Title: Conformal Blocks, Entanglement Entropy & Heavy States

Date: Oct 25, 2016 02:00 PM

URL: http://pirsa.org/16100062

Abstract: <p>We consider conformal blocks of two heavy operators and an arbitrary number of light operators in a 1+1 dimensional CFT with large central charge. Using the monodromy method, these higher-point conformal blocks are shown to factorize into products of 4-point conformal blocks in the heavy-light limit for a class of OPE channels. This result is reproduced by considering suitable worldline configurations in the bulk conical defect geometry. We apply the CFT results to calculate the entanglement entropy of an arbitrary number of disjoint intervals for heavy states. The corresponding holographic entanglement entropy calculated via the minimal area prescription precisely matches these results from CFT.</p>

Conformal Blocks, Entanglement Entropy & Heavy States

Pinaki Banerjee

The Institute of Mathematical Sciences, Chennai

October 25, 2016

The Broad Picture

- Holography has interesting universal features.
- Some particular conformal blocks in 2D CFT
  $\equiv$ geodesic lengths in asymptoticly AdS_3
- **Goal** : To show conformal blocks with two heavy & arbitrary number of light operators factorize & its dual bulk picture.
- **Application** : Relevant in the context of EE for excited states with multiple intervals.

The Plan..

- Introduction
- Boundary Computation
- The Bulk Picture
- EE : An Application
- Conclusions

What are Conformal Blocks?

- Consider a p-point correlator

$$\langle \mathcal{O}(z_1)\mathcal{O}(z_2)\mathcal{O}(z_3)\cdots\mathcal{O}(z_p)\rangle$$

- Insert $p-3$ resolutions of the identity

$$\sum_{\alpha,\beta,\xi,\ldots} \langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)|\alpha\rangle \langle\alpha|\mathcal{O}_3(z_3)|\beta\rangle \cdots \langle\zeta|\mathcal{O}_{p-1}(z_{p-1})\mathcal{O}_p(z_p)\rangle$$

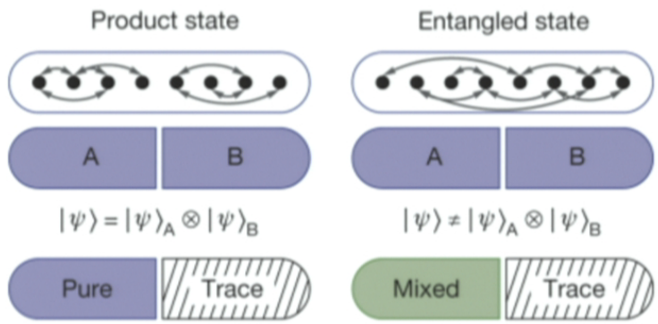
- A typical term of this sum is called conformal block

$$\mathcal{F}_p(z_i, h_i, \tilde{h}_i) := \langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)|\alpha\rangle \langle\alpha|\mathcal{O}_3(z_3)|\beta\rangle \cdots \langle\zeta|\mathcal{O}_{p-1}(z_{p-1})\mathcal{O}_p(z_p)\rangle$$

- These are building blocks of CFT correlators.

What is entanglement entropy?

- Density matrix of a state is defined as $\rho_{tot} = |\Psi\rangle\langle\Psi|$
- EE is Von Neumann entropy of reduced density matrix $\rho_A = \mathrm{Tr}_B(\rho_{tot})$



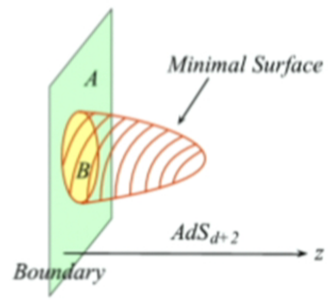
[Courtesy : R. Islam et al.]

$$S_{\partial A} = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$

- A measure of entanglement between subsystems. Vanishes for pure states.

What is entanglement entropy?

- Density matrix of a state is defined as $\rho_{tot} = |\Psi\rangle\langle\Psi|$
- EE is a geometric quantity



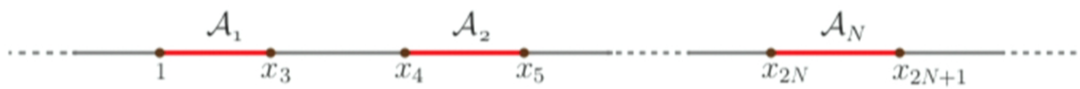
[Courtesy : T. Nishioka et al.]

$$S_{\mathcal{A}} = \frac{\min[\gamma_{\mathcal{A}}]}{4G_N}$$

- A measure of entanglement between subsystems. Vanishes for pure states.

What is entanglement entropy?

- Density matrix of a state is defined as $\rho_{tot} = |\Psi\rangle\langle\Psi|$
- Disjoint intervals in 1+1 dimensional systems



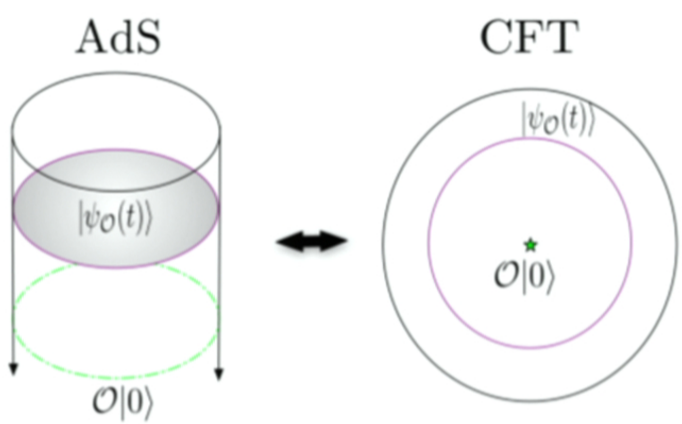
$$S_{\partial A} = -\mathrm{Tr}_A(\rho_A \log \rho_A).$$

- A measure of entanglement between subsystems. Vanishes for pure states.

Which excited states?

- Using the state-operator correspondence

$$|\psi\rangle = \mathcal{O}_H(0)|0\rangle \quad \text{and} \quad \langle\psi| = \lim_{z,\bar{z}\to\infty} \bar{z}^{2h_H} z^{2h_H} \langle 0|\mathcal{O}_H(z,\bar{z}).$$



[Courtesy : J. Kaplan]

- $\mathcal{O}_H(0)$ has very large scaling dimension. Corresponding states are heavy states.

Heavy-light correlators

- We are interested in

$$\langle \mathcal{O}_H(z_1, \bar{z}_1) \prod_{i=2}^{m+1} \mathcal{O}_L(z_i, \bar{z}_i)\, \mathcal{O}_H(z_{m+2}, \bar{z}_{m+2}) \rangle$$

- In terms of cross-ratios

$$\left\langle \mathcal{O}_H(\infty) \left[\mathcal{O}_L(1) \prod_{i=3}^{m+1} \mathcal{O}_L(x_i) \right] \mathcal{O}_H(0) \right\rangle.$$

- We work in $c \to \infty$ limit for which

$$\mathcal{F}_{(p)}(z_i, h_i, \tilde{h}_i) = \exp\left[-\frac{c}{6} f_{(p)}(z_i, h_i, \tilde{h}_i) \right].$$

Monodromy method

- Recall a typical conformal block looks like

$$\mathcal{F}_p(z_i, h_i, \tilde{h}_i) := \langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)|\alpha\rangle \langle\alpha|\mathcal{O}_3(z_3)|\beta\rangle \cdots \langle\zeta|\mathcal{O}_{p-1}(z_{p-1})\mathcal{O}_p(z_p)\rangle$$

- Let's insert an additional operator, $\hat{\psi}(z)$

$$\begin{aligned}\Psi(z, z_i) &:= \langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)|\alpha\rangle \langle\alpha|\hat{\psi}(z)\mathcal{O}_3(z_3)|\beta\rangle \cdots \langle\zeta|\mathcal{O}_{p-1}(z_{p-1})\mathcal{O}_p(z_p)\rangle \\ &\approx \psi(z, z_i) \exp\left[-\frac{c}{6} f_{(p)}(z_i, \epsilon_i, \tilde{\epsilon}_i)\right]\end{aligned}$$

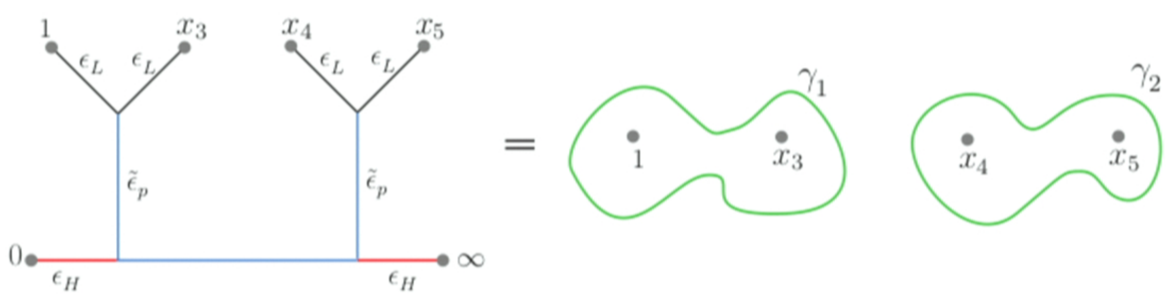
- Choose that $\hat{\psi}(z)$ obeys the null-state condition at level 2

$$\left[L_{-2} - \frac{3}{2(2h_\psi + 1)} L_{-1}^2\right]|\Psi\rangle = 0, \qquad \text{with, } h_\psi \overset{c\to\infty}{=} -\frac{1}{2} - \frac{9}{2c}$$

Monodromy method

Choice of monodromy contour = Choice of OPE channel

- We choose the contours such that each of them contains a pair of light operators within.
- This is equivalent to looking at the OPE channel in which light operators fuse in pairs.
- This choice is geared towards entanglement entropy calculations.



Monodromy method

- The monodromy conditions for all the contours form a coupled system of equations for the accessory parameters.
- Performing the exercise for 5- and 6-point blocks provides sufficient intuition to guess the solutions.

Monodromy method

The accessory parameters can now be used to obtain the conformal block

$$c_i = -\frac{\partial f_{(p)}(z_i, \epsilon_i, \tilde{\epsilon}_i)}{\partial z_i} \qquad \mathcal{F}_{(p)}(z_i, h_i, \tilde{h}_i) = \exp\left[-\frac{c}{6} f_{(p)}(z_i, \epsilon_i, \tilde{\epsilon}_i)\right]$$

Even-point conformal blocks

- The $(m+2)$-point block factorizes into a product of $m/2$ 4-point conformal blocks

$$\begin{aligned} \mathcal{F}_{(m+2)}(\{x_i\}; \epsilon_L, \epsilon_H; \tilde{\epsilon}_p) &= \prod_{\boldsymbol{\Omega}_i \mapsto \{(p,q)\}} \exp\left[-\frac{c}{6} f_{(4)}(x_p, x_q; \epsilon_L, \epsilon_H; \tilde{\epsilon}_p)\right] \\ &= \prod_{\boldsymbol{\Omega}_i \mapsto \{(p,q)\}} \mathcal{F}_{(4)}(x_p, x_q; \epsilon_L, \epsilon_H; \tilde{\epsilon}_p). \end{aligned}$$

$\boldsymbol{\Omega}_i$: Indicates the OPE channels / monodromy contours.

Caveats

This factorization is true only . . .

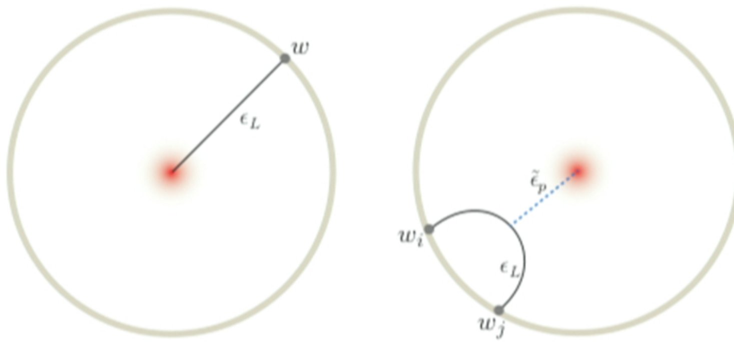
1. at large central charge.
2. in the heavy-light limit
3. for this specific choice of OPE channels
4. $\tilde{\epsilon}_p \ll \epsilon_L$

The dual geometry

- The heavy excited state is dual to the conical defect geometry

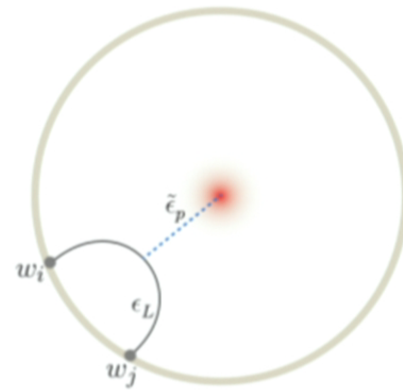
$$ds^2 = \frac{\alpha^2}{\cos^2\rho}\left(-dt^2 + \frac{1}{\alpha^2}d\rho^2 + \sin^2\rho\, d\phi^2\right), \quad \text{with } \alpha = \sqrt{1 - 24h_H/c}.$$

- The conformal blocks can be reproduced by considering lengths of suitable worldline configurations in the bulk.



4-point block from bulk

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(x_i)\mathcal{O}_L(x_j)\mathcal{O}_H(0)\rangle$$



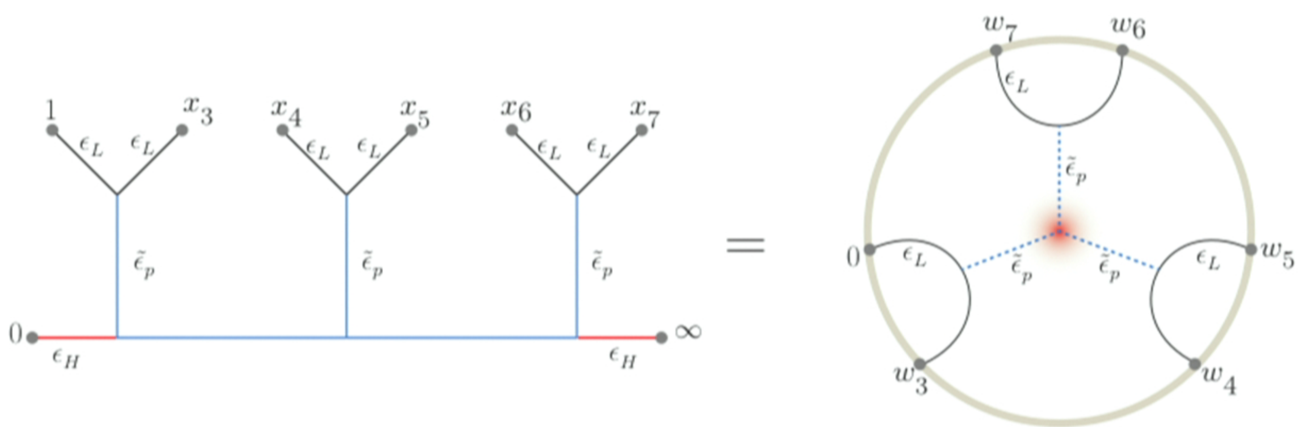
- The worldline action : $S = \epsilon_L l_L + \tilde{\epsilon}_p l_p$
- From cylinder to plane : $x_i = e^{iw_i}$ and $x_j = e^{iw_j}$

$$\mathcal{F}_{(4)}(x_i, x_j) = x_i^{-h_L} x_j^{-h_L} \times e^{-\frac{c}{6}S(w_i,w_j)}\Big|_{w_{i,j}=-i\log x_{i,j}}$$

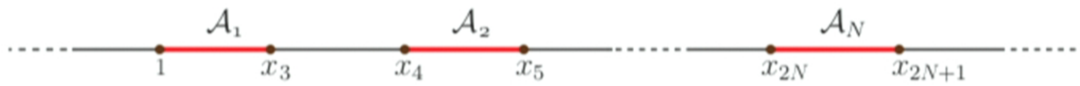
(Matches!)

Higher point block from bulk

Even-point conformal blocks



EE for excited states



- EE from Rényi entropy : $S_{\mathcal{A}}^{(n)} = \frac{1}{1-n} \log \operatorname{tr}_{\mathcal{A}} (\rho_{\mathcal{A}})^n \;\; ; \; n \to 1$
- Effectively need to compute (for $n \to 1$)

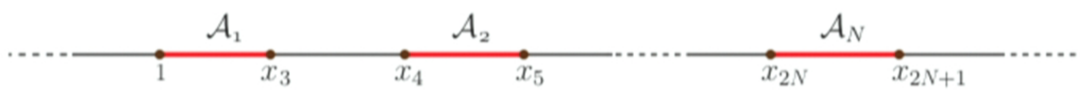
$$G_n(x_i, \bar{x}_i) = \langle \Psi | \sigma(1) \bar{\sigma}(x_3) \sigma(x_4) \bar{\sigma}(x_5) \sigma(x_6) \bar{\sigma}(x_7) \ldots \sigma(x_{2N}) \bar{\sigma}(x_{2N+1}) | \Psi \rangle$$

$$= \langle 0 | \; \Psi(\infty) \; \sigma(1) \bar{\sigma}(x_3) \prod_{i=4,6,\cdots}^{2N} \sigma(x_i) \bar{\sigma}(x_{i+1}) \; \Psi(0) \; |0\rangle$$

- Dimensions of the twist and anti-twist operators

$$h_\sigma = h_{\bar{\sigma}} = \frac{c}{24} \left(n - \frac{1}{n} \right)$$

EE for excited states



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- Dimensions of the twist and anti-twist operators

$$h_\sigma = h_{\bar{\sigma}} = \frac{c}{24} \left(n - \frac{1}{n} \right)$$

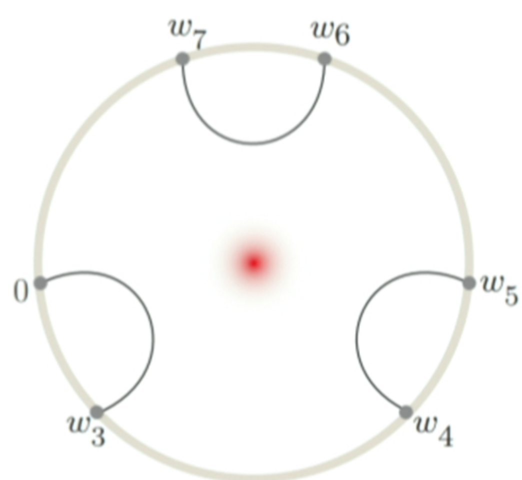
EE for excited states

- In the limit $n \to 1$

$\sigma, \bar{\sigma}$: Light operators
Ψ : Heavy operator

$$S_{\mathcal{A}} = \lim_{n \to 1} S_{\mathcal{A}}^{(n)} = \frac{c}{3} \min_i \left\{ \sum_{\widetilde{\boldsymbol{\Omega}}_i \mapsto \{(p,q)\}} \log \frac{(x_p^\alpha - x_q^\alpha)}{\alpha (x_p x_q)^{\frac{\alpha-1}{2}}} \right\}.$$

with, $\alpha = \sqrt{1 - 24 h_H / c}$



Summary

- Higher point conformal blocks are tractable in the heavy-light limit.
- These conformal blocks can be reproduced precisely from the dual gravity picture.
- This is applied to find entanglement entropy of disjoint intervals in heavy states.
- This conformal block can be rewritten in terms of geodesic lengths (bulk locality?)

Outlook

Applications

1. Tripartite information
2. Mutual information in local quenches
3. Scrambling, chaos, . . .

Extensions

1. Higher spin holography
2. One-loop corrections
3. Higher dimensions, . . .


Thank
you