Title: The Space of Quantum Field Theories

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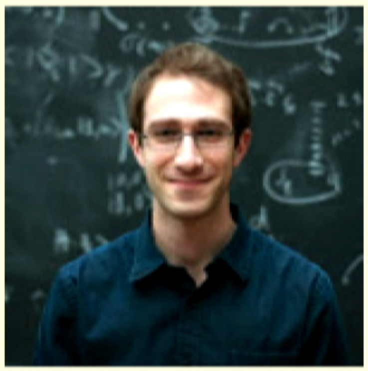
Abstract: <p>Recent explorations of the space of quantum field theories have provided novel topological and geometric information about this space. This voyage has resulted in the solution of some long-standing questions: the computation of sought-after topological invariants (Gromov-Witten invariants) by a new physics-based approach, the first instance of exact correlation functions in a four-dimensional QFT and the unearthing of the action of dualities on the basic observables of three dimensional gauge theories.</p>

- Based on work with Doroud, Le Floch, Lee, Gerchkovitz, Komargodski, Ishtiaque, Hsin, Schwimmer, Seiberg, Theisen, Karasik, Pufu

- I would like to highlight my students


Nima Doroud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Introduction

- A remarkable wealth of phenomena is captured by quantum field theory
  - Particle Physics
  - Statistical Mechanics
  - Quantum Many Body
  - Cosmology
- QFT has provided remarkable insights into mathematics
- Ideas in quantum information have yielded new perspectives on QFT
- Posterchild the universality and unity of Physics

$$\text{quantum theory} + \text{relativity} \longrightarrow \text{QFT}$$

- There are many seemingly distinct QFTs

  - Are they related?

$$\mathrm{QFT}_1 \ \rightarrow \ \mathrm{QFT}_2 ?$$

$\Longrightarrow$

explore the space of QFTs

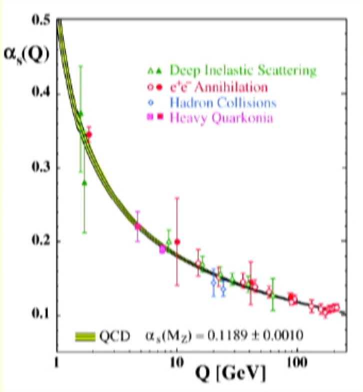
$$\mathcal{M}$$

- Given a QFT we explore $\mathcal{M}$ by deforming with local operators $\mathcal{O}_i$

$$Z[\lambda^i] = \langle e^{\int dx \sum_i \lambda^i \mathcal{O}_i(x)} \rangle$$

  - $\mathcal{O}_i$ are tangent vectors in $T\mathcal{M}$
  - λ^i are coordinates of $\mathcal{M}$

- The degrees of freedom and dynamics of a physical system depend on the energy scale at which it is probed



- This is formalized by the renormalization group equations

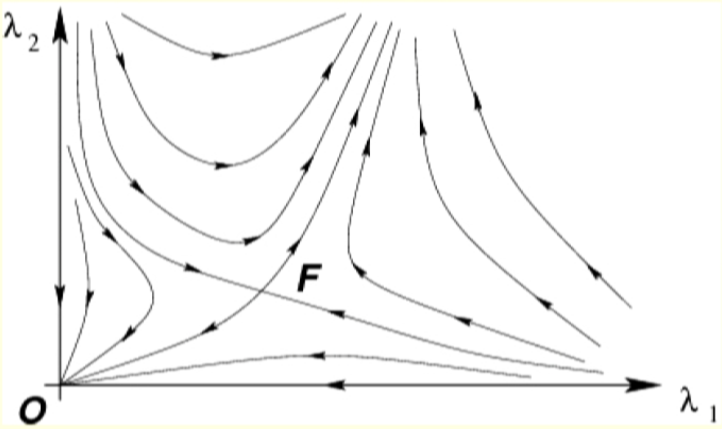
• The renormalization group equation encodes the dependence of physical quantities on the scale being probed

$$\mu\frac{\partial}{\partial\mu}\log Z[\lambda^i] + \sum_j \beta_{\lambda^j}(\lambda^i)\frac{\partial}{\partial\lambda^j}\log Z[\lambda^i] = 0$$

where

$$\mu\frac{\partial}{\partial\mu}\lambda^j \equiv \beta_{\lambda^j}(\lambda^i)$$

β_{λ^j} are vector fields in $\mathcal{M}$ controlling the RG flow: UV $\rightarrow$ IR



- Operators characterized by their effect on long distance physics
  - irrelevant: $\beta > 0$
  - relevant: $\beta < 0$
  - marginal: $\beta = 0$

Universality

- The long distance, infrared limit of any QFT described either by:

  1. Topological QFT (topological phases of matter)

  2. Scale Invariant QFT $\rightarrow$ a conformal field theory (CFT)

Invariant under scale transformations:

$$x^\mu \rightarrow \alpha\, x^\mu$$

Symmetry in the IR is enhanced

$$\text{Poincare}_d \rightarrow \text{Poincare}_d \rtimes R$$

Under reasonable conditions further enhanced to conformal symmetry

$$\text{Poincare}_d \rightarrow SO(2,d)$$

CFT

- A local QFT has a conserved energy-momentum $T_{\mu\nu}$ tensor

$$\partial^\mu T_{\mu\nu} = 0$$

- A CFT is invariant under angle-preserving coordinate transformations

$$ds^2_{\mathbb{R}^d} \to e^{2\sigma(x)} ds^2_{\mathbb{R}^d}$$

$\Longrightarrow$

$$T^\mu_\mu = 0$$

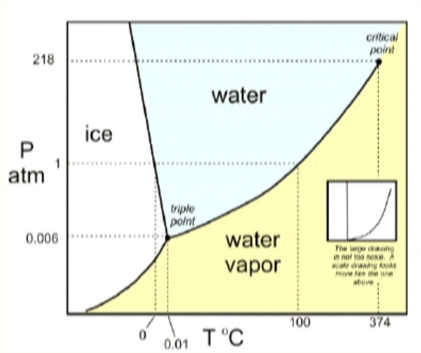
- Operators/states in a CFT organized in conformal families

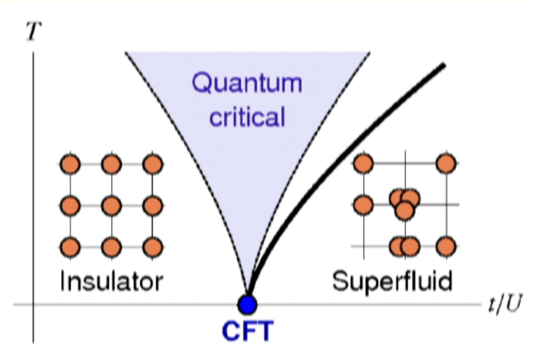
$$\text{primary operator}: \ \mathcal{O} \to e^{-\Delta\sigma(x)}\mathcal{O}$$

- Observables are the correlation functions of primaries

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \ldots \mathcal{O}_k(x_k) \rangle$$

- CFTs are directly relevant for nature, e.g:
  - Critical Phenomena (thermal fluctuations)
  - Quantum Phase Transitions (quantum fluctuations)





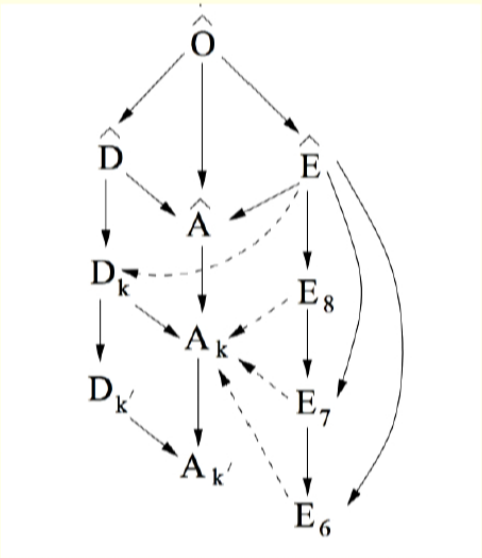
Foundational in other domains:

- Candidate for physics beyond the standard model
- String Theory
- Quantum Gravity with Anti-de-Sitter asymptopia
- Mathematics

- CFTs play a foundational role in QFT

  a QFT obtained as an RG-flow triggered by an operator in a CFT

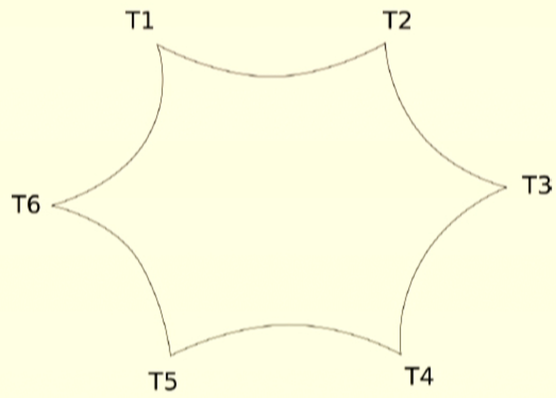
- CFTs induce an ordering in the space of all QFTs



RG flow is irreversible

$\mathcal{M}_{\text{CFT}}$

- Only coarse features of the space of QFTs are known
- Study the space of CFTs $\mathcal{M}_{\text{CFT}} \subset \mathcal{M}$ generated by marginal operators



- Alternative descriptions of the same quantum theory
- Operators transported around $\mathcal{M}_{\text{CFT}}$ acquire a Berry phase
- Physical quantities depend nontrivially on λ^I

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \ldots \mathcal{O}_k(x_k) \rangle_{\{\lambda\}}$$

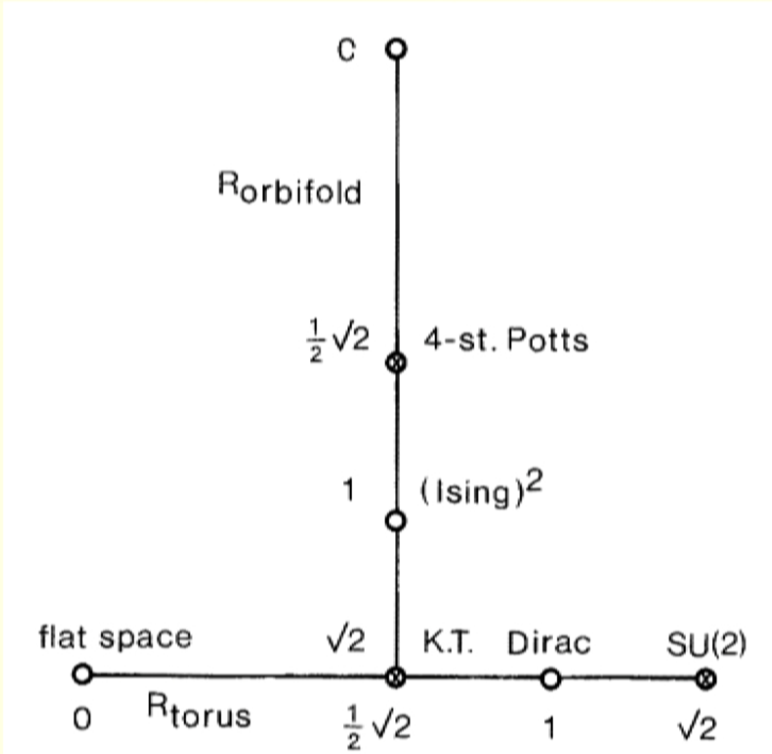
- Quantum stability of marginal operators $\{\mathcal{O}_{\mathcal{I}}\}$ is non-trivial, e.g.

$$\langle \mathcal{O}_{\mathcal{I}}(0)\mathcal{O}_{\mathcal{J}}(1)\mathcal{O}_{\mathcal{K}}(\infty)\rangle = 0$$

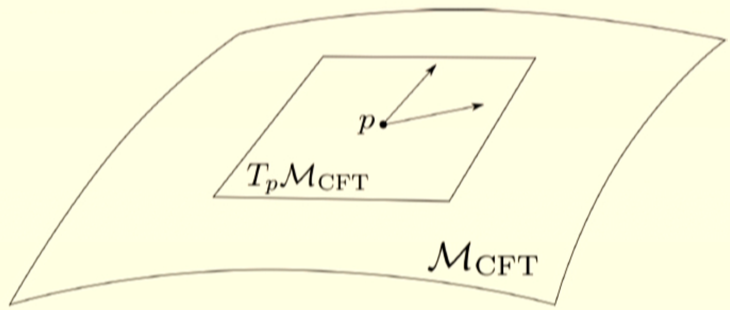
Stability typically requires:

- extra symmetries (current algebra, supersymmetry,...)
- large N

- Physically relevant example (Ashkin-Teller, XY-model,...)



- $\mathcal{M}_{\rm CFT}$ is a Riemannian manifold



- $\mathcal{M}_{\rm CFT}$ endowed with a canonical metric: Zamolodchikov metric

$$\langle \mathcal{O}_I(x)\mathcal{O}_J(x)\rangle_{\{\lambda^I\}} = \frac{\gamma_{IJ}(\lambda^K)}{x^{2d}}$$

  - Nontrivial function of the couplings λ^I
  - Riemann curvature on $\mathcal{M}_{\rm CFT}$ computed by a four-point function of $\mathcal{O}_I$'s

String Theory and Conformal Invariance

- String Theory vacua described by 2d CFTs
- Conformal Invariance on the string worldsheet $\Longrightarrow$ Spacetime physics

$$\mathcal{L} = \sqrt{g}\left[R + \mathrm{Tr}F_{\mu\nu}^2 + \bar{\Psi}\not{D}\Psi + \ldots\right]$$

- Worldsheet theory described by a nonlinear sigma model

$$S = \int_\Sigma d^2x\, G_{\alpha\beta}\partial\phi^\alpha\bar{\partial}\phi^\beta$$

where

$$\phi : \Sigma \longrightarrow X$$

- NLSM path integral subject to important non-perturbative corrections

$$\text{worldsheet instantons :} \quad \bar{\partial}\phi^\alpha = 0$$

- Deep and rich interplay between spacetime and worldsheet physics
- Consider vacua of the type $\mathbb{R}^{1,3} \times M$

1. massless fields $\longleftrightarrow$ marginal operators in the 2d CFT

$$\Phi_I \longleftrightarrow \mathcal{O}_I$$

2. spacetime interactions $\longleftrightarrow$ correlators in the 2d CFT

$$\int d^4x \sqrt{g}\, \gamma_{IJ}\, \partial\Phi^I \cdot \partial\Phi^J \longleftrightarrow \gamma_{IJ} = \langle \mathcal{O}_I(1)\mathcal{O}_J(0)\rangle$$

$\Longrightarrow$

spacetime action requires knowing the Zamolodchikov metric of 2d CFT!

- Physically and mathematically appealing vacua

$$M \text{ is a Calabi-Yau manifold}$$

$$\Updownarrow$$

2d CFT has extra symmetry: superconformal field theory (SCFT)

$\Longrightarrow$

$\mathcal{M}_{\rm CFT}$ is a Kähler manifold

$$\gamma_{I\bar{J}} = \partial_I \partial_{\bar{J}} \mathcal{K}$$

where

$$\mathcal{K} \text{ is the Kähler potential}$$

$\mathcal{K}$ determines kinetic terms, Yukawa couplings,... in spacetime

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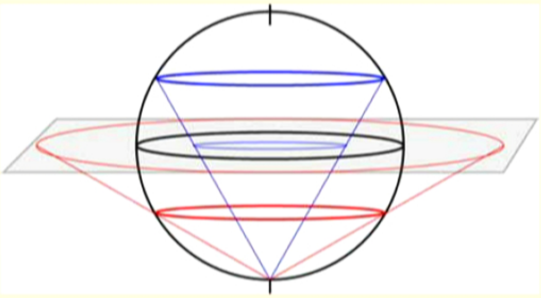
RG Approach to $\mathcal{K}$

1. Perform Weyl transformation and place CFT on a two-sphere

$$ds^2_{S^2} = e^{2\sigma(x)} ds^2_{\mathbb{R}^2} = \left(\frac{1}{1+\frac{x^2}{4r^2}} \right) ds^2_{\mathbb{R}^2}$$



2. Promote coupling constants of CFT to background fields

$$\lambda^I \to \lambda^I(x)$$

$\Longrightarrow$

this leads to a conformal anomaly

Anomalies

- An anomaly is quantum mechanical violation of a classical symmetry

$$\partial^\mu j_\mu = O(\hbar)$$

  - Anomalous violation of Ward identities

$$\delta_\sigma \ln Z = \int dx\, \mathcal{A} \neq 0$$

  - Play a fundamental role in QFT
    - restrict the space of consistent (unitary) QFTs
    - constrain RG fows (anomaly matching)
    - play an important role in observed explaining physical phenomena

Conformal Anomaly

- The background fields $g_{\mu\nu}$ and λ^I induce a conformal anomaly

$$\delta_\sigma \log Z[g_{\mu\nu}, \lambda] = \frac{c}{24\pi} \int d^2x \sqrt{g}\, R\, \delta\sigma - \frac{1}{4\pi} \int d^2x \sqrt{g}\, \gamma_{I\bar{J}} \partial\lambda^I \cdot \partial\bar{\lambda}^{\bar{J}} \delta\sigma + \ldots$$

where

- c is "central charge" of the CFT
  - measures various universal quantities of the CFT
  - determines the entanglement entropy of the CFT
- $\gamma_{I\bar{J}} = \partial_I \partial_{\bar{J}} \mathcal{K}$ is the metric on $\mathcal{M}_{\text{CFT}}$

- Physical consistency implies $[\mathcal{K}] = 0 \implies$

$$\mathcal{M}_{\text{CFT}} \text{ is non-compact}$$

- Integrating the anomaly equation yields

see also Jockers, et al

$$Z[S^2] = r^{c/3} e^{-\mathcal{K}(\lambda,\bar{\lambda})}$$

- The partition function of SCFT on S^2 computes $\mathcal{K}$!
- We have given $Z[S^2]$ a physical meaning. Measures flat space correlators

but how we compute $Z[S^2]$?

Go with the RG flow

1. Embed CFT in the renormalization group flow of a gauge theory in the UV

UV	gauge theory
	$\downarrow$
IR	CFT

Denote the sphere partition function of gauge theory by

$$Z[S^2]_{UV}$$

2. Proof that $Z[S^2]_{UV}$ is a renormalization group invariant observable

$$\frac{\partial}{\partial \mu} Z[S^2]_{UV} = 0$$

$\Longrightarrow$

$$Z[S^2] = Z[S^2]_{UV}$$

3. Compute $Z[S^2]_{UV}$ exactly. Saddle point approximation is exact!

$Z[S^2]$ as an exact function of the couplings spanning $\mathcal{M}_{\mathrm{CFT}}$

- $Z[S^2]$ admits a contour integral representation see also Benini, Cremonesi

$$Z[S^2] = \sum_B \int_{\mathfrak{t}} da\, e^{-4\pi i r \xi a + i\vartheta B} \prod_{\alpha\in\Delta} \left[\left(\frac{\alpha\cdot B}{2r}\right)^2 + (\alpha\cdot a)^2 \right] \prod_{w\in\mathbf{R}} \frac{\Gamma\left(-ir(w\cdot a) - \frac{w\cdot B}{2}\right)}{\Gamma\left(1 + ir(w\cdot a) - \frac{w\cdot B}{2}\right)}$$

  - determines exact metric in $\mathcal{M}_{\rm CFT}$
  - by a physics-based approach based on renomalization group flows
  - we also showed that

$$Z_X[S^2] = Z_Y[S^2]$$

  - this approach does not rely on mirror symmetry
  - leads to new topological invariants N_β

- Topological invariants

$\widetilde{N}_{m_0,m_1}$	$m_0=0$	$1/2$	1	$3/2$	2	$5/2$	3
$m_1=0$	–		56		0		0
$1/2$		192		896		192	
1	56		2 544		23 016		41 056
$3/2$		896		52 928		813 568	
2	0		23 016		1 680 576		35 857 016
$5/2$		192		813 568		66 781 440	
3	0		41 056		35 857 016		3 074 369 392
$7/2$		0		3 814 144		1 784 024 064	
4	0		23 016		284 749 056		96 591 652 016
$9/2$		0		6 292 096		20 090 433 088	
5	0		2 544		933 789 504		1 403 214 088 320
$11/2$		0		3 814 144		105 588 804 096	
6	0		56		1 371 704 192		10 388 138 826 968
$13/2$		0		813 568		277 465 693 248	
7	0		0		933 789 504		41 598 991 761 344
$15/2$		0		52 928		380 930 182 784	
8	0		0		284 749 056		93 976 769 192 864
$17/2$		0		896		277 465 693 248	
9	0		0		35 857 016		122 940 973 764 384
$19/2$		0		0		105 588 804 096	
10	0		0		1 680 576		93 976 769 192 864
$21/2$		0		0		20 090 433 088	
11	0		0		23 016		41 598 991 761 344
$23/2$		0		0		1 784 024 064	
12	0		0		0		10 388 138 826 968
$25/2$		0		0		66 781 440	
13	0		0		0		1 403 214 088 320
$27/2$		0		0		813 568	
14	0		0		0		96 591 652 016
$29/2$		0		0		192	
15	0		0		0		3 074 369 392
$31/2$		0		0		0	

arXiv:1208.6244

Correlators in 4d QFTs

- "Solving" a 4d QFT is currently out of reach
- No known exact correlation function of local operators in a 4d QFT

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \ldots \mathcal{O}_k(x_k) \rangle \qquad x_i \in \mathbb{R}^4$$

recently, this was accomplished in 4d CFTs with extra symmetries

$$\mathcal{N} = 2 \text{ SCFTs}: \quad T_{\mu\nu}, S^a_{\mu\alpha}, \bar{S}^a_{\mu\dot{\alpha}}$$

- supercurrents generate supersymmetry generators

$$Q^a_\alpha = \int dx\, S^a_{0\alpha} \qquad \bar{Q}^a_{\dot{\alpha}} = \int dx\, \bar{S}^a_{0\dot{\alpha}}$$

- these include supersymmetric gauge theories, e.g:

1. maximally supersymmetric Yang-Mills ($\mathcal{N} = 4$)
2. SQCD: $SU(N)$ with $N_F = 2N$

Extremal Correlators

- Study correlators in 4d $\mathcal{N}=2$ superconformal field theories

$$\left\langle \mathcal{O}_{I_1}(x_1)\mathcal{O}_{I_2}(x_2)\ldots\mathcal{O}_{I_n}(x_n)\overline{\mathcal{O}}_{\bar{J}}(y)\right\rangle_{\{\tau,\bar{\tau}\}}$$

where

$$[\overline{Q}^a_{\dot{\alpha}}, \mathcal{O}_I] = 0 \qquad\qquad [Q^a_{\alpha}, \overline{\mathcal{O}}_{\bar{J}}] = 0$$

- $\mathcal{O}_I$ generate a ring

$$\mathcal{O}_I(x)\mathcal{O}_J(0) = \mathcal{O}_I\mathcal{O}_J(0)$$

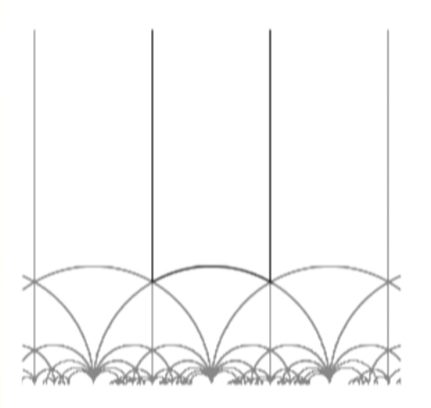
$\Longrightarrow$

correlators captured by

$$\left\langle \mathcal{O}_I(x)\overline{\mathcal{O}}_{\bar{J}}(0)\right\rangle_{\{\tau,\bar{\tau}\}} = \frac{G_{I\bar{J}}(\tau,\bar{\tau})}{|x|^{2\Delta_I}}\delta_{\Delta_I\Delta_{\bar{J}}}$$

- these correlators do not preserve Poincaré supersymmetries

- $G_{I\bar{J}}(\tau,\bar{\tau})$ when $\Delta_I = 2$ is the Zamolodchikov metric of $\mathcal{M}_{\rm CFT}$



$\{\tau\}$ are coordinates in $\mathcal{M}_{\rm CFT}$

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

- correlators afflicted with perturbative and non-perturbative corrections due to Yang-Mills instantons

can we compute $G_{I\bar{J}}(\tau,\bar{\tau})$?

Mixing anomaly

- Consider the partition function of the CFT on four-sphere S^4

$$Z[S^4] = \left\langle \exp \int dx\, d\theta \sqrt{g} \sum_I \tau^{(I)} \mathcal{O}_I(x) \right\rangle_{S^4}$$

- Since S^4 is conformal to $\mathbb{R}^4$, can $\mathbb{R}^4$ correlators be computed from $Z[S^4]$?
  - This naive expectation fails due to a novel "mixing anomaly" in S^4
  - $\mathcal{O}_I$ mixes with $\mathcal{O}_J$ on S^4 if $\Delta_I = \Delta_J + 2\mathbb{N}$. e.g:

$$\mathcal{O}_I \to \mathcal{O}_I + \alpha(\tau, \bar{\tau}) R\, \mathbb{1} \qquad\qquad \Delta_I = 2$$

$\implies$ resonance anomaly

$$\langle \mathcal{O}_I(N) \overline{\mathcal{O}}_{\bar{J}}(S) \rangle_{S^4} \neq 0$$

Extremal Correlators

- Goal is to determine

$$G_{2n}(\tau,\bar{\tau}) = \left\langle \mathcal{O}_n(\infty)\overline{\mathcal{O}}_n(0)\right\rangle_{\mathbb{R}^4}$$

where

$$\mathcal{O}_n = \left(\mathrm{Tr}(\phi^2)\right)^n$$

- By virtue of Ward identity

$$\langle \mathcal{O}_n(N)\overline{\mathcal{O}}_m(S)\rangle_{S^4} = \frac{1}{Z[S^4]}\partial_\tau^n \partial_{\bar{\tau}}^m Z[S^4]$$

- Gram-Schmidt $\Longrightarrow$ correlator written as ratio of determinants

$$G_{2n}(\tau,\bar{\tau}) = \frac{1}{Z[S^4]}\frac{\det_{(k,l)=0,\ldots,n}\left(\partial_\tau^k \partial_{\bar{\tau}}^l Z[S^4]\right)}{\det_{(k,l)=0,\ldots,n-1}\left(\partial_\tau^k \partial_{\bar{\tau}}^l Z[S^4]\right)}$$

$\Longrightarrow$ Connections with integrability

Toda Chain

- Extremal correlators acted by integrable differential equations: Toda chain

$$\partial_\tau \partial_{\bar\tau} q_n = e^{q_{n+1}-q_n} - e^{q_n - q_{n-1}} \qquad n = 1, 2, \cdots$$

where

$$q_n = \ln\left(G_{2n}\, Z[S^4]\right)$$

  - Correlators governed by a set of coupled oscillators q_n
  - Prescribed dependence of the left-most oscillator $q_0 = \ln\left(Z[S^4]\right)$
  - Extremal correlators computable from $Z[S^4]$

Comments

- Algorithm to compute extremal correlators
- Express observables in $\mathbb{R}^4$ in terms of a certain $Z[S^4]$

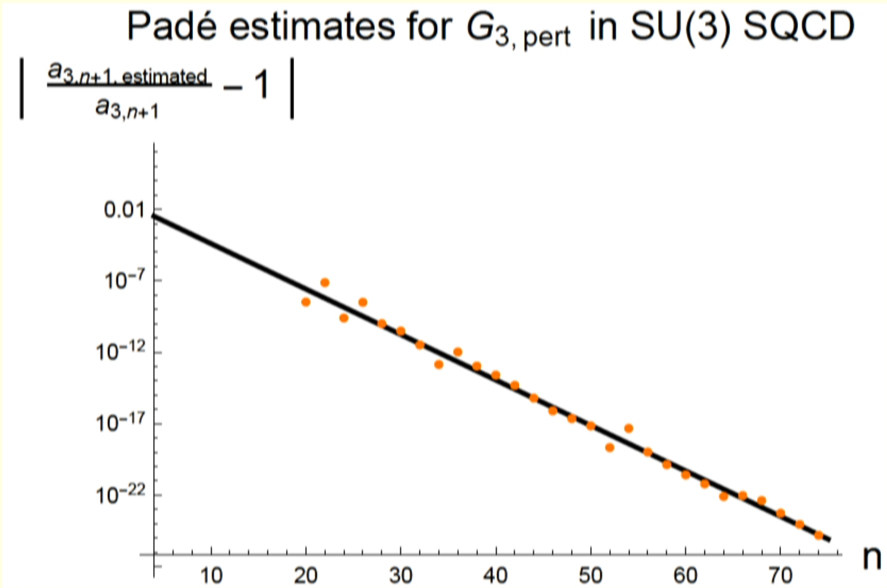
$\Longrightarrow$

Non-perturbative solution of QFT correlators in $d > 2$ at finite N

  - Connections with the conformal bootstrap program
  - Mathematical connections and predictions
  - Allow to test conjectures and ideas about large order behaviour in QFT

Resurgence

- Use our exact results to probe behaviours and conjectures in QFT, such as
  - Borel summability of the perturbative expansion
  - Convergence of the Pade approximation



Realizes a conjecture made about QCD perturbation theory Karliner

Conclusion

- The space of QFTs is vast and largely uncharted
- Anomalies, S^n and the RG-flow have given us topological and geometrical insights into $\mathcal{M}_{\text{CFT}}$
- These studies have shed new light on old questions in QFT and mathematics

- For an introduction to some of these topics, including

  - CFT

  - anomalies

$\Longrightarrow$

QFT3 course at PSI