Title: Advances in quantum query complexity

Date: Oct 26, 2016 04:00 PM

URL: http://pirsa.org/16100066

Abstract: <p>I will describe some of the recent progress in quantum query complexity, including super-quadratic separations between classical and quantum measures for total functions, a better understanding of the power of some lower bound techniques, and insight into when we should expect exponential quantum speedups for partial functions.</p>

Advances in Quantum Query Complexity

Shalev Ben-David

Query Complexity

- f = OR

	0		1	0			

- We care about the worst case
- D(f) = n deterministic queries
- R(f) = Ω(n) random queries (bounded error)
- Q(f) = $\Theta(n^{1/2})$ quantum queries (bounded error)

Query Complexity

- f = OR

	0		1	0			

- We care about the worst case
- D(f) = n deterministic queries
- R(f) = $\Omega(n)$ random queries (bounded error)
- Q(f) = $\Theta(n^{1/2})$ quantum queries (bounded error)
- We can prove these!

Questions in Query Complexity

- How much do resources like randomness and quantumness help?
  - Is there a function that can be solved with very few randomized queries, but which require a lot of deterministic queries?
- When do these resources help?
  - What structure must the functions have?
- What are good lower bound techniques for these models?

Part 1: How Much Speedup?

Partial Functions: Quantum vs Randomized

- [Simon '94]: Function f with

  $Q(f) \approx \log^2 n \qquad R(f) \approx n^{1/2}$

- [Aaronson, Ambainis '14]:
  - "Forrelation" with

    $Q(f) \approx 1 \qquad R(f) \approx n^{1/2}$

  - Give a candidate function for

    $Q(f) \approx \log n \qquad R(f) \approx n$

Partial Functions: Quantum vs Randomized

- [Simon '94]: Function f with

  $Q(f) \approx \log^2 n$ $R(f) \approx n^{1/2}$

- [Aaronson, Ambainis '14]:
  - "Forrelation" with

    $Q(f) \approx 1$ $R(f) \approx n^{1/2}$
  - Give a candidate function for

    $Q(f) \approx \log n$ $R(f) \approx n$
  - Show that 1 vs. n gap is impossible

Separations in 2015

- April 4 (Göös, Pitassi, Watson):
  - Introduced the idea of pointer functions
  - Quadratic separation between D(f) and deg(f)
- June 16 (Ambainis, Balodis, Belovs, Lee, Santha, Smotrovs):
  - Quadratic separation between D(f) and R(f)
  - Power 4 separation between D(f) and Q(f)
  - Many other separations, involving $R_0(f)$ and $Q_E(f)$
- June 26 (B.):
  - Power 2.5 separation between R(f) and Q(f)
  - Introduced cheat sheets

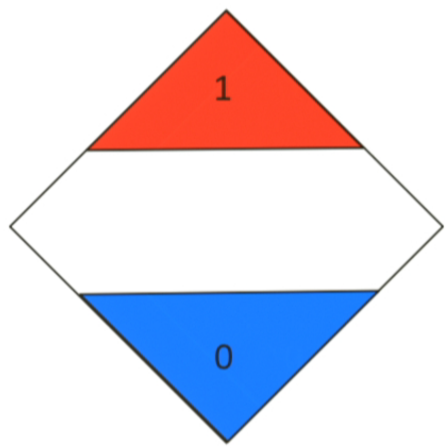
Separations in 2015

- April 4 (Göös, Pitassi, Watson):
  - Introduced the idea of pointer functions
  - Quadratic separation between D(f) and deg(f)
- June 16 (Ambainis, Balodis, Belovs, Lee, Santha, Smotrovs):
  - Quadratic separation between D(f) and R(f)
  - Power 4 separation between D(f) and Q(f)
  - Many other separations, involving $R_0(f)$ and $Q_E(f)$
- June 26 (B.):
  - Power 2.5 separation between R(f) and Q(f)
  - Introduced cheat sheets
- Nov 5 (Aaronson, B., Kothari):
  - Used cheat sheets to reprove many of the other separations
  - Power 4-o(1) separation between Q(f) and approximate degree

A Super-Grover Speedup

Turning partial functions total

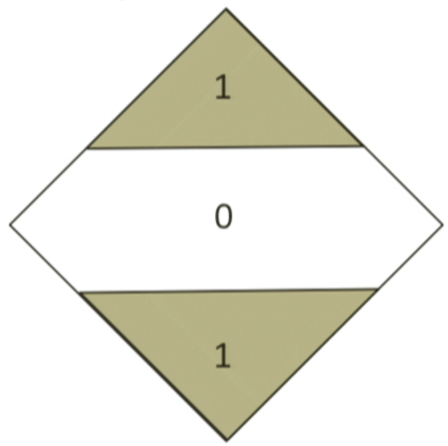
- Given a partial function f that has a good separation, how can we turn it total?
- For concreteness, set f to be f(x) = 1 if x is 2/3 ones, 0 if x is 2/3 0s ("two-thirds")



Turning partial functions total

- The problem is that the *promise* is difficult for a randomized algorithm to calculate

$$p_f(x) = \begin{cases} 1 & if\ x \in Dom(f) \\ 0 & otherwise \end{cases}$$



Cheat sheet step 1: Compose with AND-OR

- AND-OR: Is there an all-1 column?

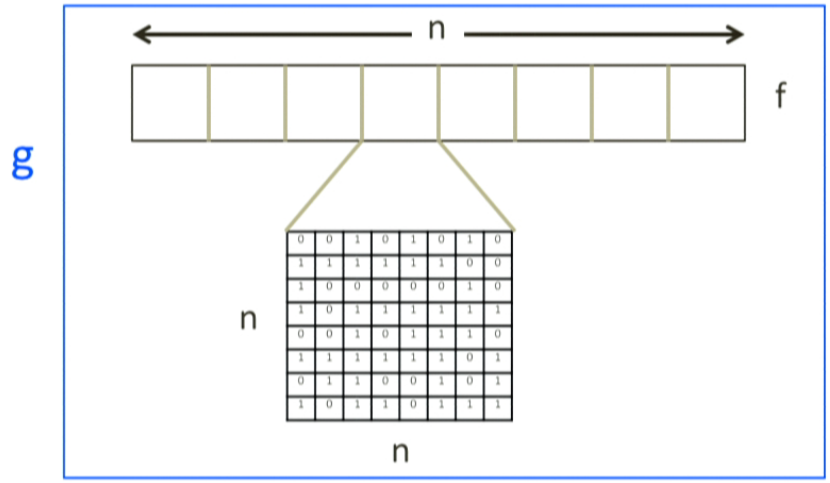
0	0	1	0	1	0	1	0
1	1	1	1	1	1	0	0
1	0	1	0	0	0	1	0
1	0	1	1	1	1	1	1
0	0	1	0	1	1	1	0
1	1	1	1	1	1	0	1
0	1	1	0	0	1	0	1
1	0	0	1	0	1	1	1

Cheat sheet step 1: Compose with AND-OR

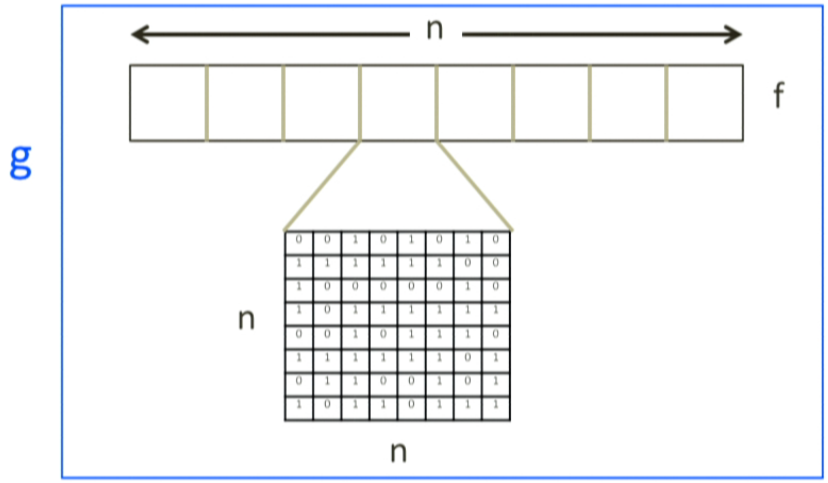
- AND-OR: Is there an all-1 column?

0	0	1	0	1	0	1	0
1	1	1	1	1	1	0	0
1	0	1	0	0	0	1	0
1	0	1	1	1	1	1	1
0	0	1	0	1	1	1	0
1	1	1	1	1	1	0	1
0	1	1	0	0	1	0	1
1	0	0	1	0	1	1	1

Cheat sheet step 1: Compose with AND-OR

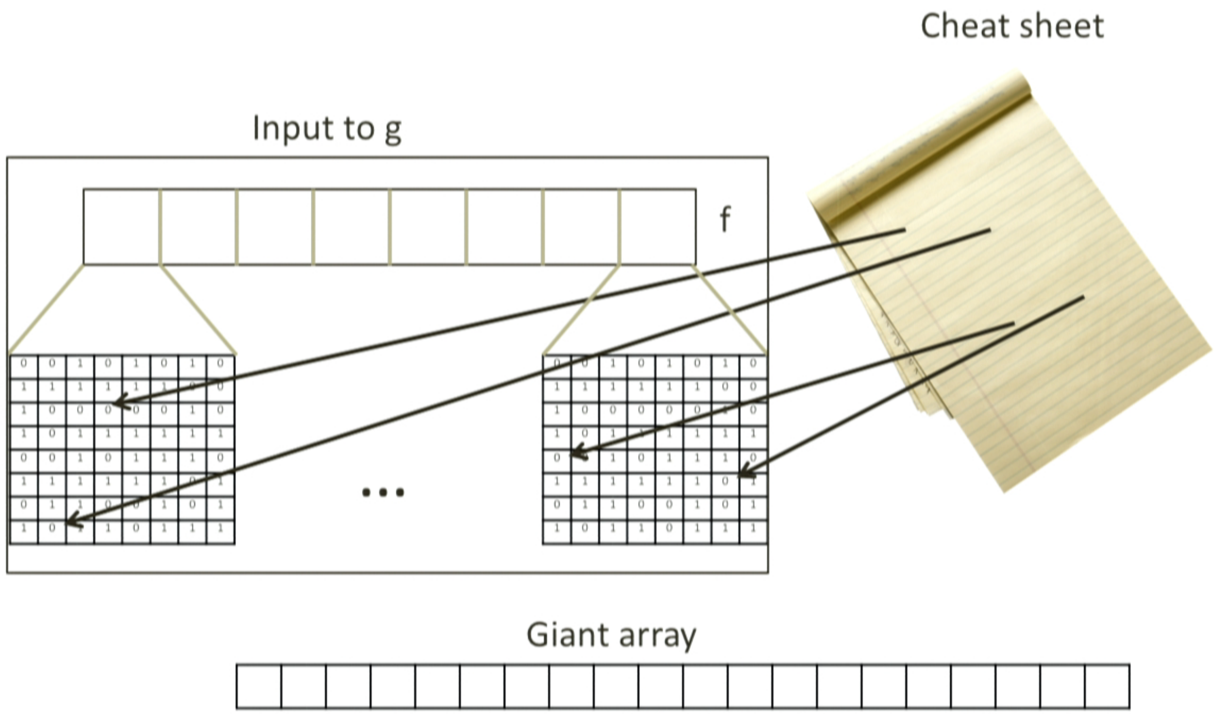


Cheat sheet step 1: Compose with AND-OR

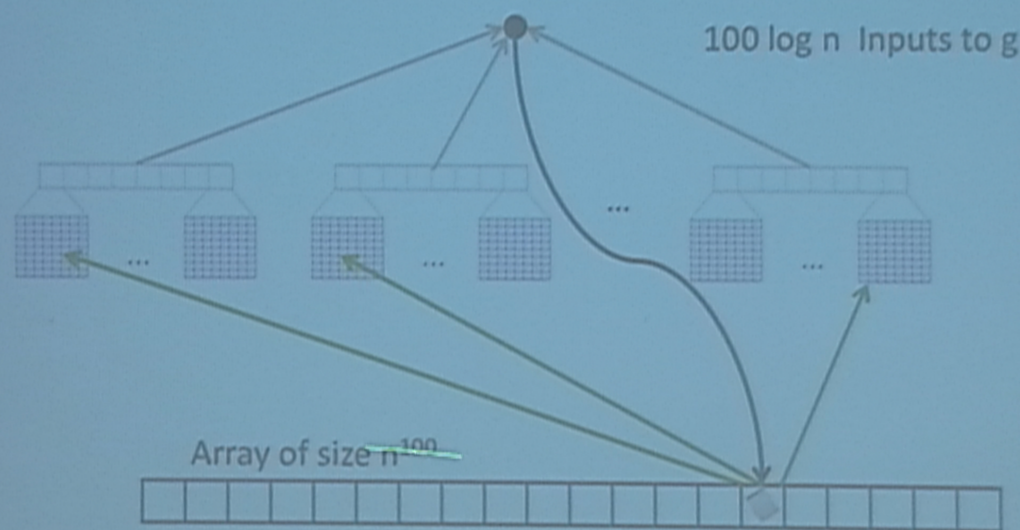


- If f = Forrelation:
  - $R(g) \approx n^{2.5}$
  - $Q(g) \approx n$

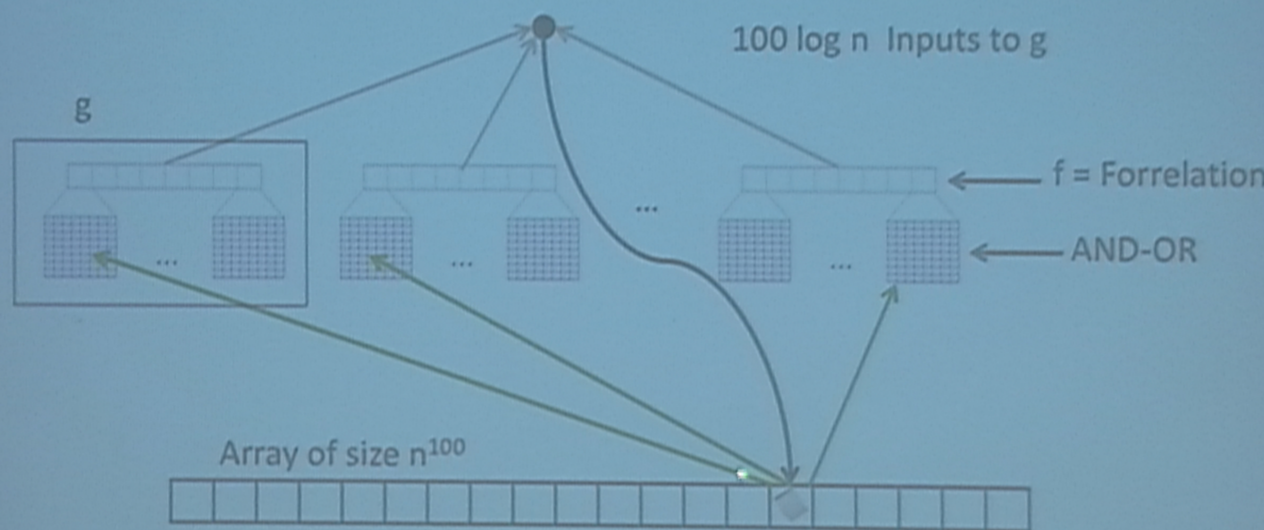
Step 2: hide a cheat sheet



Step 3: Find the cheat sheet



Cheat sheet framework



More Complexity Measures

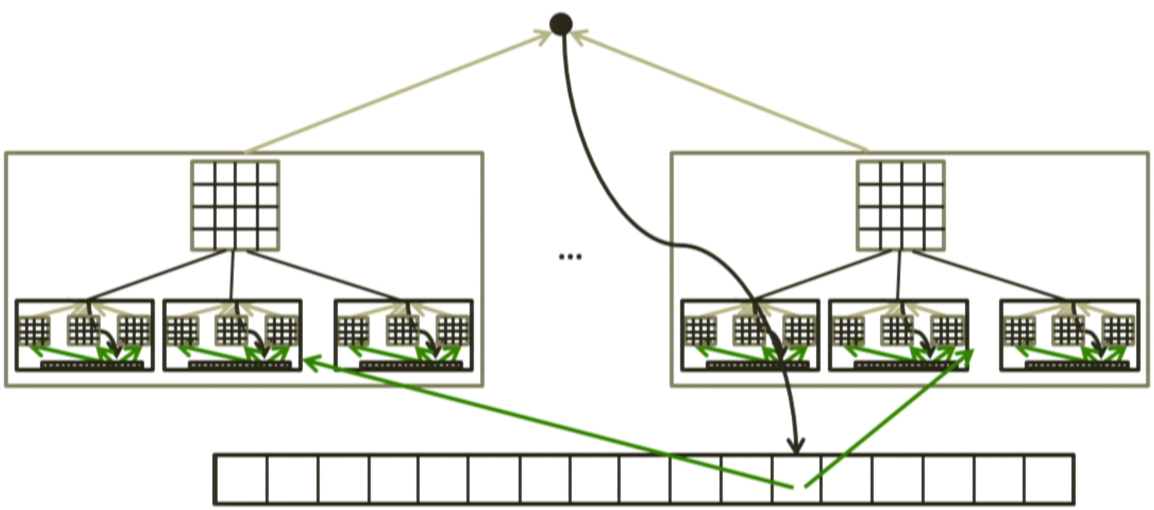
	D	R_0	R	C	RC	bs	Q_E	deg	Q	$\widetilde{\deg}$
D		2, 2 [ABB+15]	2*, 3 [ABB+15]	2, 2 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2, 3 [ABB+15]	2, 3 [GPW15]	4*, 6 [ABB+15]	4*, 6 [ABB+15]
R_0	1, 1 $\oplus$		2, 2 [ABB+15]	2, 2 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2, 3 [ABB+15]	2, 3 [GJPW15]	3, 6 [ABB+15]	4*, 6 [ABB+15]
R	1, 1 $\oplus$	1, 1 $\oplus$		2, 2 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	1.5, 3 [ABB+15]	2, 3 [GJPW15]	2.5, 6 Th. 1	4*, 6 [ABB+15]
C	1, 1 $\oplus$	1, 1 $\oplus$	1, 2 $\oplus$		2, 2 [GSS13]	2, 2 [GSS13]	1.1527, 3 [Amb13]	$\log_3 6$, 3 [NW95]	2, 4 $\wedge$	2, 4 $\wedge$
RC	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$		1.5, 2 [GSS13]	1.1527, 3 [Amb13]	$\log_3 6$, 3 [NW95]	2, 2 $\wedge$	2, 2 $\wedge$
bs	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$		1.1527, 3 [Amb13]	$\log_3 6$, 3 [NW95]	2, 2 $\wedge$	2, 2 $\wedge$
Q_E	1, 1 $\oplus$	1.3267, 2 $\bar{\wedge}$-tree	1.3267, 3 $\bar{\wedge}$-tree	2, 2 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$		2, 3 Th. 4	2, 6 $\wedge$	4*, 6 Th. 2
deg	1, 1 $\oplus$	1.3267, 2 $\bar{\wedge}$-tree	1.3267, 3 $\bar{\wedge}$-tree	2, 2 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	2*, 3 $\wedge \circ \vee$	1, 1 $\oplus$		2, 6 $\wedge$	2, 6 $\wedge$
Q	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	2, 2 Th. 3	2*, 3 Th. 3	2*, 3 Th. 3	1, 1 $\oplus$	2, 3 Th. 4		4*, 6 Th. 2
$\widetilde{\deg}$	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	7/6, 2 $\wedge \circ$ ED	7/6, 3 $\wedge \circ$ ED	7/6, 3 $\wedge \circ$ ED	1, 1 $\oplus$	1, 1 $\oplus$	1, 1 $\oplus$	

New separations

Separations we reprove

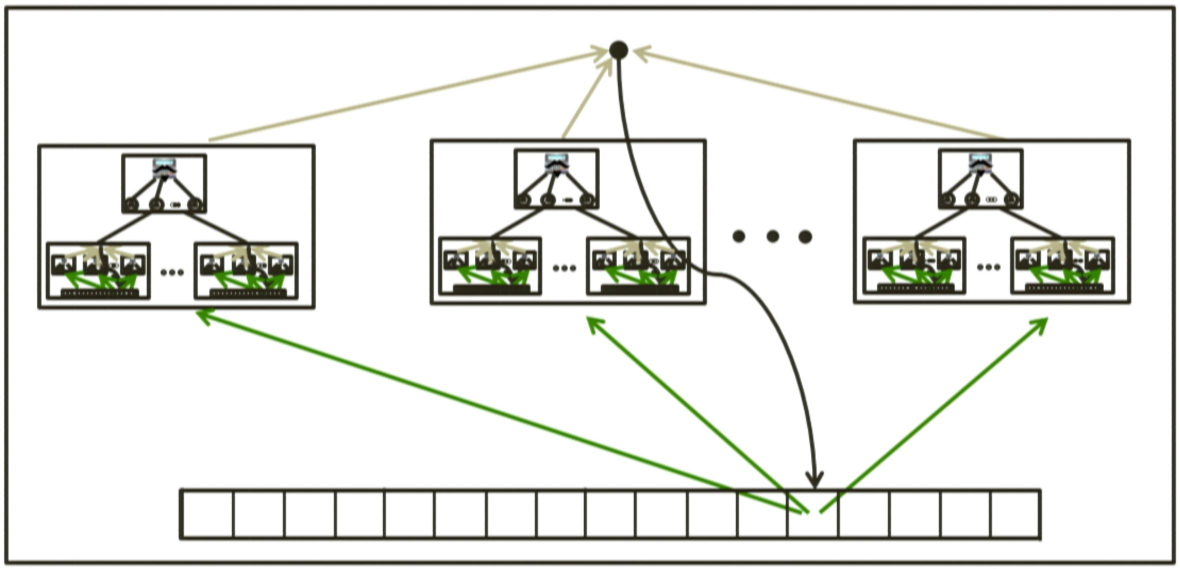
Ambainis, Kokainis, Kothari (Dec 3)

$D \approx UC^{2-o(1)}$



Ambainis, Kokainis, Kothari (Dec 3)

$Q \approx UC^{1.5-o(1)}$



Cheat Sheets in Communication Complexity

- Anshu, Belovs, B., Göös, Jain, Kothari, Lee, Santha
  - Get the 2.5 separation in communication complexity
  - Also get a power 2-o(1) separation between R and partition number
  - Main contribution: showing the lower bound for R

- Anshu, B., Garg, Jain, Kothari, Lee (coming soon)
  - Prove a "cheat sheet theorem" for quantum communication
  - Get a separation between quantum communication and approximate logrank

Cheat Sheets in Communication Complexity

- Anshu, Belovs, B., Göös, Jain, Kothari, Lee, Santha
  - Get the 2.5 separation in communication complexity
  - Also get a power 2-o(1) separation between R and partition number
  - <u>Main contribution</u>: showing the lower bound for R

- Anshu, B., Garg, Jain, Kothari, Lee (coming soon)
  - Prove a "cheat sheet theorem" for quantum communication
  - Get a separation between quantum communication and approximate logrank
  - No previous super-linear separation was known

Can we get better lower bound techniques?

- If the lower bound techniques we have all break, can we hope for better techniques?
- For quantum query complexity, there is a tight technique

Randomized Lower Bounds

- If there are no techniques, how do we prove the cheat sheet lower bounds?
- Answer: we have specialized theorems that help lower bound functions built from other functions
- Other answer: cheat sheet lower bounds were annoying and ad hoc

- B., Kothari 2016: new lower bound technique (“Sabotage Complexity”) that makes some things a bit easier

Part 2: When are exponential speedups possible?

B. 2012

- For some concrete types of promises, there cannot be an exponential quantum speedup for any function
- Permutation promise: “the input is a permutation of {1,2,…,n}”

Aaronson, B. 2015

- "You can usually sculpt speedups"
  - If you fix a function in advance and get to choose a promise afterwards
  - Then for most functions, you can get an exponential quantum speedup
  - Exact characterization ("H-index")
  - "Quantum speedups are all about the promise"

Open Problems

- Pretty much everything is still open
- What is the largest possible quantum speedup for a total function? Is it 2.5? 3? 6?
- What is the largest possible quantum speedup for a partial function? Is log n vs. n possible?

Open Problems

- Pretty much everything is still open
- What is the largest possible quantum speedup for a total function? Is it 2.5? 3? 6?
- What is the largest possible quantum speedup for a partial function? Is log n vs. n possible?
- What are some other promises on which there are or aren't exponential quantum speedups? Can we get a complete characterization?

Open Problems

- Pretty much everything is still open
- What is the largest possible quantum speedup for a total function? Is it 2.5? 3? 6?
- What is the largest possible quantum speedup for a partial function? Is log n vs. n possible?
- What are some other promises on which there are or aren’t exponential quantum speedups? Can we get a complete characterization?
- What are some better lower bound techniques for randomized algorithms?