

Title: Strings Group Meeting

Date: Oct 07, 2016 10:00 AM

URL: <http://pirsa.org/16100055>

Abstract:

$N=4$ SYM

GAUGE GROUP G

A_m , FERMIONS,

Φ^1

Φ^2 ...

Φ^6

$$\text{Tr} \left[F_{\mu\nu}^2 + (D_\mu \phi^i)^2 + [\Phi^i, \Phi^j]^2 + \text{FERMIONS} + \frac{\theta}{2\pi} F \wedge F \right]$$

$\frac{1}{g_{YM}^2}$

$$\frac{i}{g_{YM}^2} + \frac{\theta}{2\pi} = \gamma$$

$N=4$ SYM G τ



$N=4$ SYM $\vee G, \tau = -\frac{1}{\tau}$

3d N=4

B

N=4 SYM G r



N=4 SYM vG, $\gamma = -\frac{1}{r}$

$$\text{Tr} \left[F_{\mu\nu}^2 + (D_\mu \phi^i)^2 + [\Phi^c, \phi^i]^2 + \text{FERMIONS} + \frac{\theta}{2\pi} F \wedge F \right]$$

$$\frac{i}{g_m^2} + \frac{\theta}{2\pi} = \gamma$$

$$\{Q^+, Q^-\} = \text{C.P.}$$

3d N=4

B

B

$N=4$ SYM G τ

\updownarrow S-DUALITY

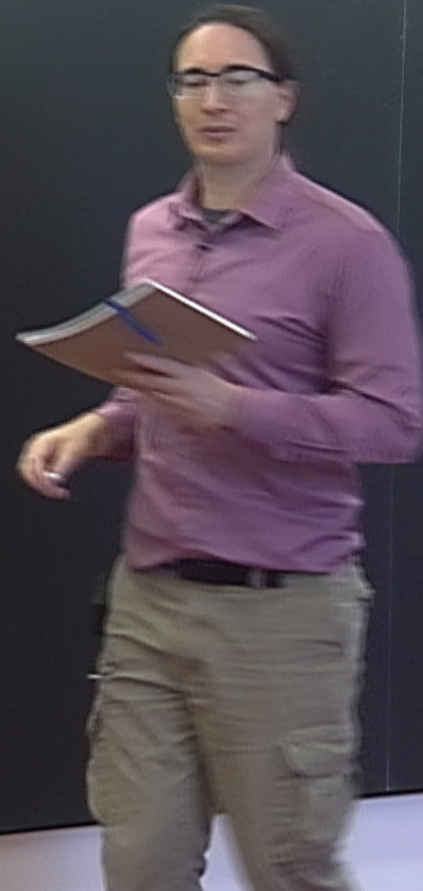
$N=4$ SYM $\checkmark G$, $\checkmark \tau = \frac{1}{\tau}$

$G \quad \gamma$

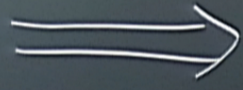
$R \times R^+ \times C$
 \Rightarrow

DUALITY

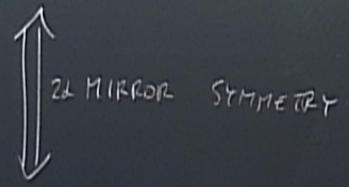
$\forall G, \quad \forall \gamma = -\frac{1}{\gamma}$



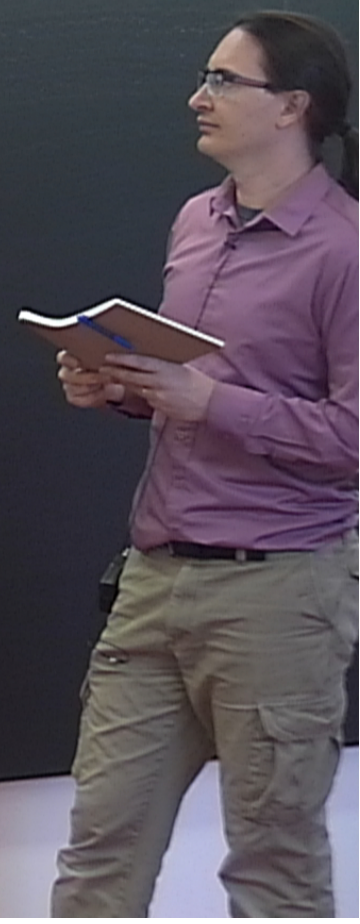
$R \times R^+ \times C$



2d σ -MODEL ON $\mathcal{M}_H(C, G)$



2d σ -MODEL ON $\mathcal{M}_H(C, \mathbb{V}G)$



$R \times R^+ \times C$
 \Rightarrow

B_{BAA}
OR
 B_{BBB}
 \Downarrow
 $\vee B_{BBB}$
OR
 $\vee B_{BAA}$

2d $(4,4)$ σ -MODEL ON $\mathcal{M}_H(C, G)$

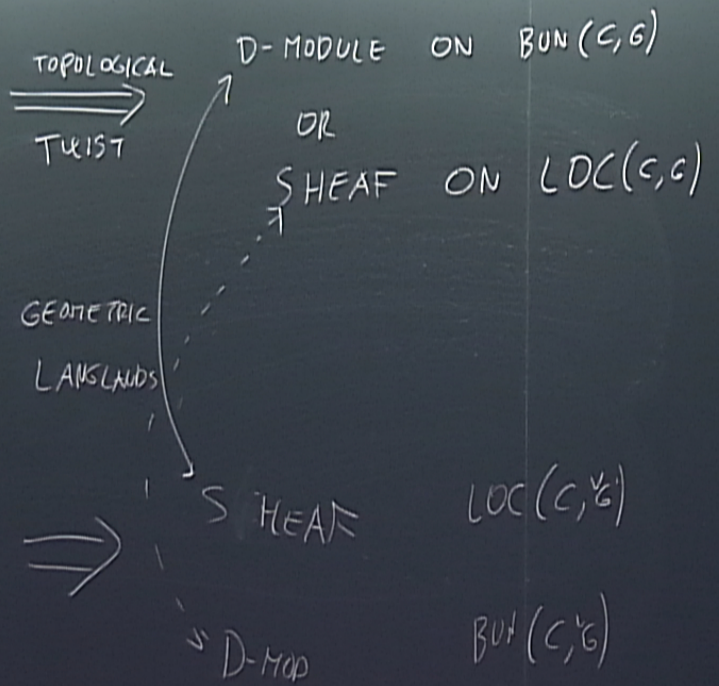
\Updownarrow 2d MIRROR SYMMETRY

2d $(4,4)$ σ -MODEL ON $\mathcal{M}_H(C, \vee G)$

L ON $\mathcal{M}_H(C, G)$

↑ ZL MIRROR SYMMETRY ↓

DEL ON $\mathcal{M}_H(C, \mathbb{G}_m)$



GAUGE GROUP G

ONS, $\Phi^1, \Phi^2, \dots, \Phi^6$

$$SO(6)_R \longrightarrow SO(3)_H \times SO(3)_C$$

EXAMPLES OF BOUND

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp} = 0$$

$$U(N) \leftrightarrow U(N)$$

$$\rightarrow SO(3)_H \times SO(3)_C$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\parallel}^{3d}$$

$S^1 \times SO(3)$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^c$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\perp}^{3d}$$

$$\phi^4 = \mu_i^{3d}$$

$$\phi^5 = \mu_i^{7d}$$

$$\phi^c = \mu_i^{7d}$$

$$\rightarrow SO(3)_H \times SO(3)_C$$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\perp}^{3d}$$

$$\phi^4 = \mu_{\perp}^{3d}$$

$$\phi^5 = \mu_{\parallel}^{3d}$$

$$\phi^6 = \mu_{\parallel}^{3d}$$

DIRICHLET

$$A_{\parallel} = 0$$

$$\phi^1 = \phi^2 = \phi^3 = 0$$

$$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^6 = 0$$

→ $SO(3)_H \times SO(3)_C$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\parallel}^{3d}$$

$$\phi^4 = \mu_i^{3d}$$

$$\phi^5 = \mu_i^{7d}$$

$$\phi^6 = \mu_i^{7d}$$

→ $SO(3)_H \times SO(3)_C$

DIRICHLET
(6 SYMMETRY!)

$$A_{\parallel} = a_{\parallel}^{3d}$$

$$\phi^1 = \phi^2 = \phi^3 = \phi^{4,5,6}$$

$$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^6 = 0$$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

$SO(6) \rightarrow SO(3) \times SO(3)$

NEUMANN BC

$F_{\perp\parallel} = 0$

$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$

$\phi^4 = 0 = \phi^5 = \phi^c$

NEUMANN BC + MATTER

$F_{\perp\parallel} = J^{3d}$

$\phi^4 = \mu_1^{3d}$

$\phi^5 = \mu_2^{3d}$

$\phi^c = \mu_3^{3d}$

DIRICHLET
(6 SYMMETRY!)

$A_{\parallel} = a_{\parallel}^{3d}$

$\phi^1 = \phi^2 = \phi^3 = \mu_{3d}$

$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^c = 0$

$U(N) \leftrightarrow U$

$N=4$ SYM GAUGE GROUP G

A_μ ,

FERMIONS,

$\bar{\Phi}^1, \Phi^2, \dots, \Phi^6$

$SO(2) \subset SO(6)$

$A_0, A_1, A_2, A_{\bar{2}}, \phi_2, \bar{\phi}_{\bar{2}}, \bar{\Phi}_2, \phi \dots \phi_4$

$SO(6) \rightarrow S$

$R \times R^+ \times C$
 \Rightarrow

$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{\bar{z}} + [\Phi_z, \Phi_{\bar{z}}] &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

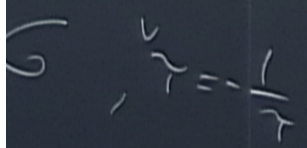
HITCHIN EQUATIONS

TOPOLOGICAL
 \Rightarrow
TWIST

GEOMETRIC
LANGSANDS

D-MO

LITY



\Rightarrow

\Rightarrow

$R \times R^+ \times C$
 \Rightarrow

$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{\bar{z}} + [\Phi_z, \Phi_{\bar{z}}] &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

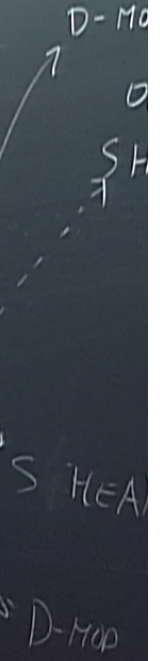
HITCHIN EQUATIONS

$$\Rightarrow \mathcal{M}_H(C, G)$$

TOPOLOGICAL
 \Rightarrow
TWIST

GEOMETRIC
LANGSANDS

\Rightarrow



$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$
 \Rightarrow

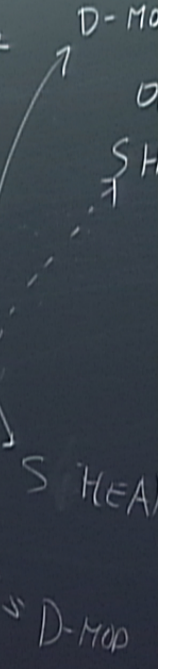
$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{z\bar{z}} + [\Phi_z, \Phi_{\bar{z}}]| &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

HITCHIN EQUATIONS

$$\Rightarrow \mathcal{M}_H(C, G)$$

TOPOLOGICAL
 \Rightarrow
 TWIST

GEOMETRIC
 LANGLANDS



HYPERKÄHLER !

$$\begin{aligned} I^2 &= -1 & IJ + JI &= 0 \\ J^2 &= -1 & & \\ K^2 &= -1 & & \end{aligned}$$

\Rightarrow

$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$
 \Rightarrow

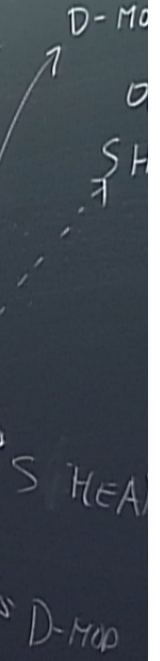
$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{z\bar{z}} + [\Phi_z, \Phi_{\bar{z}}]| &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

HITCHIN EQUATIONS

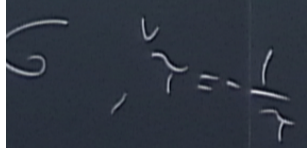
$$\hat{=} \mathcal{M}_H(C, G)$$

TOPOLOGICAL
 \Rightarrow
 TWIST

GEOMETRIC
 LANGLANDS



LITY



\Rightarrow

HYPERKÄHLER

$$(aI + bJ + cK)^2 = -1$$

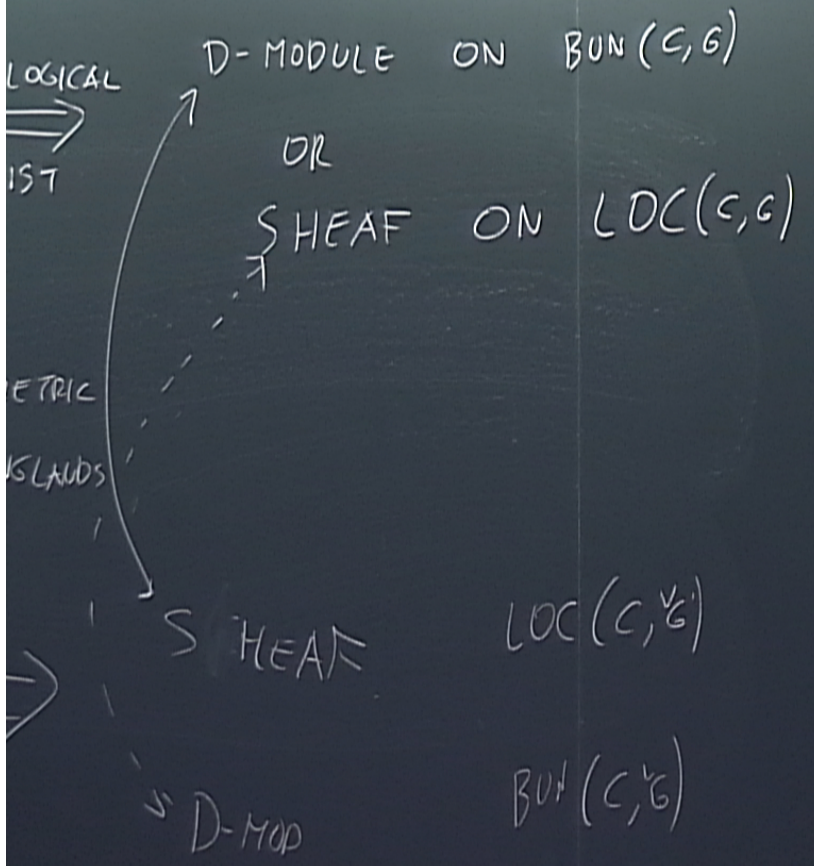
$$a^2 + b^2 + c^2 = 1$$

$$I^2 = -1$$

$$J^2 = -1$$

$$K^2 = -1$$

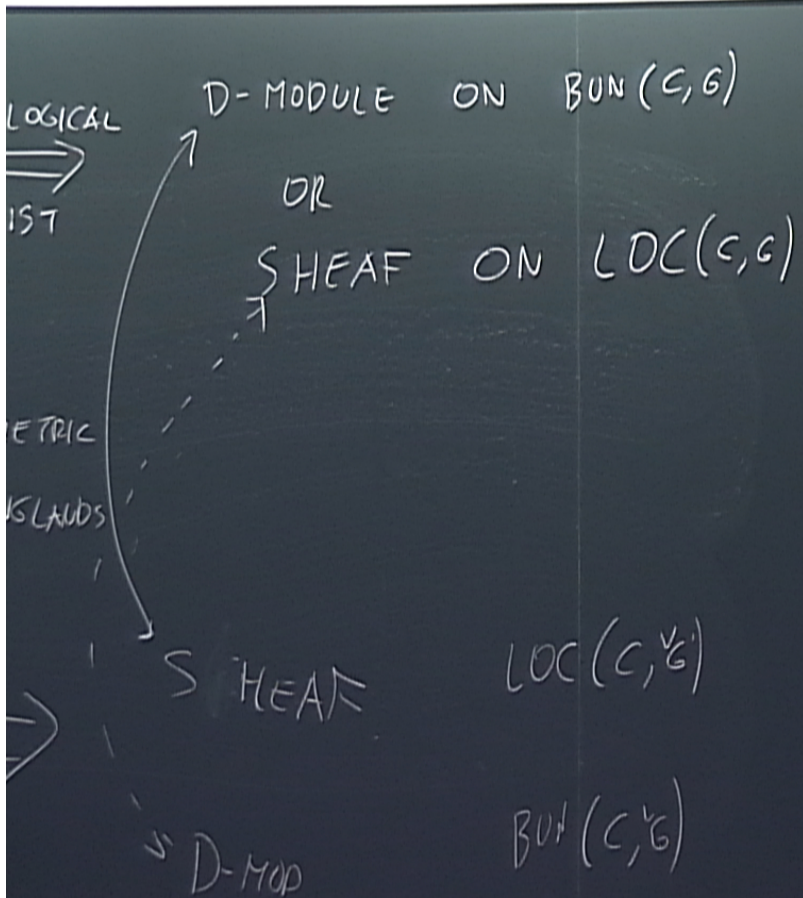
$$IJ + JI = 0$$



\mathbb{J}^3

\mathbb{R}^4 $U = x^1 + ix^2$ $V = x^3 + ix^4$

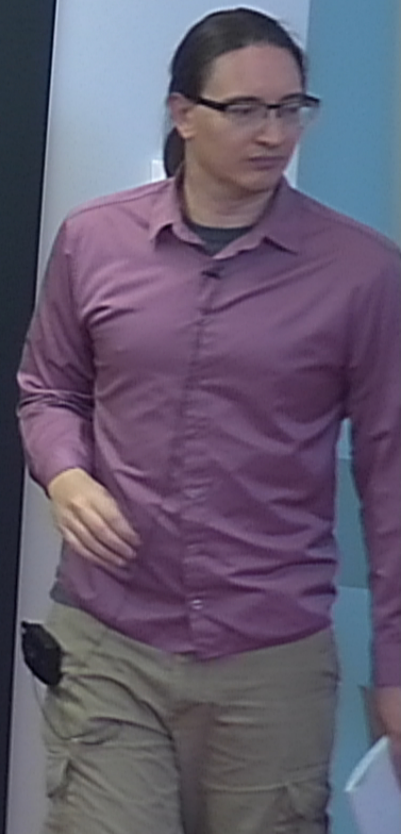
$U + 3\bar{V}$, $V - 3\bar{U}$

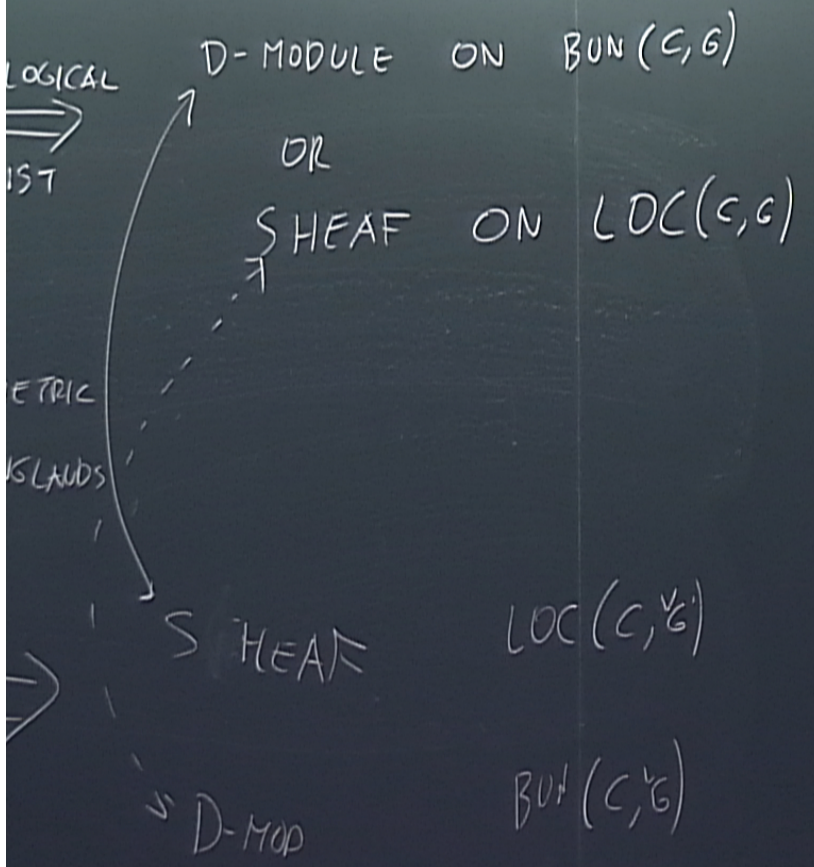


$$\mathbb{R}^4 \quad U = x^1 + ix^2 \quad V = x^3 + ix^4$$

$$U + 3\bar{V}, V - 3\bar{U}$$

$$I \frac{1 - |z|^2}{1 + |z|^2} + \frac{\text{Re } z}{1 + |z|^2} = \text{Im } z \frac{1}{1 + |z|^2} = \text{Im } z$$





$$\mathbb{R}^4 \quad U = x^1 + ix^2 \quad V = x^1 + ix^3$$

$$U + 3V, V - 3U$$

$$\mathbb{J}^3$$

$$\int \frac{1-|z|^2}{1+|z|^2} + \frac{\operatorname{Re} z}{1+|z|^2} + \frac{\operatorname{Im} z}{1+|z|^2} = \mathbb{J}^3$$

$$\mathbb{R}^2 \times \mathbb{T}^2$$

$$(x, \bar{x}, \theta_1, \theta_2)$$

$$\int x, \theta_1 - \theta_2$$

$$\mathbb{J}^3 \quad e^{\frac{x}{3} + i\theta_1 + \bar{x}\bar{\theta}}$$

$$e^{\frac{x}{3} + i\theta_2 + \bar{x}\bar{\theta}}$$

$$U(N) \xleftrightarrow{\Sigma} U(N)$$

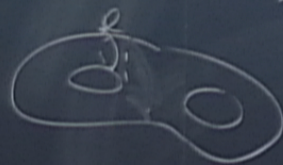
$$\phi, \bar{\phi}, A_z, A_{\bar{z}}$$

$$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$$

$$\Rightarrow$$

$$A_z^3 = A_z + \frac{\phi}{3}, \quad A_{\bar{z}}^3 = A_{\bar{z}} + \bar{\phi} \bar{3}$$

$$[D_z + \frac{\phi}{3}, D_{\bar{z}} + \bar{3}\bar{\phi}] = 0$$



$$\text{Reexp} \int \left(A_z^3 dz + A_{\bar{z}}^3 d\bar{z} \right)$$

$$\Rightarrow$$

$$\begin{cases} D_z \\ F_{z\bar{z}} \\ D_{\bar{z}} \end{cases}$$

$$U(N) \xleftrightarrow{\Sigma} U(N)$$

$$\phi, \bar{\phi}, A_z, \bar{A}_{\bar{z}}$$

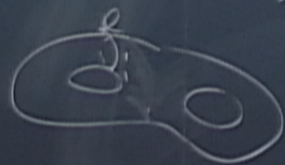
$$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$$

$$\Rightarrow \Rightarrow$$

$$A_z^3 = A_z + \frac{\phi}{3}, \quad A_{\bar{z}}^3 = A_{\bar{z}} + \bar{\phi} \bar{3}$$

$$\mathcal{M}_H^3 = \text{LOC}(C, G)$$

$$\left[D_z + \frac{\phi}{3}, D_{\bar{z}} + \bar{3}\bar{\phi} \right] = 0$$

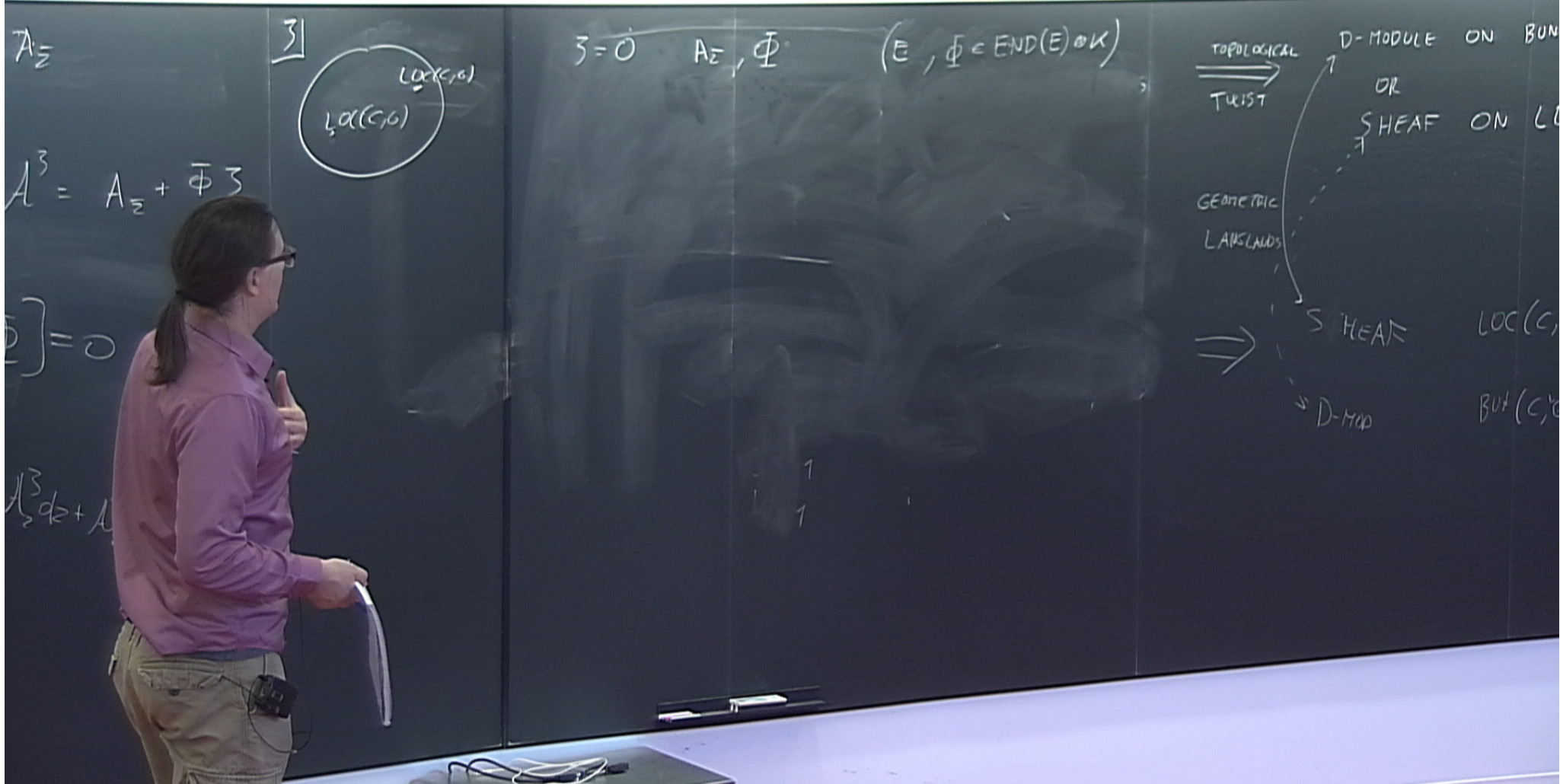


$$\text{Reexp} \int \left(A_z^3 dz + A_{\bar{z}}^3 d\bar{z} \right)$$

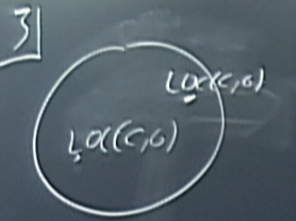
$$\Rightarrow \Rightarrow$$

$$\begin{cases} D_z \\ F_{z\bar{z}} \\ D_{\bar{z}} \end{cases}$$

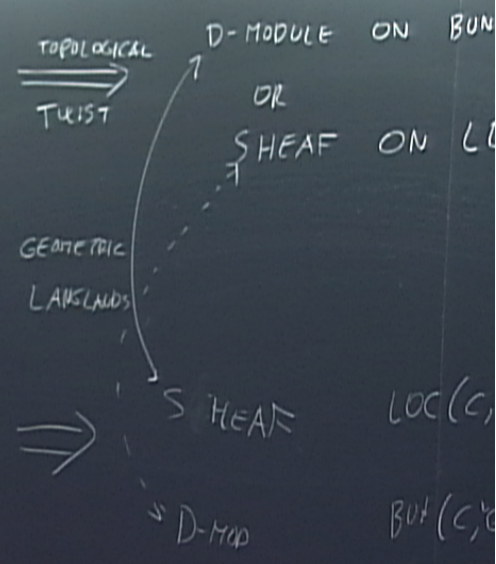
$$\phi^3 = \mu^3$$

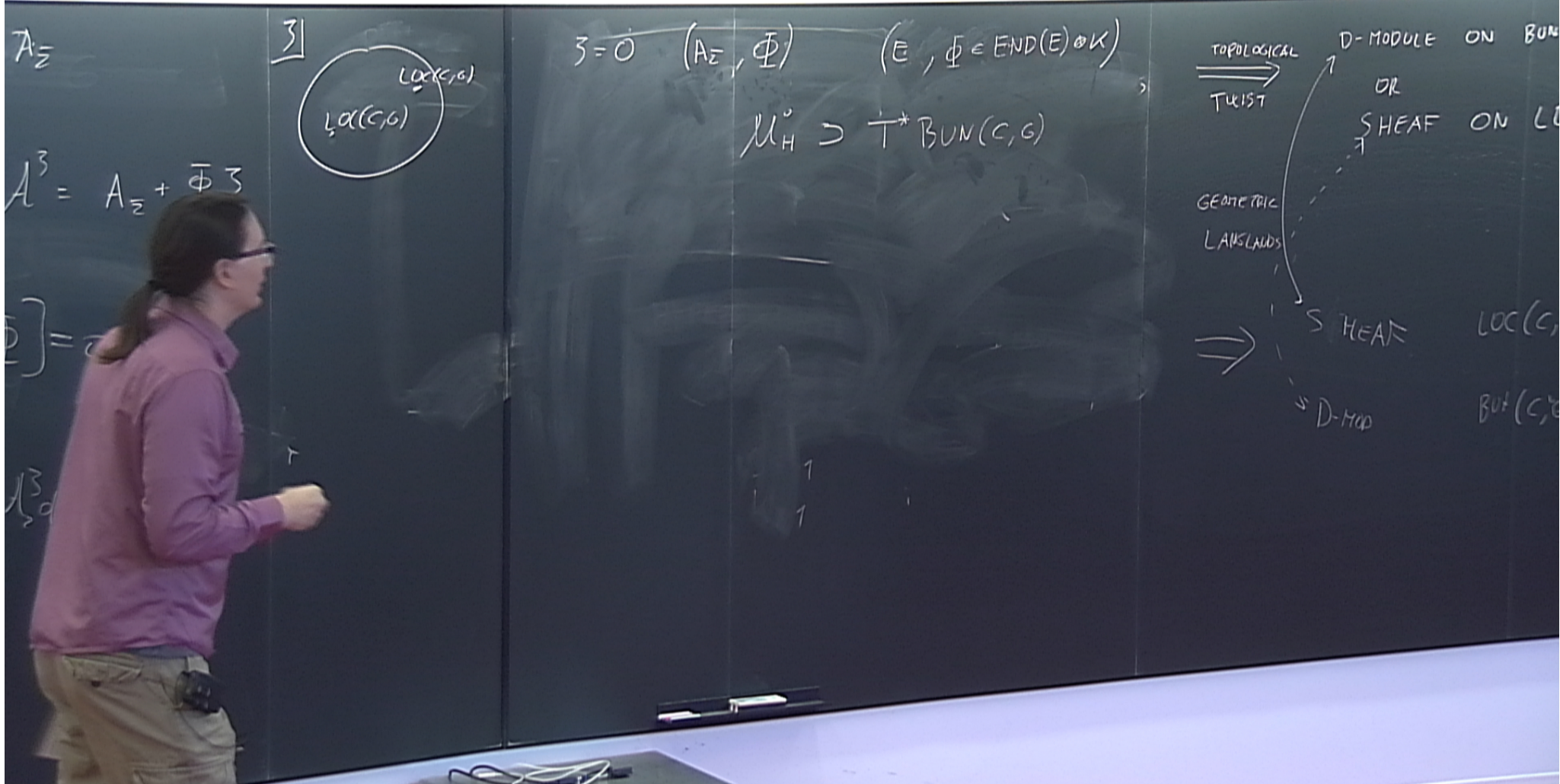


A_z
 $A^3 = A_z + \bar{\Phi} z$
 $F = 0$
 $A^3 = d + A$



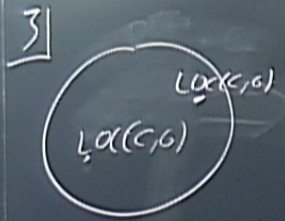
$F = 0$ A_z, Φ $(E, \Phi \in END(E) \oplus K)$





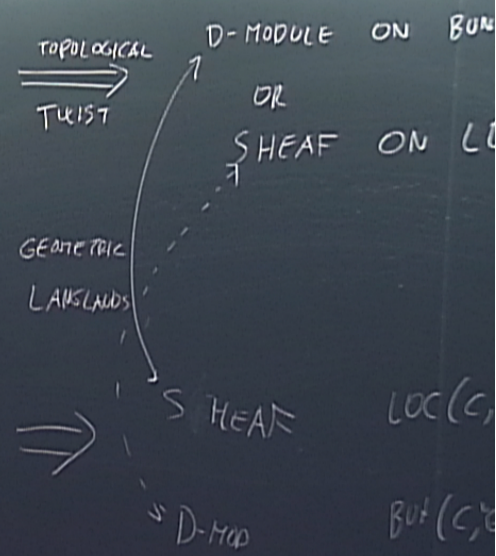
$A_{\bar{z}}$

$A^3 = A_{\bar{z}} + \bar{\Phi} \bar{z}$



$\bar{z} = 0 \quad (A_{\bar{z}}, \bar{\Phi}) \quad (E, \bar{\Phi} \in \text{END}(E) \otimes K)$

$M_H \supset T^*BUN(C, G)$

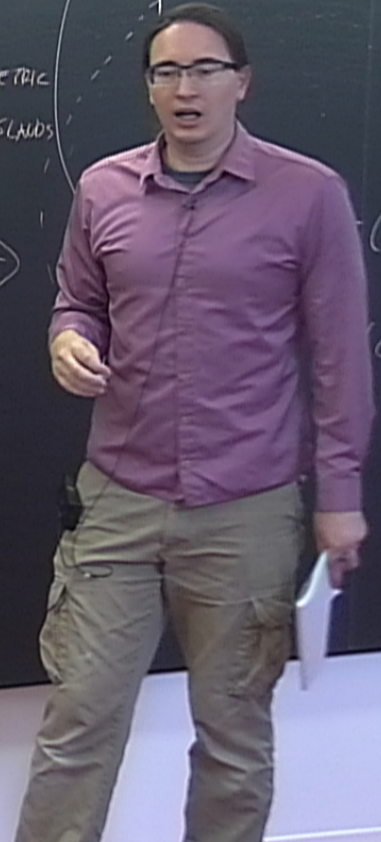


$A_{\bar{z}}$
 $\mathcal{L} = A_{\bar{z}} + \bar{\Phi} \mathcal{L}$
 $\mathcal{L} = 0$
 $dz + A_{\bar{z}} dz$

$\mathcal{L}(C, G)$
 $\mathcal{L}(C, G)$

$\mathcal{L} = 0 \quad (A_{\bar{z}}, \bar{\Phi}) \quad (E, \bar{\Phi} \in \text{END}(E) \otimes K)$
 $\mathcal{M}_H^0 \supset T^* \text{BUN}(C, G)$
 $\det(x - \bar{\Phi})$

TOPOLOGICAL TWIST \Rightarrow D-MODULE ON $\text{BUN}(C, G)$ OR SHEAF ON LOC
 GEOMETRIC LANGLANDS \Rightarrow



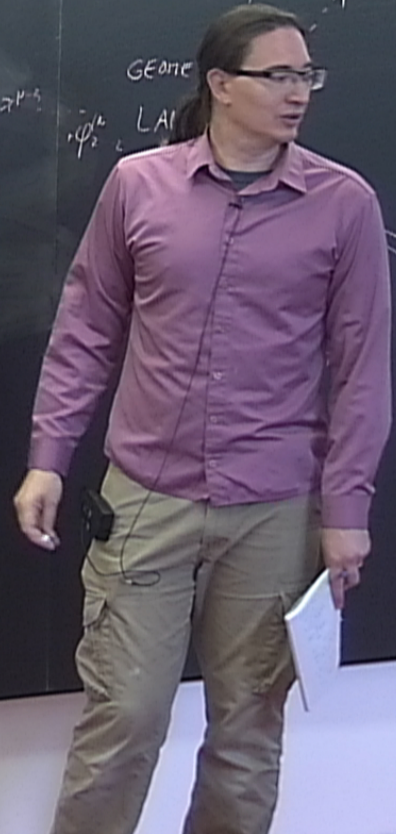
$A_{\bar{z}}$
 $\bar{\partial} \Phi = 0$
 $\mathcal{M}_H \supset \mathbb{T}^* \text{BUN}(C, G)$
 $\det(x - \Phi) = x^N + \varphi_{\bar{z}z}^{(1)} x^{N-1} + \varphi_{z\bar{z}}^{(2)} x^{N-2} + \varphi_{z\bar{z}z}^{(3)} x^{N-3}$
 $\bar{\partial} \varphi^{(1)} = 0$

[3] $\text{LOC}(C, G)$
 $\text{LOC}(C, G)$
 $U(N)$

$\bar{\partial} \Phi = 0$
 $\bar{\partial} \varphi^{(1)} = 0$

TOPOLOGICAL TWIST \rightarrow D-MODULE ON $\text{BUN}(C, G)$ OR SHEAF ON $\text{LOC}(C, G)$
 GEOMETRIC LAI

$\text{LOC}(C, G)$
 $\text{BUN}(C, G)$



$A = \bar{z}$
 $\bar{z} = A + \bar{\Phi} z$

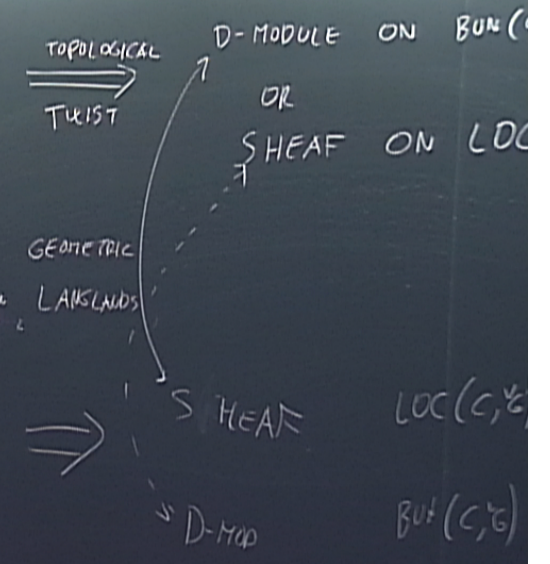
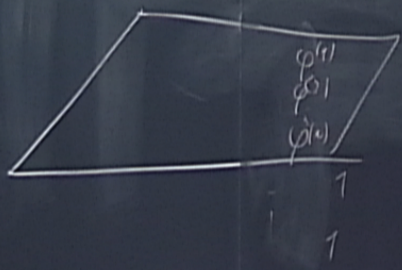
\bar{z}
 $\text{Loc}(C, G)$
 $\text{Loc}(C, G)$
 $U(N)$

$\bar{z} = 0 \quad (A, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$

$\bar{D}\Phi = 0 \quad \mathcal{M}_H \supset \mathbb{T}^* \text{BUN}(C, G)$

$$\det(x - \Phi) = x^N + \varphi_{11}^{(1)} x^{N-1} + \varphi_{22}^{(2)} x^{N-2} + \varphi_{33}^{(3)} x^{N-3}$$

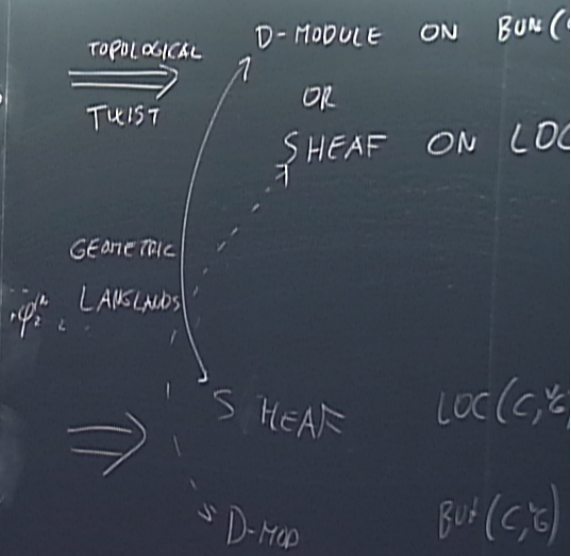
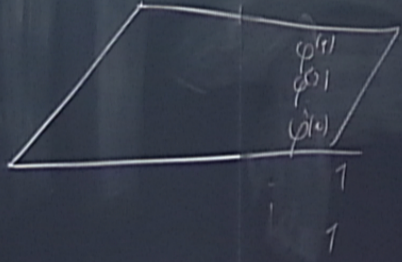
$$\bar{\partial} \varphi^{(1)} = 0$$



A_{Σ}
 $\mathcal{L} = \Lambda_{\Sigma} + \bar{\Phi} \mathcal{L}$

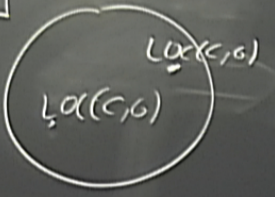
3] $LOC(C, G)$
 $LOC(C, G)$
 $U(N)$
 $P(x, z) = 0$
 $x^*(z)$

$\mathcal{L} = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$
 $\bar{\partial} \Phi = 0 \quad \mathcal{M}_H^0 \supset T^* \text{BUN}(C, G)$
 $P(x, z) = \det(x - \Phi) = x^N + \varphi_1^{(1)} x^{N-1} + \varphi_2^{(2)} x^{N-2} + \varphi_{zz}^{(3)} x^{N-3}$
 $\Phi v = xv \quad \bar{\partial} \varphi^{(1)} = 0$



A_{Σ}

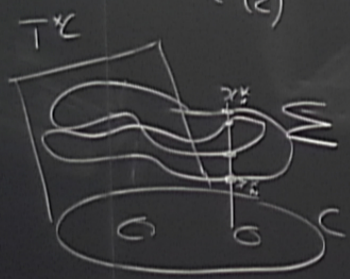
3



$A^3 = A_{\Sigma} + \bar{\Phi} \mathcal{Z}$

$U(N)$

$P(x, z) = 0$
 $x^*(z)$



$k + \mathcal{L} \rightarrow d\bar{z}$

$\bar{\mathcal{Z}} = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$

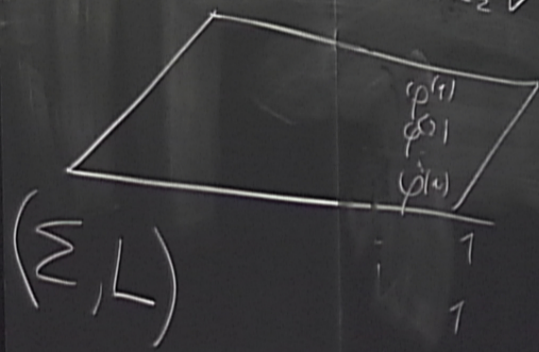
$\bar{\mathcal{D}}\Phi = 0$

$\mathcal{M}_H^0 \supset T^* \text{BUN}(C, G)$

$P(x, z) = \det(x - \bar{\Phi}) = x^M + \varphi_2^{(1)} x^{M-1} + \varphi_{22}^{(2)} x^{M-2} + \varphi_{222}^{(3)} x^{M-3}$

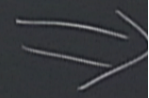
$\Phi v = x v$

$\bar{\mathcal{D}}v = a_{\Sigma} v \quad \bar{\mathcal{D}}\varphi^{(1)} = 0$



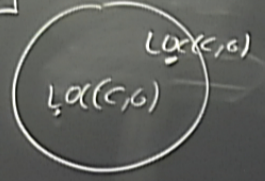
TOPOL
 TWIS

GEOMET
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A_{Σ}

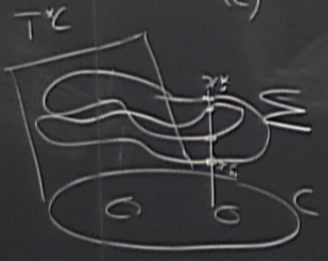
3]



$$A^3 = A_{\Sigma} + \bar{\Phi} \bar{z}$$

\bar{z}

$U(N)$
 $P(z, \bar{z}) = 0$
 $\chi^*(z)$



$$\bar{z} = 0 \quad (A_{\Sigma}, \bar{\Phi}) \quad (E, \bar{\Phi} \in \text{END}(E) \otimes K)$$

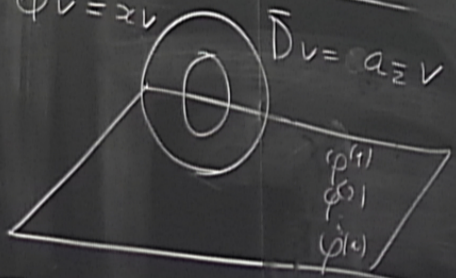
$$\bar{D}\bar{\Phi} = 0$$

$$\mathcal{M}_H^{\circ} \supset T^* \text{BUN}(C, G)$$

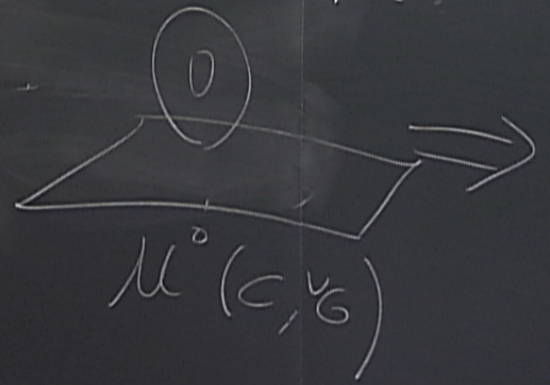
$$P(z, \bar{z}) = \det(z - \bar{\Phi}) = z^N + \varphi_z^{(1)} z^{N-1} + \varphi_{zz}^{(2)} z^{N-2} + \varphi_{zzz}^{(3)} z^{N-3}$$

$$\Phi v = z v$$

$$\bar{D}v = a_{\Sigma} v \quad \bar{\partial} \varphi^{(1)} = 0$$



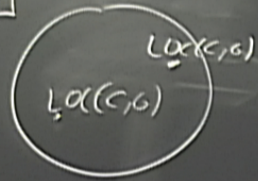
$$(\Sigma, L) \Leftrightarrow (\bar{\Phi}, A_{\Sigma}) \quad \mathcal{M}^{\circ}(C, G)$$



TOPOLOGICAL
 $\xrightarrow{\quad}$
 TWIST

GEOMETRIC
 LANGS LAUBS

3]

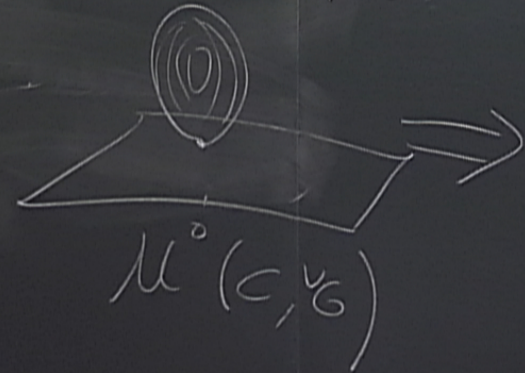
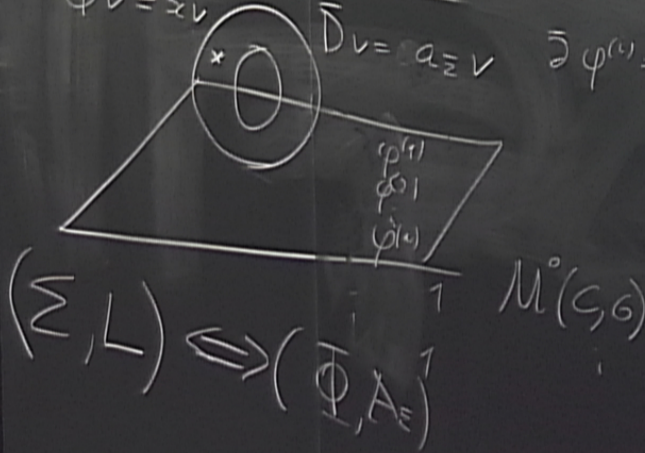


$$\bar{\partial} = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$$

$$\bar{\partial} \Phi = 0 \quad \mathcal{M}_H^{\circ} \supset T^* \text{BUN}(C, G)$$

$$P(z, z) = \det(z - \Phi) = z^N + \varphi_z^{(1)} z^{N-1} + \varphi_{zz}^{(2)} z^{N-2} + \varphi_{zzz}^{(3)} z^{N-3} \dots$$

$$\Phi v = z v \quad \bar{\partial} v = a_{z\bar{z}} v \quad \bar{\partial} \varphi^{(1)} = 0$$



TOPOLOGICAL
TWIST

GEOMETRIC
LANGSLANDS

D-MOD

SHEAF

D-MOD