Title: Tensor network state correspondence and Holography

Date: Oct 05, 2016 11:00 AM

URL: http://pirsa.org/16100044

Abstract: <p>In recent years, tensor network states have emerged as a very useful conceptual and simulation framework to study local quantum many-body systems at low energies. </p>

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<p>In this talk, I will describe how a tensor network representation of a quantum many-body ground state also encodes, in a natural way, another quantum many-body state whose properties must be related to the ground state in a systematic way. One can apply this tensor network state correspondence to the multi-scale entanglement renormalization ansatz (MERA) representation of the ground state of a one dimensional (1D) quantum lattice system to obtain a quantum many-body state of a 2D hyperbolic quantum lattice, whose boundary is the original 1D lattice. I propose that this bulk/boundary correspondence could potentially be a candidate implementation of the holographic correspondence of String theory on a lattice using the MERA.</p>

<p> </p>

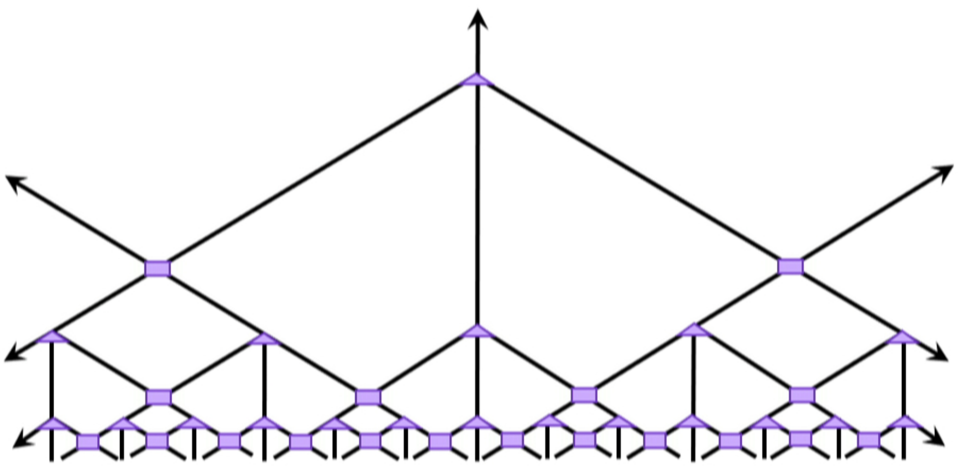
<p>For the MERA representation of a critical ground state I will show how the critical properties can be obtained from the corresponding bulk state, in particular, illustrating how point-like boundary operators are identified with extended operators in the bulk. I will also describe the entanglement and correlations in the bulk state, present numerical results that illustrate that the bulk entanglement may depend on the boundary critical charge, and describe how the bulk state can be described in terms of "holographic screens". If the boundary state has a global symmetry, the corresponding bulk state has a local gauge symmetry (described by the same group). In fact, the bulk state decomposes in terms of spin networks as they appear in lattice gauge theory, where they describe the gauge-invariant sector of the theory (here, the bulk). This decomposition also reveals entanglement between gauge degrees of freedom in the bulk, which are dual to a global symmetry at the boundary, and remaining bulk degrees of freedom which may potentially include gravitational degrees of freedom in a holographic interpretation of the MERA.</p>

Tensor network state correspondence
&
Holography

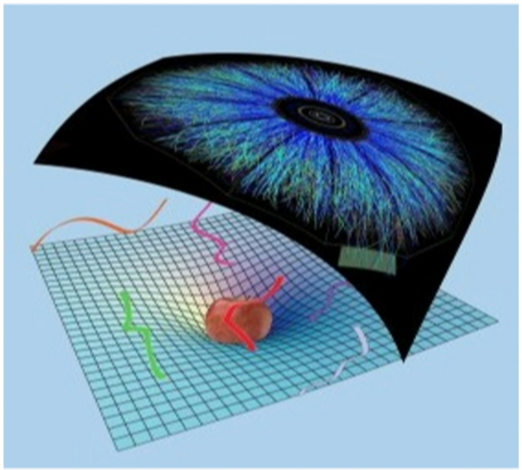
Sukhi Singh

(Institute for Quantum Optics & Quantum Information,

Austrian Academy of Sciences, Vienna)



MERA

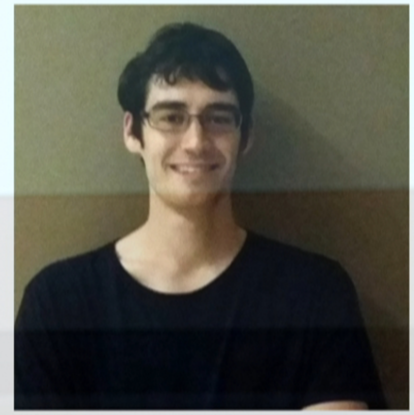


Holographic correspondence

Collaborators

A/Prof Gavin Brennen
(Macquarie University,
Sydney, Australia)

Arxiv postings to appear soon!



Nathan McMahon
(PhD student
University of Queensland,
Brisbane Australia)

- A useful lesson from applying quantum information ideas to study condensed matter systems: in many cases low energy states of local Hamiltonians can be efficiently represented by means of a tensor network.

- Several popular tensor network representations: MPS, PEPS, MERA etc.

- Basis of powerful many-body simulation algorithms e.g. DMRG, TEBD, tensor network renormalization etc.

- This talk: a tensor network representation of a quantum many-body state also encodes, in a simple and natural way, another quantum many-body state.

- The 2 states “correspond” to each other by means of the TN.

- Will apply to the MERA: RG flow of a quantum many-body ground state

- The two states are seen to live on the boundary and in the bulk of a geometry: a bulk/boundary correspondence

Goals

- A candidate implementation of (at least some general features of) the holographic correspondence on a lattice.

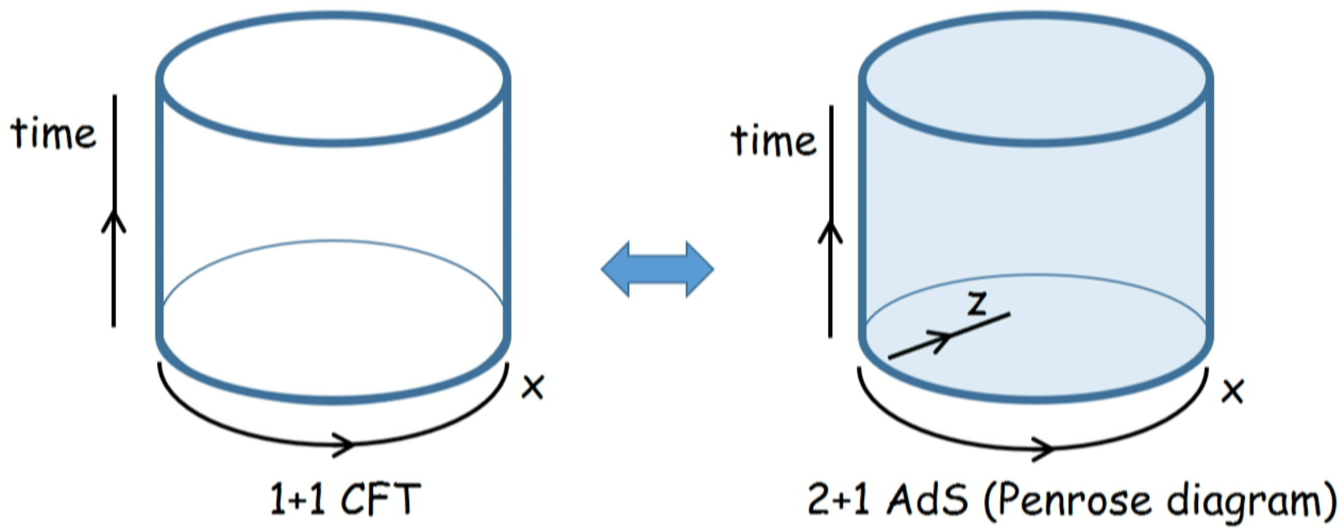
- Present a new B/B correspondence between (certain) quantum many-body states using tensor networks: potential applications beyond holography

Contents

Holographic correspondence

- Equivalence between a theory of a gravity in *D* dimensional spacetime and a quantum field theory (without gravity) in *D-1* dimensional spacetime
- QFT lives on the boundary of the *D* dimensional spacetime
- Extra dimension corresponds to the RG flow of the boundary theory
- Concrete realization: the AdS/CFT correspondence

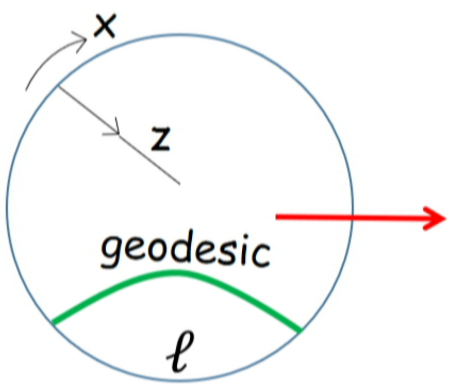
J. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) *E. Witten, Adv.Theor. Math. Phys. 2, 253 (1998)*



(Semi-)classical regime

- Strong/weak type duality
- Semi-classical regime: e.g. certain “Large N” gauge theories & large central charge CFTs

CFT ground state ⟺



Spatial slice of 2+1 AdS:
2D hyperbolic geometry

c: central charge ⟺ R: radius of curvature

J. Brown, M. Hennaux, Comm. Math. Phys., 1986

Entanglement entropy ($\propto c \log \ell$) = Length of geodesic ($\propto R \log \ell$)

S. Ryu & T. Takayanagi, PRL 96, 181602 (2006)

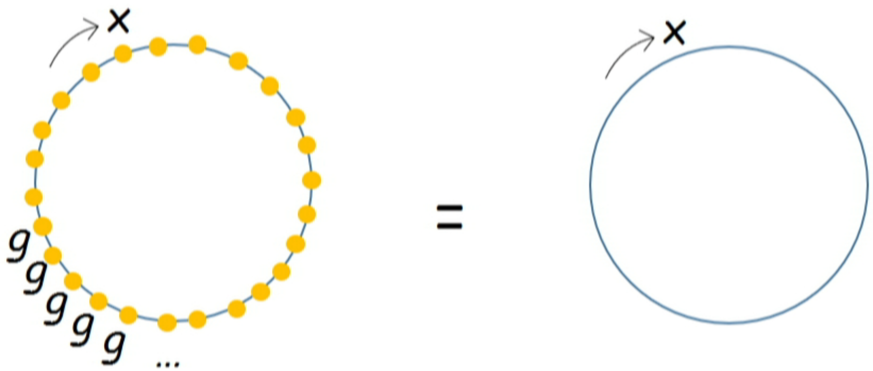
Quantum gravity regime

- Certain CFTs with small central charge are dual to quantum gravity in the bulk.
- Example: gravity dual of the Ising model

  A. Castro, M. R. Gaberdiel, T. Hartman, A. Maloney, and R. Volpato, Phys. Rev. D. 85 024032
- No description in terms of a classical geometry in the bulk.
- Quantum state at boundary dual to a quantum state in the bulk

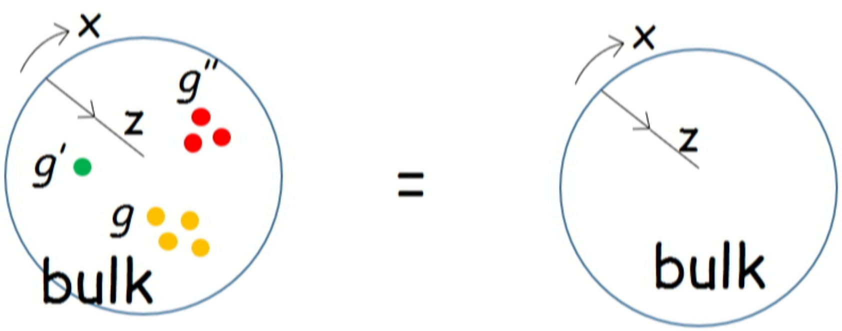
Symmetries

If boundary state has a global symmetry G = {g,g′,g′′,…}



$\forall\, g \in G$

Then the bulk state has a local symmetry



$\forall\, g, g', g'' \in G$

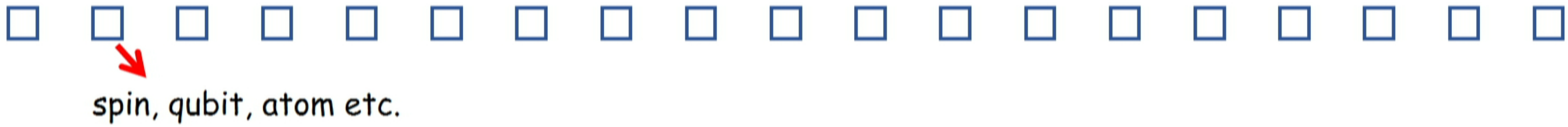
Symmetries

- If the CFT has a global symmetry, there is a corresponding gauge field in the bulk
- Gauge field is valued in the algebra of boundary symmetry group.
- In the quantum gravity regime, the bulk vacuum is entangled between the gauge and remaining bulk degrees of freedom.

Multi-scale entanglement renormalization ansatz (MERA)

G. Vidal, PRL 99, 220405 (2007)

Infinite one dimensional quantum lattice



- Hamiltonian: local, translation invariant and critical (described by a CFT in the continuum)
- Interested in the ground state

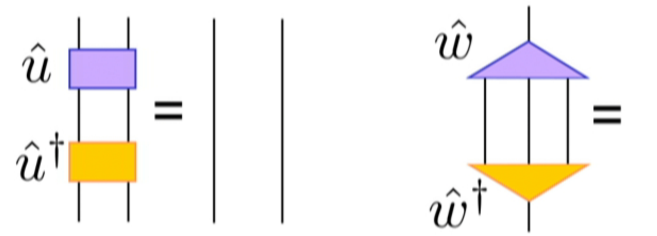
$$|\Psi\rangle = \sum_{o_1 o_2 \dots} \Psi_{o_1 o_2 \dots} \, |o_1\rangle \otimes |o_2\rangle \otimes \cdots$$

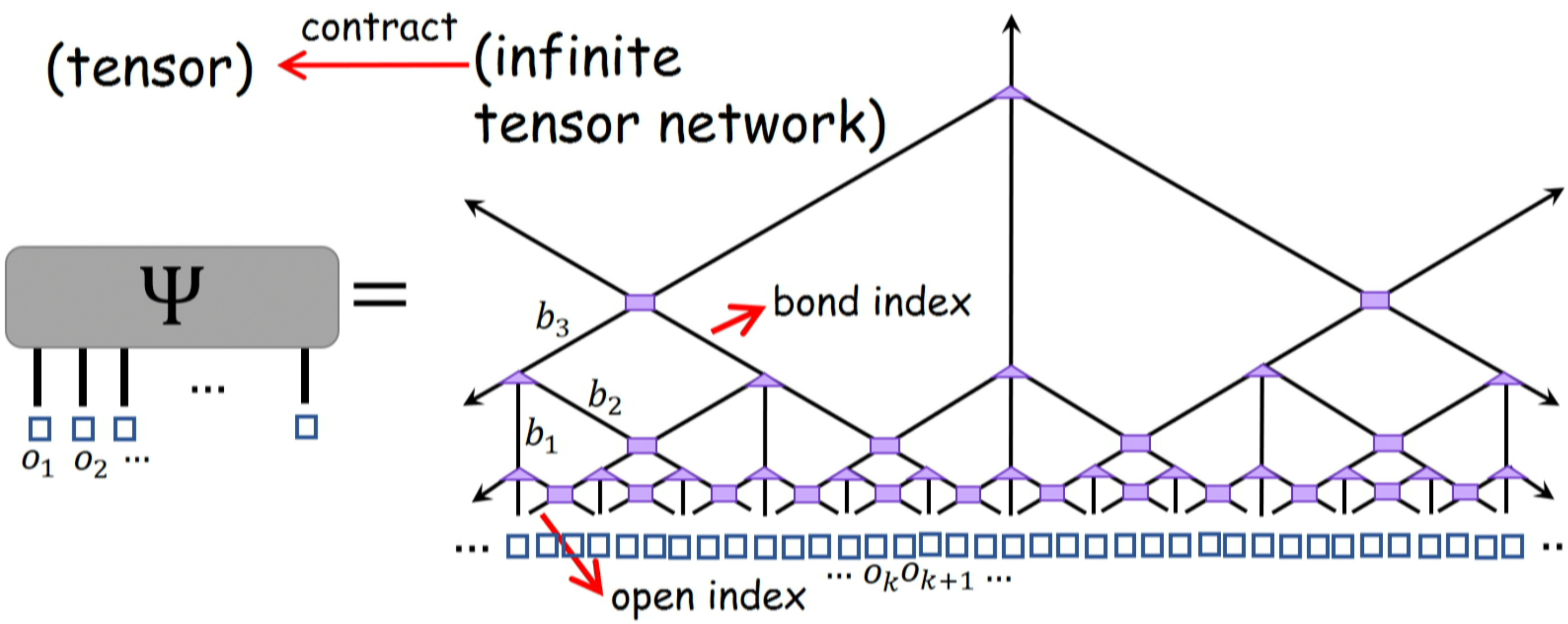
$\{|o_i\rangle\}$: Orthonormal basis on site i

Ground state can be represented as a MERA

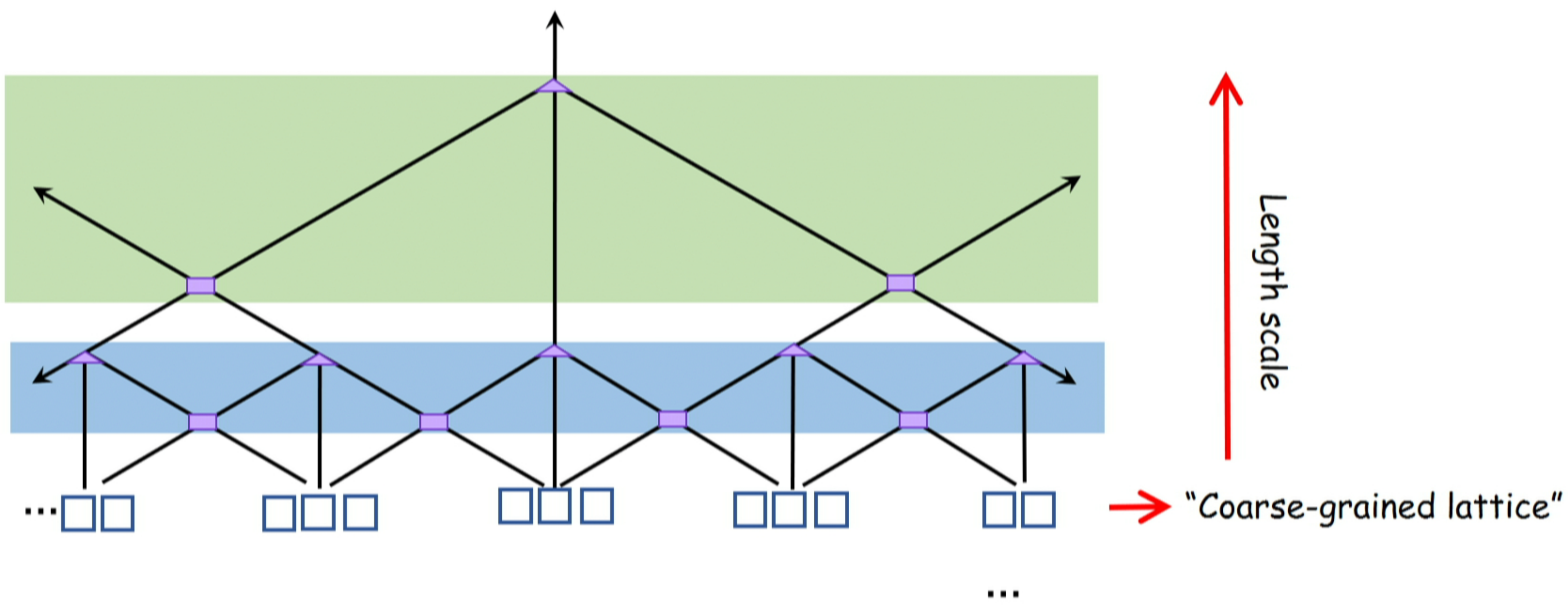
G. Vidal, PRL 99, 220405 (2007)

$$|\Psi\rangle = \sum_{o_1 o_2 \ldots} \Psi_{o_1 o_2 \ldots} \; |o_1\rangle \otimes |o_2\rangle \otimes \cdots$$



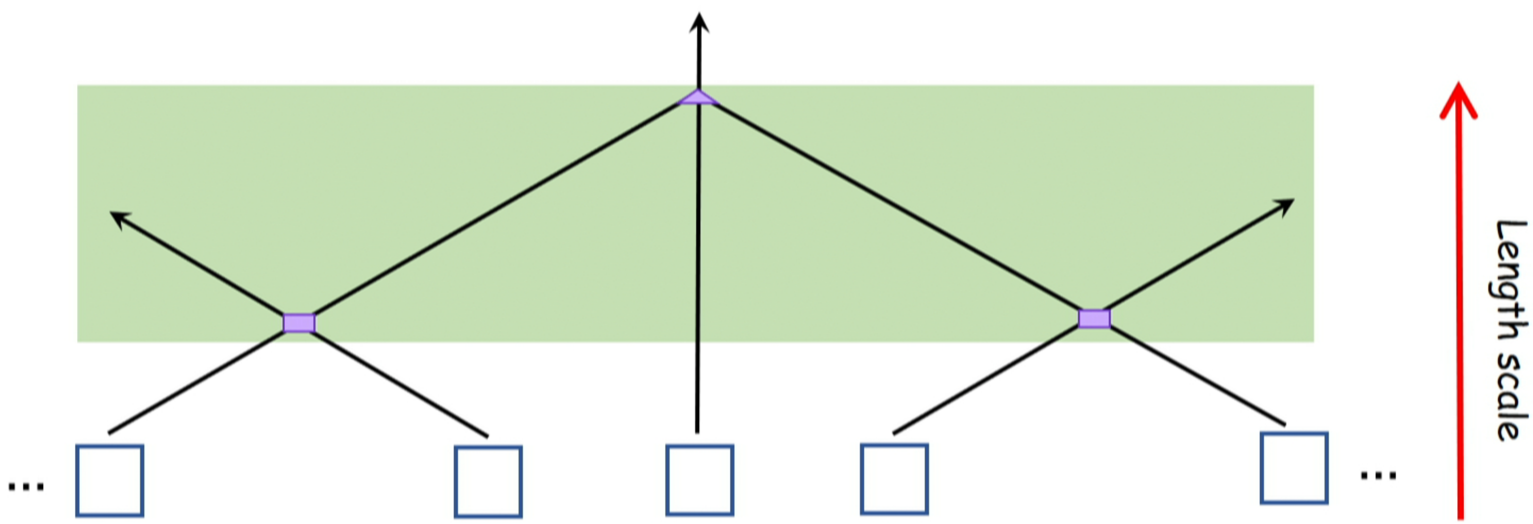


- Each layer of tensors implements a coarse-graining transformation
- Discarding bottom layers yields a coarse-grained ground state

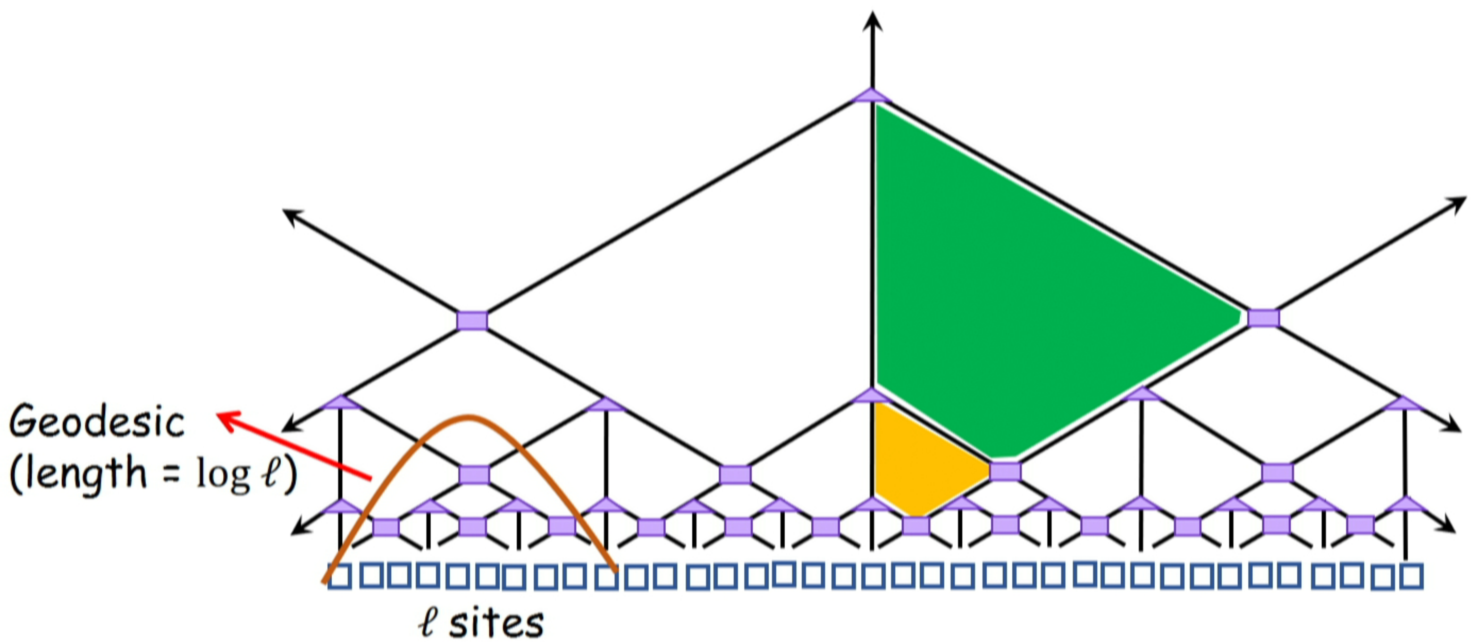


MERA encodes the RG flow of the ground state!

- Each layer of tensors implements a coarse-graining transformation
- Discarding bottom layers yields a coarse-grained ground state
- After several coarse-grainings ground state flows to a scale-invariant state



MERA has a hyperbolic geometry



The green and yellow tiles have the same area!

MERA as a realization of the holographic correspondence?

- MERA's hyperbolic geometry can be interpreted as a spatial slice of 2+1 AdS spacetime. *B. Swingle, Phys. Rev. D 86, 065007 (2012)*

- Several features of the holographic correspondence have been skteched using the MERA (e.g., Ryu-Takayanagi formula, black hole/thermal state correspondence, bulk/boundary symmetries): Mostly qualitative observations.

- But how is a dual bulk description encoded in the MERA?

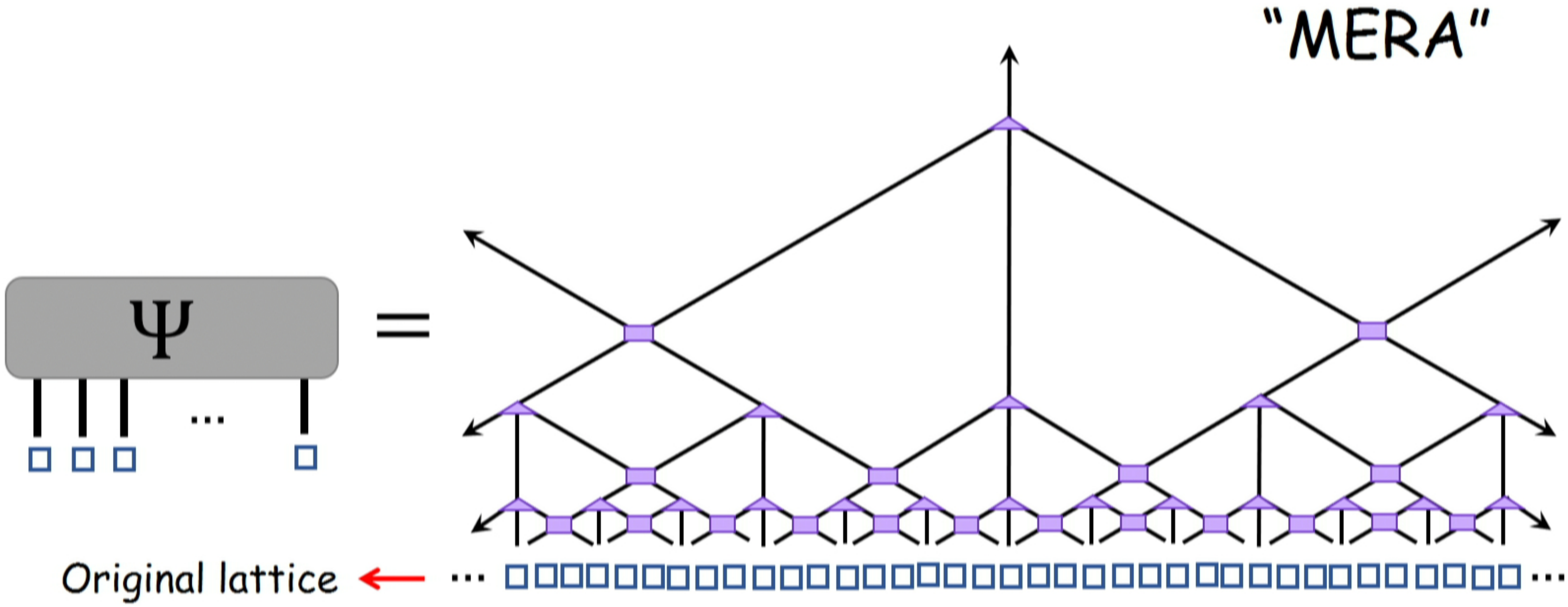
- More recently, MERA's hyperbolic geometry as 1+1 deSitter geometry
  C. Beny, New J. Phys. 15, 023020 (2013); B. Czech, L. Lamprou, S. McCandlish, J. Sully, SU-ITP-15/18, SLAC-PUB-16292.

- This talk: Independent of any specific interpretation of the MERA's geometry

A bulk/boundary correspondence from the MERA

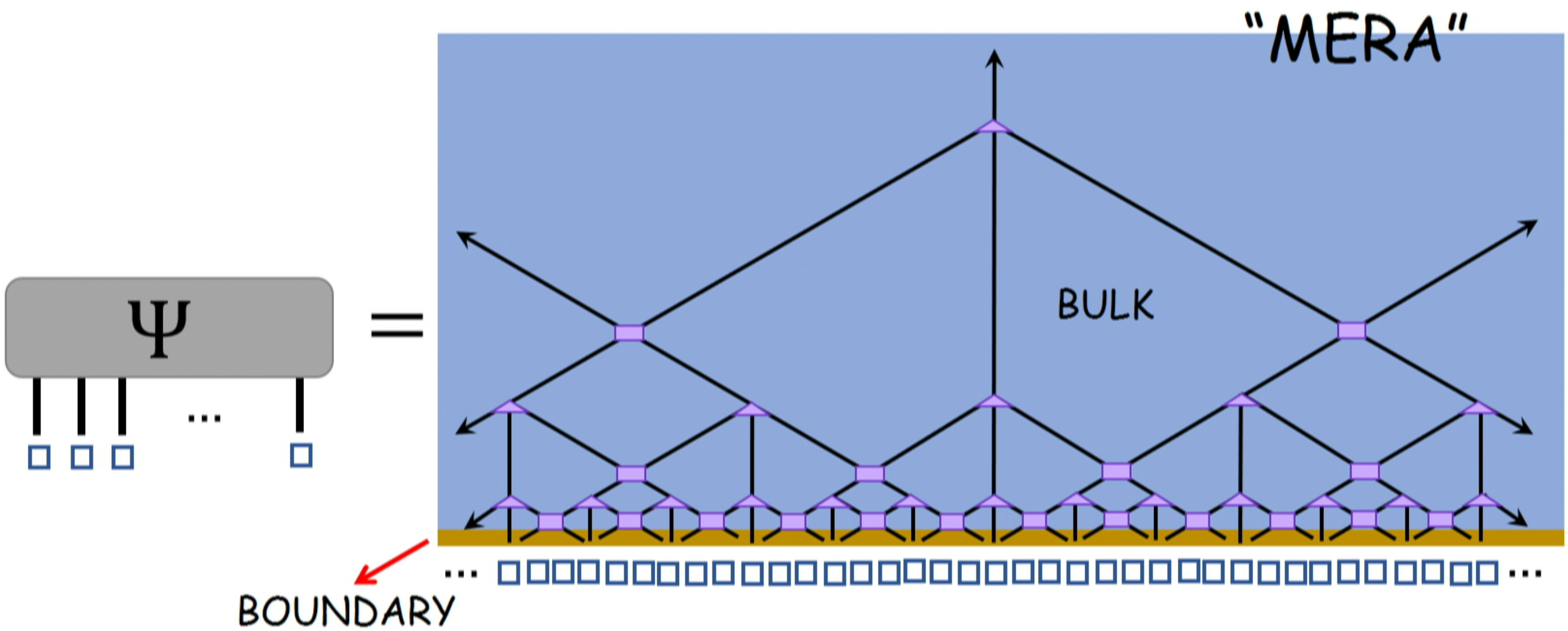
"Lifting" the MERA representation of a ground state

$$|\Psi^{\text{bound}}\rangle = \sum_{o_1 o_2 \ldots} \Psi_{o_1 o_2 \cdots} |o_1\rangle \otimes |o_2\rangle \otimes \cdots$$



Step 1: Embed the MERA in a 2D manifold with a boundary

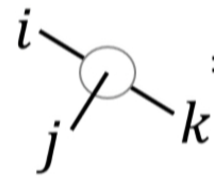
$$|\Psi^{\text{bound}}\rangle = \sum_{o_1 o_2 \dots} \Psi_{o_1 o_2 \cdots} |o_1\rangle \otimes |o_2\rangle \otimes \cdots$$



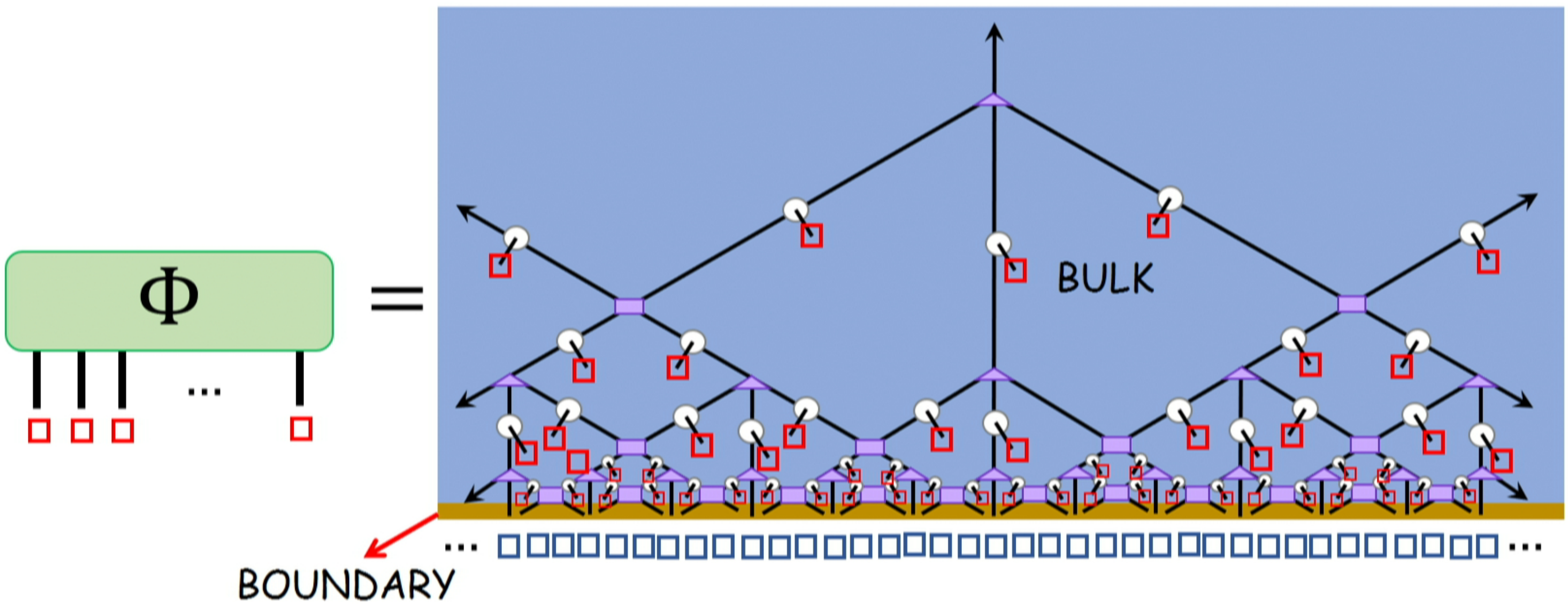
Step 3: Insert copy tensors on the bonds

$$|\Psi^{\text{bulk}}\rangle = \sum_{o_1 o_2 \dots b_1 b_2 \dots} \Phi_{o_1 o_2 \dots b_1 b_2 \dots} |o_1\rangle \otimes |o_2\rangle \otimes \cdots |b_1\rangle \otimes |b_2\rangle \otimes \cdots$$

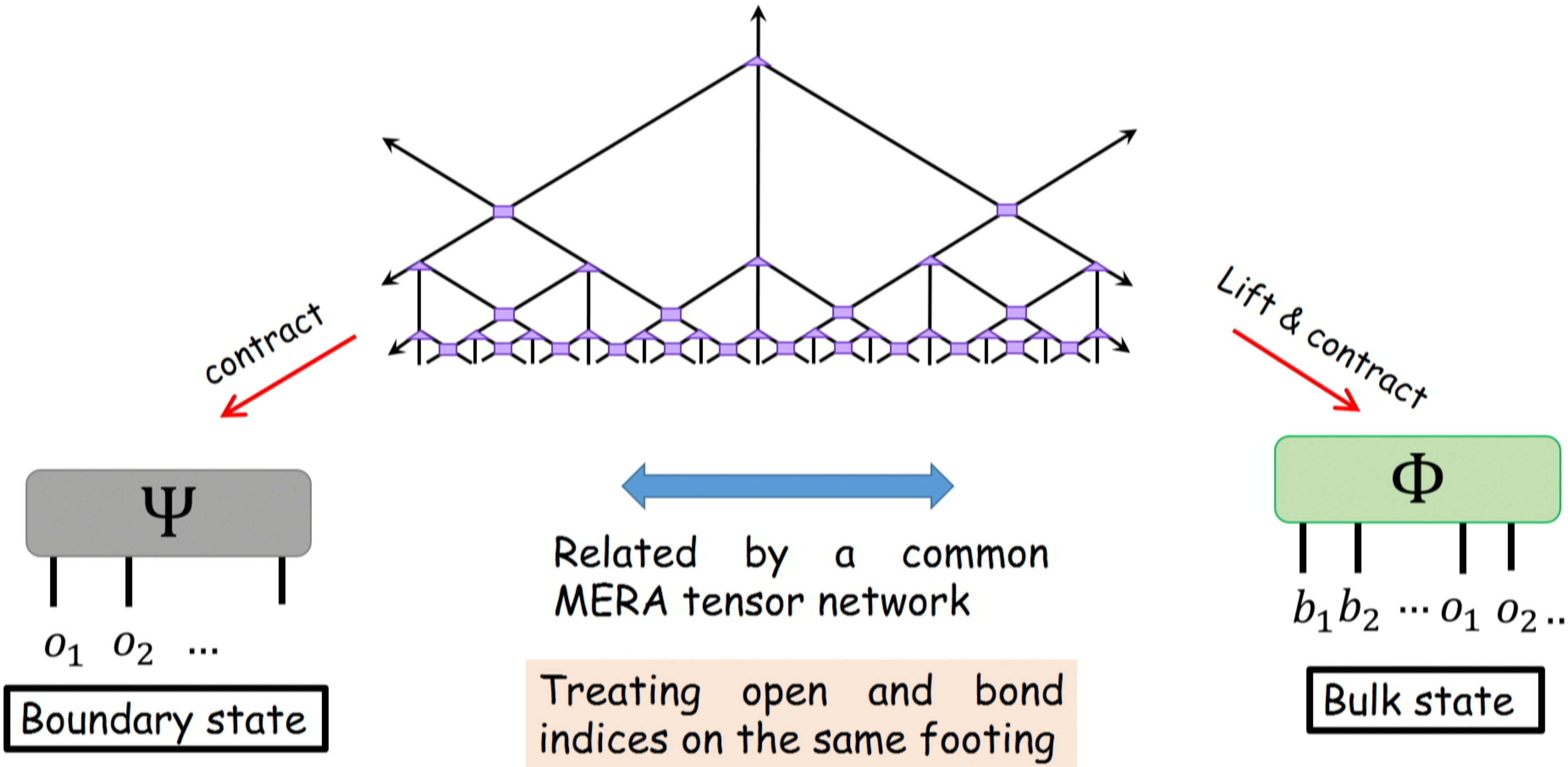
"Copy tensor"



$= \delta_{ij}\delta_{jk}$



A bulk/boundary correspondence from the MERA



Holographic interpretation

- *Proposal:* The bulk state is the holographic dual of the ground state. That is, the bulk state is a physical state of a quantum gravity theory (e.g., satisfies the diffeomorphism and Hamiltonian constraints of loop quantum gravity)

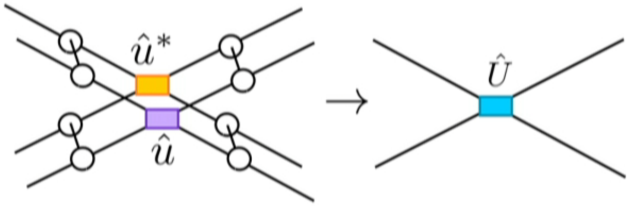
- We do not attempt to deduce a classical geometry in the bulk

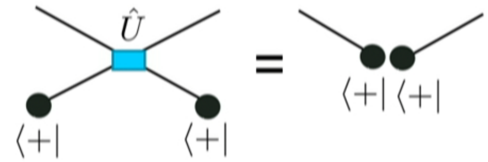
- Generally have a quantum state in the bulk, which may become a classical description in some limit (e.g. large central charge at the boundary)

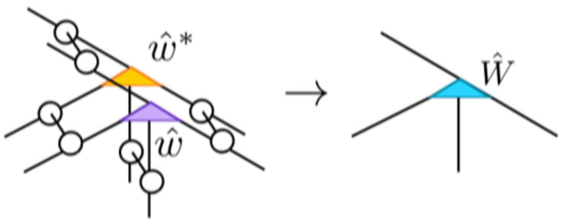
- We associate bulk degrees of freedom with the bonds, not with the tensors as in "Exact holographic mapping" by Qi, arxiv:1309:6282.

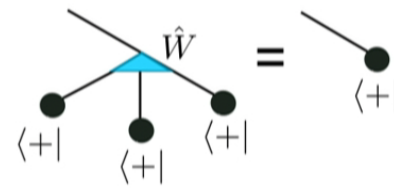
- Allows for introducing suitable gauge transformations in the bulk

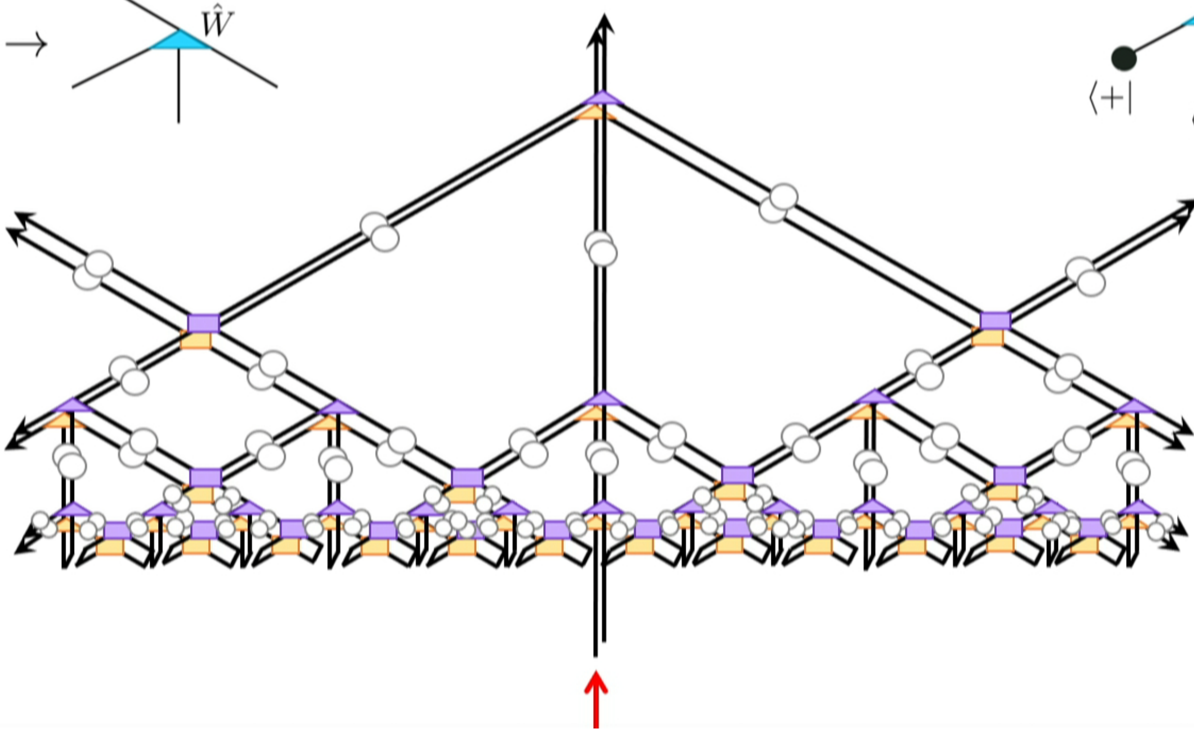
Reduced density matrix of a bulk site











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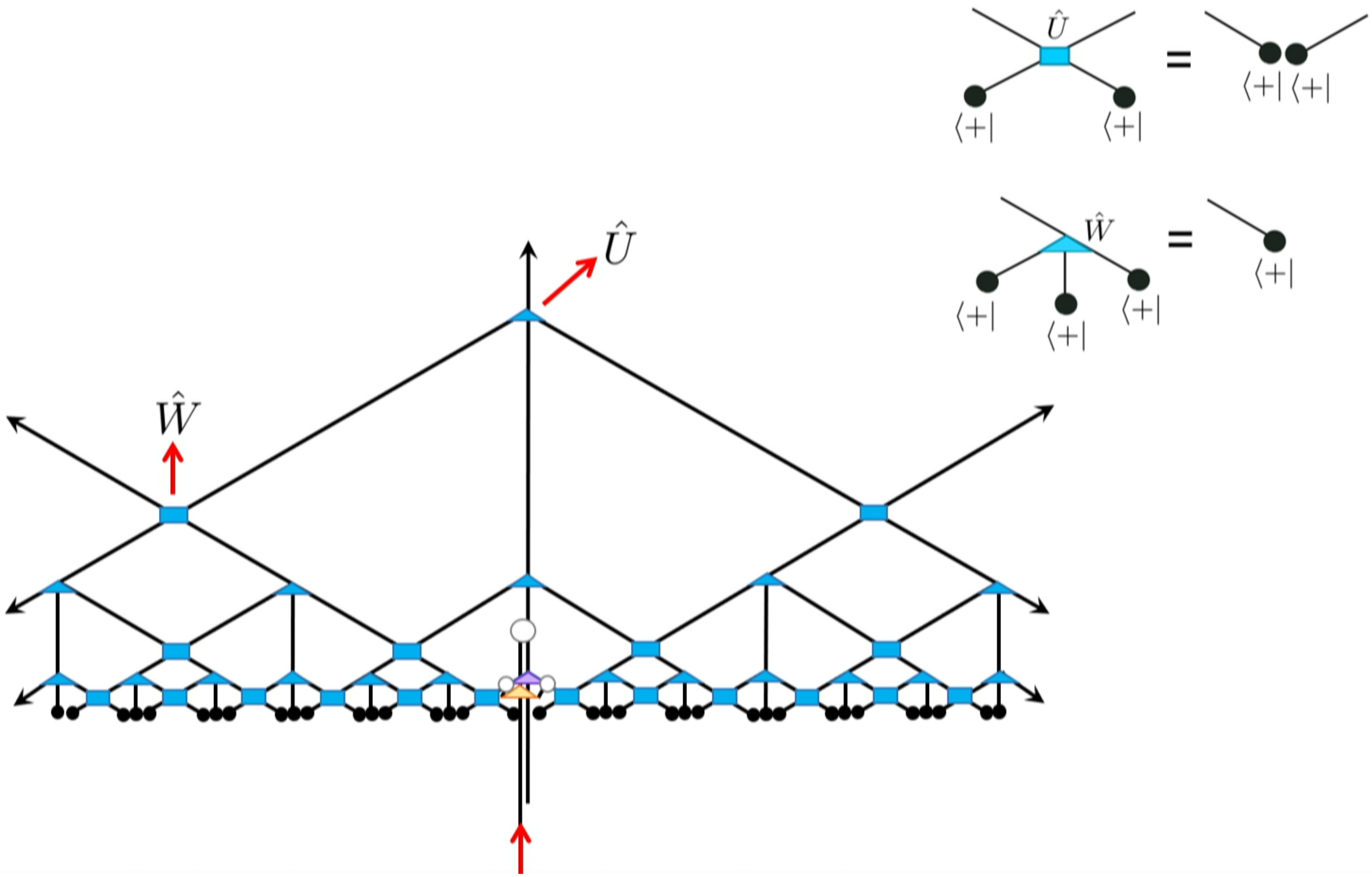

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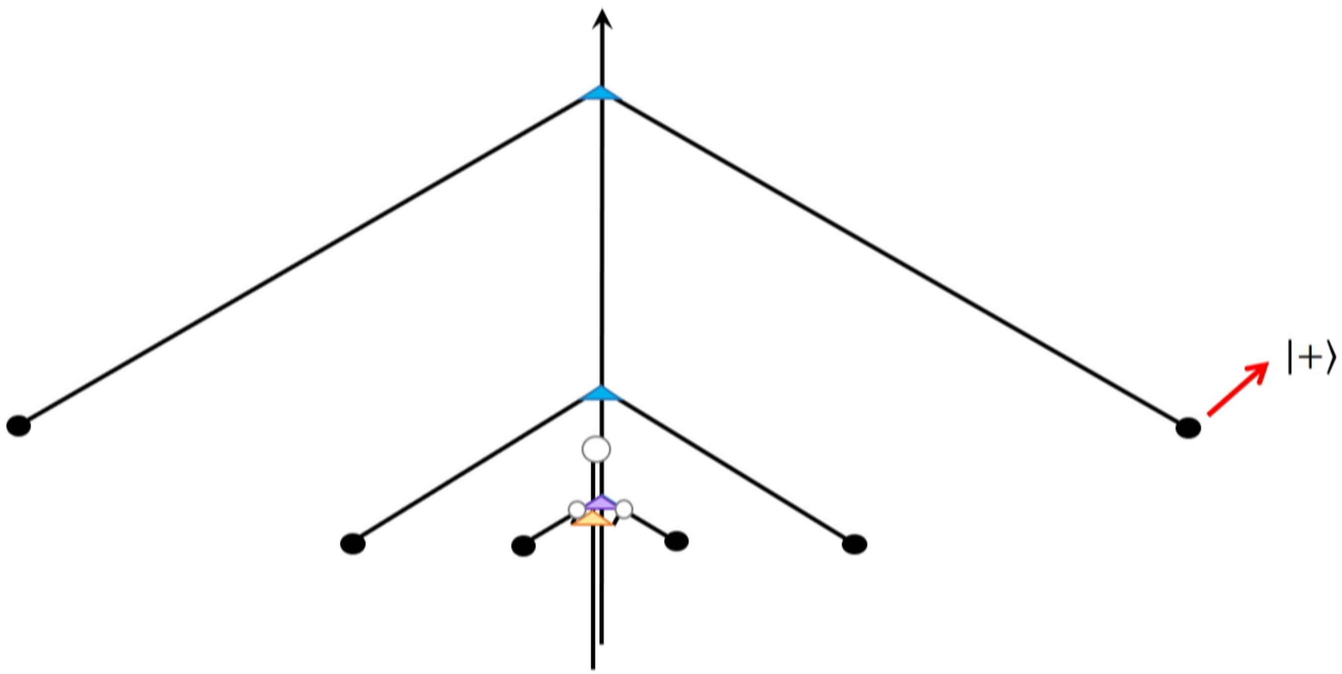
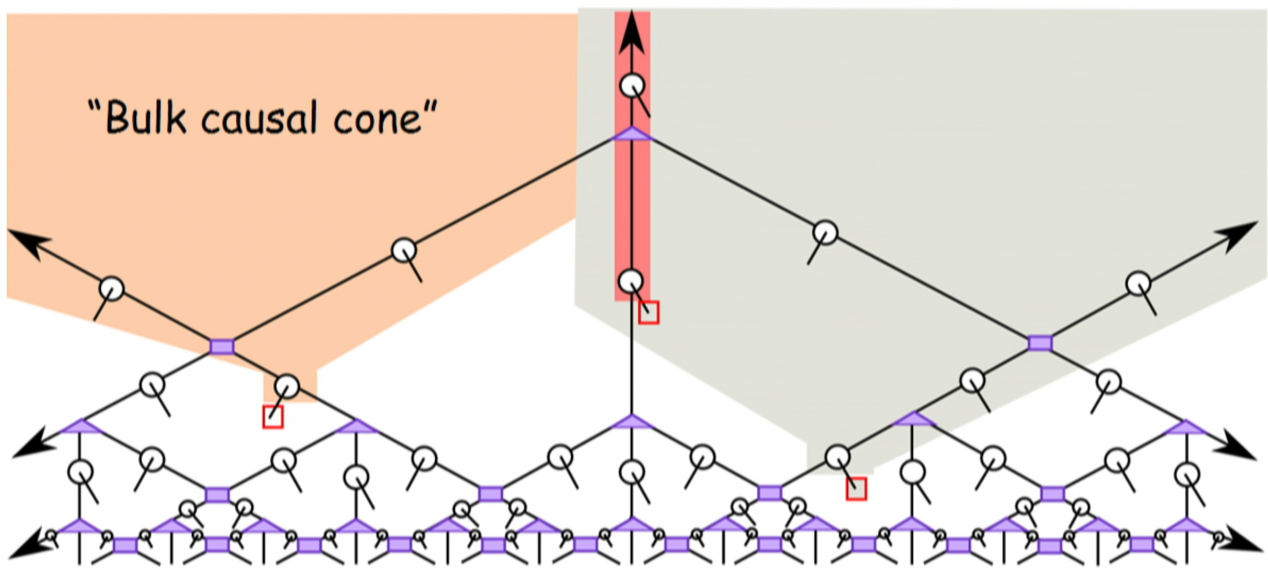


Lifted MERA has a causal cone structure

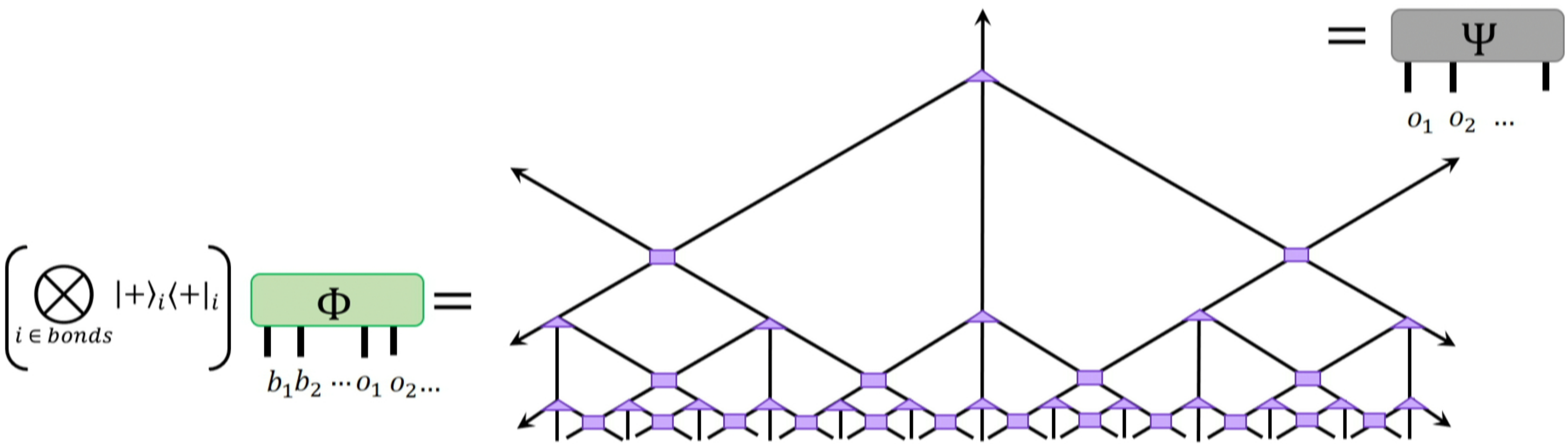


A bulk/boundary dictionary

Recovering the boundary state from the bulk state

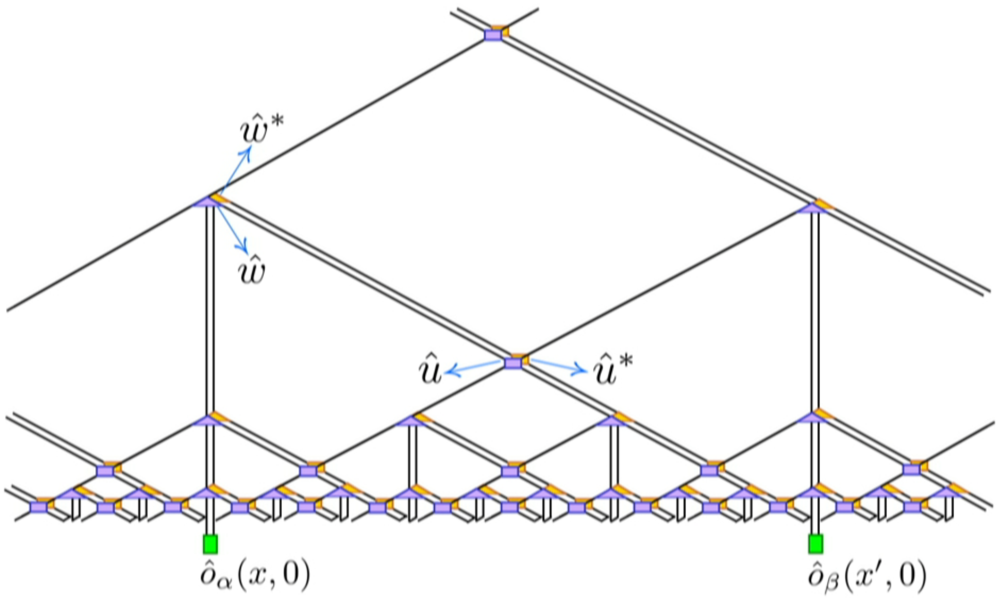
Project each bulk site to $|+\rangle = |0\rangle + |1\rangle + \cdots |\chi\rangle$

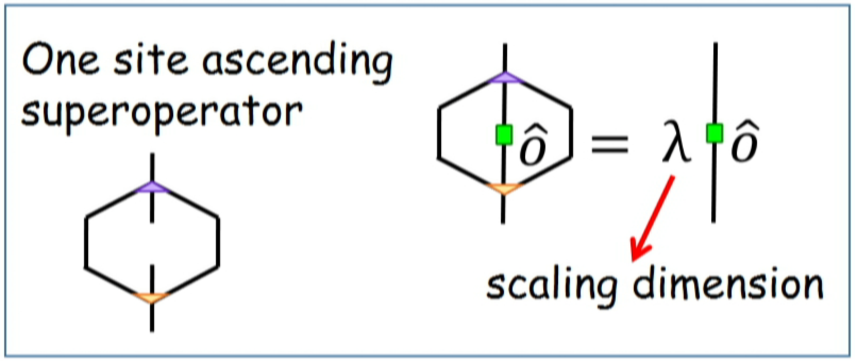
$$|\Psi^{\text{bound}}\rangle = \left(\bigotimes_{i \in bonds} |+\rangle_i \langle +|_i \right) |\Psi^{\text{bulk}}\rangle$$



A bulk/boundary dictionary for a critical ground state

$$\langle \Psi^{\text{bound}} | \, \hat{o}_\alpha(x,0) \hat{o}_\beta(x',0) | \Psi^{\text{bound}} \rangle =$$





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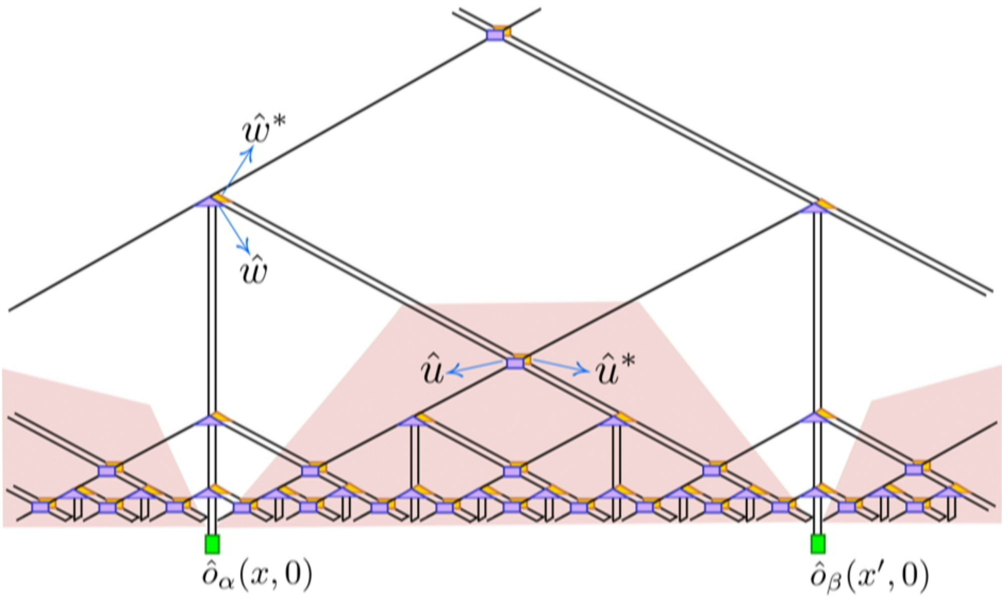
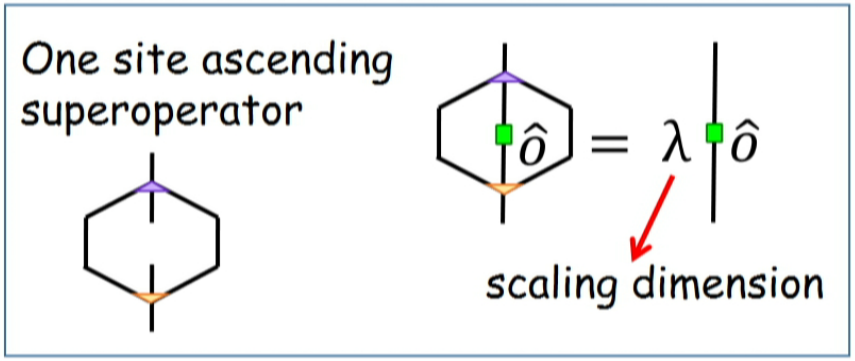
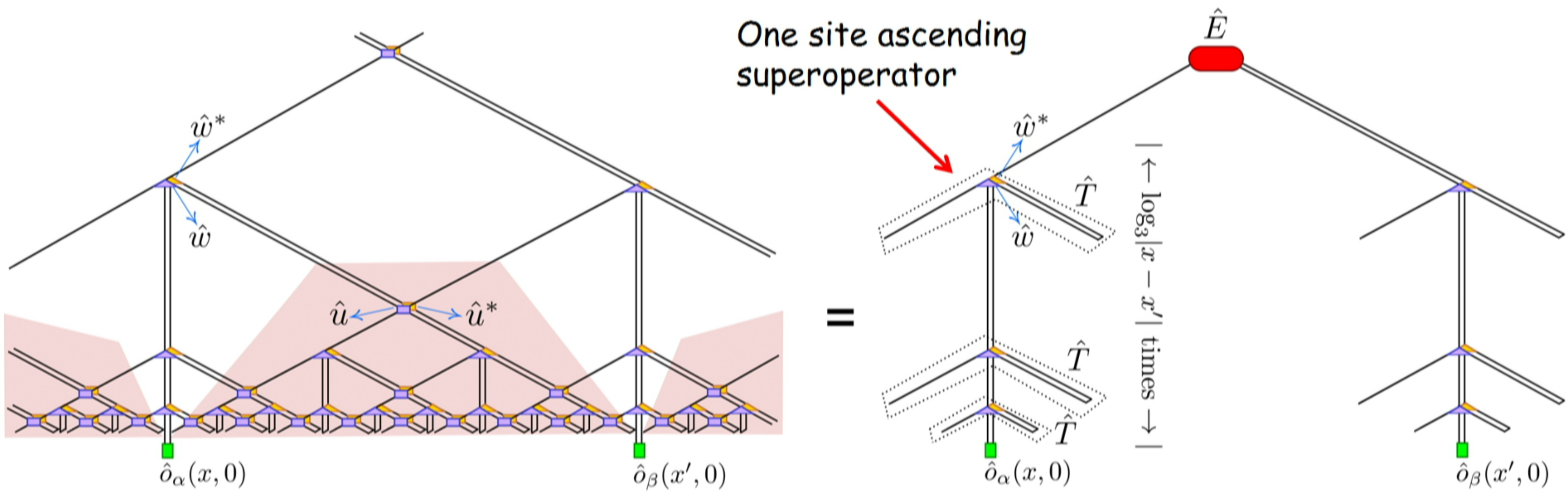




A bulk/boundary dictionary for a critical ground state

$$\langle \Psi^{\text{bound}} \big| \, \hat{o}_\alpha(x,0)\hat{o}_\beta(x',0) \big| \Psi^{\text{bound}} \rangle =$$



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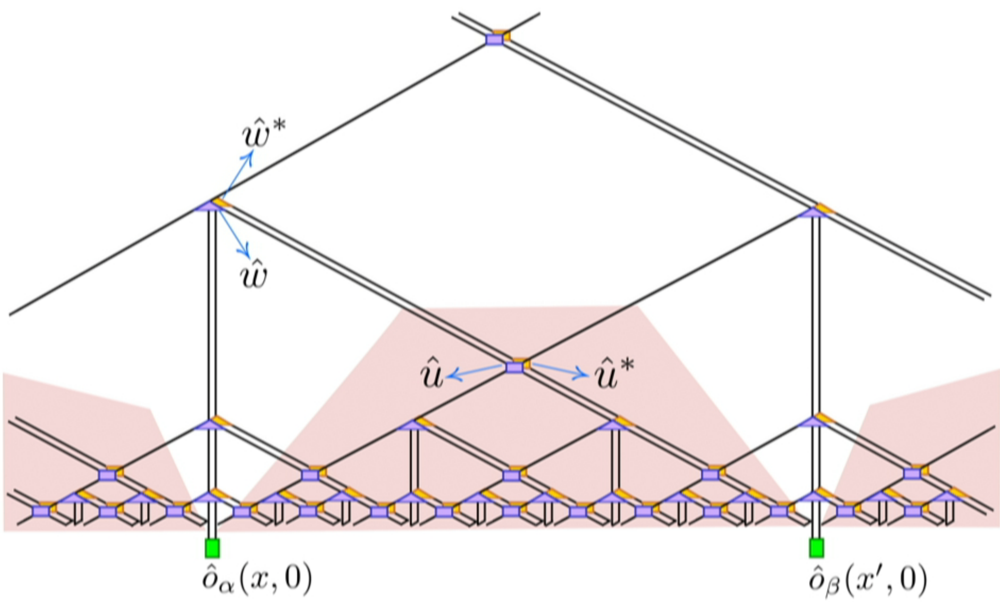
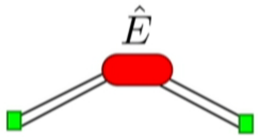
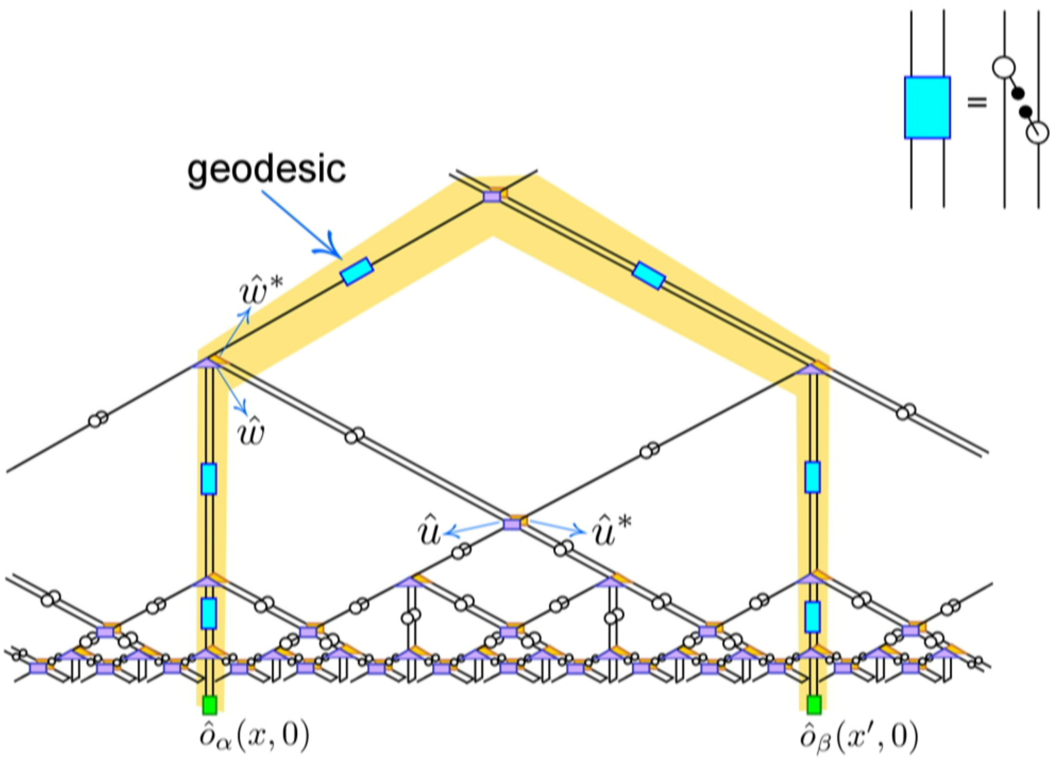


$$= \quad \lambda^p$$



A bulk/boundary dictionary for a critical ground state

Project each site on the geodesic between (x,0) and (x',0) to the $|+\rangle$ state.



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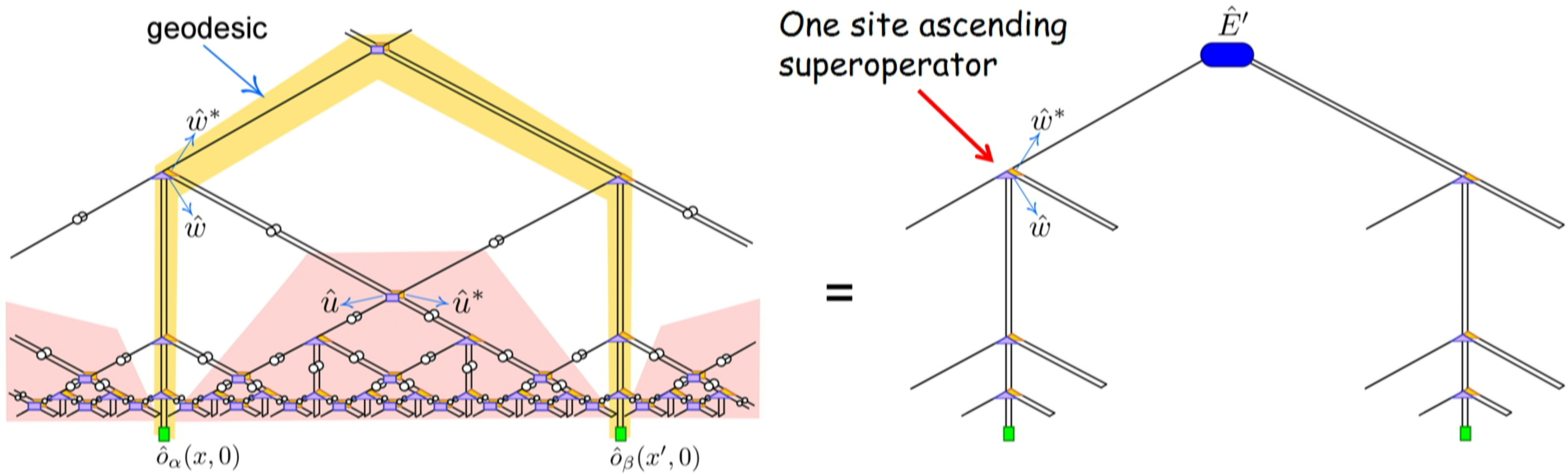

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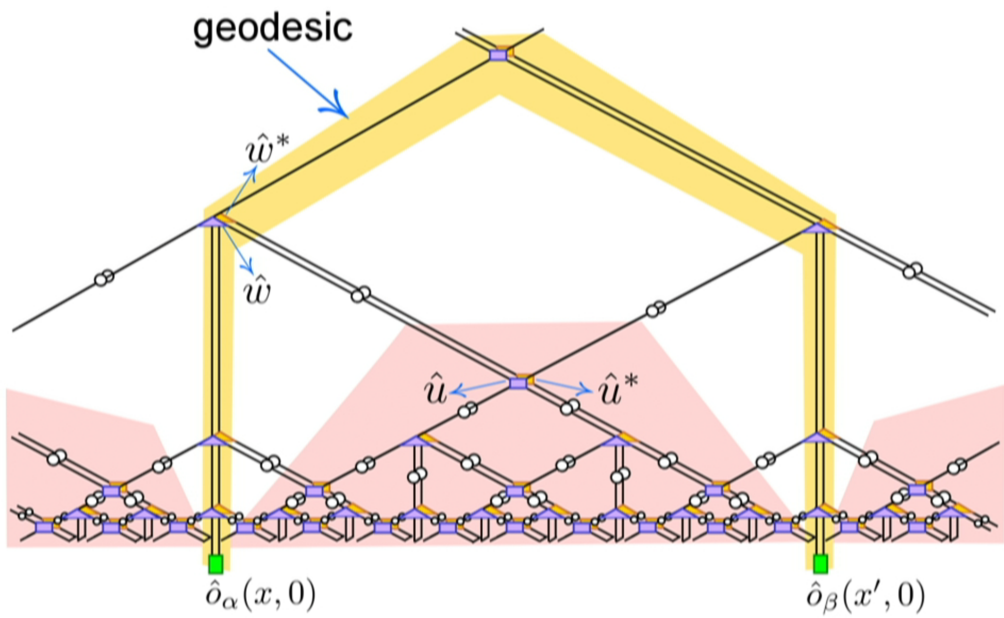

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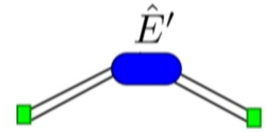

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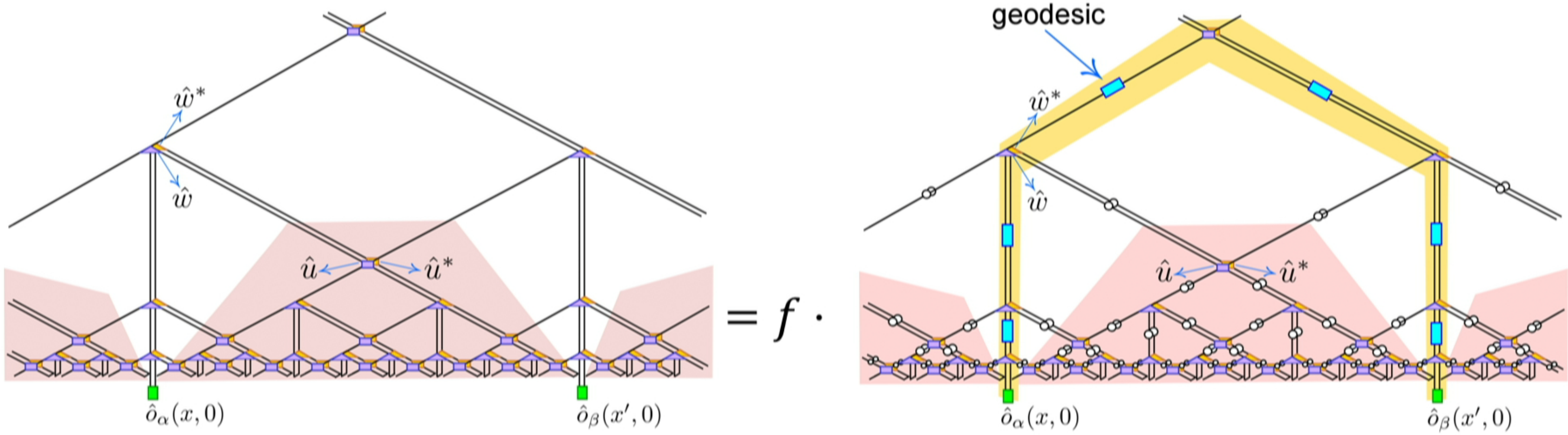
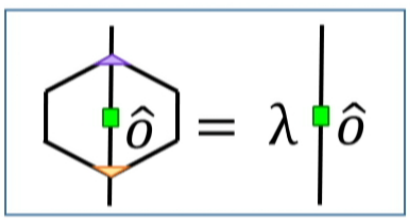
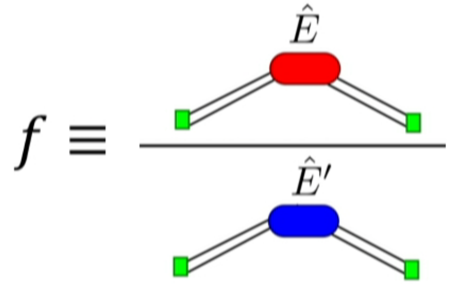
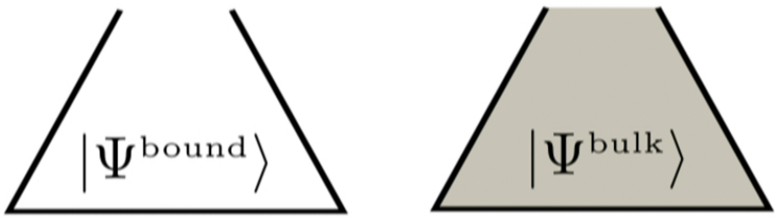




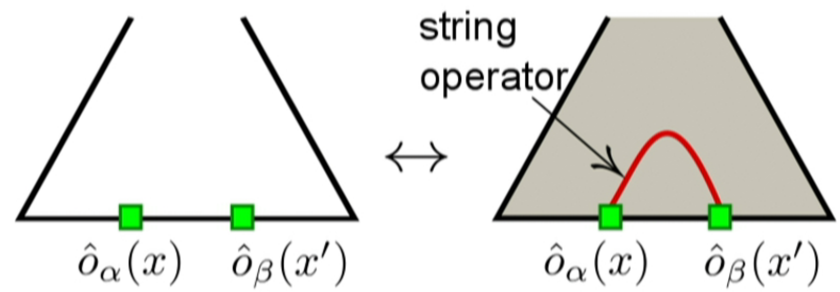


A bulk/boundary dictionary for a critical ground state

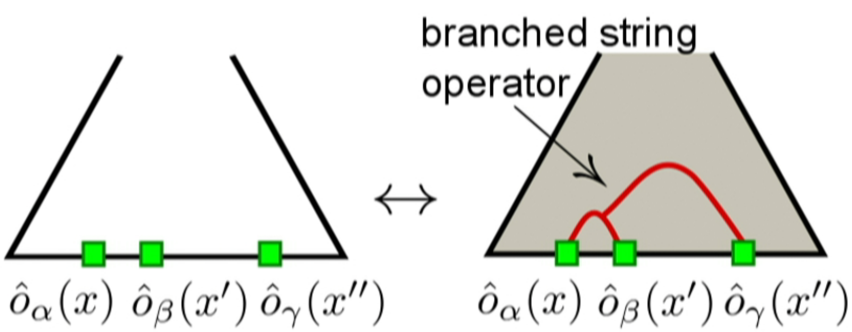


Schematic depiction of the MERA and the lifted MERA

Correlators only on special sites!

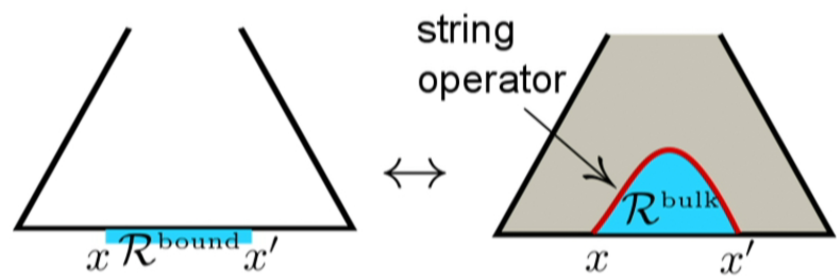


2-point correlator



3-point correlator

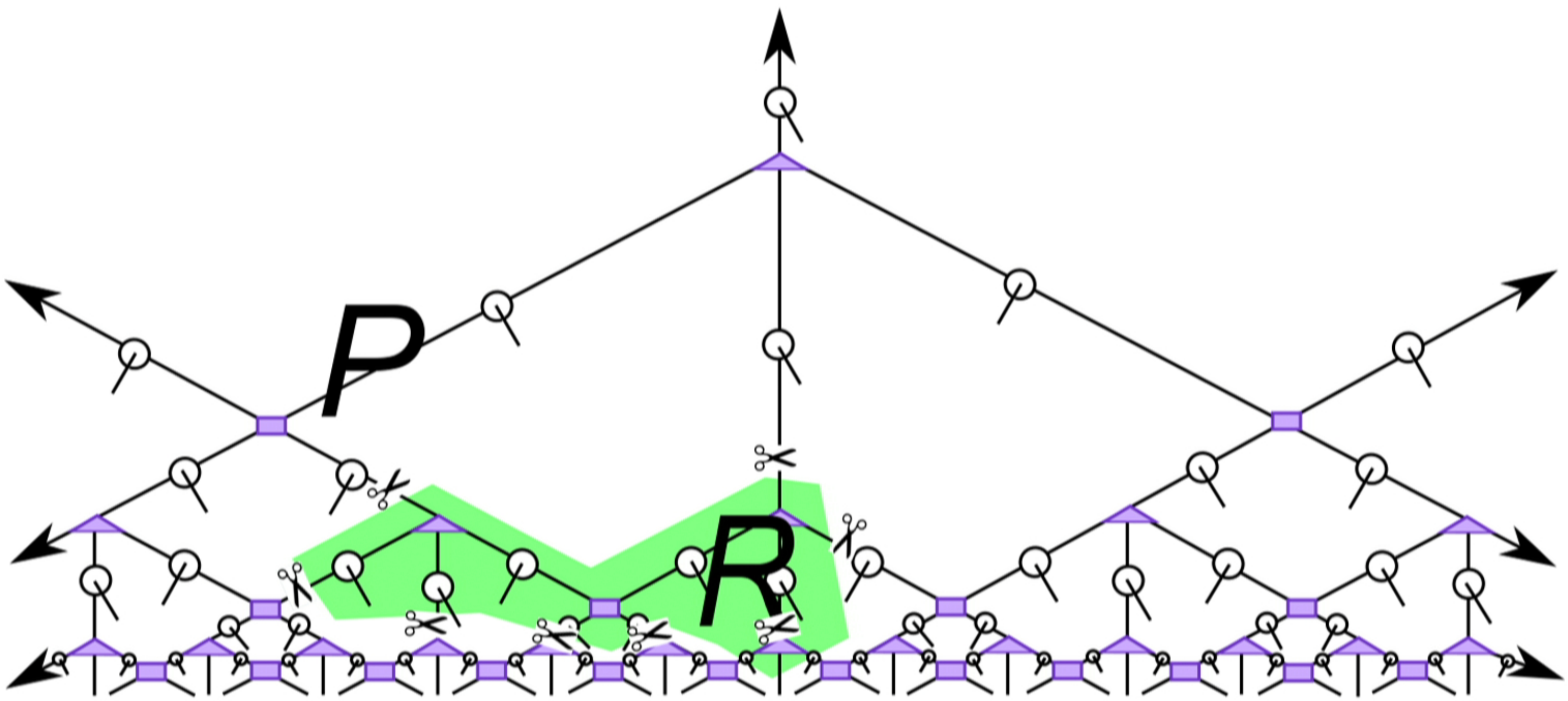
Witten diagrams?



Reyni entanglement entropy

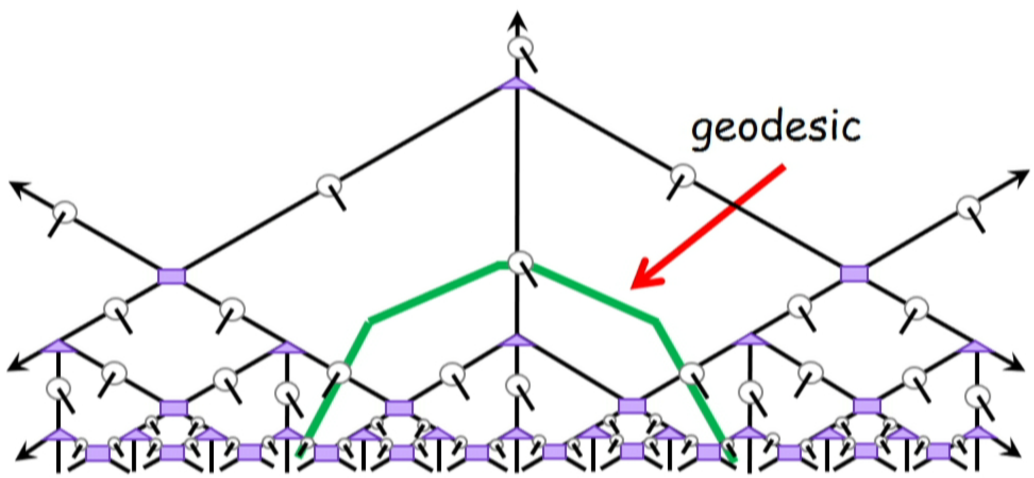
Entanglement and correlations in the bulk state

Bulk state has an "area law" entanglement



Entanglement of bulk sites along a geodesic

Different critical models	Central charges
$\hat{H}^{\mathrm{ISING}} = \sum_i \hat{\sigma}_z^i \hat{\sigma}_z^{i+1} + \hat{\sigma}_x^i,$	0.5
$\hat{H}^{\mathrm{BC}} = \sum_i -\hat{X}^i \hat{X}^{i+1} + \alpha(\hat{X}^i)^2 + \beta(\hat{Z}^i)^2,$	0.7 $(\alpha \approx 0.91 \;\; \beta \approx 0.41)$
$\hat{H}^{\mathrm{POTTS}} = \sum_i \hat{P}_x^i \hat{P}_y^{i+1} + \hat{P}_y^i \hat{P}_x^{i+1} + \hat{P}_z^i,$	0.8
$\hat{H}^{\mathrm{XXZ}} = \sum_i \hat{\sigma}_x^i \hat{\sigma}_x^{i+1} + \hat{\sigma}_y^i \hat{\sigma}_y^{i+1} + \theta \hat{\sigma}_z^i \hat{\sigma}_z^{i+1},$	1 $(-1 < \theta \leq 1)$



Entanglement of bulk sites along a geodesic

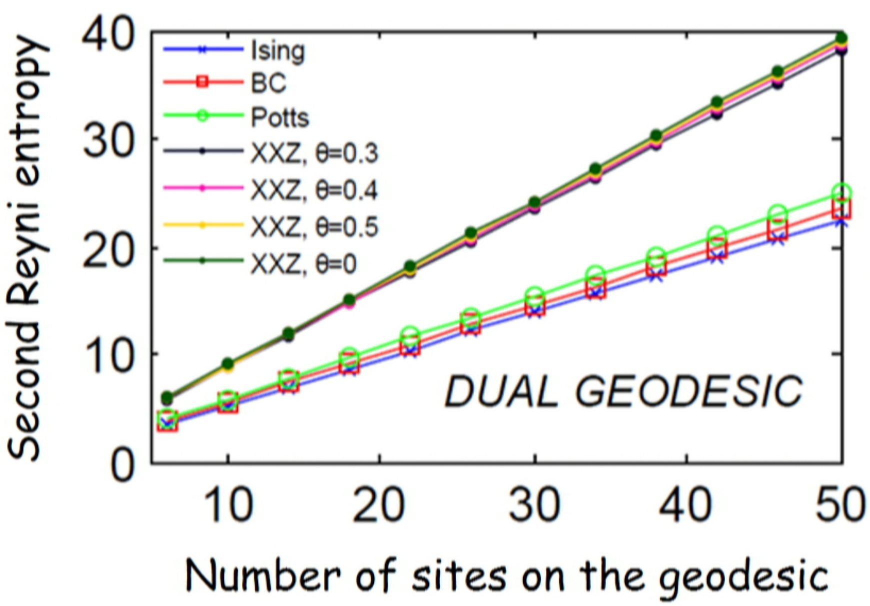


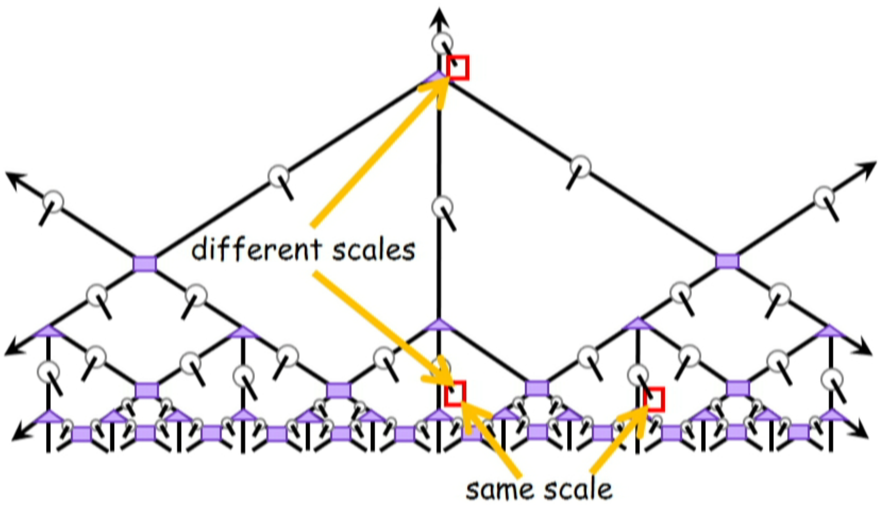
TABLE I. The boundary central charge and slope of the second Reyni entropy scaling of bulk sites lying on a dual geodesic for various 1D critical models.

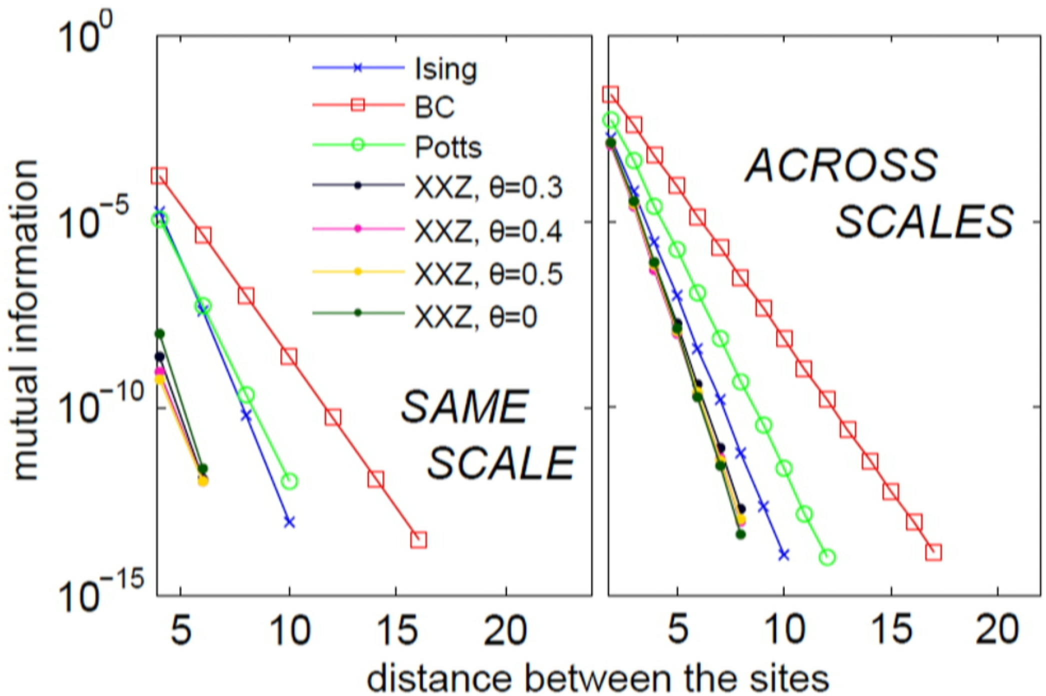
MODEL	CENTRAL CHARGE (MERA)	ENTROPY SCALING SLOPE
Ising	0.5018	0.8518
BlumeCapel	0.7052	0.8927
3-state Potts	0.8061	0.9456
XXZ, $\theta = 0.3$	1.0147	4.3971/3
XXZ, $\theta = 0.4$	1.0143	4.4592/3
XXZ, $\theta = 0.5$	1.0145	4.4982/3
XXZ, $\theta = 0$	1.0273	4.5447/3

- We find that the entanglement entropy scales with a slope that increases with central charge.
- And that for different models with the same central charge the slope is approximately equal.
- This suggests that the bulk states "knows" the central charge of the boundary.

  (Ryu-Takayanagi formula?)

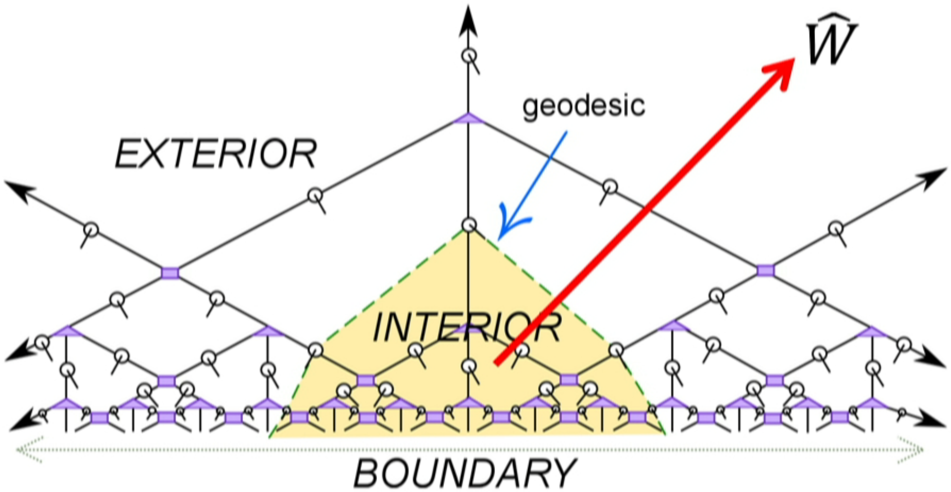
Correlation lengths in the bulk





- Finite correlation length in both directions
- Correlation lengths decreases monotonically with increase in central charge at the boundary, except for the Ising model.

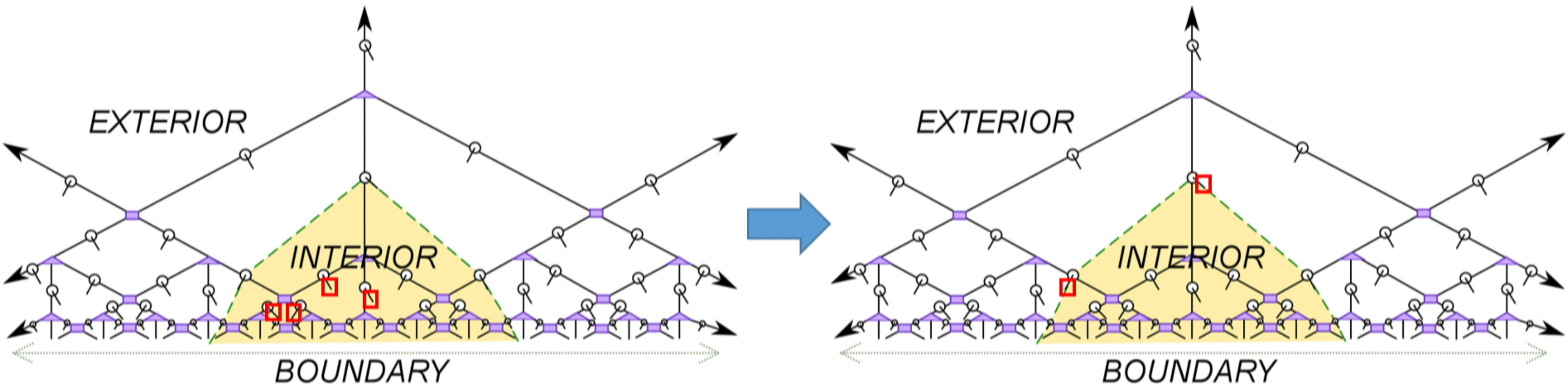
Structure of the bulk quantum information



$$\hat{\rho}^{GEO} = \widehat{W}^{\dagger}\hat{\rho}^{INT}\,\widehat{W} \qquad \widehat{W}^{\dagger}\widehat{W} = \hat{I}$$

$$S^{GEO} = S^{INT}$$

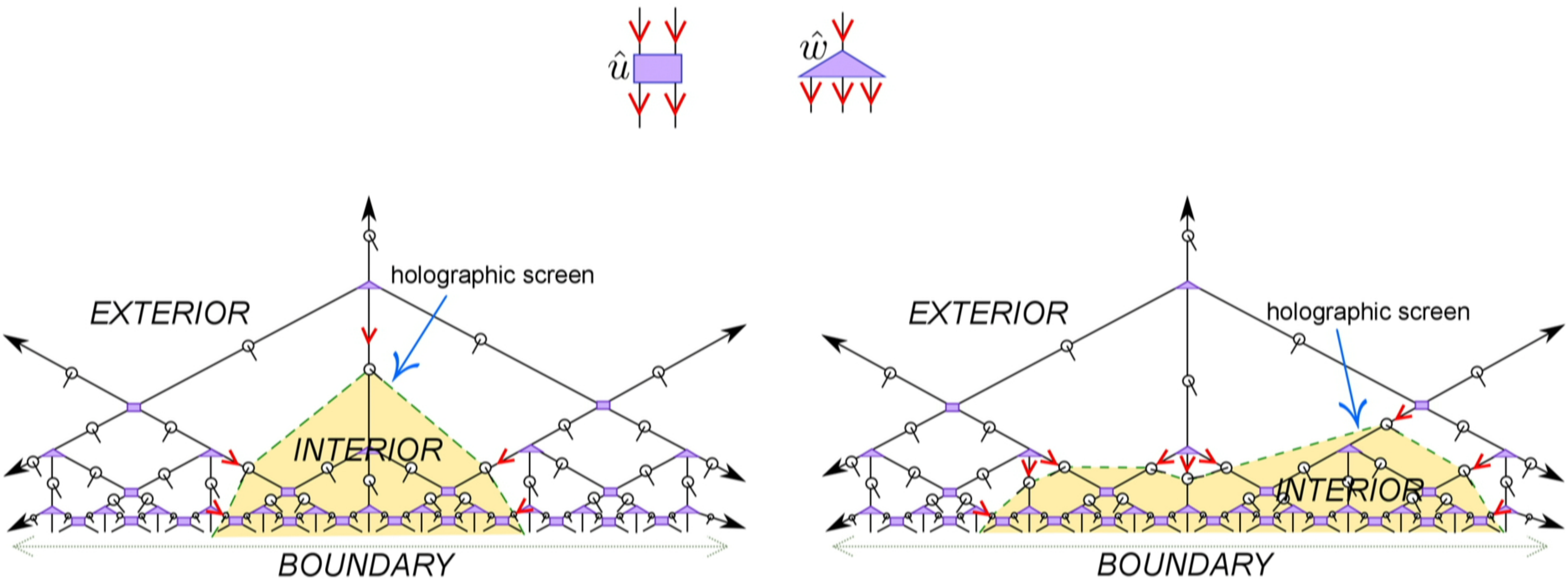
Structure of the bulk quantum information



$$\hat{O}^{GEO} = \widehat{W}\ \hat{O}^{INTERIOR}\widehat{W}^{\dagger}$$

$$\langle \hat{O}^{INTERIOR} \rangle = \langle \hat{O}^{GEO} \rangle$$

Structure of the bulk quantum information



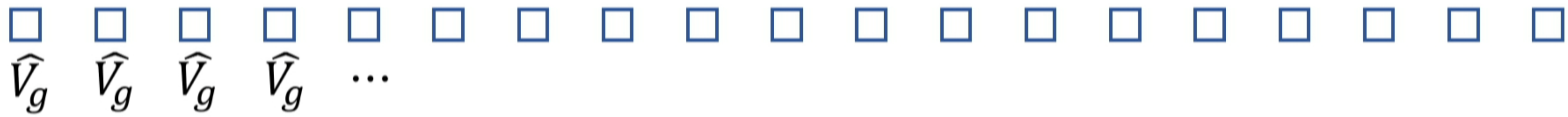
- Analog of the presence of holographic screens in the boundary description?
- Tensors located in the INTERIOR of a holographic screen in the MERA implement a local conformal transformation. *B. Czech et al, PRB 94, 085101 (2016)*

Summary of features related to holography so far

- Bulk state is derived from an RG description (MERA) of the ground state
- 2-point boundary correlators correspond to expectation value of extended operators in the bulk (Witten diagrams?)
- Bulk state can be described in terms of holographic screens (local conformal transformations of the ground state)
- Bulk entanglement shows some dependence on boundary central charge (analog of the Ryu-Takayanagi formula?)

Bulk observables
and
holographic spin networks

- Introduce action of a global symmetry G on the lattice.



$\hat{V}_g$: on-site unitary representation of the symmetry

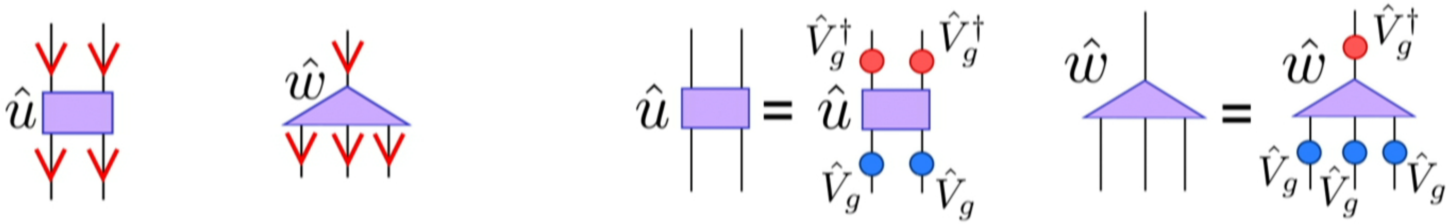
- Hamiltonian commutes with the symmetry:

$$[\hat{H}, (\otimes_{i \in sites} \hat{V}_g)] = 0, \quad \forall g \in G$$

- Ground state is invariant under the action of the symmetry:

$$(\otimes_{i \in sites} \hat{V}_g)|\Psi\rangle = |\Psi\rangle, \quad \forall g \in G$$

- Represent the ground state with a MERA made of tensors that "commute" with the symmetry.



- Symmetric tensors represent the symmetry of the ground state exactly

- And also capture the expected RG flow of the ground state.

S. Singh and G. Vidal, PRB 88, 121108(R) (2013)

- If symmetry not protected along the RG flow in a non-trivial symmetric gapped phase (e.g. Haldane phase), ground state flows to a product state.

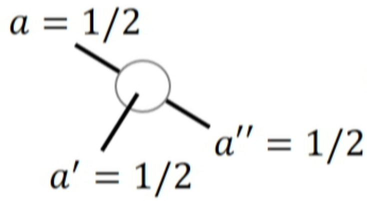
Symmetric copy tensor

- Natural basis on the indices: labelled by irreps of the symmetry

  a: total spin t_a: Degeneracy of spin a m_a: Spin projection

- We require a copy tensor "copies" the bond representation of the MERA to an open index of the lifted MERA, and also that it is symmetric.

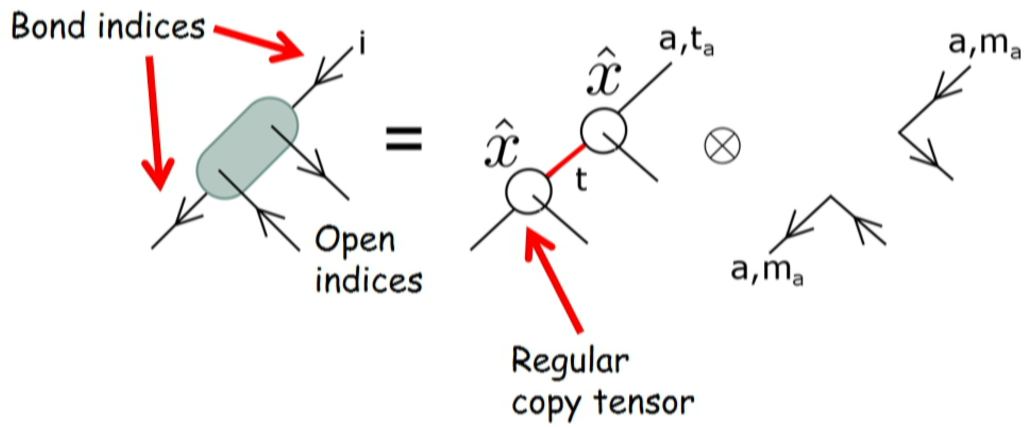
- No such tensor with three indices.



(No such symmetric tensor!)

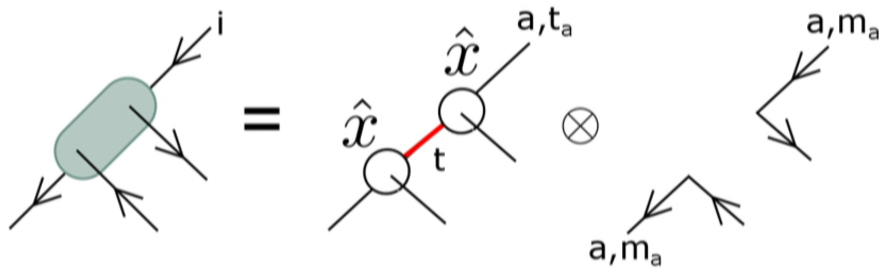
- Simplest choice is a 4-index intertwiner.

Symmetric copy tensor

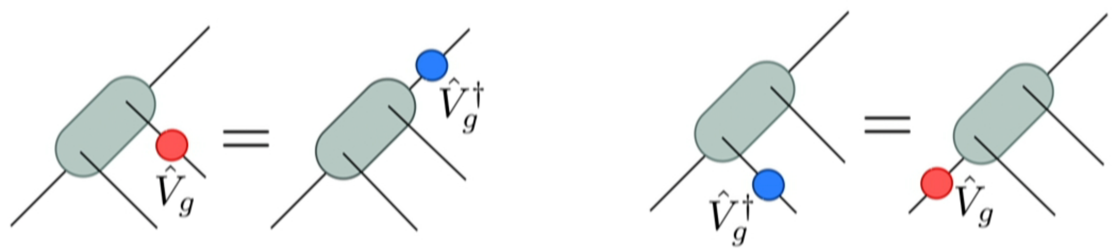


(Two bulk sites for each bond!)

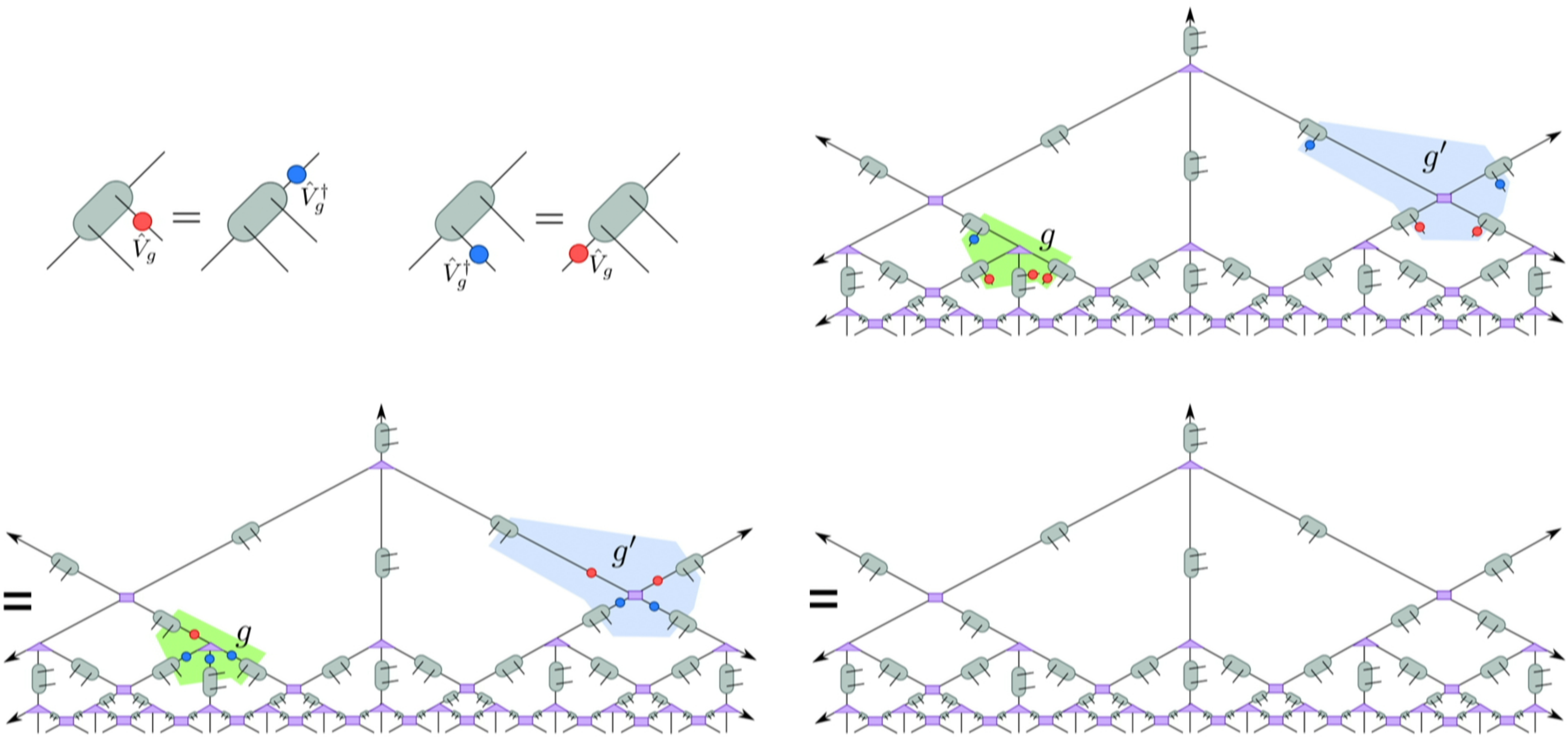
Symmetric copy tensor



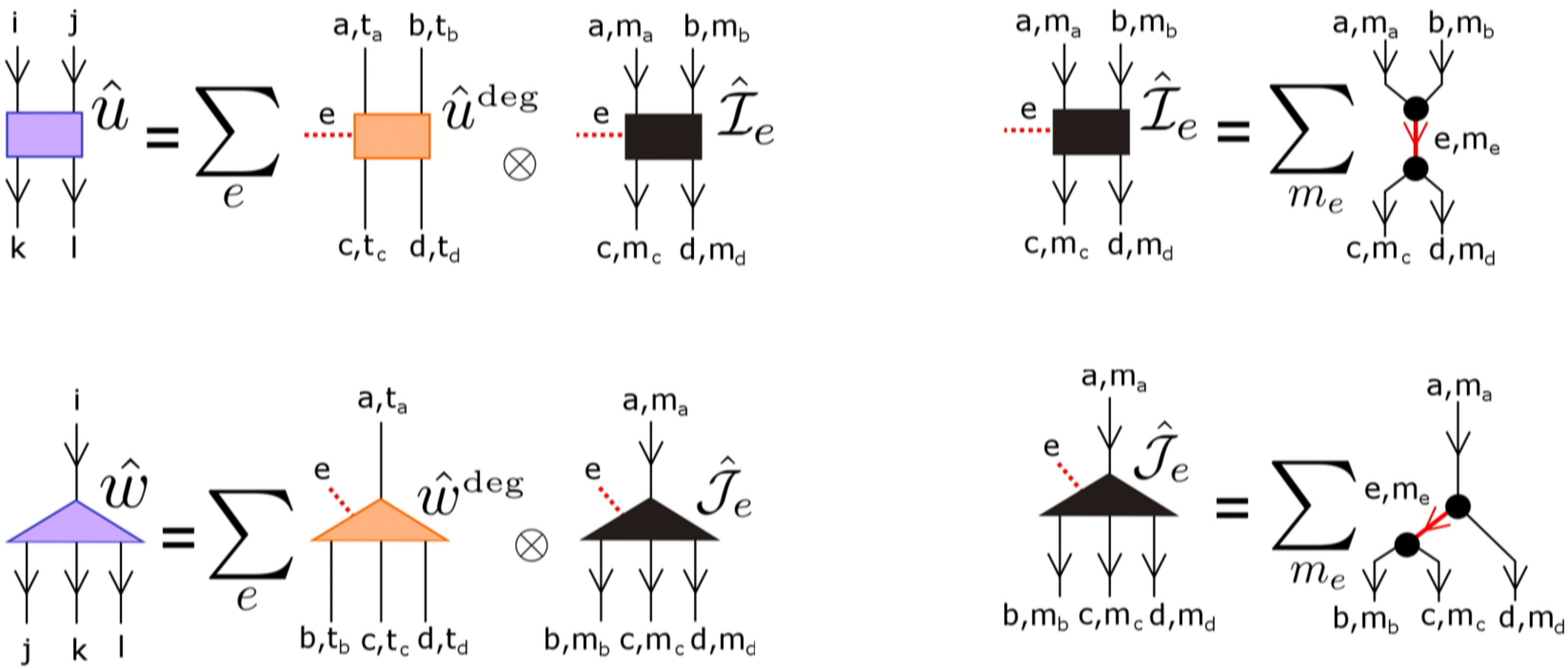
(Two bulk sites for each bond!)



Bulk state has a local symmetry!

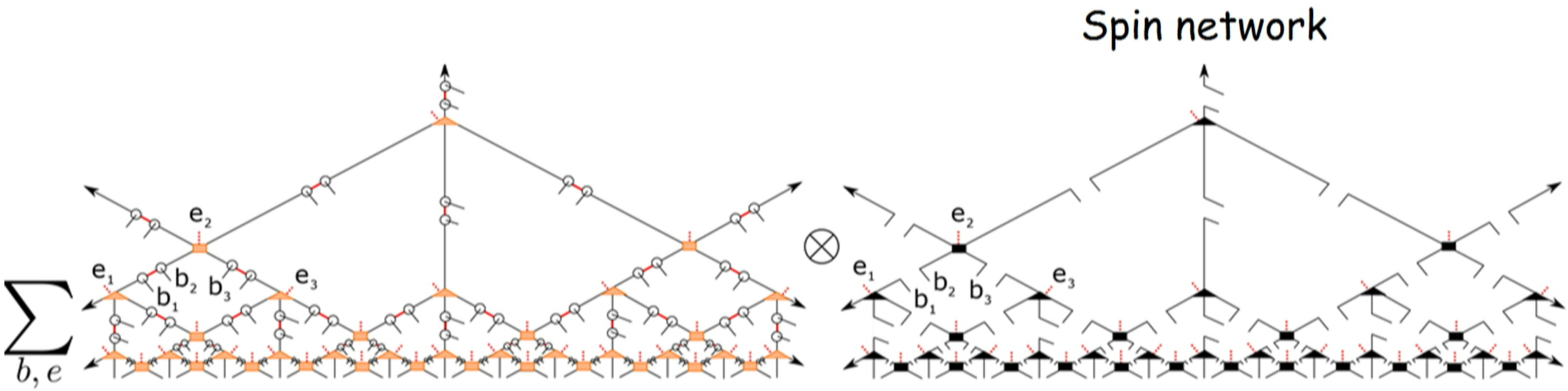


Decomposition of symmetric tensors



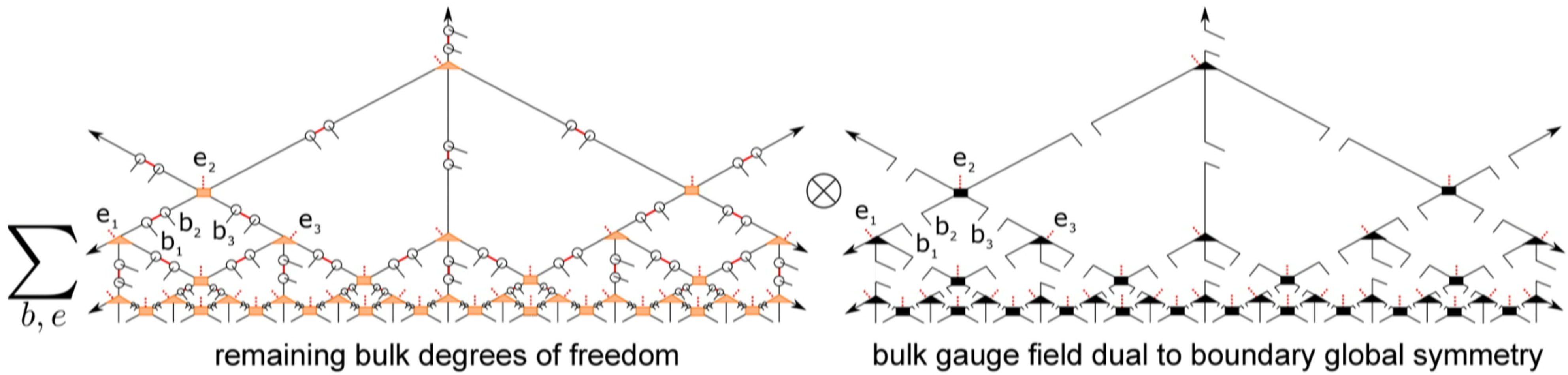
(Wigner-Eckart theorem)

Bulk state decomposed in terms of spin networks



- The spin networks label a basis in the gauge-invariant support of the bulk state in the bulk (like in lattice gauge theories)
- Decomposition allows probing of gauge-invariant bulk observables, since we have well defined quantum numbers in the bulk.

Holographic interpretation



- The decomposition separates out the gauge degrees of freedom in the bulk.
- The remaining degrees of freedom may include gravitational degrees of freedom. (For CFTs with a gravity dual.)
- Entanglement between gauge and remaining degrees of freedom

Summary of features related to holography

- Bulk state is derived from an RG description (MERA) of the ground state.
- 2-point boundary correlators correspond to expectation value of extended operators in the bulk (Witten diagrams?)
- Bulk state can be described in terms of holographic screens (corresponding to local conformal symmetry of the ground state)
- Bulk entanglement shows some dependence on boundary central charge (Ryu-Takayanagi formula?)
- Boundary global symmetry corresponds to a gauge symmetry in the bulk.
- Entanglement between gauge and remaining bulk degrees of freedom.

Outlook

- Constructed a bulk/boundary correspondence between quantum many-body states from the MERA.

- As a toy model to explore the holographic correspondence? (Of interest in quantum gravity + condensed matter physics.)

- Lifting a tensor network is treating a tensor network as a spin network: associating degrees of freedom with the bonds.

- Bulk/boundary correspondence can also relate together states belonging to different types of quantum phases of matter: a possible correspondence between different quantum orders?

- Applied to other tensor networks: MPS, PEPS etc?

Thanks!