Title: Fault-tolerant quantum error correction with non-abelian anyons

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Abstract: <p>Non-abelian anyons have drawn much interest due to their suspected existence in two-dimensional condensed matter systems and for their potential applications in quantum computation. In particular, a quantum computation can in principle be realized by braiding and fusing certain non-abelian anyons. These operations are expected to be intrinsically robust due to their topological nature. Provided the system is kept at a
 temperature T lower than the spectral gap, the density of thermal excitations is suppressed by an exponential Boltzman factor. In contrast to the topological protection however, this thermal protection is not scalable: thermal excitations do appear at constant density for any non-zero temperature and so their presence is unavoidable as the size of the computation increases. Thermally activated anyons can corrupt the encoded
 data by braiding or fusing with the computational anyons.

In the present work, we generalize a fault-tolerant scheme introduced by Harrington for the toric-code to the setting of non-cyclic modular anyons. We prove that the quantum information encoded in the fusion space of a non-abelian anyon system can be preserved for arbitrarily long times with poly-log overhead. In particular, our model accounts for noise processes which lead to the creation of anyon pairs from the vacuum, anyon diffusion, anyon fusion as well as errors in topological charge measurements.
Related Arxiv #: arXiv:1607.02159</p>

Fault-Tolerant Quantum Error Correction for Non-Abelian Anyons

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Waterloo, On, November 2016

Topological quantum computation

- 2D system with degenerate ground state and spectral gap Δ.
- Degeneracy is caused by
  - Manifold topology, such as punctures, genus.
  - Pinned excitations (modified Hamiltonian).
- Excitations carry *topological charges* $a,\ b,\ c, \ldots$.
  - Topological charge of a region cannot be changed by finite depth circuit.
  - Equivalence class between excitations (topological).
- Excitations can
  - Split $c \to a + b$ (including creation $1 \to a + \overline{a}$).
  - Be displaced or diffuse.
  - Fuse $a + b \to c$ (including disappear $a + \overline{a} \to 1$).
- Unitary transformations on the ground space can be realized through these topological operations, including braiding.
  - These are robust due to topological nature: larger systems are less error prone.
- Some anyon models support universal quantum computation.

Finite tempreature

- Thermally activated anyons will appear at finite density $e^{-\Delta/T}$.
- Thermally activated anyons can fuse and braid with computational anyons, corrupting the information.
- No scalable thermal protection: errors probability increases with system size.

Error model

- Pair creation:
  - With probability q_a a pair $(a, \bar{a})$ with trivial joint charge is added on neighboring sites.
  - Rate $\sum_a q_a = q$.
  - After a pair creation, the topological charge at every lattice site is projected onto a superselection sector with appropriate probabilities.
- Measurement errors:
  - A measurement reports the correct charge with probability $1 - p$ and charge a with probability p_a.
  - Rate $\sum_a p_a = p$.
- Charge displacement:
  - Not explicitly modeled, implied by pair creation.

QEC with non-abelian anyons

- QEC suppresses single round of pair creation (no other error).

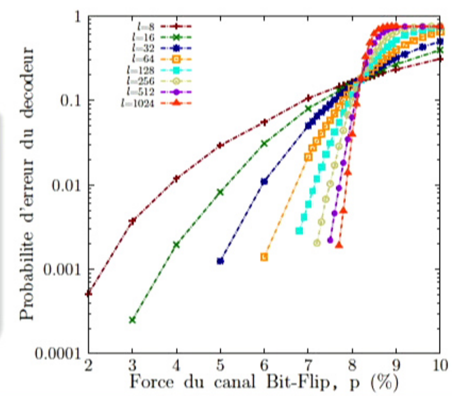
Main challenge: decoding problem

Statistically infer the most likelly particle worldline homology given a perfect snapshot of their location.

- Decoders have been devised and thresholds observed for:
  - Ising anyons — Brell, Burton, Dauphinais, Flammia, & DP, *PRX* 2014
  - $\Phi - \Lambda$ models — Wootton, Burri, Iblisdir, & Loss & DP, *PRX* 2014
  - Fibonacci anyons — Burton, Brell, & Flammia, arXiv:1506.03815

Threshold

Below a critical noise rate, the probability of inferring the right worldline homology increases with system size.



FTQC with abelian anyons

Fault tolerance

Correcting errors with noisy instruments (faulty measurements).

- Particularity of abelian anyons:
  - Fusion is deterministic, $a + b \to c(a, b)$.
  - Braid produce *commuting* unitary transformations on the ground space.
- Toric code has been widely studied.
  - Has a threshold at about 1%.
  - Decoding needs to analyse a space-time block of (faulty) measurement history.

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Gacs' positive rates proof

- A CA operates with local update rules on a lattice of finite state systems.
- Can a 1D CA perform a reliable computation in the presence of errors?
  - Positive rate conjecture asserts it is impossible.
  - Phase transition in 1D nearest-neighbour, translationally invariant system with finite-dimensional local systems.
  - In 1D, it is not possible to locally decide what side of a domain wall contains an error.
- Peter Gács found a counter example.
  - CA corrects small local errors.
  - CA is programmed to simulate itself: The effective computation performed by Q-cell colony is identical to the base cell computation.
  - Use percolation theory to show that errors organize in hierarchical manner.
  - Level-k errors are corrected by level-k update rules.

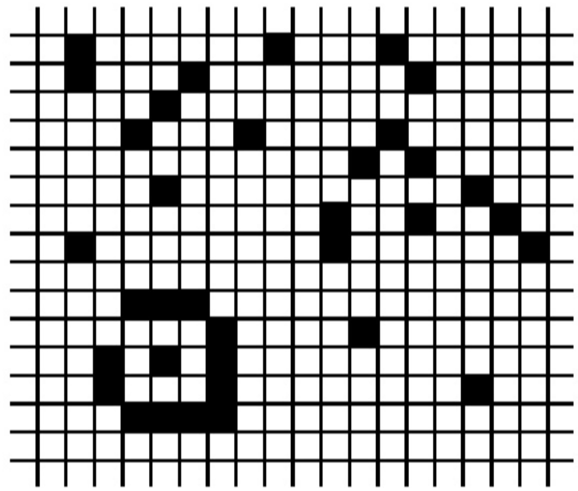
Harrington's toric code

- Errors in the 2D toric code manifest themselves much like classical errors in a 1D system.
- In his PhD thesis, Jim Harrington builds on Gács' ideas to construct a CA decoder for the toric code.
  - Hierarchical error organisation.
  - Square of Q^2 cells form colonies.
  - Local update rules at different levels.
  - Does not attempt to implement the high-level update rules locally (neither do we).
- Proves threshold $\approx 10^{-11}$.
  - Numerically $\approx 10^{-3}$.

Error classification

Here $Q = 5$ and $a = 1$.
All errors

.

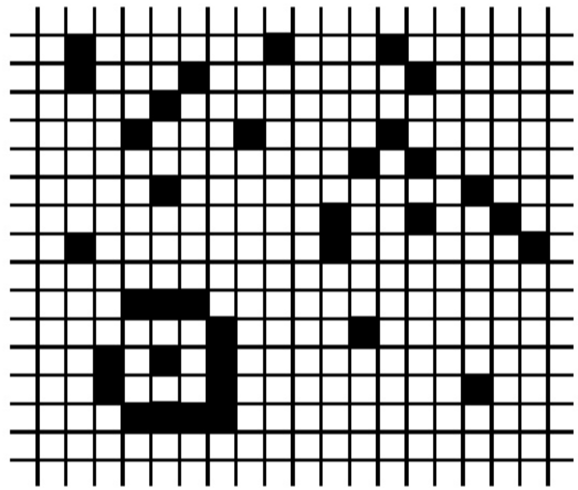


Theorem

The error rate of level n noise is $\epsilon_n \leq (Q^4 4p)^{2^n}$.

Error classification

Here $Q = 5$ and $a = 1$.
Level 0 candidate errors
Fit in a $(Q^0 + 1) \times (Q^0 + 1)$ box



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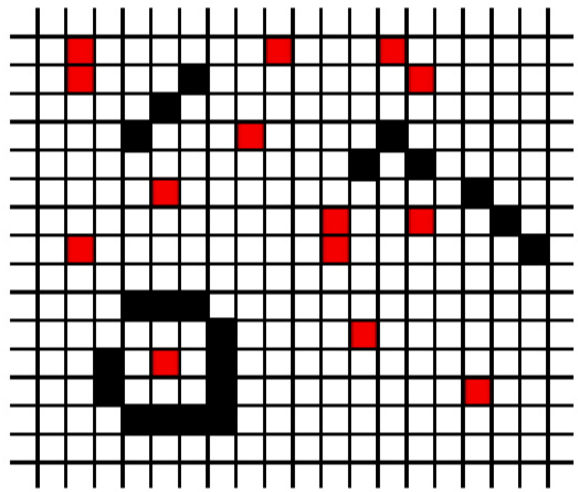

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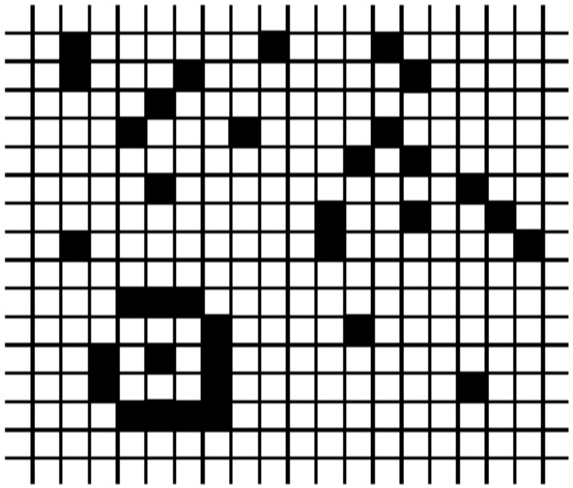

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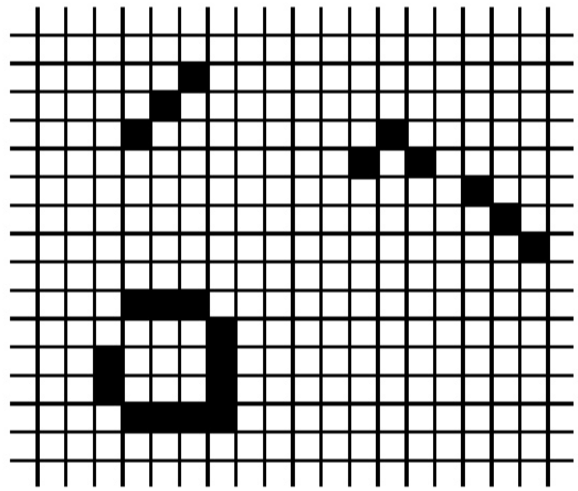
Errors without level-0 noise

.



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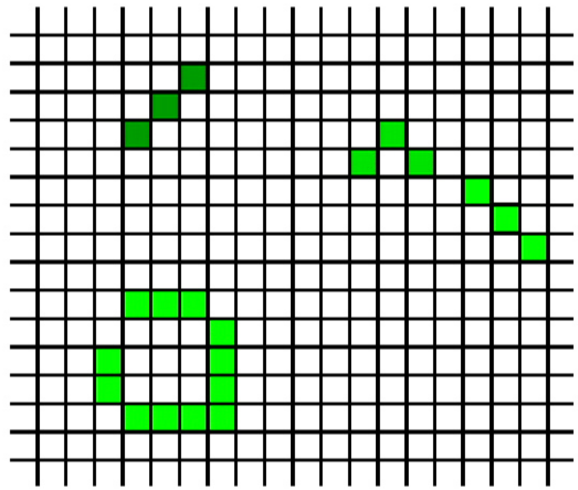

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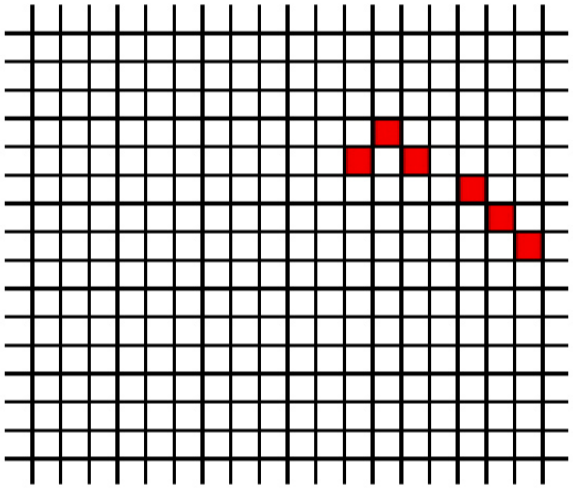

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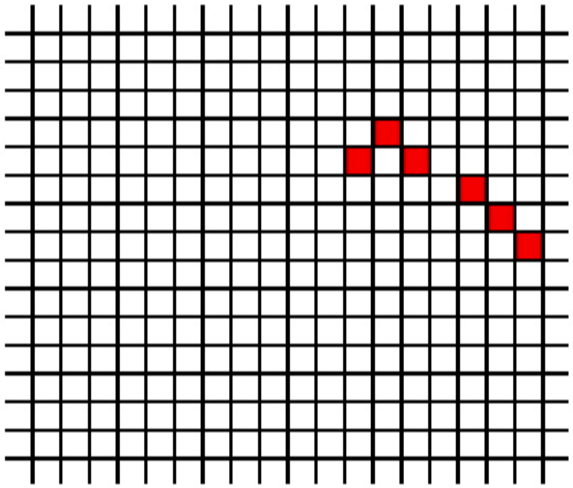


Theorem

The error rate of level n noise is $\epsilon_n \leq (Q^4 4p)^{2^n}$.
In 2+1 dimensions, $\epsilon_n \leq (Q^4 U^2 4(p+q))^{2^n}$.

CA rules

- Lattice is divided into colonies of $Q \times Q$ cells.
- Syndromes need to persist to trigger a correction operation.
- Neighbouring detected charges are fused together.
- If no neighbor, detected charge is moved towards the colony center.
- Every U time-steps, clusters of $Q \times Q$ colonies are organized into super colonies.
- The transition rules are applied at the level of super colonies, and so forth.

Harrington's main result

Provided constants a, b, Q, and U are chosen carefully, level-k actual errors are corrected by level-k transition rules.

Fusion

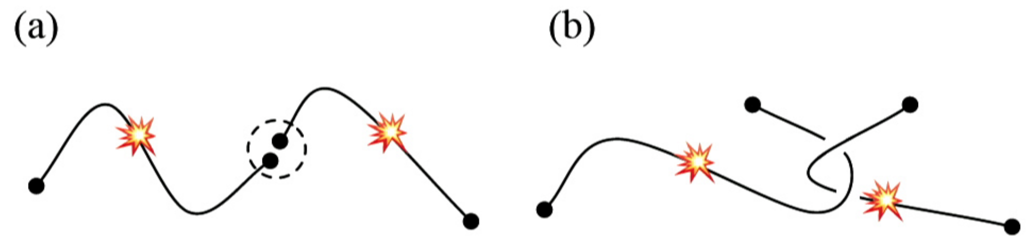
- 2+1 dimension picture breaks down.
  - You don't know the outcome of a fusion process until it happens.
  - Error correction must be a dynamical process.
- Fusion make error correction irreversible processes.

Trajectory domain of error E

The set of sites whos amplitude of supporting a certain topological charge at time t is changed by the presence of E form $\mathcal{D}_t(E)$.

Braid

- Errors can build up through fusion or braiding.



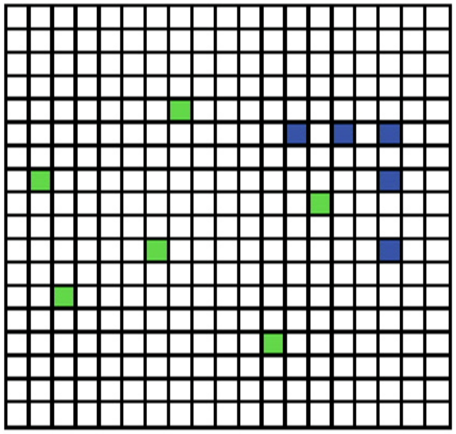
- Only fusion for abelian anyons.
- There are more ways to braid than to fuse
    - This additional entropic contribution could favour a disordered phase.

Outline

1. Motivation & Background
2. Fault-tolerance using local CA
3. Problems with non-abelian anyons
   - Intrinsic problems
   - Problems with proof technique
4. Results

Interactions between actual errors

- Actual errors can interact via error correction.



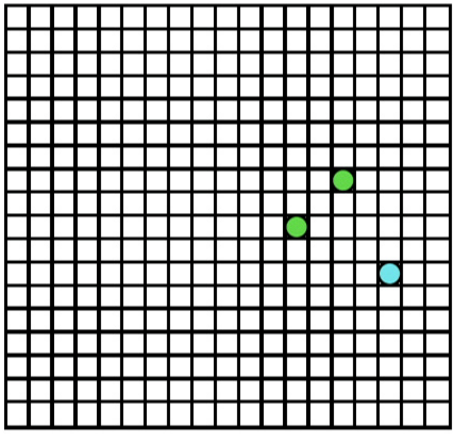
- The idea that level-k errors are handeled by level-k transition rules breaks down.

Causally linked cluster $C_t(E)$

The set of errors that have, directly or indirectly, interacted with error E prior to time t.

Interactions between actual errors

- Actual errors can interact via error correction.



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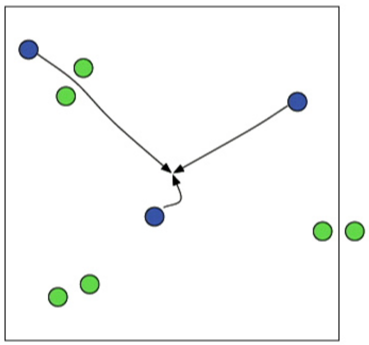
How to tell if a syndrom is right or wrong?

- The syndrome of a level-k colony should represent the total charge contained in that colony.
  - Fluctuates due to low-level errors along the boundary.
- Consider only anyons causally-connected to errors of level k.
  - Depends on the path used to remove the low-level anyons.

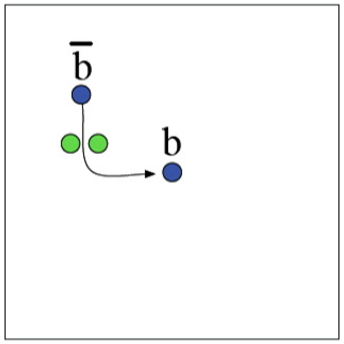


Valid level-k syndrome: total charges of level-k anyons

- Use path prescribed by CA.
- If depends on low-level error, that error becomes causally linked

Success

- Error correction of a level-k error can fail due to lower level errors.

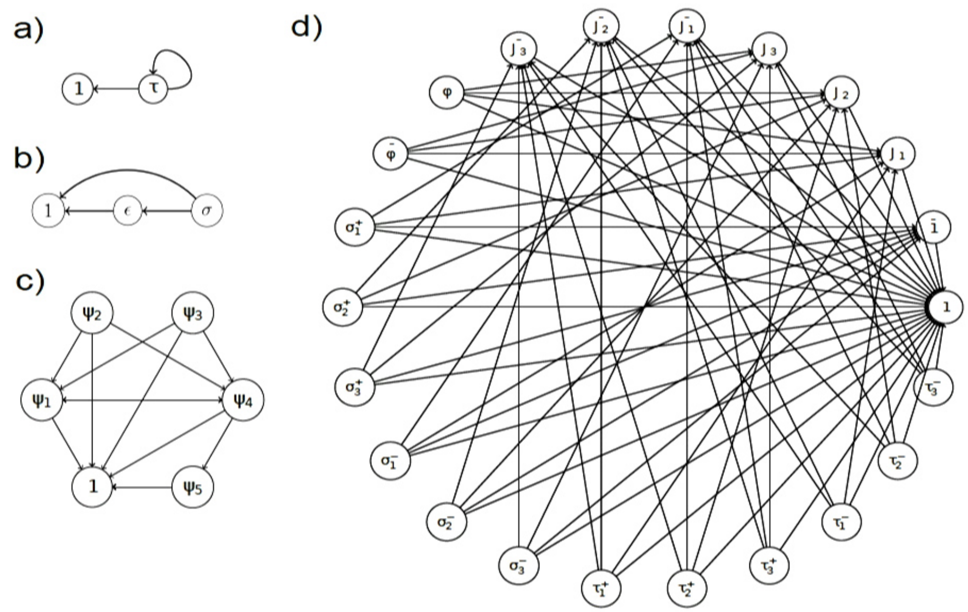


- Fusion $a + \overline{a} \rightarrow b$.
- Fusion $b + \overline{b} \rightarrow c$.

Acyclic anyon model

The directed graph $(a, \overline{a}) \rightarrow (b, \overline{b})$ iff $N^{b}_{a\overline{a}} > 0$ is acyclic .

Non-cyclic anyons



a) Fibonacci (Cyclic)
b) Ising
c) $SO(2)_5$ (Cyclic)
d) Quantum double of quaternion group $D(\overline{H})$.

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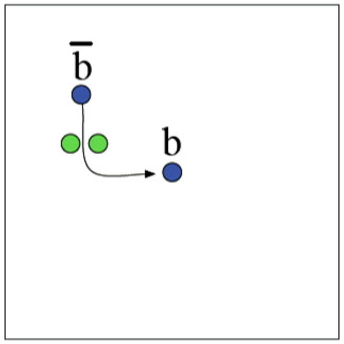

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Main result

Theorem

If $\mathcal{A}$ is non-cylic, then there exists a critical value $p_c > 0$ such that if $p + q < p_c$, for any number of time steps T and any $\epsilon > 0$, there exists a linear system size $L = Q^n \in \mathcal{O}(\log \frac{1}{\epsilon})$ such that with probability of at least $1 - \epsilon$, the encoded quantum state can in principle be recovered after T time steps.

- Threshold lower-bound $p_c \geq 2.7 \times 10^{-20} \times (3D + 1)^{-4}$ where $D = \text{Diam}(G_{\mathcal{A}})$.

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- Threshold lower-bound $p_c \geq 2,7 \times 10^{-20} \times (3D + 1)^{-4}$ where $D = \mathrm{Diam}(G_{\mathcal{A}})$.
  - Ising Anyons, $D = 2$, we get $p_c \geq 1,1 \times 10^{-23}$

Numerics

- Simulating the dynamics of non-abelian anyons is BQP-complete.
  - QEC can be simulated, but verifying success is hard.
- Ising anyons generate Clifford operations: can be simulated.
- Use two-stage perfect matching (σ then ψ) periodically to check if information is in principle still present.

Brell, Burton, Dauphinais, Flammia, & DP, *PRX* 2014

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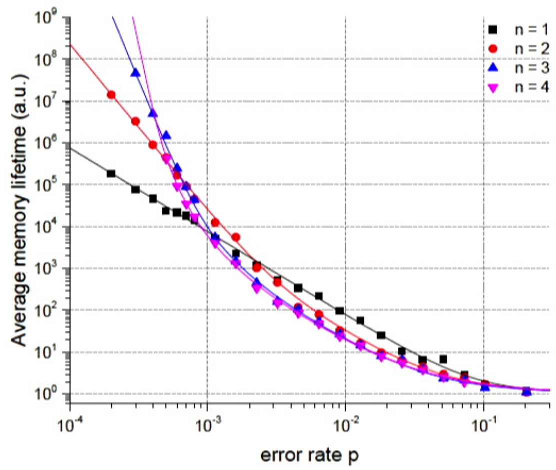

- $Q = 3$ & $U = 49$.
- Consistent with $p_c \approx 2 \times 10^{-4}$.

Summary & Outlook

- Topological quantum computation requires error correction at finite temperature.
- Non-abelian anyons pose many additional challenges.
- We adapted a CA fault-tolerant scheme to non-cyclic anyonic models.
  - Threshold: 10^{-23} (Th) 10^{-4} (Num).
- Cyclic anyon models?
  - We cannot put a hard cutoff on the time required to correct a level-k error for cyclic anyons.
  - The error may become causally linked with another level-k error.
  - Perhaps a more sophisticated analysis can evaluate the probability that this occurs.
- Improvements?
  - Algorithm is oblivious to fusion rules.

Institue Quantique @ Sherbrooke

In 2015, we received substantial grants to continue groundbreaking research at the frontier of quantum information and quantum materials, and to go from quantum science to quantum technologies.

- $35 M from Canada First Research Excellence Fund (operations).
- $8M from Canadian Foundation for Innovation (instruments).

We are looking for talented

- Graduate students
- Postdocs
- Visiting faculty/scientists

Talk to me if you have any interest.