Title: Building a Gauge-Invariant Holographic Dictionary

Date: Oct 18, 2016 02:00 PM

URL: http://pirsa.org/16100060

Abstract: <p>What natural CFT quantities can â€œseeâ€• in the interior of the bulk AdS in a diffeomorphism invariant way? And how can we use them to learn about the emergence of local bulk physics? Inspired by the Ryu-Takayanagi relation, we construct a class of simple non-local operators on both sides of the duality and demonstrate their equivalence. Integrals of free bulk fields along geodesics/minimal surfaces are dual to what we will call â€œOPE blocksâ€•: Individual conformal family contributions to the OPE of local operators. Our findings can be utilized to reconstruct local bulk operators and unify a number of previously disconnected AdS/CFT results. We extend this kinematic correspondence to incorporate gravitational interactions in AdS3 by relating geodesic operators to the Virasoro OPE blocks, hence providing a natural CFT prescription for â€œgravitational dressingâ€•. We conclude with discussion of preliminary results on an interesting CFT structure that generalizes our dictionary to include arbitrary local bulk interactions. </p>

Building a Gauge-Invariant Holographic Dictionary

Lampros Lamprou
Stanford University

Perimeter Institute, October 2016

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Based on:
[1604.03110], [1608.06282] B.Czech, L.L., S.McCandlish, B.Mosk, J.Sully
L.L., S.McCandlish *to appear*

See also: [1606.03307] J.de Boer, F.Haehl, M. Heller, R.Myers

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Philosophical Overture

Gravity **disfavors locality**:

- Diffeomorphism invariance.
- Black hole resonances in high energy scattering.
- Holographic bound.

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- What mathematics describe their **dynamics**?
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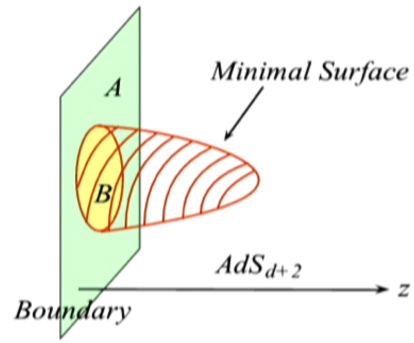
Our goal: Make progress via a **gauge-invariant** approach to the **AdS/CFT dictionary**.

A Clue from AdS/CFT

New approach to observables in gravity: **Ryu-Takayanagi** relation.



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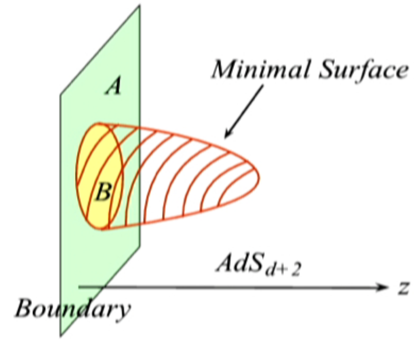

Entanglement entropy = Area of minimal surfaces

Has all the right properties:

a. **Probes geometry** of bulk **interior.**
b. **Diff invariant** – makes reference only to the boundary.
c. **Natural non-local** object in the **CFT.**

A Gauge-Invariant Dictionary

Apply RT philosophy to the **operator dictionary**:

- Identify **natural CFT operators** dual to **extended bulk probes**.
- Reconstruct the familiar **local bulk fields** and describe their **dynamics**.

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Apply RT philosophy to the **operator dictionary:**

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- Reconstruct the familiar **local bulk fields** and describe their **dynamics**.

- **Ultimate goal:** What is the **CFT principle** responsible for the approximate **bulk locality** at low energies?

Our Approach

- **Step 1:** Gauge-invariant dictionary for **free bulk fields.**
  ("A Stereoscopic Look into the Bulk")

- **Step 2:** Include **gravitational** interactions in **2+1** dimensions.
  (See also Sam's talk in a couple of weeks!)

- **Step 3:** Incorporate arbitrary **local interactions**.
  (In progress…)

Geodesic Operators

- **Geodesics** and **minimal surfaces** are special bulk objects.
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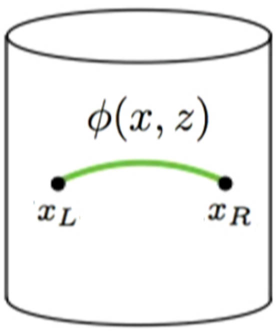

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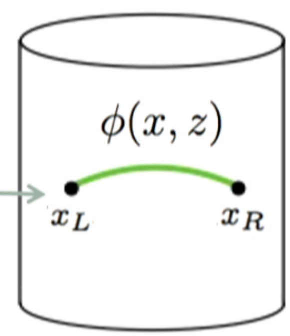
Radon Transform

Parameterized by 2 space-like separated points!



Global OPE Blocks

- We are looking for a **natural CFT bi-local**.
- The **OPE** provides with an **organizational principle**.
- Product of any two operators is expanded as:

$$\mathcal{O}_i(x)\,\mathcal{O}_j(0) \;=\; \sum_k C_{ijk}\,|x|^{\Delta_k-\Delta_i-\Delta_j}\,(1+b_1\,x^\mu\partial_\mu+b_2\,x^\mu x^\nu\partial_\mu\partial_\nu+\ldots)\,\mathcal{O}_k(0)$$

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Global OPE Blocks

- **OPE blocks** admit an **integral representation**.

$$\mathcal{B}_k\left(x,y\right) = \int_{\diamond_{xy}} dw d\bar{w}\, G_k\left(w,\bar{w};\, x,y\right) \mathcal{O}_k\left(w,\bar{w}\right)$$

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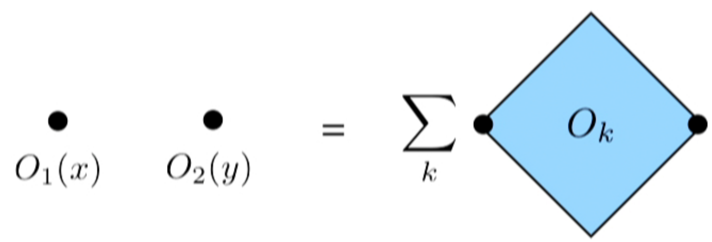
- Smear the primary operator $\mathcal{O}_k(w,\bar{w})$ over the **causal diamond** defined by the two points.



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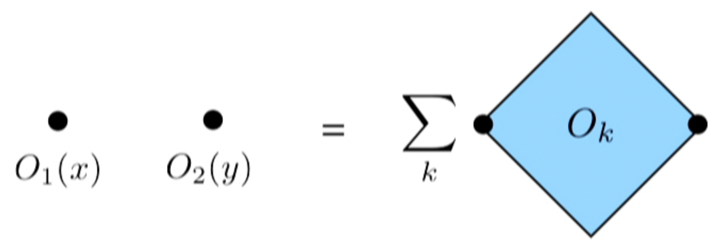
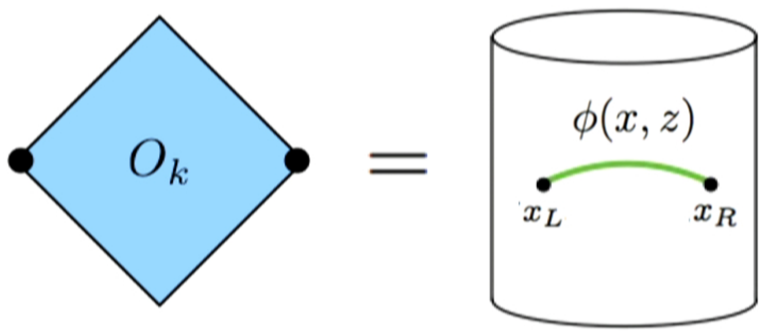


- Smearing function: $G_k(w,\bar{w};x,y) = \dfrac{\langle \mathcal{O}_H(x)\mathcal{O}_H(y)\tilde{\mathcal{O}}_k(w,\bar{w})\rangle}{\langle \mathcal{O}_H(x)\mathcal{O}_H(y)\rangle}$

Gauge-Invariant Dictionary 1.0

OPE Blocks are dual to **geodesic operators**:

$$\mathcal{B}_{h,\bar{h}}(x_L, x_R) = \tilde{\phi}(\gamma_{x_L x_R}) = \int_{\gamma_{x_L x_R}} ds\ \phi(x,z)\Big|_{\gamma}$$



Kinematic Space

- **Geodesic operators** and **OPE blocks** are **bi-local** functions on the boundary.

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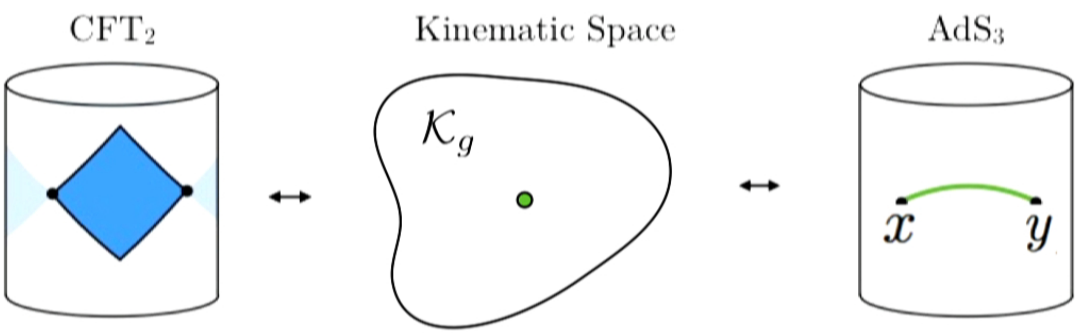

- This is **kinematic space** and it has **geometric** structure.

Kinematic Space

- Utilize **conformal symmetry** to define a **metric** on kinematic space.

$$ds^2 = \frac{I_{\mu\nu}(x-y)}{|x-y|^2} dx^\mu dy^\nu \quad \text{with:} \quad I_{\mu\nu} \equiv \eta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}$$

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- For a **CFT$_2$** kinematic space decomposes to a product of **two de Sitter** spaces.

$$ds^2 = \underbrace{\frac{du_1 du_2}{|u_2 - u_1|^2}}_{dS_2} + \underbrace{\frac{dv_1 dv_2}{|v_2 - v_1|^2}}_{dS_2}$$

Proof of the Dictionary

Utility of kinematic space: **OPE blocks** and **geodesic operators** behave as **free kinematic fields**!

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- Blocks are **eigenvectors** of the **conformal Casimirs:**

$$\left[L^2, \mathcal{B}_k(x,y)\right] = C_k \mathcal{B}_k(x,y)$$

where: $C_k = (\Delta_k + \ell_k)(\Delta_k + l_k - 2)$

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Redundancy: Not all functions on kinematic space are consistent Radon transforms!

We need **constraints**: John's equations.

$$\left(\Box_{\mathrm{d}S_2} - \bar{\Box}_{\mathrm{dS}_2}\right)\tilde{\phi} = 0$$

Proof of the Dictionary

- **One last step**: Boundary conditions.
- Determined by the coincident limit.

OPE Block

$$\mathcal{B}_k\left(x,y\right) \rightarrow \ \left|x-y\right|^{\Delta_k} \mathcal{O}_k\left(x\right)$$

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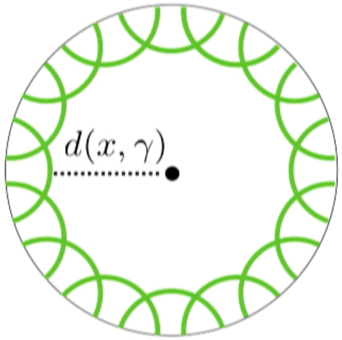
- Both objects obey the **same equations** with the **same boundary conditions**.

$$\mathcal{B}_k\,(x,y) = \;\tilde{\phi}(\gamma) = \int\; ds\;\phi(x,z)\Big|_{\gamma}$$

Local Bulk Operators

- Use geodesic operators to **reconstruct** the **local** operators: **Inverse Radon transform**.

$$f(x) = -\frac{1}{\pi}\int_0^\infty \frac{dp}{\sinh p}\,\frac{d}{dp}\left(\underset{d(x,\gamma)=p}{\text{average}}\ \tilde{f}_g(\gamma)\right)$$



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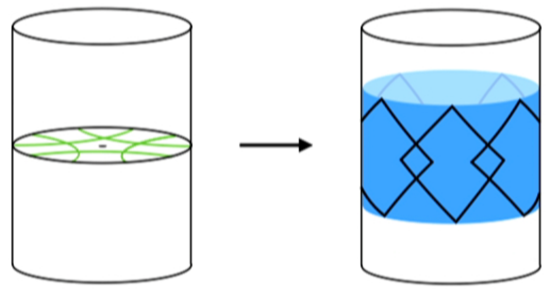
- Manifestly **diffeomorphism invariant**.
- Reconstruction **point** defined by **set of geodesics** that cross it.

Local Bulk Operators

- **Result**: Integral over the **spacelike** separated **boundary region**:

$$\phi(\rho = 0) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dt \int_{0}^{2\pi} d\theta \, K_\Delta(t) \; \mathcal{O}_\Delta(t, \theta)$$



$$K_\Delta(t) = -\frac{k}{\pi} (\cos t)^{\Delta - 2} \log \cos t$$

- This matches the **HKLL global smearing** function.

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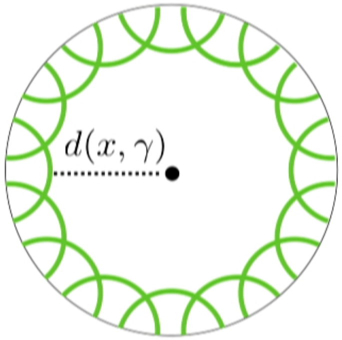

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- **The idea**: Repackaging our data by utilizing symmetries can manifest connections previously invisible.
- Indeed: It helps us "tidy up" seemingly disconnected holographic results:

a) **Conformal blocks** as **geodesic Witten diagrams.**

b) **First law of entanglement entropy** and **linearized Einstein's equations.**

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Recall: $\mathcal{O}_i(x)\,\mathcal{O}_j(0) = |x|^{-\Delta_i-\Delta_j}\sum_k C_{ijk}\mathcal{B}_k^{ij}(x,0)$

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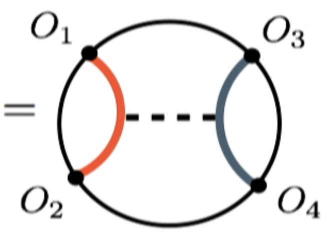
$\langle 0|\mathcal{B}_k(x_1,x_2)\,\mathcal{B}_k(x_3,x_4)\,|0\rangle = g_{k|1234}(u,v) =$ 

[Hijano, Kraus, Perlmutter, Snively]

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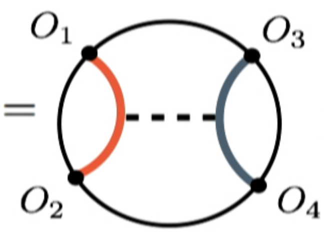
$\langle 0|\mathcal{B}_k(x_1,x_2)\,\mathcal{B}_k(x_3,x_4)|0\rangle = g_{k|1234}(u,v) =$ 

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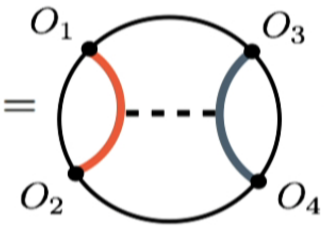

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$\delta S = \delta\langle H_{\text{mod}}\rangle$

$H_{\text{mod}} = -\frac{1}{6}\mathcal{B}_{T_{00}}$

$(\Box_{\mathcal{K}} + 2d)\delta H = -\int_\gamma (\Box_{AdS} - 2d)\delta g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0$

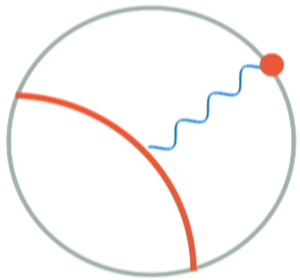
[Faulkner, Guica, Hartman, Lashkari, McDermont, Myers, Swingle, Van Raamsdonk]

STEP 2: GRAVITATIONAL DRESSING

Geodesic Operators and Virasoro OPE Blocks

A Glimpse of Interactions

- Without interactions: $\tilde{\phi}_\Delta(\gamma) = \mathcal{B}_\Delta(\gamma)$



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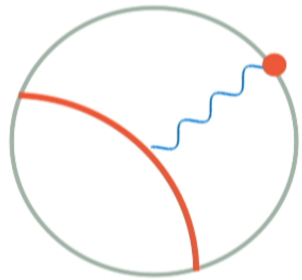

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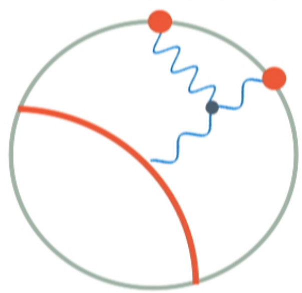

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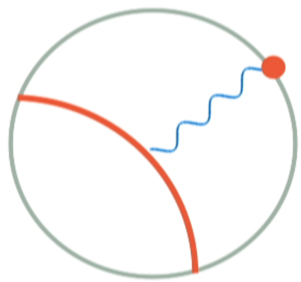

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Multi-trace Blocks

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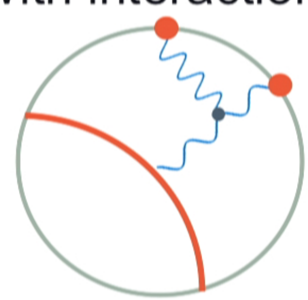

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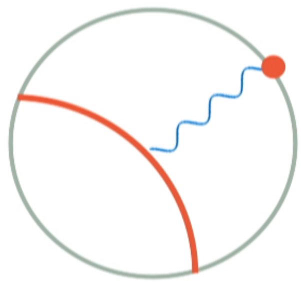

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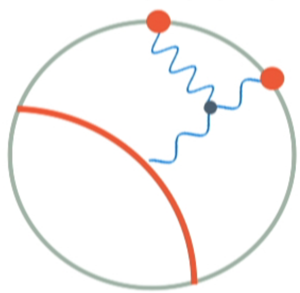

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- What **CFT principle** determines these **"dressed blocks"**?

Virasoro OPE Blocks

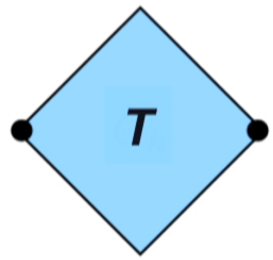
- **OPE** of 2D CFTs is organized in **larger block structures.**
- These are fixed by **local** conformal symmetry.

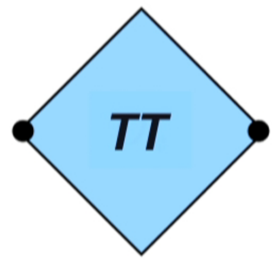
$$O_\Delta(x_1)O_\Delta(x_2) = \mathcal{V}_{\mathbb{I}}^{\Delta\Delta} + \sum_k c_k^{\Delta\Delta}\mathcal{V}_k^{\Delta\Delta}$$

- We will call them **Virasoro OPE blocks**.
- They contain the contributions from all local descendants.

Virasoro OPE Blocks

Identity Block: $$\mathcal{V}_{\mathbb{I}}^{\Delta\Delta}(x_1, x_2) = \frac{\mathbb{I}}{x_{12}^{2\Delta}}\left(1 + \frac{2\Delta}{c}\mathcal{B}_T + \#\mathcal{B}_{T\partial^n T} + \cdots\right)$$

$$= \frac{1}{x_{12}^{2\Delta}}\Bigg(1 +$$


$$+$$


$$+ \cdots \Bigg)$$

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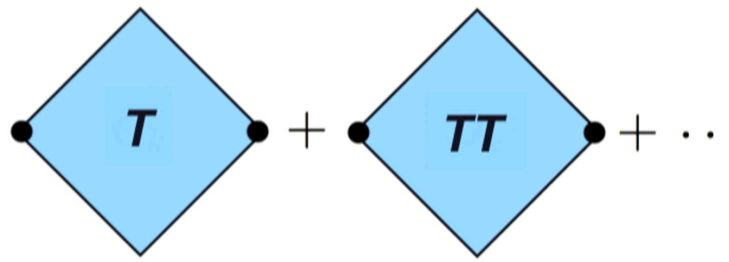
Identity Block: $\mathcal{V}_{\mathbb{I}}^{\Delta\Delta}(x_1, x_2) = \frac{\mathbb{I}}{x_{12}^{2\Delta}} \left(1 + \frac{2\Delta}{c}\mathcal{B}_T + \#\mathcal{B}_{T\partial^n T} + \cdots\right)$

$$= \frac{1}{x_{12}^{2\Delta}}\Bigg(1 + \quad\Bigg)$$



Arbitrary family: $\mathcal{V}_{k}^{\Delta\Delta}(x_1, x_2) = \frac{1}{x_{12}^{2\Delta}} \left(\mathcal{B}_k + \#\mathcal{B}_{T\partial^n \mathcal{O}_k} + \cdots\right)$

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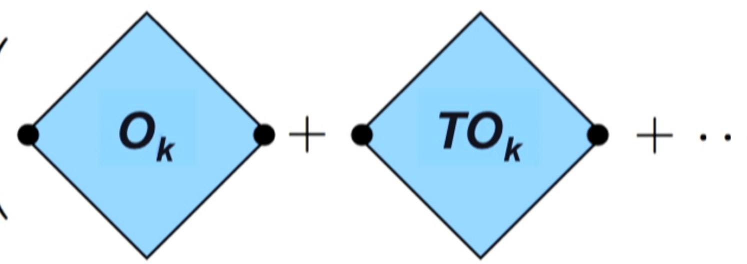

Gauge Invariant Dictionary 2.0

- What I would like to claim:

$$\mathcal{V}_{\mathbb{I}}^{\Delta\Delta}(x_1, x_2) = e^{-\frac{6\Delta}{c}\,\ell(x_1,x_2)}$$

$$\mathcal{V}_{k}^{\Delta\Delta}(x_1, x_2) = e^{-\frac{6\Delta}{c}\,\ell(x_1,x_2)}\tilde{\phi}_k(x_1, x_2)$$

Semi-classical States

- **Semi-classical states:** CFT states with well-defined **energy**.

$$\langle T \rangle \sim \mathcal{O}(c) \qquad \frac{\langle T^n \rangle - \langle T \rangle^n}{\langle T \rangle^n} \sim \mathcal{O}\left(\frac{1}{c^k}\right)$$

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Sketch of the proof

- **Semi-classical** states are characterized **only** by their energy density:

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- If **geodesic operators** and **Virasoro blocks** transform the same way then the **proof follows** from previous discussion.

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- Equation for length: $\dfrac{\partial^2 \omega}{\partial u_1 \partial u_2} = \dfrac{c}{6} e^{-\frac{12}{c}\omega} + J(u_1, u_2)$

[J.de Boer, F.Haehl, M. Heller, R.Myers]

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- Equation for geodesic operators:

$$e^{-\frac{12}{c}\omega} \partial_{u_1} \partial_{u_2} \tilde{\phi}_k + m_k^2 \tilde{\phi}_k = 0$$

- This is a Liouville theory!

Taking stock...

- **Geodesic integrals** of free fields are dual to **global OPE blocks**.

- **Gravitational dressing** for geodesic operators becomes the "**Virasoro dressing**" for the OPE.

- These operators have a life of their own: Can be described as **fields on kinematic space** in terms of a **Liouville theory**.

Looking ahead…

- Can we think of this Liouville theory as a quantum theory?
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- Can we think of this Liouville theory as a quantum theory?
- What **CFT principle** defines the **dressed blocks** for **arbitrary interactions**?
- Is there a **useful** kinematic space description for them?
- Can we use this dictionary to "**see**" inside **black holes**?
- Can this teach us how to think of **operators** in gravity on **flat** and/or **de Sitter** space?

Bartek Czech

Sam McCandlish

Thank you!

Benjamin Mosk

James Sully