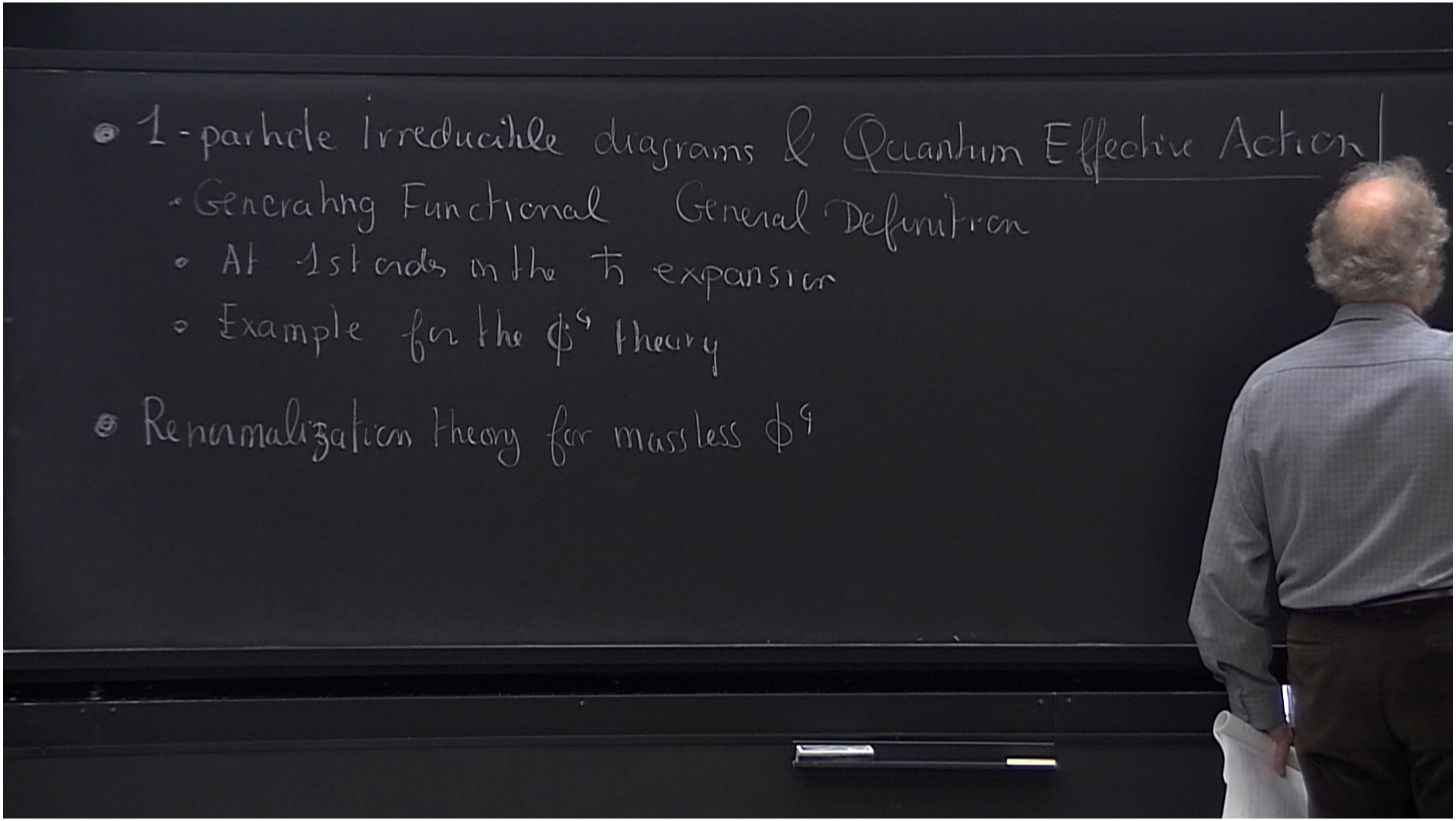
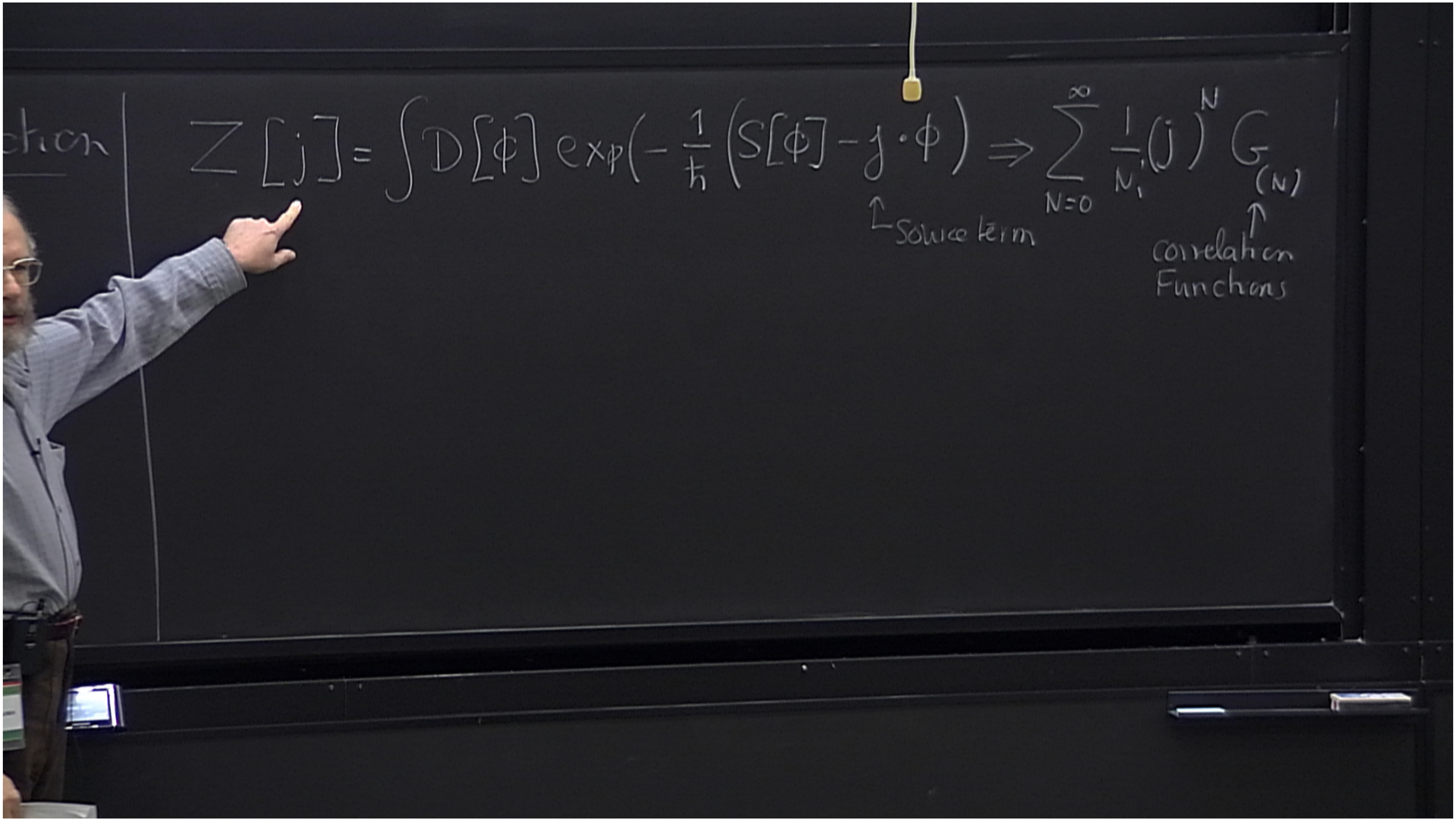
Title: PSI 2016/2017 Quantum Field Theory II - Lecture 7

Date: Nov 15, 2016 09:00 AM

URL: http://pirsa.org/16110007

Abstract:


1-particle irreducible diagrams & Quantum Effective Action
Generating Functional General Definition
At 1st order in the $\hbar$ expansion
Example for the ϕ^4 theory
Renormalization theory for massless ϕ^4


$Z[j] = \int D[\phi] \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right) \Rightarrow \sum_{N=0}^{\infty} \frac{1}{N!}(j)^N G_{(N)}$
Source term
correlation Functions

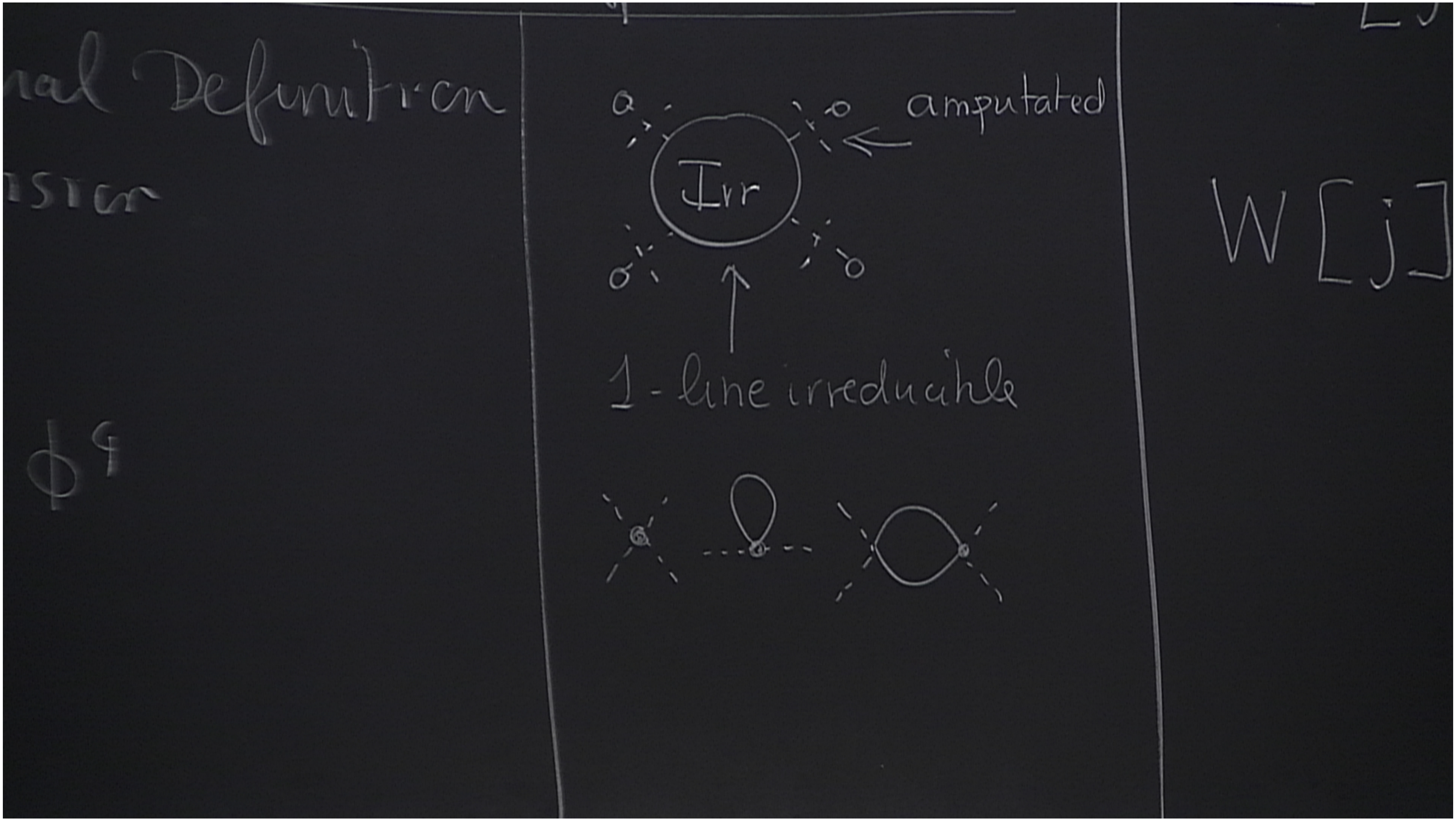
ction

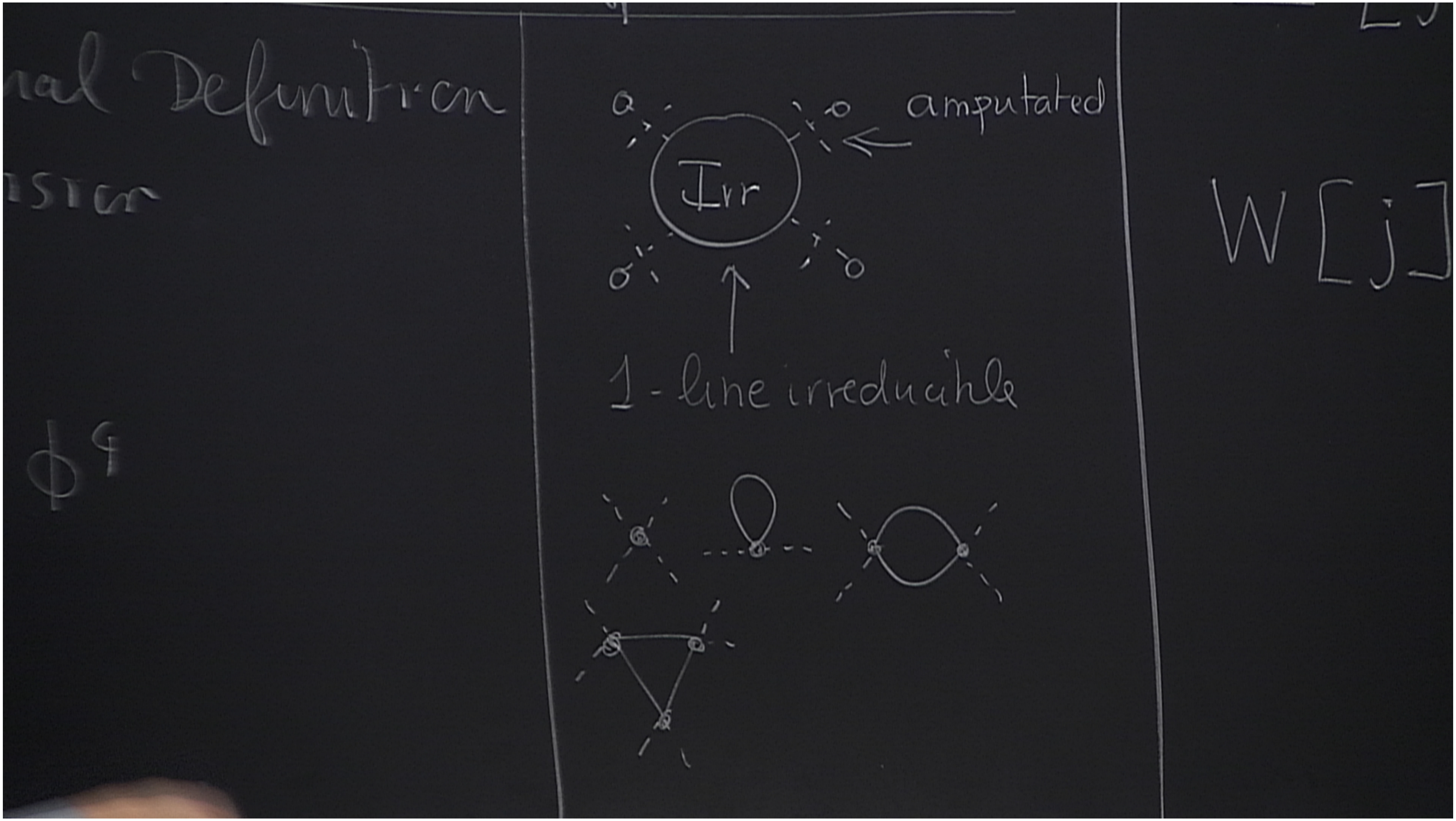
$$Z[j] = \int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right) \Rightarrow \sum_{N=0}^{\infty} \frac{1}{N!}(j)^N G_{(N)}$$

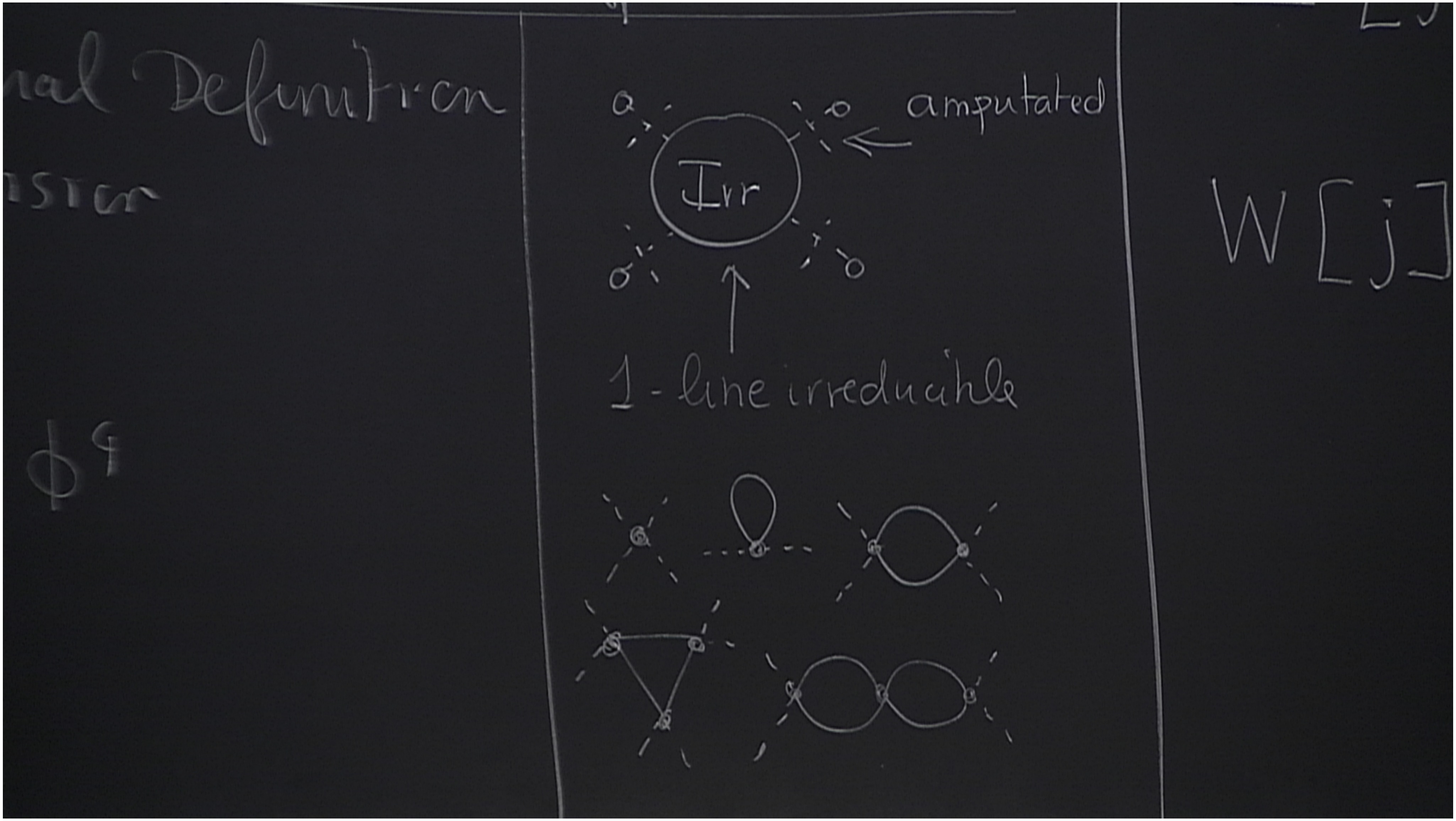
source term

correlation Functions

$$W[j] = \hbar \,\mathrm{Log}\left[Z[j]\right] \Rightarrow \sum_{N} \frac{1}{N!}(j)^N G^{\text{CONNECTED}}_{(N)}$$


amputated
Irr
1-line irreducible
W[j]
ϕ^4


Definition
Irr
amputated
1-line irreducible
ϕ4
W[j]


Definition
amputated
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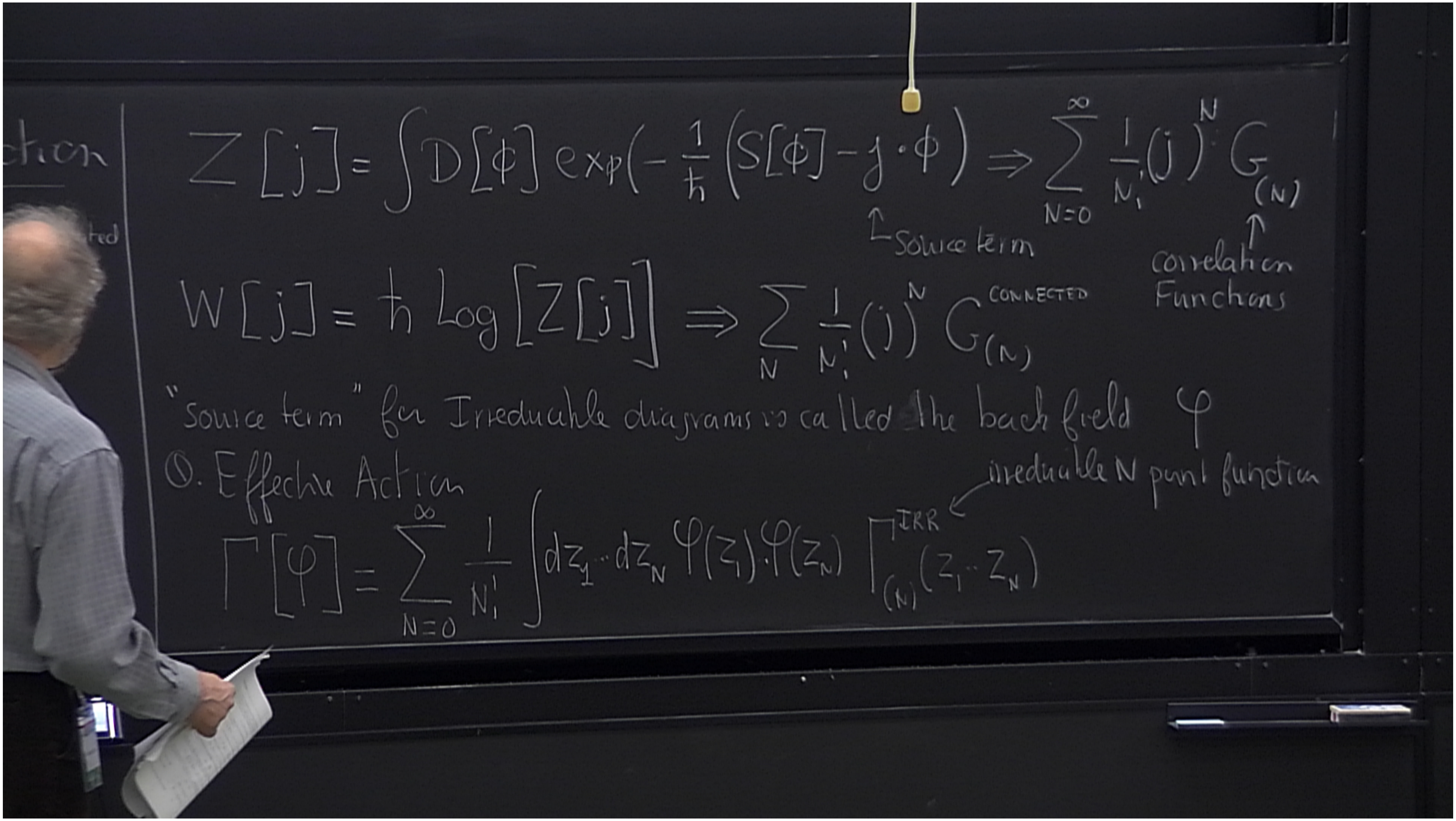
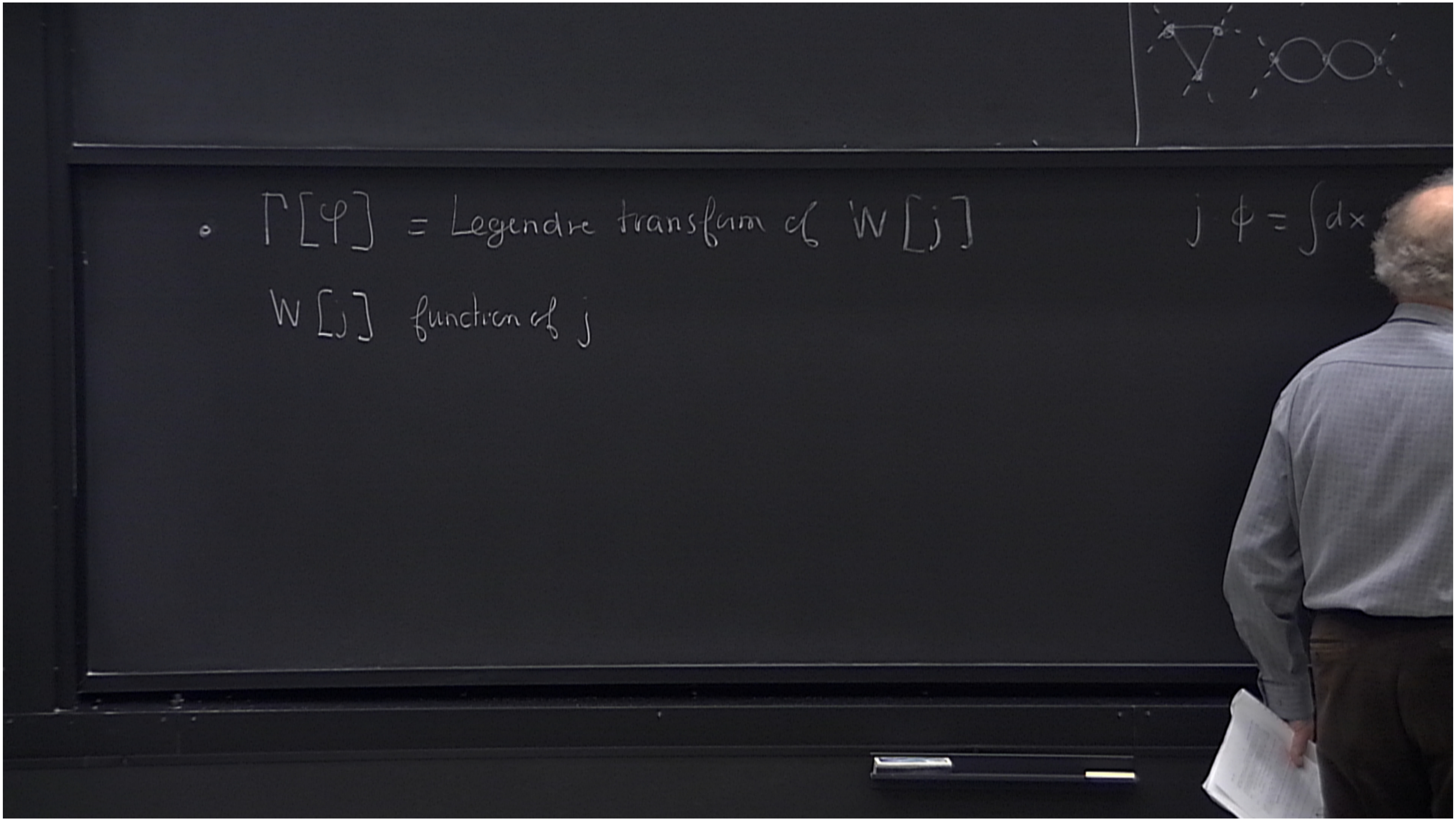
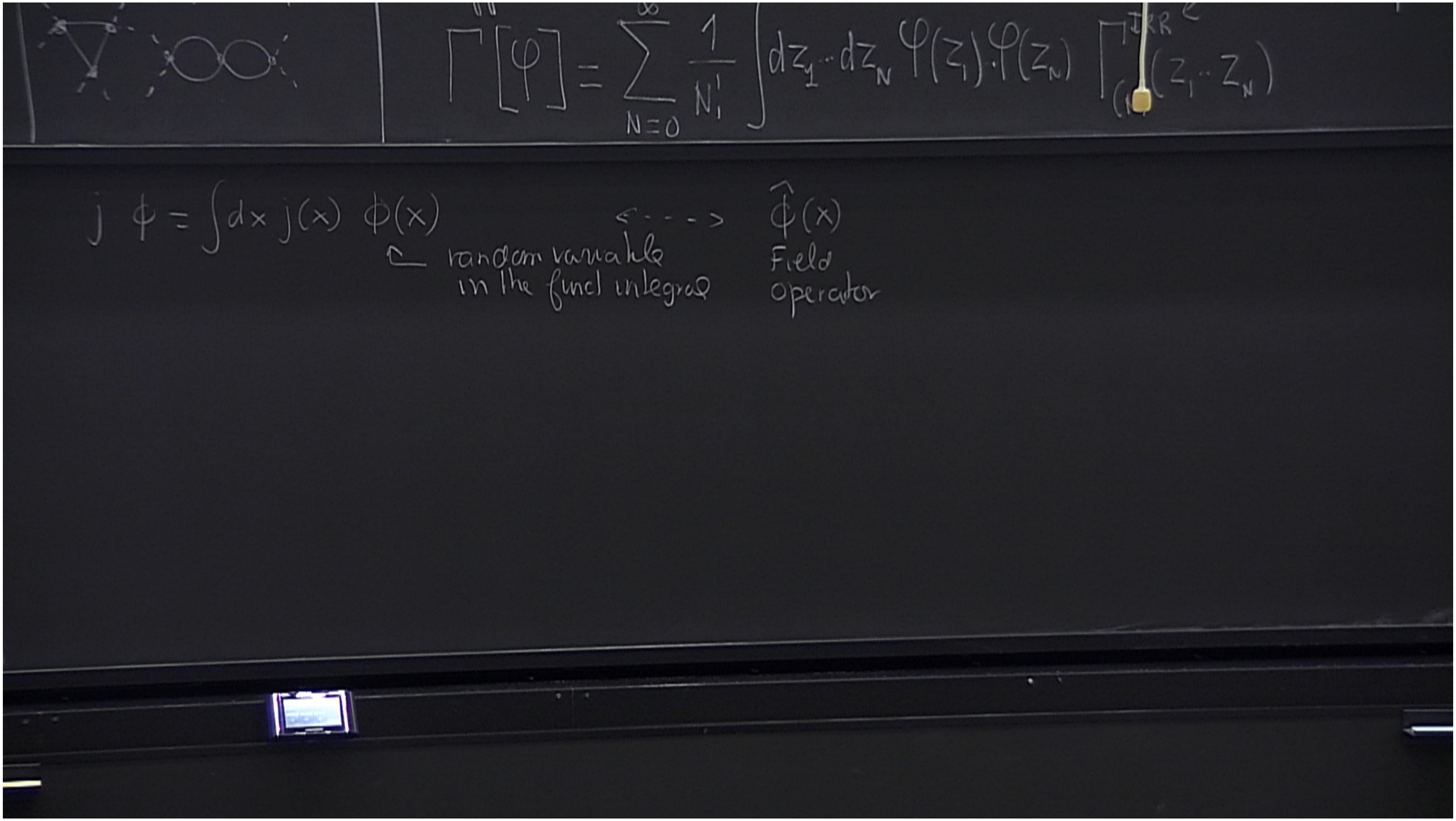

ction
ted
$Z[j] = \int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right) \Rightarrow \sum_{N=0}^{\infty} \frac{1}{N!} (j)^N G_{(N)}$
Source term
correlation Functions
$W[j] = \hbar \,\mathrm{Log}\left[Z[j]\right] \Rightarrow \sum_{N} \frac{1}{N!} (j)^N G_{(N)}^{\mathrm{CONNECTED}}$
"source term" for Irreducible diagrams is called the back field φ
①. Effective Action
irreducible N point function
$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N\, \varphi(z_1) \cdots \varphi(z_N)\, \Gamma^{\mathrm{IRR}}_{(N)}(z_1 \cdots z_N)$


Γ[φ] = Legendre transform of W[j]
W[j] function of j
j φ = ∫dx



$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N \, \varphi(z_1) \cdots \varphi(z_N) \, \Gamma^{IRR}_{(N)}(z_1 \cdots z_N)$
$j\phi = \int dx \, j(x) \, \phi(x) \quad \longleftrightarrow \quad \hat{\phi}(x)$
random variable
in the funct integral
Field
operator

• $\Gamma[\varphi]$ = Legendre transform of $W[j]$ $\qquad j\phi = \int dx\, j(x)$

$W[j]$ function of j. $\quad \dfrac{\delta W[j]}{\delta j(x)}$

$\Gamma[\varphi$

$\Gamma[\varphi] =$ Legendre transform of $W[j]$

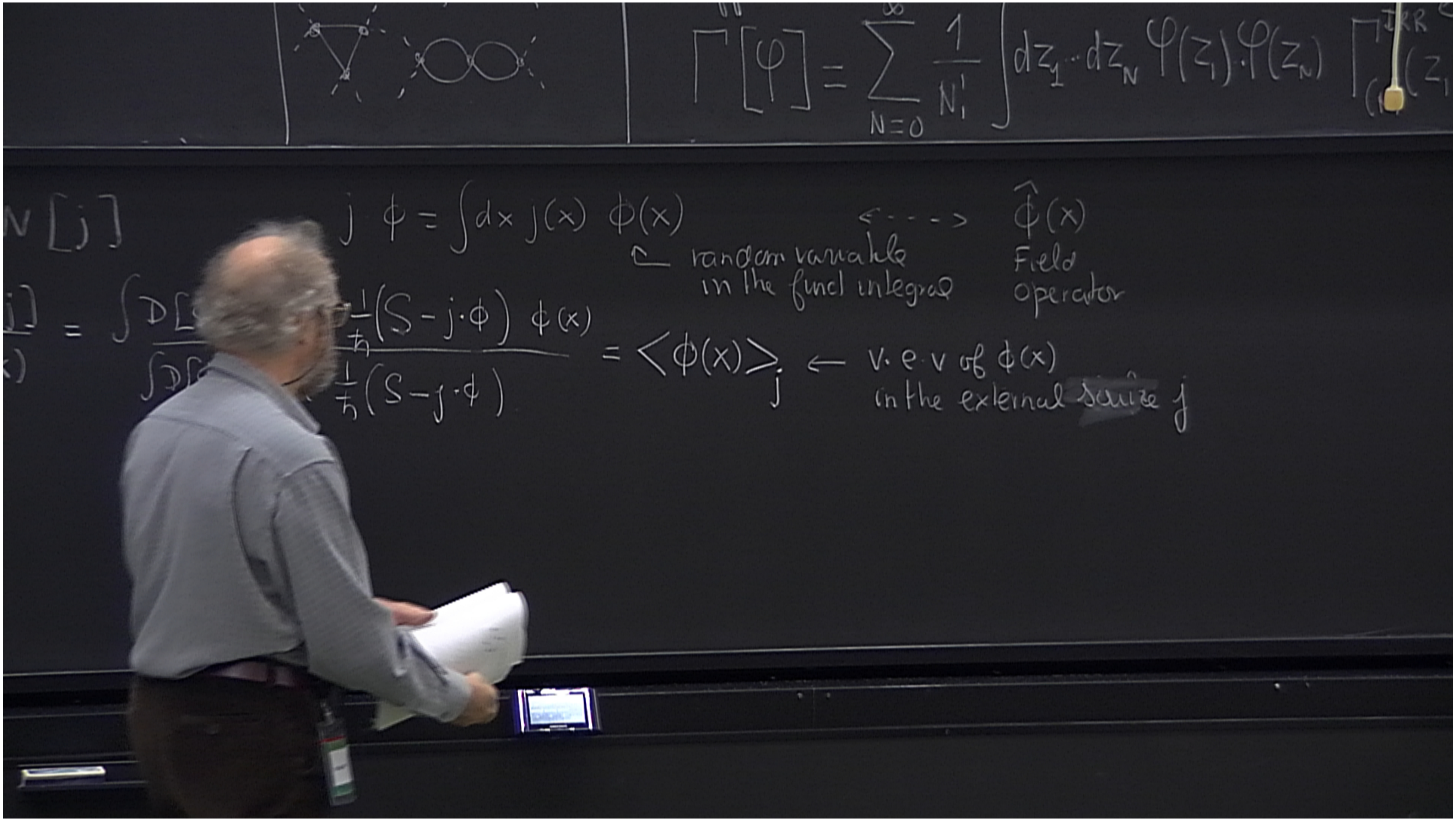
$j \cdot \phi = \int dx \, j(x) \, \phi(x)$

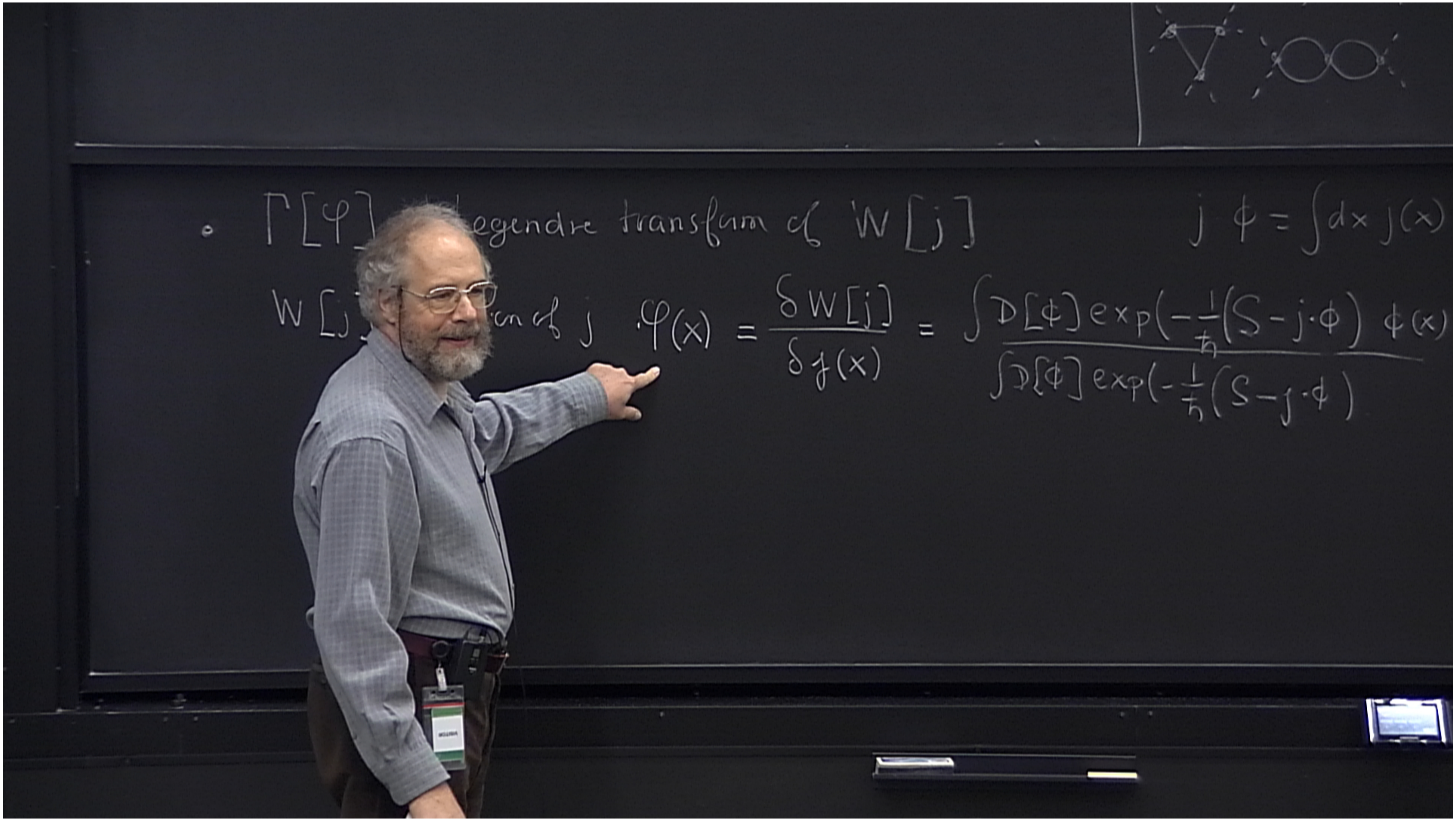
ran

in

$W[j]$ function of j.

$$\frac{\delta W[j]}{\delta j(x)} = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S - j\cdot\phi)\right) \phi(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S - j\cdot\phi)\right)} = \langle \ldots$$


Γ[φ] = Σ_{N=0}^∞ 1/N! ∫dz_1 ⋯ dz_N φ(z_1)⋯φ(z_N) Γ^{IRR}
j φ = ∫dx j(x) φ(x)
φ̂(x)
random variable
in the funcl integral
Field
operator
= ⟨φ(x)⟩_j
v.e.v of φ(x)
in the external source j


Γ[φ]
egendre transfum of W[j]
j φ = ∫dx j(x)
W[j]
φ(x) = δW[j] / δj(x) = ∫D[φ] exp(− 1/ħ (S − j·φ)) φ(x) / ∫D[φ] exp(− 1/ħ (S − j·φ))

$$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N \, \varphi(z_1) \cdots \varphi(z_N) \, \Gamma^{IRR}_{(N)}(z_1 \cdots z_N)$$

$x \, j(x) \, \phi(x)$ <- - - -> $\hat{\phi}(x)$

random variable in the funct integral

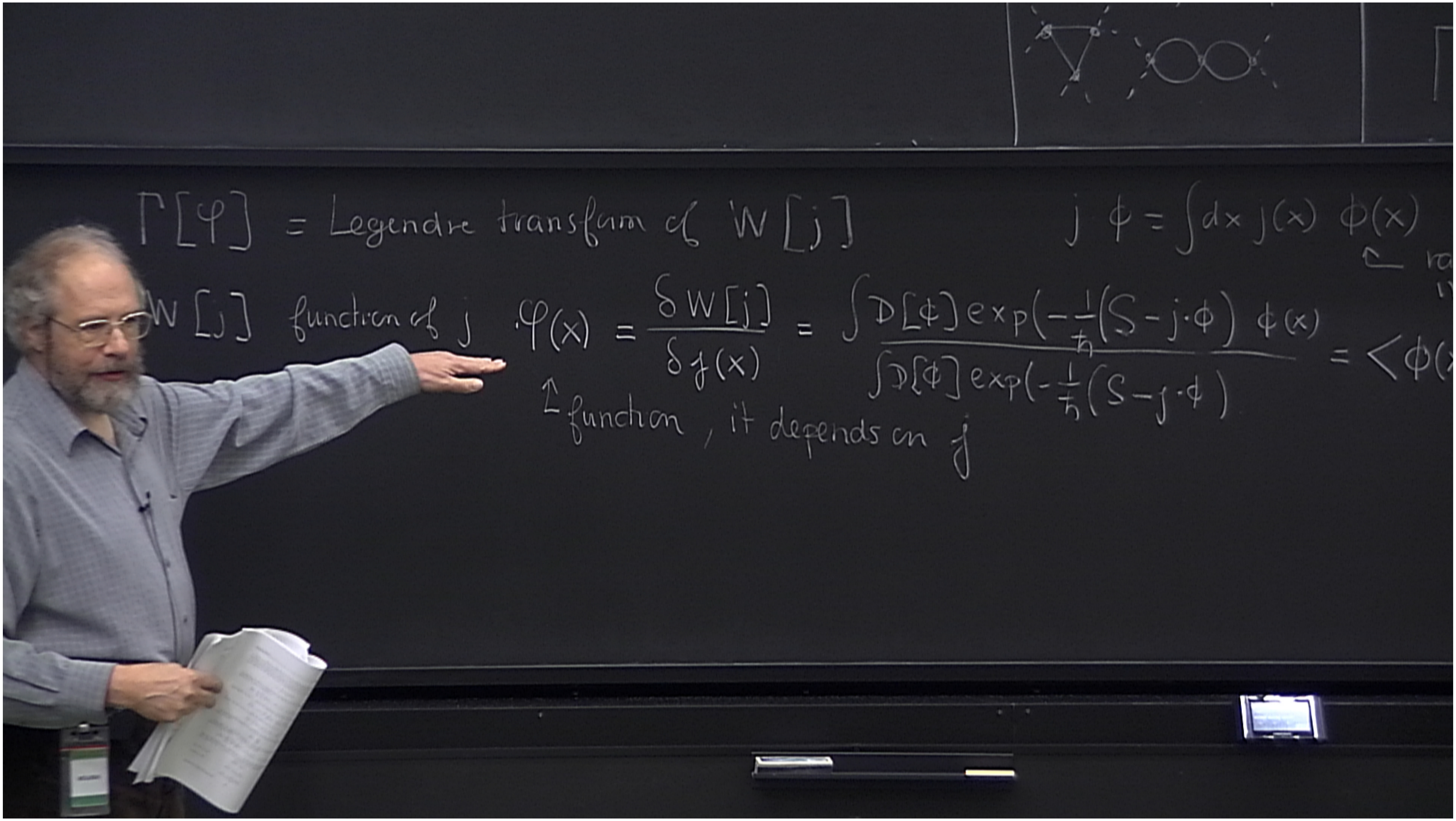
Field operator

ϕ = \phi

φ = \varphi

$\phi(x)$

$= \langle \phi(x) \rangle_j \leftarrow$ v.e.v of $\phi(x)$ in the external ~~source~~ j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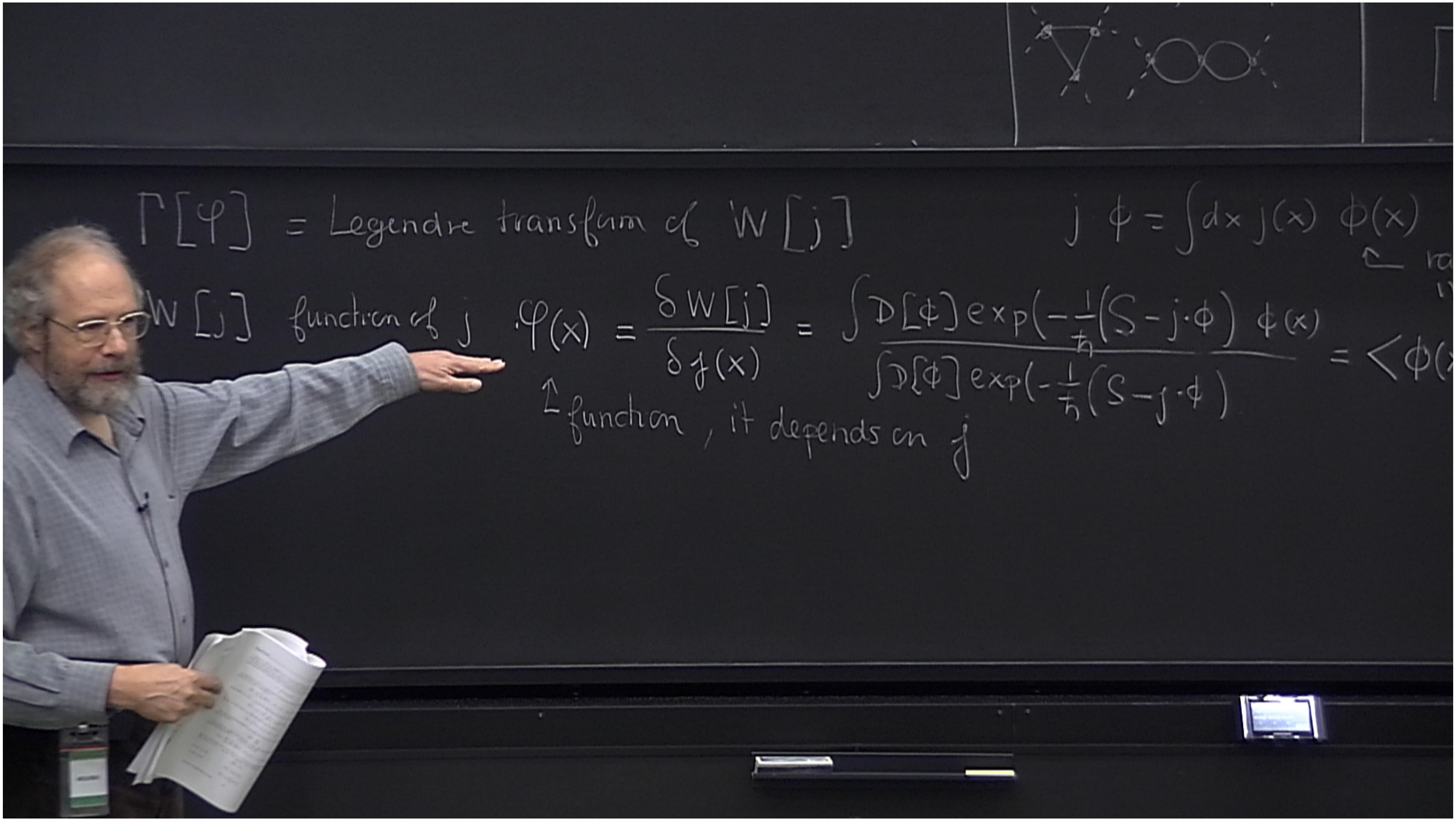
j φ = ∫dx j(x) φ(x)
W[j] function of j
φ(x) = δW[j] / δj(x) = ∫D[φ] exp(−1/ħ (S − j·φ)) φ(x) / ∫D[φ] exp(−1/ħ (S − j·φ) = ⟨φ(
function, it depends on j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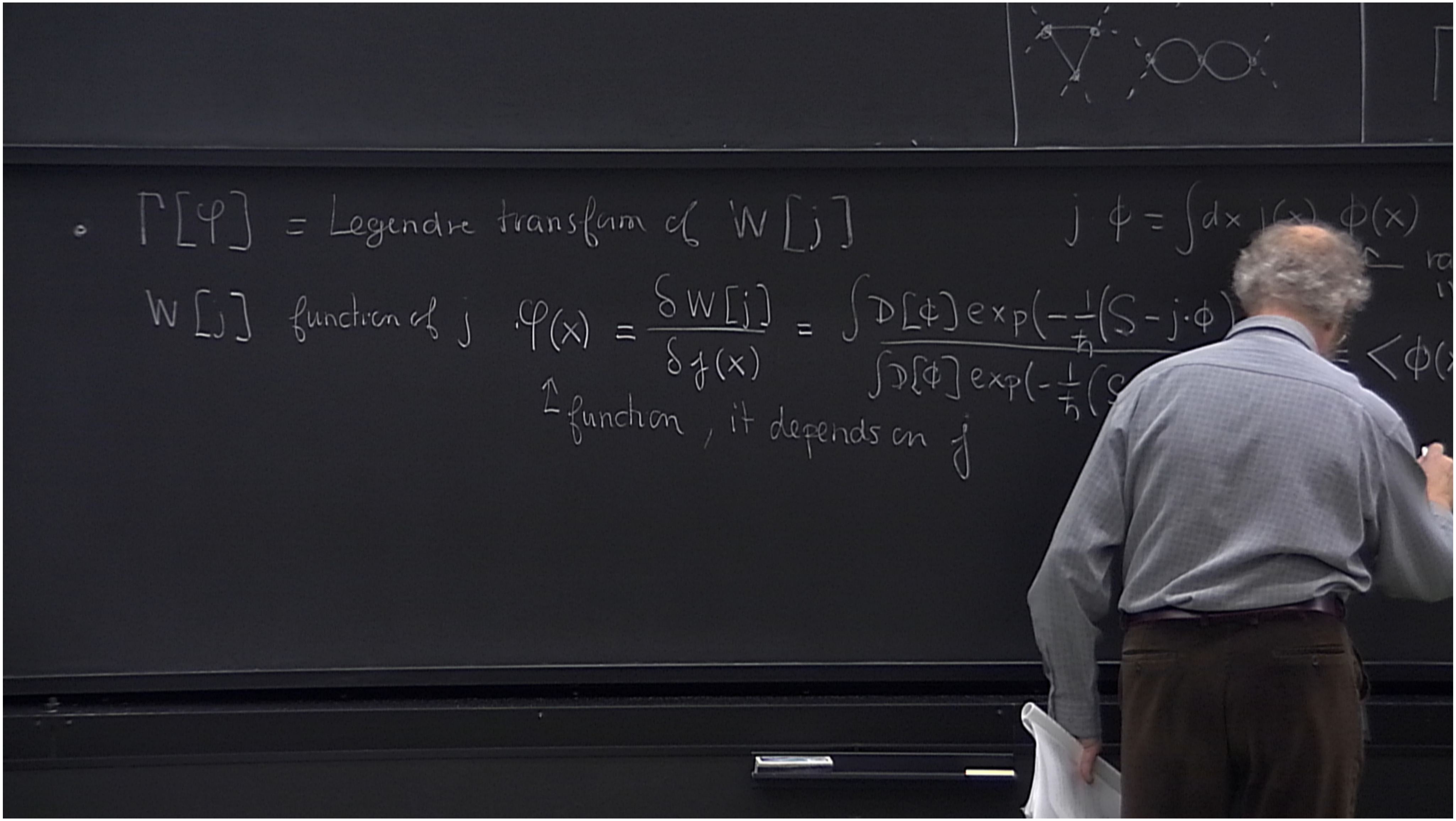

Γ[φ] = Legendre transform of W[j]
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φ(x) = δW[j] / δj(x) = ∫D[ϕ] exp(−1/ħ (S − j·ϕ)) / ∫D[ϕ] exp(−1/ħ (S
function, it depends on j

$$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N$$

...re transform of $W[j]$

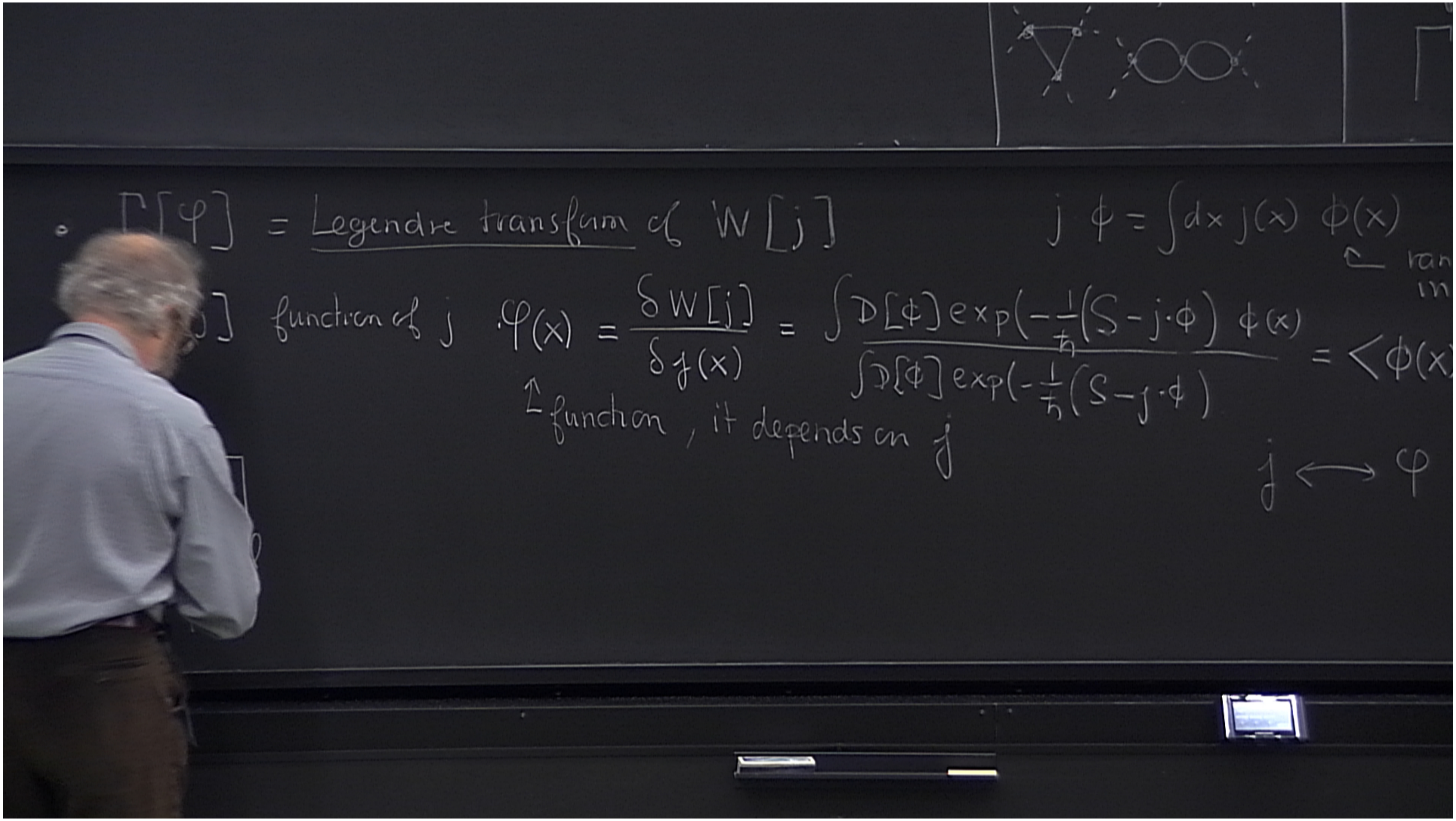
$$j \cdot \phi = \int dx \, j(x) \, \phi(x)$$

random variable in the funct integral $\longleftrightarrow$ $\hat{\phi}(x)$ Field operator

$$j \quad \varphi(x) = \frac{\delta W[j]}{\delta j(x)} = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S - j\cdot\phi)\right) \phi(x)}{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S - j\cdot\phi)\right)} = \langle \phi(x) \rangle_j \leftarrow \text{v.e.v of } \phi(x) \text{ in the external sou...}$$

function, it depends on j

$$j \longrightarrow \varphi$$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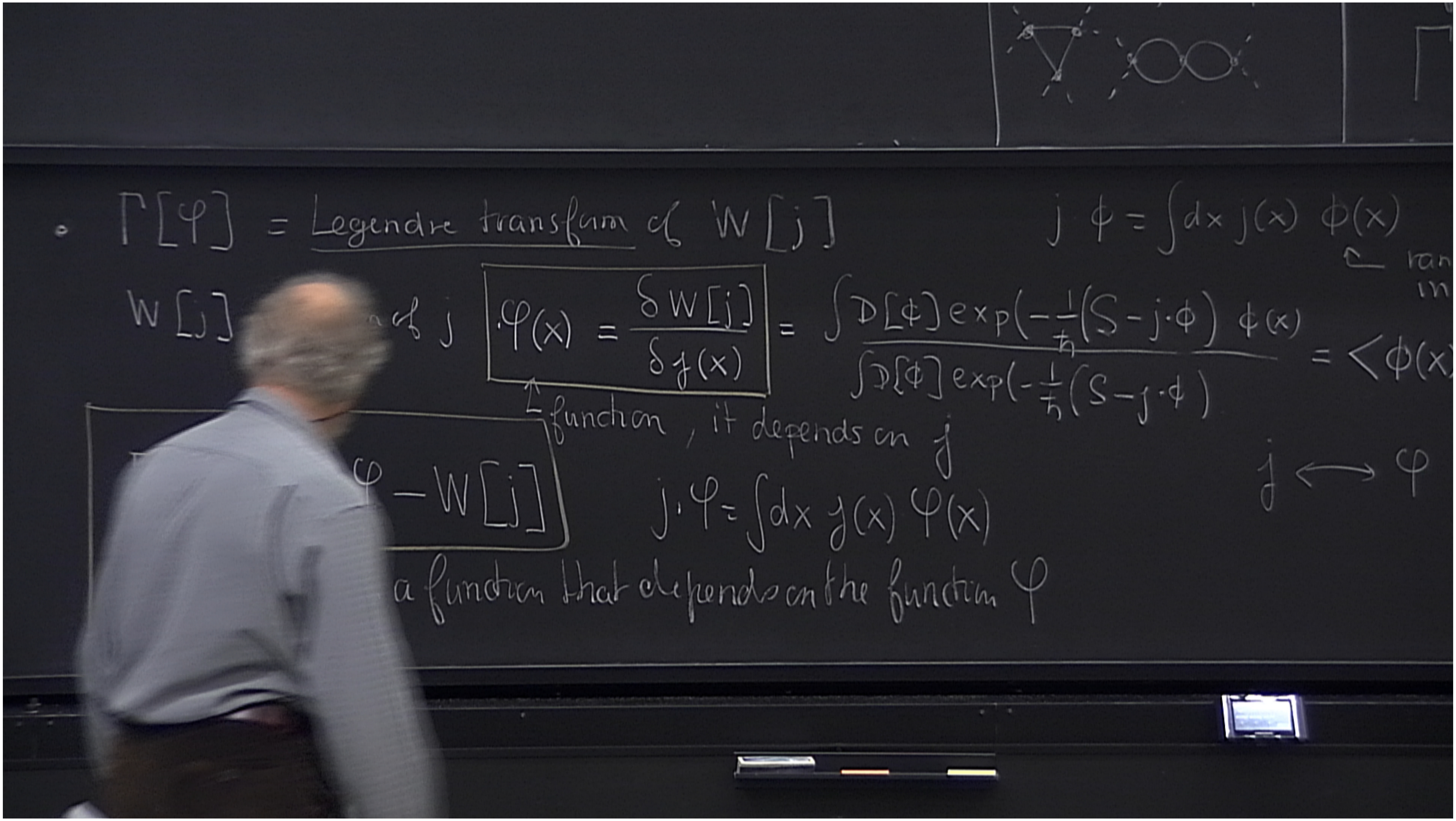
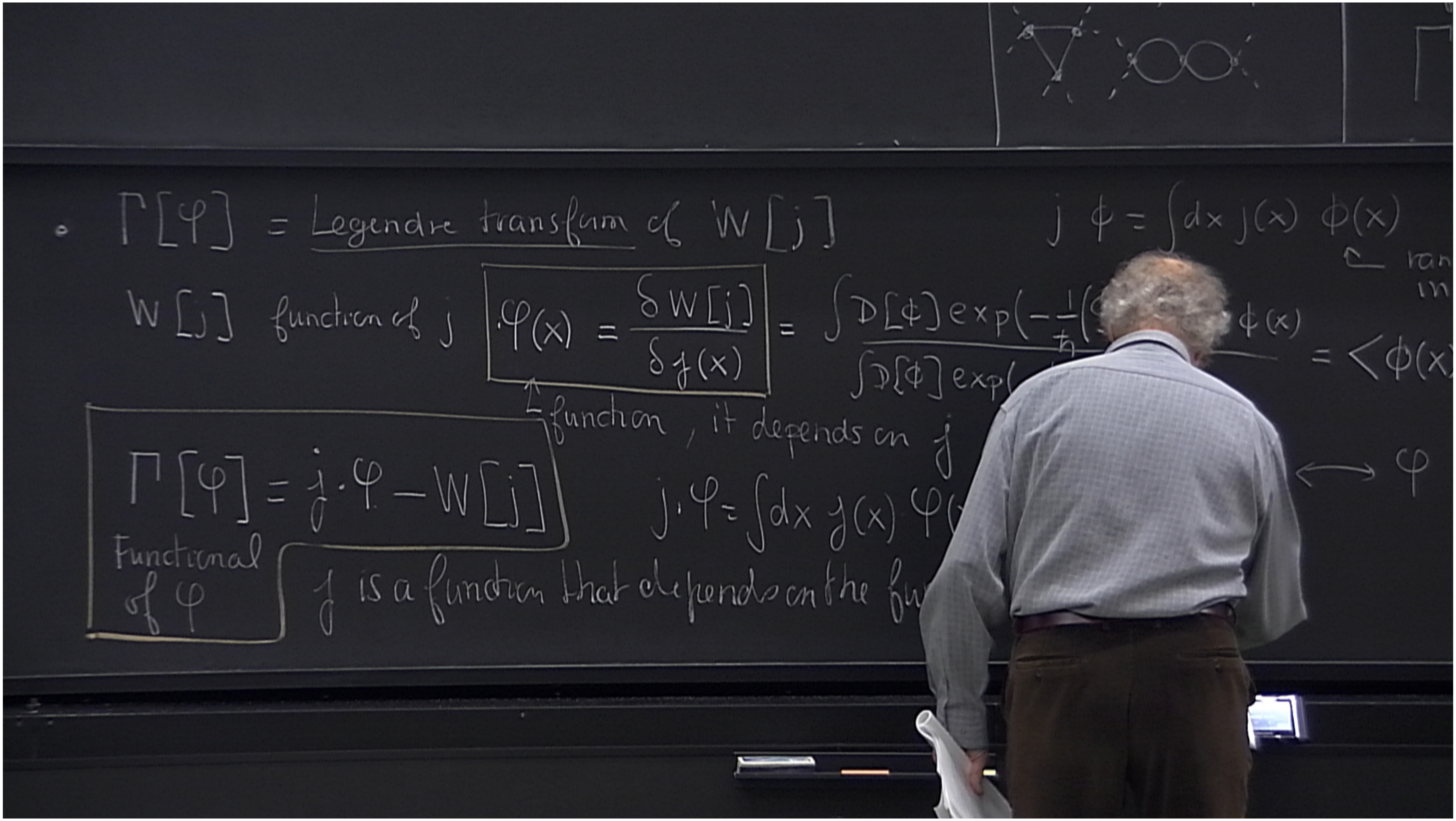
j φ = ∫dx j(x) φ(x)
function of j
φ(x) = δW[j] / δj(x) = ∫D[φ] exp(−1/ħ (S − j·φ)) φ(x) / ∫D[φ] exp(−1/ħ (S − j·φ)) = <φ(x)
function, it depends on j
j ⟷ φ



Γ[φ] = Legendre transform of W[j]
j φ = ∫dx j(x) φ(x)
φ(x) = δW[j] / δj(x) = ∫D[φ] exp(−1/ħ(S − j·φ) φ(x) / ∫D[φ] exp(−1/ħ(S − j·φ)
function, it depends on j
j ⟷ φ
−W[j]
j·φ = ∫dx j(x) φ(x)
a function that depends on the function φ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W[j] function of j
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function, it depends on j
Γ[φ] = j·φ − W[j]
j·φ = ∫dx j(x) φ(x)
Functional of φ
j is a function that depends on the fu

$$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N \, \varphi(z_1)\cdots\varphi(z_N) \, \Gamma^{IRR}_{(1}(z_1 \cdots z_N)$$

$j \cdot \phi = \int dx \, j(x) \, \phi(x)$ <- - - -> $\hat{\phi}(x)$

random variable in the funct integral

Field operator

ϕ = \phi

φ = \varphi

$$\frac{\int e^{-\frac{1}{\hbar}(S - j\cdot\phi)} \, \phi(x)}{\int e^{-\frac{1}{\hbar}(S - j\cdot\phi)}} = \langle \phi(x) \rangle_j \leftarrow$$

v.e.v of $\phi(x)$ in the external source j

$$j \longleftrightarrow \varphi$$

φ φ background field

$$\Gamma[\varphi] = \sum_{N=0}^{\infty} \frac{1}{N!} \int dz_1 \cdots dz_N \, \varphi(z_1) \cdots \varphi(z_N) \, \Gamma^{IRR}(z_1 \cdots z_N)$$

$j\,\phi = \int dx\, j(x)\, \phi(x)$ $\quad \longleftarrow \cdots \longrightarrow \quad$ $\hat{\phi}(x)$

random variable in the funcl integral — Field operator

$\phi =$ \phi

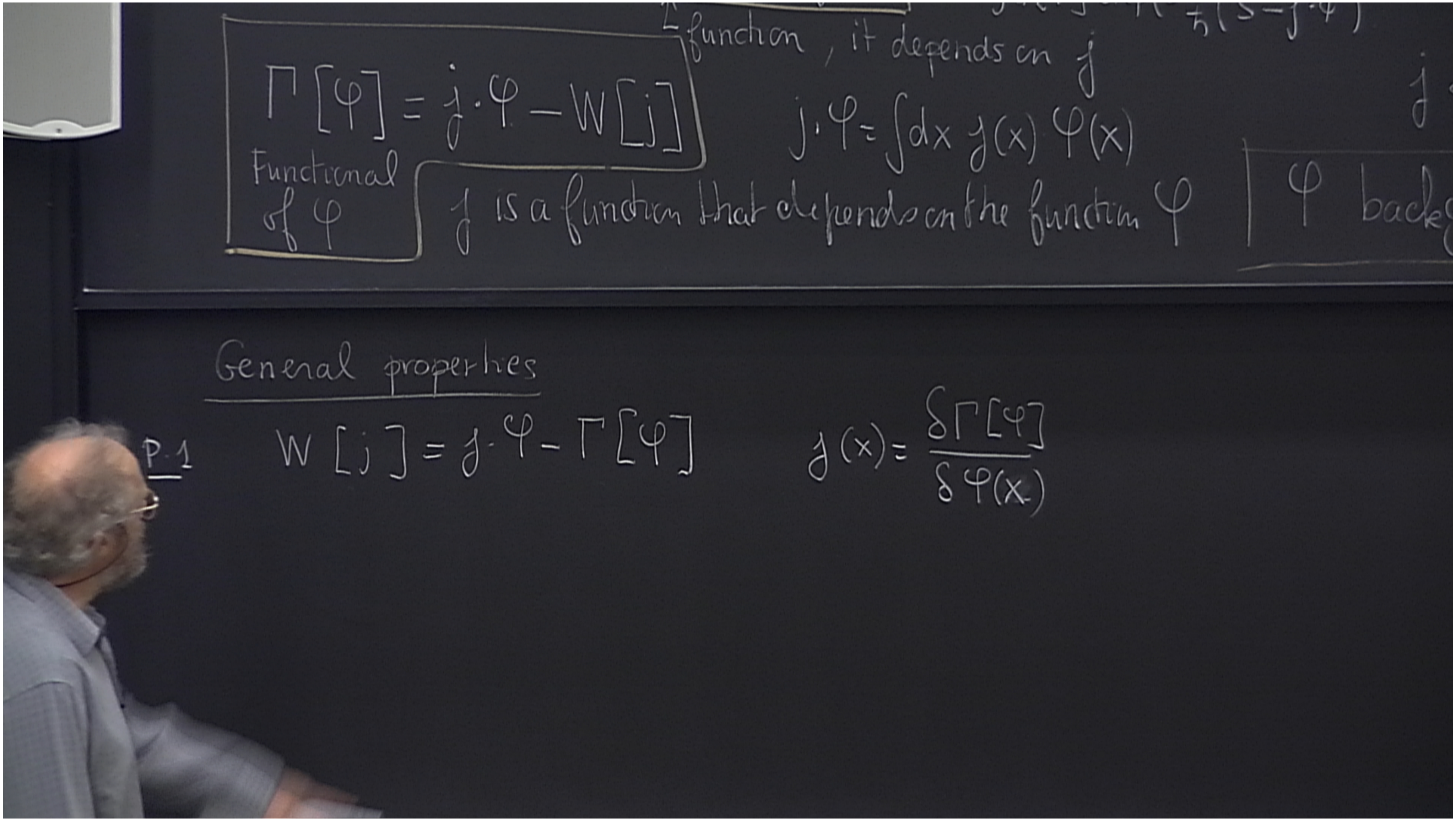
$\varphi =$ \varphi

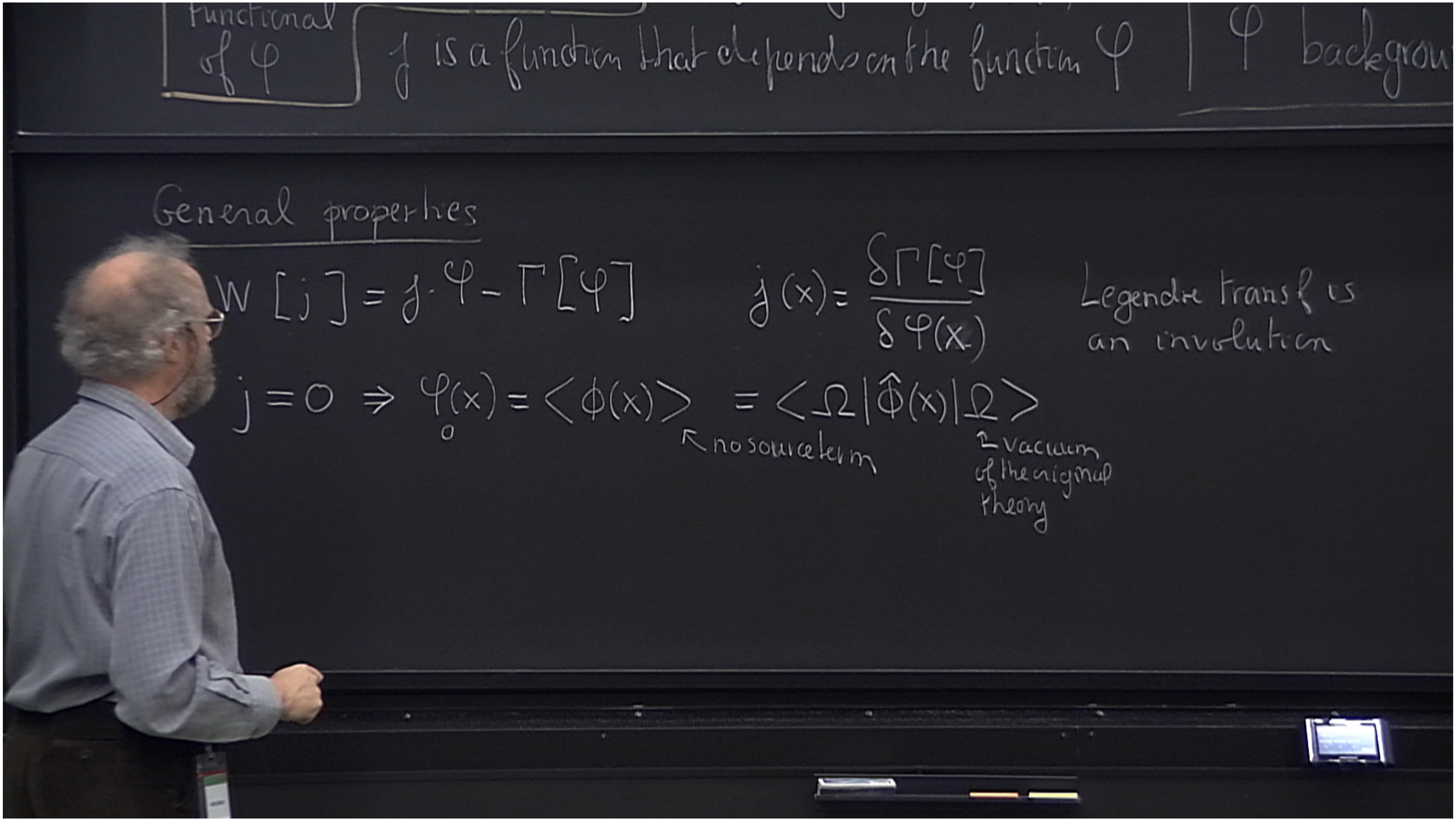
$$\frac{\cdots e^{-\frac{1}{\hbar}(S - j\cdot\phi)}\, \phi(x)}{\cdots e^{-\frac{1}{\hbar}(S - j\cdot\phi)}} = \langle \phi(x) \rangle_j \quad \leftarrow \text{v.e.v of } \phi(x) \text{ in the external source } j$$

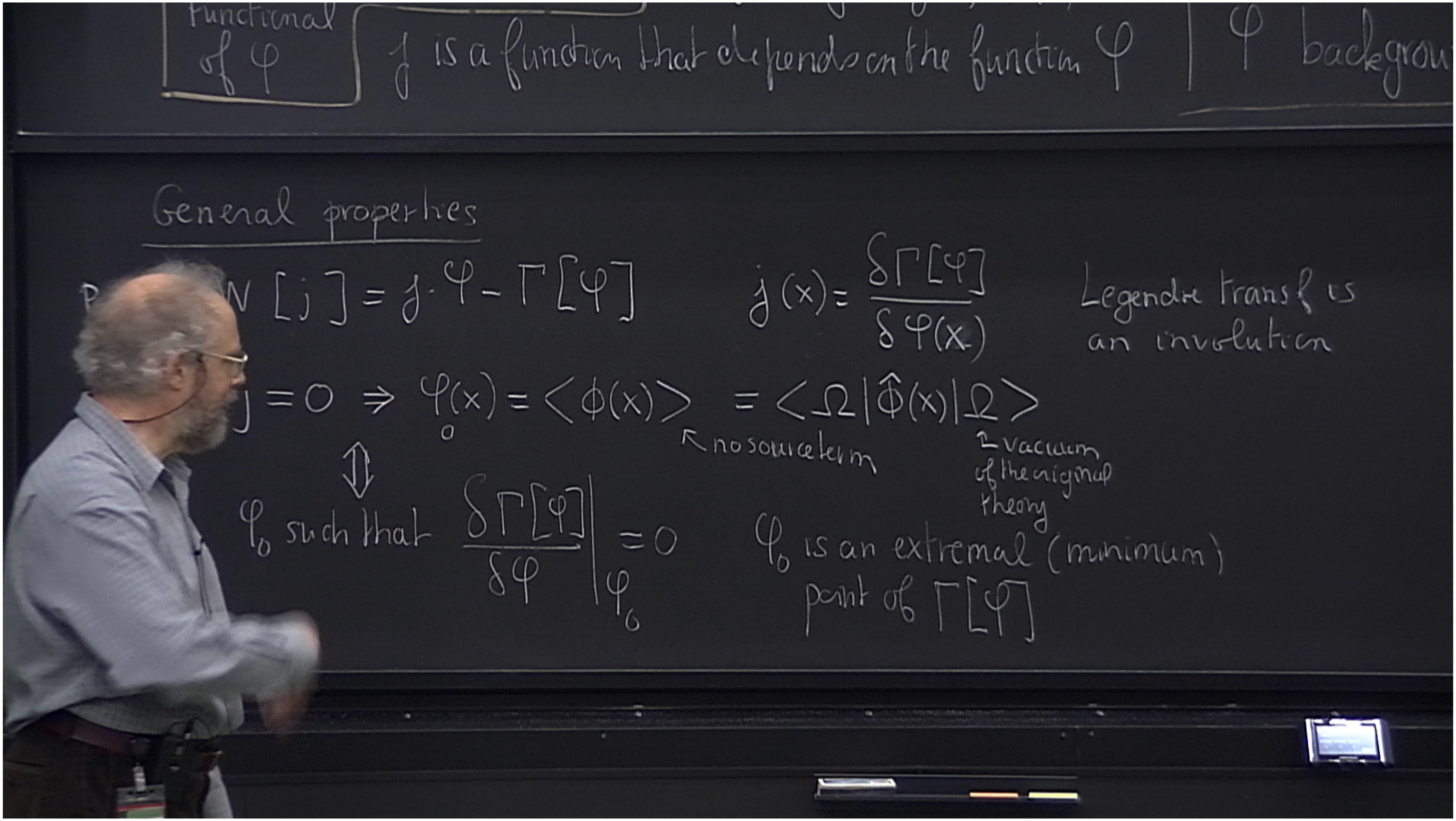
$$j \longleftrightarrow \varphi$$

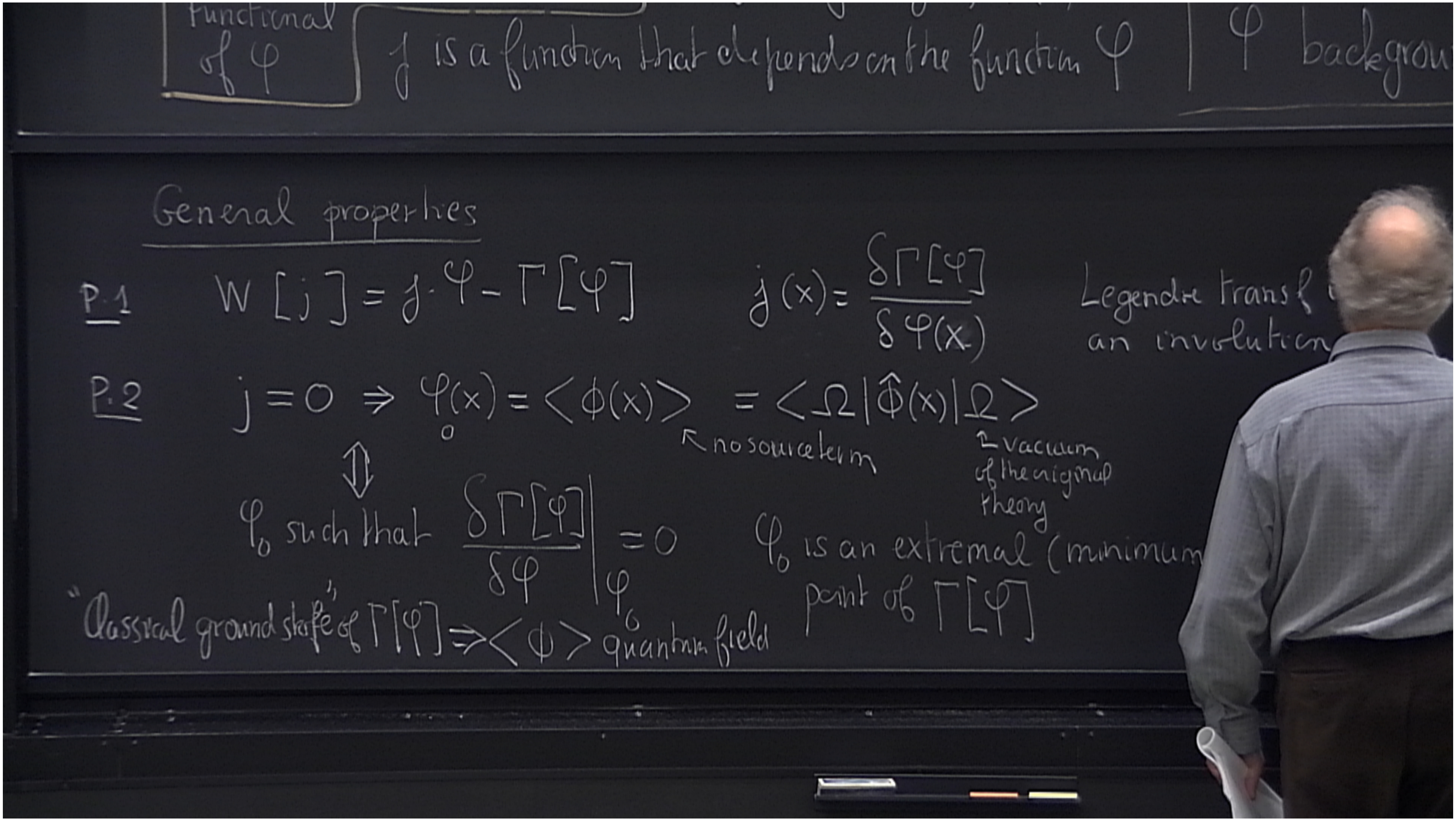
$\Gamma[\varphi]$ is the quantum effective action

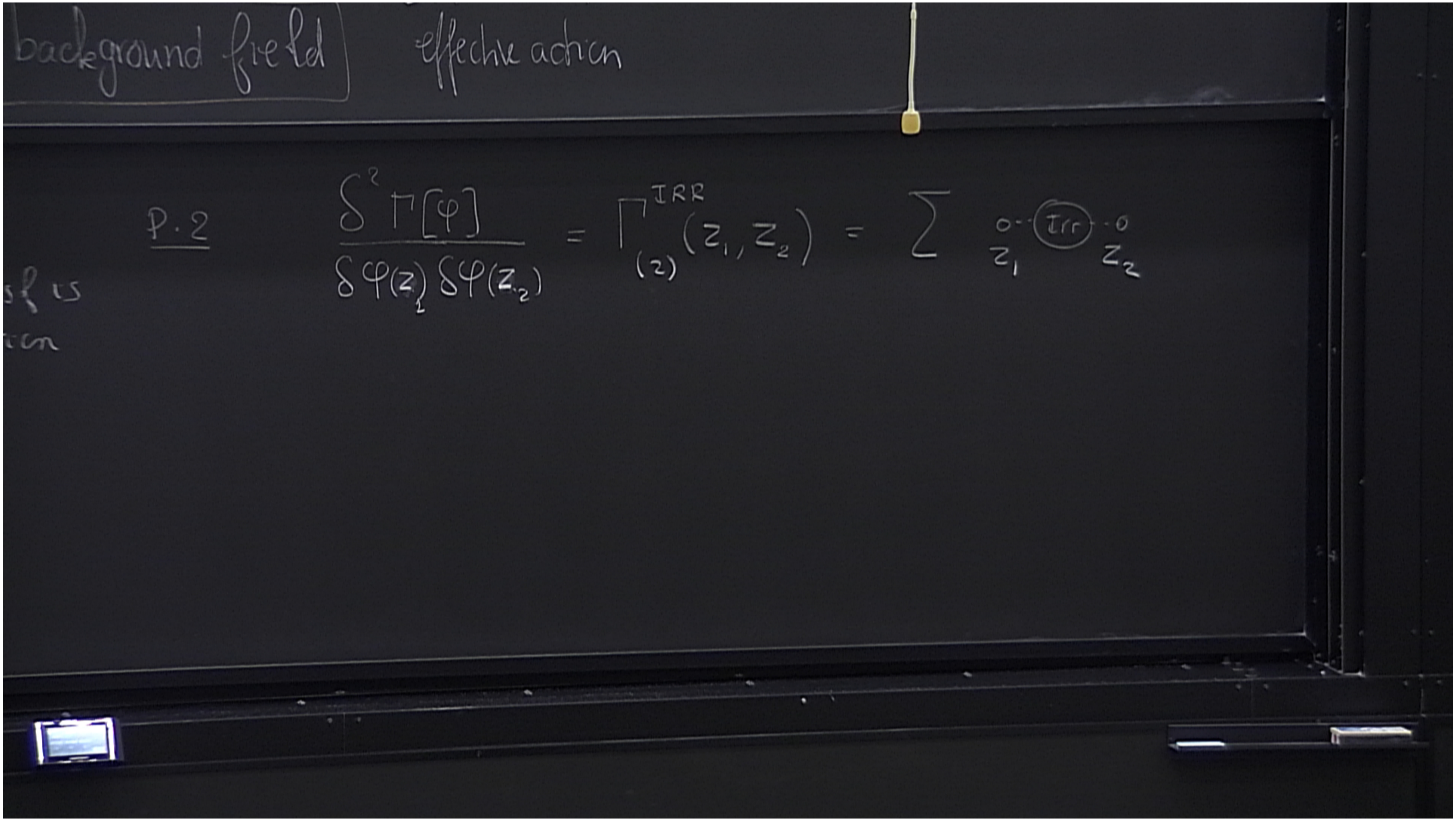
φ | $\boxed{\varphi \text{ background field}}$

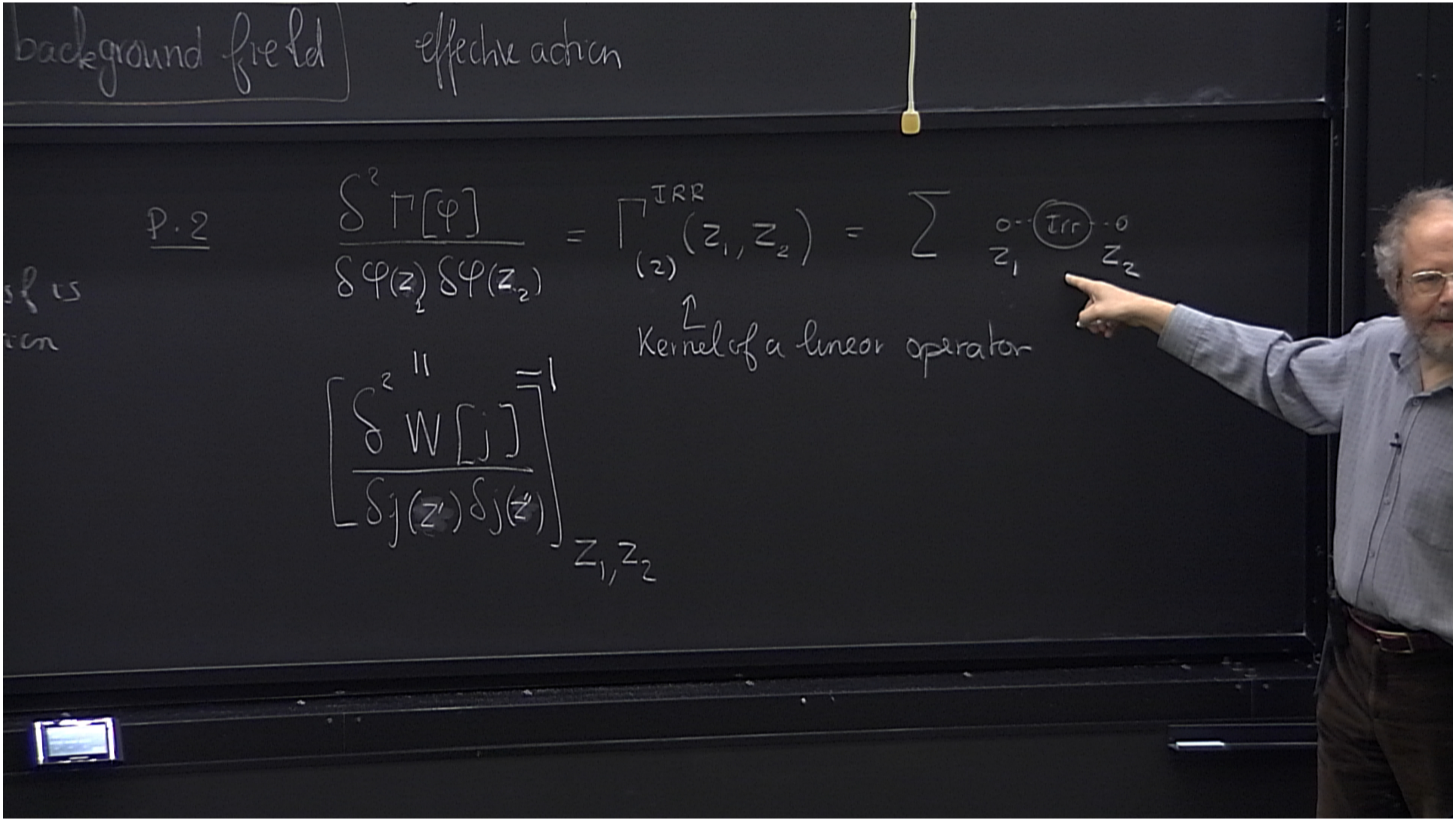

function , it depends on j
Γ[φ] = j·φ − W[j]
j·φ = ∫dx j(x) φ(x)
Functional of φ
j is a function that depends on the function φ
General properties
P.1 W[j] = j·φ − Γ[φ]
j(x) = δΓ[φ] / δφ(x)


Functional of φ
j is a function that depends on the function φ
φ backgrou
General properties
$W[j] = j \cdot \varphi - \Gamma[\varphi]$
$j(x) = \frac{\delta \Gamma[\varphi]}{\delta \varphi(x)}$
Legendre transf is an involution
$j = 0 \Rightarrow \varphi_0(x) = \langle \phi(x) \rangle = \langle \Omega | \hat{\Phi}(x) | \Omega \rangle$
no source term
vacuum of the original theory


Functional of φ
j is a function that depends on the function φ
φ background
General properties
W [j] = j · φ − Γ[φ]
j(x) = δΓ[φ] / δφ(x)
Legendre transf is an involution
j = 0 ⇒ φ0(x) = <ϕ(x)> = <Ω|Φ̂(x)|Ω>
no source term
Ω vacuum of the original theory
φ0 such that δΓ[φ]/δφ |φ0 = 0
φ0 is an extremal (minimum) point of Γ[φ]


Functional of φ
j is a function that depends on the function φ
φ background
General properties
P.1 W[j] = j·φ − Γ[φ]   j(x) = δΓ[φ]/δφ(x)
Legendre transf
an involution
P.2 j = 0 ⇒ φ₀(x) = <ϕ(x)> = <Ω|ϕ̂(x)|Ω>
no source term
vacuum of the original theory
φ₀ such that δΓ[φ]/δφ |φ₀ = 0
φ₀ is an extremal (minimum
point of Γ[φ]
"Classical ground state" of Γ[φ] ⇒ <ϕ> quantum field


background field
effective action
P. 2
$\frac{\delta^2 \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)} = \Gamma^{IRR}_{(2)}(z_1, z_2) = \sum$ Irr z_1 z_2


background field
effective action
P. 2
$\frac{\delta^2 \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)} = \Gamma^{IRR}_{(2)}(z_1, z_2) = \sum$ Irr
Kernel of a linear operator
$\left[\frac{\delta^2 W[j]}{\delta j(z')\,\delta j(z'')}\right]^{-1}_{z_1, z_2}$

background field   effective action

P. 2

$$\frac{\delta^2 \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)} = \Gamma^{IRR}_{(2)}(z_1, z_2) = \sum \underset{z_1}{\circ} \cdots \mathrm{Irr} \cdots \underset{z_2}{\circ}$$

Kernel of a linear operator

$$\left[\frac{\delta^2 W[j]}{\delta j(z')\,\delta j(z'')}\right]^{-1}_{z_1, z_2}$$

$$\Rightarrow \int dx_2\, \Gamma^{IRR}_{(2)}(z_1, z_2) \cdot G^{CONN}_{(2)}(z_2, z_3) = \delta(z_1 - z_3)$$

background field | effective action

...s of us
...ion

P.2

Irr 2pt function is the inverse of the connected 2pt function

$$\frac{\delta^2 \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)} = \Gamma^{IRR}_{(2)}(z_1, z_2) = \sum \; \underset{z_1}{o} - - \text{Irr} - - \underset{z_2}{o}$$

Kernel of a linear operator

$$\| \qquad \left[\frac{\delta^2 W[j]}{\delta j(z')\,\delta j(z'')}\right]^{-1}_{z_1, z_2}$$

$$\Rightarrow \int dx_2\, \Gamma^{IRR}_{(2)}(z_1, z_2) \cdot G^{CONN}_{(2)}(z_2, z_3) = \delta(z_1 - z_3)$$

$\mathcal{O} \longrightarrow$ Kernel $\mathcal{O}(z_1, z_2)$

acts on functions $\varphi(z_1)$

$\mathcal{O}$

$$\varphi \longrightarrow \varphi = \mathcal{O} \cdot \varphi$$

$$\varphi(z_1) = \int dz_2 \, \mathcal{O}(z_1, z_2) \cdot \varphi(z_2)$$

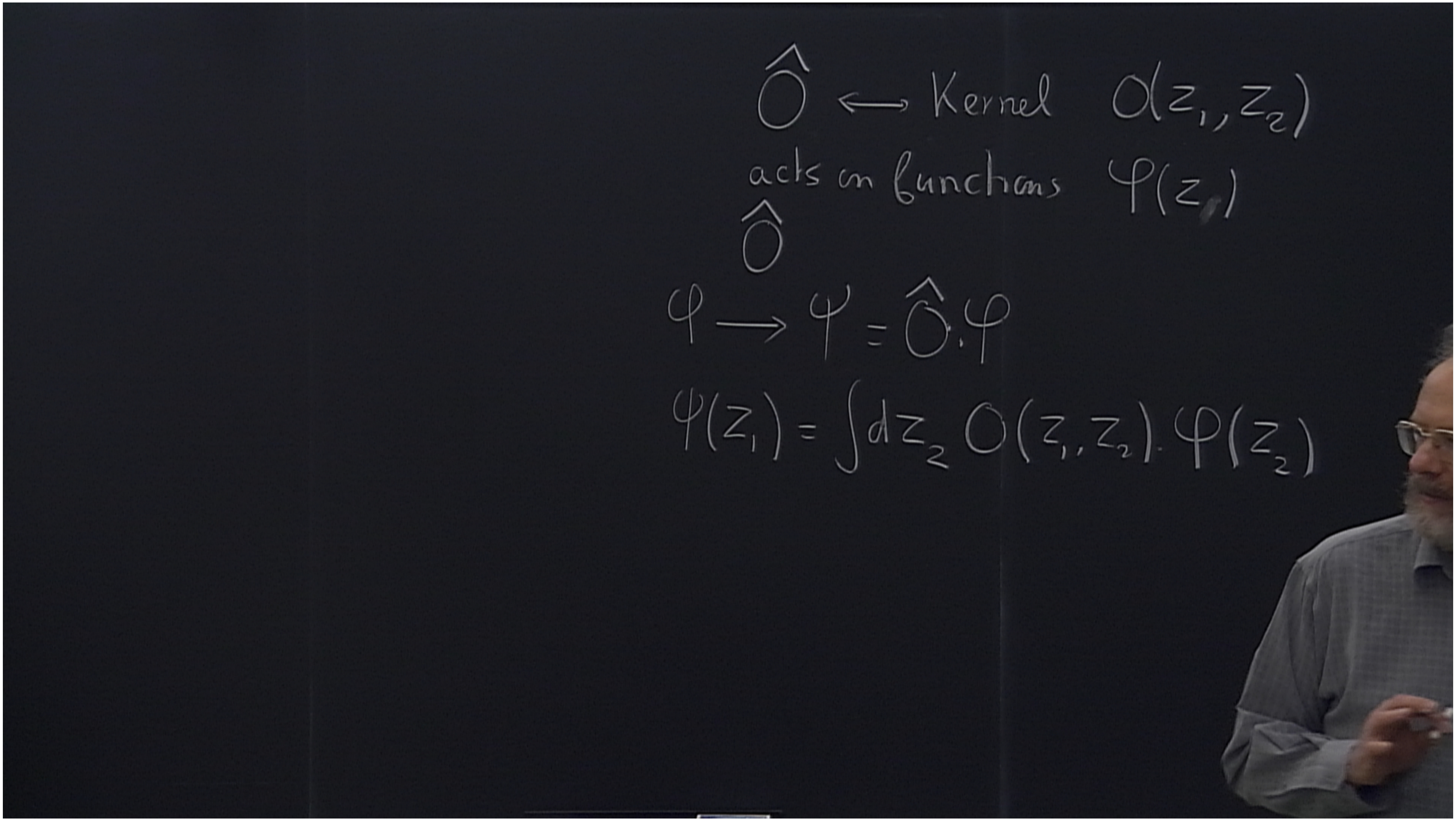
$\hat{O} \longrightarrow$ Kernel $O(z_1, z_2)$

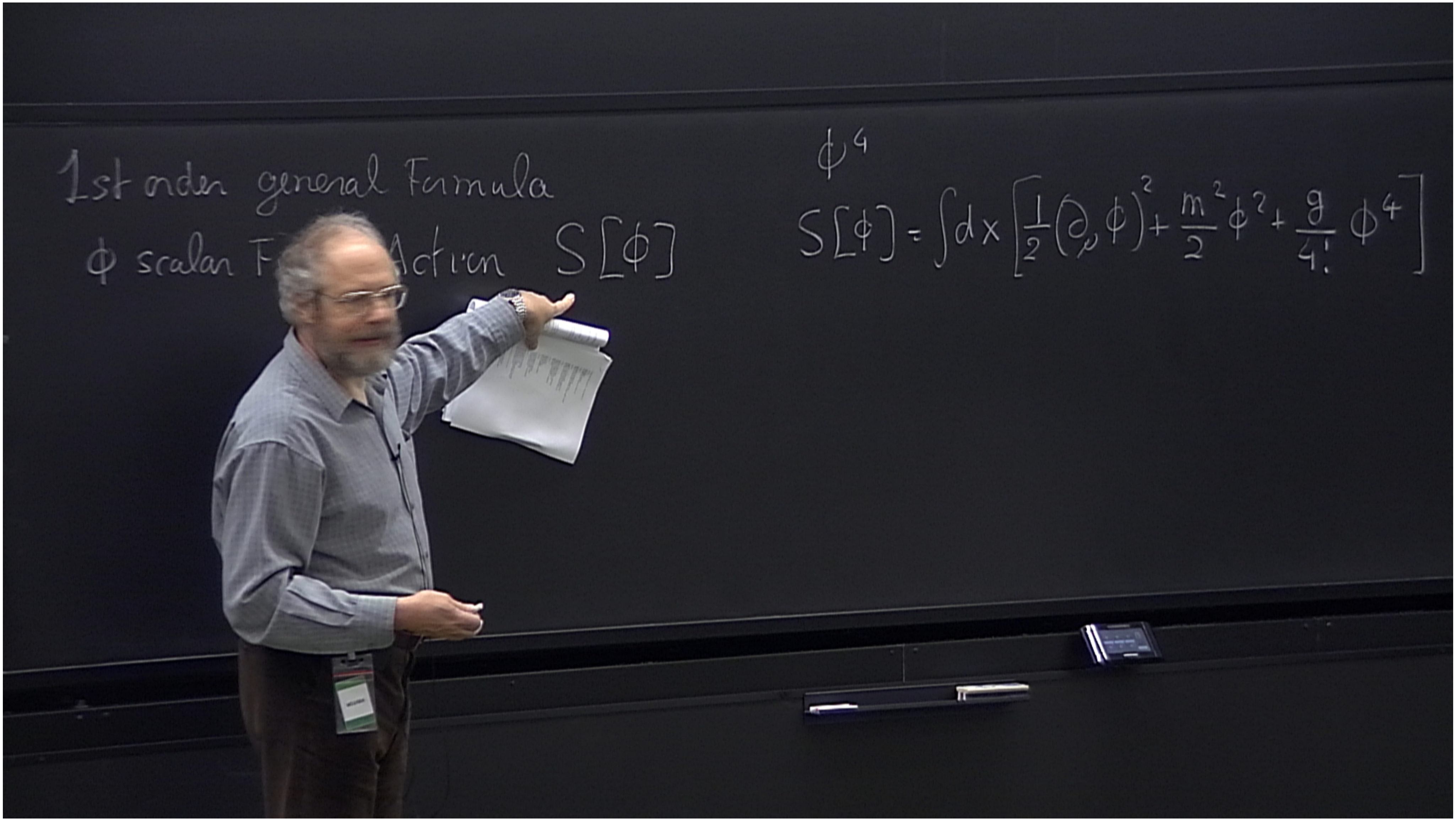
acts on functions $\varphi(z_1)$

$\hat{O}$

$$\varphi \longrightarrow \psi = \hat{O}\cdot\varphi$$

$$\psi(z_1) = \int dz_2\, O(z_1, z_2)\cdot\varphi(z_2)$$


$\hat{O} \longleftrightarrow$ Kernel $O(z_1, z_2)$
acts on functions $\varphi(z_1)$
$\hat{O}$
$\varphi \longrightarrow \psi = \hat{O} \cdot \varphi$
$\psi(z_1) = \int dz_2 \, O(z_1, z_2) \cdot \varphi(z_2)$


1st order general Formula
φ scalar F Action S[φ]
φ^4
S[φ] = ∫dx [1/2 (∂_ν φ)^2 + m^2/2 φ^2 + g/4! φ^4]

1st order general Formula

ϕ scalar Field, Action $S[\phi]$

2nd derivative of $S[\phi]$

$$\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)} =$$

ϕ^4

$$S[\phi] = \int dx \left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4\right]$$

1st order general Formula

ϕ scalar Field, Action $S[\phi]$

2nd derivative of $S[\phi]$

$$\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\,\delta\phi(x_2)} = S''(x_1, x_2; \phi)$$

function

functional

Kernel of the Hessian operator S''

ϕ^4

$$S[\phi] = \int dx \left[\frac{1}{2}(\partial_\nu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$$

1st order general Formula

ϕ scalar Field, Action $S[\phi]$

2nd derivative of $S[\phi]$

$$\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\,\delta\phi(x_2)} = S''(x_1, x_2; \phi)$$

function ← functional

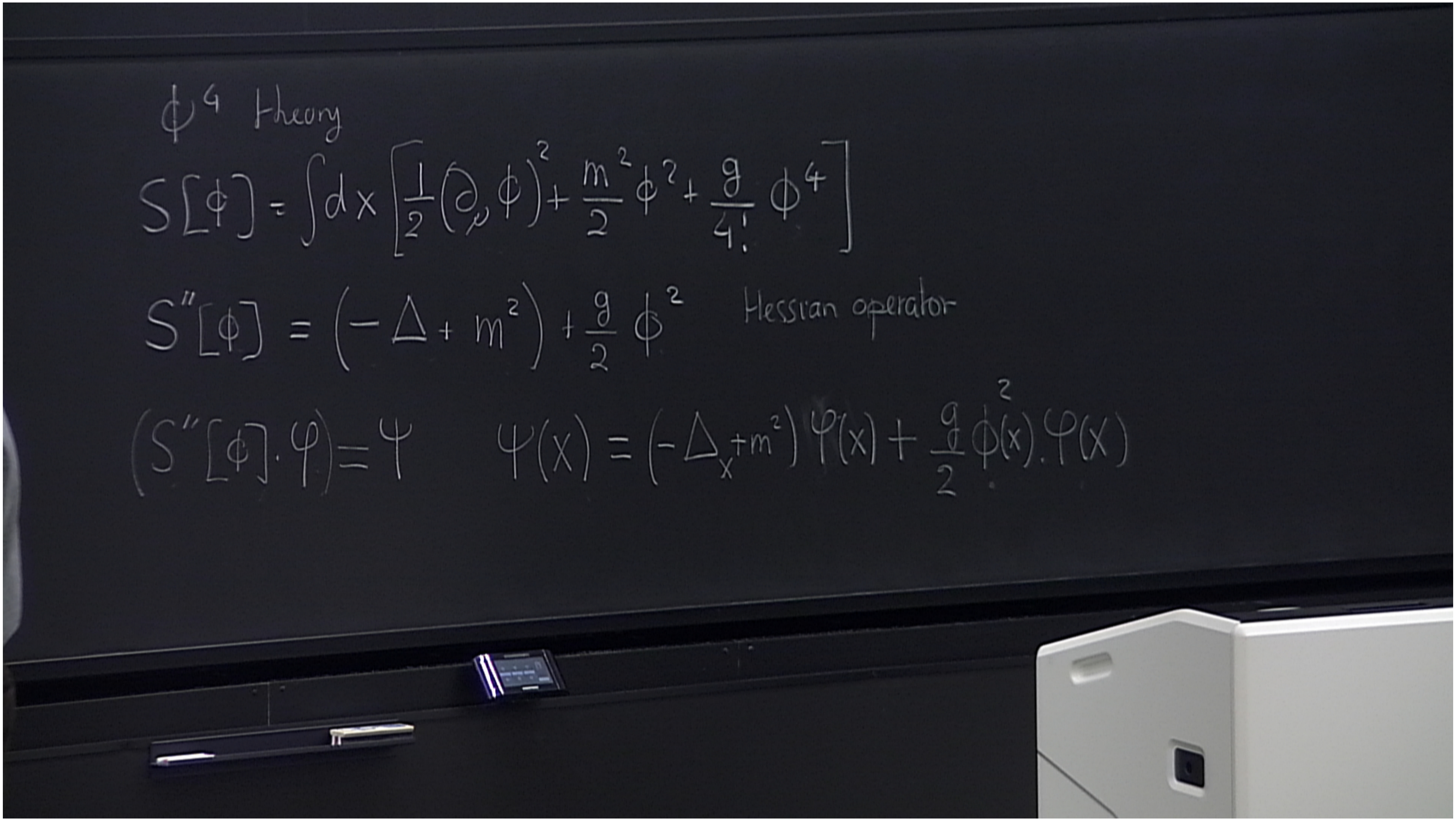
Kernel of the Hessian operator S''

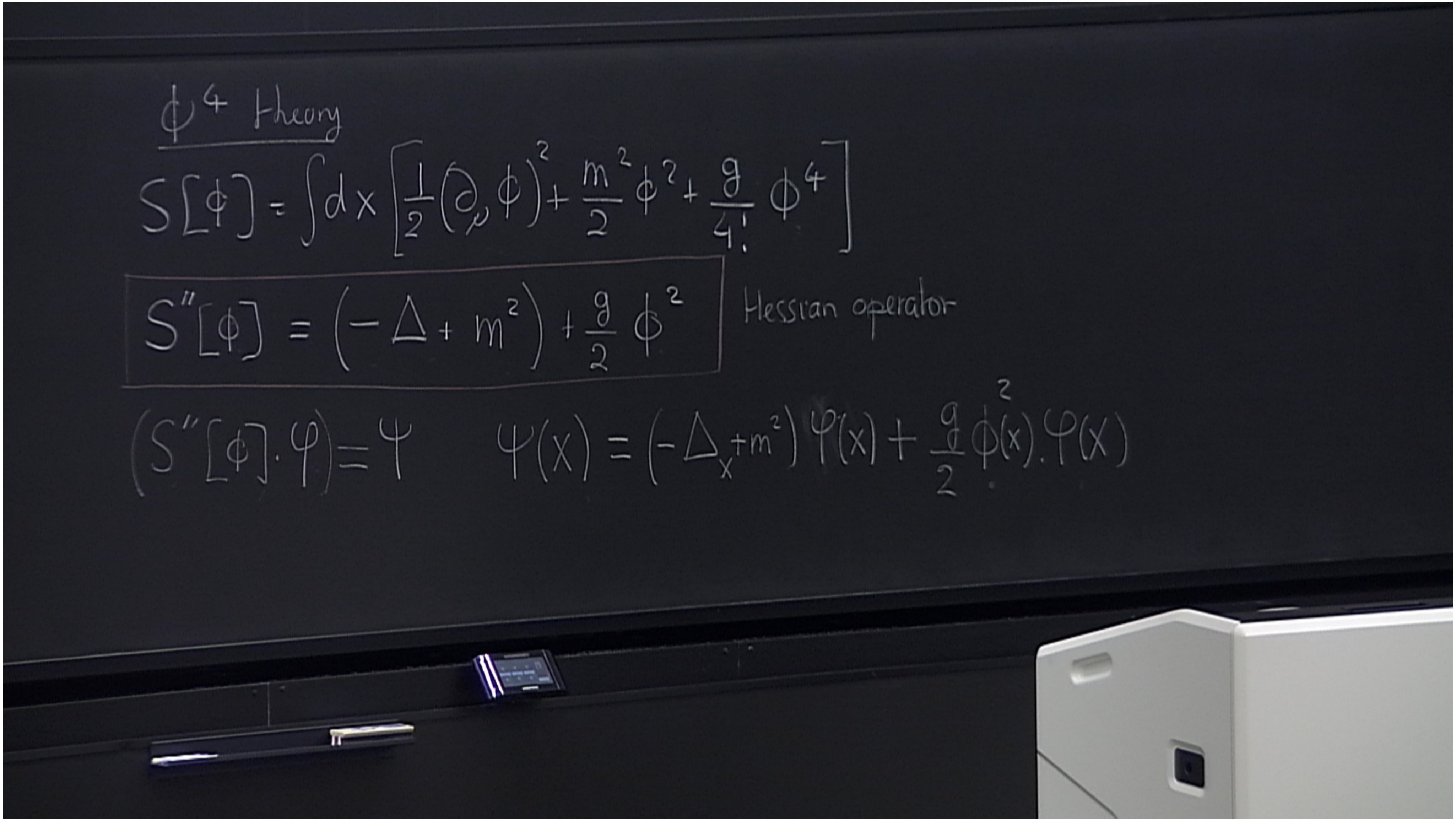
ϕ^4 theory

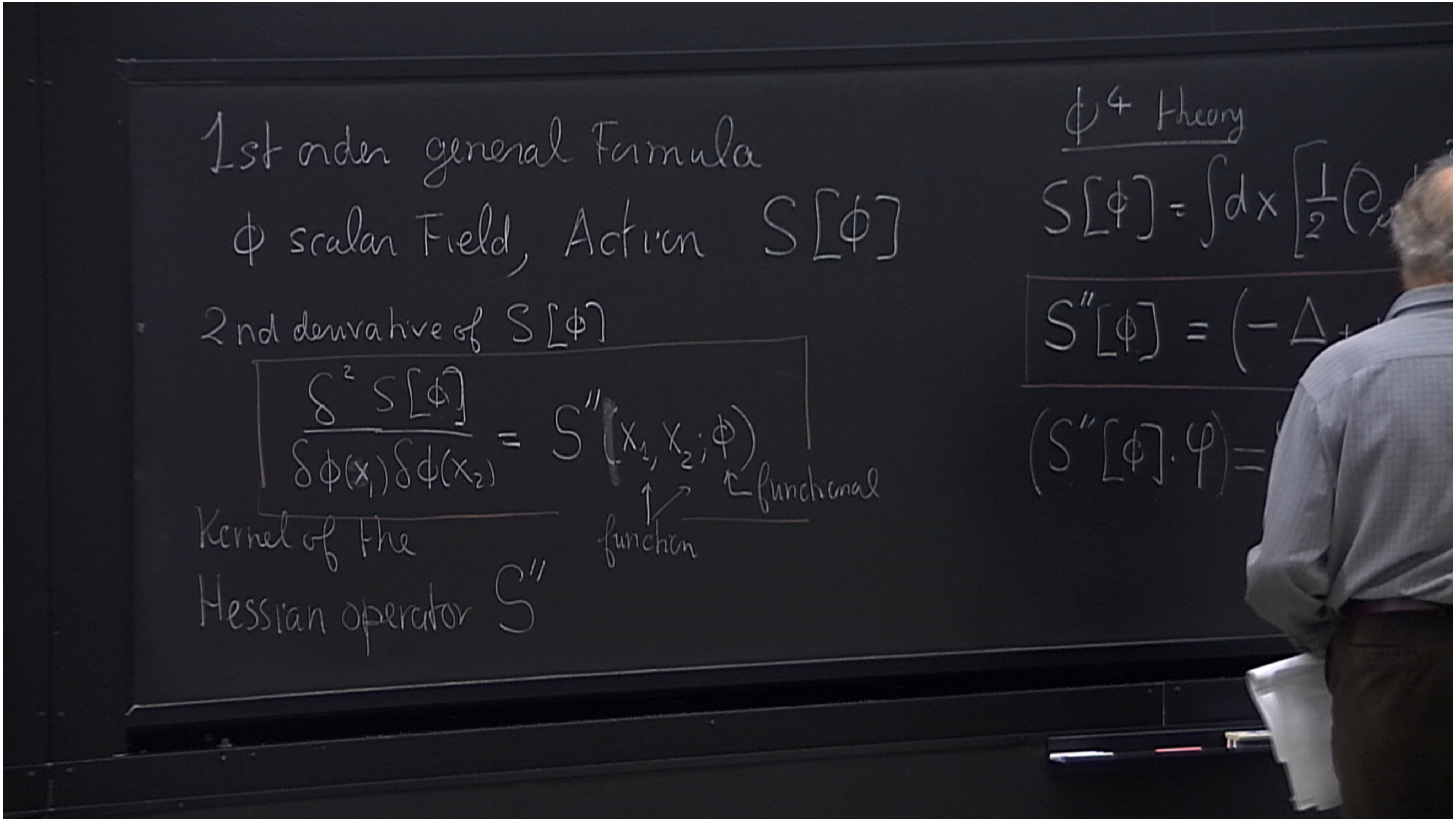
$$S[\phi] = \int dx \left[\frac{1}{2}(\partial_\nu \phi)^2\right.$$

$$S''[\phi] = \left(-\Delta + m^2\right)$$

$$(S''[\phi]\cdot\varphi) = \psi \qquad \psi($$


ϕ^4 theory
$S[\phi] = \int dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$
$S''[\phi] = (-\Delta + m^2) + \frac{g}{2}\phi^2$ Hessian operator
$(S''[\phi]\cdot\varphi) = \Psi$ $\Psi(x) = (-\Delta_x + m^2)\varphi(x) + \frac{g}{2}\phi^2(x)\cdot\varphi(x)$


ϕ^4 theory
$S[\phi] = \int dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$
$S''[\phi] = (-\Delta + m^2) + \frac{g}{2}\phi^2$
Hessian operator
$(S''[\phi]\cdot\varphi) = \psi \qquad \psi(x) = (-\Delta_x + m^2)\varphi(x) + \frac{g}{2}\phi^2(x)\cdot\varphi(x)$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2nd derivative of $S[\phi]$
$\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)} = S''(x_1, x_2; \phi)$
function
functional
Kernel of the Hessian operator S''
ϕ^4 theory
$S[\phi] = \int dx \Big[\frac{1}{2}(\partial_\mu$
$S''[\phi] = (-\Delta +$
$(S''[\phi]\cdot\varphi) =$

Hessian operator S''

Choosen action $S[\phi]$; $\hbar$ is "small"

$$\Gamma[\varphi] =$$

function

Hessian operator S''

Choosen action $S[\phi]$, $\hbar$ is "small", use the saddle point approximation

$$\Gamma[\varphi] = \underset{\uparrow}{S}[\varphi]$$

classical theory

$\hbar$ is "small", use the saddle point approximation + fluctuations

$\hbar \, \frac{1}{2} \log (\det ($

Hessian operator S''

Choosen action $S[\phi]$, $\hbar$ is "small", use the saddle point approximat

$$\Gamma[\varphi] = \underset{\substack{\uparrow \\ \text{classical theory}}}{S[\varphi]} + \hbar \, \frac{1}{2} \log\left(\det\left(S''[\varphi]\right)\right) + o(\hbar^2)$$

log of the determinant of the Hessian operator

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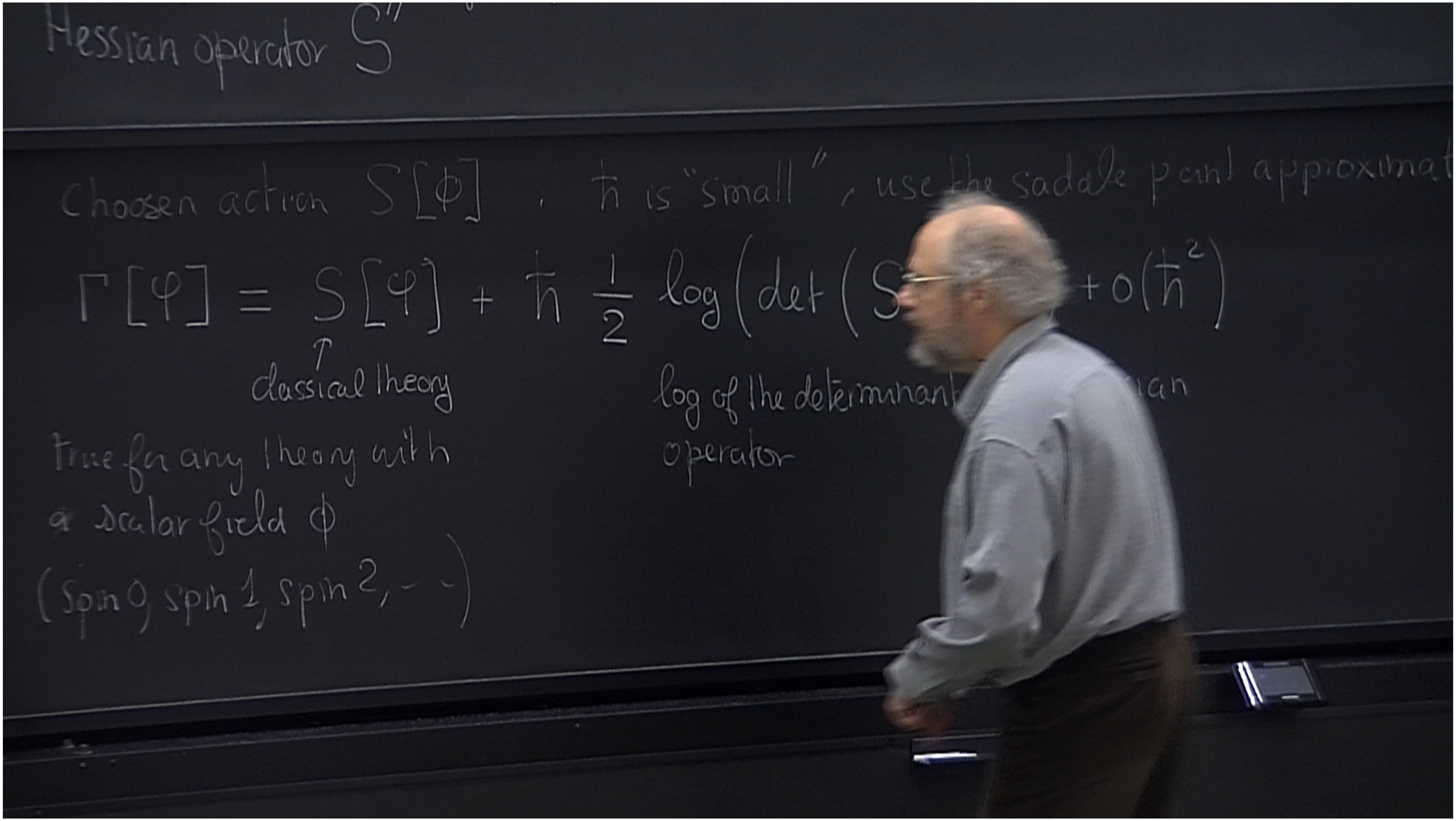
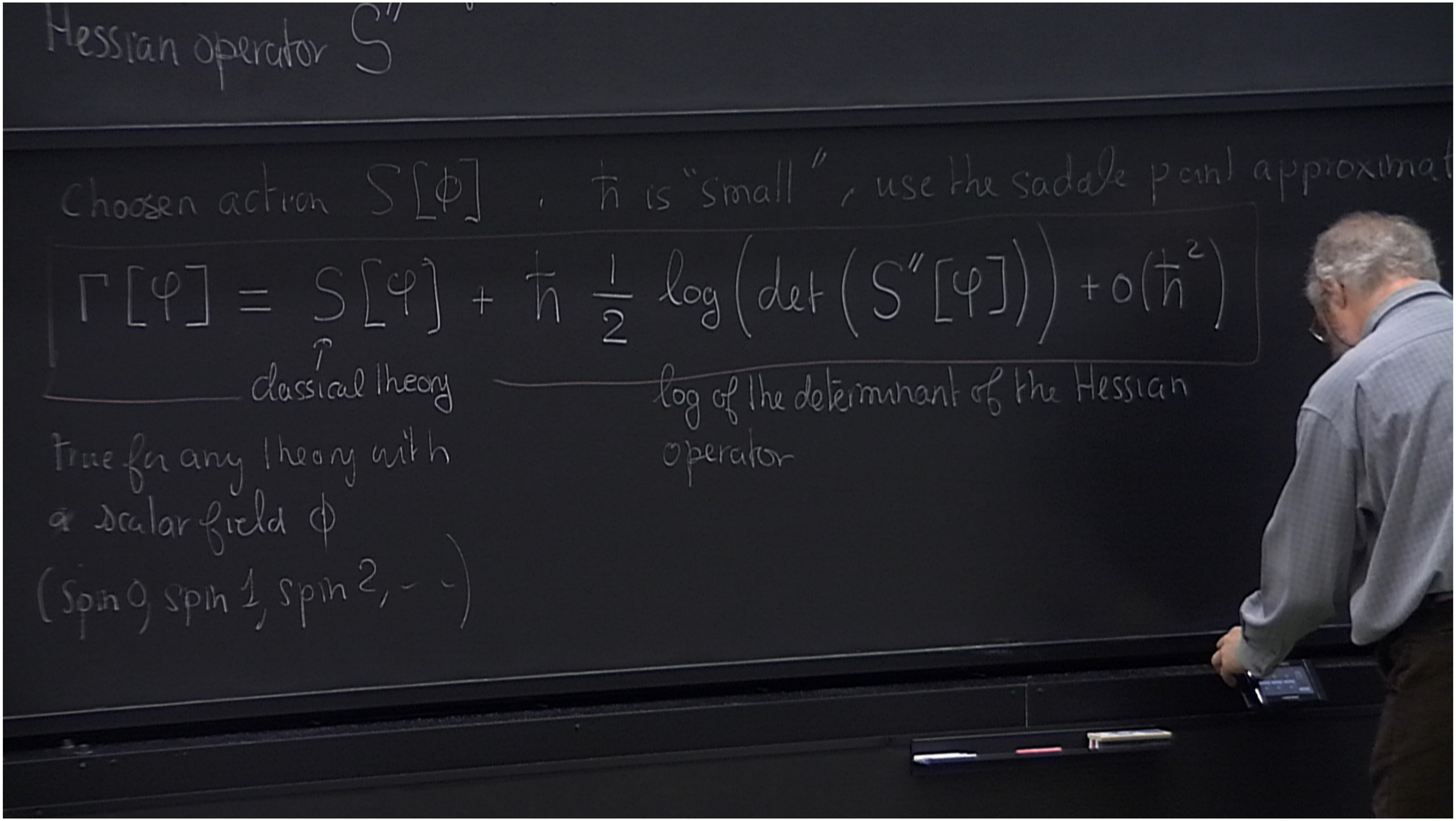
log of the determinant of the Hessian operator

True for any theory with a scalar field ϕ


Hessian operator S''
Choosen action S[φ] , ħ is "small", use the saddle point approximat
Γ[φ] = S[φ] + ħ 1/2 log(det(S
+ O(ħ²))
classical theory
log of the determinant
operator
true for any theory with
a scalar field φ
(spin 0, spin 1, spin 2, - -)


Hessian operator S''
Choosen action $S[\phi]$, $\hbar$ is "small", use the saddle point approximat
$\Gamma[\varphi] = S[\varphi] + \hbar \frac{1}{2} \log(\det(S''[\varphi])) + o(\hbar^2)$
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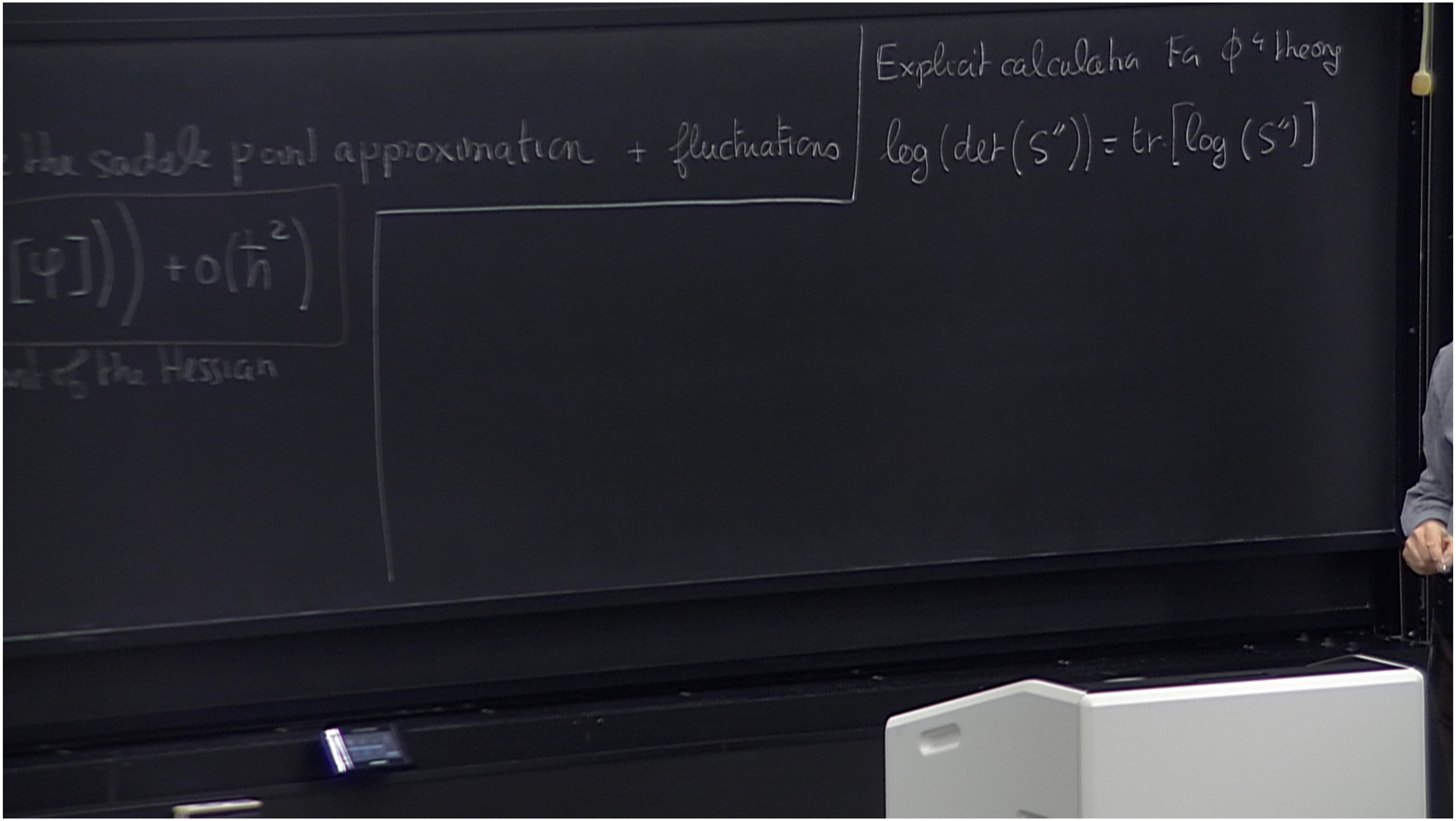
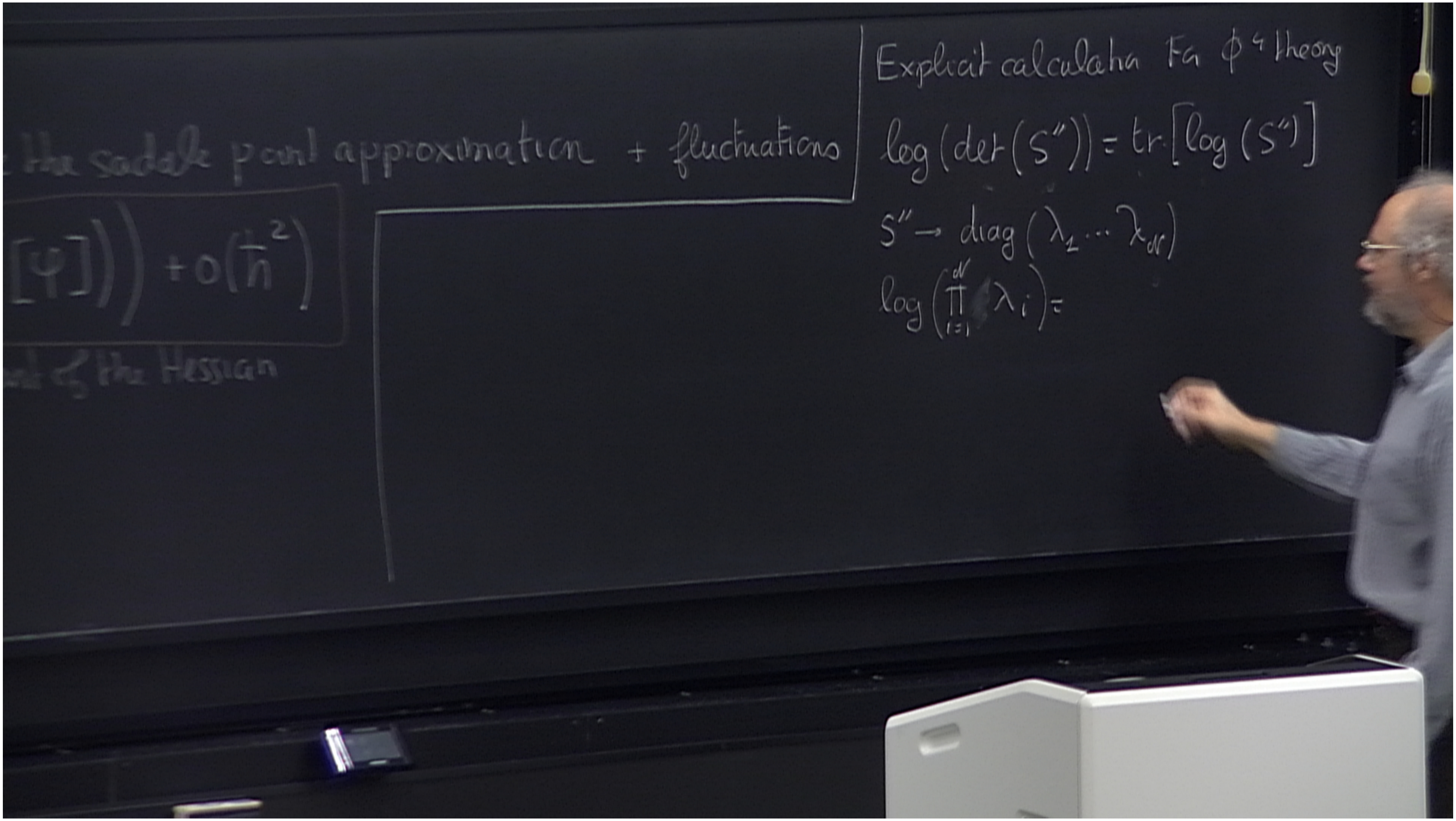
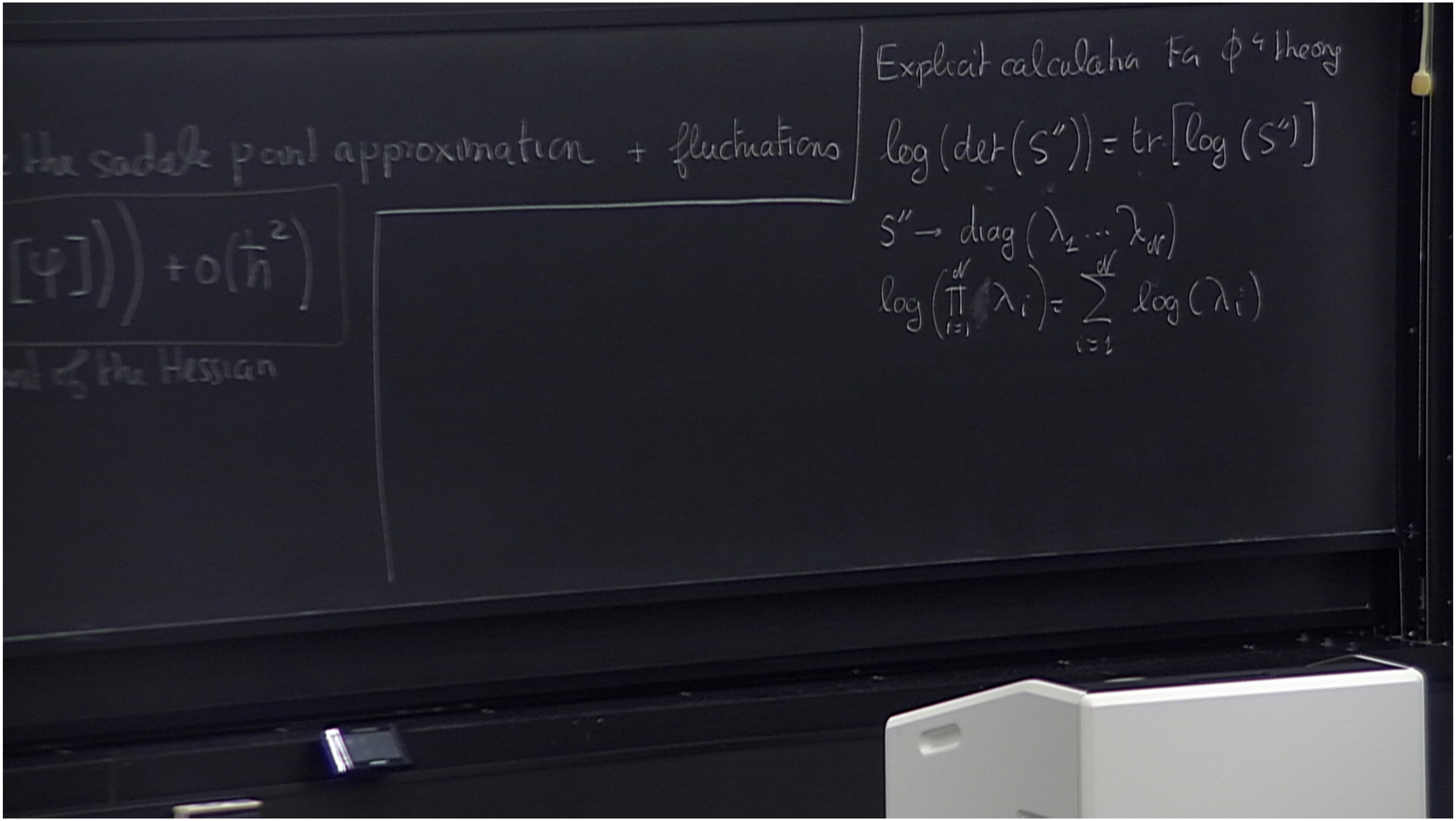

the saddle point approximation + fluctuations
Explicit calculation for ϕ^4 theory
$\log(\det(S'')) = \mathrm{tr.}[\log(S'')]$
$[\varphi])) + O(\hbar^2)$
of the Hessian



the saddle point approximation + fluctuations
[ψ])) + O(ħ²)
of the Hessian
Explicit calculation For φ⁴ theory
log(det(S'')) = tr.[log(S'')]
S'' → diag(λ₁ … λ_N)
log(∏_{i=1}^{N} λ_i) =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Explicit calculation for ϕ^4 theory
$\log(\det(S'')) = \mathrm{tr}[\log(S'')]$
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Choosen action $S[\phi]$; $\hbar$ is "small", use the saddle point app

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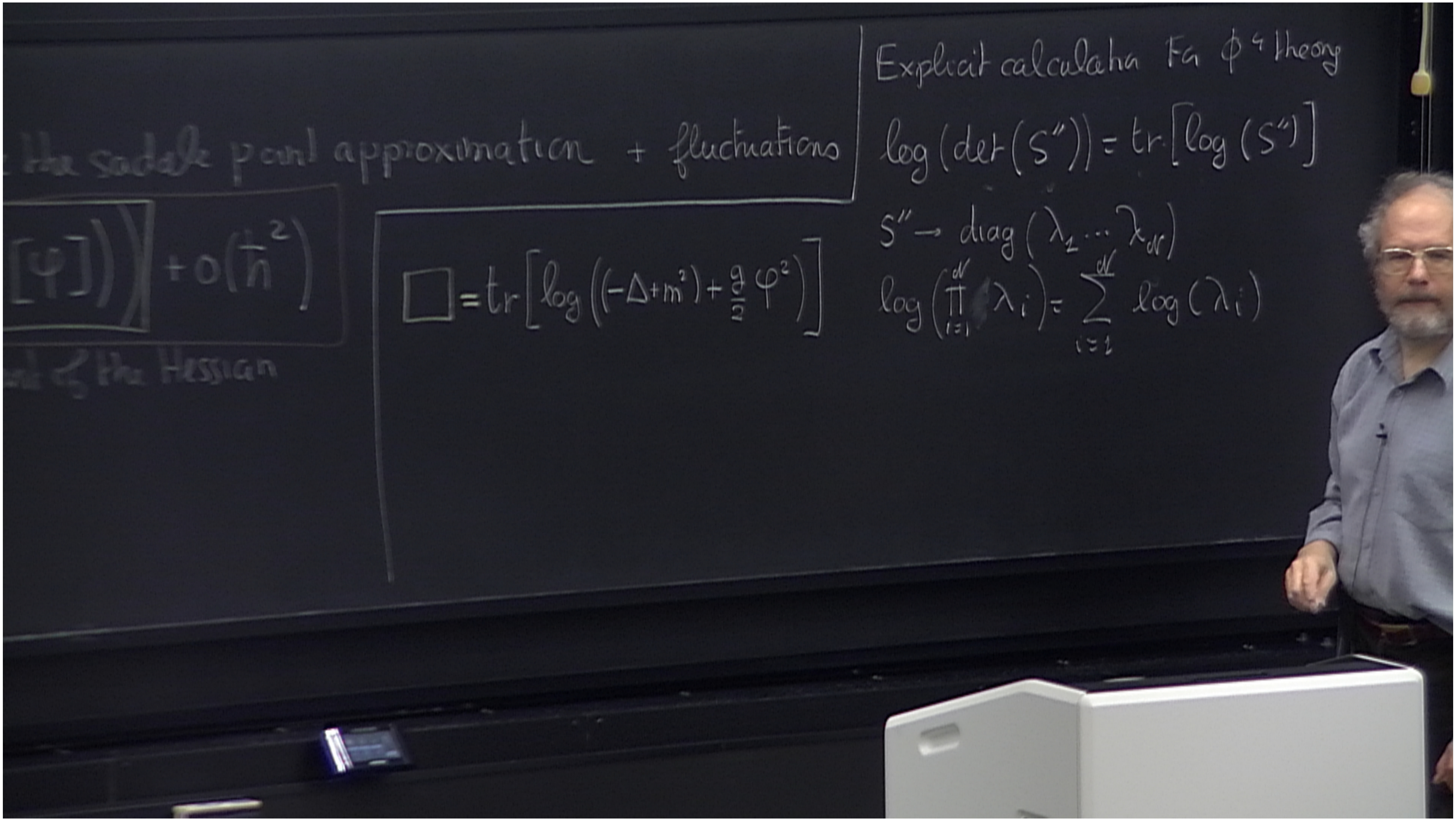
log of the determinant of the Hessian operator

True for any theory with a scalar field ϕ

(spin 0, spin 1, spin 2, ...)



the saddle point approximation + fluctuations
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Explicit calculation for ϕ^4 theory
$\log(\det(S'')) = \mathrm{tr}[\log(S'')]$
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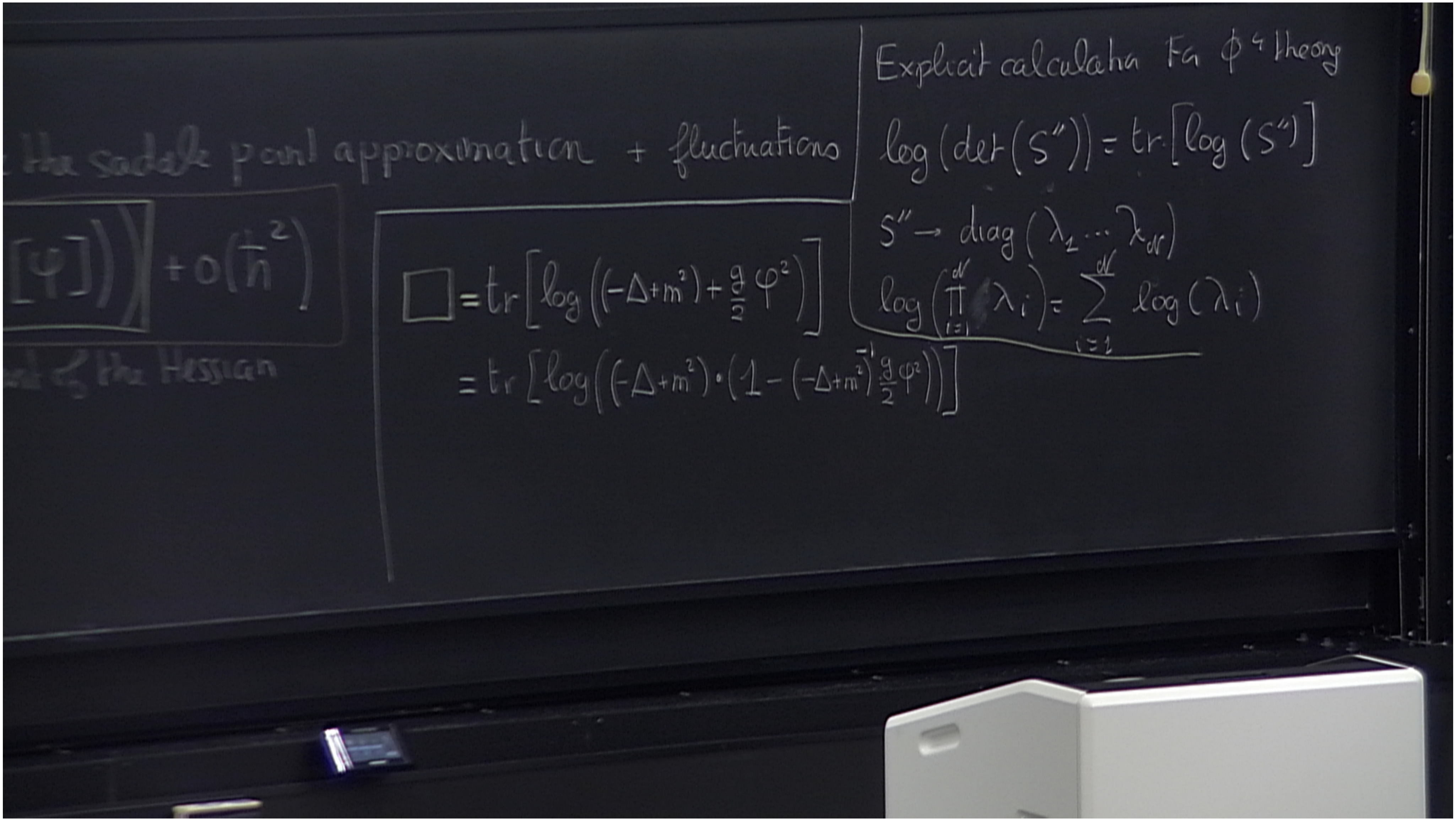
$\log(\prod_{i=1}^{N} \lambda_i) = \sum_{i=1}^{N} \log(\lambda_i)$
$\square = \mathrm{tr}[\log((-\Delta + m^2) + \frac{g}{2}\varphi^2)]$



the saddle point approximation + fluctuations
$[\varphi]])) + O(\hbar^2)$
of the Hessian
$\square = \mathrm{tr}\left[\log\left((-\Delta+m^2)+\frac{g}{2}\varphi^2\right)\right]$
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approximation + fluctuations

Explicit calculation for ϕ^4 theory

$$\log(\det(S'')) = \mathrm{tr}\left[\log(S'')\right]$$

$$S'' \rightarrow \mathrm{diag}(\lambda_1 \dots \lambda_N)$$

$$\log\left(\prod_{i=1}^{N} \lambda_i\right) = \sum_{i=1}^{N} \log(\lambda_i)$$

$$\det(A \cdot B) = \det(A) \cdot \det(B)$$

$$\square = \mathrm{tr}\left[\log\left((-\Delta + m^2) + \frac{g}{2}\varphi^2\right)\right]$$

$$= \mathrm{tr}\left[\log\left((-\Delta + m^2)\cdot\left(1 + (-\Delta + m^2)^{-1}\frac{g}{2}\varphi^2\right)\right)\right]$$

l approximation + fluctuations

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$$= \mathrm{tr}\left[\log(-\Delta+m^2)\right] + \mathrm{tr}\left(\log\left(1+(-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right)$$

Free theory

expansion in g

Explicit calculation for ϕ^4 theory

$$\log(\det(S'')) = \mathrm{tr}\left[\log(S'')\right]$$

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approximation + fluctuations

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$$= \underset{\uparrow \text{ Free theory}}{\mathrm{tr}\left[\log(-\Delta+m^2)\right]} + \boxed{\underset{\text{expansion in } g}{\mathrm{tr}\left(\log\left(1+(-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right)}}$$

Explicit calculation for ϕ^4 theory

$$\log(\det(S'')) = \mathrm{tr}\left[\log(S'')\right]$$

$$S'' \to \mathrm{diag}(\lambda_1 \dots \lambda_{\mathcal{N}})$$

$$\log\left(\prod_{i=1}^{\mathcal{N}} \lambda_i\right) = \sum_{i=1}^{\mathcal{N}} \log(\lambda_i)$$

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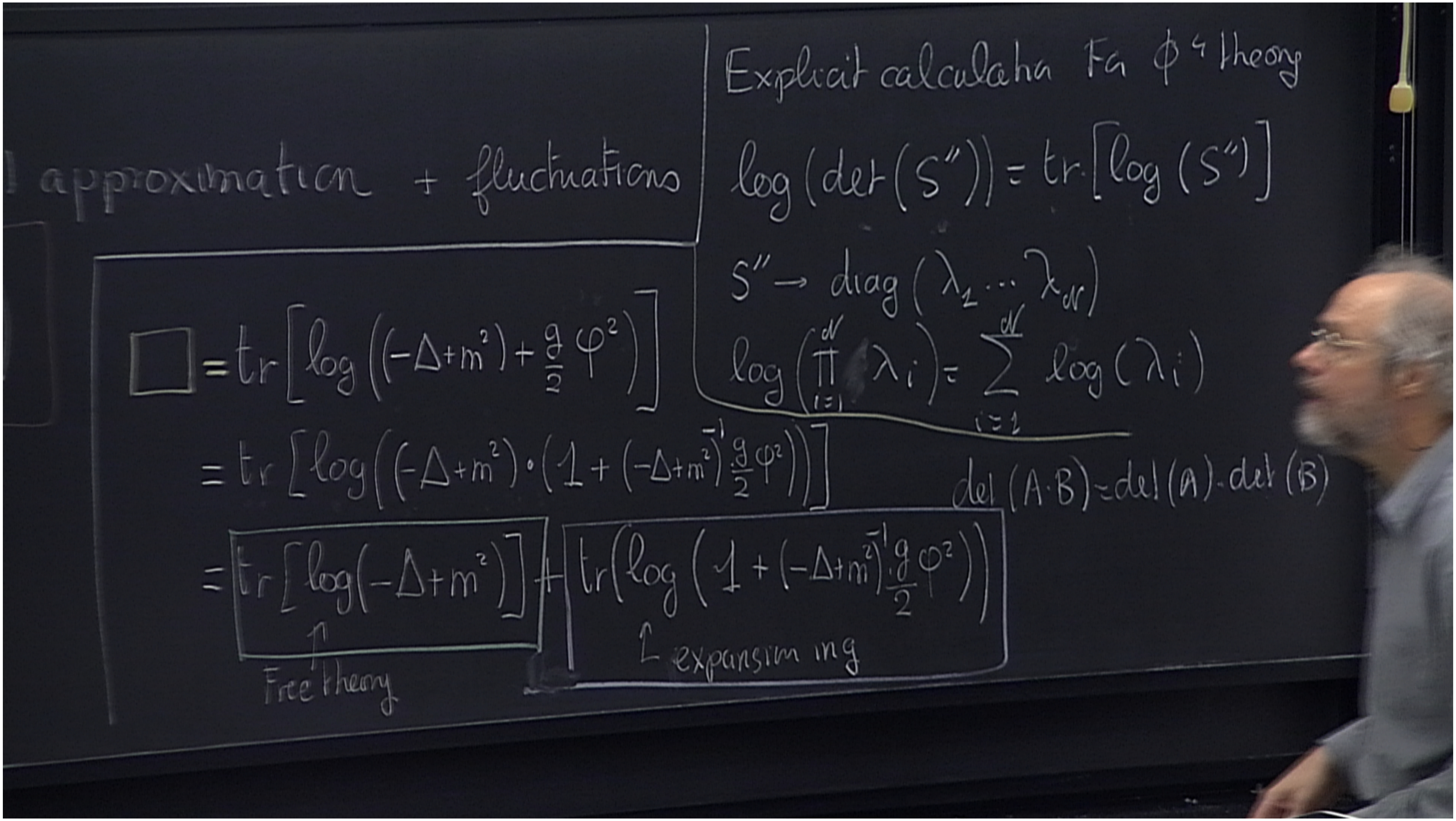
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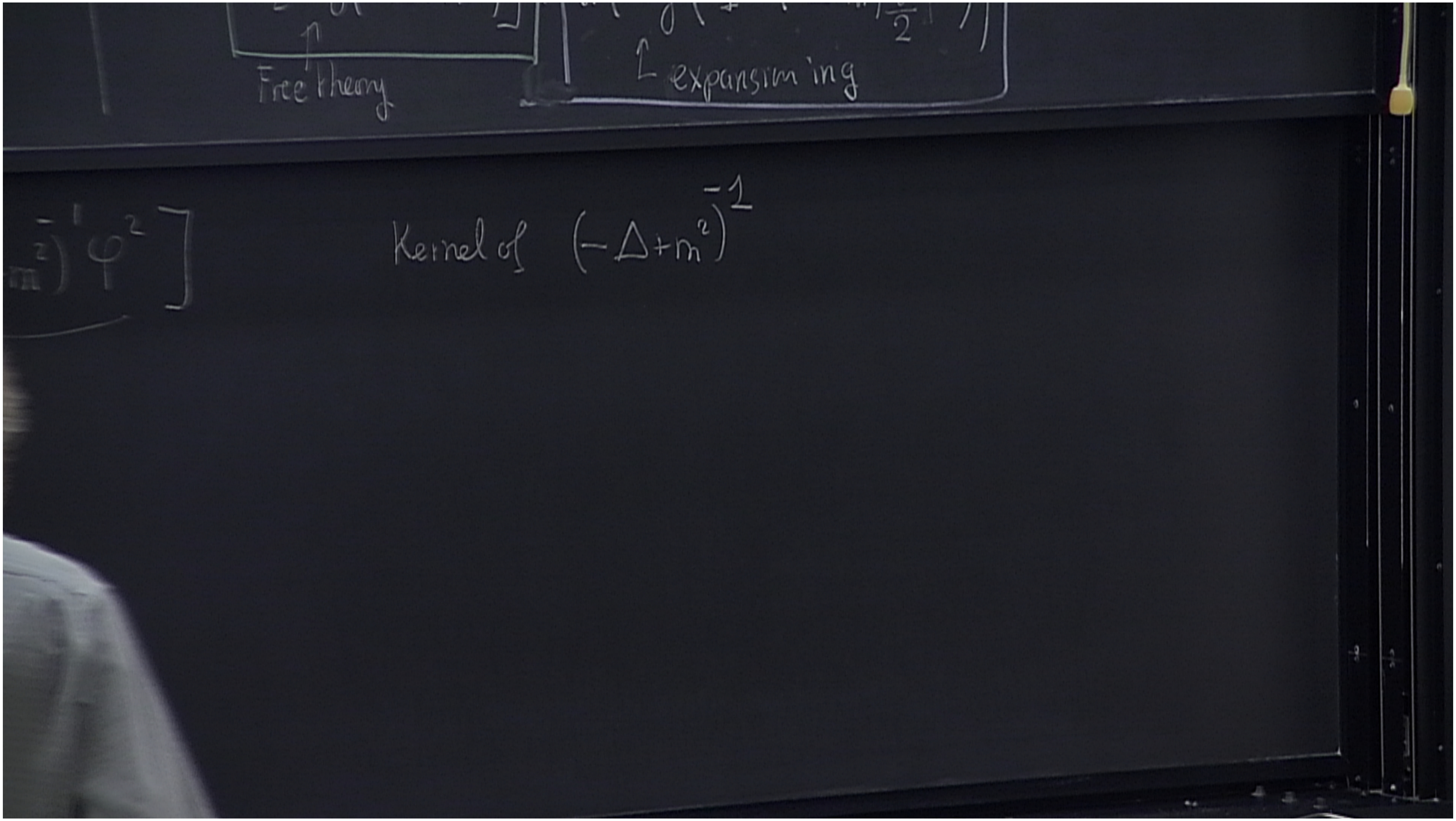
Free theory
expansion in g

(spin 0, spin 1, spin 2, ...)

$$\Box = \sum_{k=1}^{\infty} (g/2)^k \frac{(-1)^{k-1}}{k} \, \mathrm{tr}\Big[\underbrace{(-\Delta+m^2)^{-1}\varphi^2 (-\Delta+m^2)^{-1}\varphi^2 \cdots\cdots (-\Delta+m^2)^{-1}\varphi^2}_{k\text{-times}} \Big]$$


Free theory
expansion ing
Kernel of (−Δ+m²)⁻¹

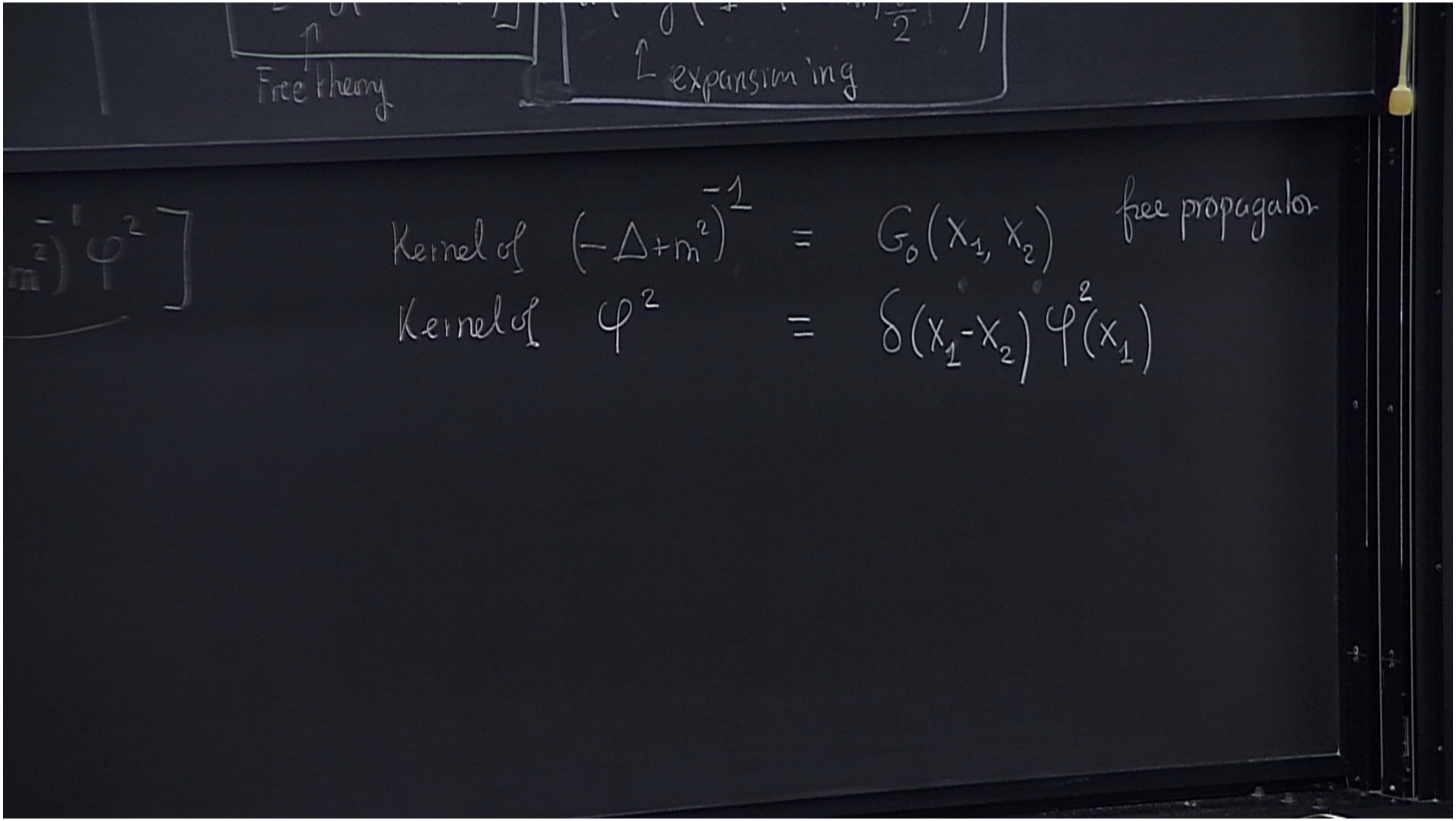
Free theory

expansion in g

$m^2)^{-1}\varphi^2\,]$

Kernel of $(-\Delta+m^2)^{-1} = G_0(x_1, x_2)$ free propagator

Kernel of $\varphi^2 = \delta(x_1-x_2)\,\varphi^2(x_1)$



Free theory
expansion in g
Kernel of $(-\Delta+m^2)^{-1} = G_0(x_1, x_2)$ free propagator
Kernel of $\varphi^2 = \delta(x_1-x_2)\,\varphi^2(x_1)$

(spin 0, spin 1, spin 2, ...)

$$\square = \sum_{K=1}^{\infty} (g/2)^K \frac{(-1)^{K-1}}{K} \, \mathrm{tr}\Big[\underbrace{\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_1 \qquad x_2} \underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_2 \qquad x_3} \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_N \qquad x_{N+1}=x_1}}_{K\text{-times}} \Big]$$

$$\int dx_1 \, dx_2 \ldots dx_N$$

(spin 0, spin 1, spin 2, ...)

$$\Box = \sum_{K=1}^{\infty} (g/2)^{K} \frac{(-1)^{K-1}}{K}\, \mathrm{tr}\Big[\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \qquad x_2}\varphi^2\, \underbrace{\underbrace{(-\Delta+m^2)^{-1}}_{x_2 \qquad x_3}\varphi^2 \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}}_{x_N \qquad x_{N+1}=x_1}\varphi^2}_{K\text{-times}}\Big]$$

$$\int dx_1\, dx_2 \ldots dx_N\; G_0(x_1-x_2)\,\varphi^2(x_2)\, G_0(x_2-x_3)\,\varphi^2(x_3)\cdots$$

Free theory

$$\frac{(-1)^{K-1}}{K}\,\mathrm{tr}\Big[\underbrace{\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \quad x_2}\varphi^2\,\underbrace{(-\Delta+m^2)^{-1}}_{x_2 \quad x_3}\varphi^2\cdots\cdots\underbrace{(-\Delta+m^2)^{-1}}_{x_N \quad x_{N+1}=x_1}\varphi^2}_{K\text{-times}}\Big]$$

Kernel of $(-\Delta+m^2)^{-1}$

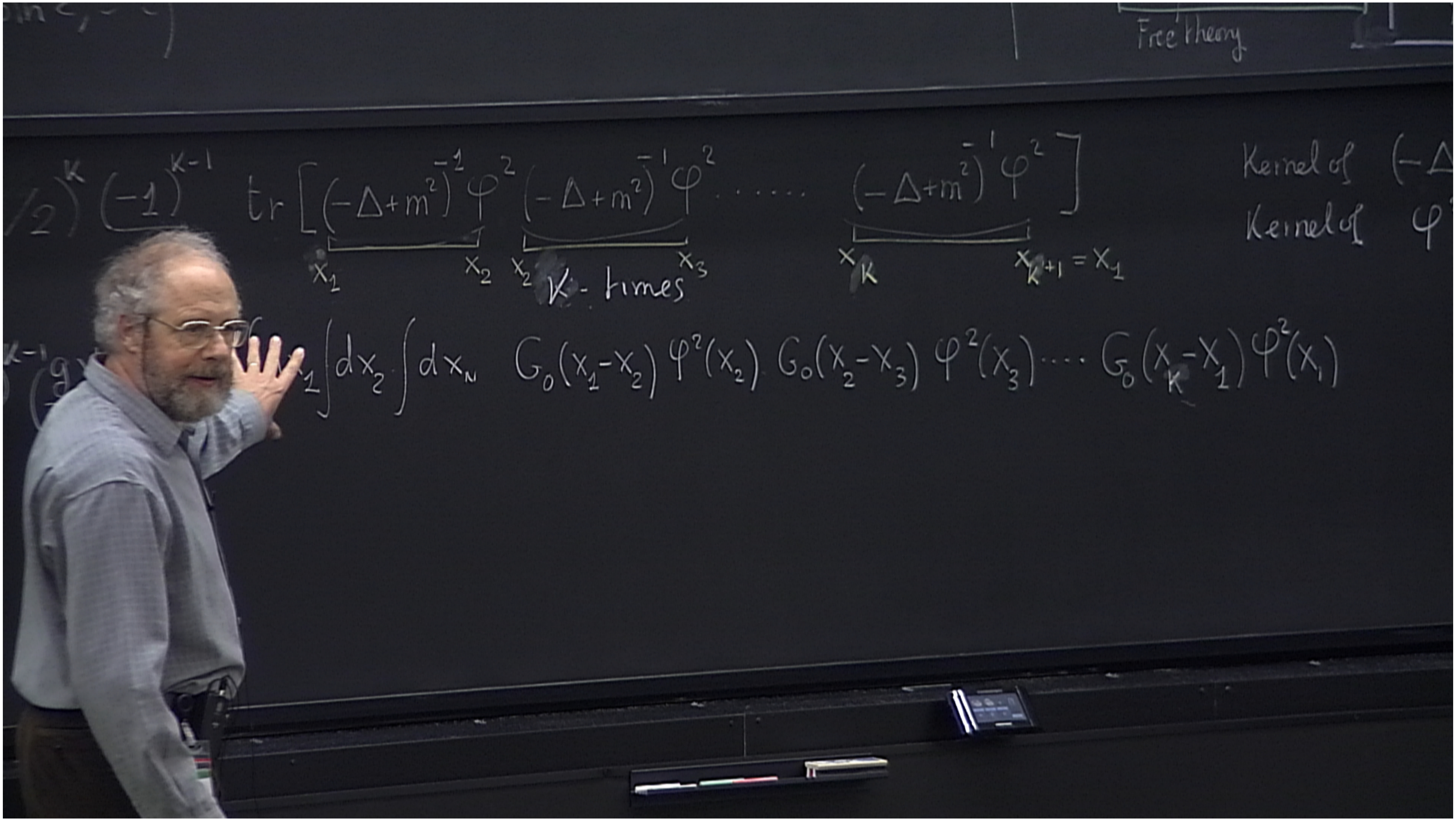
Kernel of φ^2

$$\int dx_1\int dx_2\int dx_N \; G_0(x_1-x_2)\,\varphi^2(x_2)\,G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_N-x_1)\,\varphi^2(x_1)$$

(spin 0, spin 1, spin 2, ...)

$$\square = \sum_{K=1}^{\infty} (g/2)^K \frac{(-1)^{K-1}}{K} \, \mathrm{tr}\Big[\underbrace{\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \quad x_2} \varphi^2 \underbrace{(-\Delta+m^2)^{-1}}_{x_2 \quad x_3} \varphi^2 \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}}_{x_N \quad x_{N+1}} \varphi^2}_{K\text{-times}} \Big]$$

$$= \sum_{K=1}^{\infty} (-1)^{K-1} \left(\frac{g}{2}\right)^K \cdot \frac{1}{K} \int dx_1 \int dx_2 \int dx_N \; G_0(x_1 - x_2)\, \varphi^2(x_2)\, G_0(x_2 - x_3)\, \varphi^2(x_3) \cdots$$


Free theory
tr [(−Δ+m²)⁻¹ φ² (−Δ+m²)⁻¹ φ² ...... (−Δ+m²)⁻¹ φ²]
K - times
dx₂ ∫ dx_N G₀(x₁−x₂) φ²(x₂) G₀(x₂−x₃) φ²(x₃) .... G₀(x_K−x₁) φ²(x₁)
Kernel of (−Δ
Kernel of φ

Free theory

expansion in g

$$\ldots m^2)^{-1} \varphi^2 \Big]$$

$x_{k+1} = x_1$

$$\varphi^2(x_3) \cdots G_0(x_k - x_1)\, \varphi^2(x_1)$$

Kernel of $(-\Delta + m^2)^{-1} = G_0(x_1, x_2)$ free propagator

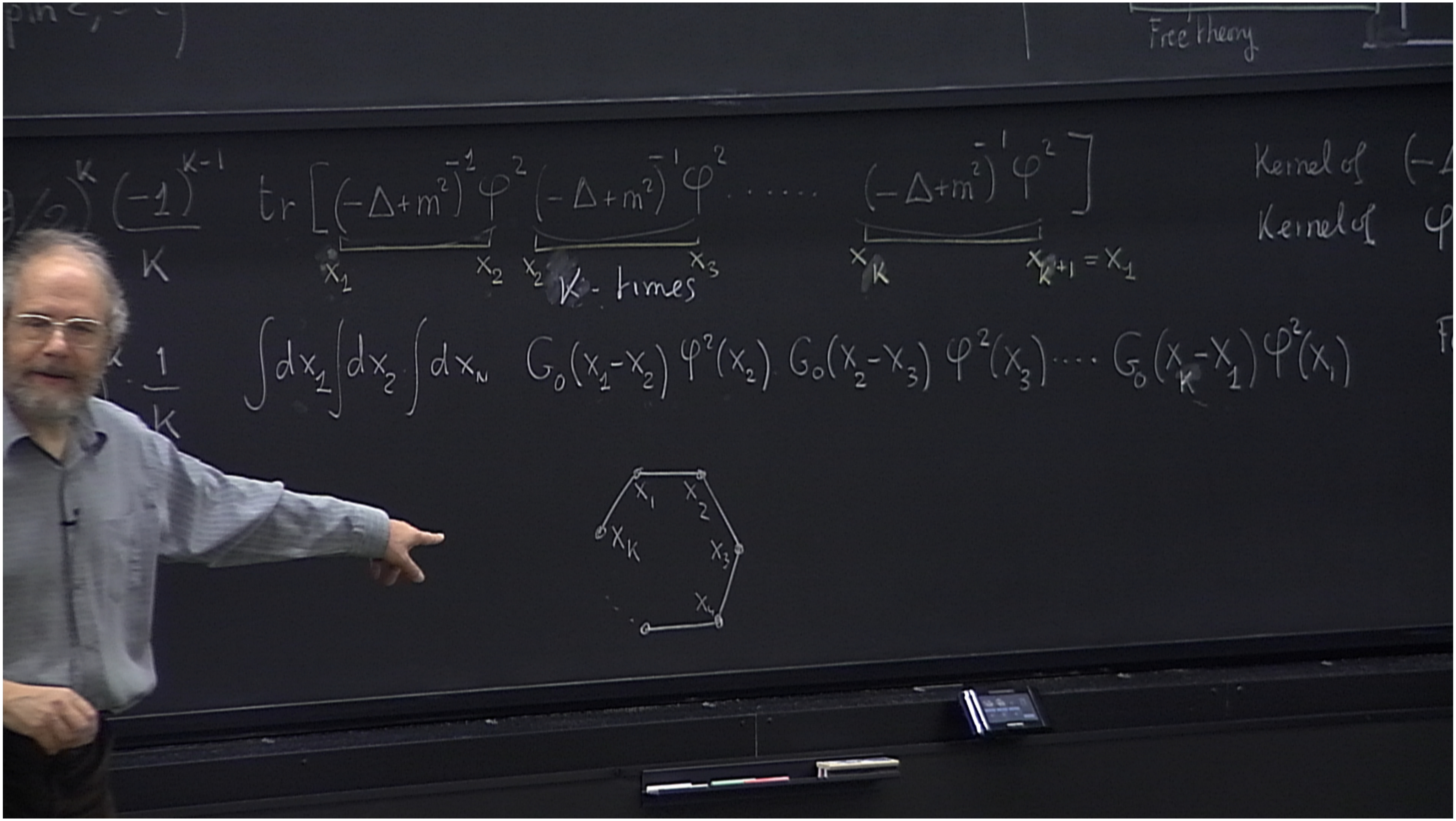
Kernel of $\varphi^2 = \delta(x_1 - x_2)\, \varphi^2(x_1)$

Feynman diagrammatic representation

$$(-\Delta + m^2)\,(\varphi^2)\,(-\Delta + m^2)\,(\varphi^2)$$
$$x_1 \quad y_1 \; y_1 \quad x_2 \; x_2 \quad y_2 \; y_2 \quad x_3$$

$$\int dx_1\, dy_1\, dx_2\, dy_2 \ldots\; G_0(x_1 - y_1)\; \delta(y_1 - x_2)\, \varphi^2(x_2)\, G_0(x_2 - y_2)\, \delta(y_2 - x_3)\, \varphi^2(x_3) \ldots$$

$$\int dx_1\, dx_2 \ldots\; G_0(x_1 - x_2)\, \varphi^2(x_2) \ldots$$


Free theory
$\frac{(-1)^{K-1}}{K}$ $\mathrm{tr}\left[\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_1 \quad x_2}\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_2 \quad x_3}\cdots\cdots\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_K \quad x_{K+1}=x_1}\right]$
K - times
Kernel of
Kernel of
$\frac{1}{K}$ $\int dx_1 \int dx_2 \int dx_N \; G_0(x_1-x_2)\,\varphi^2(x_2)\,G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_K-x_1)\,\varphi^2(x_1)$
x_1 x_2 x_3 x_4 x_K

Free theory

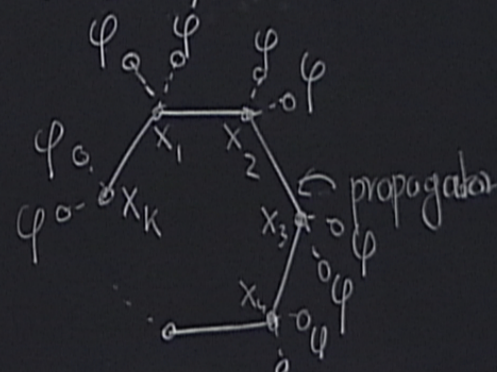
$$\left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K}\ \mathrm{tr}\Big[\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \quad x_2}\varphi^2\ \underbrace{(-\Delta+m^2)^{-1}}_{x_2 \quad x_3}\varphi^2 \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}}_{x_K \quad x_{K+1}=x_1}\varphi^2\Big]$$

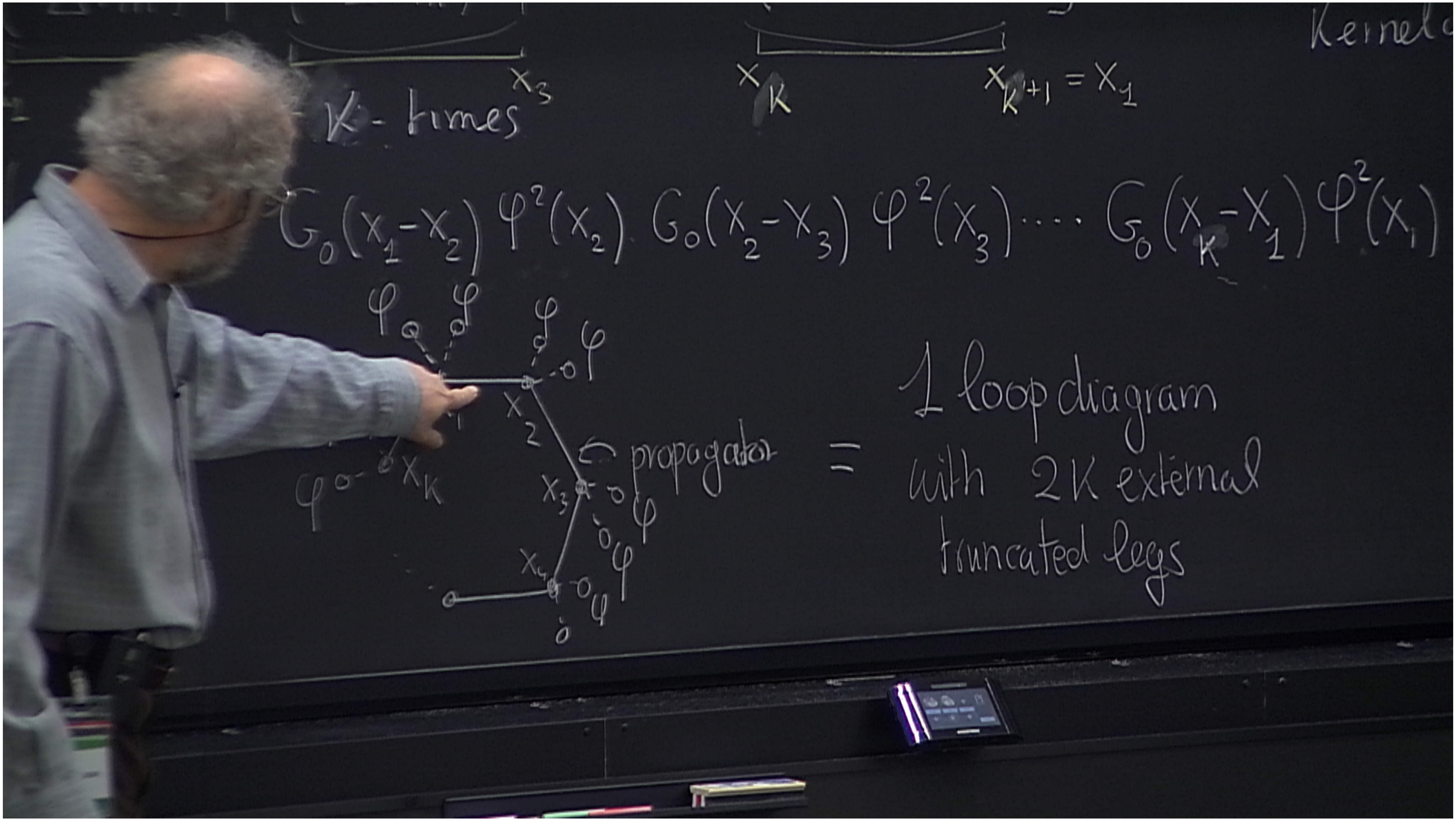
K - times

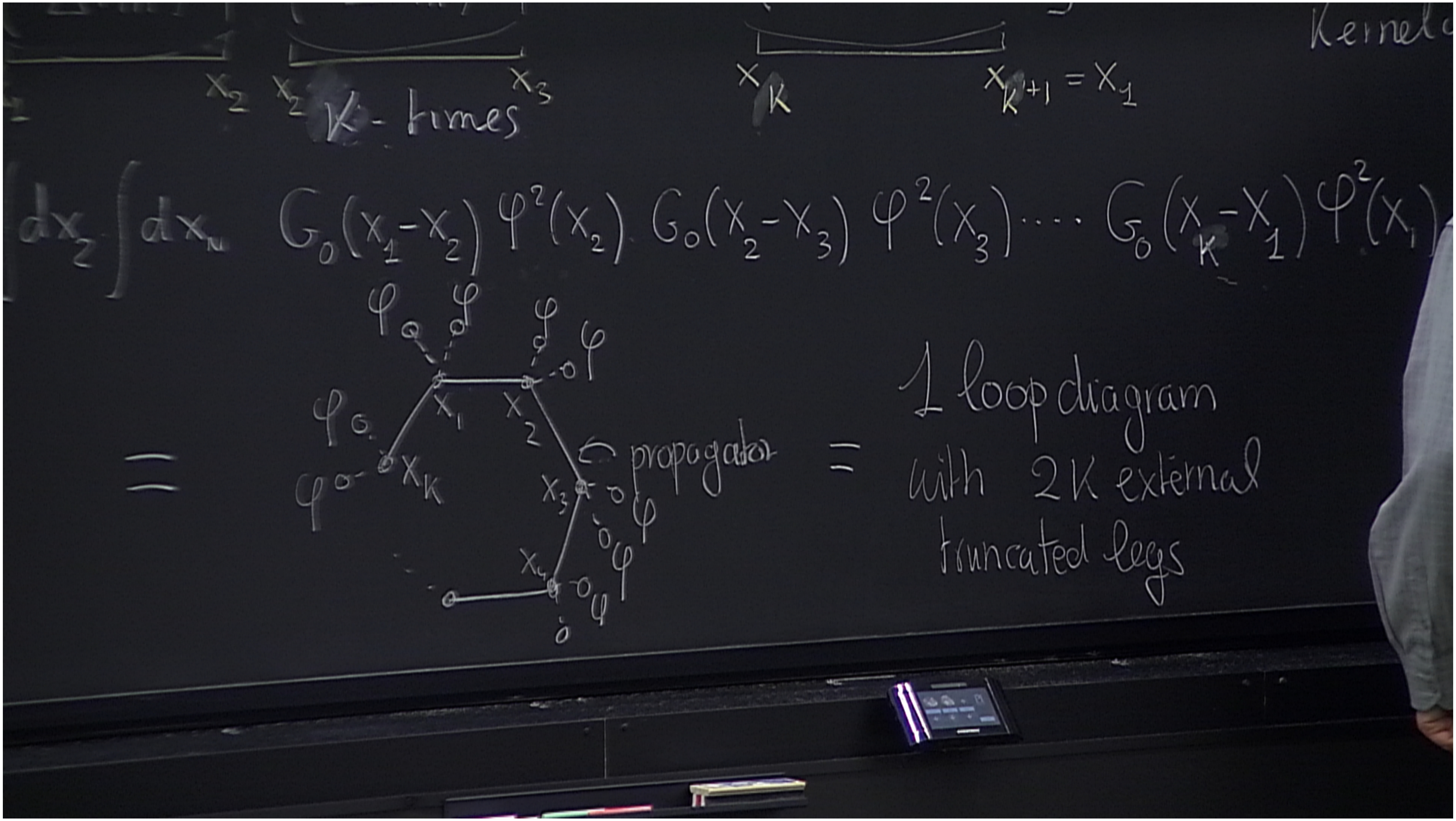
Kernel of $(-\Delta$

Kernel of φ

$$(-1)^{K-1}\left(\frac{g}{2}\right)^K \cdot \frac{1}{K} \int dx_1 \int dx_2 \int dx_N\ G_0(x_1-x_2)\,\varphi^2(x_2)\, G_0(x_2-x_3)\,\varphi^2(x_3) \cdots G_0(x_K-x_1)\,\varphi^2(x_1)$$




K- times
x_3
x_K
$x_{K+1} = x_1$
$G_0(x_1-x_2)\,\varphi^2(x_2)\,G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_K-x_1)\,\varphi^2(x_1)$
propagator
=
1 loop diagram
with 2K external
truncated legs


Kernel
K - times
$x_{K+1} = x_1$
$\int dx_2 \int dx_K \; G_0(x_1 - x_2)\,\varphi^2(x_2)\; G_0(x_2 - x_3)\,\varphi^2(x_3) \cdots G_0(x_K - x_1)\,\varphi^2(x_1)$
propagator
1 loop diagram
with 2K external
truncated legs

$$\Big[\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_1\quad x_2}\;\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_2\quad x_3}\cdots\cdots\underbrace{(-\Delta+m^2)^{-1}\varphi^2}_{x_K\quad x_{K+1}=x_1}\Big]$$

K-times

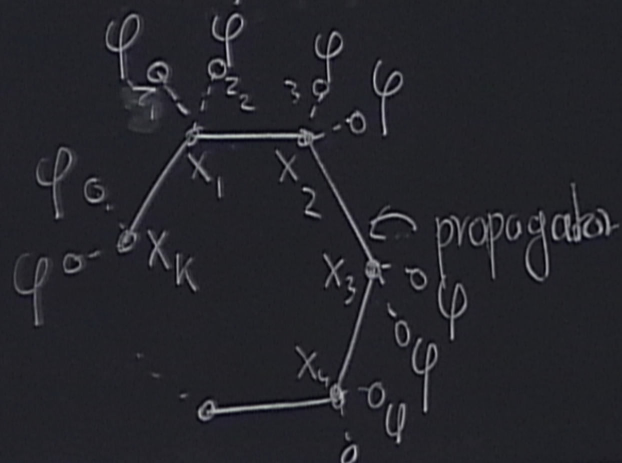
Kernel of $(-\Delta+$

Kernel of φ^2

$$\int dx_2 \int dx_n \; G_0(x_1-x_2)\,\varphi^2(x_2)\, G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_K-x_1)\,\varphi^2(x_1)$$

Fey

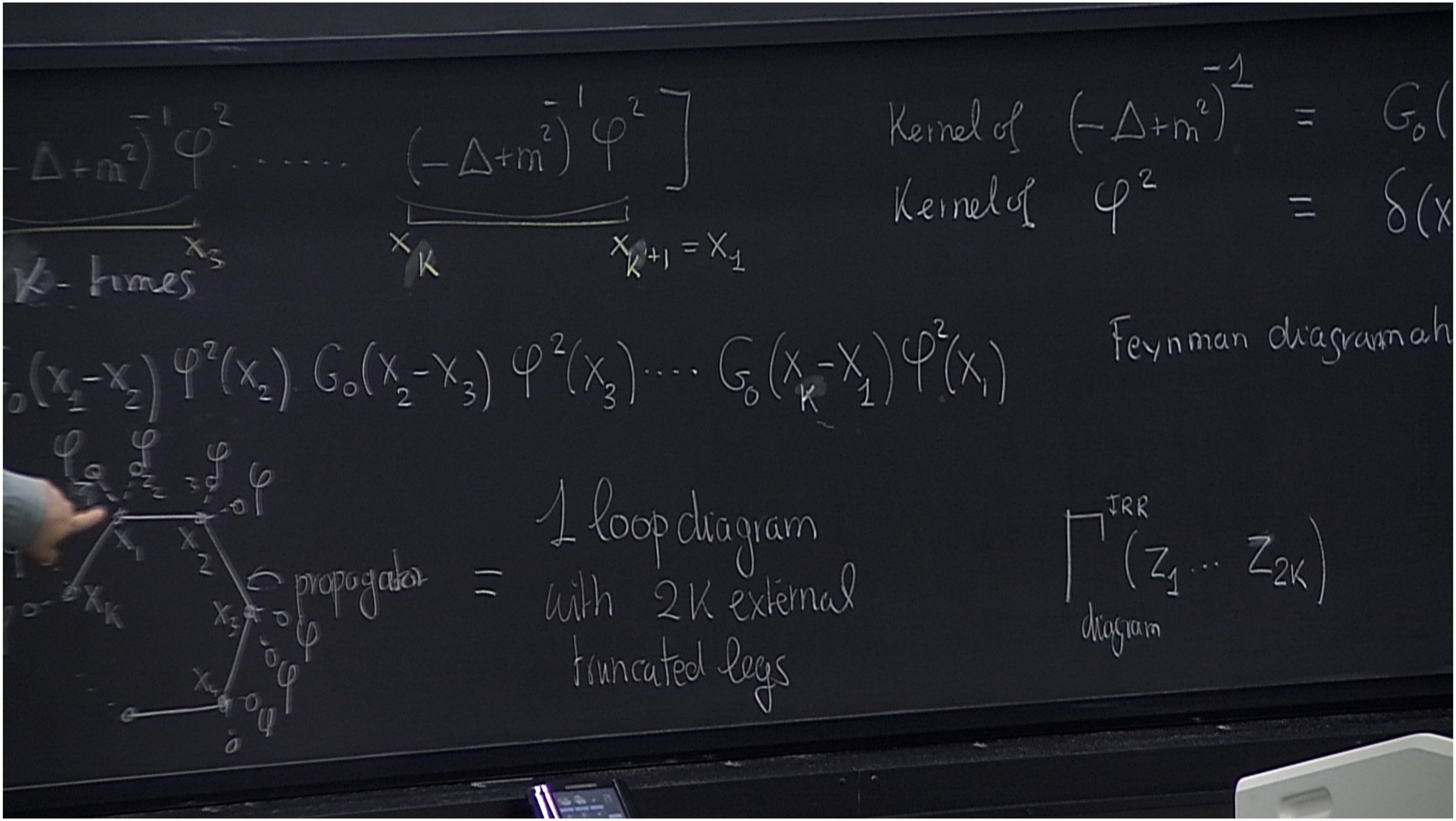
$=$

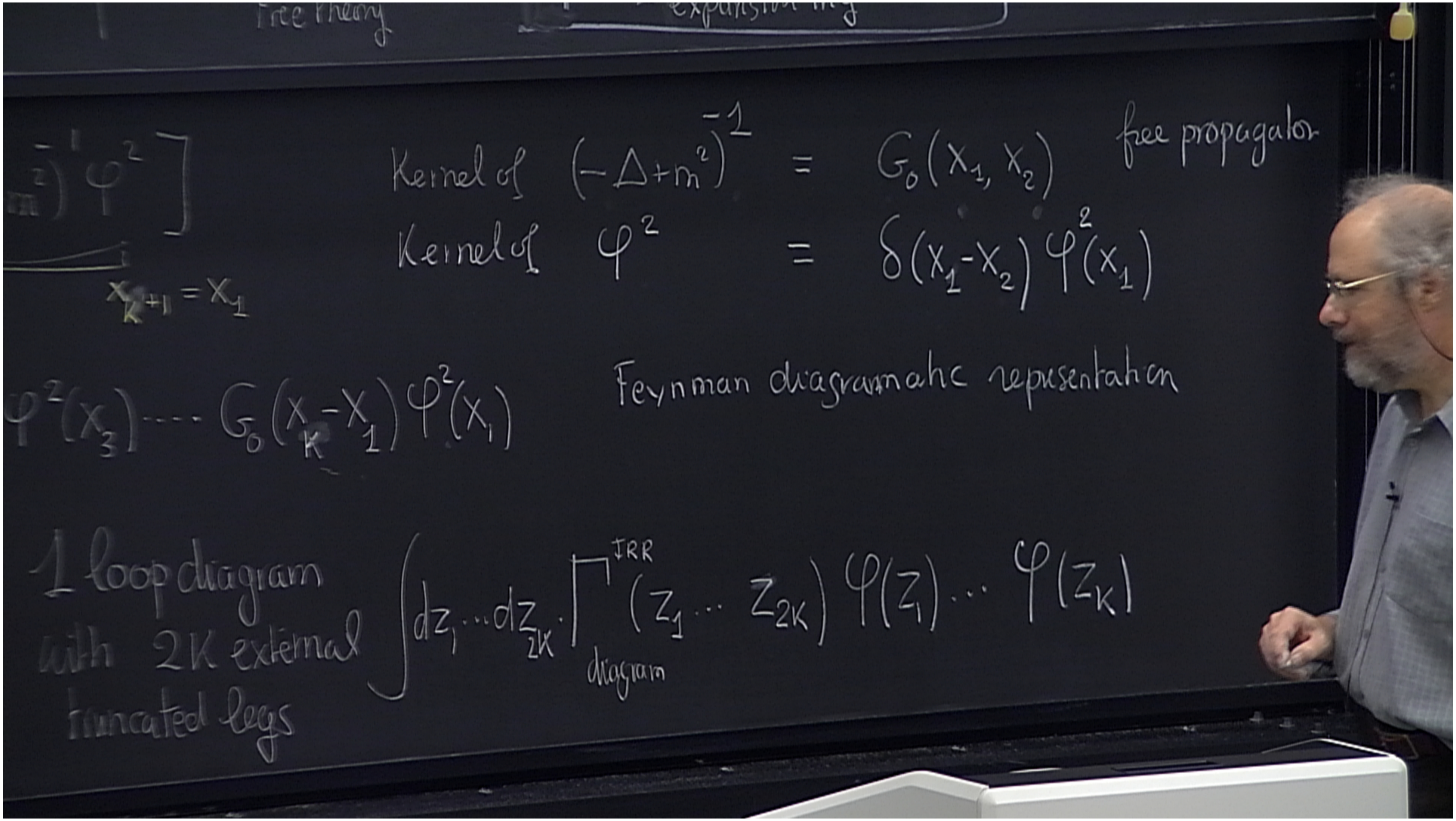


$=$ 1 loop diagram with $2K$ external truncated legs

Γ^{IR}

dia


$(-\Delta+m^2)^{-1}\varphi^2 \cdots\cdots (-\Delta+m^2)^{-1}\varphi^2]$
x_3
k-times
x_k $x_{k+1} = x_1$
Kernel of $(-\Delta+m^2)^{-1}$ = $G_0($
Kernel of φ^2 = $\delta(x$
Feynman diagram ah
$G_0(x_1-x_2)\,\varphi^2(x_2)\,G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_k-x_1)\,\varphi^2(x_1)$
x_1 x_2 x_3 x_4 x_k
φ
propagator
= 1 loop diagram
with 2K external
truncated legs
$\Gamma^{IRR}_{\text{diagram}}(z_1 \cdots z_{2K})$


Free theory
expansion
Kernel of $(-\Delta + m^2)^{-1} = G_0(x_1, x_2)$ free propagator
Kernel of $\varphi^2 = \delta(x_1 - x_2)\,\varphi^2(x_1)$
$x_{k+1} = x_1$
$\varphi^2(x_3) \cdots G_0(x_k - x_1)\,\varphi^2(x_1)$
Feynman diagrammatic representation
1 loop diagram with 2K external truncated legs
$\int dz_1 \cdots dz_{2K} \cdot \Gamma^{IRR}_{diagram}(z_1 \cdots z_{2K})\,\varphi(z_1) \cdots \varphi(z_K)$

(spin 0, spin 1, spin 2, ...)

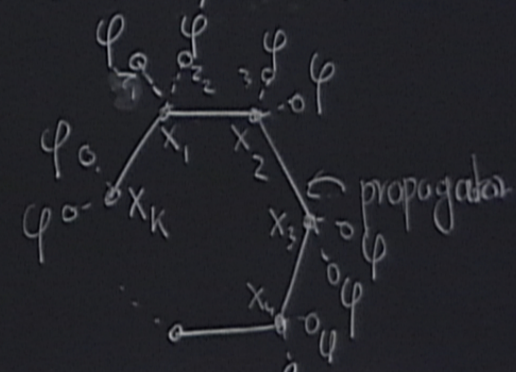
$$\boxed{} = \sum_{K=1}^{\infty} (g/2)^K \frac{(-1)^{K-1}}{K}\, \mathrm{tr}\Big[\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \qquad x_2}\varphi^2 \underbrace{(-\Delta+m^2)^{-1}}_{x_2 \qquad x_3}\varphi^2 \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}}_{x_K}\varphi \ldots$$

K - times

$$= \sum_{K=1}^{\infty} (-1)^{K-1}\left(\frac{g}{2}\right)^K \cdot \frac{1}{K} \int dx_1 \int dx_2 \int dx_N \; G_0(x_1-x_2)\,\varphi^2(x_2)\, G_0(x_2-x_3)\,\varphi^2(x_3 \ldots$$

symmetry Factor
for this 1 loop diagram

$=$



$=$ 1 lo...
with...
four...

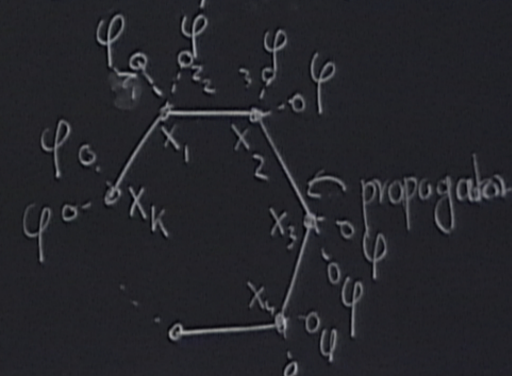
(spin 0, spin 1, spin 2, ...)

$$\boxed{} = \sum_{K=1}^{\infty} (g/2)^{K} \frac{(-1)^{K-1}}{K}\, \mathrm{tr}\Big[\underbrace{(-\Delta+m^2)^{-1}}_{x_1 \qquad x_2}\varphi^2 \underbrace{(-\Delta+m^2)^{-1}}_{x_2 \qquad x_3}\varphi^2 \cdots\cdots \underbrace{(-\Delta+m^2)^{-1}}_{x_K}\varphi$$

K - times

$$= \sum_{K=1}^{\infty} (-1)^{K-1}\left(\frac{g}{2}\right)^{K} \cdot \frac{1}{K} \int dx_1 \int dx_2 \int dx_N \; G_0(x_1-x_2)\,\varphi^2(x_2)\, G_0(x_2-x_3)\,\varphi^2(x_3$$

symmetry Factor
for this 1 loop diagram =



= 1 lo
with
four

$\underbrace{(-\Delta+m^2)^{-1}\varphi^2}\,]$

$x_k \qquad\qquad x_{k+1}=x_1$

Kernel of $(-\Delta+m^2)^{-1} = G_0($

Kernel of $\varphi^2 = \delta(x$

$(x_2)\,G_0(x_2-x_3)\,\varphi^2(x_3)\cdots G_0(x_k-x_1)\,\varphi^2(x_1)$

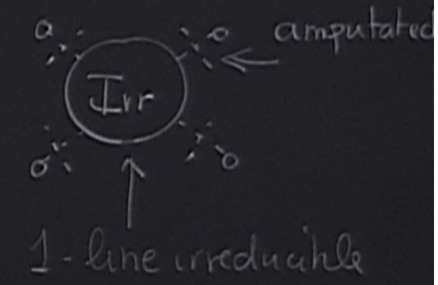
Feynman diagram ah

irreducible

propagator = 1 loop diagram with $2K$ external truncated legs $= \int dz_1\cdots dz_{2K}\cdot \Gamma^{IRR}_{\text{diagram}}(z_1\cdots z_{2K})\,\varphi(z_1)$

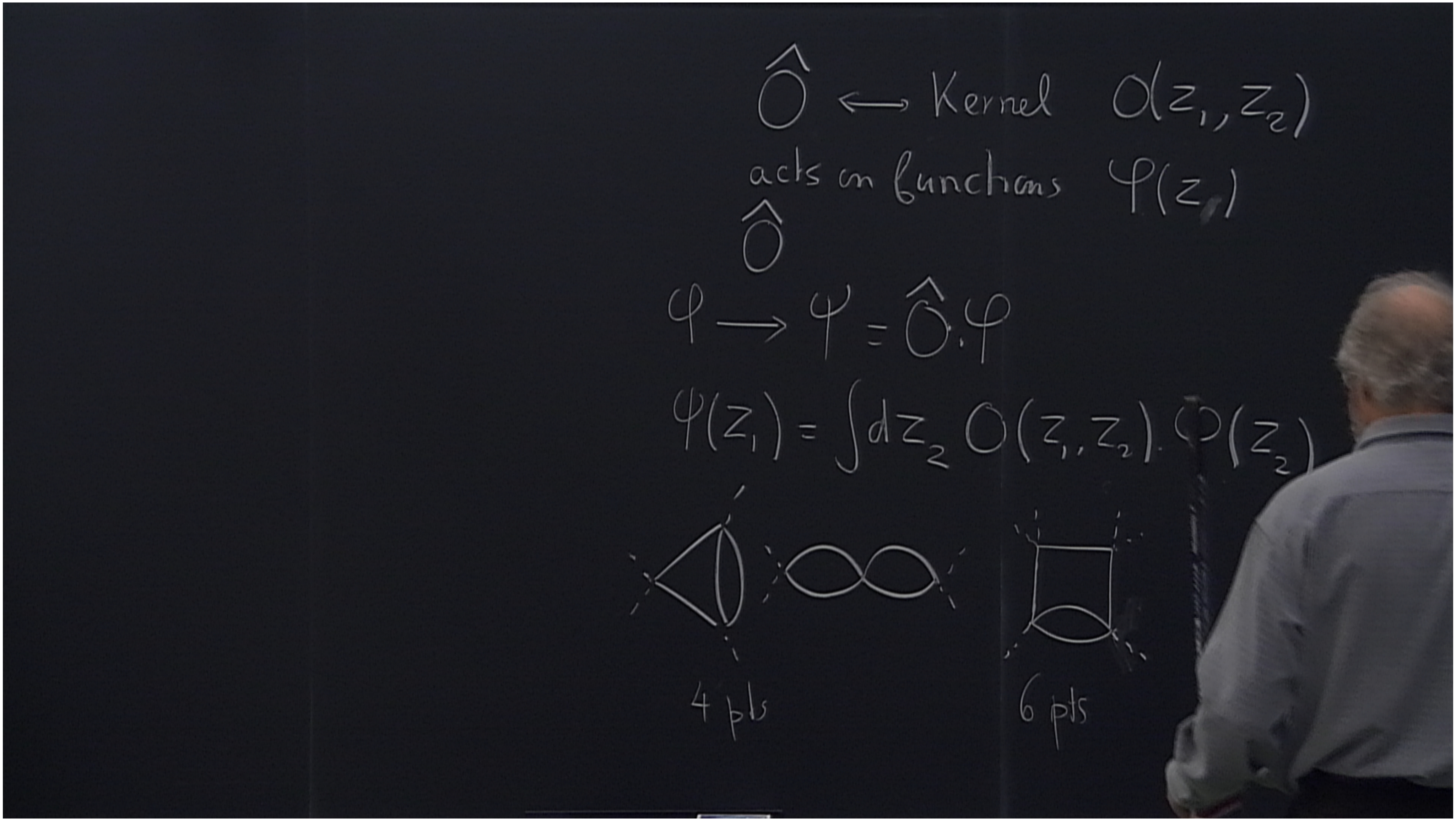
Functional | Γ is a function that depends on the function φ | φ back...

- 1-particle irreducible diagrams & Quantum Effective Action
  - Generating Functional General Definition
  - At 1st order in the $\hbar$ expansion
  - Example for the ϕ^4 theory



- Renormalization theory for massless ϕ^4

$\hbar^B$ in $\Gamma[\varphi] \longrightarrow$ irreducible B-loop diagrams


$\hat{O} \longleftrightarrow$ Kernel $O(z_1, z_2)$
acts on functions $\varphi(z_1)$
$\hat{O}$
$\varphi \longrightarrow \psi = \hat{O} \cdot \varphi$
$\psi(z_1) = \int dz_2\, O(z_1, z_2) \cdot \varphi(z_2)$
4 pts
6 pts

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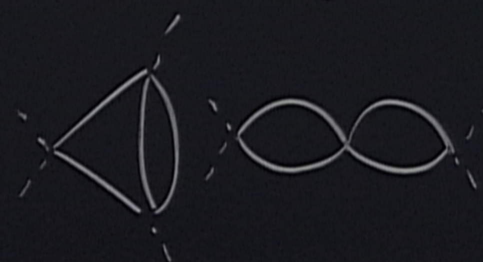
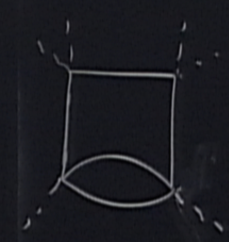


4 pts

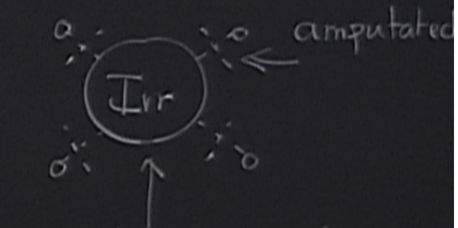


6 pts

$\hbar^2$

Functional | Γ is a function that depends on the function φ | φ back...

- 1-particle irreducible diagrams & Quantum Effective Action
  - Generating Functional General Definition
  - At 1st order in the $\hbar$ expansion
  - Example for the ϕ^4 theory



- Renormalization theory for massless ϕ^4

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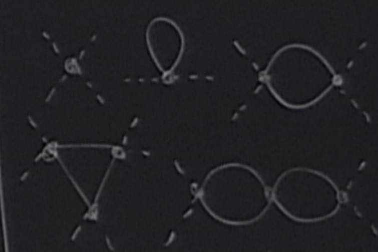

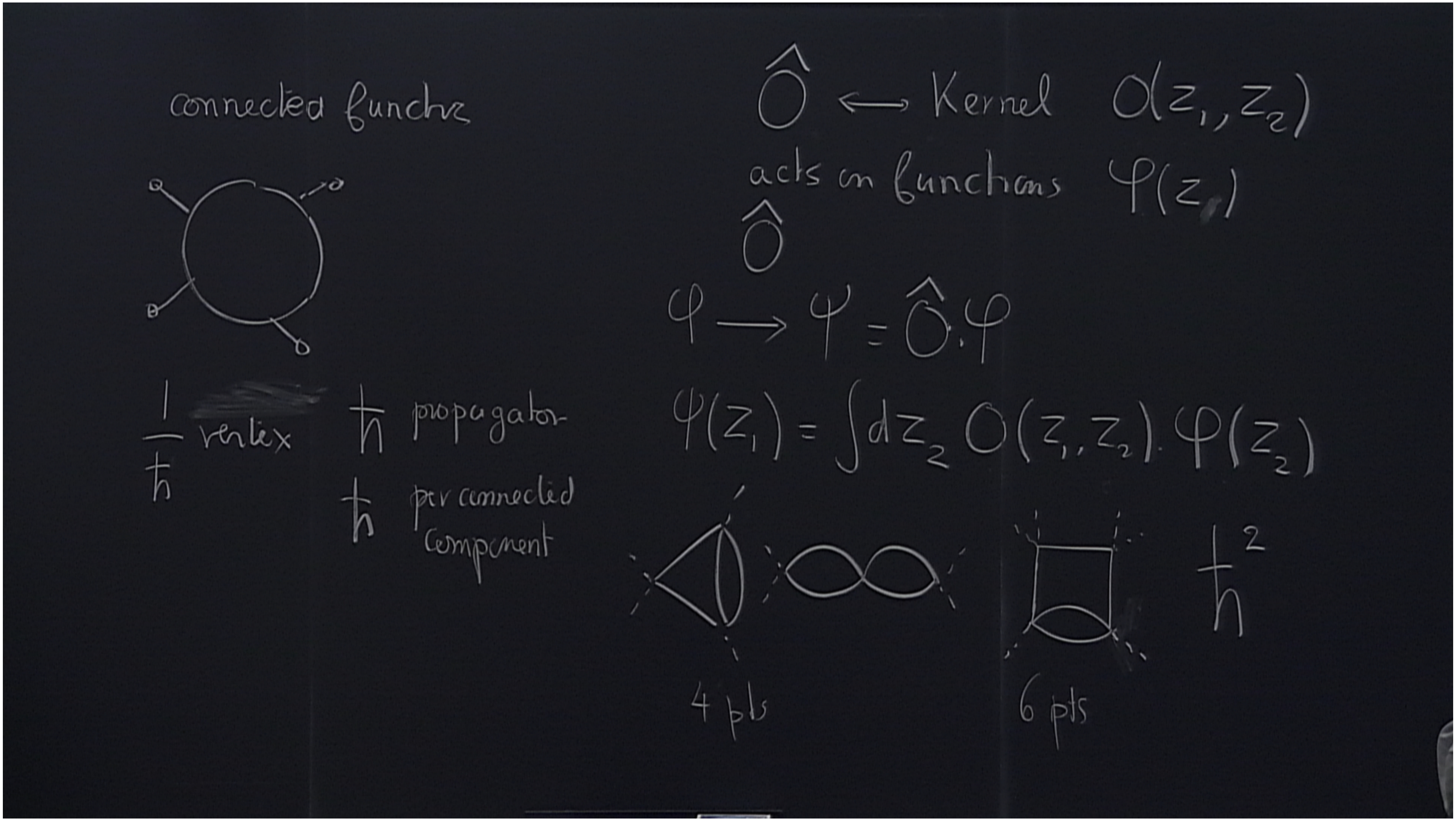
connected functn
$\frac{1}{\hbar}$ vertex
$\hbar$ propagator
$\hbar$ per connected component
$\hat{O} \longleftrightarrow$ Kernel $O(z_1, z_2)$
acts on functions $\varphi(z_1)$
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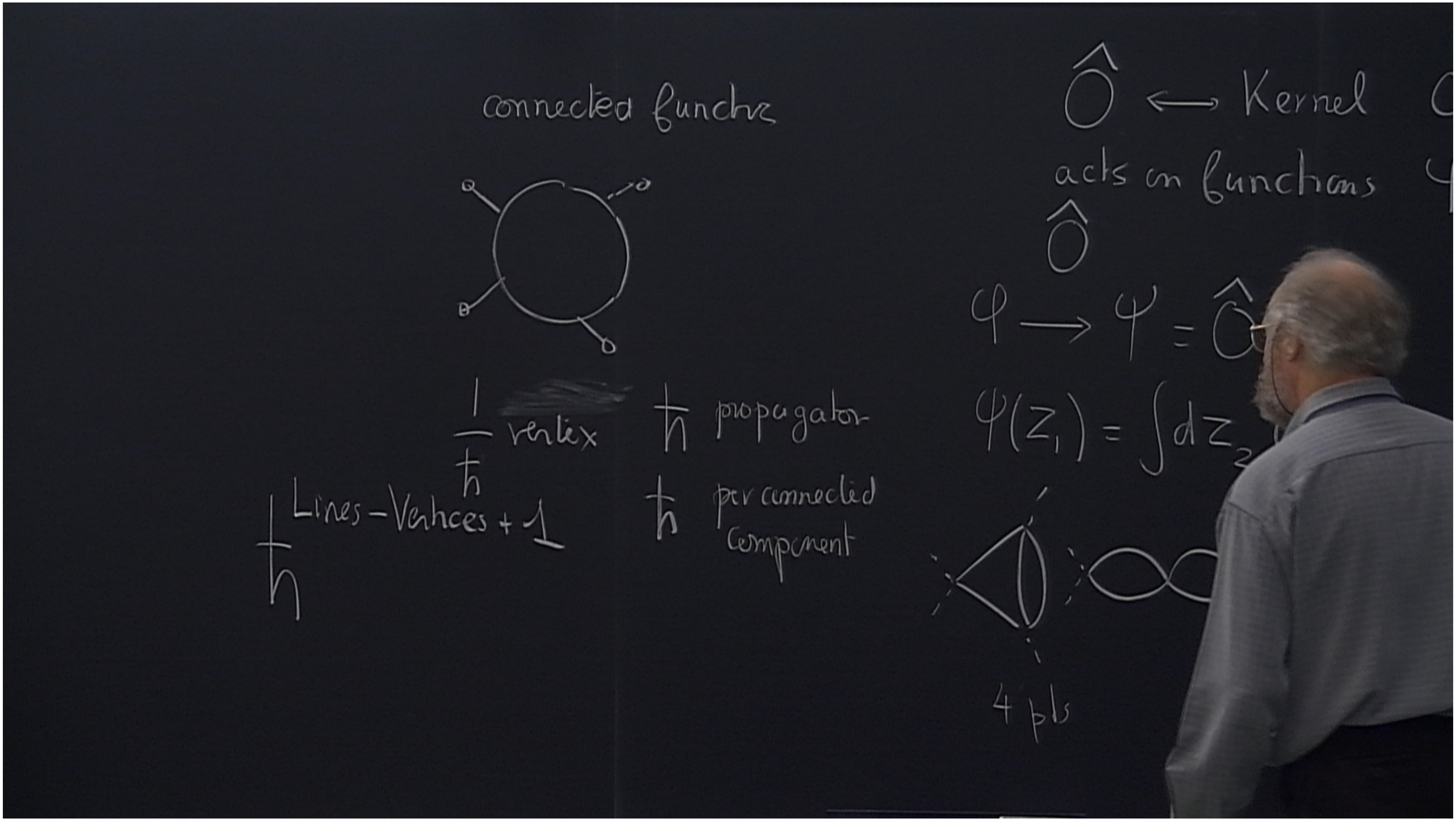
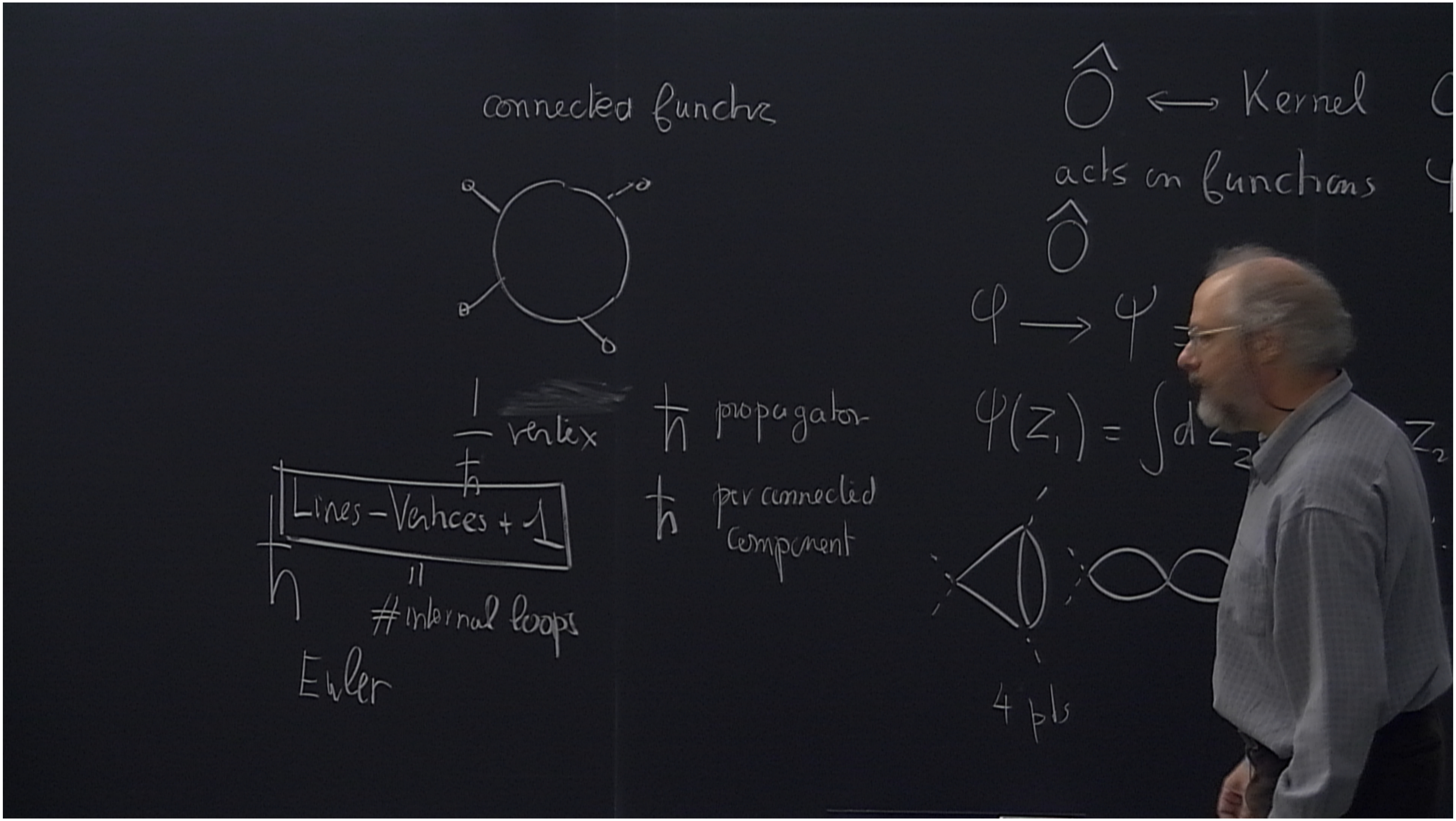
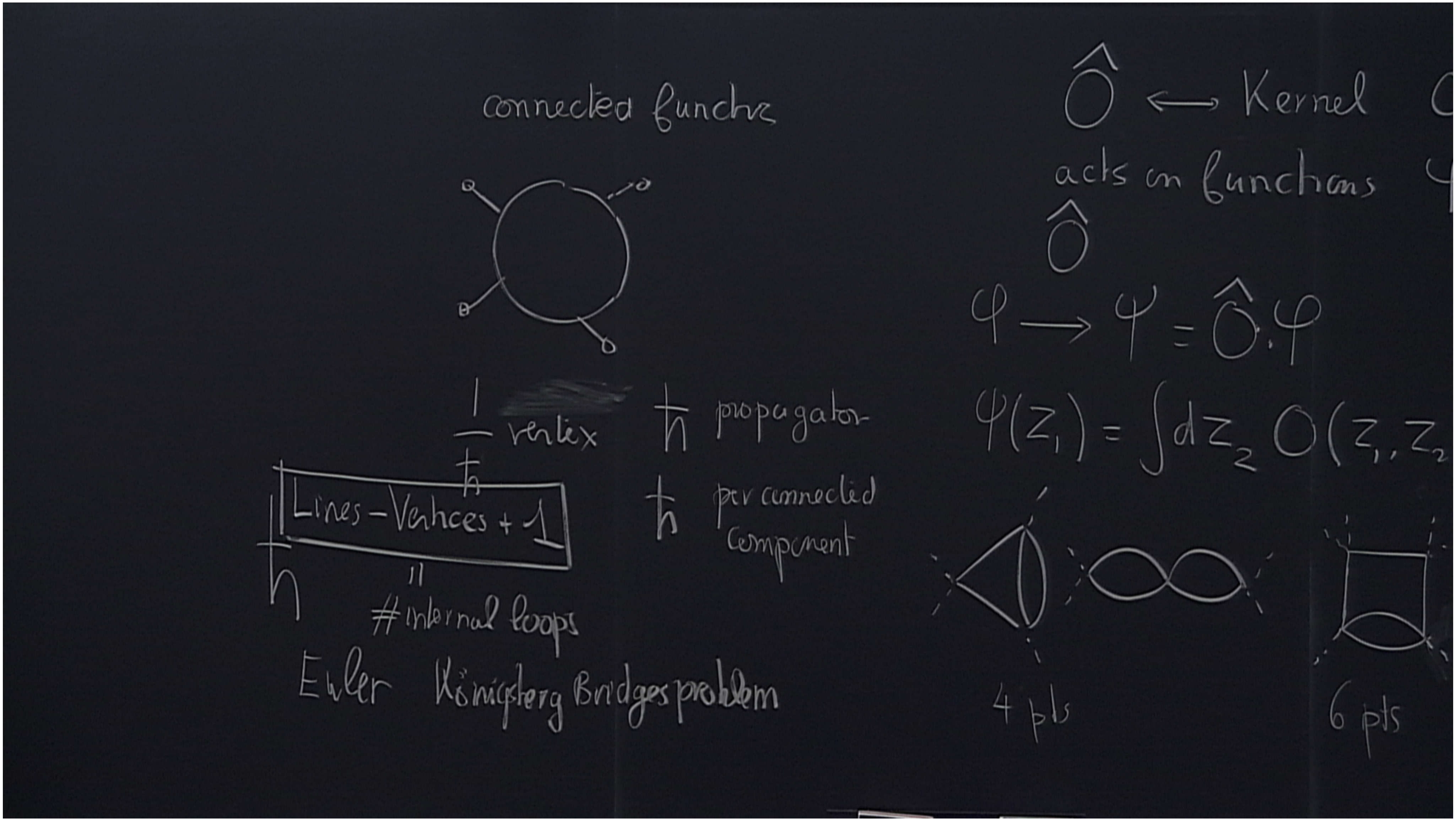
4 pts
6 pts
$\hbar^2$



connected functio
1/ħ vertex
ħ propagator
ħ Lines − Vertices + 1
ħ per connected component
Ô ⟷ Kernel
acts on functions
Ô
φ → ψ = Ô
ψ(z1) = ∫dz2
4 pts


connected functions
vertex
propagator
per connected component
Lines − Vertices + 1
internal loops
Euler
Kernel
acts on functions
4 pts


connected functn
$\frac{1}{\hbar}$ vertex
$\hbar$ propagator
$\hbar$ per connected component
$\hbar^{\boxed{\text{Lines} - \text{Vertices} + 1}}$
$=$ # internal loops
Euler Königsberg Bridges problem
$\hat{O} \longleftrightarrow$ Kernel O
acts on functions φ
$\hat{O}$
$\varphi \longrightarrow \psi = \hat{O}\cdot\varphi$
$\psi(z_1) = \int dz_2\, O(z_1, z_2$
4 pts
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Functional | 1 is a function that depends on the function φ | φ back...

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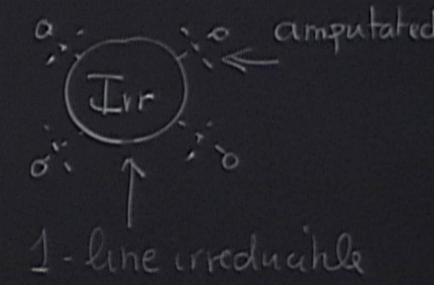
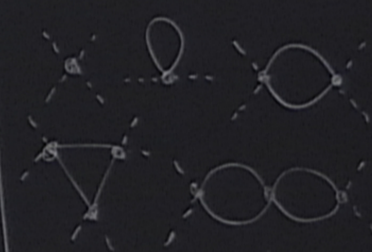
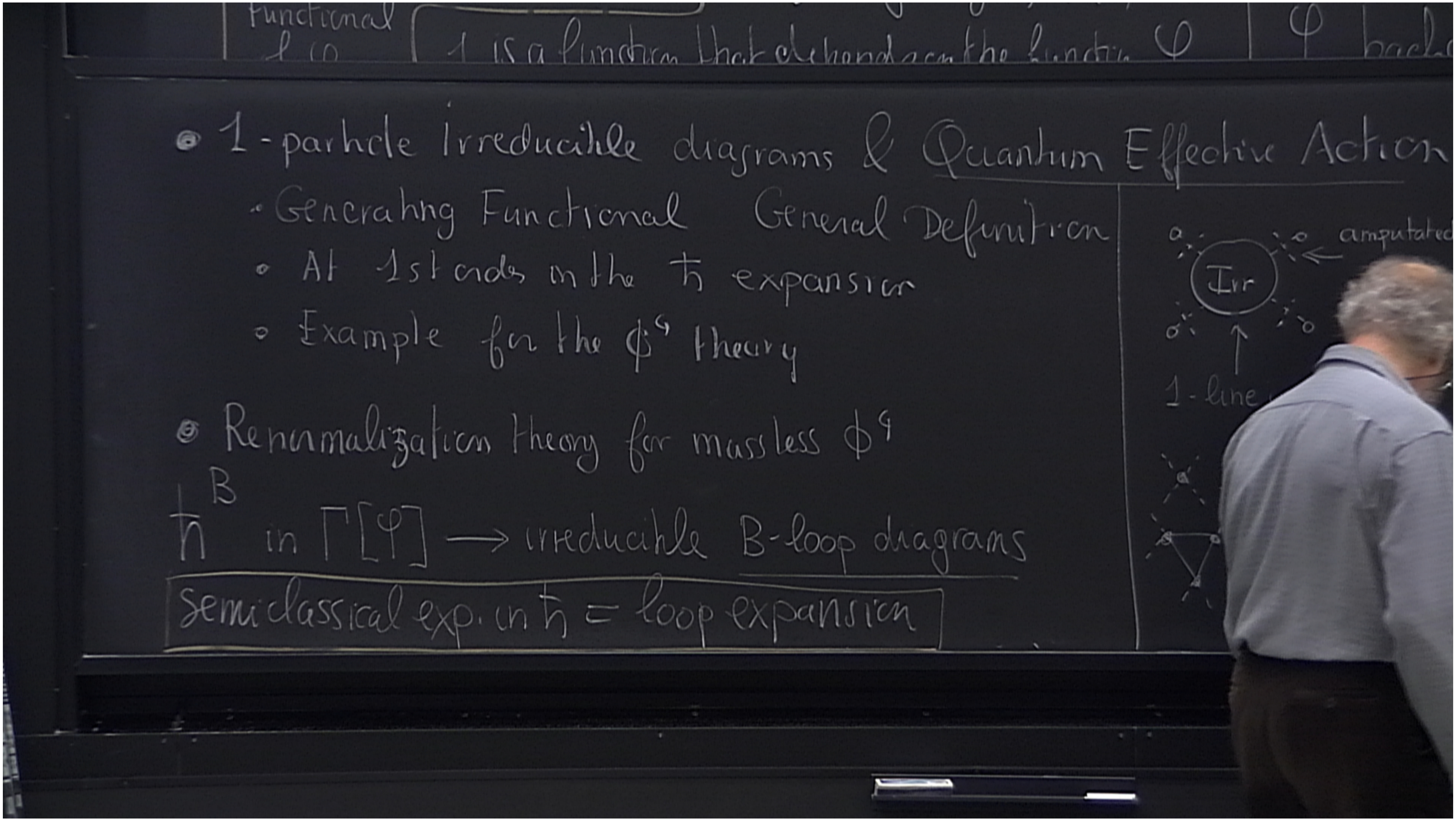




- Renormalization theory for massless ϕ^4

$\hbar^B$ in $\Gamma[\varphi] \longrightarrow$ irreducible B-loop diagrams

semiclassical exp. in $\hbar$ = loop expansion


Functional
1 is a function that depends on the function φ
φ
1-particle irreducible diagrams & Quantum Effective Action
Generating Functional General Definition
At 1st order in the ħ expansion
Example for the φ⁴ theory
Renormalization theory for massless φ⁴
ħ^B in Γ[φ] → irreducible B-loop diagrams
Semiclassical exp. in ħ = loop expansion
amputated
Irr
1-line

classical ground state of $\Gamma[\varphi] \Rightarrow \langle \phi \rangle$ quantum field

point of $\Gamma[\varphi]$

Renormalisation : introduction

$\Leftarrow$ UV singularities / divergences

UV regulator. - Lattice

- momentum scale

... field point of $\Gamma[\varphi]$

...ction $\Leftarrow$ UV singularities /divergences

UV regulator. - Lattice

- momentum scale $|k| < \Lambda$

Poincaré

Unitarity

... field   point of $\Gamma[\varphi]$

...tion $\Leftarrow$ UV singularities / divergences

UV regulator. – Lattice $\#$ Poincaré !

– momentum scale $|k| < \Lambda$ Unitarity !

field point of $\Gamma[\varphi]$

tion $\Leftarrow$ UV singularities / divergences

UV regulator. – Lattice – momentum scale $|k| < \Lambda$

Poincaré !

Unitarity !

defines the theory

$\downarrow$

get rid of it ?

"Classical ground state" of $\Gamma[\varphi] \Rightarrow \langle \phi \rangle$ quantum field   point of $\Gamma[\varphi]$

Renormalisation: introduction

$\Leftarrow$ UV singularities / divergences

UV regulator. - Lattice
- momentum

defines the theory
$\downarrow$
get rid of it ?

Renormalized parameters (observable quantities)

"Bare parameters" (needed in order to construct a regularized theory)

2 point Function irreducible

$$\underset{1}{\circ}\!\!\overset{p_1\rightarrow \quad \leftarrow p_2}{-\!\!\!+\!\!\!+\!\!\!-}\!\!\underset{2}{\circ} + g\,\frac{1}{2}\ \underset{1}{\circ}\!-\!\!\!\overset{\bigcirc}{\bullet}\!\!\!-\!\underset{2}{\circ} + O(g^2) = \Gamma^{IRR}_{(2)}(z_1, z_2) \xrightarrow{F.T} (2\pi)^D \delta(p_1+p_2)\,\hat{\Gamma}_{(2)}(p_1) \qquad \hat{\Gamma}_{(2)}$$

Φ > quantum field   point of $\Gamma[\varphi]$

ntroduction $\Leftarrow$ UV singularities / divergences

(observable quantities)

eded in order to construct regularized theory)

ducible

UV regulator. - Lattice

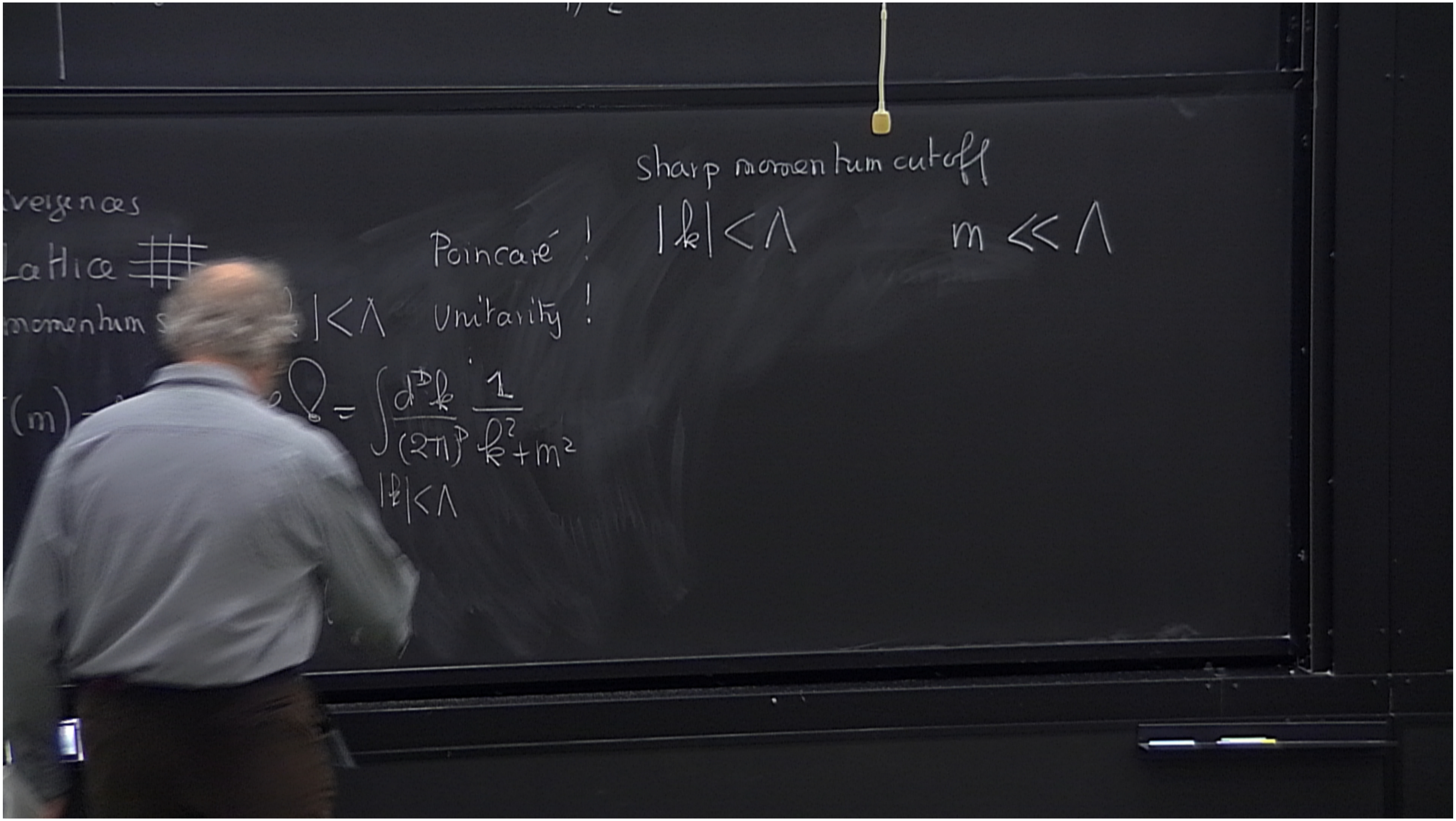
- momentum scale $|k| < \Lambda$

defines the theory $\downarrow$ get rid of it ?

Poincaré ! Unitarity !

$T(m)$ = Amplitude of "tadpole" $= \int_{|k|<\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2}$

$$g^2) = \Gamma^{IRR}_{(2)}(z_1, z_2) \xrightarrow{F.T} (2\pi)^D \delta(p_1+p_2)\, \hat{\Gamma}_{(2)}(p_1) \qquad \hat{\Gamma}_{(2)}(p) = p^2 + m^2 + \frac{g}{2} T(m)$$


Sharp momentum cutoff
$|k| < \Lambda$
$m \ll \Lambda$
Poincaré !
Unitarity !
$= \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m^2}$
$|k| < \Lambda$

Sharp momentum cutoff

ivergences

Lattice

momentum scale $|k| < \Lambda$

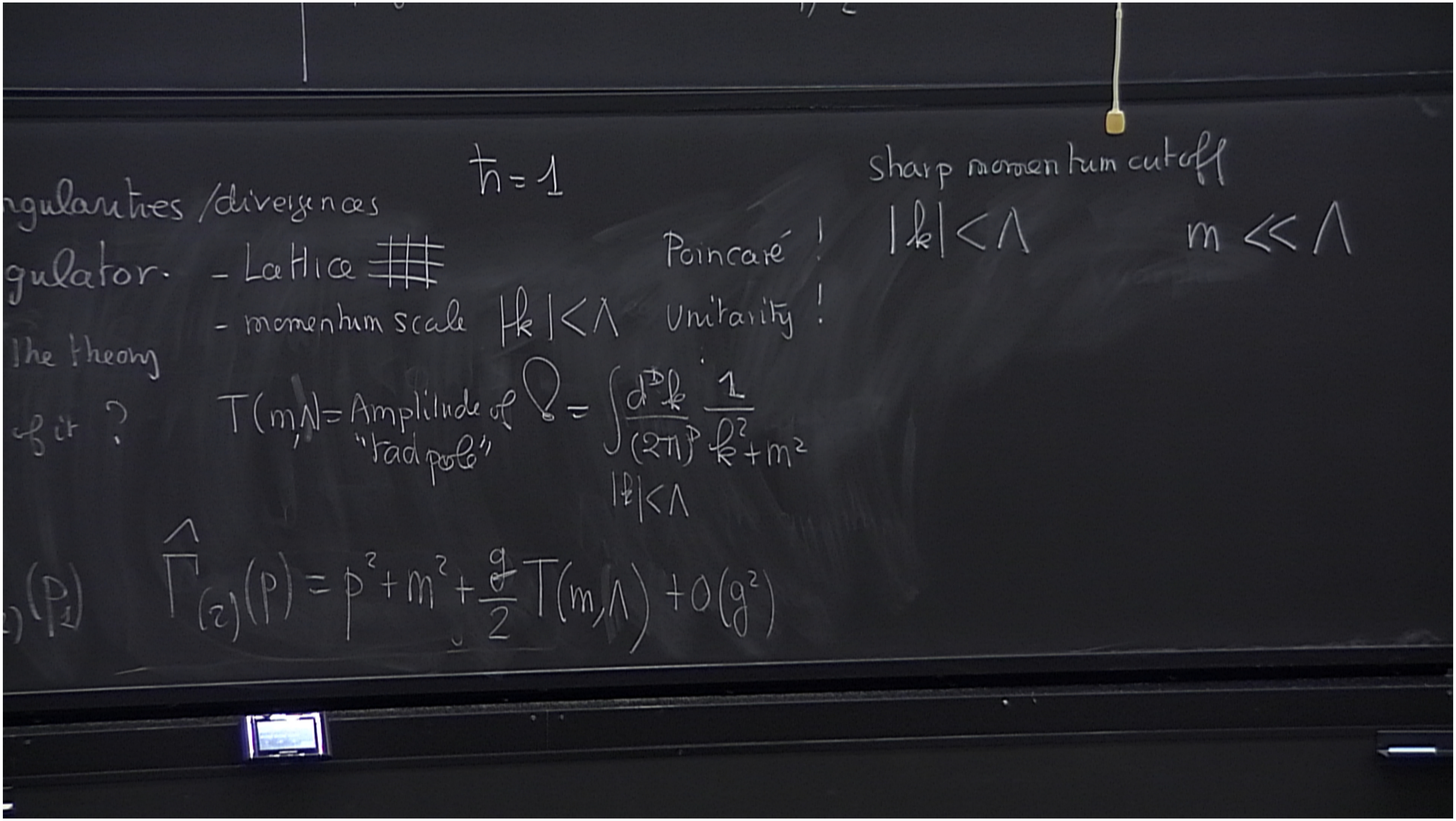
Poincaré !

Unitarity !

$|k| < \Lambda$ $\qquad m \ll \Lambda$

$T(m,\Lambda)$ = Amplitude of "tadpole" $= \int_{|k|<\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2}$

$(p) = p^2 + m^2 + \frac{g}{2} T(m,\Lambda)$



$\hbar = 1$
Sharp momentum cutoff
ngularities /divergences
$|k| < \Lambda$
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gulator. - Lattice
Poincaré !
- momentum scale $|k| < \Lambda$
Unitarity !
The theory
of it ?
$T(m,\Lambda)$ = Amplitude of "tadpole" $= \int_{|k|<\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2}$
$\hat{\Gamma}_{(2)}(p) = p^2 + m^2 + \frac{g}{2} T(m,\Lambda) + O(g^2)$

divergences

$\hbar = 1$ simpler

- Lattice
- momentum scale $|k| < \Lambda$

Poincaré !

Unitarity !

Sharp momentum cutoff

$|k| < \Lambda$ $\qquad m \ll \Lambda$

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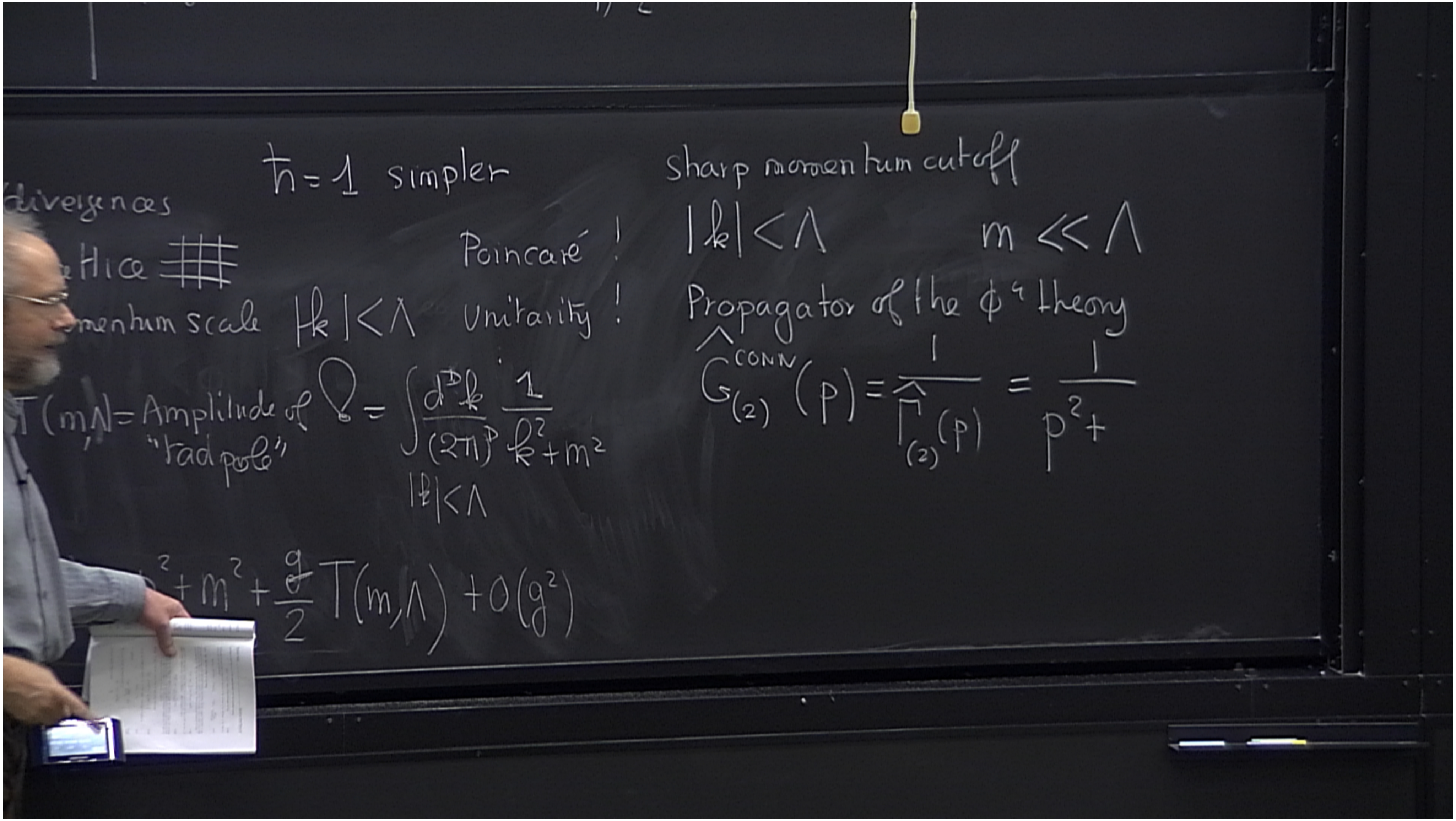
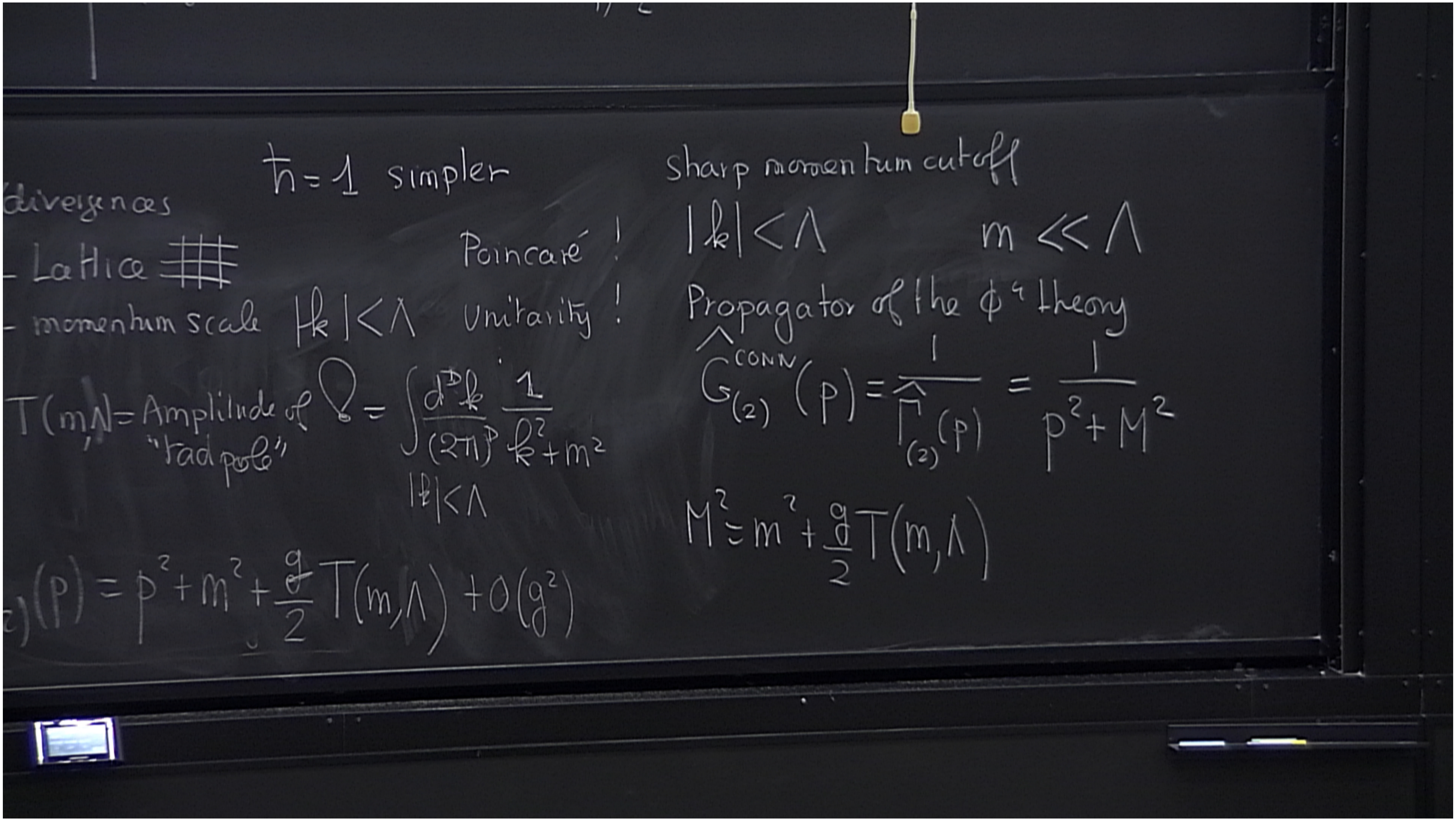
$$(P) = p^2 + m^2 + \frac{g}{2} T(m,\Lambda) + O(g^2)$$


ħ = 1 simpler
divergences
lattice
momentum scale |k| < Λ
Poincaré !
Unitarity !
T(m,Λ) = Amplitude of "tadpole" = ∫_{|k|<Λ} d^D k/(2π)^D 1/(k² + m²)
² + m² + g/2 T(m,Λ) + O(g²)
Sharp momentum cutoff
|k| < Λ
m ≪ Λ
Propagator of the φ⁴ theory
Ĝ^CONN_(2)(p) = 1/Γ̂_(2)(p) = 1/(p² +


$\hbar = 1$ simpler
divergences
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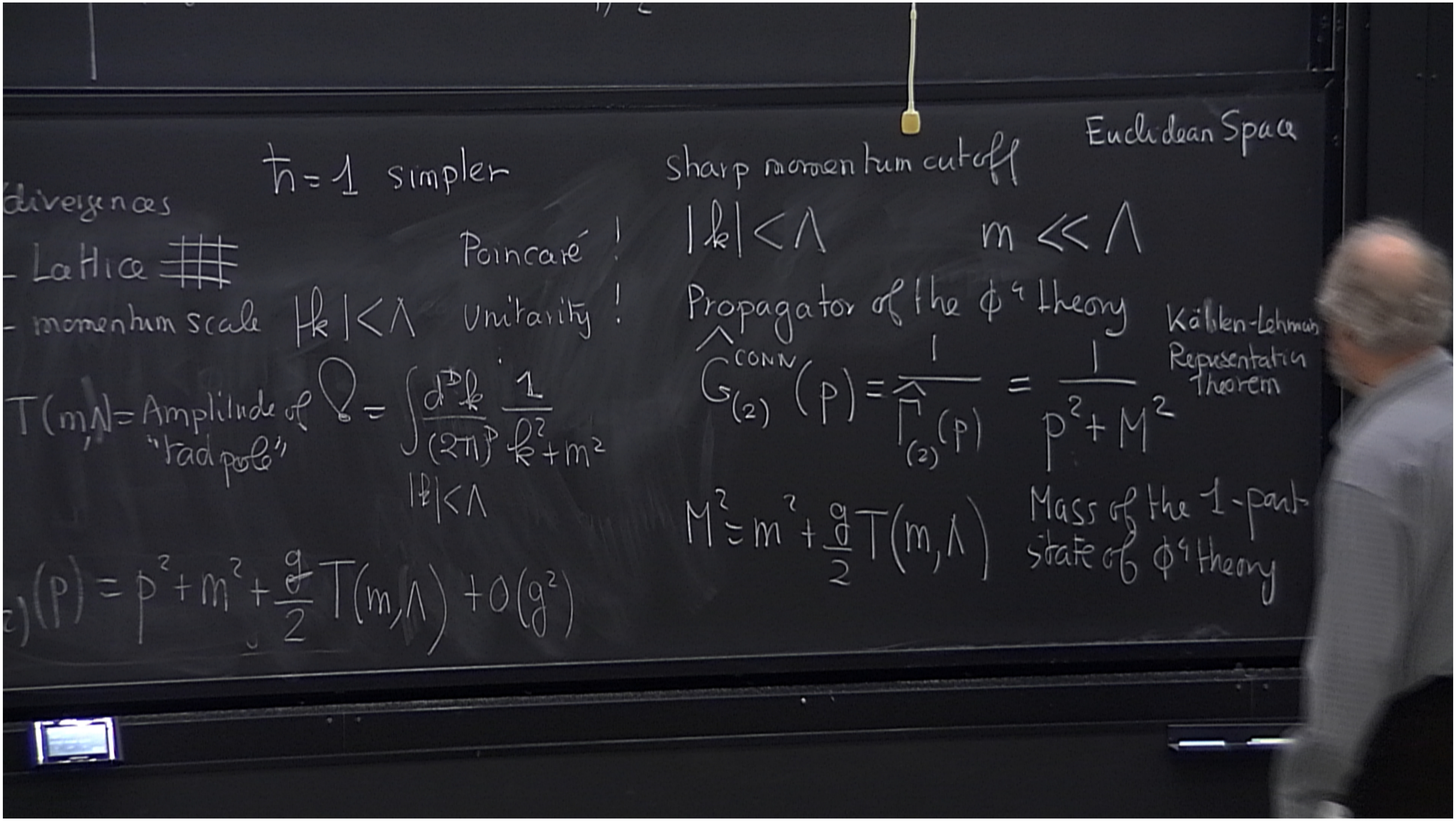
$(p) = p^2 + m^2 + \frac{g}{2} T(m,\Lambda) + O(g^2)$
Sharp momentum cutoff
$|k| < \Lambda$
$m \ll \Lambda$
Propagator of the ϕ^4 theory
$\hat{G}^{\mathrm{CONN}}_{(2)}(p) = \frac{1}{\hat{\Gamma}_{(2)}(p)} = \frac{1}{p^2 + M^2}$
$M^2 = m^2 + \frac{g}{2} T(m,\Lambda)$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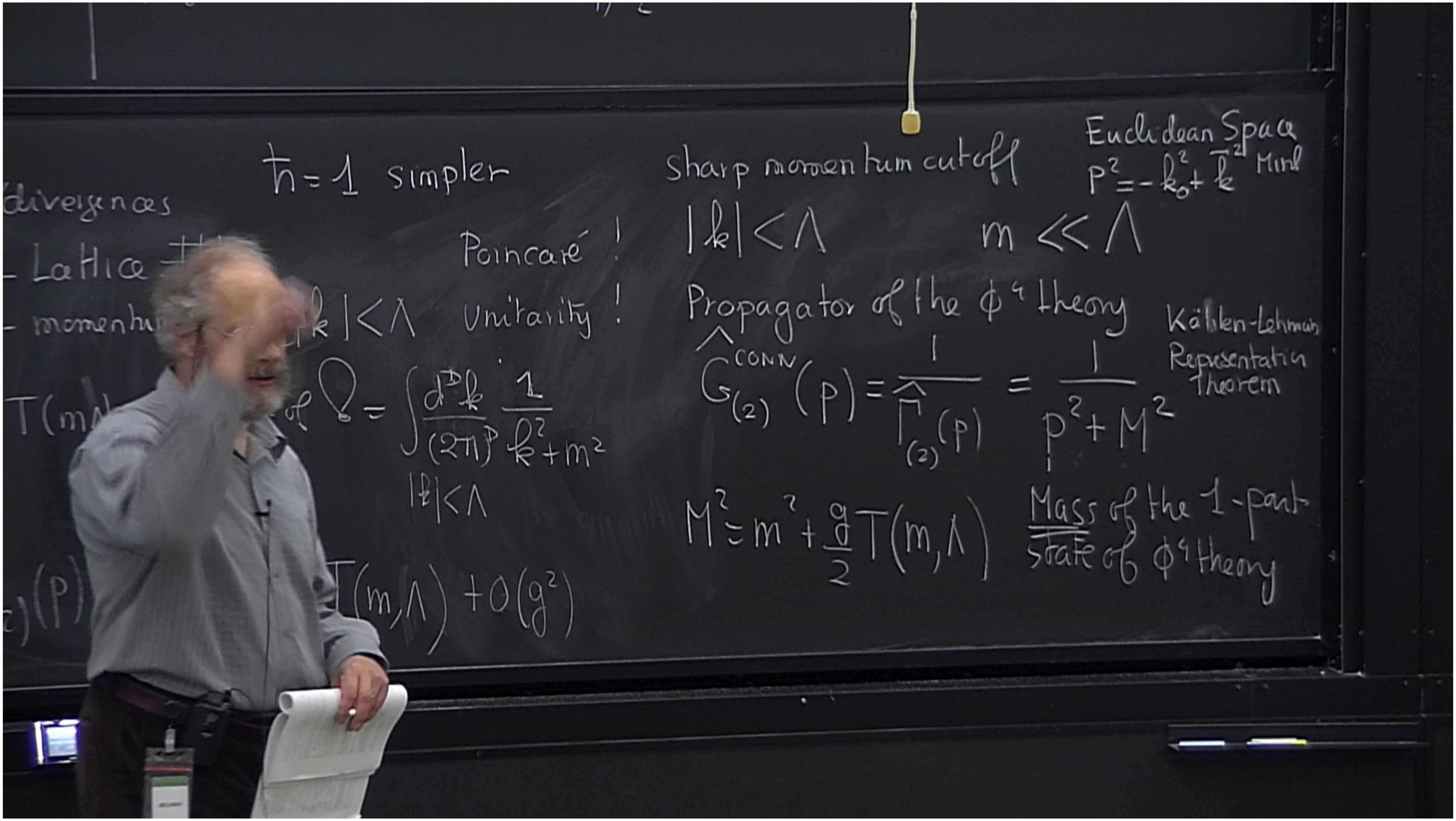
Sharp momentum cutoff
Euclidean Space
$|k| < \Lambda$
$m \ll \Lambda$
Propagator of the ϕ^4 theory
$\hat{G}^{CONN}_{(2)}(p) = \frac{1}{\hat{\Gamma}_{(2)}(p)} = \frac{1}{p^2 + M^2}$
Källén-Lehmann Representation Theorem
$M^2 = m^2 + \frac{g}{2} T(m,\Lambda)$
Mass of the 1-part. state of ϕ^4 theory


ħ = 1 simpler
divergences
Lattice
momentum
Poincaré !
Unitarity !
|k| < Λ
Sharp momentum cutoff
|k| < Λ
m ≪ Λ
Euclidean Space
Propagator of the φ⁴ theory
Källen-Lehmann Representation Theorem
M² = m² + g/2 T(m,Λ)
Mass of the 1-part. state of φ⁴ theory
T(m,Λ) + O(g²)

divergences

- Lattice
- momentum scale $|k|<\Lambda$

$\hbar = 1$ simpler

Poincaré !

Unitarity !

$T(m,\Lambda)$ = Amplitude of "tadpole" $= \int_{|k|<\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2}$

$$\Gamma_{(2)}(p) = p^2 + m^2 + \frac{g}{2} T(m,\Lambda) + O(g^2)$$

Sharp momentum cutoff

$|k| < \Lambda$ $\qquad m \ll \Lambda$

Euclidean Space

$p^2 = -k_0^2 + \vec{k}^2$ Mink

Propagator of the ϕ^4 theory

$$\hat{G}^{\mathrm{CONN}}_{(2)}(p) = \frac{1}{\hat{\Gamma}_{(2)}(p)} = \frac{1}{p^2 + M^2}$$

Källen-Lehmann Representation Theorem

$$M^2 = m^2 + \frac{g}{2} T(m,\Lambda)$$

physical mass

Mass of the 1-part. state of ϕ^4 theory

divergences

- Lattice
- momentum scale $|k| < \Lambda$

$\hbar = 1$ simpler

Poincaré !

Unitarity !

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Sharp momentum cutoff

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Propagator of the ϕ^4 theory

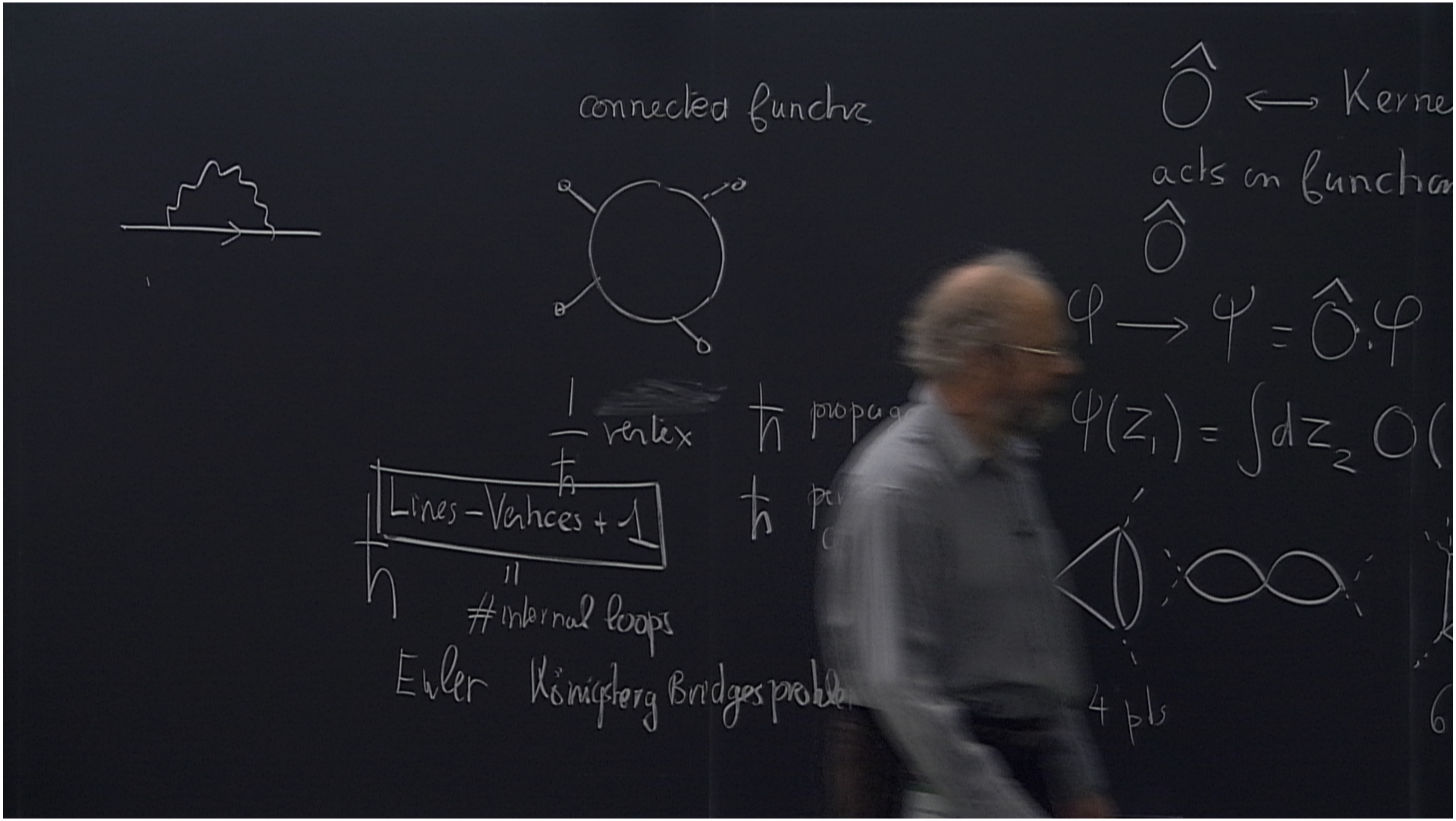
$$\hat{G}^{\mathrm{CONN}}_{(2)}(p) = \frac{1}{\hat{\Gamma}_{(2)}(p)} = \frac{1}{p^2 + M^2}$$

Källen-Lehmann Representation Theorem

$$M^2 = m^2 + \frac{g}{2} T(m,\Lambda)$$

M^2: physical mass; $\frac{g}{2} T(m,\Lambda)$: radiative correction

Mass of the 1-particle state of ϕ^4 theory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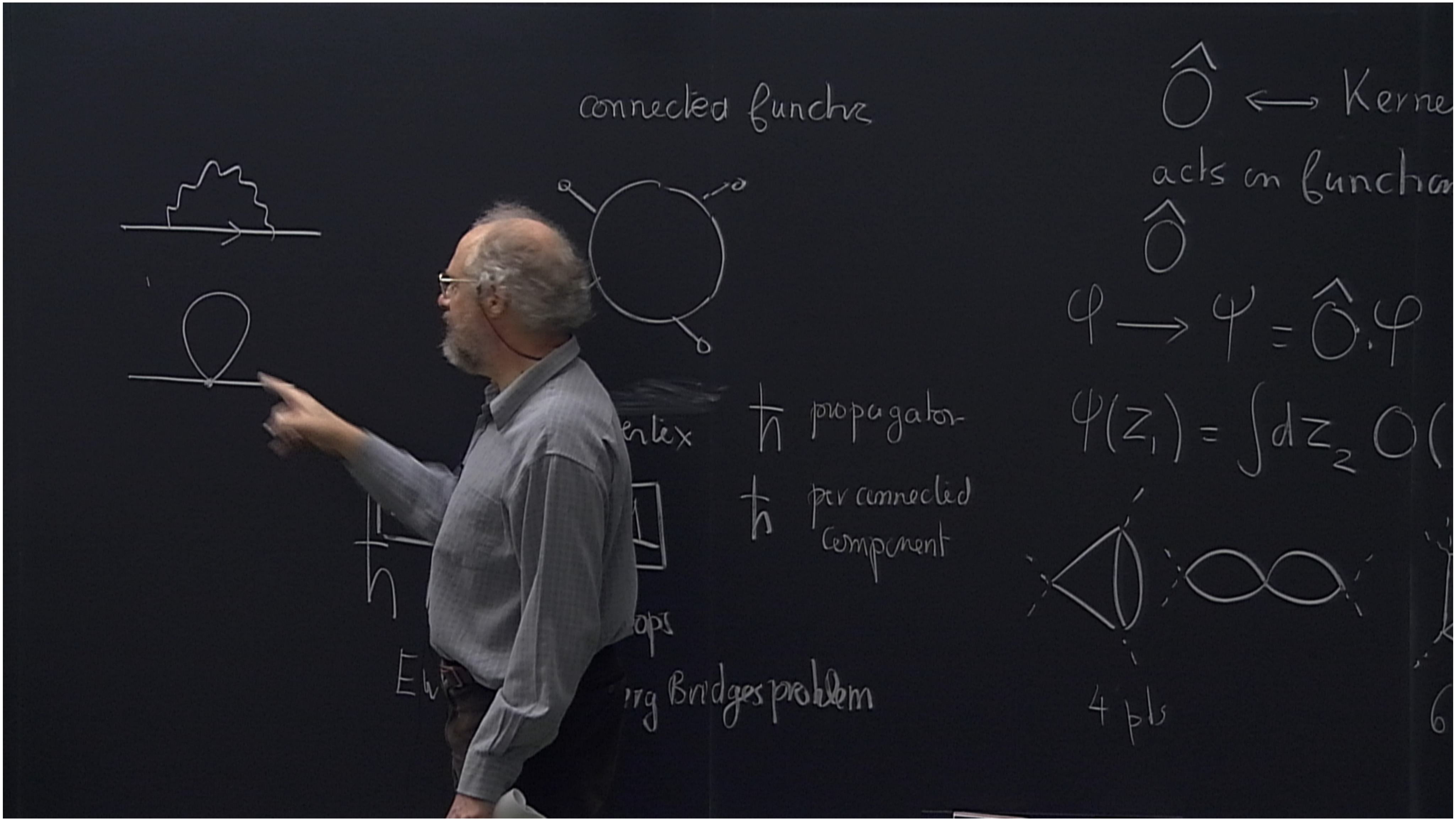
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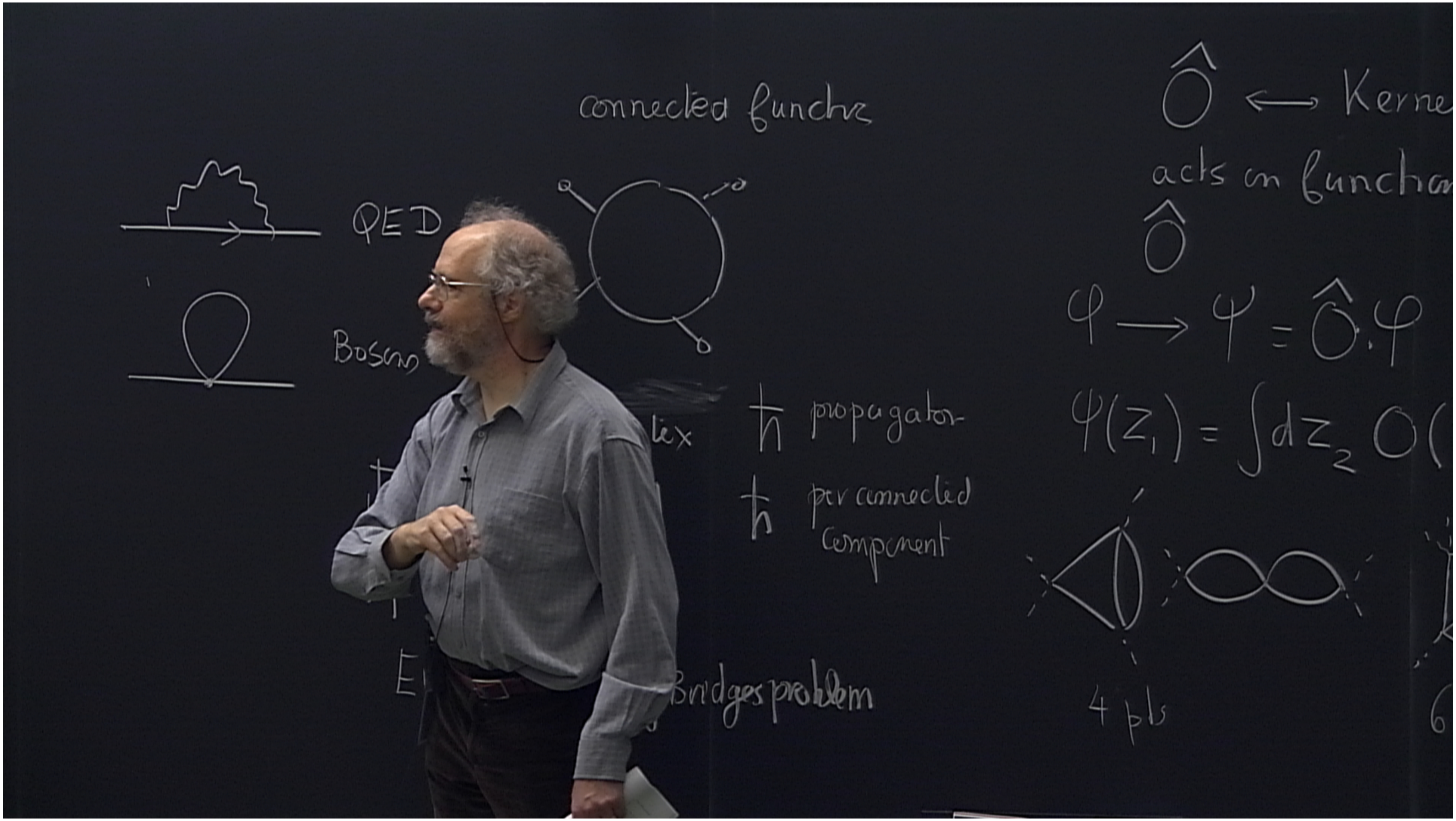
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