Title: Weinberg Soft Theorems from Weinberg Adiabatic Modes

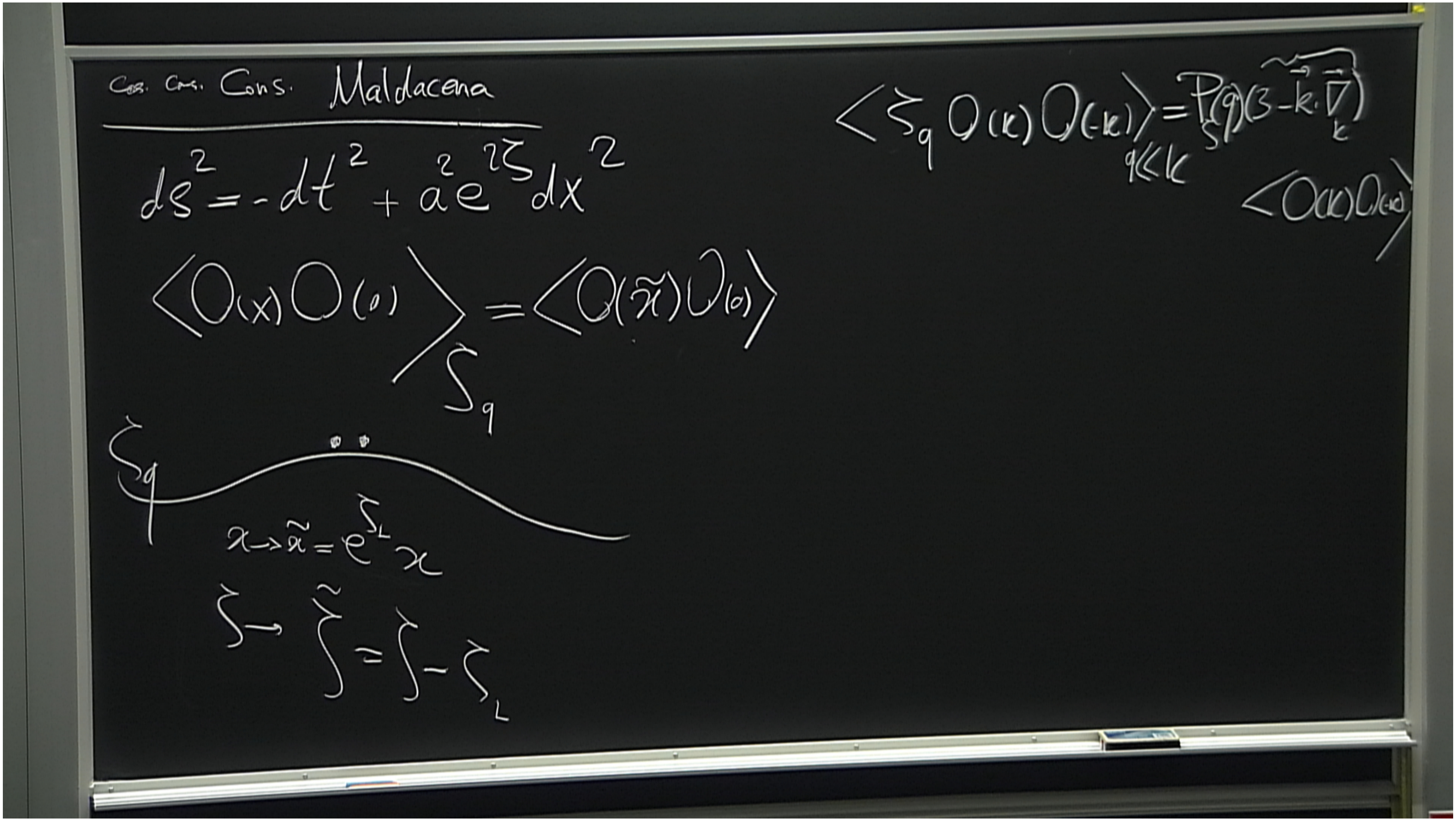
Date: Nov 08, 2016 01:00 PM

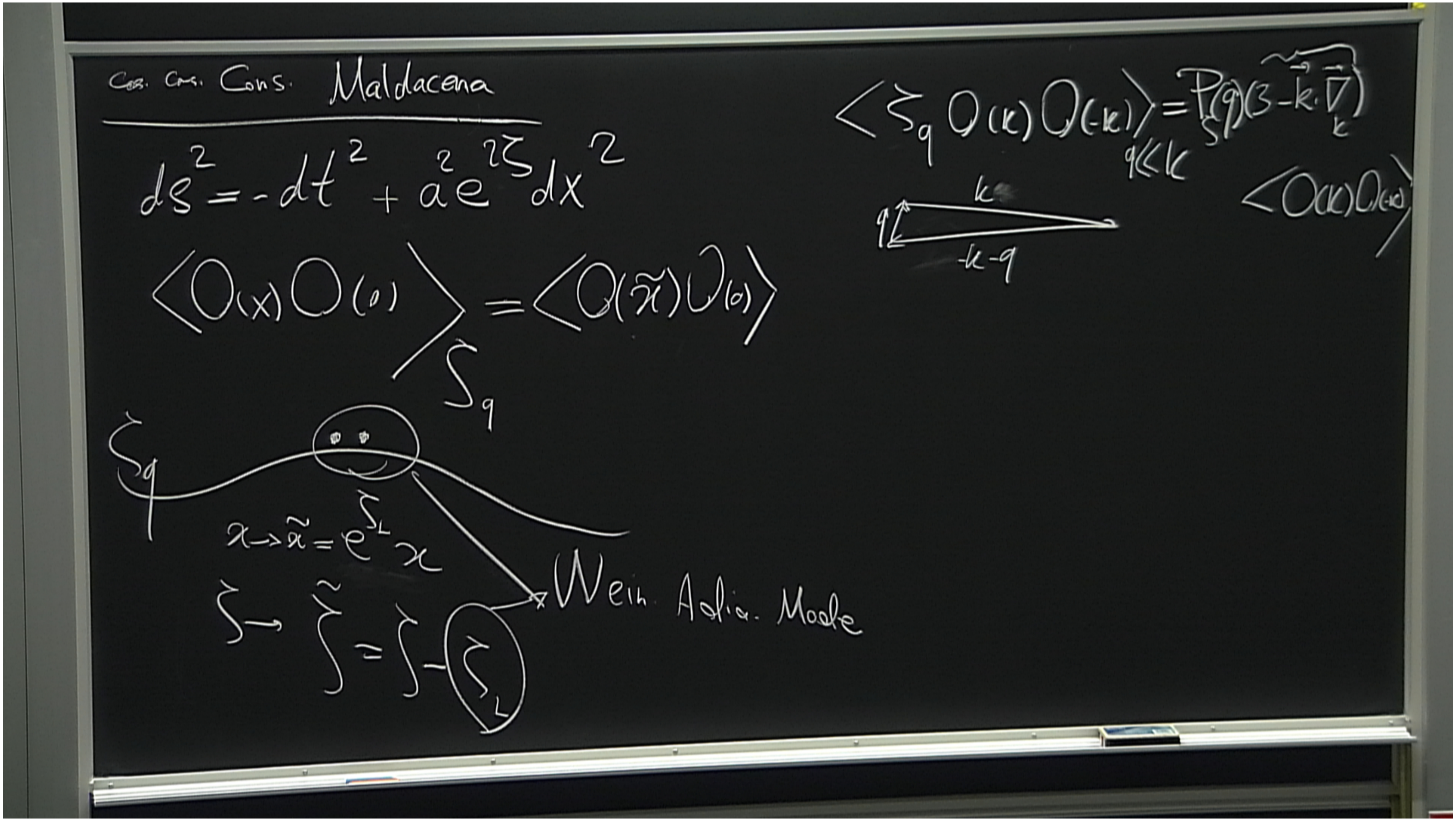
URL: http://pirsa.org/16110020

Abstract: <p>Soft theorems for the scattering of low energy photons and gravitons and cosmological consistency conditions on the squeezed-limit correlation functions are both understood to be consequences of invariance under large gauge transformations. I apply the same method used in cosmology -- based on the identification of an infinite set of "adiabatic modes" and the corresponding conserved currents -- to derive flat space soft theorems for electrodynamics and gravity. I discuss how the recent derivations based on the asymptotic symmetry groups (BMS) can be continued to a finite size sphere surrounding the scattering event. Finally, I comment on Hawking-Perry-Strominger proposal of "soft hair on black holes".</p>

Weinberg soft th. from Wein. adiabatic modes. w/ M. Simonovič, M. Porrati

$$
\begin{tikzcd}
\text{Cosm. Cons. Cond.} & \text{BMS asym. Mink. (large diff.)} \arrow[d, "\text{Strominger et. al.}"] \\
\text{Wein. adia. modes large diff.} \arrow[u] \arrow[r] & \text{Wein. soft photon/gr. th.}
\end{tikzcd}
$$


Cos. Cos. Cons. Maldacena
$ds^2 = -dt^2 + a^2 e^{2\zeta} dx^2$
$\langle O(x) O(0) \rangle_{\zeta_q} = \langle O(\tilde{x}) O(0) \rangle$
ζ_q
$x \to \tilde{x} = e^{\zeta_L} x$
$\zeta \to \tilde{\zeta} = \zeta - \zeta_L$
$\langle \zeta_q O(k) O(-k) \rangle = P_\zeta(q)(3 - \vec{k} \cdot \vec{\nabla}_k) \langle O(k) O(-k) \rangle$
$q \ll k$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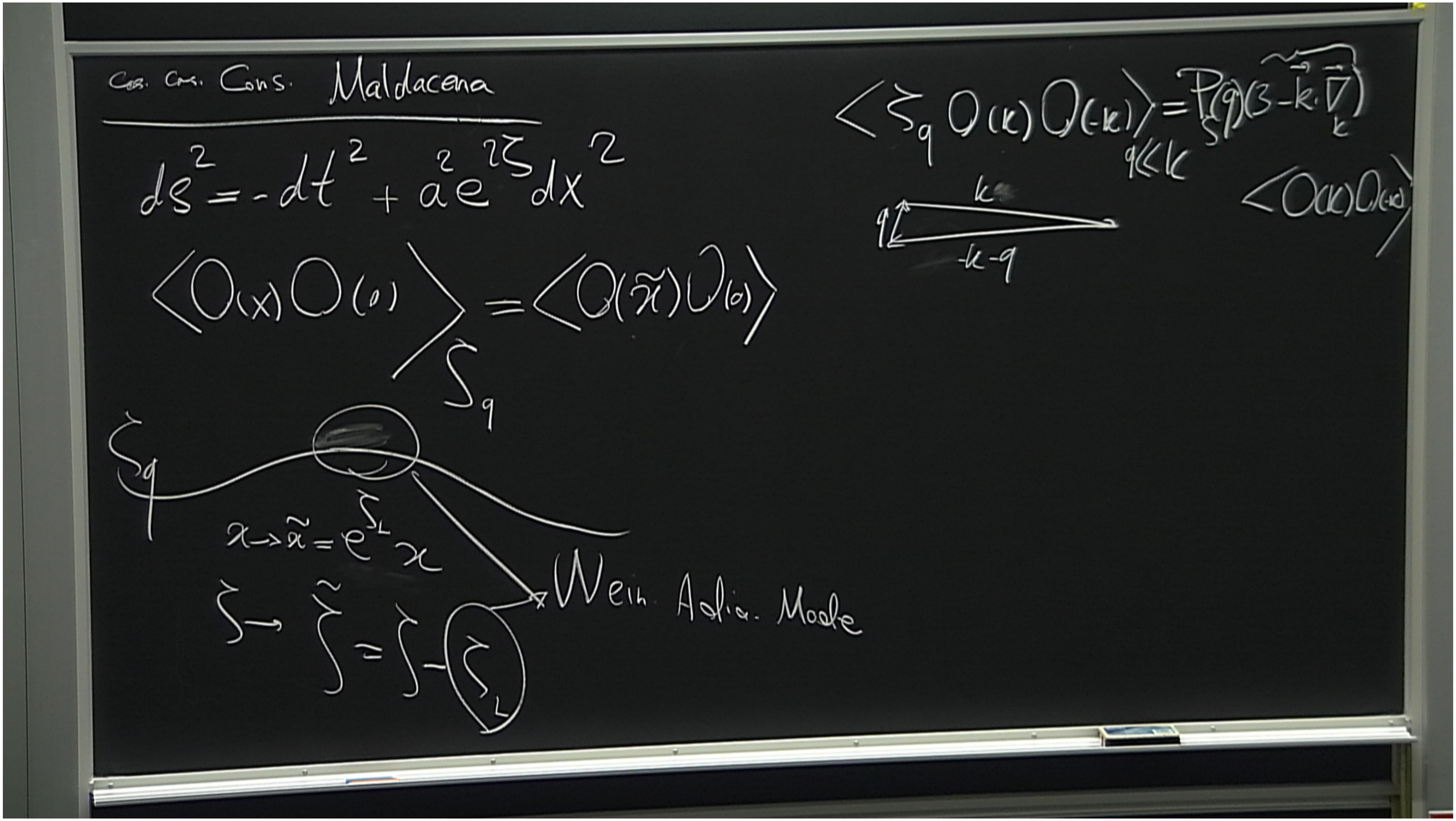
q
$-k-q$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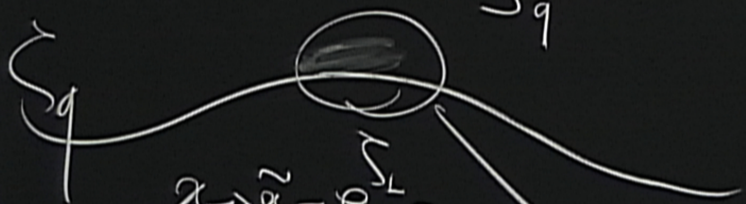
Cos. Cos. Cons. Maldacena

$$ds^2 = -dt^2 + a^2 e^{2\zeta} dx^2$$

$$\langle O(x)\, O(0) \rangle_{\zeta_q} = \langle O(\tilde{x})\, O(0) \rangle$$

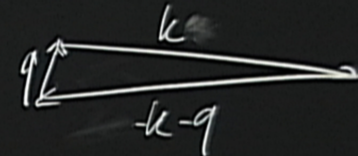


$x \to \tilde{x} = e^{\zeta_L} x$

$\zeta \to \tilde{\zeta} = \zeta - \zeta_L$ → Wein. Adia. Mode

$$\langle \zeta_q\, O(k)\, O(-k) \rangle = P_\zeta(q)\,(3 - \vec{k}\cdot\vec{\nabla}_k)\, \langle O(k)\, O(-k) \rangle$$

$q \ll k$



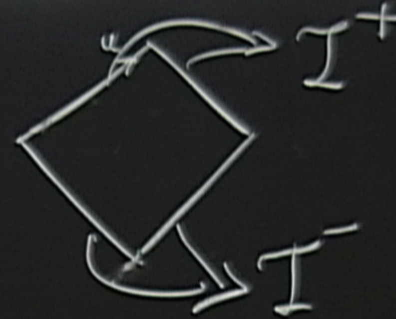
BMS

$$ds^2 = -du^2 - 2du\,dr + r^2 d\Omega^2$$

$BMS_{\pm}$, Lorentz + $\xi^u(\hat{r})$

$$Q_{\pm} = Q_S + Q_H$$

EM, $\alpha_{\pm}(\hat{r})$



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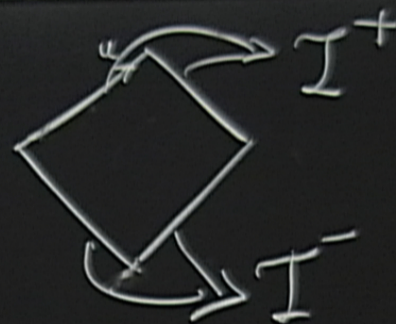
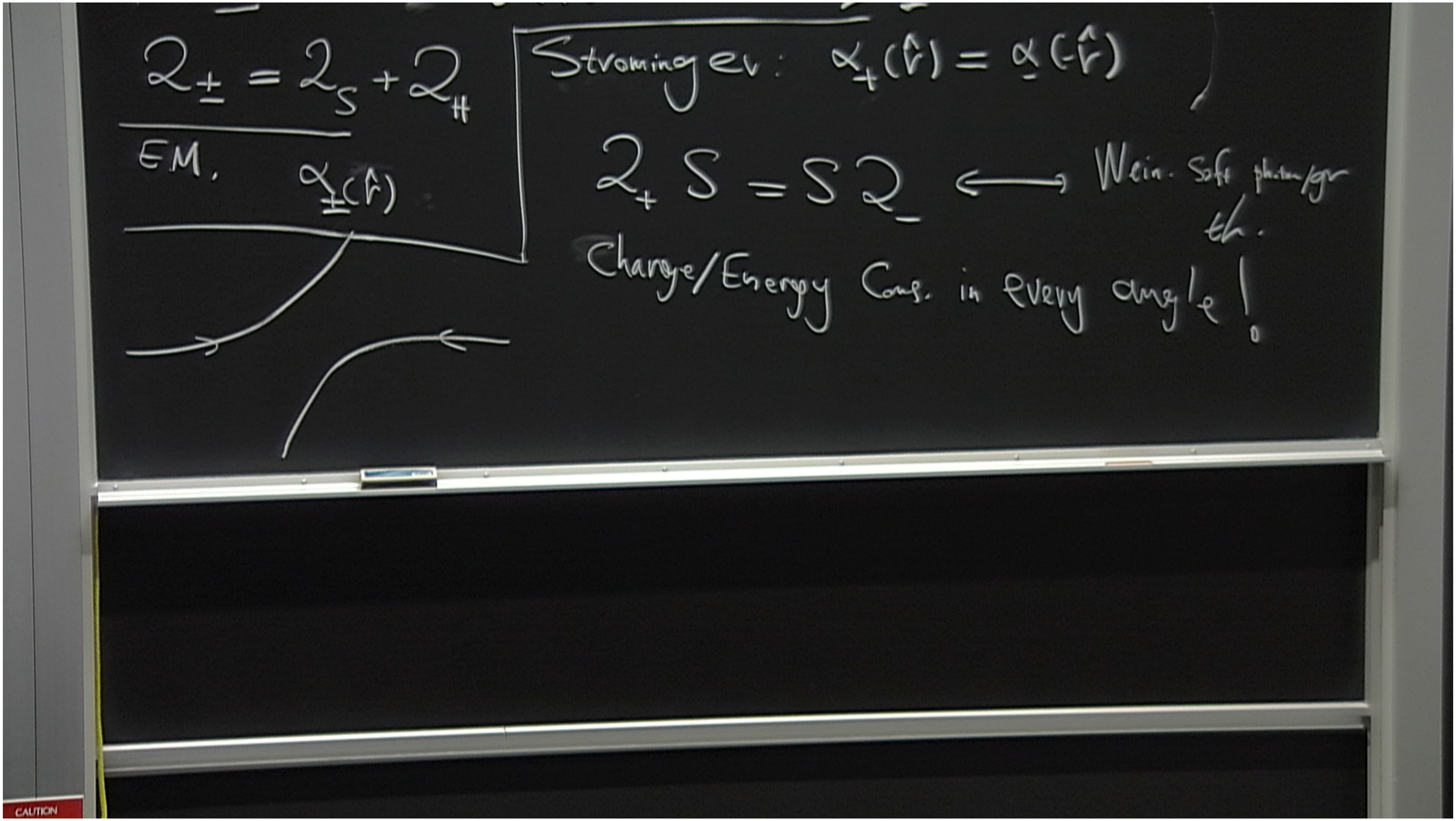


$Q_\pm = Q_S + Q_H$

EM. $\alpha_\pm(\hat{r})$

Strominger: $\alpha_+(\hat{r}) = \alpha_-(-\hat{r})$

$Q_+ S = S Q_-$ $\longleftrightarrow$ Wein. soft photon/gr th.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$Q_+ S = S Q_-$ ⟷ Wein. Soft photon/gr th.
Charge/Energy Cons. in every angle!

BMS

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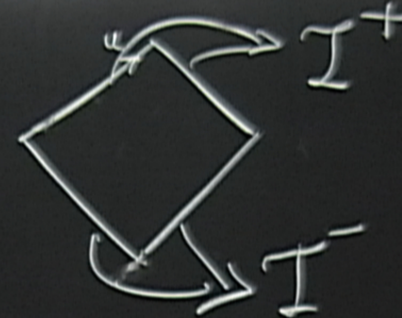

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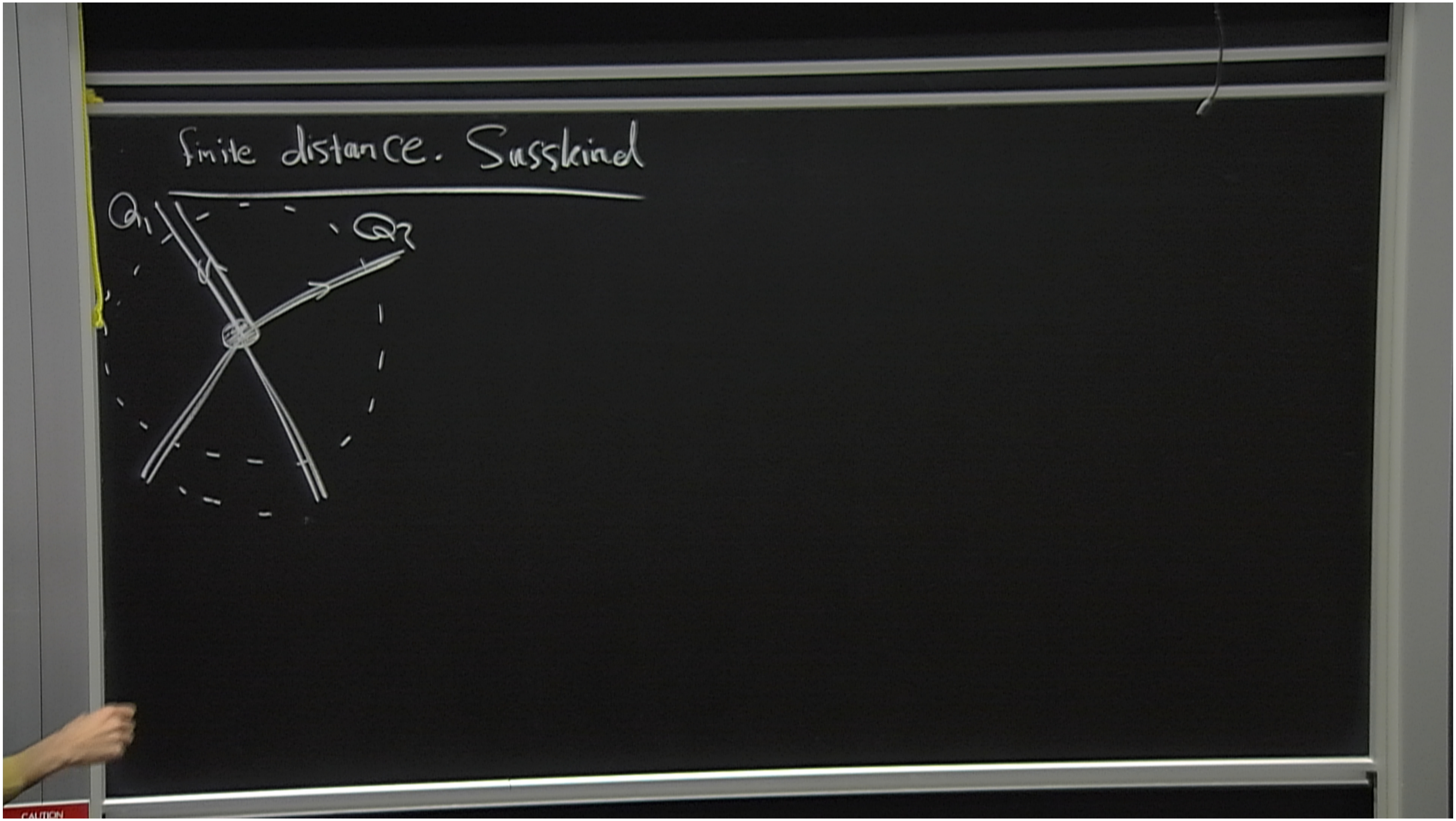
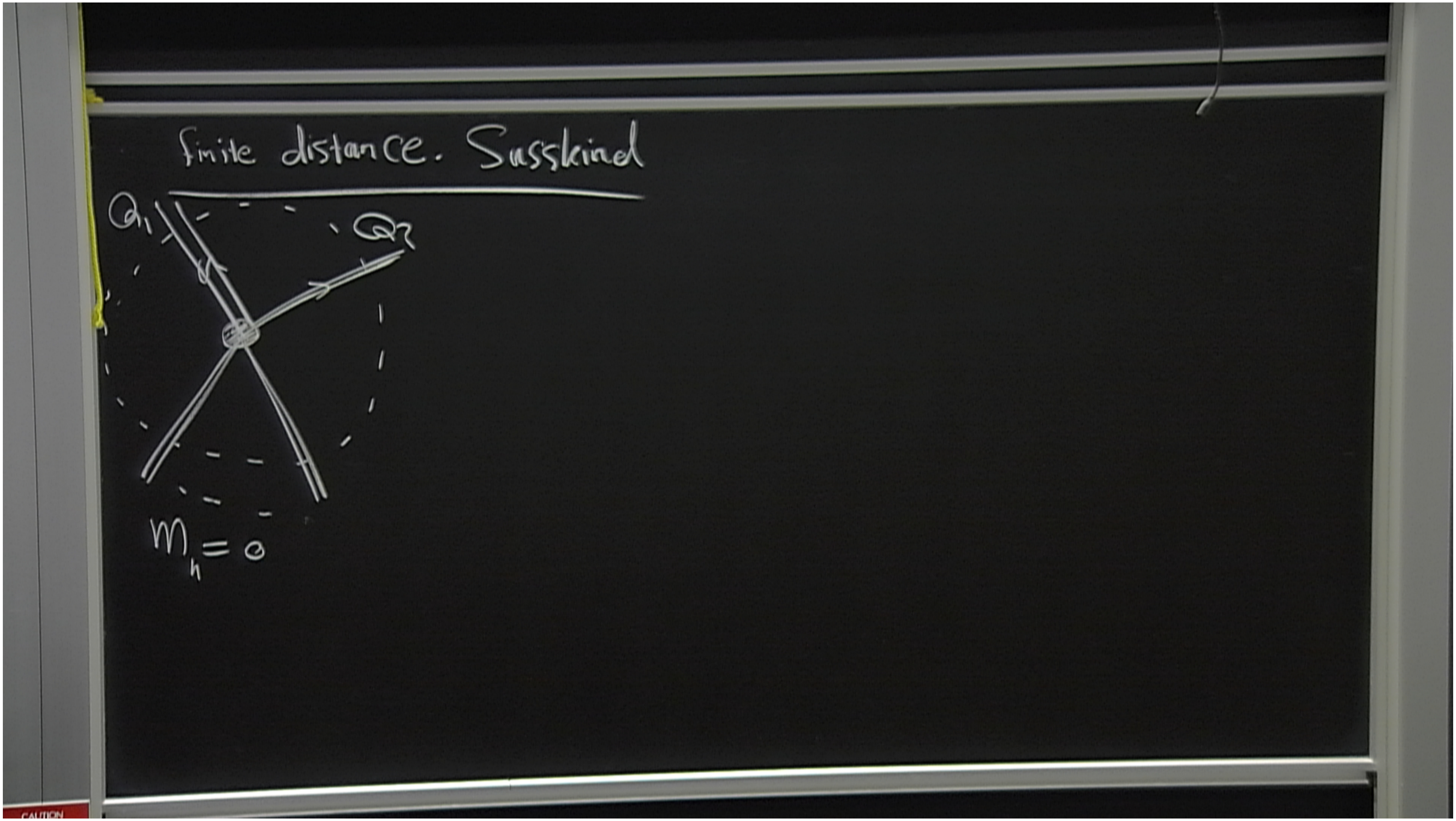
Charge/Energy Cons. in every angle!


finite distance. Susskind
Q_1
Q_2


finite distance. Susskind
Q_1
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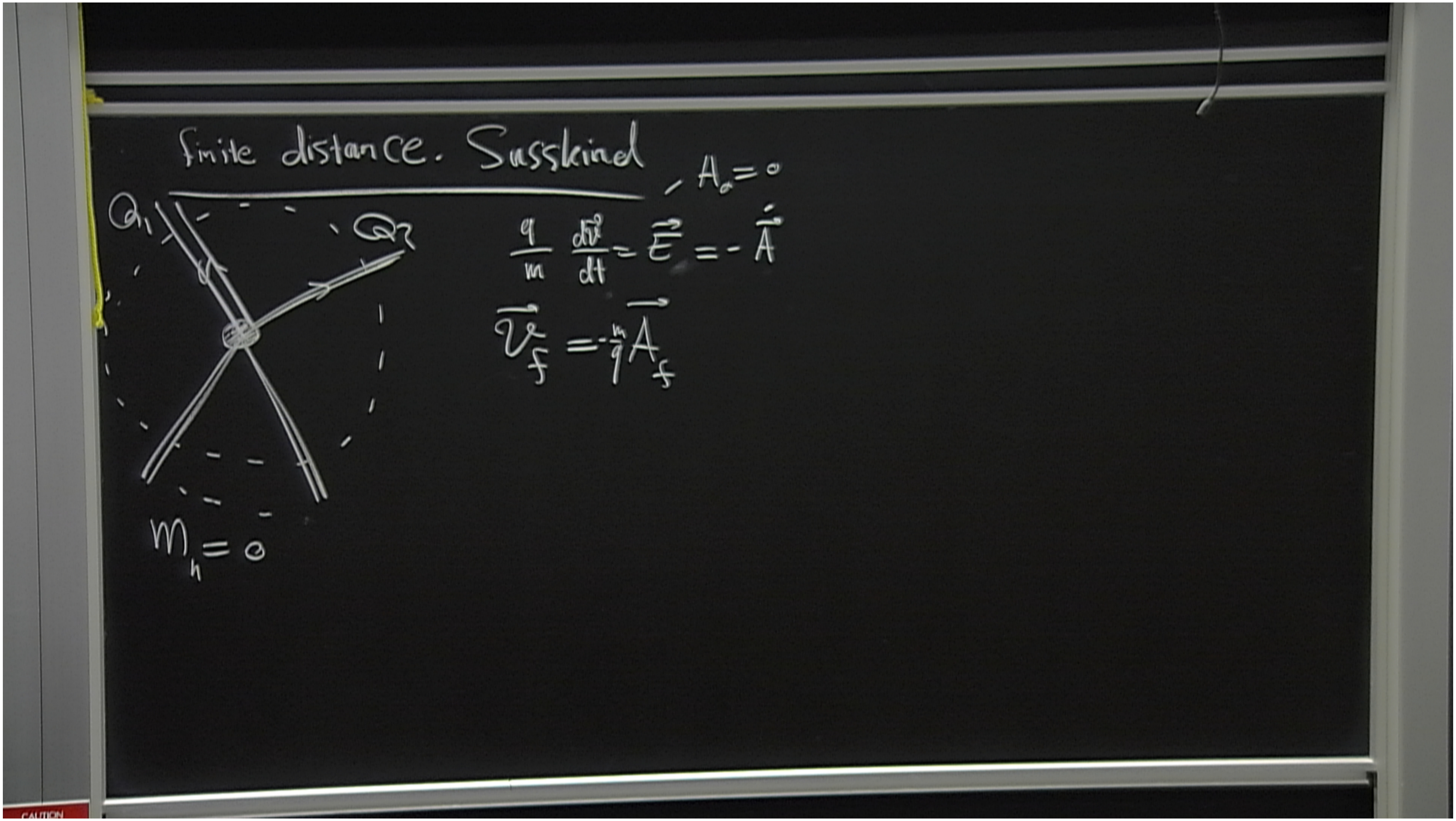
$m_h = 0$


finite distance. Susskind
$A_0 = 0$
Q_1
Q_2
$\frac{q}{m}\frac{d\vec{v}}{dt} = \vec{E} = -\dot{\vec{A}}$
$\vec{v}_f = -\frac{m}{q}\vec{A}_f$
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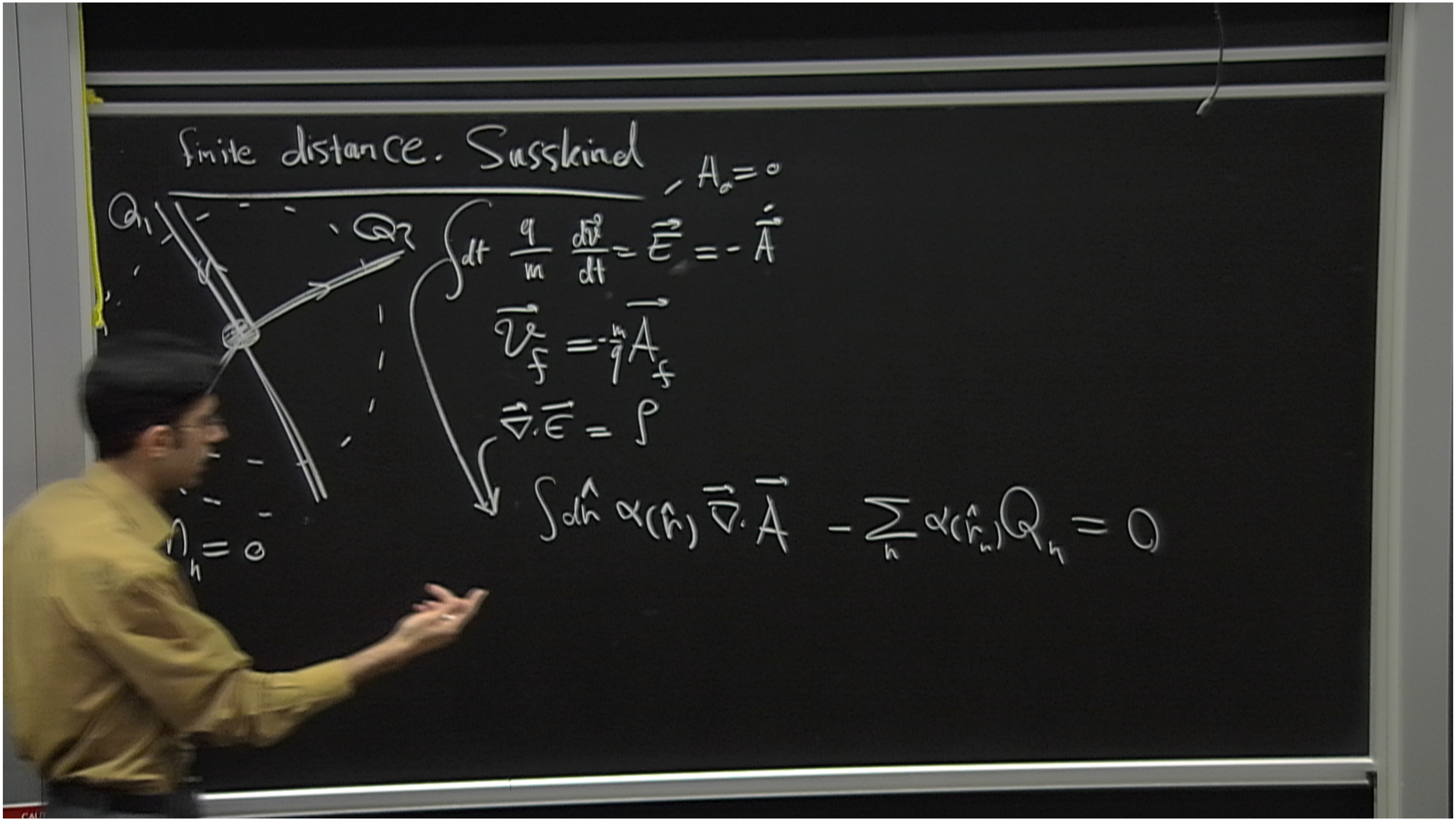
finite distance. Susskind
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$\int dt \, \frac{q}{m} \frac{d\vec{v}}{dt} = \vec{E} = -\dot{\vec{A}}$
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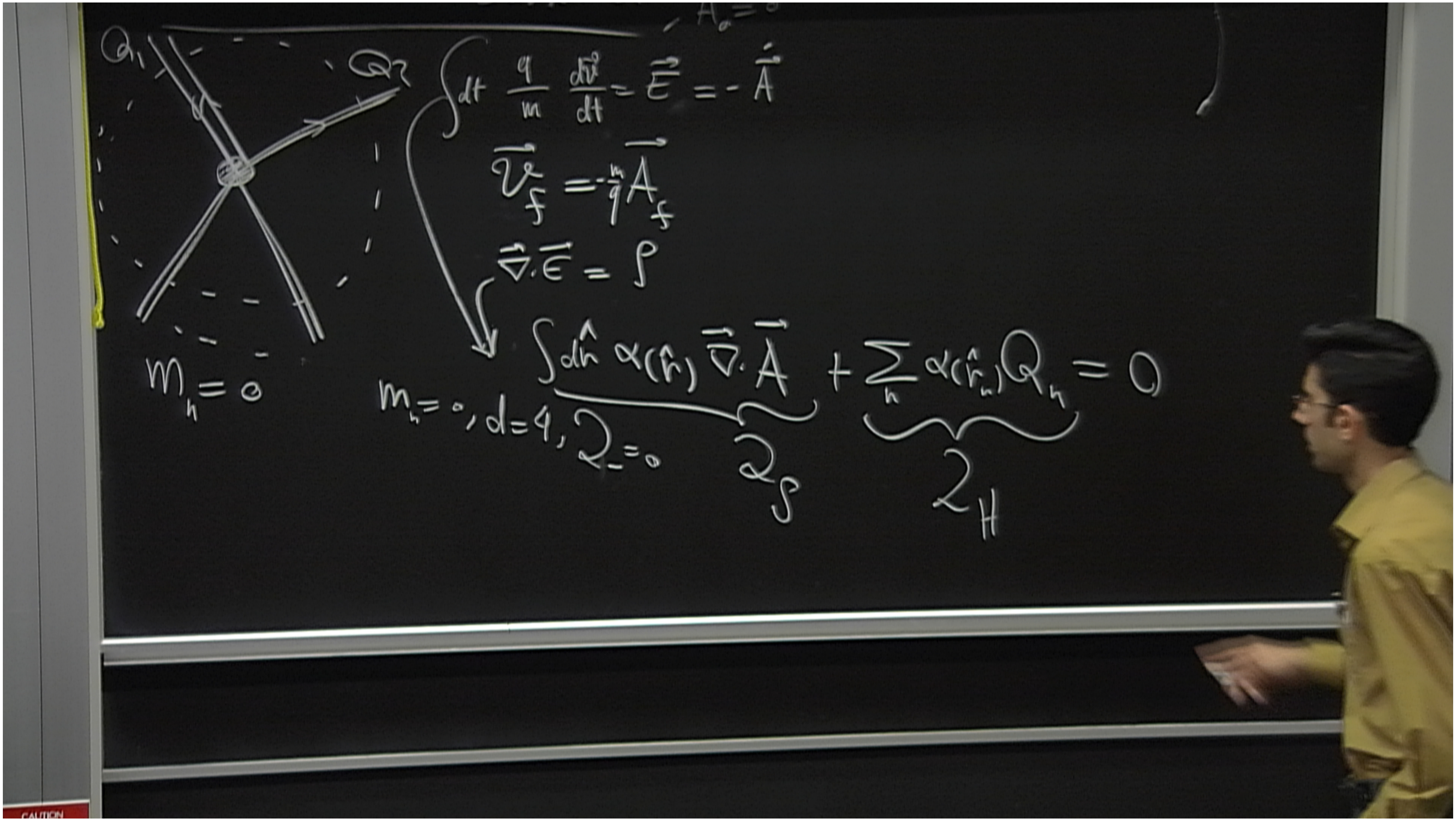
$\vec{\nabla} \cdot \vec{E} = \rho$
$\int d\hat{n} \, \alpha(\hat{n}) \, \vec{\nabla} \cdot \vec{A} - \sum_n \alpha(\hat{n}_n) Q_n = 0$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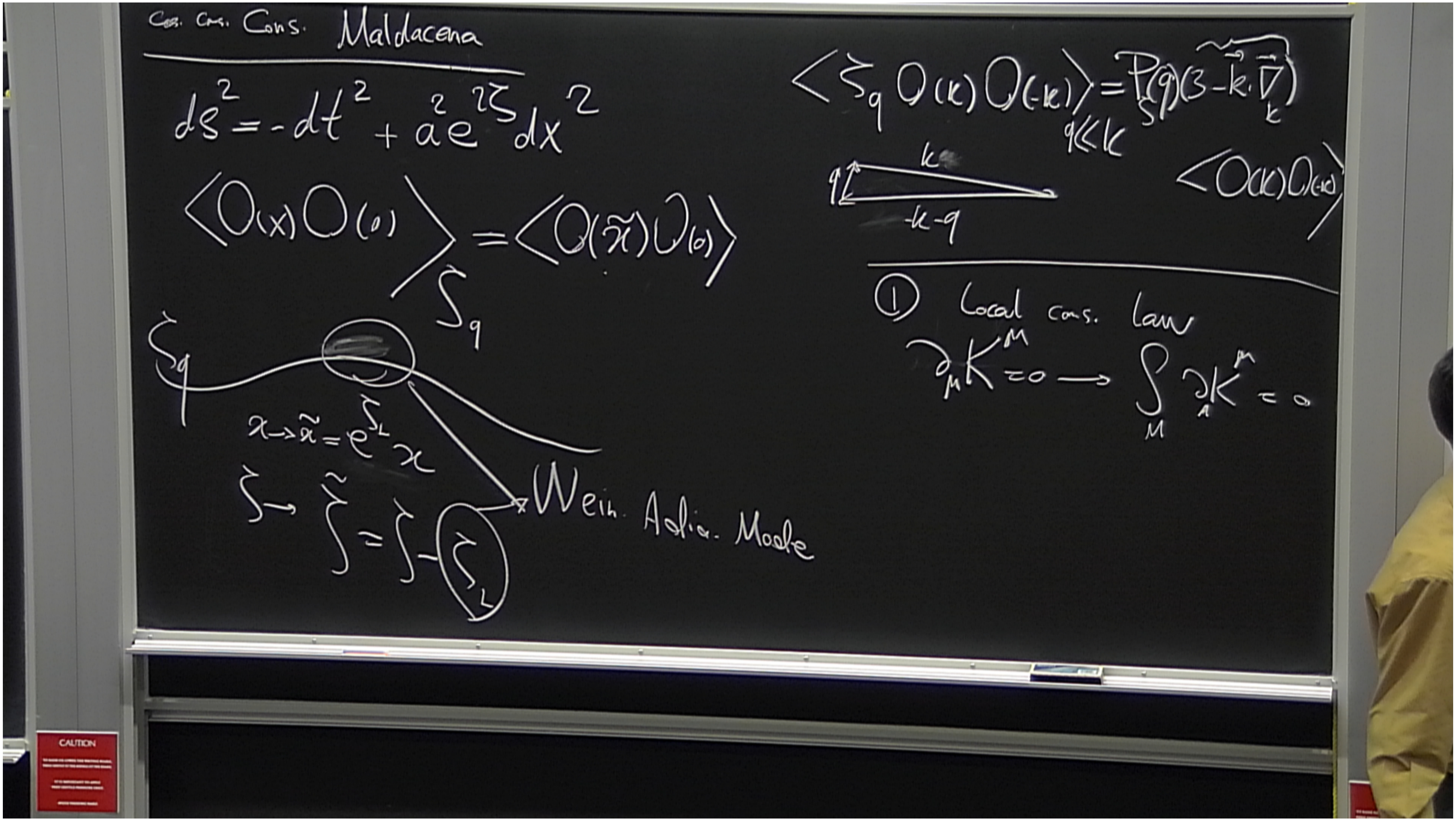
$\vec{\nabla}\cdot\vec{E} = \rho$
$m_n = 0$
$\underbrace{\int d\hat{n}\, \alpha(\hat{n})\, \vec{\nabla}\cdot\vec{A}}_{Q_S} + \underbrace{\sum_n \alpha(\hat{r}_n) Q_n}_{Q_H} = 0$
$m_n = 0,\ d = 4,\ Q_- = 0$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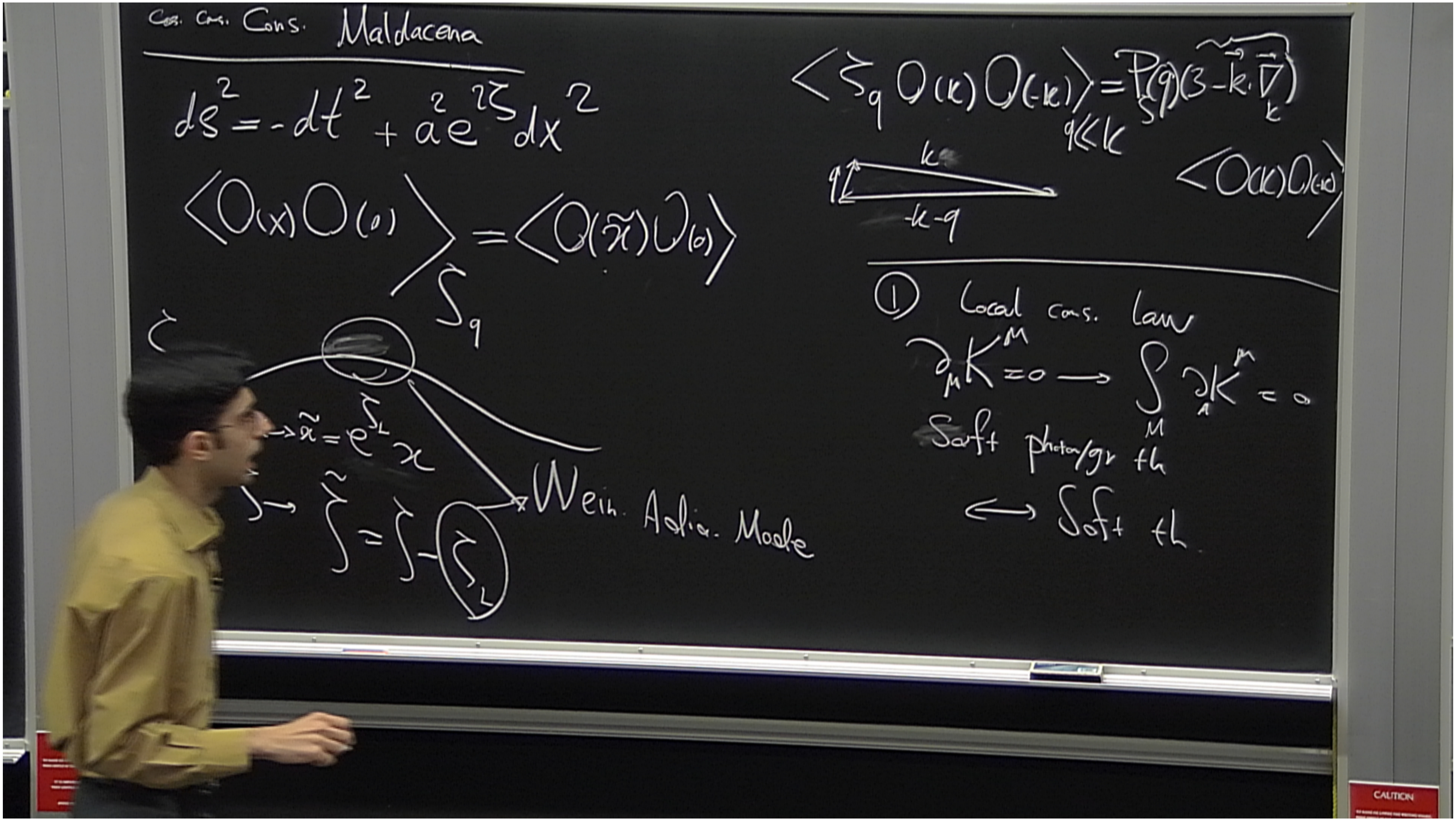
① Local cons. law
$\partial_\mu K^\mu = 0 \longrightarrow \int_M \partial_\mu K^\mu = 0$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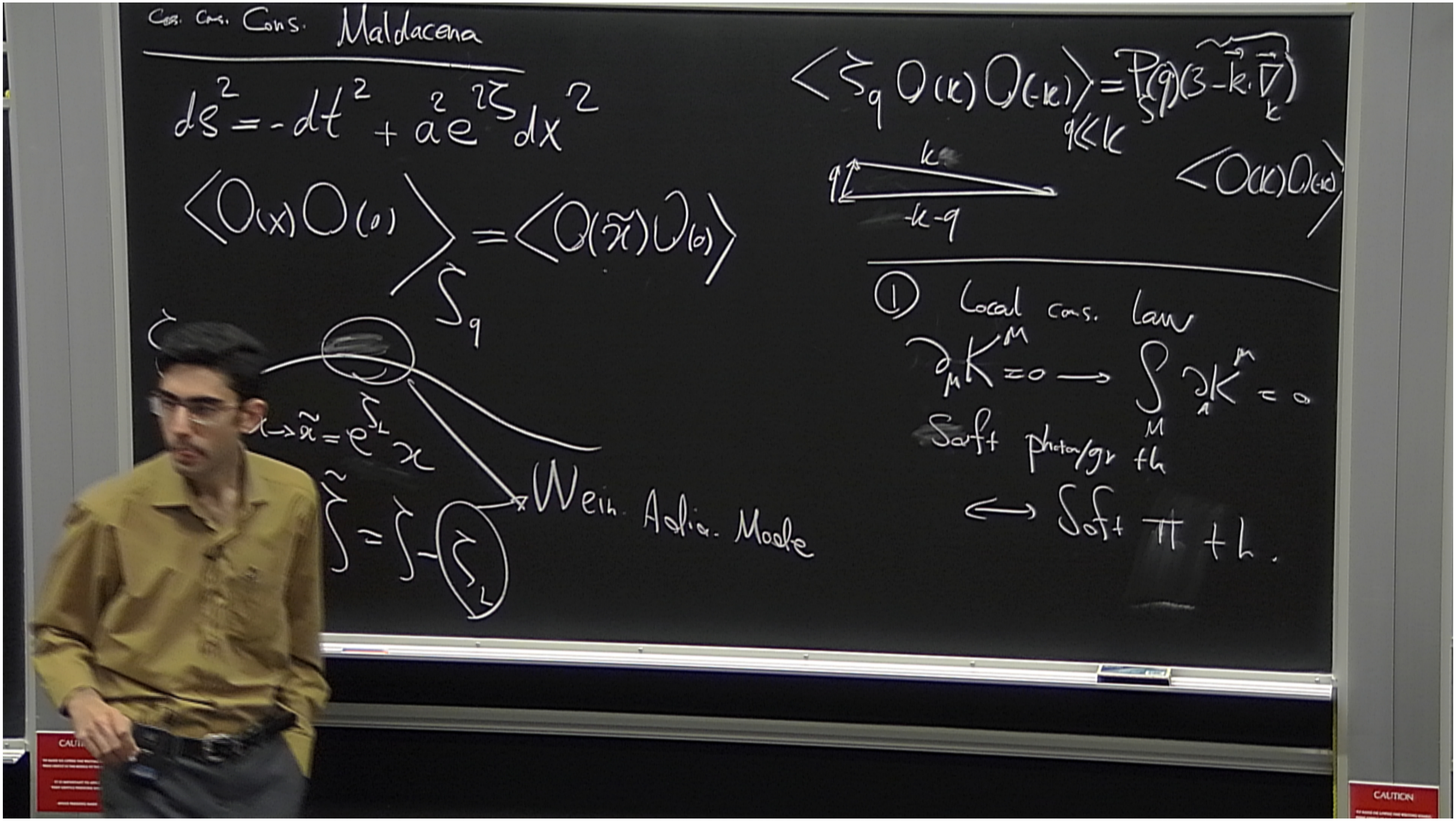
Soft photon/gr th
$\longleftrightarrow$ Soft th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CAUTION

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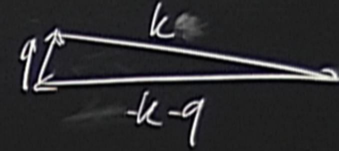
Wein. Adia. Mode

$$\langle \zeta_q\, O(k)\, O(-k) \rangle \underset{q \ll k}{=} P_\zeta(q)\left(3 - \vec{k}\cdot\vec{\nabla}_k\right) \langle O(k)\, O(-k) \rangle$$



$O(q^0)$

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Soft photon/gr th

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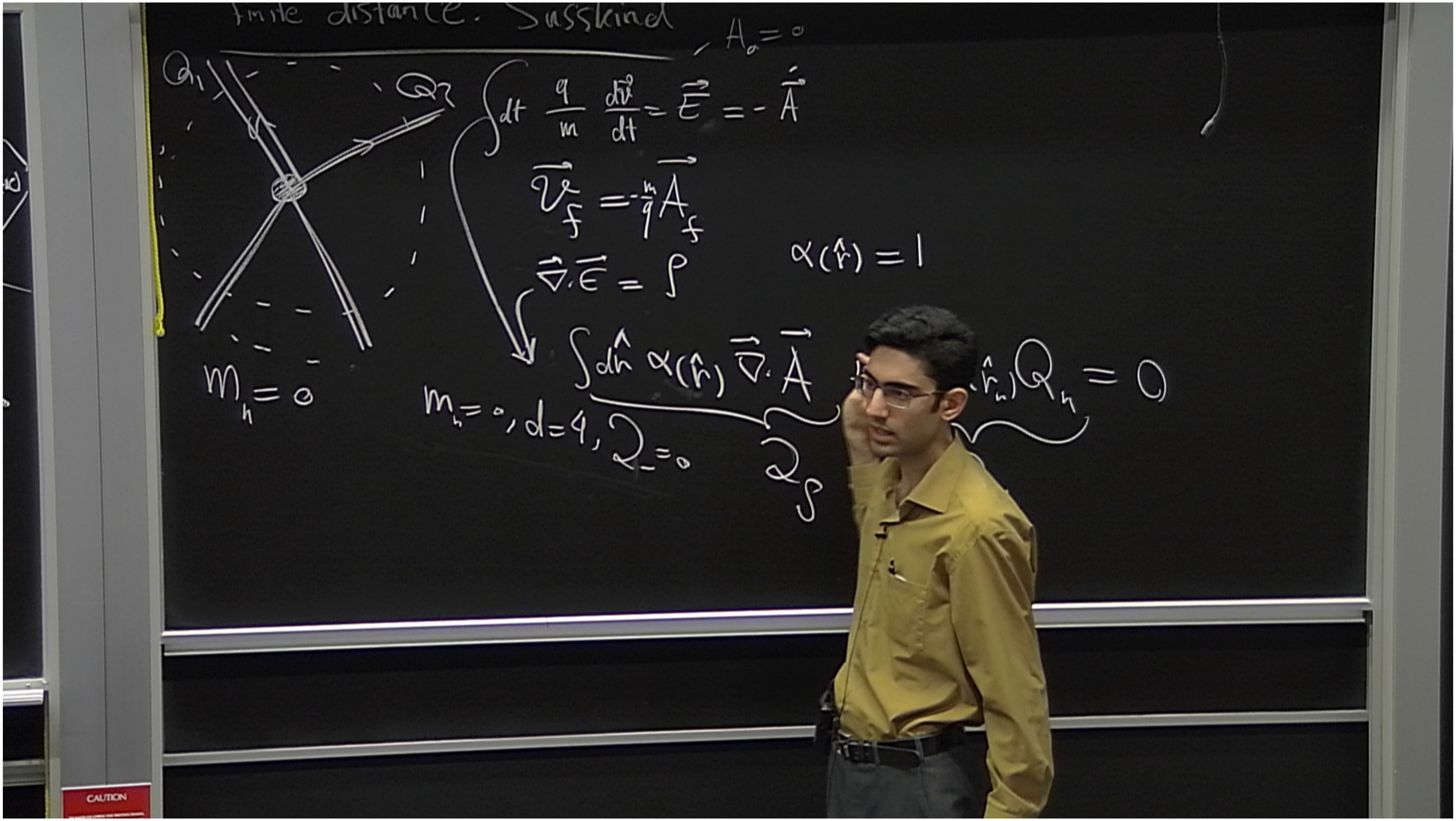
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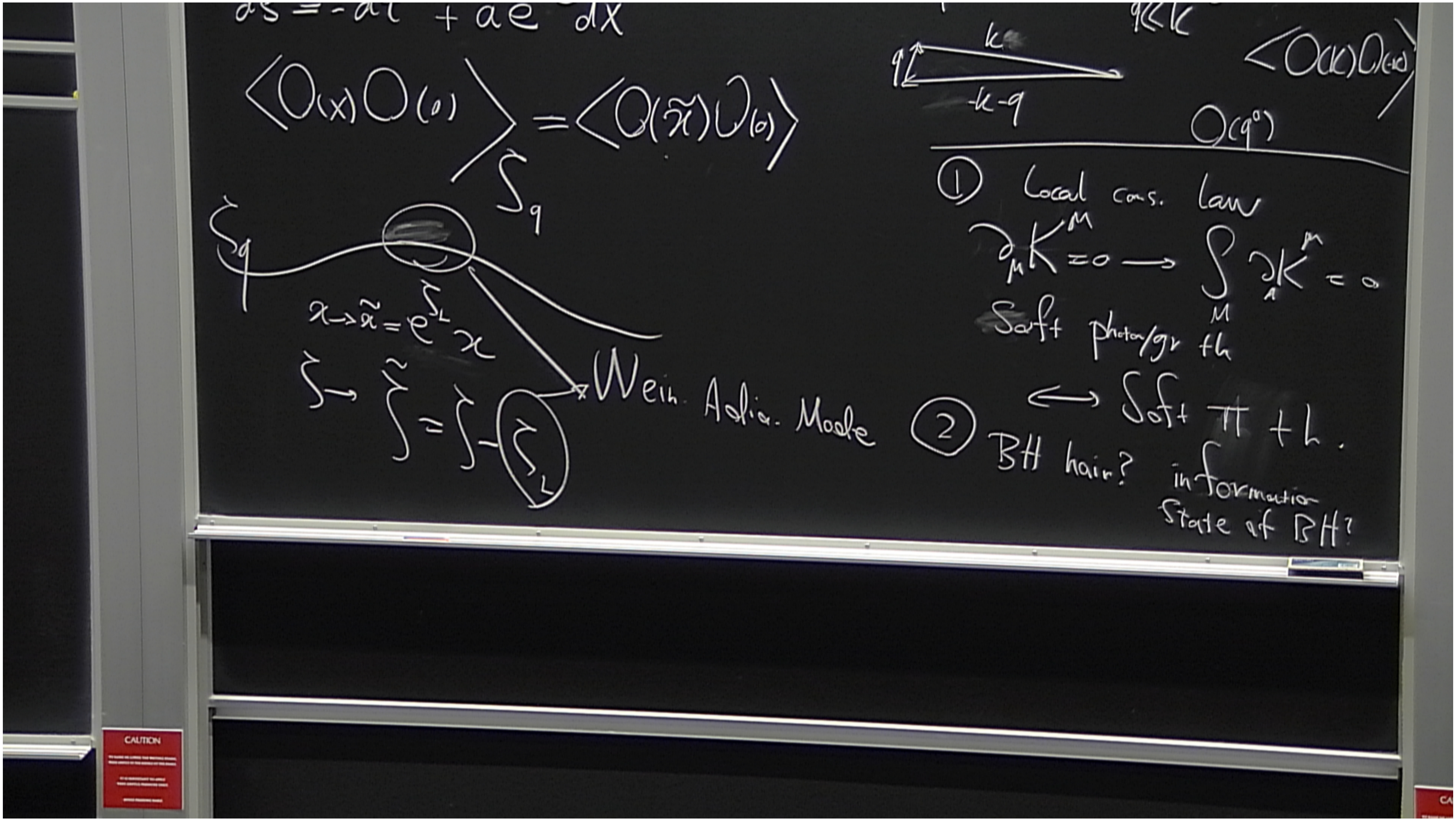
$\vec{\nabla}\cdot\vec{E} = \rho$
$\alpha(\hat{r}) = 1$
$m_n = 0$
$m_n = 0, \; d = 4, \; \mathcal{Q}_- = 0$
$\int d\hat{n} \; \alpha(\hat{r}) \; \vec{\nabla}\cdot\vec{A}$
$(\hat{r}_n) Q_n = 0$
$\mathcal{Q}_S$


⟨O(x)O(0)⟩ = ⟨O(x̃)O(0)⟩
x → x̃ = e^{ζ_L} x
ζ → ζ̃ = ζ - ζ_L
Wein. Adia. Mode
q≪k
k
-k-q
O(q⁰)
① Local cons. law
∂_μ K^μ = 0 → ∫_M ∂_μ K^μ = 0
Soft photon/gr th
⟷ Soft π th.
② BH hair? information
State of BH?
CAUTION

Soft π th.

$\pi \to \pi + c$

$J^\mu = \partial^\mu \pi + \ldots$

Soft π th: $\int e^{iq\cdot x}\, \partial_\mu \langle \text{out} | J^\mu | \text{in} \rangle = 0$

Soft π th.

$\pi \to \pi + c$

$J^\mu = \partial^\mu \pi + \dots$

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$\pi(x)$

Soft π th.

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$J^{\mu} = \partial^{\mu}\pi + \ldots$

Soft π th: $\int e^{iq\cdot x}\, \partial_{\mu} \langle out | J^{\mu} | in \rangle = 0$

$\pi(x)$

Adia. Modes EM

1) Gauge fix. ex. $A_0 = 0$

2) $A_i = \nabla_i \alpha\,, \quad \dot{\alpha} = 0$

3) Locally unobservable configuration

$$A_i(0,\vec{x}) = \sum a_{i\, i_1 \ldots i_n} x^{i_1} \cdots x^{i_n}$$

$\partial = \partial\pi + \ldots$

Soft π th: $\int e^{iq\cdot x}\, \partial_\mu \langle out | J^\mu | in \rangle \neq 0$

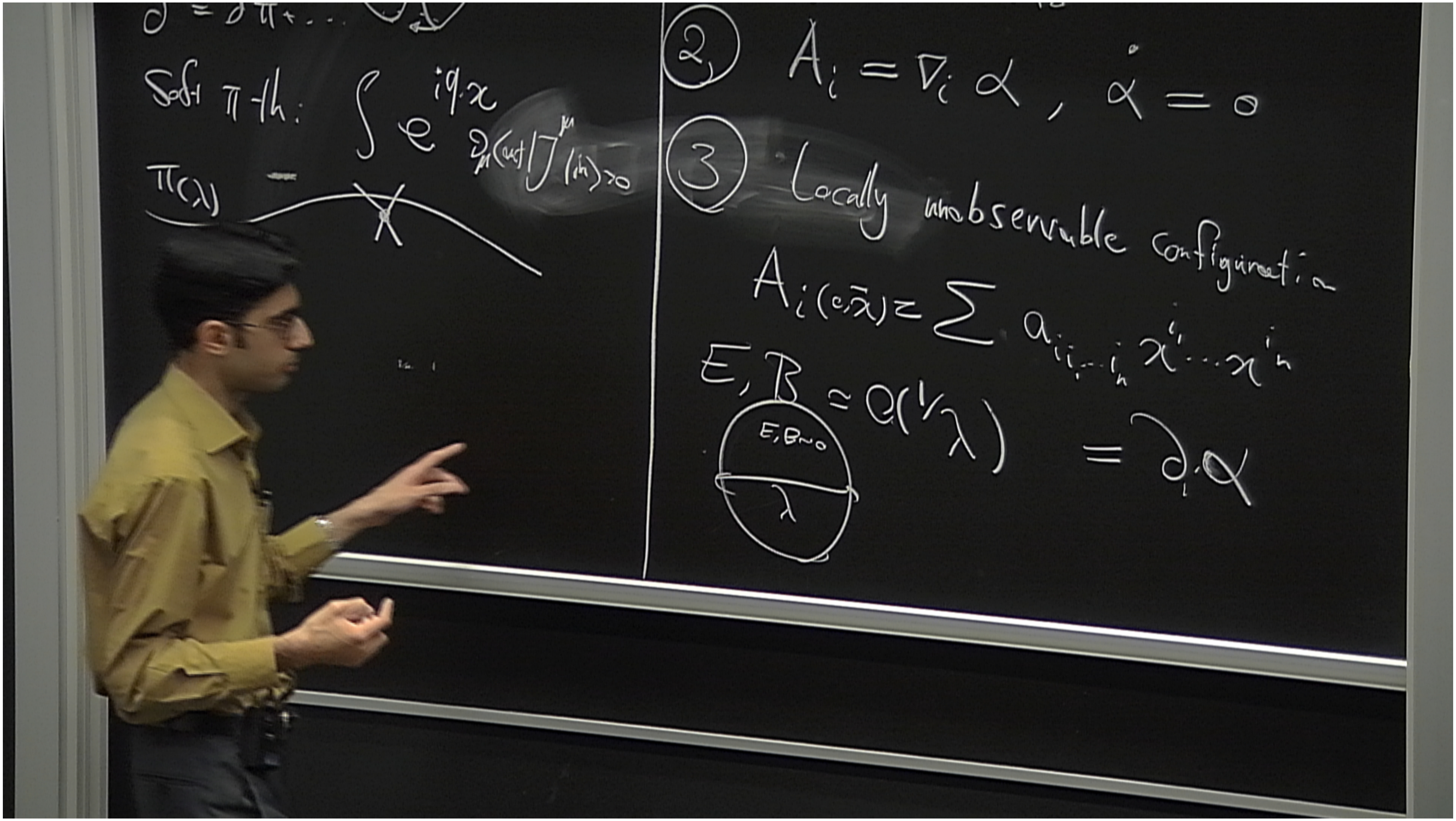
$\pi(\lambda)$ —

② $A_i = \nabla_i \alpha\,, \quad \dot{\alpha} = 0$

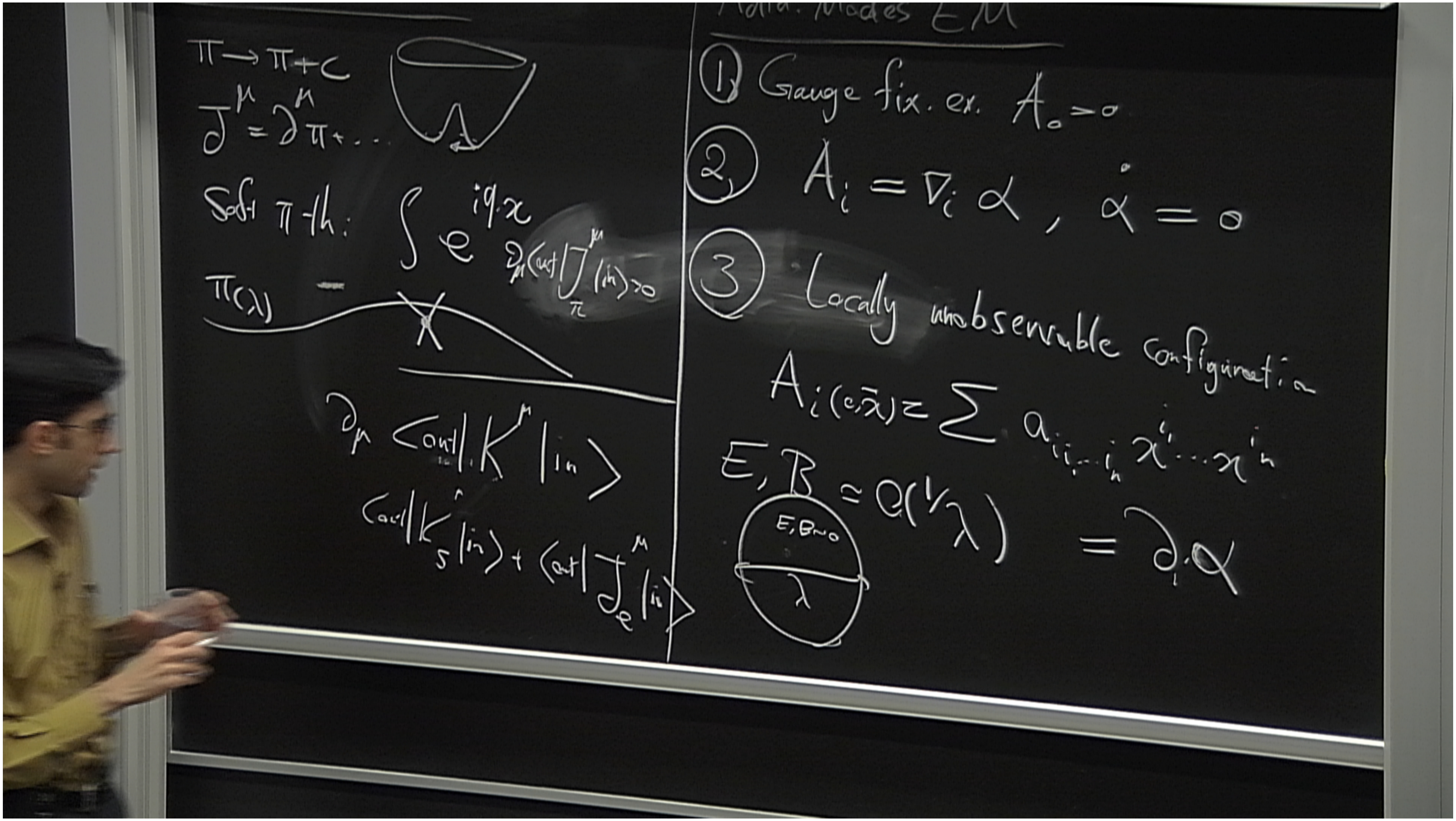
③ Locally unobservable configuration

$A_i(0,\vec{x}) = \sum a_{i_1 \ldots i_n} x^{i_1} \cdots x^{i_n}$

$E, B \simeq O(1/\lambda) \quad = \partial_i \alpha$


Soft π th:
π(λ)
2 $A_i = \nabla_i \alpha$, $\dot{\alpha} = 0$
3 Locally unobservable configuration
$A_i(0,\vec{x}) = \sum a_{i\, i_1 \dots i_n} x^{i_1} \dots x^{i_n}$
$E, B \simeq O(1/\lambda) = \partial_i \alpha$
E,B~0
λ

$$K^{\mu} = \underbrace{\partial_i \alpha F^{i\mu}}_{K_S} + \underbrace{\alpha J^{\mu}}_{K_S} \xrightarrow{\text{on-shell}} K^{\mu} = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$$


π → π + c
Soft π-th:
① Gauge fix. ex. A₀ = 0
② Aᵢ = ∇ᵢ α , α̇ = 0
③ Locally unobservable configuration
E, B ≃ O(1/λ)
E, B ~ 0
λ

$$K^{\mu} = \underbrace{\partial_i \alpha\, F^{i\mu}}_{K_s} + \underbrace{\alpha\, J^{\mu}}_{K_s} \xrightarrow{\text{on shell}} K^{\mu} = \partial_{\nu}(\alpha F^{\mu\nu}) = *d*(\alpha F)$$

Soft hair on BH, HPS

1- $\vec{P}$

$$K^\mu = \underbrace{\partial_i \alpha F^{i\mu}}_{K_s} + \underbrace{\alpha J^\mu}_{K_S} \xrightarrow{\text{on shell}} K^\mu = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$$

Soft hair on BH, HPS

1-IS $\vec{P}$ a BH hair? yes.

$$K^\mu = \underbrace{\partial_i \alpha F^{i\mu}}_{K_S} + \underbrace{\alpha J^\mu}_{K_S} \xrightarrow{\text{on shell}} K^\mu = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$$

Soft hair on BH, HPS

1 - Is $\vec{P}$ a BH hair? yes.

$$\vec{P}_{\text{early}} + \vec{P}_{\text{late}} = 0$$

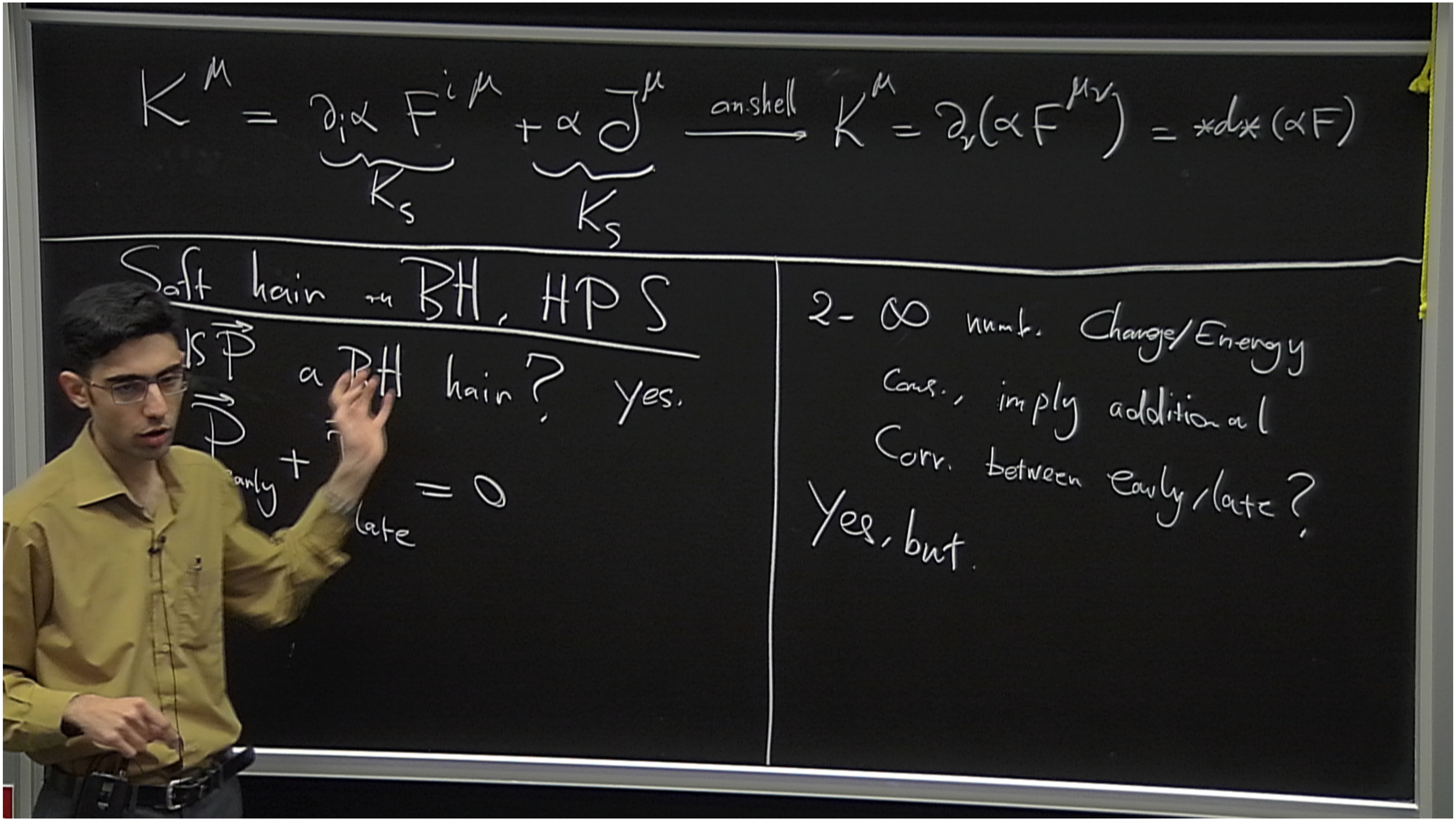
$$K^\mu = \underbrace{\partial_i \alpha\, F^{i\mu}}_{K_s} + \underbrace{\alpha\, J^\mu}_{K_S} \xrightarrow{\text{on shell}} K^\mu = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$$

Soft hair on BH, HPS

1- Is $\vec{P}$ a BH hair? yes.

$$\vec{P}_{early} + \vec{P}_{late} = 0$$

2- ∞ numb. Charge/Energy Cons., imply additional Corr. between early/late?


$K^\mu = \underbrace{\partial_i \alpha F^{i\mu}}_{K_s} + \underbrace{\alpha J^\mu}_{K_s} \xrightarrow{\text{on shell}} K^\mu = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$
Soft hair on BH, HPS
Is $\vec{P}$ a BH hair? yes.
$\vec{P}_{early} + \vec{P}_{late} = 0$
2- ∞ numb. Charge/Energy
Cons., imply additional
Corr. between early/late?
Yes, but.

$$K^\mu = \underbrace{\partial_i \alpha F^{i\mu}}_{K_s} + \underbrace{\alpha J^\mu}_{K_S} \xrightarrow{\text{on shell}} K^\mu = \partial_\nu(\alpha F^{\mu\nu}) = *d*(\alpha F)$$

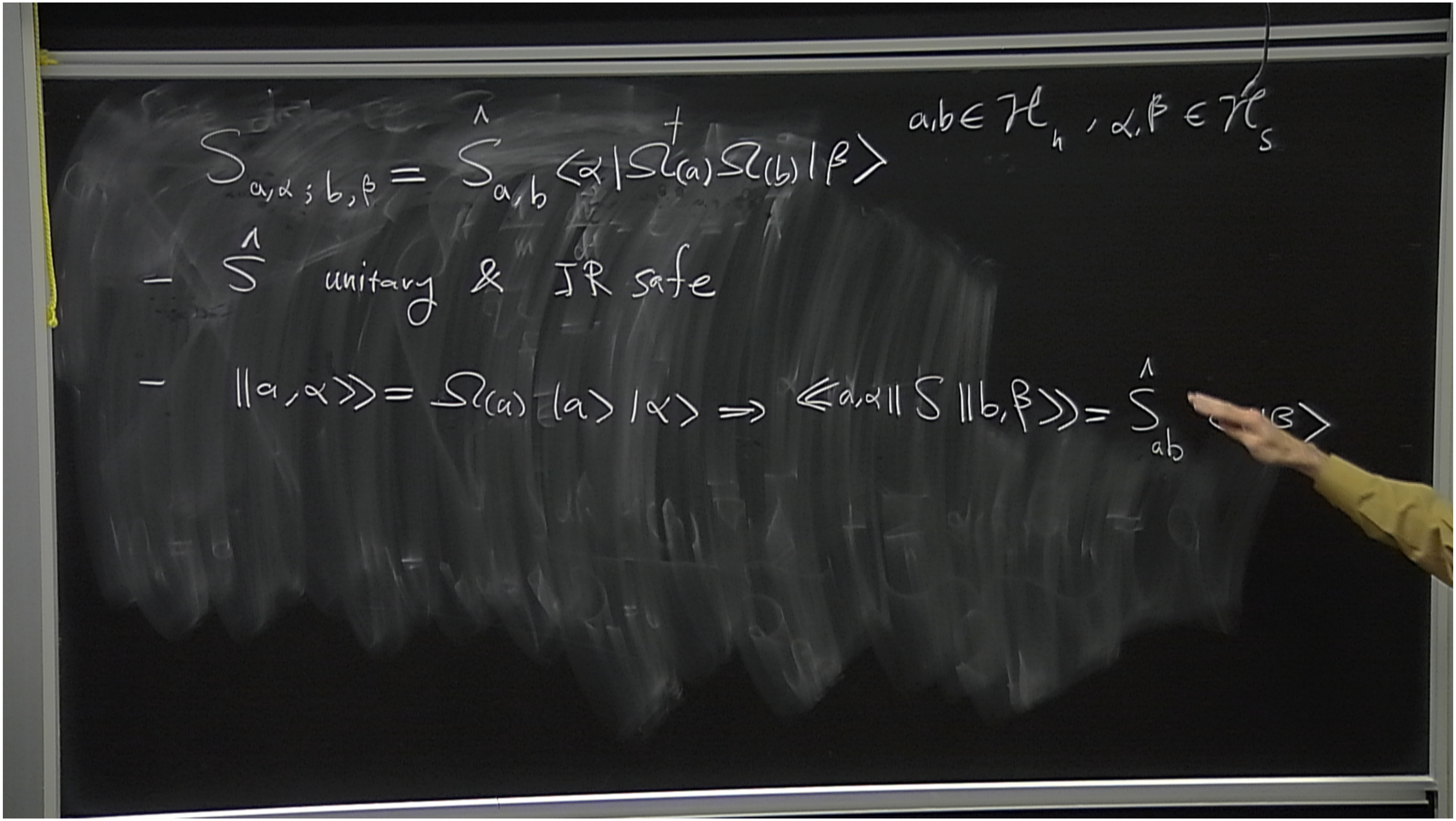
Soft hair on BH, HPS

1- Is $\vec{P}$ a BH hair? yes.

$$\vec{P}_{early} + \vec{P}_{late} = 0$$

2- ∞ numb. Charge/Energy Cons., imply additional Corr. between early/late?

Yes, but ∞ many additional ambiguity in BH


$S_{a,\alpha;b,\beta} = \hat{S}_{a,b} \langle \alpha | \Omega^\dagger(a) \Omega(b) | \beta \rangle$
$a,b \in \mathcal{H}_h \,,\, \alpha,\beta \in \mathcal{H}_s$
$- \ \hat{S}$ unitary & IR safe
$- \ \|a,\alpha\rangle\rangle = \Omega(a)|a\rangle|\alpha\rangle \Rightarrow \langle\langle a,\alpha \| S \| b,\beta\rangle\rangle = \hat{S}_{ab}$

$$S_{a,\alpha;b,\beta} = \hat{S}_{a,b} \langle\alpha| \Omega^\dagger(a)\,\Omega(b) |\beta\rangle \qquad a,b \in \mathcal{H}_h\,,\ \alpha,\beta \in \mathcal{H}_s$$

- $\hat{S}$ unitary & IR safe

- $\|a,\alpha\rangle\rangle = \Omega(a)\,|a\rangle|\alpha\rangle \Rightarrow \langle\langle a,\alpha\| S \|b,\beta\rangle\rangle = \hat{S}_{ab}\,\langle\alpha|\beta\rangle$