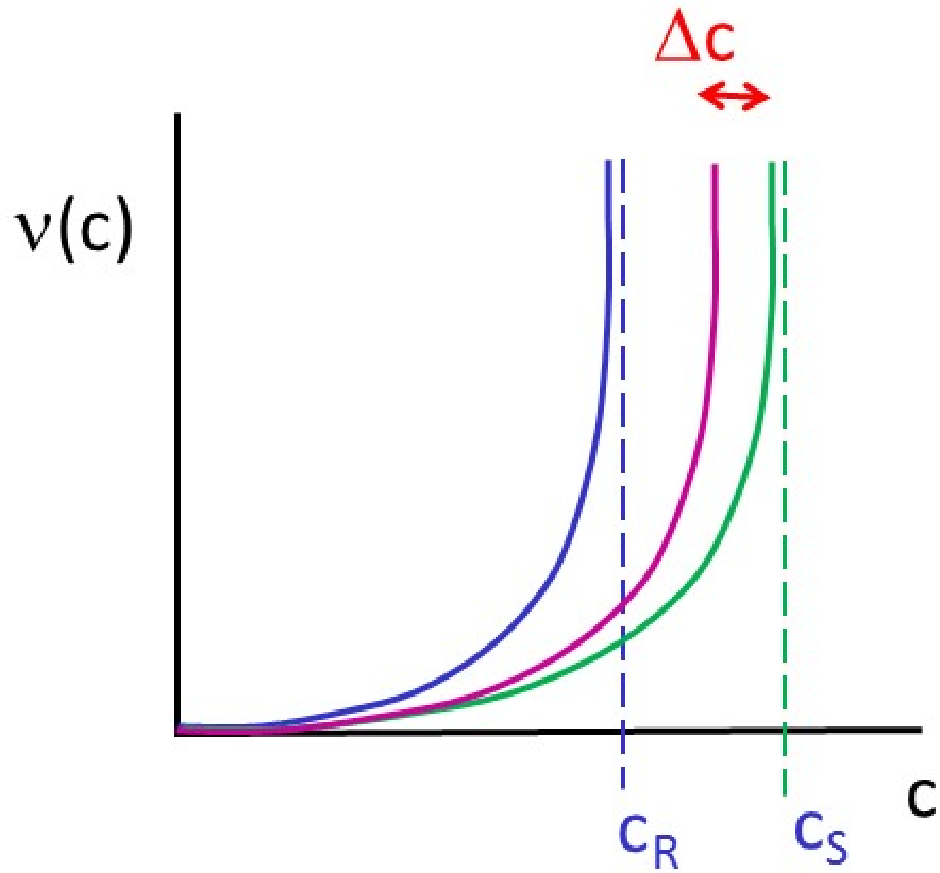
Dense Suspensions: Contact Engineering





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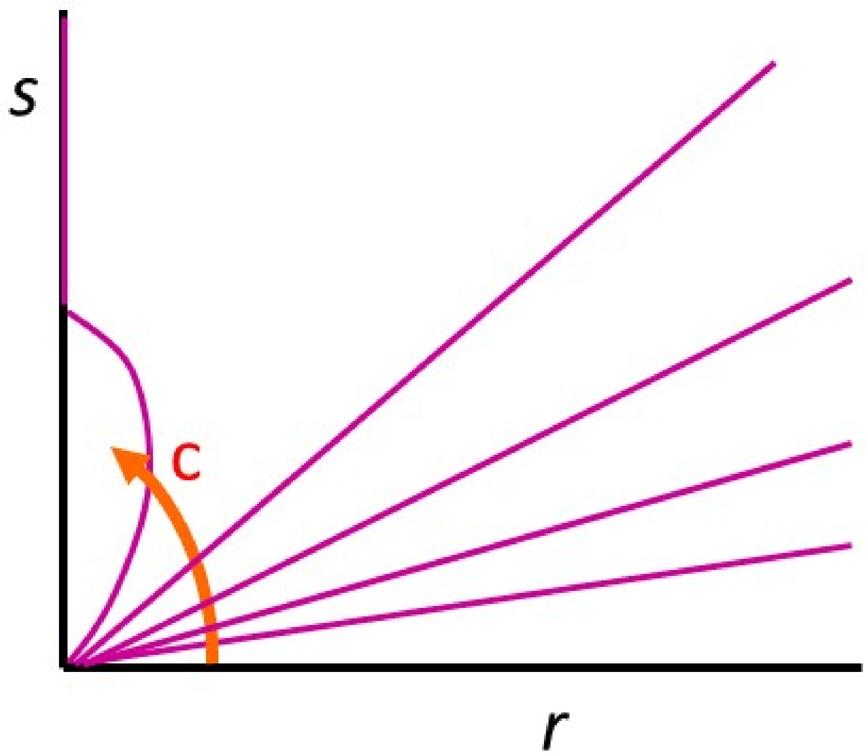
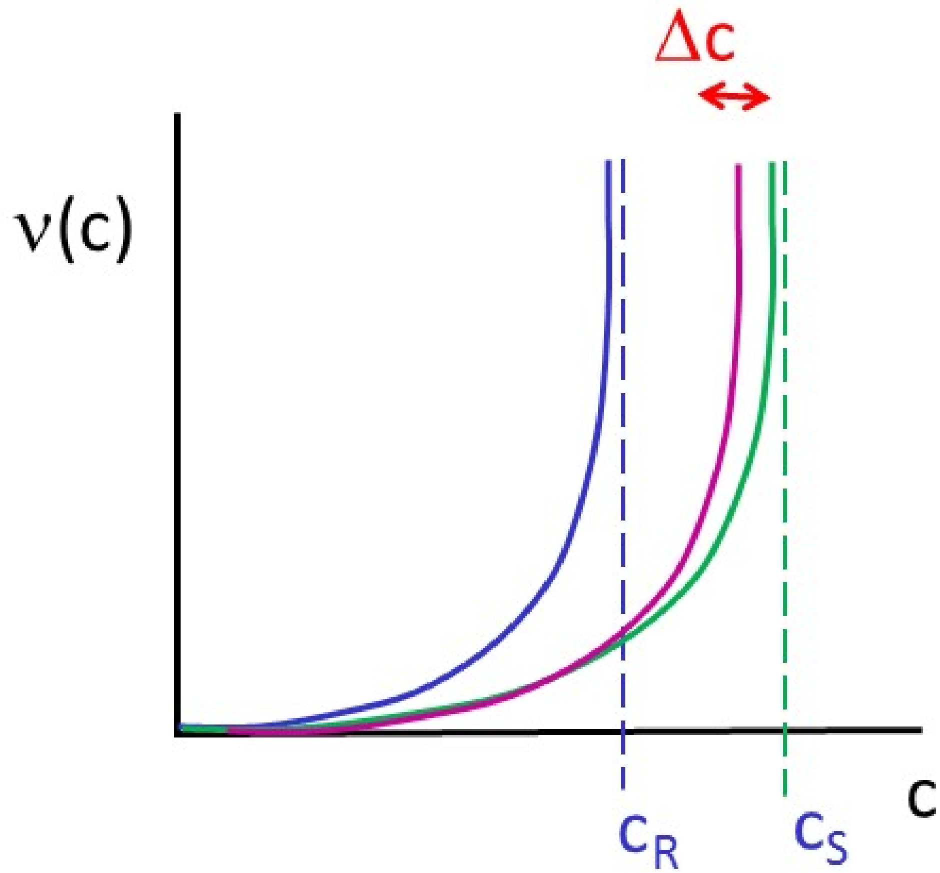
Dense Suspensions: Contact Engineering





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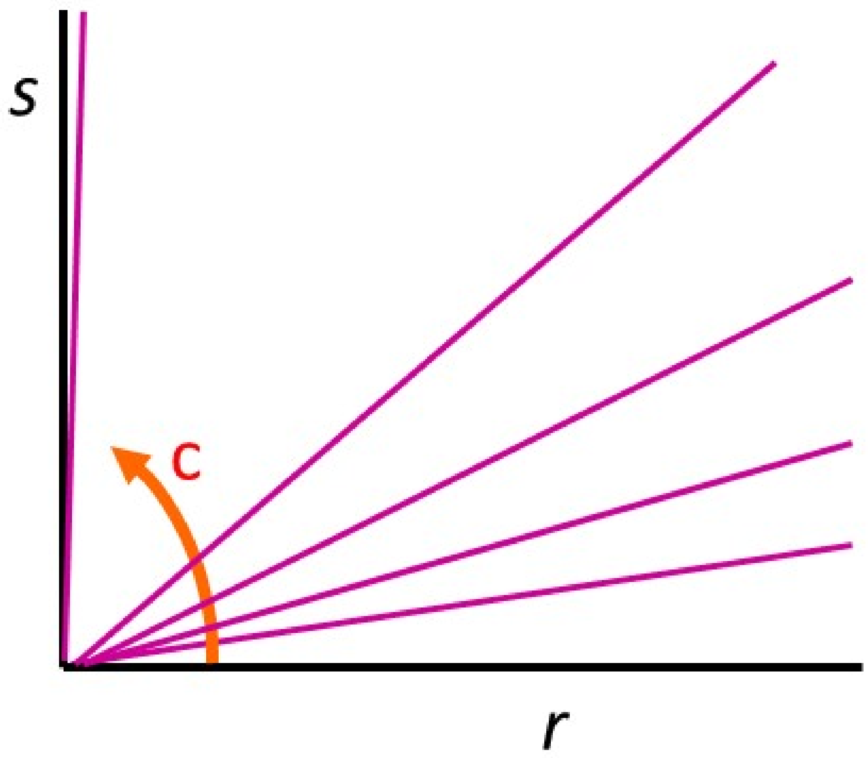
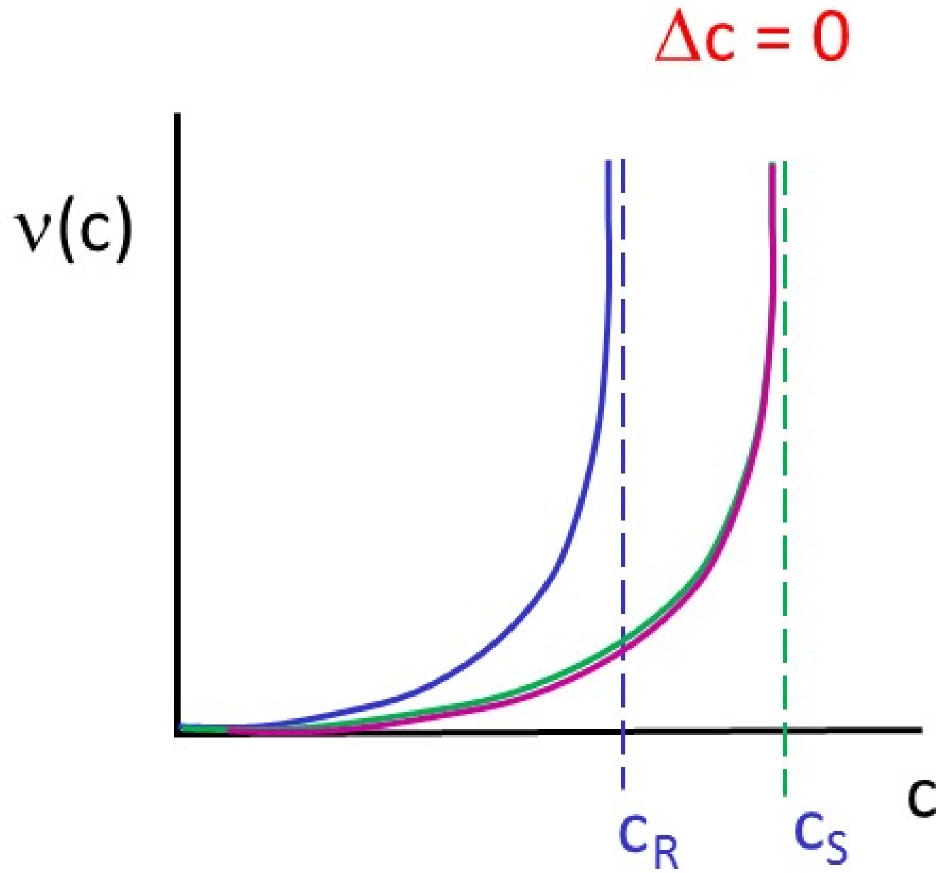
Dense Suspensions: Contact Engineering





restore control by reducing friction:
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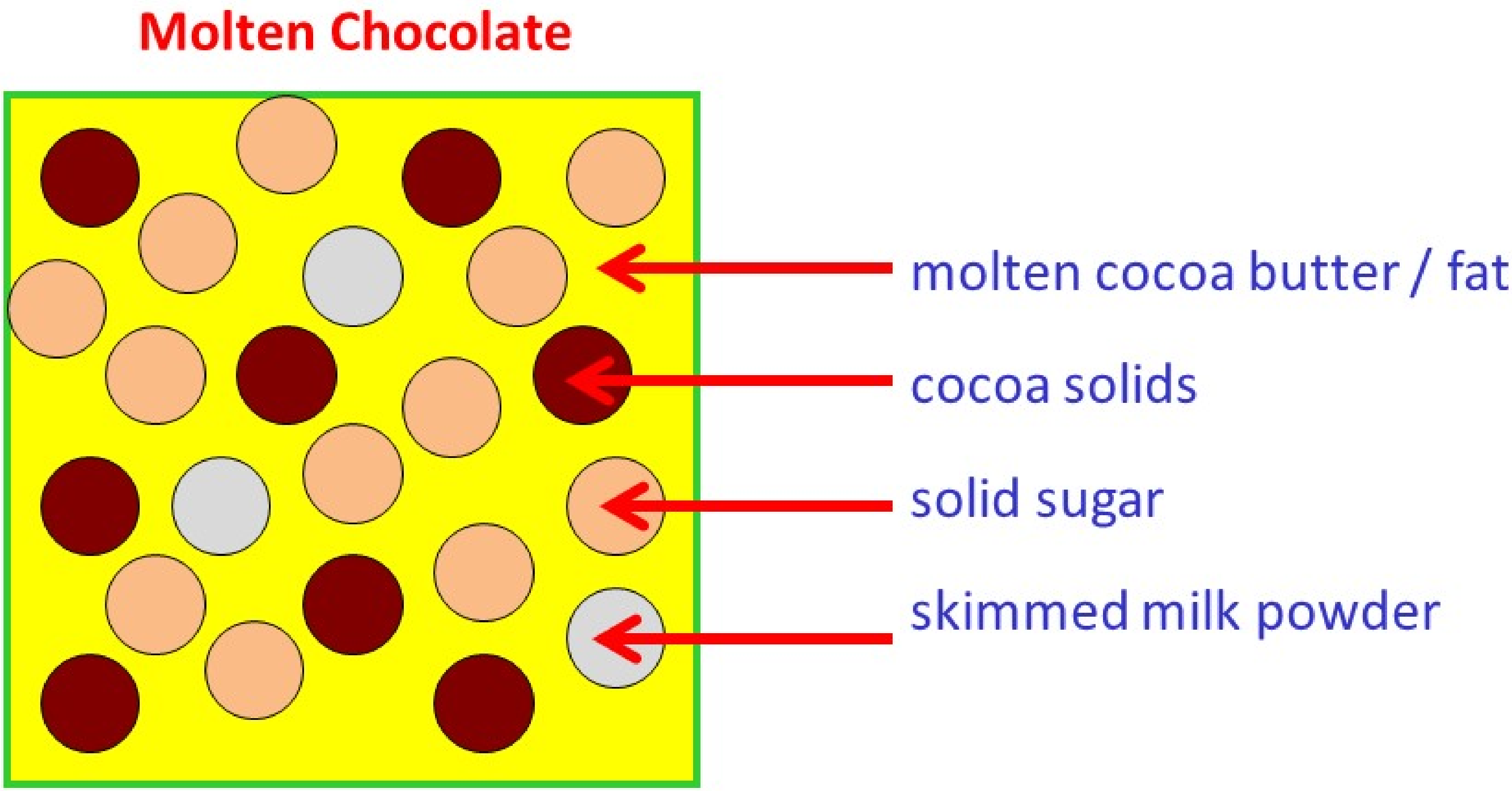
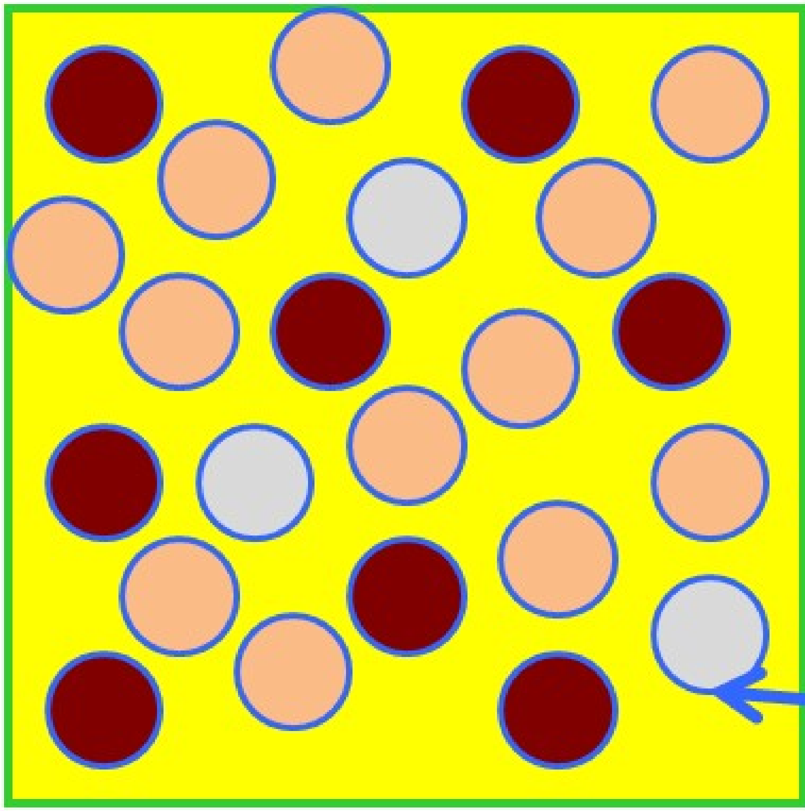
Dense Suspensions: Contact Engineering

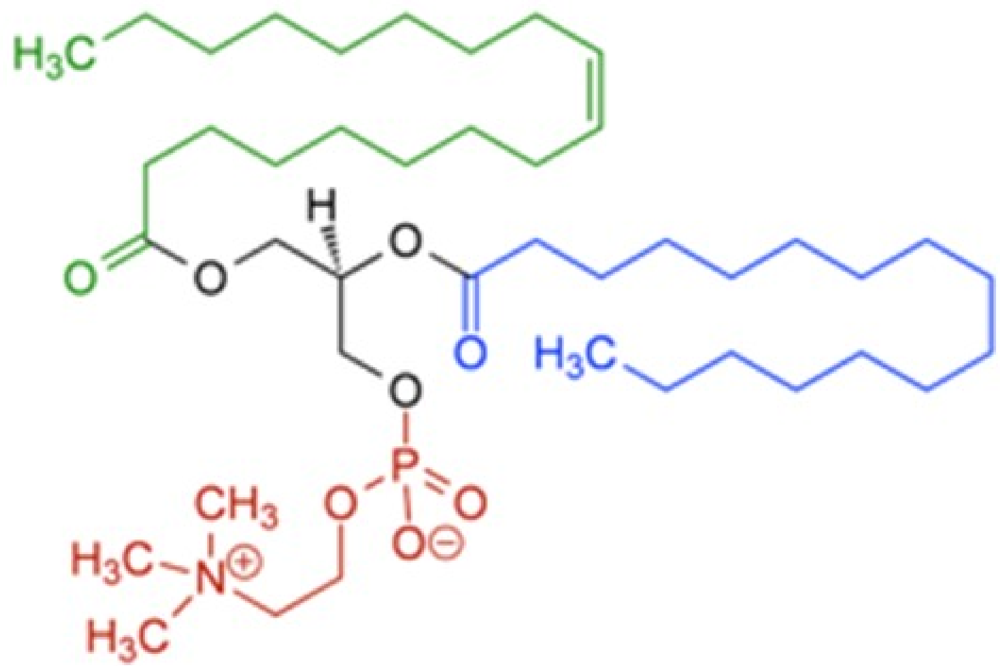
Dense Suspensions: Contact Engineering



Dense Suspensions: Contact Engineering

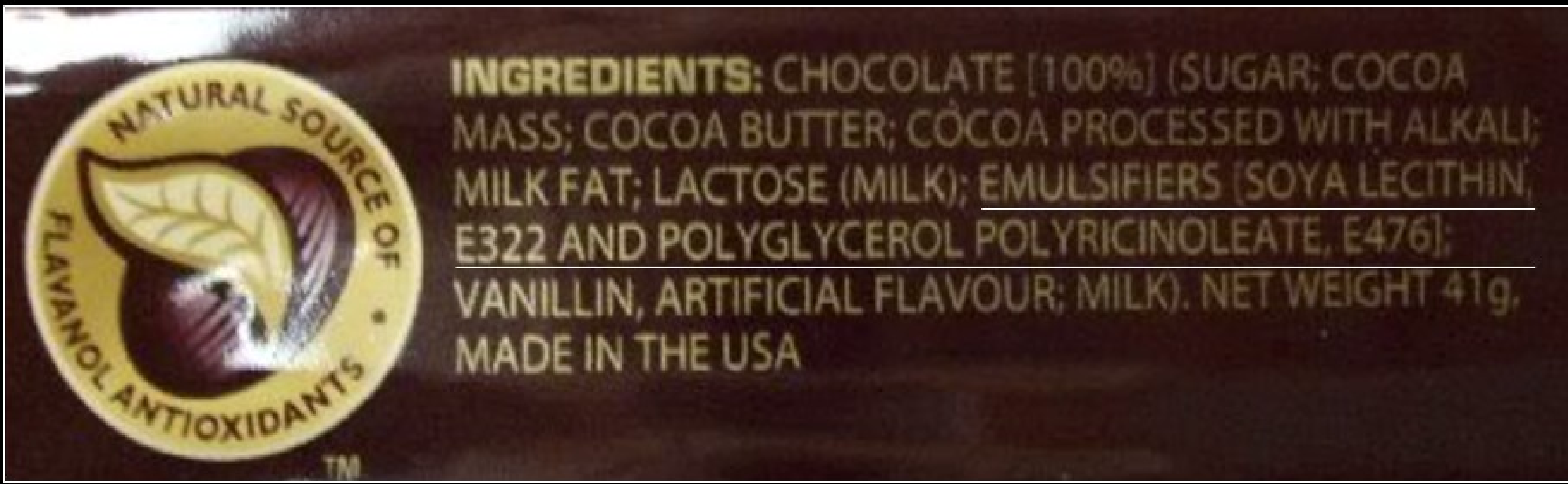
Molten Chocolate





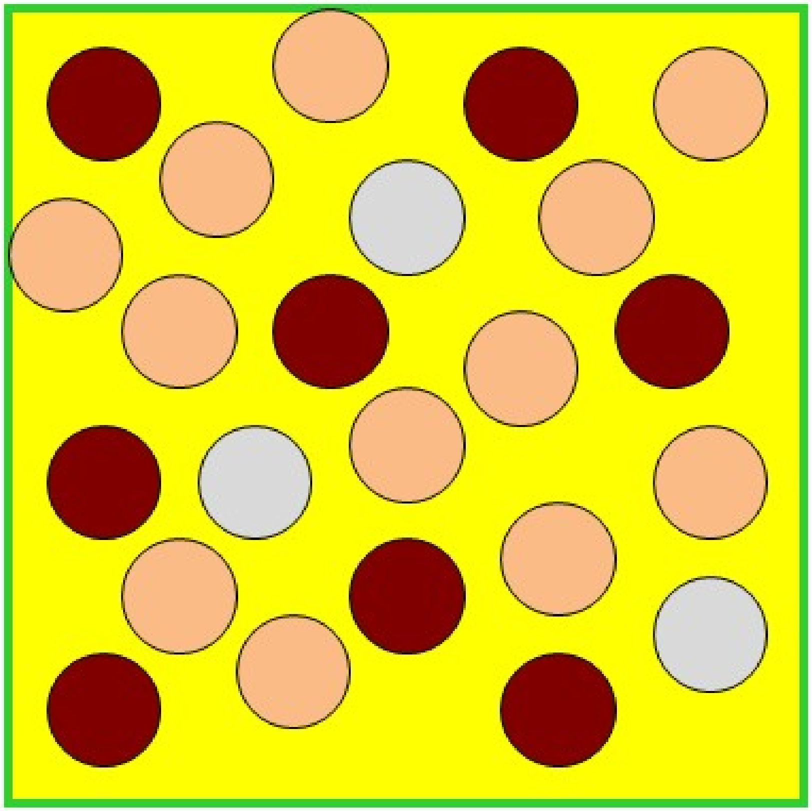
lecithin/PGPR coating

restore control by reducing friction:
absorb slippery molecules


NATURAL SOURCE OF FLAVANOL ANTIOXIDANTS
TM
INGREDIENTS: CHOCOLATE [100%] (SUGAR; COCOA MASS; COCOA BUTTER; COCOA PROCESSED WITH ALKALI; MILK FAT; LACTOSE (MILK); EMULSIFIERS [SOYA LECITHIN, E322 AND POLYGLYCEROL POLYRICINOLEATE, E476]; VANILLIN, ARTIFICIAL FLAVOUR; MILK). NET WEIGHT 41g, MADE IN THE USA

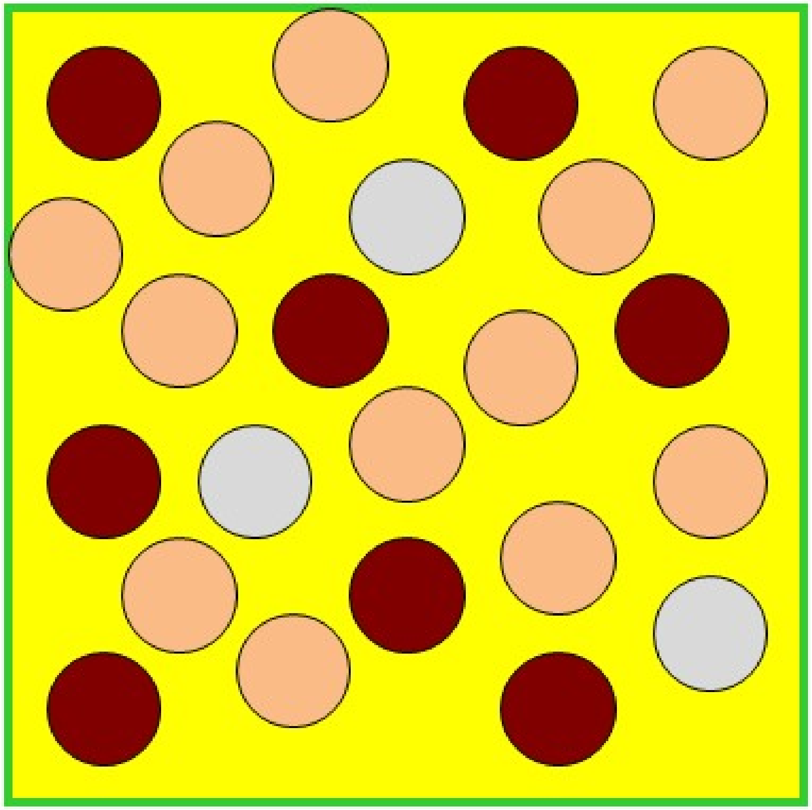
Dense Suspensions: Contact Engineering

Molten Chocolate



Dense Suspensions: Contact Engineering

Other Materials



Setting Cement

Ceramic Pastes

Wastewater Slurries

Oil Well Fluids

Emulsion Paints

Li-ion Battery Electrodes

.... etc.

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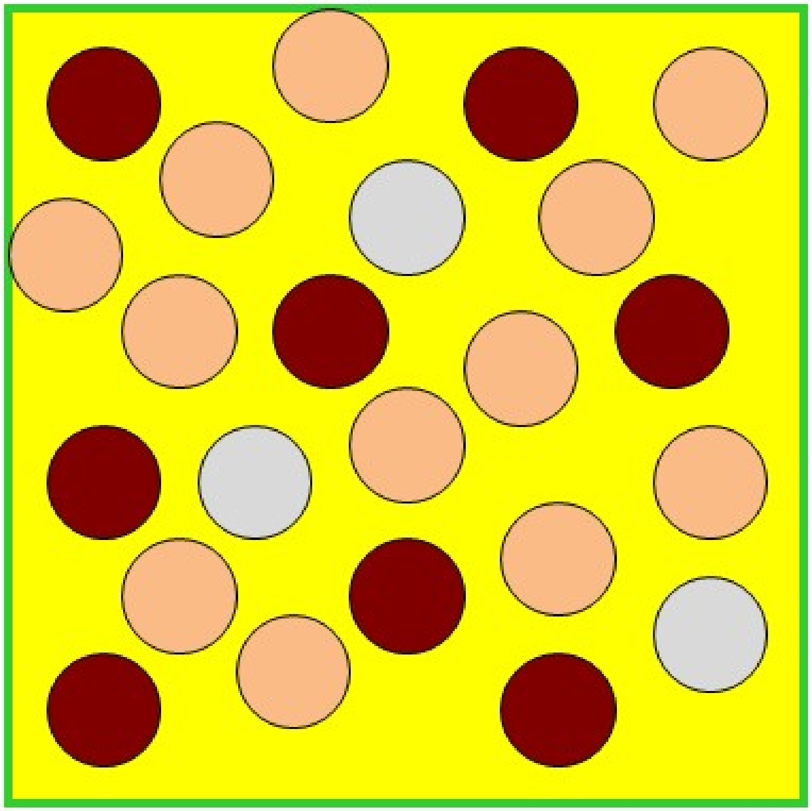

Setting Cement

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restore control by reducing friction:
absorb slippery molecules

BULLET-PROOF CUSTARD:

Fluids that stop flowing when you push them too hard

- Soft matter: what and why?
- Dense suspensions under flow
- Shear-thickening suspensions
    - What we are trying to explain
    - How we explain it
- How is this useful?

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Thanks for listening!