Title: Chiral algebra and BPS spectrum of Argyres-Douglas theories from M5-branes.

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Abstract: <p>We study chiral algebras associated with Argyres-Douglas theories engineered from M5 brane. For the theory engineered using 6d (2,0) type J theory on a sphere with a single irregular singularity (without mass parameter), its chiral algebra is the minimal model of W algebra of JJ type. For the theory engineered using an irregular singularity and a regular full singularity, its chiral algebra is the affine Kac-Moody algebra of JJ type. We can obtain the Schur and Hall-Littlewood index of these theories by computing the vacua character of the corresponding chiral algebra.</p>

Chiral Algebra and BPS Spectrum of Argyres-Douglas theories

Wenbin Yan
CMSA, Harvard University
Perimeter Institute, Oct 11 2016

Based on:
D. Xie, WY and S-T Yau, 1604.02155
J. Song, D. Xie and WY, work in progress

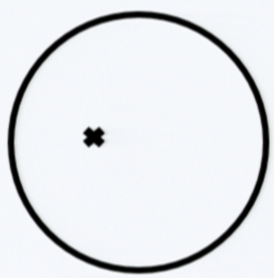
AD theories from M5 branes

Intrinsically **strongly coupled**, original examples are isolated fixed points of certain supersymmetric gauge theories (pure SU(3)) [AD95]

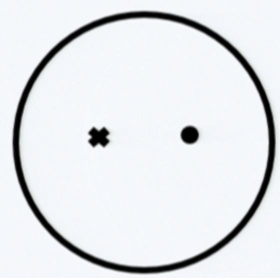
- Coulomb branch operators: fractional dimension
- Wall-crossing phenomenon

M5 brane construction:

- Compactify 6d (2,0) type-J theory on a sphere with irregular puncture + at most one regular puncture (SCFT) [GMN09, BMT12, Xie13, WX15]



$\times R^{3,1}$



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AD theories from M5 branes

- 6d (2,0) theory on Riemann surface and Hitchin system
- Sphere with irregular puncture
  - Irregular puncture: $\Phi \sim \frac{T}{z^{2+k/b}} + \cdots$
  - Classified by $J^b[k]$, J=ADE,
- Sphere with irregular puncture and regular puncture
  - Irregular puncture: $\Phi \sim \frac{T}{z^{2+k/b}} + \cdots$
  - Regular puncture: Y
  - Classified by $(J^b[k], Y)$, J=ADE
- k, b are integers, b usually has a few choices, k may be integers coprime with b

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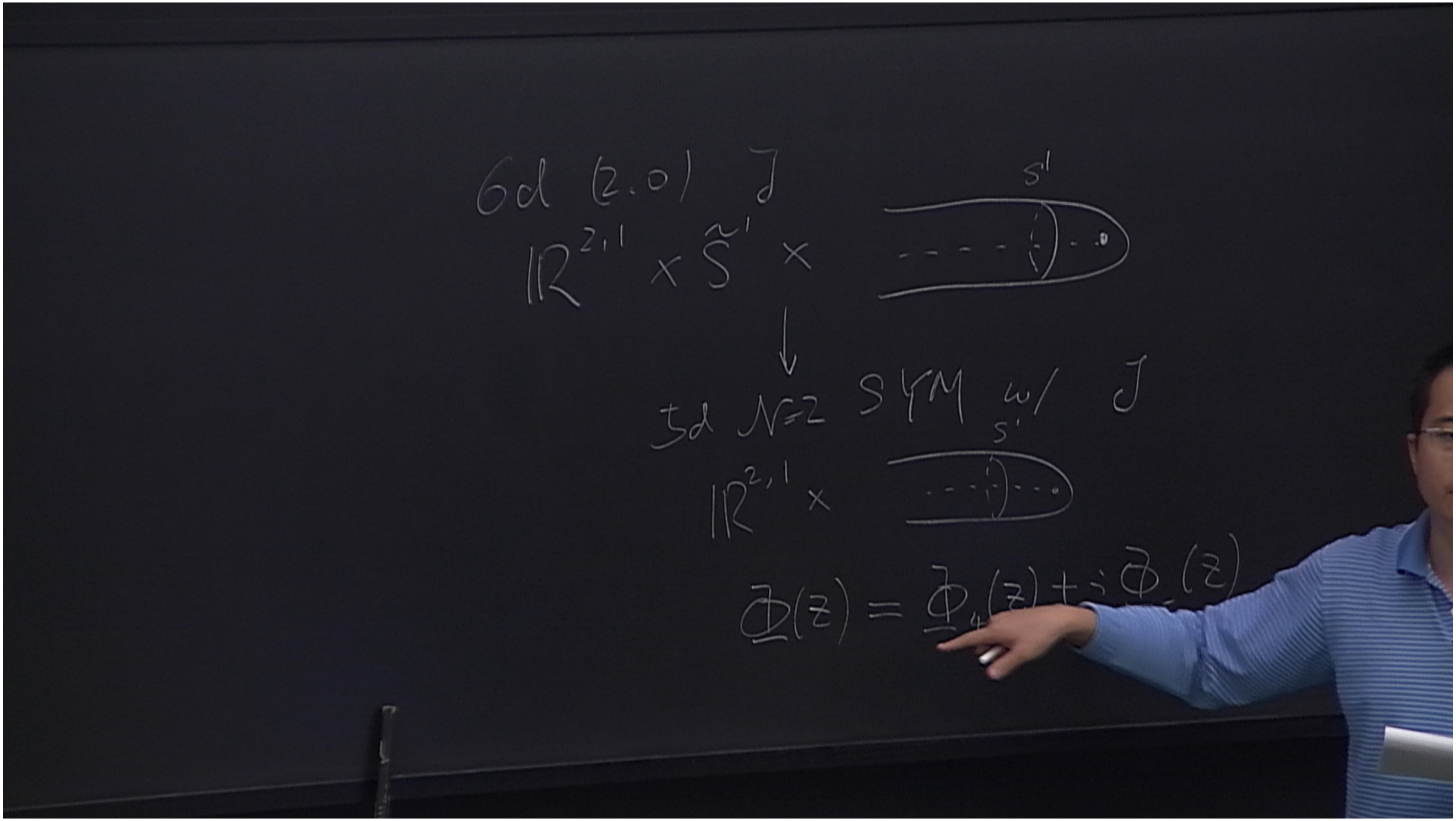
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S'
IR^{2,1} × S̃' ×
3d N=2 SYM w/ J
S'
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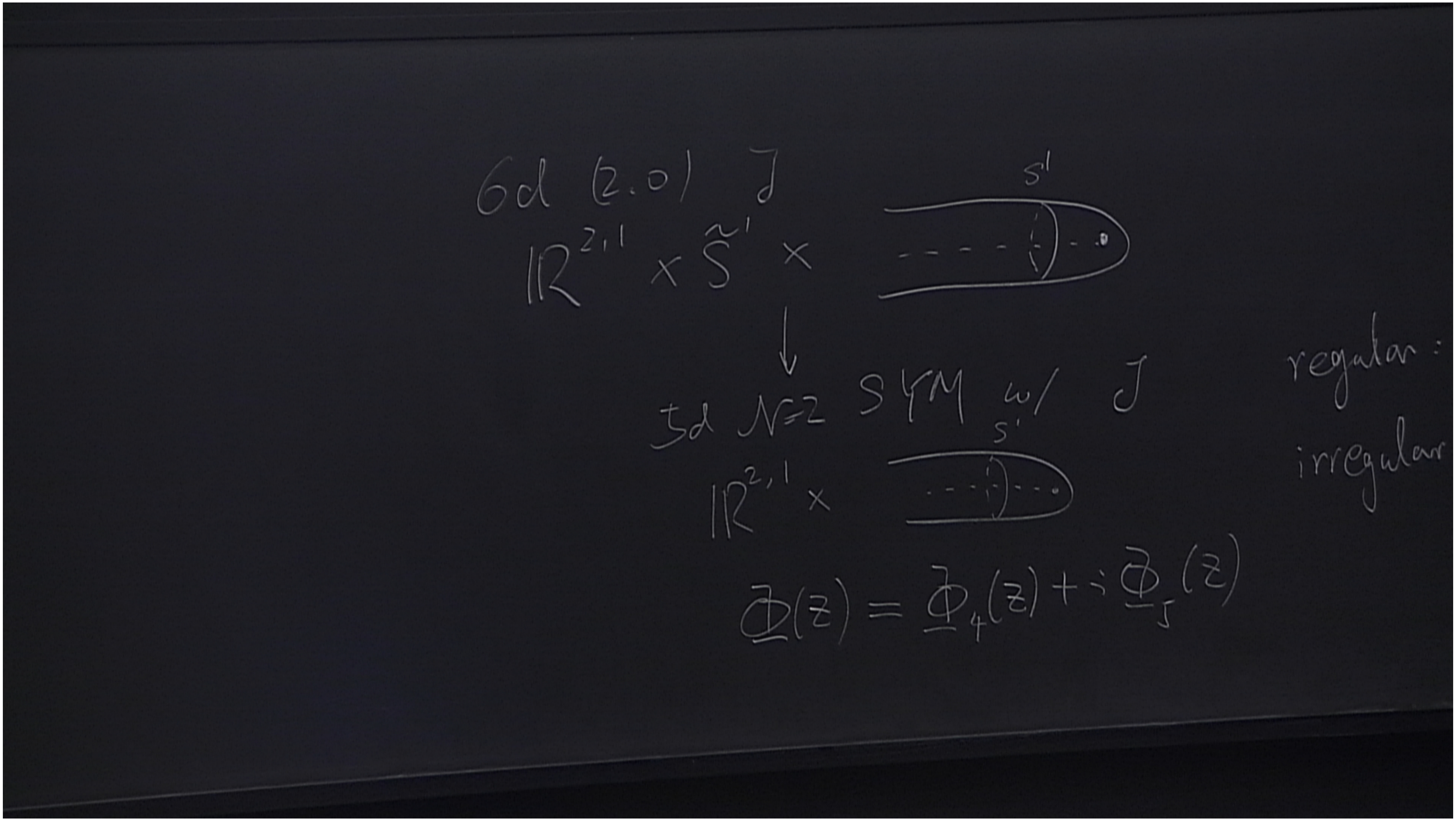
6d (2,0) J
S'
IR^{2,1} × S̃' ×
3d N=2 SYM w/ J
S'
IR^{2,1} ×
Φ(z) = Φ_4(z) + i Φ_5(z)
regular:
irregular

BPS spectrum of AD theories

- Our tool: (limit of) superconformal index

  Refined Witten index, only counts states saturate certain BPS condition, Invariant under RG

- Schur index for N=2 SCFTs

  $I = Tr(-1)^F q^{E-R}$, trace over BPS states $E - j_1 - j_2 - 2R = 0$, $r + j_1 - j_2 = 0$

  $\hat{C}$ (Stress tensor, higher spin current,...), $\hat{B}$ (moment maps,...),...

  No Coulomb branch operators

- Usually easy to compute for Lagrangian theories or theories dual to Lagrangian theories. Need new ways for AD theories. Guess work involved

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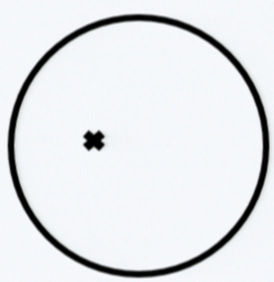
Schur index and chiral algebra

Two important properties

1. $c_{2d} = -12c_{4d}$, $k_{2d} = -k_F$
2. The (normalized) vacuum character of 2d chiral algebra is the 4d Schur index

AD theories without flavor symmetry

- 6d (2,0) ADE theory compactified on a sphere with irregular puncture
- Irregular puncture: $\Phi \sim \frac{T}{z^{2+k/b}} + \cdots$
- Classified by $J^b[k]$, J=ADE



$\times R^{3,1}$

AD theories without flavor symmetry

- AD theory $J^b[k]$
- 2d chiral algebra: **diagonal coset model**
- $A = \frac{g_l \oplus g_1}{g_{l+1}}$, with g=J, $l = -\frac{hk-b}{k}$
- $c_{2d} = \frac{l \dim J}{l+h} + \frac{\dim J}{1+h} - \frac{(l+1)\dim J}{l+1+h} = -12c_{4d}$
- h is dual Coexter number
- Schur index = vacuum character

$A_{N-1}^{N-1}[k]$	$\frac{(Nk-N+1)(N+k+Nk-1)}{12(N-1+k)}$
$D_N^N[k]$	$\frac{((N-1)2k-N)(N+k(2N-1))}{12(k+N)}$
$E_6^9[k]$	$\frac{(4k-3)(13k+9)}{6(9+k)}$
$E_7^{18}[k]$	$\frac{7(k-1)(19k+18)}{12(18+k)}$
$E_8^{30}[k]$	$\frac{2(k-1)(30+31k)}{3(30+k)}$
$E_8^{20}[k]$	$\frac{(3k-2)(20+31k)}{3(20+k)}$

AD theories without flavor symmetry: Example

- $A_{N-1}^{N}[k] = (A_{N-1}, A_{k-1})$
- 2d chiral algebra: W(k,k+N) minimal model
- Schur index = vacuum character = $PE\left[\frac{(q-q^k)(q-q^N)}{(1-q)^2(1-q^{k+N})}\right] = PE\left[\frac{(q^2+\cdots+q^N)}{1-q} - \cdots\right]$, with $PE[x] = \exp[\sum_{n=1}^{\infty}\frac{1}{n}x^n]$
- **Symmetric under the exchange of k and N**, two realization leads to the same SCFT
- The theory has N-1 operators with E-R=2,..., N. q^2 represents the Stress tensor.
- One can also read the relations between operators

AD theories with flavor symmetry

- Consider only full puncture F
- Classified by $(J^b[k], \mathrm{F})$
- Kac-Moody algebra A_{-k_F}
- A=J, $c_{2d} = -\frac{k_F \dim A}{h-k_F} = -12\, c_{4d}$
- $k_F = h - \frac{b}{b+k}$
- Schur index = vacuum character (Kac-Wakimoto)

Theory	c_{4d}	k_F
$(A_{N-1}^{N}[k], F)$	$\frac{1}{12}(N+k-1)(N^2-1)$	$\frac{N(N+k-1)}{N+k}$
$(A_{N-1}^{N-1}[k], F)$	$\frac{(N+1)[N^2+N(k-2)+1]}{12}$	$\frac{(N-1)^2+kN}{N+k-1}$
$(D_{N}^{2N-2}[k], F)$	$\frac{1}{12}N(2N-1)(2N+k-3)$	$\frac{(2N-2)(2N+k-3)}{2N-2+k}$
$(D_{N}^{N}[k], F)$	$\frac{(2N-1)[2k(N-1)+N(2N-3)]}{12}$	$\frac{2k(N-1)+N(2N-3)}{N+k}$

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$(E_6^{12}[k], F)$	$\frac{13(k+11)}{2}$	$\frac{12(k+11)}{k+12}$
$(E_6^{9}[k], F)$	$\frac{13}{6}(33+4k)$	$12 - \frac{9}{k+9}$
$(E_6^{8}[k], F)$	$\frac{13}{4}(22+3k)$	$12 - \frac{8}{k+8}$
$(E_7^{18}[k], F)$	$\frac{133}{12}(17+k)$	$\frac{18(k+17)}{k+18}$
$(E_7^{14}[k], F)$	$\frac{19}{12}(119+9k)$	$18 - \frac{14}{k+14}$
$(E_8^{30}[k], F)$	$\frac{62}{3}(29+k)$	$\frac{30(k+29)}{k+30}$
$(E_8^{24}[k], F)$	$\frac{31}{6}(116+5k)$	$30 - \frac{24}{k+24}$
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AD theories with flavor symmetry

- Classified by $(J^b[k], \mathrm{F})$
- Kac-Moody algebra A_{-k_F}
- A=J, $c_{2d} = -\frac{k_F \dim A}{h - k_F} = -12\, c_{4d}$
- At b=h, we have

$$I = PE[\frac{q - q^{b+k}}{(1-q)(1-q^{b+k})} \chi^F_{adj}(z)]$$

AD theories with flavor symmetry: Examples

$A_4^5[-3] = D_2[SU(5)]$, Kac-Moody algebra is $su(5)_{-\frac{5}{2}}$

Schur index = $PE\left[\frac{q}{1-q^2}\chi_{adj}^{SU(5)}\right] = PE[\frac{q(1-q+q^2-q^3+\cdots)}{1-q}\mathbf{24} + q^2 - q^2]$

- Higgs branch operators O (moment maps) in adjoint (**24**) rep of SU(5); Stress tensor; etc
- $O_{\mathbf{24+1}}^2 = 0$ which is the Joseph relation of the Higgs branch

Structure of the Higgs branch is **encoded** in the Schur index

- Direct obtain the Higgs branch from the chiral algebra

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Higgs branch from Chiral Algebra

AD theory $(J^b[k], \mathrm{F})$, Kac-Moody algebra J_{-k_F}

The Higgs branch is determined by the associated variety X of th Kac-Moody algebra J_{-k_F}

- For $k_{2d} = -h + \frac{p}{q}$, $p \geq h$, associated variety X is certain nilpotent orbit $\bar{O}_q$

  Details see Arakawa[2015], Beem-Rastelli

In our case, for b=h, the Higgs branch is determined by the nilpotent orbit $\bar{O}_q$

theory	k_{2d}	admissible?
$(A_{N-1}^N[k], F)$	$-N + \frac{N}{N+k}$	o
$(A_{N-1}^{N-1}[k], F)$	$-N + \frac{N-1}{N+k+1}$	x
$(D_N^{2N-2}[k], F)$	$-(2N-2) + \frac{2N-2}{2N-2+k}$	o
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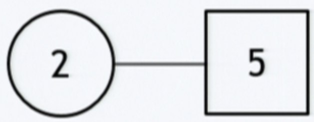
Higgs branch from Chiral Algebra

- $\left(J^b[k], \mathrm{F}\right)$, Kac-Moody algebra J_{-k_F}
- For $k_{2d} = -h + \frac{p}{q}$, $p \geq h$
  - $q < h$, nilpotent orbit is given by the table (Arakawa[2015])
  - $q \geq h$, nilpotent orbit is just the principal nilpotent orbit $\bar{O}_{prin}$
- $\left(J^h[k], \mathrm{F}\right)$, $k = -h + \frac{h}{h+k}$
- $k \geq 0$, **all AD theories have the same Higgs branch**

g	q	$\bar{O}_q$
sl_n	any	$(q, \dots, q, s), 0 \leq s \leq q-1$
so_{2n}	odd	$(q, \dots, q, s), 0 \leq s \leq q,$ $s\ odd, number\ of\ q\ odd$ $(q, \dots, q, s, 1), 0 \leq s \leq q-1,$ $s\ odd, number\ of\ q\ even$
	even	$(q+1, q, \dots, q, s), 0 \leq s \leq q-1,$ $s\ odd, number\ of\ q\ even$ $(q+1, q, \dots, q, q-1, s, 1), 0 \leq s \leq q-1,$ $s\ odd, number\ of\ q\ even$

Higgs branch from Chiral Algebra

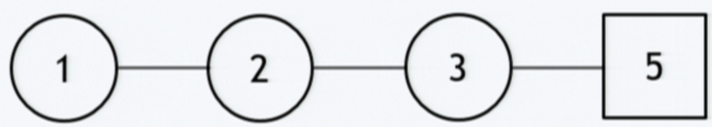
- $A_4^5[-3] = D_2[SU(5)]$, Kac-Moody algebra is $SU(5)_{-\frac{5}{2}}$, $k = -5 + \frac{5}{2}$, $q = 2$
  - Moment maps O in adjoint (**24**) rep of SU(5)
  - $O^2_{\mathbf{24+1}} = 0$ which is the Joseph relation of the chiral ring of Higgs branch
- The nilpotent orbit is given by (2,2,1)



- 2-instanton ADHM quiver for the gauge group SU(5)

Higgs branch from Chiral Algebra

- $A_4^5[-1]$, Kac-Moody algebra is $SU(5)_{-\frac{15}{4}}$, $\quad k = -5 + \frac{5}{4}, \quad q = 4$
- The nilpotent orbit is given by (4,1)



- The Higgs branch of the above quiver gives the nilpotent orbit

Summary

- Understand the BPS spectrum of AD theories using the index
  - Read off protected operators (current, Higgs branch operators…) and their relations
- Compute the index: chiral algebra vs. TQFT
  - AD theories without flavor symmetry: diagonal coset model
  - AD theoires with flavor symmetry: Kac-Moody algebra
  - Higgs branch = associated varieties of Kac-Moody algebra

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Future

- Generalization
  - Arbitrary puncture: Drinfeld-Sokolov reduction
  - Twisted case: AD⇒BC
- Beyond spectrum:
  - Correlation functions between BPS operators
  - Combining with bootstrap and more constraints on AD theories [LL15]
- Thank you