Title: Inflation, Quantum Gravity, and the Weak Gravity Conjecture

Date: Oct 11, 2016 01:00 PM

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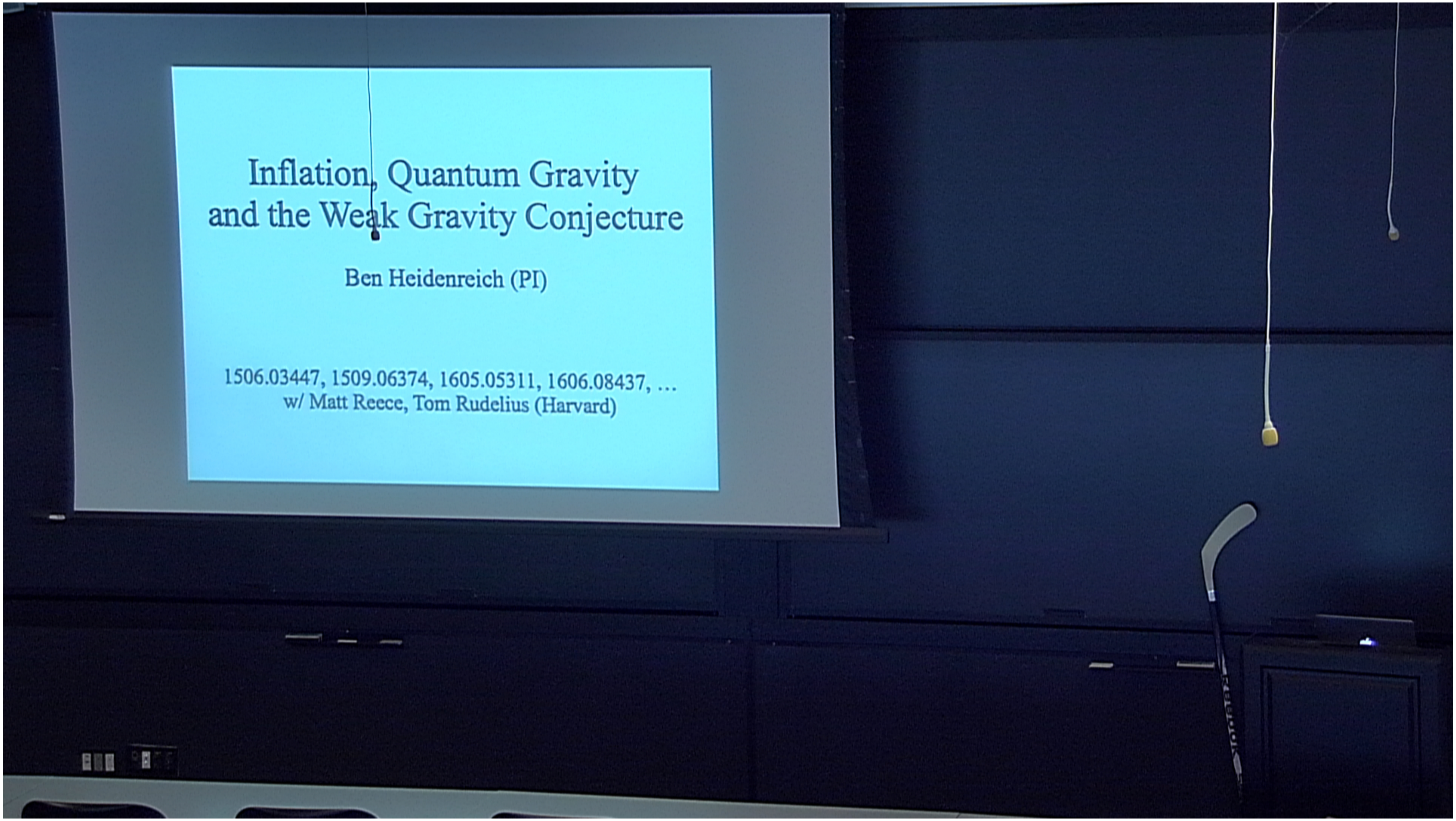
Abstract: <p>Lack of fine tuning in effective field theory does not ensure that a particular scenario is natural or even realizable in a UV complete theory of quantum gravity. Large field axion inflation appears natural from the effective field theory perspective, but I argue that it is tuned from a quantum gravity perspective. The argument is based on the Weak Gravity Conjecture (WGC), a conjectural universal feature of quantum gravity that is present in all known string theory examples. In the process, I highlight recent progress in understanding the WGC and related conjectures about the charged spectrum of quantum gravity and discuss other potential applications, both formal and phenomenological.</p>


Inflation, Quantum Gravity
and the Weak Gravity Conjecture
Ben Heidenreich (PI)
1506.03447, 1509.06374, 1605.05311, 1606.08437, …
w/ Matt Reece, Tom Rudelius (Harvard)

Axion Inflation and the Swampland

The eta problem

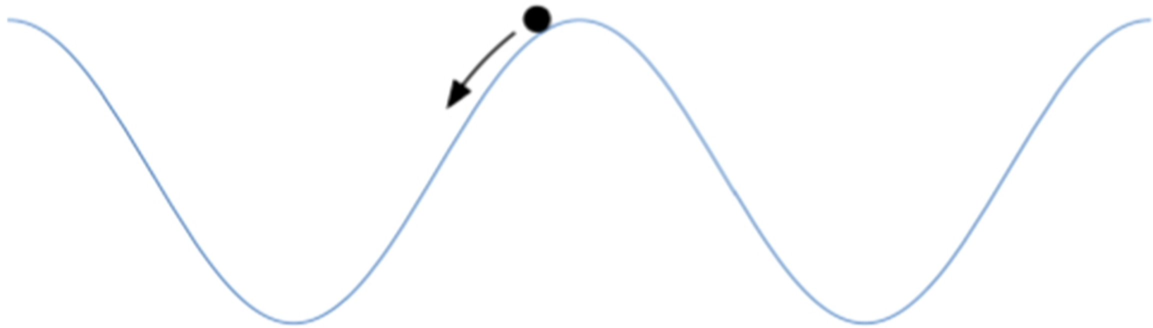
$$\varepsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = M_{\rm pl}^2 \frac{V''}{V} \qquad \varepsilon, |\eta| \ll 1$$

$$V \to \left(1 + \alpha \frac{\phi^2}{M_{\rm pl}^2}\right) V \quad \Longrightarrow \quad \eta \to \eta + 2\alpha$$

Slow roll inflation is UV sensitive!

Natural inflation

<u>V protected by shift symmetry</u>



$$V = V_0\left(1 - \cos\frac{\phi}{f}\right) \qquad \phi \to \phi + 2\pi f$$

Needs $f \gtrsim 5-10\ M_{\mathrm{pl}}$ (Planck results)

2/34

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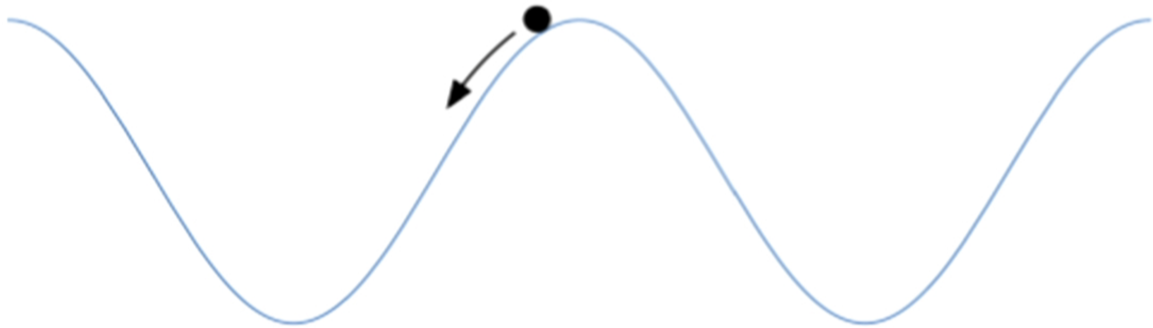

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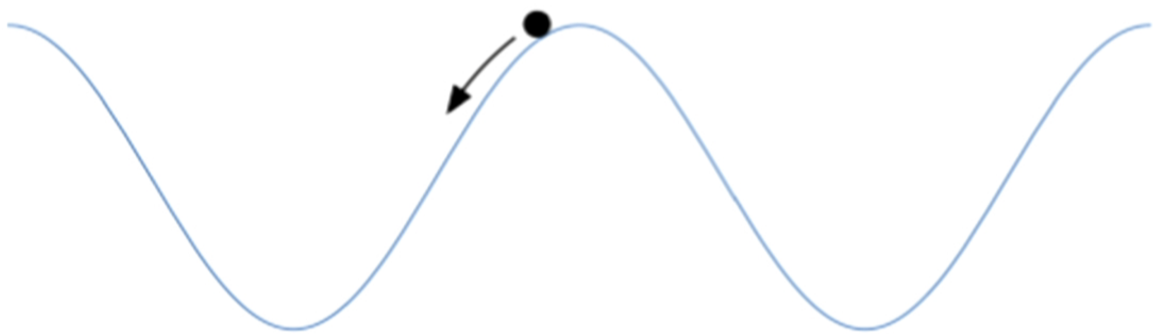

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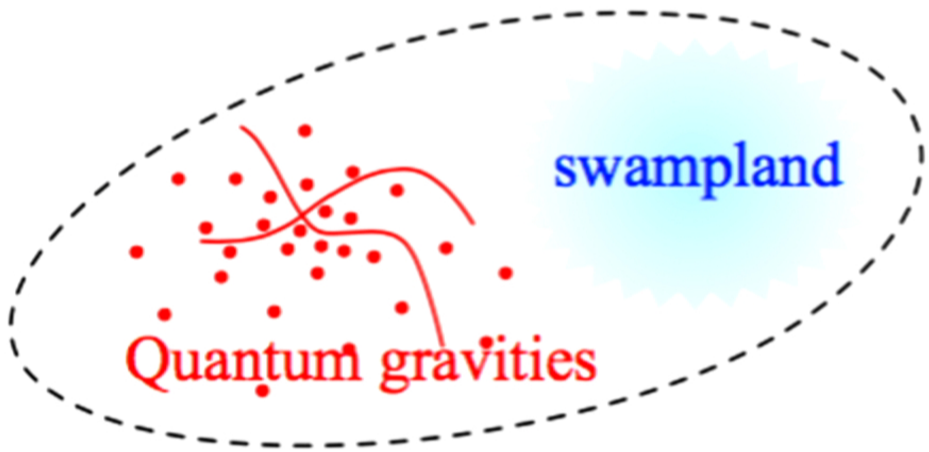
Swampland

In string theory $f \lesssim M_{\rm pl}$!

(Banks, Dine, Fox, Gorbatov '03)

Gravitational Effective Field Theories



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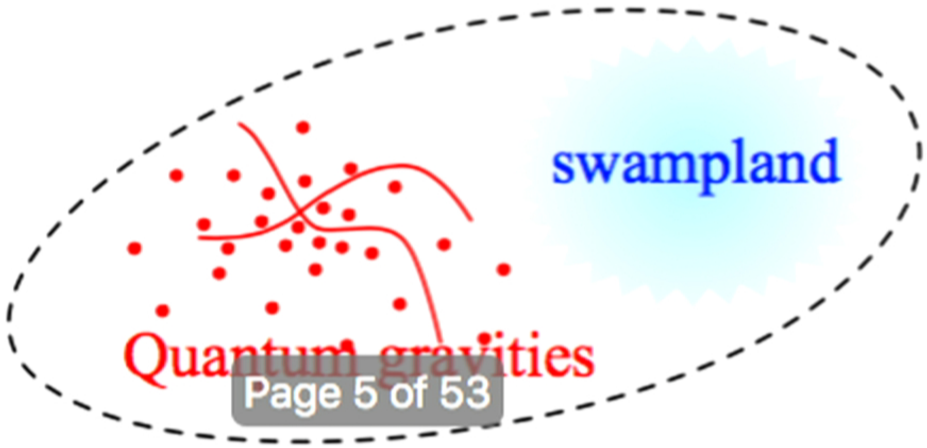
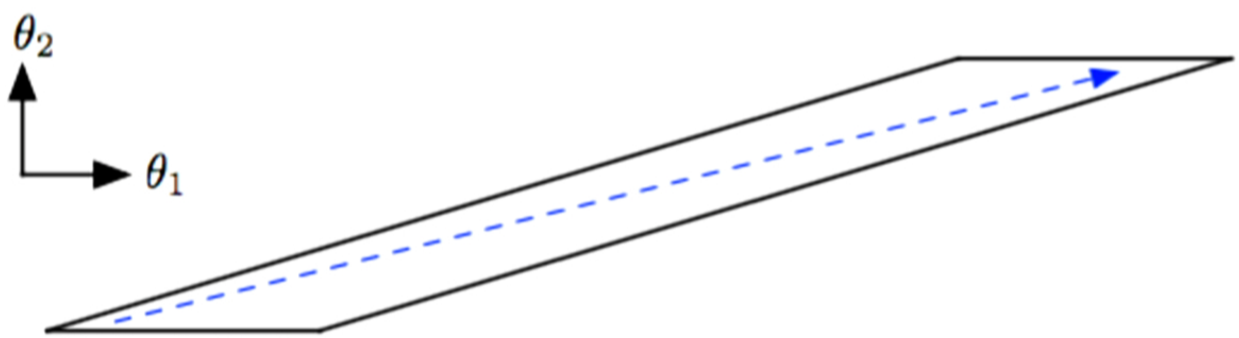


Alignment

Kim, Nilles, Peloso '04



$$V = V_1(1 - \cos\theta_2) + V_2\left[1 - \cos(\theta_1 - N\theta_2)\right]$$

$$f_{\text{eff}} \sim N f_1$$

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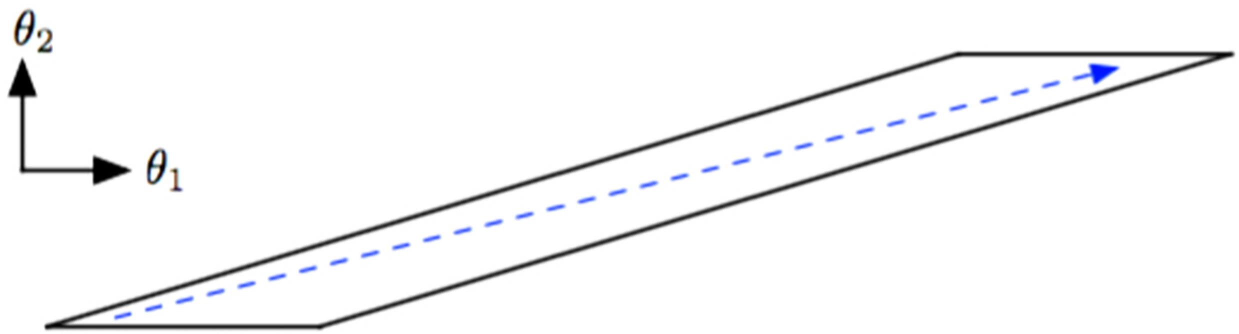
Alignment

Kim, Nilles, Peloso '04



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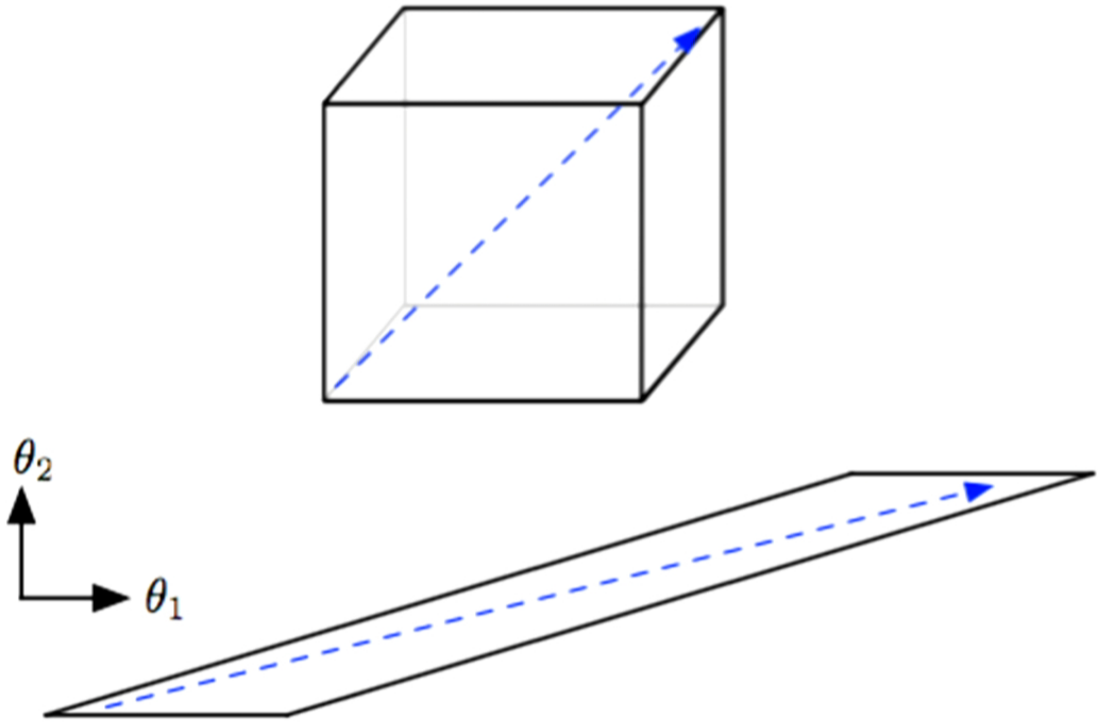
Are these models in the swampland?



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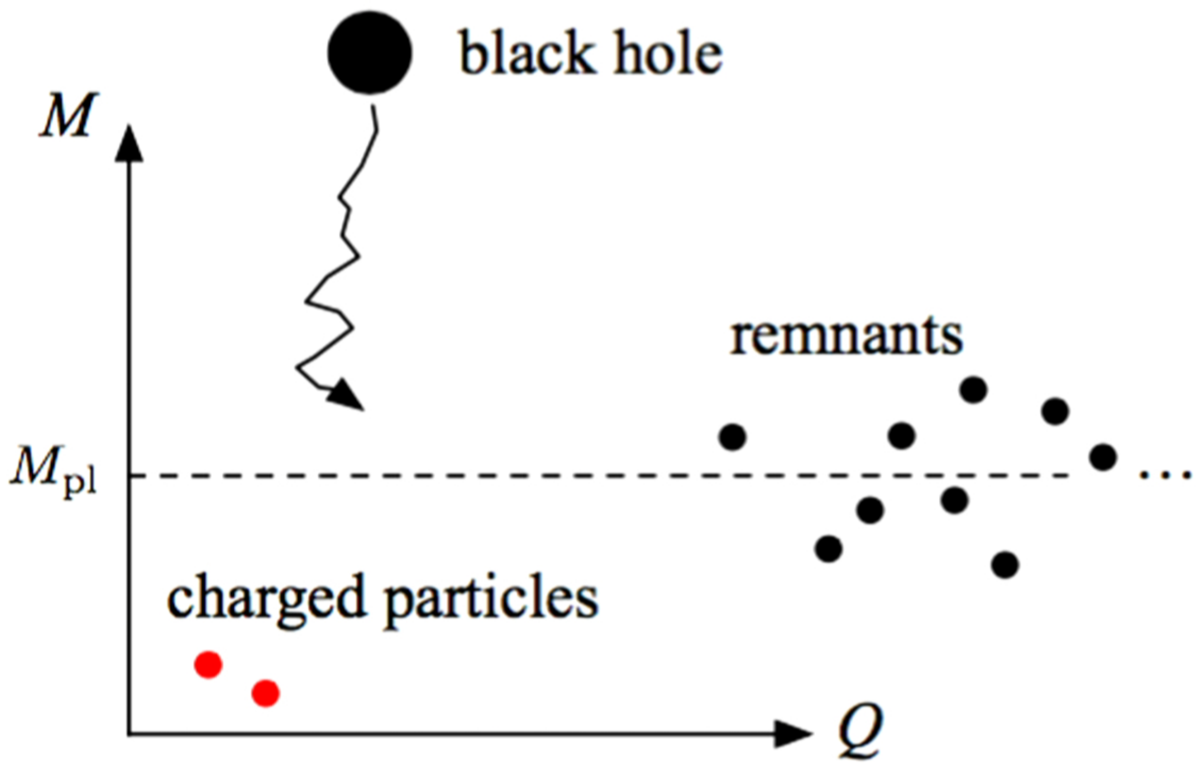
I: No global symmetries



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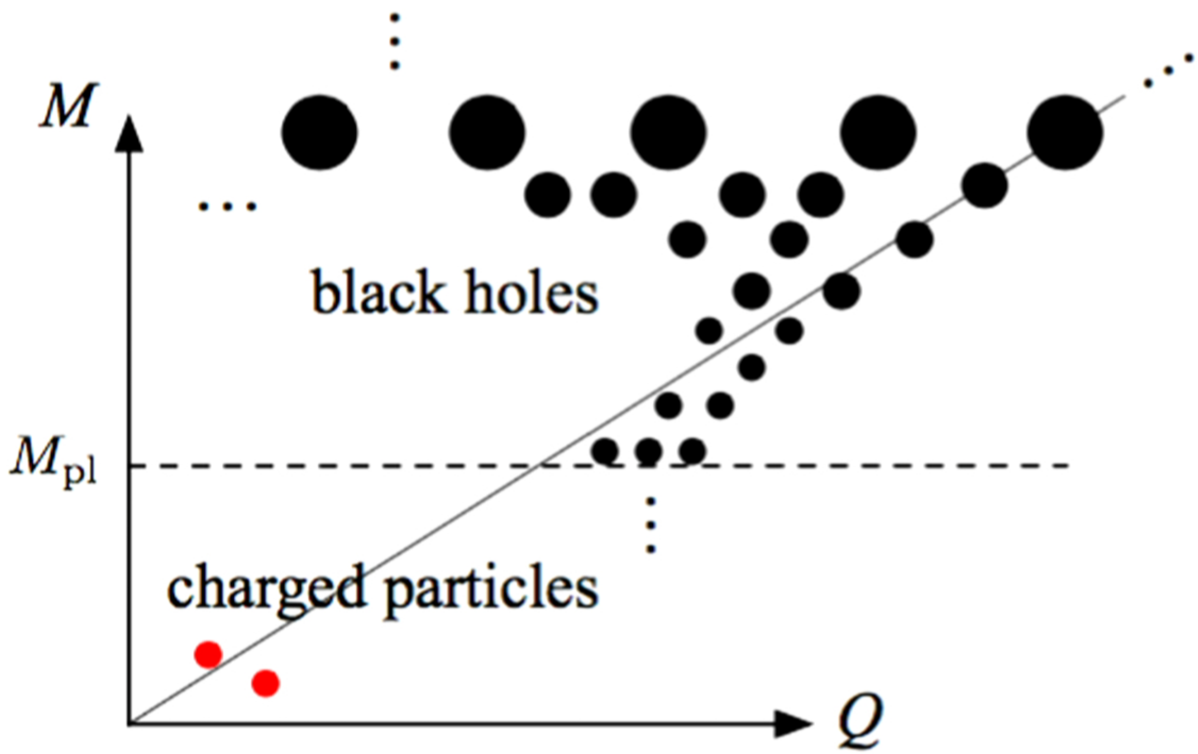
II: Completeness



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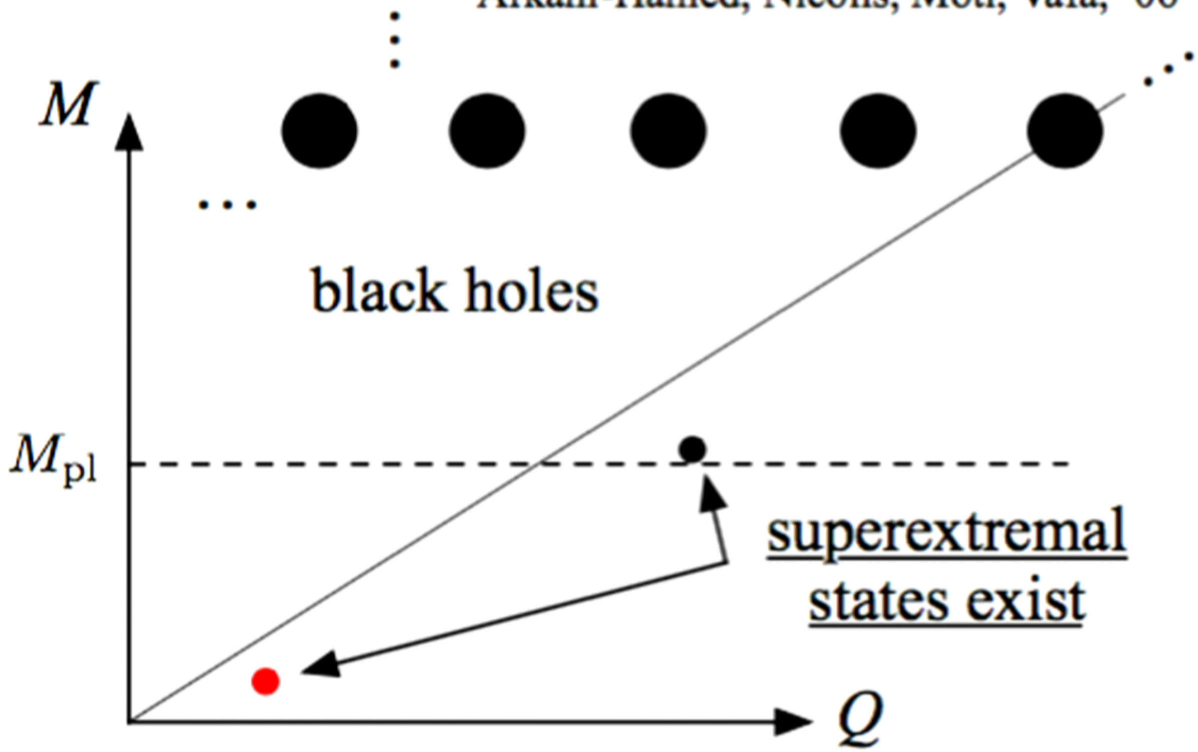
III: "Gravity is the weakest force"

Arkani-Hamed, Nicolis, Motl, Vafa, '06



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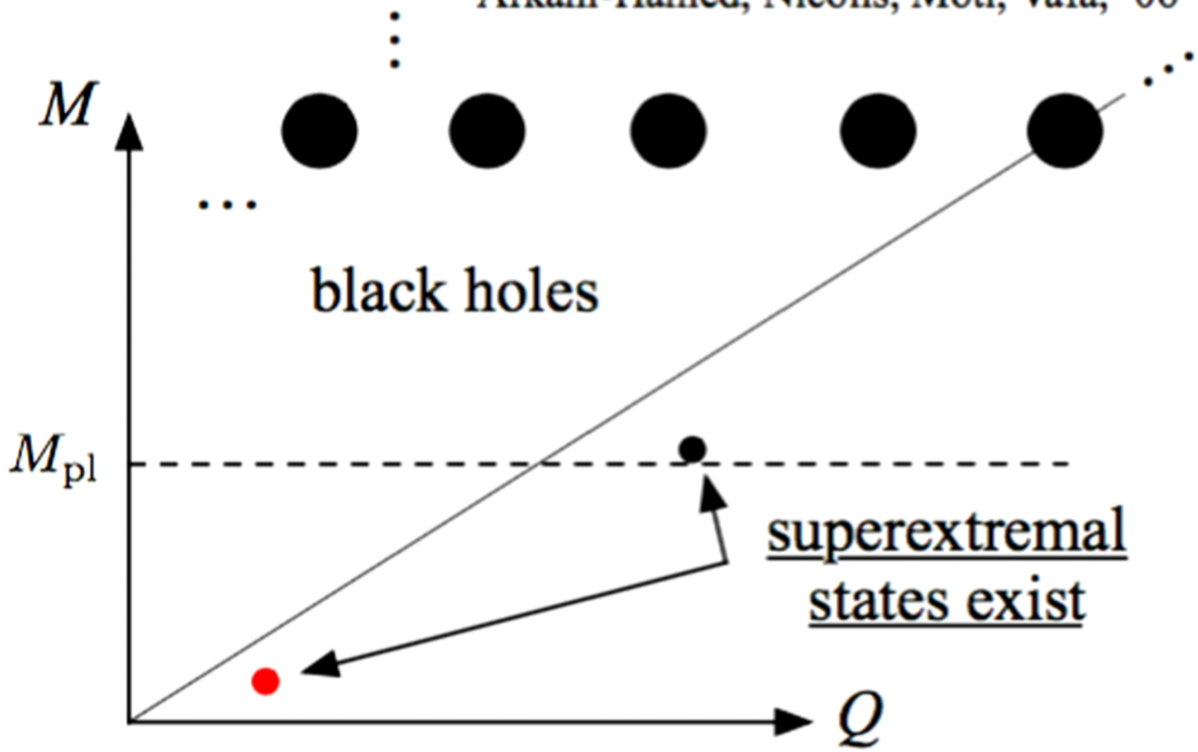

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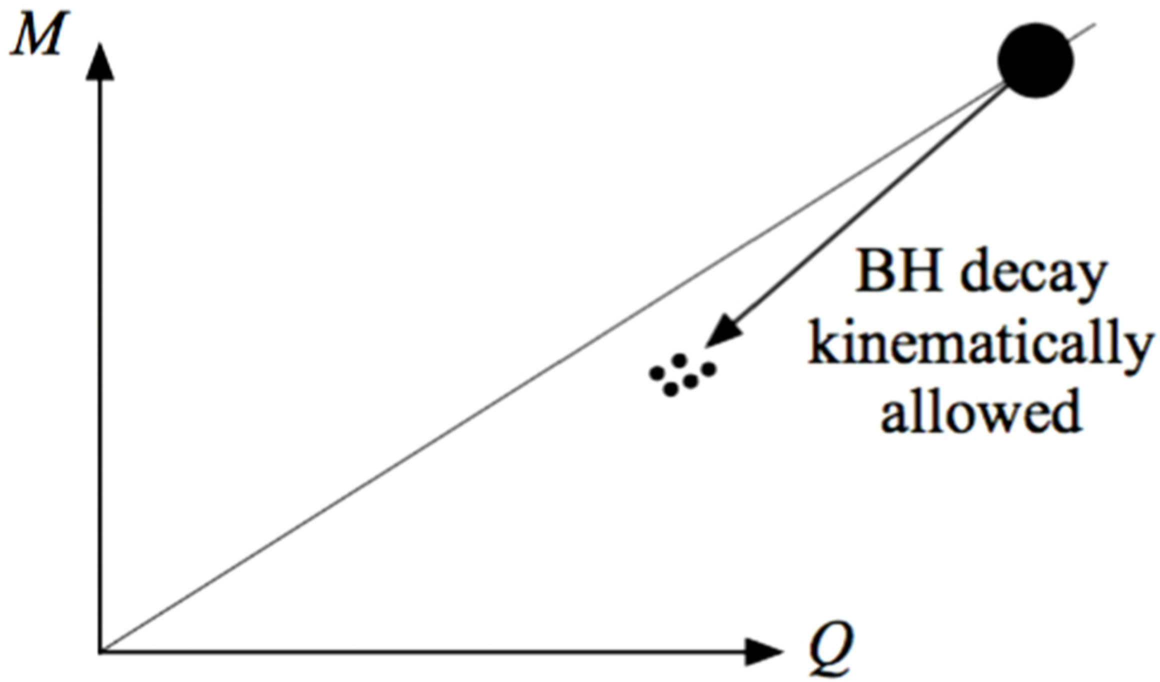

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III: "Gravity is the weakest force"

Arkani-Hamed, Nicolis, Motl, Vafa, '06

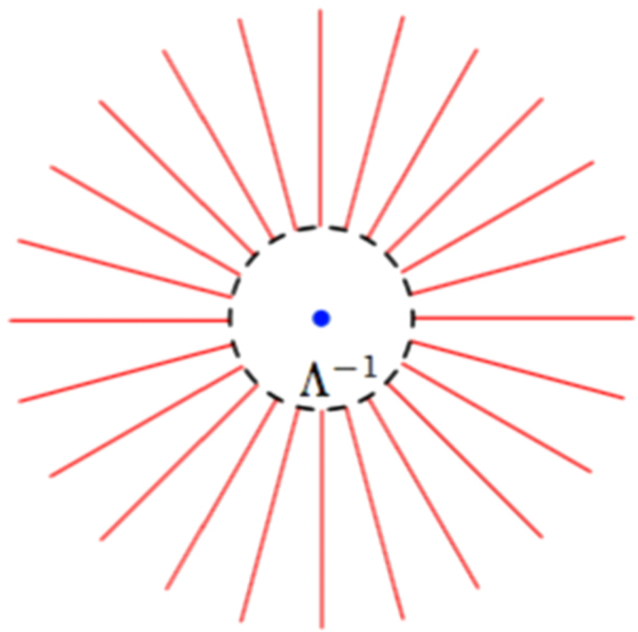
Monopole self-energy

$$m \gtrsim \frac{Q^2}{e^2}\Lambda$$

WGC

$$m < \sqrt{2}\,\frac{Q}{e}M_{\rm pl}$$

$$\Longrightarrow \boxed{\Lambda \lesssim e M_{\rm pl}}$$



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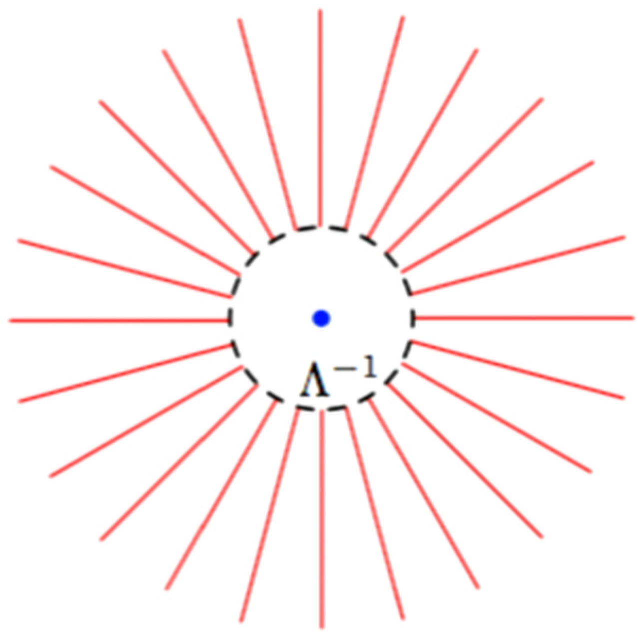

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Why?

- True in all known string theory examples
- Lower bound on *g*:

Very weakly coupled gauge theory

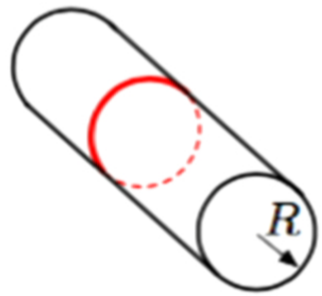
$\approx$

Global symmetry

Axion inflation and the WGC

"Extra natural"

(Arkani-Hamed, Cheng, Creminelli, Randall, '03)



$$\phi = f \oint A$$

$$f = \frac{1}{\sqrt{2\pi R} e_5}$$

$$\text{WGC: } S = 2\pi R m \leq \sqrt{\frac{3}{2}} \frac{Q M_{\mathrm{pl}}}{f}$$

$$S \gtrsim 1 \implies f_{\mathrm{eff}} \lesssim M_{\mathrm{pl}} \qquad (f_{\mathrm{eff}} \equiv f/Q)$$

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Small action loophole

de la Fuente, Saraswat, Sundrum '14

$S \ll 1$ looks uncontrolled…

$$\ldots\text{but} \quad V_n \sim \frac{1}{n^5} e^{-nS} \cos(n\phi/f_{\text{eff}})$$

"Better" argument using magnetic WGC(?)

$$1 \lesssim 2\pi R \Lambda \lesssim 2\pi R e M_{\text{pl}} = \frac{M_{\text{pl}}}{f}$$

$$\Longrightarrow f \lesssim M_{\text{pl}}$$

11/34

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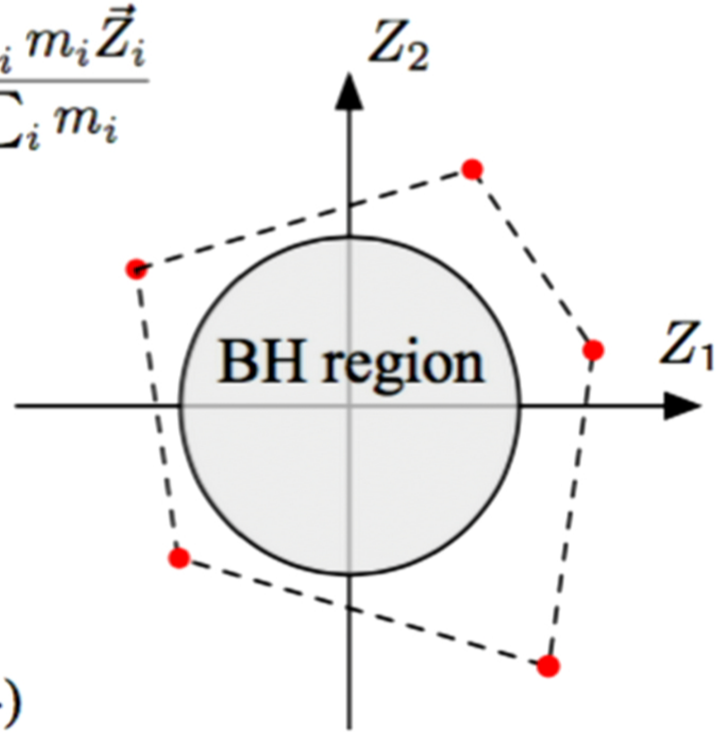
Exploring the WGC

WGC with multiple photons $\vec{Z} \equiv \vec{Q}/m$

$$\vec{Z}_{\rm BH} = \underbrace{\left(\frac{\sum_i m_i}{M_{\rm BH}}\right)}_{\leq 1} \frac{\sum_i m_i \vec{Z}_i}{\sum_i m_i}$$



kinematics of BH decay

(Cheung, Remmen '14)

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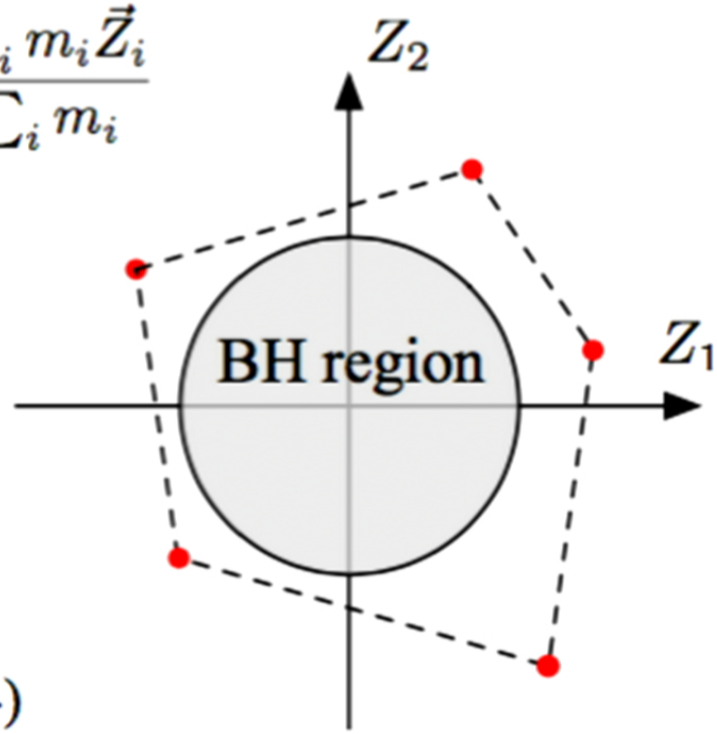


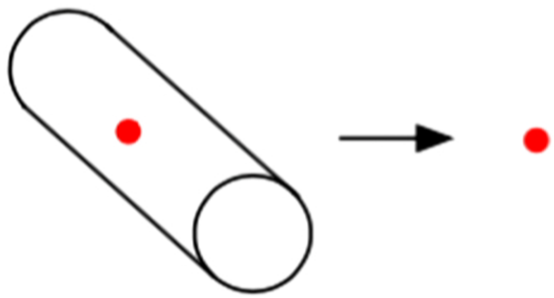
<u>kinematics</u>
<u>of BH decay</u>

(Cheung, Remmen '14)

Black holes in $D > 4$ $\qquad \gamma e^2 Q^2 \leq \kappa^2 M^2$

(e.g., BH, Reece, Rudelius '15)

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left(R - \frac{1}{2}(\nabla\phi)^2\right) - \frac{1}{2e^2}\int d^D x\sqrt{-g}e^{-\alpha\phi}F_2^2$$



$$\gamma = \left[\alpha^2 + \frac{D-3}{D-2}\right]^{-1}$$

$$\gamma_D(\alpha) > \gamma_{D+1}(\alpha)$$

$$\frac{1}{e_D^2} = \frac{2\pi R}{e_{D+1}^2} \qquad \frac{1}{\kappa_D^2} = \frac{2\pi R}{\kappa_{D+1}^2}$$

WGC bound weaker after DR!

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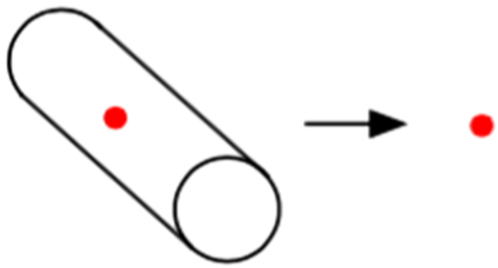
We ignored the radion $\lambda(x)$

$$\mathrm{d}s^2_{D+1} = e^{\frac{\lambda}{D-2}}\mathrm{d}s^2_D + e^{-\lambda}\mathrm{d}y^2$$

$$\alpha^2_D = \alpha^2_{D+1} + \underbrace{\frac{2}{(D-1)(D-2)}}_{\text{radion coupling}}$$



$$\gamma_D(\alpha_D) = \gamma_{D+1}(\alpha_{D+1})!$$

Preserves WGC bound

Renormalization

SUSY: WGC bound preserved by D.R.

$$\gamma_D(\alpha_D) = \gamma_{D+1}(\alpha_{D+1})$$

~~SUSY~~: WGC bound weakened by D.R.

$$\gamma_D(0) > \gamma_{D+1}(0)$$

Moduli stabilization:

WGC bound weakened

$$\gamma_D(0) > \gamma_D(\alpha)$$

UV WGC ⟶ IR WGC

Kaluza-Klein theory

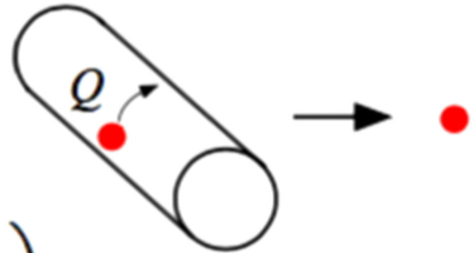
$$\mathrm{d}s^2_{D+1} = e^{\frac{\lambda}{D-2}}\mathrm{d}s^2_D + e^{-\lambda}(\mathrm{d}y + RB)^2$$

$$\frac{1}{e^2_{\mathrm{KK}}} = \frac{R^2}{2\kappa^2_D} \qquad \alpha_{\mathrm{KK}} = \sqrt{\frac{2(D-1)}{D-2}}$$

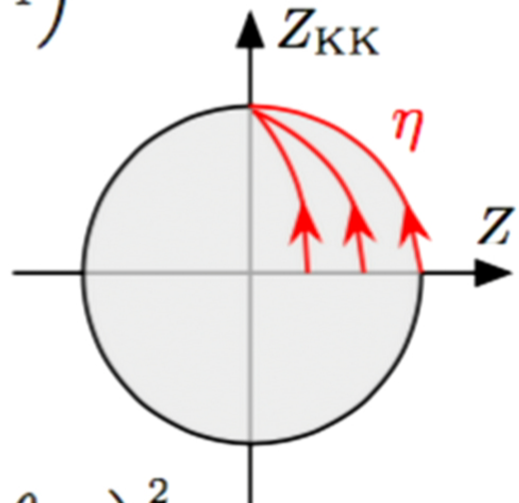
$$M^2 \geq \frac{Q^2_{\mathrm{KK}}}{R^2} \quad \text{v.} \quad m^2_D = m^2_{D+1} + \frac{Q^2_{\mathrm{KK}}}{R^2}$$

WGC saturated by graviton KK modes!

Boosted black holes



$$A_1^{(D+1)} = A_1^{(D)} + \frac{A_0}{2\pi}\left(\frac{\mathrm{d}y}{R} + B_1\right)$$

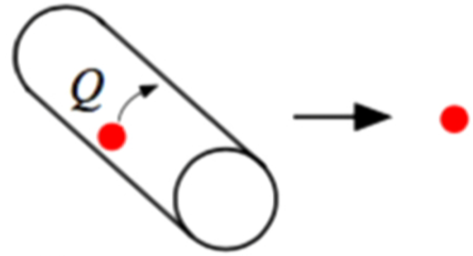


ϕ, λ, A_0 all contribute

$$t \to (\cosh\eta)t + (\sinh\eta)y$$
$$y \to (\sinh\eta)t + (\cosh\eta)y$$

$$M^2 \geq \frac{\gamma e_D^2}{\kappa_D^2}Q^2 + \frac{1}{R^2}\left(Q_{\mathrm{KK}} - \frac{\theta}{2\pi}Q\right)^2$$

WGC with boosts



$$M^2 \geq \frac{\gamma e_D^2}{\kappa_D^2} Q^2 + \frac{1}{R^2}\left(Q_{\rm KK} - \frac{\theta}{2\pi} Q\right)^2$$

v.

$$m_D^2 = m_{D+1}^2 + \frac{1}{R^2}\left(Q_{\rm KK} - \frac{\theta}{2\pi} Q\right)^2$$

$$\vec{Z} = \frac{(\mu Z_{D+1}, xR)}{\sqrt{\mu^2 + x^2}}$$

$$\mu \equiv m_{D+1} R$$

$$x \equiv Q_{\rm KK} - \frac{\theta}{2\pi} Q$$

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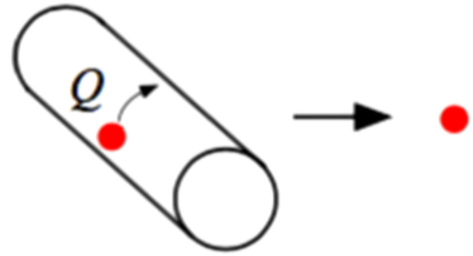
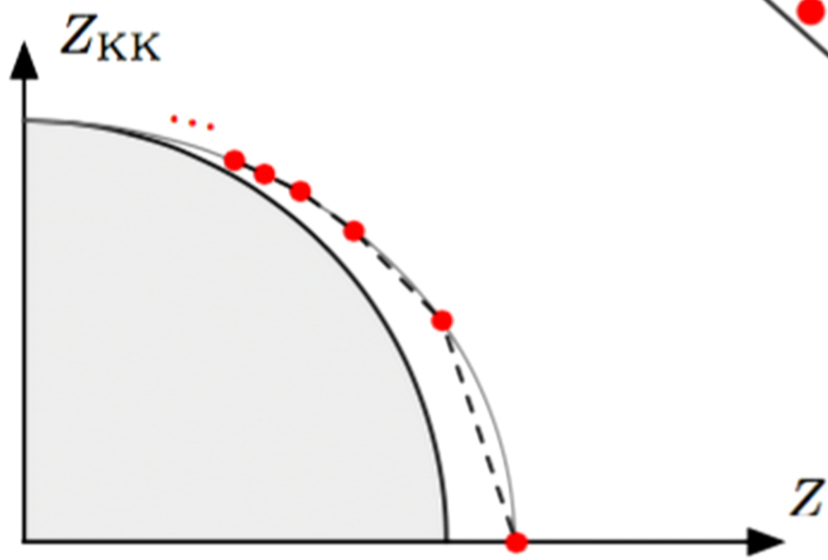
WGC with boosts II





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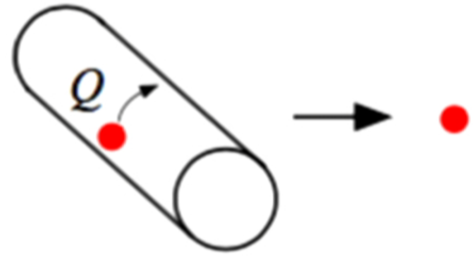

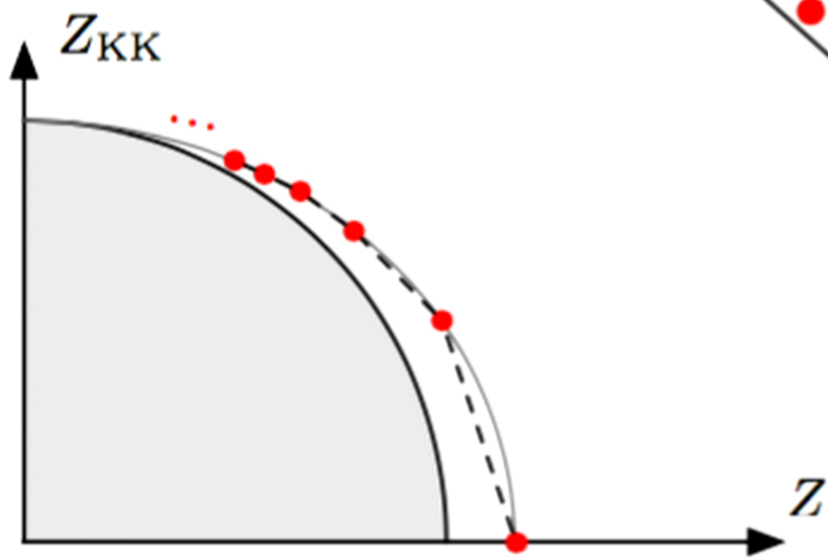

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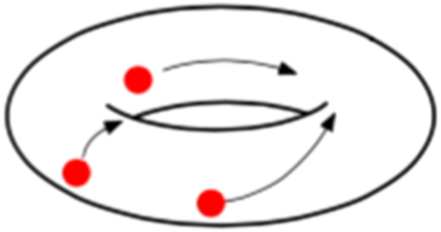
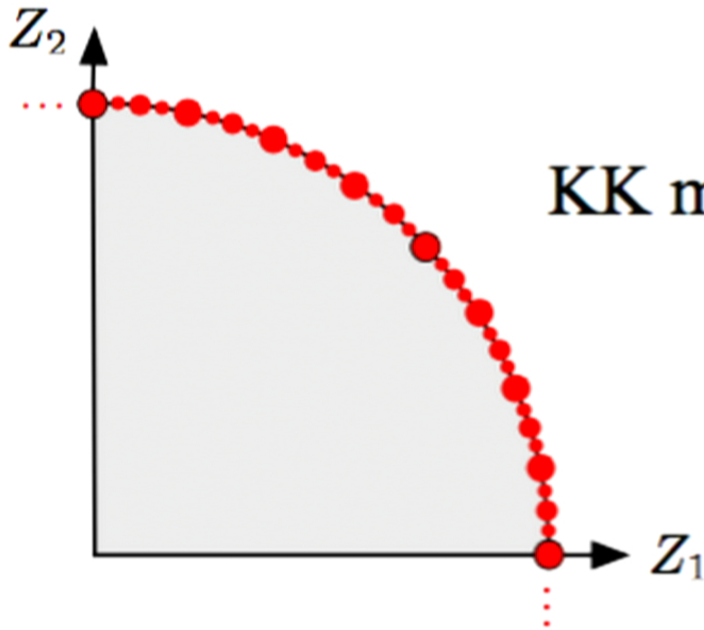
Kaluza Klein, revisited

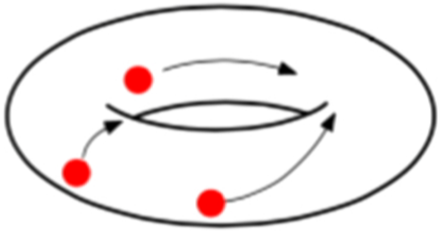


$$m^2 = \frac{Q_1^2}{R_1^2} + \frac{1}{R_2^2}\left(Q_2 - \frac{\theta}{2\pi}Q_1\right)^2$$



KK modes are dense!

Kaluza Klein, revisited



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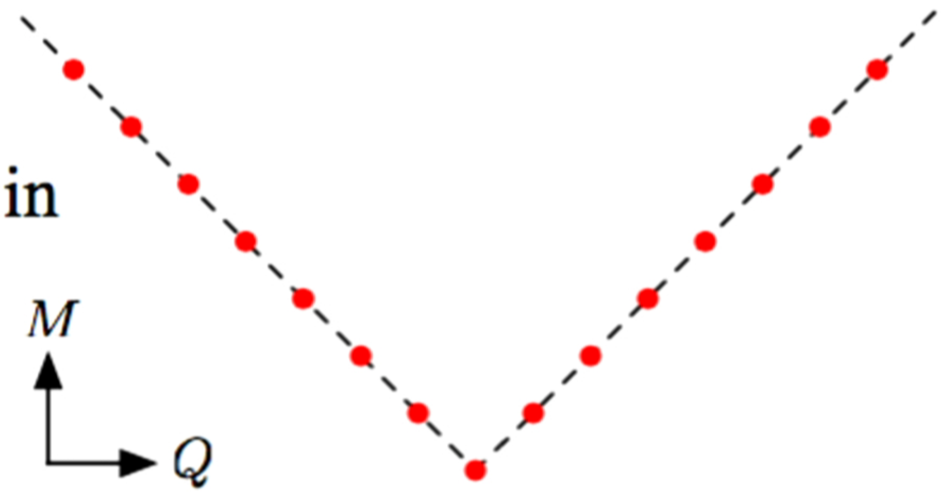
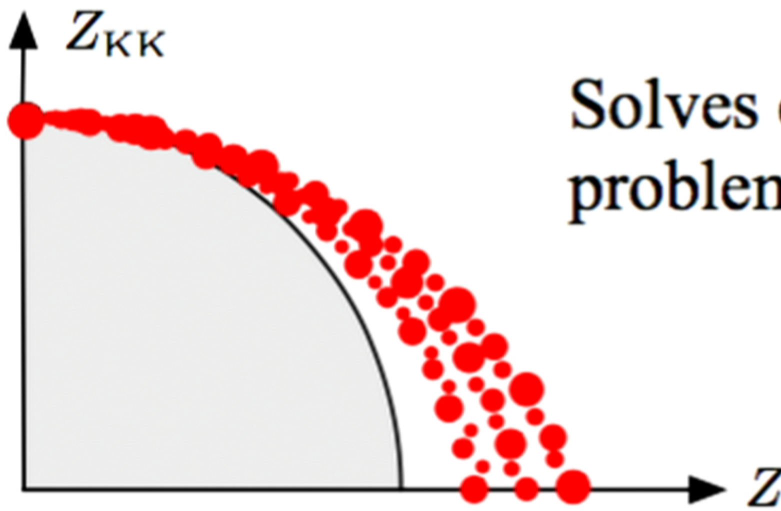
spectrum in
$D+1$



The Lattice Weak Gravity Conjecture

For every $Q \in \Gamma_Q$, $\exists$ a charged particle with $Z \geq Z_{\text{ext}}$



Solves consistency problem!

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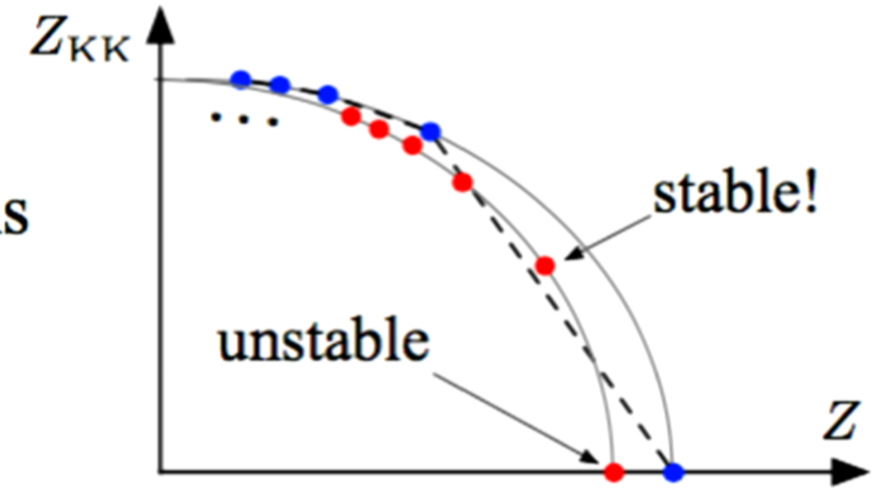
LWGC constrains unstable particles



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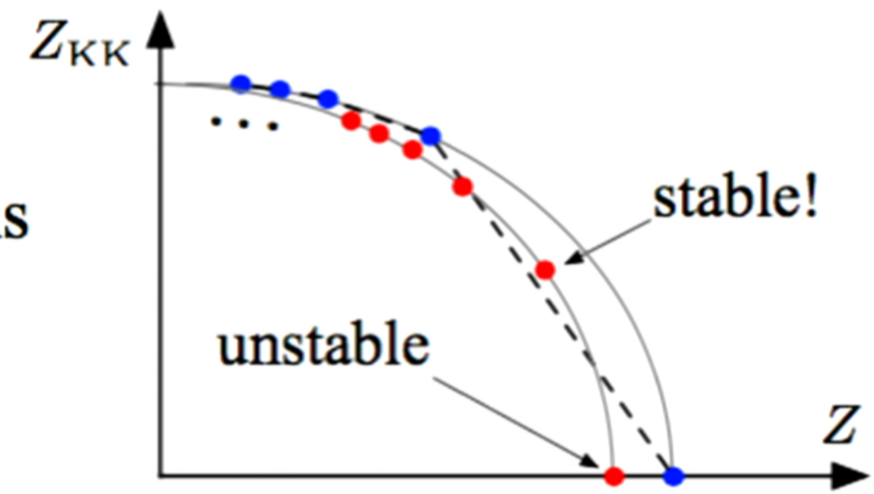

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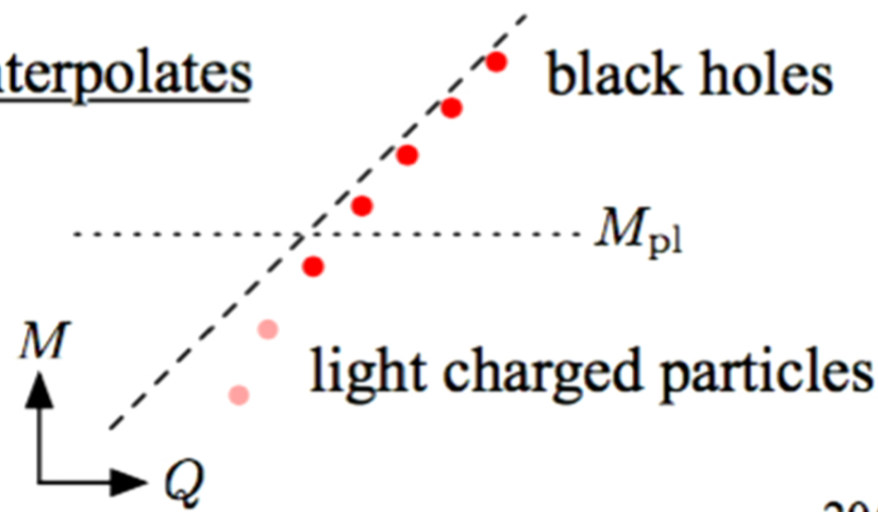
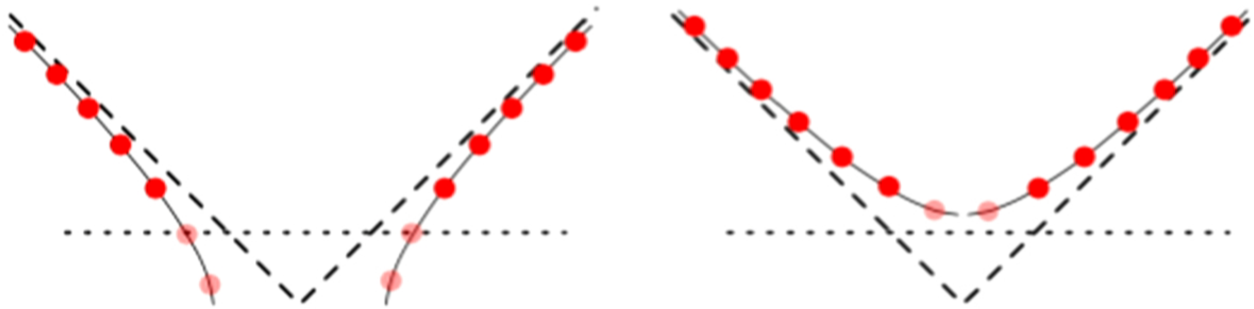
Smoothly interpolates



Black Hole Prediction

$$\mathcal{L}_4^{D=4} = a_1(F_{\mu\nu}F^{\mu\nu})^2 + a_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + a_3 F_{\mu\nu}F_{\rho\sigma}C^{\mu\nu\rho\sigma}$$



$$a_1 - \frac{1}{2}a_3 > 0 \qquad\qquad a_1 - \frac{1}{2}a_3 < 0$$

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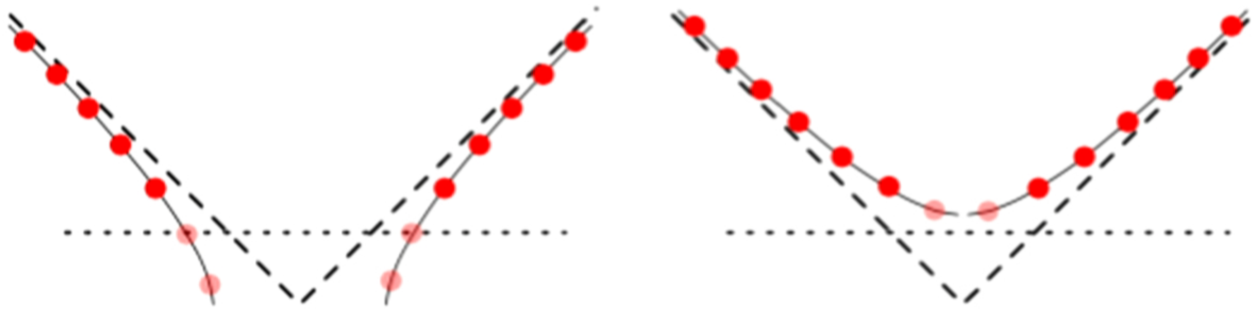

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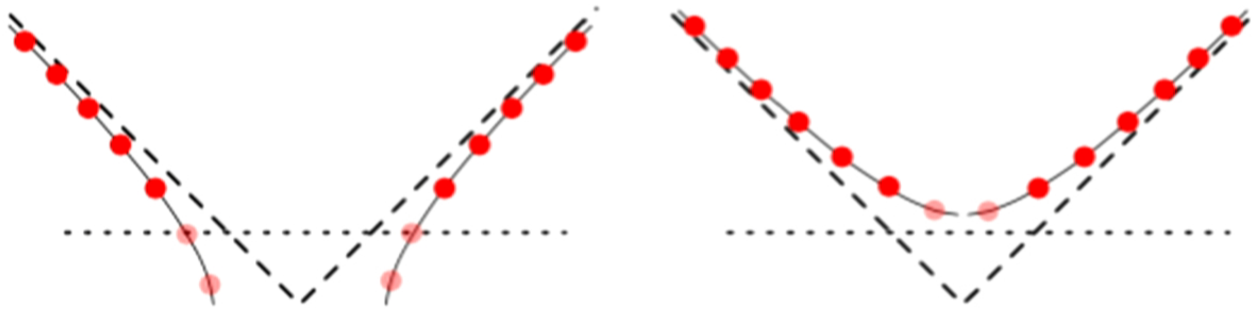

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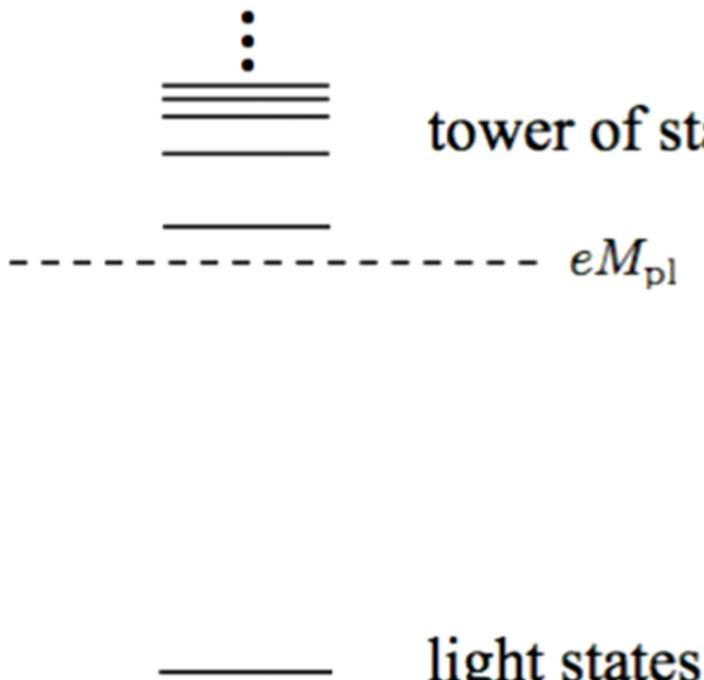
The magnetic WGC revisited



<u>LWGC explains the "cutoff"</u>

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The LWGC is not universal

$\underline{T^3/\mathbb{Z}_2}$

$$\mathbb{Z}_2 : \theta_w \to \theta_w + \pi, \theta_y \to \theta_y + \pi$$
$$\mathbb{Z}_2' : \theta_w \to -\theta_w, \theta_z \to \theta_z + \pi$$

$$\phi(x^\mu, \theta_w, \theta_y, \theta_z) = \sum \phi_{n_w,n_y,n_z}(x^\mu)\, e^{in_w\theta_w + in_y\theta_y + in_z\theta_z}$$

$$\mathbb{Z}_2 : \quad \phi_{n_w,n_y,n_z}(x) = (-1)^{n_w+n_y}\, \phi_{n_w,n_y,n_z}(x),$$
$$\mathbb{Z}_2' : \quad \phi_{n_w,n_y,n_z}(x) = (-1)^{n_z}\, \sigma(\phi)\phi_{-n_w,n_y,n_z}(x)$$

$$n_y \in 2\mathbb{Z}+1 : \quad m^2 \geq \frac{n_y^2}{R_y^2} + \frac{n_z^2}{R_z^2} + \frac{1}{R_w^2}$$

subextremal!

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The LWGC is not universal

$\underline{T^3/\mathbb{Z}_2}$

$$\mathbb{Z}_2 : \theta_w \to \theta_w + \pi, \theta_y \to \theta_y + \pi$$
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subextremal!

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The sub-Lattice WGC (sLWGC)

Generalization: $\frac{T^N \times \widehat{M}_k}{G} \quad G \subset U(1)^N$

$\Gamma_G^* \subseteq \mathbb{Z}^N$ populated by extremal states

> $\exists\ \Gamma_{\text{ext}} \subseteq \Gamma_Q$ of finite index such that for every $Q \in \Gamma_{\text{ext}}$, $\exists$ a charged particle with $Z \geq Z_{\text{ext}}$

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The sub-Lattice WGC (sLWGC)

Follows from worldsheet modular invariance* in perturbative string theory!

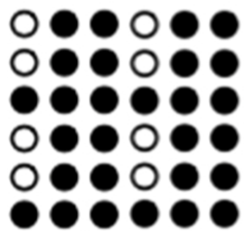
> $\exists\ \Gamma_{\rm ext} \subseteq \Gamma_Q$ of finite index such that for every $Q \in \Gamma_{\rm ext}$, $\exists$ a charged particle with $Z \geq Z_{\rm ext}$

The density of extremal sites

Index of $\Gamma_{\rm ext}$ can be large (e.g., 256)

…but fraction of (super)extremal sites

$\gtrsim 1/2$

Ex: 

(Multiple choices of $\Gamma_{\rm ext}$)

$\overline{\text{sLWGC}}$

$\Gamma_{\not{\rm ext}} \subset \Gamma$

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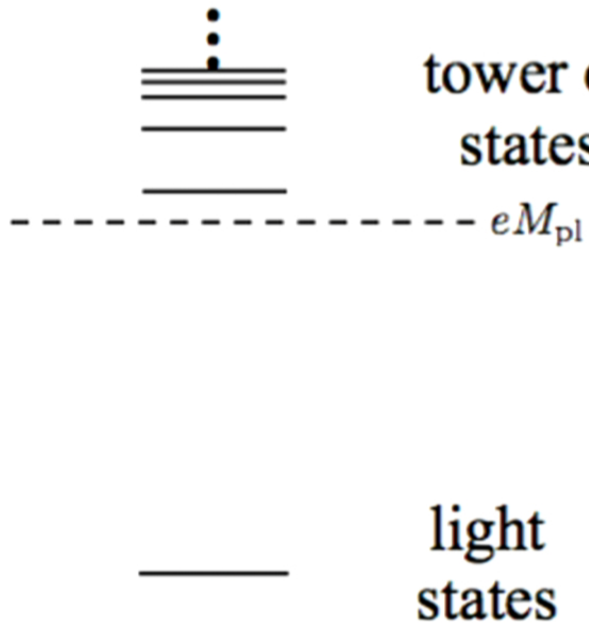
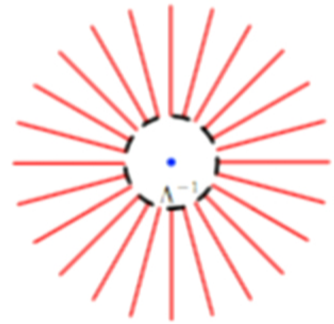
The density of extremal sites





O(1) fraction of (super)extremal states may be required to prevent monopole tuning

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Closing the small action loophole

$$V = \frac{3(-1)^F}{4\pi^2}\frac{1}{(2\pi R)^4}\sum_{n=1}^{\infty}\frac{1}{n^5}f(2\pi Rmn)\cos(Qn\theta),$$

$$f(x) = \left(1 + x + \frac{1}{3}x^2\right)e^{-x}$$

$$\downarrow$$

$$V \sim \frac{3}{4\pi^2}\frac{1}{(2\pi R)^4}\sum_{Q=1}^{\infty} f\left(\frac{M_{\rm pl}}{fZ}Q\right)\cos(Q\theta),$$

$$Z \equiv \frac{QM_{\rm pl}}{fS} = \frac{QM_{\rm pl}}{f2\pi Rm} \gtrsim 1$$

$\Longrightarrow\ f \lesssim M_{\rm pl}$ to suppress harmonics

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Closing the small action loophole

Conclusion essentially unchanged w/o strict LWGC

…so long as large fraction of (super)extremal sites

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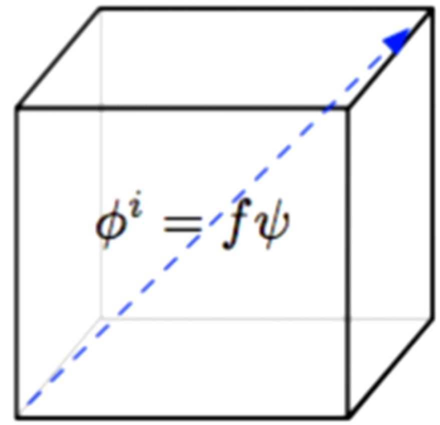
Isotropic N-flation

$$V = V_0 \sum_{Q=1}^{\infty} A_Q \cos(Q\psi)$$

$$A_Q \gtrsim \sum_{\vec{Q}} \delta_{\sum_i Q_i = Q} e^{-\frac{M_{\rm pl}}{f}|\vec{Q}|}$$

$$\implies A_n \gtrsim \sum_{p=0}^{\infty} \binom{N}{n+p,\, p} e^{-\frac{M_{\rm pl}}{f}\sqrt{n+2p}}$$



$Q_i \in \{0, \pm 1\}$

<u>"subleading" Q's can dominate</u>

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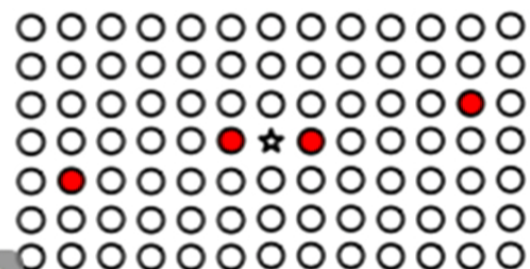
Alignment

Harder to constrain

Need leading instantons $(1,0)$ and $(N,1)$ with

$$S_{(1,0)} \lesssim \frac{M_{\rm pl}}{f} - \#\log N$$

Get $f_{\rm eff} \lesssim \# \dfrac{N}{\log N}$



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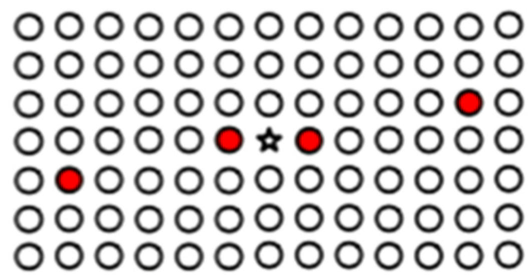

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Concluding Diversions