

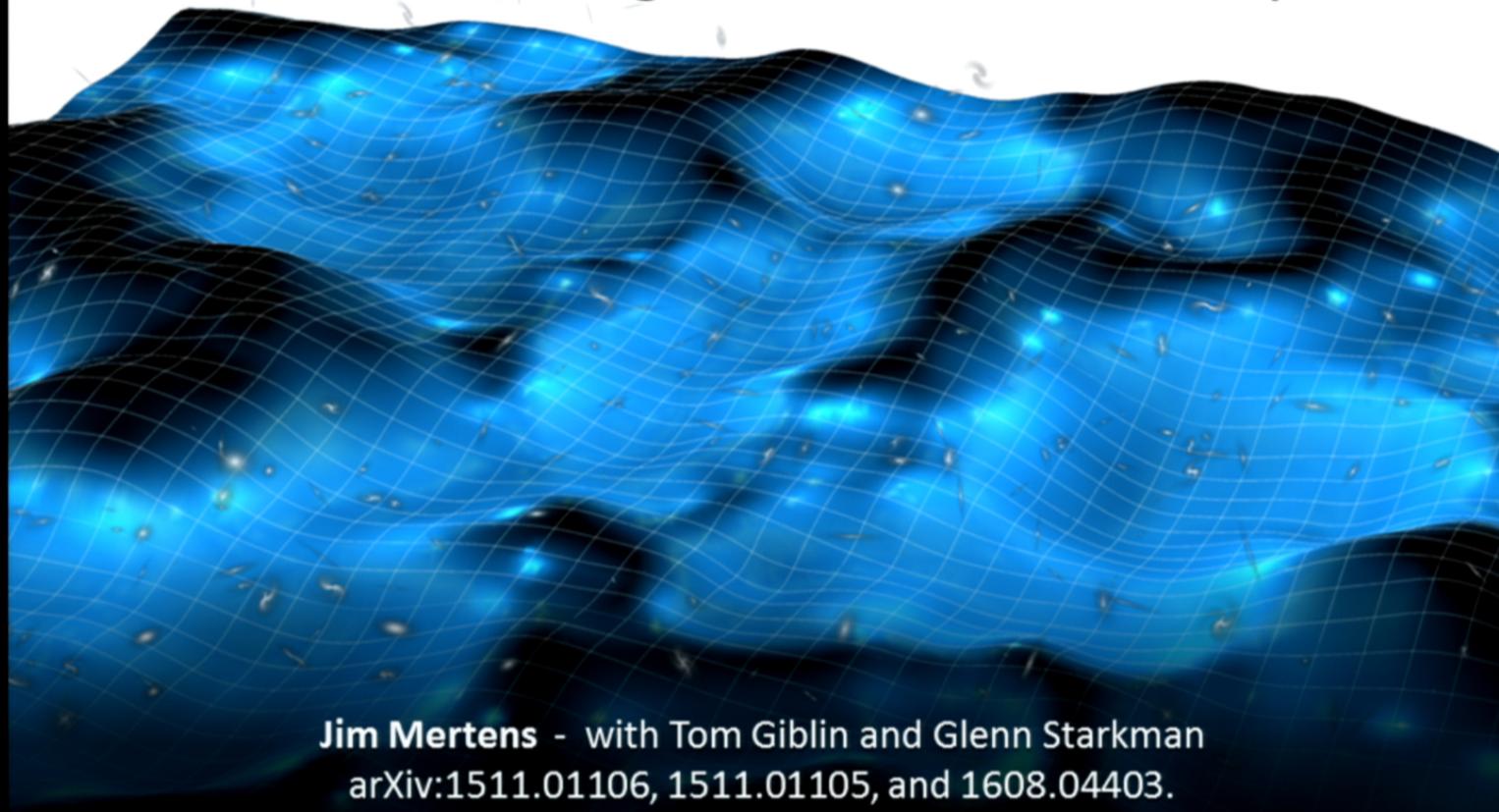
Title: Modeling Deviations from Homogeneity in an Inhomogeneous Universe Using Numerical Relativity

Date: Oct 11, 2016 11:00 AM

URL: <http://pirsa.org/16100052>

Abstract: <p>It has long been wondered to what extent the observable properties of an inhomogeneous universe will be measurably different from a corresponding FLRW model. Here, we use tools from numerical relativity to study the properties of photons traversing an inhomogeneous universe. We evolve the full, unconstrained Einstein field equations for a spacetime containing dust, with a spectrum of long-wavelength density perturbations similar to the observed one. We then integrate the optical scalar equations along paths through this numerical spacetime, with all paths terminating at an observer situated similarly to ourselves, and construct the resulting Hubble diagrams.</p>

Computing Observables in an Inhomogeneous Universe using Numerical Relativity



Jim Mertens - with Tom Giblin and Glenn Starkman
arXiv:1511.01106, 1511.01105, and 1608.04403.



Computing Observables in an Inhomogeneous Universe using Numerical Relativity

Most observations in Λ CDM cosmology rely on a number of approximations. How important are corrections from GR?

- 1) Do nonperturbative effects affect overall or background spacetime dynamics?
 - No definite conclusions have been made about the importance
eg, arxiv:1506.06452 vs arxiv:1505.07800
 - Desirable to phrase as an observational rather than mathematical question
- 2) How large are corrections to the local evolution & observables (optical properties) of an FLRW + Newtonian approximation?
 - Again, no definite conclusions, just hints from perturbative work.
eg, 1509.01699 (Adamek et al.)

Tools from numerical relativity can help!

Numerical Relativity in Cosmology

- **Phase Transitions & Primordial GWs**

Centrella, Matzner, Rothman, Wilson (1980s)

Rezzolla, Miller, Pantano (1995)

Bastero-Gil, Macias-Perez, Santos (2010)

Wainwright, Johnson, Peiris, Aguirre, Lehner, Liebling (2014)

- **Inflation**

Centrella, Wilson, Kurki-Suonio, Laguna, Matzner (1980s-1990s)

Bastero-Gil, Tristram, Macias-Perez, Santos (2007)

East, Kleban, Linde, Senatore, Kearney, Shakya, Yoo, Zurek (2015, 2016)

Braden, Johnson, Peiris, Aguirre, Clough, Lim, DiNunno, Fischler, Flauger, Paban (2016)

- **Late-universe**

Anninos (1998)

Bentivegna, Korzynski, Yoo, Abe, Nakao, and Takamori Okawa (2012-2014)

Torres, Alcubierre, Diez-Tejedor, Núñez, Rekier, Cordero-Carrion, Füzfa, Macorra (2014-2015)

Giblin, Starkman, Mertens, Bentivegna, Bruni (2015-2016)



The BSSN Formalism (and some modifications)

Code Implementation & Tests

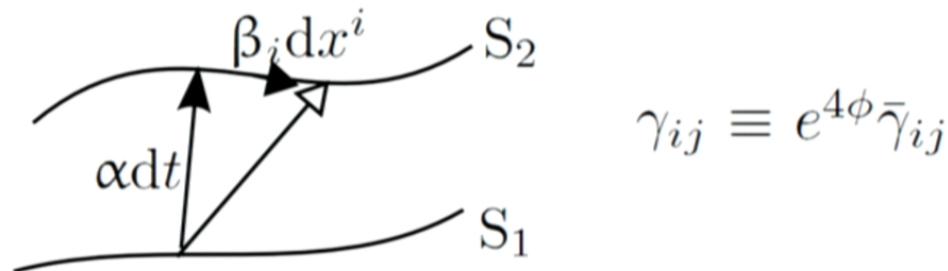
Late-Universe Cosmology:

- Initial Conditions
- Inhomogeneous EdS/“Dust” evolution
- Optical Properties & Observables

The BSSN Formalism

Introduced to address issues of numerical stability in 3+1 formulations. The metric is written as:

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{lk}\beta^l\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$



The BSSN Formalism

Gravity

$$\left\{ \begin{array}{l} \partial_t \phi = -\frac{1}{6} K \\ \partial_t \bar{\gamma}_{ij} = -2 \bar{A}_{ij} \\ \partial_t K = \bar{A}_{ij} \bar{A}^{ij} + \frac{1}{3} K^2 + 4\pi(\rho + S) \\ \partial_t \bar{A}_{ij} = e^{-4\phi} (R_{ij} - 8\pi S_{ij})^{TF} + K \bar{A}_{ij} - 2 \bar{A}_{il} \bar{A}_j^l \\ \\ \partial_t \bar{\Gamma}^i = 2 \bar{\Gamma}_{jk}^i \bar{A}^{jk} - \frac{4}{3} \bar{\gamma}^{ij} \partial_j K - 16\pi \bar{\gamma}^{ij} S_j + 12 \bar{A}^{ij} \partial_j \phi \end{array} \right.$$

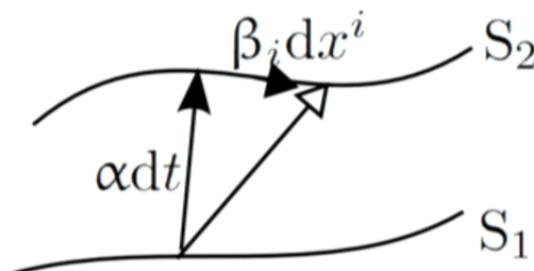
CDM

$$\left\{ \begin{array}{l} \partial_t (\gamma^{1/2} \rho_0) = 0 \end{array} \right.$$

The BSSN Formalism

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$

$$\mathcal{M}^i \equiv \bar{D}_j (e^{6\phi} \tilde{A}^{ij}) - \frac{2}{3} e^{6\phi} \bar{D}^i K - 8\pi e^{6\phi} S^i = 0$$



Code specs | Reference Metric

$$\partial_t \phi = -\frac{1}{6}K$$

$$\partial_t \bar{\gamma}_{ij} = -2\tilde{A}_{ij}$$

$$\partial_t K = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 4\pi(\rho + S)$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}(R_{ij} - 8\pi S_{ij})^{TF} + K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}_j^l$$

$$\partial_t \phi_{FRW} = -\frac{1}{6}K_{FRW}$$

$$\partial_t K_{FRW} = \frac{1}{3}K_{FRW}^2 + 4\pi(\rho_{FRW} + S_{FRW})$$

$$\begin{aligned}\Delta\phi &= \phi - \phi_{FRW} \\ \Delta K &= K - K_{FRW} \\ \Delta\rho &= \rho - \rho_{FRW} \\ \Delta\bar{\gamma}_{ij} &= \bar{\gamma}_{ij} - \bar{\delta}_{ij}\end{aligned}$$

$$\partial_t \Delta\phi = -\frac{1}{6}\Delta K$$

$$\partial_t \Delta\bar{\gamma}_{ij} = -2\tilde{A}_{ij}$$

$$\partial_t \Delta K = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}\Delta K(\Delta K + 2K_{FRW}) + 4\pi(\Delta\rho + \Delta S)$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}(R_{ij} - 8\pi S_{ij})^{TF} + K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}_j^l$$

Code specs | Reference Metric

$$\partial_t \phi = -\frac{1}{6}K$$

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$$\partial_t \Delta K = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}\Delta K(\Delta K + 2K_{FRW}) + 4\pi(\Delta\rho + \Delta S)$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}(R_{ij} - 8\pi S_{ij})^{TF} + K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}_j^l$$

Code specs | Constraint Damping

Improvements beyond BSSN include constraint damping:

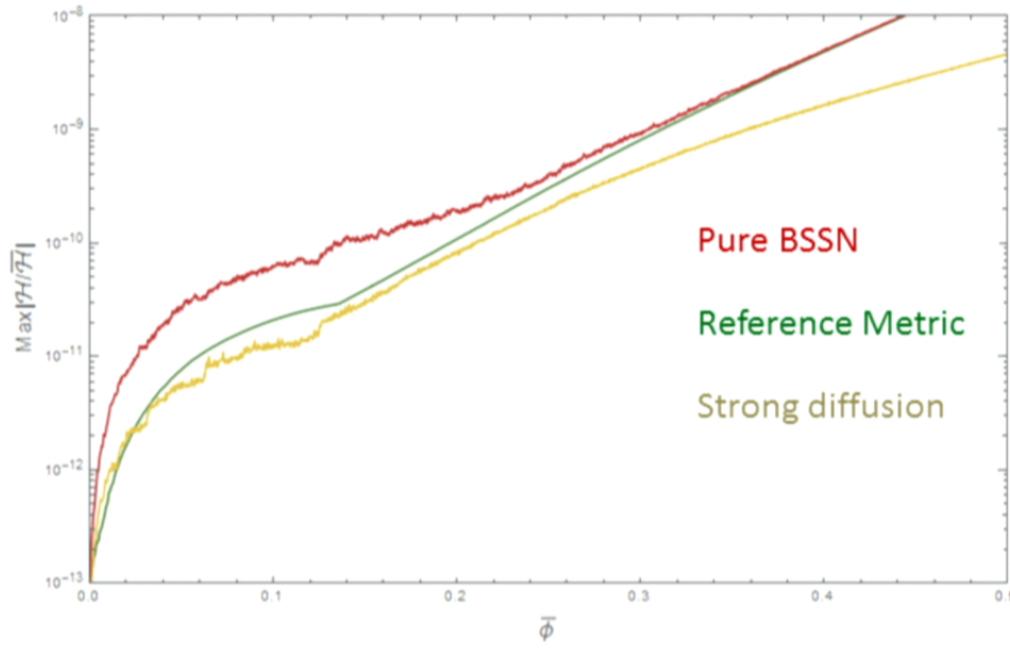
- Z4c terms – lead to constraint damping when coupled to puncture gauge (arxiv:0912.2920)
 - Minimal modification to BSSN that leads to constraint damping
- Duez et al. modifications (arxiv:gr-qc/0401076)

$$\partial_t \phi = \dots + 0.1 c_H \Delta t \mathcal{H}$$

$$\partial_t \bar{\gamma}_{ij} = \dots + 0.5 c_H \bar{\gamma}_{ij} \Delta t \mathcal{H}$$

$$\partial_t \tilde{A}_{ij} = \dots - 1.0 c_H \tilde{A}_{ij} \Delta t \mathcal{H}$$

Code specs | Beyond BSSN

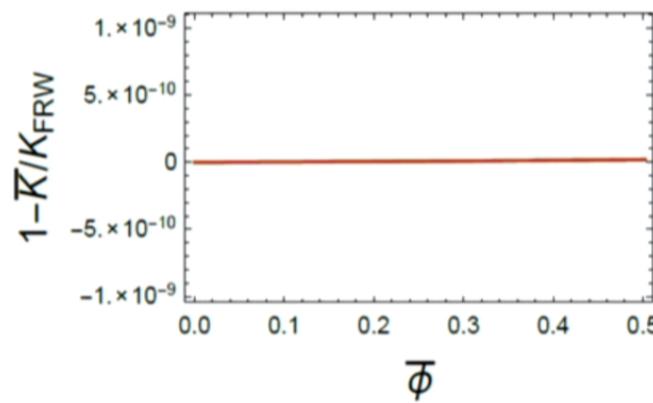
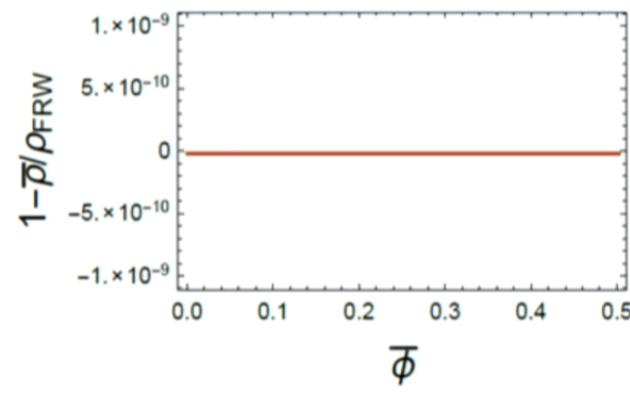


Cosmological run, low modes only

Code Tests: FRW

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$

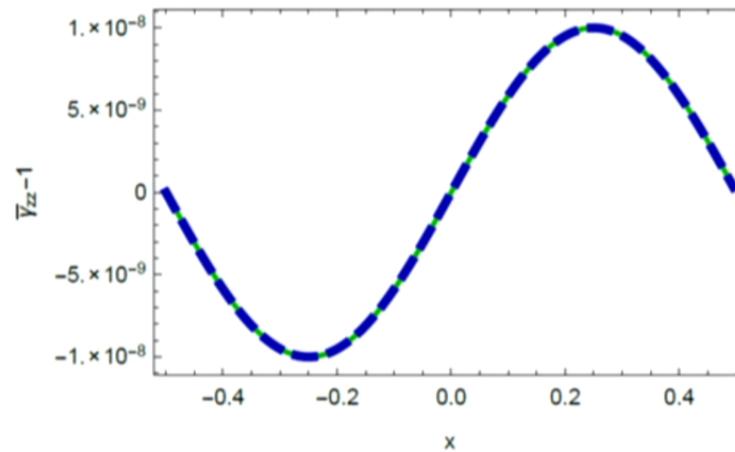
FRW Behavior



Code Tests

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$

Linearized Wave Behavior

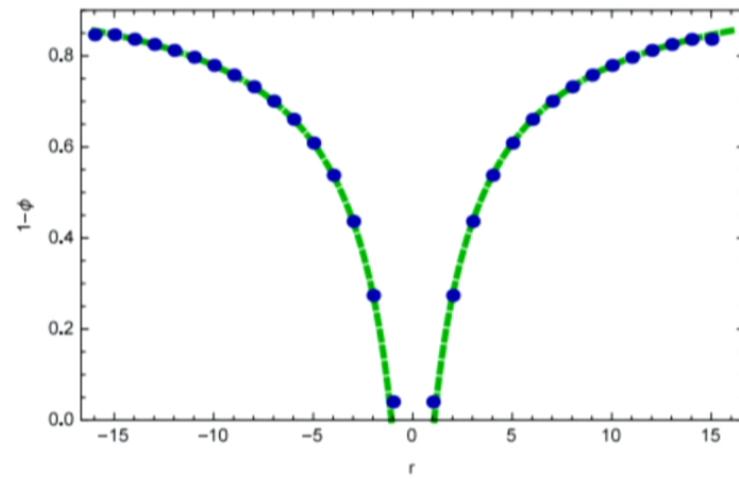


Code Tests

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$

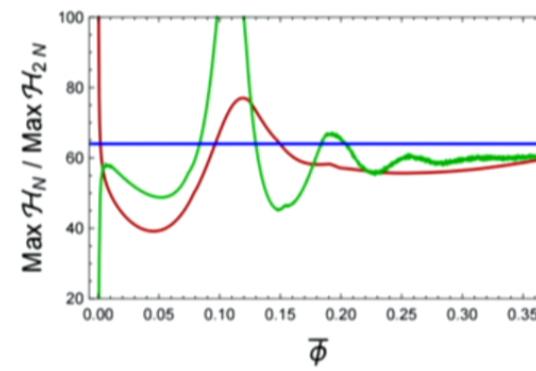
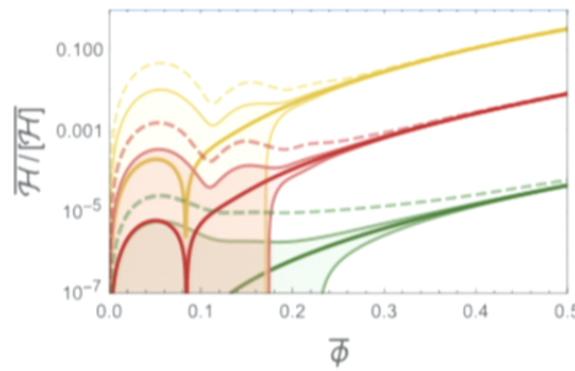


Black Hole Behavior



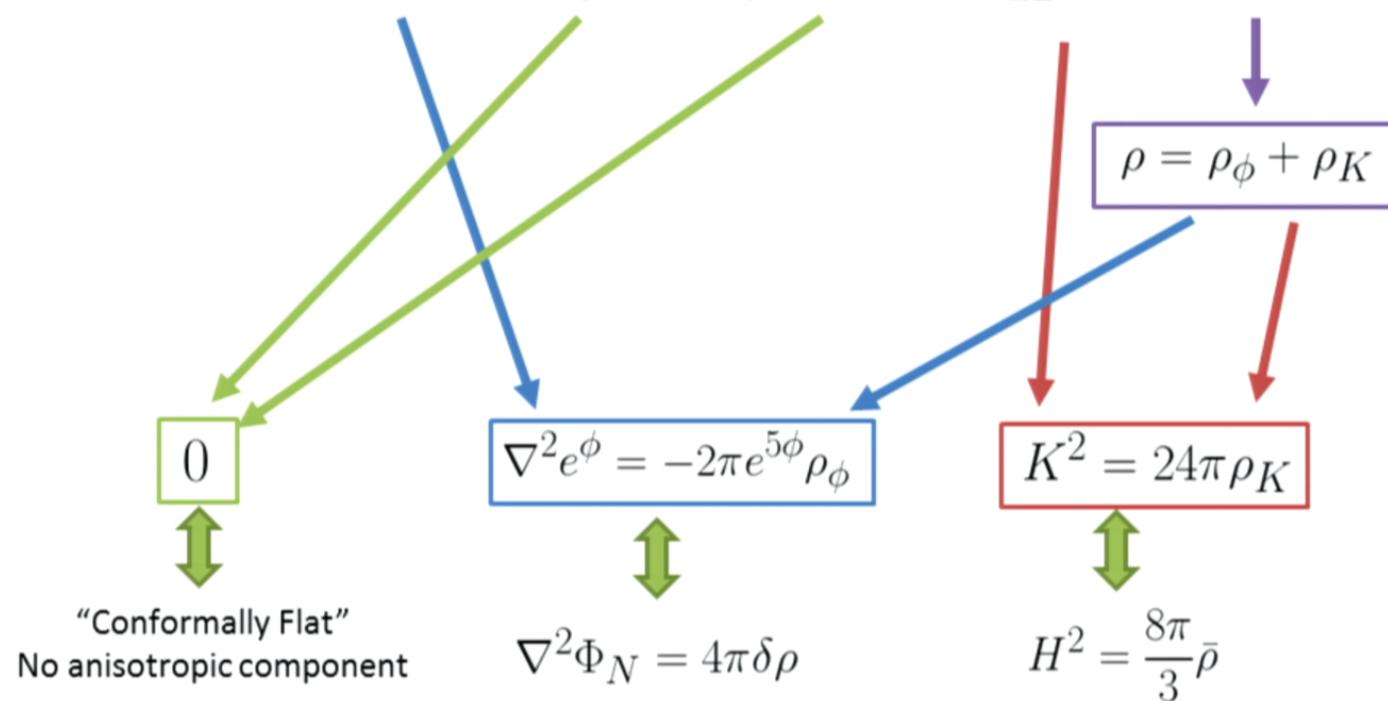
Error Quantification

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$



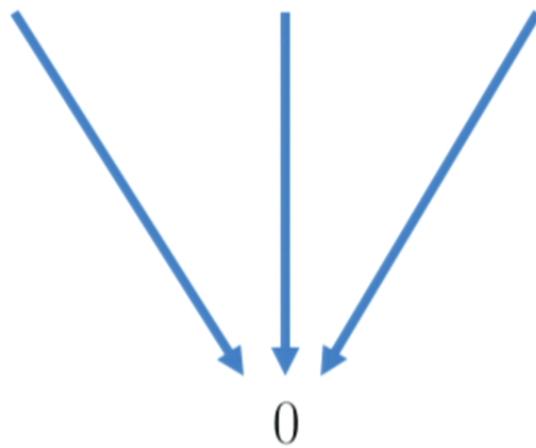
Cosmological Initial Conditions in GR

$$\mathcal{H} \equiv \bar{\gamma}^{ij} \bar{D}_i \bar{D}_j e^\phi - \frac{e^\phi}{8} \bar{R} + \frac{e^{5\phi}}{8} \tilde{A}_{ij} \tilde{A}^{ij} - \frac{e^{5\phi}}{12} K^2 + 2\pi e^{5\phi} \rho$$



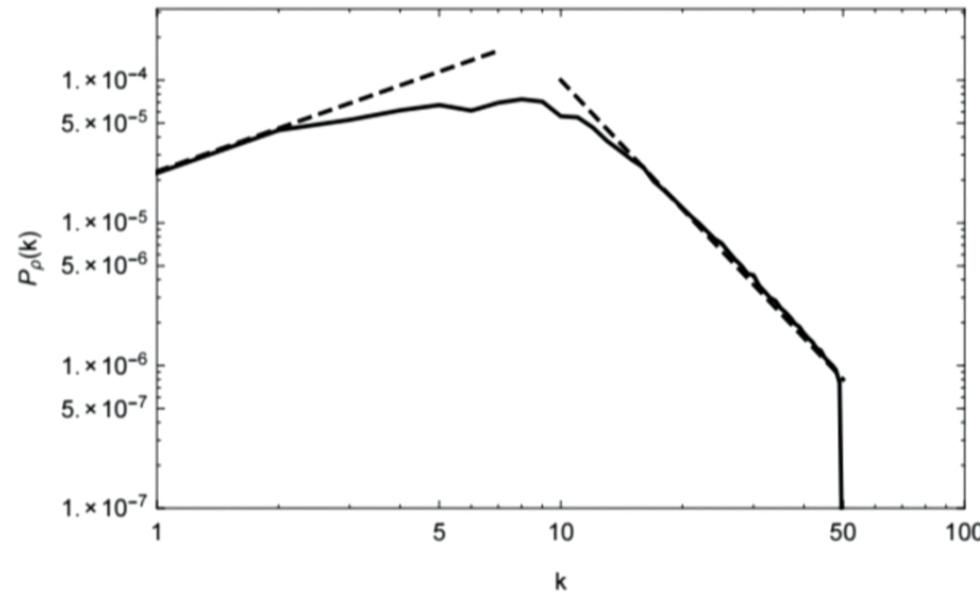
Setting Cosmological Initial Conditions

$$\mathcal{M}^i \equiv \bar{D}_j(e^{6\phi}\tilde{A}^{ij}) - \frac{2}{3}e^{6\phi}\bar{D}^iK - 8\pi e^{6\phi}S^i = 0$$



Setting Cosmological Initial Conditions

$$\nabla^2 e^\phi = -2\pi e^{5\phi} \rho_\phi$$



Raytracing | Following Null Geodesics

Need to convert geodesic equation into a 3+1 form

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$



$$\frac{d^2x^\mu}{dt^2} = \Gamma_{\alpha\beta}^0 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \frac{dx^\mu}{dt} - \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}$$

Raytracing | Following Null Geodesics

$$q^\mu = \frac{dx^\mu}{dt} = \alpha (n^\mu + V^\mu) \quad p^\mu = E (n^\mu + V^\mu)$$

$$\frac{d^2x^\mu}{dt^2} = \Gamma_{\alpha\beta}^0 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \frac{dx^\mu}{dt} - \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}$$

$$\frac{dX^i}{dt} = NV^i - \beta^i$$

$$\frac{dV^i}{dt} = NV^j [V^i (\partial_j \ln N - K_{jk} V^k) + 2K^i{}_j - {}^3\Gamma^i_{jk} V^k] - \gamma^{ij} \partial_j N - V^j \partial_j \beta^i$$

$$\frac{dE}{dt} = E(NK_{jk} V^j V^k - V^j \partial_j N)$$

(arXiv:1208.3927)

Raytracing | Following Null Geodesics

Distance measures: evolving angular diameter distance

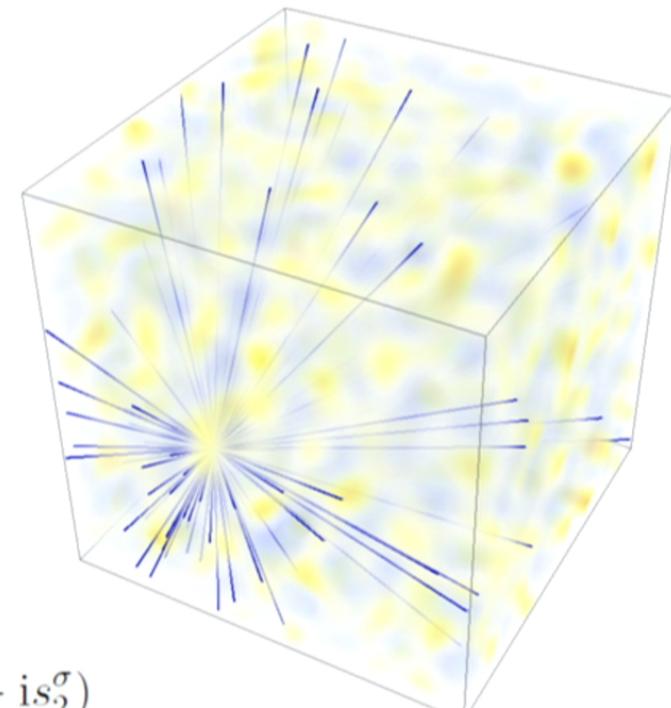
$$\frac{d^2 D_A}{dv^2} = (\mathcal{R} - |\sigma|^2) D_A$$

$$\frac{dD_A^2 \sigma}{dv} = D_A^2 \mathcal{W}$$

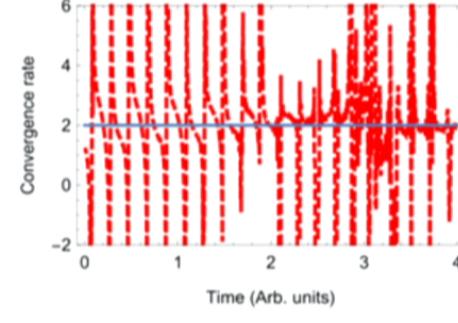
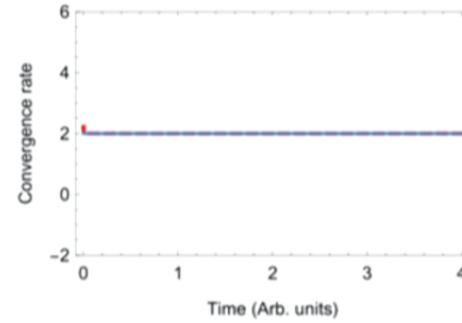
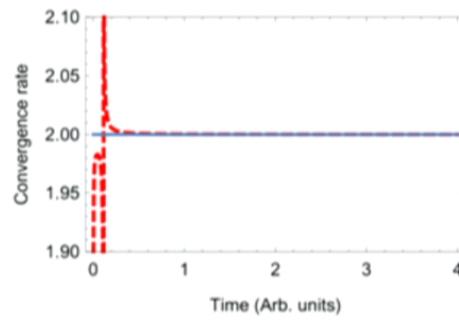
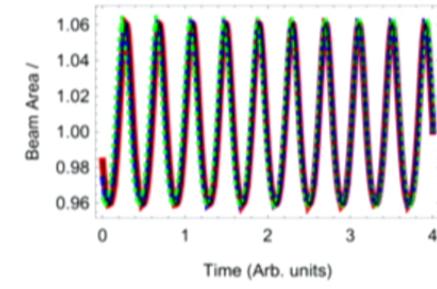
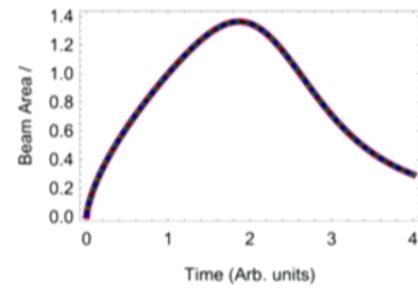
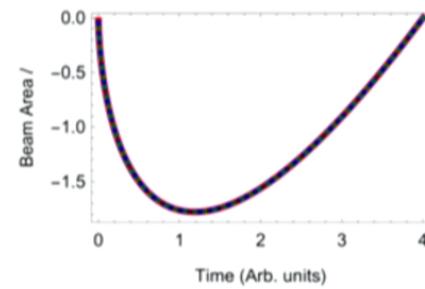
$$D_A = \frac{\ell}{\theta}$$

$$\mathcal{R} = -4\pi G T_{\mu\nu} k^\mu k^\nu \leq 0$$

$$\mathcal{W} \equiv -\frac{1}{2} C_{\mu\nu\rho\sigma} (s_1^\mu - i s_2^\mu) k^\nu k^\rho (s_1^\sigma - i s_2^\sigma)$$



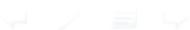
Code Tests | Raytracing



FRW

Kasner

Sinusoidal

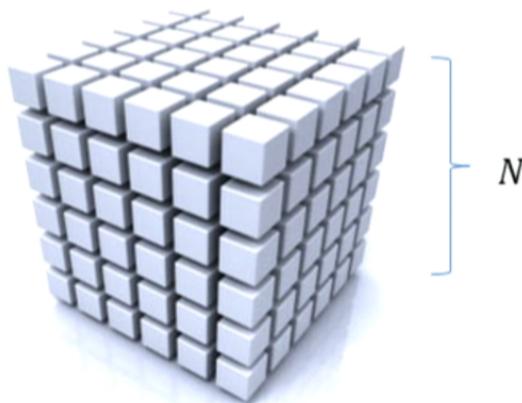


Cosmological Evolution

Comparisons are made in synchronous gauge.

- Comparisons phrased in terms of the FRW
(exact solution)
- Better: compute & compare observable quantities

Running a BSSN simulation



Running on a regular lattice

“Usual” Finite difference
schemes, RK4 integrator

Periodic boundary conditions

$$N \cdot \Delta x \sim H^{-1}$$

Cosmological Evolution

$$\partial_t K = \bar{A}_{ij} \bar{A}^{ij} + \frac{1}{3} K^2 + 4\pi(\rho + S)$$

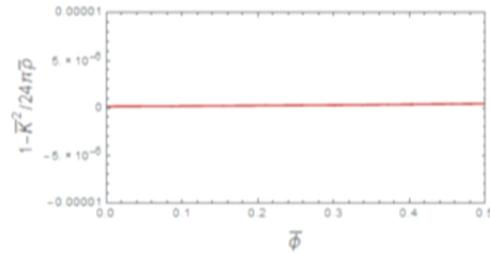
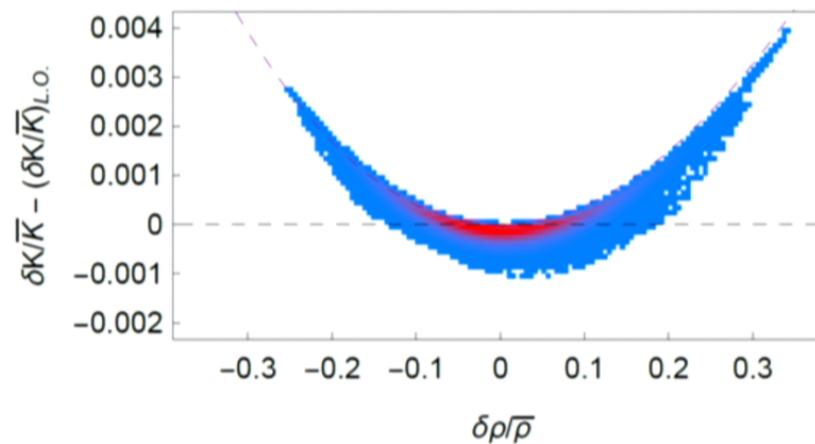
$$\partial_t \tilde{D} = \partial_t (\gamma^{1/2} \rho_0) = 0$$



$$\partial_t \delta\rho = \bar{\rho} \delta K + \bar{K} \delta\rho$$

$$\partial_t \delta K = \frac{2}{3} \bar{K} \delta K + 4\pi \delta\rho.$$

$$\bar{K}^2 = 24\pi \bar{\rho}$$



(Eg, arxiv:astro-ph/9506072)

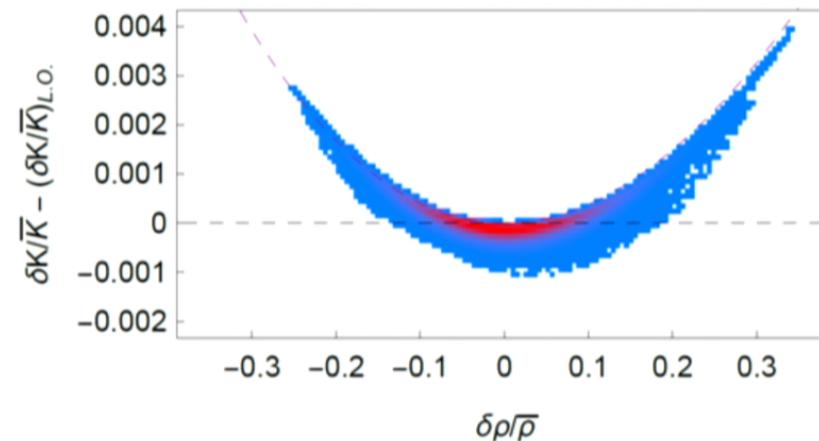
Cosmological Evolution

$$\partial_t K = \bar{A}_{ij} \bar{A}^{ij} + \frac{1}{3} K^2 + 4\pi(\rho + S)$$

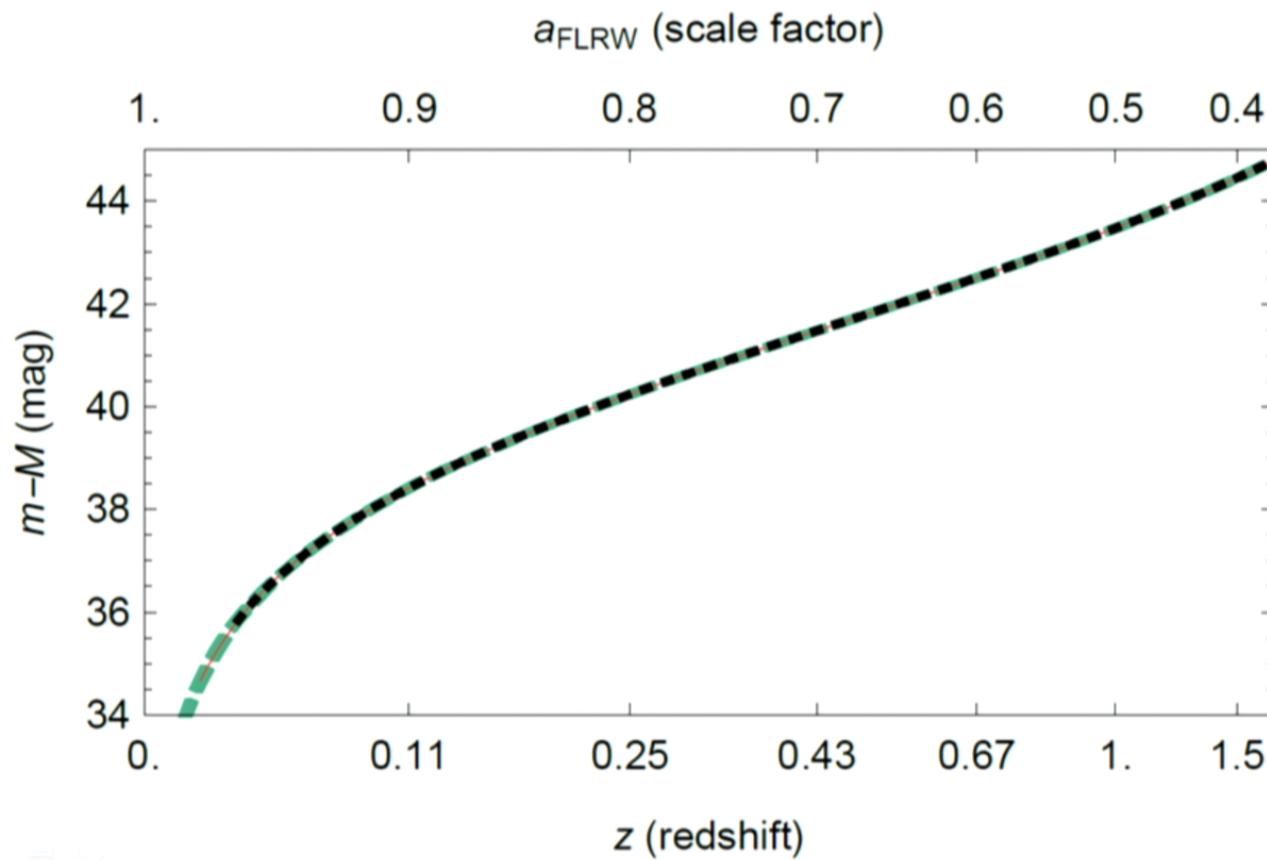
$$\partial_t \tilde{D} = \partial_t (\gamma^{1/2} \rho_0) = 0$$



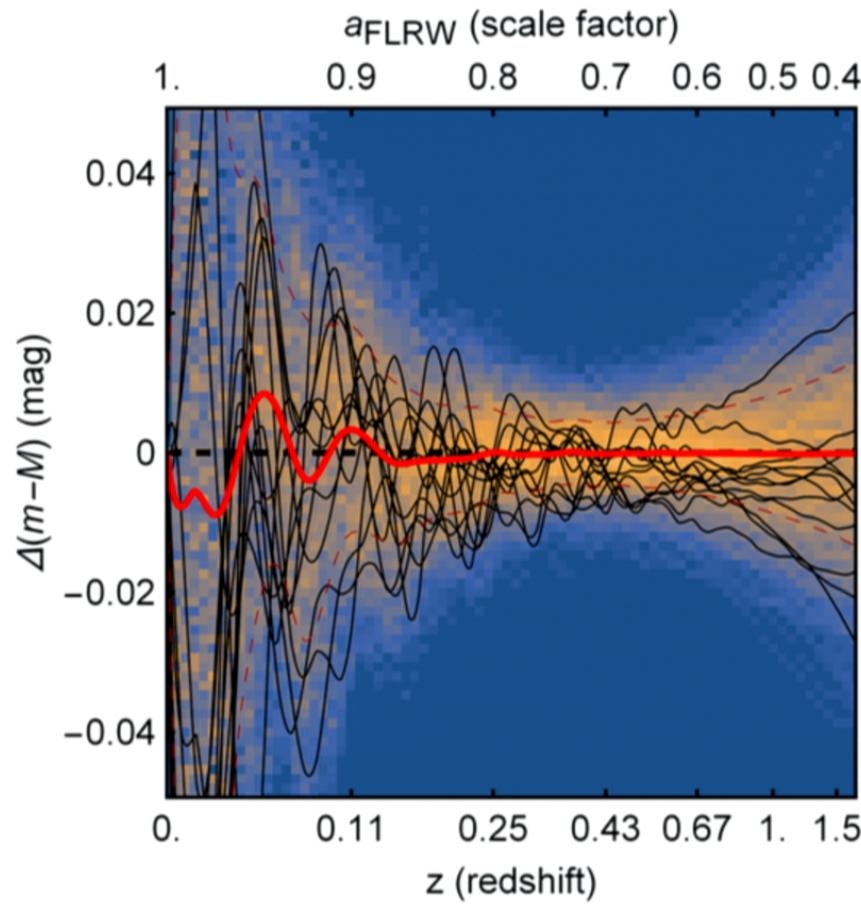
$$\begin{aligned}\partial_t K &= \frac{1}{3} K^2 + 4\pi\rho \\ \partial_t \rho &= K\rho\end{aligned}$$



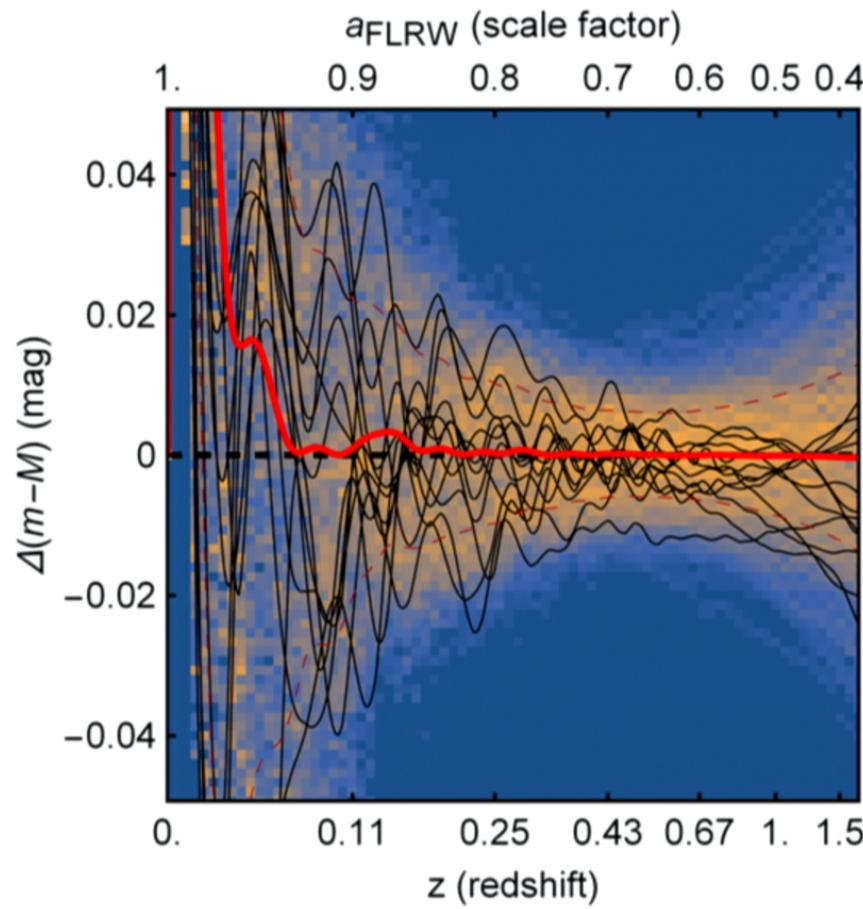
Hubble Diagrams



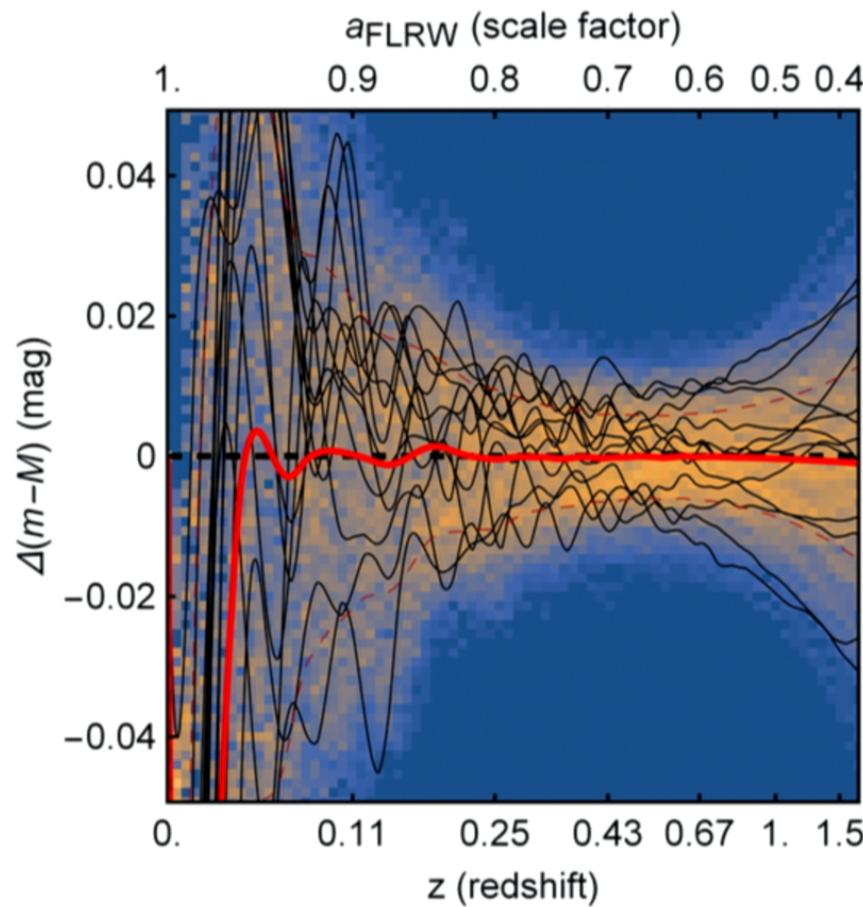
Hubble Diagrams



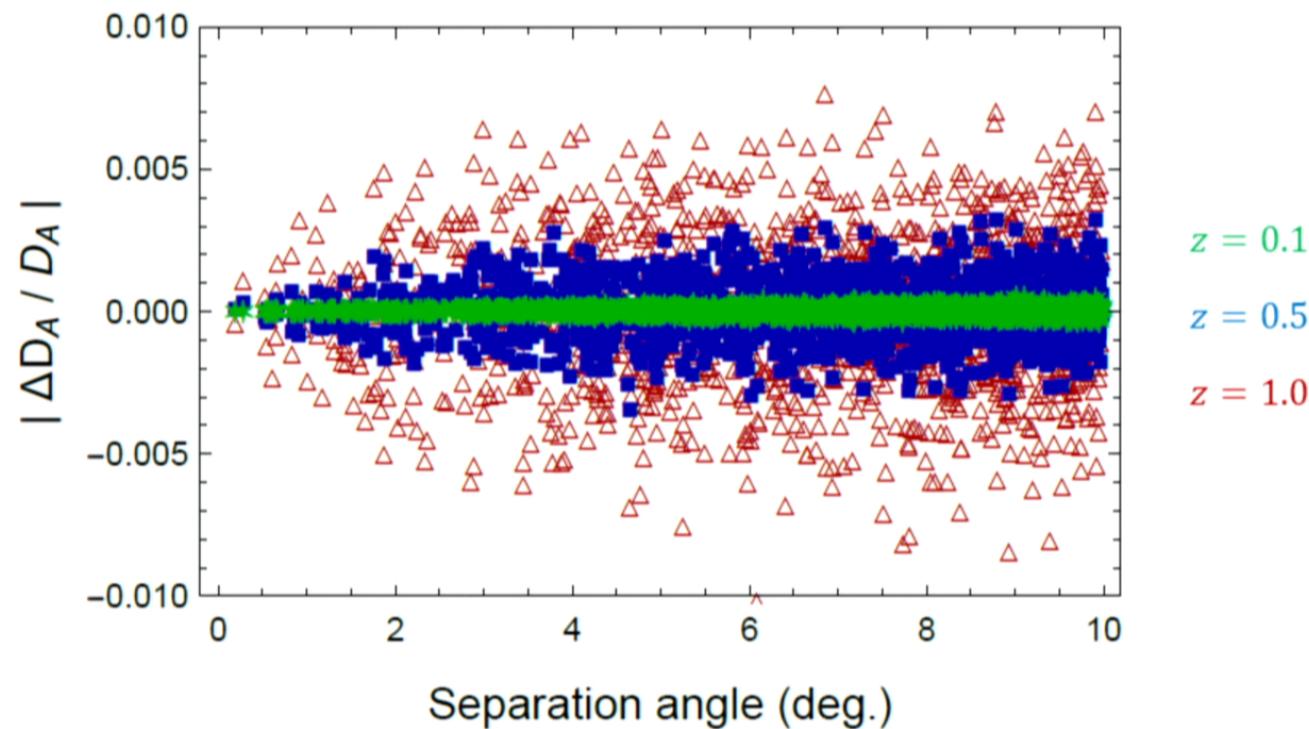
Hubble Diagrams



Hubble Diagrams



Small Beam Approximation Violations

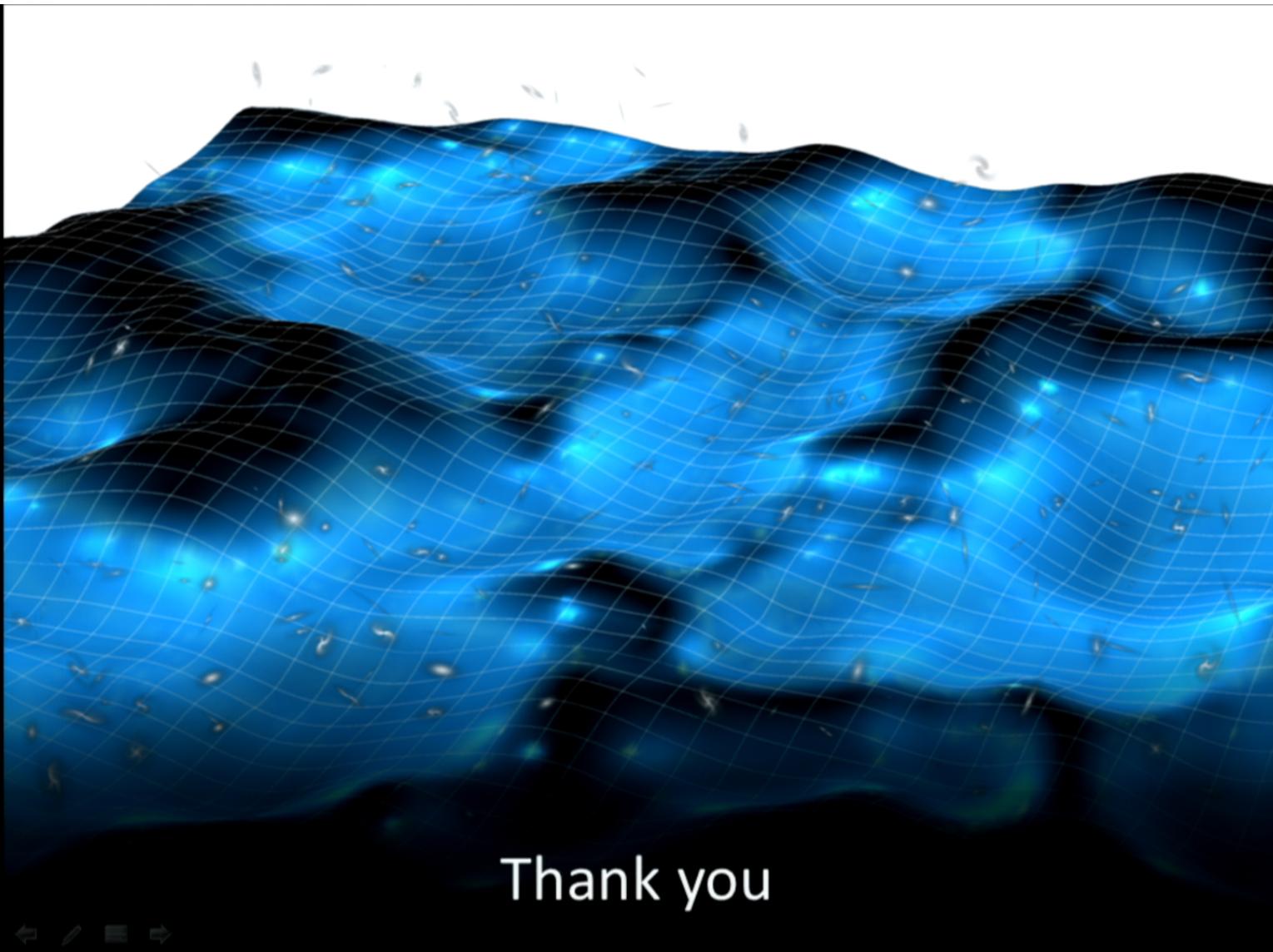


Future Goals

We have proof-of-principle that we can make accurate cosmological measurements in a General Relativistic framework.

In order to model a more realistic universe, we must work on resolving smaller scales – stay tuned!





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