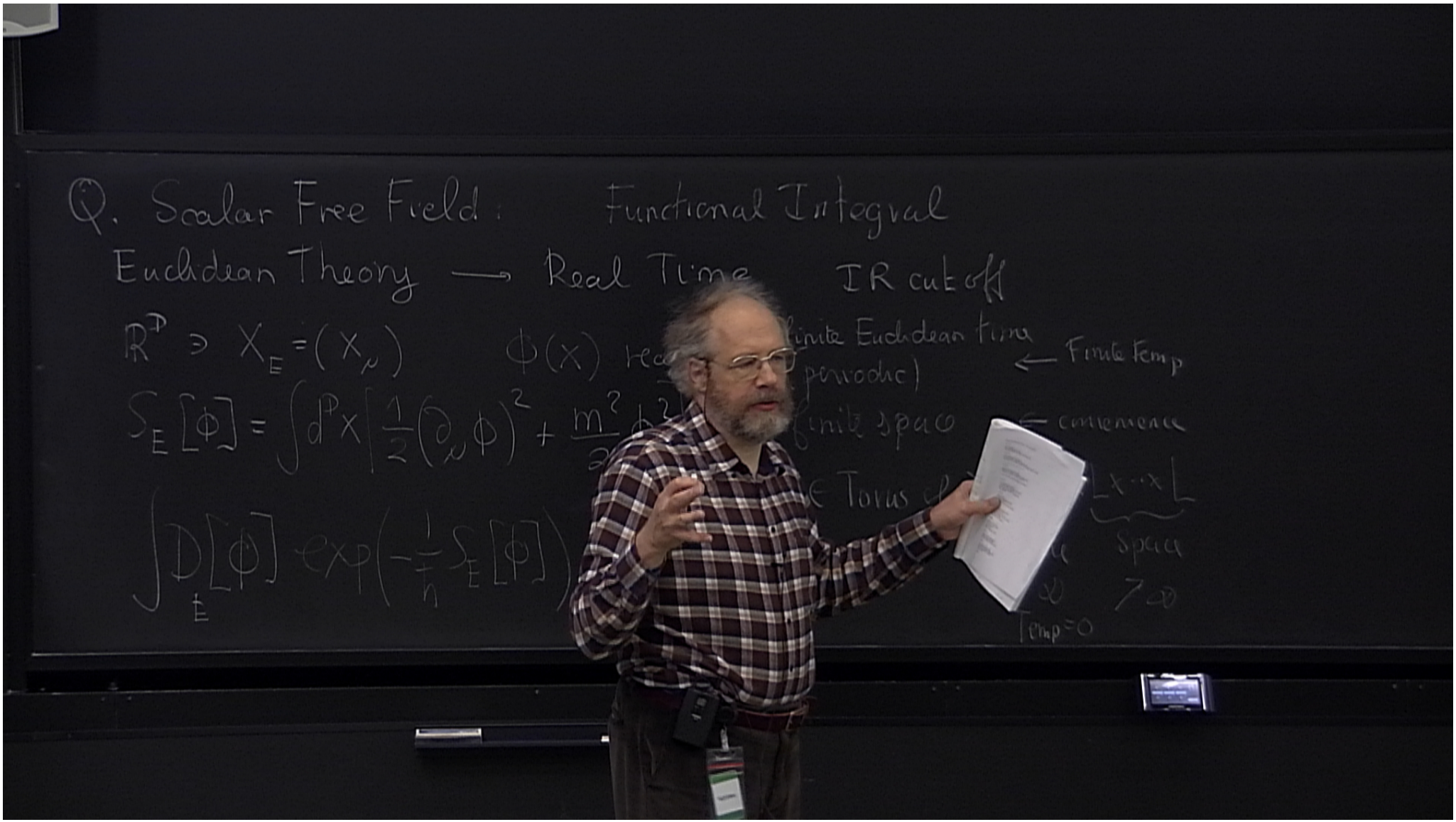
Title: PSI 2016/2017 Quantum Field Theory II - Lecture 4

Date: Nov 10, 2016 09:00 AM

URL: http://pirsa.org/16110004

Abstract:


Q. Scalar Free Field: Functional Integral
Euclidean Theory → Real Time
IR cutoff
Finite Temp
periodic)
finite space
Torus
space
Temp=0

Q. Scalar Free Field: Functional Integral

Euclidean Theory $\longrightarrow$ Real Time

IR cutoff

$\mathbb{R}^D \ni X_E = (X_\mu)_{\mu=0,\dots,D-1}$ $\phi(X)$ real

$$S_E[\phi] = \int d^D x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 \right]$$

$$\int_E D[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right) \dots$$

finite Euclidean time (periodic) $\leftarrow$ Finite Temp

finite space $\leftarrow$ convenience

$X \in$ Torus of size $\underbrace{L}_{\text{E. time}} \times \underbrace{L \times \cdots \times L}_{\text{Space}}$

$\nearrow \infty$ Temp $= 0$ $\quad \nearrow \infty$

nal Integral

ne

IR cutoff

finite Euclidean time (periodic) ← Finite Temp

finite space ← convenience

$X \in$ Torus of size $\underbrace{L}_{\text{E. time}} \times \underbrace{L \times \cdots \times L}_{\text{Space}}$

$\nearrow \infty$ $\nearrow \infty$

Temp = 0

UV. cutoff

$i \in (i_0 \cdots i_{D-1})$ integers

$X_i = i \cdot \epsilon$

lattice

$\epsilon = \Delta\tau = \Delta x$

$X_0 \quad X_1 \cdots X_{D-1}$

$\partial_\mu \phi \to \nabla_\mu \phi = \dfrac{\phi(X_{i+e_\mu}) - \phi(X_i)}{\epsilon}$ Finite difference

e_μ = unit vector in direction μ = $(0 \cdots \underset{\mu}{1} \cdots 0)$

tegral

IR cut off

Euclidean time ← Finite Temp

nodic)

ite space ← convenience

Torus of size $\underbrace{L}_{\text{E. time}} \times \underbrace{L \times \cdots \times L}_{\text{Space}}$

$\nearrow \infty$ Temp $= 0$ $\quad \nearrow \infty$

UV. cut off

$i \in (i_0 \cdots i_{D-1})$ integers

$x_i = i \cdot \epsilon$

$\epsilon = \Delta\tau = \Delta x$

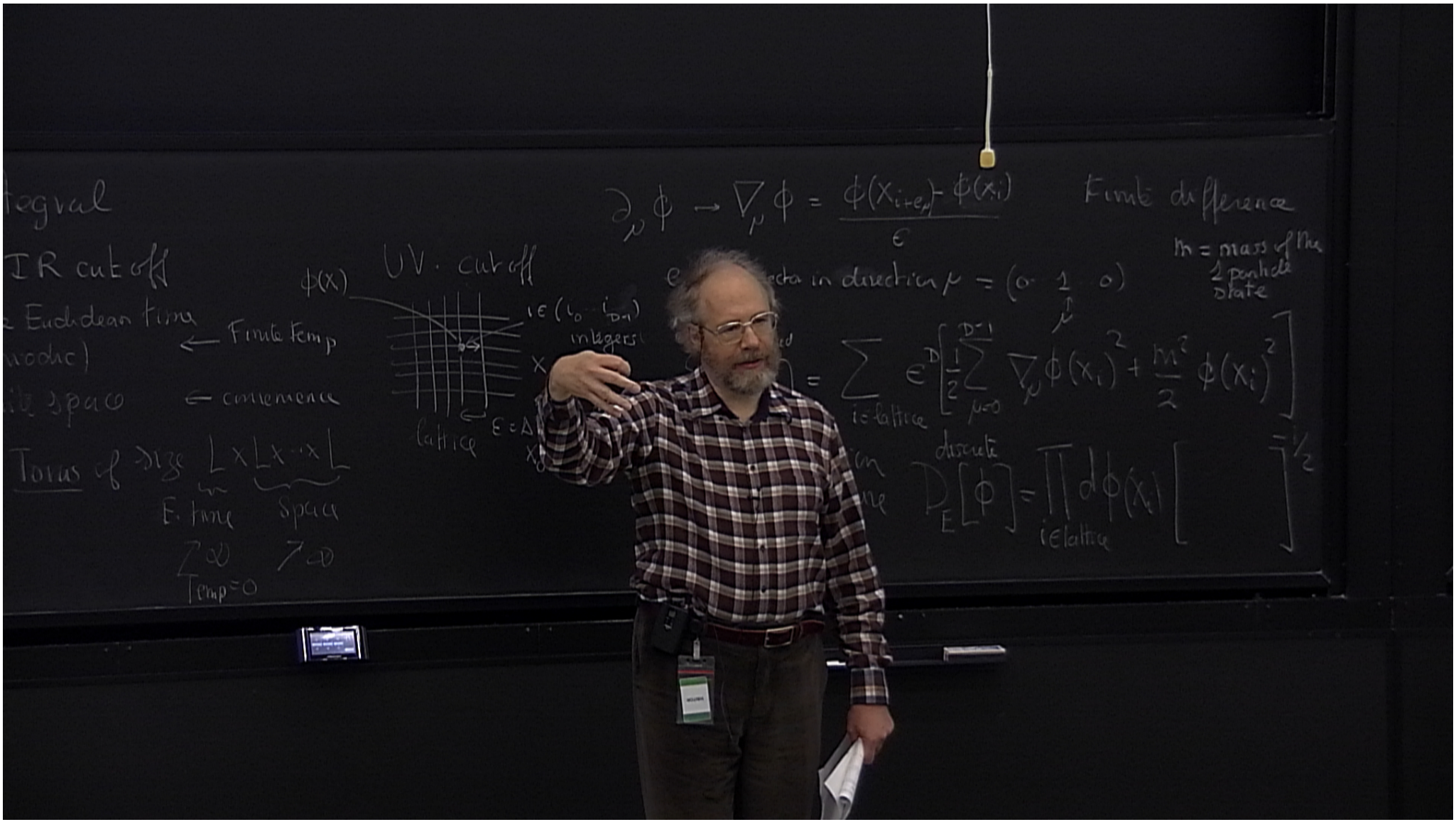
lattice $\quad x_0 \quad x_1 \cdots x_{D-1}$

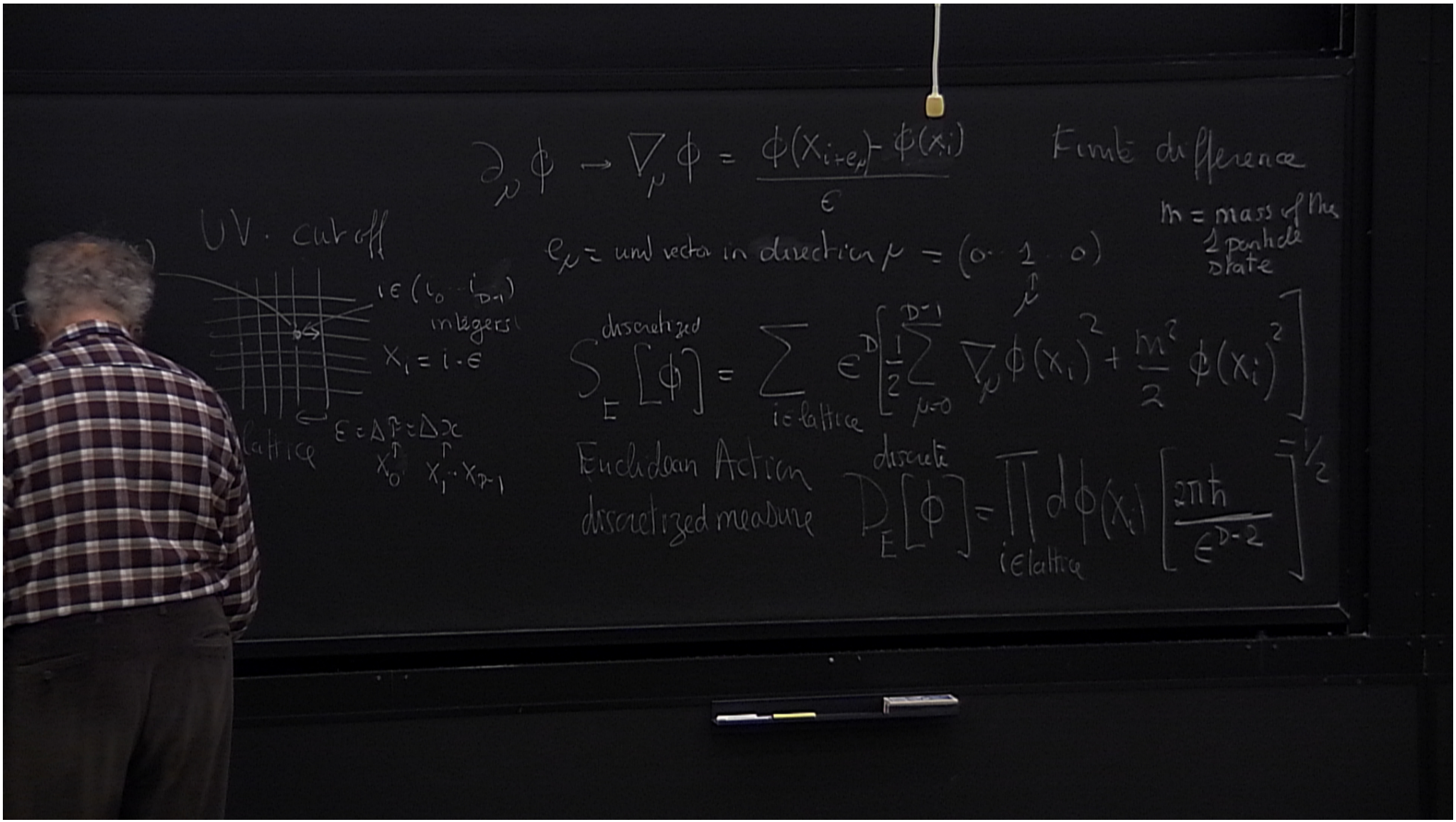
$$\partial_\mu \phi \to \nabla_\mu \phi = \frac{\phi(x_{i+e_\mu}) - \phi(x_i)}{\epsilon}$$ Finite difference

e_μ = unit vector in direction μ = $(0 \cdots \underset{\mu}{1} \; 0)$

$$S_E^{\text{discretized}}[\phi] = \sum_{i \in \text{lattice}} \epsilon^D \left[\frac{1}{2} \sum_{\mu=0}^{D-1} \nabla_\mu \phi(x_i)^2 + \frac{m^2}{2} \phi(x_i)^2 \right]$$

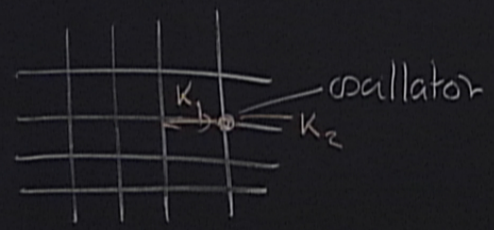
Euclidean Action


IR cut off
UV. cut off
Finite Temp
convenience
Finite difference
integers
lattice
E. time
Space
Temp=0
discrete
i∈lattice


$\partial_\mu \phi \to \nabla_\mu \phi = \frac{\phi(x_{i+e_\mu}) - \phi(x_i)}{\epsilon}$
Finite difference
m = mass of the 1 particle state
UV. cutoff
e_μ = unit vector in direction μ = $(0 \cdots 1 \cdots 0)$
μ
$i \in (i_0 \cdots i_{D-1})$
integers
$x_i = i \cdot \epsilon$
$\epsilon = \Delta\tau = \Delta x$
x_0 $x_1 \cdots x_{D-1}$
lattice
discretized
$S_E[\phi] = \sum_{i \in \text{lattice}} \epsilon^D \left[\frac{1}{2} \sum_{\mu=0}^{D-1} \nabla_\mu \phi(x_i)^2 + \frac{m^2}{2} \phi(x_i)^2 \right]$
Euclidean Action
discretized measure
discrete
$D_E[\phi] = \prod_{i \in \text{lattice}} d\phi(x_i) \left[\frac{2\pi\hbar}{\epsilon^{D-2}} \right]^{-1/2}$

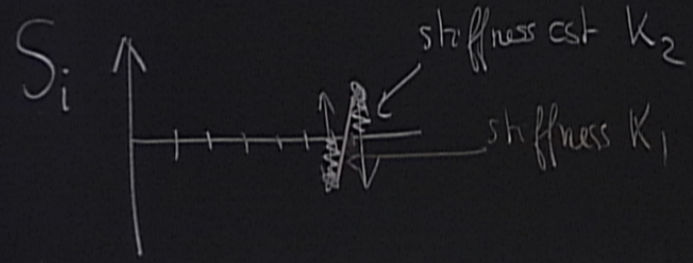
* Functional integral is a Gaussian Integral

* Statistical Mechanics D-dim lattice (hypercubic)



coupled harmonic oscillators (linearily)

S_i continuous spin variable $\in \mathbb{R}$ classical
at each site $i \in \mathbb{Z}^D$



$$E[\{S_i\}] = \sum_{\substack{\text{sites} \\ i}} \frac{K_2}{2} S_i^2 + \sum_{\substack{\text{links} \\ e=\langle i,j\rangle}} \frac{K_1}{2}(S_i - S_j)^2$$

local term linear coupling term

$\exp\left(-\frac{1}{\hbar} S_E[\phi]\right) \ldots$

E. time $\nearrow \infty$ Temp $= 0$ Space $\nearrow \infty$

discretized measure

ntegral is a Gaussian Integral

l Mechanics

D-dim lattice (hypercubic)

coupled harmonic oscillators (linearily)

oscillator

K_2

stiffness cst K_2

stiffness K_1

S_i continuous spin variable $\in \mathbb{R}$ classical

at each site $i \in \mathbb{Z}^D$

$$E[\{S_i\}] = \sum_{\text{sites}} \frac{K_2}{2} S_i^2 + \sum_{\text{links}} \frac{K_1}{2}(S_i - S_j)^2$$

S_i = value of spin variable for the spin S_i located at site i

local term

$\ell = \langle i,j \rangle$

linear coupling term

At finite temperature T_{emp}

$$Z = \sum_{\{S_i\}} \exp\left(-\frac{1}{k_B T} E[S]\right) \Longleftrightarrow$$

$\{S_i\}$ ← sum over config

$$\int \prod_{i \text{ sites}} dS_i$$

Ising like model but with continuous spins (classical) $\Leftrightarrow$ elastic lattice (classical phonons)

$x \in$ Torus of size $L \times L \times \cdots \times L$

E. time $\to \infty$, Temp $= 0$; Space $\to \infty$

$x_0 \quad x_1 \cdots x_{D-1}$

Euclidean Action

discretized measure

$$D_E[\phi] = \prod_{i \in \text{lattice}} d\phi(x_i) \left[\frac{2\pi\hbar}{\epsilon^{D-2}}\right]^{-1/2}$$

2

(hypercubic)

oscillators (linearly)

variable $\in \mathbb{R}$ classical

$\in \mathbb{Z}^D$

$$= S_i^2 + \sum_{\substack{\text{links} \\ \partial = \langle ij \rangle}} \frac{K_1}{2}(S_i - S_j)^2$$

term linear coupling term

At finite temperature T_{emp}

$$Z = \sum_{\{S_i\}} \exp\left(-\frac{1}{k_B T} E[S]\right) \Longleftrightarrow \int \prod_i d\phi(x_i) \cdot e^{-\frac{1}{\hbar} S_E[\phi]}$$

$\{S_i\} \leftarrow$ sum over config

$$\| \quad \int \prod_{i \text{ sites}} dS_i$$

statistical Lattice Space

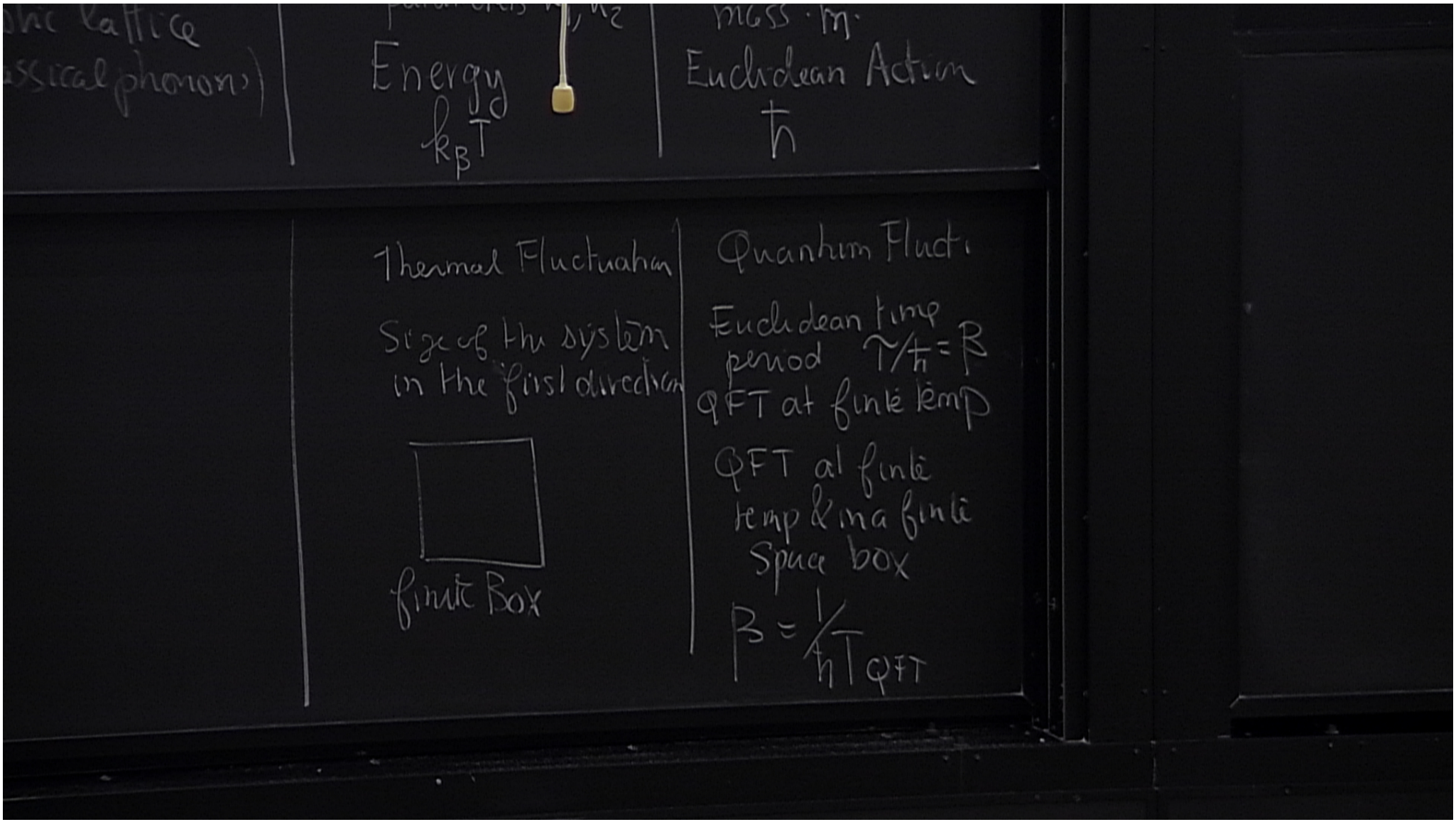
local order para meter S_i

QFT

Euclidean Space time $1 + (D-1)$ dim.

Field $\phi(x_i)$

Ising like model but with continuous spins (classical) $\Longleftrightarrow$ elastic lattice (classical phonons)


Energy
$k_B T$
Euclidean Action
$\hbar$
Thermal Fluctuation
Size of the system in the first direction
finite Box
Quantum Fluct.
Euclidean time period $\tau/\hbar = \beta$
QFT at finite temp
QFT at finite temp & in a finite space box
$\beta = 1/\hbar T_{QFT}$

$(S_i - S_j)^2$

$\prod_{i \text{ sites}} dS_i$

Ising like model but with continuous spins (classical) $\Leftrightarrow$ elastic lattice (classical phonons)

local order parameter S_i

parameters K_1, K_2

Energy $k_B T$

Field $\phi(x_i)$

parameters in the Hamilt. mass m

Euclidean Action $\hbar$

2 dim statistical model $\Updownarrow$ 1 dim Quantum system

$i \to \hat{S}_i$ operator

Classical Statistical model in D dimension $\Updownarrow$ Quantum Field Theory in D-1 dim space relativistic

1 time direction

Thermal Fluctuation

Size of the system in the first direction

finite Box

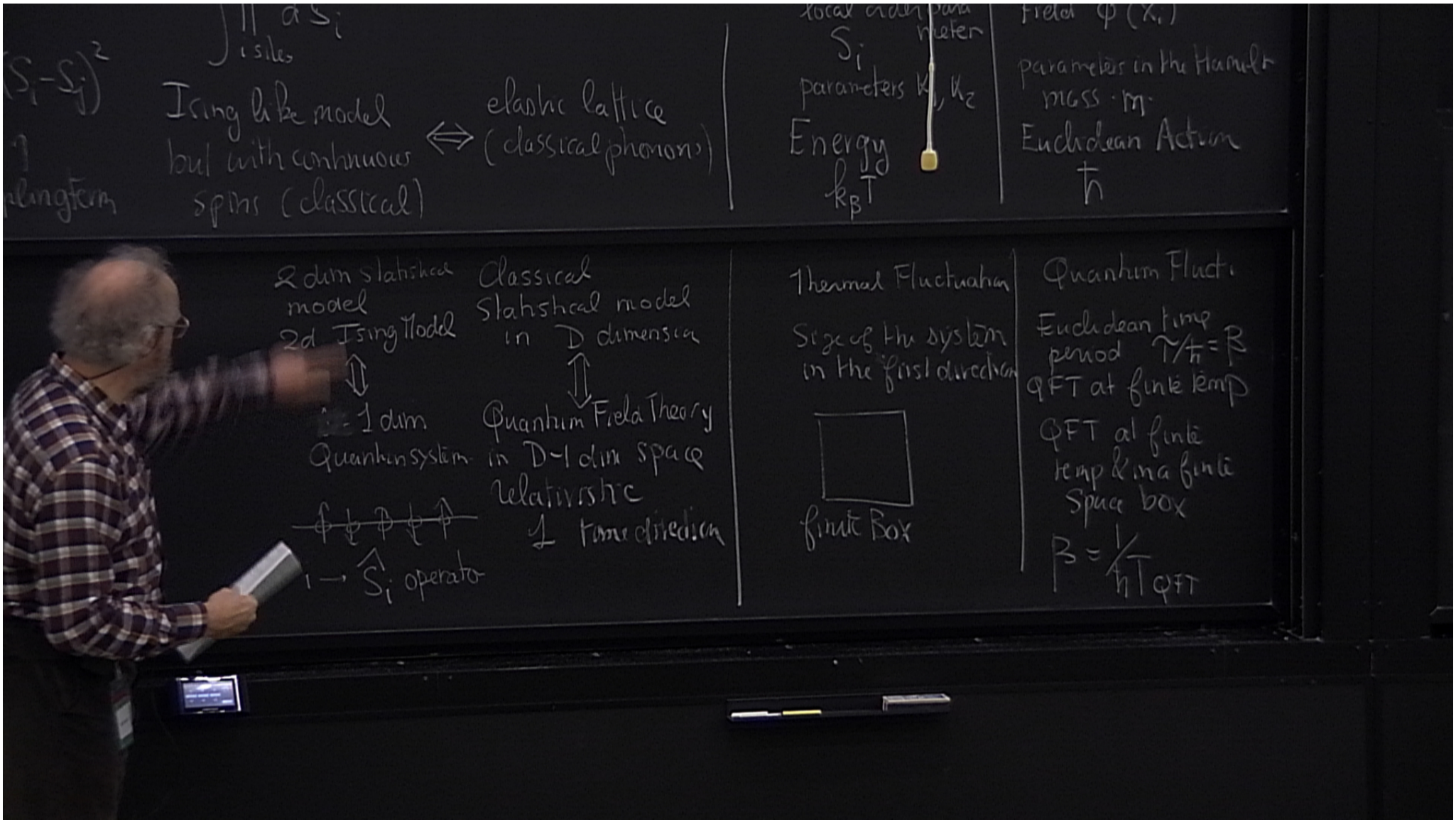
Quantum Fluct.

Euclidean time period $\tau/\hbar = \beta$

QFT at finite temp

QFT at finite temp & in a finite space box

$\beta = \frac{1}{\hbar T_{QFT}}$


Ising like model
but with continuous
spins (classical)
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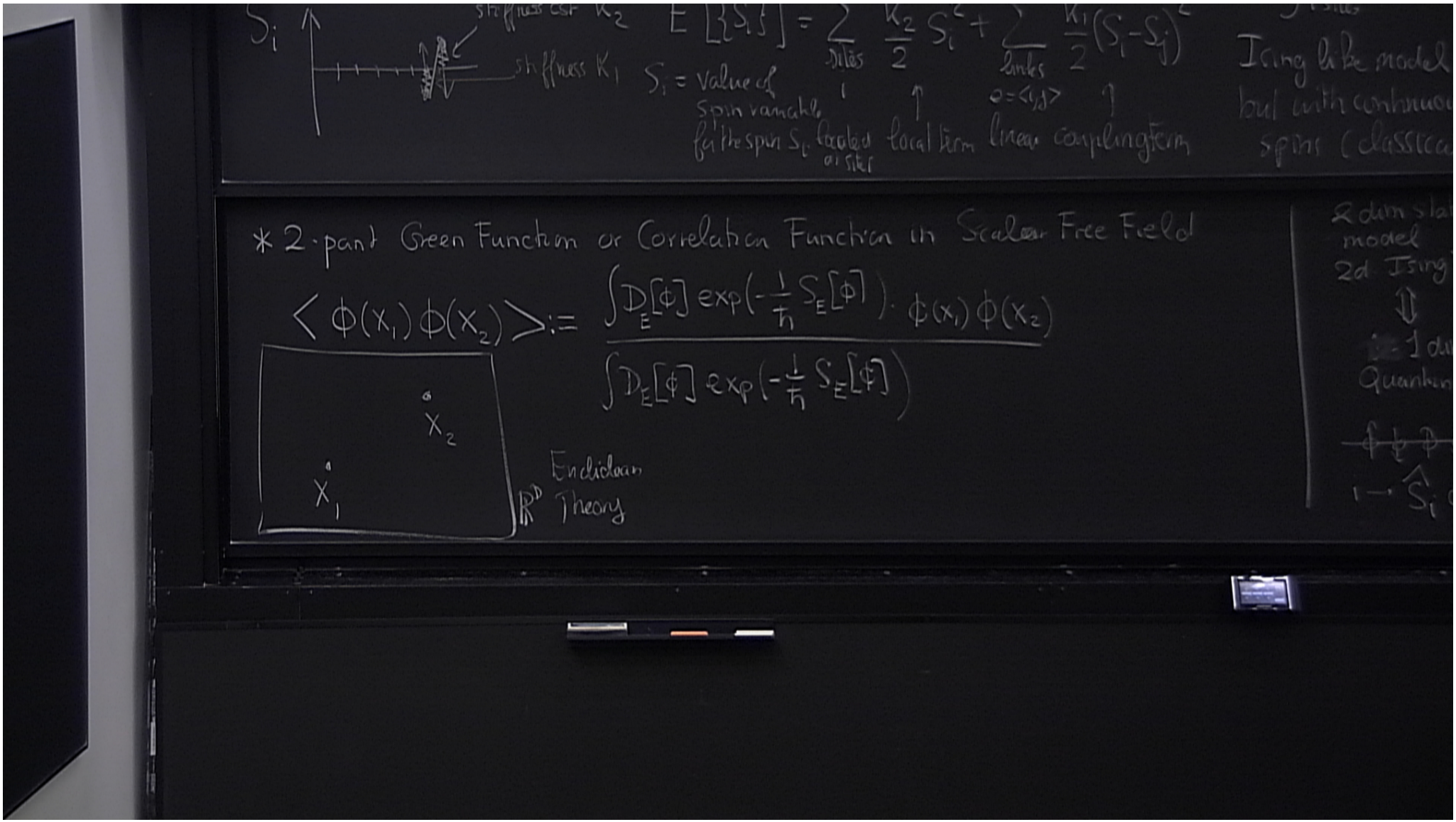
local order parameter
S_i
parameters K_1, K_2
Energy
$k_B T$
Field $\phi(x_i)$
parameters in the Hamilt
mass · m
Euclidean Action
$\hbar$
2 dim statistical
model
2d Ising Model
1 dim
Quantum system
$\hat{S}_i$ operator
Classical
Statistical model
in D dimension
Quantum Field Theory
in D-1 dim space
relativistic
1 time direction
Thermal Fluctuation
Size of the system
in the first direction
finite Box
Quantum Fluct.
Euclidean time
period $T/\hbar = \beta$
QFT at finite temp
QFT at finite
temp & in a finite
space box
$\beta = 1/\hbar T_{QFT}$


S_i
stiffness K_2
stiffness K_1
$E[\{S_i\}] = \sum_{\text{sites}} \frac{K_2}{2} S_i^2 + \sum_{\text{links}} \frac{K_1}{2}(S_i - S_j)^2$
S_i = value of spin variable for the spin S_i located at site i
local term
$e = \langle i,j \rangle$
linear coupling term
Ising like model but with continuous spins (classica
* 2-point Green Function or Correlation Function in Scalar Free Field
$\langle \phi(x_1)\phi(x_2) \rangle := \frac{\int D_E[\phi] \exp(-\frac{1}{\hbar} S_E[\phi]) \cdot \phi(x_1)\phi(x_2)}{\int D_E[\phi] \exp(-\frac{1}{\hbar} S_E[\phi])}$
x_2
x_1
$\mathbb{R}^D$
Euclidean Theory
2 dim stat model
2d Ising
1 du
Quantum
$\hat{S}_i$

S_i = value of spin variable for the spin S_i located at site

local term

$e = \langle i,j \rangle$

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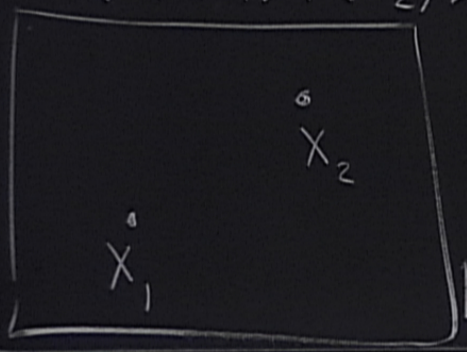
* 2-point Green Function or Correlation Function in Scalar Free Field

$$\langle \phi(x_1)\phi(x_2) \rangle := \frac{\int D_E[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right) \cdot \phi(x_1)\phi(x_2)}{\int D_E[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right)}$$

Laplace operator

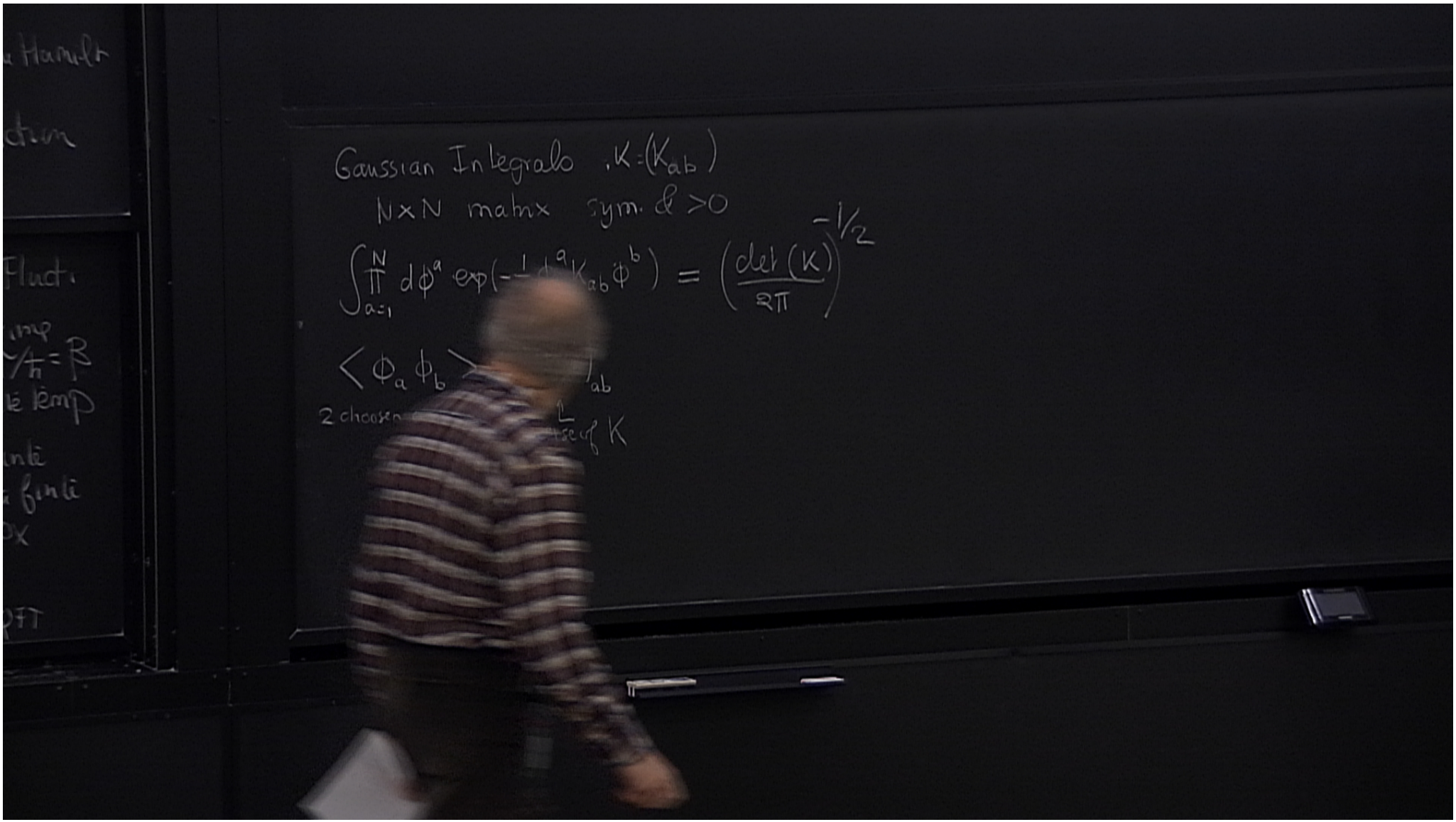
$$\Delta = \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\mu}$$



Euclidean Theory $\mathbb{R}^D$

Gaussian Integral $S_E[\phi] = \int d^D x \, \frac{1}{2} \phi \left(-\Delta + m_1^2\right) \phi$

+ boundary terms


Gaussian Integrals , K = (K_ab)
N×N matrix sym. & >0
∫ ∏_{a=1}^{N} dφ^a exp(−½ φ^a K_ab φ^b) = (det(K)/2π)^{−1/2}
⟨φ_a φ_b⟩ = (K^{−1})_ab
2 choosen
inverse of K

Gaussian Integrals $K = (K_{ab})$

$N \times N$ matrix sym. & > 0

$\longleftrightarrow$ $K = -\Delta + m^2$ linear differential operator $\phi \longrightarrow K\phi = (-\Delta + m^2)\phi$

$$\int \prod_{a=1}^{N} d\phi^a \exp\left(-\frac{1}{2}\phi^a K_{ab} \phi^b\right) = \left(\frac{\det(K)}{2\pi}\right)^{-1/2}$$

$$\langle \phi_a \phi_b \rangle = (K^{-1})_{ab} = G_{ab} \qquad K_{ab} G_{bc} = \delta_{ac}$$

2 choosen a & b

inverse of K

$\longleftrightarrow \quad K = -\Delta + m^2$ linear differential operator $\quad \phi \longrightarrow K\phi = (-\Delta + m^2)\phi$

$$\left(\frac{\det(K)}{2\pi}\right)^{-1/2}$$

$$\langle \phi(x_1)\,\phi(x_2)\rangle = G(x_1, x_2) = \langle x_2 | \frac{1}{-\Delta + m^2} | x_1 \rangle$$

Kernel of the operator $(-\Delta + m^2)^{-1}$

Bra-Ket notation of QM

$$K_{ab}\, G_{bc} = \delta_{ac}$$

Identity operator

$$\phi \longrightarrow \mathbb{1}\cdot\phi = \phi$$

$$\phi(x) = \int dy\, \mathbb{1}(x,y)\cdot\phi(y)$$

general operator $\mathcal{O}$

$$\phi \longrightarrow \psi = \mathcal{O}\cdot\phi$$

Kernel: definition

$$\psi(x) = \int dy\, \mathcal{O}(x,y)\,\phi(y)$$

Identity operator

$$\phi \longrightarrow \mathbb{1}\cdot\phi = \phi$$

$$\phi(x) = \int d^D y \, \mathbb{1}(x,y)\cdot\phi(y)$$

$$\mathbb{1}(x,y) = \delta(x-y)$$

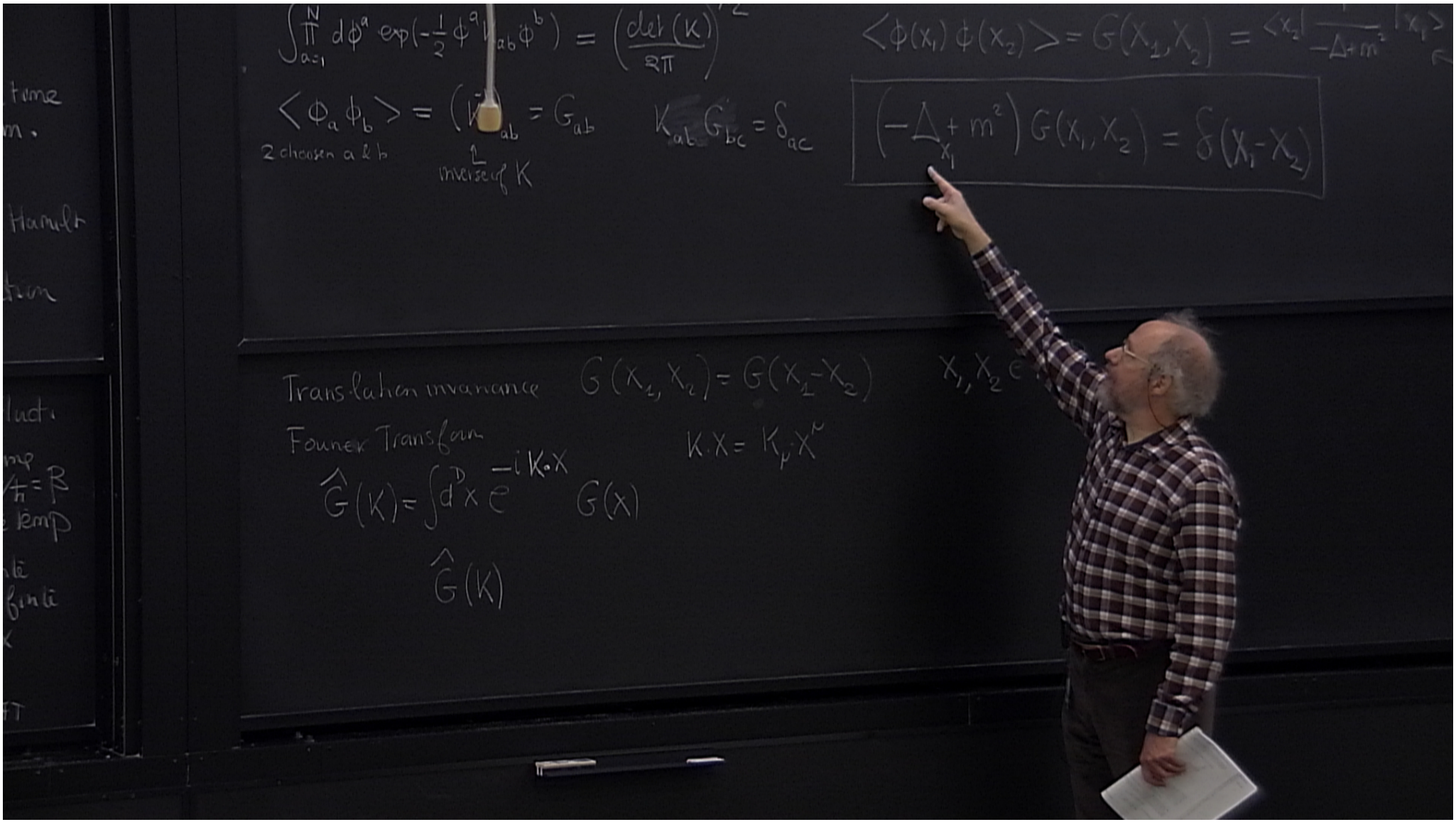
Dirac δ-function in D-dim.

general operator O

$$\phi \longrightarrow \psi = O\cdot\phi$$

Kernel: definition

$$\psi(x) = \int dy \, O(x,y)\,\phi(y)$$


$\int \prod_{a=1}^{N} d\phi^a \exp(-\frac{1}{2}\phi^a K_{ab}\phi^b) = \left(\frac{\det(K)}{2\pi}\right)^{1/2}$
$\langle \phi_a \phi_b \rangle = (K^{-1})_{ab} = G_{ab}$
2 choosen a & b
inverse of K
$K_{ab} G_{bc} = \delta_{ac}$
$\langle \phi(x_1)\phi(x_2) \rangle = G(x_1, x_2) = \langle x_2 | \frac{1}{-\Delta + m^2} | x_1 \rangle$
$(-\Delta_{x_1} + m^2) G(x_1, x_2) = \delta(x_1 - x_2)$
Translation invariance $G(x_1, x_2) = G(x_1 - x_2)$
Fourier Transform
$K \cdot X = K_\mu X^\mu$
$\hat{G}(K) = \int d^D x \, e^{-iK\cdot X} G(x)$
$\hat{G}(K)$

inverse of K

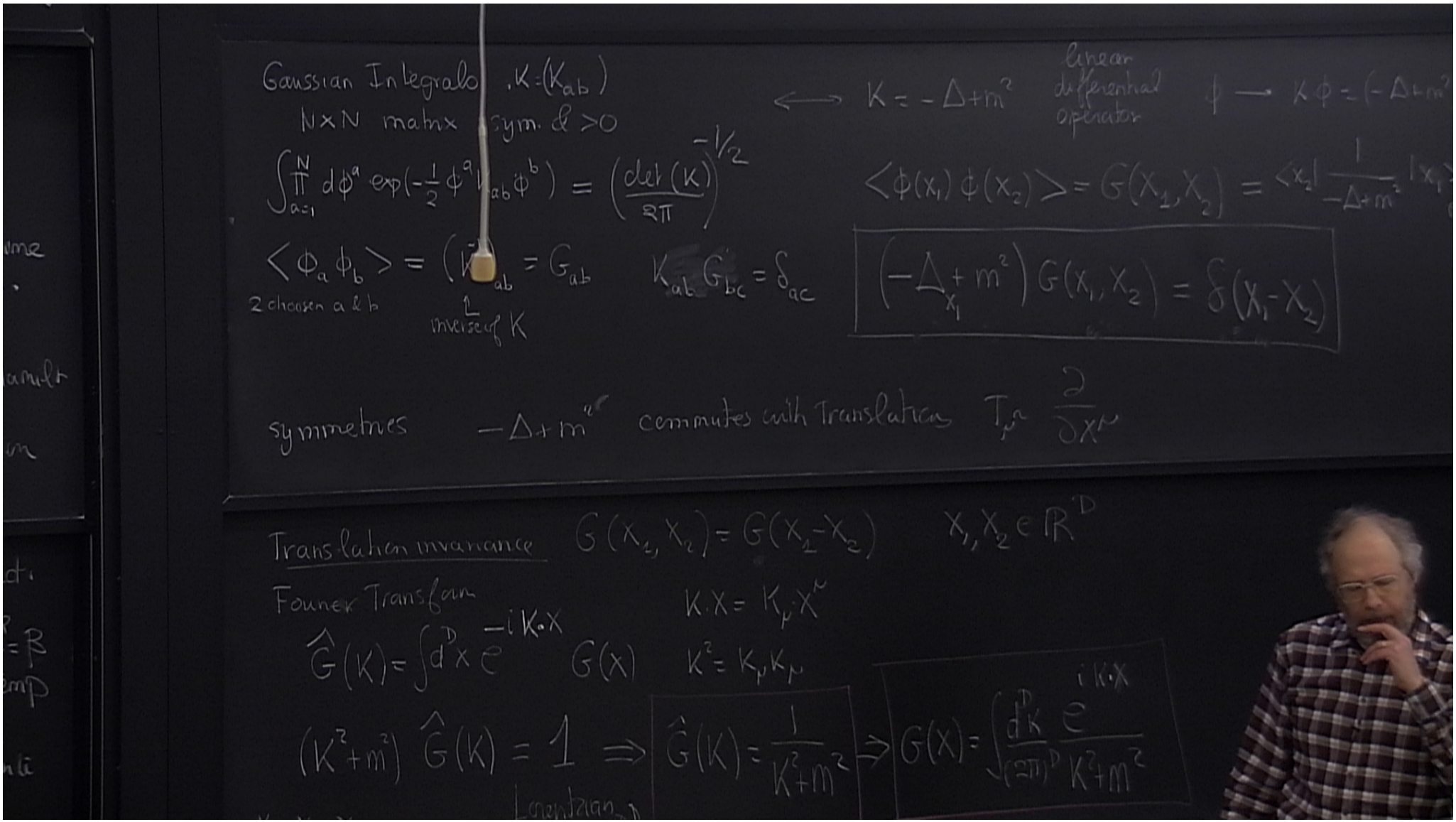
Translation invariance $G(x_1, x_2) = G(x_1 - x_2)$ $\quad x_1, x_2 \in \mathbb{R}^D$

Fourier Transform $\quad K \cdot X = K_\mu X^\mu$

$$\hat{G}(K) = \int d^D x \, e^{-iK\cdot X} \, G(x) \qquad K^2 = K_\mu K_\mu$$

$$(K^2 + m^2)\,\hat{G}(K) = 1 \Rightarrow \boxed{\hat{G}(K) = \frac{1}{K^2+m^2}} \Rightarrow \boxed{G(x) = \int \frac{d^D K}{(2\pi)^D} \frac{e^{iK\cdot X}}{K^2+m^2}}$$

Lorentzian distrib in $\mathbb{R}^D$


Gaussian Integralo .K=(K_ab)
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⟨φ_a φ_b⟩ = (K^{-1})_ab = G_ab
K_ab G_bc = δ_ac
2 choosen a & b
inverse of K
⟷ K = -Δ+m²
linear differential operator
φ → Kφ = (-Δ+m²
⟨φ(x_1) φ(x_2)⟩ = G(x_1, x_2) = ⟨x_2| 1/(-Δ+m²) |x_1⟩
(-Δ_{x_1} + m²) G(x_1, x_2) = δ(x_1 - x_2)
symmetries -Δ+m² commutes with Translations T_μ ∂/∂x^μ
Translation invariance G(x_1, x_2) = G(x_1 - x_2)
x_1, x_2 ∈ R^D
Fourier Transform
K·X = K_μ X^μ
Ĝ(K) = ∫ d^D x e^{-iK·X} G(X)
K² = K_μ K_μ
(K²+m²) Ĝ(K) = 1 ⇒ Ĝ(K) = 1/(K²+m²) ⇒ G(X) = ∫ d^D K/(2π)^D e^{iK·X}/(K²+m²)

Gaussian integrals $K=(K_{ab})$

$N\times N$ matrix sym. & >0

$$\int \prod_{a=1}^{N} d\phi^a \exp\left(-\frac{1}{2}\phi^a K_{ab}\phi^b\right) = \left(\frac{\det(K)}{2\pi}\right)^{-1/2}$$

$$\langle \phi_a \phi_b \rangle = (K^{-1})_{ab} = G_{ab} \qquad K_{ab} G_{bc} = \delta_{ac}$$

2 choosen a & b

inverse of K

$\longleftrightarrow$ $K = -\Delta + m^2$ differential operator $\phi \longrightarrow K\phi = (-\Delta+m^2)\phi$

$$\langle \phi(x_1)\phi(x_2)\rangle = G(x_1,x_2) = \langle x_2 | \frac{1}{-\Delta+m^2} | x_1 \rangle$$

Kernel of the operator

Bra-Ket notation of ϕ

$$\boxed{\left(-\Delta_{x_1} + m^2\right) G(x_1,x_2) = \delta(x_1-x_2)}$$

symmetries $-\Delta+m^2$ commutes with Translations $T_\mu = \frac{\partial}{\partial x^\mu}$

Translation invariance $G(x_1,x_2) = G(x_1-x_2)$ $\quad x_1, x_2 \in \mathbb{R}^D$

Fourier Transform

$$\hat{G}(K) = \int d^D x\, e^{-iK\cdot X} G(X) \qquad K\cdot X = K_\mu X^\mu \qquad K^2 = K_\mu K_\mu$$

$$(K^2+m^2)\,\hat{G}(K) = 1 \Rightarrow \boxed{\hat{G}(K) = \frac{1}{K^2+m^2}} \Rightarrow \boxed{G(X) = \int \frac{d^D K}{(2\pi)^D} \frac{e^{iK\cdot X}}{K^2+m^2}}$$

$X = X_1 - X_2$

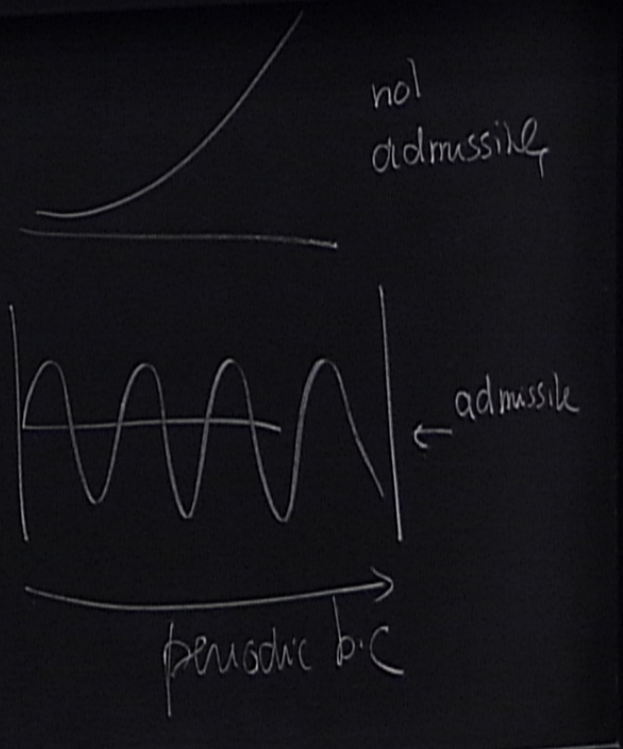
Lorentzian distrib in $\mathbb{R}^D$

e. state $e^{iK\cdot X}$

e v $\quad K^2+m^2 > 0$ if $m>0$

e-state $e^{iK.X}$

e.v $K^2+m^2>0$ if $m>0$



$$\int \prod_{a=1}^{N} d\phi^a \exp\left(-\frac{1}{2}\phi^a K_{ab}\phi^b\right) = \left(\frac{\det(K)}{2\pi}\right)^{\frac{1}{2}}$$

$$\langle \phi(x_1)\phi(x_2)\rangle = G(x_1,x_2) = \langle x_2 | \frac{1}{-\Delta+m^2} | x_1 \rangle$$

← Bra-Ket notation of QM

$$\langle \phi_a \phi_b \rangle = (K^{-1})_{ab} = G_{ab} \qquad K_{ab} G_{bc} = \delta_{ac}$$

2 choosen a & b

inverse of K

$$\boxed{\left(-\Delta_{x_1} + m^2\right) G(x_1,x_2) = \delta(x_1 - x_2)}$$

symmetries $\quad -\Delta + m^2$ commutes with Translations $T_\mu = \frac{\partial}{\partial x^\mu}$

Translation invariance $\quad G(x_1,x_2) = G(x_1 - x_2) \qquad x_1, x_2 \in \mathbb{R}^D$

e. state $\quad e^{iK\cdot X}$

e V $\quad K^2 + m^2 > 0$ if $m > 0$

Fourier Transform $\qquad K\cdot X = K_\mu X^\mu$

$$\hat{G}(K) = \int d^D x\, e^{-iK\cdot X} G(X) \qquad K^2 = K_\mu K_\mu$$

2 point function in position space

$$(K^2 + m^2)\,\hat{G}(K) = 1 \Rightarrow \boxed{\hat{G}(K) = \frac{1}{K^2+m^2}} \Rightarrow \boxed{G(X) = \int \frac{d^D K}{(2\pi)^D} \frac{e^{iK\cdot X}}{K^2+m^2}} = \frac{1}{2\pi}\left(\frac{2\pi|X|}{m}\right)^{\frac{2-D}{2}} K_{\frac{D-2}{2}}(|X|\cdot m)$$

2nd King Bessel

$X = X_1 - X_2$

Lorentzian distrib in $\mathbb{R}^D$

$$(K^2+m^2)\,\hat{G}(K) = 1 \Rightarrow \boxed{\hat{G}(K) = \frac{1}{K^2+m^2}} \Rightarrow \boxed{G(X) = \int \frac{d^D K}{(2\pi)^D}\frac{e^{\cdots}}{K^2+m^2}} = \frac{1}{2\pi}\left(\frac{2\pi|X|}{m}\right) K_{D}$$

$X = X_1 - X_2$

Lorentzian distrib in $\mathbb{R}^D$

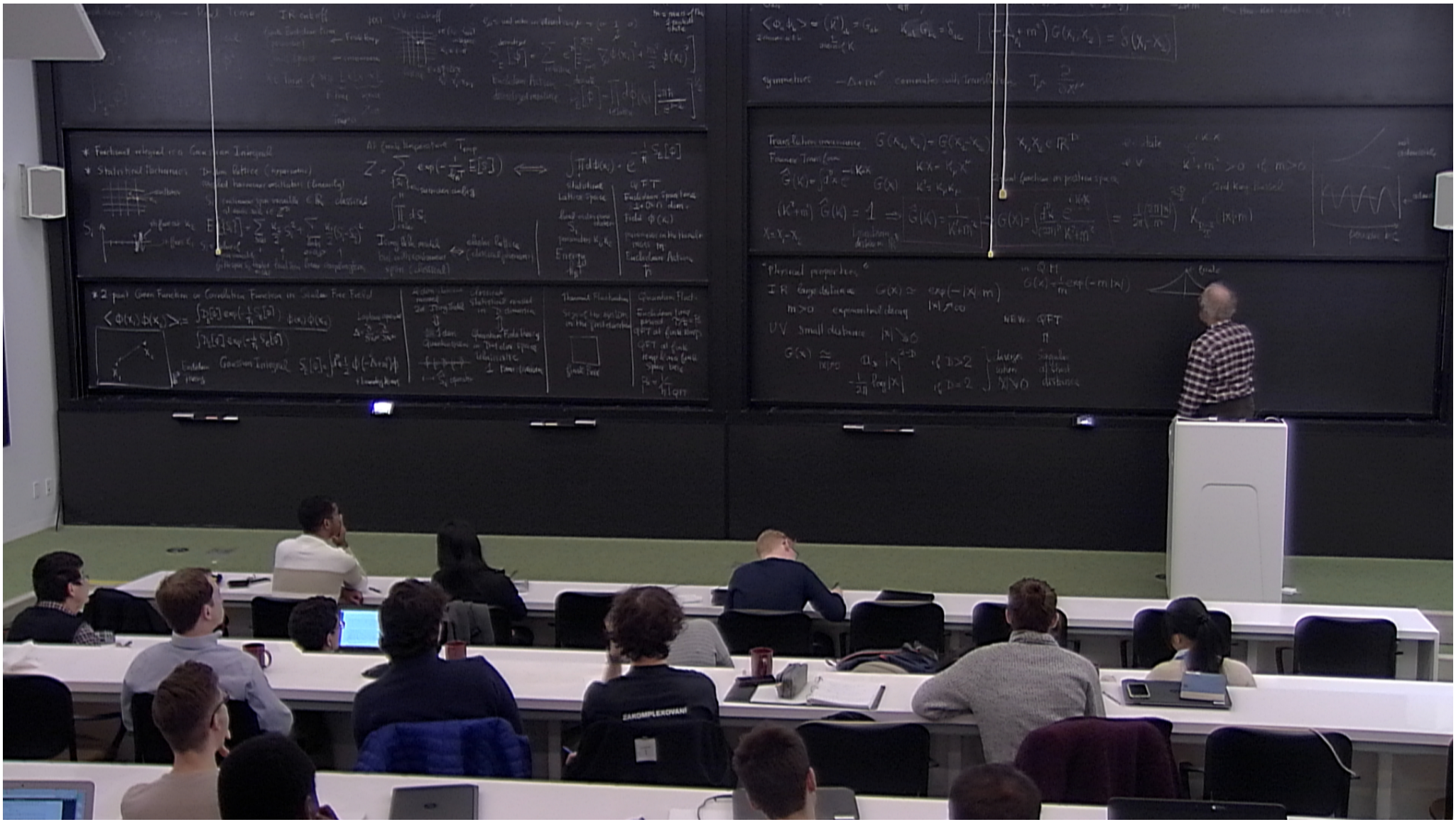
"Physical properties"

I.R large distance $\quad G(X) \simeq \exp(-|X|\cdot m)$ $\quad |X| \nearrow \infty$

$m > 0$ exponential decay

UV small distance. $|X| \searrow 0$

$$G(X) \underset{|X|\nearrow 0}{\simeq} \begin{cases} a_D \cdot |X|^{2-D} & \text{if } D>2 \\ -\frac{1}{2\pi}\log|X| & \text{if } D=2 \end{cases} \Bigg\} \text{ diverges when } |X| \searrow 0$$


Physical properties
symmetries
Q.F.T
Energy
Euclidean Action

$$(K^2+m^2)\,\hat{G}(K) = 1 \Rightarrow \boxed{\hat{G}(K) = \frac{1}{K^2+m^2}} \Rightarrow \boxed{G(X) = \int \frac{d^D K}{(2\pi)^D}\frac{e^{\;}}{K^2+m^2}} = \frac{1}{2\pi}\left(\frac{2\pi|X|}{m}\right) K_D$$

$X = X_1 - X_2$

Lorentzian distrib in $\mathbb{R}^D$

"Physical properties"

in Q.M

$G(x) = \frac{1}{m}\exp(-m|x|)$

I.R large distance $\quad G(x) \simeq \exp(-|x|\cdot m)$, $|x| \nearrow \infty$

$m > 0$ exponential decay

NEW: QFT

$\Uparrow$

Singular at short distance

UV small distance. $|x| \searrow 0$

$$G(x) \underset{|x|\nearrow 0}{\simeq} \begin{cases} a_D \cdot |x|^{2-D} & \text{if } D>2 \\ -\frac{1}{2\pi}\log|x| & \text{if } D=2 \end{cases}$$

diverges when $|x| \searrow 0$

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Singular at short distance

Back to Minkowski & real time

$\mathbb{R}^D \ni X_E = (\tau, \vec{x})$ $\quad \tau = it$ $\quad X = (t, \vec{x})$

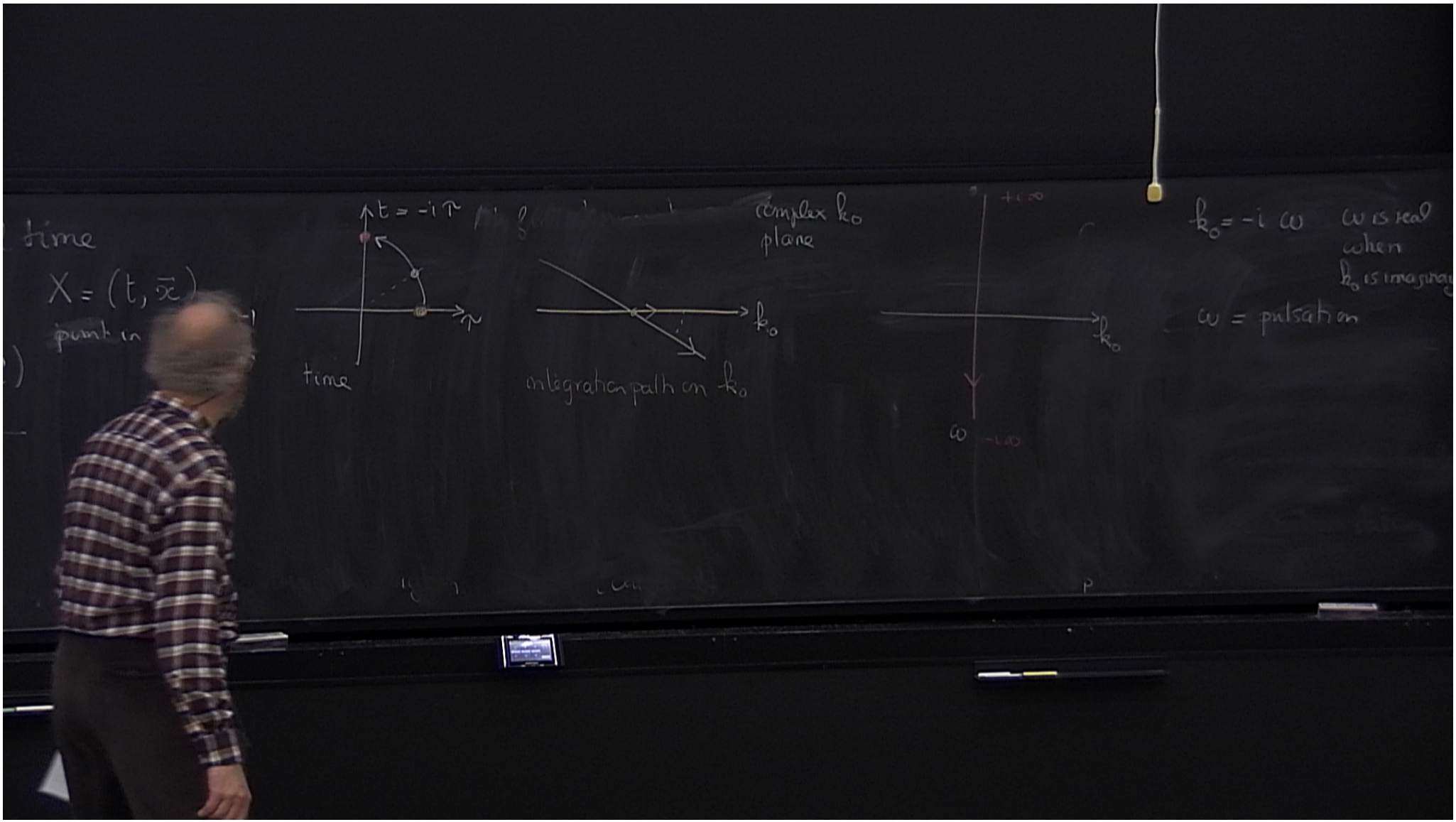
point in $\mathbb{M}^{1,D-1}$

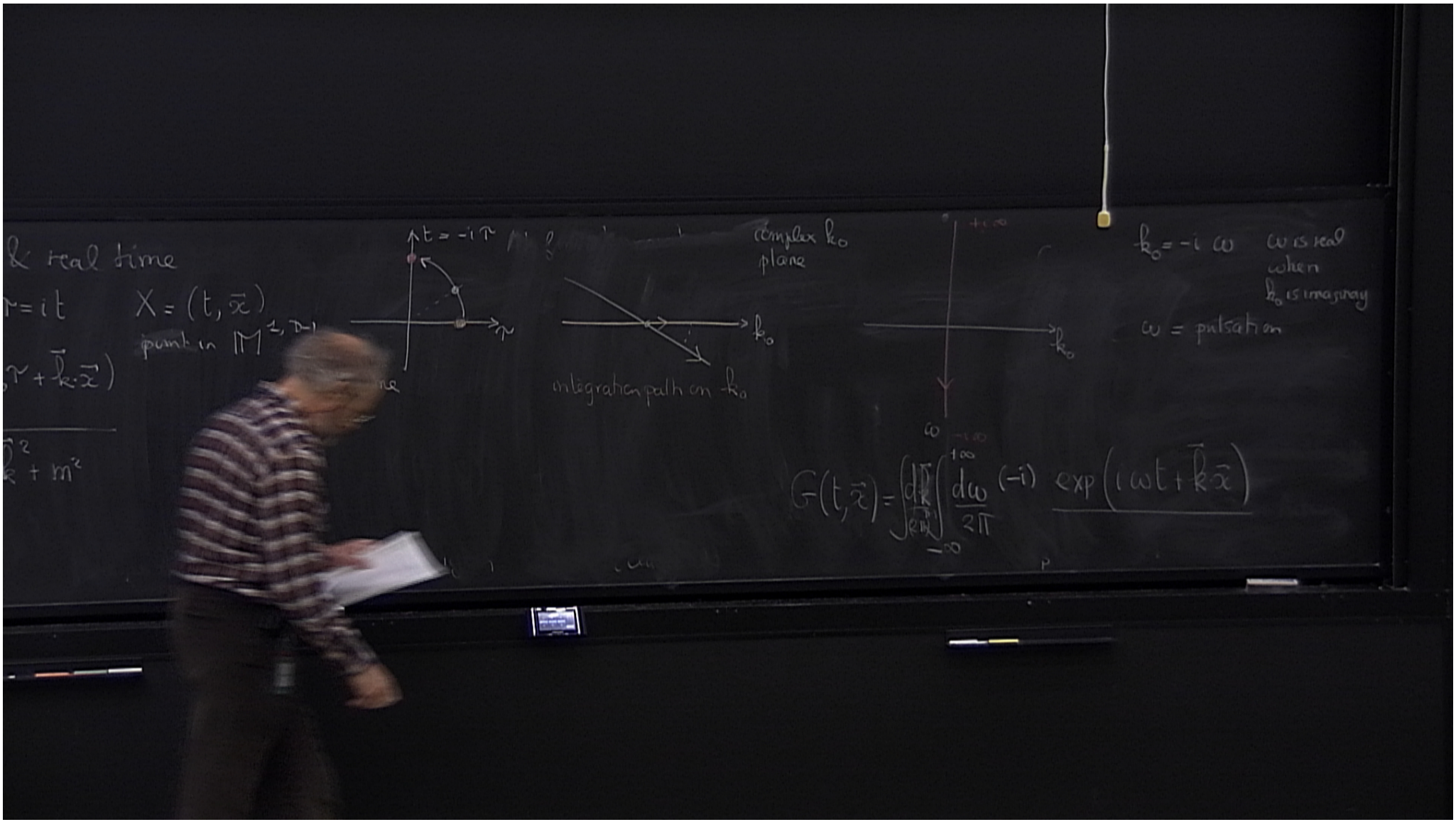
time

$$G(X_E) = \int \frac{dk_0\, d^{D-1}\vec{k}}{(2\pi)^D} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$$

momenta

$K_E = (k_0, \vec{k})$


time
X = (t, x̄)
t = -iτ
τ
time
complex k₀ plane
k₀
integration path on k₀
+i∞
k₀
ω
-i∞
k₀ = -i ω
ω is real when k₀ is imaginary
ω = pulsation


& real time
τ = i t
X = (t, x)
point in M
t = -i τ
complex k0 plane
integration path on k0
k0 = -i ω
ω is real when k0 is imaginary
ω = pulsation
G(t, x) = ∫ d³k/(2π)³ ∫ dω/2π (-i) exp(iωt + k·x)

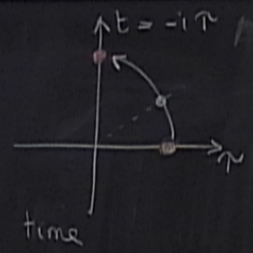
& real time

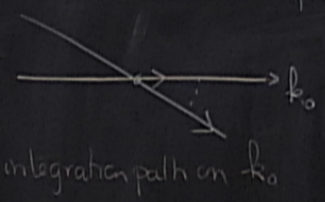
$\tau = it$ $\quad X = (t, \vec{x})$

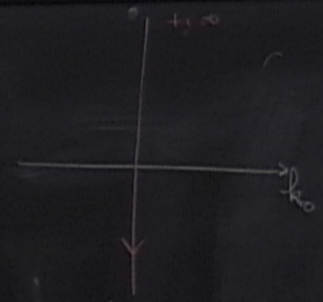
point in $\mathbb{M}^{1,D-1}$

$(\tau + \vec{k}\cdot\vec{x})$

$\vec{k}^2 + m^2$







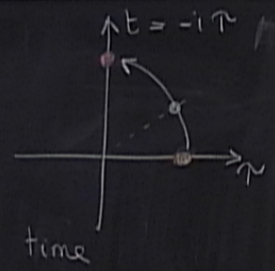
$k_0 = -i\omega$ $\quad$ ω is real when k_0 is imaginary

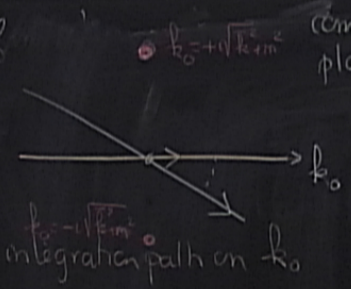
ω = pulsation

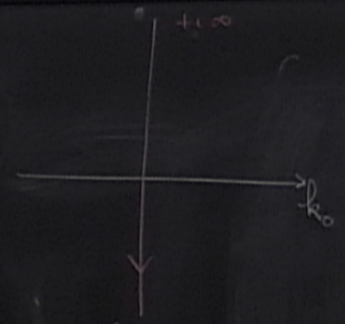
$$G(t,\vec{x}) = \int \frac{d^{D-1}\vec{k}}{(2\pi)^{D-1}} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} (-i) \frac{\exp(i\omega t + \vec{k}\cdot\vec{x})}{-\omega^2 + \vec{k}^2 + m^2}$$

$= (t, \vec{x})$

$\in \mathbb{M}^{1,D-1}$





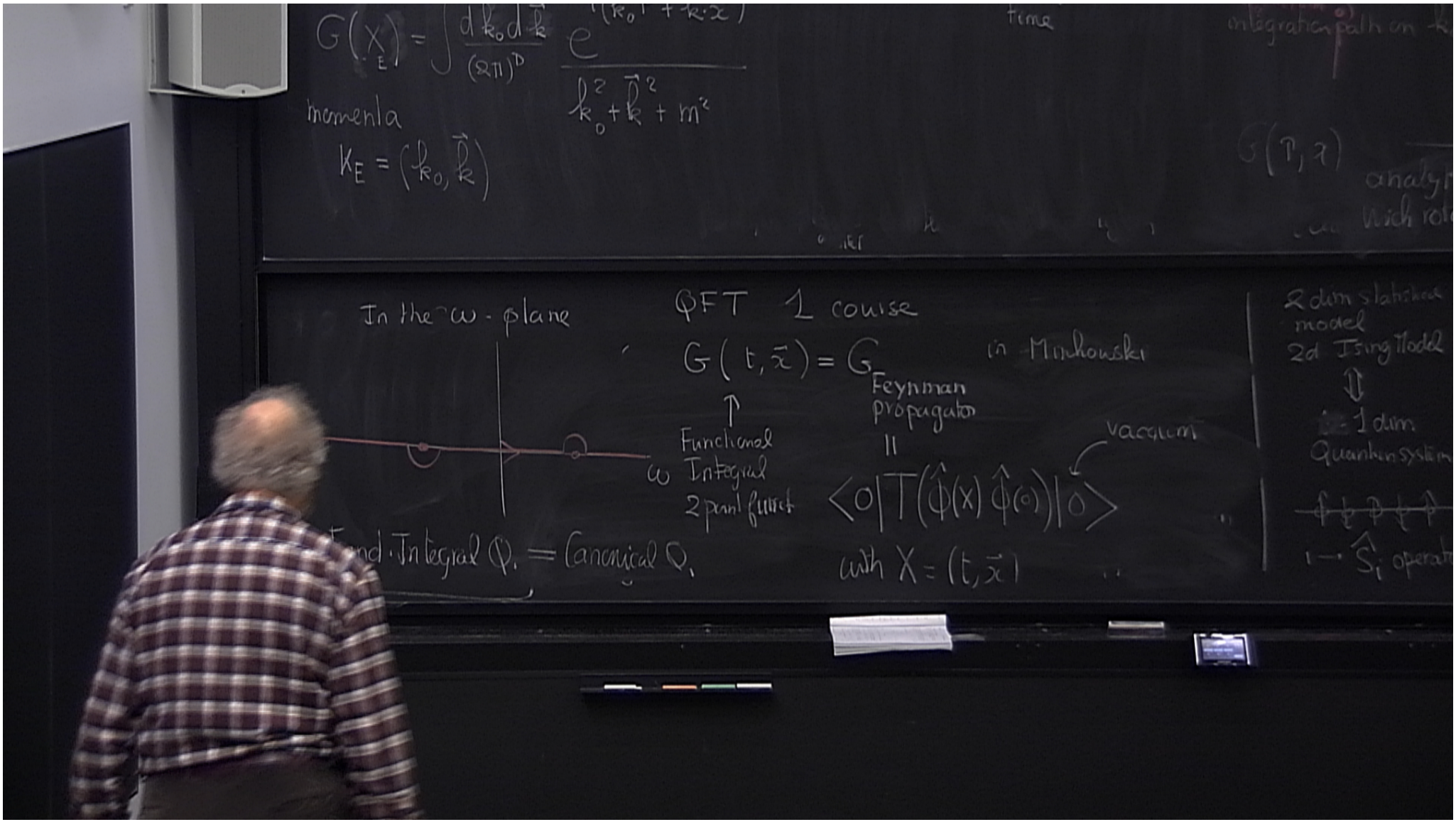


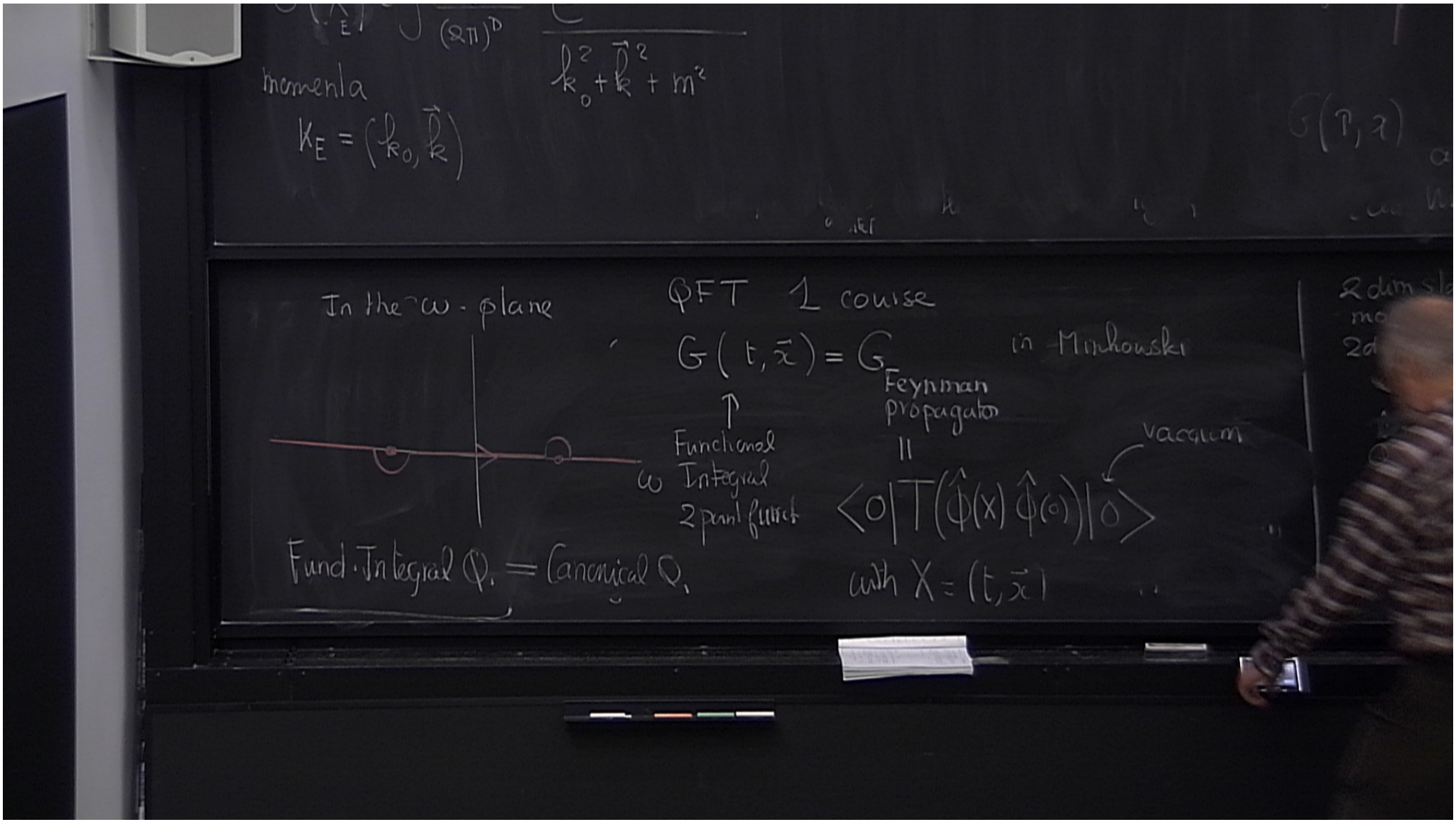
$k_0 = -i\,\omega$ ω is real when k_0 is imaginary

ω = pulsation

But poles at $\omega = \pm\sqrt{\vec{k}^2+m^2}$

$$G(\tau, \vec{x}) \xrightarrow[\text{with rotating}]{\text{analytic}} G(t, \vec{x}) = \int \frac{d^{D-1}\vec{k}}{(2\pi)^{D-1}} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} (-i) \frac{\exp\left(i(\omega t + \vec{k}\cdot\vec{x})\right)}{-\omega^2 + \vec{k}^2 + m^2}$$


time
integration path on k
momenta
In the ω - plane
QFT 1 course
in Minkowski
Feynman propagator
Functional Integral
2 point function
vacuum
2 dim statistical model
2d Ising Model
1 dim Quantum system
Canonical Q.


momenta
$K_E = (k_0, \vec{k})$
$k_0^2 + \vec{k}^2 + m^2$
In the ω - plane
ω
Fund. Integral Q. = Canonical Q.
QFT 1 course
$G(t, \vec{x}) = G$ in Minkowski
Feynman propagator
Functional Integral
2 point funct
$\langle 0 | T(\hat{\phi}(x)\, \hat{\phi}(0)) | 0 \rangle$
vacuum
with $X = (t, \vec{x})$