

Title: Strings Group Meeting

Date: Oct 07, 2016 10:00 AM

URL: <http://pirsa.org/16100055>

Abstract:

$N=4$ SYM

GAUGE GROUP G

A_m ,

FERMIONS,

Φ^1

Φ^2

...

Φ^6

$$\text{Tr} \left[F_{\mu\nu}^2 + (D_\mu \phi^i)^2 + [\Phi^i, \Phi^j]^2 + \text{FERMIONS} + \frac{\theta}{2\pi} F \wedge F \right]$$

 $\frac{1}{g_{YM}^2}$

$$\frac{i}{g_{YM}^2} + \frac{\theta}{2\pi} = \gamma$$

$N=4$ SYM G τ



$N=4$ SYM \sqrt{G} , $\tau = -\frac{1}{\tau}$

3d N=4

B

N=4 SYM G r



N=4 SYM \checkmark G, \checkmark r = -1/r

$$\int_{\mathcal{M}} \sqrt{|g_{\mu\nu}|} \left[F_{\mu\nu}^2 + (D_\mu \phi^i)^2 + [\Phi^i, \phi^j]^2 + \text{FERMIONS} + \frac{\theta}{2\pi} F \wedge F \right]$$

$$g_{\mu\nu} \quad \frac{i}{g_m^2} + \frac{\theta}{2\pi} = \gamma$$

$$\{Q^+, Q^-\} = \text{C.P.}$$

3d N=4

B



B

$N=4$ SYM G τ

S-DUALITY

$N=4$ SYM $\checkmark G$, $\checkmark \tau = \frac{1}{\tau}$

$G \quad \gamma$

$R \times R^+ \times C$
 \Rightarrow

DUALITY

$v^* G \quad \gamma^* = -\frac{1}{\gamma}$



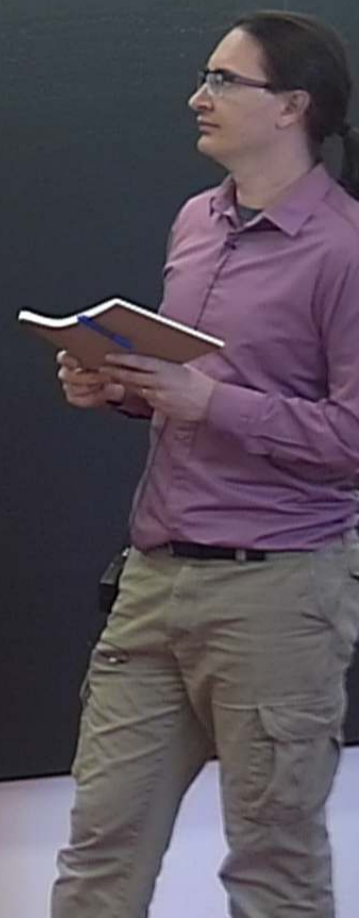
$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$
 \Rightarrow

2d σ -MODEL ON $\mathcal{M}_H(C, G)$

\Updownarrow 2d MIRROR SYMMETRY

2d σ -MODEL ON $\mathcal{M}_H(C, V_G)$

\Rightarrow



$R \times R^+ \times C$
 \Rightarrow

B_{BAA}
OR
 B_{BBB}
 \Downarrow
 $\vee B_{BBB}$
OR
 $\vee B_{BAA}$

2d $(4,4)$ σ -MODEL ON $\mathcal{M}_H(C, G)$

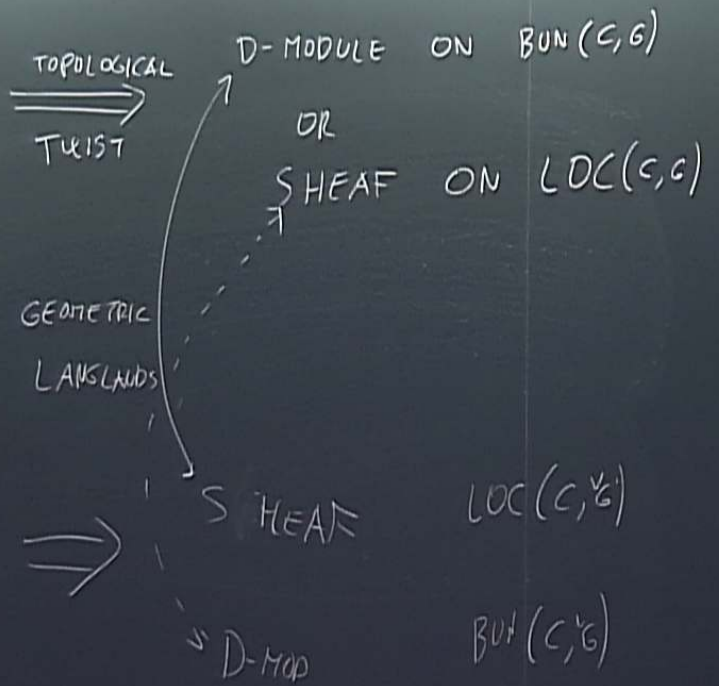
\Updownarrow 2d MIRROR SYMMETRY

2d $(4,4)$ σ -MODEL ON $\mathcal{M}_H(C, \vee G)$

L ON $\mathcal{M}_H(C, G)$

↑ 2d MIRROR SYMMETRY ↓

DEL ON $\mathcal{M}_H(C, \mathbb{G}_m)$



GAUGE GROUP G

ONS, $\Phi^1, \Phi^2, \dots, \Phi^6$

$$SO(6)_R \longrightarrow SO(3)_H \times SO(3)_C$$

EXAMPLES OF BOUND

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp//} = 0$$

$$U(N) \leftrightarrow U(N)$$

$$\rightarrow SO(3)_H \times SO(3)_C$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\perp\parallel}^{3d}$$

$SU(3)_H \times SO(3)_C$

$$U(N) \leftrightarrow U(N)$$



EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^c$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\perp}^{3d}$$

$$\phi^4 = \mu_i^{3d}$$

$$\phi^5 = \mu_i^{3d}$$

$$\phi^c = \mu_i^{3d}$$

$$\rightarrow SO(3)_H \times SO(3)_C$$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

$$U(N) \leftrightarrow U(N)$$

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\perp\parallel}^{3d}$$

$$\phi^4 = \mu_{\perp}^{3d}$$

$$\phi^5 = \mu_{\parallel}^{3d}$$

$$\phi^6 = \mu_{\perp}^{3d}$$

$$\rightarrow SO(3)_H \times SO(3)_C$$

DIRICHLET

$$A_{\parallel} = 0$$

$$\phi^1 = \phi^2 = \phi^3 = 0$$

$$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^6 = 0$$

EXAMPLES OF BOUNDARY CONDITIONS

NEUMANN B.C.

$$F_{\perp\parallel} = 0$$

$$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$$

$$\phi^4 = 0 = \phi^5 = \phi^6$$

NEUMANN B.C. + MATTER

$$F_{\perp\parallel} = J_{\parallel}^{3d}$$

$$\phi^4 = \mu_i^{3d}$$

$$\phi^5 = \mu_i^{3d}$$

$$\phi^6 = \mu_i^{3d}$$

→ $SO(3)_H \times SO(3)_C$

DIRICHLET
(6 SYMMETRY!)

$$A_{\parallel} = a_{\parallel}^{3d}$$

$$\phi^1 = \phi^2 = \phi^3 = \mu_i^{3d}$$

$$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^6 = 0$$

$$U(N) \leftrightarrow U(N)$$

EXAMPLES OF BOUNDARY CONDITIONS

$SO(6) \rightarrow SO(3) \times SO(3)$

NEUMANN BC

$F_{\perp\parallel} = 0$

$D_{\perp}\phi^1 = 0 = D_{\perp}\phi^2 = D_{\perp}\phi^3$

$\phi^4 = 0 = \phi^5 = \phi^c$

NEUMANN BC + MATTER

$F_{\perp\parallel} = T^{3d}$

$\phi^4 = \mu_1^{3d}$

$\phi^5 = \mu_2^{3d}$

$\phi^c = \mu_3^{3d}$

DIRICHLET
(G SYMMETRY!)

$A_{\parallel} = a_{\parallel}^{3d}$

$\phi^1 = \phi^2 = \phi^3 = \mu_{3d}$

$D_{\perp}\phi^4 = D_{\perp}\phi^5 = D_{\perp}\phi^c = 0$

$U(N) \leftrightarrow U$

$N=4$ SYM GAUGE GROUP G

A_M ,

FERMIONS,

$\bar{\Phi}^1, \bar{\Phi}^2, \dots, \bar{\Phi}^6$

$SO(2) \subset SO(6)$

$A_0, A_1, A_2, A_{\bar{2}}, \phi_2, \bar{\Phi}_{\bar{2}}, \bar{\Phi}_3, \phi \dots \phi_4$

$SO(6) \rightarrow S$

$R \times R^+ \times C$
 \Rightarrow

$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{\bar{z}} + [\Phi_z, \Phi_{\bar{z}}] &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

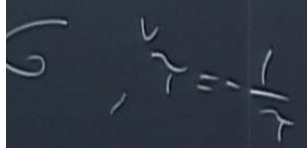
HITCHIN EQUATIONS

TOPOLOGICAL
 \Rightarrow
TWIST

GEOMETRIC
LANGSANDS

D-MO

LITY



\Rightarrow

\Rightarrow

$R \times R^+ \times C$
 \Rightarrow

$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{\bar{z}} + [\Phi_z, \Phi_{\bar{z}}] &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

HITCHIN EQUATIONS

$$\Rightarrow \mathcal{M}_H(C, G)$$

TOPOLOGICAL
 \Rightarrow
TWIST

GEOMETRIC
LANGLANDS

\Rightarrow

D-MOD
SHEAF
D-MOD

$R \times R^+ \times C$
 \Rightarrow

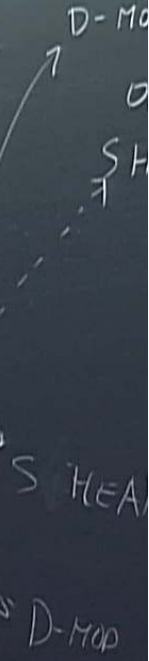
$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{\bar{z}} + [\Phi_z, \Phi_{\bar{z}}]| &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

HITCHIN EQUATIONS

$$\Rightarrow \mathcal{M}_H(C, G)$$

TOPOLOGICAL
 \Rightarrow
TWIST

GEOMETRIC
LANGLANDS



HYPERKÄHLER !

$$\begin{aligned} I^2 &= -1 & IJ + JI &= 0 \\ J^2 &= -1 \\ K^2 &= -1 \end{aligned}$$

$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$
 \Rightarrow

$$\begin{aligned} D_{\bar{z}} \Phi_z &= 0 \\ |F_{z\bar{z}} + [\Phi_z, \Phi_{\bar{z}}]| &= 0 \\ D_z \Phi_{\bar{z}} &= 0 \end{aligned}$$

HITCHIN EQUATIONS

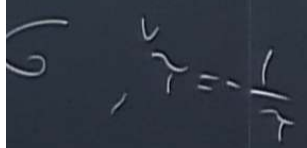
$\Rightarrow \mathcal{M}_H(C, G)$

TOPOLOGICAL
 \Rightarrow
 TWIST

GEOMETRIC
 LANGLANDS



LITY



\Rightarrow

HYPERKÄHLER

$$(aI + bJ + cK)^2 = -1$$

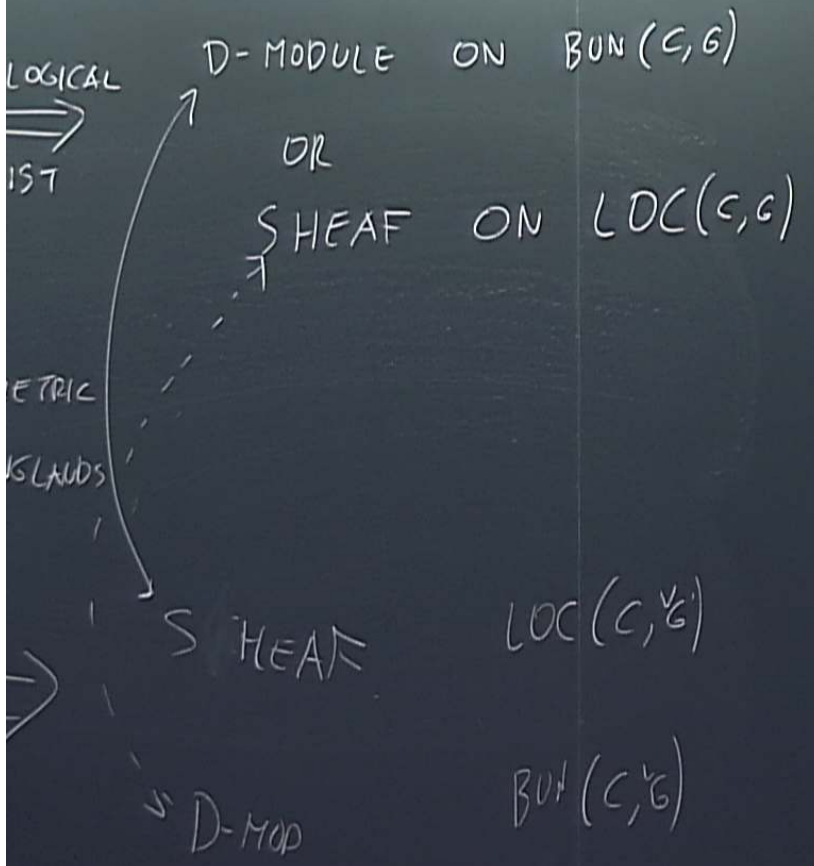
$a^2 + b^2 + c^2 = 1$

$$I^2 = -1$$

$$J^2 = -1$$

$$K^2 = -1$$

$$IJ + JI = 0$$



$$\mathbb{R}^4 \quad U = x^1 + ix^2 \quad V = x^3 + ix^4$$

$$U + 3\bar{V}, V - 3\bar{U}$$

LOGICAL
 \Rightarrow
 IST

D-MODULE ON $BUN(C, G)$
 OR
 SHEAF ON $LDC(C, G)$

METRIC
 ISLANDS

SHEAF $LDC(C, G')$
 \Rightarrow
 D-MOD $BUN(C, G')$

\mathbb{R}^4 $U = z^1 + iz^2$ $V = z^3 + iz^4$
 $U + 3\bar{V}$, $V - 3\bar{U}$

\mathbb{J}^3

$$\mathbb{I} \frac{1 - |z|^2}{1 + |z|^2} + \frac{\text{Re } z \mathbb{J}}{1 + |z|^2} + \frac{\text{Im } z \mathbb{K}}{1 + |z|^2} = \mathbb{J}^3$$


LOGICAL
LIST

METRIC
ISLANDS

→

D-MODULE ON $BUN(C, G)$

OR

SHEAF ON $LOC(C, G)$

SHEAF

$LOC(C, G)$

D-MOD

$BUN(C, G)$

R^4 $U = x^1 + ix^2$ $V = x^1 + ix^1$

$U + 3V, V - 3U$

J^3

$$I \frac{1 - |z|^2}{1 + |z|^2} + \frac{\text{Re } z}{1 + |z|^2} J + \frac{\text{Im } z}{1 + |z|^2} K = J^3$$

$R^2 \times T^2$

$(x, \bar{x}, \theta_1, \theta_2)$

$I \quad x, \theta_1 - \tau \theta_2$

$J^3 \quad e^{\frac{x}{3} + i\theta_1 + \bar{x}3}$

$e^{\frac{\tau x}{3} + i\theta_2 + \bar{\tau} \bar{x}3}$

$$U(N) \xleftrightarrow{\Sigma} U(N)$$

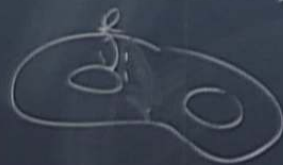
$$\phi, \bar{\phi}, A_z, A_{\bar{z}}$$

$$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$$

$$\Rightarrow$$

$$A_z^3 = A_z + \frac{\phi}{3}, \quad A_{\bar{z}}^3 = A_{\bar{z}} + \bar{\phi} 3$$

$$[D_z + \frac{\phi}{3}, D_{\bar{z}} + 3\bar{\phi}] = 0$$



$$\text{Res} \int_{\Sigma} A_z^3 dz + A_{\bar{z}}^3 d\bar{z}$$

$$\Rightarrow$$

$$\begin{cases} D_z \\ F_{z\bar{z}} \\ D_{\bar{z}} \end{cases}$$

$$U(N) \xleftrightarrow{\Sigma} U(N)$$

$$\phi, \bar{\phi}, A_z, \bar{A}_{\bar{z}}$$

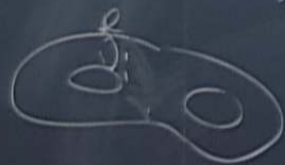
$$\mathbb{R} \times \mathbb{R}^+ \times \mathbb{C}$$

$$\Rightarrow$$

$$A_z^3 = A_z + \frac{\phi}{3}, \quad A_{\bar{z}}^3 = A_{\bar{z}} + \bar{\phi} \bar{3}$$

$$\mathcal{M}_H^3 = \text{LOC}(C, G)$$

$$\left[D_z + \frac{\phi}{3}, D_{\bar{z}} + \bar{3}\bar{\phi} \right] = 0$$

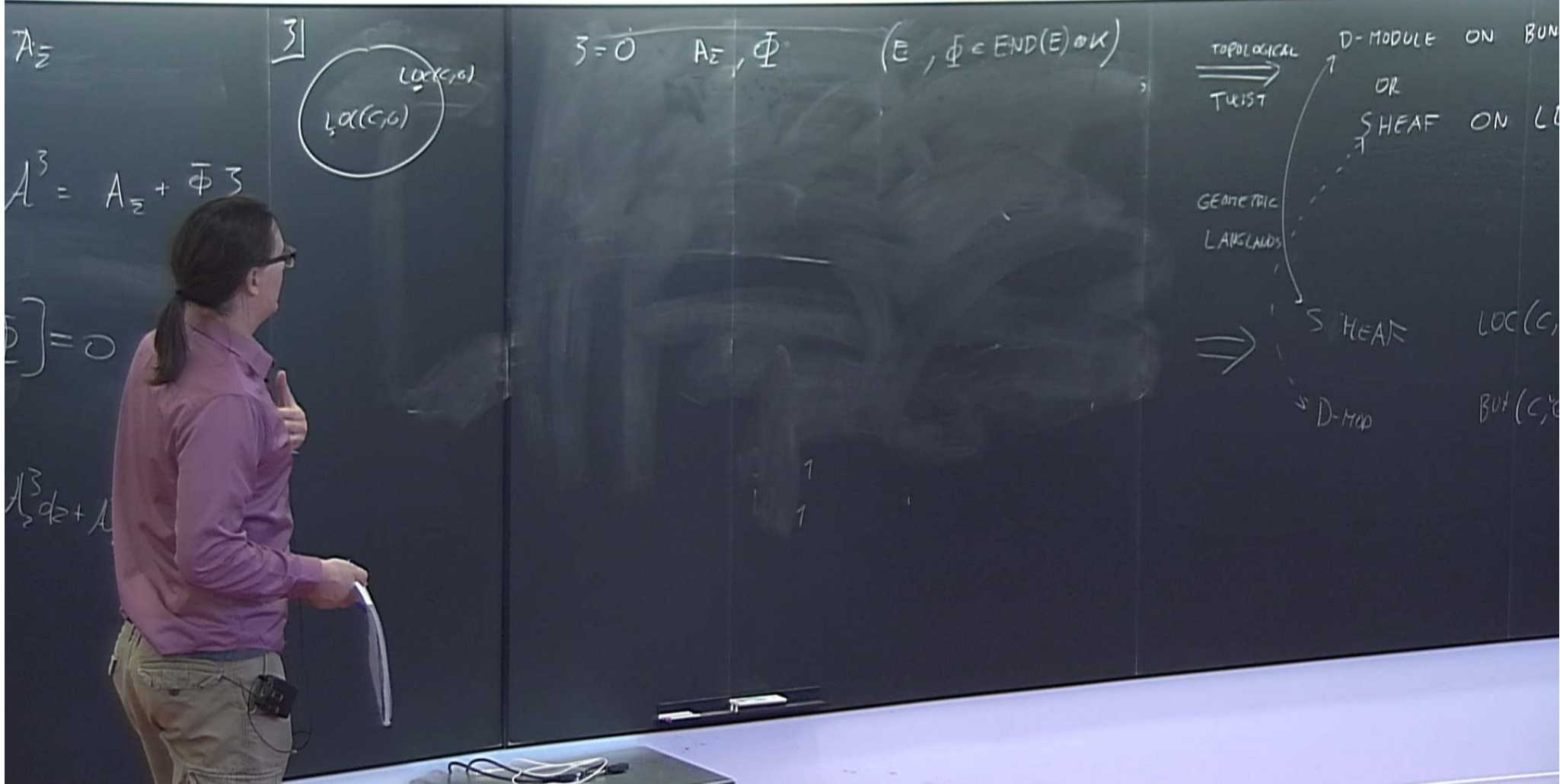


$$\text{Reexp} \int_{\Sigma} A_z^3 dz + A_{\bar{z}}^3 d\bar{z}$$

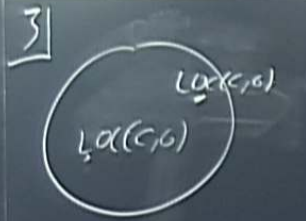
$$\Rightarrow$$

$$\begin{cases} D_z \\ F_{z\bar{z}} \\ D_{\bar{z}} \end{cases}$$

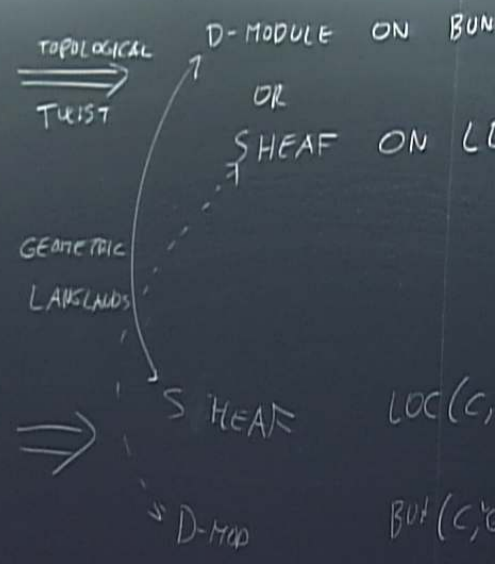
$$\phi^3 = \dots$$

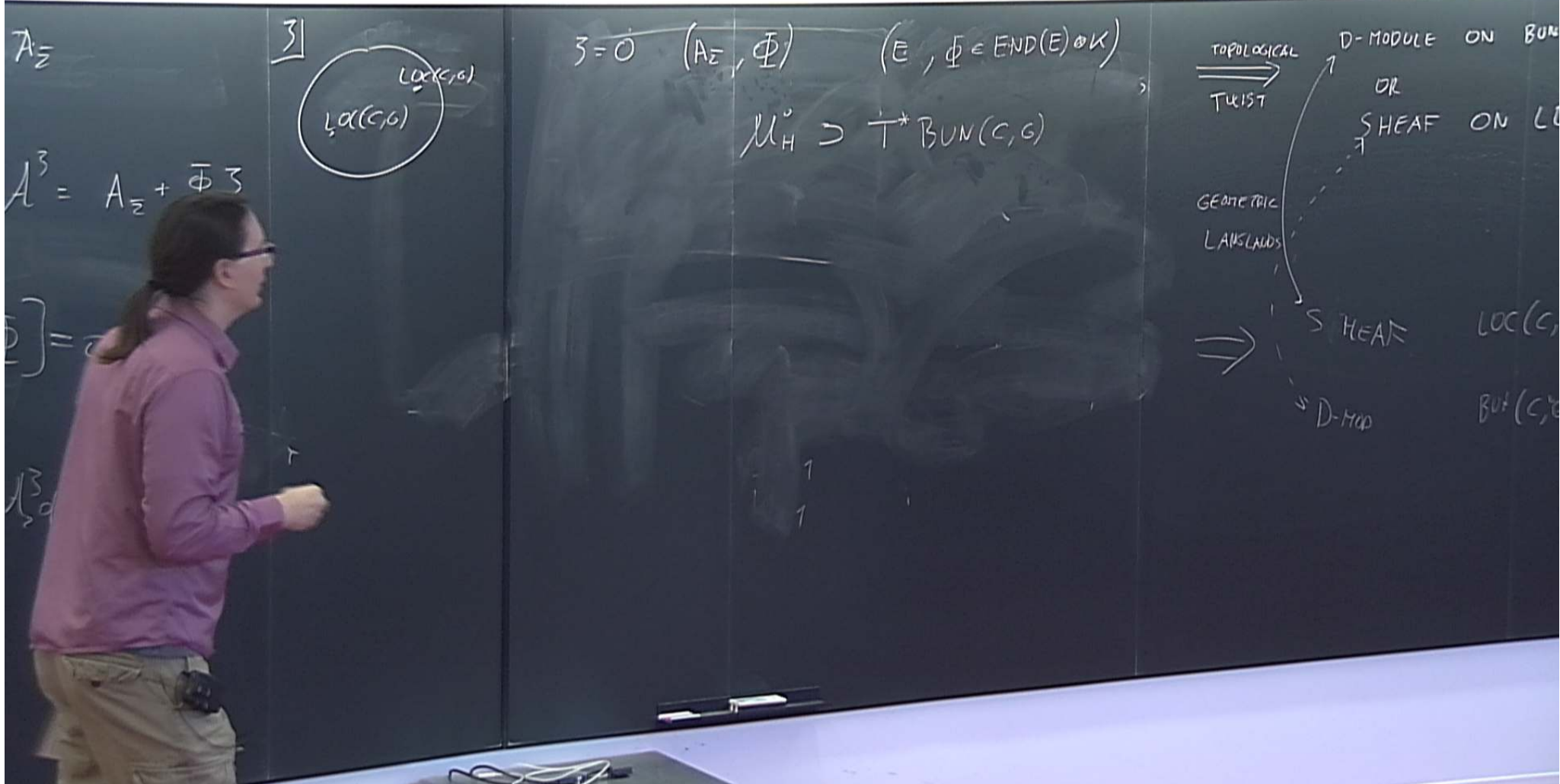


A_z
 $A^3 = A_z + \bar{\Phi} z$
 $\bar{\Phi} z = 0$
 $A^3 = d + A$

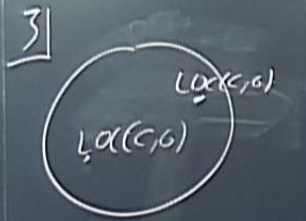


$3 = 0 \quad A_z, \Phi \quad (E, \Phi \in \text{END}(E) \oplus K)$

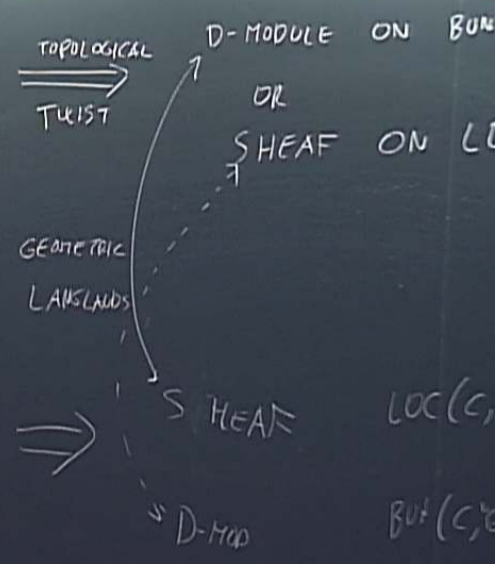




A_z
 $A^3 = A_z + \bar{\Phi} z$
 $\Phi = \dots$
 $\mathcal{M}_H^3 \supset \dots$



$3=0 \quad (A_z, \Phi) \quad (E, \Phi \in \text{END}(E) \otimes K)$
 $\mathcal{M}_H \supset T^* \text{BUN}(C, G)$

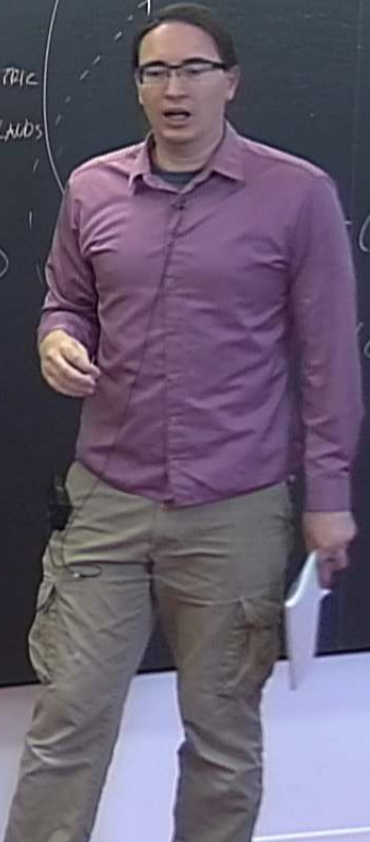


$A_{\bar{z}}$
 $\mathcal{L} = A_{\bar{z}} + \bar{\Phi} \mathcal{L}$
 $\mathcal{L} = 0$
 $dz + A_{\bar{z}} dz$

$\mathcal{L}(C, G)$
 $\mathcal{L}(C, G)$

$\mathcal{L} = 0 \quad (A_{\bar{z}}, \bar{\Phi}) \quad (E, \bar{\Phi} \in \text{END}(E) \oplus K)$
 $\mathcal{M}_H^0 \supset \mathbb{T}^* \text{BUN}(C, G)$
 $\det(x - \bar{\Phi})$

TOPOLOGICAL TWIST \Rightarrow D-MODULE ON $\text{BUN}(C, G)$ OR SHEAF ON LOC
 GEOMETRIC LANGLANDS \Rightarrow



$\mathbb{3} = 0 \quad (A_{\bar{z}}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$
 $\bar{\partial}\Phi = 0 \quad \mathcal{M}_H \supset \mathbb{T}^* \text{BUN}(C, G)$
 $\det(x - \Phi) = x^N + \varphi_{\bar{z}z}^{(1)} x^{N-1} + \varphi_{z\bar{z}}^{(2)} x^{N-2} + \varphi_{z\bar{z}z}^{(3)} x^{N-3}$
 $\bar{\partial}\varphi^{(1)} = 0$

TOPOLOGICAL TWIST \rightarrow D-MODULE ON $\text{BUN}(C, G)$ OR SHEAF ON $\text{LOC}(C, G)$
 GEOMETRIC LAI

$\text{LOC}(C, G)$
 $\text{BUN}(C, G)$

$\mathbb{3} = A_{\bar{z}} + \bar{\Phi}\mathbb{3}$
 $\mathbb{3} = 0$
 $\mathbb{3} dz + A_{z\bar{z}} d\bar{z}$

$\text{LOC}(C, G)$
 $\text{BUN}(C, G)$



$\mathbb{Z} = 0 \quad (A_{\mathbb{Z}}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$
 $\bar{\partial}\Phi = 0 \quad \mathcal{M}_H^0 \supset \mathbb{T}^* \text{BUN}(C, G)$
 $\det(x - \Phi) = x^N + \varphi_{11}^{(1)} x^{N-1} + \varphi_{22}^{(2)} x^{N-2} + \varphi_{33}^{(3)} x^{N-3}$
 $\bar{\partial}\varphi^{(1)} = 0$

TOPOLOGICAL TWIST \Rightarrow D-MODULE ON $\text{BUN}(C, G)$ OR SHEAF ON $\text{LOC}(C, G)$
 GEOMETRIC LANGLANDS \Rightarrow SHEAF \Rightarrow D-MOD

$\text{LOC}(C, G)$
 $\text{BUN}(C, G)$

$U(N)$

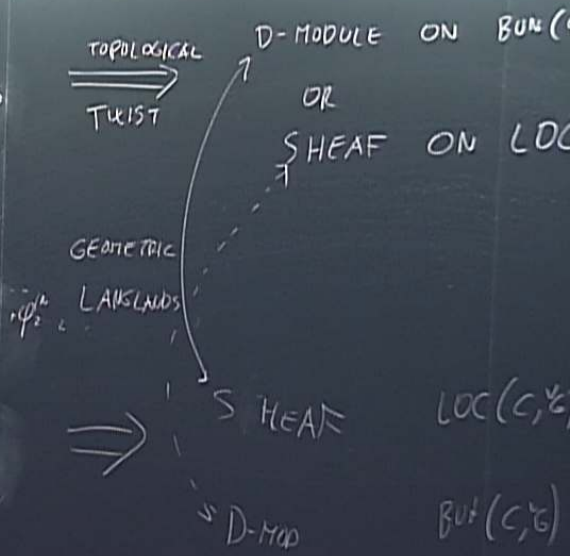
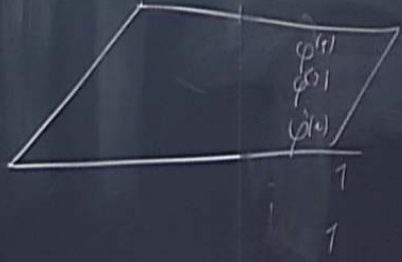
$\varphi^{(1)}$
 $\varphi^{(2)}$
 $\varphi^{(3)}$

1
 1

A_{Σ}
 $\mathcal{L} = \Lambda_{\Sigma} + \bar{\Phi} \mathcal{L}$

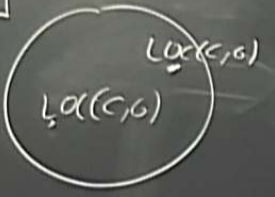
3] $\text{Loc}(C, G)$
 $\text{Loc}(C, G)$
 $U(N)$
 $P(x, z) = 0$
 $x^*(z)$

$\mathcal{L} = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi = \text{END}(E) \oplus K)$
 $\bar{\partial} \Phi = 0 \quad \mathcal{M}_H^0 \supset T^* \text{BUN}(C, G)$
 $P(x, z) = \det(x - \Phi) = x^N + \varphi_{11}^{(1)} x^{N-1} + \varphi_{22}^{(2)} x^{N-2} + \varphi_{33}^{(3)} x^{N-3}$
 $\Phi v = xv \quad \bar{\partial} \varphi^{(1)} = 0$



A_{Σ}

3

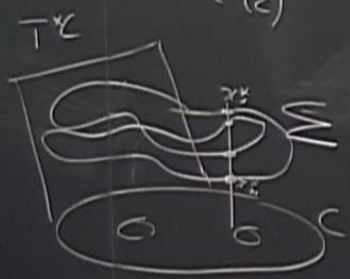


$$A^3 = A_{\Sigma} + \bar{\Phi} \zeta$$

$U(N)$

$$P(x, z) = 0$$

$$x^*(z)$$



$$k + \Delta_{\Sigma} = d_{\Sigma}$$

$$\zeta = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$$

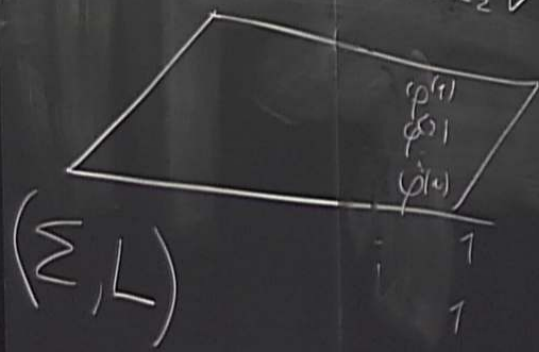
$$\bar{\partial}\Phi = 0$$

$$\mathcal{M}_H^0 \supset T^* \text{BUN}(C, G)$$

$$P(x, z) = \det(x - \bar{\Phi}) = x^M + \varphi_2^{(1)} x^{M-1} + \varphi_{22}^{(2)} x^{M-2} + \varphi_{222}^{(3)} x^{M-3}$$

$$\Phi v = x v$$

$$\bar{\partial}v = a_{\Sigma} v \quad \bar{\partial}\varphi^{(1)} = 0$$



TOPOL
TWIS

GEOMET
LANGU

$A_{\bar{z}}$

$A^3 = A_{\bar{z}} + \bar{\Phi} \bar{z}$

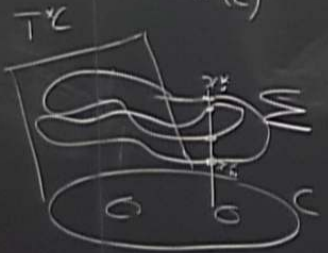
3]

$L\alpha(C, G)$

$U(N)$

$P(z, \bar{z}) = 0$

$\chi^*(z)$



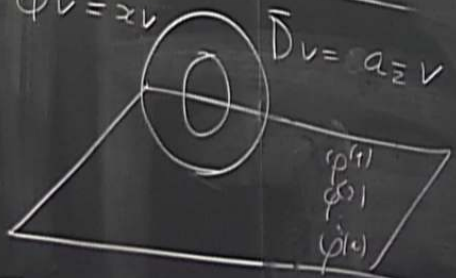
$\bar{z} = 0 \quad (A_{\bar{z}}, \bar{\Phi}) \quad (E, \bar{\Phi} \in \text{END}(E) \otimes K)$

$\bar{D}\bar{\Phi} = 0$

$\mathcal{M}_H^{\circ} \supset T^* \text{BUN}(C, G)$

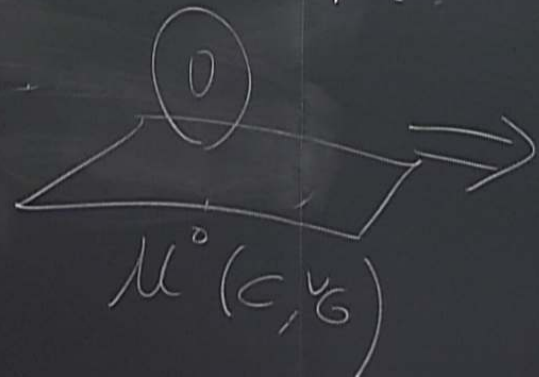
$P(z, \bar{z}) = \det(z - \bar{\Phi}) = z^N + \varphi_{\bar{z}}^{(1)} z^{N-1} + \varphi_{\bar{z}\bar{z}}^{(2)} z^{N-2} + \varphi_{\bar{z}\bar{z}\bar{z}}^{(3)} z^{N-3}$

$\bar{D}v = a_{\bar{z}} v \quad \bar{\partial} \varphi^{(1)} = 0$



$(\Sigma, L) \Leftrightarrow (\bar{\Phi}, A_{\bar{z}})$

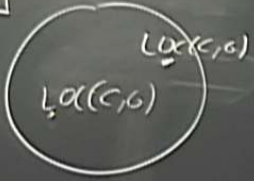
$\mathcal{M}^{\circ}(C, G)$



TOPOLOGICAL
TWIST

GEOMETRIC
LANGSMAUDS

3]

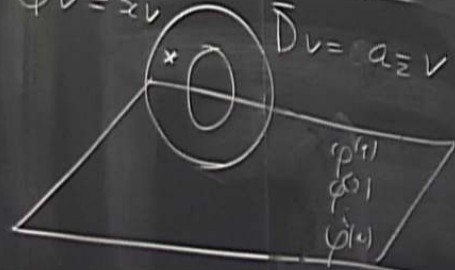


$$\bar{\partial} = 0 \quad (A_{\Sigma}, \Phi) \quad (E, \Phi \in \text{END}(E) \oplus K)$$

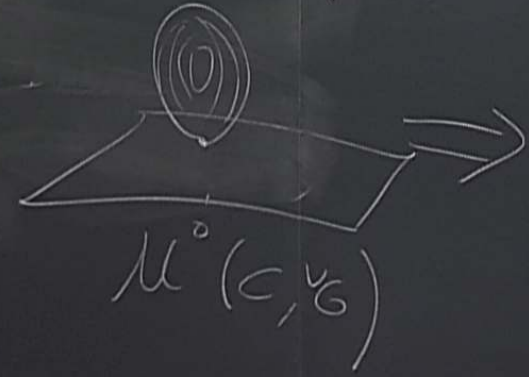
$$\bar{\partial} \Phi = 0 \quad \mathcal{M}_H^{\circ} \supset T^* \text{BUN}(C, G)$$

$$P(z, z) = \det(z - \Phi) = z^N + \varphi_z^{(1)} z^{N-1} + \varphi_{zz}^{(2)} z^{N-2} + \varphi_{zzz}^{(3)} z^{N-3}$$

$$\Phi v = z v \quad \bar{\partial} v = a_{\Sigma} v \quad \bar{\partial} \varphi^{(1)} = 0$$



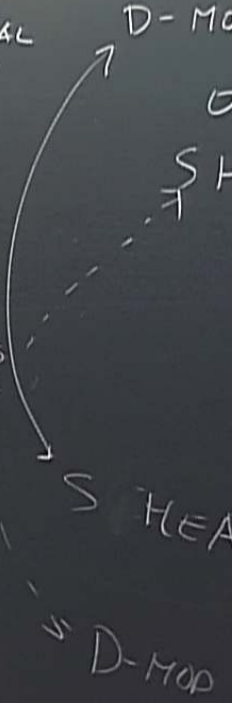
$$(\Sigma, L) \Leftrightarrow (\Phi, A_{\Sigma}) \quad \mathcal{M}^{\circ}(C, G)$$



$$\mathcal{M}^{\circ}(C, G)$$

TOPOLOGICAL
TWIST

GEOMETRIC
LANGLANDS



$\bar{\partial} + \bar{\partial} \bar{\partial}$

$U(N)$

$$P(z, z) = 0$$

