Title: PSI 2016/2017 Quantum Field Theory II - Lecture 14

Date: Nov 24, 2016 09:00 AM

URL: http://pirsa.org/16110014

Abstract:

①: Gauge Fixing & Gauge Fixed Action.

$$Z = \int \mathcal{D}[A] \exp\left(i S_{YM}[A]\right)$$

Gauge Invariant

$A \longrightarrow A_g$

$A \in \mathcal{A}$ Gauge Potentials

$g \in \mathcal{G}$ Gauge Group

Gauge Fixing Condition $F[A] = 0$

$A \in \mathcal{A}$ unique $g = g_F[A] \in \mathcal{G}$ such that

$$F[A_{g_F[A]}] = 0$$

Action.

Gauge Fixing Condition $F[A] = 0$

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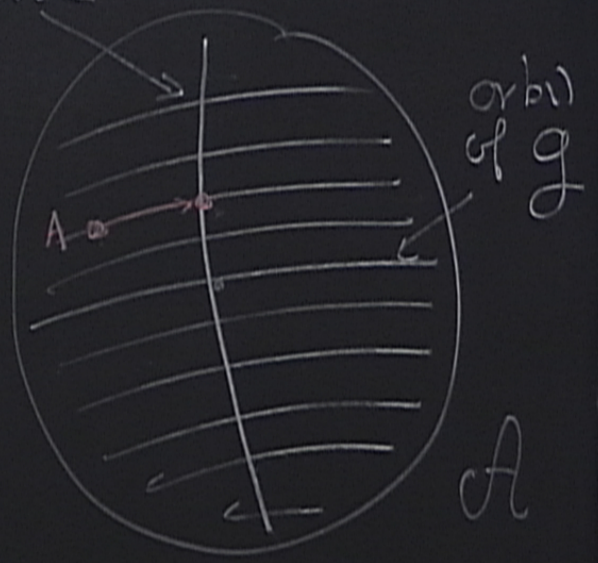
$$F[A_{g_F[A]}] = 0$$

- Trick 1

$$1 = \int_{\mathcal{G}} d\mu(g)\, \delta[g - g_F[A]]$$

$d\mu(g)$: Haar Measure

- Transverse to the Orbits
- Unique intersection



Condition $F[A] = 0$

$g = g_F[A] \in \mathcal{G}$ such that

$F[A_{g_F[A]}] = 0$

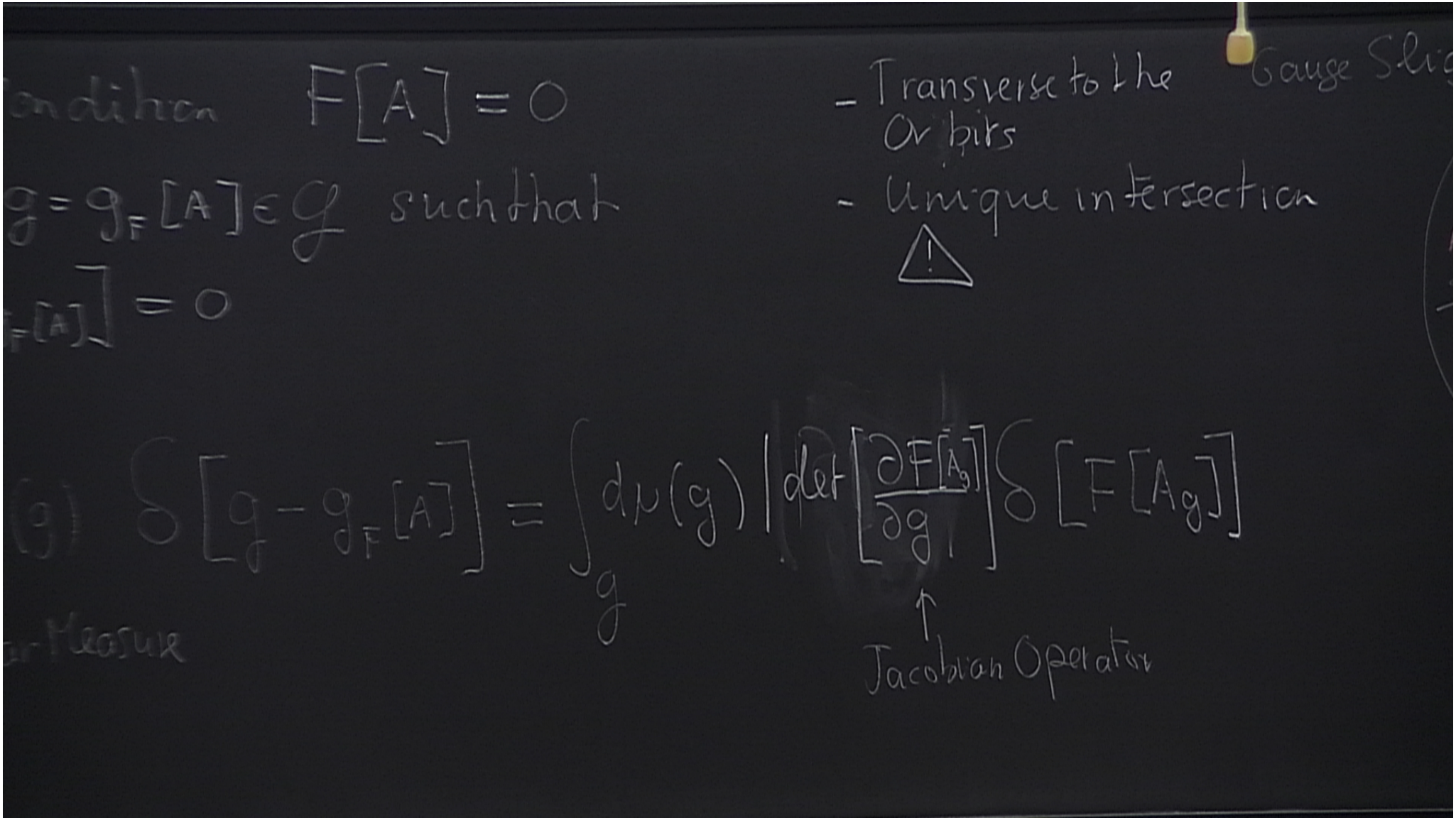
- Transverse to the Orbits
- Unique intersection



Gauge Slice

(g) $\delta[g - g_F[A]] = \int_{\mathcal{G}} d\nu(g)\, \delta[F[A_g]]$

Measure


Condition $F[A] = 0$
$g = g_F[A] \in \mathcal{G}$ such that
$F[A_{g_F[A]}] = 0$
- Transverse to the Orbits
- Unique intersection
Gauge Slice
$\int_{\mathcal{G}} [dg]\, \delta[g - g_F[A]] = \int_{\mathcal{G}} d\mu(g)\, |\det[\frac{\partial F[A_g]}{\partial g}]|\, \delta[F[A_g]]$
Jacobian Operator

(2) Invert the integrals

$$Z = \int_{\mathcal{G}} d\nu(g) \int_{\mathcal{A}} \mathcal{D}[A] \exp(i S_{YM}[A]) \left| \mathrm{Det}\left[\frac{\partial F[A_g]}{\partial g}\right] \right| \delta(F[A_g])$$

Change of variable : use A_g istead of A

$$Z = \left(\int_{\mathcal{G}} d\nu(g)\right) \left[\int_{\mathcal{A}} \mathcal{D}[A] \exp(i S_{YM}[A])\right] \left| \mathrm{Det}\left[\frac{\partial F[A_g]}{\partial g}\right]_A \right| \delta(F[A])$$

$\mathrm{Vol}(\mathcal{G})$ factors out

Det. Function of A

The integrals

$$\nu(g) \int_{\mathcal{A}} \mathcal{D}[A]\, \exp(i S_{YM}[A])\, \left| \mathrm{Det}\left[\frac{\partial F[A_g]}{\partial g}\right] \right| \delta(F[A_g])$$

variable : use A_g istead of A

$$\nu(g) \int_{\mathcal{A}} \mathcal{D}[A]\, \exp(i S_{YM}[A])\, \left| \mathrm{Det}\left[\frac{\partial F[A_g]}{\partial g}\right]_A \right| \delta(F[A])$$

Gauge Fixed Functional Integral

Det. Function of A

factors out

Trick 2. Fadeev-Popov "Ghost Fields" to represent the determinant

- Perturbation Theory & Funct. Integral
- Canonical Quantization & Hilbert Space Representation

Trick 2. Fadeev-Popov "Ghost Fields" to represent the determinant

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$$J[A] = \left.\frac{\partial F[A_g]}{\partial g}\right|_{g=Id}$$

Jacobian Operator

c Ghost
$\bar{c}$ Anti-Ghost.

$$\mathrm{Det}[J[A]] = \int \mathcal{D}[\bar{c}]\mathcal{D}[c] \exp\left(i \bar{c} \cdot J[A] c\right)$$

Coupling J

Conjugate Grassmann Fields

Trick 3. $\delta[F[A]] = \int D[\Lambda] \exp(i \Lambda \cdot F[A])$

Λ is a Lagrange Multiplier Field

Gauge Fixed Actra $S_{\substack{\text{gauge} \\ \text{fixed}}} = S_{SX}[A] + \bar{C} \mathcal{J}[A] C + \Lambda \cdot F[A]$

$A, C, \bar{C}, \Lambda$

Feynman Gauge $F = \partial^\mu A_\mu - \epsilon \qquad \partial^\mu A^a_\mu(x) - \epsilon^a(x) = F^a(x)$

What is J ? $\quad A \to A_g$ with $g = \mathbb{1} + i\alpha(x) \qquad \alpha(x) = \alpha^a(x)\, t_a$

$$J^{ab}_F(x,y) = \frac{\delta F^a(x)}{\delta \alpha^b(y)}$$

Kernel of J = Kernel of a Diff Operator

$$\partial_\mu D^\mu$$

$A_\mu - \epsilon \quad \partial^\nu A^a_\mu(x) = \epsilon^a(x) = F^a(x) \quad a = 1,2,3$ Adj Rep $SU(2)$

μ Lorentz

A_g with $g = \mathbb{1} + i\alpha(x) \quad \alpha(x) = \alpha^a(x) t_a \quad x \in \mathbb{M}$ space

Γ = Kernel of a Diff Operator $\partial_\nu \cdot D^\nu \quad D_\nu := \partial_\nu - i[A_\mu, \cdot]$

Feynman Gauge $F = \partial^\mu A_\mu - \epsilon$ $\partial^\mu A^a_\mu(x) = \epsilon^a(x) = F^a(x$

What is J ? $A \to A_g$ with $g = \mathbb{1} + i\alpha(x)$ $\alpha(x) = \alpha^a(x) t_a$

$$J^{ab}_P(x,y) = \frac{\delta F^a(x)}{\delta \alpha^b(y)}$$ Kernel of J = Kernel of a Diff Operator $\partial_\mu D^\mu$

In terms of components

$$J_{ab} = \delta_{ab}\Delta + \epsilon_{abc} A^c_\mu \partial^\mu + \epsilon_{abc}\left(\partial^\mu A^c_\mu\right)$$

$SU(2)$ $f_{abc} = \epsilon_{abc}$

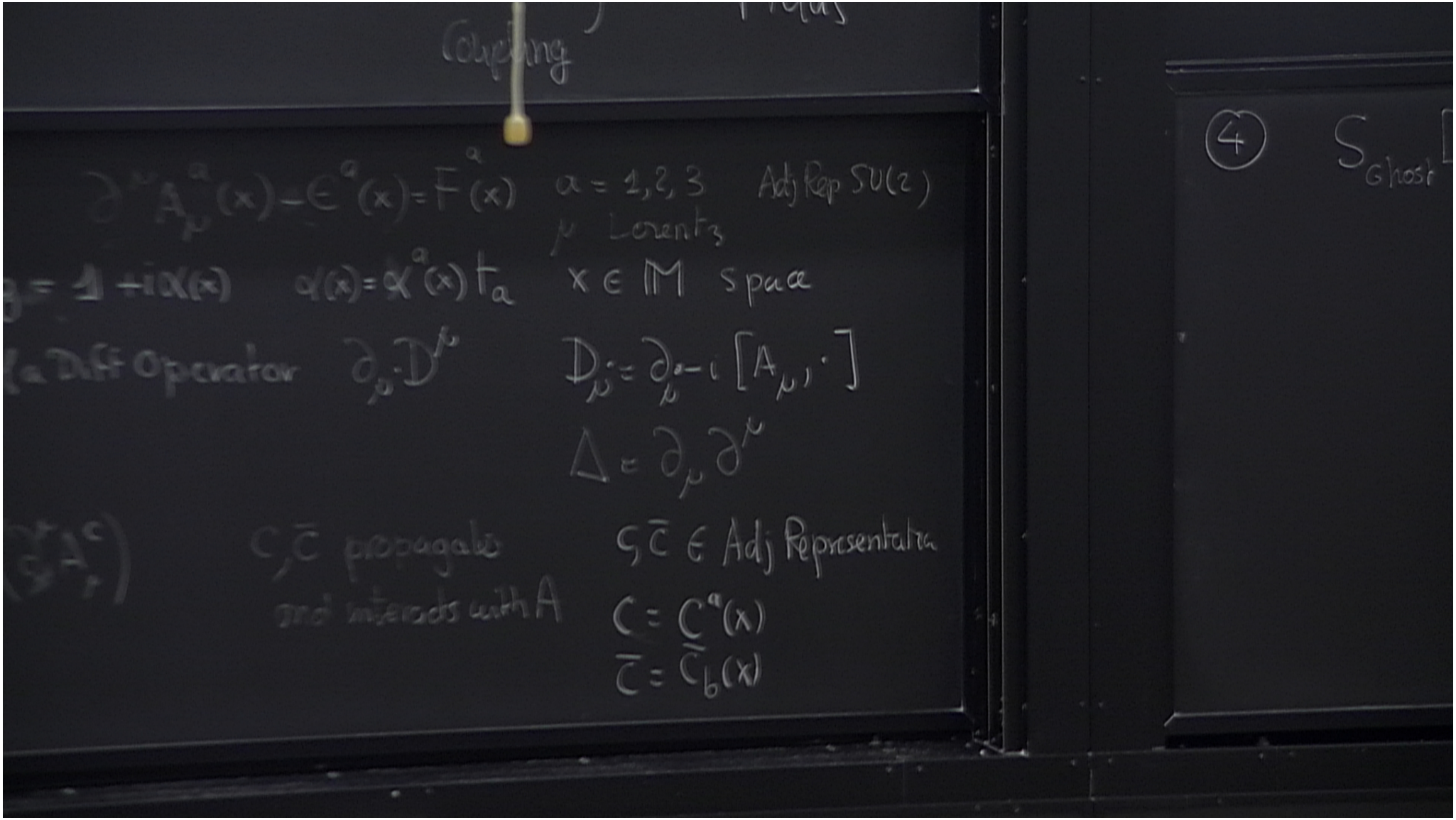
with $g = \mathbb{1} + i\alpha(x)$ $\alpha(x) = \alpha^a(x) t_a$ $x \in M$

kernel of a Diff Operator $\partial_\mu \cdot D^\mu$ $D_\mu = \partial_\mu$

$\Delta =$

$-\epsilon_{abc}\left(\partial^\mu A^c_\mu\right)$

$c, \bar{c}$ propagate and interacts with A


Coupling
$\partial^\mu A^a_\mu(x) - \epsilon^a(x) = F^a(x)$ $a = 1,2,3$ Adj Rep SU(2)
μ Lorentz
$g = 1 + i\alpha(x)$ $\alpha(x) = \alpha^a(x) t_a$ $x \in \mathbb{M}$ space
(a Diff Operator $\partial_\mu D^\mu$ $D_\mu = \partial_\mu - i[A_\mu, \cdot]$
$\Delta = \partial_\mu \partial^\mu$
$c, \bar{c}$ propagate $c, \bar{c} \in$ Adj Representation
and interacts with A
$c = c^a(x)$
$\bar{c} = \bar{c}_b(x)$
(4) S_{Ghost}

μ Lorentz

$\alpha(x) = \alpha^a(x)\, t_a$ $\quad x \in \mathbb{M}$ space

$\partial_\mu \cdot D^\mu$ $\quad D_\mu \cdot = \partial_\mu \cdot - i\,[A_\mu, \cdot\,]$

$\Delta = \partial_\mu \partial^\mu$

$\bar{C}$ propagates $\quad c, \bar{c} \in$ Adj Representation

and interacts with A $\quad C = C^a(x)$

$\bar{C} = \bar{C}_b(x)$

$\Lambda]\ \exp(i\Lambda \cdot F[A])$

tiplier Field

$A] + \bar{c}\, J[A]\, C + \Lambda \cdot F[A]$

(maybe some sign "typos"

feynman

What is J

$J^{a}_{b}(x,y) = \frac{\delta F^{a}}{\delta \alpha^{b}}$

In terms of co

$J_{ab} = \delta$

$C(a)$ C

④ $S_{Ghost}[c, \bar{c}, A] = \int dx \; \bar{c}_a \, \delta^a_b (-\Delta) c^b + \epsilon_{abc} \left(\partial^\mu \bar{c}_a(x)\right) A^b_\mu c^c$

Propagator c ---→--- $\bar{c}$

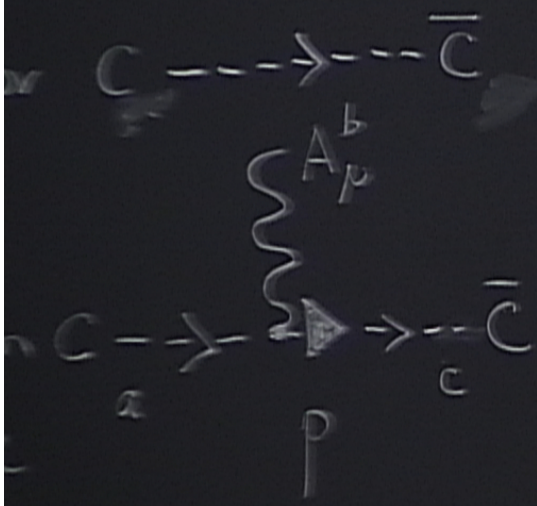
④ $S_{Ghost}[C, \bar{C}, A] = \int dx \; \bar{C}_a \, \delta^a_b (-\Delta) C^b + \epsilon_{abc}$

Propagator of Ghost $C \dashrightarrow \bar{C}$ $\quad \frac{1}{p^2} \cdot \delta_{ab}$

Interaction $C \dashrightarrow \bar{C}$ with A^b_μ $\quad \epsilon_{abc} \, p_\mu$

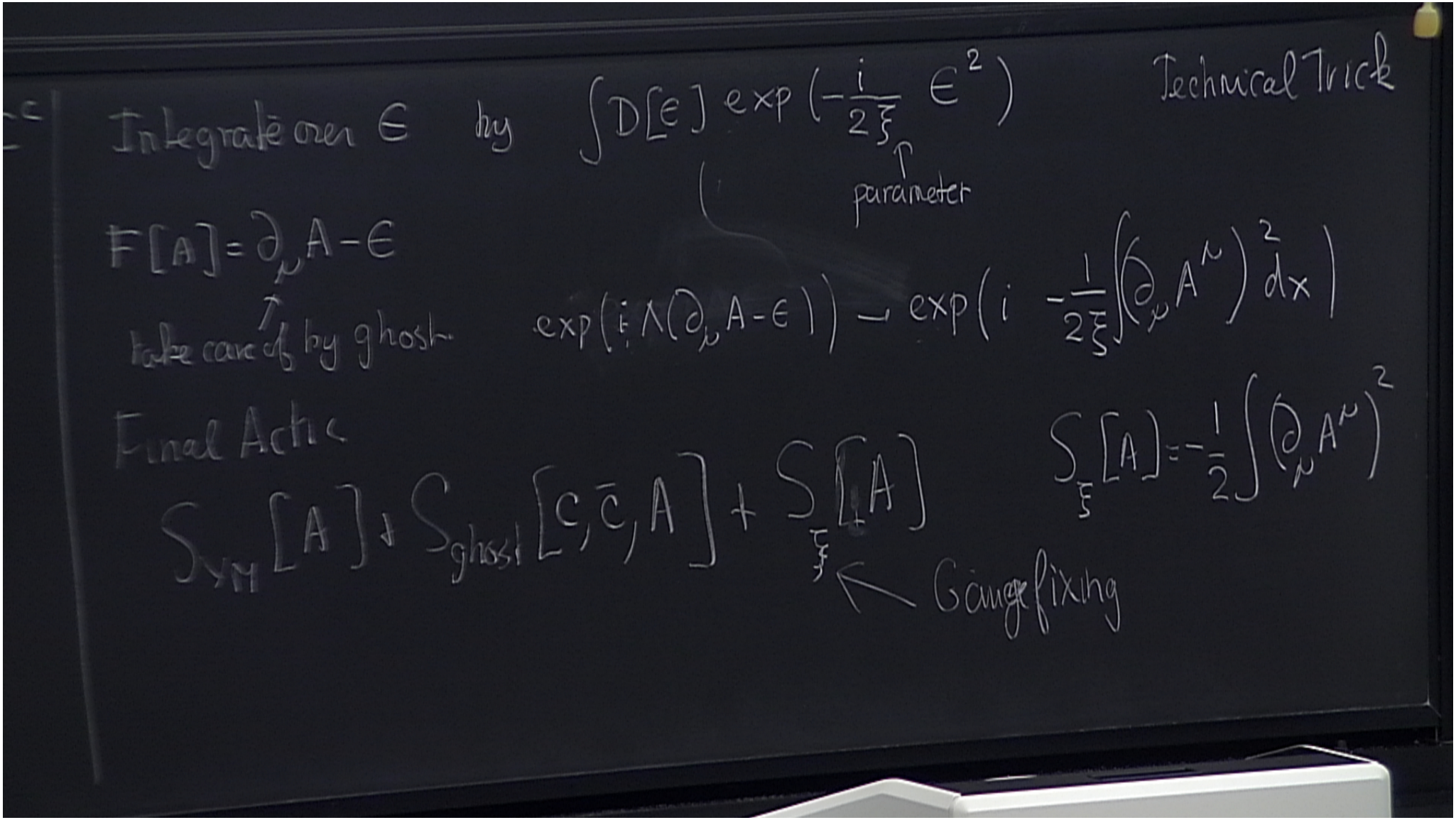
(a, c, p)

$$C, \bar{C}, A] = \int dx \; \bar{C}_a \, \delta^a_b (-\Delta) C^b + \epsilon_{abc} \left(\partial^\mu \bar{C}_a(x) \right) A^b_\mu \cdot C^c$$



$$\frac{1}{p^2} \cdot \delta_{ab}$$

$$\epsilon_{abc} \, p_\mu$$


Integrate over ϵ by ∫D[ϵ] exp(−i/2ξ ϵ²)
Technical Trick
parameter
F[A] = ∂_μ A − ϵ
take care of by ghost
exp(i ∫ Λ(∂_μ A − ϵ)) → exp(i −1/2ξ ∫ (∂_μ A^μ)² dx)
Final Action
S_YM[A] + S_ghost[c, c̄, A] + S_ξ[A]
S_ξ[A] = −1/2 ∫ (∂_μ A^μ)²
Gauge fixing

$$Z = \int D[A] \cdot \left| \det \left[J[A] \right] \right| \delta \left[F_{\text{Lorentz}}[A] - \epsilon \right]$$

indept of ϵ

Add here

choice of gauge

$$1 = \int D[\epsilon] \exp(-\epsilon^2)$$

$$\int D[\epsilon] \exp(-\epsilon^2) \delta \left[F_{\text{Lorentz}} - \epsilon \right] = \exp\left(-\left(F_{\text{Lorentz}} \right)^2 \right)$$

$$F_{\text{Lorentz}} = \partial_\mu A^\nu$$

$$\int dx \left(\partial_\mu A^\nu(x) \right)^2$$

$$\exp\left(-\left(F_{\text{Landau}}\right)^2\right)$$ — compact notation

$$\int dx \left(\partial_\mu A^a_\nu(x) \cdot \partial^\nu A^a_\nu(x)\right) \quad x,\ a$$

$$\epsilon^2 = \int dx\, \epsilon^a(x)\, \epsilon_a(x)$$

$\langle 0 | O_1 \dots O_N | 0 \rangle$

polarizatia e

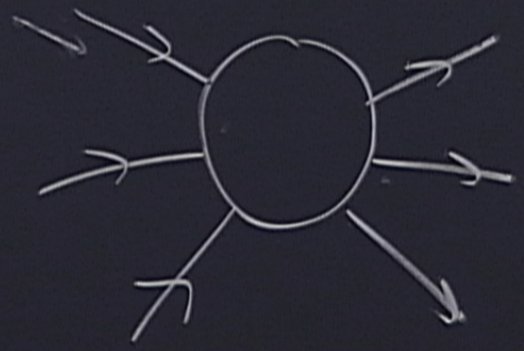
$k^2 = 0$ $\quad e \cdot k = 0$

Gauge Invariant

LSZ

independent of F

in particular of ξ



With this Gauge Fixing : Gauge Action is

$S_{YM}[A] + S_{\xi}[A]$

$S_{YM}[A] = \frac{1}{2g}\int d^4x \, \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu}\right]$

ξ is fixed (gauge fixi

$S_{\xi} + S_{YM} = S_{(2)} + S_{(3)} + S_{(4)}$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$

$S_{(2)} = \frac{1}{2} A_\mu \left(D^{-1}{}^{\mu\nu} - \frac{1}{\xi}\partial^\mu\partial^\nu\right) A_\nu$ $+\frac{1}{\xi} k_\mu k_\nu$

remove the zero mode ;
I does the job

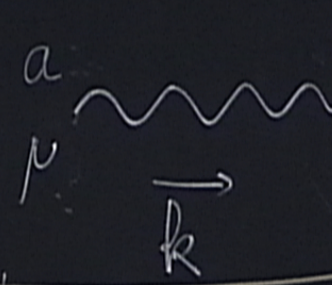
Maxwell

$D^{\mu\nu} = \partial_\mu \partial_\nu - \delta_{\mu\nu}\Delta$ $\quad D^{-1}{}^{\mu\nu}_{\mu\nu} = -k_\mu k_\nu + \delta_{\mu\nu} k^2$ $\quad k_\mu D_{\mu\nu} = 0$ zeromo

ameter)

$$S_\xi[A] = \int dx \frac{1}{2\xi} \partial_\mu A^\mu \partial_\nu A^\nu$$

Feynman Rules Gauge Field



Propagator

$$\delta_{ab} \frac{-i}{k^2 - i0_+} \left(h_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)$$

Feynman Propagator

Prescription to go around the pole

Propagation of a transverse mode

$A_\mu(k) = \epsilon_\mu$ with $\epsilon_\mu k^\mu = 0$

$\epsilon^\mu G_{\mu\nu}(k) = \frac{-i}{k^2} \epsilon_\nu$ does not depend on ξ

QED-like

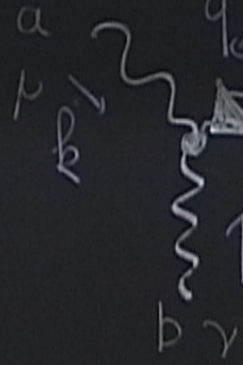
momenta

col

② Couplings between the gauge Fields

$S_{(3)}$ $\partial A \cdot A \cdot A$

3 gauge fields vertex



k g

$\epsilon_{abc}\left(h^{\mu\nu}\right.$

$S_{(4)}$ $AAAA$

4 gauge field vertex



g^2

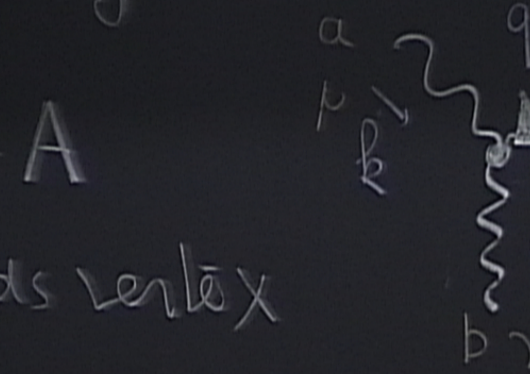
...een the gauge Fields

momenta

coupling constant

A-A-A

...se fields vertex



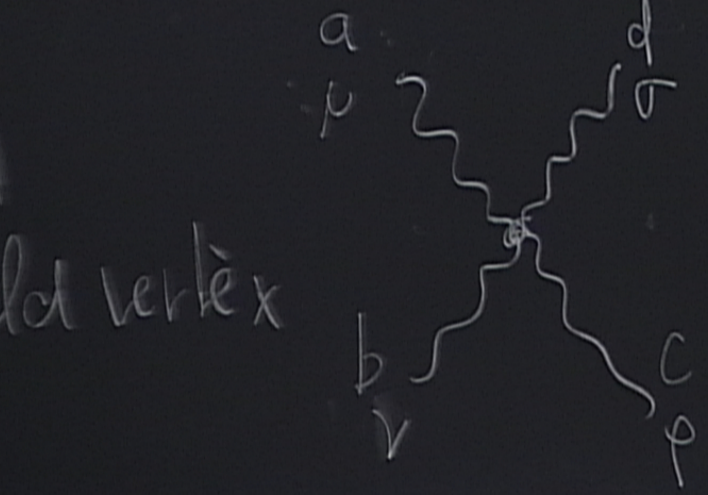
k g

$k+p+q=0$

$\epsilon_{abc}\left(h^{\mu\nu}(k-p)^{\rho} + \text{//} + \text{//}\right)$

A A A

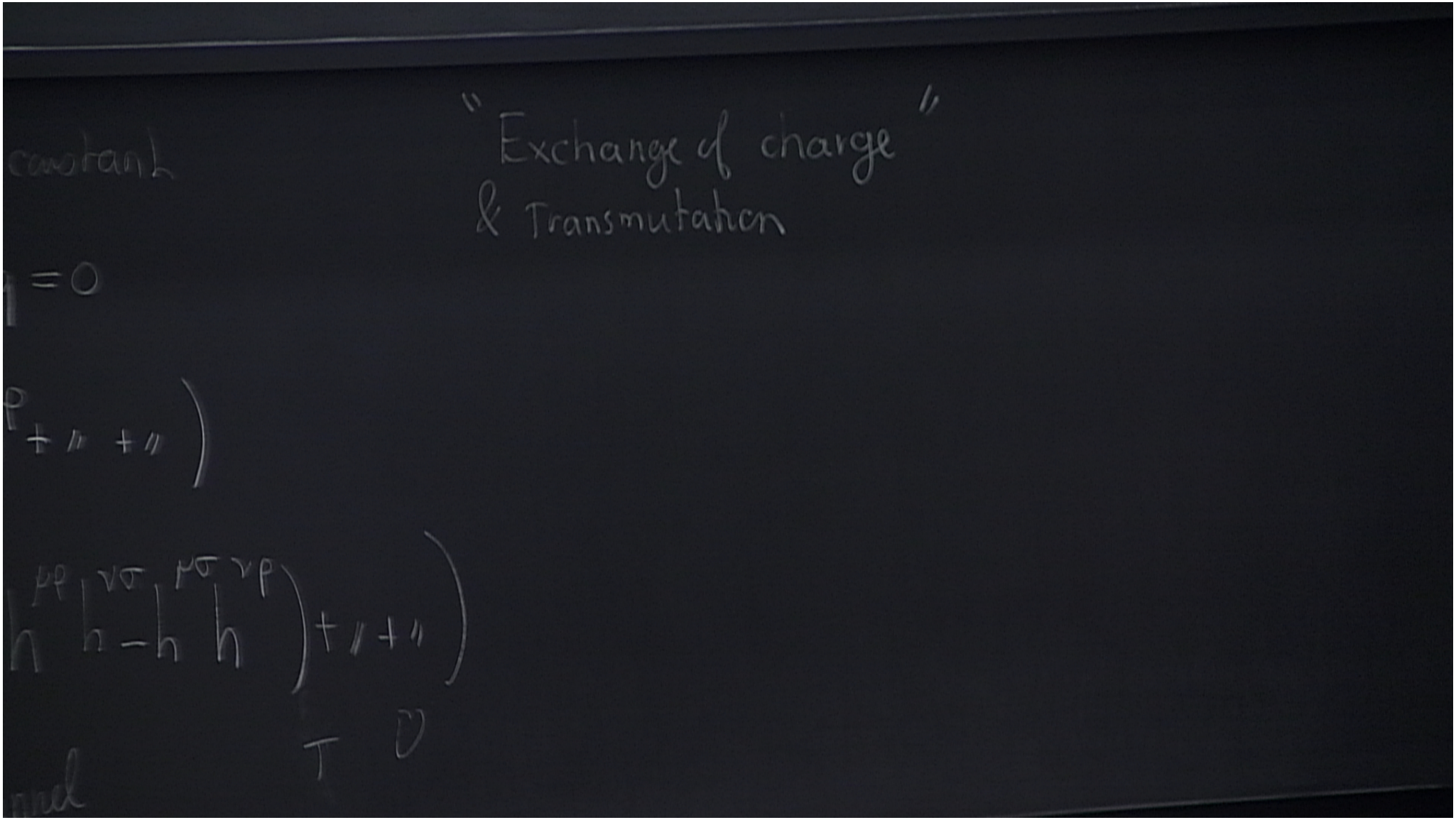
...se field vertex



$g^2\left(\epsilon_{abe}\epsilon_{cde}\left(h^{\mu\rho}h^{\nu\sigma} - h^{\mu\sigma}h^{\nu\rho}\right) + \text{//} + \text{//}\right)$

S-channel

T U


"Exchange of charge"
& Transmutation
constant
=0

charge "

Coupling with Matter Dirac Fermions

$\psi, \bar{\psi} \in$ Fundamental Representation

$\psi = \psi^i_\alpha(x)$ $i = 1, 2$ Fund. of $SU(2)$

$\alpha = 1..4$ Dirac Indices Spinor

Action $S_{Fermions} = \int dx\ \bar{\psi}(i \not{D} - m)\psi$

$\not{D} = \gamma^\mu D_\mu$ D_μ the covariant derivation.

charge

$\psi, \bar{\psi} \in$ Fundamentar Repesentahon

$\psi = \psi^i_\alpha(x)$ $i = 1, 2$ Fund. of $SU(2)$

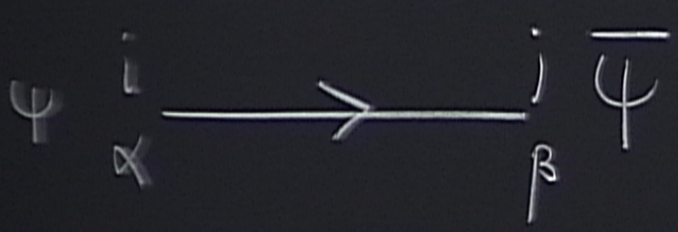
$\alpha = 1..4$ Dirac Induces Spinor

Action $S_{Fermions} = \int dx\ \bar{\psi}(i \not{D} - m)\psi$

$\not{D} = \gamma^\mu D_\mu$ D_μ the covariant derivation.

$D_\mu = \partial_\mu - i A_\mu$ on Fund. Repesentahs

Fermion Propagator

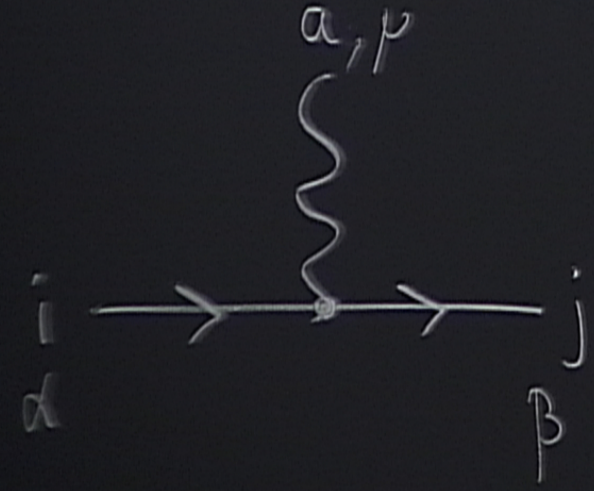


$$\left(\frac{i}{\not{p} - m}\right)_{\alpha\beta} \cdot \delta_{ij}$$

charge is conserved

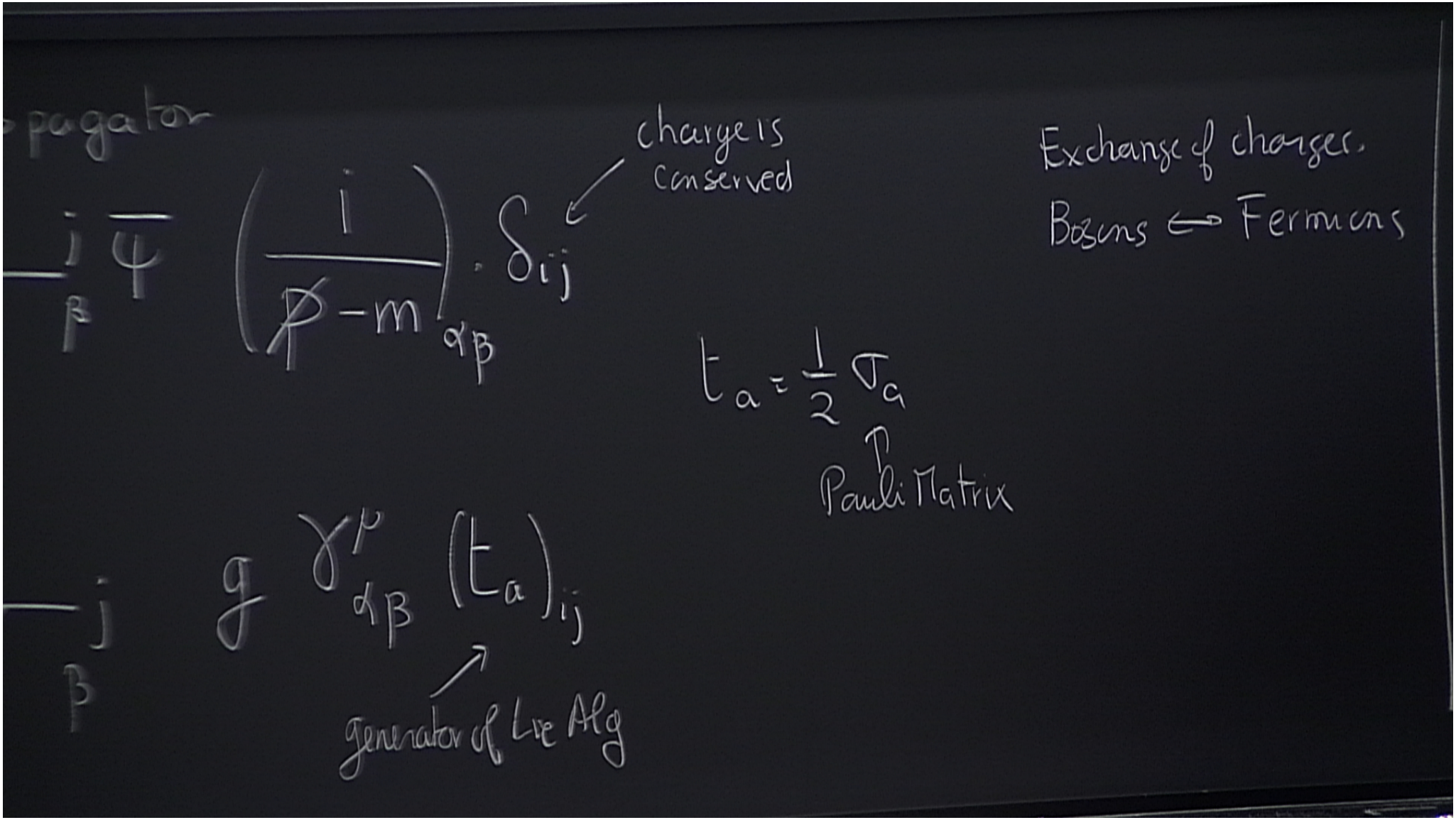
$$t_a = \frac{1}{2}\sigma_a$$

Pauli Matrix



$$g\,\gamma^{\mu}_{\alpha\beta}\,(t_a)_{ij}$$

generator of Lie Alg


pagator
charge is conserved
Exchange of charges.
Bosons ⟷ Fermions
$\frac{i}{\not p - m}_{\alpha\beta} \cdot \delta_{ij}$
$t_a = \frac{1}{2}\sigma_a$
Pauli Matrix
$g\,\gamma^\mu_{\alpha\beta}\,(t_a)_{ij}$
generator of Lie Alg