Title: The loop gravity string

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Abstract: <p>In this talk we present the study of canonical gravity in finite regions for which we introduce a generalisation of the Gibbons-Hawking boundary term including the Immirzi parameter. We study the canonical formulation on a spacelike hypersuface with a boundary sphere and show how the presence of this term leads to a new type of degrees of freedom coming from the restoration of the gauge and diffeomorphism symmetry at the boundary. In the presence of a loop quantum gravity state, these boundary degrees of freedom localize along a set of punctures on the boundary sphere. We demonstrate that these degrees of freedom are effectively described by auxiliary strings with a 3-dimensional internal target space attached to each puncture. We show that the string currents represent the local frame field, that the string angular momenta represent the area flux and that the string stress tensor represents the two dimensional metric on the boundary of the region of interest. Finally, we show that the commutators of these broken diffeomorphisms charges of quantum geometry satisfy at each puncture a Virasoro algebra with central charge c = 3. This leads to a description of the boundary degrees of freedom in terms of a CFT structure with central charge proportional to the number of loop punctures. The boundary SU(2) gauge symmetry is recovered via the action of the U(1) 3 Kac-Moody generators (associated with the string current) in a way that is the exact analog of an infinite dimensional generalization of the Schwinger spin-representation.</p>

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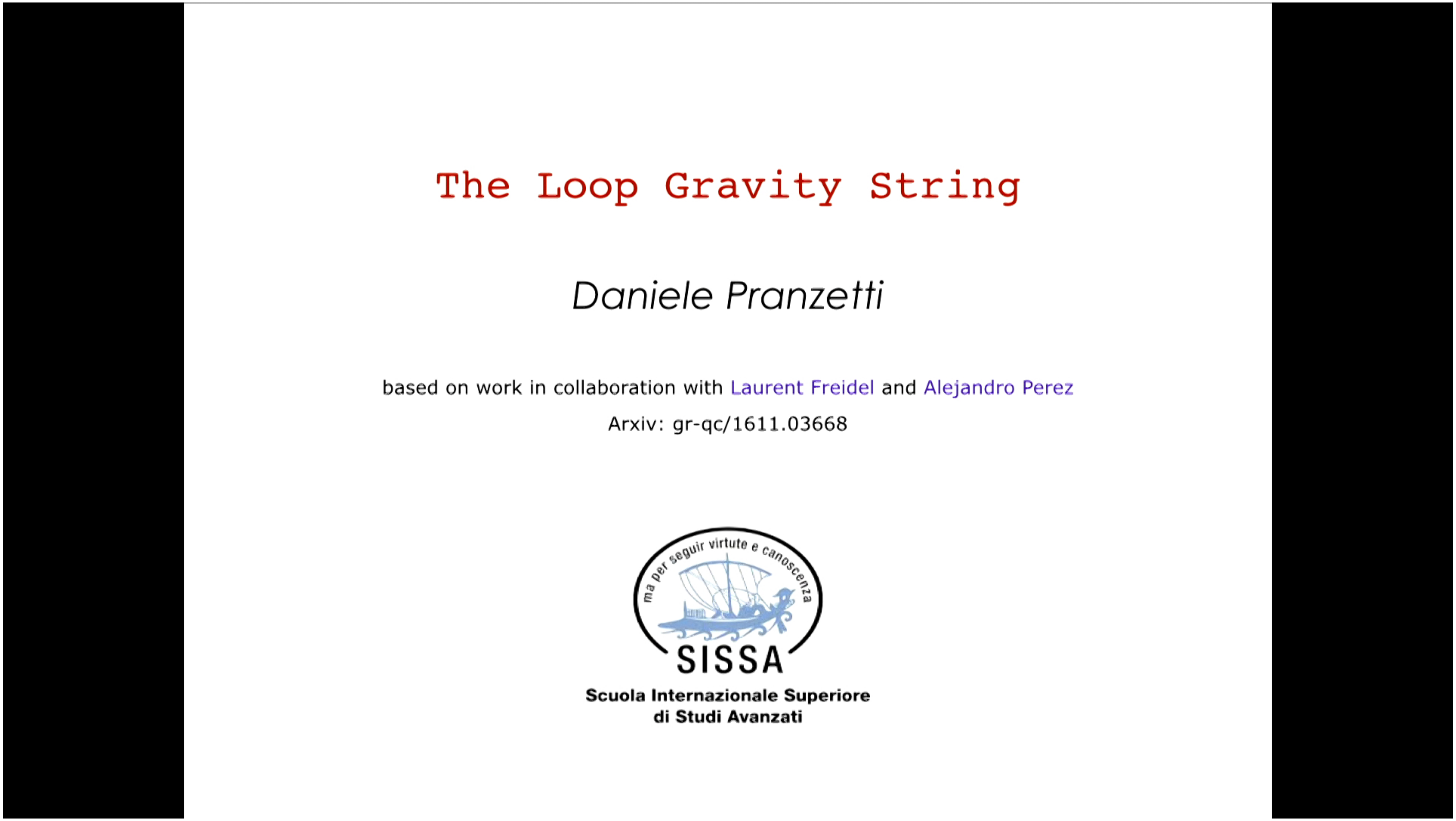
<p>(Based on the joint work with Laurent Freidel and Alejandro Perez arXiv:1611.03668)</p>



The Loop Gravity String

Daniele Pranzetti

based on work in collaboration with Laurent Freidel and Alejandro Perez

Arxiv: gr-qc/1611.03668

SISSA
Scuola Internazionale Superiore
di Studi Avanzati

A couple of old ideas:

- ✦ Boundaries break gauge symmetries and new degrees of freedom appear when trying to restore them

[Benguria, Cordero, Teitelboim '77]; [Smolin '95]; [Teitelboim '95]; [Carlip '99]...

recently revisited in [Freidel, Perez '15], [Donnelly, Freidel '16]

- ✦ CFT degrees of freedom naturally dwell around punctures

[Witten '89]; [Moore, Seiberg '89]

explored in the context of LQG in [Ghosh, Pranzetti '14]

The new degrees of freedom that arise from the presence of a boundary are physical:

- They represent the set of all possible boundary conditions that need to be included in order to reconstruct the expectation value of all gravity observables

- They correspond to partial observables, which could represent detectors on a boundary or physical boundary conditions

- They are needed in the reconstruction of the total Hilbert space in terms of the Hilbert space for the subsystems (edge states/soft modes): They encode entanglement between subsystems

- They also represent the degrees of freedom that one needs in order to couple the subsystem to another system in a gauge invariant manner

Ingredients

- A generalized Gibbons-Hawking-York boundary term

- The new boundary degrees of freedom organize themselves under the representation of the conformal symmetry group. Local conformal invariance realized through the thickening of the spin network links into spin-tubes

originally postulated in [Smolin '95], [Major, Smolin '95]
and recently resurfaced in the context of spin foam amplitudes computation in [Haggard, Han, Kaminski, Riello '14]

- Background geometry assumption: the tangential curvature of the connection vanishes everywhere on the boundary except at the location of a given set of punctures. Motivated also by the new, dual vacuum of loop gravity

hinted by [Bianchi '09], established at the semiclassical level by [Freidel, Ziprick '13],
implemented in the quantum theory by [Dittrich, Geiller '14]

Phase space analysis

➢ We want to consider the canonical structure of general relativity in the first order formalism on a 3d slice that possesses a 2d boundary punctured by spin network links.

We start from a formulation of gravity on a manifold $M \times \mathbb{R}$ with a boundary two sphere S^2

3d space-like hypersurface

$$S = \frac{1}{2\gamma\kappa}\left[\int_{M\times\mathbb{R}} E^{IJ}\wedge F_{IJ}(\omega) + \int_{S^2\times\mathbb{R}} e_I \wedge \mathrm{d}_A e^I\right]$$

$\kappa = 8\pi G\,,$ $\gamma :$ Barbero-Immirzi parameter, e^I : Frame field, ω^{IJ} : 4d spin connection

$$A^{IJ} := (\omega^{IJ} + \gamma * \omega^{IJ})$$ Boundary 'connection'

$$E^{IJ} = [\underbrace{(e^I \wedge e^J)}_{\text{gives the Holst term which vanishes on-shell}} + \gamma * (e^I \wedge e^J)]|_{bulk}$$ Flux defined by the simplicity constraint

- Notice that, due to the boundary term, the action is differentiable for arbitrary variations of the fields

The boundary action term:

$$S_{bound} = \frac{1}{\gamma}\int_{S\times\mathbb{R}} e_I \wedge \mathrm{d}_A e^I = \int_{S\times\mathbb{R}} - * \omega^{IJ} \wedge (e_I \wedge e_J) + \frac{1}{\gamma} e_I \wedge d_\omega e^I$$

- By choosing a Lorentz gauge where one of the tetrad is fixed to be the normal to the boundary, it is easy to see that the first component is simply given by the integral of the well known Gibbons-Hawking-York boundary density:

$$*\omega^{IJ} \wedge (e_I \wedge e_J) \quad \rightarrow \quad 2\sqrt{h}K\,,$$

where
h = determinant of the induced metric on the boundary
K = trace of the boundary extrinsic curvature

- The second one is a new addition to the standard boundary term of the metric formulation, which vanishes on shell due to the torsion free condition (Cartan eq.):

$\gamma^{-1} e_I \wedge d_\omega e^I$ is a natural complement to the Holst term in the bulk action $\gamma^{-1} F_{IJ}(\omega) \wedge e^I \wedge e^J$

- Time gauge: $\mathrm{d}n^I = 0\,, \quad \overset{\text{normal to } M}{\overset{\swarrow}{n}} = e^I n_I \qquad A^i = \overset{\text{Spin connection}}{\overset{\searrow}{\Gamma^i}} + \gamma \overset{\text{Extrinsic curvature}}{\overset{\downarrow}{K^i}}\,, \qquad \Sigma_i = \frac{1}{2}\epsilon_{ijk}\, \overset{\text{coframe fields tangent to } M}{\overset{\searrow}{e^j}} \wedge e^k$

Symplectic 2-form: $$\Omega = \Omega_M + \Omega_{S^2} = \frac{1}{\kappa\gamma}\int_M (\delta A^i \wedge \delta\Sigma_i) + \frac{1}{2\kappa\gamma}\int_{S^2} (\delta e_i \wedge \delta e^i)$$

The extended phase space

Poisson brackets: $\{A^i_a(x), \Sigma^j_{bc}(y)\} = \kappa\gamma\delta^{ij}\epsilon_{abc}\delta^3(x-y)$ $\quad \{e^i_a(x), e^j_b(y)\} = \kappa\gamma\,\delta^{ij}\epsilon_{ab}\delta^2(x-y)$

bulk phase space

boundary phase space

The preservation of the gauge and diffeomorphism symmetry in the presence of the boundary imposes the validity of additional boundary constraints

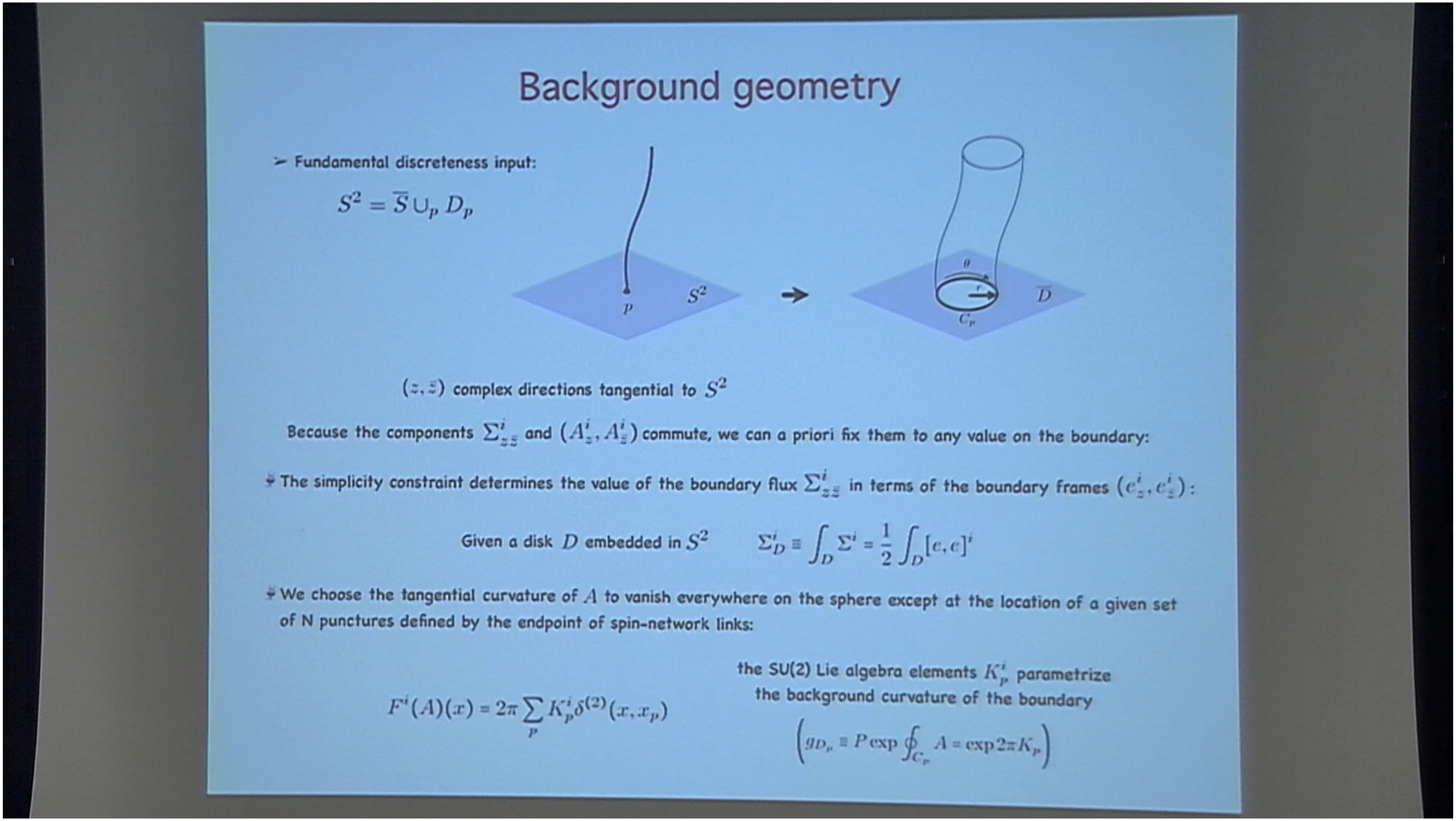
Boundary constraints:

- Boundary Gauss law $\Sigma_i|_{bulk} = \frac{1}{2}[e,e]_i|_{boundary}$

Boundary simplicity constraint: Matching of bulk and boundary area elements
(the requirement of gauge invariance replaces the boundary condition $\delta h = 0$)

Initially, e commutes with all the bulk fields A and Σ, and Σ commutes with itself:

it is the simplicity constraint, which enables the preservation of SU(2) symmetry in the presence of a boundary, that leads to the flux non-commutativity already at the classical level



Background geometry

- Fundamental discreteness input:

$$S^2 = \overline{S} \cup_p D_p$$

$(z, \bar{z})$ complex directions tangential to S^2

Because the components $\Sigma^i_{z\bar{z}}$ and $(A^i_z, A^i_{\bar{z}})$ commute, we can a priori fix them to any value on the boundary:

- The simplicity constraint determines the value of the boundary flux $\Sigma^i_{z\bar{z}}$ in terms of the boundary frames $(e^i_z, e^i_{\bar{z}})$:

Given a disk D embedded in S^2 $\quad \Sigma^i_D \equiv \int_D \Sigma^i = \frac{1}{2} \int_D [e, e]^i$

- We choose the tangential curvature of A to vanish everywhere on the sphere except at the location of a given set of N punctures defined by the endpoint of spin-network links:

$$F^i(A)(x) = 2\pi \sum_p K^i_p \delta^{(2)}(x, x_p)$$

the SU(2) Lie algebra elements K^i_p parametrize the background curvature of the boundary

$$\left(g_{D_p} \equiv P \exp \oint_{C_p} A = \exp 2\pi K_p \right)$$

Algebra of boundary constraints

Hamiltonian generators associated to the boundary constraints:

The Poisson bracket is related to the symplectic structure via $\{F, G\} = \Omega(\delta_F, \delta_G)$

where δ_F is the Hamiltonian variation generated by F, $\Omega(\delta_F, \delta) = \delta F$

The two generators associated with the Gauss and diffeo constraints are obtained from the symplectic structure through

$$\Omega(\delta_\alpha, \delta) = \delta G_D(\alpha), \qquad \Omega(\delta_\varphi, \delta) = \delta S_D(\varphi)$$

➥ $$G_D(\alpha) \equiv \frac{1}{\kappa\gamma}\left(\frac{1}{2}\int_D \alpha^i [e,e]_i - \int_M \mathrm{d}_A \alpha^i \wedge \Sigma_i\right), \quad S_D(\varphi) \equiv \frac{1}{\kappa\gamma}\left(\int_D \mathrm{d}_A \varphi^i e_i + \int_M F_i(A) \wedge [e,\varphi]^i\right)$$

boundary extension of the Gauss constraint

By integrating by parts the bulk term we see that it imposes the Gauss Law $\mathrm{d}_A \Sigma^i = 0$ and the boundary simplicity constraint $\Sigma_i = 1/2[e,e]_i$

also generator of internal rotations

$$\delta_\alpha e_i = [\alpha, e]_i$$

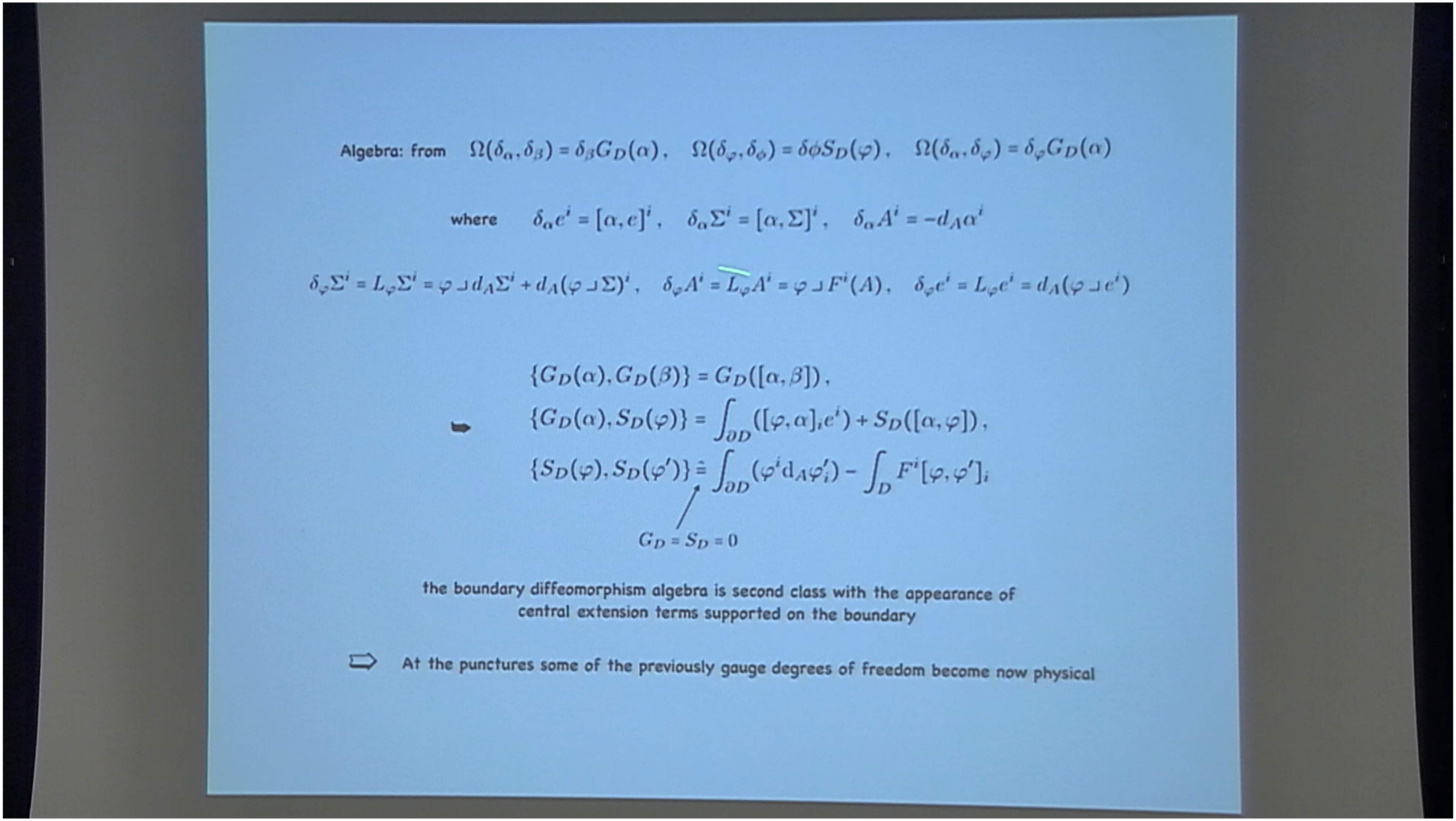
boundary extension of tangent diffeos generator

where $\varphi^i = \varphi^a e^i_a$ and

$\varphi = \varphi^a \partial_a$ is a vector tangent to M

also generator of the transformations

$$\delta_\varphi e^i = \mathrm{d}_A \varphi^i = L_\varphi e^i$$



Algebra: from $\Omega(\delta_\alpha, \delta_\beta) = \delta_\beta G_D(\alpha)\,, \quad \Omega(\delta_\varphi, \delta_\phi) = \delta\phi S_D(\varphi)\,, \quad \Omega(\delta_\alpha, \delta_\varphi) = \delta_\varphi G_D(\alpha)$

where $\delta_\alpha e^i = [\alpha, e]^i\,, \quad \delta_\alpha \Sigma^i = [\alpha, \Sigma]^i\,, \quad \delta_\alpha A^i = -d_A \alpha^i$

$$\delta_\varphi \Sigma^i = L_\varphi \Sigma^i = \varphi \lrcorner d_A \Sigma^i + d_A(\varphi \lrcorner \Sigma)^i\,, \quad \delta_\varphi A^i = L_\varphi A^i = \varphi \lrcorner F^i(A)\,, \quad \delta_\varphi e^i = L_\varphi e^i = d_A(\varphi \lrcorner e^i)$$

$$\begin{aligned} &\{G_D(\alpha), G_D(\beta)\} = G_D([\alpha, \beta])\,, \\ \Rightarrow\quad &\{G_D(\alpha), S_D(\varphi)\} = \int_{\partial D}([\varphi, \alpha]_i e^i) + S_D([\alpha, \varphi])\,, \\ &\{S_D(\varphi), S_D(\varphi')\} \,\hat{=}\, \int_{\partial D}(\varphi^i \mathrm{d}_A \varphi'_i) - \int_D F^i[\varphi, \varphi']_i \end{aligned}$$

$G_D = S_D = 0$

the boundary diffeomorphism algebra is second class with the appearance of central extension terms supported on the boundary

⇨ At the punctures some of the previously gauge degrees of freedom become now physical

Kac-Moody charges

- Our goal now is to study the quantisation of this boundary system in the presence of the background fields

Boundary charges: $Q_D(\varphi) \equiv \frac{1}{\sqrt{2\pi\kappa\gamma}} \int_D \mathrm{d}_A \varphi^i \wedge e_i$ where $\varphi = \varphi^i \tau_i$ SU(2)-valued field

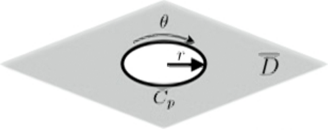
→ $Q_{\overline{D}}(\varphi) \hat{=} - \sum_p Q_p(\varphi)$ and $Q_p(\varphi) \hat{=} \frac{1}{\sqrt{2\pi\kappa\gamma}} \oint_{C_p} \varphi^i e_i$

↗ on-shell of $\mathrm{d}_A e_i = 0$

By means of the PB $\{e^i_a(x), e^j_b(y)\} = \kappa\gamma\, \delta^{ij} \epsilon_{ab} \delta^2(x,y)$ and solving the condition $F^i(A) = 2\pi K^i \delta(x)$

in the neighborhood of the puncture and fixing the gauge freedom: $A = K_p \mathrm{d}\theta$

(in this gauge the gauge field is constant and the fields are periodic)



$$\Rightarrow \quad \{Q_p(\varphi), Q_{p'}(\psi)\} = \frac{\delta_{pp'}}{2\pi} \oint_{C_p} (\varphi^i \mathrm{d}\psi_i - K^i_p [\varphi, \psi]_i \mathrm{d}\theta)$$

By defining the modes $Q^j_n \equiv Q(\tau^j e^{i\theta n})$, where $[\tau^i, \tau^j] = \epsilon^{ijk} \tau_k$ anti-hermitian basis

$$➡ \quad \{Q^i_n, Q^j_m\} = -i \left(n\delta^{ij} + K^{ij} \right) \delta_{n+m} \quad \text{, where} \quad K^{ij} \equiv -i\epsilon^{ijk} K_k$$

✦ In the case where the curvature vanishes we simply get $\{Q^i_n, Q^j_m\} = -in\delta^{ij}\delta_{n+m}$

☞ $U(1)^3$ Kac-Moody algebra with central extension equal to 1

✦ In the presence of curvature, we obtained a three dimensional abelian Kac-Moody algebra twisted by K :

Let us work in a complex basis: $\tau^a = (\tau^3, \tau^+, \tau^-)$, $\tau^\pm = (\tau^1 \mp i\tau^2)/\sqrt{2}$, $[\tau^3, \tau^\pm] = \pm i\tau^\pm$, $[\tau^+, \tau^-] = i\tau^3$

where $K = k\tau_3$ and $K^{a\bar{b}}$ is diagonal, with $(a = 3, +, -)$, $\bar{a} = (3, -, +)$

then the twisted Kac-Moody algebra can then be written compactly as $(\{\cdot,\cdot\} \to -i[\cdot,\cdot])$:

$$\boxed{[Q^a_n, Q^b_m] = \delta^{a\bar{b}}(n + k^a)\delta_{n+m}}$$

where $k^a := (0, +k, -k)$, $\sum_a k_a = 0$

A Kac-Moody algebraic structure follows directly from the gravitational symplectic structure when distributional configurations (punctures) are considered

The theory associated with k and with $k + 1$ are **equivalent**.
This equivalence corresponds to the fact that at the quantum level the connection is compactified, a fact that here is derived completely naturally in the continuum.

- We will restrict in the following to $k \in \mathbb{Z}/N$ for some integer N

Virasoro generators

- Generator of boundary diffeomorphisms along a vector field $v^a \partial_a$ tangent to S^2:

$$\Omega_D(L_v, \delta) = \delta T_D \quad \text{, where} \quad L_v e^i := v \lrcorner \mathrm{d}_A e^i + \mathrm{d}_A (v \lrcorner e^i)$$

$$\rightarrow \quad T_D = \frac{1}{2\kappa\gamma} \int_D L_v e^i \wedge e_i \hat{=} \frac{1}{2\kappa\gamma} \oint_{\partial D_p} (v \lrcorner e^i) e_i$$

We can introduce the modes $L_n^{(p)} := T_{D_p}(\exp(i\theta n)\partial_\theta)$, explicitly

$$L_n = \frac{1}{2\pi} \oint e^{i\theta n} \, T_{\theta\theta} \, \mathrm{d}\theta \quad \text{, where} \quad T_{\theta\theta} = \frac{\pi e^i_\theta e_{\theta i}}{\kappa\gamma}$$

The SET modes can be obtained from the Kac-Moody modes Q^a_n through the Sugawara construction:

At the quantum level $L_n = \frac{1}{2} \sum_a \sum_{m \in \mathbb{Z}} : Q^a_m Q^{\bar{a}}_{n-m} :$, where $: Q^a_n Q^b_m : = \begin{cases} Q^b_m Q^a_n & \text{if } n + k^a > 0 \\ Q^a_n Q^b_m & \text{if } n + k^a \leq 0 \end{cases}$

$$\Rightarrow \quad [L_n, L_m] = (n - m) L_{n+m} + \frac{c}{12} n(n^2 - 1) \delta_{n+m,0}$$

On the completely algebraic level, we obtain a Virasoro algebra with $c = 3$

$$[L_n, Q^a_m] = -(m + k^a) Q^a_{n+m}$$

the currents are primary fields of weight 1 twisted by k

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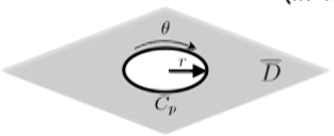

$$\Rightarrow \quad \{Q_p(\varphi), Q_{p'}(\psi)\} = \frac{\delta_{pp'}}{2\pi} \oint_{C_p} \left(\varphi^i \mathrm{d}\psi_i - K^i_p[\varphi,\psi]_i \mathrm{d}\theta\right)$$

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Intertwiner

Can we recover the SU(2) local symmetry algebra generated by Σ^i for transformations that affects only the boundary modes while leaving the background fields invariant?

Yes, when the curvature is integer-valued. Concretely, when $k = 0$, we introduce

$$M^i = \epsilon^i{}_{jk} \sum_{n \neq 0} \frac{Q^j_n Q^k_{-n}}{2n} \quad \rightarrow \quad [M^i, M^j] = \epsilon^{ij}{}_k \, M^k$$

Infinite dimensional analog of the Schwinger representation of the generators of rotations: a new representation of the su(2) Lie algebra generators in terms of the U(1)^3 Kac-Moody ones

quanta of Flux at each puncture

$$M^i_p = \int_{D_p} \Sigma^i$$

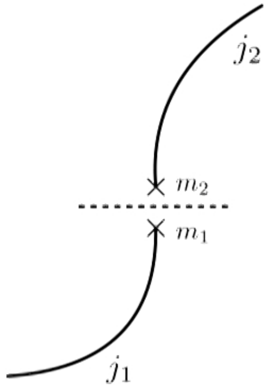
$[L_n, M^i] = 0$ ➥ Each puncture carries a representation of $Vir \times \mathrm{SU}(2)$

New boundary symmetry group whose associated charges represent new boundary observables

- In general $\sum_p L^p_n = -L^{\overline{D}}_n$, $\sum_p M^p = -M^{\overline{D}}$ → violation of the closure constraint

New vacuum that allows for curvature in the bulk: `Virasoro' intertwiner

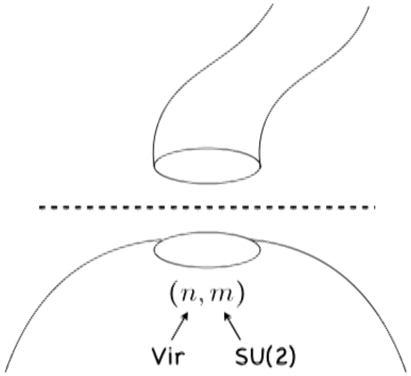
SU(2) boundary symmetry



$$\rightarrow \quad S_E = \log(d_j)$$

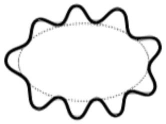
[Donnelly '08]

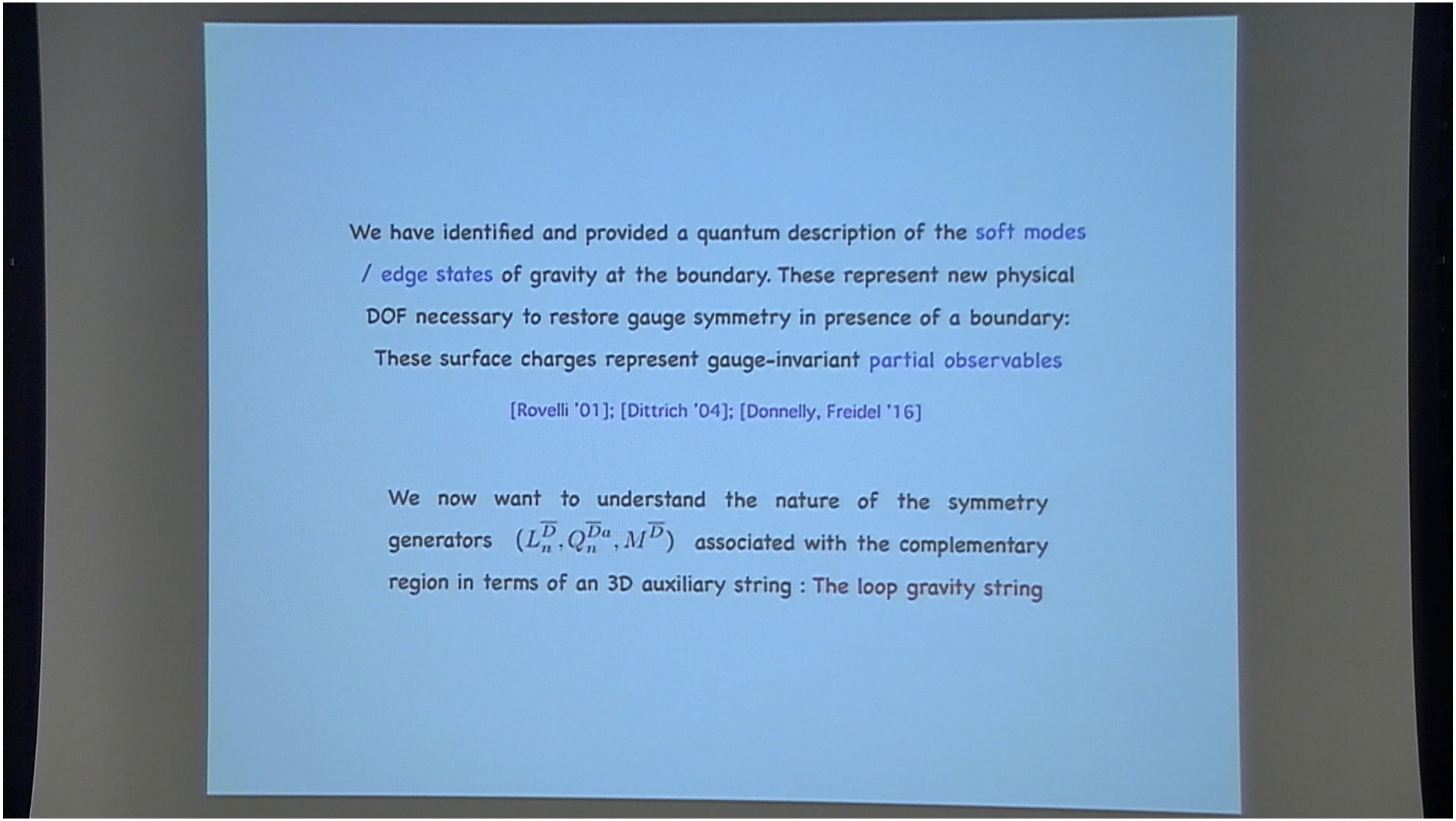
Vir x SU(2) boundary symmetry



The shapes of the circle (the edge modes) encode entanglement

$$\rightarrow \quad S_E = S(\Delta_0(j))$$

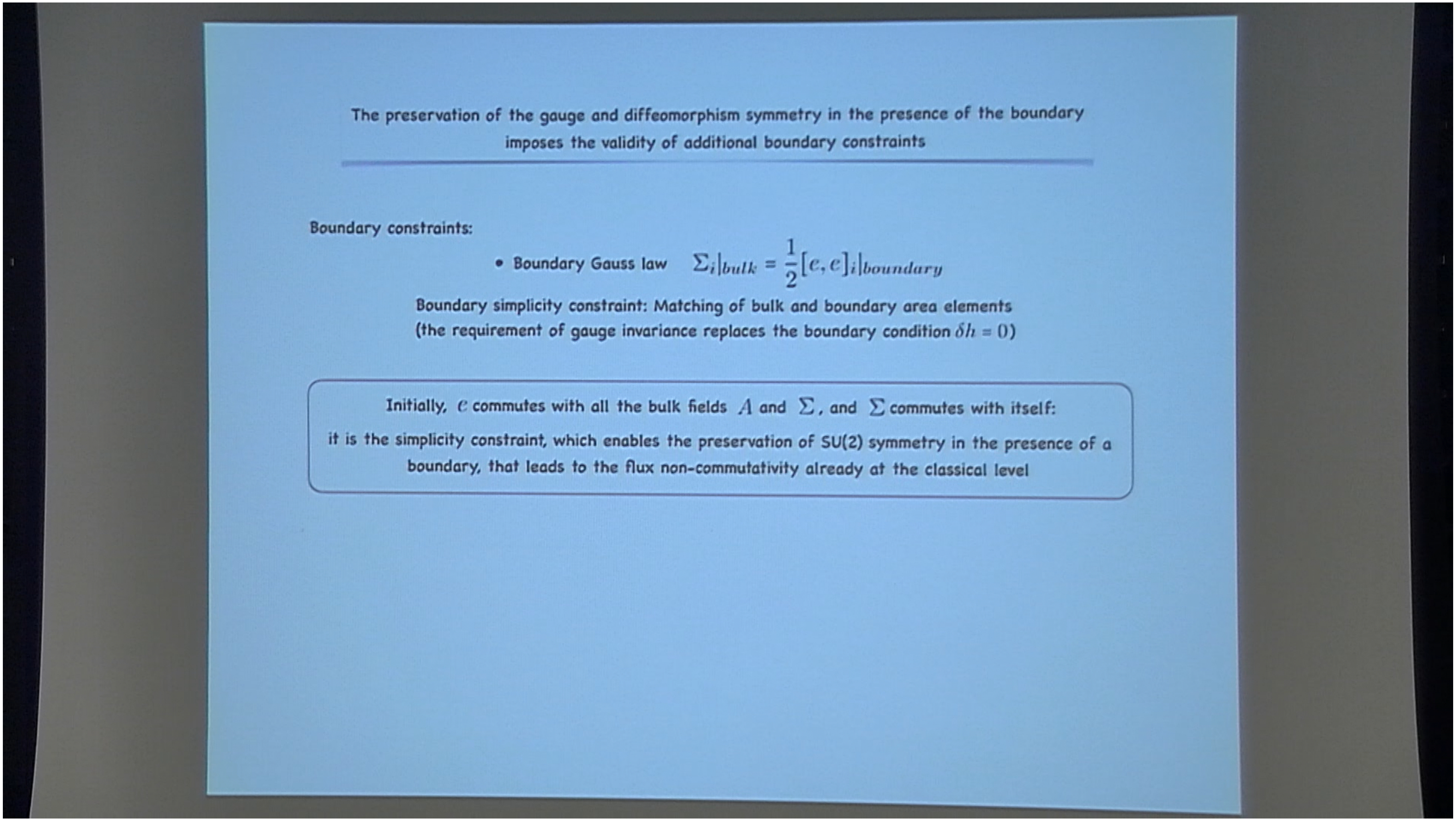




We have identified and provided a quantum description of the soft modes / edge states of gravity at the boundary. These represent new physical DOF necessary to restore gauge symmetry in presence of a boundary: These surface charges represent gauge-invariant partial observables

[Rovelli '01]; [Dittrich '04]; [Donnelly, Freidel '16]

We now want to understand the nature of the symmetry generators $(L_n^{\overline{D}}, Q_n^{\overline{D}a}, M^{\overline{D}})$ associated with the complementary region in terms of an 3D auxiliary string : The loop gravity string



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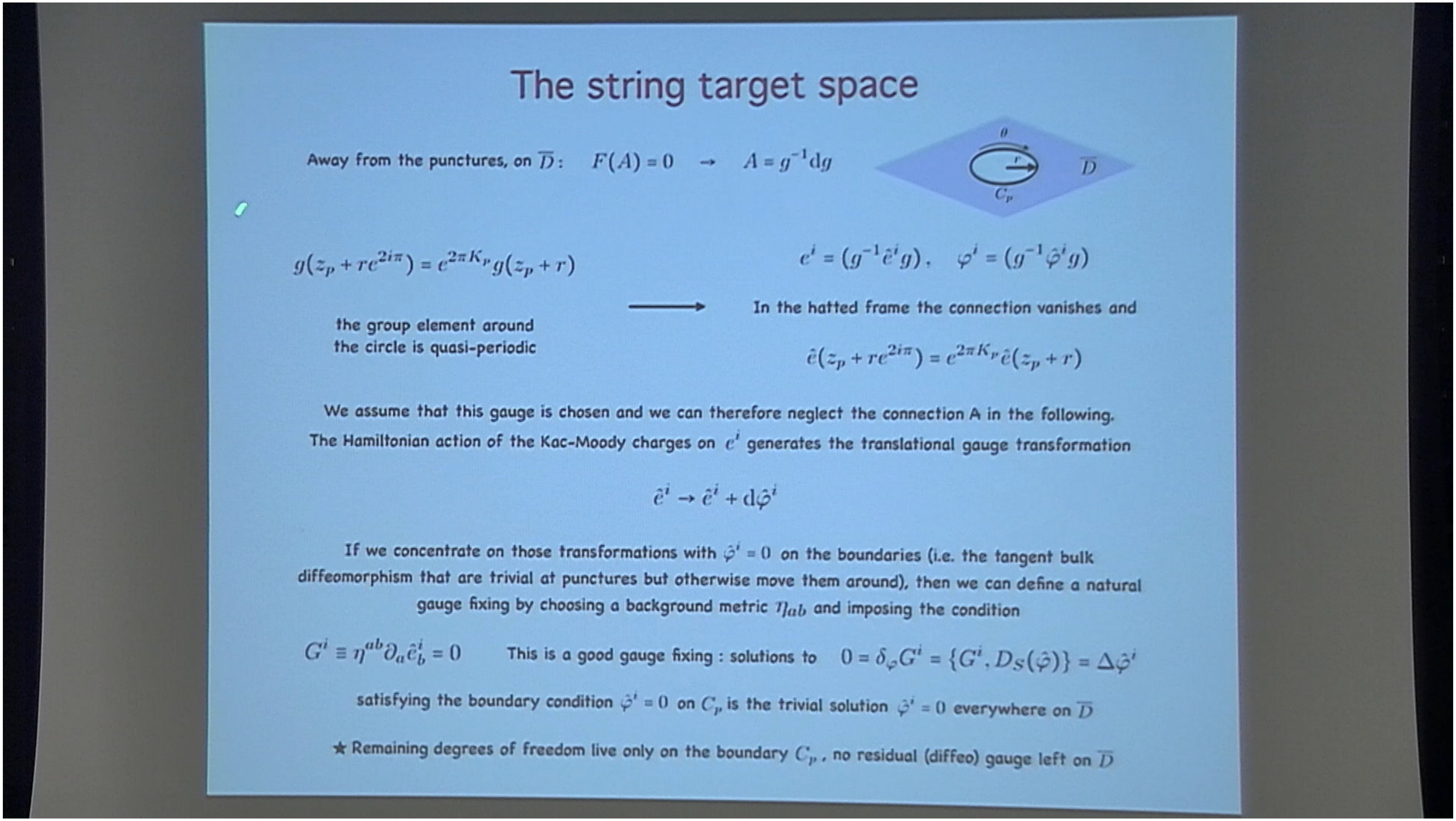


The string target space

Away from the punctures, on $\overline{D}$: $F(A) = 0 \rightarrow A = g^{-1}\mathrm{d}g$

$$g(z_p + re^{2i\pi}) = e^{2\pi K_p} g(z_p + r)$$

the group element around
the circle is quasi-periodic

$\longrightarrow$

$$e^i = (g^{-1}\hat{e}^i g)\,, \quad \varphi^i = (g^{-1}\hat{\varphi}^i g)$$

In the hatted frame the connection vanishes and

$$\hat{e}(z_p + re^{2i\pi}) = e^{2\pi K_p}\hat{e}(z_p + r)$$

We assume that this gauge is chosen and we can therefore neglect the connection A in the following.

The Hamiltonian action of the Kac-Moody charges on e^i generates the translational gauge transformation

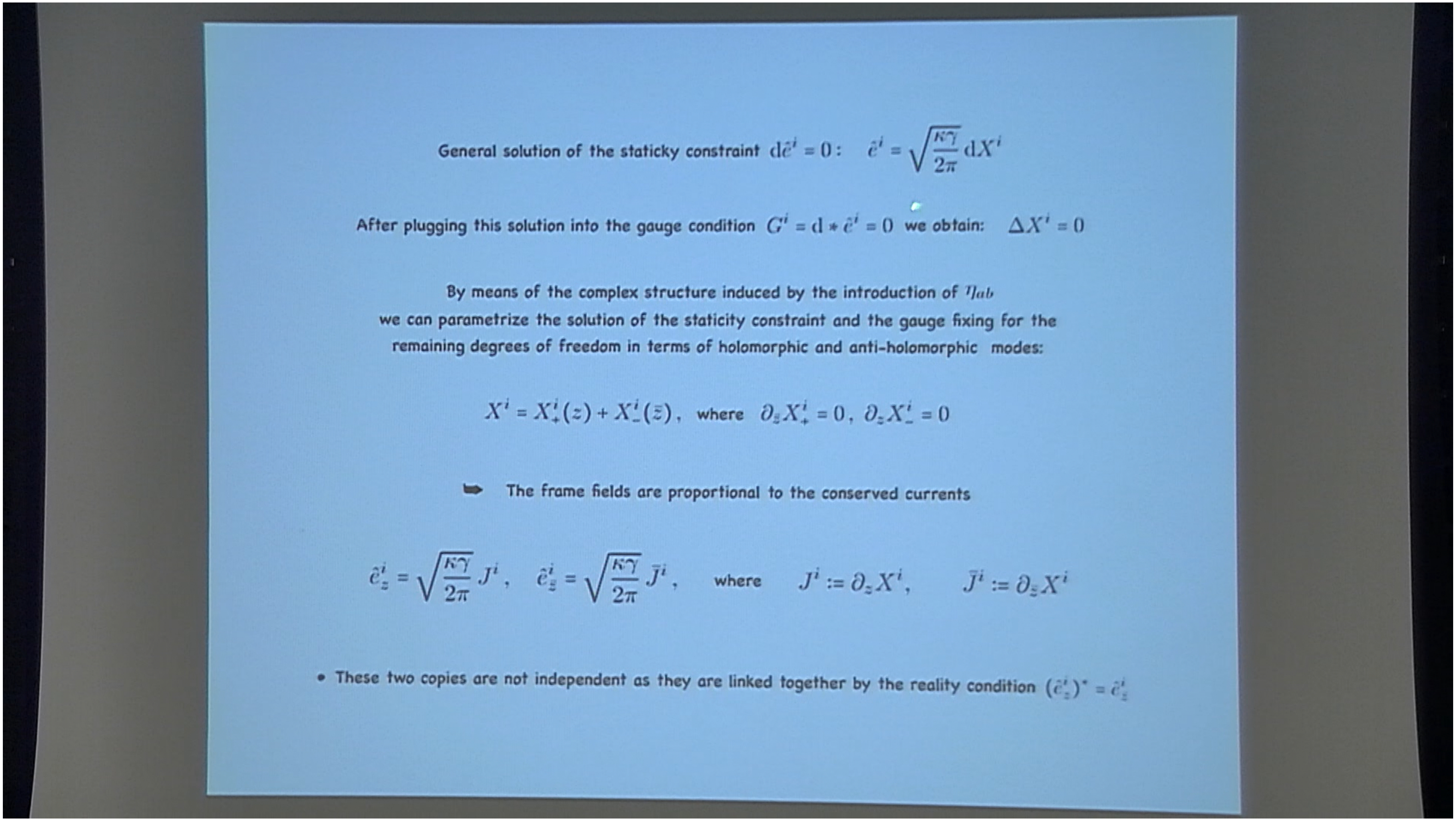
$$\hat{e}^i \rightarrow \hat{e}^i + \mathrm{d}\hat{\varphi}^i$$

If we concentrate on those transformations with $\hat{\varphi}^i = 0$ on the boundaries (i.e. the tangent bulk diffeomorphism that are trivial at punctures but otherwise move them around), then we can define a natural gauge fixing by choosing a background metric η_{ab} and imposing the condition

$G^i \equiv \eta^{ab}\partial_a \hat{e}^i_b = 0$ This is a good gauge fixing : solutions to $0 = \delta_\varphi G^i = \{G^i, D_S(\hat{\varphi})\} = \Delta\hat{\varphi}^i$

satisfying the boundary condition $\hat{\varphi}^i = 0$ on C_p is the trivial solution $\hat{\varphi}^i = 0$ everywhere on $\overline{D}$

★ Remaining degrees of freedom live only on the boundary C_p, no residual (diffeo) gauge left on $\overline{D}$



General solution of the staticky constraint $\mathrm{d}\hat{e}^i = 0$: $\hat{e}^i = \sqrt{\frac{\kappa\gamma}{2\pi}}\,\mathrm{d}X^i$

After plugging this solution into the gauge condition $G^i = \mathrm{d} * \hat{e}^i = 0$ we obtain: $\Delta X^i = 0$

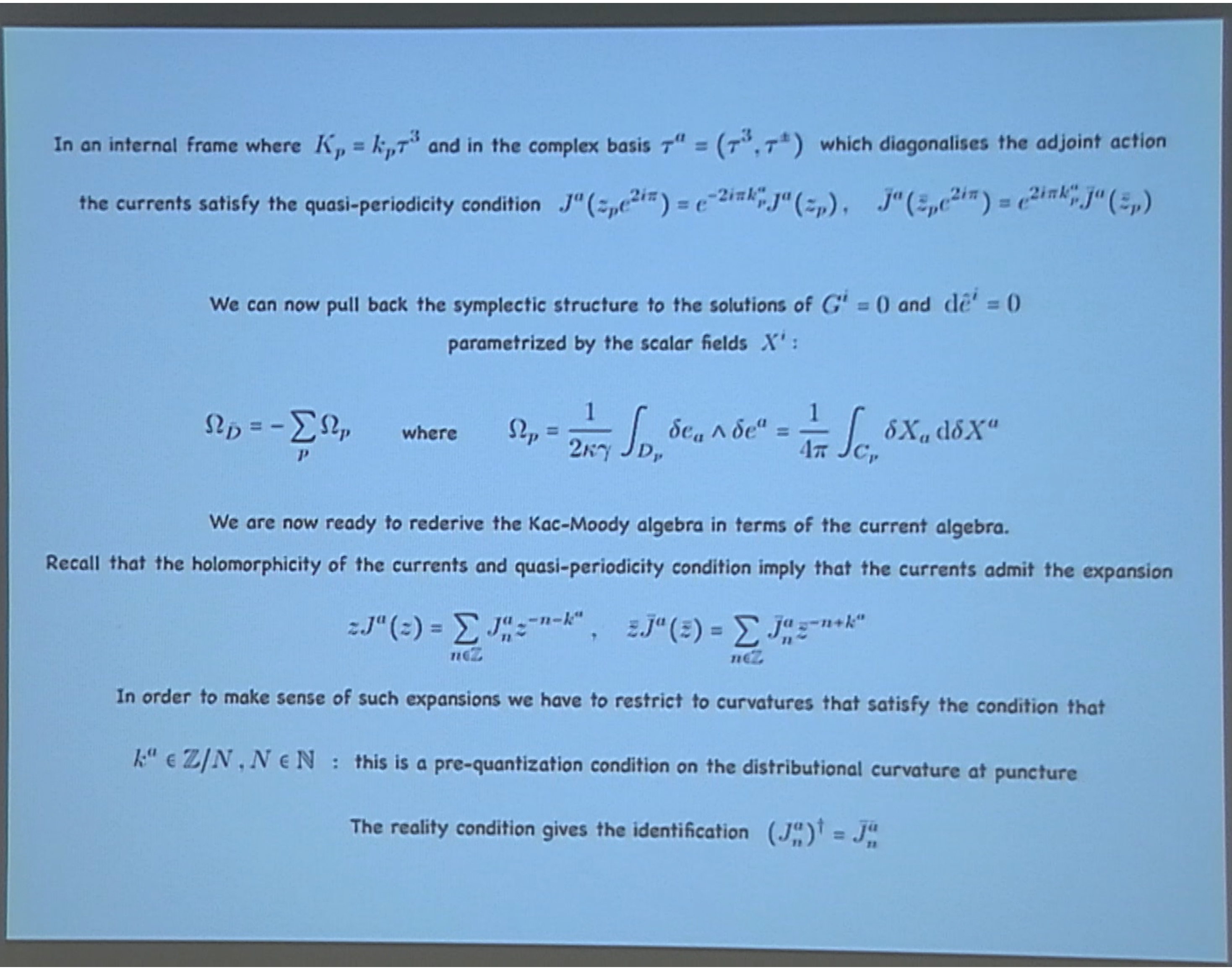
By means of the complex structure induced by the introduction of η_{ab}
we can parametrize the solution of the staticity constraint and the gauge fixing for the
remaining degrees of freedom in terms of holomorphic and anti-holomorphic modes:

$$X^i = X^i_+(z) + X^i_-(\bar{z}), \quad \text{where} \quad \partial_{\bar{z}} X^i_+ = 0, \ \partial_z X^i_- = 0$$

➡ The frame fields are proportional to the conserved currents

$$\hat{e}^i_z = \sqrt{\frac{\kappa\gamma}{2\pi}}\, J^i, \quad \hat{e}^i_{\bar{z}} = \sqrt{\frac{\kappa\gamma}{2\pi}}\, \bar{J}^i, \qquad \text{where} \qquad J^i := \partial_z X^i, \qquad \bar{J}^i := \partial_{\bar{z}} X^i$$

- These two copies are not independent as they are linked together by the reality condition $(\hat{e}^i_z)^* = \hat{e}^i_{\bar{z}}$



In an internal frame where $K_p = k_p \tau^3$ and in the complex basis $\tau^a = (\tau^3, \tau^\pm)$ which diagonalises the adjoint action

the currents satisfy the quasi-periodicity condition $J^a(z_p e^{2i\pi}) = e^{-2i\pi k_p^a} J^a(z_p)\,, \quad \bar{J}^a(\bar{z}_p e^{2i\pi}) = e^{2i\pi k_p^a} \bar{J}^a(\bar{z}_p)$

We can now pull back the symplectic structure to the solutions of $G^i = 0$ and $\mathrm{d}\hat{e}^i = 0$

parametrized by the scalar fields X^i :

$$\Omega_D = -\sum_p \Omega_p \qquad \text{where} \qquad \Omega_p = \frac{1}{2\kappa\gamma} \int_{D_p} \delta e_a \wedge \delta e^a = \frac{1}{4\pi} \int_{C_p} \delta X_a \, \mathrm{d}\delta X^a$$

We are now ready to rederive the Kac-Moody algebra in terms of the current algebra.

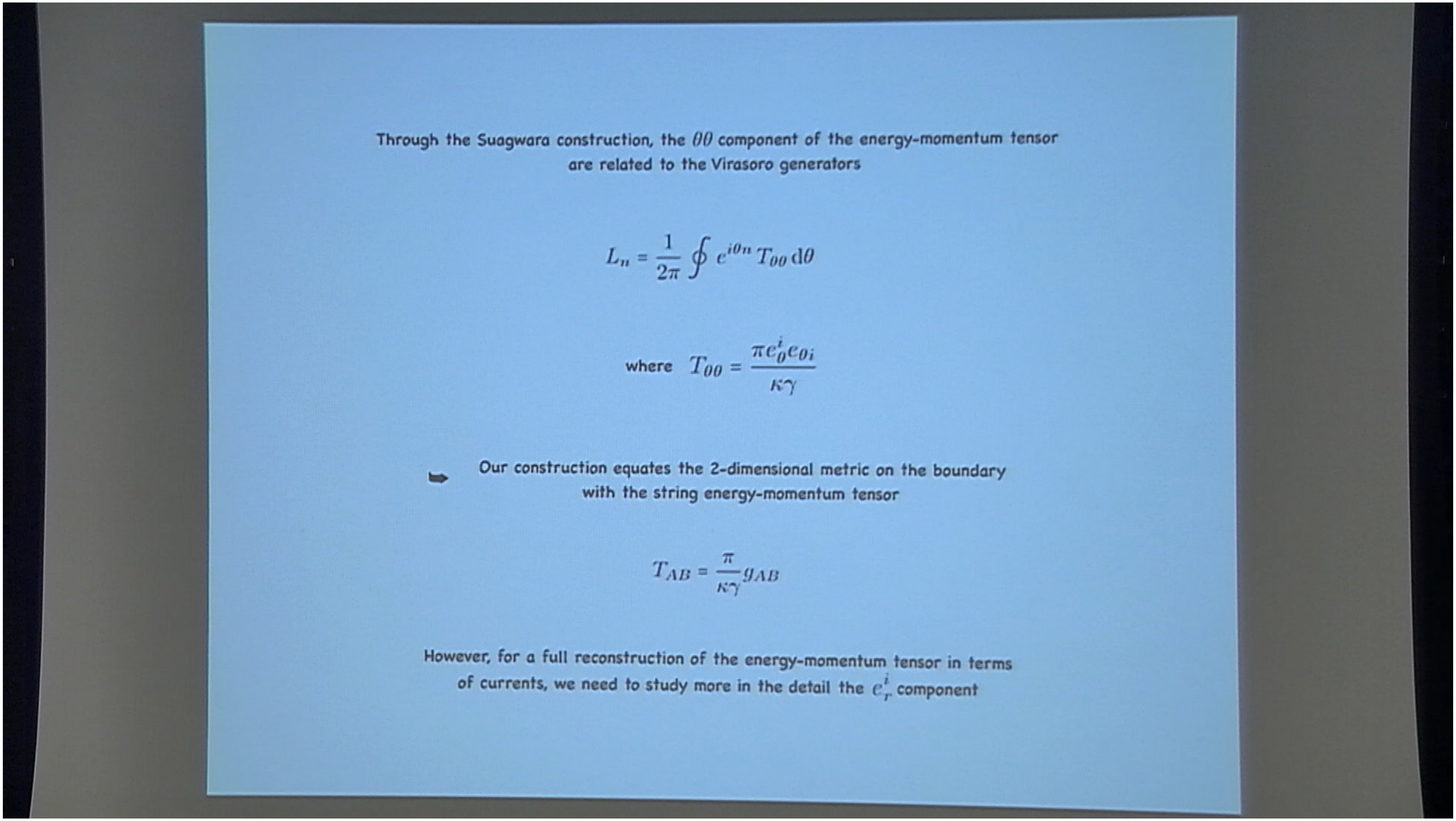
Recall that the holomorphicity of the currents and quasi-periodicity condition imply that the currents admit the expansion

$$z J^a(z) = \sum_{n \in \mathbb{Z}} J_n^a z^{-n-k^a}\,, \quad \bar{z} \bar{J}^a(\bar{z}) = \sum_{n \in \mathbb{Z}} \bar{J}_n^a \bar{z}^{-n+k^a}$$

In order to make sense of such expansions we have to restrict to curvatures that satisfy the condition that

$k^a \in \mathbb{Z}/N\,, N \in \mathbb{N}$: this is a pre-quantization condition on the distributional curvature at puncture

The reality condition gives the identification $(J_n^a)^\dagger = \bar{J}_n^a$



Through the Suagwara construction, the 00 component of the energy-momentum tensor are related to the Virasoro generators

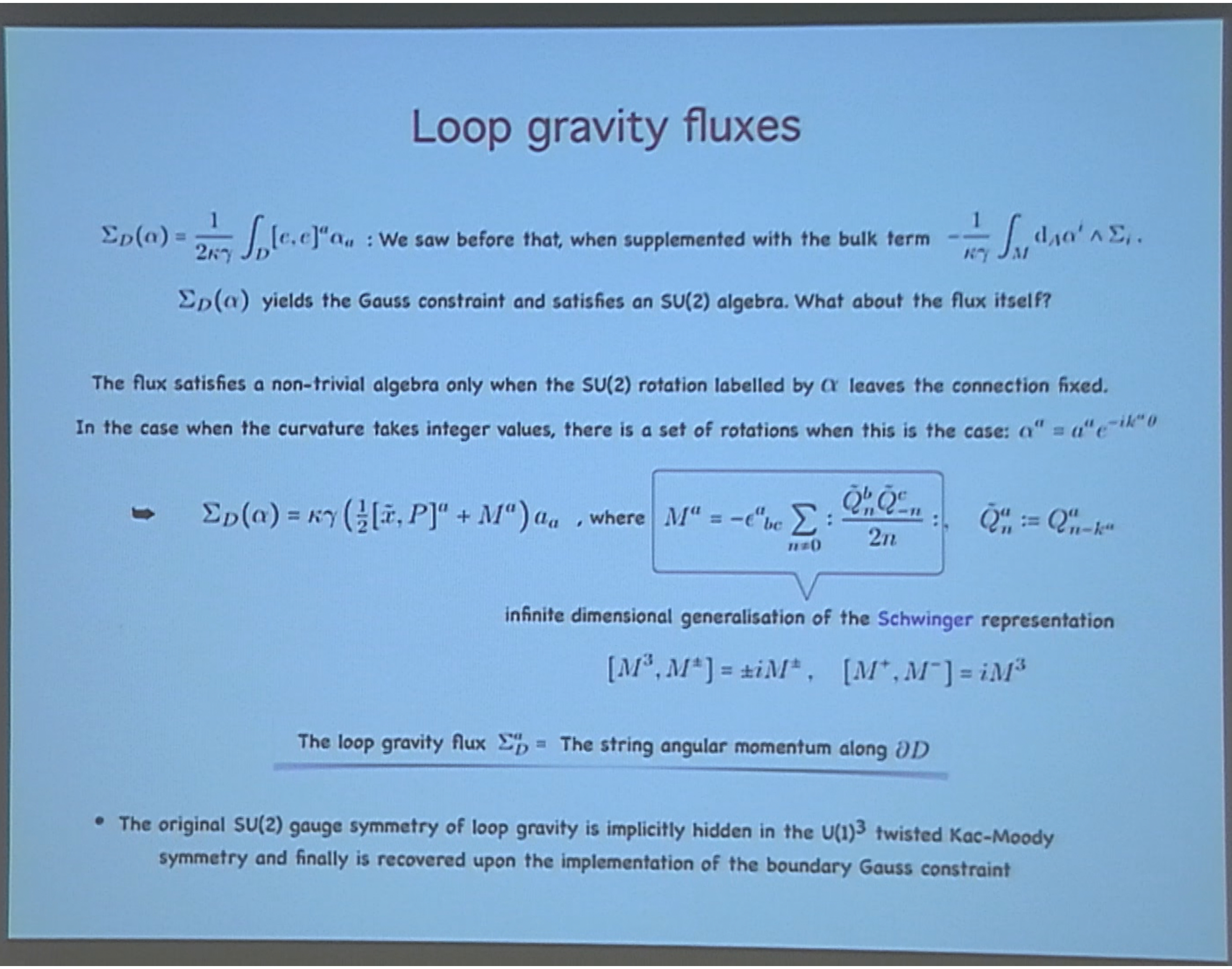
$$L_n = \frac{1}{2\pi} \oint e^{i0n}\, T_{00}\, \mathrm{d}0$$

where $T_{00} = \dfrac{\pi e^{i}_{0} e_{0i}}{\kappa\gamma}$

➨ Our construction equates the 2-dimensional metric on the boundary with the string energy-momentum tensor

$$T_{AB} = \frac{\pi}{\kappa\gamma} g_{AB}$$

However, for a full reconstruction of the energy-momentum tensor in terms of currents, we need to study more in the detail the e^{i}_{r} component



Loop gravity fluxes

$\Sigma_D(\alpha) = \frac{1}{2\kappa\gamma}\int_D [e,e]^a \alpha_a$: We saw before that, when supplemented with the bulk term $-\frac{1}{\kappa\gamma}\int_M \mathrm{d}_A\alpha^i \wedge \Sigma_i$.

$\Sigma_D(\alpha)$ yields the Gauss constraint and satisfies an SU(2) algebra. What about the flux itself?

The flux satisfies a non-trivial algebra only when the SU(2) rotation labelled by α leaves the connection fixed.

In the case when the curvature takes integer values, there is a set of rotations when this is the case: $\alpha^a = a^a e^{-ik^a\theta}$

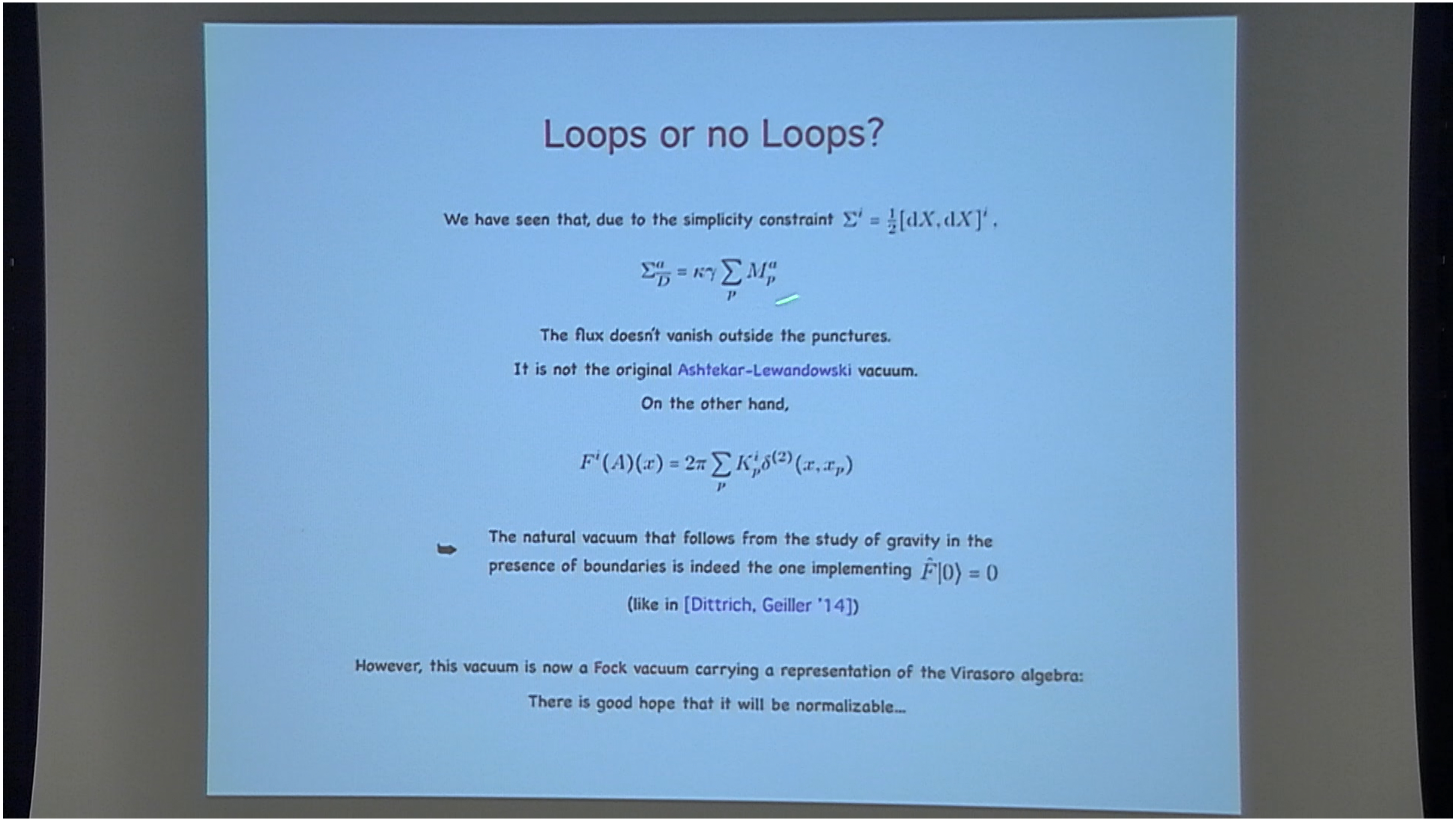
➡ $\Sigma_D(\alpha) = \kappa\gamma\left(\frac{1}{2}[\tilde{x}, P]^a + M^a\right)a_a$, where $\boxed{M^a = -\epsilon^a{}_{bc}\sum_{n\neq 0} : \frac{\tilde{Q}^b_n \tilde{Q}^c_{-n}}{2n} :}$, $\tilde{Q}^a_n := Q^a_{n-k^a}$

infinite dimensional generalisation of the Schwinger representation

$$[M^3, M^\pm] = \pm i M^\pm, \quad [M^+, M^-] = iM^3$$

The loop gravity flux Σ^a_D = The string angular momentum along ∂D

- The original SU(2) gauge symmetry of loop gravity is implicitly hidden in the U(1)^3 twisted Kac-Moody symmetry and finally is recovered upon the implementation of the boundary Gauss constraint



Loops or no Loops?

We have seen that, due to the simplicity constraint $\Sigma^i = \frac{1}{2}[\mathrm{d}X, \mathrm{d}X]^i$,

$$\Sigma^a_D = \kappa\gamma \sum_p M^a_p$$

The flux doesn't vanish outside the punctures.

It is not the original Ashtekar-Lewandowski vacuum.

On the other hand,

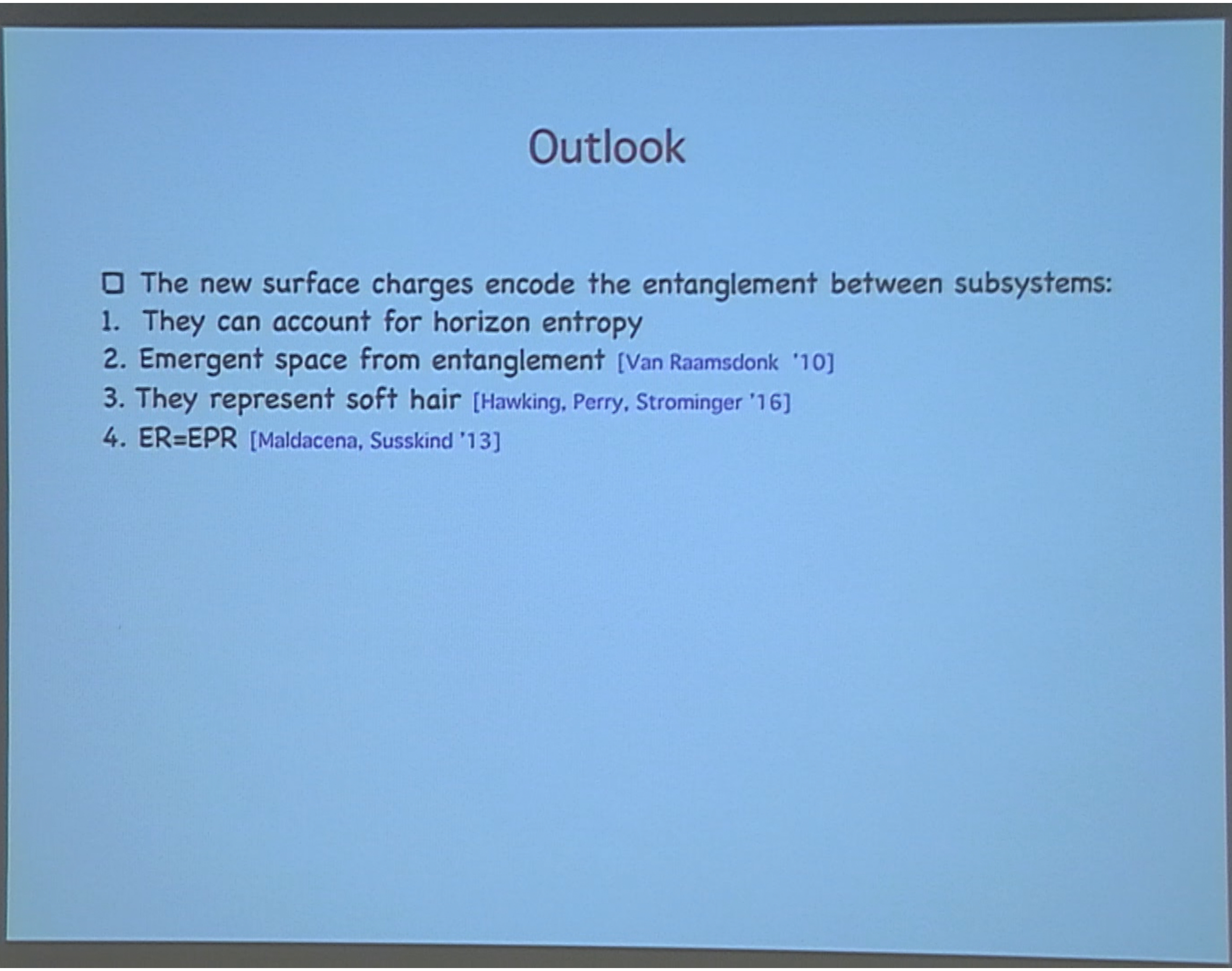
$$F^i(A)(x) = 2\pi \sum_p K^i_p \delta^{(2)}(x, x_p)$$

➡ The natural vacuum that follows from the study of gravity in the presence of boundaries is indeed the one implementing $\hat{F}|0\rangle = 0$

(like in [Dittrich, Geiller '14])

However, this vacuum is now a Fock vacuum carrying a representation of the Virasoro algebra:

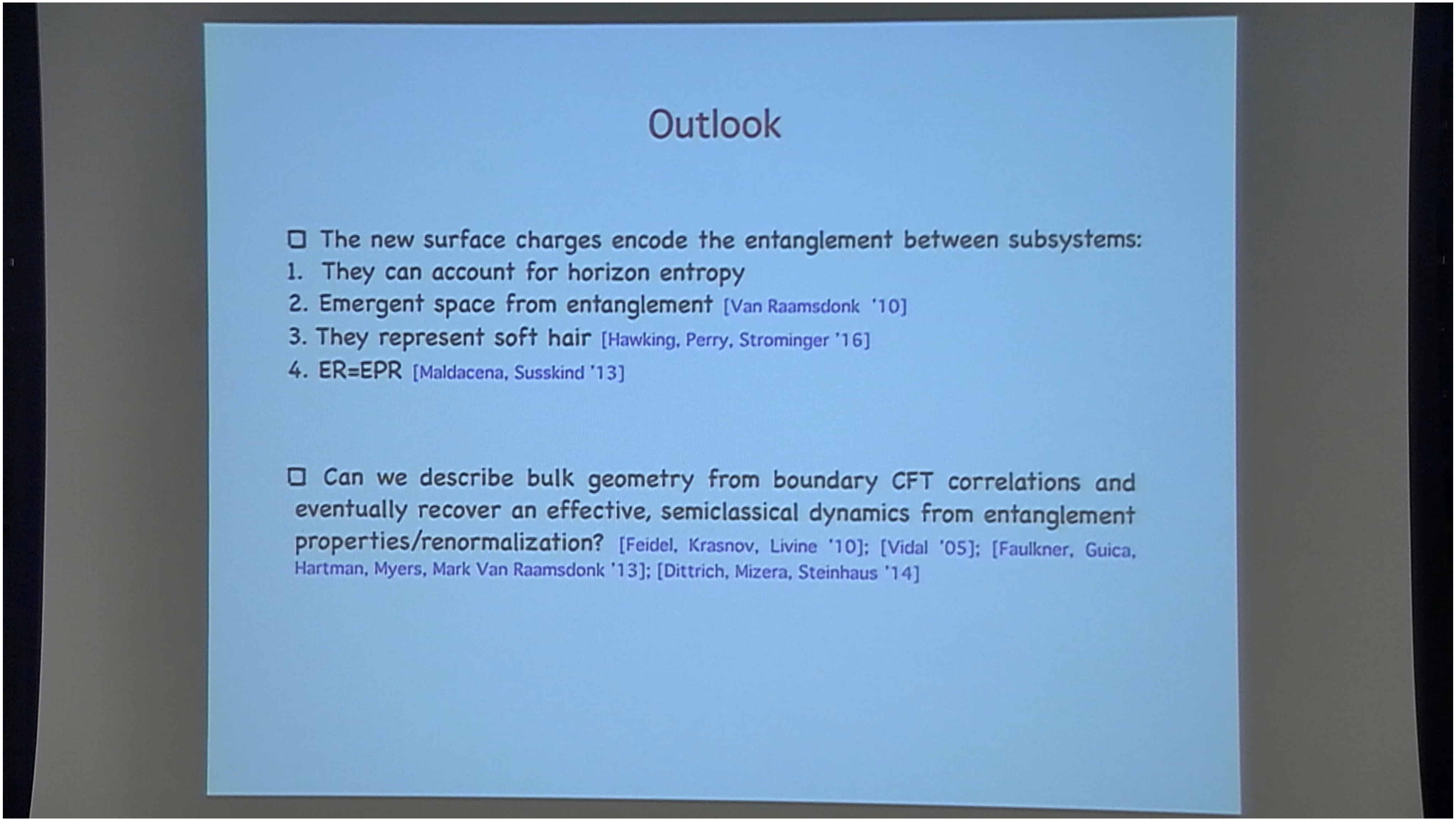
There is good hope that it will be normalizable...



Outlook

☐ The new surface charges encode the entanglement between subsystems:

1. They can account for horizon entropy
2. Emergent space from entanglement [Van Raamsdonk '10]
3. They represent soft hair [Hawking, Perry, Strominger '16]
4. ER=EPR [Maldacena, Susskind '13]



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☐ Can we describe bulk geometry from boundary CFT correlations and eventually recover an effective, semiclassical dynamics from entanglement properties/renormalization? [Feidel, Krasnov, Livine '10]; [Vidal '05]; [Faulkner, Guica, Hartman, Myers, Mark Van Raamsdonk '13]; [Dittrich, Mizera, Steinhaus '14]

Through the Suagwara construction, the $\theta\theta$ component of the energy-momentum tensor are related to the Virasoro generators

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