Title: A Stereoscopic Look into the Bulk

Date: Nov 08, 2016 02:00 PM

URL: http://pirsa.org/16110031

Abstract: <p>Abstract: We present the foundation for a holographic dictionary with depth perception. The dictionary consists of natural operators associated with CFT bilocals whose duals are simple, diffeomorphism-invariant bulk operators. These objects admit a description as fields in kinematic space, a phase space for such probes. The framework of kinematic space allows for conceptually simple derivations of many results known in the literature, including linearized Einsteinâ€™s equations, the relationship between conformal blocks and geodesic Witten diagrams, and the CFT representation of bulk local operators. Reference: https://arxiv.org/abs/1604.03110</p>

EFT

Successfully classifies the predictions of qualitatively different models (often in the form of consistency conditions):

- Einstein gravity – vacuum scalars – minimal slow-roll inflation:

$$\gamma \sim \frac{H}{M_{\rm pl}}, \qquad \zeta \sim \frac{H}{\sqrt{\epsilon} M_{\rm pl}} \quad \text{(notation)}$$

- Attractor single-field inflation (Maldacena's squeezed limit consistency)

$$f_{\rm NL,loc} = \frac{\langle \zeta(\mathbf{q}) \zeta_{\mathbf{k}} \zeta_{-\mathbf{k}} \rangle_{q \ll k}}{P_\zeta(q) P_\zeta(k)} = -\frac{5}{6}(n_s - 1)$$

There are EFTs of multifield, supersymmetric, solid, . . . inflation.

Could the microphysics become relevant in a profoundly different way?

A Stereoscopic Look into the Bulk

Sam McCandlish
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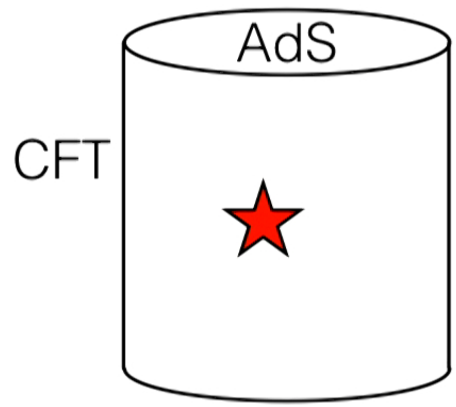
[1604.03110] [1608.06282] with Bartek Czech, Lampros Lamprou, Ben Mosk, James Sully

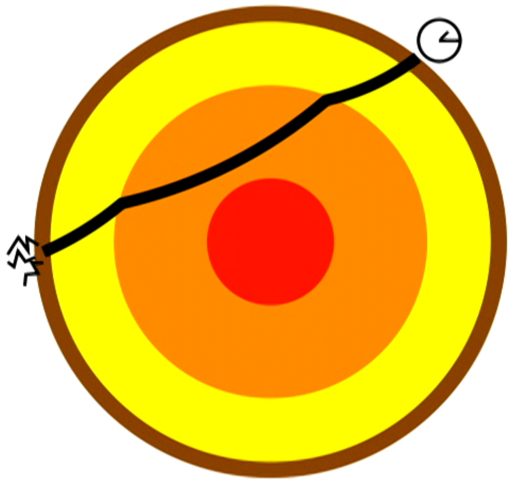
Related work: [1606.03307] by Jan de Boer, Felix Haehl, Michal Heller, Robert Myers
[1604.07373] [1610.08952] by Bruno Carneiro da Cunha, Monica Guica

How do we see into the bulk?



How to determine the structure of an AdS universe?

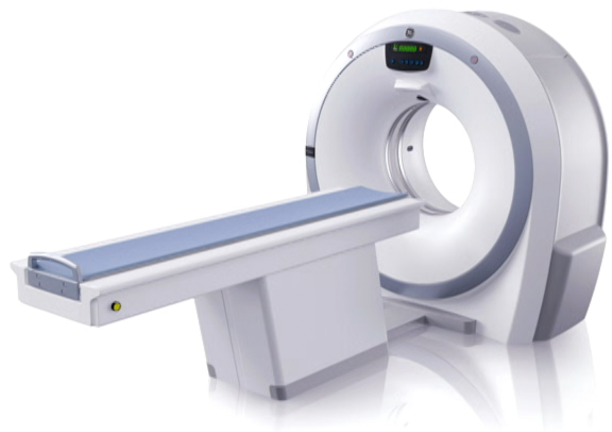
How do we see into the bulk?



How to determine the inner structure of the Earth?

Dig for a long time…
or use **travel time tomography**

How do we see into the bulk?



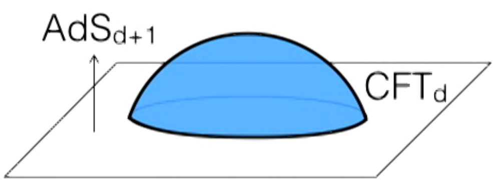
How to determine the inner structure of a person?

Invasive surgery…
or use **X-ray tomography**

How do we see into the bulk?

- The building blocks of quantum gravity are likely to be **extended probes** rather than local fields
- If AdS = CFT, these probes should be visible in the CFT
- **We know some** already:
  - Quantum entropy, Wilson loops, scalar eigenvalues

$$S_{\rm CFT} = A_{\rm min} + S_{\rm bulk} + \cdots$$



- But how do extended probes underly a **local** theory?
- We will use **bulk reconstruction** to understand this question

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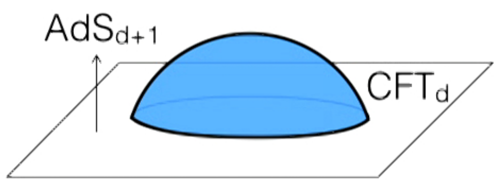


- But how do extended probes underly a **local** theory?
- We will use **bulk reconstruction** to understand this question

Bulk Reconstruction

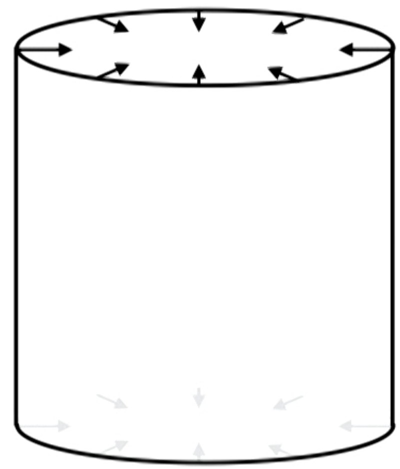
- Let ϕ be a free field in AdS

$$\Box\phi = m^2\phi$$

- The AdS/CFT dictionary relates ϕ to a CFT operator O

$$\phi(z,x) \sim z^{\Delta}\mathcal{O}(x)$$

- How do we solve this boundary value problem?



A **non-standard** Cauchy problem!

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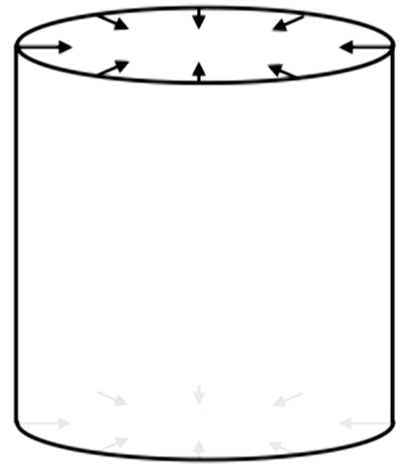


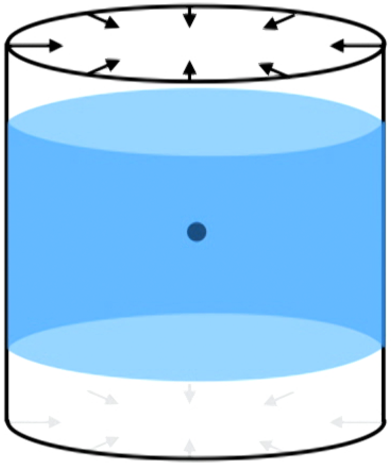
A **non-standard** Cauchy problem!

Bulk Reconstruction

- Standard HKLL construction: bulk local operator is a **smeared boundary operator**

$$\phi\,(z,x) = \int dx'\,K\,(x'\,|\,z,x)\,\mathcal{O}\,(x')$$

- The smearing function K is determined by "brute force" from the bulk mode expansion
- Some remaining questions:
  - How is a local operator constructed from gauge-invariant observables?
  - **How does a smeared operator "see into the bulk"?**



A non-standard Cauchy problem!

$$\Box\phi = m^2\phi$$

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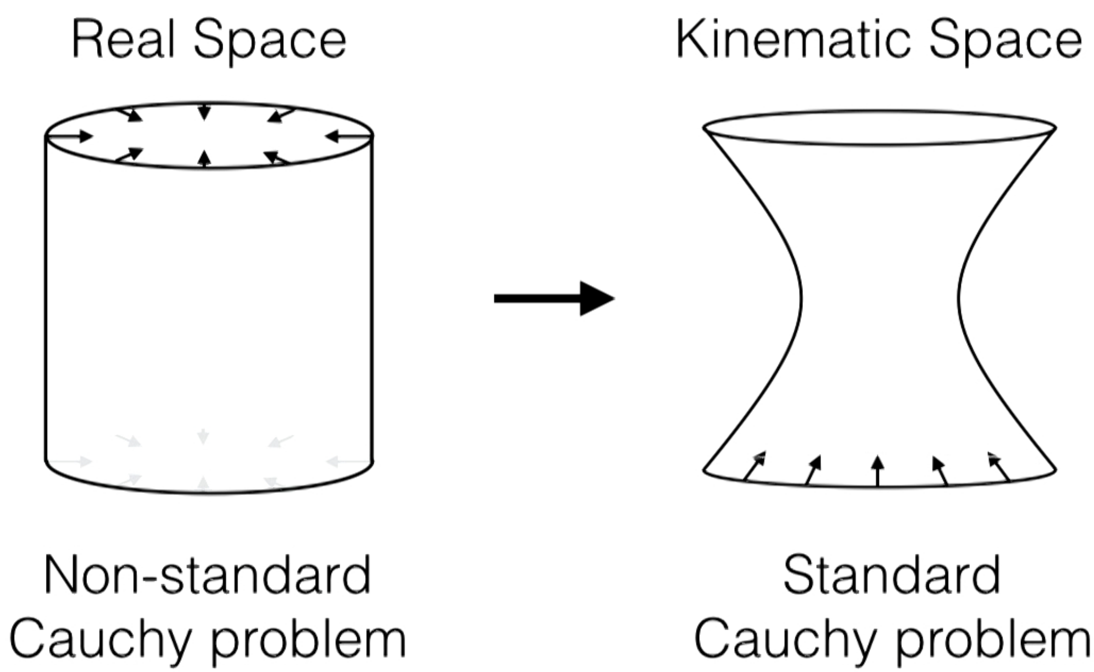
Bulk Reconstruction

- A detour through **kinematic space** will provide insight!



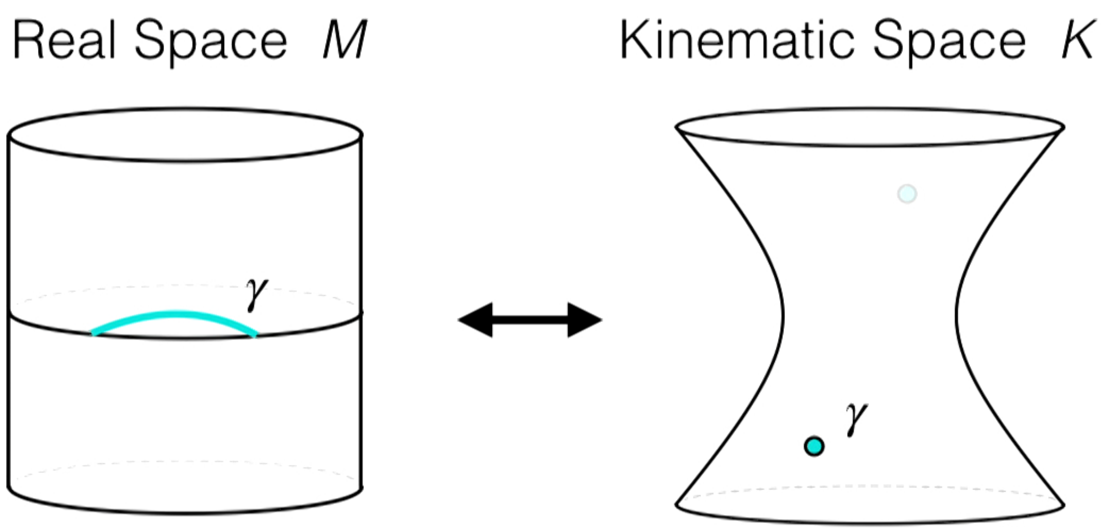
Plan

- Kinematic space: a phase space for gravitational probes
  - Geometric structure, Fields
  - Local bulk operators
  - Connection to the CFT operator product

- Generalizations and applications:
  - Modular Hamiltonian and Einstein's equations
  - Higher dimensions, surface operators
  - Interactions and dressed operators

Integral Geometry

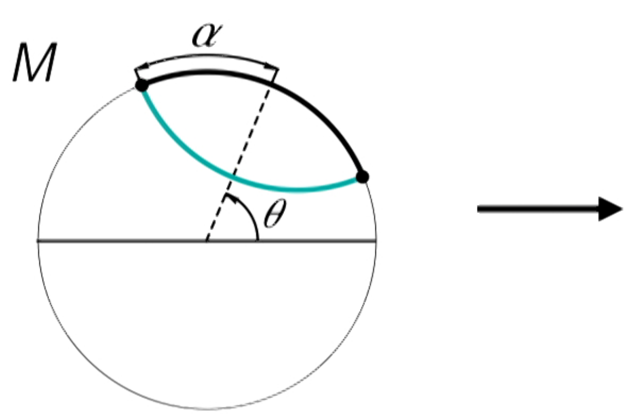
- **Kinematic space** *K*: the space of oriented spacelike geodesics in a manifold *M*
- A point in *K* corresponds to a geodesic in *M*
- Think of it as a phase space of natural bulk observables

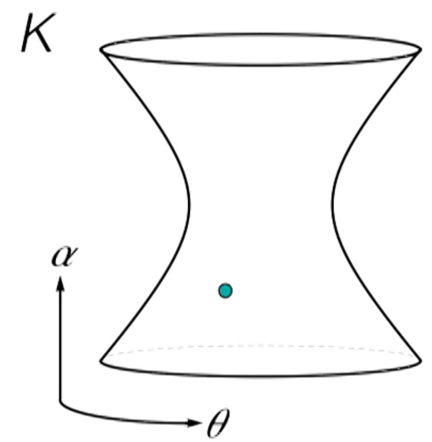


The Geodesic X-Ray Transform

- Example: let *M* be $\mathbb{H}_2$
- Parameterize geodesics by midpoint $\boldsymbol{\theta}$ and opening angle $\boldsymbol{\alpha}$

$$f(x) \longrightarrow Rf(\alpha,\theta) = \int_{\gamma_{\alpha,\theta}} ds\, f(x)$$





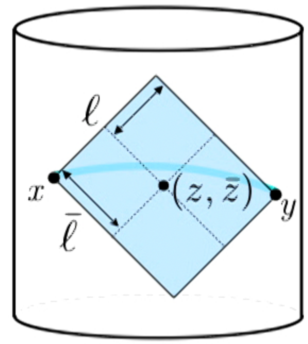
Structure of Kinematic Space

- We want to find an equation of motion for $R\phi(\gamma)$
- First, we must fix a metric on kinematic space - a **distance function** on the **space of geodesics**
- Kinematic Space for AdS_n or H_n is a highly symmetric space
  - No distinguished geodesics: all spacelike geodesics are related by symmetry
- We can fix a **unique metric** on *K* using this symmetry

Structure of Kinematic Space

- What is this metric space?

$$ds^2 = \frac{1}{(x-y)^2}\left(\eta_{\mu\nu} - 2\frac{(x-y)_\mu (x-y)_\nu}{(x-y)^2}\right) dx^\mu dy^\nu$$



- It is (2*d*)-dimensional - *d* space, *d* time

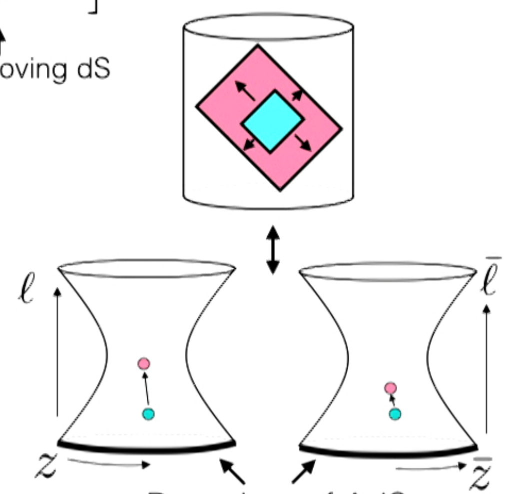
- For **AdS_3**, it is **dS_2 x dS_2**: $ds^2 = \frac{1}{2}\left[\frac{-d\ell^2 + dz^2}{\ell^2} + \frac{-d\bar{\ell}^2 + d\bar{z}^2}{\bar{\ell}^2}\right]$

  Left-moving dS　　Right-moving dS

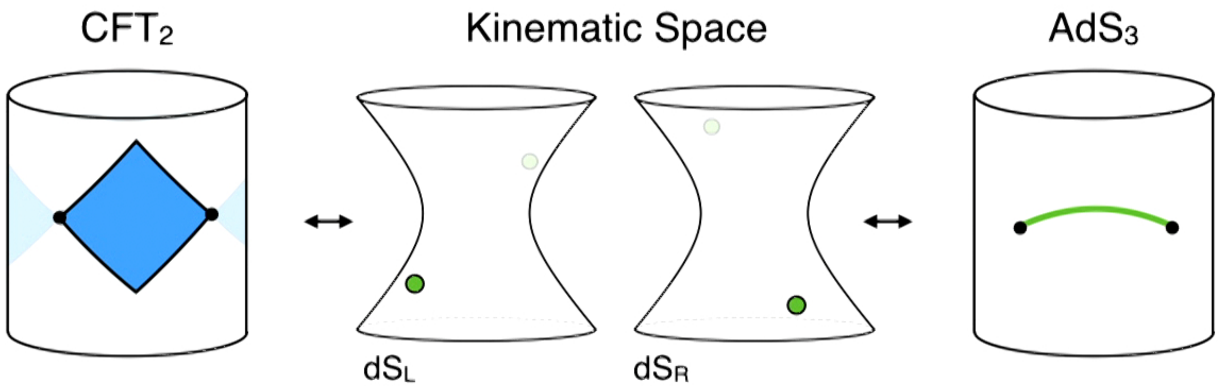
- For Hyperbolic 2-space, it is the diagonal dS_2

- The **causal structure** is determined by **containment** of boundary causal diamonds

- The **asymptotic past** of *K* is the **boundary** of AdS



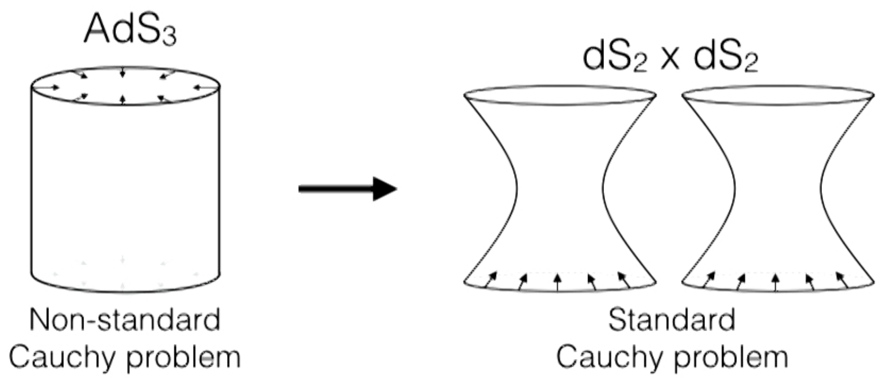
Structure of Kinematic Space



Intertwining Property

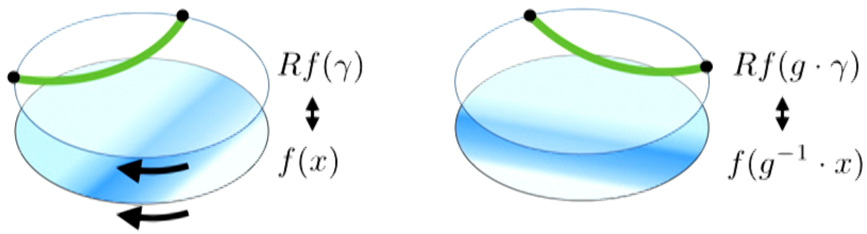
- We want to find **two** equations of motion for $R\phi(\gamma)$
- A Laplace equation: $R\Box_{AdS_3} = -\Box_{dS\times dS}R$
- A constraint equation: $\left(\Box_{dS_L} - \Box_{dS_R}\right) Rf = 0$

$$\Box_{AdS_3}\phi = m^2\phi \longrightarrow \begin{array}{l} \Box_{dS\times dS}\; R\phi = -m^2\; R\phi \\ \left(\Box_{dS_L} - \Box_{dS_R}\right) R\phi = 0 \end{array}$$



Intertwining Property

- A shift in a function by an AdS isometry can be compensated for by an inverse isometry in kinematic space



- This yields an **intertwining relation** between the differential operators representing these isometries:

$$L^{(\gamma)}_{AB} RF = -RL^{(x)}_{AB} f$$

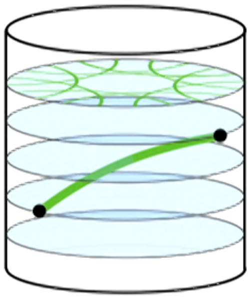
- Applying this twice, we find that the **Laplacians intertwine**

$$R\Box_{AdS_3}\phi = -\Box_{dS\times dS}R\phi$$

Intertwining Property

$$L^{(\gamma)}_{AB} RF = -RL^{(x)}_{AB} f$$

- Can also show: $(\Box_{dS_L} - \Box_{dS_R})\, Rf = 0$
- Comes from a **redundancy** in the X-ray transform
- f is a function of 3 variables, but
  Rf is a function of 4 variables
- The **constraint** reduces this dimensionality by 1
- E.g., can determine boosted geodesics in terms of unboosted geodesics
- Similar result in flat space: “John’s equation”



The Operator Product Expansion

- Consider a CFT in *d* dimensions.
- There are two equivalent bases of operators:

Local basis: $\partial_\mu \partial_\nu \cdots O_k(x)$

Global basis: $O_k(x) \quad \forall x$

- We can expand a **product** of local primary operators in a local basis – the **operator product expansion**

$$O_1(x)\, O_2(y) = \frac{1}{|x-y|^{\Delta_1+\Delta_2}} \sum_k C_{12k} \underbrace{|x-y|^{\Delta_k} \left(1 + \#\partial + \#\partial^2 + \ldots\right) O_k(z)}_{\text{The OPE Block } \mathcal{B}_k^{12}(x,y)}$$

The **OPE Block** $\mathcal{B}_k^{12}(x,y)$

The Operator Product Expansion

- The OPE in the **local basis**:

$$O_1(x)\, O_2(y) = \frac{1}{|x-y|^{\Delta_1+\Delta_2}} \sum_k C_{12k} \underbrace{|x-y|^{\Delta_k} \left(1 + \#\partial + \#\partial^2 + \ldots\right) O_k(z)}_{\mathcal{B}_k^{12}(x,y)}$$

- The OPE in the **global basis**:

$$O_1(x)\, O_2(y) = \sum_k C_{12k} \int d^d z \left\langle O_1(x)\, O_2(y)\, \tilde{O}_k(z) \right\rangle O_k(z)$$

- A "shadow operator" with $\tilde{\Delta}_k = d - \Delta_k$ gives the correct conformal transformation properties. [Ferrara, Gatto, Grillo, Parisi 1972; Simmons-Duffin 2014]
- But what about the coincident limit $x \to y$?
  - There are issues with "shadow" contributions…

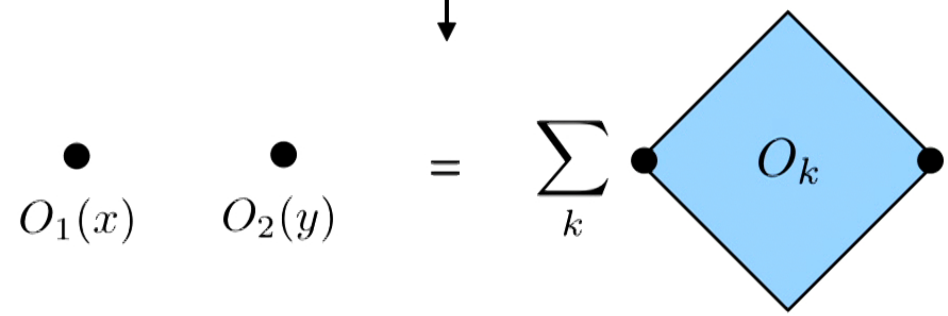
The Operator Product Expansion

- The fix for d = 2: integrate over a **causal diamond**

$$\mathcal{B}_k^{12}\left(x,y\right) = \left|x-y\right|^{\Delta_1+\Delta_2} \int_{\diamond} d^d z \left\langle O_1\left(x\right) O_2\left(y\right) \tilde{O}_k\left(z\right)\right\rangle O_k\left(z\right)$$

$$O_1(x)O_2(y) \quad = \quad \sum_k C_{12k}\mathcal{B}_k^{12}\left(x,y\right)$$

$$\downarrow$$



Conformal Blocks

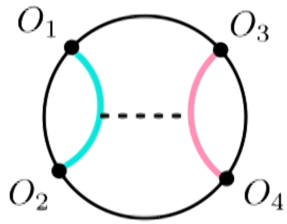
- The two-point **correlator** of OPE blocks is a **conformal block**

$$g_k(u,v) = \frac{\langle O_1 O_2 \mathcal{P}_k O_3 O_4 \rangle}{x_{12}^{-\Delta_1-\Delta_2} x_{34}^{-\Delta_3-\Delta_4}} = \langle \mathcal{B}_k(x_1,x_2)\,\mathcal{B}_k(x_3,x_4)\rangle$$

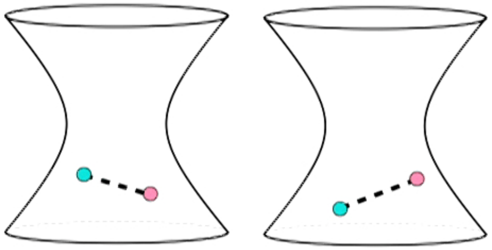
- A global conformal block is a **geodesic Witten diagram**

[Hijano, Kraus, Perlmutter, Snively 2015]

$$g_k(u,v) = \int_{\gamma_{12}} ds \int_{\gamma_{34}} ds' G^k_{bb}(s,s')$$

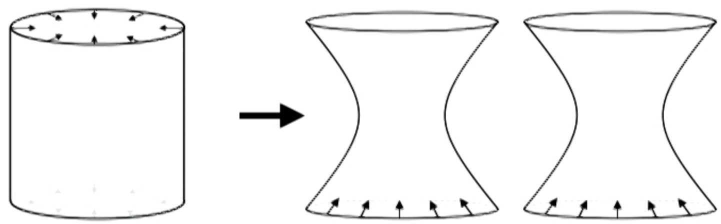


- It is also a **kinematic space propagator**



What have we learned?

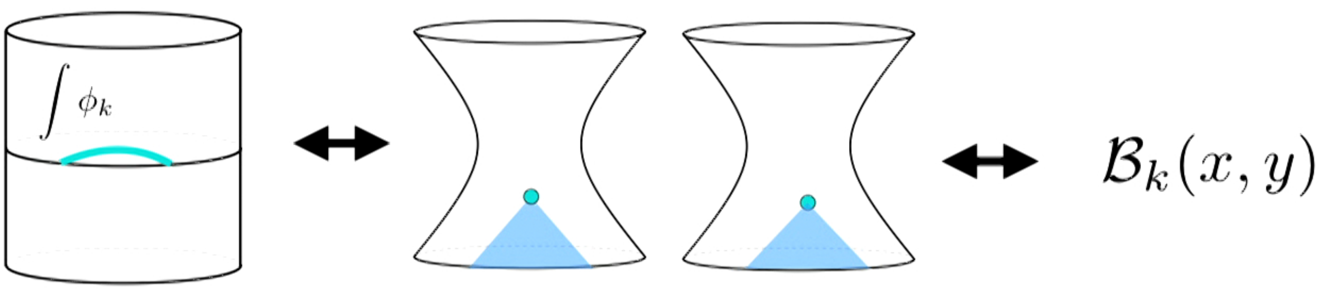
Bulk equations of motion **intertwine** with kinematic space equations of motion



$\Box_{AdS_3} \phi = m^2 \phi$ $\qquad$ $\Box_{dS \times dS} \; R\phi = -m^2 \; R\phi$

We construct a local bulk field using **geodesic operators**

Geodesic operators and OPE blocks are both **kinematic space fields**



A holographic dictionary with **depth perception**

Generalizing Ryu-Takayanagi

- What about OPE blocks for tensor CFT operators?

$$\mathcal{B}_T(x,y) = |x-y|^{2\Delta} \int d^2z \left\langle O(x)\, O(y)\, \tilde{T}^{\mu\nu}(z) \right\rangle T_{\mu\nu}(z)$$

- This is the vacuum **modular Hamiltonian**: [Casini, Huerta, Myers]

("entanglement first law")

$$\mathcal{B}_T(x,y) \propto 2\pi \int_x^y dz \frac{(z-x)(y-z)}{(y-x)} T_{00}(z) = H_{\mathrm{mod}} \quad = \delta S$$

- Define a **transverse X-ray transform** analogous to the scalar version:

$$\delta A = \frac{1}{2} \int_\gamma ds\, \delta g_{\mu\nu} \hat{v}^\mu \hat{v}^\nu$$

- Both satisfy the same equations of motion with the same boundary conditions:

$(\Box_{dS} + 2) H_{\mathrm{mod}} = 0$ [de Boer, Myers, Heller, Neiman] $\qquad H_{\mathrm{mod}}(x,y) \to T_{00}(x)$

$(\Box_{dS} + 2)\, \delta A = R[\delta G_{00}] = 0 \qquad \delta A(x,y) \to T_{00}(x)$

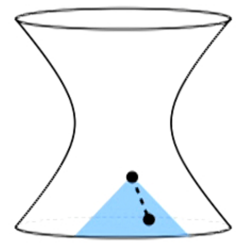
- A derivation perturbative Ryu-Takayanagi / JLMS formula – the kinematic dictionary **generalizes the RT formula**!

Generalizing Ryu-Takayanagi

- What if we allow perturbations of the stress tensor?

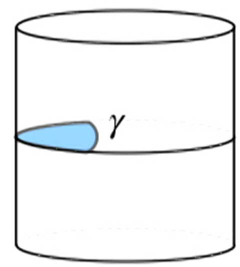
$$(\Box_{dS} + 2)\, H_{\text{mod}} = 0$$

$$(\Box_{dS} + 2)\, \delta A = R\,[\delta G_{00}] = \boxed{-R\,[\delta T_{00}]}$$



- We need to correct the kinematic dictionary:

$$H_{\text{mod}} = \delta A + X \qquad\qquad (\Box_{dS} + 2)\, X = R\,[\delta T_{00}]$$



- Solve using a kinematic space propagator:

$$X = \int_{\triangle} G_{dS}\,(\gamma, \gamma')\, R\,[\delta T_{00}]\, d^2\gamma' = \int_{\text{slice}} \delta T_{\mu\nu}\xi^{\mu} d\Sigma^{\nu}$$

- This is the bulk vacuum modular Hamiltonian

$$H_{\text{mod}} = \delta A + \delta S_{\text{bulk}}$$

Generalizing Ryu-Takayanagi

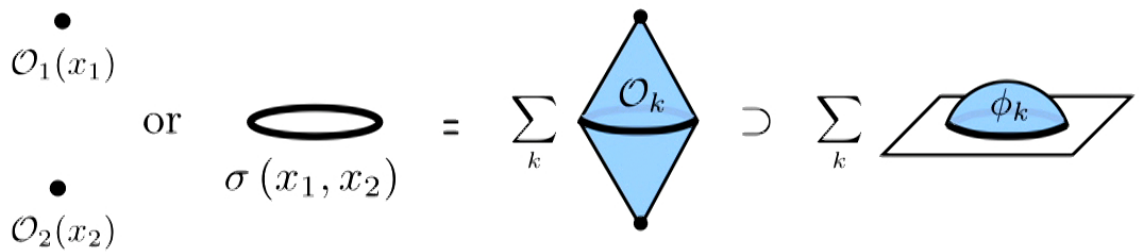
AdS		CFT
Equations of motion	$\longleftrightarrow$	OPE block equations
$\Box_{AdS_3}\phi = m^2\phi$	$\longleftrightarrow$	$\left(\Box_{dS\times dS} + m^2\right) R\phi = 0$ $R\phi = \mathcal{B}_\phi$
Einstein equations	$\longleftrightarrow$	Entropy equation of motion
$G_{\mu\nu} = T_{\mu\nu}$	$\longleftrightarrow$	$(\Box_{dS\times dS} + 2)\,\delta A = -R[\delta T_{00}]$ $S_{\text{CFT}} = A_{\text{min}} + S_{\text{bulk}}$

Can we generalize the quantum entropy?

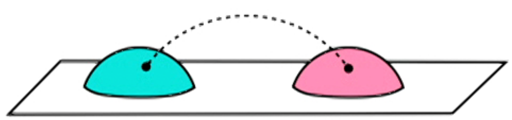
[Nozaki, Numasawa, Prudenziati, Takayanagi], [Faulkner, Guica, Hartman, Myers, van Raamsdonk]
[de Boer, Myers, Heller, Neiman], [Lashkari, McDermott, van Raamsdonk], [Swingle, van Raamsdonk], and more

Higher Dimensions

- What do we do in higher dimensions?
- **Timelike separated** points correspond to **minimal surfaces**
- Define the **Radon transform** of a function - its integral over the surface
- The previous discussion holds - a diamond-smeared boundary operator is a **bulk surface operator**
- OPE blocks for timelike bilocals or **CFT surface operators** are the same



- Surface operator correlators have a **conformal block expansion**



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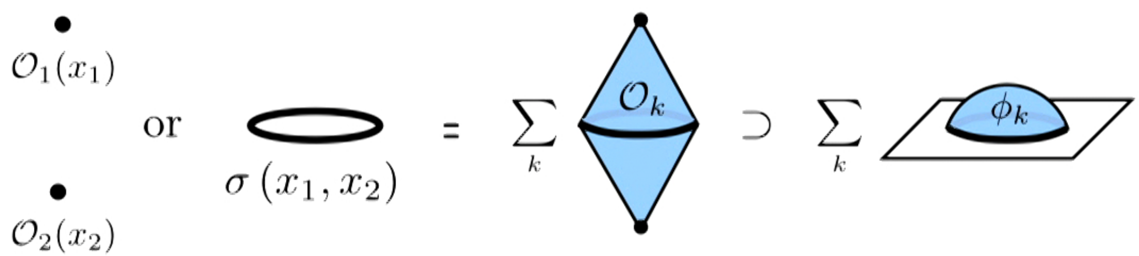

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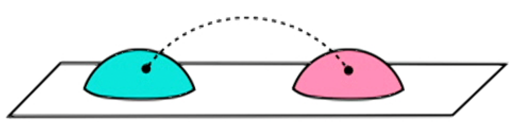

Beyond the Vacuum

- More generally, the bilocals encode a **world-line path integral**

$$\langle \phi(x)\,\phi(x')\rangle = \int d\mathcal{P} e^{iS_{\mathrm{wl}}[\mathcal{P}]}$$

- In the geometric optics limit, this implies the OPE contains **world-line operators**

$$\frac{O(x)\,O(x')}{\langle O(x)\,O(x')\rangle} = 1 + i\delta S_{\mathrm{wl}} + \cdots$$

- The multi-particle operators resum to produce the dependence on state

Bartek Czech

Lampros Lamprou

Thank you!

Benjamin Mosk

James Sully