Title: A Modern Approach to Superradiance

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URL: http://pirsa.org/16110032

Abstract: <p>We introduce a simple and modern discussion of rotational superradiance based on quantum field theory. We work with an effective theory valid at scales much larger than the size of the spinning object responsible for superradiance. Within this framework, the probability of absorption by an object at rest completely determines the superradiant amplification rate when that same object is spinning. We first discuss in detail superradiant scattering of spin 0 particles with orbital angular momentum l = 1, and then extend our analysis to higher values of orbital angular momentum and spin. Along the way, we provide a simple derivation of vacuum frictionâ€”a â€œquantum torqueâ€• acting on spinning objects in empty space. Our results apply not only to black holes but to arbitrary spinning objects. We also discuss superradiant instability due to formation of bound states and, as an illustration, we calculate the instability rate Î“ for bound states with massive spin 1 particles.</p>

Outline:

- What is superradiance? What is it not
- Why think about it now?
- Build an EFT of spinning objects
- …but first review SSB and coset construction
- Superradiance from the EFT perspective
- Bound states (+ LIGO + beyond the SM physics)

What is (rotational) Superradiance?

For relativists: Superradiance in the context of a kerr black holes

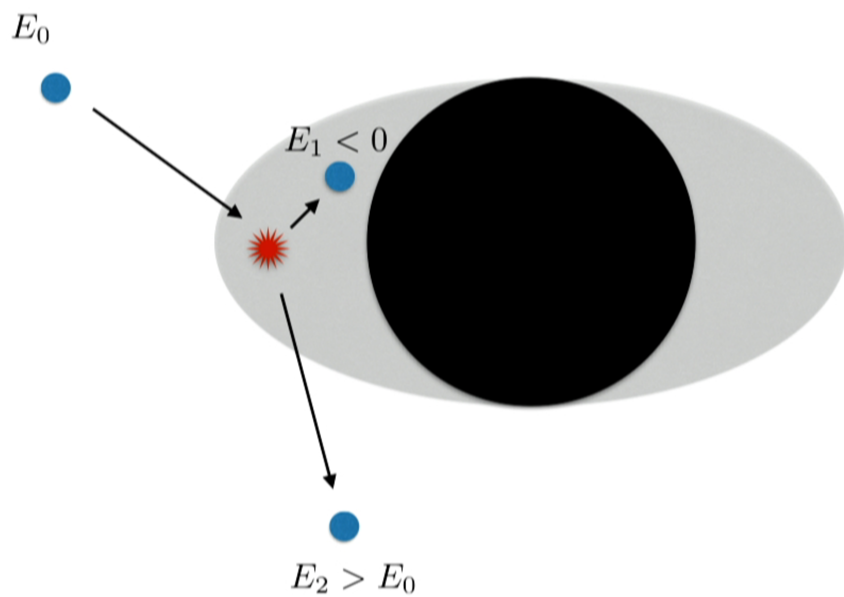
kerr solution ⇨ add linearized perturbations

What is (rotational) Superradiance?

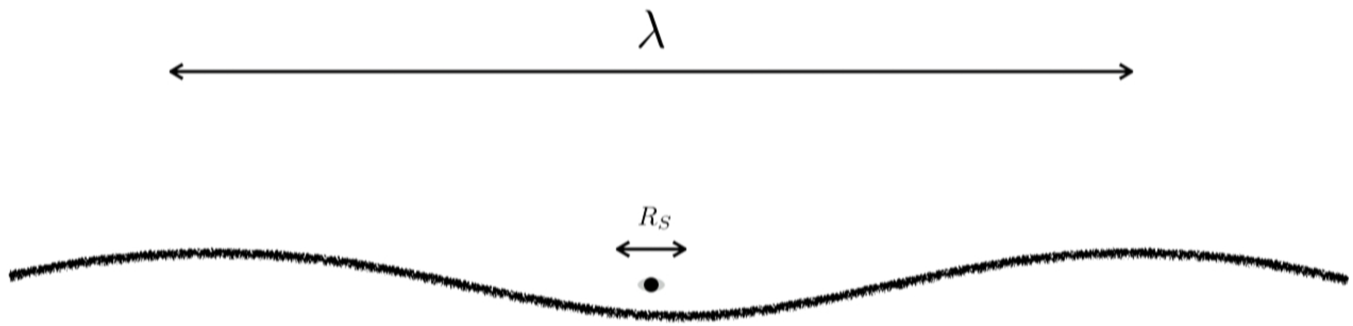
Solve Teukolsky equations and find something incredible

⇒ AMPLIFICATION for modes that satisfy the superradiant condition

Penrose process



(rotational) superradiance

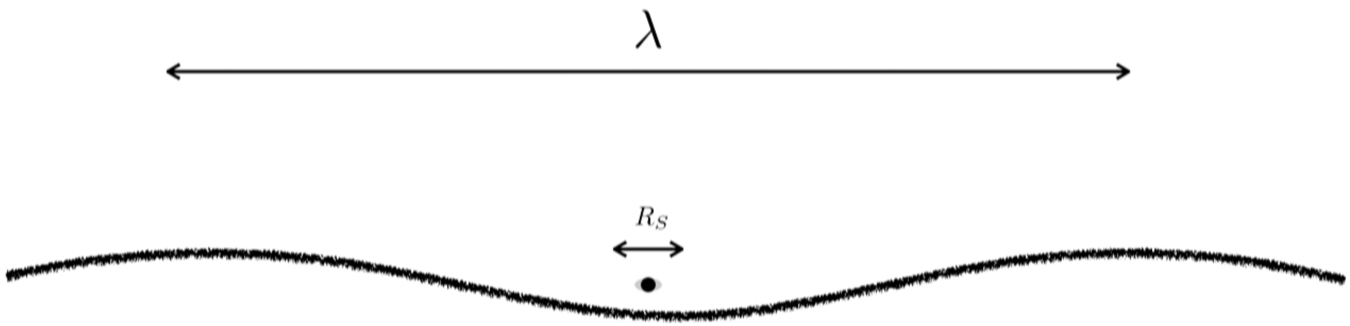


$$\lambda \gg R_S$$

$$|\omega, \ell, m\rangle \to (1 + A)\,|\omega, \ell, m\rangle$$

$$A < 0 \quad \Rightarrow \quad \text{Absorbtion}$$
$$A > 0 \quad \Rightarrow \quad \text{Amplification}$$

(rotational) superradiance



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$$\begin{array}{lcl} A < 0 & \Rightarrow & \text{Absorbtion} \\ A > 0 & \Rightarrow & \text{Amplification} \end{array}$$

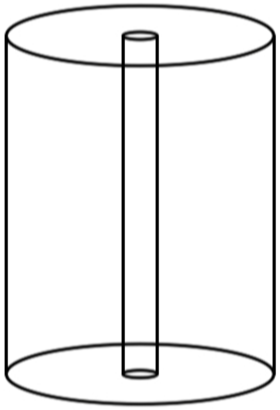
Two Claims:

1) Superradiance is a consequence of **rotation** + **dissipation**

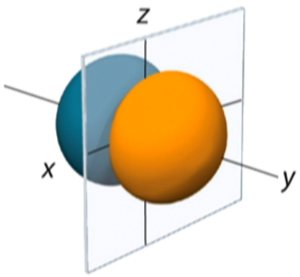
2) Superradiance follows from a tension between **absorption** + **stimulated emission**

Why care now?

- Black hole physics mined for the purposes of AdS/CFT — boundary acts as a mirror



- Axions and the like motivate thinking about light particles interacting with rotating BHs — mass acts as a mirror*



$(\ell = 1)$

*A. Arvanitaki, M. Baryakhtar, R. Lasenby and friends
hep-th/0905.4720 + hep-ph/1411.2263 + 1604.03958

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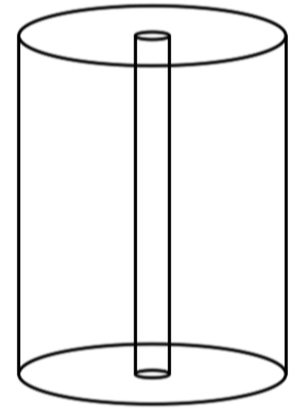

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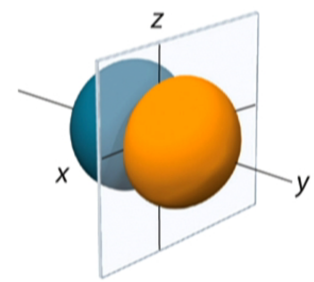

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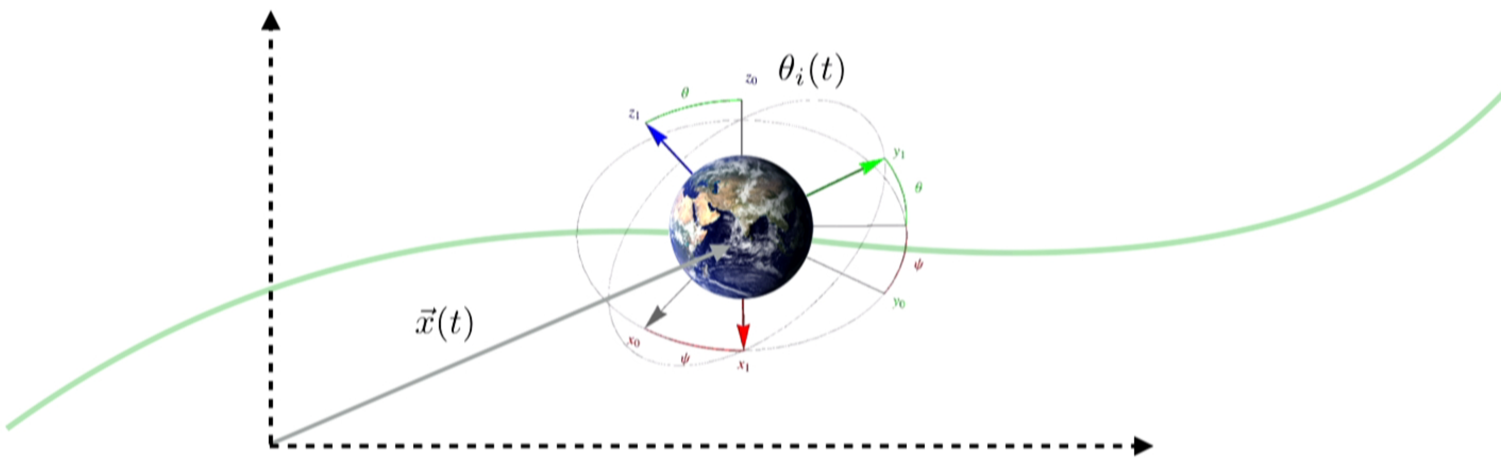
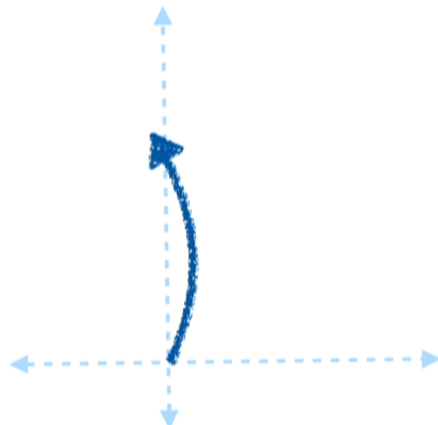
An EFT for (small) spinning objects*:

Want theory with $\mathrm{DOF} = \{\vec{x}(t), \theta_i(t)\}$ ⟹ Lorentz Invariance?



*Details can be found in L. Delacrétaz, **S. Endlich**, A. Monin, R. Penco, F. Riva: **hep-th/1405.7384**

Point particle:



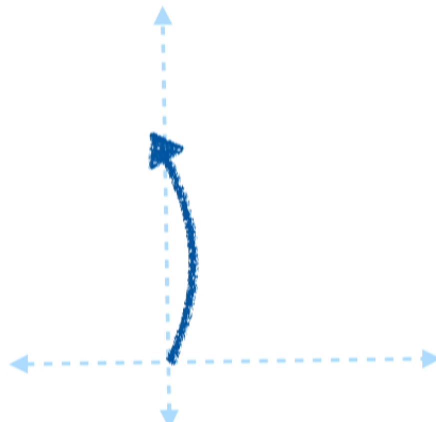
Add small perturbations:

$$(t(\lambda), \vec{x}(\lambda))$$

While obeying the constraint: $$\eta_{\mu\nu} \frac{dx^\mu(\lambda)}{d\lambda} \frac{dx^\nu(\lambda)}{d\lambda} = -1$$

$$-m \int d\tau \longrightarrow \int dt \left(-m + \frac{m}{2} v^2 + \cdots \right)$$

Point particle:



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how can we arrive at this directly?

SSB of spacetime symmetries—more recent literature:

Interesting (and rich) story

- Neilson and Chadha (1976)
- Schäfer, Son, Stephanov, Toublan, and Verbaarschot hep-ph/0108210 (2001)
- Low and Manohar hep-th/0110285 (2002)
- Watanabe and Brauner 1109.6327 (2011)
- Nicolis, and Piazza 1112.5174 (2011)
- Watanabe and Murayama 1203.0609 (2012)
- Hidaka 1203.1494 (2012)
- Nicolis, Penco, Piazza, and Rosen 1306.1240 (2013)
- Endlich, Nicolis, and Penco 1310.2272 (2013)
- Endlich, Nicolis, and Penco 1311.6491 (2013)
- Brauner, Endlich, Monin, and Penco 1407.7730 (2014)
- Nicolis, Penco, Piazza, and Rattazzi 1501.03845 (2015)
- etc.

“CliffsNotes” of that story using CCWZ(+V)*:

1. Identify symmetry breaking pattern:

$$G \to H$$

2. Build objects out of Goldstones that transform linearly

$$\mathcal{D}\pi \overset{g\in G}{\longrightarrow} h(g,\pi)\mathcal{D}\pi$$

3. Eliminate redundant** d.o.f.

$$\mathcal{D}\pi^A = 0 \longrightarrow \pi^B(\partial\pi^A)$$

4. Build a Lagrangian out of H invariant objects using $\{\mathcal{D}\pi, A\}$

*Callan, Coleman, Wess and Zumino + Volkov

**but not always; so-called “Inverse Higgs constraint”

In essence:

$$G \to H$$

$$\mathcal{L}(\partial\pi, \nabla)$$

invariant under full

$$G$$

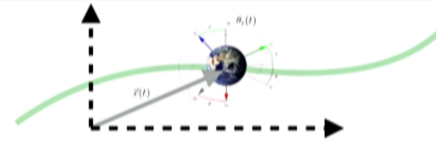
with Goldstones transforming non-linearly

+ missing ingredient: “external field coupling”

schematically $\tilde{R} \equiv \Omega^{-1}(\pi) \cdot R \implies \tilde{R} \xrightarrow{g \in G} h\tilde{R}$

w/ $\Omega(\pi) \equiv e^{i\pi^I T_I}$

An EFT for spinning objects:



1.

$$\text{Unbroken} \;=\; \begin{cases} P_0 & \text{time translations} \\ \bar{J}_{ij} = S_{ij} + J_{ij} & \text{spatial rotations} \end{cases}$$

$$\text{Broken} \;=\; \begin{cases} P_i & \text{spacial translations} \\ J_{ab} & \text{rotations and boosts} \end{cases}$$

w/ $S \subset SO(d)$

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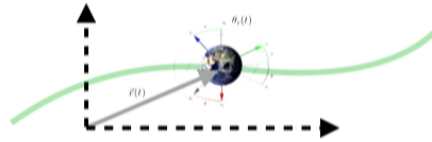

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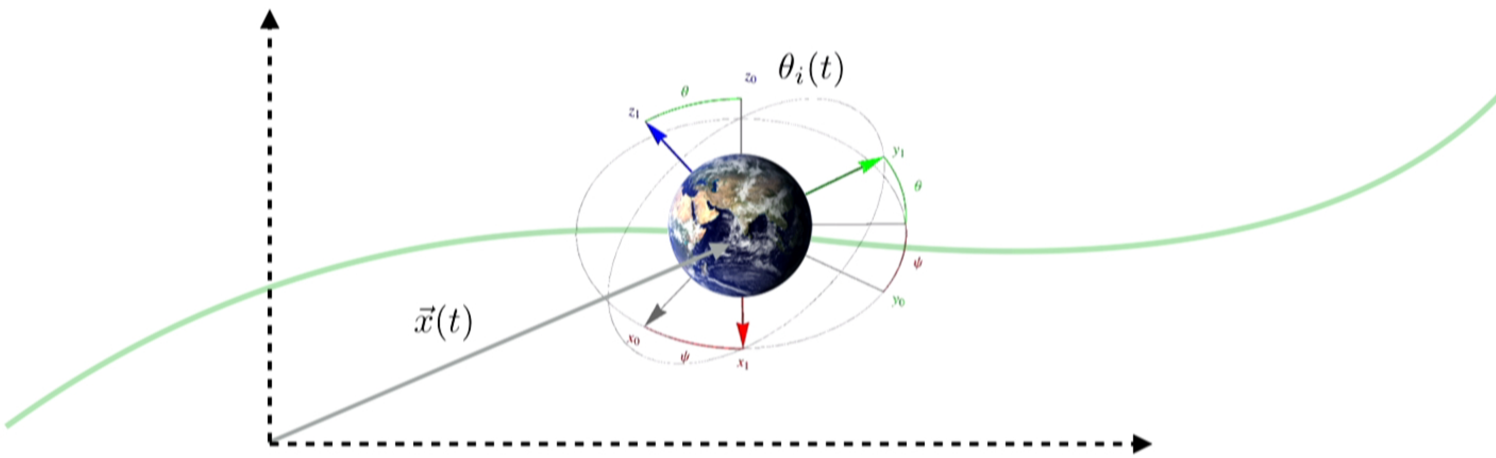
2 -> 4.

$$S = \int d\tau \left(-m + \frac{I_{ijkl}}{4} \mathcal{D}\alpha^{ij} \mathcal{D}\alpha^{kl} + \cdots \right)$$

w/ $\mathcal{D}\alpha = \Lambda^{-1} d_\tau \Lambda + u^\mu \Lambda^{-1} \omega_\mu \Lambda = \Omega$ **angular velocity**

$$\Lambda(\alpha) = \Lambda(\eta)\Lambda(\theta)$$

Missing light modes **are** dissipation



$$\{\vec{x}(t), \theta_i(t)\} + \{\psi_1(t), \psi_2(t), \cdots, \psi_\infty(t)\}$$

MUST have support at $\omega \to 0$

Superradiance is then just a **general** consequence of the fact that the spectral density of bosonic operators is an odd function of its argument, and that it is positive for positive arguments.

$$\Phi_{out} < \Phi_{in} \quad \text{when} \quad \omega - m\Omega > 0$$
$$\Phi_{out} > \Phi_{in} \quad \text{when} \quad \omega - m\Omega < 0$$

$$\frac{\Phi_{\rm out} - \Phi_{\rm in}}{\Phi_{\rm in}} = P_{\rm em} - P_{\rm abs} = -\frac{k^4}{6\pi\, v\, \omega}\, \rho(\omega - m\Omega) \qquad (\ell = 1)$$

w/ $\Delta(\omega) - \Delta(-\omega) \equiv \rho(\omega)$ **density of states**

Higher moments:

Leading interaction given by: $$H_{\rm int} = \partial^{I_1} \cdots \partial^{I_\ell} \phi \, R_{I_1}{}^{J_1} \cdots R_{I_\ell}{}^{J_\ell} \mathcal{O}_{J_1 \cdots J_\ell}$$

very similar calculation as before but with $$V_I^m \to V_{I_1 \cdots I_\ell}^m$$

$$\frac{\Phi_{\rm out} - \Phi_{\rm in}}{\Phi_{\rm in}} = P_{\rm em} - P_{\rm abs} = -\frac{\ell! \, k^{2\ell+2}}{2\pi(2\ell+1)!! v\omega} \, \rho_\ell(\omega - m\Omega) \, .$$

Higher spin similar…

Matching at low energies:

Density of states admits
low frequency expansion*:

$$\rho(\omega) \simeq \gamma\,\omega + O(\omega^3) \qquad \text{with} \qquad \gamma > 0$$

Extract this parameter from numerical simulations or analytical calculations in a simple, idealized problem

*More on dissipation **S. Endlich**, A. Nicolis, R. Porto, and J. Wang: **hep-th/1211.6461**

Bound states

More concretely

$$\hat{\phi} = \sum_{n\ell m} \frac{1}{\sqrt{2E_{n\ell m}}} \left\{ \hat{a}_{n\ell m}\, f_{n\ell m}(r,\theta,\varphi) e^{-iE_{n\ell m}t} + \hat{a}^{\dagger}_{n\ell m}\, f^{*}_{n\ell m}(r,\theta,\varphi) e^{iE_{n\ell m}t} \right\} + \cdots$$

To leading order in PN $\quad f_{n\ell m}(r,\theta,\varphi) = R_{n\ell}(r) Y_{\ell m}(\theta,\varphi)$

satisfy usual Hydrogenic Schrödinger Eqn w/ $\quad E^2_{n\ell m} \simeq \mu^2 \left[1 - \frac{\alpha^2}{n^2}\right] \qquad \ell + 1 \leqslant n$

Gravitational fine structure constant $\quad \boxed{\alpha \equiv GM\mu} \quad \ll 1$

*Arvanitaki, Dimopoulos, Dubovsky, Kaloper, and March-Russell: **hep-th/0905.4720**
and e.g. recent paper by Hui, Ostriker, Tremaine, and Witten: **astro-ph/1610.08297**

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Bound states — superradiant instability

Just as before compute $X_i + (n,\ell,m) \to X_f \qquad \ell = 1$

w/ $\quad H_{\rm int} = \partial^I \phi \, R_I{}^J \mathcal{O}_J$

Things follow as before, but now with

$$\Delta\Gamma = \Gamma_{\rm em} - \Gamma_{\rm abs} \simeq \left(\frac{GM\mu^2}{2}\right)^5 \frac{\rho(m\Omega - \mu)}{2\pi\mu} \simeq \left(\frac{GM\mu^2}{2}\right)^5 \frac{(m\Omega-\mu)\gamma}{2\pi\mu}$$

$$P_{\rm abs} = \int \frac{d\omega}{2\pi} \frac{\Delta(\omega)}{2\mu} \left| \int dt\, \partial^I f_{n\ell m}(r=0) R_I{}^J(t) e^{i(\omega-\mu)t} \right|^2$$

$$\Gamma_{\rm abs} = \frac{1}{2\pi\mu}\left(\frac{GM\mu^2}{2}\right)^5 \Delta(\mu - m\Omega) \qquad (n=2,\ \ell=1)$$

similarly $\quad \Gamma_{\rm em} = \frac{P_{\rm em}}{T} \simeq \frac{1}{2\pi\mu}\left(\frac{GM\mu^2}{2}\right)^5 \Delta(m\Omega - \mu) \qquad (n=2,\ \ell=1)$

Bound states — superradiant instability

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Using the matching we have from before

$$\Delta\Gamma \simeq \frac{(GM\mu)^9}{6}\Omega$$

Outlook/more things to talk about:

- bound states of **massive vectors** $\Delta\Gamma_{BH}^{vector} \sim (GM\mu)^7\Omega$
- use EFT to do **hard** computations (higher order in PN, mode mixing etc.)
- useful for general objects and for BHs in a regime where numerics is difficult (long time scales)
- LIGO observables and tests of fundamental physics

Just the beginning!