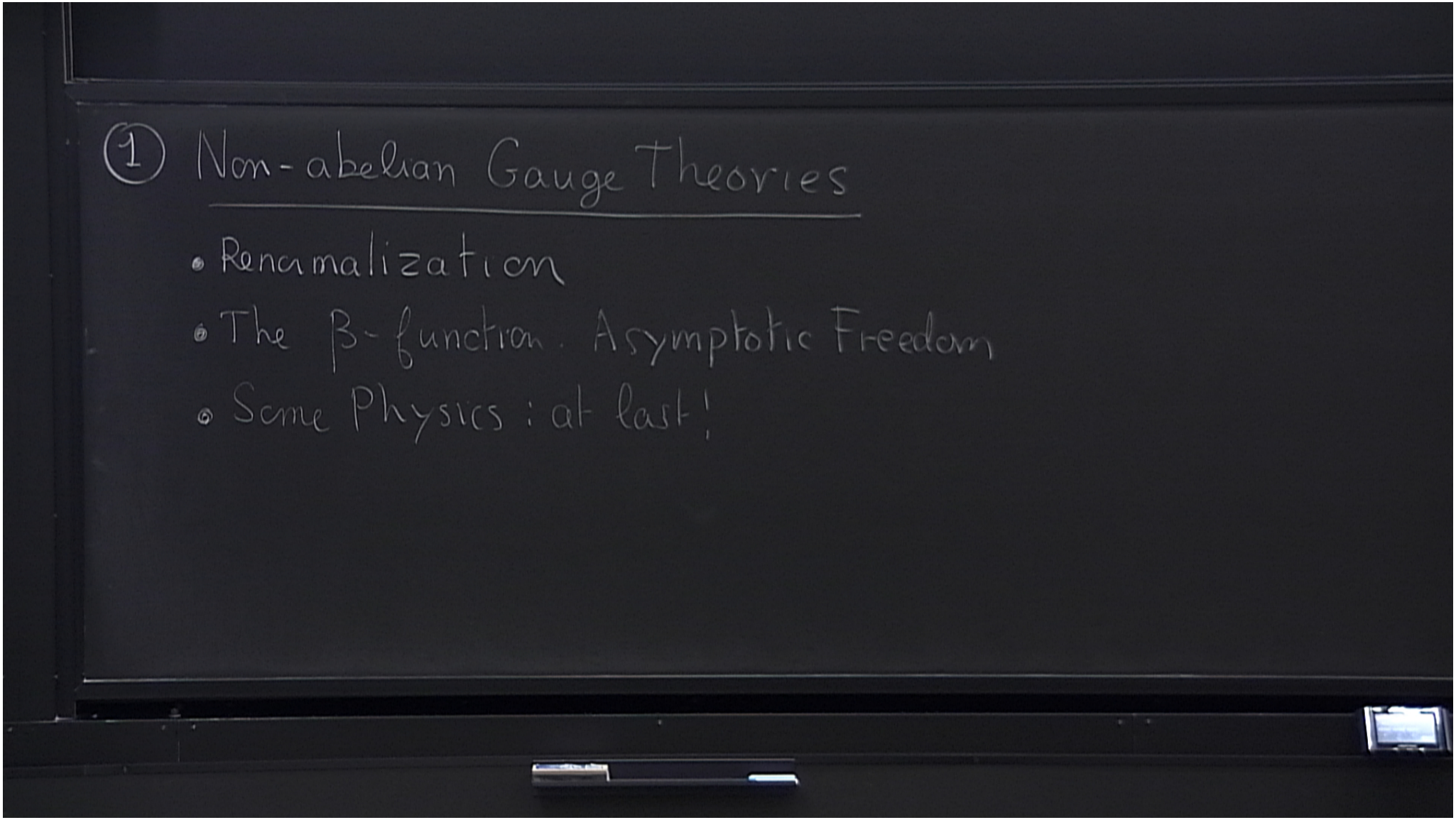
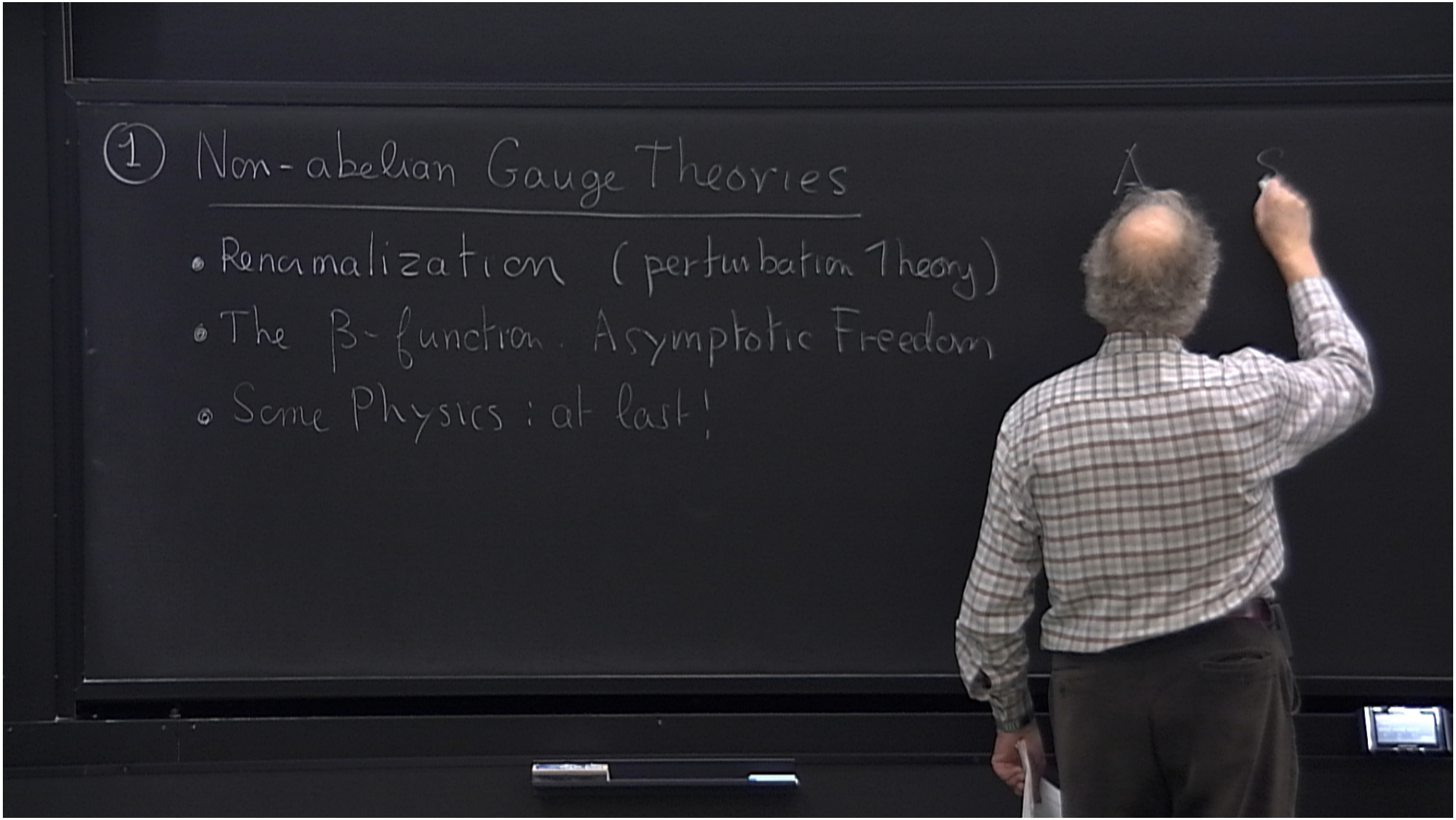
Title: PSI 2016/2017 Quantum Field Theory II - Lecture 15

Date: Nov 25, 2016 09:00 AM

URL: http://pirsa.org/16110015

Abstract:


1 Non-abelian Gauge Theories
• Renormalization
• The β-function. Asymptotic Freedom
• Some Physics : at last!


1 Non-abelian Gauge Theories
• Renormalization (perturbation Theory)
• The β-function. Asymptotic Freedom
• Some Physics : at last!

$A_\mu$ $SU(2)$ or $G$ Lie Group

heory) $S_{YM}[A] = -\frac{1}{2g^2}\int d^4x\, Tr\left[F_{\mu\nu} F^{\mu\nu}\right]$

edom $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

heory)

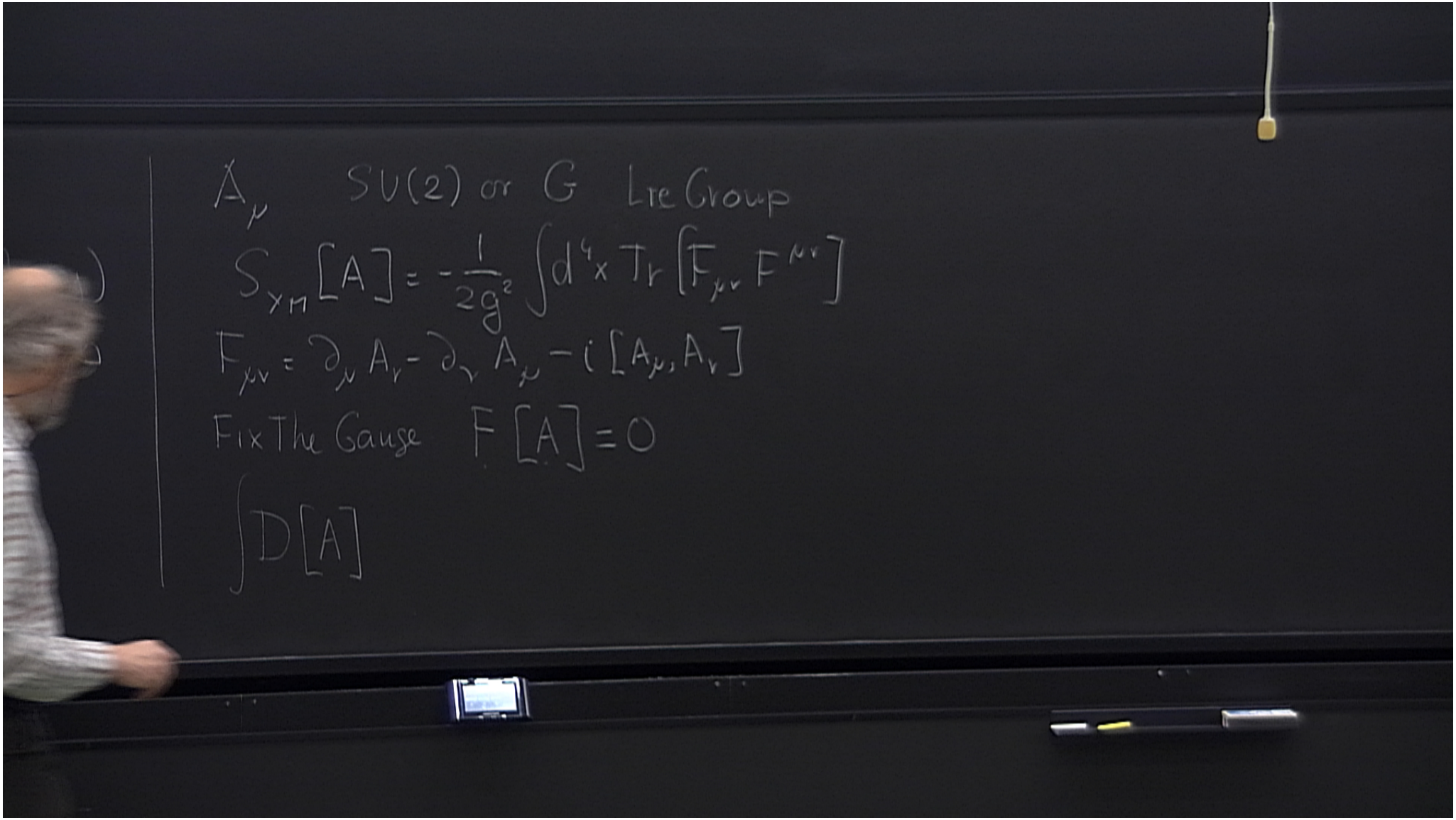
edom

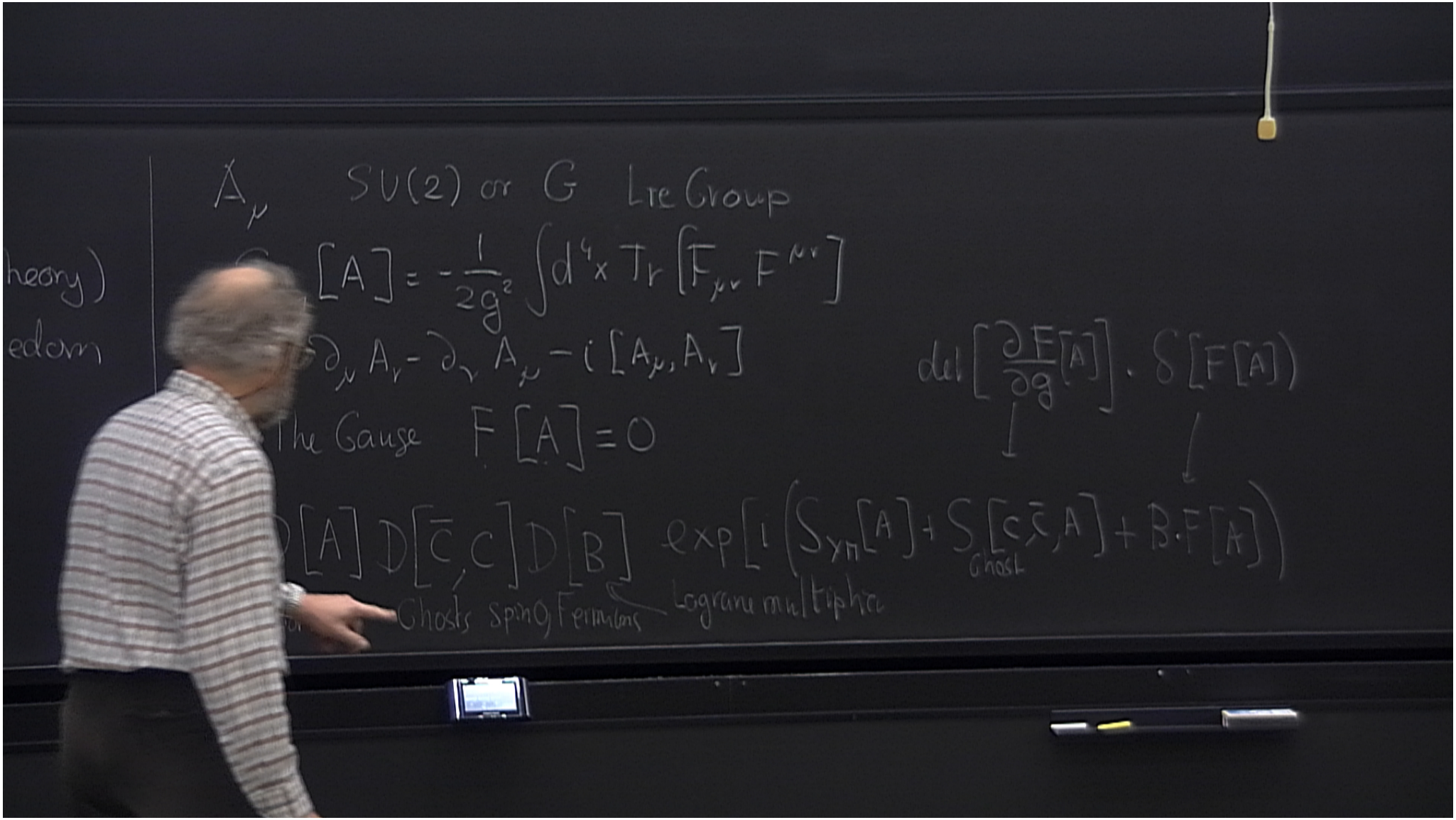
$A_\mu$ $SU(2)$ or $G$ Lie Group

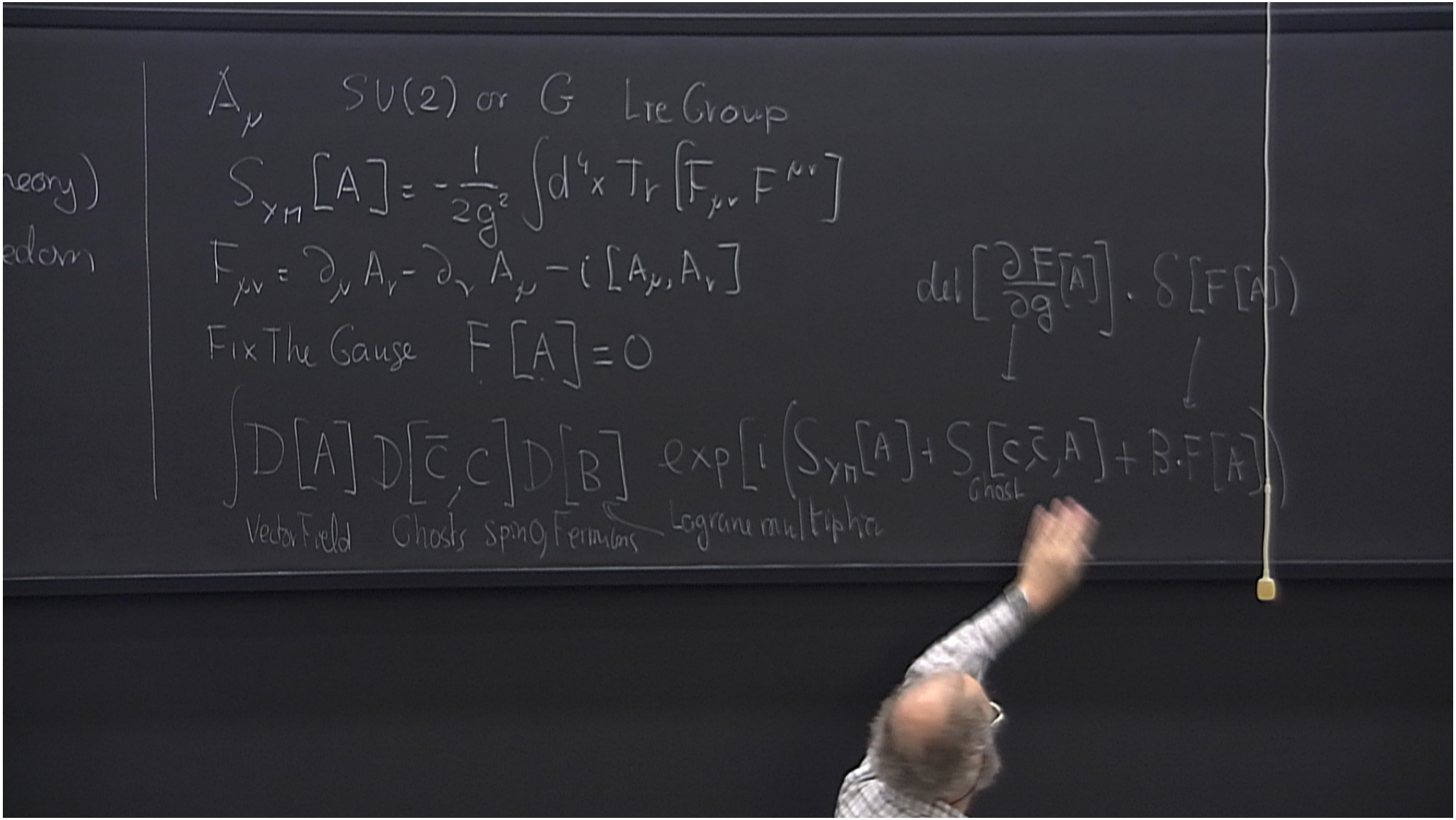
$$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x \, Tr\left[F_{\mu\nu} F^{\mu\nu}\right]$$

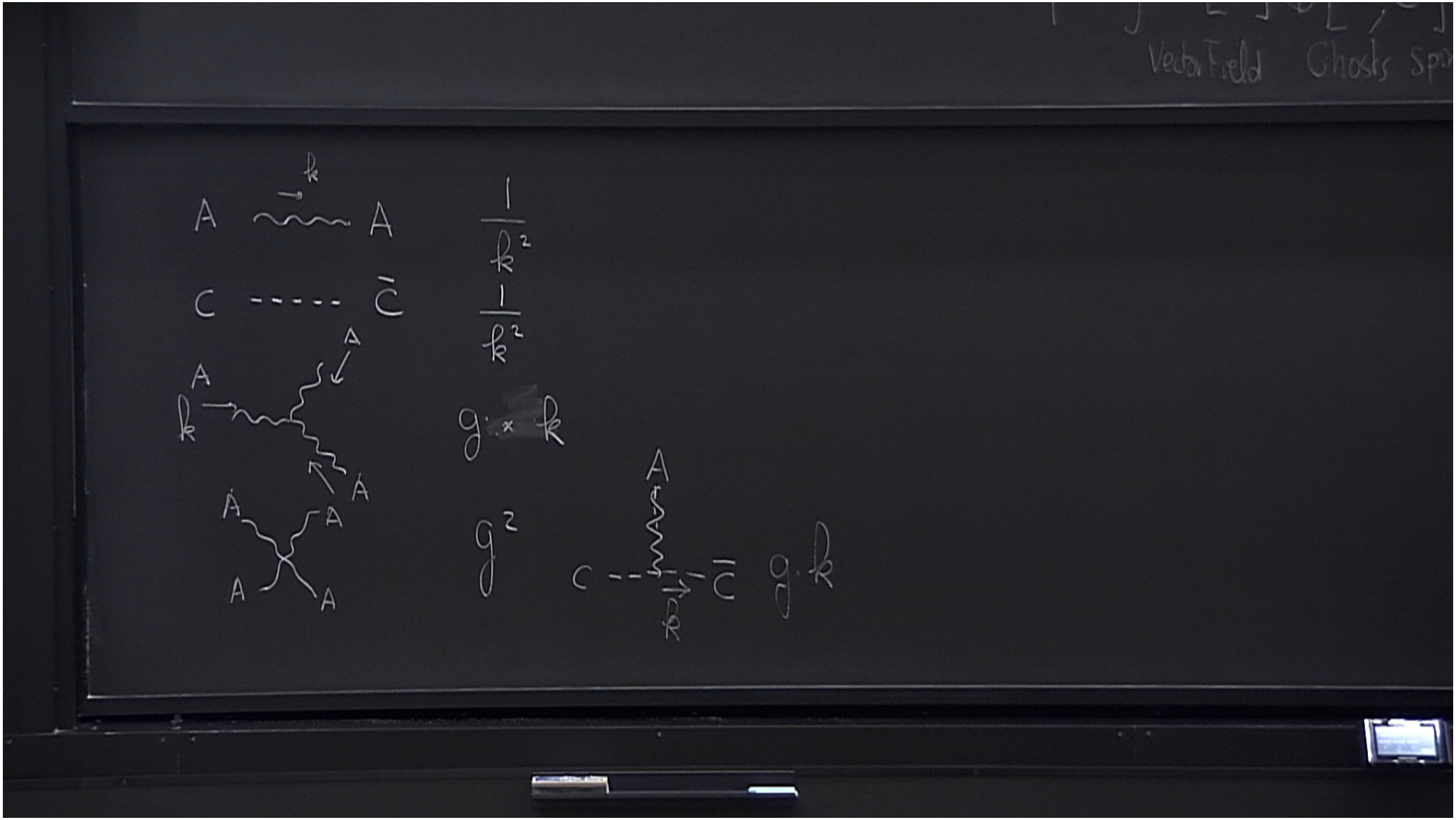
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

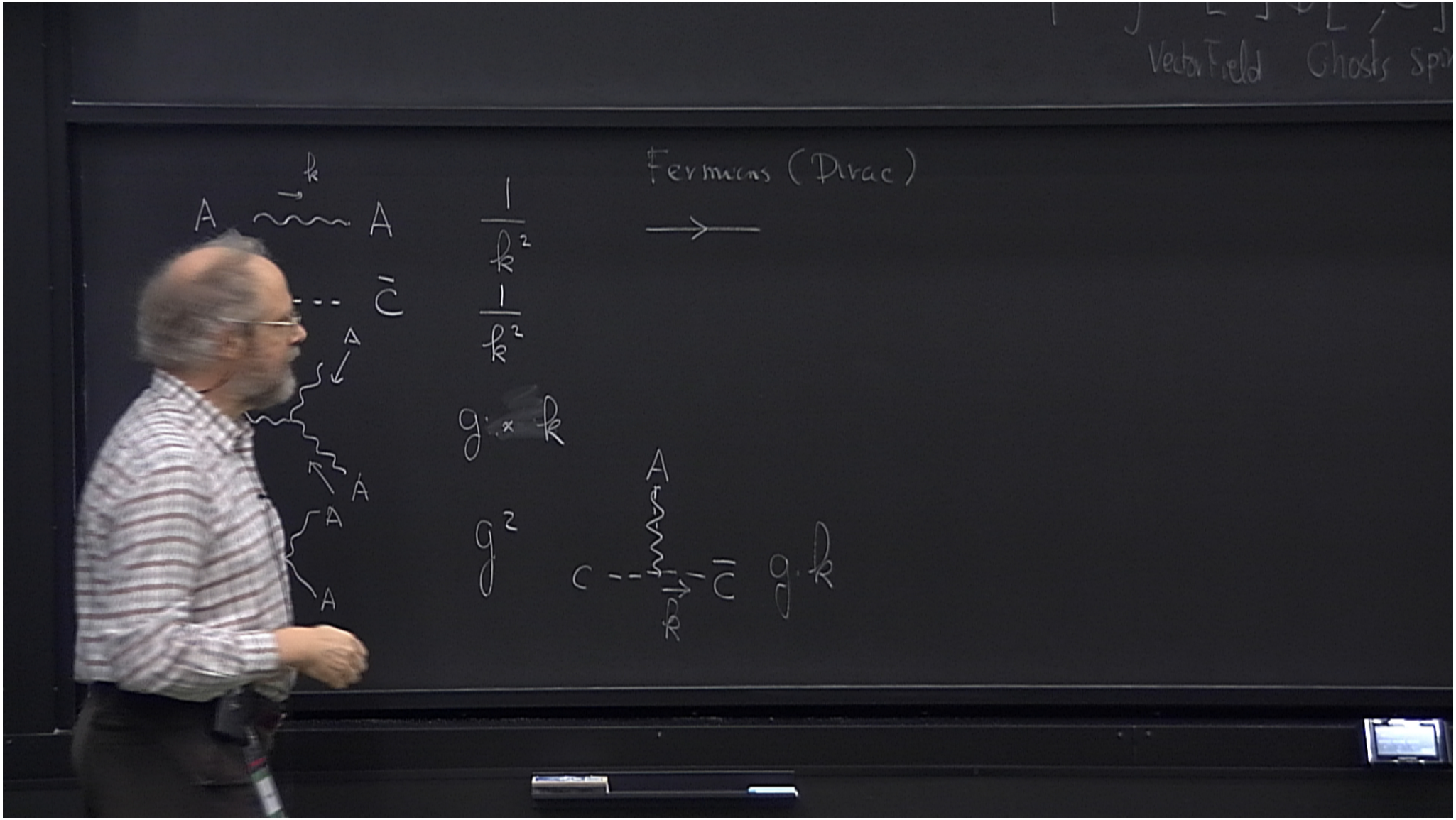
Fix The Gauge $F[A] = 0$

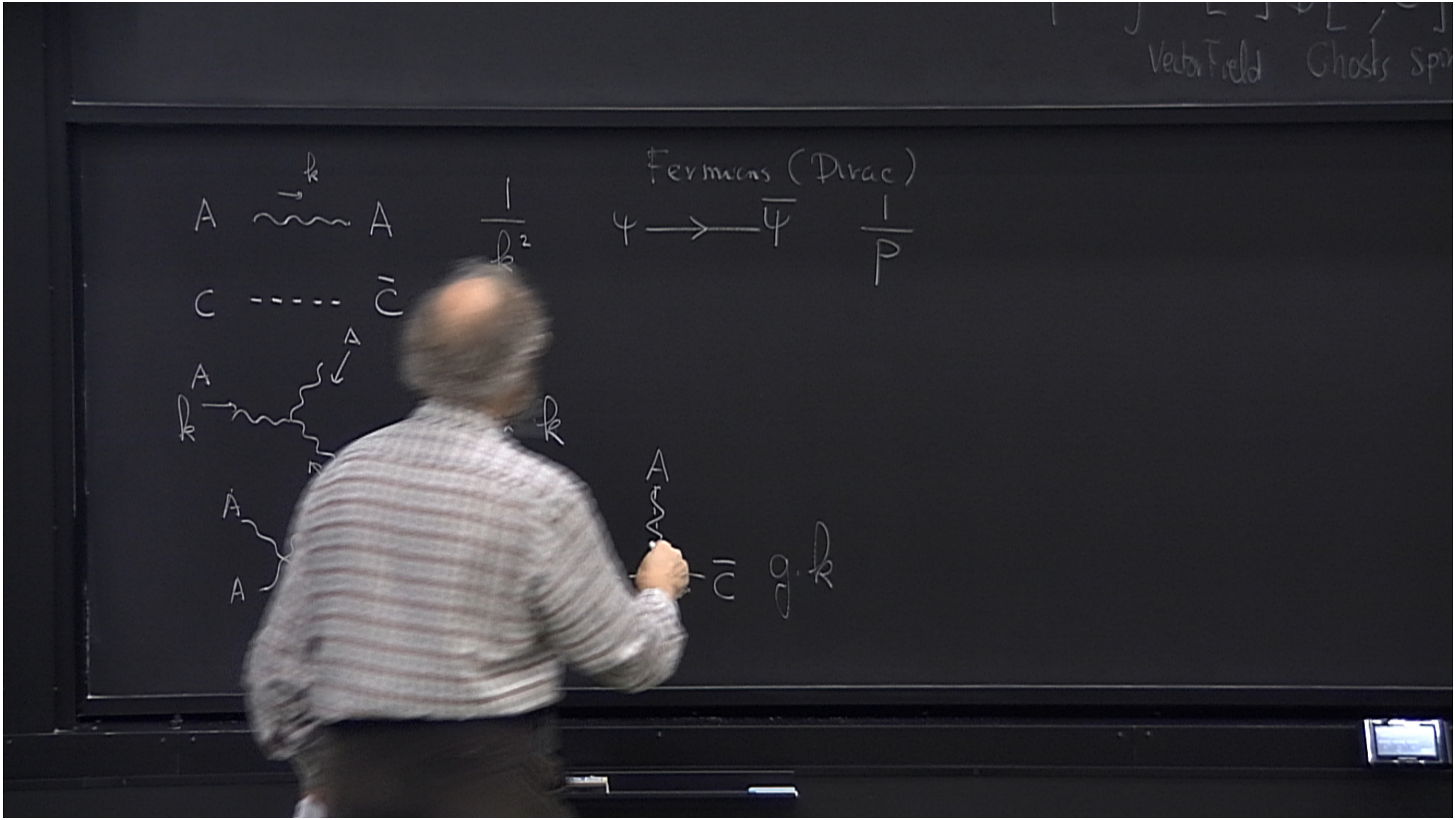

$A_\mu$ SU(2) or G Lie Group
$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x\, Tr\left[F_{\mu\nu} F^{\mu\nu}\right]$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
Fix The Gauge $F[A] = 0$
$\int D[A]$

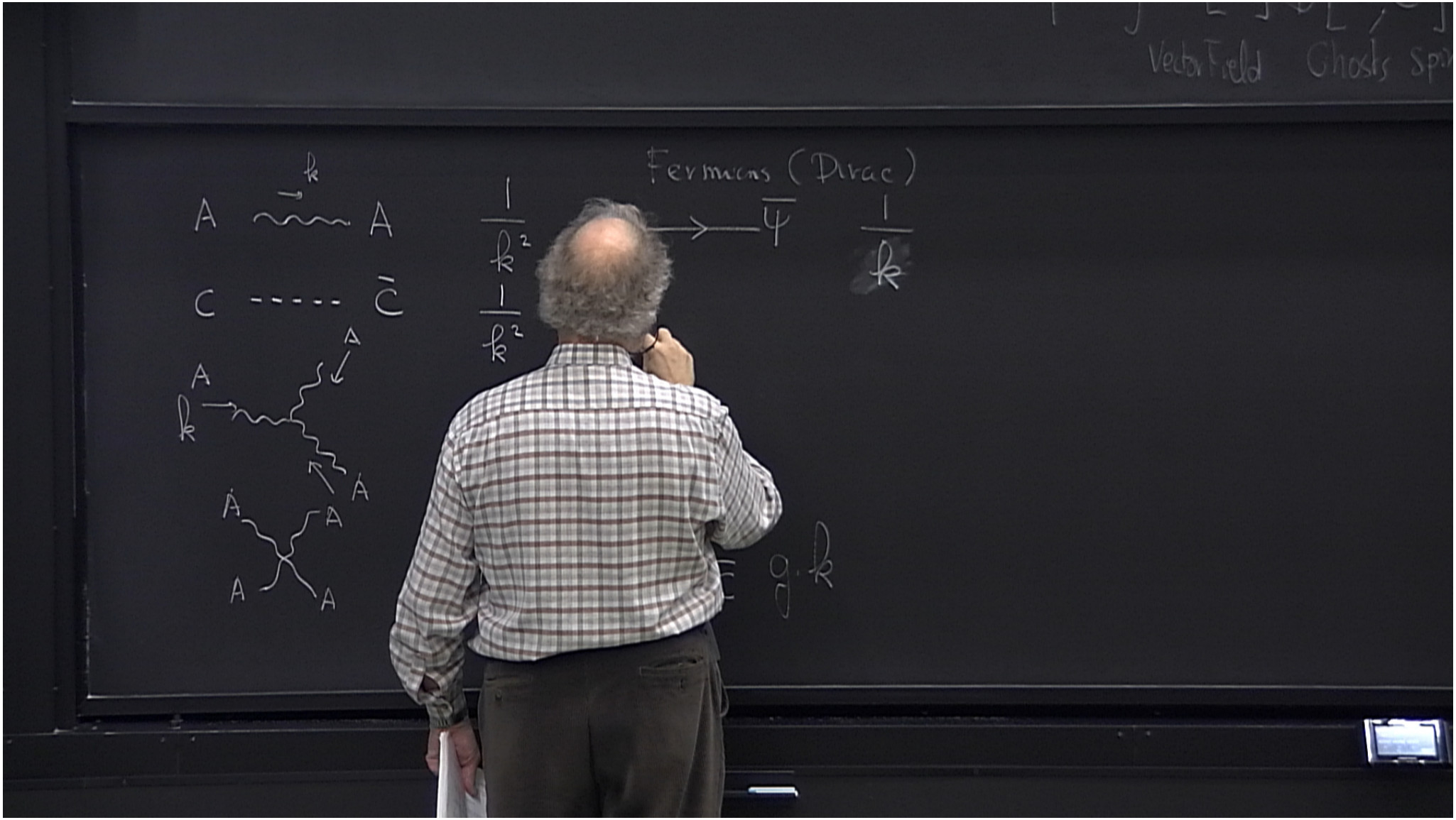

$A_\mu$ SU(2) or G Lie Group
$[A] = -\frac{1}{2g^2}\int d^4x\, Tr\left[F_{\mu\nu} F^{\mu\nu}\right]$
$\partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
The Gauge $F[A] = 0$
$\det\left[\frac{\partial F}{\partial g}[A]\right] \cdot \delta[F[A]]$
$[A]\, D[\bar{C}, C]\, D[B]\, \exp\left[i\left(S_{YM}[A] + S_{Ghost}[c, \bar{c}, A] + B \cdot F[A]\right)\right]$
Ghosts Spin 0 Fermions
Lagrange multipliers

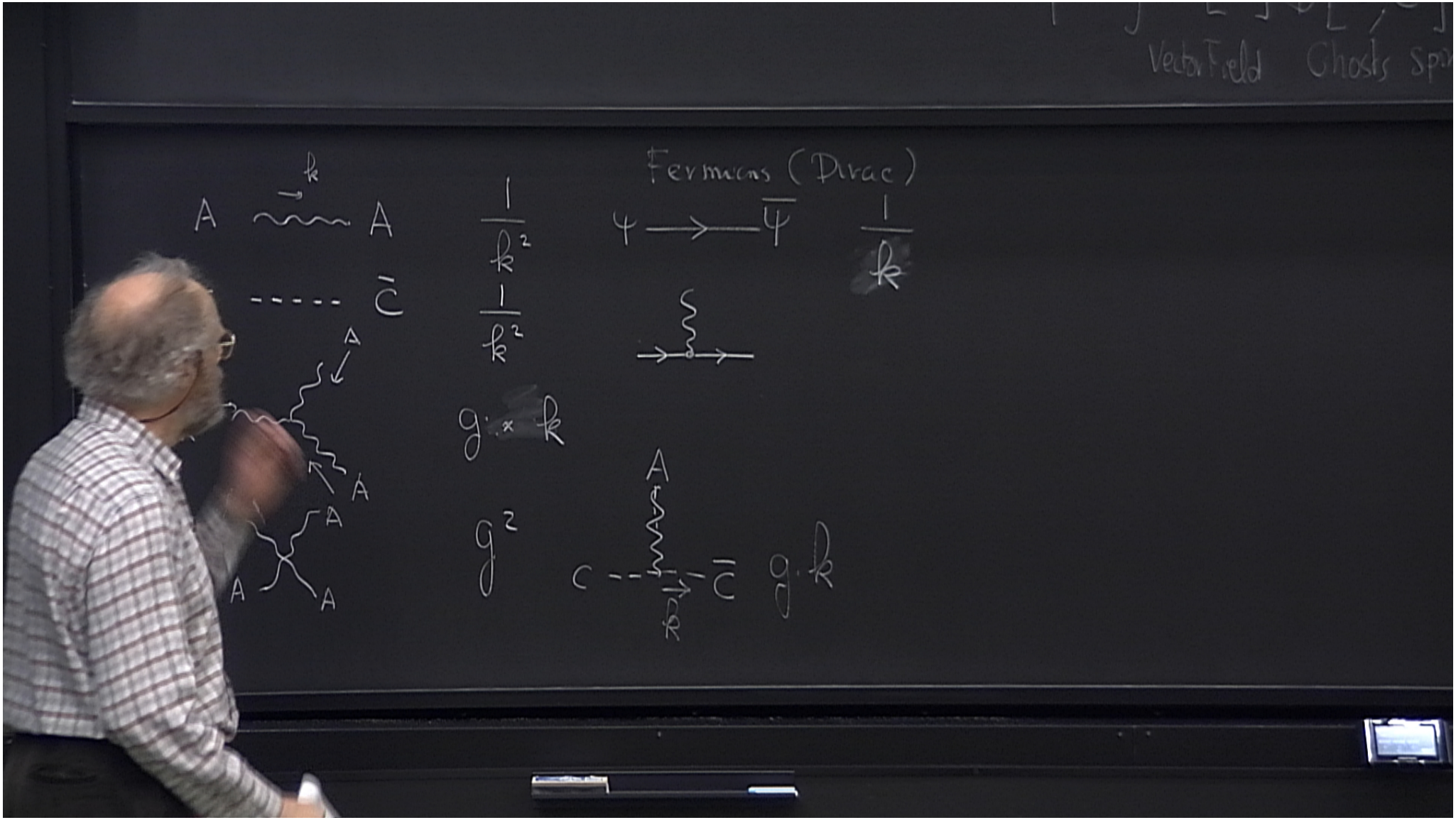

$A_\mu$ SU(2) or G Lie Group
$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x\, Tr[F_{\mu\nu}F^{\mu\nu}]$
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Fix The Gauge $F[A] = 0$
$\det\left[\frac{\partial F}{\partial g}[A]\right] \cdot \delta[F[A]]$
$\int D[A]\, D[\bar{C}, C]\, D[B]\, \exp\left[i\left(S_{YM}[A] + S_{Ghost}[C, \bar{C}, A] + B \cdot F[A]\right)\right]$
Vector Field Ghosts spin 0 Fermions Lagrange multipliers

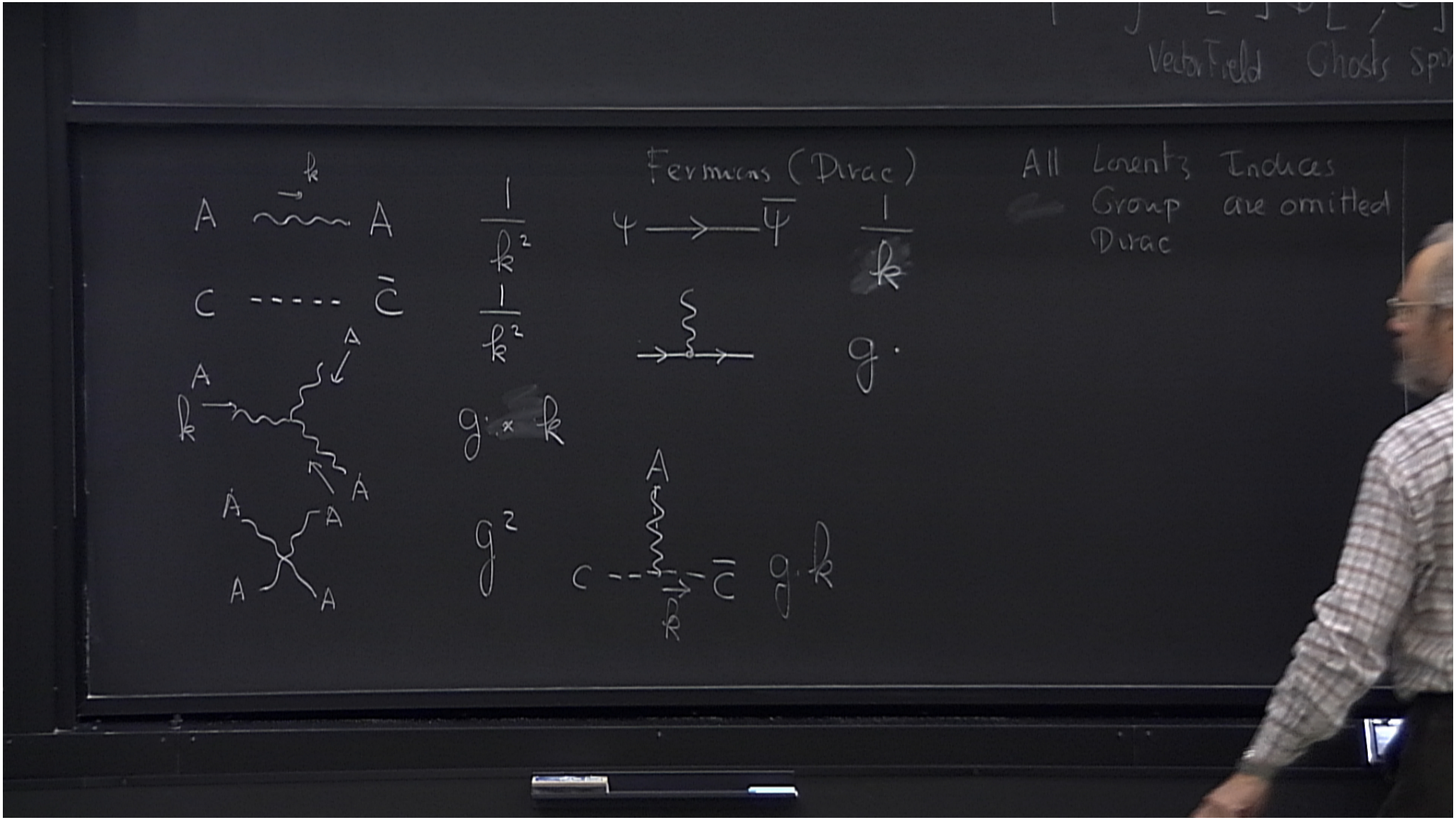

Vector Field Ghosts
A
A
k
C
C̄
$\frac{1}{k^2}$
$\frac{1}{k^2}$
g · k
$g^2$
g · k

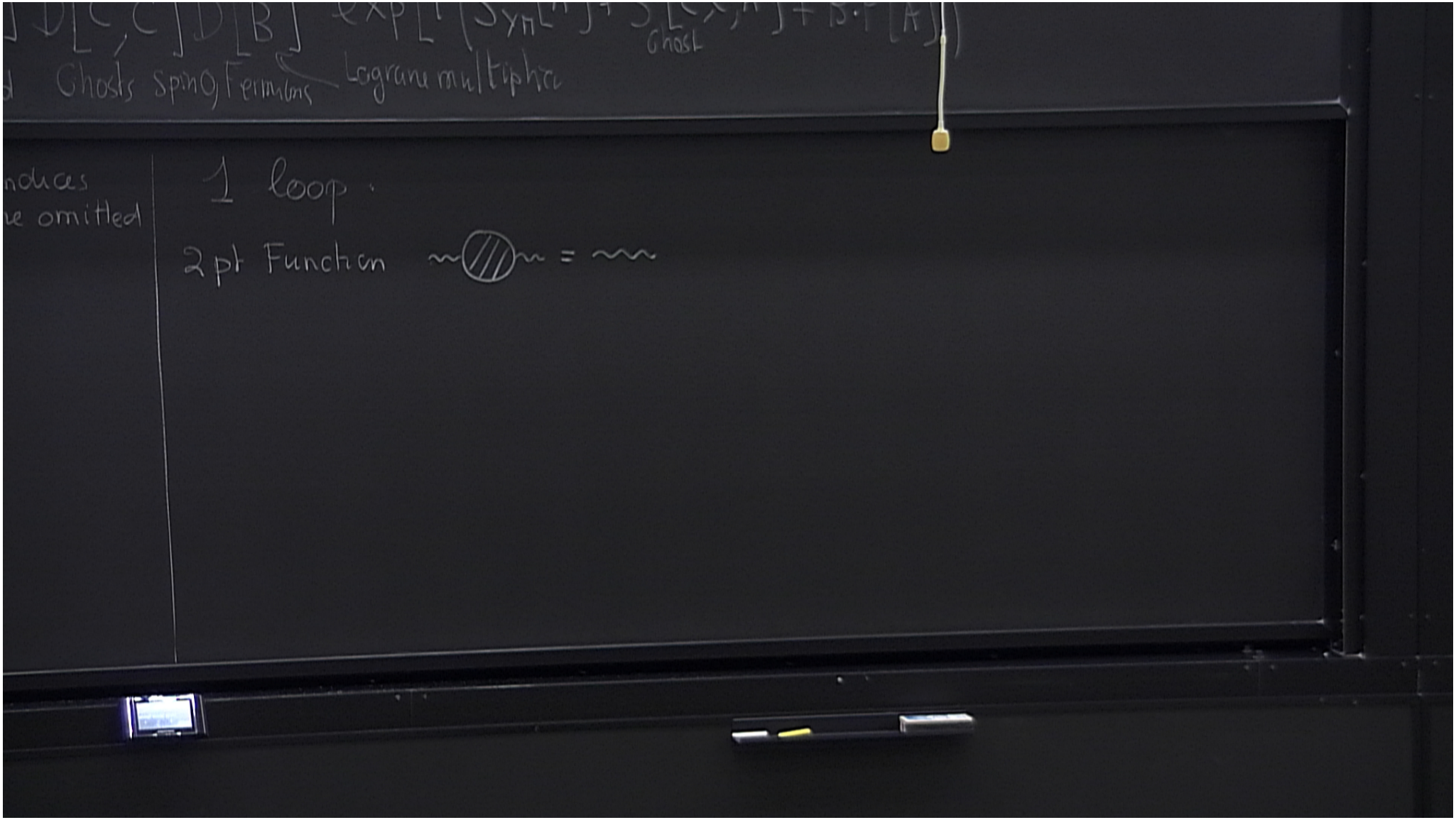

Vector Field Ghosts
Fermions (Dirac)
A
A
k
$\frac{1}{k^2}$
$\frac{1}{k^2}$
$\bar{c}$
A
A
A
A
$g \cdot k$
$g^2$
A
c
$\bar{c}$
k
$g \cdot k$

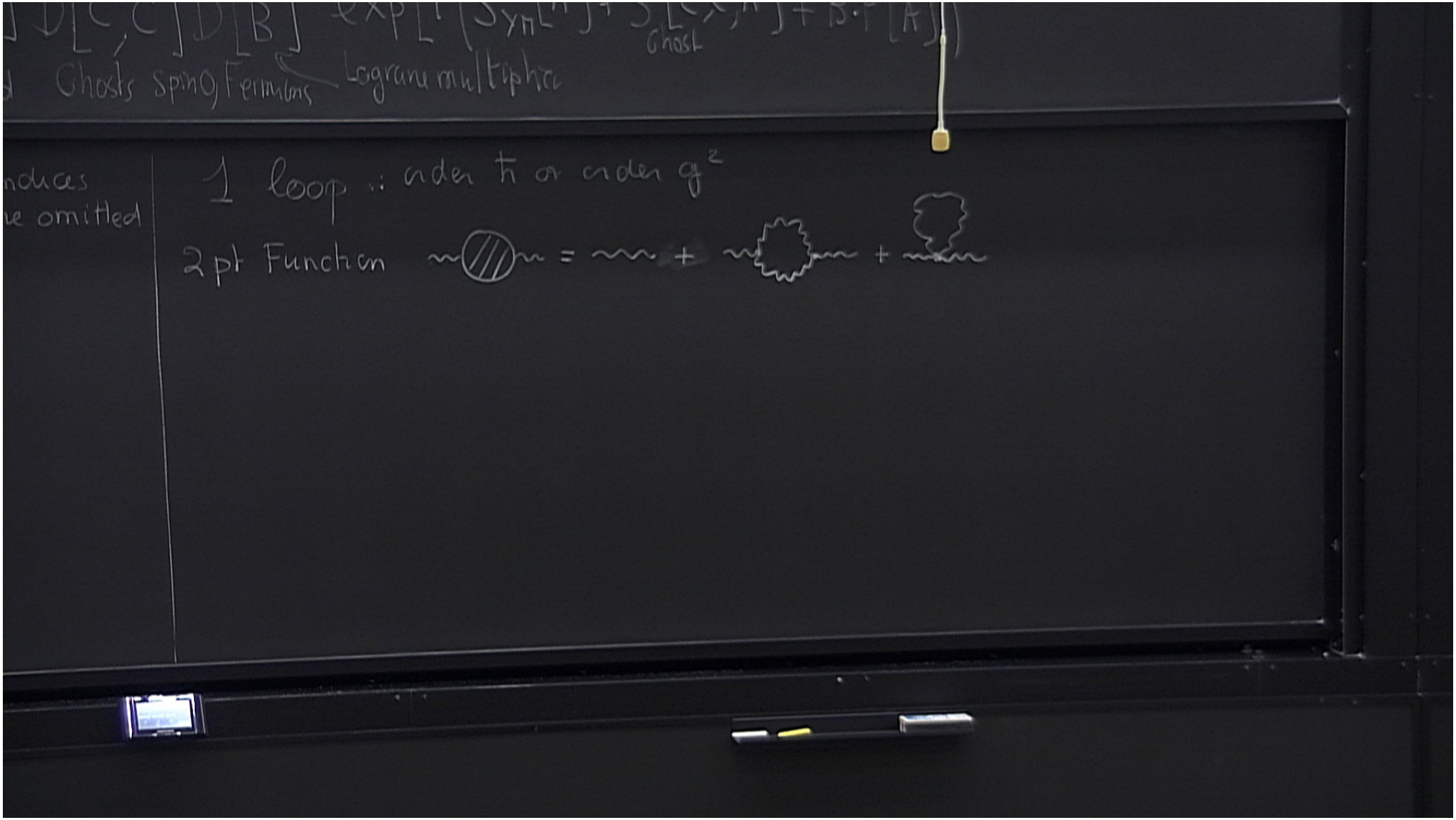

Vector Field Ghosts
Fermions (Dirac)
A A
C C̄
ψ ψ̄
k
1/k²
1/p
g·k

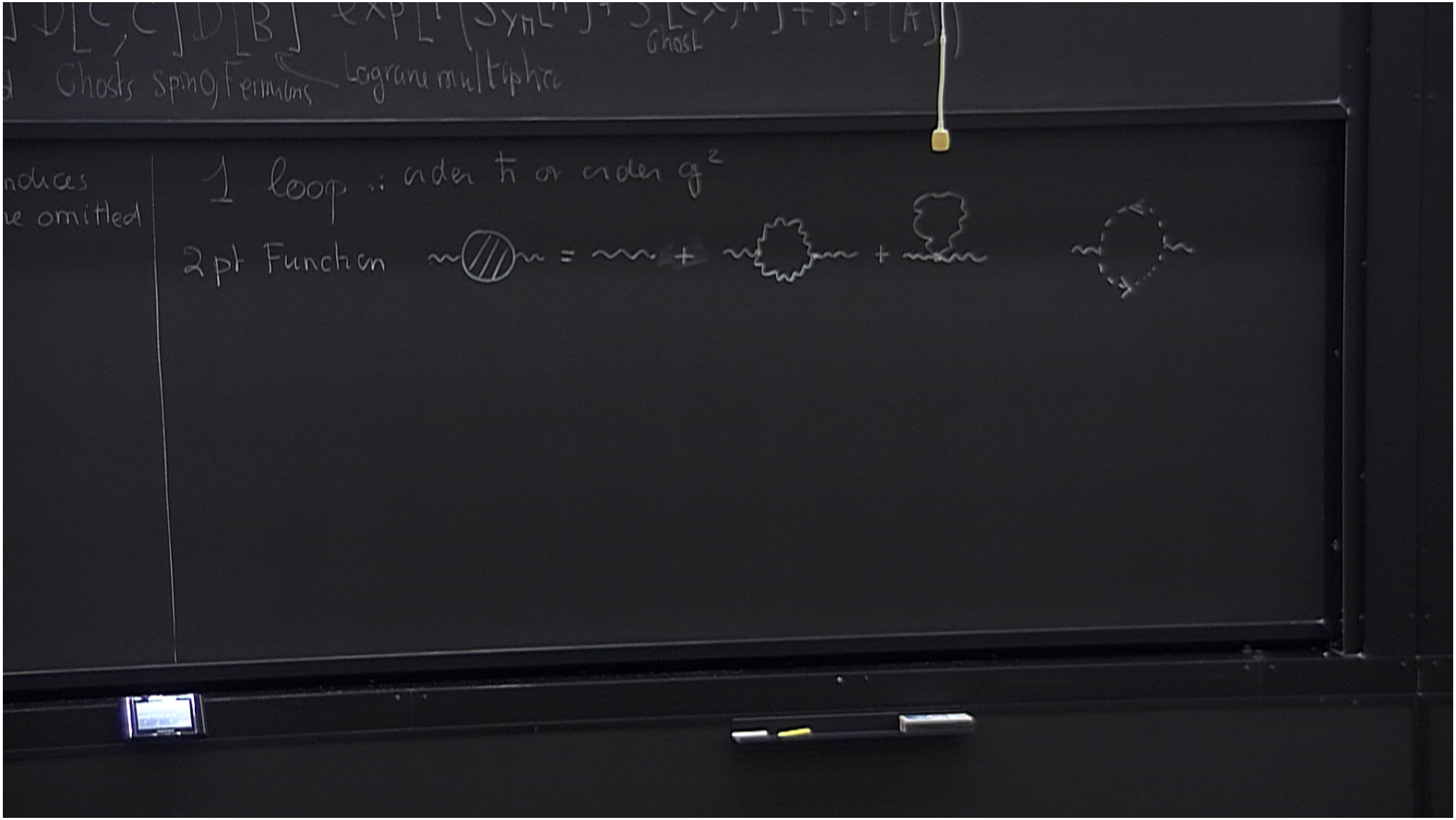

Vector Field Ghosts
Fermions (Dirac)
A
A
k
C
C̄
k
A
A
A
A
A
A
A
A
1/k²
1/k²
ψ̄
1/k
g·k


Vector Field Ghosts
Fermions (Dirac)
A
A
k
$\frac{1}{k^2}$
$\frac{1}{k^2}$
$\bar{c}$
$\psi$
$\bar{\psi}$
$\frac{1}{k}$
$g \cdot k$
$g^2$
c
$\bar{c}$
$g \cdot k$

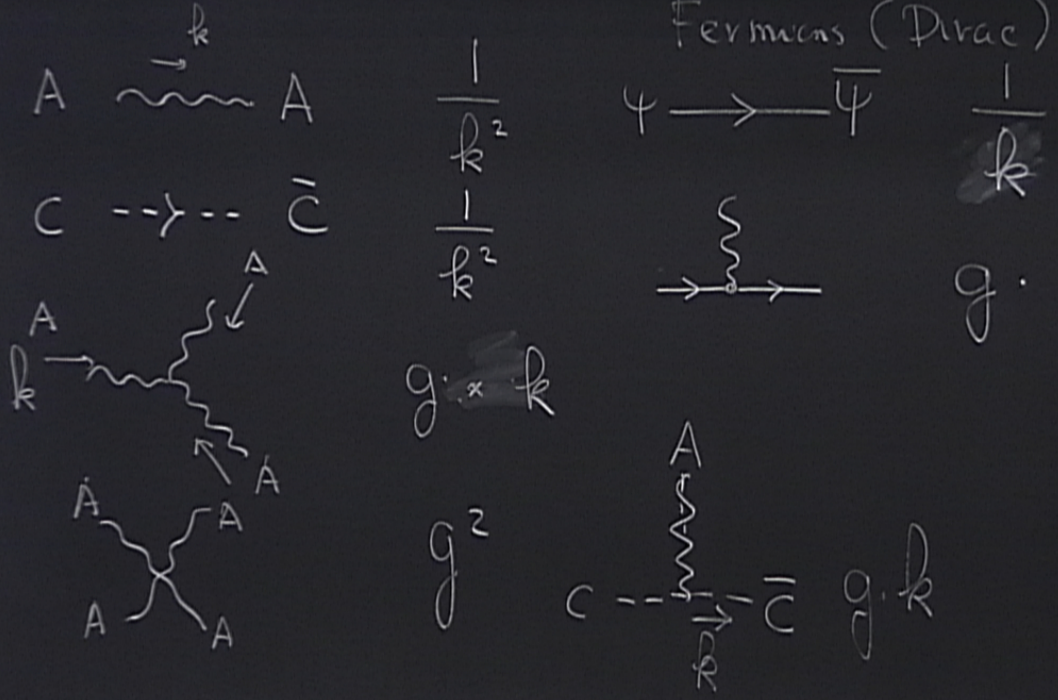

Vector Field Ghosts
Fermions (Dirac)
All Lorentz Indices
Group are omitted
Dirac
A
A
k
C
C̄
g
g²
g·k


Ghosts spin0 Fermions
Lagrange multipliers
Ghost
1 loop:
2pt Function

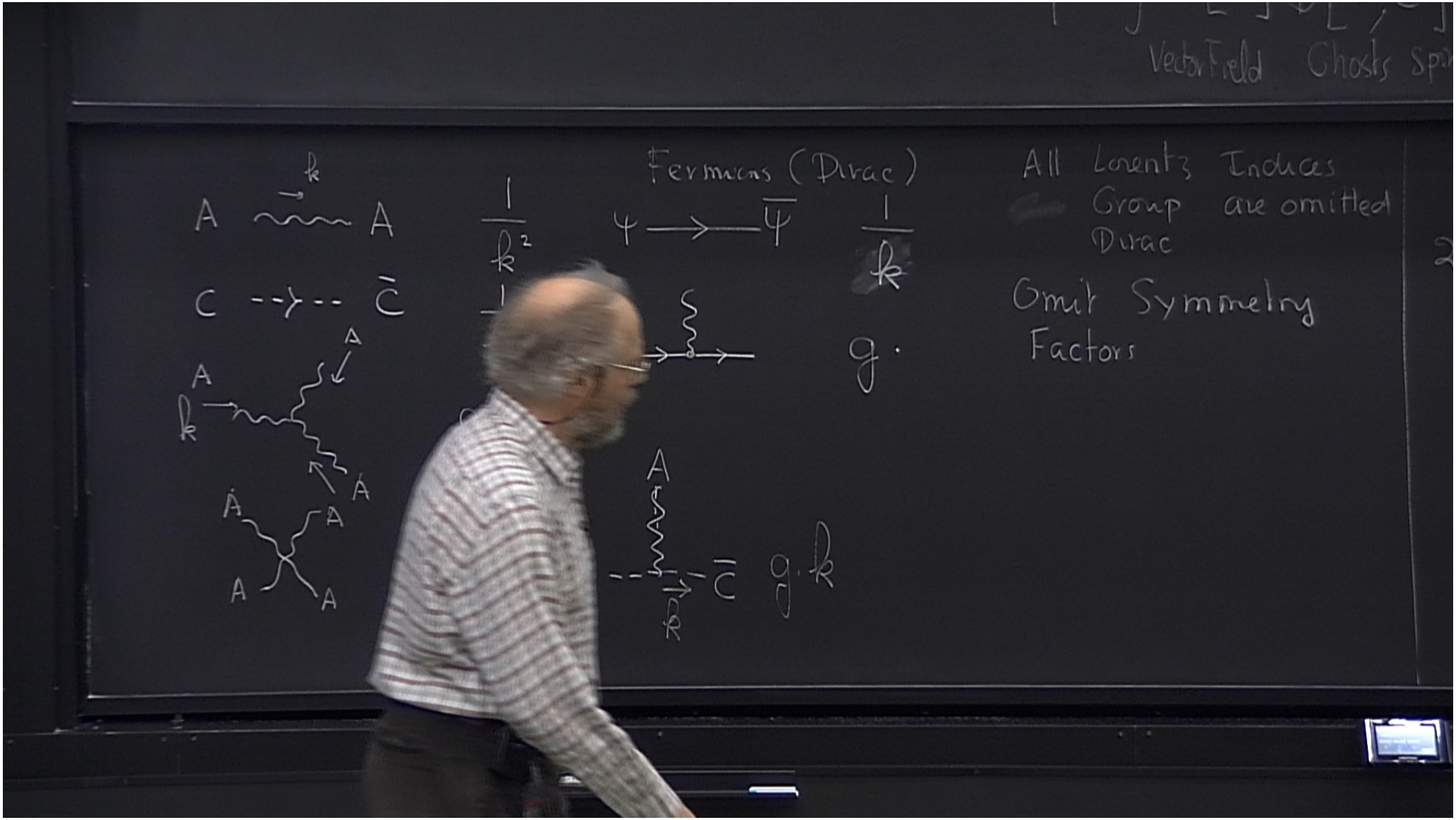

Ghosts spin 0 Fermions
Lagrange multipliers
Ghost
1 loop : order ħ or order g²
2pt Function

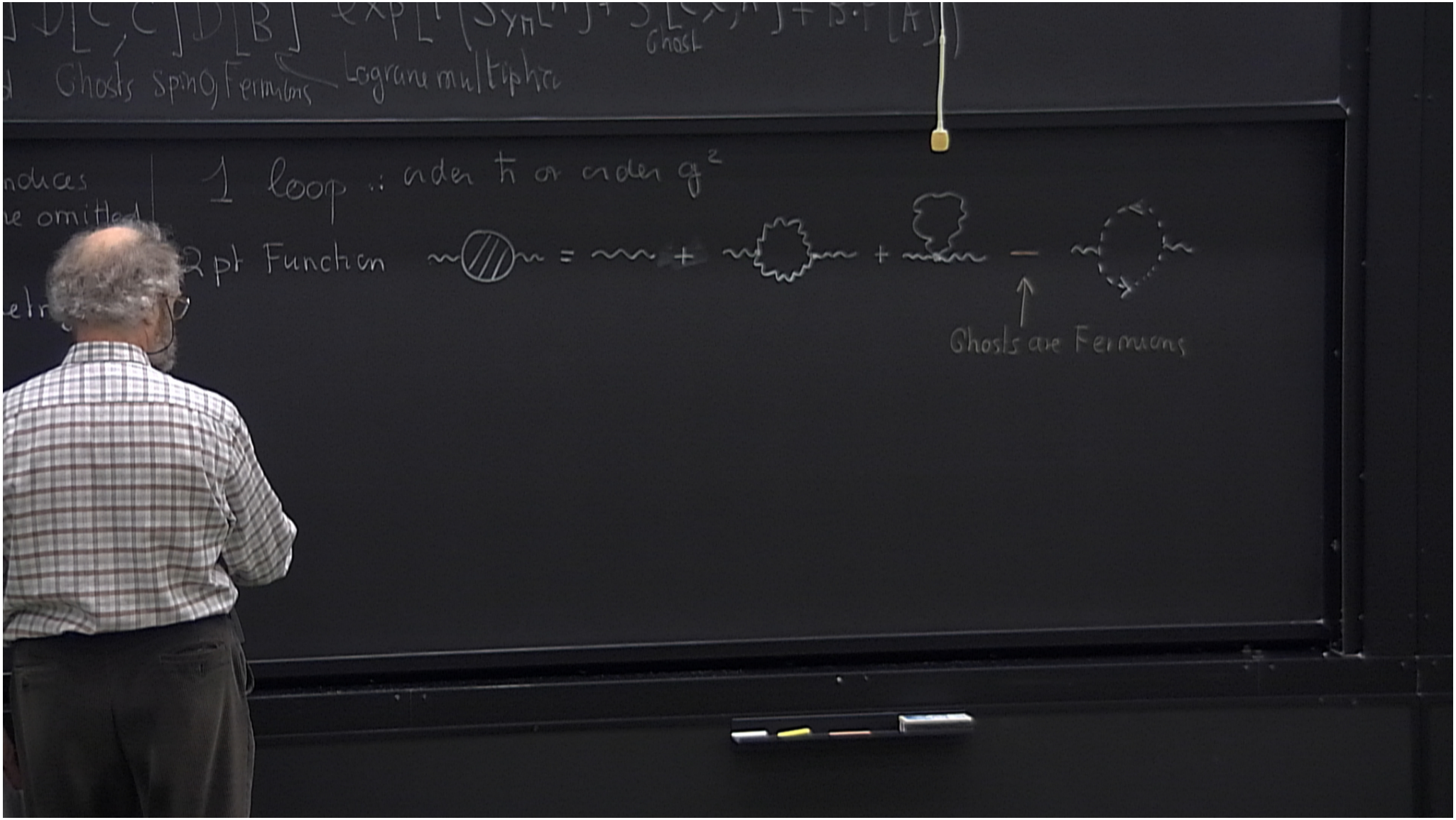

Ghosts spin 0 Fermions
Lagrange multipliers
Ghost
1 loop : order $\hbar$ or order $g^2$
2pt Function

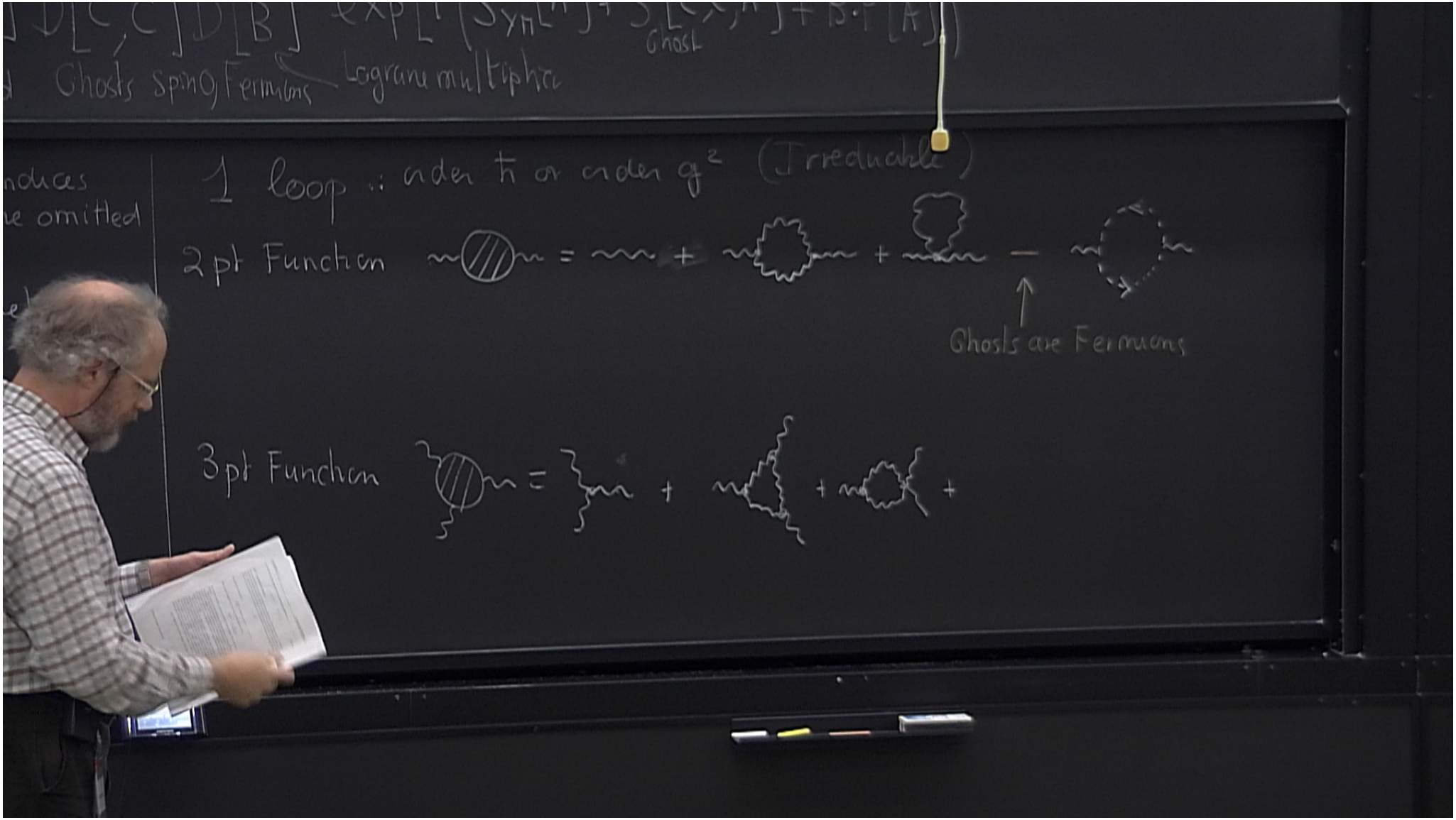
Vector Field Ghosts Sp

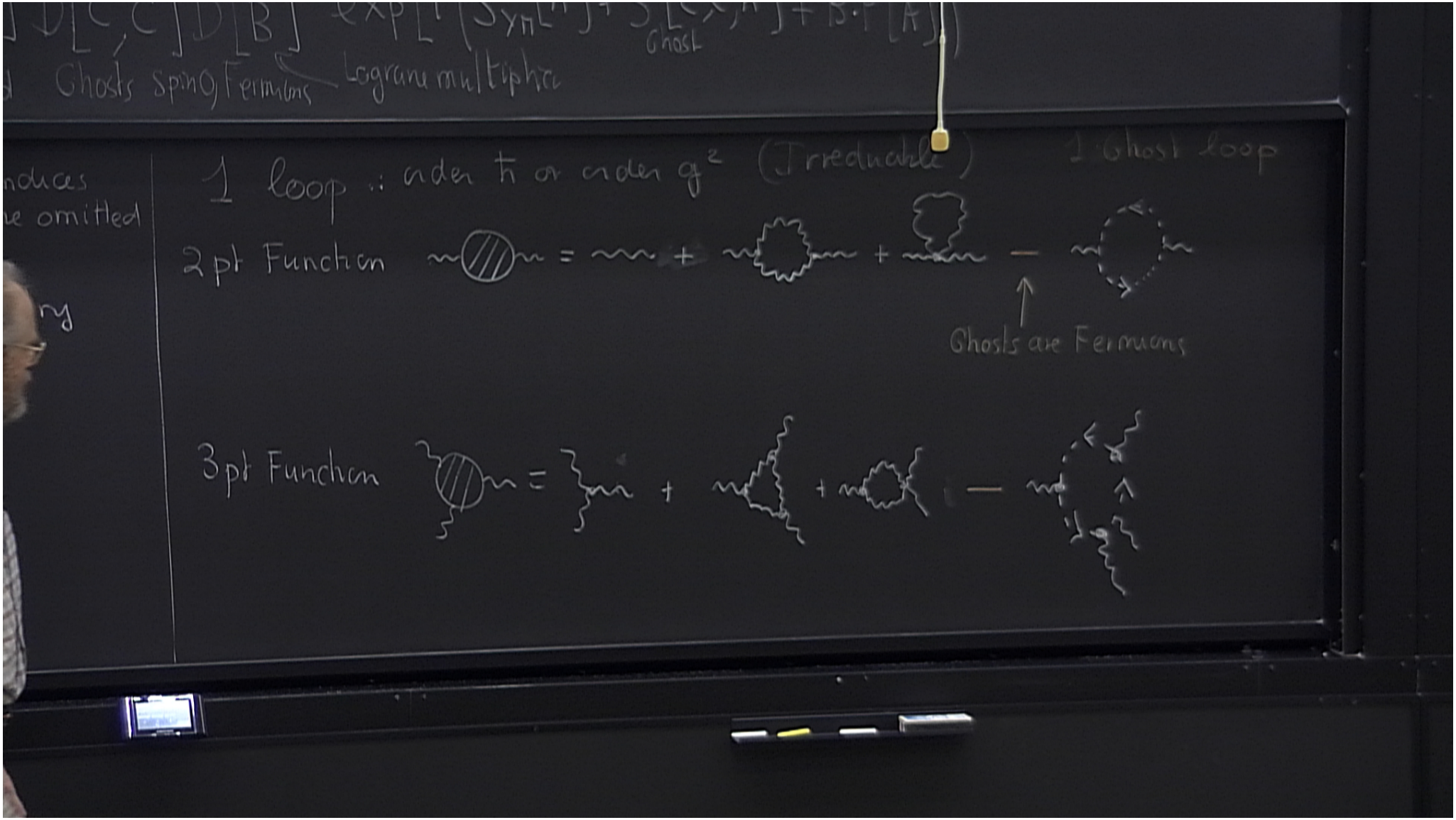


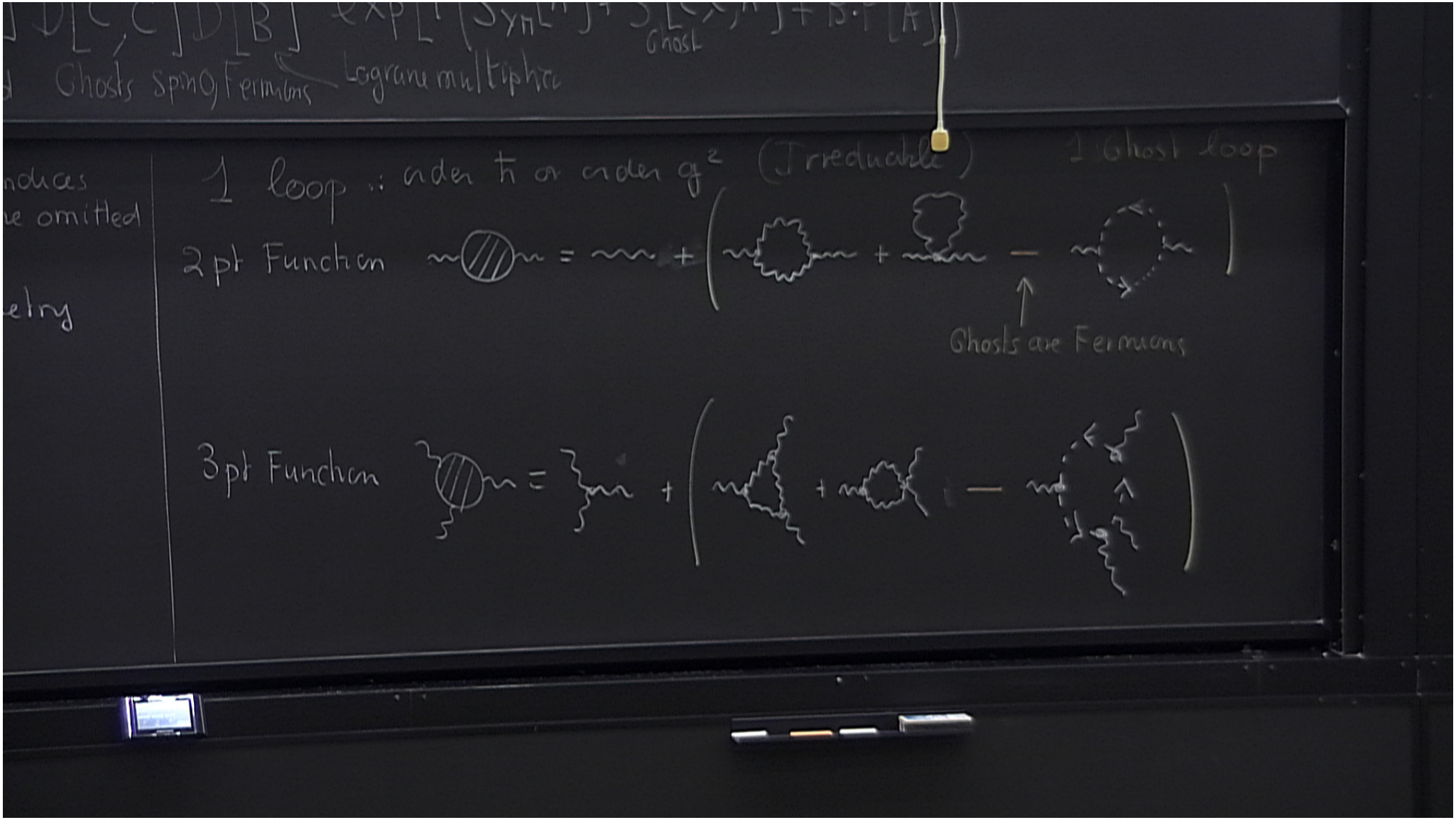

All Lorentz Indices
Group are omitted
Dirac

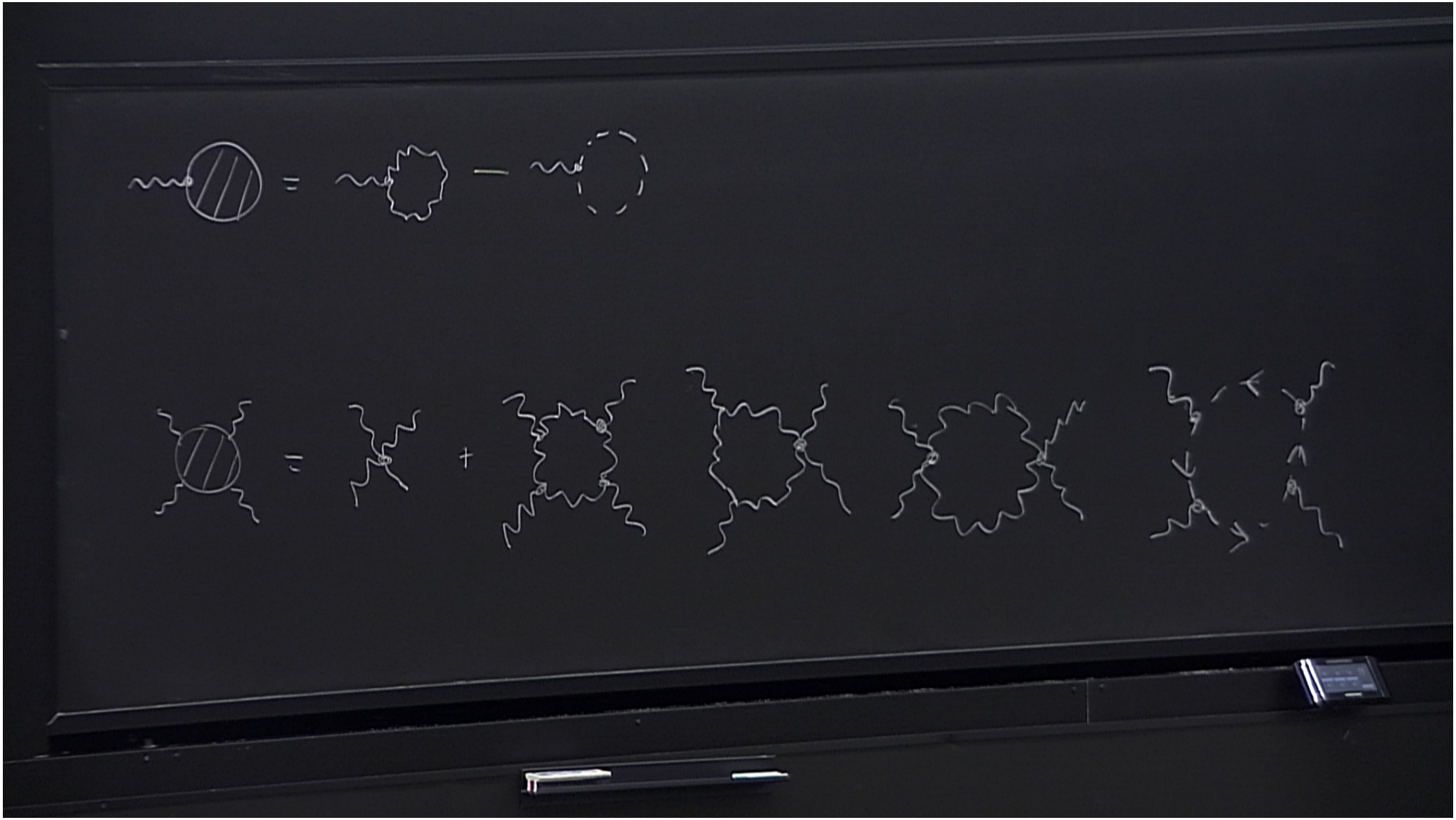

Vector Field Ghosts
Fermions (Dirac)
All Lorentz Indices
Group are omitted
Dirac
Omit Symmetry
Factors
A
A
k
$\frac{1}{k^2}$
C
$\bar{C}$
$\psi$
$\bar{\psi}$
$\frac{1}{k}$
g·
g·k

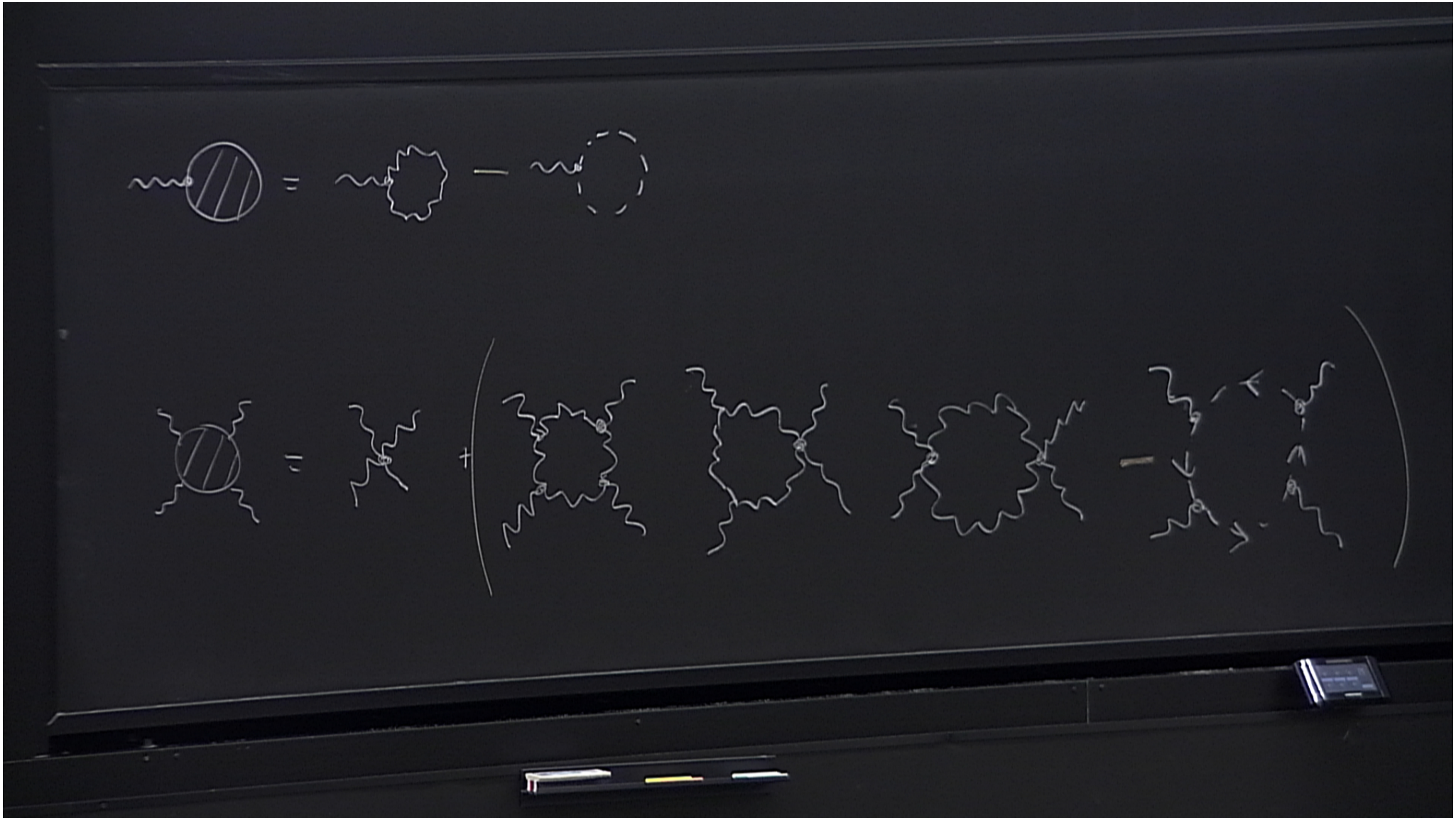

Ghosts spin 0, Fermions
Lagrange multiplier
Ghost
1 loop : order ħ or order g²
2pt Function
Ghosts are Fermions

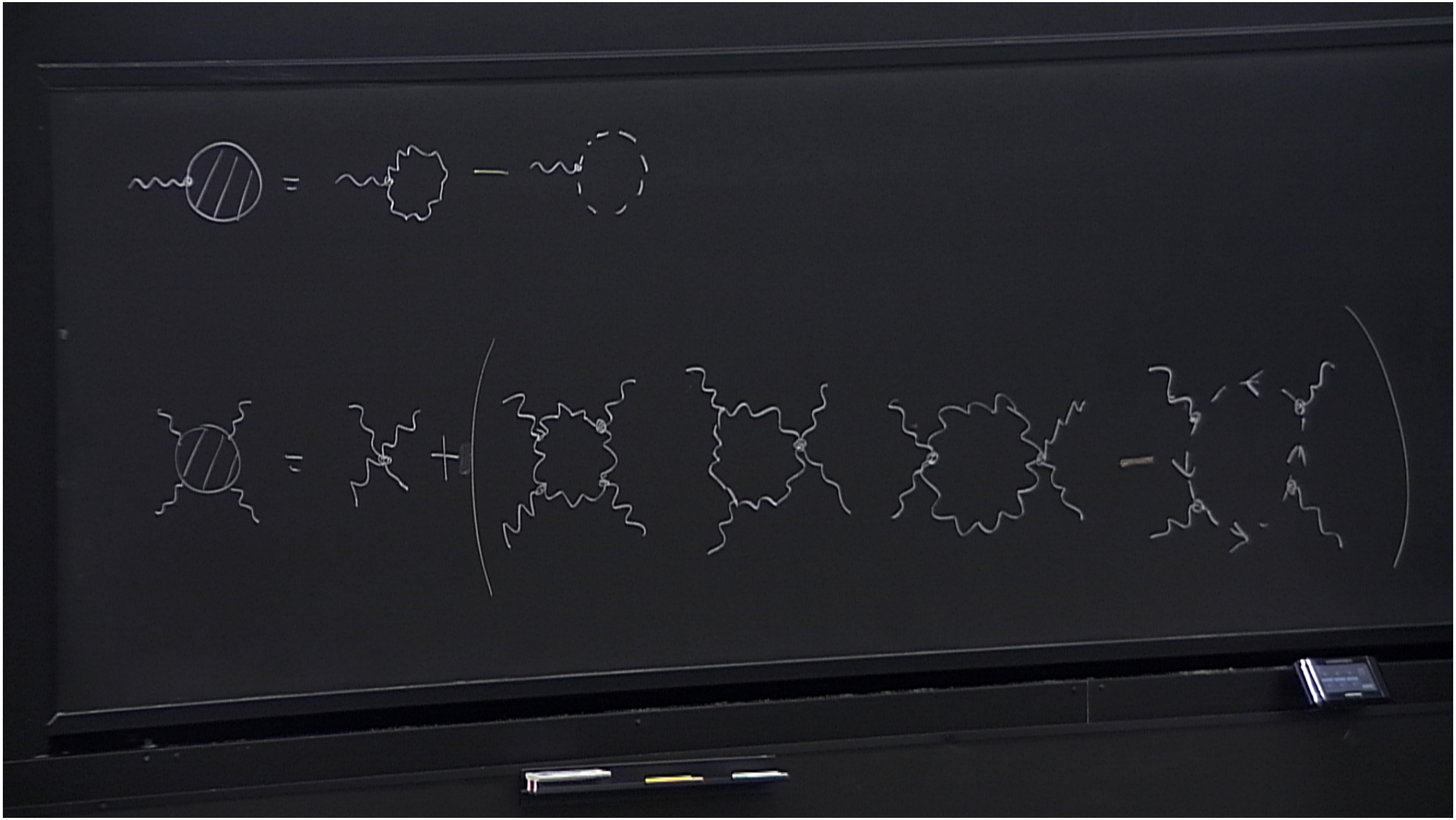

Ghosts spin 0 Fermions
Lagrange multipliers
Ghost
1 loop : order ħ or order g²
(Irreducible)
2pt Function
=
+
+
−
Ghosts are Fermions
3pt Function
=
+
+
+

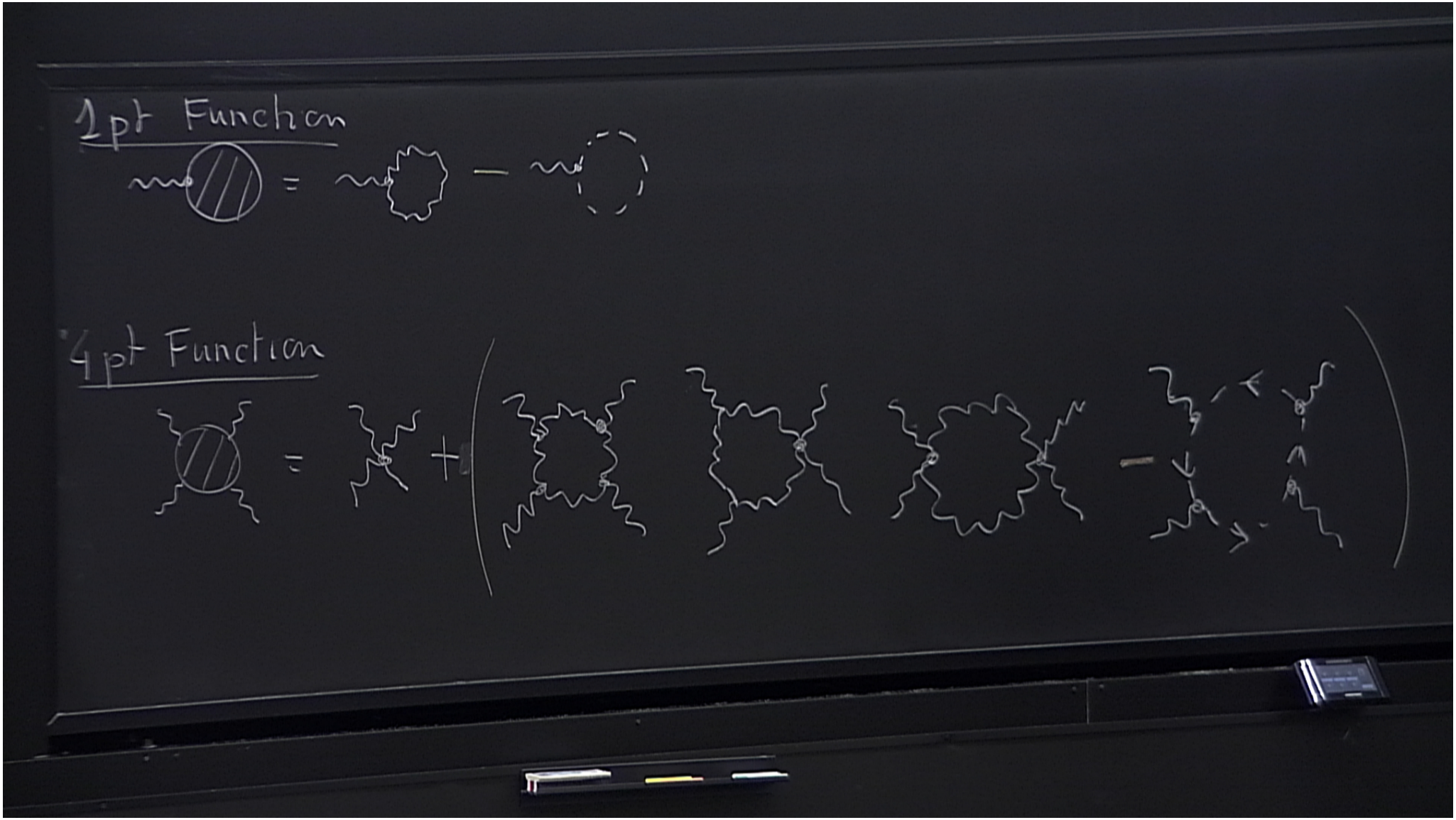

Ghosts spin 0 Fermions
Lagrange multipliers
Ghost
1 loop : order ħ or order g²
(Irreducible)
1 Ghost loop
2pt Function
Ghosts are Fermions
3pt Function

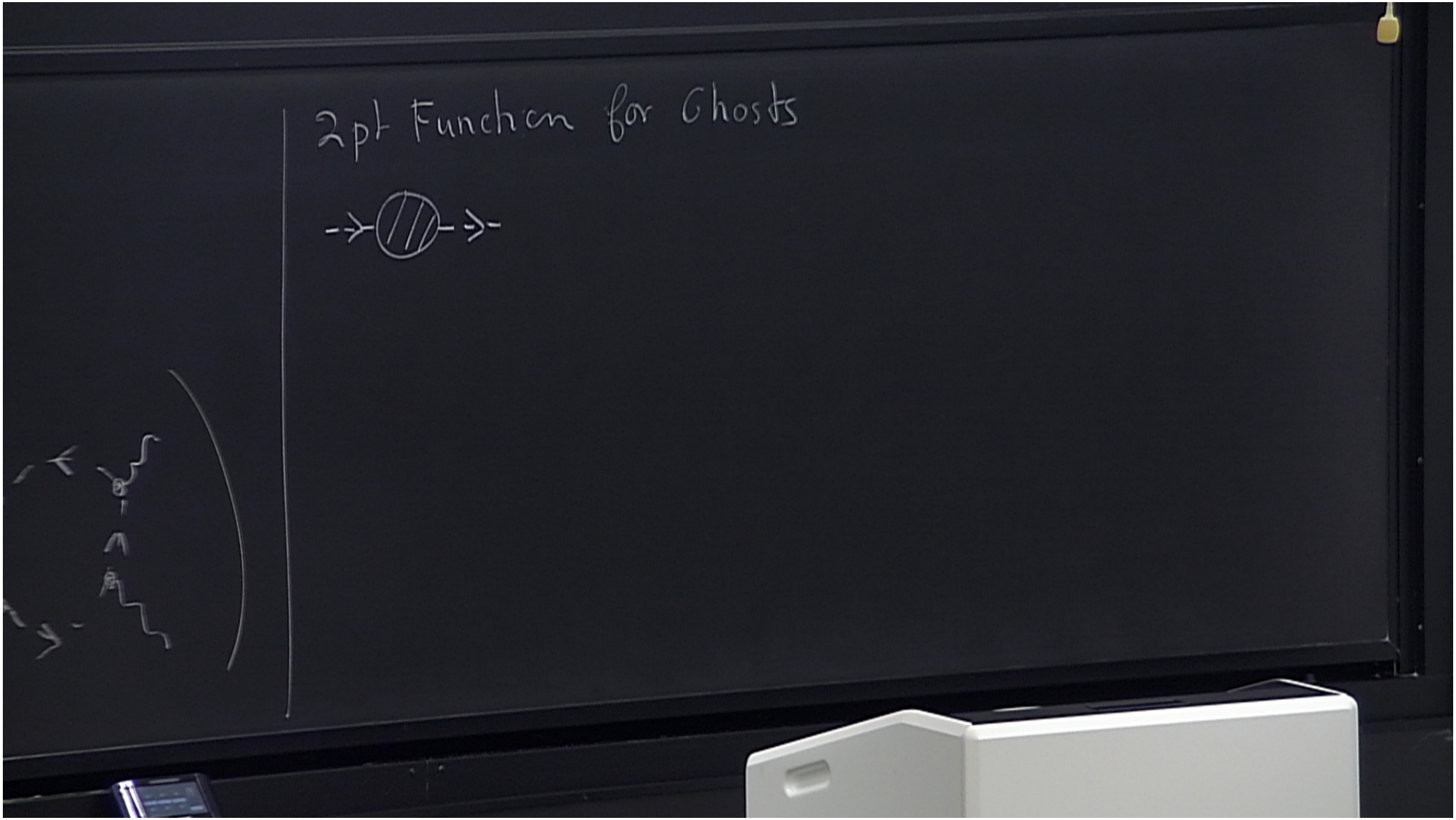

Ghosts spin 0, Fermions
Lagrange multipliers
Ghost
1 loop : order ħ or order g²
(Irreducible)
1 Ghost loop
2pt Function
Ghosts are Fermions
3pt Function

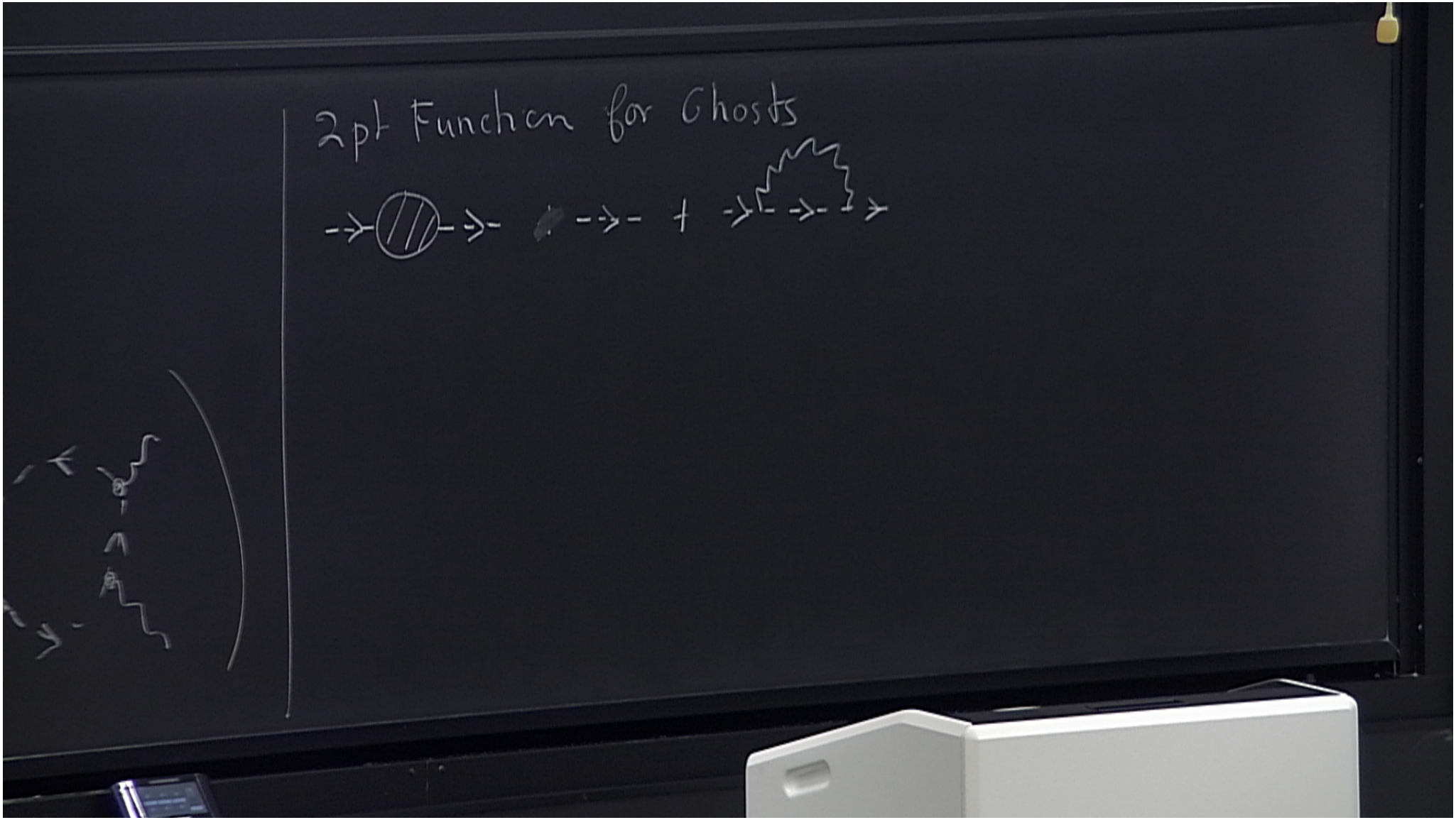




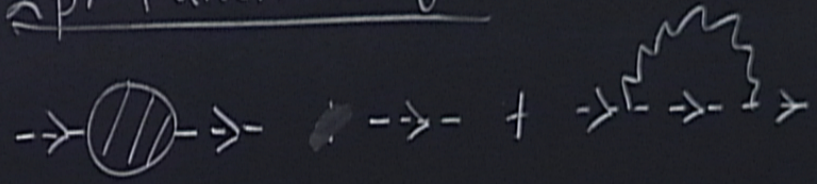



1pt Function
4pt Function

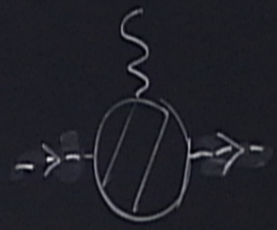

2pt Function for Ghosts

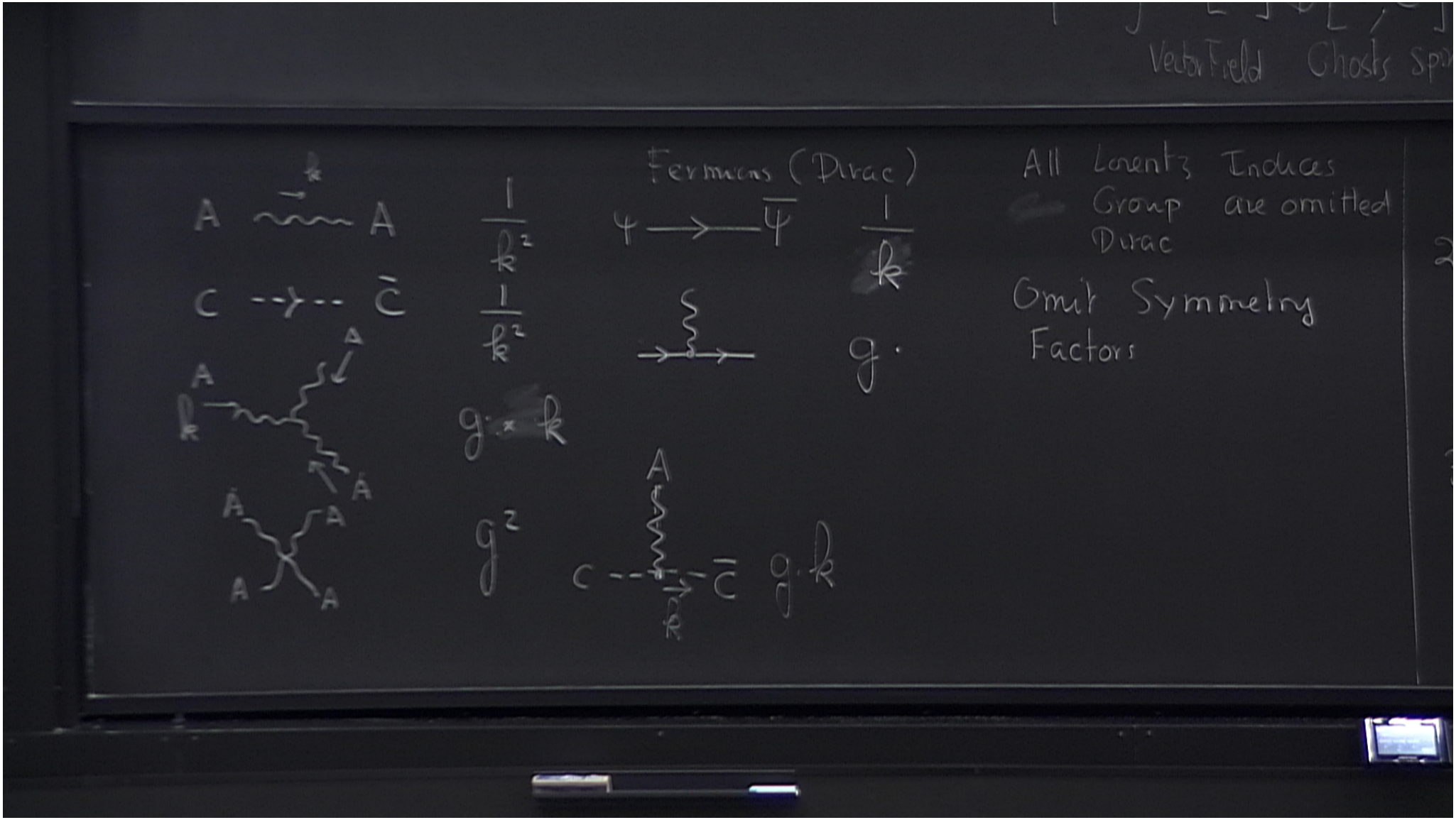

2pt Function for Ghosts

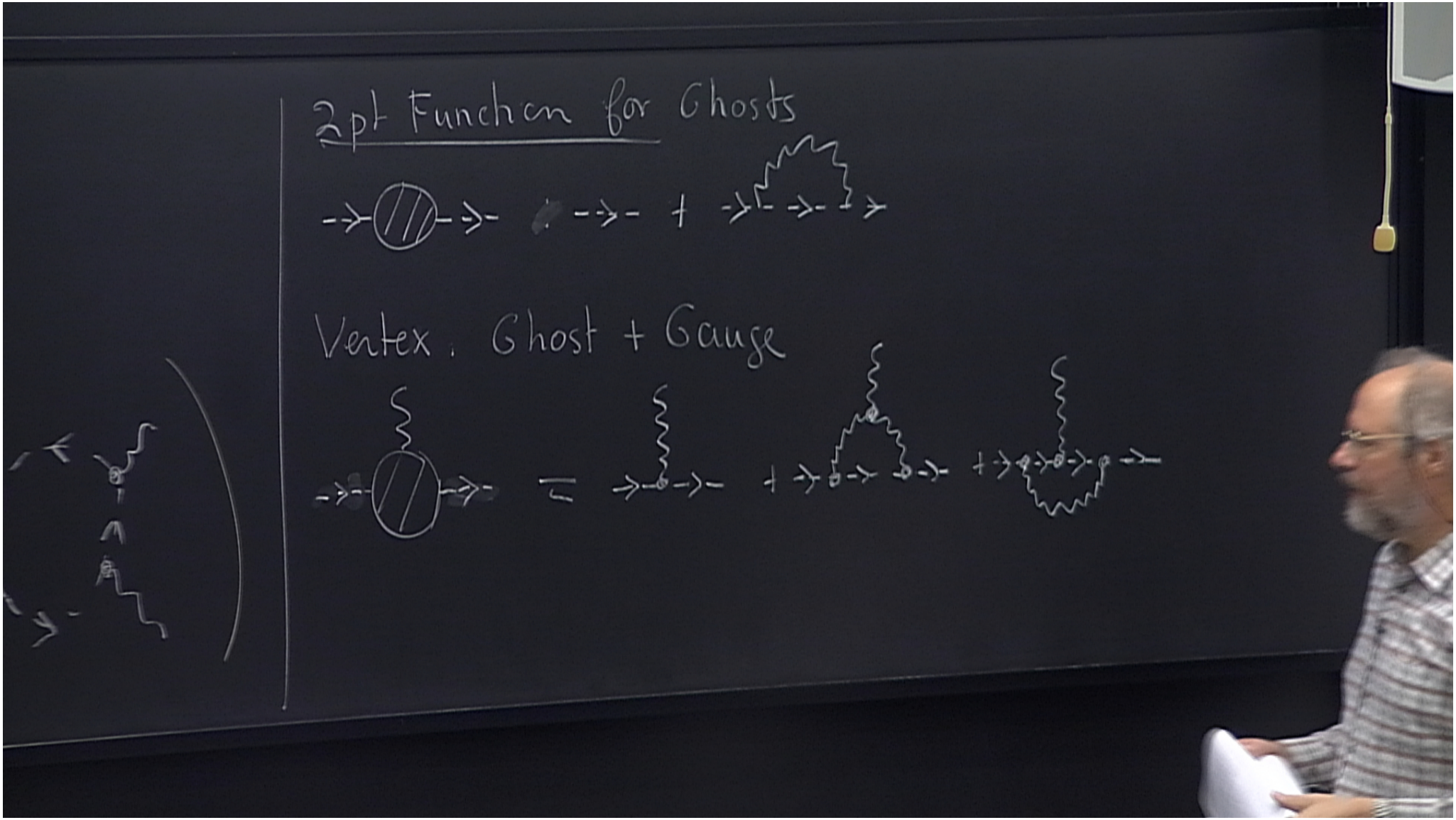
2pt Function for Ghosts



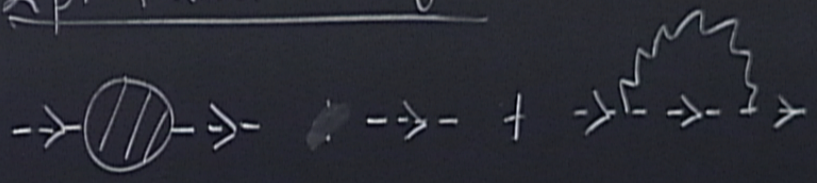
Vertex: Ghost + Gauge



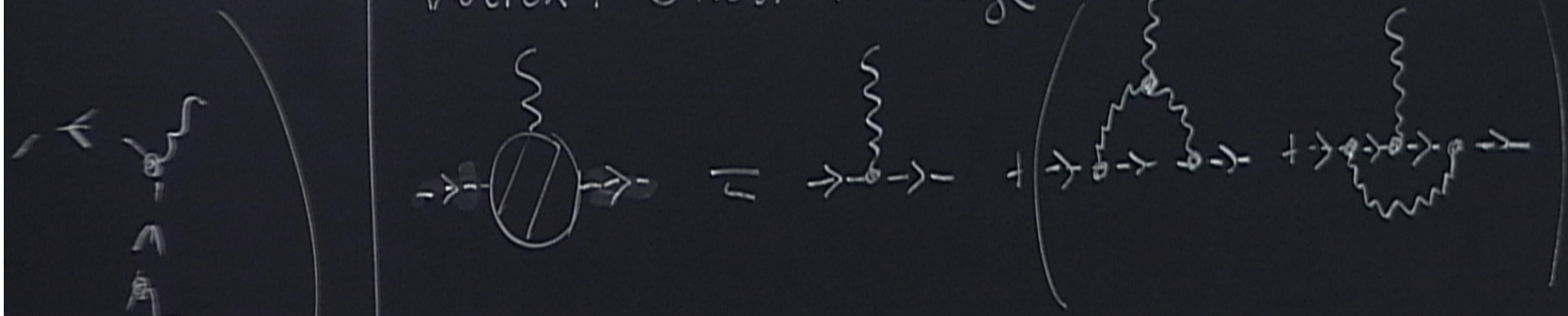

Vector Field Ghosts
Fermions (Dirac)
All Lorentz Indices
Group are omitted
Dirac
Omit Symmetry
Factors
A A
$\frac{1}{k^2}$
$\psi \to \bar{\psi}$
$\frac{1}{\not{k}}$
c $\bar{c}$
$\frac{1}{k^2}$
$g \cdot$
$g \cdot k$
$g^2$
$g \cdot k$

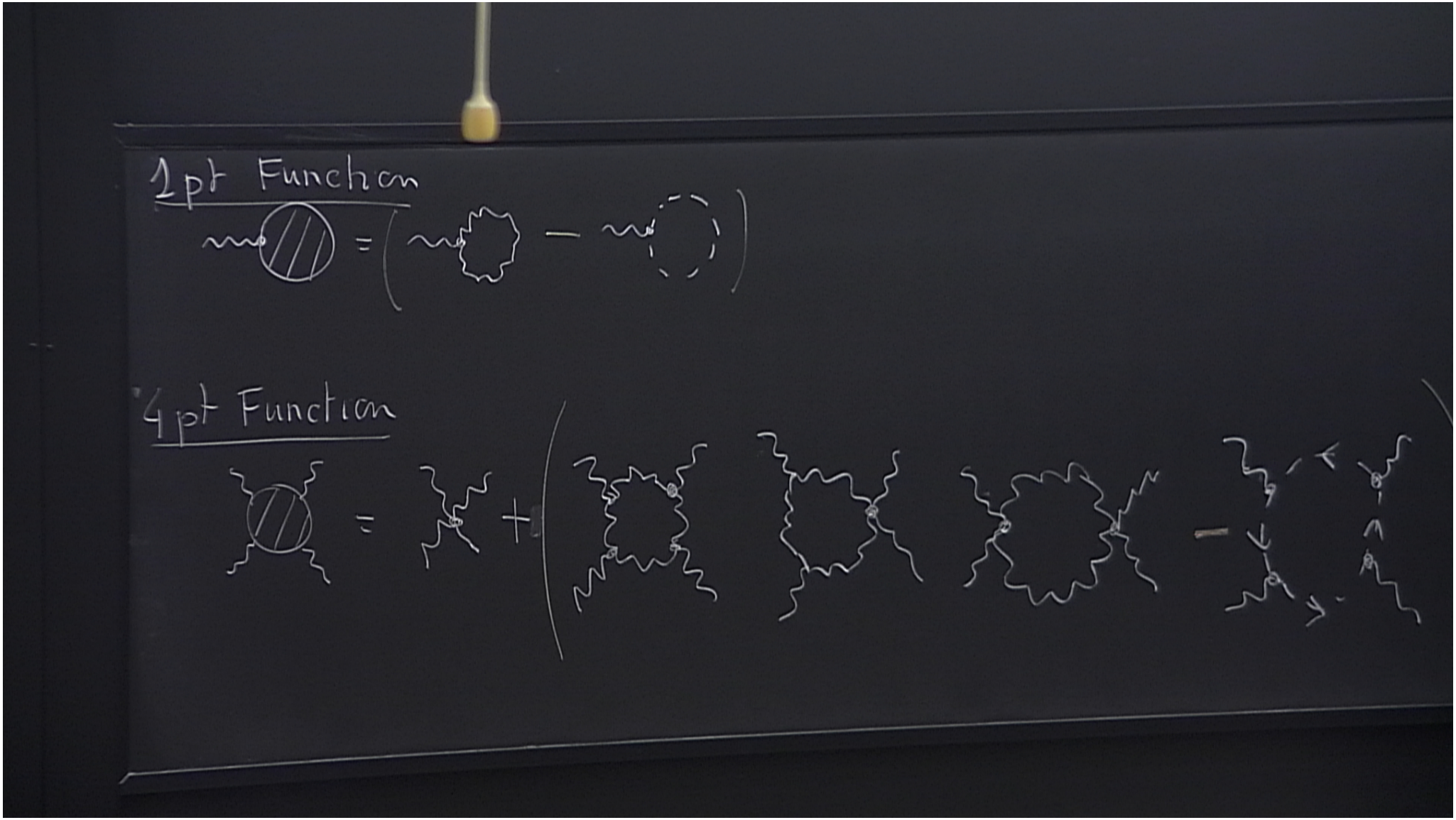

2pt Function for Ghosts
Vertex. Ghost + Gauge

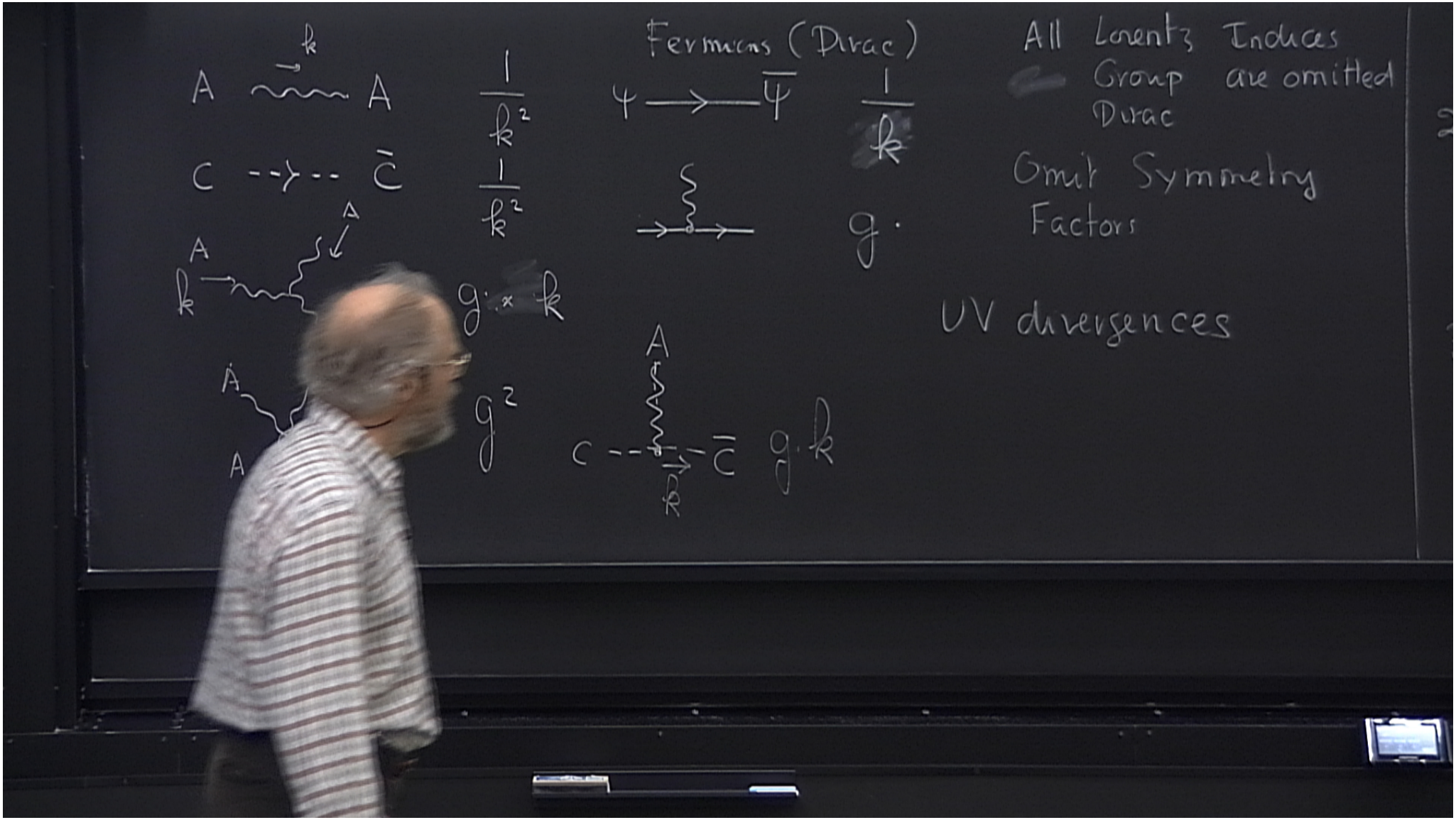
## 2pt Function for Ghosts



## Vertex: Ghost + Gauge




1pt Function
=
4pt Function
=
+
-


Fermions (Dirac)
All Lorentz Indices
Group are omitted
Dirac
Omit Symmetry
Factors
UV divergences

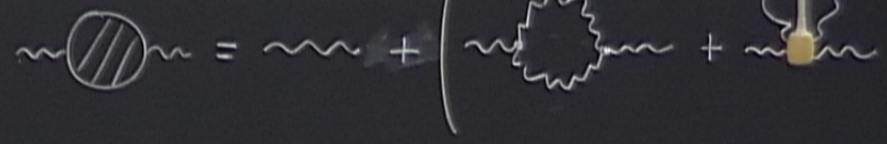
All Lorentz Indices
Group are omitted
Dirac

Omit Symmetry Factors

UV divergences in $D=4$

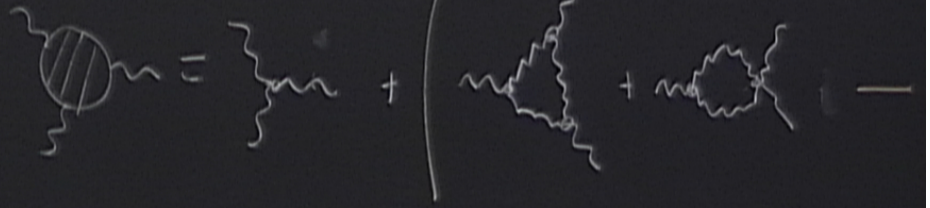
1 loop : order $\hbar$ or order $g^2$ (Irreducible)

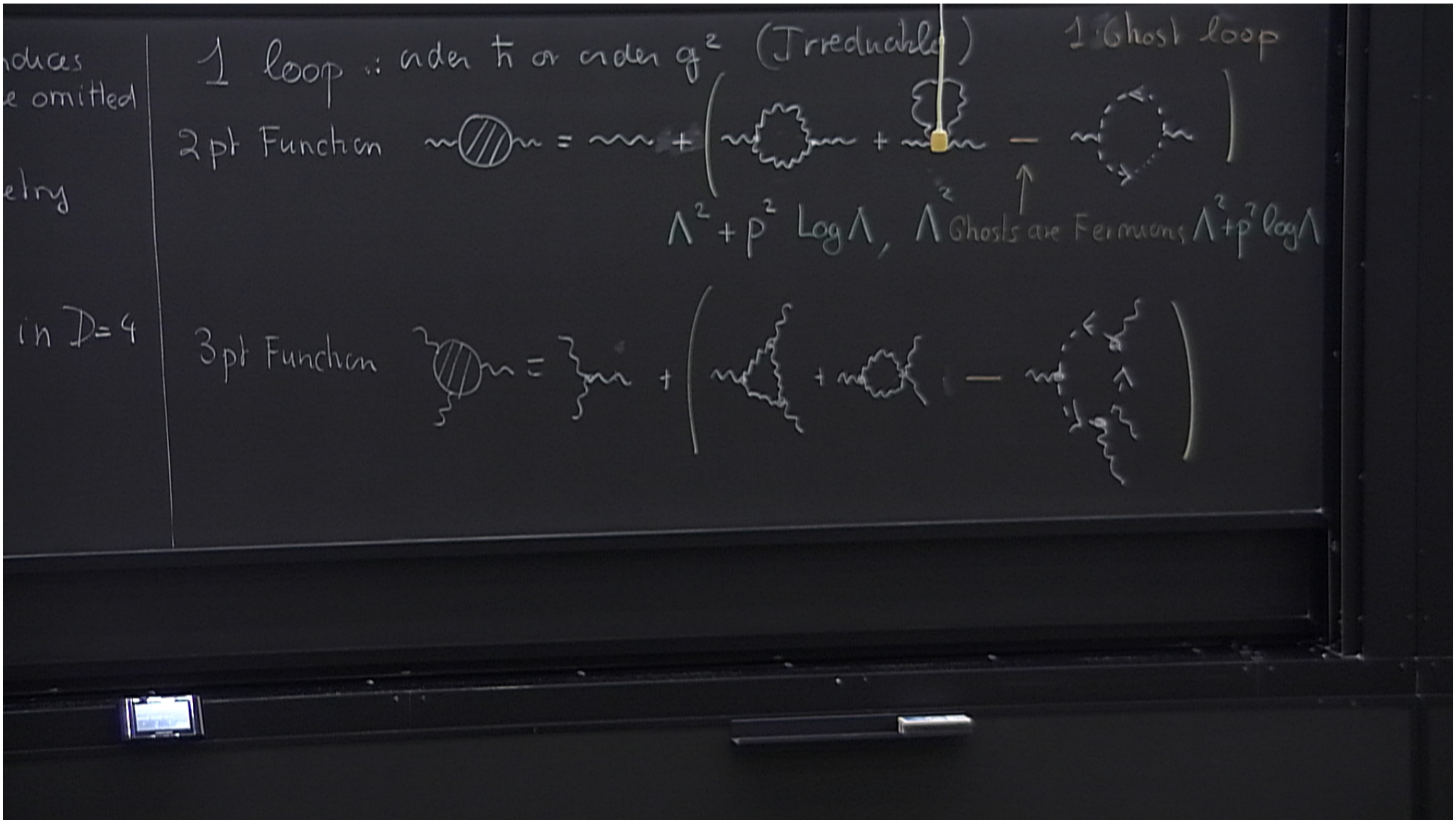
2pt Function

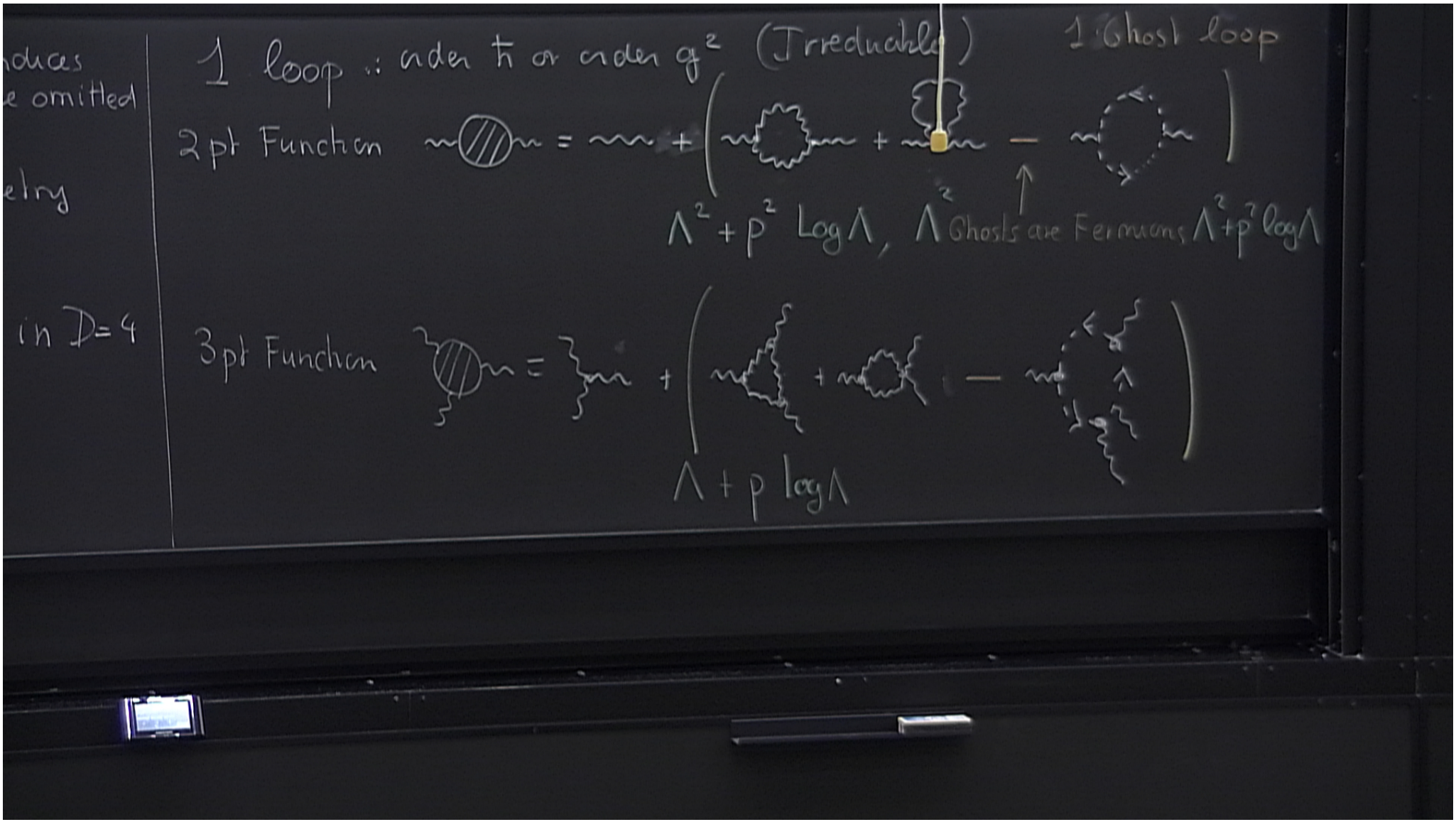


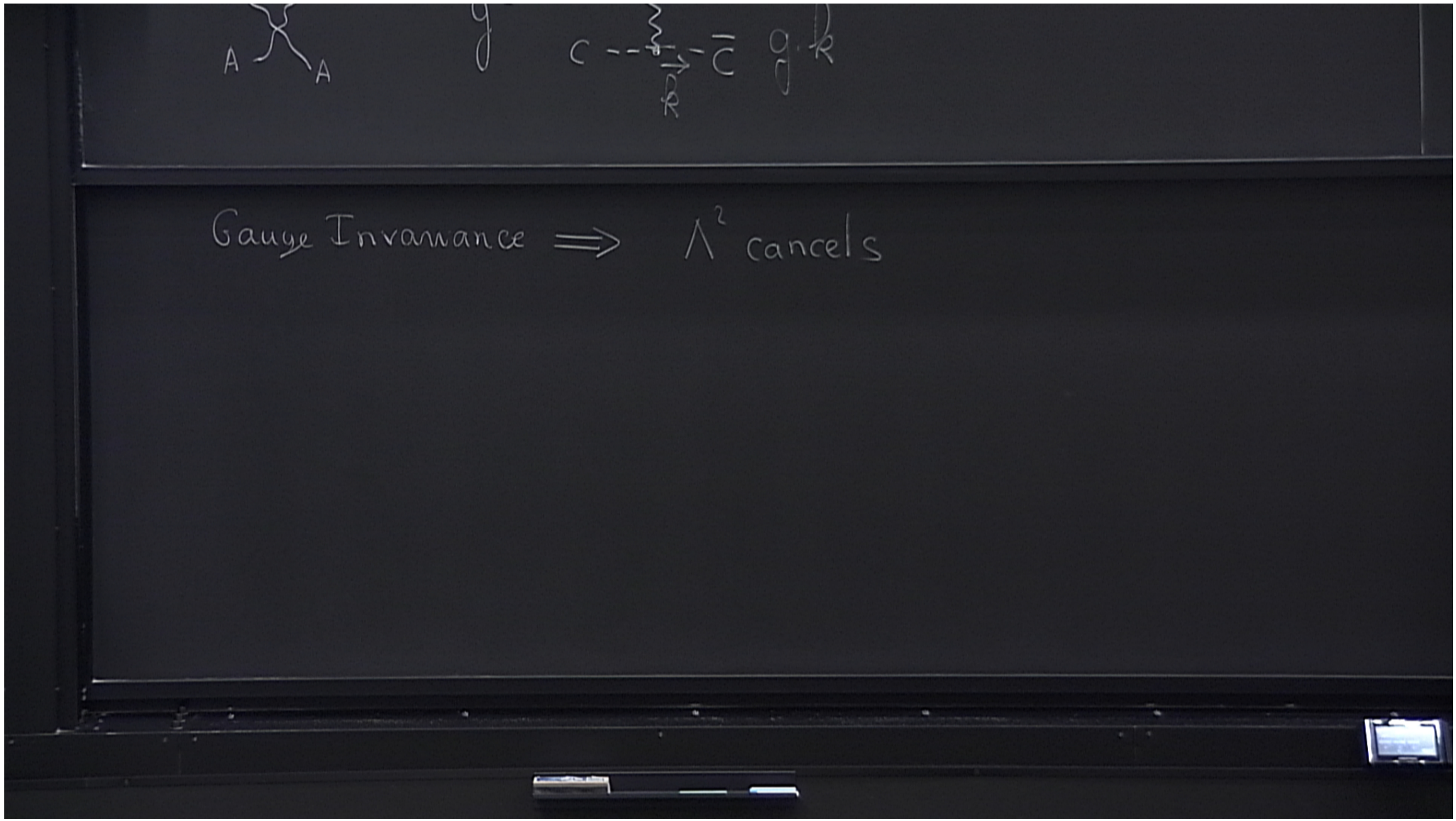
Ghost

3pt Function

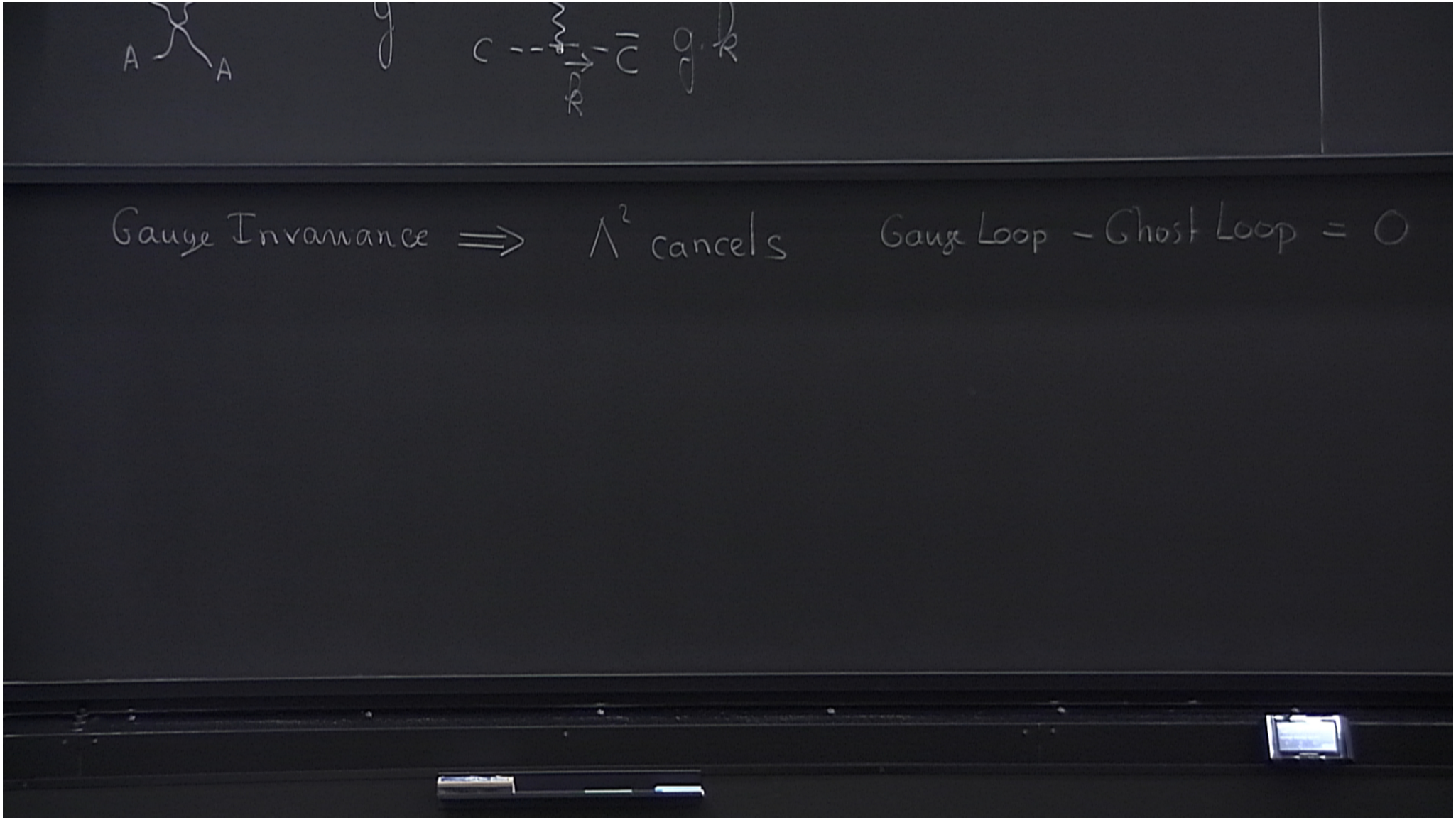



1 loop : order ħ or order g² (Irreducible)
1 Ghost loop
2pt Function
Λ² + p² Log Λ, Λ²
Ghosts are Fermions, Λ² + p² log Λ
3pt Function

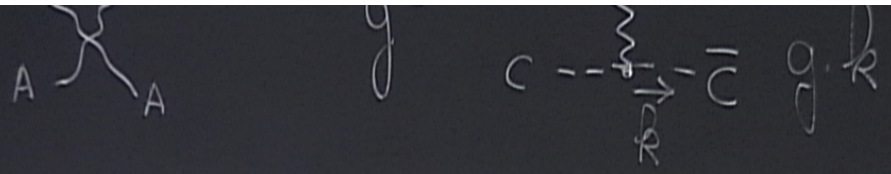

1 loop : order ħ or order g²
(Irreducible)
1 Ghost loop
2pt Function
Λ² + p² Log Λ, Λ²
Ghosts are Fermions Λ² + p² log Λ
3pt Function
Λ + p log Λ




Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels

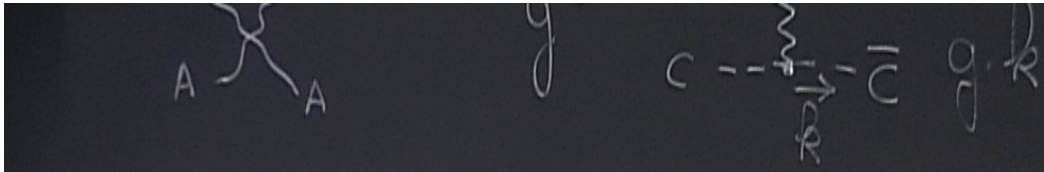



Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels  Gauge Loop − Ghost Loop = 0



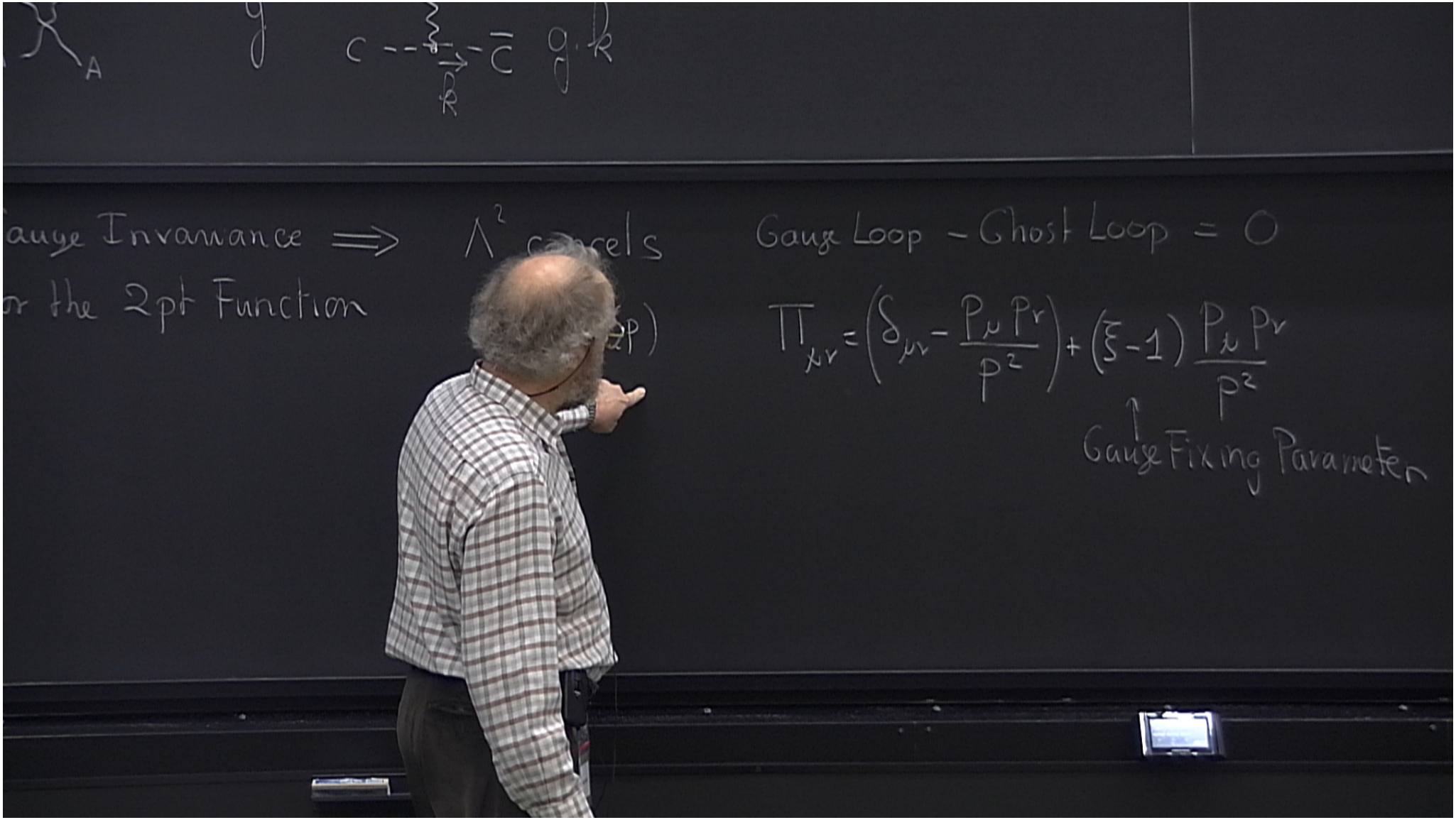

Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels  Gauge Loop − Ghost Loop = 0

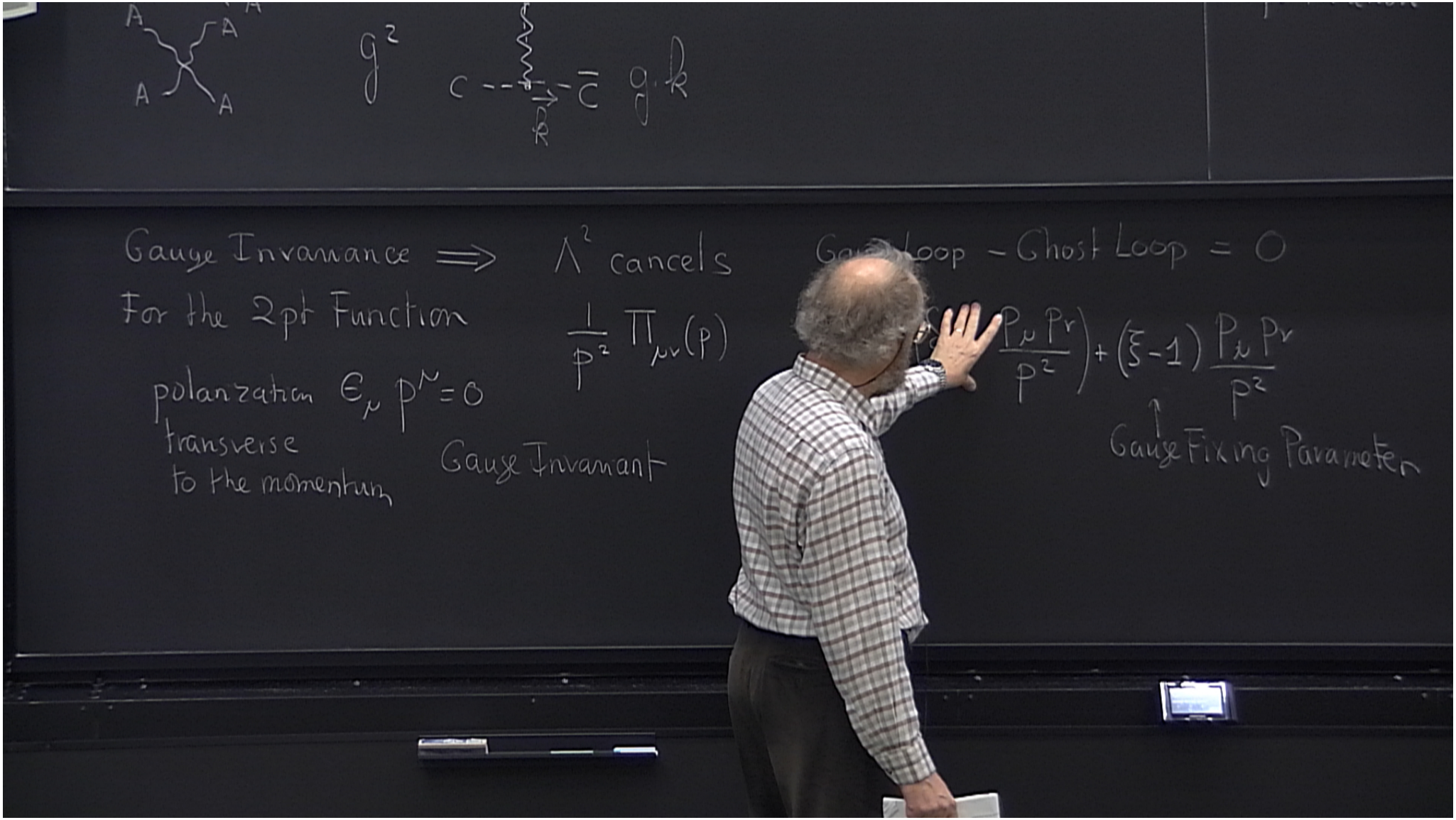
For the 2pt Function

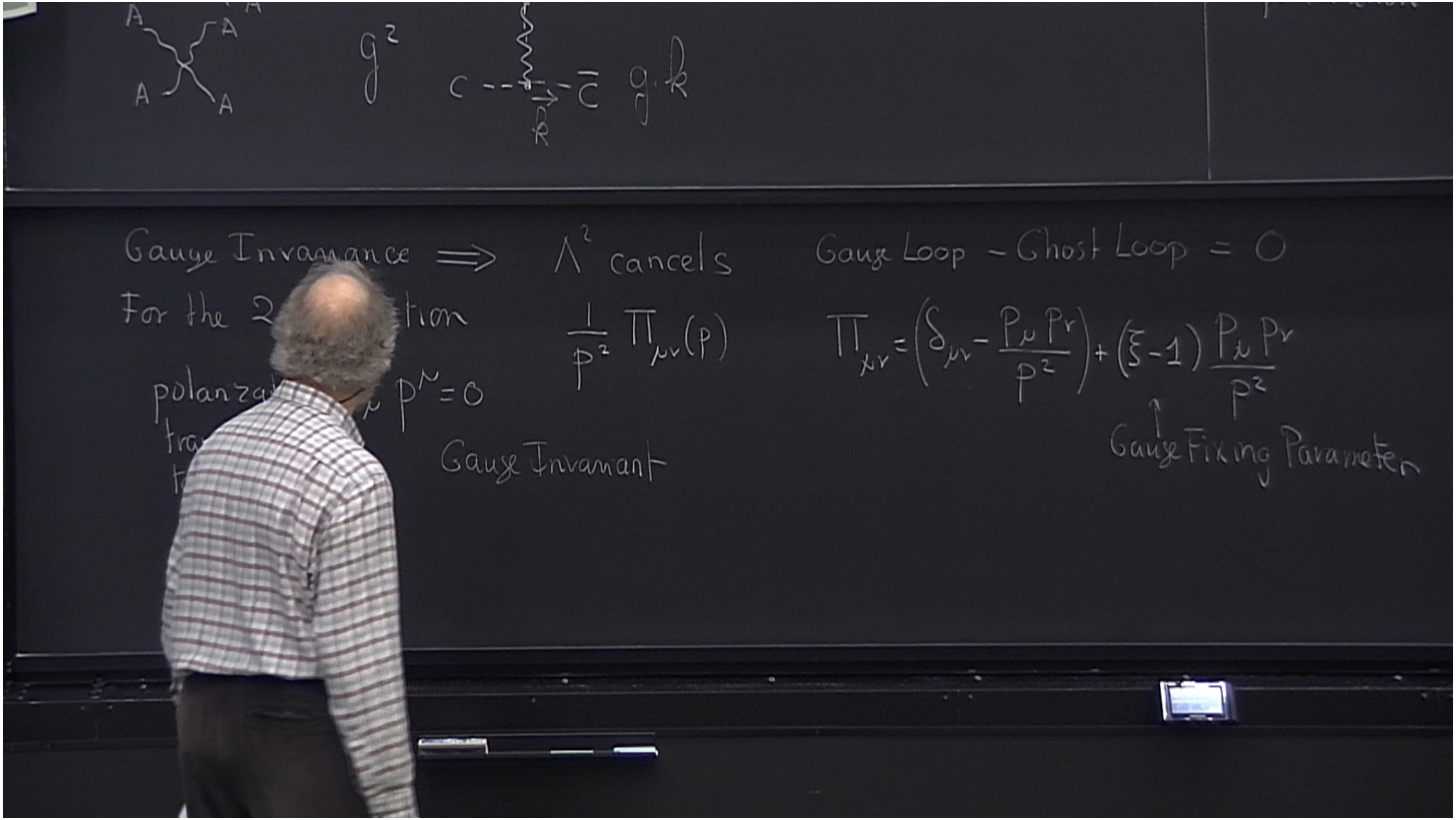


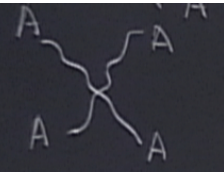

Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels  Gauge Loop − Ghost Loop = 0

For the 2pt Function  $\frac{1}{p^2}\Pi_{\mu\nu}(p)$  $\Pi_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$

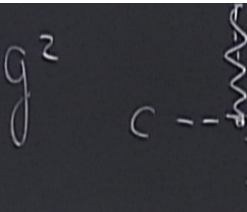

A
c
$\bar{c}$
q
k
Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels
for the 2pt Function
Gauge Loop − Ghost Loop = 0
$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + (\xi - 1)\frac{p_\mu p_\nu}{p^2}$
Gauge Fixing Parameter


A
A
A
A
$g^2$
c
$\bar{c}$
$g \cdot k$
$k$
Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels
For the 2pt Function
$\frac{1}{p^2}\Pi_{\mu\nu}(p)$
Ghost Loop = 0
$\frac{p_\mu p_\nu}{p^2}$
$+(\xi-1)\frac{p_\mu p_\nu}{p^2}$
Gauge Fixing Parameter
polarization $\epsilon_\mu p^\mu = 0$
transverse
to the momentum
Gauge Invariant


A
A
A
A
$g^2$
c
$\bar{c}$
$g \cdot k$
$k$
Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels
Gauge Loop − Ghost Loop = 0
For the 2
tion
$\frac{1}{p^2}\Pi_{\mu\nu}(p)$
$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + (\xi - 1)\frac{p_\mu p_\nu}{p^2}$
polariza
$p^\nu = 0$
tra
Gauge Invariant
Gauge Fixing Parameter




$g^2$




$g \cdot k$

Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels  Gauge Loop − Ghost Loop = 0

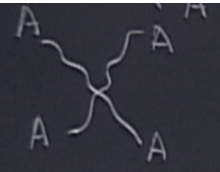
For the 2pt Function  $\frac{1}{p^2}\Pi_{\mu\nu}(p)$  $\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + (\xi - 1)\frac{p_\mu p_\nu}{p^2}$

polarization $\epsilon_\mu p^\mu = 0$
transverse
to the momentum

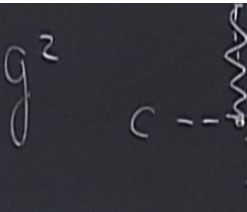
Gauge Invariant

↑ Gauge Fixing Parameter

2pt Function




$g^2$




$g \cdot k$

Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels

For the 2pt Function $\frac{1}{p^2}\Pi_{\mu\nu}(p)$

polarization $\epsilon_\mu p^\mu = 0$

transverse to the momentum

Gauge Invariant

Gauge Loop − Ghost Loop = 0

$$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + (\xi - 1)\frac{p_\mu p_\nu}{p^2}$$

↑ Gauge Fixing Parameter

$\Lambda + p \log \Lambda$

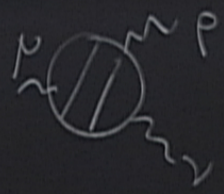
ost Loop = 0

$\left(\frac{P_r}{\quad}\right) + (\xi - 1)\frac{P_\nu P_r}{p^2}$

↑ Gauge Fixing Parameter

3pt Function.

Gauge Invariance ; Ward Identities



$\Lambda + p \log \Lambda$

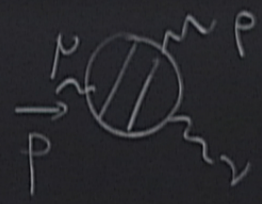
ost Loop = 0

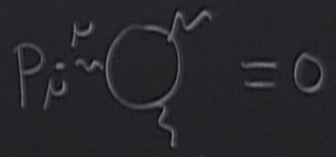
$\left(\frac{P_r}{\xi}\right) + (\xi - 1)\frac{P_\mu P_\nu}{p^2}$

↑ Gauge Fixing Parameter

3pt Function.

Gauge Invariance ; Ward Identities



$\Lambda + p \log \Lambda$

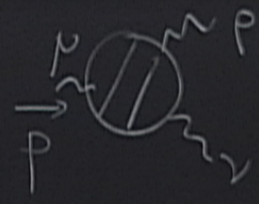
host Loop = 0

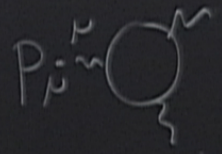
$$\left(\frac{P_r}{\xi}\right) + (\xi - 1)\frac{P_\mu P_\nu}{p^2}$$

↑ Gauge Fixing Parameter

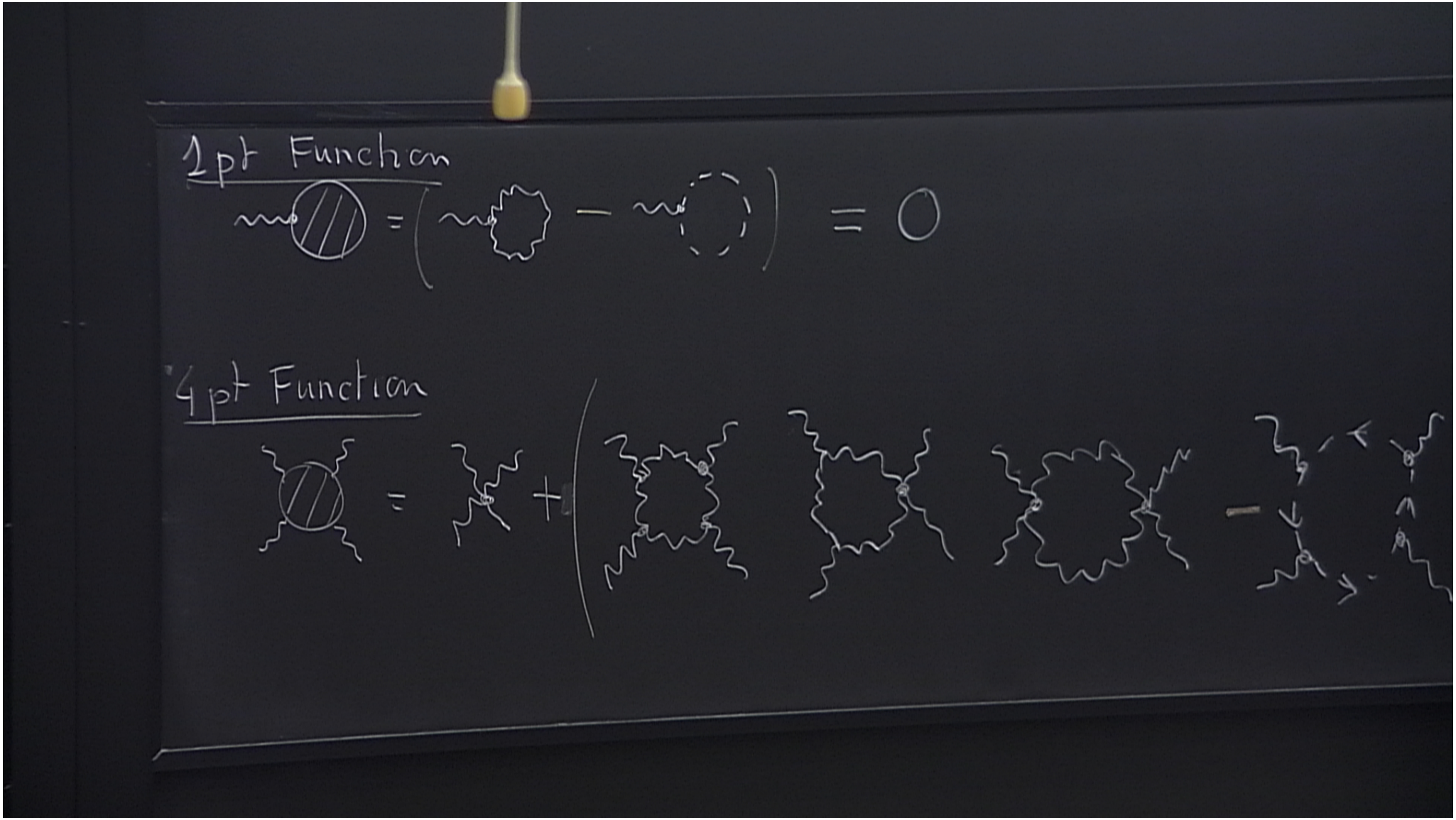
3pt Function.

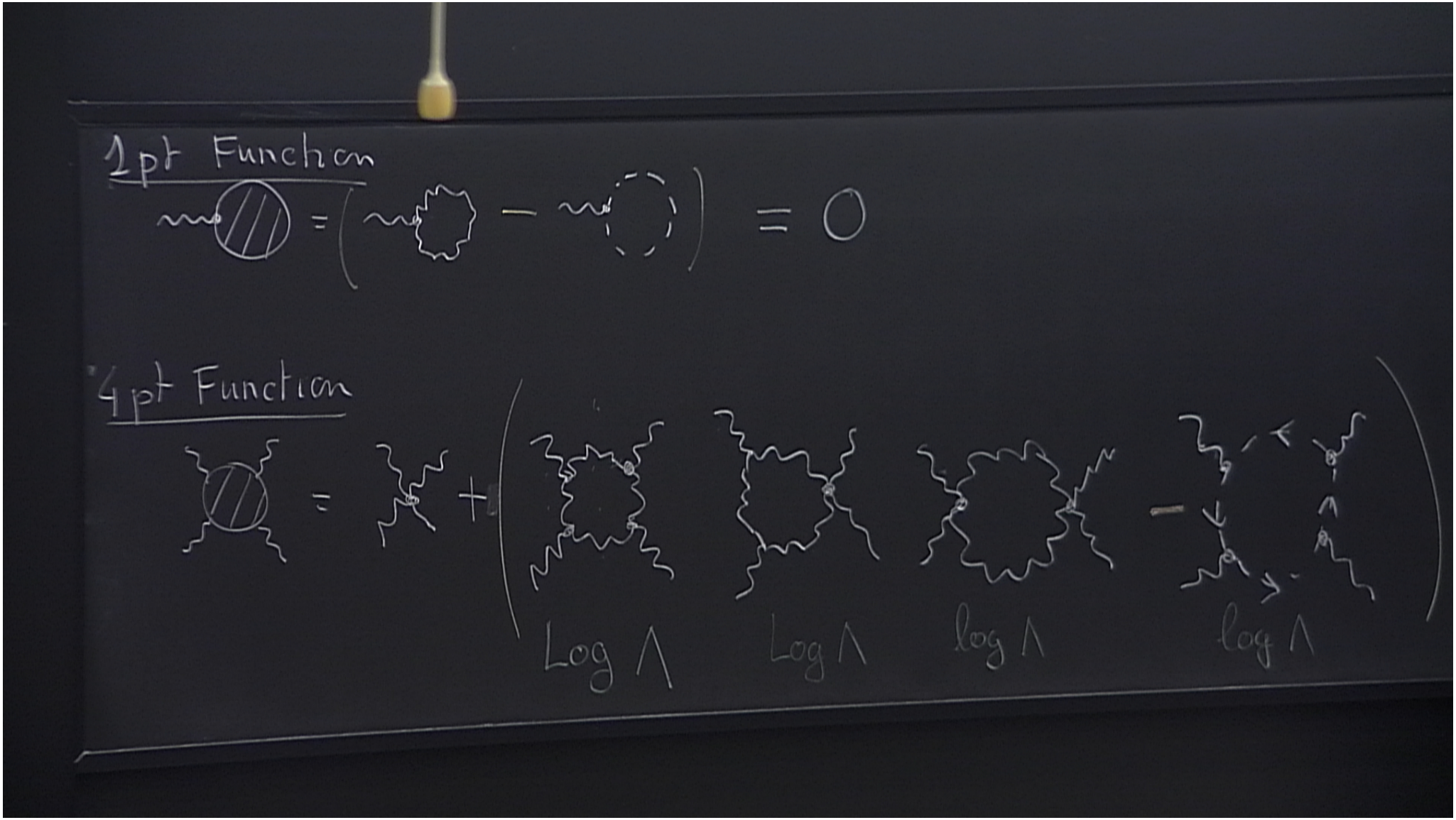
Gauge Invariance : Ward Identities

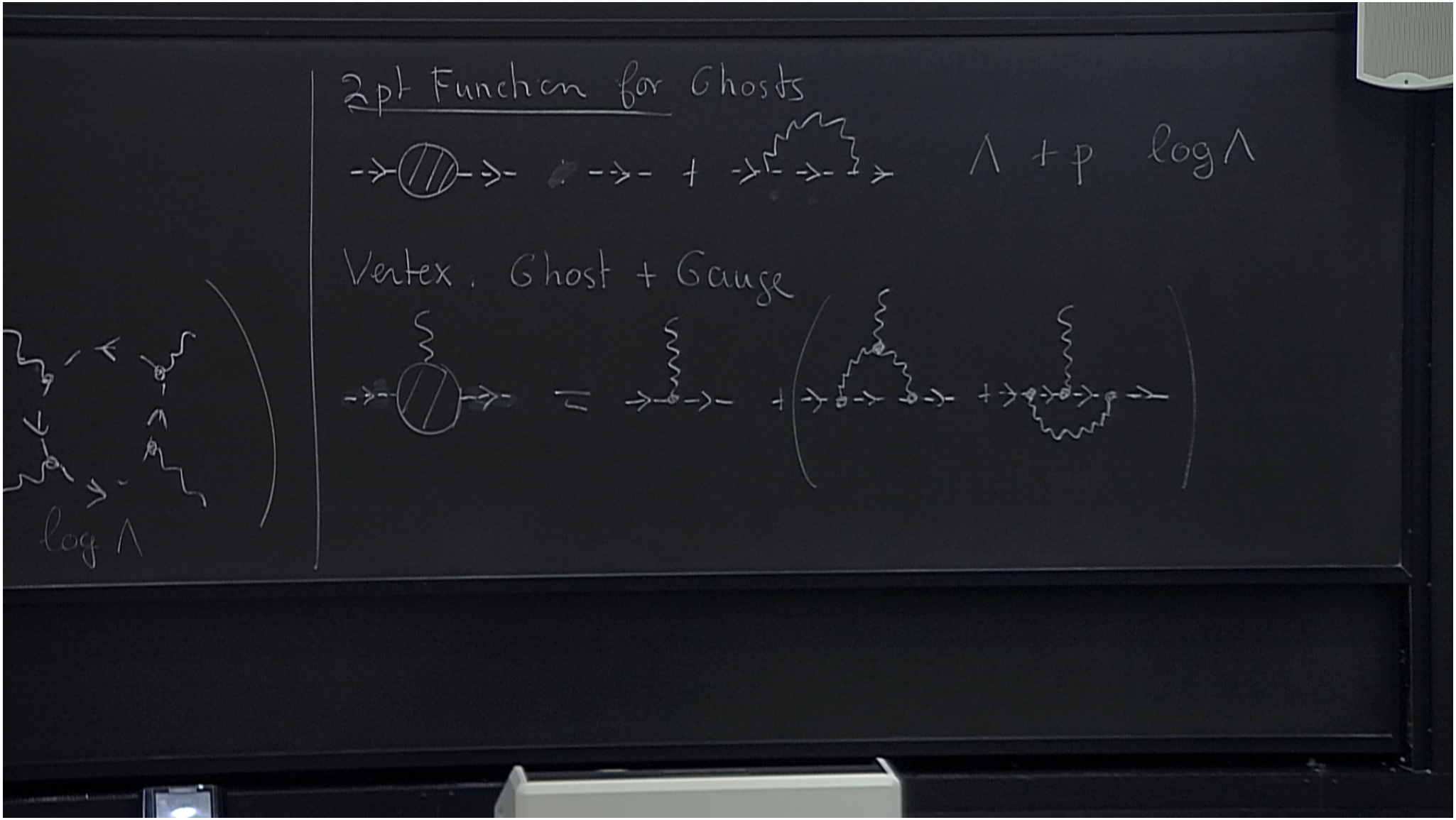


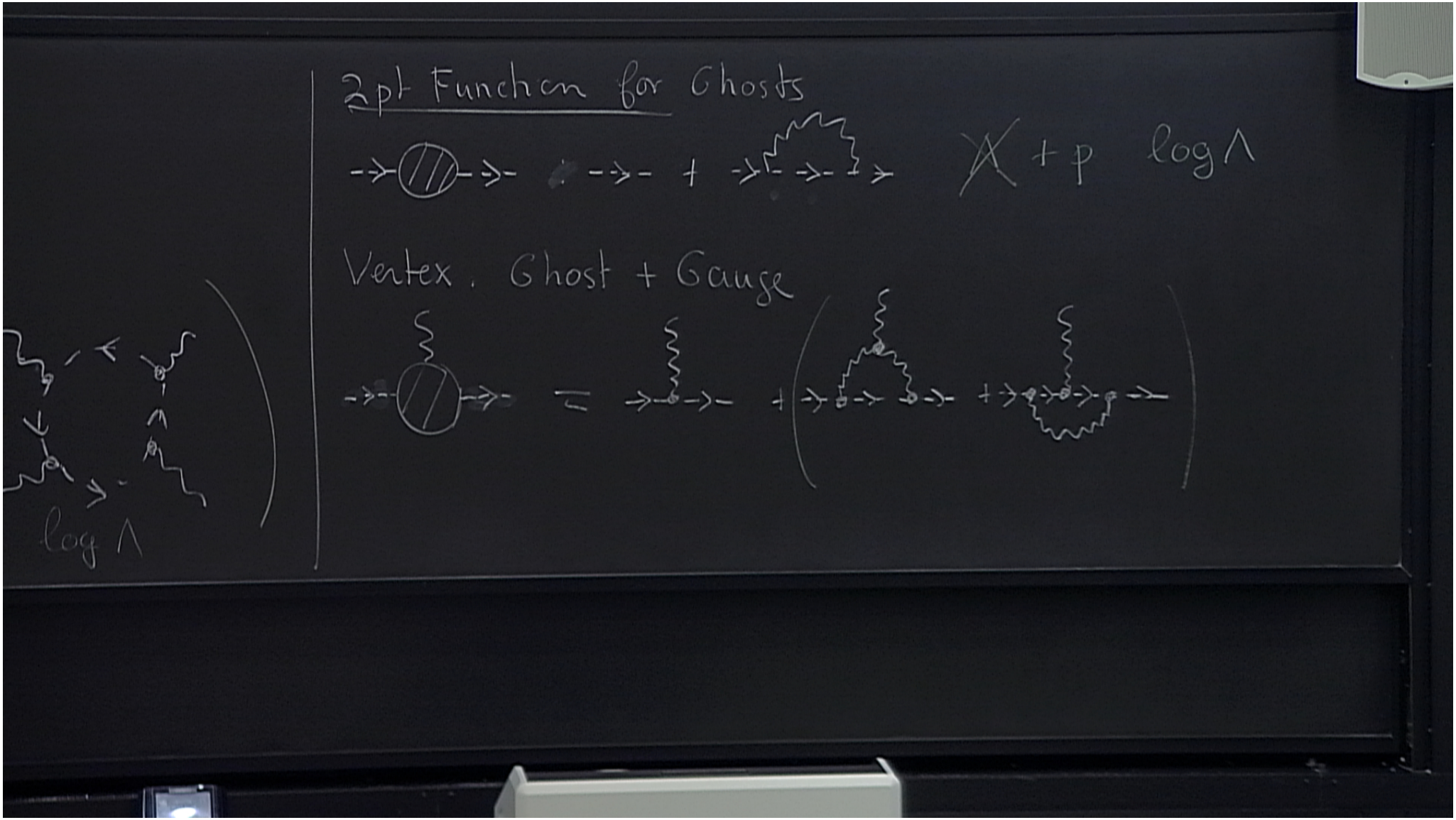



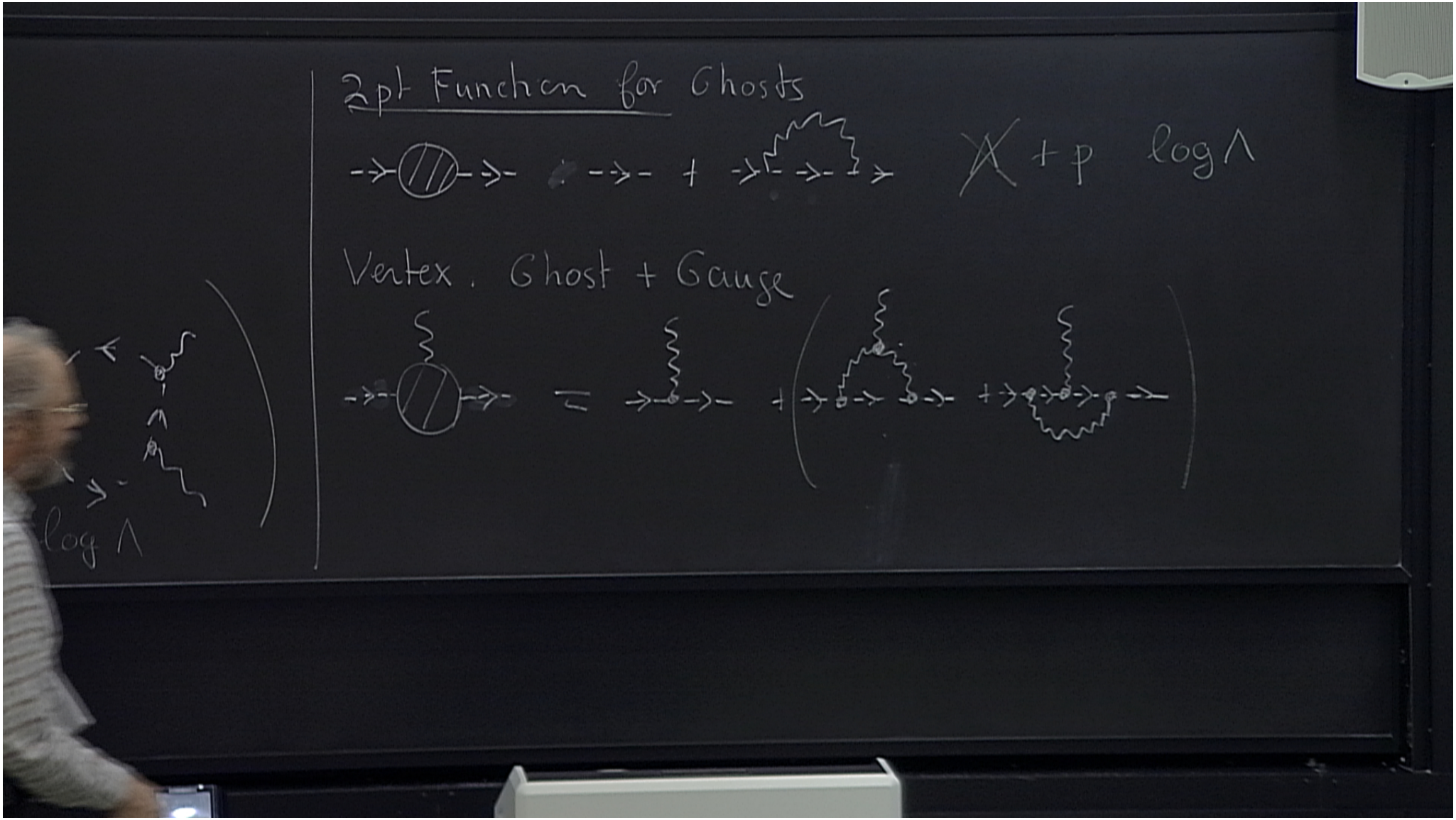

$= 0 =$ Gauge loop − Ghost Loop

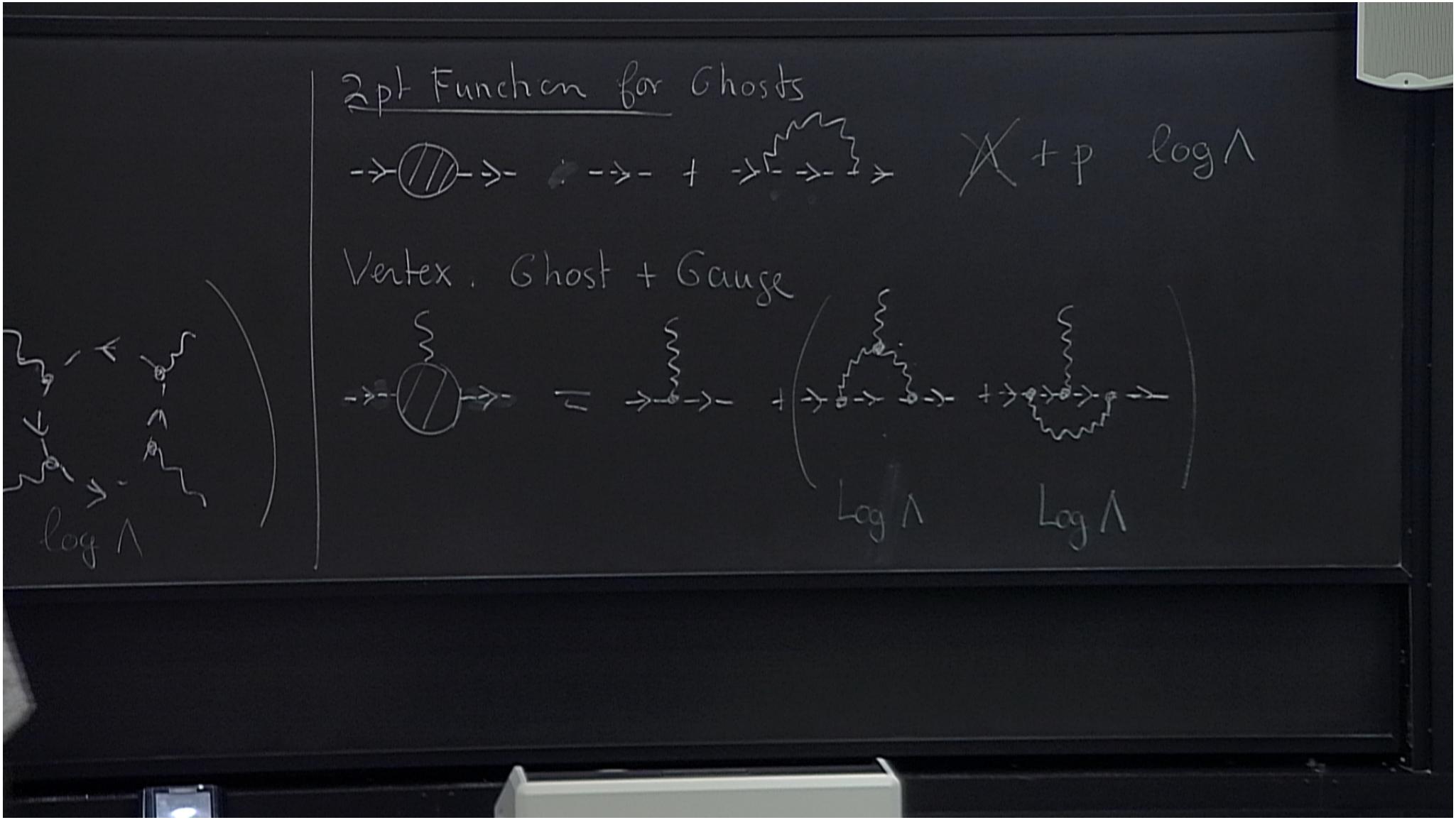

1pt Function
= ( − ) = 0
4pt Function
=
+
−


1pt Function
= 0
4pt Function
Log Λ
Log Λ
log Λ
log Λ


2pt Function for Ghosts
Λ + p  log Λ
Vertex. Ghost + Gauge
log Λ


2pt Function for Ghosts
+ p log Λ
Vertex. Ghost + Gauge
log Λ


2pt Function for Ghosts
+ p log Λ
Vertex. Ghost + Gauge
log Λ


2pt Function for Ghosts
+ p log Λ
Vertex. Ghost + Gauge
log Λ
Log Λ
Log Λ

$\Lambda + p \log \Lambda$

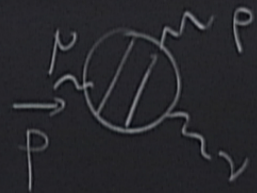
st Loop = 0

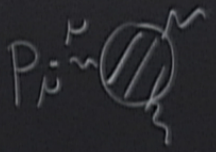
$+ (\xi - 1) \frac{P_\nu P_\nu}{P^2}$

↑ Gauge Fixing Parameter

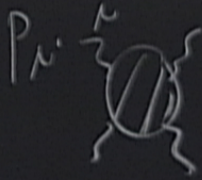
3pt Function.

Gauge Invariance : Ward Identities

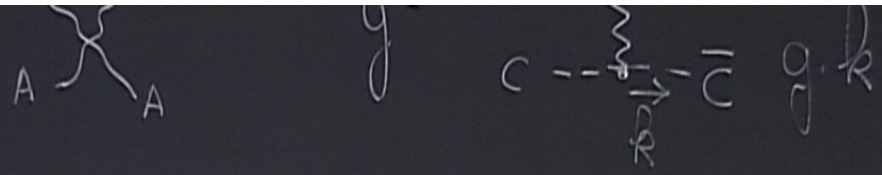



$= 0 =$ Gauge loop − Ghost Loop




$= 0$




② 

③ Gauge Invariance $\Longrightarrow$ $\Lambda^2$ cancels

For the 2pt Function $\frac{1}{p^2}\Pi_{\mu\nu}(p)$

polarization $\epsilon_\mu p^\mu = 0$

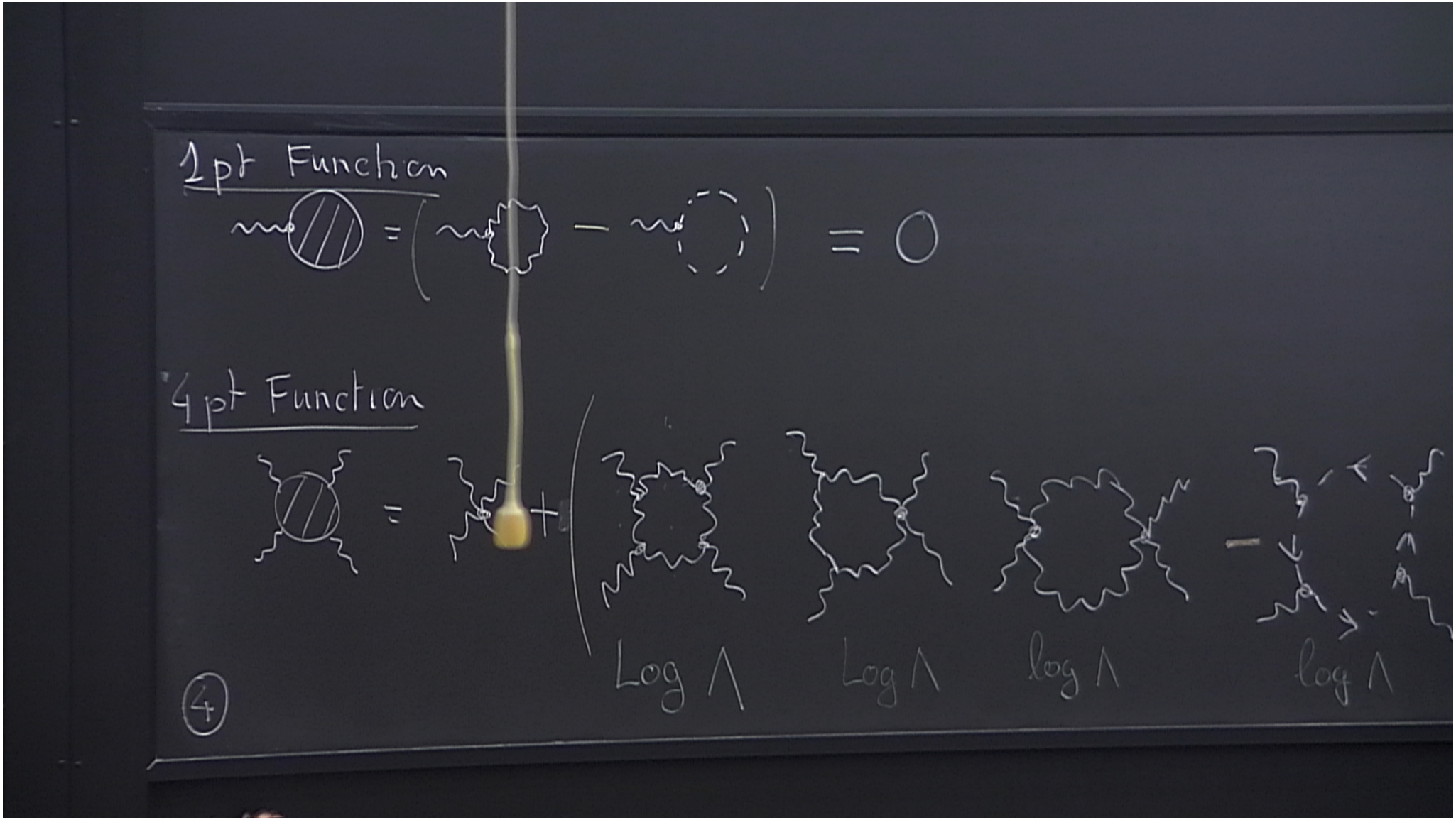
transverse to the momentum

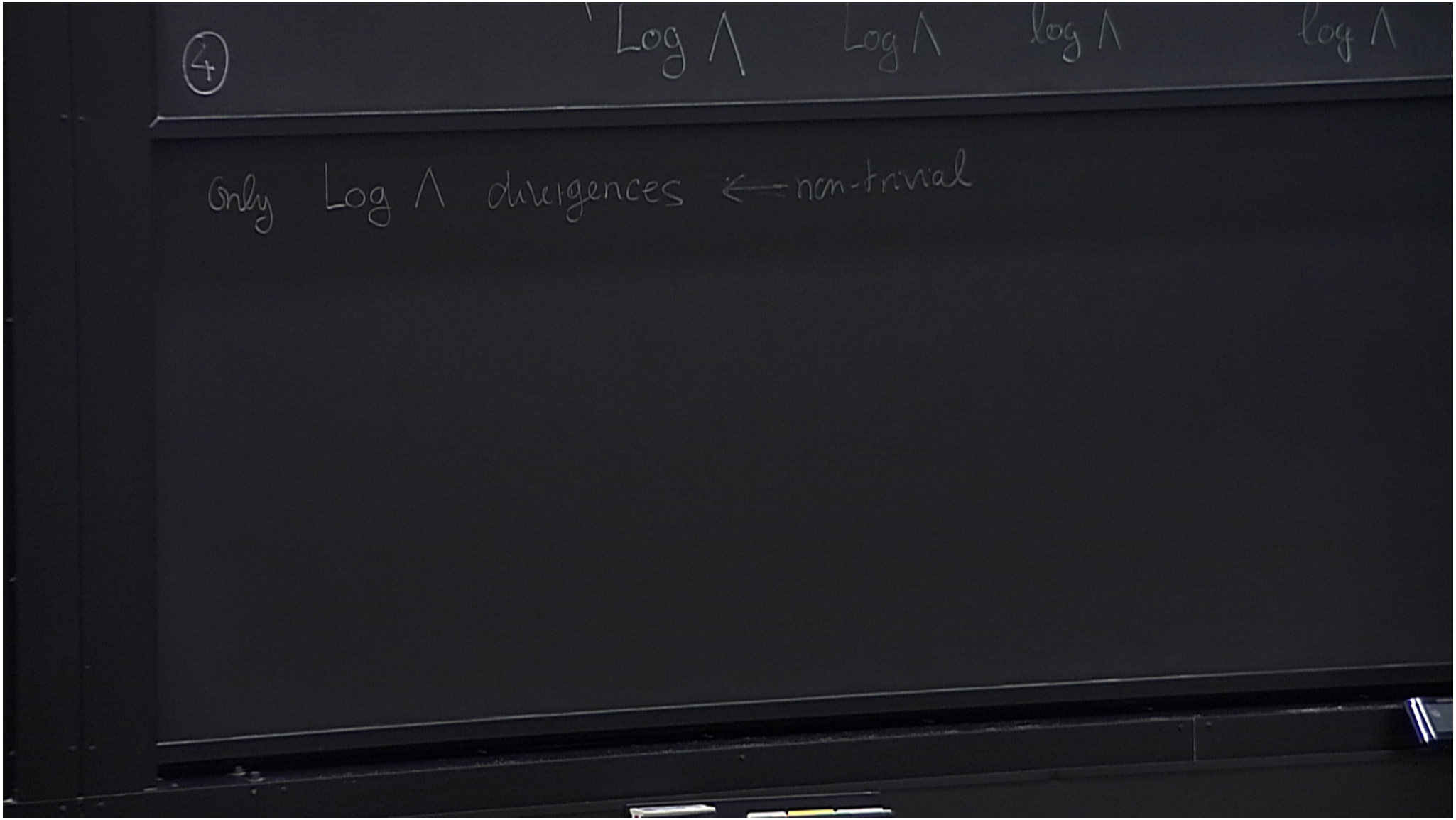
Gauge Invariant

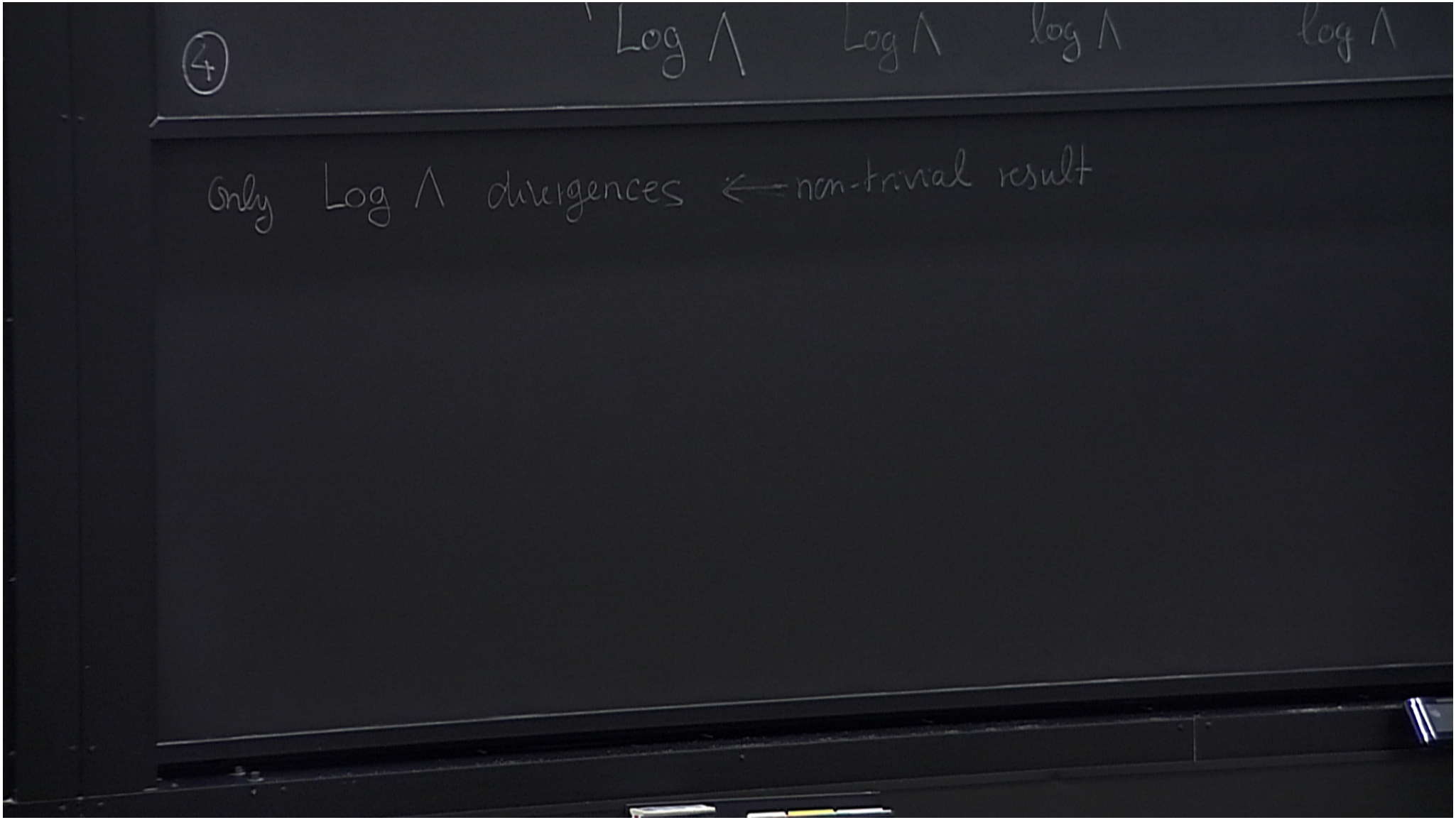
Gauge Loop − Ghost Loop =

$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + (\xi - 1)\frac{p}{}$

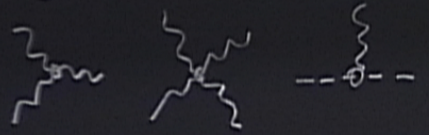
↑ Gauge Fix


1pt Function
= ( − ) = 0
4pt Function
=
+
Log Λ
Log Λ
log Λ
log Λ
4


④
Log Λ
Log Λ
log Λ
log Λ
Only Log Λ divergences ⟸ non-trivial


④
Log Λ
Log Λ
log Λ
log Λ
Only Log Λ divergences ⟸ non-trivial result

④ Log Λ Log Λ log Λ log Λ

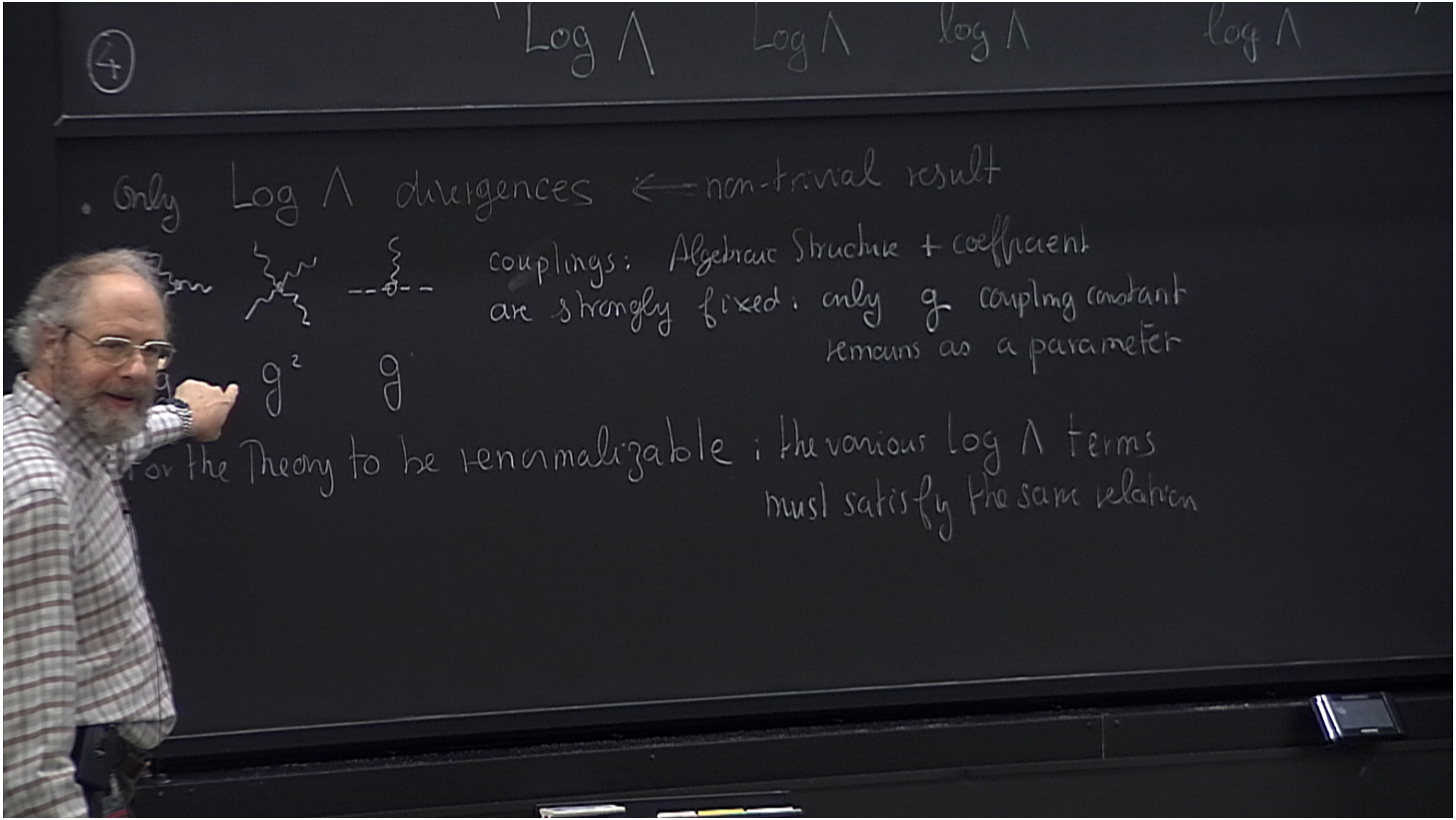
- Only Log Λ divergences ⟵ non-trivial result
-  couplings: Algebraic Structure + coefficient

④ Log Λ Log Λ log Λ log Λ

- Only Log Λ divergences ⟸ non-trivial result
-  couplings: Algebraic Structure + coefficient are strongly fixed: only $g$ coupling constant remains as a parameter

$g$ $g^2$ $g$

- For the Theory to be renormalizable

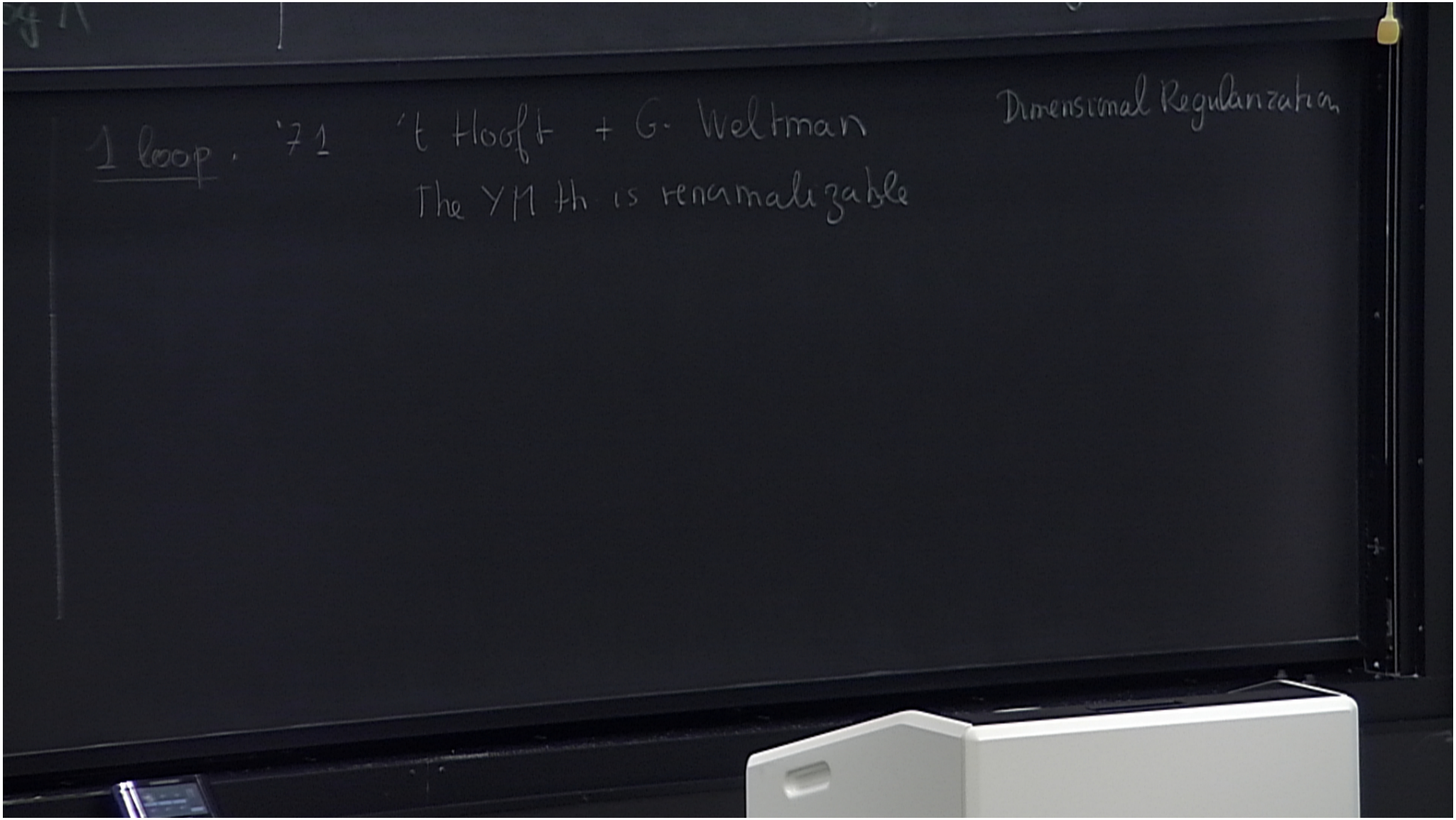

④
Log Λ Log Λ log Λ log Λ
• Only Log Λ divergences ⟸ non-trivial result
couplings: Algebraic Structure + coefficient
are strongly fixed: only g coupling constant
remains as a parameter
g² g
for the Theory to be renormalizable : the various log Λ terms
must satisfy the same relation

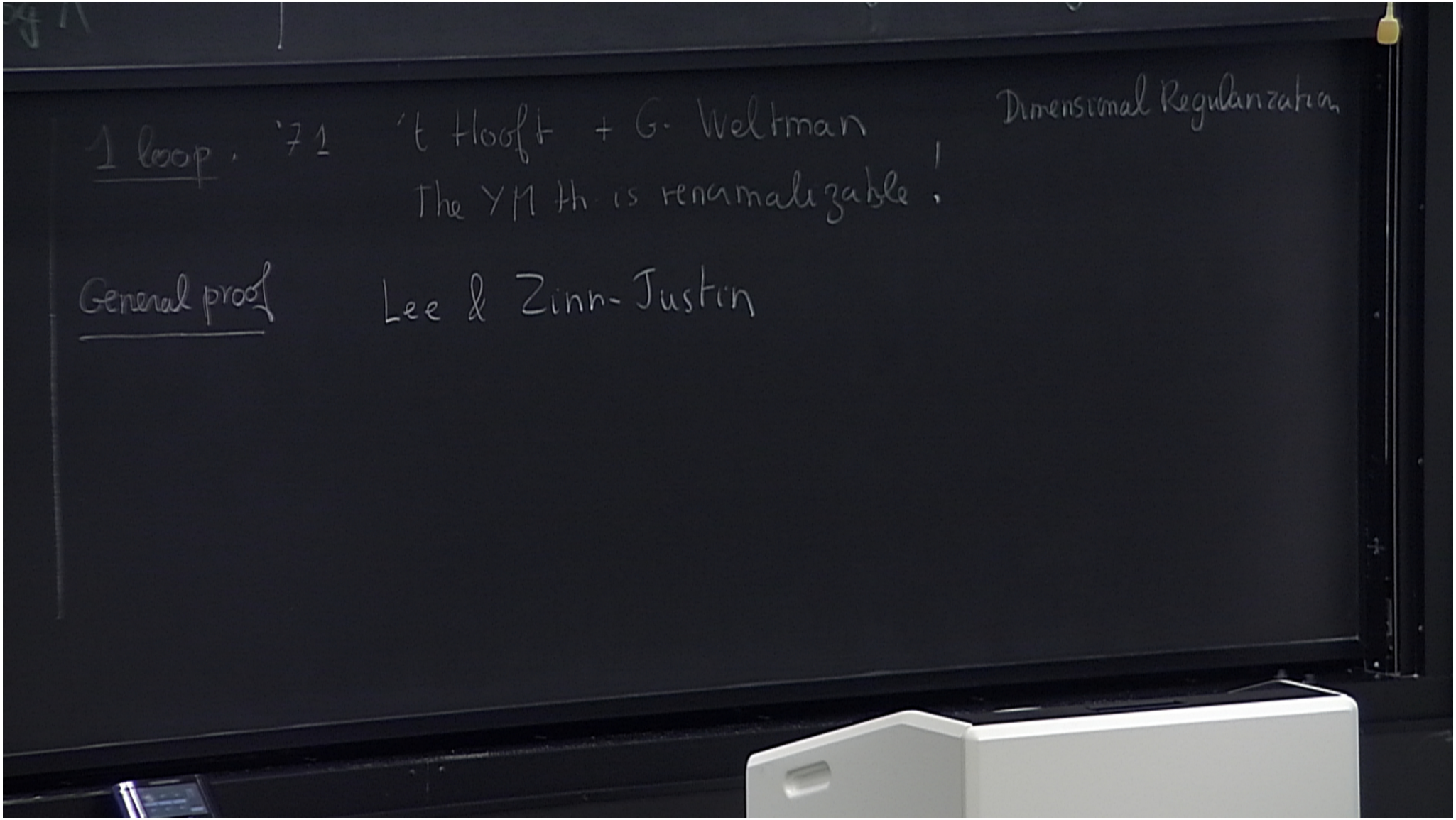
④ Log Λ Log Λ log Λ log Λ

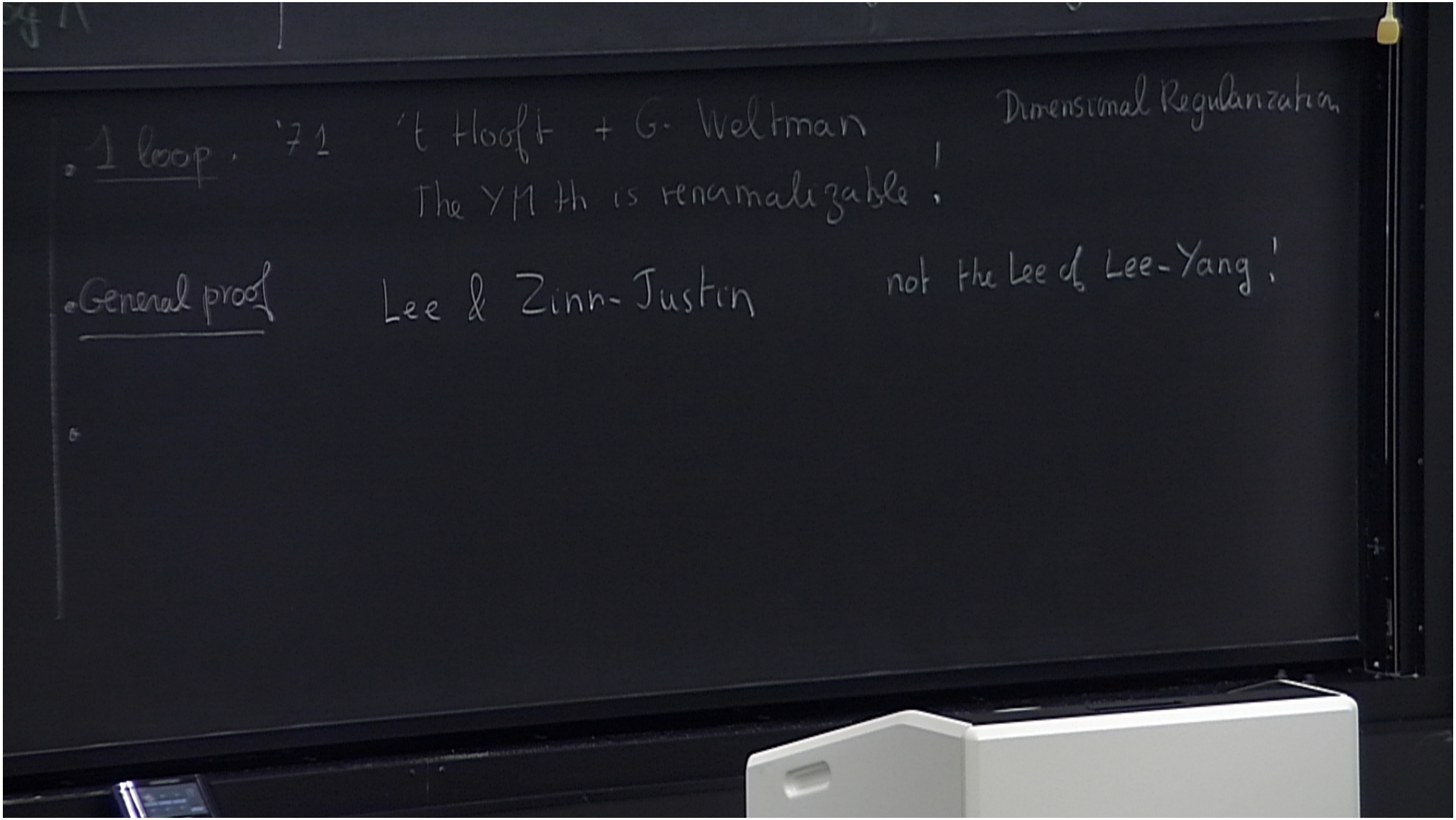
- Only Log Λ divergences ⟸ non-trivial result
-  couplings: Algebraic Structure + coefficient are strongly fixed: only $g$ coupling constant remains as a parameter

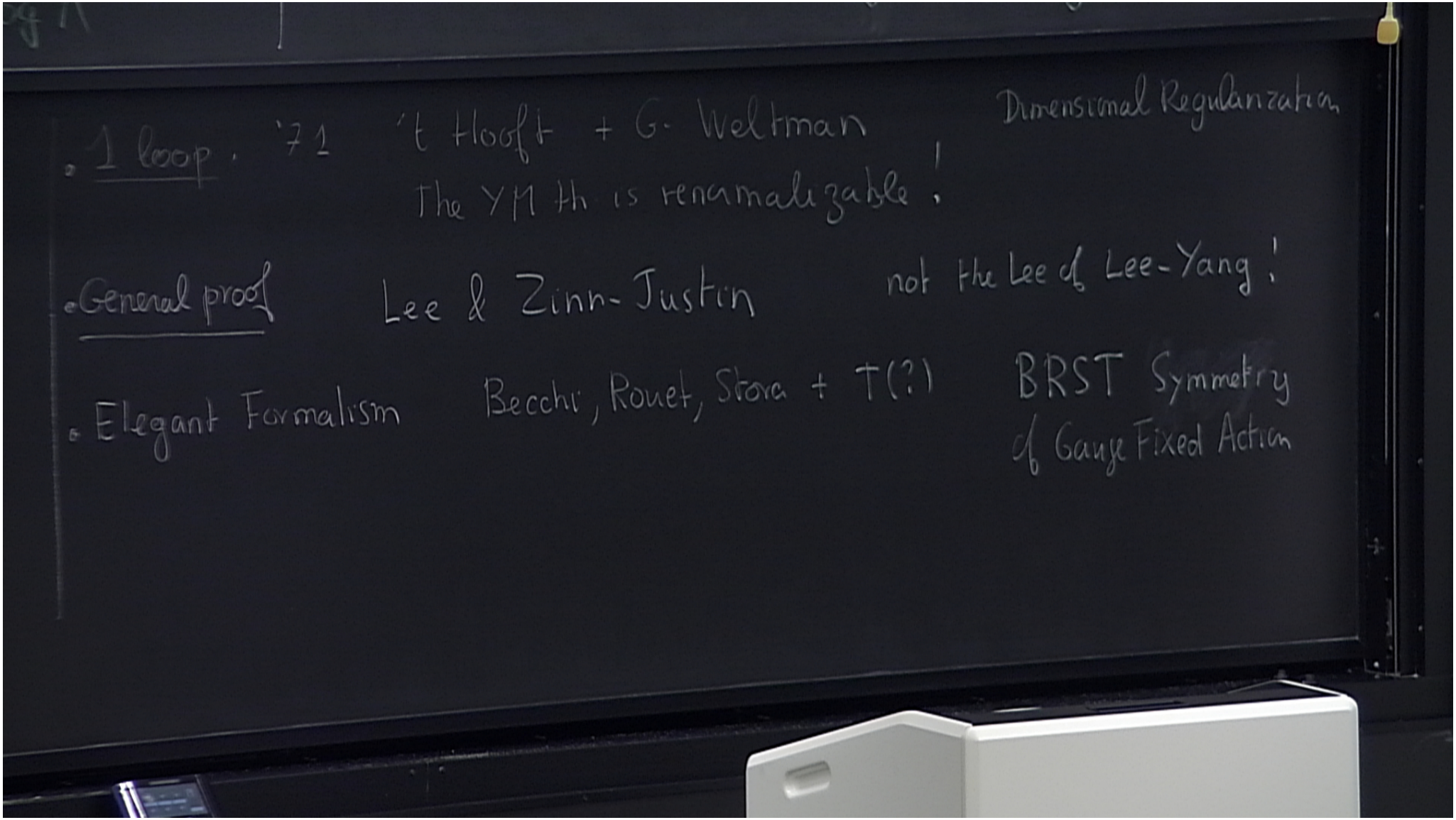
$g$ $g^2$ $g$

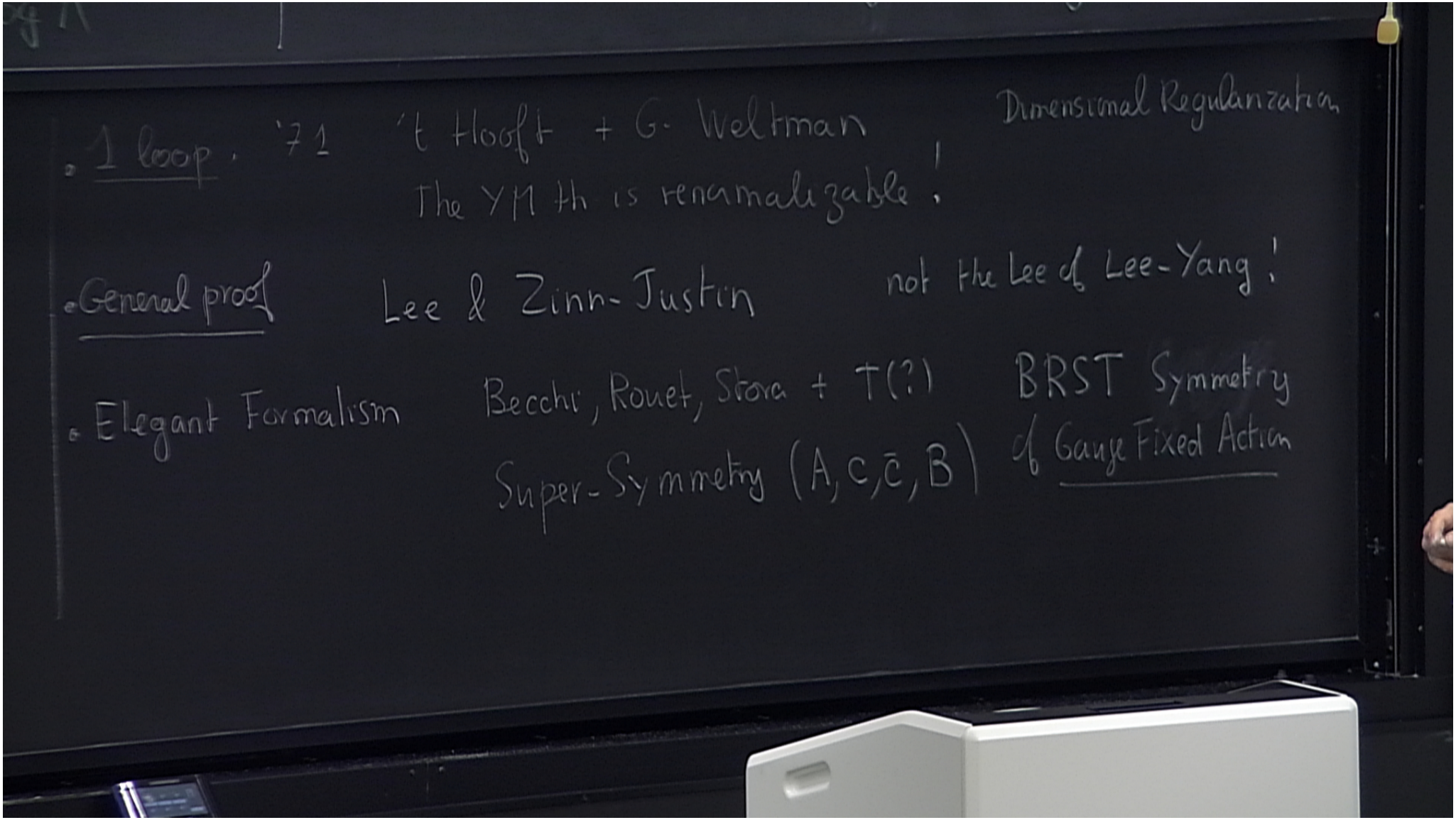
- For the Theory to be renormalizable: the various log Λ terms must satisfy the same relation
- Ward Identities are still satisfied once one introduce counterterms

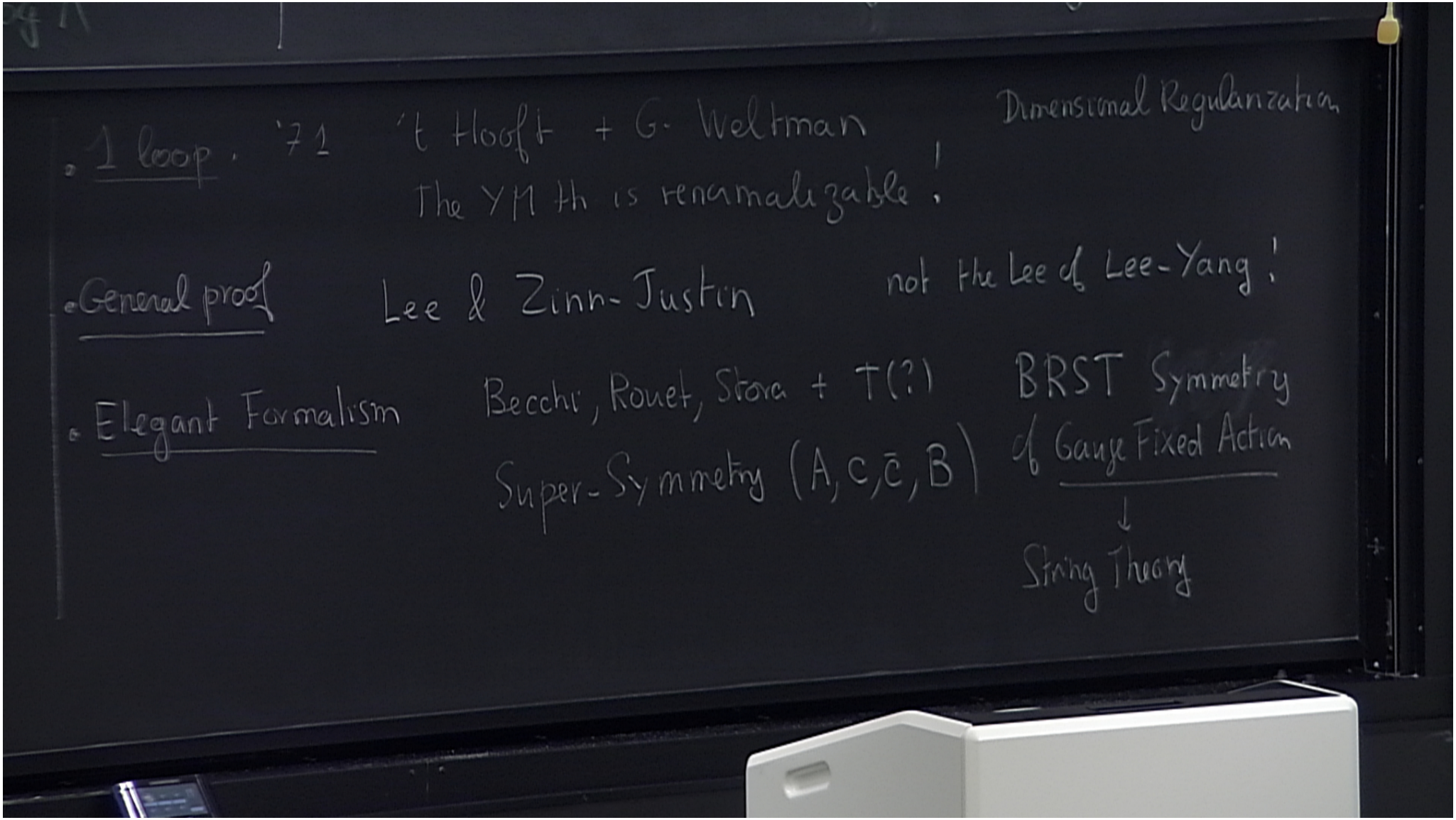

1 loop. '71 't Hooft + G. Weltman
The YM th is renormalizable
Dimensional Regularization


1 loop. '71 't Hooft + G. Weltman
Dimensional Regularization
The YM th is renormalizable!
General proof Lee & Zinn-Justin


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not the Lee of Lee-Yang!


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Dimensional Regularization
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Elegant Formalism Becchi, Rouet, Stora + T(?)
BRST Symmetry
of Gauge Fixed Action


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Dimensional Regularization
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BRST Symmetry
Super-Symmetry (A, c, c̄, B)
of Gauge Fixed Action


• 1 loop. '71 't Hooft + G. Weltman
The YM th is renormalizable!
Dimensional Regularization
• General proof Lee & Zinn-Justin
not the Lee of Lee-Yang!
• Elegant Formalism Becchi, Rouet, Stora + T(?)
BRST Symmetry
Super-Symmetry (A, c, c̄, B)
of Gauge Fixed Action
↓
String Theory

④ Log Λ Log Λ log Λ log Λ

- Only Log Λ divergences ⟸ non-trivial result
-  couplings: Algebraic Structure + coefficient are strongly fixed: only $g$ coupling constant remains as a parameter

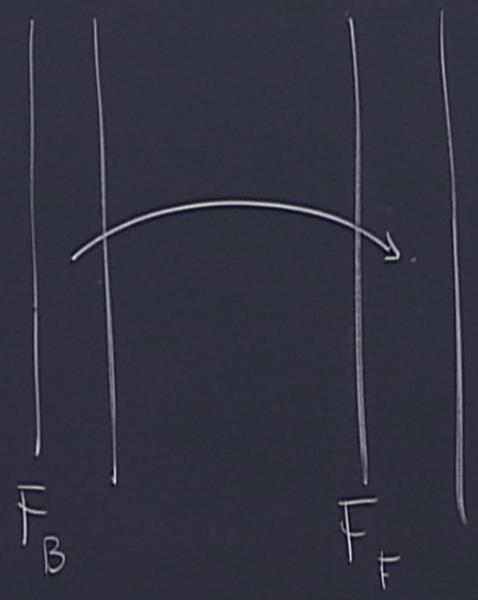
$g$ $g^2$ $g$

- For the Theory to be renormalizable: the various Log Λ terms must satisfy the same relation
- Ward Identities are still satisfied once we introduce counterterms

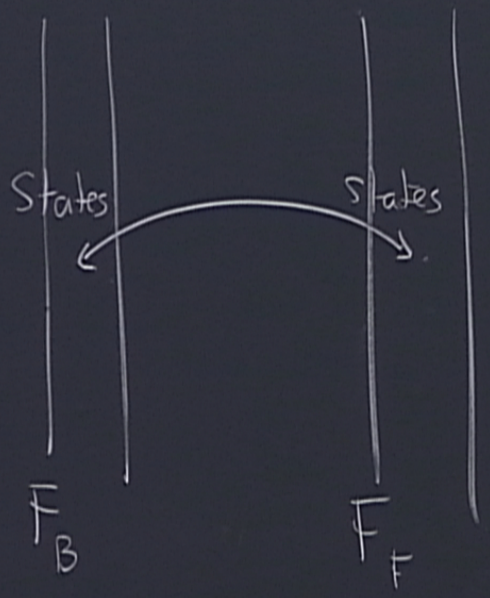
- 1 loop
- General
- Elegan

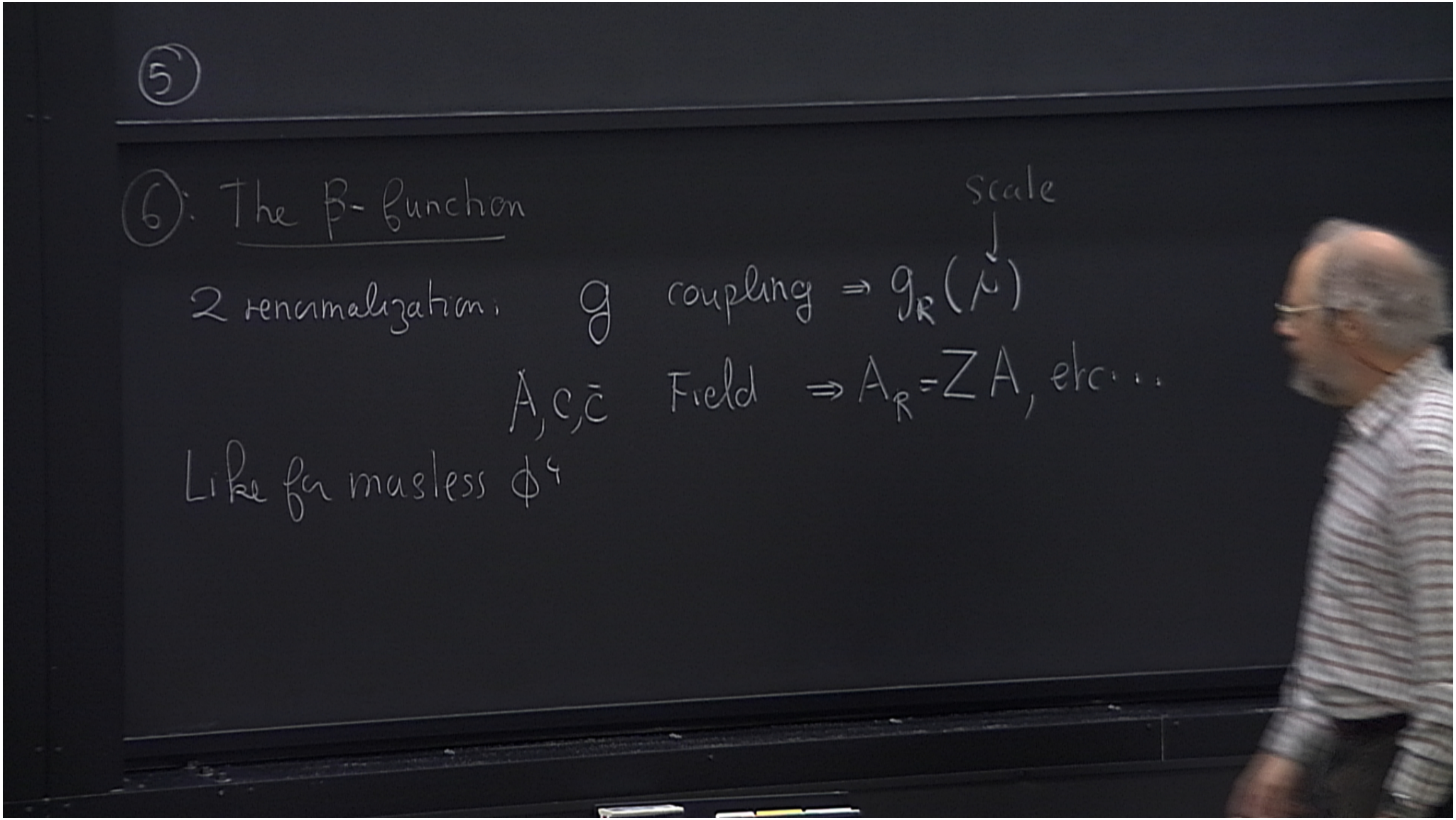
⑤

$\mathcal{H} = \mathcal{H}_{\text{Bosons}} \otimes \mathcal{H}_{\text{Fermions}}$



$\mathcal{H} = \mathcal{H}_{\text{Bosons}} \otimes \mathcal{H}_{\text{Fermions}}$




5
6: The β-function
scale
2 renormalization: g coupling ⇒ $g_R(\mu)$
A, c, $\bar{c}$ Field ⇒ $A_R = ZA$, etc...
Like for masless $\phi^4$

(5)

(6): The β-function

2 renormalization: g coupling $\Rightarrow g_R(\mu)$ (scale $\mu$)

at 1-loop $A, c, \bar{c}$ Field $\Rightarrow A_R = ZA$, etc...

Like for massless $\phi^4$ g 1 loop

$\phi$ only at 2 loop

(5)

(6): The β-function

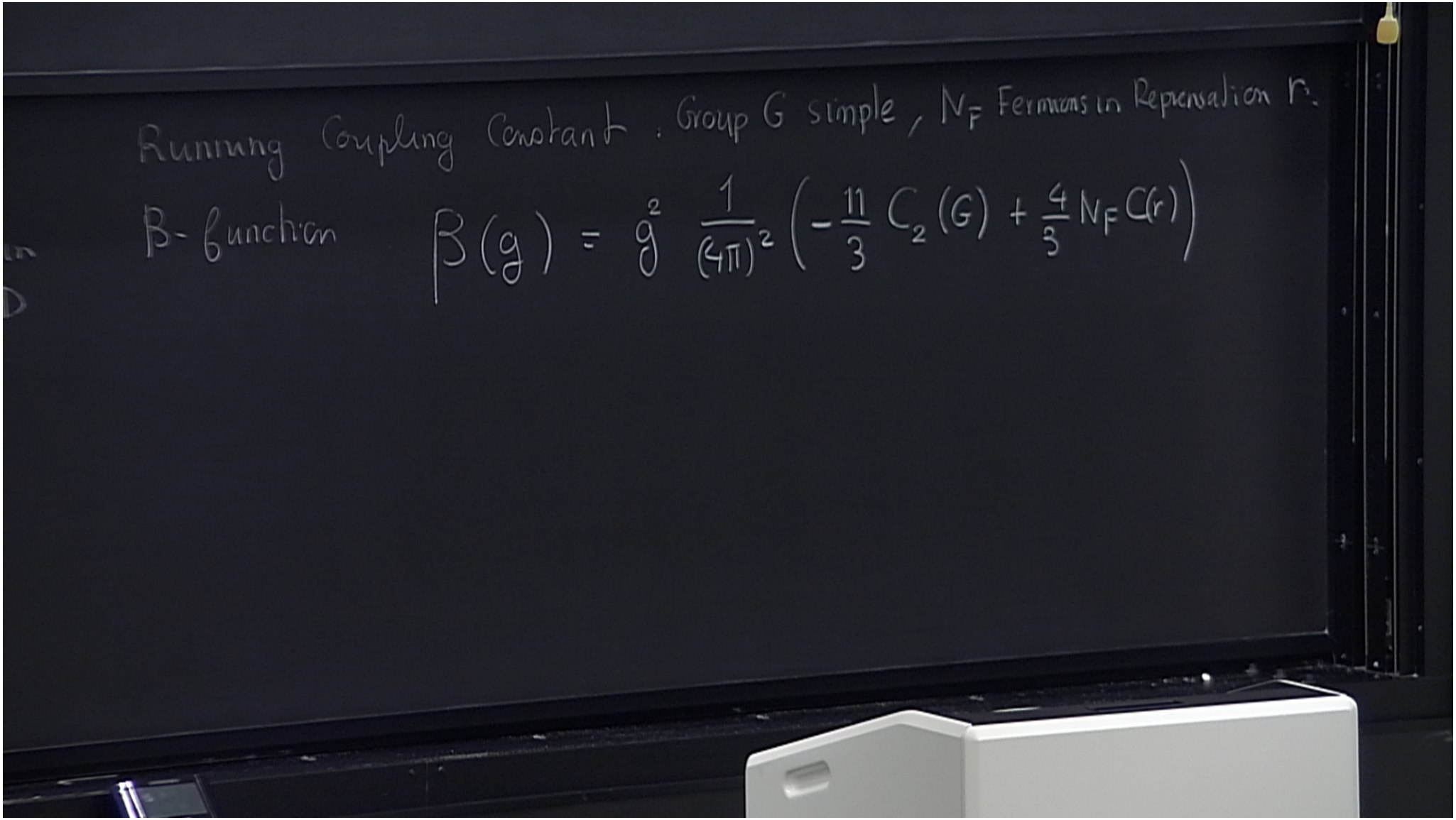
2 renormalization: g coupling ⇒ $g_R(\mu)$ (scale $\mu$) — Like in QED

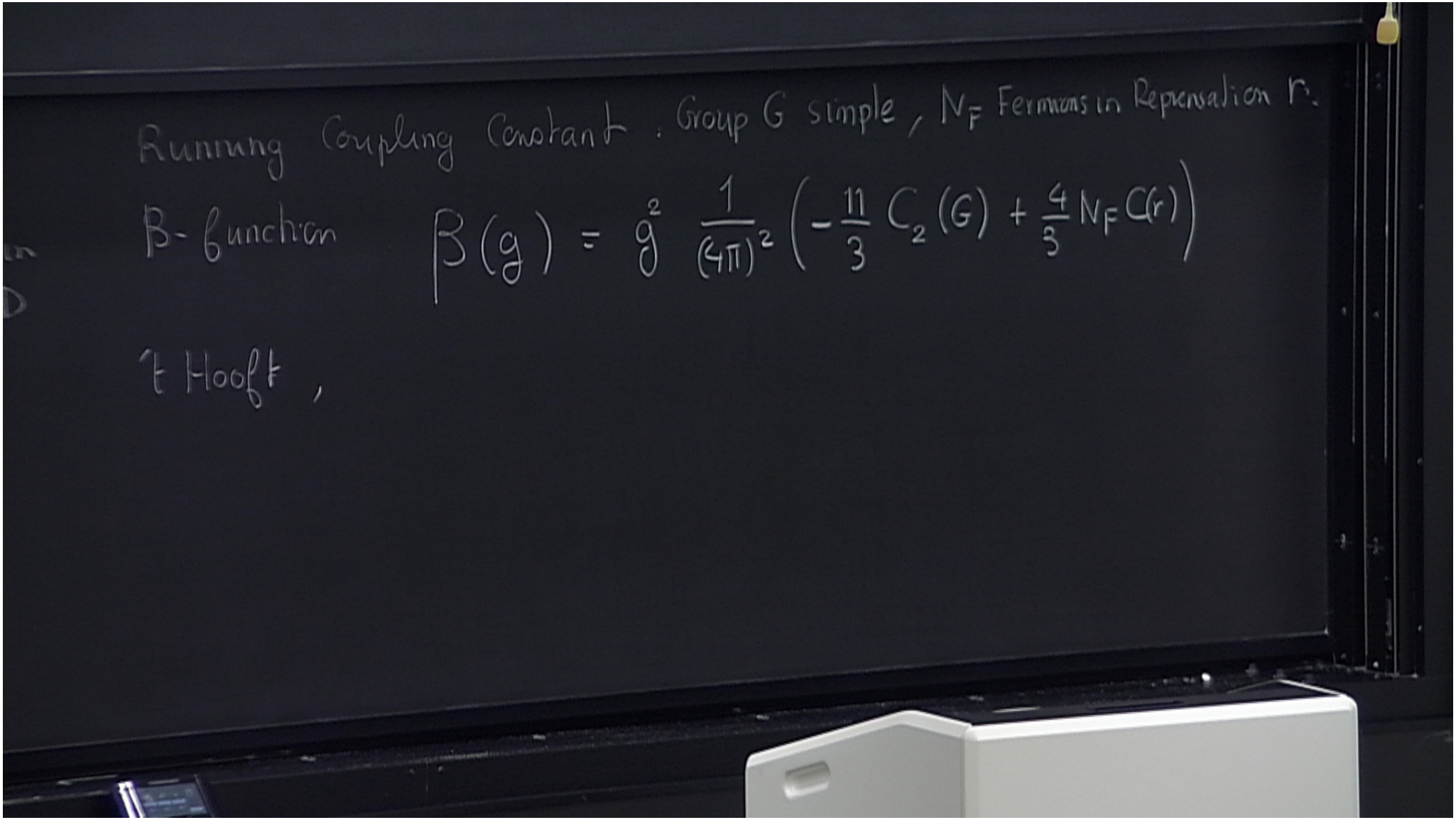
1-loop

$A, c, \bar{c}$ Field ⇒ $A_R = ZA$, etc...

Like for massless $\phi^4$: g 1 loop

$\phi$ only at 2 loop


Running Coupling Constant : Group G simple, $N_F$ Fermions in Representation r.
β-function $\beta(g) = g^2 \frac{1}{(4\pi)^2}\left(-\frac{11}{3}C_2(G) + \frac{4}{3}N_F C(r)\right)$

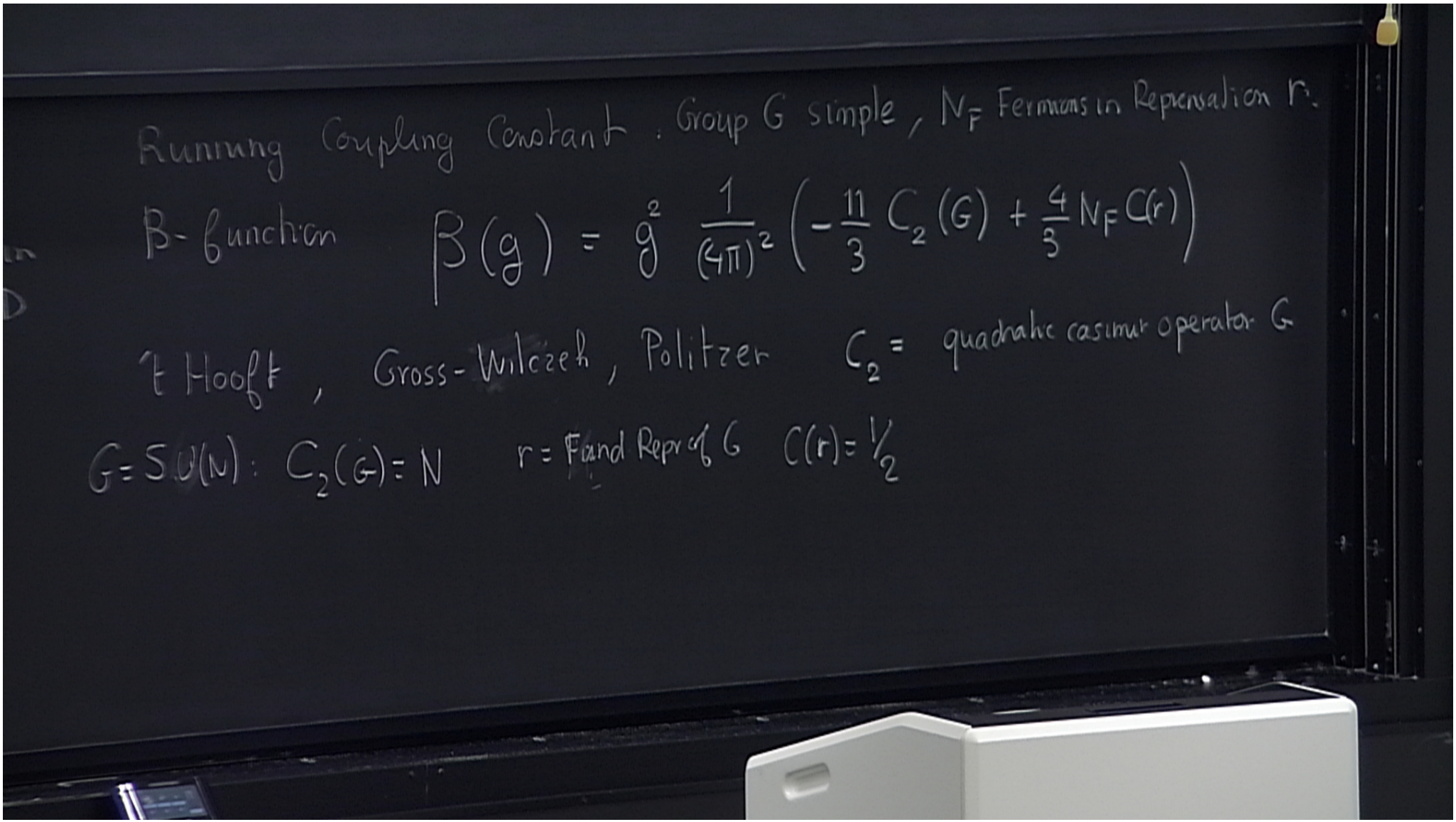

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't Hooft,

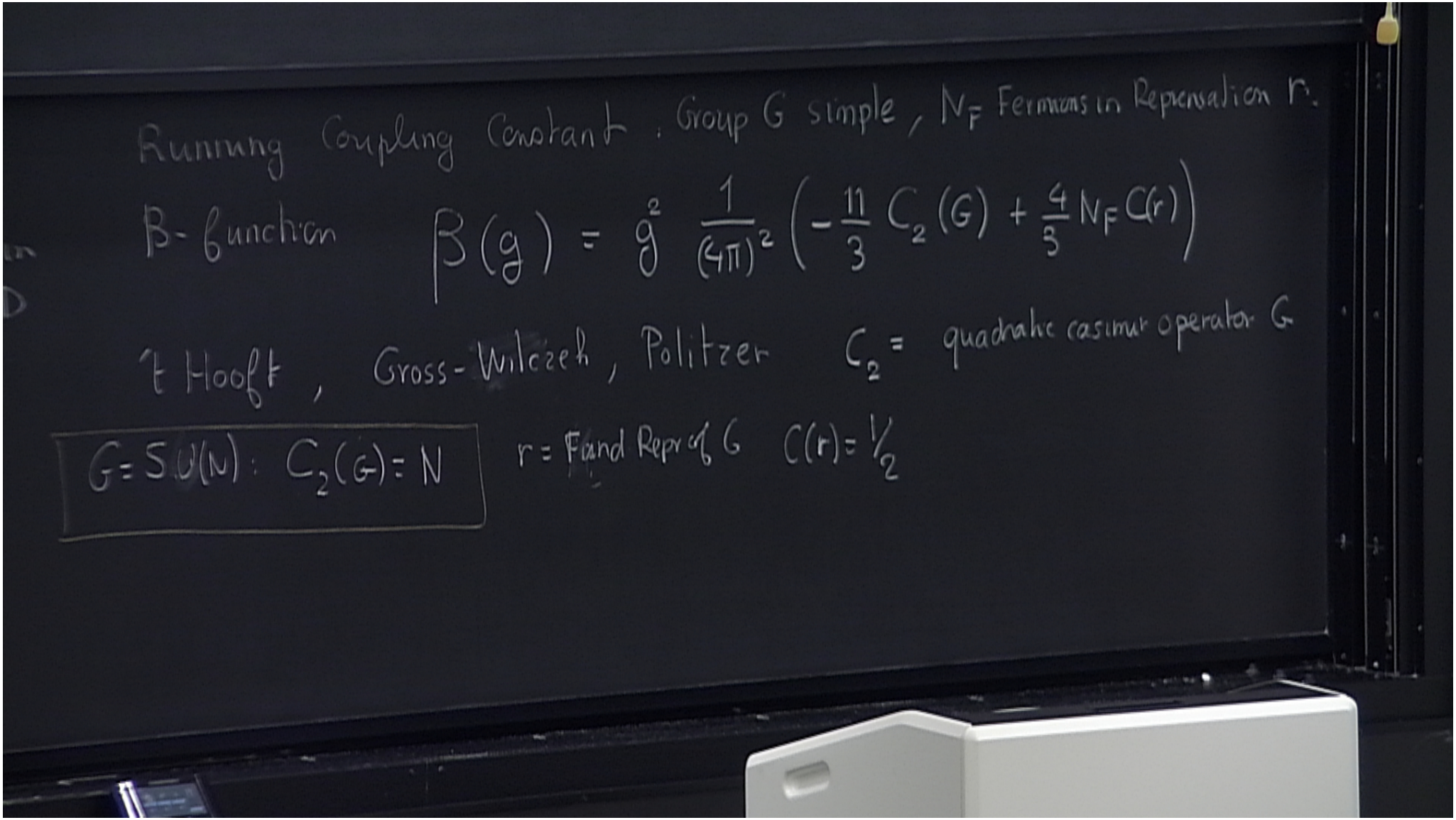
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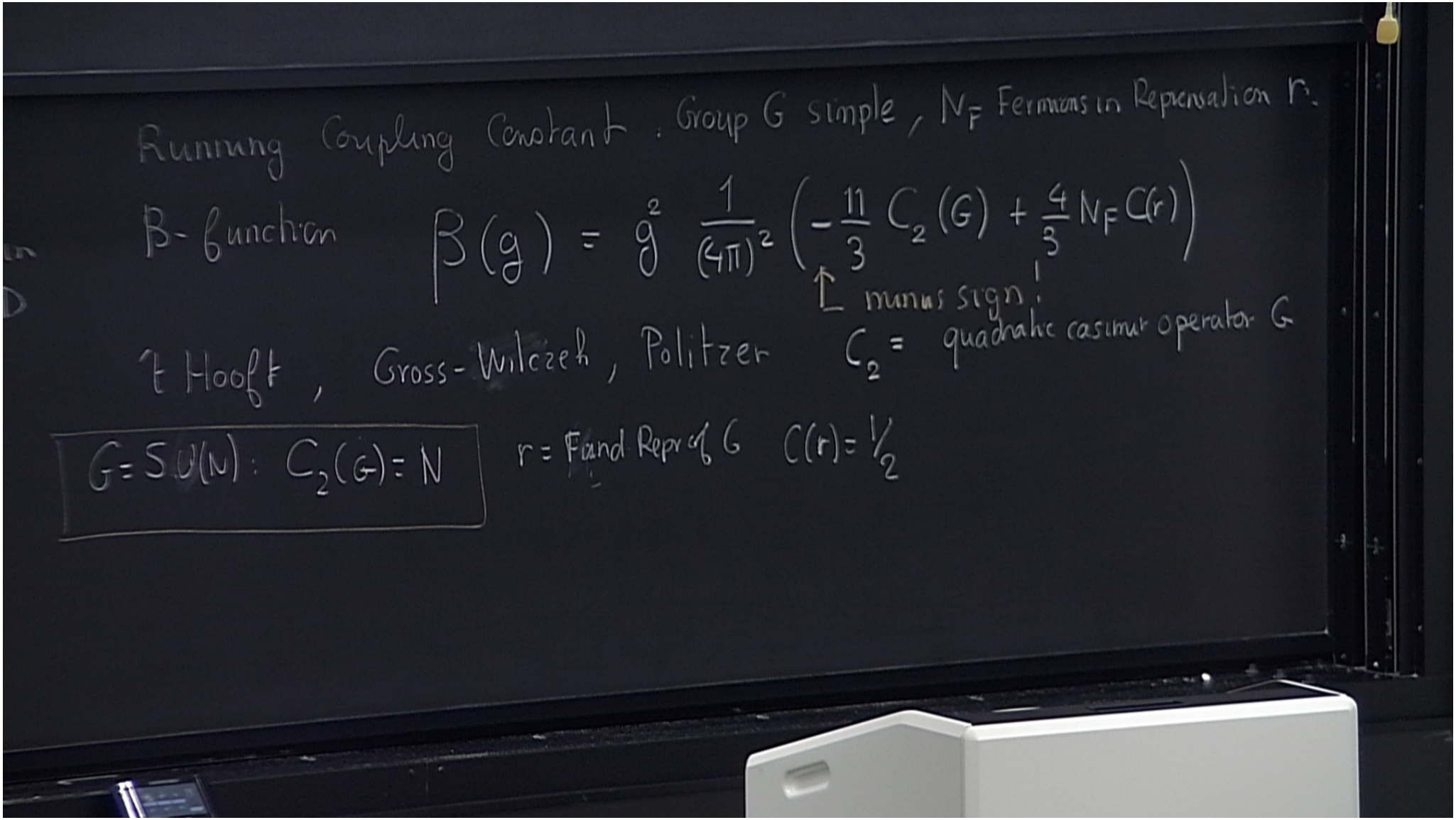
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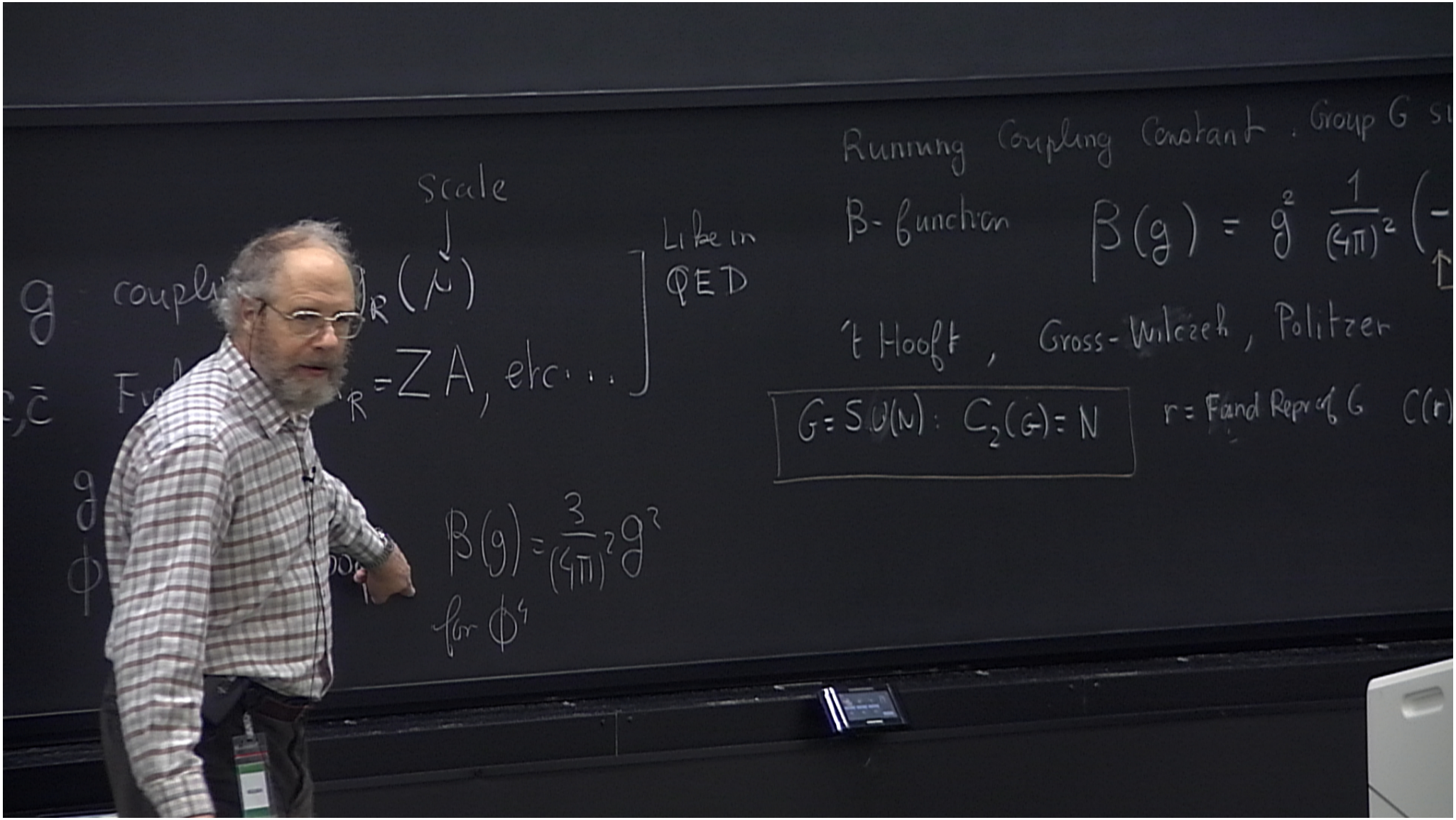
't Hooft, Gross-Wilczek, Politzer $C_2$ = quadratic casimir operator G

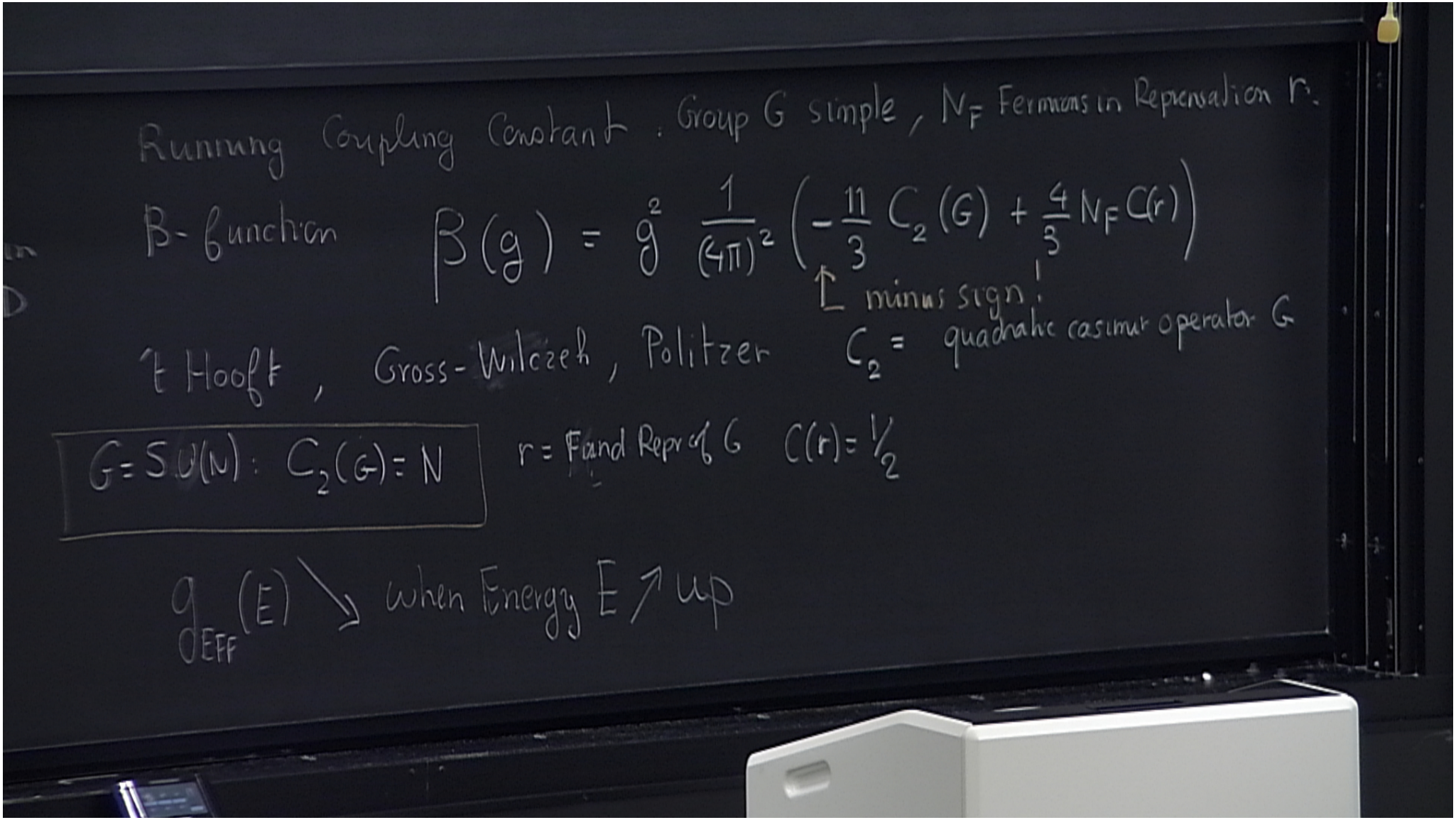
$G = SU(N)$: $C_2(G) = N$ r = Fund Repr of G $C(r) = 1/2$

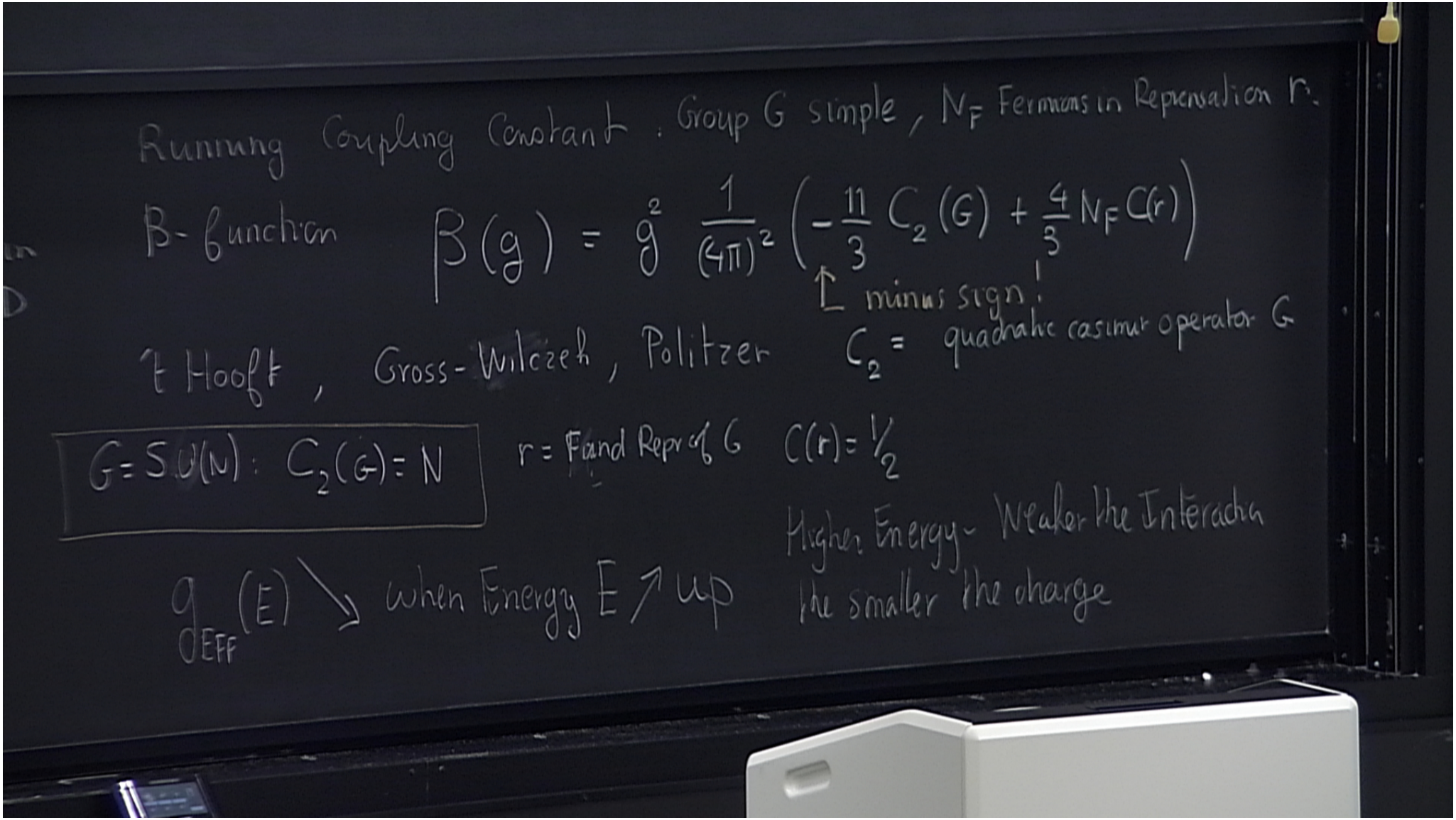


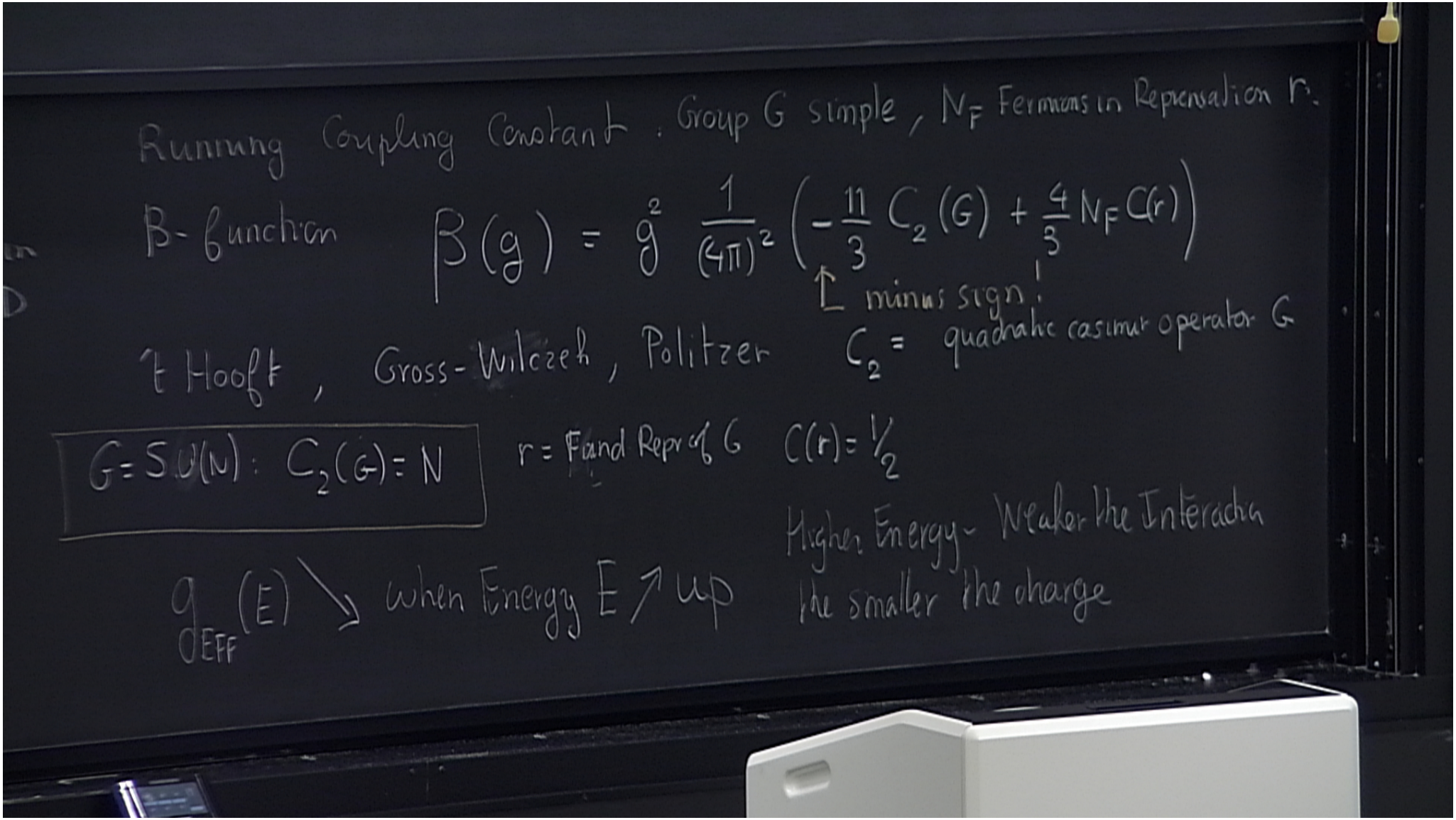

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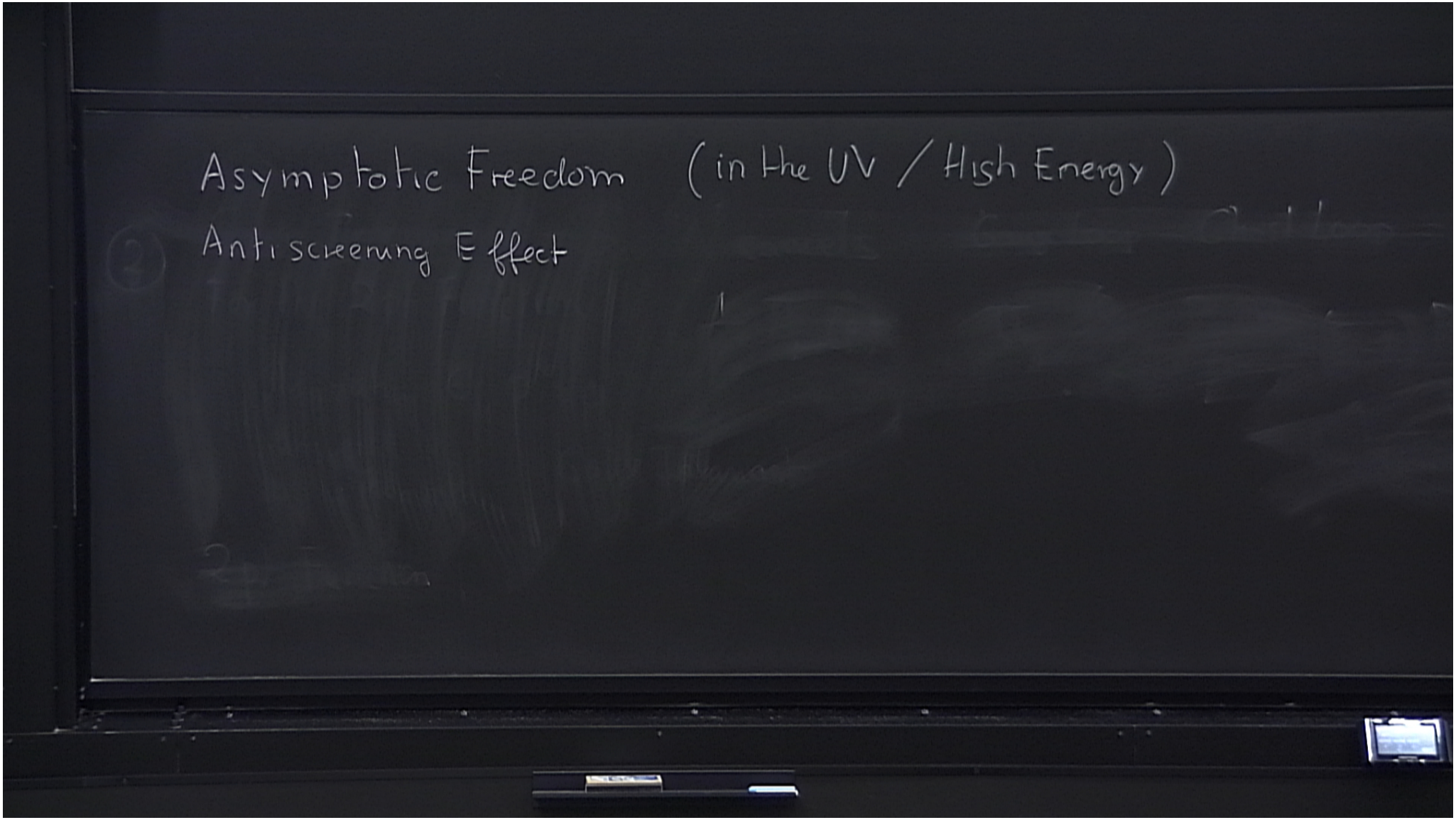

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minus sign!
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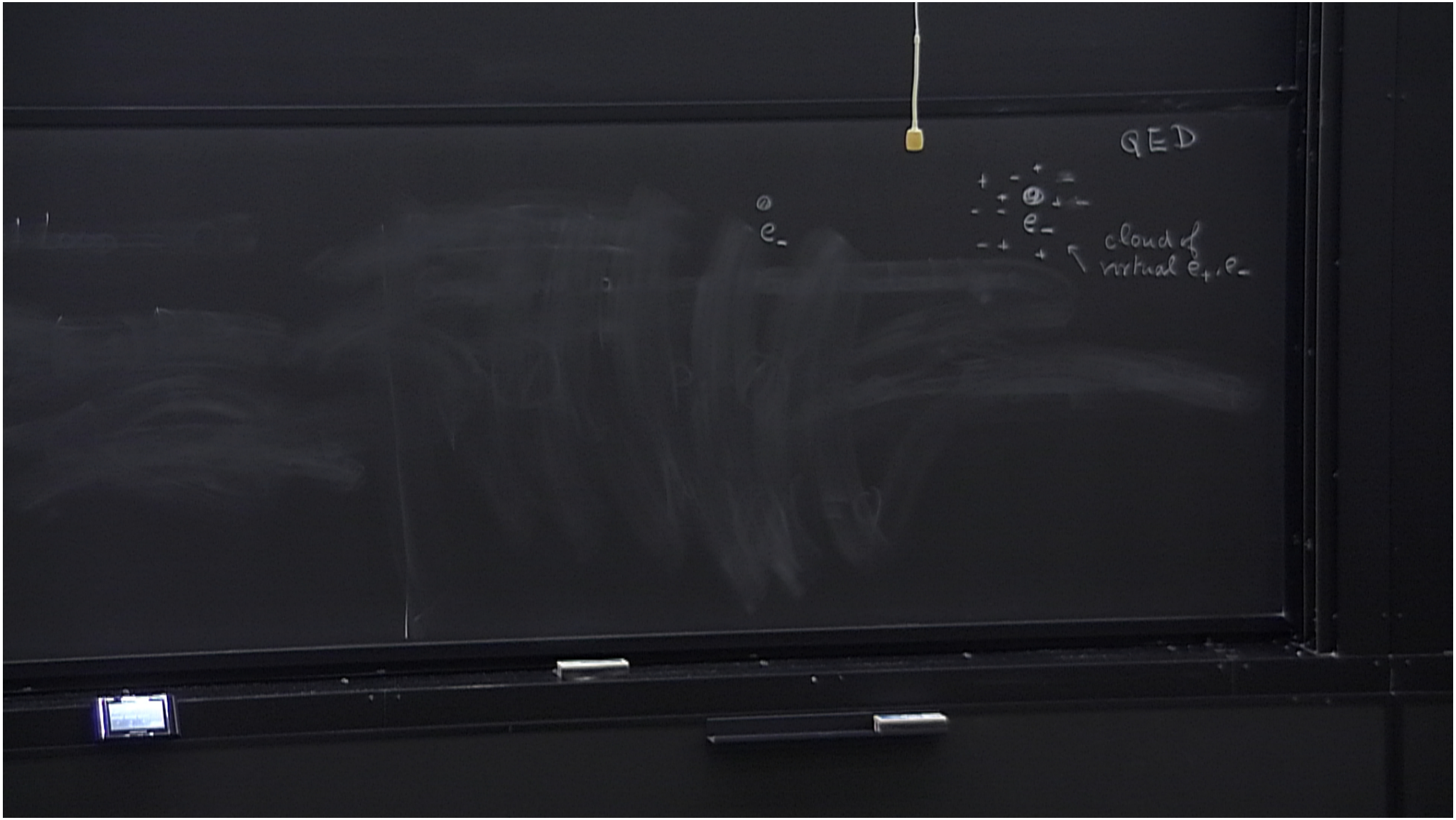

Scale
Like in QED
Running Coupling Constant . Group G
β-function
't Hooft , Gross-Wilczek , Politzer
G = SU(N) : C₂(G) = N
r = Fund Repr of G
β(g) = 3/(4π)² g²
for φ⁴

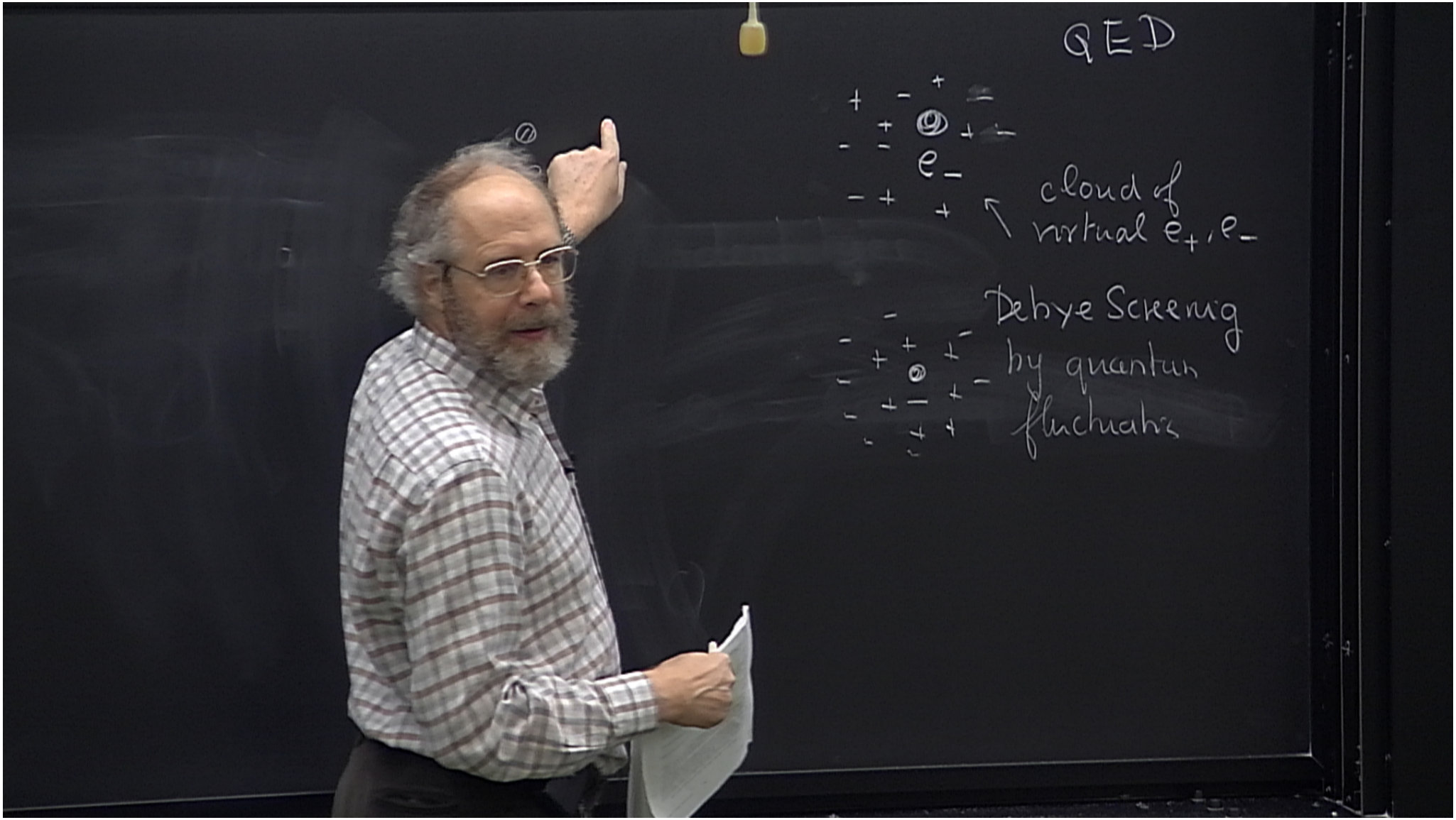

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$g_{EFF}(E)$ ↘ when Energy E ↗ up

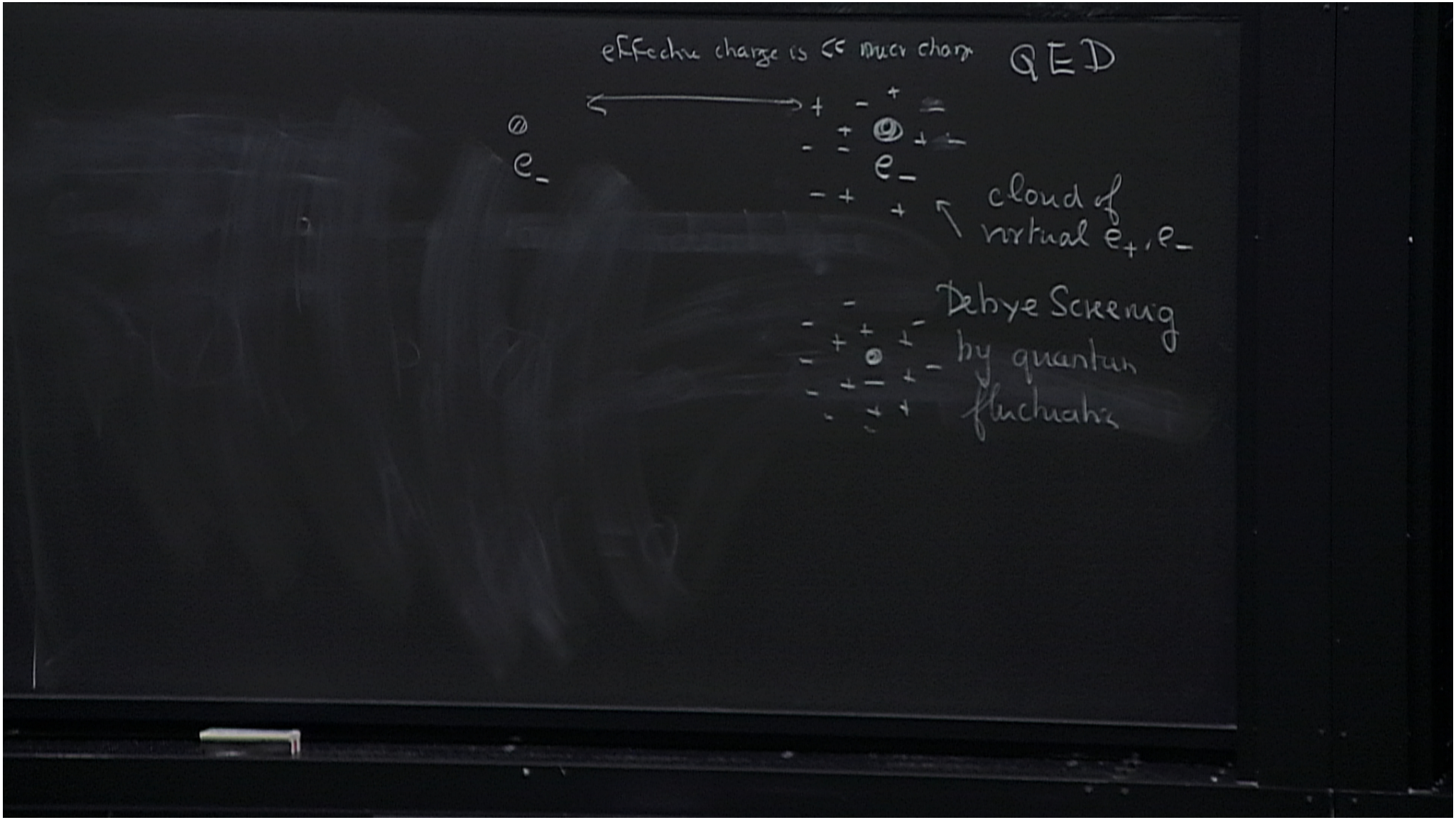

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Higher Energy - Weaker the Interaction
the smaller the charge

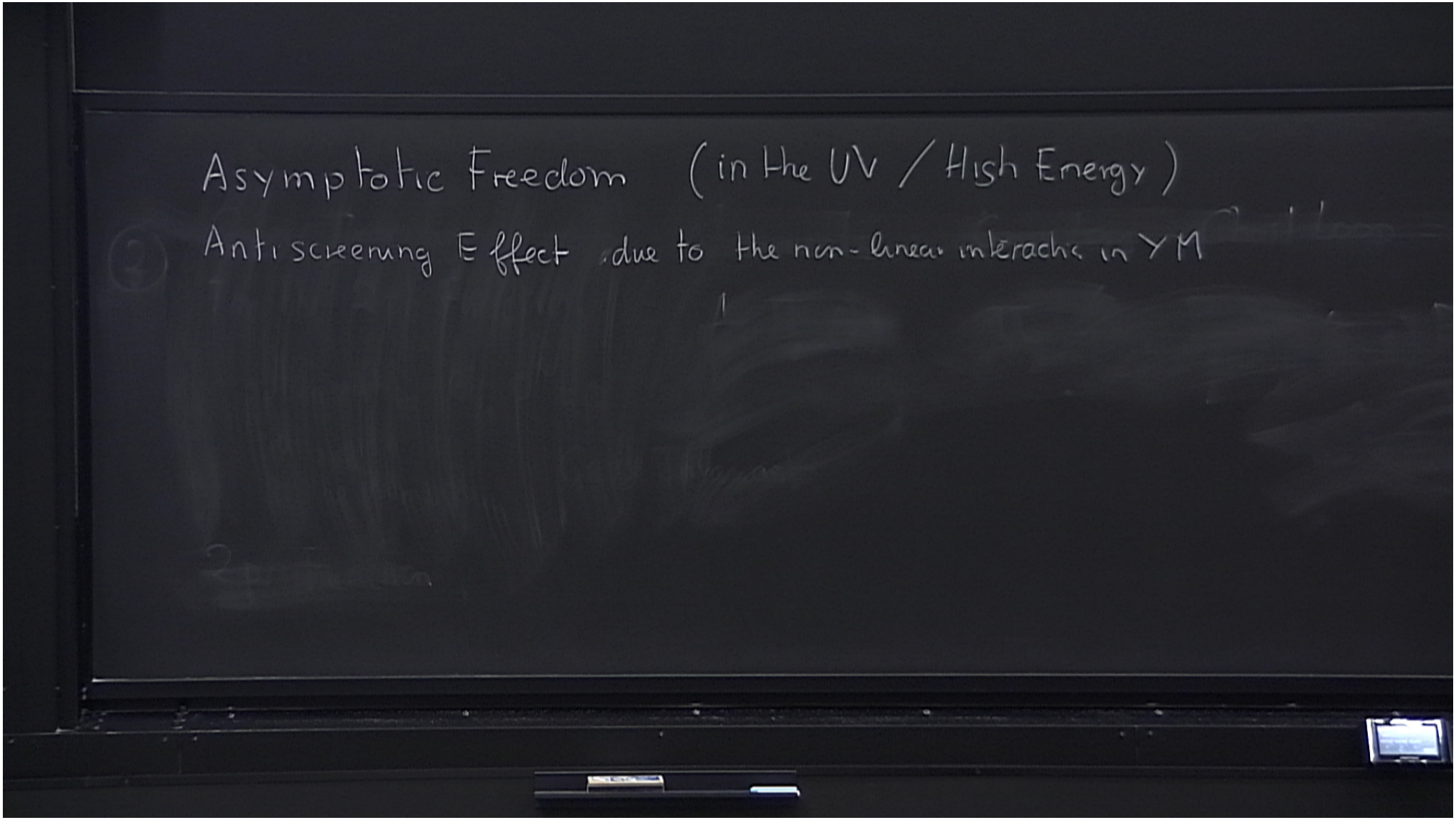

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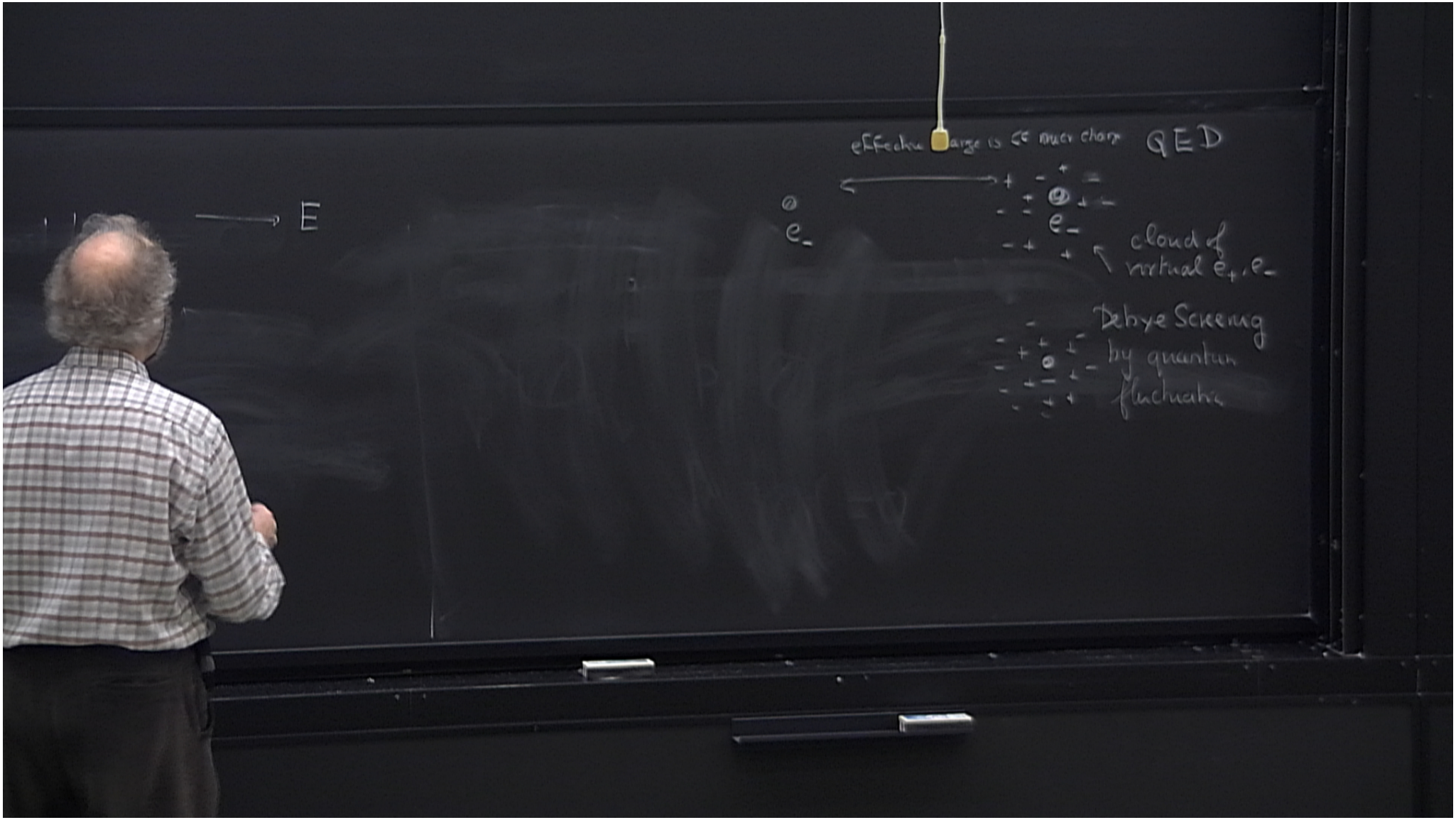

Asymptotic Freedom (in the UV / High Energy)
Antiscreening Effect

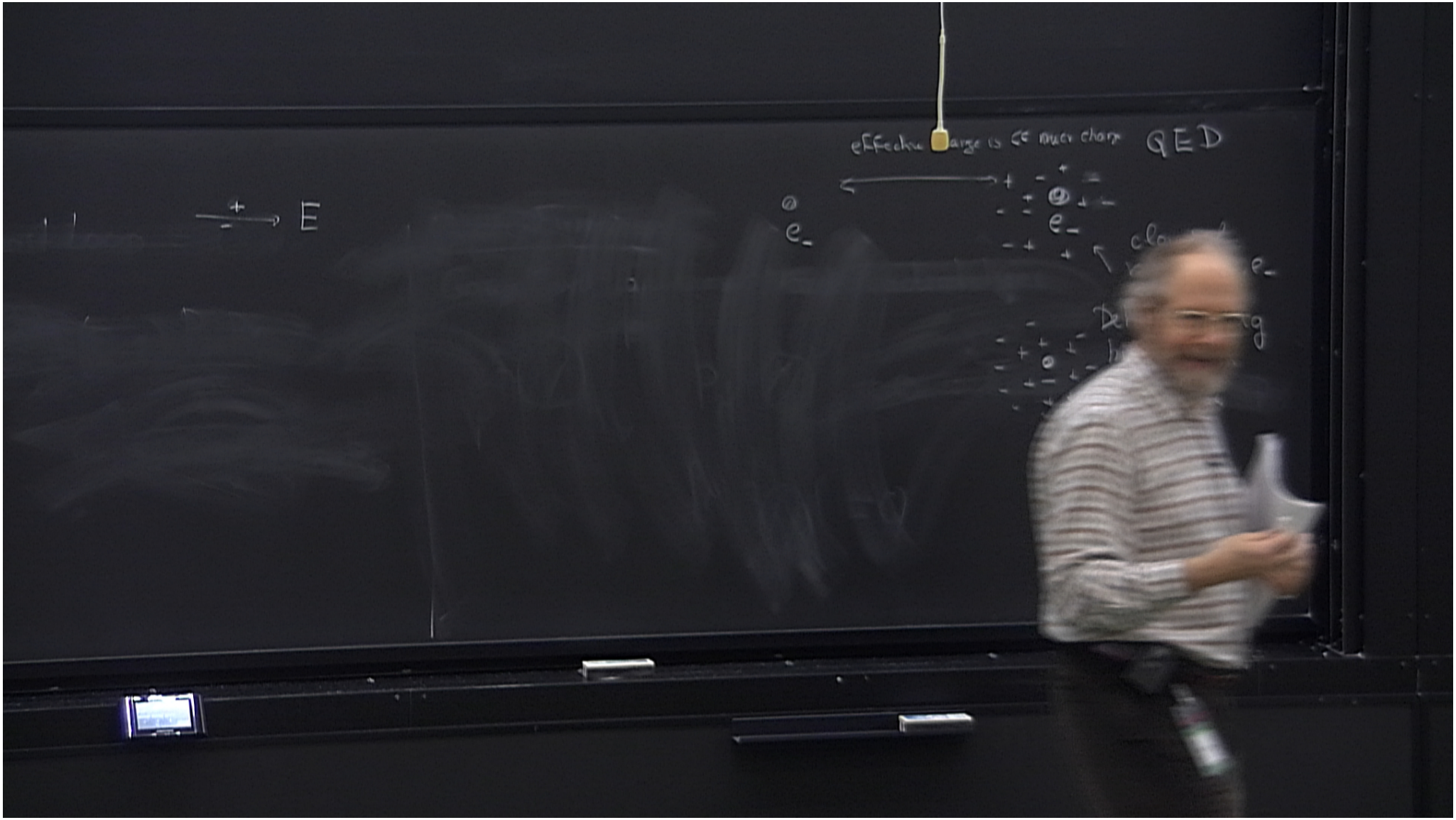

QED
e_
e_
cloud of
virtual e+, e_

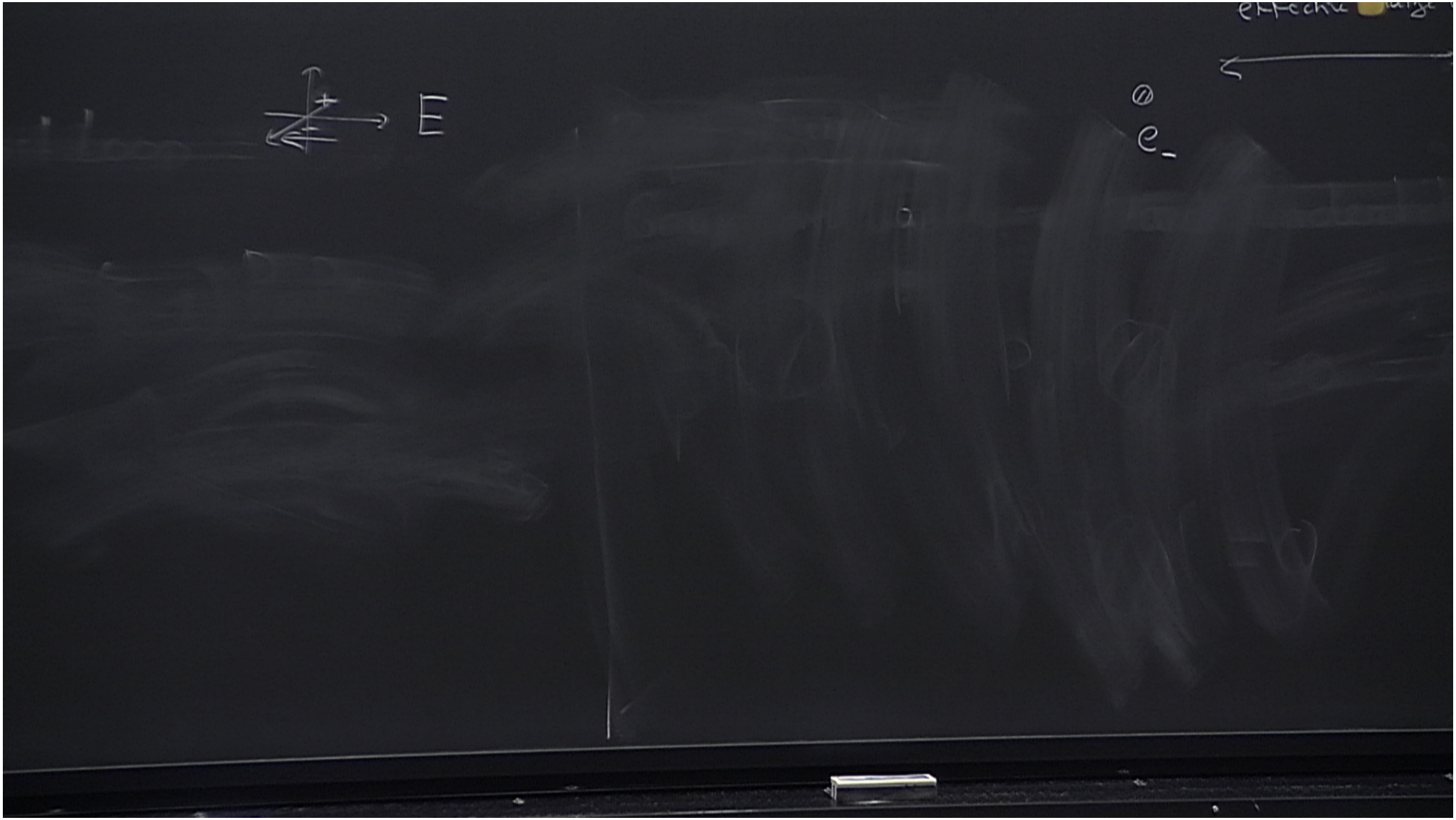

QED
e−
cloud of
virtual e+, e−
Debye Screenig
by quantum
fluctuations

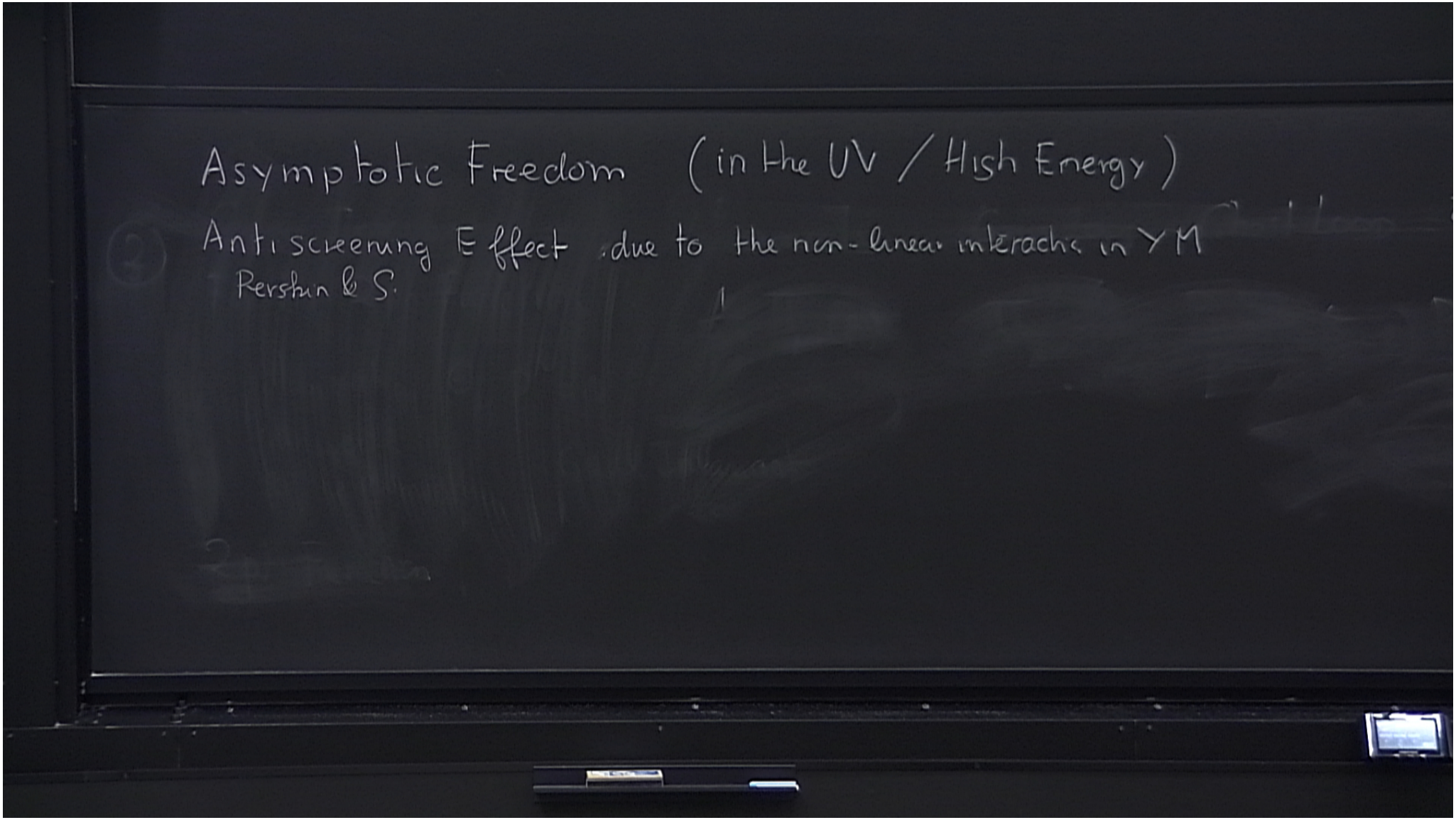

effective charge is << bare charge
QED
e_
e_
cloud of virtual e+, e_
Debye Screenig by quantum fluctuatio


Asymptotic Freedom (in the UV / High Energy)
Antiscreening Effect due to the non-linear interactions in YM


E
e_
effective charge is ee much charge
QED
cloud of
virtual e+,e-
Debye Screening
by quantum
fluctuations


E
e-
QED


E
e_

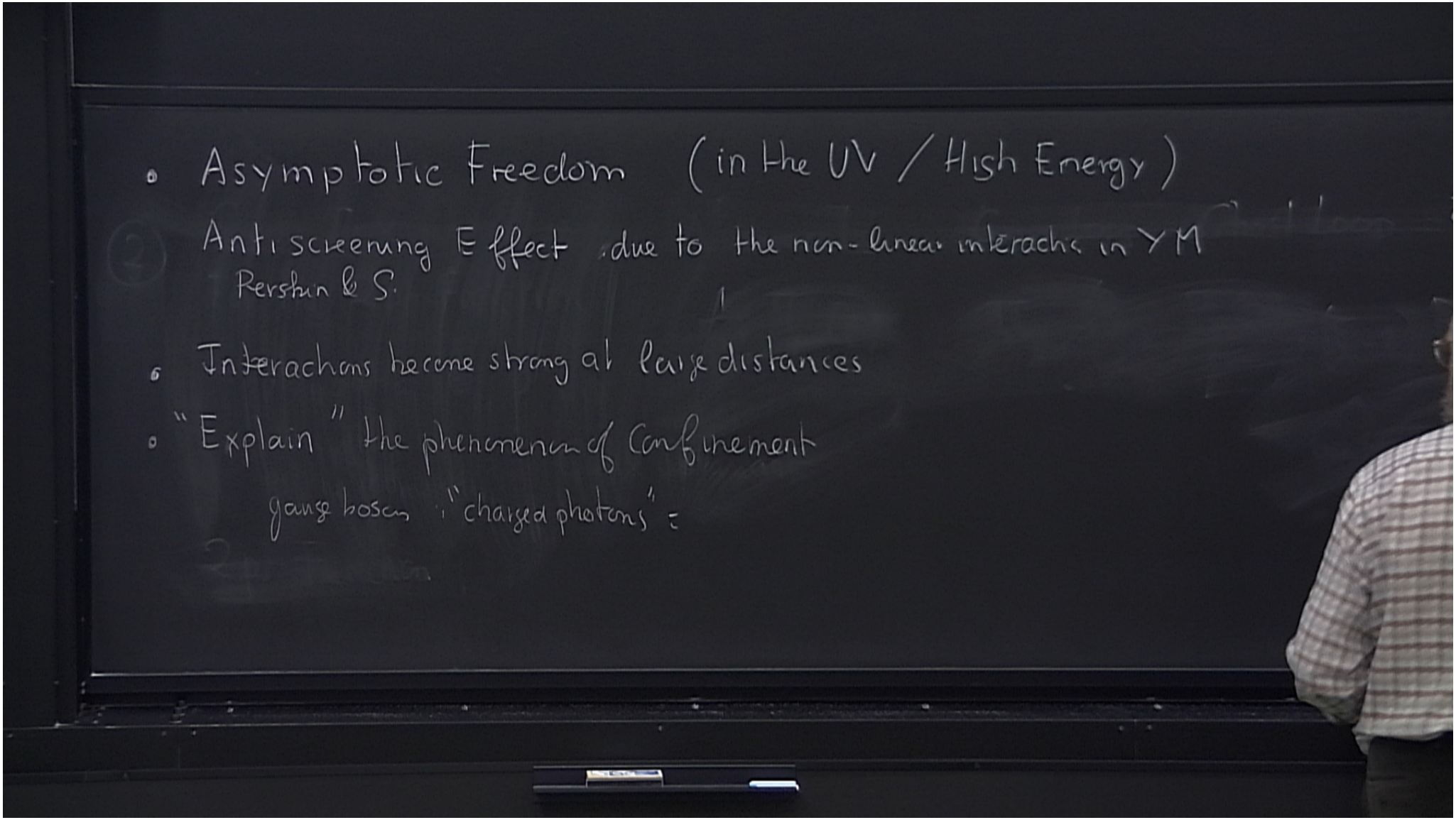

Asymptotic Freedom (in the UV / High Energy)
Anti screening Effect : due to the non-linear interactions in YM
Perturbation S.

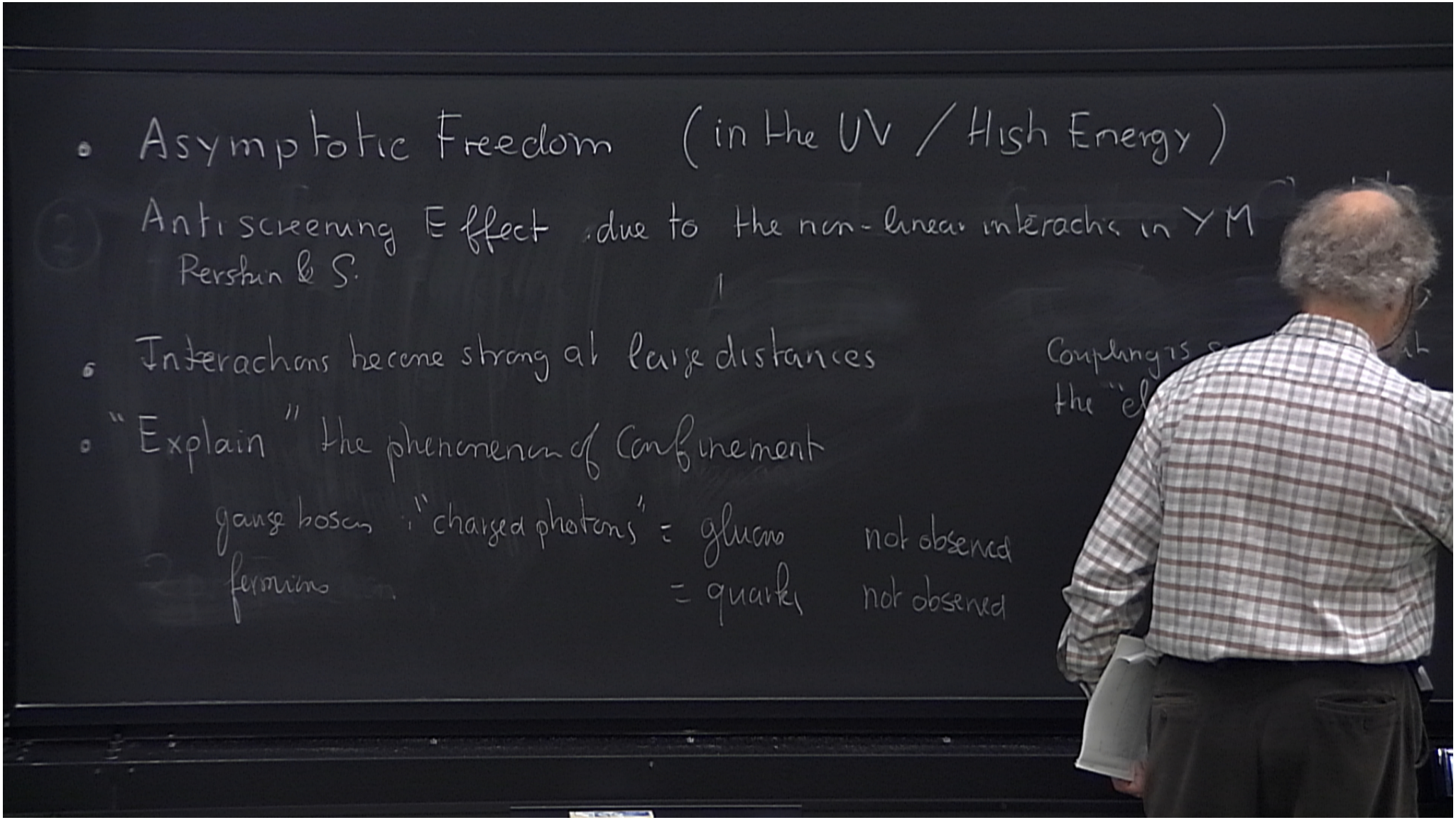
- Asymptotic Freedom (in the UV / High Energy)

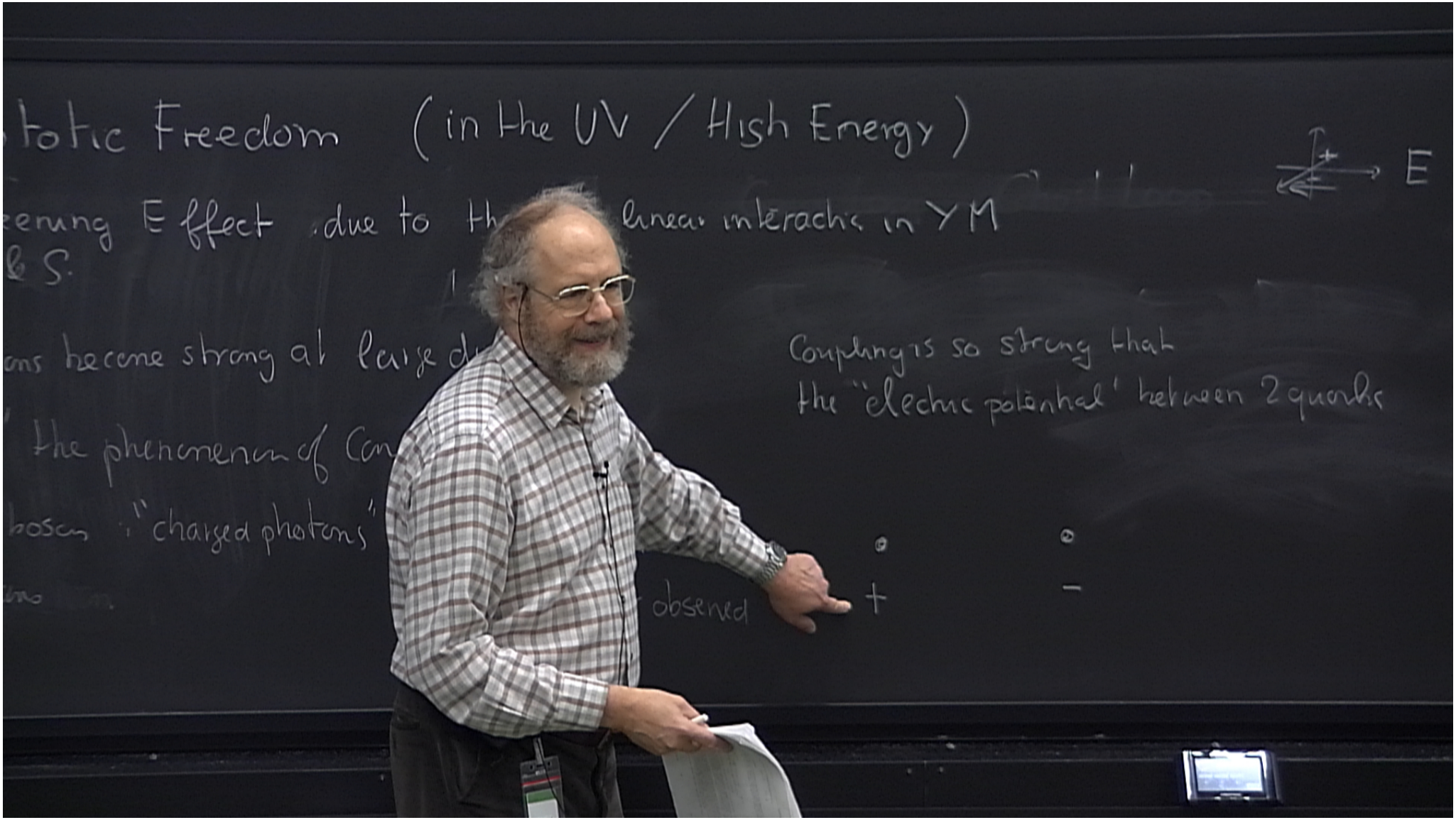
  Anti screening Effect due to the non-linear interactions in YM
  Perturbation & S.

- Interactions become strong at large distances

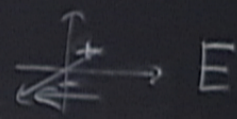
- Asymptotic Freedom (in the UV / High Energy)
  Anti screening Effect due to the non-linear interactions in YM
  Perturbation Theory
- Interactions become strong at large distances
- "Explain" the phenomenon of Confinement


• Asymptotic Freedom (in the UV / High Energy)
Anti screening Effect : due to the non-linear interactions in YM
Perturbation S.
• Interactions become strong at large distances
• "Explain" the phenomenon of Confinement
gauge bosons : "charged photons" :


• Asymptotic Freedom (in the UV / High Energy)
Anti screening Effect : due to the non-linear interactions in YM
Perstun & S.
• Interactions become strong at large distances
• "Explain" the phenomenon of Confinement
gauge boson : "charged photons" = gluons not observed
fermions = quarks not observed


totic Freedom (in the UV / High Energy)
E
Coupling is so strong that
the "electric potential" between 2 quarks
+
−

totic Freedom (in the UV / High Energy)



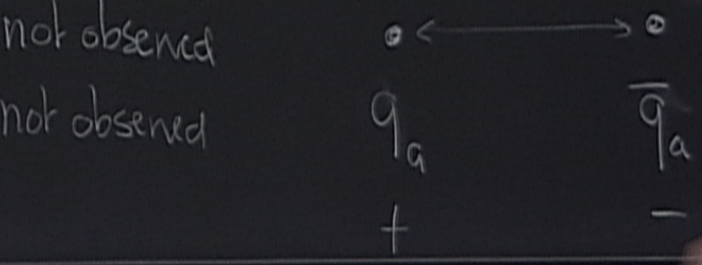

eening Effect due to the non-linear interactions in YM
& S.

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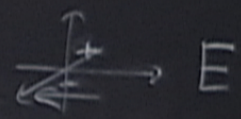
Coupling is so strong that
the "electric potential" between

the phenomenon of Confinement

bosons "charged photons" = gluons not observed
= quarks not observed



totic Freedom (in the UV / High Energy)



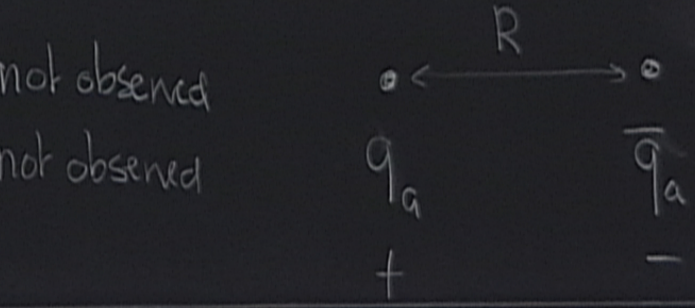

eening E flat due to the non-linear interactions in YM
& S.

ons become at large distances

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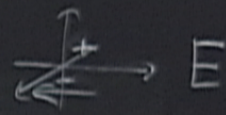
the ... of Confinement

...gluons" = gluons not observed
= quarks not observed



(in the UV / High Energy)

to the non-linear interactions in YM



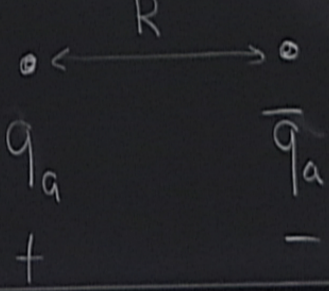

distances

Coupling is so strong that
the "electric potential" between 2 quarks

confinement

gluons not observed

quarks not observed

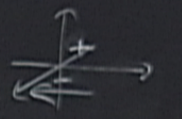



$V(R) \sim \frac{1}{R}$ at small distance

$\sim R$ at large distance

UV / High Energy)

n-linear interactions in YM

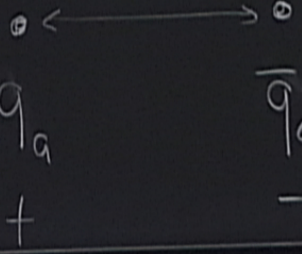



es

Coupling is so strong that
the "electric potential" between 2 quarks

not observed

not observed

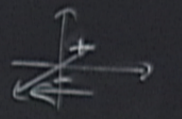



$V(R) \simeq \frac{1}{R}$ at short distance

$\sim R$ at large distance like a Spring

UV / High Energy )

n-linear interactions in YM

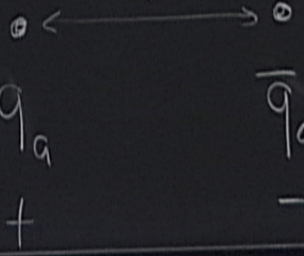



es

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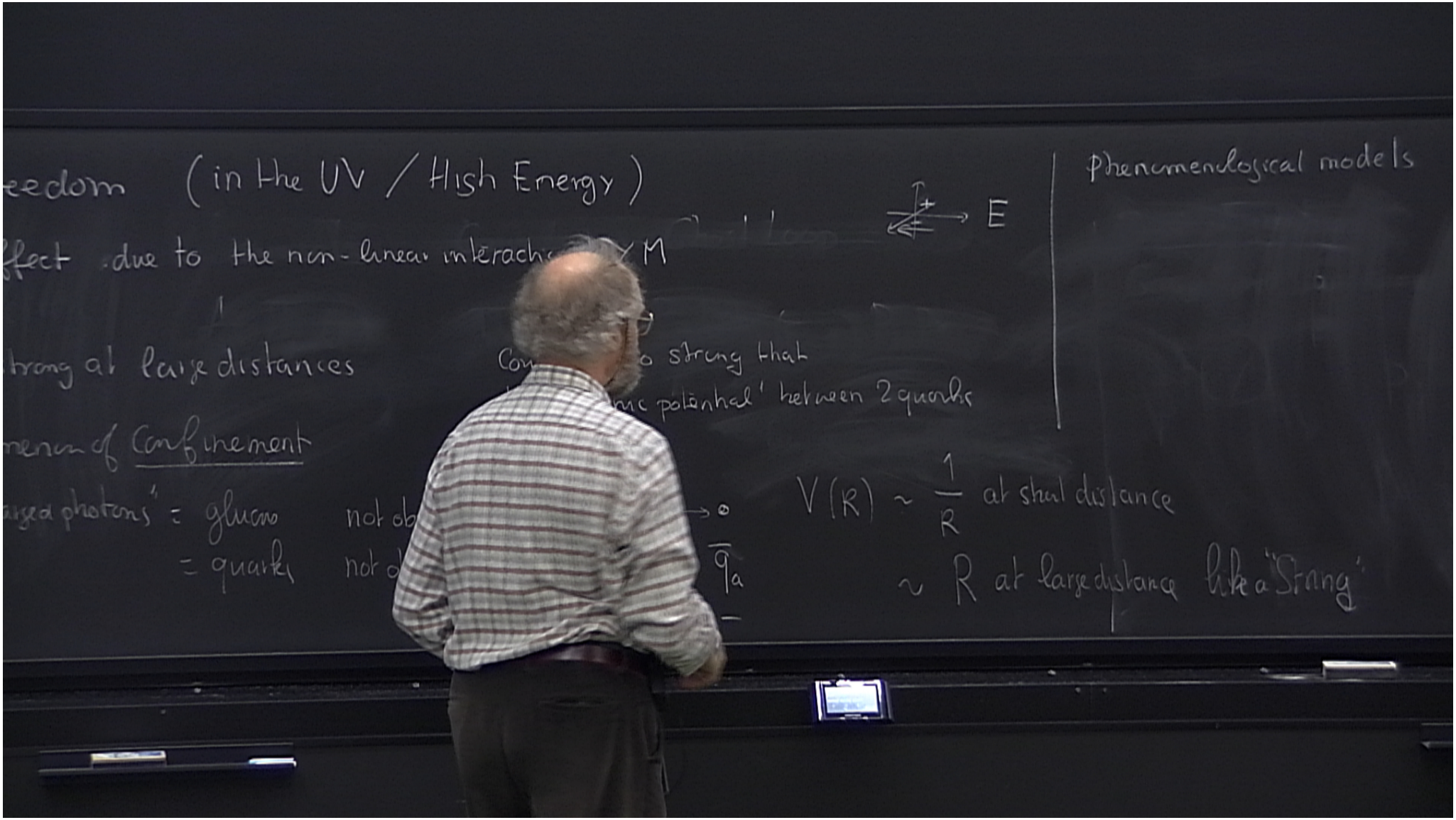
not observed

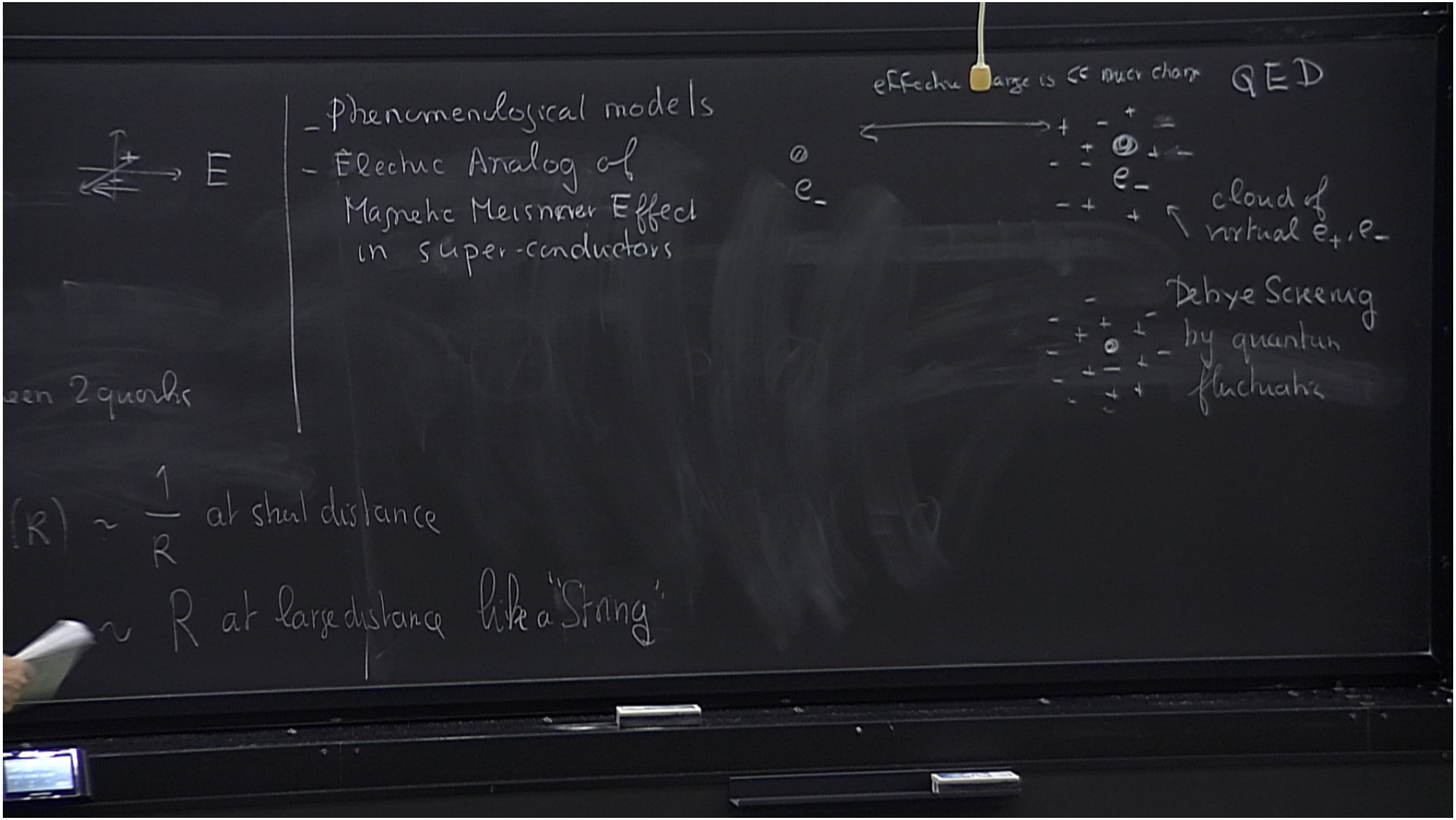
not observed



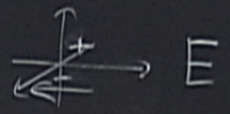

$V(R) \sim \frac{1}{R}$ at short distance

$\sim R$ at large distance like a "String"


eedom (in the UV / High Energy)
ffect due to the non-linear interac M
trong at large distances
Co o strong that
nic potential' between 2 quarks
nenon of Confinement
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$\bar{q}_a$
E
Phenomenological models
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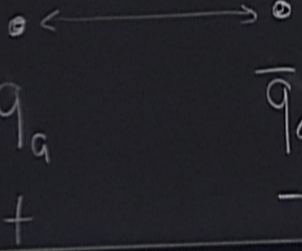

E
- Phenomenological models
- Electric Analog of Magnetic Meisnner Effect in super-conductors
e-
effective charge is ce much charge
QED
cloud of virtual e+, e-
Debye Screenig by quantum fluctuatio
een 2 quarks
(R) ~ 1/R at shal distance
~ R at large distance like a "String"

h Energy)




erachic in YM

Coupling is so strong that
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$V(R) \sim \frac{1}{R}$ at small distance

$\sim R$ at large distance like a "String"

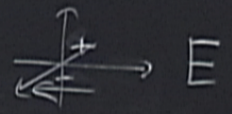
- Phenomenological models
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- Monte-Carlo and String Coupling Calculation in Lattice Gauge Theories

effective



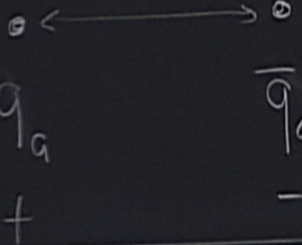


h Energy)




erachic in YM

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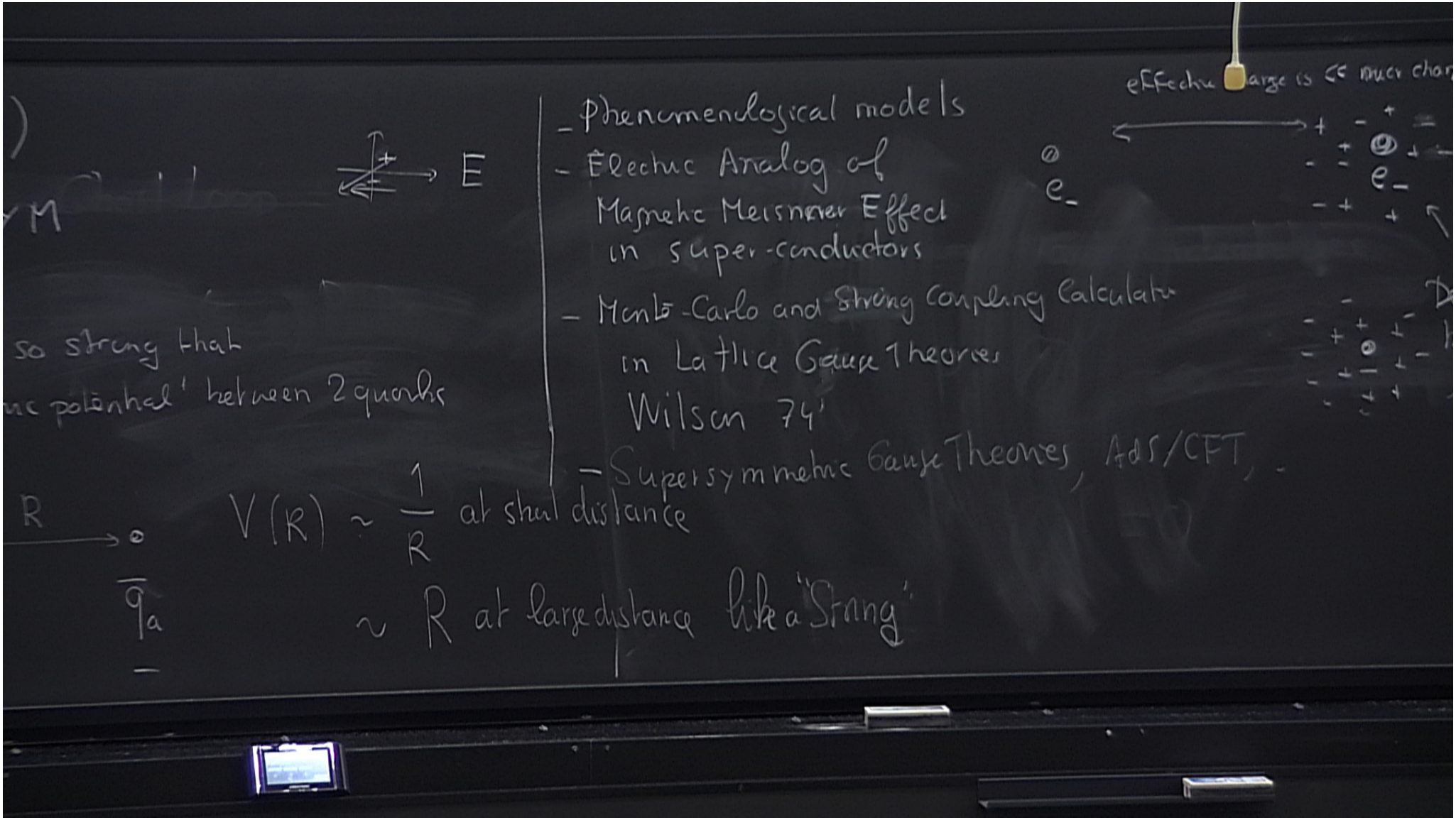
$\sim R$ at large distance like a "String"

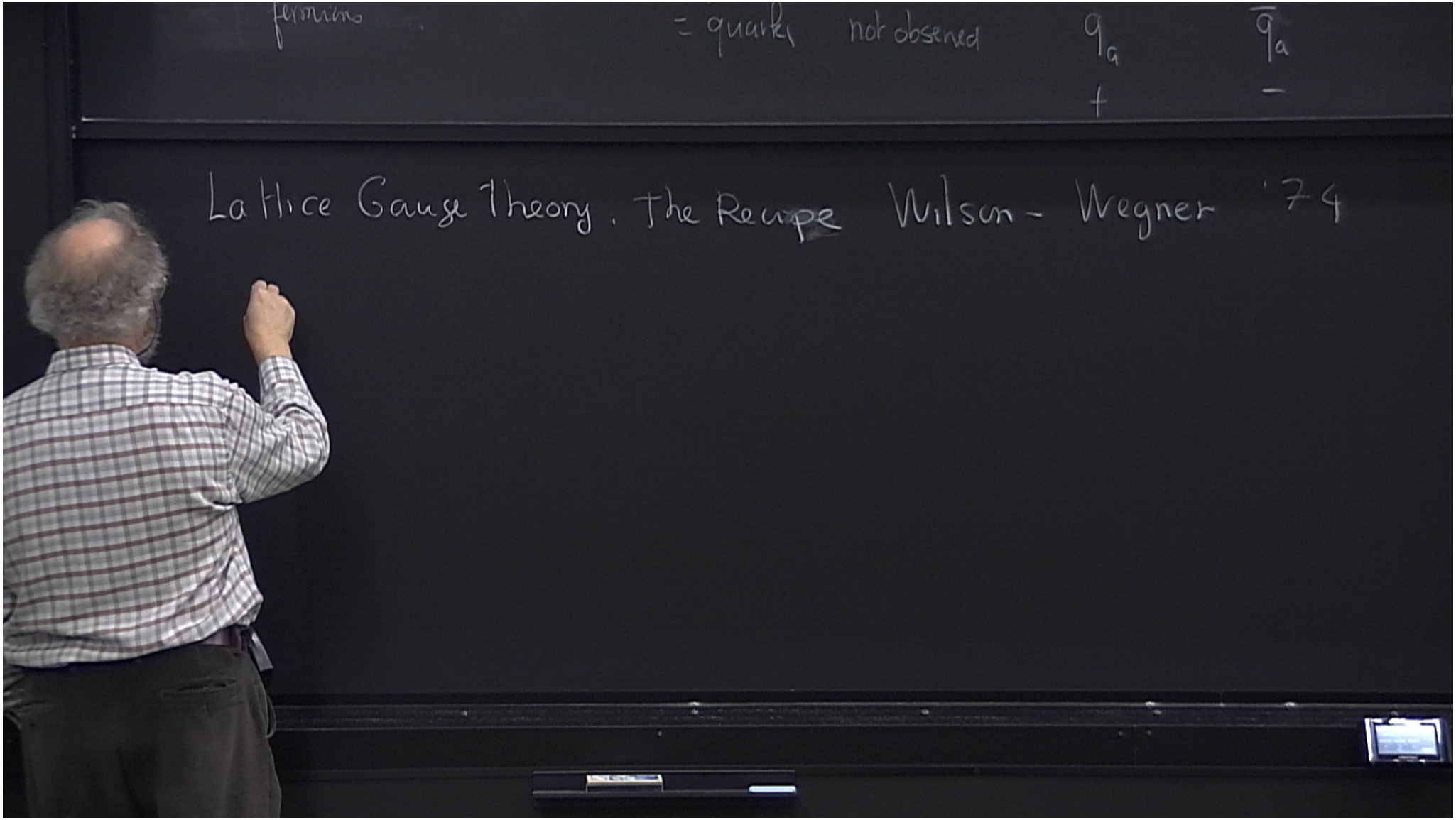
- Phenomenological models
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effectiv






E
- Phenomenological models
- Electric Analog of Magnetic Meisnner Effect in super-conductors
- Monte-Carlo and Strong Coupling Calculatu in Lattice Gauge Theories Wilson 74'
- Supersymmetric Gauge Theories, AdS/CFT,
so strong that
nc potential' between 2 quarks
R → ∞
V(R) ≃ 1/R at short distance
q̄a
~ R at large distance like a "String"

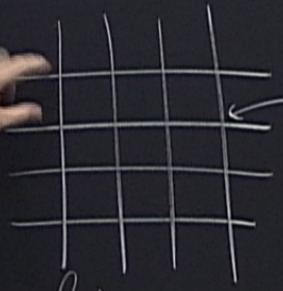

fermions
= quarks not observed
$q_a$
+
$\bar{q}_a$
−
Lattice Gauge Theory. The Recipe Wilson - Wegner '74

fermions = quarks not observed $q_a$ $\bar{q}_a$

+ −

Lattice Gauge Theory. The Recipe Wilson - Wegner '74

Scalar Field $\phi(x)$ ← Ising Model



site $i \in \mathbb{Z}^D$ → variable $S_i = \pm 1$

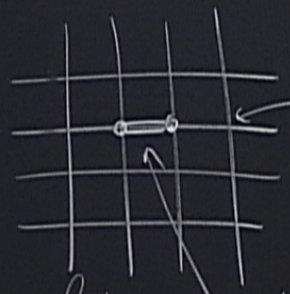
$\phi_i \in \mathbb{R}$

lattice

fermions = quarks not observed $q_a$ (+) $\bar{q}_a$ (−)

Lattice Gauge Theory. The Recipe Wilson - Wegner '74

Scalar Field $\phi(x)$ ← Ising Model



site $i \in \mathbb{Z}^D$ → variable

$(\partial\phi)^2 \rightarrow (\phi_i$

lattice edge

= quarks not observed $q_a$ $\bar{q}_a$ $\sim R$ at large

+ −

a Hice Gauge Theory. The Recipe Wilson - Wegner '74

Scalar F… (x) ← Ising Model

… site $i \in \mathbb{Z}^D$ → variable $S_i = \pm 1$

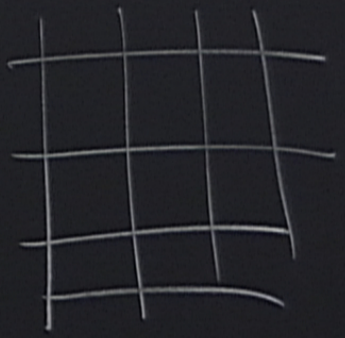
$\phi_i \in \mathbb{R}$

$(\partial\phi)^2 \to (\phi_i - \phi_j)^2$ edge $(i,j)$

Vector Field $A_\mu(x)dx^\mu = A(x)$

"

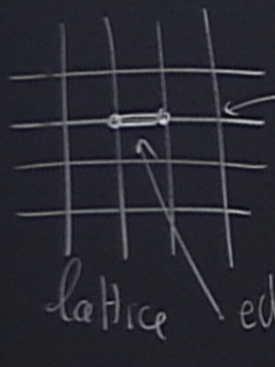
1 Form Connection



fermions = quarks not observed not observed $q_a$ $\bar{q}_a$ $+$ $-$ $\sim R$ at large distance

Lattice Gauge Theory. The Recipe Wilson - Wegner '74

Scalar Field $\phi(x)$ ← Ising Model



site $i \in \mathbb{Z}^D$ → variable

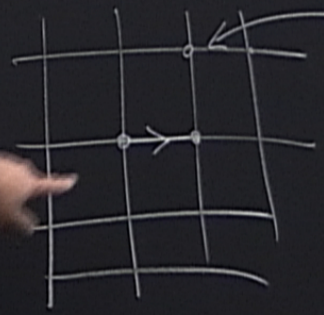
lattice edge

$(\partial\phi)^2 \to (\phi_i - \phi$

Vector Field $A_\mu(x)dx^\mu = A(x)$

"

1 Form Connection



gauge variable associated to edge $e$

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

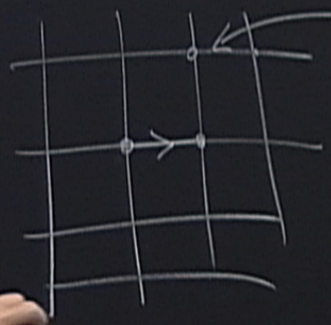
served $q_a$ $\bar{q}_a$ $R$

$+$ $-$ $\sim R$ at large distance like a "String"

Wilson - Wegner '74

Vector Field $A_\mu(x)dx^\mu = A(x)$

" 1 Form Connection



gauge variable

associated to edge $e$

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

gauge transformation

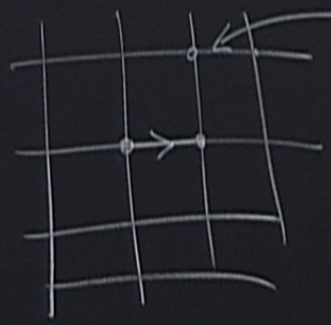
each site $i$ $\quad u(i) \in G$

served $q_a$ $\bar{q}_a$ $\sim R$ at large distance like a "String"

Wilson - Wegner '74

Vector Field $A_\mu(x)dx^\mu = A(x)$

1 Form Connection



gauge variable associated

$\vec{e} \to g(e)$

$\overleftarrow{e} \to g^{-1}(e)$

gauge transformation

each site $i$ $\quad u(i) \in G$

$g(e) \to g_u(e)$

local gauge trans

$= \pm 1$

$\in \mathbb{R}$

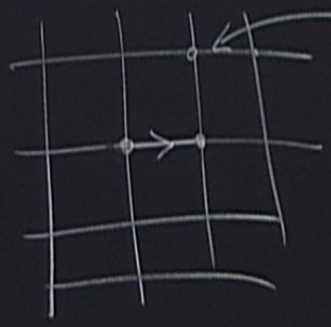
served $q_a$ $\bar{q}_a$ $\sim R$ at large distance like a "String"

+ −

Wilson - Wegner '74

Vector Field $A_\mu(x)dx^\mu = A(x)$

"1 Form Connection"



gauge variable associated to edge $e$

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

gauge transformation

each site $i$ $\quad u(i) \in G$

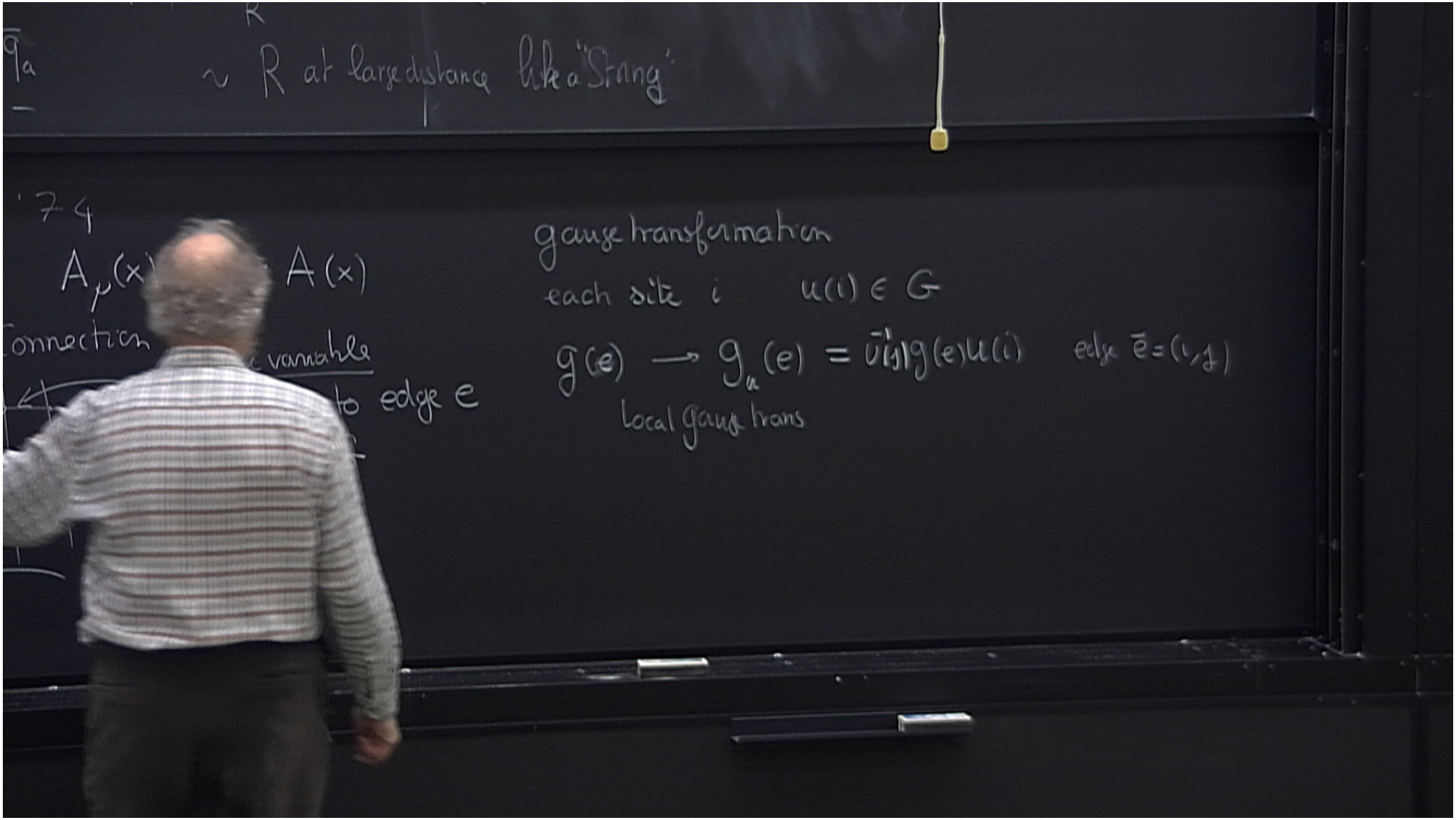
$g(e) \to g_u(e) = u(i)g(e)u(i)$ edge $e$

local gauge trans

$= \pm 1$

$\in \mathbb{R}$

$(i,j)$


R
$\bar{q}_a$
$\sim$ R at large distance like a "String"
74
$A_\mu(x)$
$A(x)$
connection
variable
to edge e
gauge transformation
each site i
$u(i) \in G$
$g(e) \longrightarrow g_u(e) = \bar{u}(i) g(e) u(i)$
edge $\vec{e} = (i,j)$
local gauge trans

$q_a$ $\sim R$ at large distance like a "String"

7-4

$A_\mu(x)dx^\mu = A(x)$

gauge transformation

each site $i$ $\quad u(i) \in G$

$g(e) \longrightarrow g_u(e) = u^{-1}(i)\, g(e)\, u(i)$ $\quad$ edge $\vec{e} = (i,j)$

local gauge trans

nnection

gauge variable

associated to edge $e$

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

$g(e) = 1 + i\, A_\mu(x)dx^\mu(\text{edge})$

$dx^\mu = A(x)$

gauge variable
associated to edge e

$e \to g(e) \in G$

$\bar{e} \to g^{-1}(e)$

gauge transformation

each site $i$ $\quad u(i) \in G$

$g(e) \longrightarrow g_u(e) = u^{-1}(i)\, g(e)\, u(i)$ $\quad$ edge $\bar{e} = (i, j)$

Local gauge trans $\quad$ Gauge Invariant observable

$g(e) = 1 + i\, A_\mu(x)\, dx^\mu(\text{edge})$

Recipe Wilson - Wegner '74

Model

variable $S_i = \pm 1$

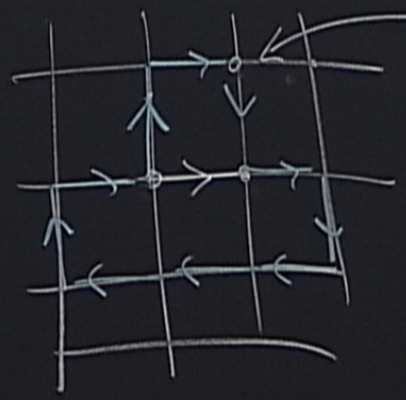
$\phi_i \in \mathbb{R}$

$(\phi_i - \phi_j)^2$ edge $(i,j)$

Vector Field $A_\mu(x) dx^\mu = A(x)$

"

1 Form Connection



gauge variable

associated to edge $e$

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

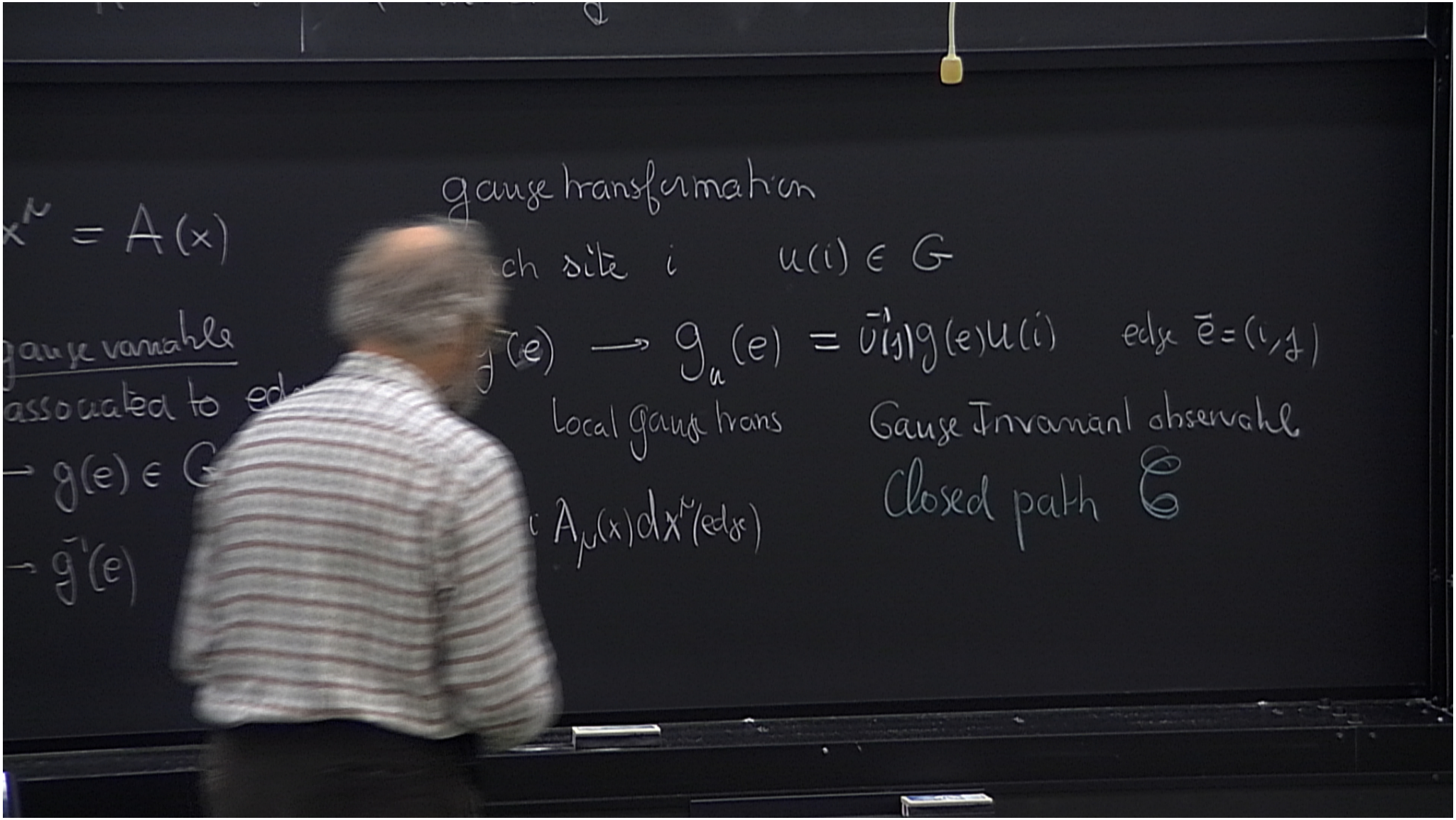
gauge h

each s

$g(e)$

Lo

$g(e) = 1 + \epsilon A_\mu$


gauge transformation
ch site i u(i) ∈ G
Local gauge trans
Gauge Invariant observable
Closed path

$x^\mu = A(x)$

gauge variable

associated to edge $e$

$\to g(e) \in G$

$\to g^{-1}(e)$

gauge transformation

each site $i$ $\quad u(i) \in G$

$g(e) \longrightarrow g_u(e) = u^{-1}(i)\, g(e)\, u(i)$ $\quad$ edge $\vec{e} = (i,j)$

Local gauge trans $\quad$ Gauge Invariant observable

$g(e) = 1 + i\, A_\mu(x)\, dx^\mu(\text{edge})$

Closed path $\mathcal{C}$

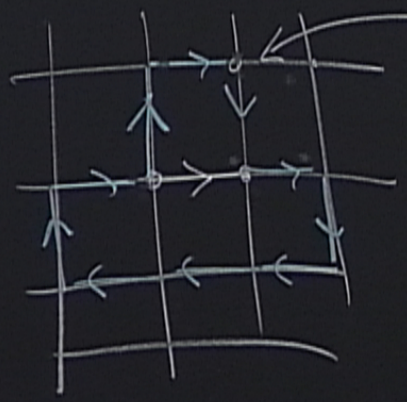
$$\prod_{\text{path}}^{\rightarrow} g(\vec{e})$$

Wilson - Wegner '74

Vector Field $A_\mu(x)dx^\mu = A(x)$

"

1 Form Connection



gauge variable associated to edge e

$\vec{e} \to g(e) \in G$

$\overleftarrow{e} \to g^{-1}(e)$

$S_i = \pm 1$

$\phi_i \in \mathbb{R}$

edge $(i,j)$

gauge transform

each site $i$

$g(e) \to g_u$

local gauge

$g(e) = 1 + i A_\mu(x)dx^\mu(e$

gauge transformation

each site $i$ $\quad u(i) \in G$

$g(e) \longrightarrow g_u(e) = \bar{U}_{ij} g(e) U(i)$ $\quad$ edge $\vec{e} = (i,j)$

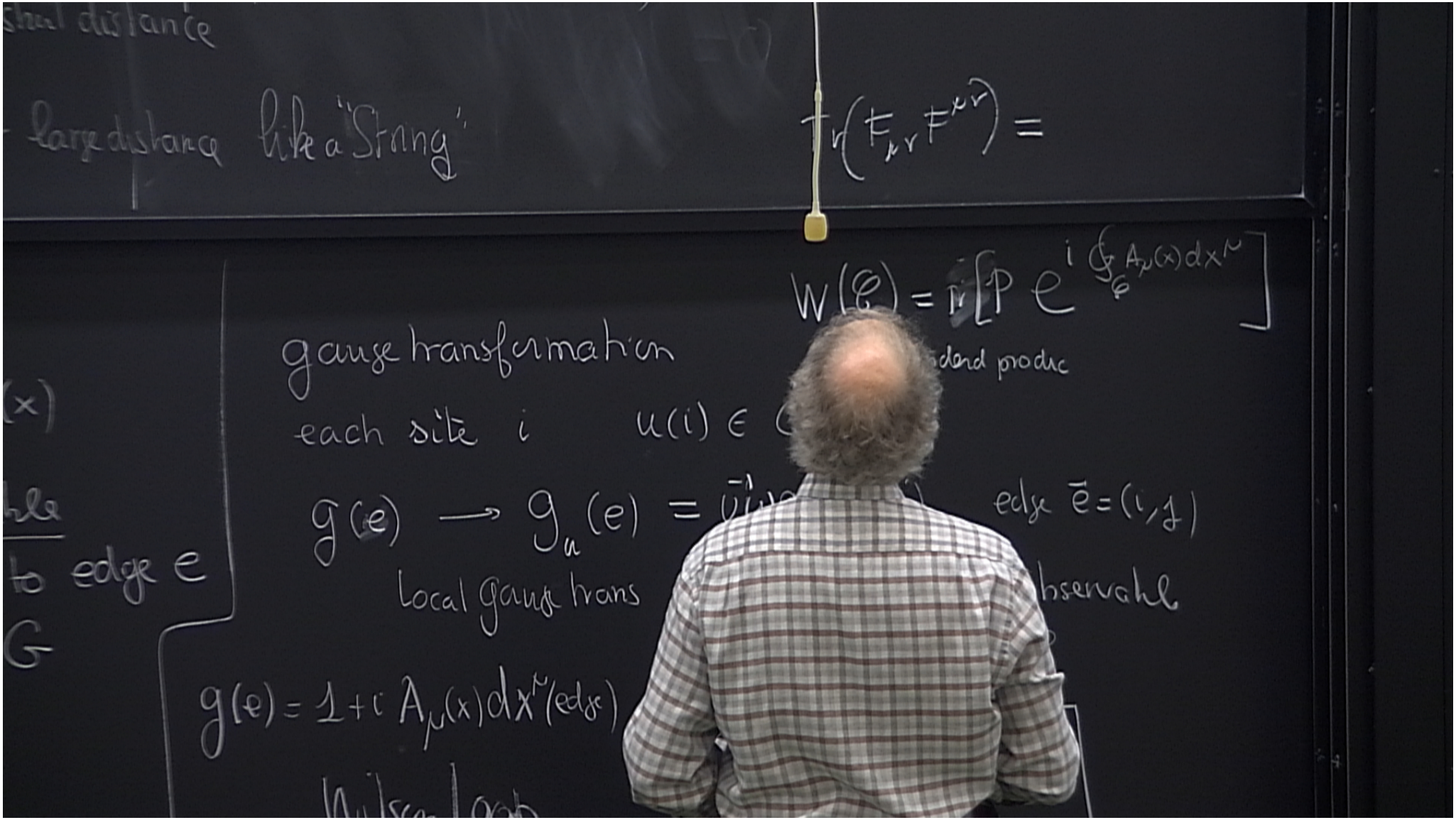
local gauge trans $\quad$ Gauge Invariant observable

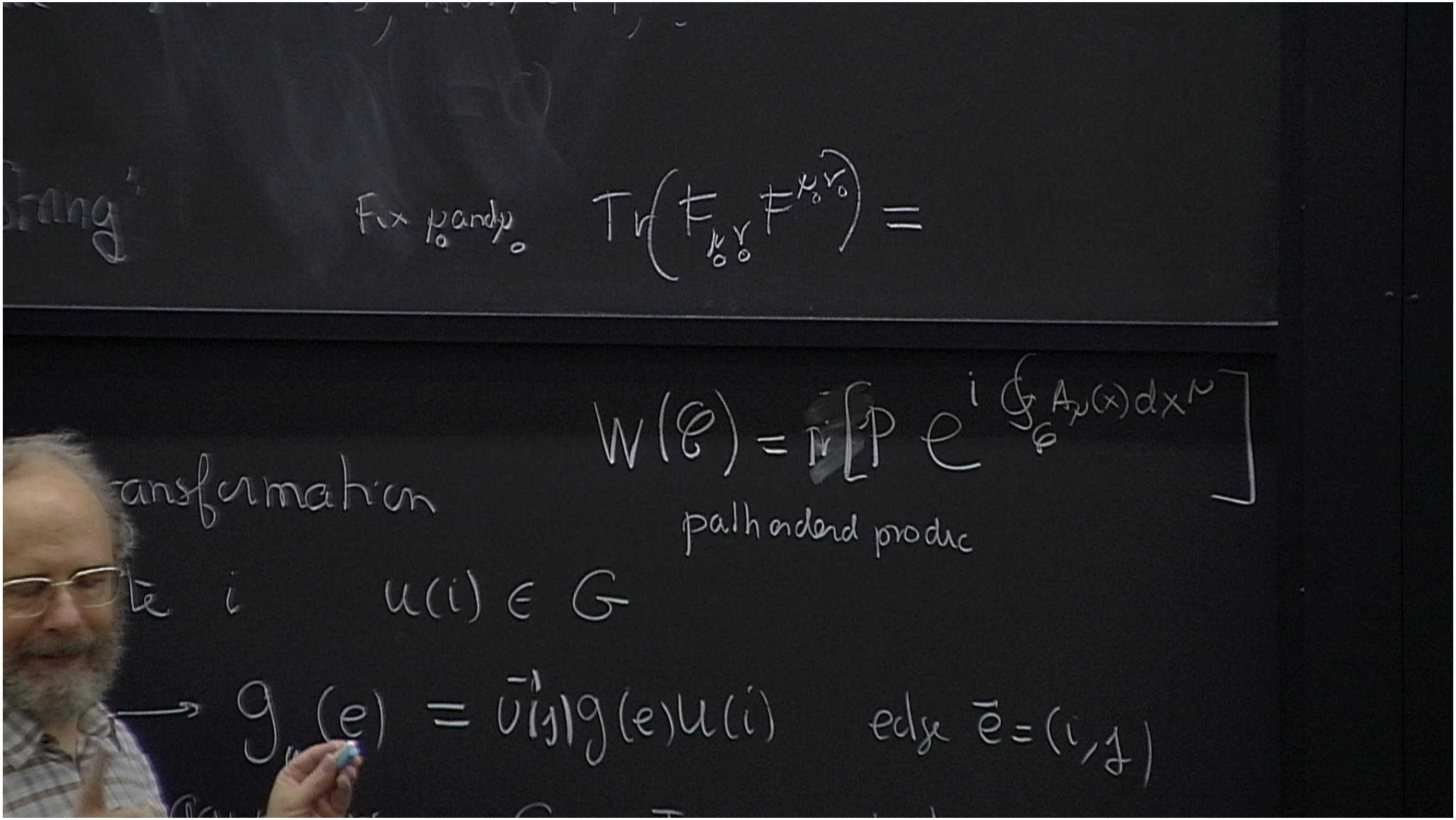
$g(e) = 1 + i A_\mu(x) dx^\mu (\text{edge})$

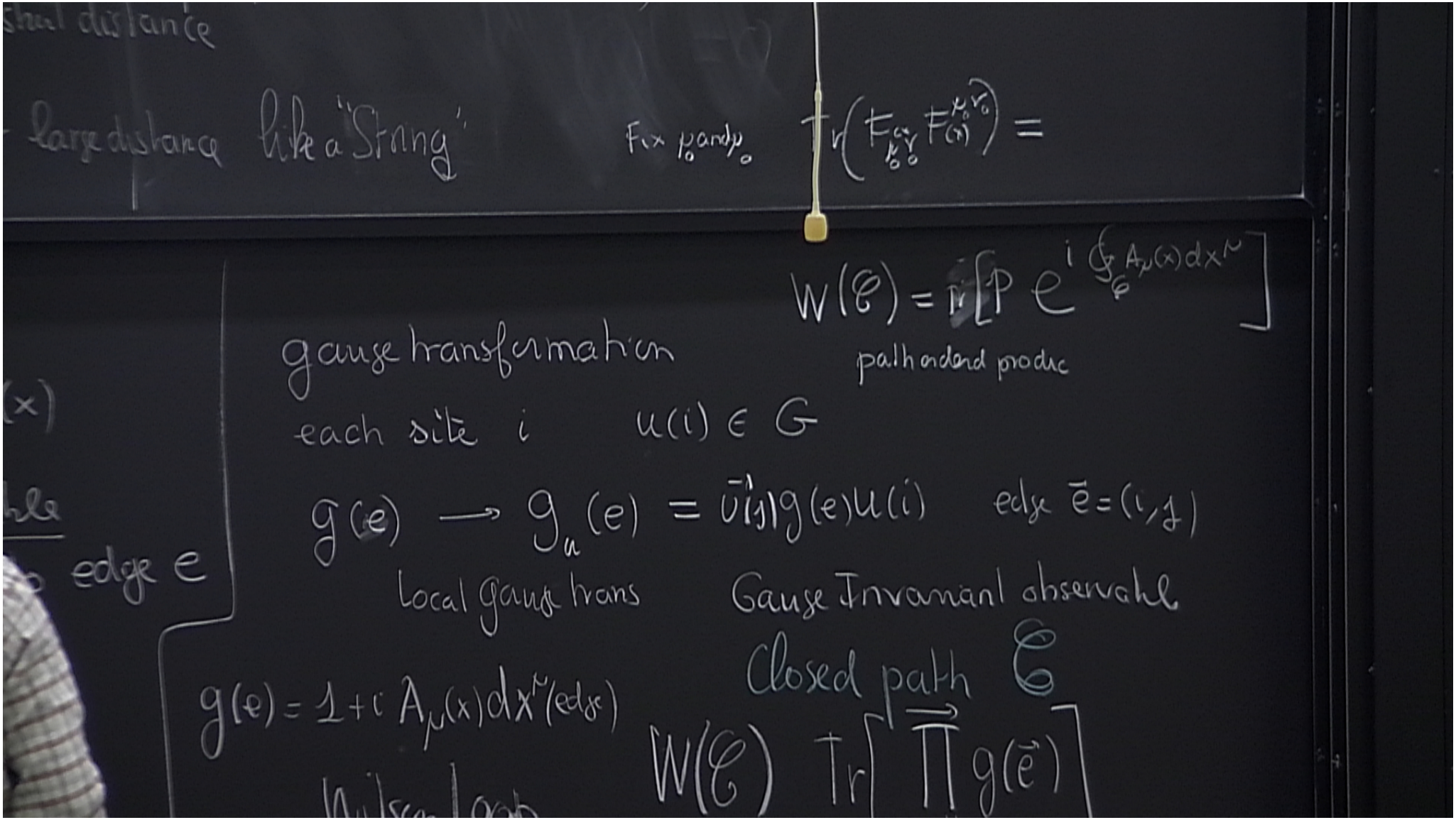
Wilson Loop

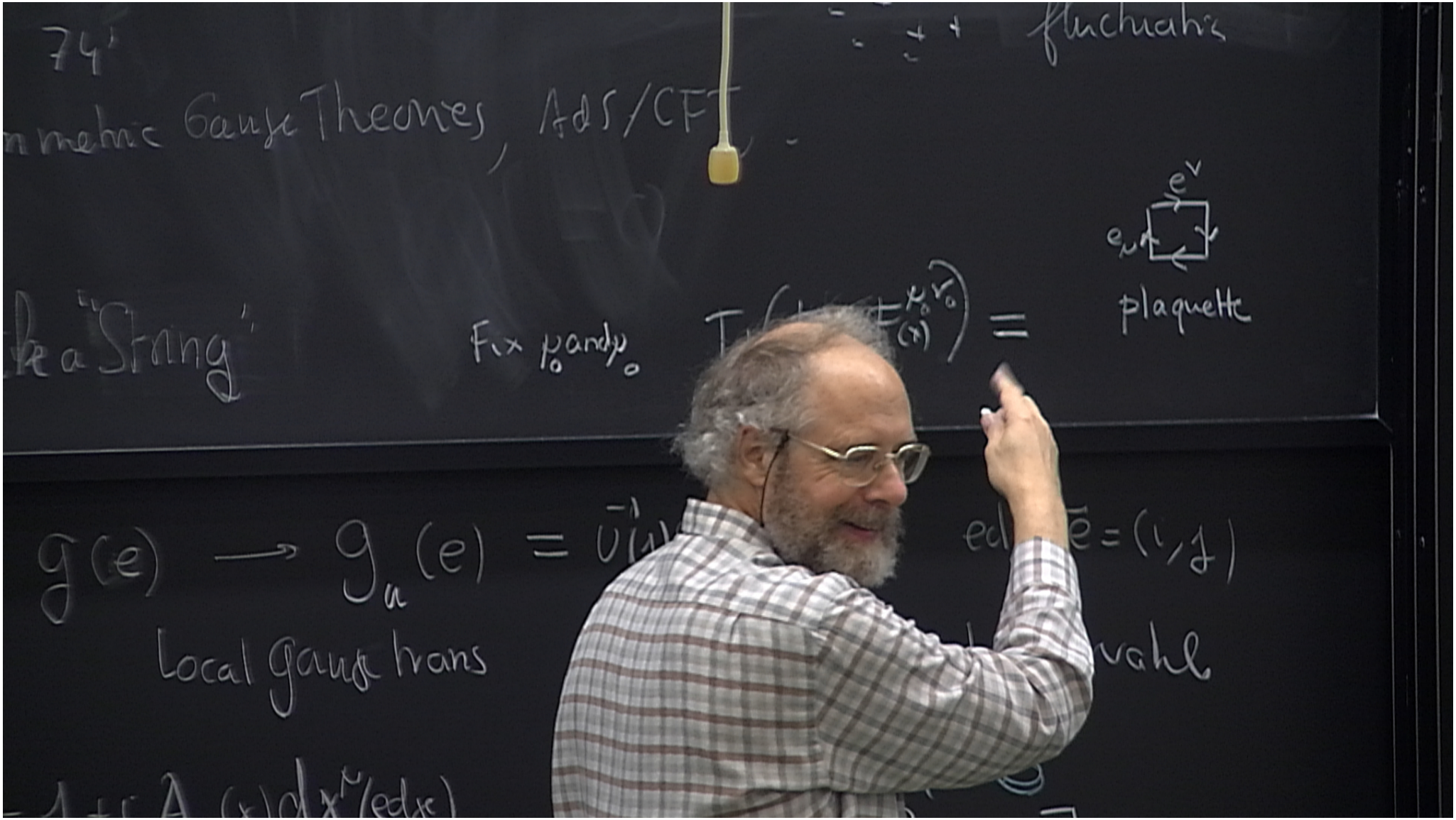
Closed path $\mathcal{C}$

$W(\mathcal{C}) \; \mathrm{Tr}\left[ \prod_{\text{path}} g(\vec{e}) \right]$


distance
large distance like a "String"
$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) =$
$W(e) = \mathrm{Tr}\left[P\, e^{i\oint_e A_\mu(x)dx^\mu}\right]$
gauge transformation
each site i
$g(e) \longrightarrow g_u(e) =$
edge $\vec{e} = (i,j)$
local gauge trans
observable
$g(e) = 1 + i\, A_\mu(x)dx^\mu$(edge)
to edge e
G


Fix $\mu_0$ and $\nu_0$
$\mathrm{Tr}\left(F_{\mu_0\nu_0} F^{\mu_0\nu_0}\right) =$
$W(\mathcal{C}) = \mathrm{Tr}\left[P\, e^{i \oint_{\mathcal{C}} A_\mu(x)\, dx^\mu}\right]$
path ordered produc
ansformation
e i $u(i) \in G$
$\longrightarrow g_u(e) = u_{(i)}^{-1} g(e) u(i)$
edge $\vec{e} = (i, j)$


distance
large distance like a "String"
Fix $\mu_0$ and $\nu_0$
$\mathrm{Tr}\left(F_{\mu_0\nu_0}^{a} F_{(x)}^{\mu_0\nu_0}\right) =$
$W(\mathcal{C}) = \mathrm{Tr}\left[P\, e^{i \oint_{\mathcal{C}} A_\mu(x) dx^\mu}\right]$
path ordered produc
gauge transformation
each site $i$ $\quad u(i) \in G$
$g(e) \longrightarrow g_u(e) = u^{-1}_{(i)} g(e) u(i)$ $\quad$ edge $e = (i,j)$
Local gauge trans
Gauge Invariant observable
$g(e) = 1 + i A_\mu(x) dx^\mu(\text{edge})$
Closed path $\mathcal{C}$
$W(\mathcal{C})$ $\mathrm{Tr}\left[\overrightarrow{\prod} g(\vec{e})\right]$
edge e


74'
fluctuations
Gauge Theories, AdS/CFT
plaquette
Fix
Local gauge trans

74'

fluctuations

mmetric Gauge Theories, AdS/CFT

e^ν

e^μ

plaquette

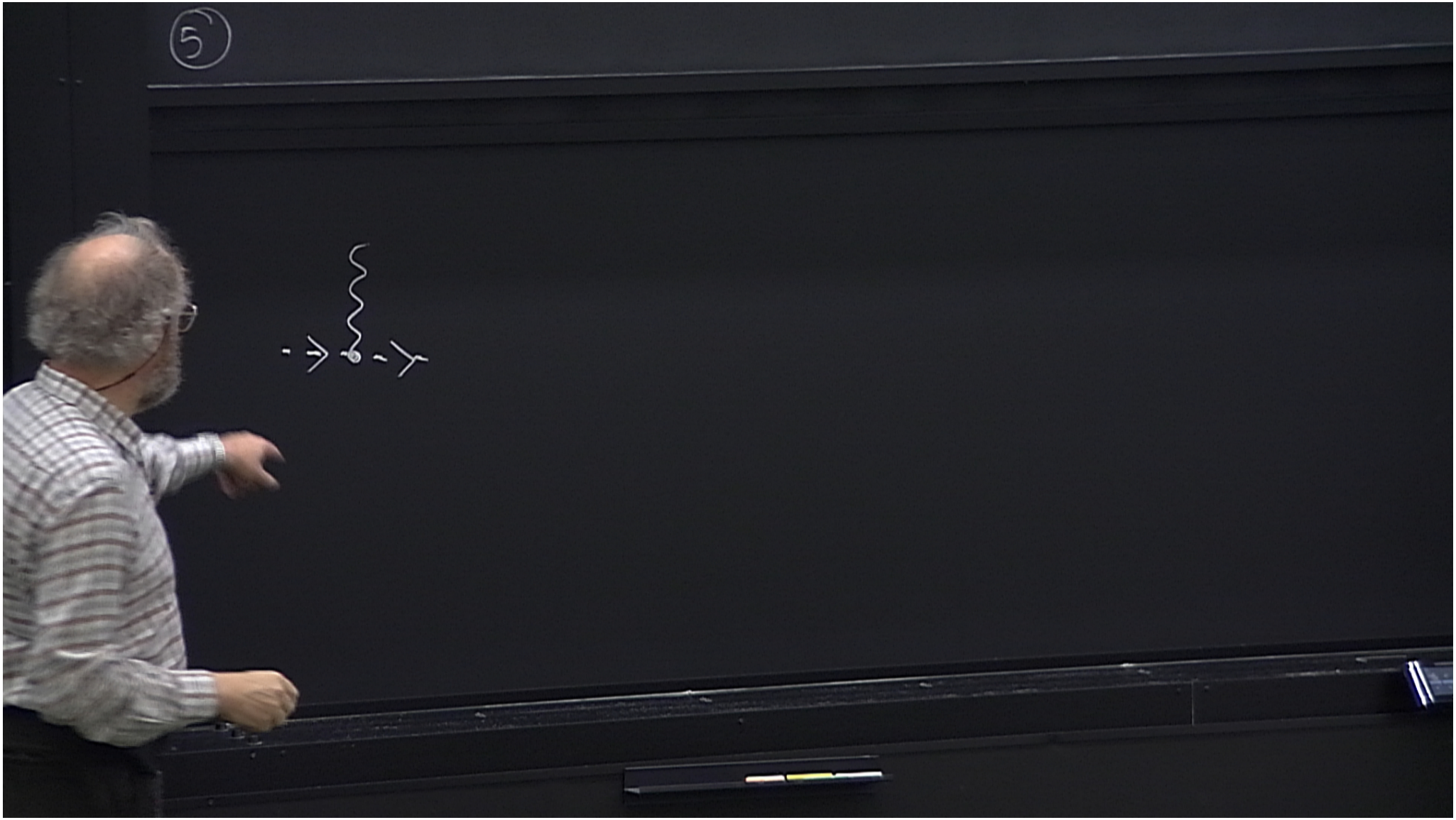
ke a "String"

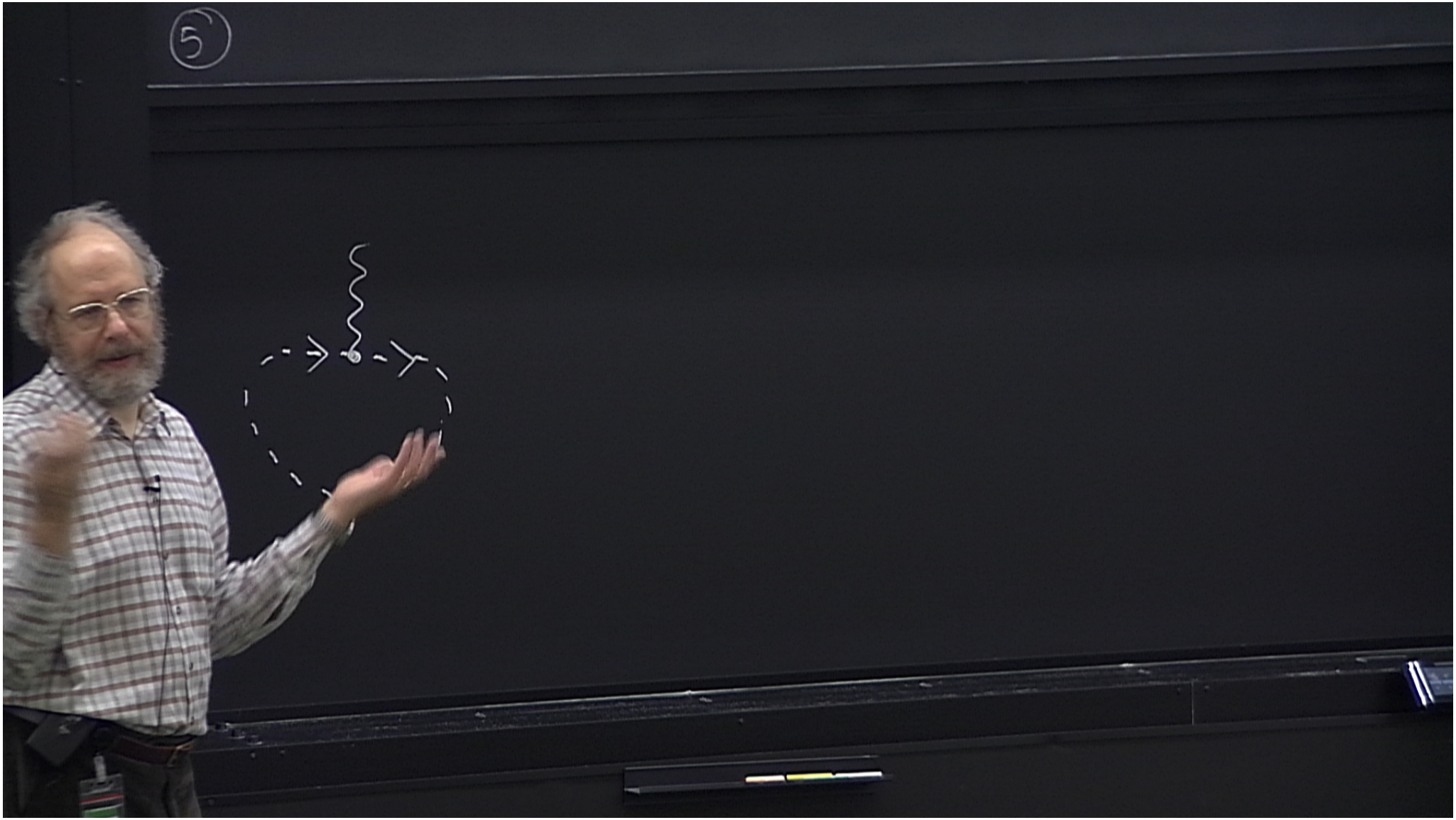
Fix $\mu_0$ and $\nu_0$ $\quad Tr\left(F_{\mu_0\nu_0}(x)\, F^{\mu_0\nu_0}(x)\right) =$

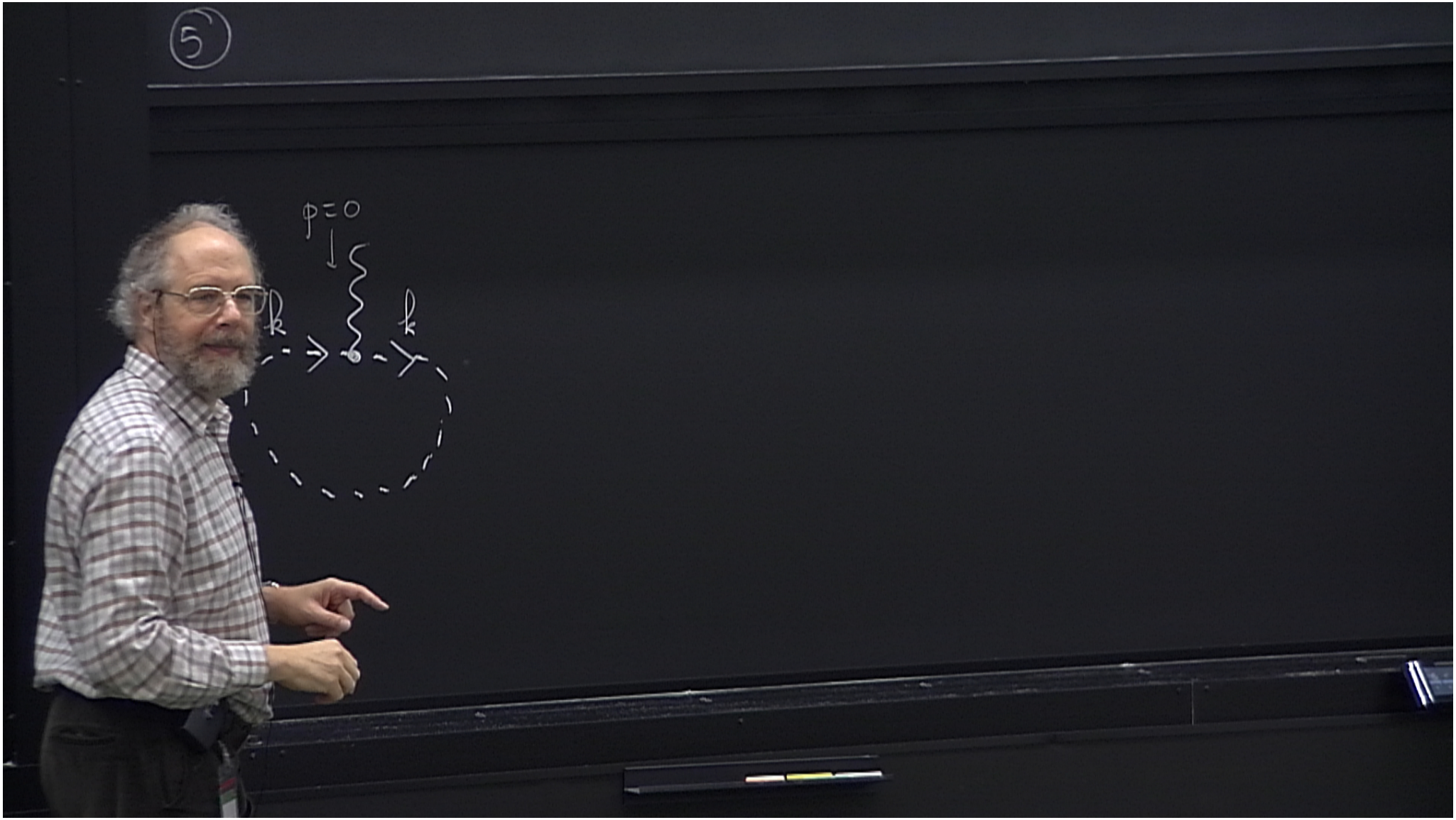
$g(e) \longrightarrow g_u(e) = \bar{v}(i)\, g(e)\, u(j) \qquad e \,\mathrm{d}x\, \bar{e} = (i,j)$

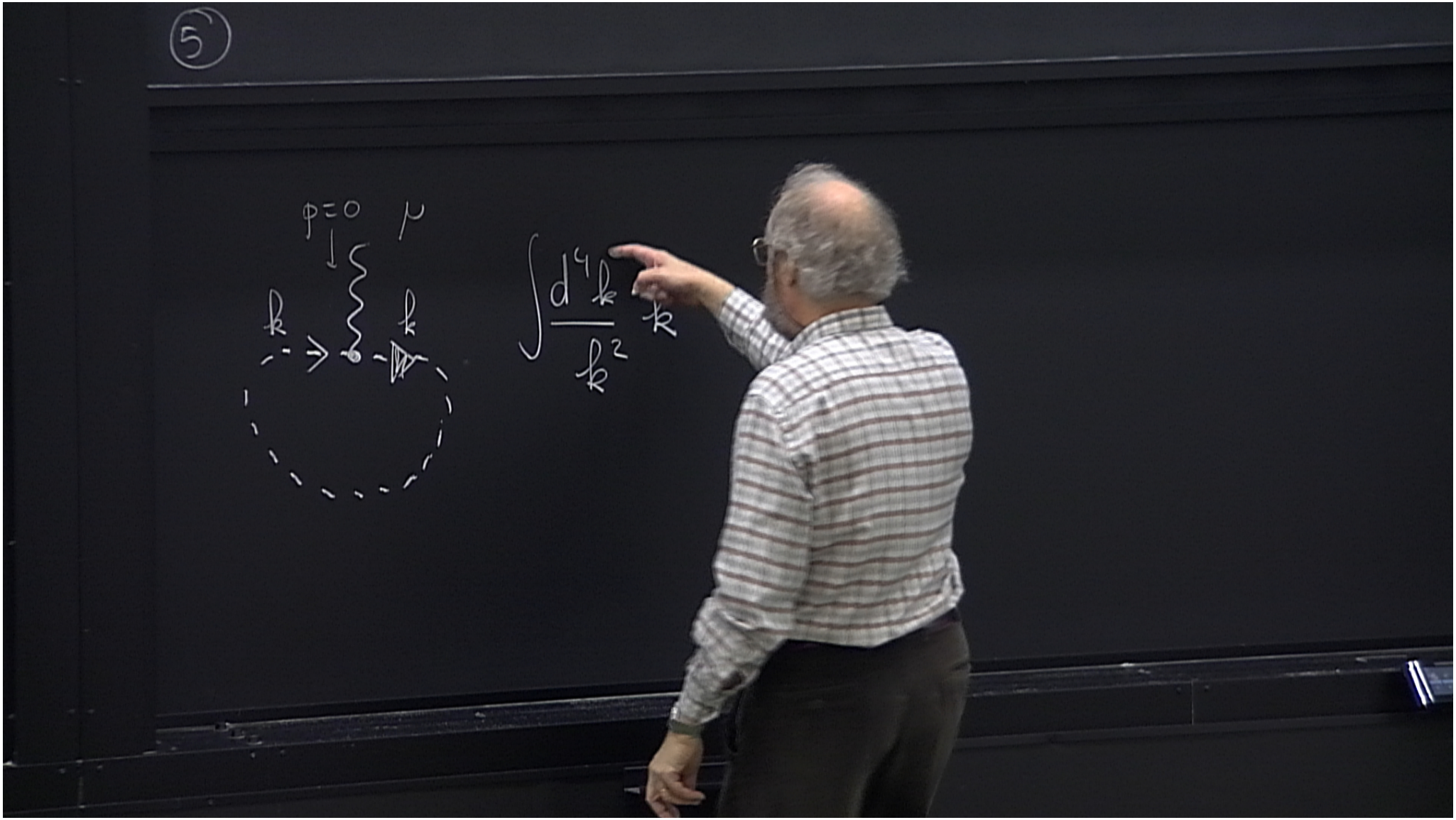
Local gauge trans $\qquad$ Gauge Invariant observable

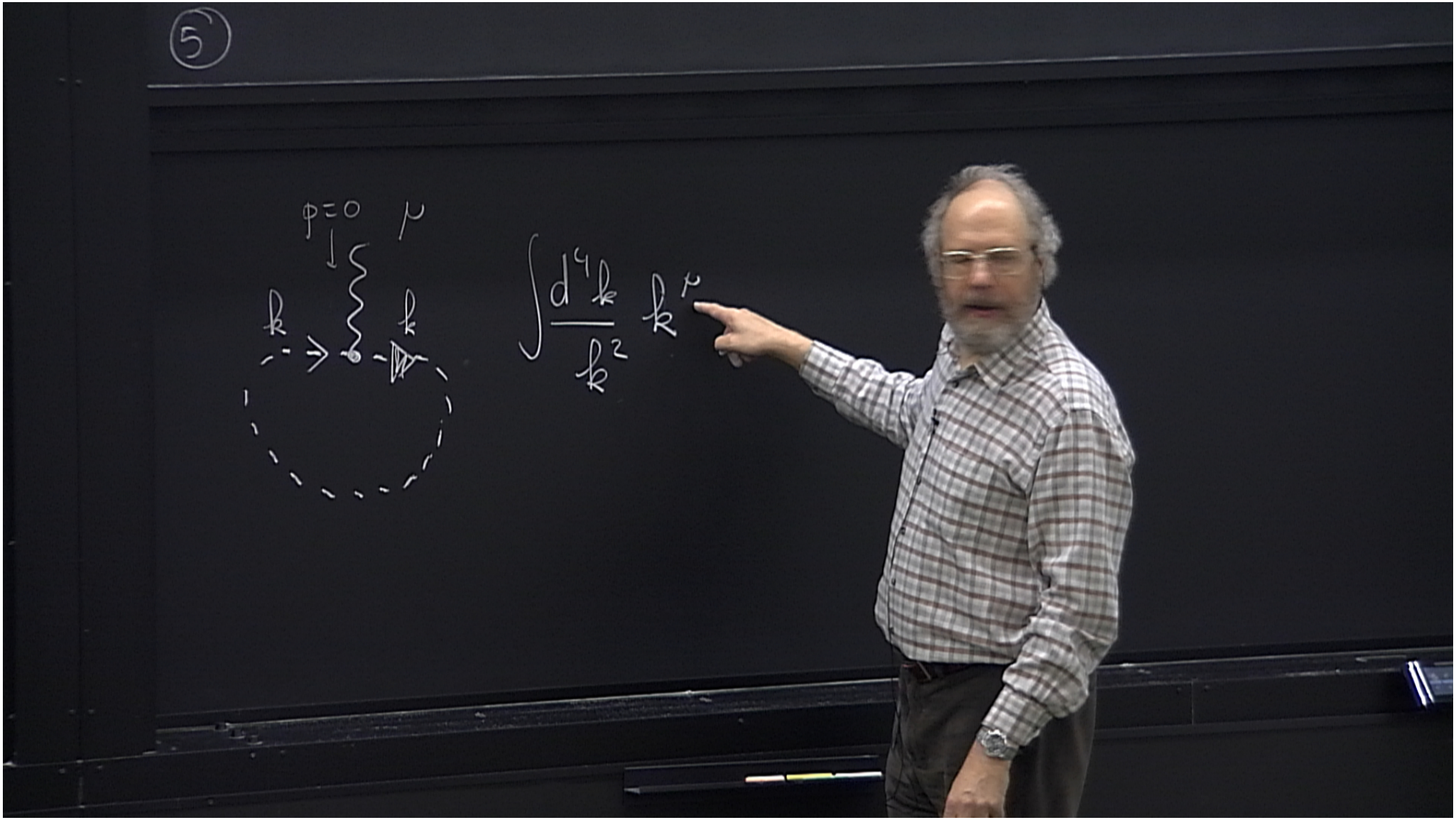
$1 + i A_\mu(x)\, \mathrm{d}x^\mu (e\,\mathrm{d}x)$ $\qquad$ Closed path $\mathcal{C}$

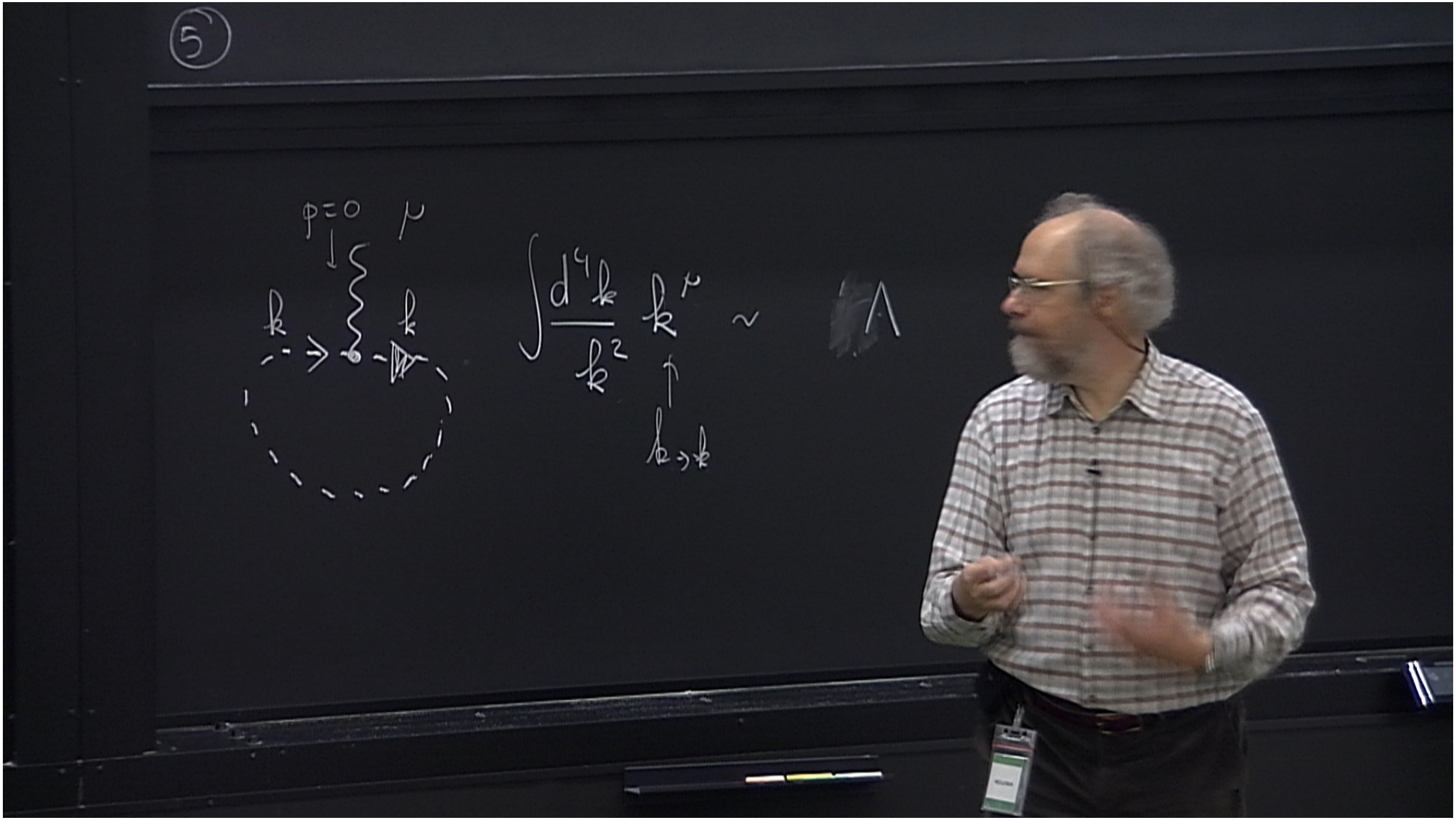

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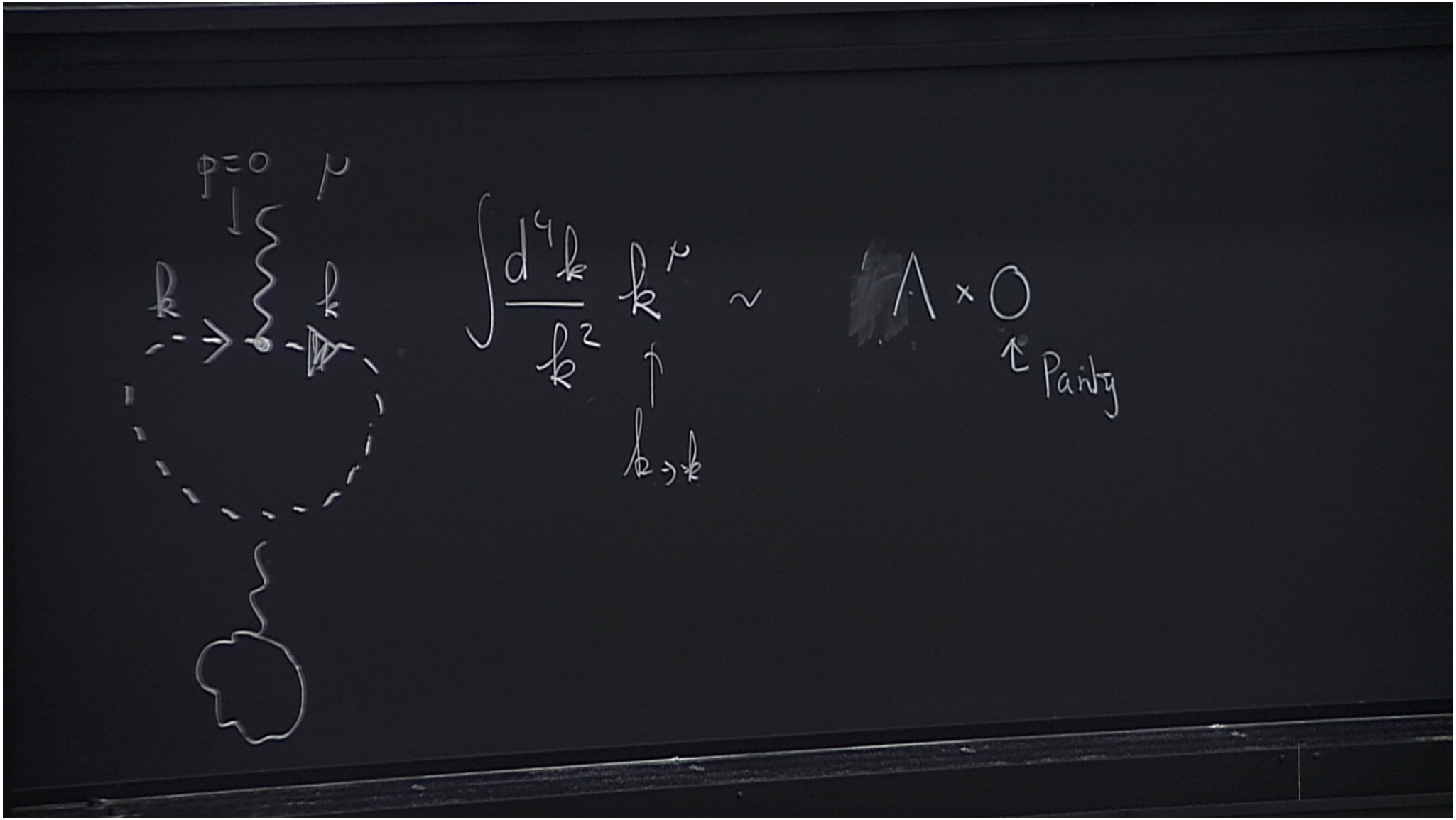

5


5
p=0
k
k


5
p=0
μ
k
k
k
∫ d⁴k / k² k


5
p=0
μ
k
k
$\int \frac{d^4k}{k^2} k^\mu$


5
p=0
μ
k
k
∫ d⁴k/k² k^μ ~ Λ
k→k


p=0
μ
k
k
$\int \frac{d^4k}{k^2} k^\mu \sim \Lambda \times 0$
$k \to -k$
Parity