Title: Thoughts on quantum gravity and the superposition principle

Date: Oct 27, 2016 02:30 PM

URL: http://pirsa.org/16100067

Abstract: <p>In this talk, I will address a major conceptual and technical concern of non-perturbative quantum gravity: the quantum superposition of causal structures of space-times. I will discuss a class of theories that can address the problem, their flaws, and their relation to general relativity.</p>

Thoughts on quantum gravity and the superposition principle

Henrique Gomes

Perimeter Institute for Theoretical Physics

October 27, 2016

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- Or, a gravitational action that is pertubatively renormalizable (around a classical background)? Weyl gravity and Horava-Lifschitz are pertubatively renormalizable.✓
- Should it also resolve singularities? Have only spin-2 gravitons? Etc.

Is non-renormalizability our only obstacle?

Non-renormalizability more damning for gravity: can't meaningfully separate scales (in field-independent way). ['t Hooft '15]

How to classify 'big' and 'small' without reference to the metric itself? Is this the fundamental problem?

E.g. Weyl gravity, $S = \int d^4x \sqrt{g} C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu}$, [Stelle '77]

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And what about symmetry content/physical dofs? E.g. Horava-Lifschitz is perturbatively UV finite.[Barvinsky et al '15] But it has an extra scalar graviton dof.

Is non-renormalizability our only obstacle?

Although they reflect more profound incompatibilities between the principles of quantum mechanics and those of gravity, non-renormalizability, non-unitarity, no well-defined continuous classical limit, wrong physical degrees of freedom, etc, are more technical issues.

But to me there seems to be a more glaring conceptual incompatibility.

Conceptual obstacles to a non-perturbative definition.

Particles should source gravitational fields through T_{ab}. Uncertainty, $\Delta x \Delta p \geq \hbar/2$, implies superposition of states. But how do we quantum superpose gravity?

More importantly, quantum superposition of $g_{\mu\nu}$'s would carry their light-cones. What does that even mean?

Can it mean anything? For instance, conservation of probability requires a fixed causal structure. Does one or the other need to give in, at a fundamental level?

E.g. relativistic QFT axiom: $[\mathcal{A}, \mathcal{B}] = 0$ for $\mathcal{A}, \mathcal{B}$ operators with space-like separated support.

But 'space-like' presumes a definite value for the very field we are trying to quantize. What can replace this axiom?

Superposing causality

In GR, the causal structure is there also off-shell (it is independent of whether the metric satisfies the Einstein equations). It is kinematical.

The proposal I would like to put forward is to have 'causality' be a dynamical, and not kinematical, structure.

In 3+1 GR, initial data is given on spatial hypersurface, Σ, for which $g^{\mu\nu}v_\mu v_\nu > 0$, $v \in T\Sigma$.

- Replace 'space-like' by '*a priori* causal separation', on a given manifold M. Require causal relations to only emerge dynamically, and for semi-classical approximations.

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This will allow us to make sense of the quantum superposition of causal structures in the path integral formalism, as interference effects.

Where we begin

Suppose the **core principles** a quantum gravity theory should embody are:

- A quantum superposition principle. ✓
- A geometrical origin of gravitational forces.✓

To translate the core principles we employ the following axioms:

1. A closed topological manifold M, encoding a weak notion of locality.
2. The kinematic field space (with causally disconnected components)

   $$\text{Riem} := \{g \in C^\infty_+(T^*M \otimes_S T^*M)\}$$

(more generally, for Euclidean signature fields ϕ on M, we call $\mathcal{M}$ the corresponding field space)

3. An action on curves on $\mathcal{M}$: $S(\gamma)$, for $\gamma : I \to \mathcal{M}$, respecting the given gauge symmetry group $\mathcal{G}$. Defines $\tilde{S}([\gamma])$, for $[\gamma] : I \to \mathcal{M}/\mathcal{G}$. (we will discuss $\mathcal{G}$, and $\mathcal{M}/\mathcal{G}$ soon).

Can now define a propagator:[1]

$$W([\phi_1],[\phi_2]) := A \int_{[\phi_1]}^{[\phi_2]} \mathcal{D}[\gamma] \exp\left[i\tilde{S}[\gamma(\lambda)]/\hbar\right]$$

I want to explore consequences of this definition for our notions of superposition of space-times.

[1] Unlike in 3+1 GR, where refoliations must be fixed on initial and final field configurations even before one can write the propagator, here it acts between gauge-invariant initial and final field configurations.

My job during the rest of this talk

1. How to replace superposition for interference, and time by relational data (in QM).
2. What theories have the correct number of degrees of freedom, with a well-defined quotient space $\mathcal{M}/\mathcal{G}$?
3. Quantum interference effects, for gravity, a simple example.
4. A notion of space-time can emerge.

Time allowing (it probably won't), I will show that,

5. With a preferred initial point, ϕ^*, notions of records and quantum operators can be defined.

Relational dynamics and QM

The passage of time can only be told by the relations of things.

Choose a subsystem $q^0 = t$, and have it act as "clock" for another subsystem, q^i.[Page &Wooters '83].

In the end, want to calculate relational probabilities:

$$P(t \in [t_o - \Delta t, t_o + \Delta t], q^i \in [q_o - \Delta q^i, q_o + \Delta q^i])$$

So the configuration, $q = (t, q^i) \in Q$, contains the relational information we want to probe.

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"Indeed, all measurements of quantum mechanical systems could be made to reduce eventually to position and time measurements (e.g., the position of a needle on a meter or time of flight of a particle). [...] a theory formulated in terms of position measurements is complete enough to describe all phenomena."
- Feynman and Hibbs

Relational dynamics and QM II

What does a quantum mechanics theory based on relational dynamics look like? In QM:

$$G(x_1, t_1|x_2, t_2) = \langle x_1|\hat{U}(t_1, t_2)|x_2\rangle$$

where $\hat{U}$ is unitary time-evolution. Then

$$|\psi(x, t)\rangle = \int dx' G(x, t|x', t')|\psi(x', t')\rangle$$

Relational transfer matrix $W(q_1, q_2)$, with no specific variable for time has been studied in non-relativistic particle mechanics:

$$W(q_1, q_2) = \langle q_1|\hat{P}|q_2\rangle, \text{ where } \hat{P} = \int d\tau e^{-i\hat{H}\tau}$$

Semi-classical path-integral approximation

For extremal paths $\gamma_{\rm cl}^{\alpha}$, between ϕ_i, ϕ_f, the Van Vleck determinant is:

$$\Delta_{\gamma_{\rm cl}} \;:=\; \det\left(\frac{\delta^2 S_{\gamma_{\rm cl}}(\phi_i,\phi_f)}{\delta\phi_i(x)\delta\phi_f(y)}\right) = \det\left(\frac{\delta\phi_f(y)}{\delta\pi_i}\right)^{-1}$$

Gives relation between initial infinitesimal volume around ϕ_i, and final volume around ϕ_f, as transported by classical paths.

(related to Lyapunov exponents.[Schullman '93])

Semi-classical approximation is then:

$$W_{\rm cl}(\phi_i,\phi_f) \;=\; \sum_\alpha (\Delta_\alpha)^{1/2} \exp\left(i\, S_{\gamma_{\rm cl}^{\alpha}}(\phi_i,\phi_f)/\hbar\right)$$

$$|W_{\rm cl}|^2 \;=\; \sum_\alpha \Delta_\alpha + \underbrace{2\sum_{\alpha\neq\nu} |\Delta_\alpha \Delta_\nu|^{1/2} \cos\left(\frac{S_{\gamma_{\rm cl}^{\alpha}} - S_{\gamma_{\rm cl}^{\nu}}}{\hbar}\right)}_{\text{Interference effects}}$$

Outline

Gauge-symmetries in Riem

Kinematic field space of acausal fields (*not 'space-like'*):

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Up to two derivatives of the metric, generators related only to two types of such symmetries:

1. Local scale transformations (size is relative).
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Implementing only the second leaves spin-0 graviton physical.

Together they imply spin-2 physical dofs (TT momenta).

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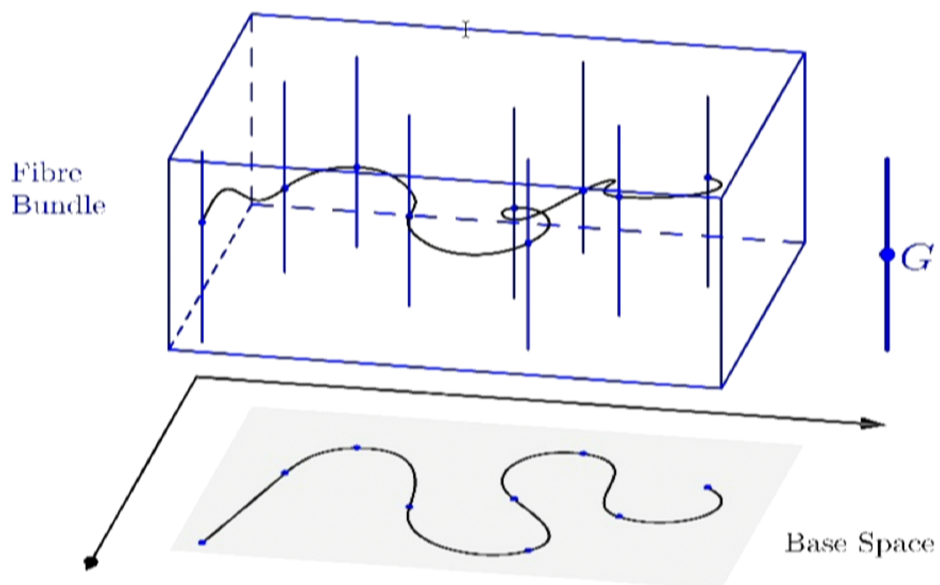
Principal fiber bundle picture

We have the space of 3-metrics, Riem, and a group action on it Diff $\ltimes \mathcal{C}$ (not groupoid). Have analogous structures to gauge theory ensuring invariance. [HG '11, '16, HG and A. Riello '16]



Riemannian gravity

Want to be able to write $W(q_1, q_2)$ in path integral formalism, in Riem$= \mathcal{M}$. Find simple action for paths in $\mathcal{M}$.

For $u, v \in T_g\mathcal{M}$, field-space metric given by:

$$\langle v,\, u\rangle_g := \int d^3x\, G^{abcd}(v_{ab}u_{cd})$$

such that inner product is $\mathcal{G}$-invariant (i.e. fibers are Killing directions in field space).

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Examples of field space-geometries, invariant wrt Diff:

- The canonical supermetric: $G^{abcd} = \sqrt{g}\, g^{ac} g^{bd}$.
- Or one that couples neighbors: $G^{abcd} = \sqrt{g}\sqrt{R^2 + \Lambda^2}\, g^{ac} g^{bd}$.

Riemannian gravity and field space geometry

Can have our actions be given by the length functional in field space, with respect to $\langle\cdot,\cdot\rangle$:

$$S[\gamma] = L[\gamma] := \int_\gamma dt \left(\int d^3x \, G^{abcd}\,\dot{g}_{ab}\dot{g}_{cd}\right)^{1/2} = \int dt|\gamma'|$$

- Example: geodesic equation for canonical supermetric,

$$\ddot{g}_{ab} = \dot{g}_{ac}\dot{g}^{c}{}_{b} + \frac{1}{4}\dot{g}_{cd}\dot{g}^{cd}g_{ab} - \frac{1}{2}\dot{g}^{c}_{c}\dot{g}_{ab}$$

We will get back to how they compare with the dynamical part of the Einstein equations.

Riemannian gravity and field space geometry

Explicit solution of geodesic equation in canonical supermetric case:[Freed & Grossier '89] Given $g^0_{ab} \in \mathcal{M}$ and $h_{ab} \in T_g\mathcal{M}$, let

$h^T_{ab} = h_{ab} - \frac{1}{3} h g_{ab}$, with $h = h^c_c$. Then the geodesic starting out at g^0_{ab} in the direction of h_{ab} is given by:

$$g_{ab}(t) = g^0_{ac} e^{(A(t)\delta^c_b + B(t) h^c_{Tb})}$$

where

$$A(t) = \frac{2}{3} \ln((1 + \frac{t}{4} h)^2 + \frac{3}{8} h^T_{cd} h^{cd}_T t^2)$$

$$B(t) = \frac{4}{\sqrt{3 h^T_{cd} h^{cd}_T}} \text{arctg}\left(\frac{\sqrt{3 h^T_{cd} h^{cd}_T}\, t}{4 + th} \right)$$

Riemannian gravity and semi-classical path integral.

$$S[\gamma] = E[\gamma] := \int dt \int d^3x \, \underbrace{G^{abcd} \, (\dot{g}_{ab} - \mathcal{L}_\xi g_{ab})(\dot{g}_{cd} - \mathcal{L}_\xi g_{cd})}_{\text{connection 1-form in Riem (doesn't come from space-time)}}$$

For quite general cases, geodesic equation, exponential maps, Jacobi fields, curvature, etc, have been computed.
[Freed & Grossier '89, Michor & Gil-Medrano '91, Michor et al '97]

Quantities of interest become geometrical. For example, Van-Vleck matrix:

$$\frac{\delta^2 S_{\gamma_{\rm cl}}(\phi_i, \phi_f)}{\delta\phi_i(x)\delta\phi_f(y)} = J(\phi_i, \phi_f)^{-1}$$

Where J is the Jacobi matrix.[2] Calculate everything needed for semi-classical path-integral with classical geometry of $\mathcal{M}$.

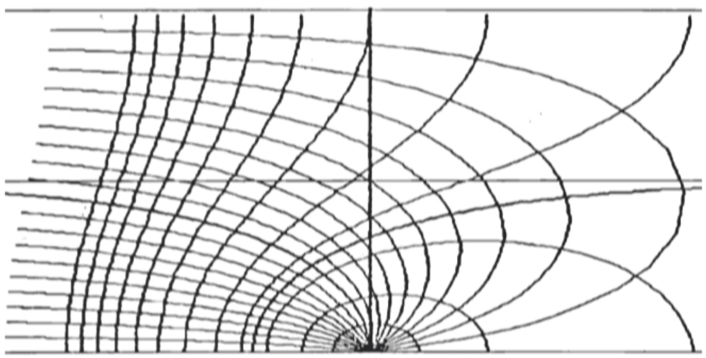
[2] Jacobi fields - geodesic variations of $\gamma_{\rm cl}$ - are uniquely defined by its values at ϕ_i and ϕ_f, through the two-point tensor matrix J.[C. DeWitt '75]

Interference effects for gravity: an example

For canonical supermetric, can compute geodesics $g(t)$, and Van-Vleck determinant explicitly.

But there geodesics with different initial velocities don't intersect.

Imposing reflecting boundary conditions at singular strata of Riem/Diff,[B. DeWitt '81] can get interference pattern of geometries explicitly.



(some properties of the conformal geodesic model)

- For pure gravity, unique up to third derivatives of the metric.
- Gravitational momenta are transverse-traceless.✓
- Simple gauge-structure allows standard Fadeev-Popov quantization (BFV not necessary). Transparent BRST symmetry.
- For spin-1, only $\nabla_{[a}A_{b]}$ can appear containing derivatives of A_a, for spin-0 field ψ, only $\nabla_a\psi$.
- The (naively) linearized equations of motion around a spherical metric, have modified dispersion relations:

$$\ddot{h}_{ab} = \Lambda \frac{g}{\eta_o} \frac{1}{r^4} \left(4\nabla^2 h_{ab} - \frac{7}{4} r^2 \nabla^4 h_{ab} + r^4 \nabla^6 h_{ab} \right)$$

The Bairlein-Sharp-Wheeler formulation of GR

The BSW action for curves in Riem:

$$S_{\mathrm{BSW}} = \int dt \int d^3x \sqrt{g} \sqrt{\underbrace{(R - 2\Lambda)}_{V} \underbrace{G^{abcd} \dot{g}_{ab} \dot{g}_{cd}}_{T}}$$

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Hamiltonian vanishes identically, but momenta not invertible for $\dot{g}_{ab}$, two constraints:

$$H = G_{abcd} \frac{\pi^{ab}\pi^{cd}}{\sqrt{g}} - \sqrt{g}(R - 2\Lambda) = 0 \quad \text{and} \quad H_a = \nabla_b \pi^a_b = 0$$

Gets back the GR (ADM) Hamiltonian:

$$H_{\text{ADM}} = \int d^3x \, (N(x)H(x) + N^a(x)H_a(x))$$

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BSW eom =ADM Ham. eq. for $\dot{\pi}^{ab}$, with the substitution

$$N \to \sqrt{\frac{T}{V}} \quad \text{and then} \quad N\, G^{abcd}\dot{g}_{ab}\sqrt{g} \to \pi^{cd}$$

What is duration? Mach:'Time is change'

In GR, duration is given geometrically, by proper time, even if no dofs are changing along a worldline.

In QM, no operator for time. Want to measure time relationally, by replacing $H \rightarrow H_{\text{clock}} + H_{\text{sys}}$, along with the states. One system measures 'time' for another.

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So far, the solutions to our purely relational models give only the curve $g(t)$ in Riem. This does not a space-time make.

$$ds^2 = -N^2 dt^2 + g_{ab}(N^a dt + dx^a)(N^b dt + dx^b).$$

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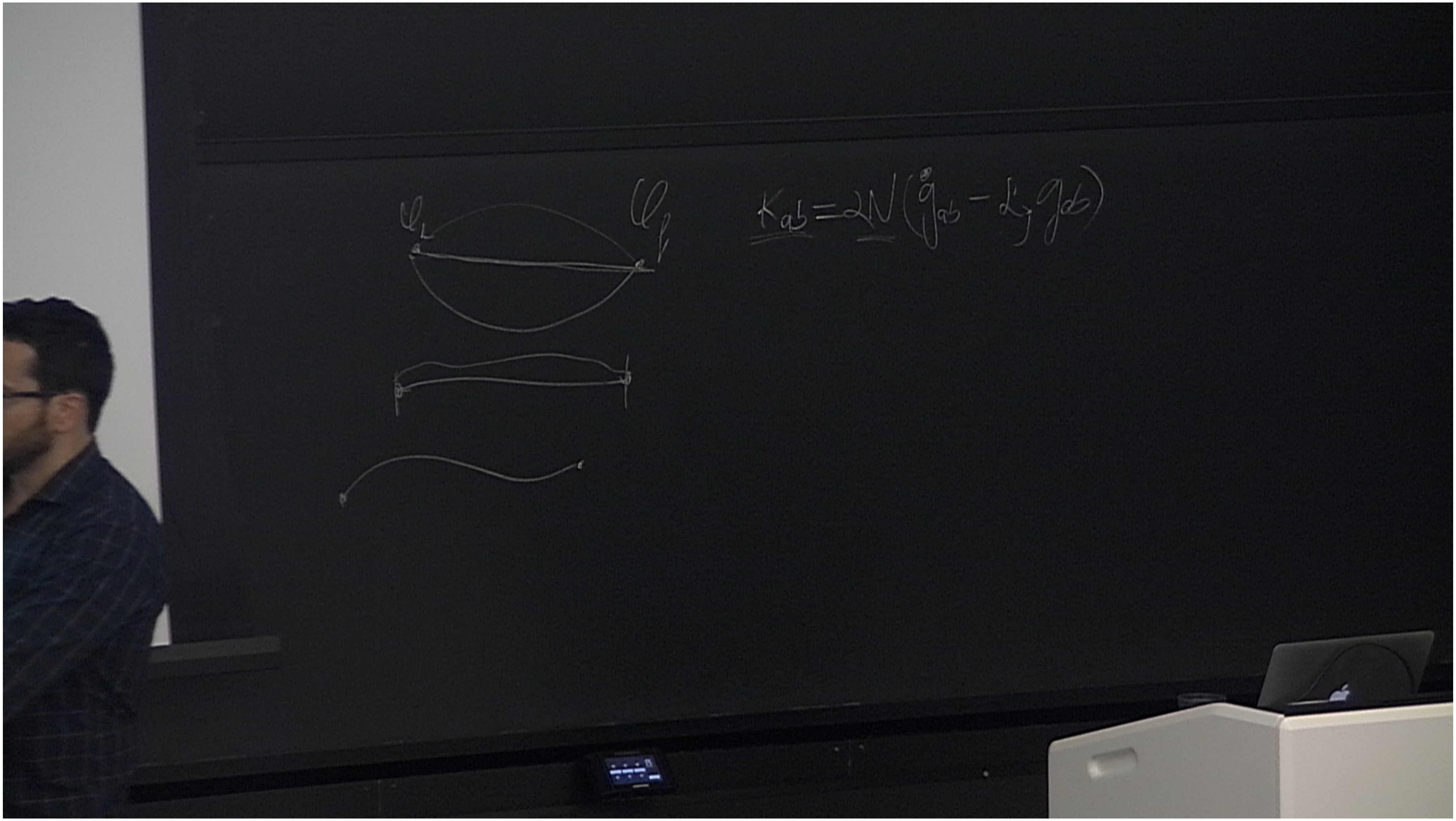
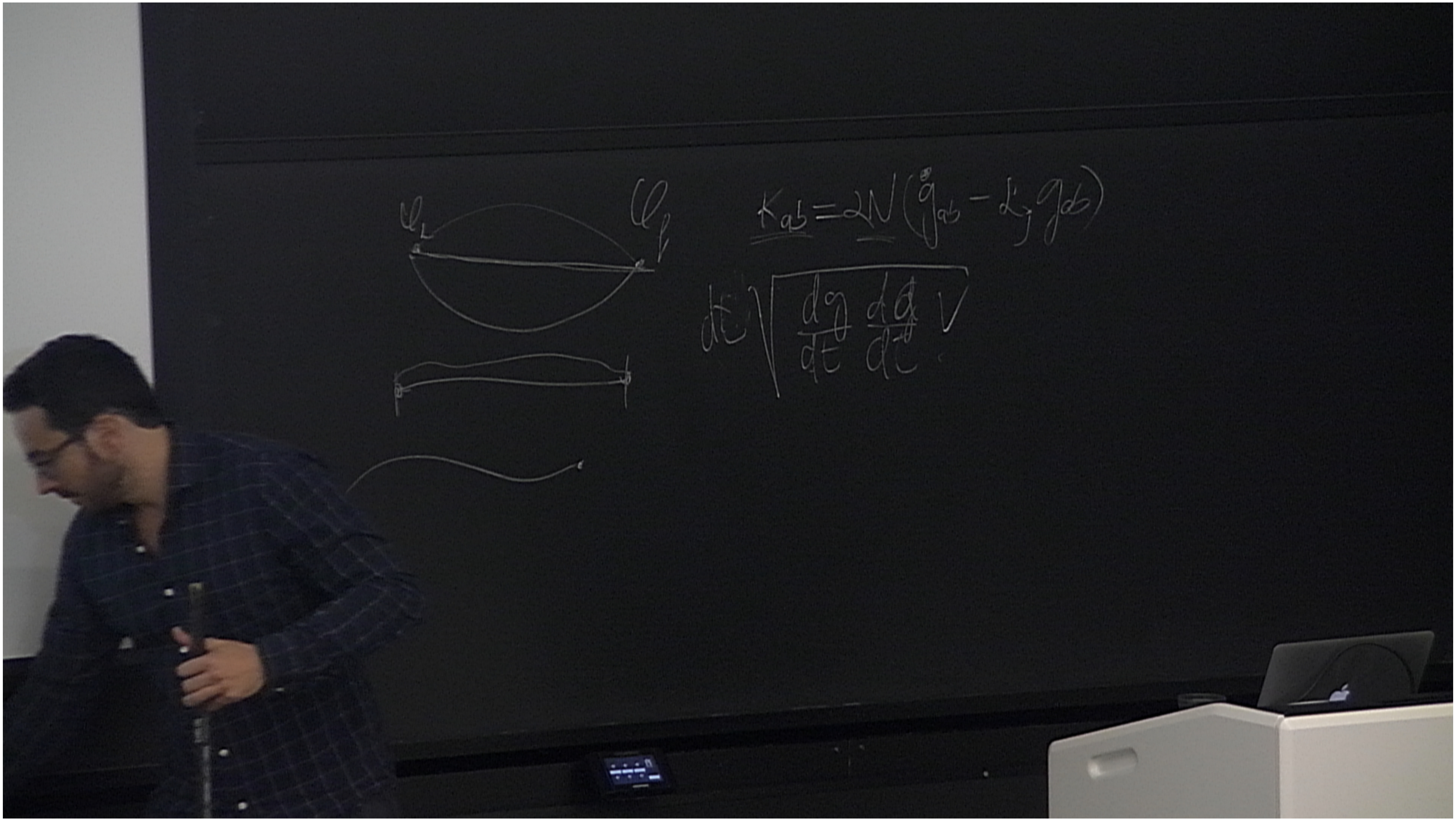
Equivalently, need to stipulate what measures time for what, making gravity parallel to relational QM.





Two speculative proposals

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Ideally, measure time explicitly, by evolution of dofs in Riem$/\mathcal{G}$. But too complex. Need a substitute for proper time of GR.

Assume $F[\phi, \dot{\phi}; x)dt$ measures local change, with F such that the duration doesn't depend on different global reparametrization of t. Example, local contribution to action: $F(\phi, \dot{\phi}) = \sqrt{TV}$.

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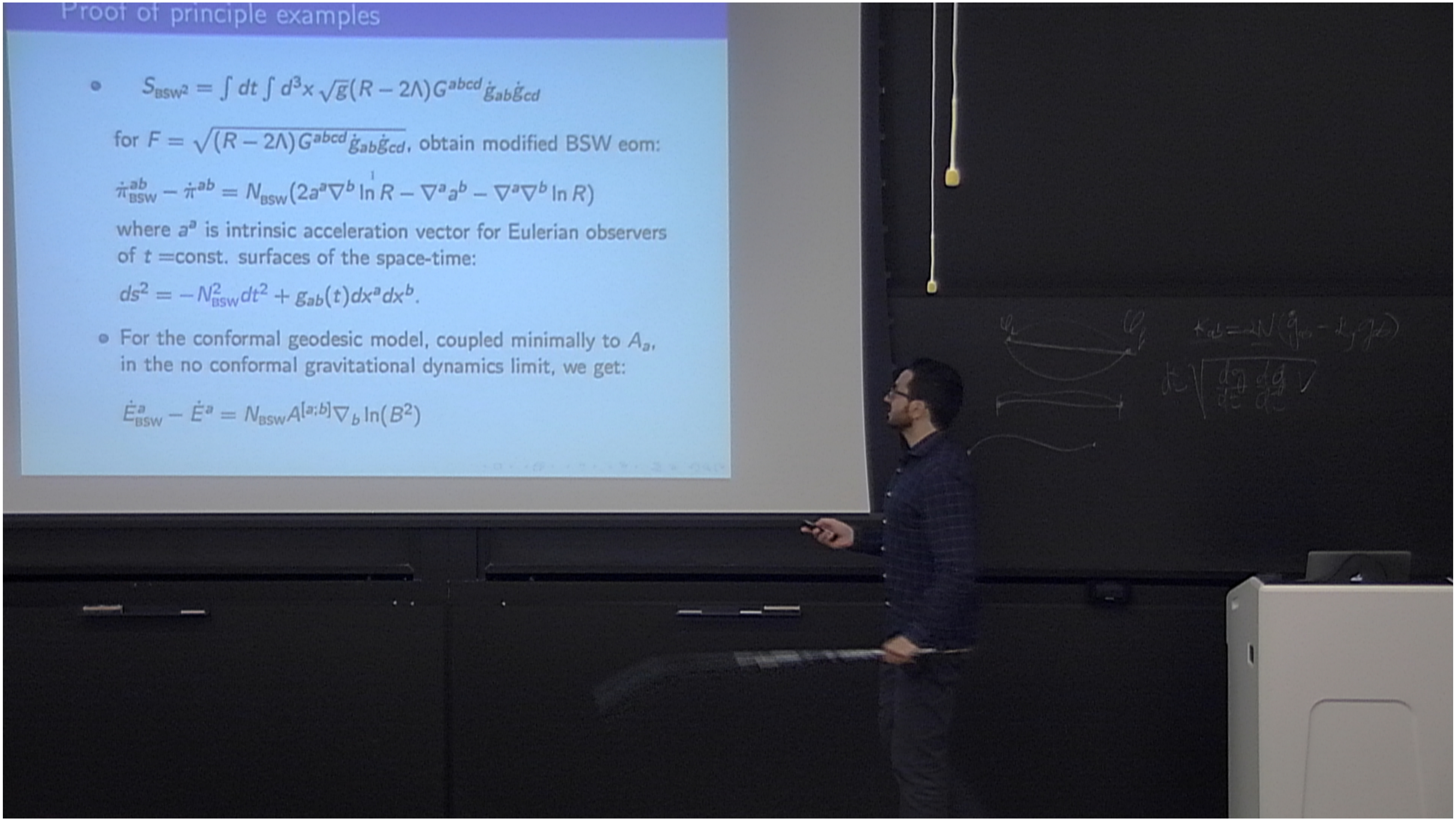
Then, two tentative ways to implement 'relational time':

- Replace $dt \to F\,dt$, in the eom of the theory.
- Replace $dt \to F\,dt$, in the action of the theory (changes it).


Proof of principle examples
$S_{\mathrm{BSW}^2} = \int dt \int d^3x \sqrt{g}(R-2\Lambda)G^{abcd}\dot{g}_{ab}\dot{g}_{cd}$
for $F = \sqrt{(R-2\Lambda)G^{abcd}\dot{g}_{ab}\dot{g}_{cd}}$, obtain modified BSW eom:
$\dot{\pi}^{ab}_{\mathrm{BSW}} - \dot{\pi}^{ab} = N_{\mathrm{BSW}}(2a^a\nabla^b \ln R - \nabla^a a^b - \nabla^a\nabla^b \ln R)$
where a^a is intrinsic acceleration vector for Eulerian observers of t =const. surfaces of the space-time:
$ds^2 = -N^2_{\mathrm{BSW}}dt^2 + g_{ab}(t)dx^a dx^b$.
For the conformal geodesic model, coupled minimally to A_a, in the no conformal gravitational dynamics limit, we get:
$\dot{E}^a_{\mathrm{BSW}} - \dot{E}^a = N_{\mathrm{BSW}}A^{[a;b]}\nabla_b \ln(B^2)$

New principles for modifying gravity

If we replace F in the BSW^2 action, we get back BSW.

Although I don't understand what any of this means, I believe these theories can represent a new way of modifying gravity.

More principled, and order of magnitudes simpler than modifications in the literature. E.g. Horndeski theories.

Moreover, it's easier to keep:

- Correct gravitational dofs.
- Explicit unitarity.
- Renormalization...?

And as a last resort, we could drop locality of the Lagrangian. Then we get arc-length-parametrized shape dynamics.[Koslowski '16]

Outline

1. Introduction
2. Brief intro to quantum mechanics without Time
3. Gravitational models prior to space-time
4. Relation to GR
5. Conclusions

Conclusions

- The quantum superposition principle and space-time causality are not good together.
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- The missing ingredient is a general relational replacement for proper time. In examples, we showed that in principle the emergent models can approximate GR and EM.