

Title: PSI 2016/2017 Quantum Field Theory II - Lecture 6

Date: Nov 14, 2016 09:00 AM

URL: <http://pirsa.org/16110006>

Abstract:

Perturbation Theory : Structure ϕ^4 theory (Euclidean)

$$S[\phi] = \int d^D x \frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4$$

Expansion as a series (formal) in $\sum_k g^k \dots$

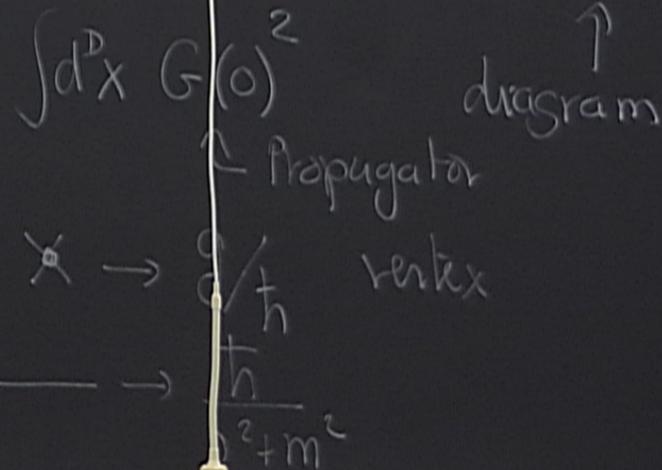
$$\left\langle \underbrace{\phi(z)}_{\text{external}} \cdot \underbrace{\phi(z)}_{\text{external}} \int d^D x \underbrace{\phi(x)}_{\text{internal}} \dots \right\rangle \leftarrow \text{Free theory}$$

Wick Th \rightarrow Sum of amplitudes \leftrightarrow Feynman Diagrams (Graphs)

$$Z = \left\langle \exp\left(-\frac{g}{\hbar^4} \int d^D x \phi^4(x)\right) \right\rangle_0 = 1 + -\frac{g}{\hbar} \frac{1}{8} \cdot \text{diagram}$$

2 pt function, (position space)

$$\langle \phi(x_1) \phi(x_2) \rangle$$



$$\langle \phi(z_1) \phi(z_2) \rangle_0 - \frac{g}{\hbar} \frac{1}{4!} \int d^D x_1 \langle \phi(z_1) \phi(z_2) \phi^4(x_1) \rangle_0 + o(g^2)$$

$$- g \hbar \frac{1}{8} 8 + o(g^2)$$

↓ Wick

$$3 \quad \langle \phi(z_1) \phi(z_2) \rangle_0 \left(\langle \phi(x_1)^2 \rangle_0 \right)^2$$

$$4 \times 3 \quad \langle \phi(z_1) \phi(x_1) \rangle_0 \langle \phi(z_2) \phi(x_1) \rangle_0 \langle \phi(x_1)^2 \rangle_0$$

$$\begin{aligned}
 \langle \phi(z_1) \phi(z_2) \rangle &= \frac{\langle \phi(z_1) \phi(z_2) \cdot \exp\left(-\frac{g}{\hbar \cdot 4!} \int d^D x \phi^4(x)\right) \rangle_0}{\langle \exp\left(-\frac{g}{\hbar \cdot 4!} \int d^D x \phi^4(x)\right) \rangle_0} \\
 &= \frac{\text{numerator}}{\text{denominator}}
 \end{aligned}$$

↑
interacting theory

numerator

$$\hbar \left(\begin{array}{c} \circ \text{---} \circ \\ z_1 \quad z_2 \end{array} - g \hbar^2 \left(\frac{1}{8} \begin{array}{c} \circ \text{---} \circ \\ z_1 \quad z_2 \end{array} \bigcirc_{x_1} + \frac{1}{2} \begin{array}{c} \circ \text{---} \circ \\ z_1 \quad z_2 \end{array} \bigcirc_{x_1} \right) + O(g^2) \right)$$

denominator

$$1 - g \hbar \frac{1}{8} \bigcirc_{x_1}$$

$$\frac{\langle \int d^D x \phi^4(x) \rangle_0}{4!} = \frac{\langle \phi(z_1) \phi(z_2) \rangle_0 - \frac{g}{\hbar} \frac{1}{4!} \int d^D x_1 \langle \dots \rangle_0}{4!}$$

$$\frac{\langle \phi^4(x) \rangle_0}{4!} = \left(\frac{1}{2} \left(\text{diagram with loop} \right) + O(g^2) \right)$$

$$1 - g \hbar \frac{1}{8} \text{diagram} + O(g^2)$$

Rule: disconnected pieces in a diagram

the amplitudes factorises

3

4 x 3

$\langle \phi(z_1) \phi(z_2) \rangle = \frac{\langle \phi(z_1) \phi(z_2) \cdot \exp(-\frac{g}{\hbar \cdot 4!} \int d^D x \phi^4(x)) \rangle_0}{\langle \exp(-\frac{g}{\hbar \cdot 4!} \int d^D x \phi^4(x)) \rangle_0}$

↑
interacting theory

$\frac{1}{\hbar} \left(\text{diagram with two external legs } z_1, z_2 \text{ and a loop } x_1 \right) = g \hbar \left(\frac{1}{8} \left(\text{diagram with two external legs } z_1, z_2 \text{ and a loop } x_1 \right) + \frac{1}{2} \left(\text{diagram with two external legs } z_1, z_2 \text{ and a loop } x_1 \right) \right) + O(g^2)$

$1 - g \hbar \left(\text{diagram with two external legs } z_1, z_2 \text{ and a loop } x_1 \right) + O(g^2)$

$$\begin{aligned}
 \langle \dots \rangle &= \underbrace{\text{---}}_{z_1 \quad z_2} - g \frac{1}{2} \underbrace{\text{---}}_{z_1 \quad x_1 \quad z_2} \\
 & \quad \underbrace{\hspace{10em}}_{G_0(z_1 - z_2)} \quad \underbrace{\hspace{10em}}_{\int d^D x_1 G_0(z_1 - x_1) G_0(x_1 - z_2) G_0(0)}
 \end{aligned}$$

$$G_{(2)}(z_1, z_2) = \langle \phi(z_1) \phi(z_2) \rangle = \underbrace{\text{---} \text{---}}_{z_1 \quad z_2} - g$$

"Green function"

"vacuum" diagram ∞ has disappeared

$$G_0(z_1 - z_2)$$

This is a general feature (all orders)

Interacting theory

vacuum diagrams

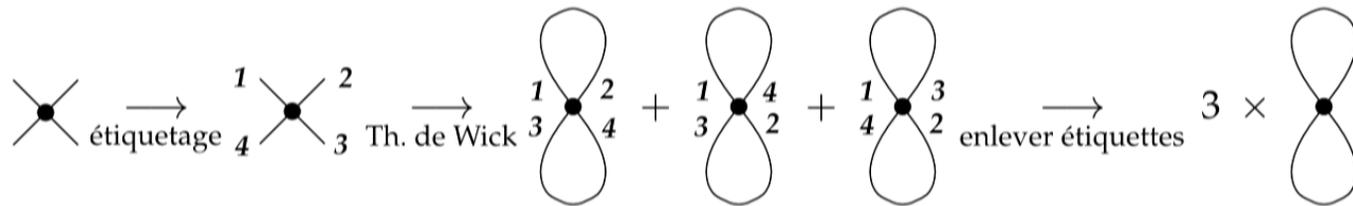
$$N = 0, K = 1$$

$$\frac{1}{4!} \int_{x_1} \langle \Phi^4(x_1) \rangle_0 = \frac{1}{8} \text{ (diagram of a figure-eight loop)}$$

$$N = 0, K = 2$$

$$\frac{1}{2! (4!)^2} \int_{x_1} \int_{x_2} \langle \Phi^4(x_1) \Phi^4(x_2) \rangle_0 = \frac{1}{128} \text{ (diagram of two separate figure-eight loops)} + \frac{1}{16} \text{ (diagram of two figure-eight loops connected by a line)} + \frac{1}{48} \text{ (diagram of two figure-eight loops connected by two lines)}$$

Origin of the combinatorial factor



$$\frac{3}{4!} = \frac{1}{8}$$

$$\begin{aligned}
 \frac{\text{Number of inequivalent labellings of } G}{\text{total number of labellings}} &= \frac{1}{\text{number of equivalent labellings}} \\
 &= \frac{1}{\text{order of the symmetry group of } G}
 \end{aligned}$$

2 points diagrams

$$N = 2, K = 0$$

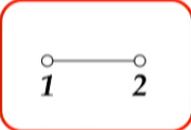
$$\langle \Phi(z_1)\Phi(z_2) \rangle_0 = \begin{array}{c} \circ \text{---} \circ \\ \mathbf{1} \quad \mathbf{2} \end{array}$$

$$N = 2, K = 1$$

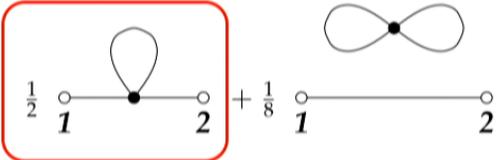
$$\frac{1}{4!} \int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1) \rangle_0 = \frac{1}{2} \begin{array}{c} \circ \text{---} \circ \\ \mathbf{1} \quad \mathbf{2} \end{array} \begin{array}{c} \text{loop} \\ \bullet \end{array} + \frac{1}{8} \begin{array}{c} \circ \text{---} \circ \\ \mathbf{1} \quad \mathbf{2} \end{array} \begin{array}{c} \text{figure-eight} \\ \bullet \end{array}$$

2 points diagrams

$$N = 2, K = 0$$

$$\langle \Phi(z_1)\Phi(z_2) \rangle_0 = \text{diagram}$$


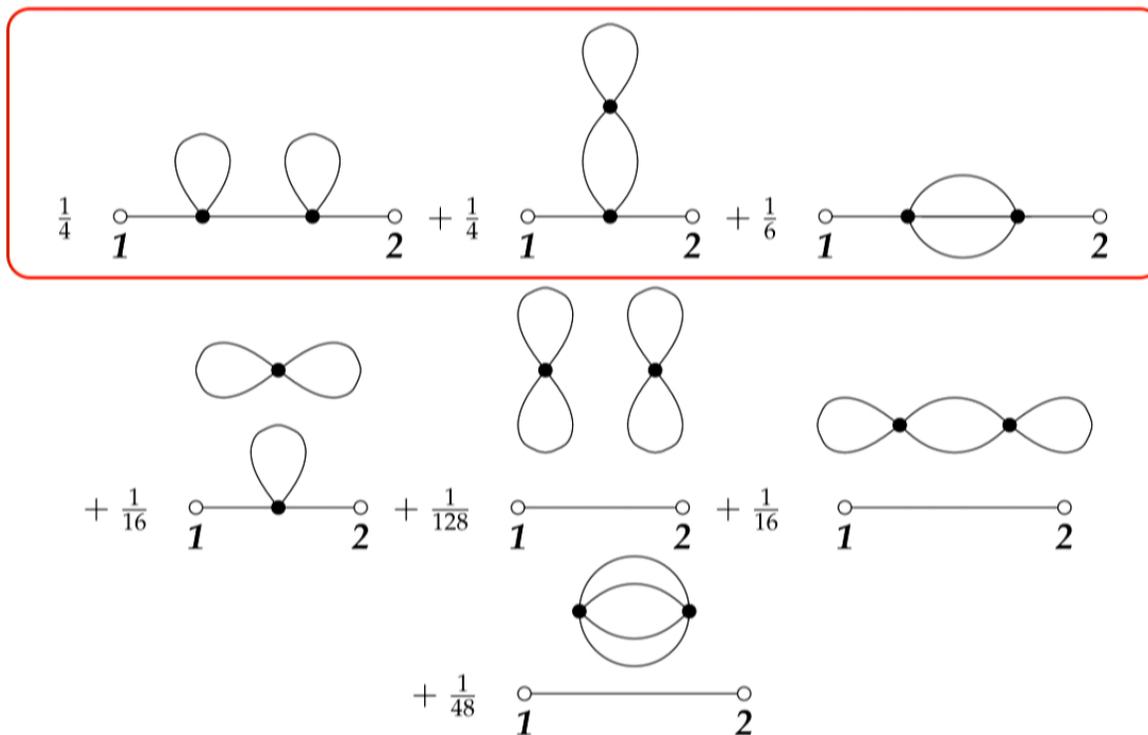
$$N = 2, K = 1$$

$$\frac{1}{4!} \int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1) \rangle_0 = \text{diagram} + \frac{1}{8} \text{diagram}$$


2 points diagrams (continued)

$$N = 2, K = 2$$

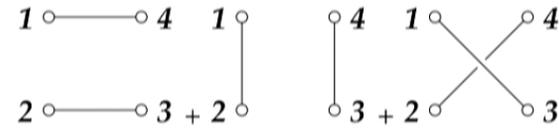
$$\frac{1}{2!(4!)^2} \int_{x_1} \int_{x_2} \langle \Phi(z_1) \Phi(z_2) \Phi^4(x_1) \Phi^4(x_2) \rangle_0 =$$



4 points diagrams

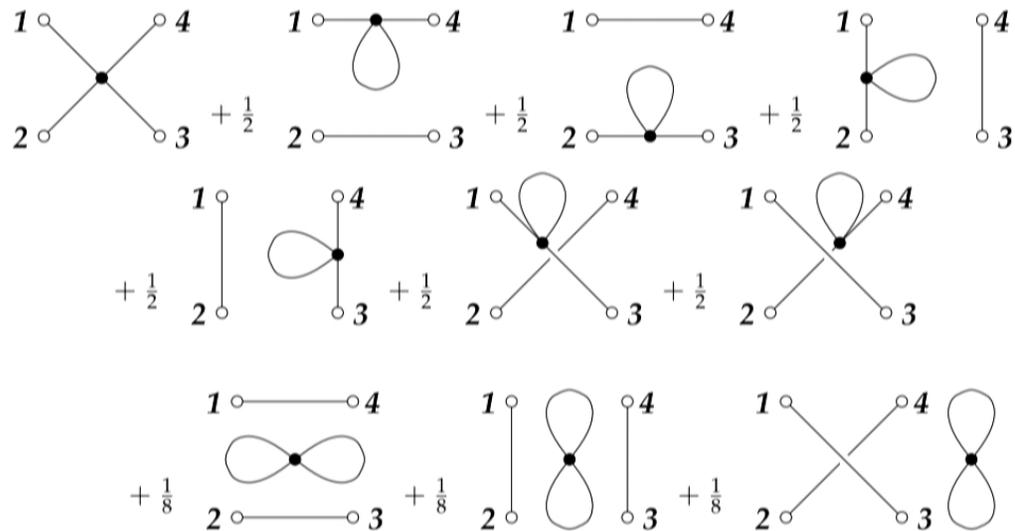
$$N = 4, K = 0$$

$$\langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4) \rangle_0 =$$



$$N = 4, K = 1$$

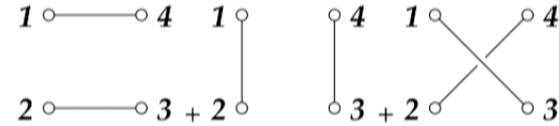
$$\frac{1}{4!} \int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\Phi^4(x_1) \rangle_0 =$$



4 points diagrams

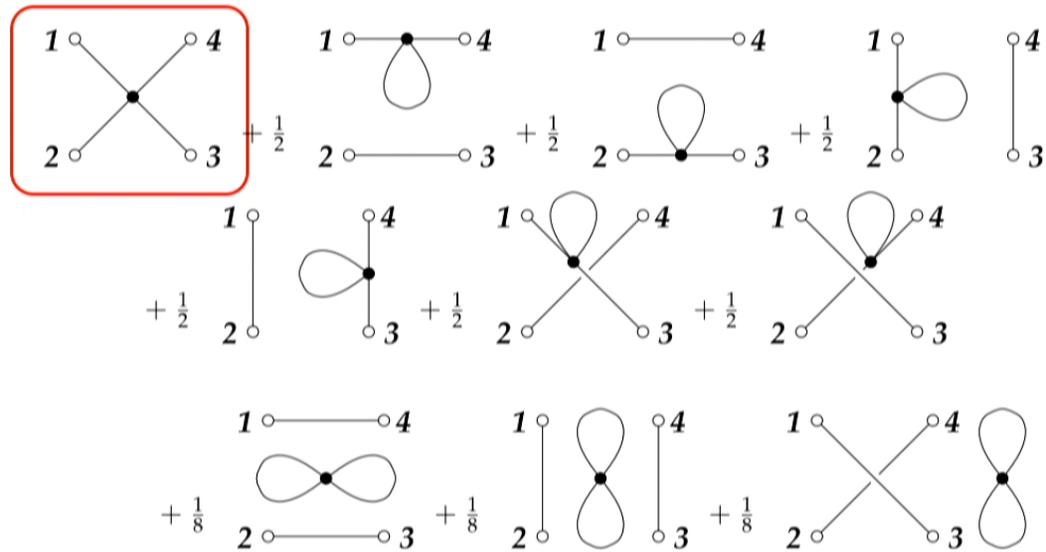
$$N = 4, K = 0$$

$$\langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4) \rangle_0 =$$



$$N = 4, K = 1$$

$$\frac{1}{4!} \int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\Phi^4(x_1) \rangle_0 =$$



Connected vacuum diagrams

$$\frac{1}{2} \text{circle} - g \frac{1}{8} \text{figure-eight} + g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

2 points function = **connected 2 points function**

$$\text{line } \frac{1}{i} \text{---} \frac{1}{2} - g \frac{1}{2i} \text{line with loop} + g^2 \left(\frac{1}{4i} \text{line with two loops} + \frac{1}{4i} \text{line with figure-eight} + \frac{1}{6i} \text{line with bubble} \right)$$

Connected vacuum diagrams

$$\frac{1}{2} \text{circle} - g \frac{1}{8} \text{figure-eight} + g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

2 points function = **connected 2 points function**

$$\text{line } 1-2 - g \frac{1}{2} \text{line with loop} + g^2 \left(\frac{1}{4} \text{line with two loops} + \frac{1}{4} \text{line with figure-eight} + \frac{1}{6} \text{line with bubble} \right)$$

4 points function (up to order 1)

$$\begin{aligned}
 & \left(\begin{array}{c} 1 \circ \text{---} \circ 4 \\ 2 \circ \text{---} \circ 3 \end{array} + \begin{array}{c} 1 \circ \\ | \\ \circ 3 \end{array} + \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \circ 2 \quad \circ 3 \end{array} \right) - g \left(\begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} \right) + \frac{1}{2} \begin{array}{c} 1 \circ \text{---} \circ 4 \\ | \\ \bullet \\ | \\ \circ 2 \text{---} \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \text{---} \circ 4 \\ | \\ \bullet \\ | \\ \circ 2 \text{---} \circ 3 \end{array} \\
 & + \frac{1}{2} \begin{array}{c} 1 \circ \\ | \\ \bullet \\ | \\ \circ 2 \end{array} \begin{array}{c} \circ 4 \\ | \\ \bullet \\ | \\ \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \\ | \\ \bullet \\ | \\ \circ 2 \end{array} \begin{array}{c} \circ 4 \\ | \\ \bullet \\ | \\ \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} \right) + \dots
 \end{aligned}$$

4 points function (up to order 1)

$$\begin{aligned}
 & \left(\begin{array}{c} 1 \circ \text{---} \circ 4 \\ 2 \circ \text{---} \circ 3 \end{array} + \begin{array}{c} 1 \circ \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \begin{array}{c} 1 \circ \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \circ 2 \quad \circ 3 \end{array} \right) - g \left(\begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} \right) \\
 & + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} \right) + \dots
 \end{aligned}$$

Connected 4 points function (up to order 2)

$$\begin{aligned}
 & -g \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \circ 2 \quad \circ 3 \end{array} + g^2 \left(\begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} \right) \\
 & + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ | \\ \circ 2 \quad \circ 3 \end{array} \right) + \dots
 \end{aligned}$$

4 points function (up to order 1)

$$\begin{aligned}
 & \left(\begin{array}{c} 1 \circ \text{---} \circ 4 \\ 2 \circ \text{---} \circ 3 \end{array} + \begin{array}{c} 1 \circ \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \begin{array}{c} 1 \circ \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \circ 2 \quad \circ 3 \end{array} \right) - g \left(\begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \text{---} \circ 4 \\ | \\ \bullet \\ | \\ \circ 2 \text{---} \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \text{---} \circ 4 \\ | \\ \bullet \\ | \\ \circ 2 \text{---} \circ 3 \end{array} \right) \\
 & + \frac{1}{2} \begin{array}{c} 1 \circ \\ | \\ \bullet \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \\ | \\ \bullet \\ | \\ 2 \circ \end{array} \begin{array}{c} \circ 4 \\ | \\ \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} \right) + \dots
 \end{aligned}$$

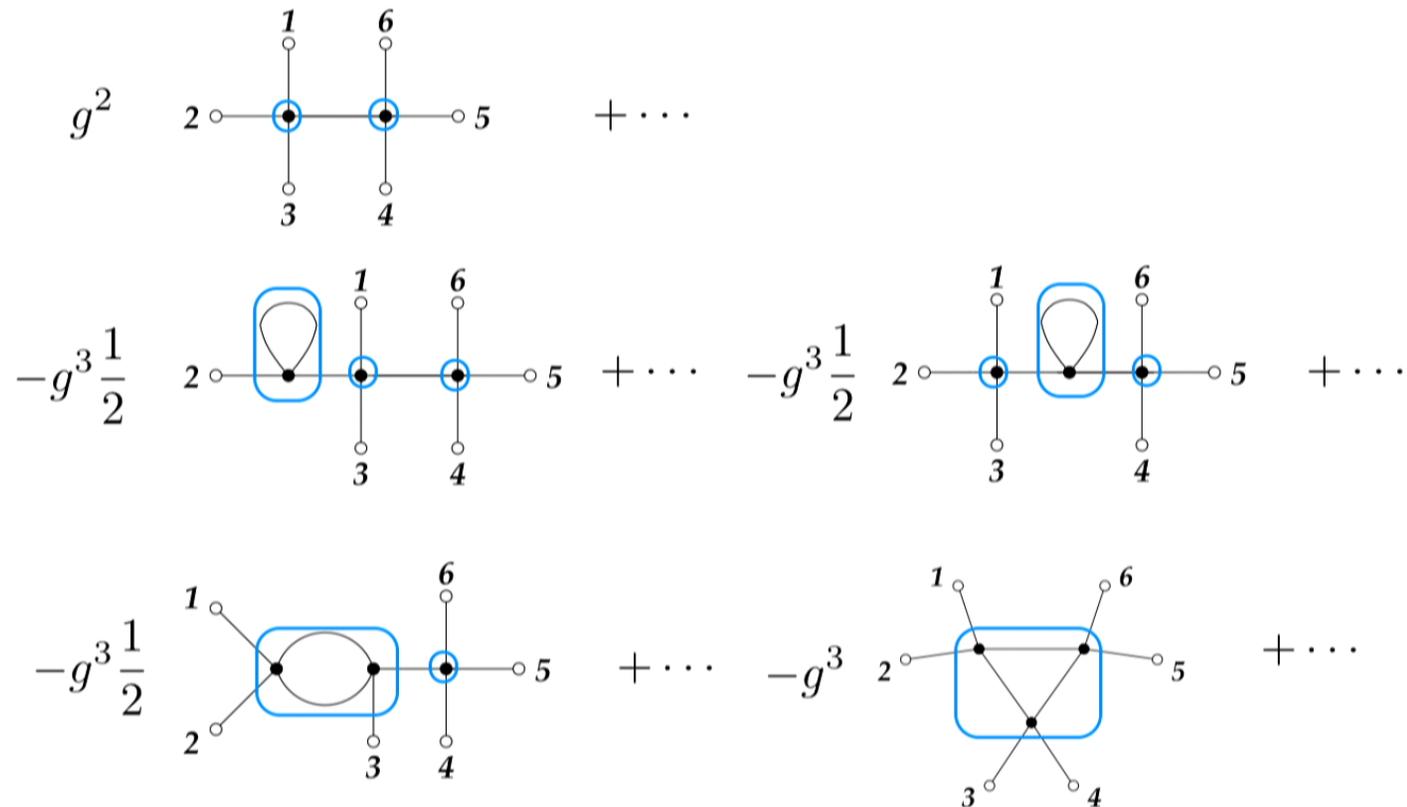
Connected 4 points function (up to order 2)

$$\begin{aligned}
 & -g \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ 2 \circ \quad \circ 3 \end{array} + g^2 \left(\frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \right. \\
 & \left. + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \circ \quad \circ 4 \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \circ 2 \quad \circ 3 \end{array} \right) + \dots
 \end{aligned}$$

Connected 6 points function (up to order 3)

$$\begin{aligned}
 &g^2 \left[\begin{array}{c} 1 \\ \circ \\ | \\ \bullet \\ \text{---} \bullet \\ | \\ \circ \\ 3 \end{array} \quad \begin{array}{c} 6 \\ \circ \\ | \\ \bullet \\ \text{---} \circ \\ 5 \\ | \\ \circ \\ 4 \end{array} \right] + \dots \\
 &-g^3 \frac{1}{2} \left[\begin{array}{c} 1 \\ \circ \\ | \\ \bullet \\ \text{---} \bullet \\ | \\ \circ \\ 3 \end{array} \quad \begin{array}{c} 6 \\ \circ \\ | \\ \bullet \\ \text{---} \circ \\ 5 \\ | \\ \circ \\ 4 \end{array} \right] + \dots \quad -g^3 \frac{1}{2} \left[\begin{array}{c} 1 \\ \circ \\ | \\ \bullet \\ \text{---} \bullet \\ | \\ \circ \\ 3 \end{array} \quad \begin{array}{c} 6 \\ \circ \\ | \\ \bullet \\ \text{---} \circ \\ 5 \\ | \\ \circ \\ 4 \end{array} \right] + \dots \\
 &-g^3 \frac{1}{2} \left[\begin{array}{c} 1 \\ \circ \\ \diagdown \\ \bullet \\ \text{---} \bullet \\ \diagup \\ \circ \\ 2 \end{array} \quad \begin{array}{c} 6 \\ \circ \\ | \\ \bullet \\ \text{---} \circ \\ 5 \\ | \\ \circ \\ 3 \end{array} \right] + \dots \quad -g^3 \left[\begin{array}{c} 1 \\ \circ \\ \diagdown \\ \bullet \\ \text{---} \bullet \\ \diagup \\ \circ \\ 2 \end{array} \quad \begin{array}{c} 6 \\ \circ \\ \diagdown \\ \bullet \\ \text{---} \circ \\ 5 \\ \diagup \\ \circ \\ 4 \end{array} \right] + \dots
 \end{aligned}$$

Connected 6 points function (up to order 3)



One Particle Irreducible (1PI) functions

Irreducible vacuum diagrams = vacuum self energy
 (= connected vacuum diagrams for Φ^4)

$$\Gamma^{(0)} = -\frac{1}{2} \text{circle} + g \frac{1}{8} \text{figure-eight} - g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

to be explained later 

Irreducible 2 points function = inverse of the 2 points function

$$\Gamma^{(2)} = \text{propagator} + g \frac{1}{2} \text{propagator with loop} - g^2 \left(\frac{1}{4} \text{propagator with two loops} + \frac{1}{6} \text{propagator with three loops} \right) + \dots$$

with amputated legs

$$\begin{aligned} \text{propagator} &= (-\Delta + m^2)_{z_1 z_2} \\ &= p^2 + m^2 \end{aligned} \qquad \begin{aligned} \text{delta} &= \delta(z - x) \\ &= 1 \end{aligned}$$

Conduction Functions / Connected / Irreducible

Feynman Diagram $G \rightarrow$ Amplitude/Integral

N external vertices
 K internal vertices

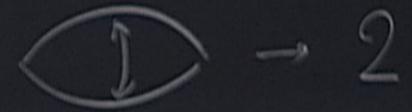
$$I_G(z_1 \dots z_N) = \int \prod_{i=1}^K d^P x_i \prod_{\text{lines}} G_0(\cdot, \cdot)$$

\uparrow propagators $\nwarrow \nearrow$ may be a Z or a X

symmetry factor

$$C(G) = \frac{\text{\# ways to construct } G \text{ out of Wick Theorem}}{K! (4!)^K} = \frac{1}{\text{\# of elements of the sym. group of } G}$$

/Irreducible



Amplitude/Integral

N external vertices
K internal vertices

$$K_i \prod_{\text{lines}} G_0(\dots) \begin{matrix} \nearrow \\ \searrow \end{matrix} \text{ may be a Z or a X}$$

propagators

$$\frac{G}{\text{em}} = \frac{1}{\# \text{ of elements of the sym group of } G}$$

Correlation Function ∞ N points

$$G_{(N)}(z_1, \dots, z_N) := \sum_{K=0}^{\infty} (-g)^K \sum_{\text{diagrams } G} C(G) I_G(z_1, \dots, z_N)$$

with N external vertices
K internal vertices

Connected correlation function

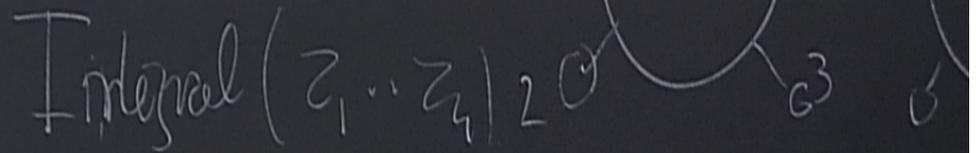
$$G_{(N)}^{\text{CONN}}(z_1, \dots, z_N) := \sum_K (-g)^K \sum_{G \text{ CONNECTED}} C(G) I_G(z_1, \dots, z_N)$$

Correlations Functions \leftarrow Connected Functions

$$G_{(4)}(z_1, \dots, z_4) = G_{(4)}^{\text{CONN}}(z_1, z_4) + G_{(2)}^{\text{CONN}}(z_1, z_2) \cdot G_{(2)}^{\text{CONN}}(z_3, z_4) + \text{"} + \text{"}$$

$$G_{(2)}(z_1, z_2) = G_{(2)}^{\text{CONN}} \quad \# \text{ vertices}$$

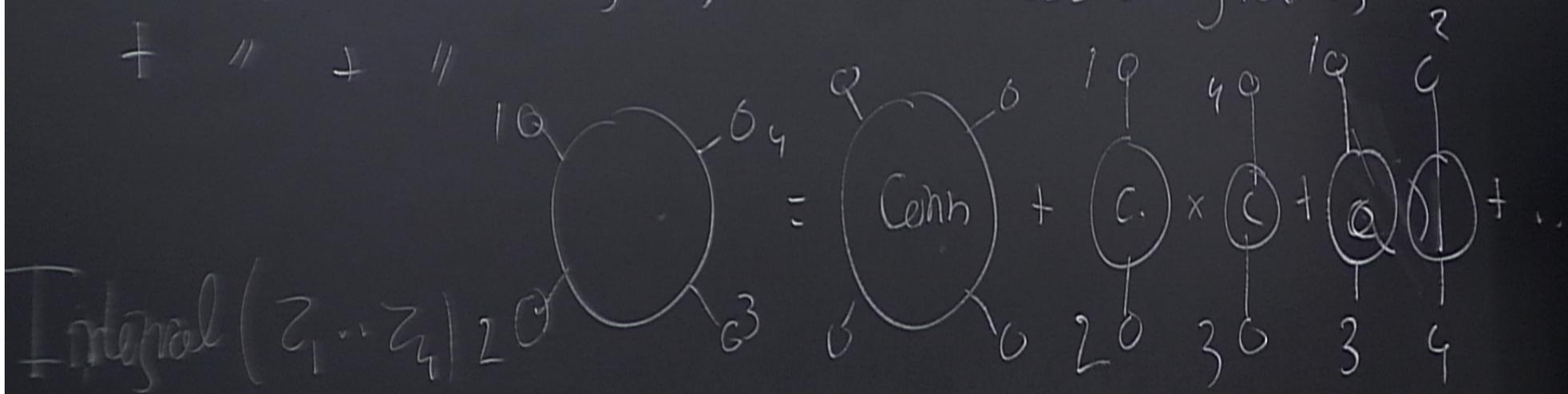
$$G_{(4)}^{\text{CONN}}(z_1, \dots, z_4) = \sum_{\text{connected diagrams}} g$$

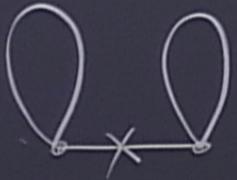


→ Connected Functions ← Irreducible
 "Functions"

$$G_{(2)}^{(2)}(z_1, z_2) \cdot G_{(2)}^{(2)}(z_3, z_4) + \dots$$

less diagrams



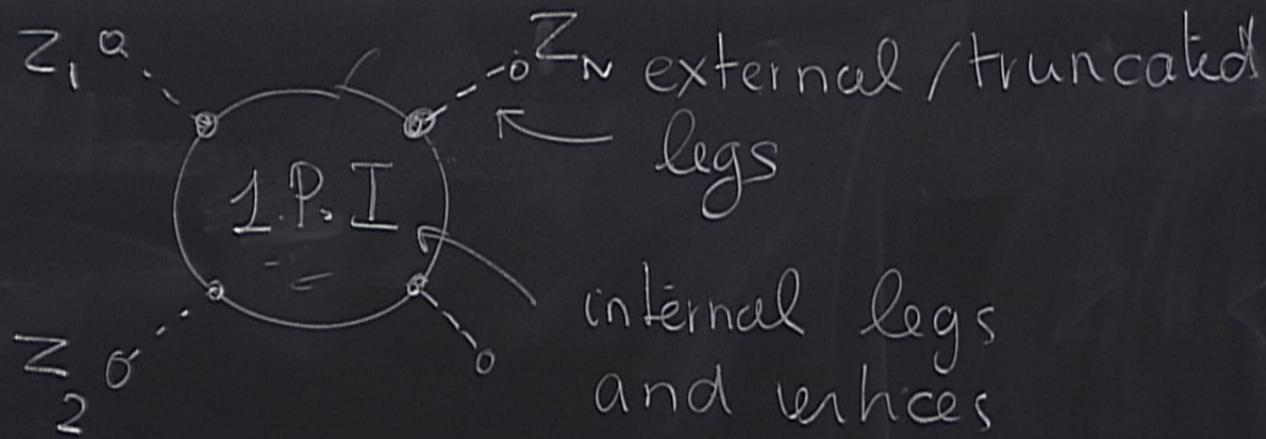


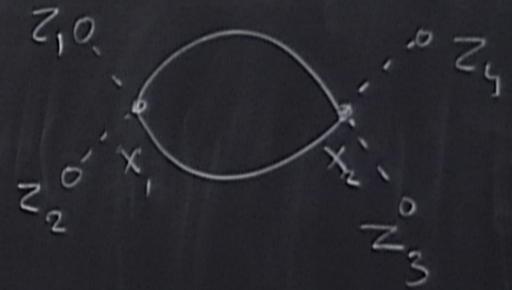
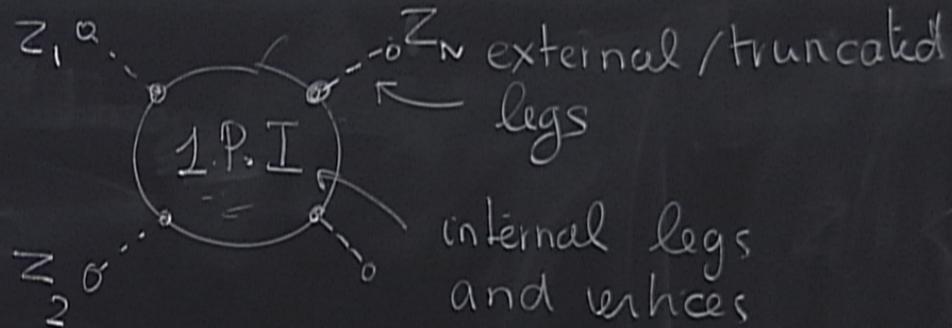
remove one line
 Drays connecteq



1 line irreducible diagrams
 1 particle irreducible //

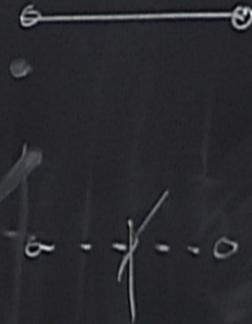
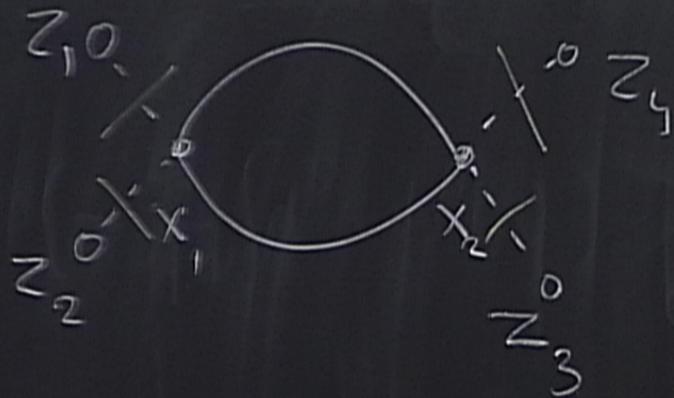
1PI





$$G_0(x, x) \prod \delta(z - x)$$

external/truncated lines



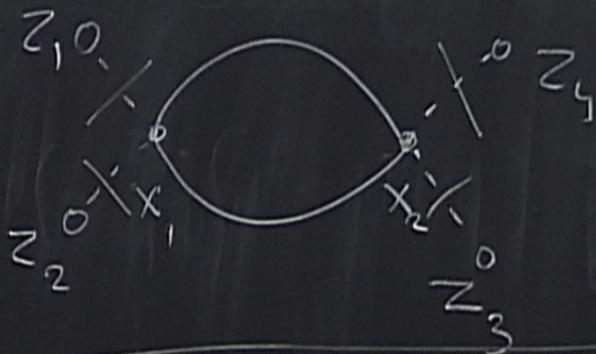
Fourier transform

$$\frac{1}{p^2 + m^2}$$

$$\int_{\mathbb{D}} dx_1 \int_{\mathbb{D}} dx_2 G_0(x_1 - x_2)^2 \delta(z_1 - x_1) \delta(z_2 - x_1) \delta(z_3 - x_2) \delta(z_4 - x_2)$$

nal/truncated

2 legs
vertices



Fourier transform

$$\frac{1}{p^2 + m^2}$$

$$1$$

$$I_{\text{IR}}(z_1, z_4) = \int d^D x_1 d^D x_2 G_0(x_1 - x_2)^2 \delta(z_1 - x_1) \delta(z_2 - x_1) \delta(z_3 - x_2) \delta(z_4 - x_2)$$

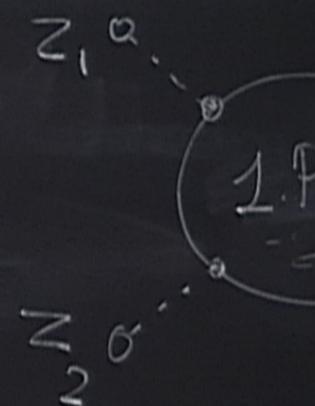
$$= \delta(z_1 - z_2) \delta(z_3 - z_4) G_0(z_1 - z_3)^2$$

Position Space

Irreducible Function

1 P. I graph G

$$I_G^{IRR}(z_1, \dots, z_N) = \int \prod_{\text{Internal vertices } i=1, \dots, K} dx_i \prod_{\text{Internal lines}} G_0(x_i, x_j) \prod_{\text{external/truncated lines}} \delta(z_i - x_i)$$



N -part irreducible Function

$$\Gamma_{(N)}(z_1, \dots, z_N) = \sum_K (-1)^{K-1} g^K \sum_{\text{1 P. I Graphs } N \text{ ext. vertices } K \text{ int. vertices}} C(G) I_G^{IRR}(z_1, \dots, z_N)$$

Irreducible 2 point function

$$\Gamma_{(2)}(z_1, z_2) =$$

$$g \frac{1}{2} \left[\text{diagram} \right] = g \frac{1}{2} \delta(z_1 - z_2) G_0(0)$$

The diagram shows two vertices connected by a dashed line, with a loop attached to the central vertex.

$$- g^2 \left(\frac{1}{8} \right) \left[\text{diagram} \right] = - g^2 \delta(z_1 - z_2)$$

The diagram shows two vertices connected by a dashed line, with a figure-eight loop attached to the central vertex.

function

$$g \frac{1}{2} \text{---} \downarrow \delta(z_1 - z_2) G_0(z_2) \quad g^2 \left(\frac{1}{8} \text{---} \downarrow \delta(z_1 - z_2) \int d^4x G(x)^2 \cdot G_0(z_2) + \frac{1}{6} \text{---} \downarrow [G_0(z_1 - z_2)]^3 \right) + \dots$$

Irreducible 2 point function

$$\Gamma_{(2)}(z_1, z_2) = \text{---} \uparrow \text{---}$$

twice truncated propagator

$$g \frac{1}{2} \text{---} \uparrow \text{---}$$

$\delta(z_1 - z_2) G_0(0)$

$$g^2 \left(\frac{1}{8} \text{---} \uparrow \text{---} + \frac{1}{6} \text{---} \uparrow \text{---} \right)$$

$\delta(z_1 - z_2) \int d^d x G_0(x)$

IRREDUCIBLE

CONV



positive space

$$o_1 - | - o_2 = (-\Delta + m^2)_{z_1} G_0(z_1 - z_2) = \delta(z_1 - z_2) \rightarrow 1$$

$$o_1 - | - o_2 = (-\Delta + m^2)_{z_1}^2 G_0(z_1 - z_2) = (-\Delta + m^2)_{z_1} \delta(z_1 - z_2)$$

distribution

$$o_1 - | - o_2 \rightarrow p^2 + m^2$$

Irreducible 2 point function

$$\Gamma_{(2)}(z_1, z_2) = \text{tree} + g \frac{1}{2} \text{loop} - g^2 \left(\frac{1}{8} \text{self-energy} + \frac{1}{6} \text{tadpole} \right) + \dots$$

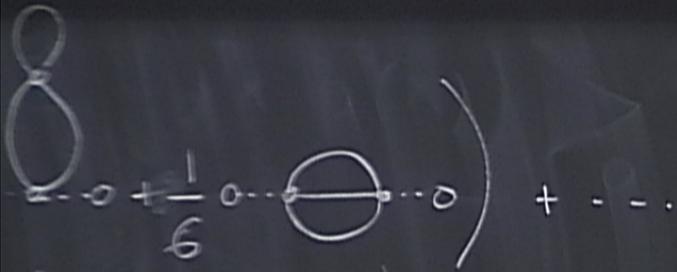
↑ twice truncated propagator
↓ $\delta(z_1 - z_2) G_0(0)$
↓ $\delta(z_1 - z_2) \int dx G(x)^2 \cdot G_0(0)$

FT

$$\hat{\Gamma}_{(2)}(p) = -p^2 + m^2 + g \frac{1}{2} G_0(0) - g^2 \left[\frac{1}{8} \text{self-energy} + \frac{1}{6} \text{tadpole} \right]$$

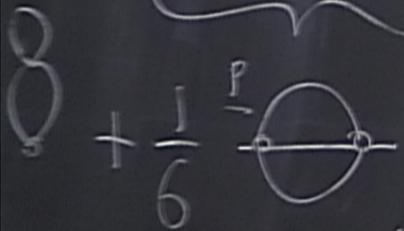
Function of P non trivial

new K-integrals



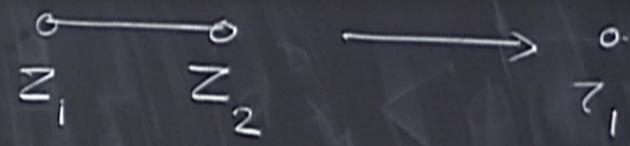
↓

$$z_1 - z_2 \int dx G(x)^2 \cdot G_0(z_1 - z_2) \rightarrow [G_0(z_1 - z_2)]^3$$



Function of P non-trivial

CONW



positive space

$$z_1 - z_2 = (-\Delta + m^2) G_0(z_1 - z_2)$$

$$z_1 - z_2 = (-\Delta + m^2)^2 z_1 - z_2$$

IRREDUCIBLE

CONV



positive space

$$\begin{matrix} o & - & + & - & o \\ 1 & & & & 2 \end{matrix} = (-\Delta + m^2)_{z_1} G_0(z_1 - z_2) = \delta(z_1 - z_2) \rightarrow 1$$

$$\begin{matrix} o & - & + & - & o \\ 1 & & & & 2 \end{matrix} = (-\Delta + m^2)_{z_1}^2 G_0(z_1 - z_2) = (-\Delta + m^2)_{z_1} \delta(z_1 - z_2)$$

distribution

$$\begin{matrix} o & - & + & - & o \\ - & p^2 & & & \end{matrix}$$

$$\hookrightarrow p^2 + m^2$$

Generating Function

Func.
Int.

$$Z[j] = \int D[\phi] \exp\left(-\frac{i}{\hbar} \int \mathcal{L}[\phi] + \int \phi j\right)$$

Functional of source term j

$Z[j] \leftarrow$ Correlation Functions

expand in j

Diagram
matic

$$\frac{Z[j]}{Z[0]} = \sum_{N=0}^{\infty} \frac{\hbar^{-N}}{N!} \int \prod_{a=1}^N dz_a j(z_a) G_{(N)}^{(j)}(z_1, \dots, z_N)$$

$]-j \cdot \phi)$

$j(x)$ source term

$$j \cdot \phi = \int d^D x j(x) \phi(x)$$

Generating Function for the connected correlations

$$Z[j] = \sum_{N=0}^{\infty} \frac{\hbar^{1-N}}{N!} \int \prod_{a=1}^N dz_a j(z_a) G_{(N)}^{\text{CONN}}(z_1, \dots, z_N)$$

Diagrammatic
Definition

$$\frac{i}{\hbar} (S[\phi] - j \cdot \phi)$$

$j(x)$ source term

$$j \cdot \phi = \int d^D x j(x) \phi(x)$$

Generating Function for the connected correlations

$$W[j] = \sum_{N=0}^{\infty} \frac{\hbar^{1-N}}{N!} \int \prod_{a=1}^N dZ_a j(z_a) G_{(N)}^{\text{CONN}}(z_1, \dots, z_N)$$

Diagrammatic
Definition

$$W[j] = \hbar \cdot \text{Log}(Z[j])$$

Funct. Int.
def.

$$j \cdot \phi = \int d^D x j(x) \phi(x)$$

the connected correlations

$$\frac{1-N}{1} \int \prod_{a=1}^N dz_a j(z_a) G_{(N)}^{\text{CONN}}(z_1, \dots, z_N)$$

Diagrammatic
Definition

$$g(z[j])$$

Tomorrow
Irreducible \Rightarrow UV divergens

Funct. Int.
def.