Title: Supersymmetric Field Theories for Mathematicians

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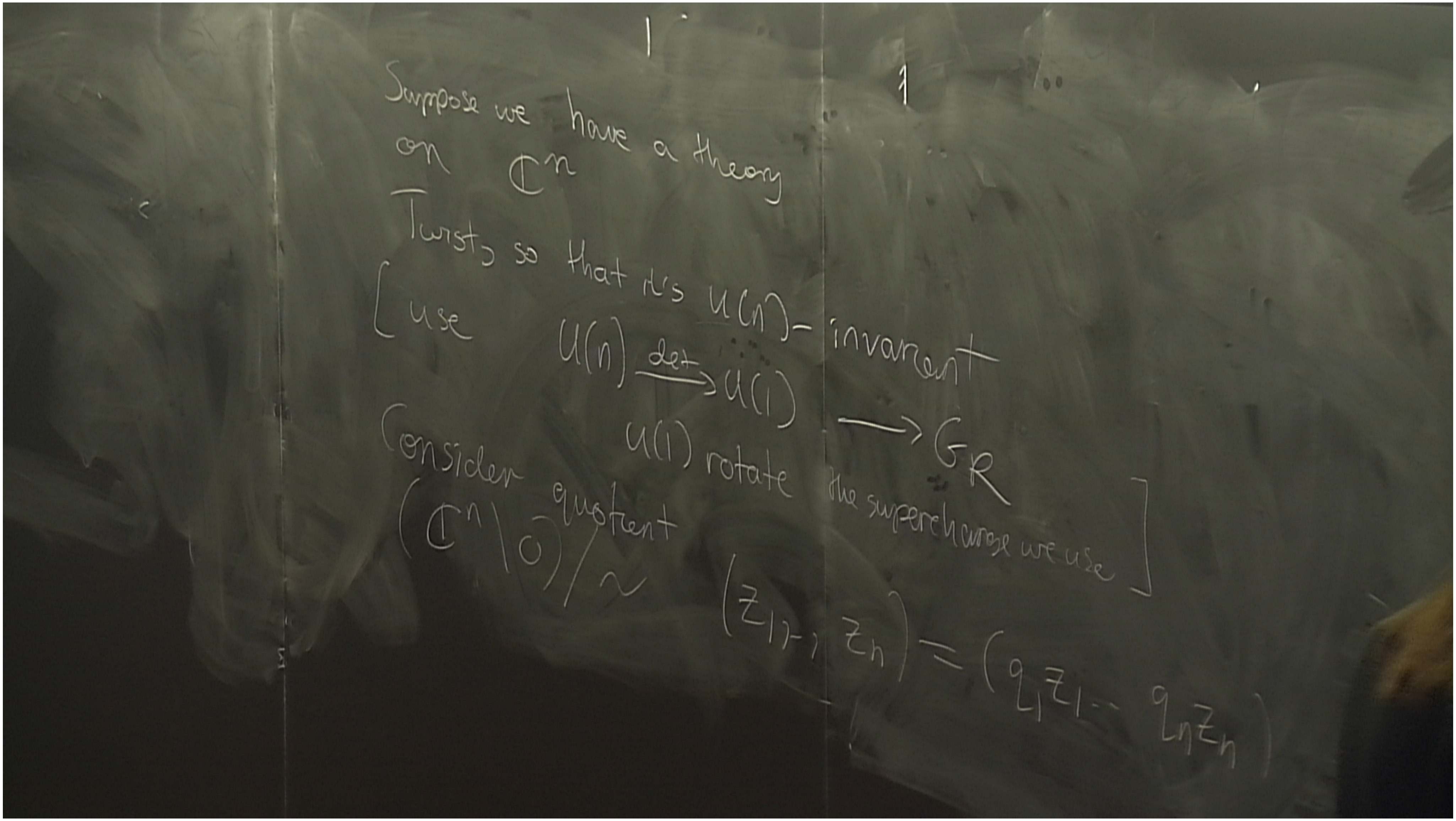
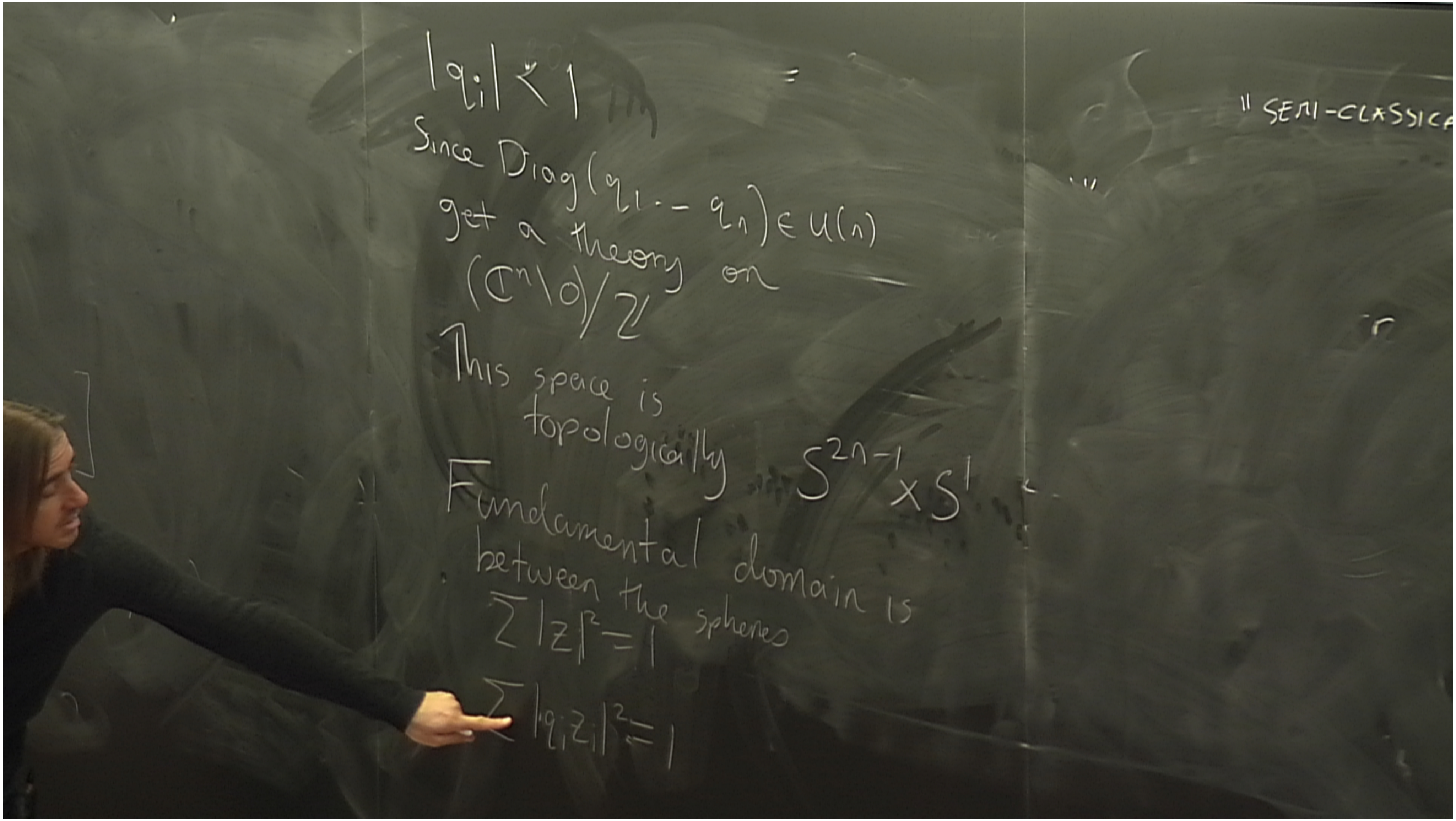
URL: http://pirsa.org/16110021

Abstract:


Suppose we have a theory
on ℂ^n
Twist, so that it's U(n)-invariant
[use U(n) det→ U(1) → G_R
U(1) rotate the supercharge we use]
Consider quotient
(ℂ^n \ 0)/~
(z_1, ..., z_n) = (q_1 z_1 ... q_n z_n)


$|q_i| < 1$
Since Diag$(q_1 \ldots q_n) \in U(n)$
get a theory on
$(\mathbb{C}^n \setminus 0)/\mathbb{Z}$
This space is
topologically $S^{2n-1} \times S^1$
Fundamental domain is
between the spheres
$\sum |z_i|^2 = 1$
$\sum |q_i z_i|^2 = 1$
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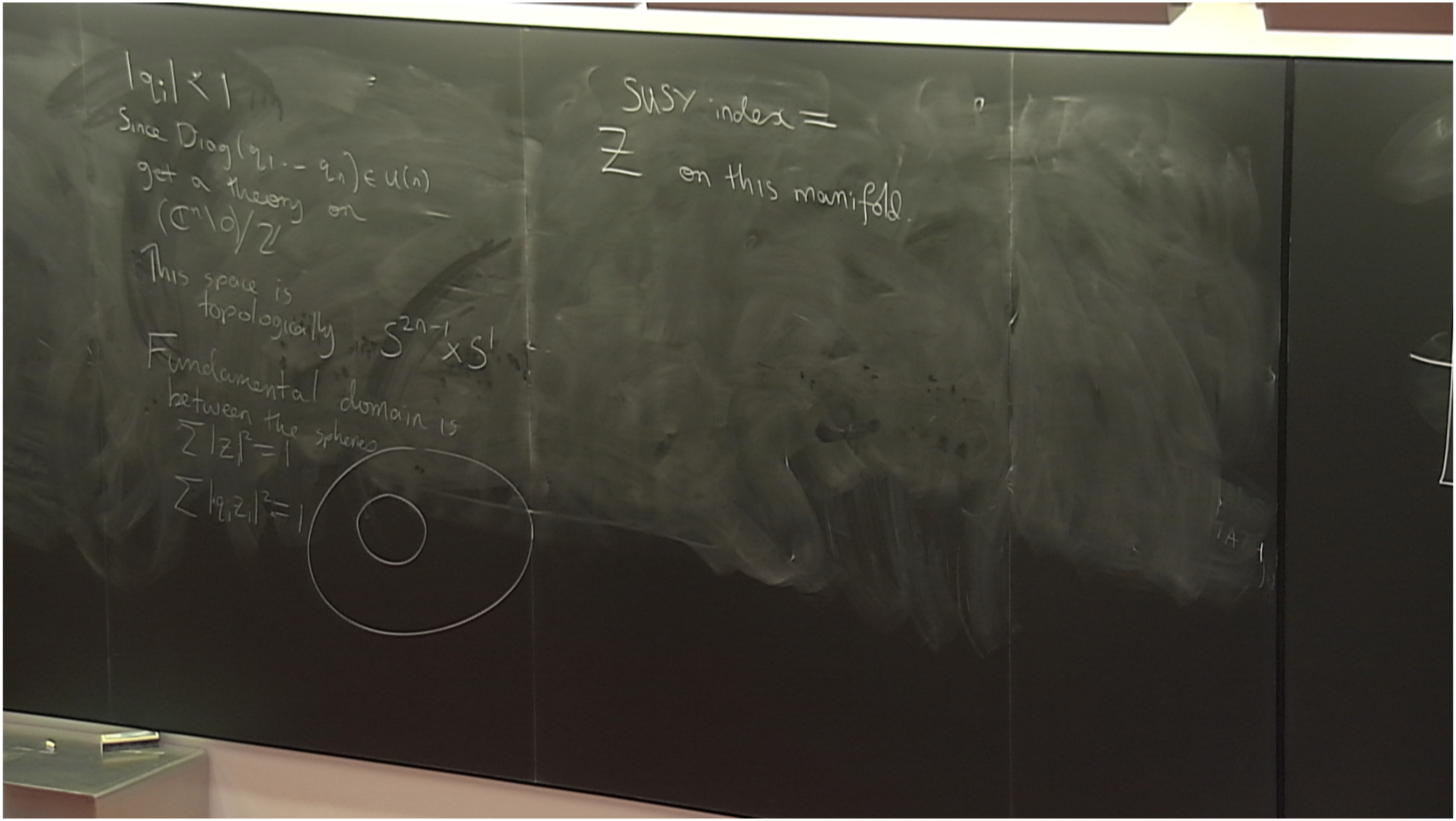
|q_i| < 1
Since Diag(q_1 .. q_n) ∈ U(n)
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(ℂ^n \ 0)/ℤ
This space is
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Fundamental domain is
between the spheres
Σ |z|^2 = 1
Σ |q_i z_i|^2 = 1
SUSY index =
Z on this manifold.

Suppose we have a theory on $\mathbb{C}^n$

Twist, so that it's $U(n)$-invariant

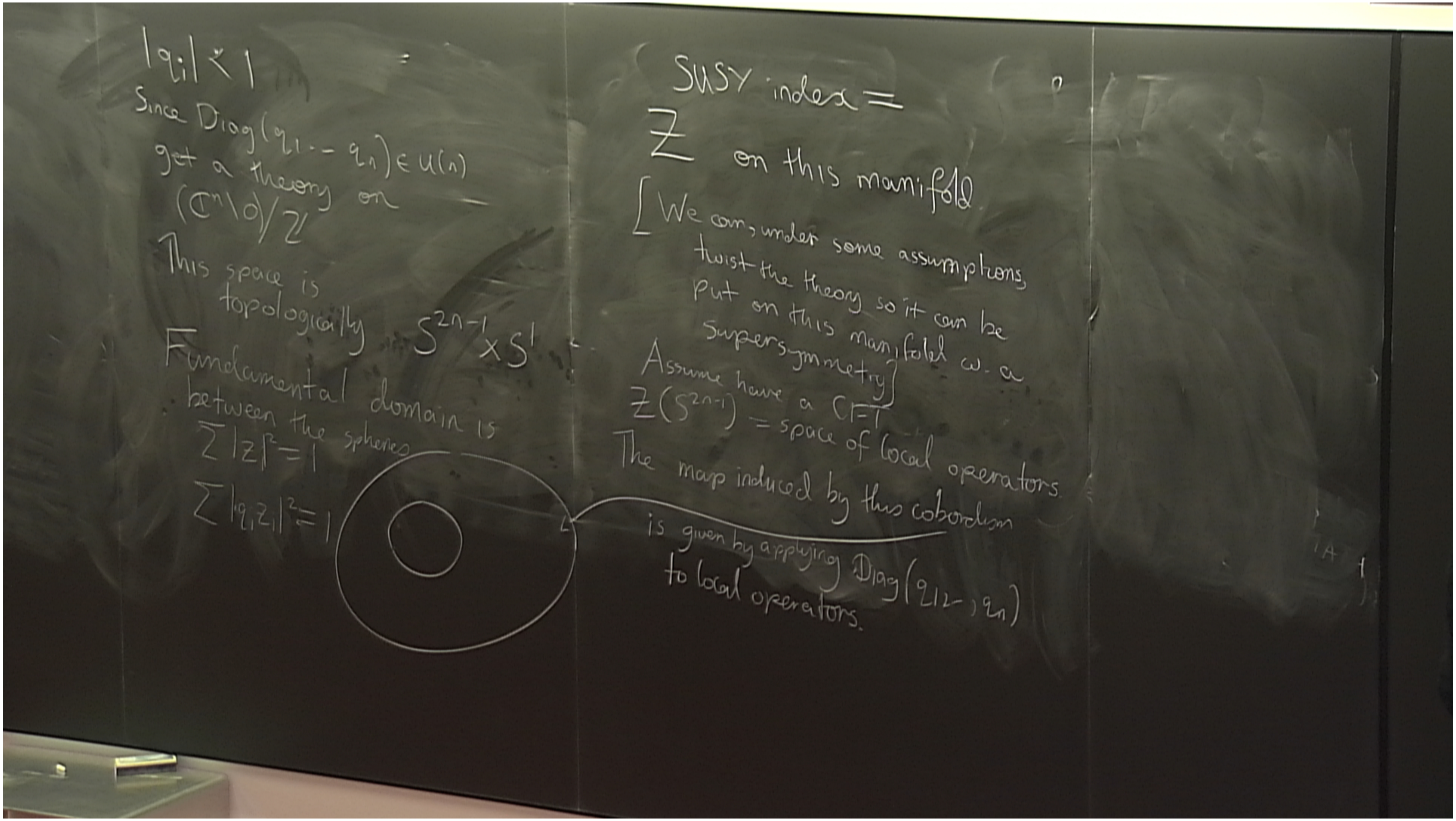
[use $GL(n,\mathbb{C}) \xrightarrow{\det} U(1) \longrightarrow G_R$

$U(1)$ rotate the supercharge we use]

Consider quotient

$$(\mathbb{C}^n \setminus 0)/\sim$$

$$(z_1, \dots, z_n) = (q_1 z_1 \dots q_n z_n)$$



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This space is
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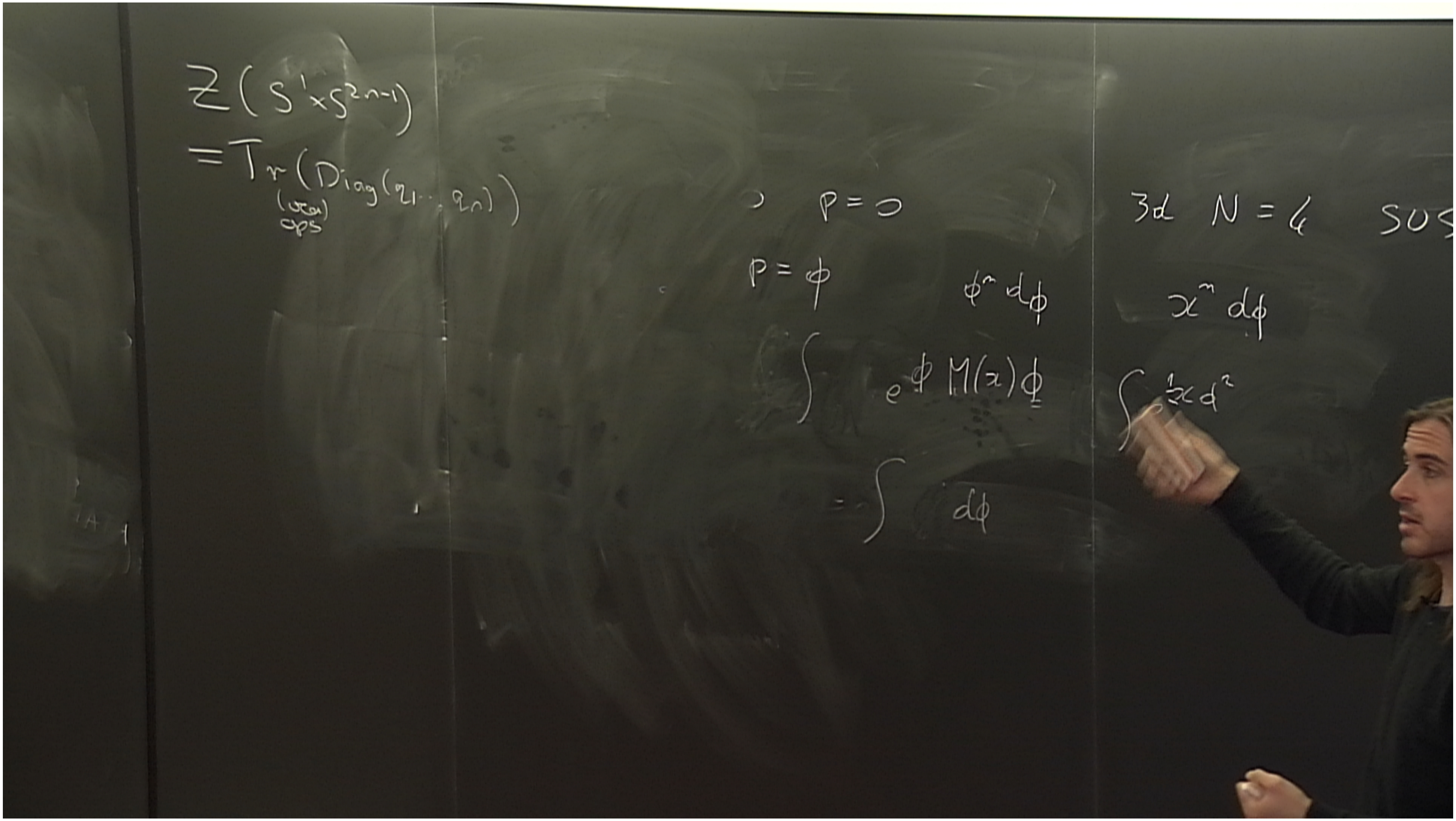
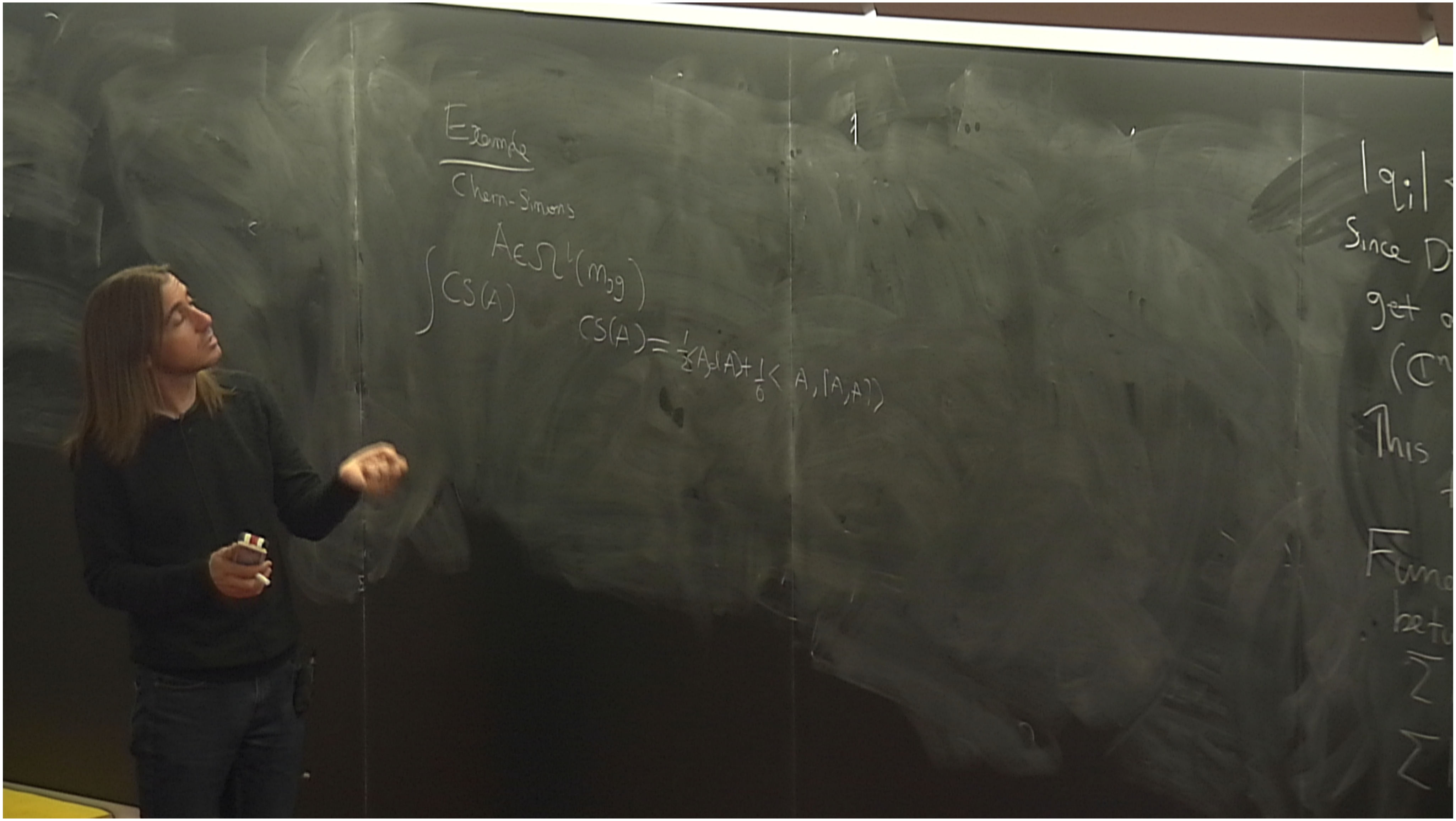
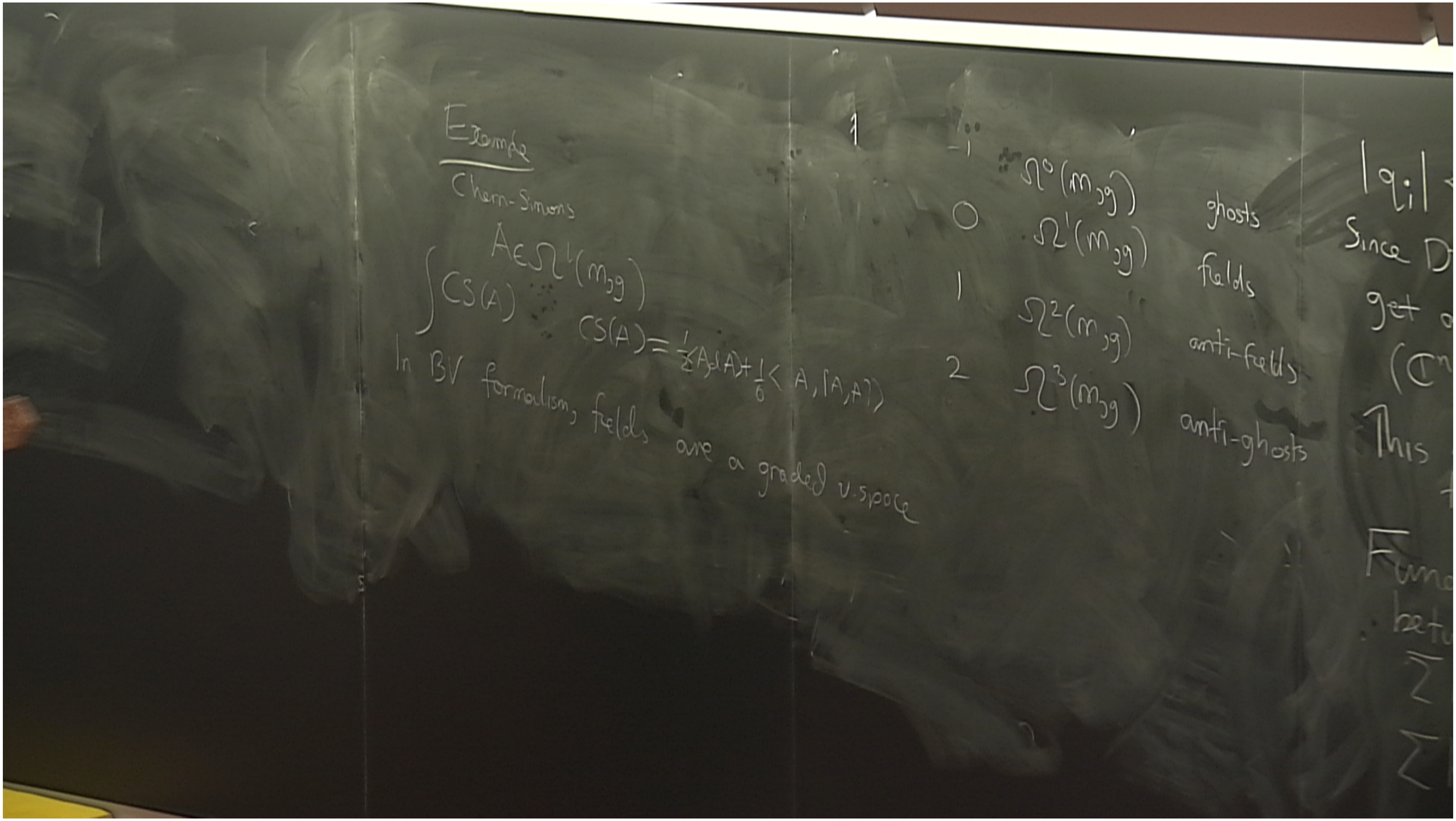
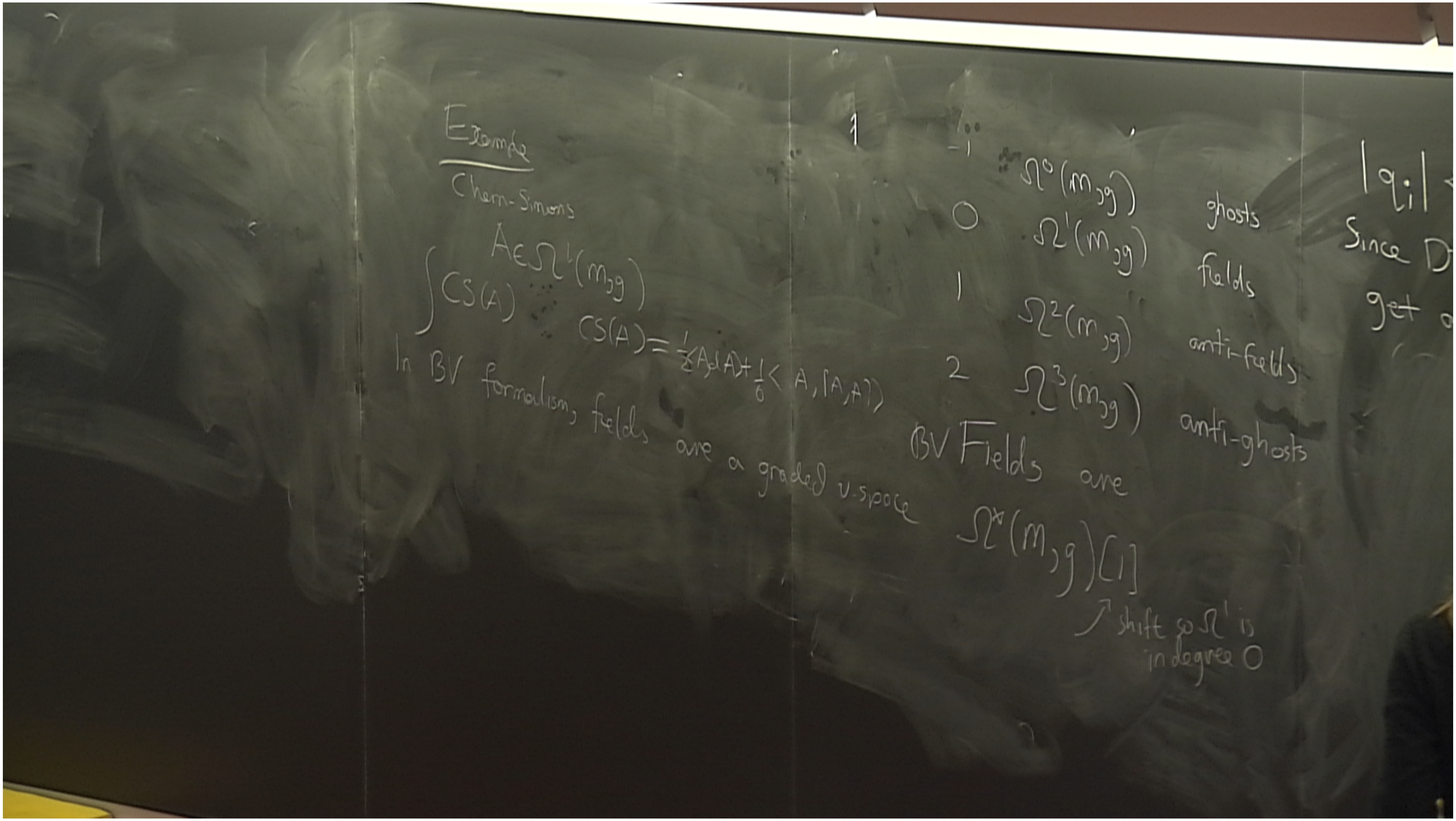
Fundamental domain is
between the spheres
Σ |z_i|^2 = 1
Σ |q_i z_i|^2 = 1
SUSY index =
Z on this manifold
[We can, under some assumptions,
twist the theory so it can be
put on this manifold w. a
supersymmetry]
Assume have a CFT
Z(S^{2n-1}) = space of local operators
The map induced by this cobordism
is given by applying Diag(q_1, .. , q_n)
to local operators.

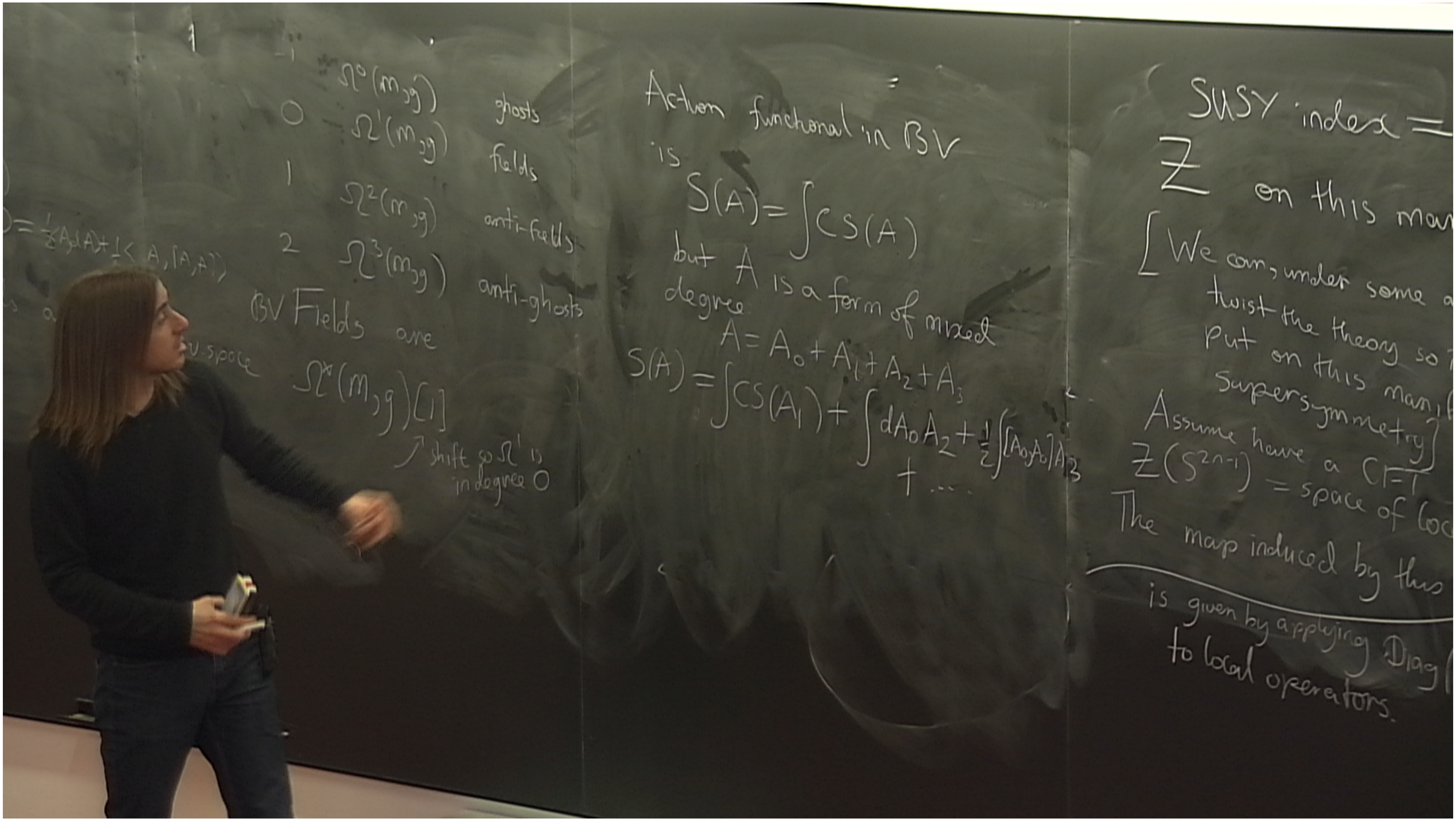

Z(S^1×S^{2n-1})
=Tr(Diag(q1.. qn))
ops
p = 0
3d N = 4 SUS
p = φ
φ^n dφ
x^n dφ
e φ M(x) φ
dφ

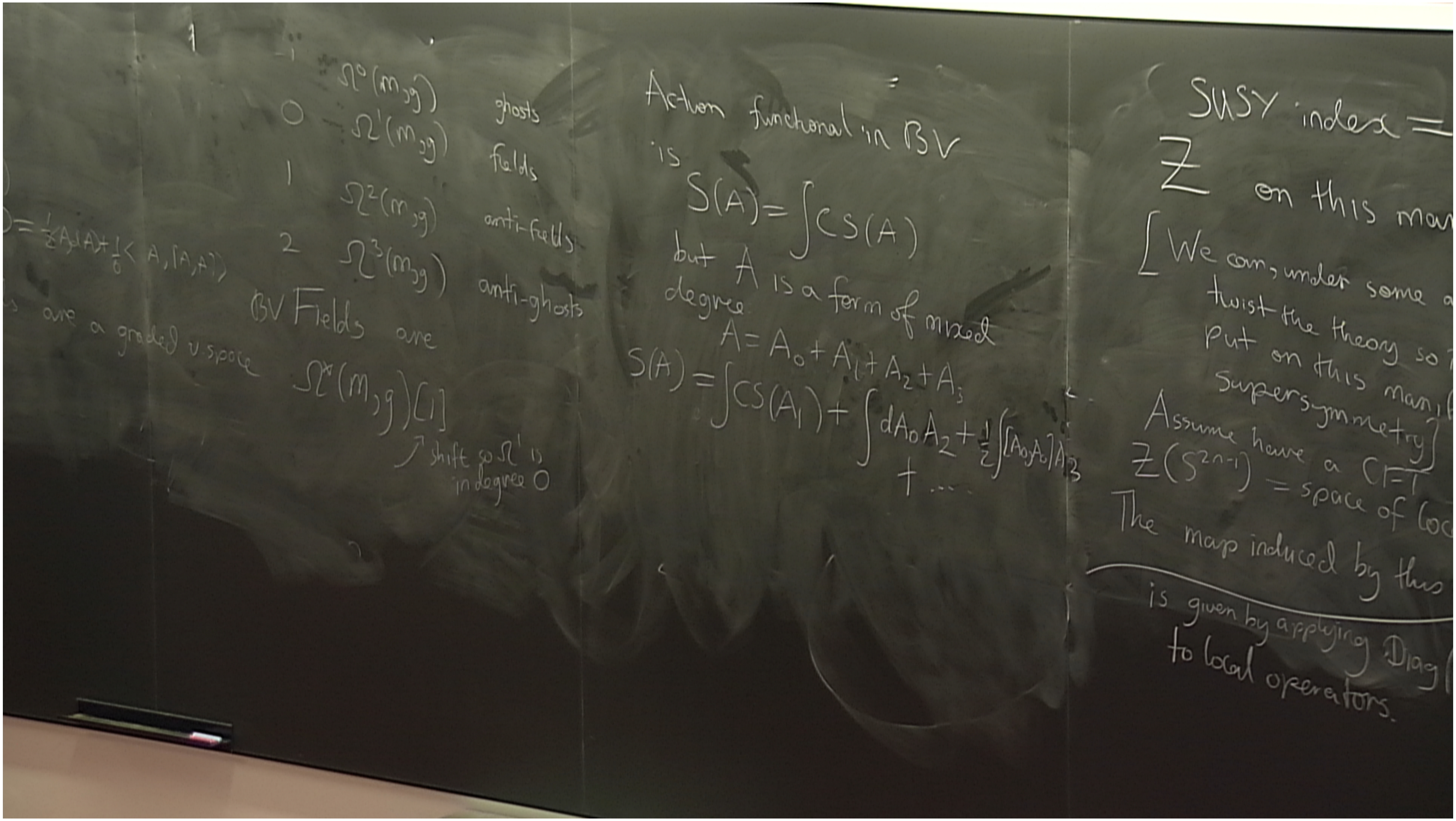

Example
Chern-Simons
A ∈ Ω¹(M, g)
∫ CS(A)
CS(A) = 1/2 ⟨A, dA⟩ + 1/6 ⟨A, [A, A]⟩
Since
get
This



Example
Chern-Simons
$A \in \Omega^1(M, \mathfrak{g})$
$\int CS(A)$
$CS(A) = \frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle A, [A,A]\rangle$
In BV formalism, fields are a graded v. space
-1 $\Omega^0(M,\mathfrak{g})$ ghosts
0 $\Omega^1(M,\mathfrak{g})$ fields
1 $\Omega^2(M,\mathfrak{g})$ anti-fields
2 $\Omega^3(M,\mathfrak{g})$ anti-ghosts
$|q_i|$
Since
get
This


Example
Chern-Simons
A ∈ Ω¹(M, g)
∫ CS(A)
CS(A) = ½⟨A, dA⟩ + ⅙⟨A, [A, A]⟩
In BV formalism, fields are a graded v.space
-1 Ω⁰(M, g) ghosts
0 Ω¹(M, g) fields
1 Ω²(M, g) anti-fields
2 Ω³(M, g) anti-ghosts
BV Fields are
Ω*(M, g)[1]
shift so Ω¹ is in degree 0
Since D
get


Ω⁰(M,g) ghosts
0 Ω¹(M,g) fields
1 Ω²(M,g) anti-fields
2 Ω³(M,g) anti-ghosts
BV Fields are
Ω*(M,g)[1]
shift so Ω¹ is in degree 0
Action functional in BV is
S(A) = ∫ CS(A)
but A is a form of mixed degree
A = A0 + A1 + A2 + A3
S(A) = ∫ CS(A1) + ∫ dA0 A2 + ½ ∫ [A0, A0] A3 + ...
SUSY index =
Z on this man
[We can, under some
twist the theory so
Put on this man
Supersymmetry]
Assume have a CFT
Z(S²ⁿ⁻¹) = space of loc
The map induced by this
is given by applying Diag
to local operators.


-1 Ω^0(M,g) ghosts
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1 Ω^2(M,g) anti-fields
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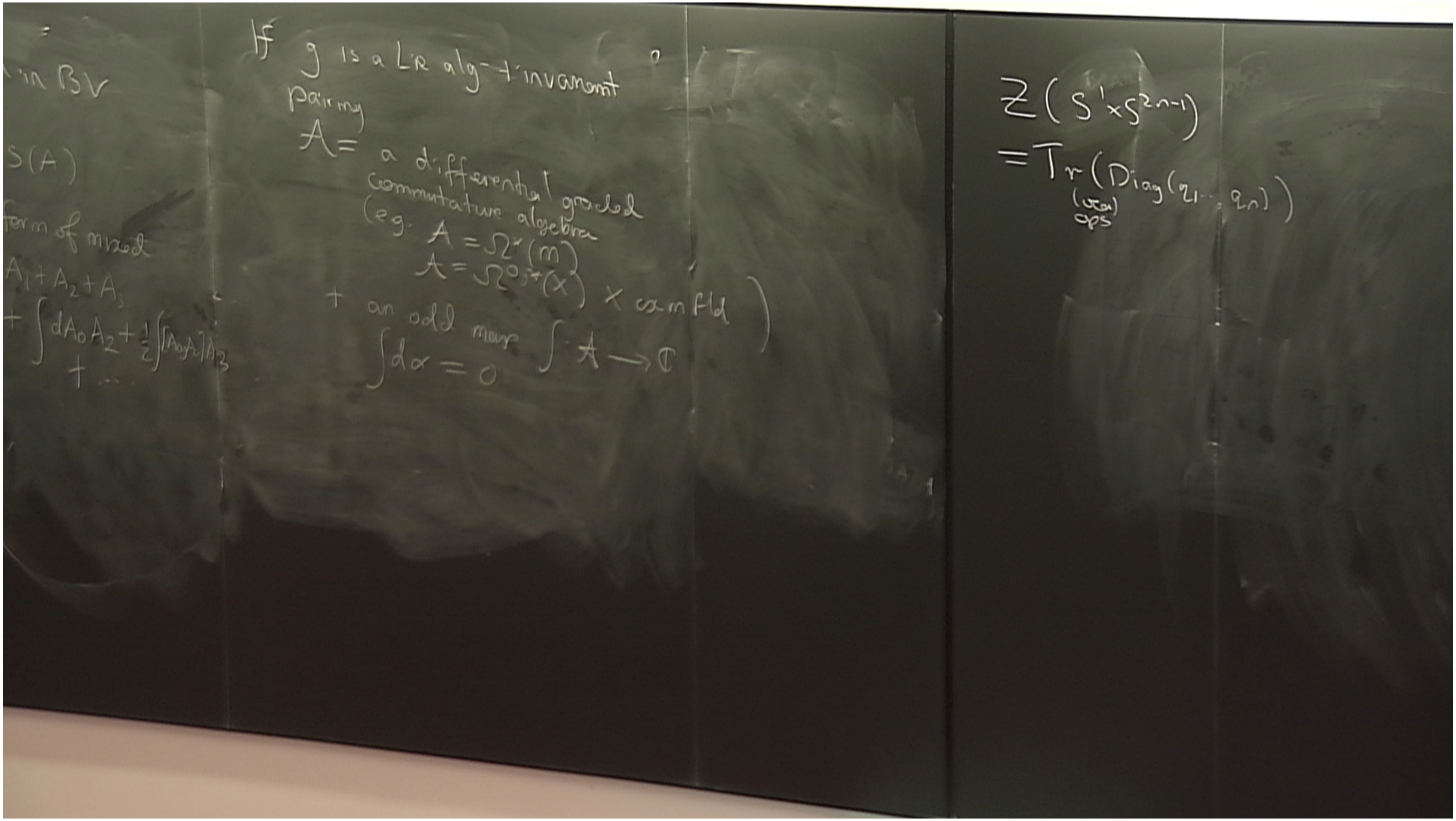
BV Fields are
are a graded v.space Ω*(M,g)[1]
shift so Ω^1 is in degree 0
Action functional in BV is
S(A) = ∫CS(A)
but A is a form of mixed degree
A = A0 + A1 + A2 + A3
S(A) = ∫CS(A1) + ∫dA0 A2 + 1/2∫[A0,A0]A3 + ...
SUSY index =
Z on this man
[We can, under some
twist the theory so
Put on this man,
supersymmetry]
Assume have a CFT
Z(S^2n-1) = space of loc
The map induced by this
is given by applying Diag
to local operators.


in BV
S(A)
form of mixed
A1 + A2 + A3
+ ∫dA0 A2 + 1/2 ∫[A0,A0]A3
+ ...
If g is a Lie alg + invariant pairing
A = a differential graded commutative algebra
(e.g. A = Ω*(M)
A = Ω0,*(X) X cx mfld)
+ an odd map ∫ A → ℂ
∫dα = 0
Z(S1 × S2n-1)
= Tr(Diag(q1 ... qn))
(local) ops

in BV

S(A)

form of mixed

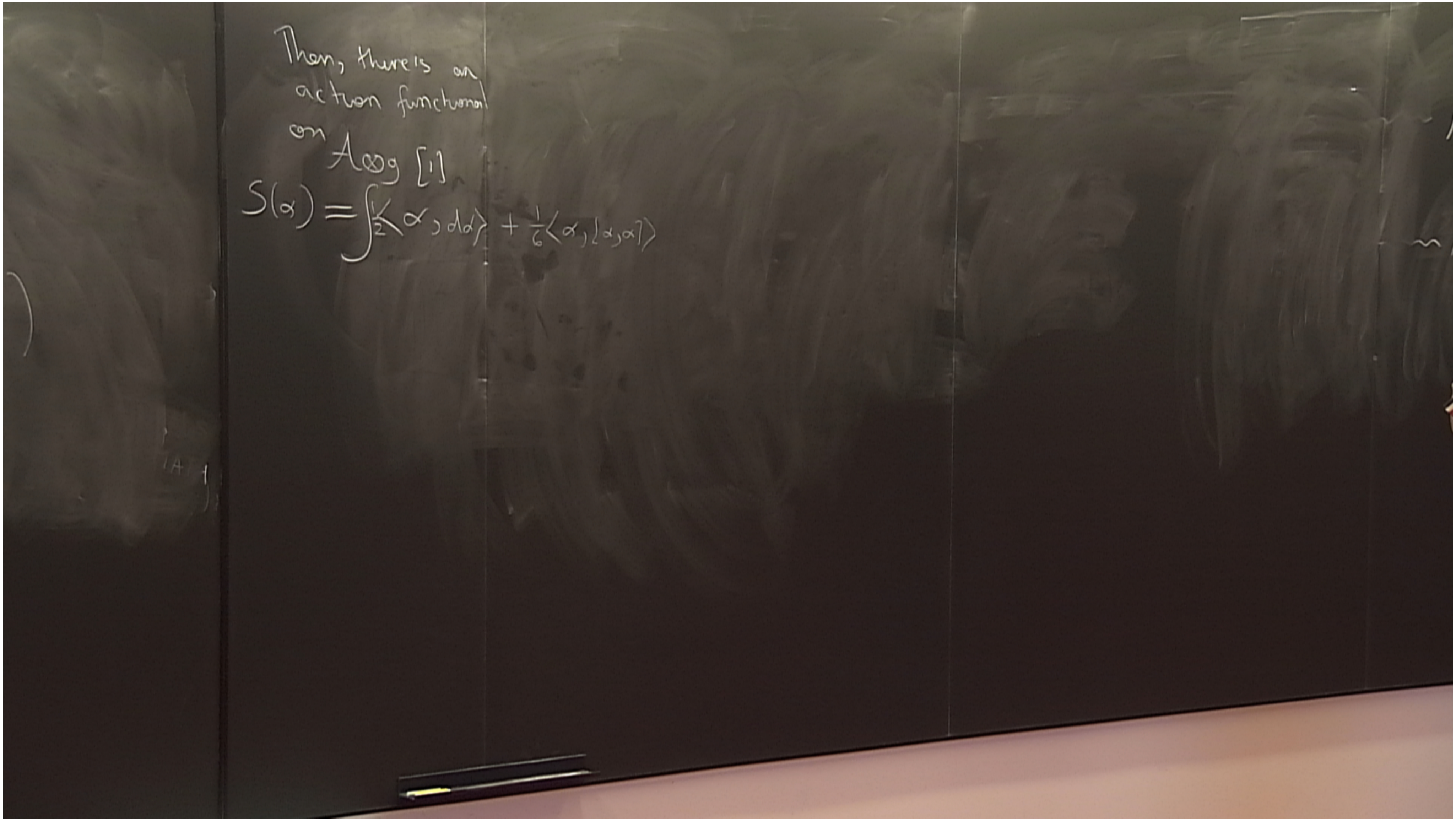
$A_1 + A_2 + A_3$

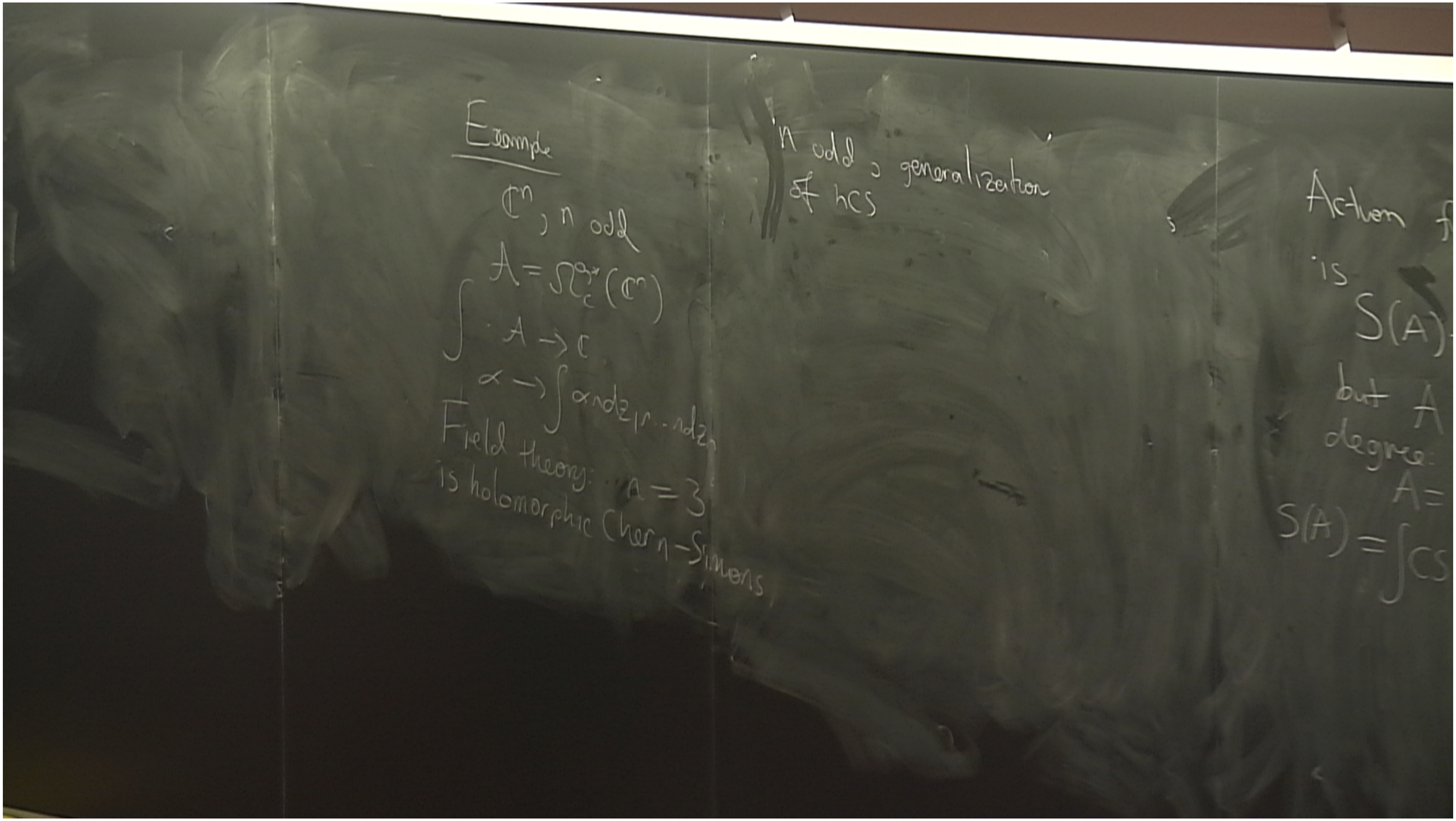
$+ \int dA_0 A_2 + \frac{1}{2}\int [A_0, A_1] A_3$
$+ \cdots$

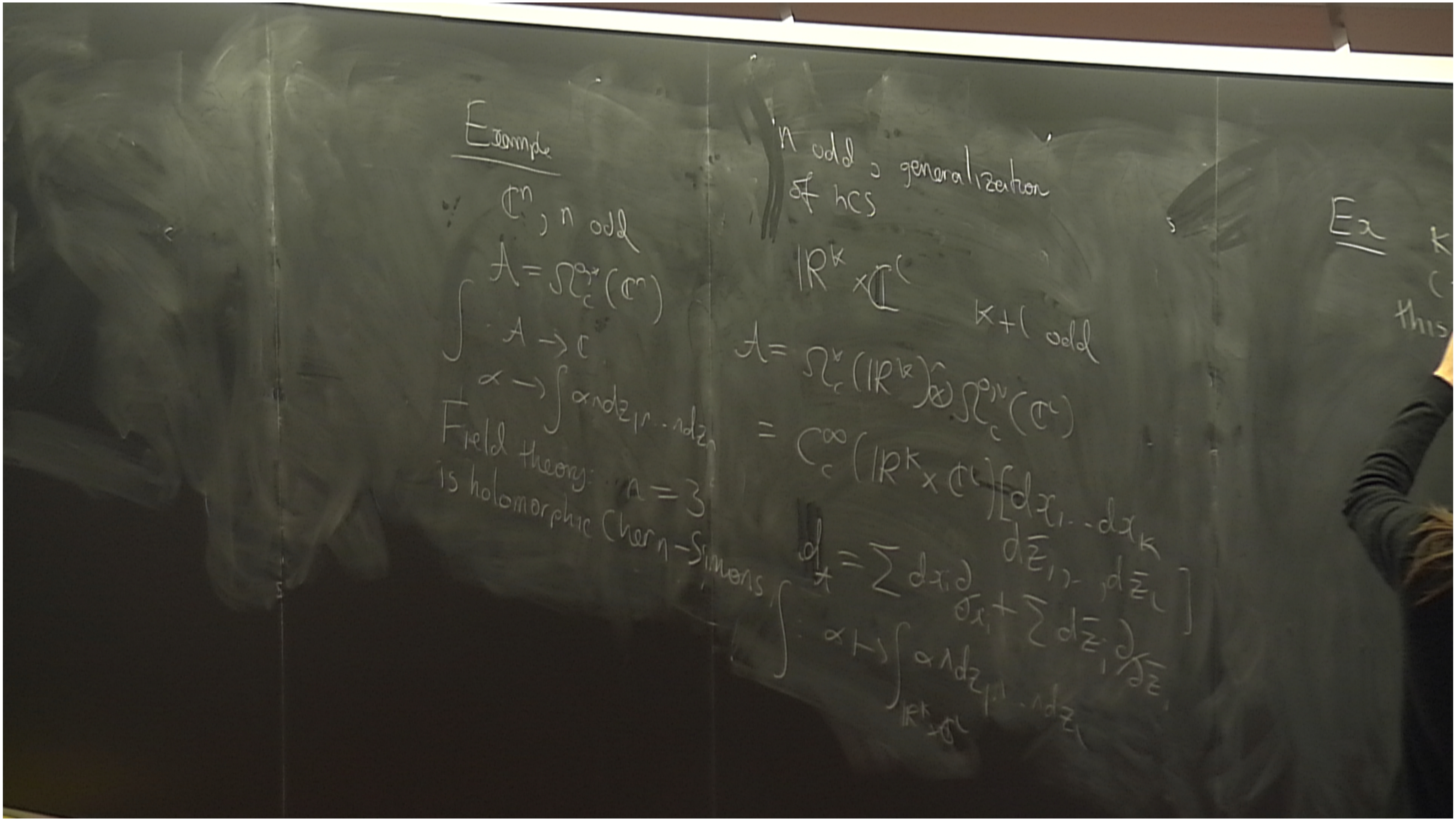
If g is a Lie alg + invariant pairing

$A =$ a differential graded commutative algebra
(e.g. $A = \Omega^*(M)$
$A = \Omega^{0,*}(X)$ X cx mfld)
+ an odd map $\int A \to \mathbb{C}$
$\int d\alpha = 0$

$Z(S^1 \times S^{2n-1})$
$= \mathrm{Tr}(\mathrm{Diag}(q_1 \ldots q_n))$
(local) ops


Then, there is an
action functional
on $\mathcal{A} \otimes g[1]$
$S(\alpha) = \int \frac{1}{2}\langle \alpha, d\alpha \rangle + \frac{1}{6}\langle \alpha, [\alpha, \alpha] \rangle$


Example
ℂn, n odd
A = Ω0,*c(ℂn)
∫ : A → ℂ
α → ∫ α∧dz1∧...∧dzn
Field theory: A = 3
is holomorphic Chern-Simons
n odd, generalization of hCS
Action f
is
S(A)
but A
degree
A =
S(A) = ∫CS


Example
C^n, n odd
A = Ω_c^{0,*}(C^n)
∫ : A → C
α ↦ ∫ α∧dz_1∧…∧dz_n
Field theory: n = 3
is holomorphic Chern-Simons
n odd, generalization of hCS
R^k × C^l   k+l odd
A = Ω_c^*(R^k) ⊗̂ Ω_c^{0,*}(C^l)
= C_c^∞(R^k × C^l)[dx_1…dx_k, dz̄_1,…,dz̄_l]
d_A = Σ dx_i ∂/∂x_i + Σ dz̄_i ∂/∂z̄_i
∫ : α ↦ ∫_{R^k×C^l} α∧dz_1∧…∧dz_l
Ex

odd ⊃ generalization of hcs

$\mathbb{R}^k \times \mathbb{C}^l$ $k+l$ odd

$\Omega^*_c(\mathbb{R}^k) \hat{\otimes} \Omega^{0,*}_c(\mathbb{C}^l)$

$C^\infty_c(\mathbb{R}^k \times \mathbb{C}^l)[dx_1, \dots, dx_k, d\bar{z}_1, \dots, d\bar{z}_l]$

$d_* = \sum dx_i \partial_{x_i} + \sum d\bar{z}_i \partial_{\bar{z}_i}$

$\int : \alpha \mapsto \int_{\mathbb{R}^k \times \mathbb{C}^l} \alpha \wedge dz_1 \wedge \dots \wedge dz_l$

Ex $k=2$
$l=1$

this is the theory related to Yangian/integrable models

If $\mathfrak{g}$ is a Lie alg + invariant pairing

$A =$ a differential graded commutative algebra

(e.g. $A = \Omega^*(M)$
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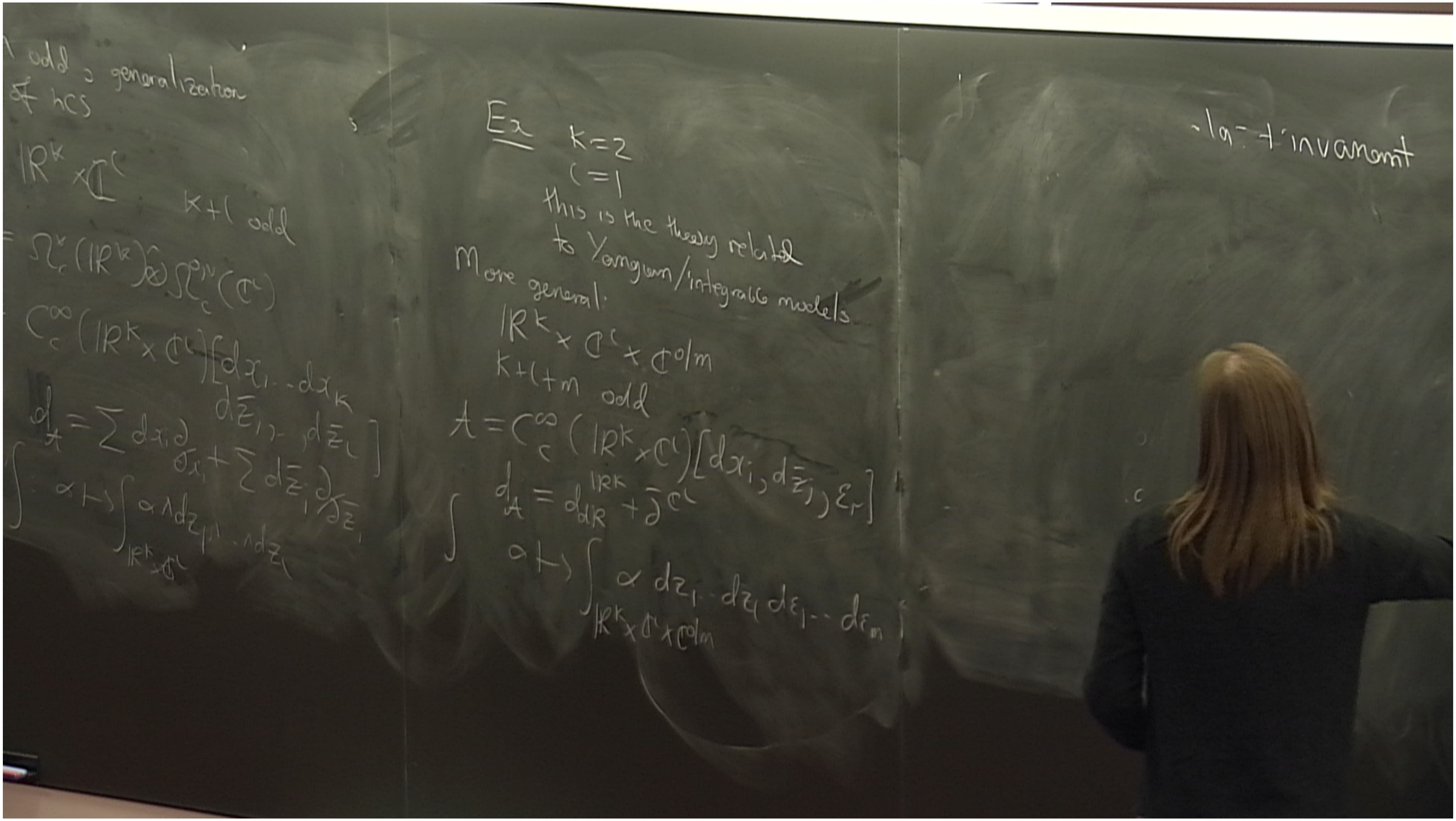
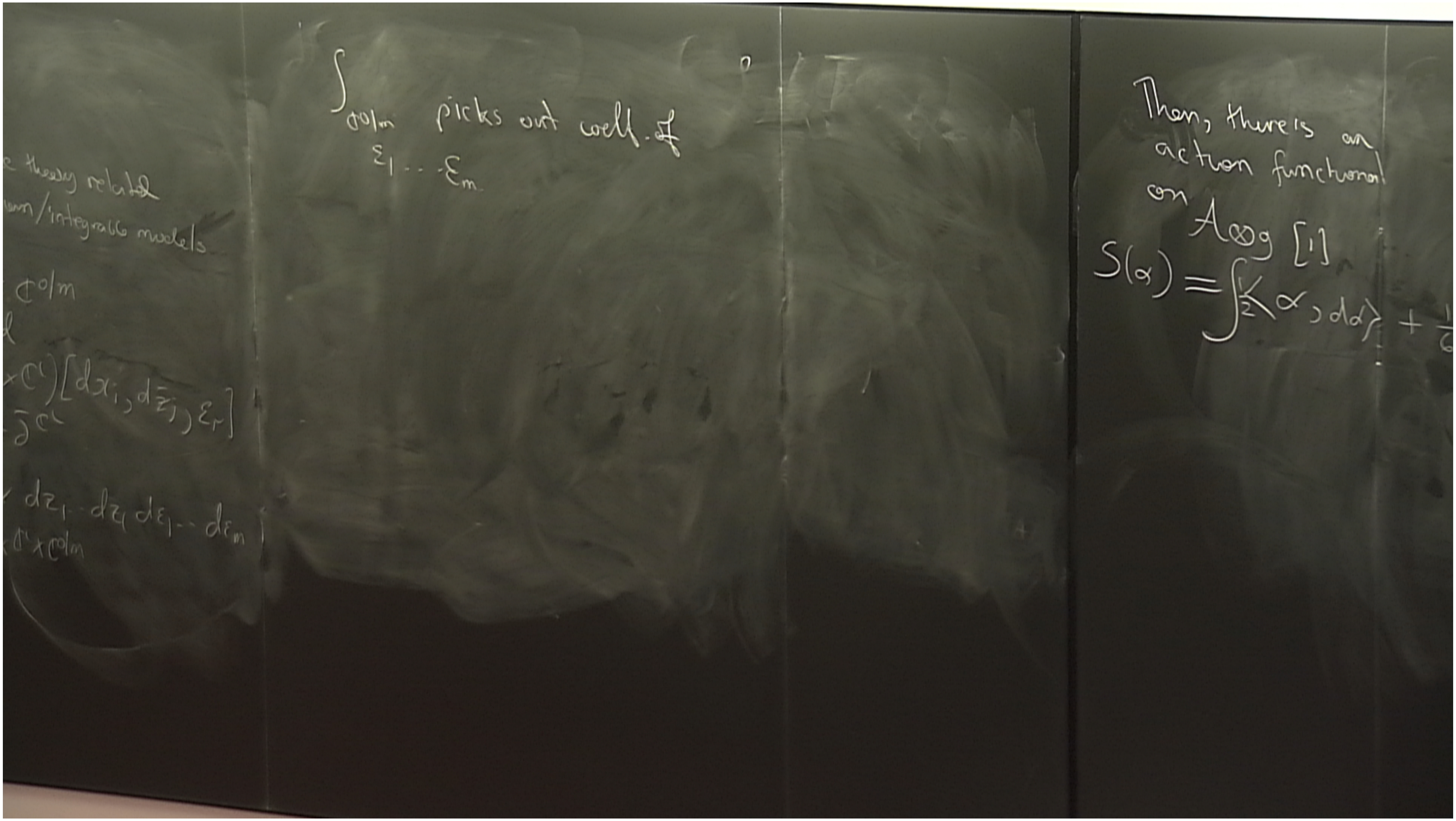
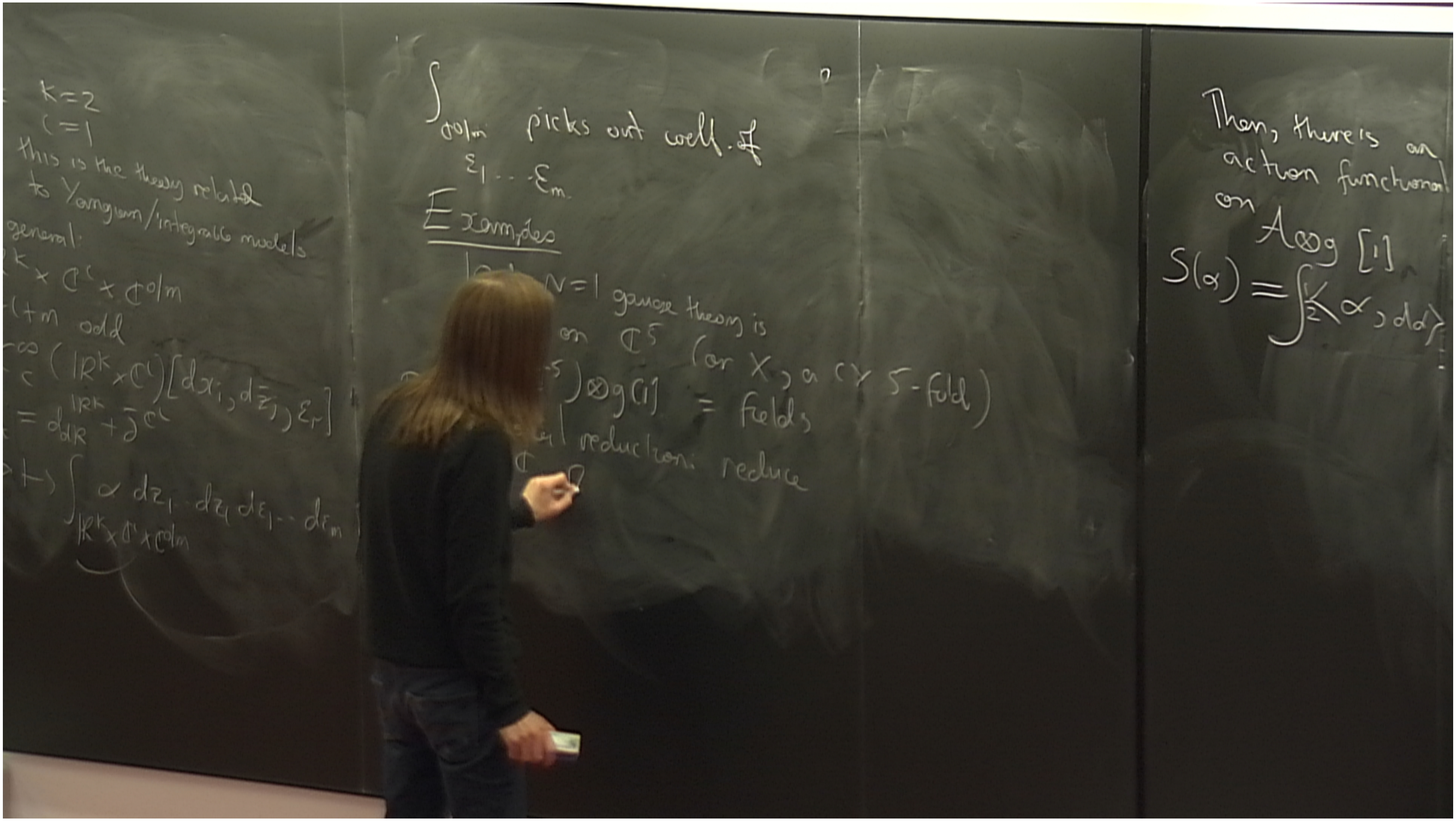

odd, generalization
of hCS
Ex k=2
l=1
this is the theory related
to Yangian/integrable models.
More general:
k+l+m odd
invariant


theory related
/integrable models
$\mathbb{C}^{0|m}$
$[dx_i, d\bar{z}_j, \varepsilon_r]$
$dz_1 \ldots dz_i \, d\varepsilon_1 \ldots d\varepsilon_m$
$\mathbb{C}^1 \times \mathbb{C}^{0|m}$
$\int_{\mathbb{C}^{0|m}}$ picks out coeff. of $\varepsilon_1 \ldots \varepsilon_m$
Then, there is an action functional on $\mathcal{A} \otimes \mathfrak{g}[1]$
$S(\alpha) = \int \frac{1}{2} \langle \alpha, d\alpha \rangle + \frac{1}{6}$


k=2
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this is the theory related
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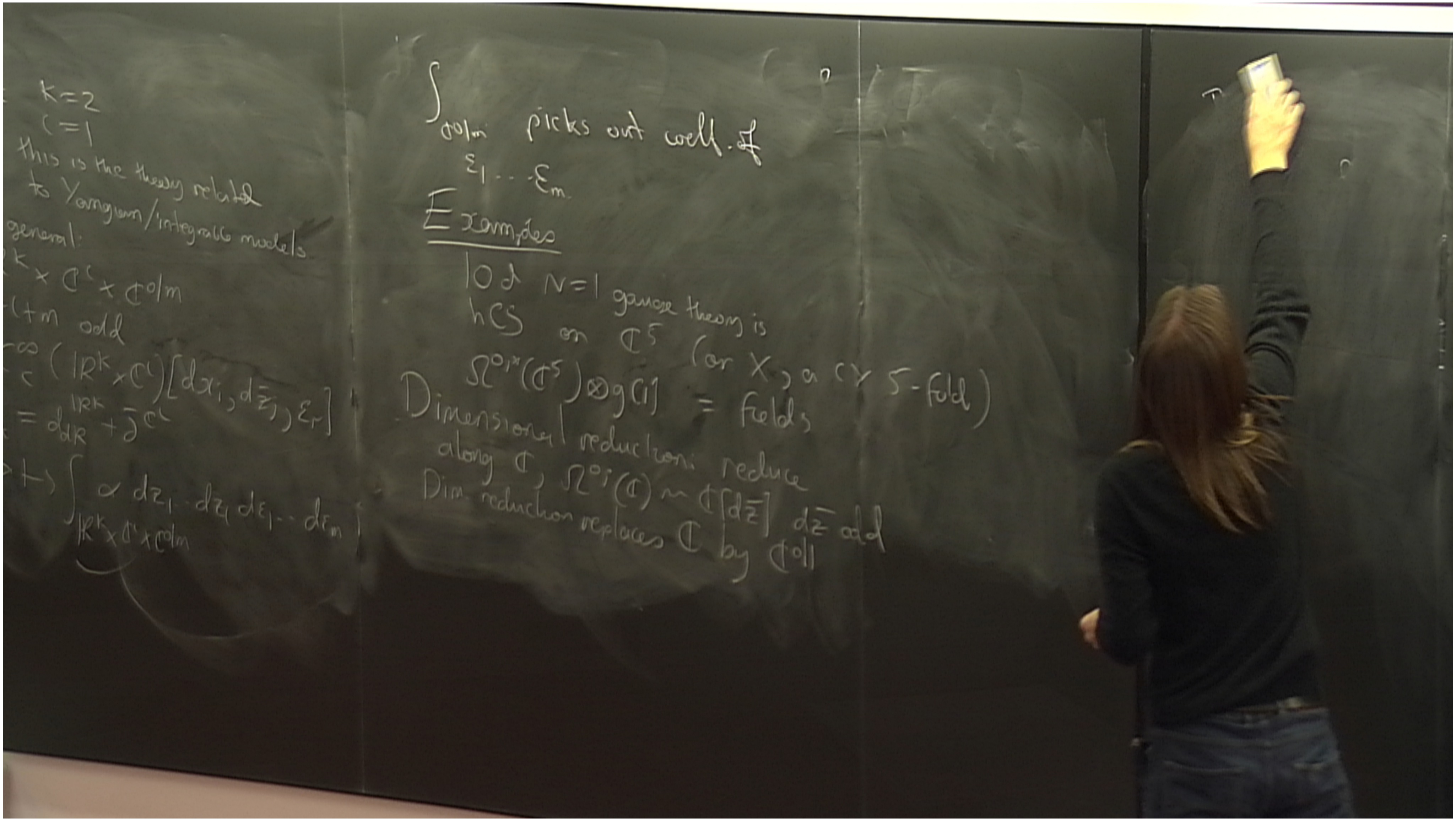
general:
$\mathbb{R}^k \times \mathbb{C}^l \times \mathbb{C}^{0|m}$
l+m odd
$\int_{\mathbb{C}^{0|m}}$ picks out coeff. of
$\varepsilon_1 \dots \varepsilon_m$
Examples
$\mathcal{N}=1$ gauge theory is
on $\mathbb{C}^5$ (or X, a CY 5-fold)
$\otimes \mathfrak{g}[1]$ = fields
reduction: reduce
Then, there is an
action functional
on $\mathcal{A} \otimes \mathfrak{g}[1]$
$S(\alpha) = \int \frac{1}{2} \langle \alpha, d\alpha \rangle$


k=2
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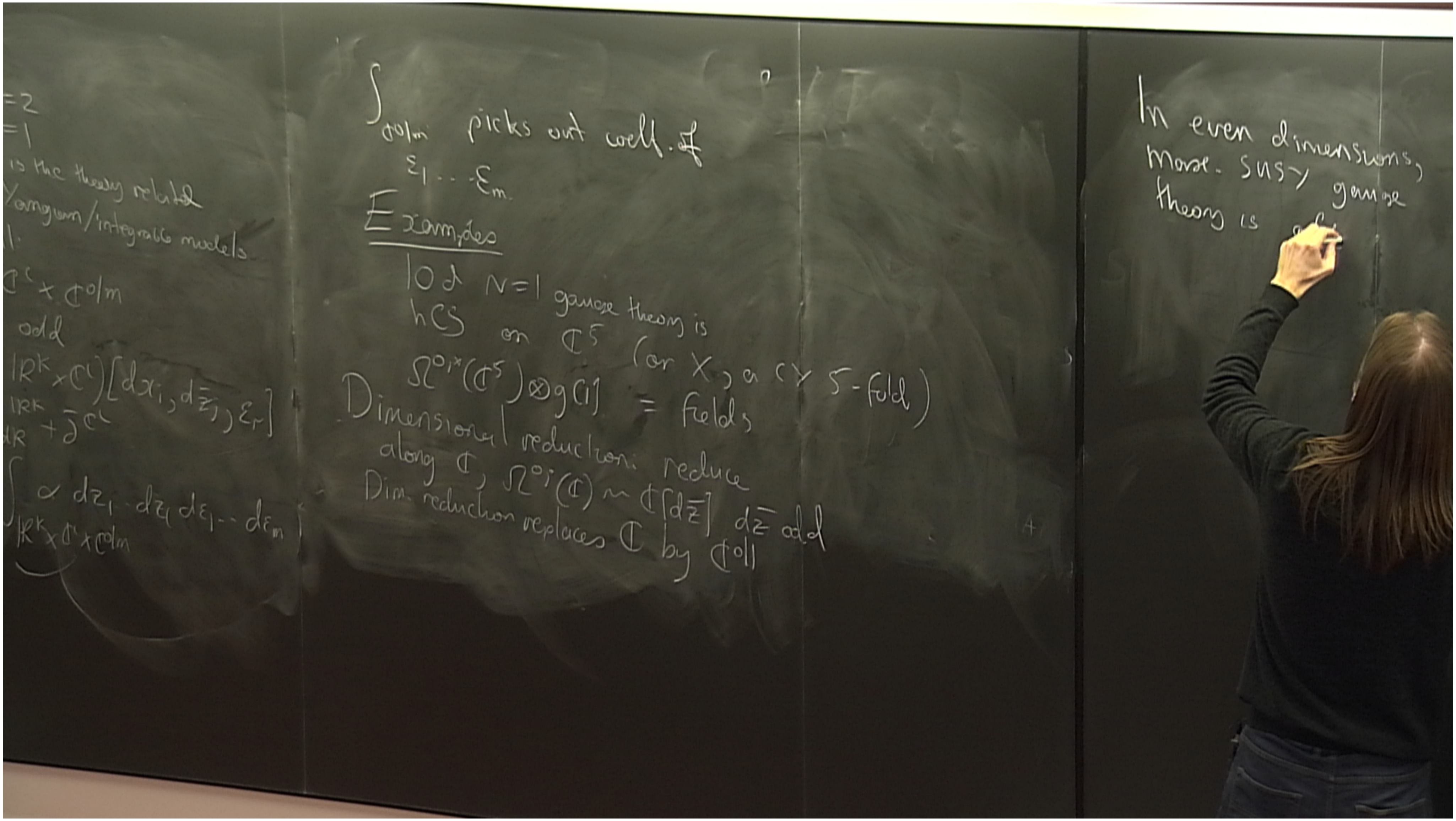
general:
ℂ^l × ℂ^{0|m}
l+m odd
(ℝ^k × ℂ^l)[dx_i, dz̄_j, ε_r]
= d_{ℝ^k} + ∂̄^{ℂ^l}
α dz_1 .. dz_l dε_1 .. dε_m
ℝ^k × ℂ^l × ℂ^{0|m}
∫_{ℂ^{0|m}} picks out coeff. of
ε_1 ... ε_m
Examples
10d N=1 gauge theory is
hCS on ℂ^5 (or X, a CY 5-fold)
Ω^{0,*}(ℂ^5) ⊗ g[1] = fields
Dimensional reduction: reduce
along ℂ, Ω^{0,i}(ℂ) ~ ℂ[dz̄] dz̄ odd
Dim. reduction replaces ℂ by ℂ^{0|1}



picks out coeff. of
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10d N=1 gauge theory is
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Dim. reduction replaces
by
In even dimensions,
max. susy gauge
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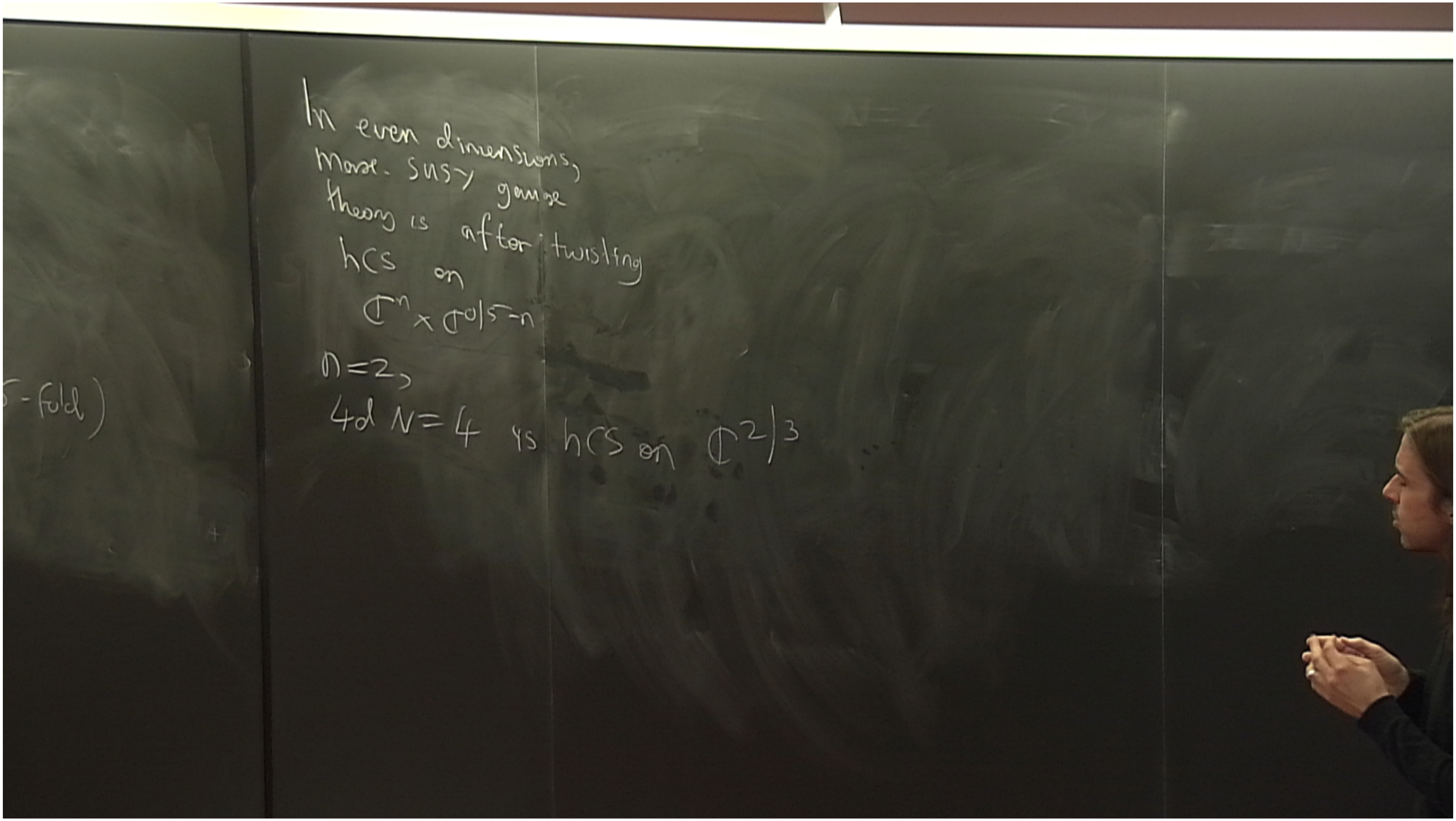
is the theory related
Yangian/integrable models
odd


In even dimensions,
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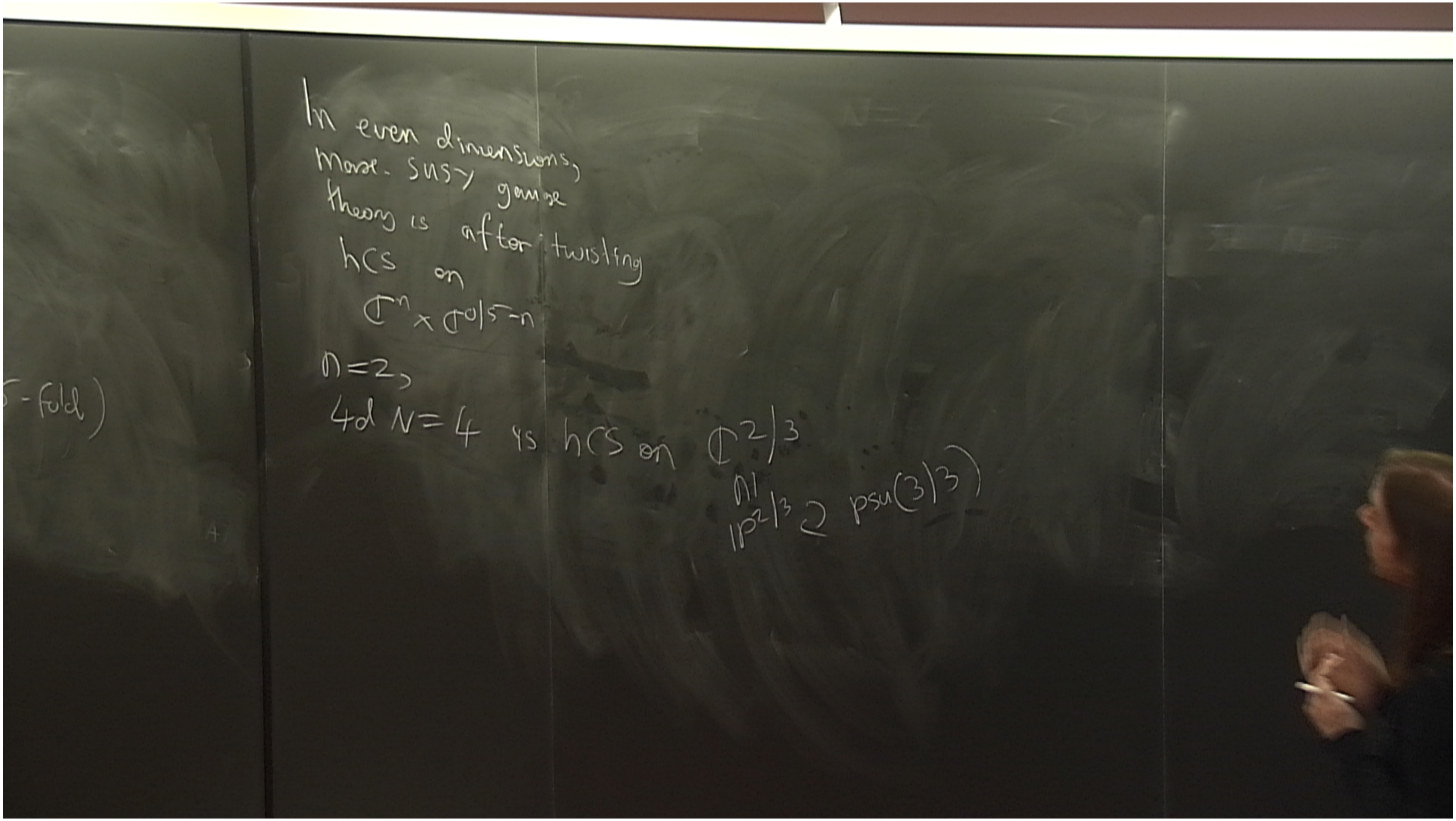
theory is after twisting
hCS on
$\mathbb{C}^n \times \mathbb{C}^{0|5-n}$
$n=2$,
4d $N=4$ ys hCS on $\mathbb{C}^{2|3}$
5-fold)


In even dimensions, max. susy gauge theory is after twisting
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$n=2$,
4d $N=4$ ys hCS on $\mathbb{C}^{2|3}$
$\mathbb{P}^{2|3} \supseteq psu(3|3)$
5-fold)