Title: Resurgence in Chern-Simons theory

Date: Oct 04, 2016 02:00 PM

URL: http://pirsa.org/16100043

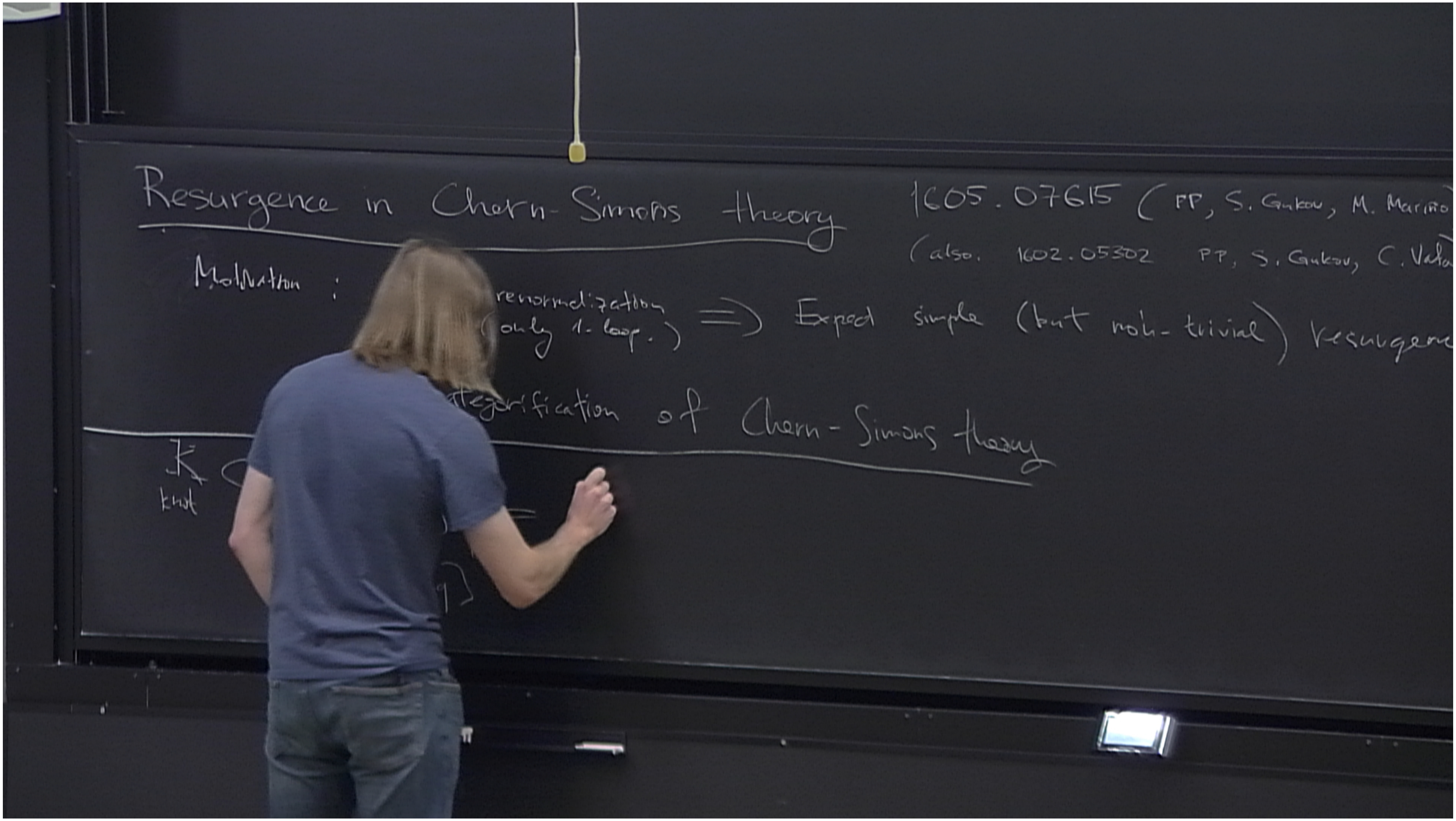
Abstract: <p>In my talk I will consider resurgence properties of Chern-Simons
theory on compact 3-manifolds. I will also describe what role
resurgence plays in the problem of categorification of Chern-Simons
theory, that is the problem of generalizing Khovanov homology of knots
to compact 3-manifolds.</p>

Resurgence in Chern-Simons theory

1605.07615 (PP, S. Gukov, M. Mariño)

(also 1602.05302 PP, S. Gukov, C. Vafa)

Motivation: ① No renormalization (only 1-loop.) $\Longrightarrow$ Expect simple (but non-trivial) resurgence properties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② Categorification of Chern-Simons theory

Chern-Simons action $S(A) = \int A\,dA + \frac{2}{3} A^3$

$G = SU(2)$

$\underset{\text{knot}}{K} \subset S^3$

$$J[K](q) = \int_{SU(2)\text{ connections on } S^3 / \text{gauge}} DA\, e^{2\pi i k_0 S(A)}\, \mathrm{Tr}_\square P \exp i \oint_K A$$

$\hat{Z}[q]$

$k = k_0 + 2$ $\qquad q = e^{\frac{2\pi i}{k}}$

$$\underset{\text{pert}}{\sim} \sum_{n \geq 0} \frac{c_n}{k^n} = J[K](q)$$

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$\underset{\text{pert}}{\sim} \sum_{n\geq 0}$

(FP, S. Gukov, M. Mariño)

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non-trivial) resurgence properties

Chern-Simons action $S(A) = \int A dA + \frac{2}{3} A^3$

$G = SU(2)$

$k = k_0 + 2$ $q = e^{\frac{2\pi i}{k}}$

$\sum_{n \geq 0} \frac{c_n}{k^n} = J[K](q)$

Khovanov homology $\quad \mathcal{H}[K] = \bigoplus_{i,j} \mathcal{H}^{i,j}$

$$J[K](q) = \sum_{i,j} (-1)^i q^j \dim \mathcal{H}^{i,j}$$

Khovanov homology $\mathcal{H}[K] = \bigoplus_{i,j} \mathcal{H}^{i,j}$

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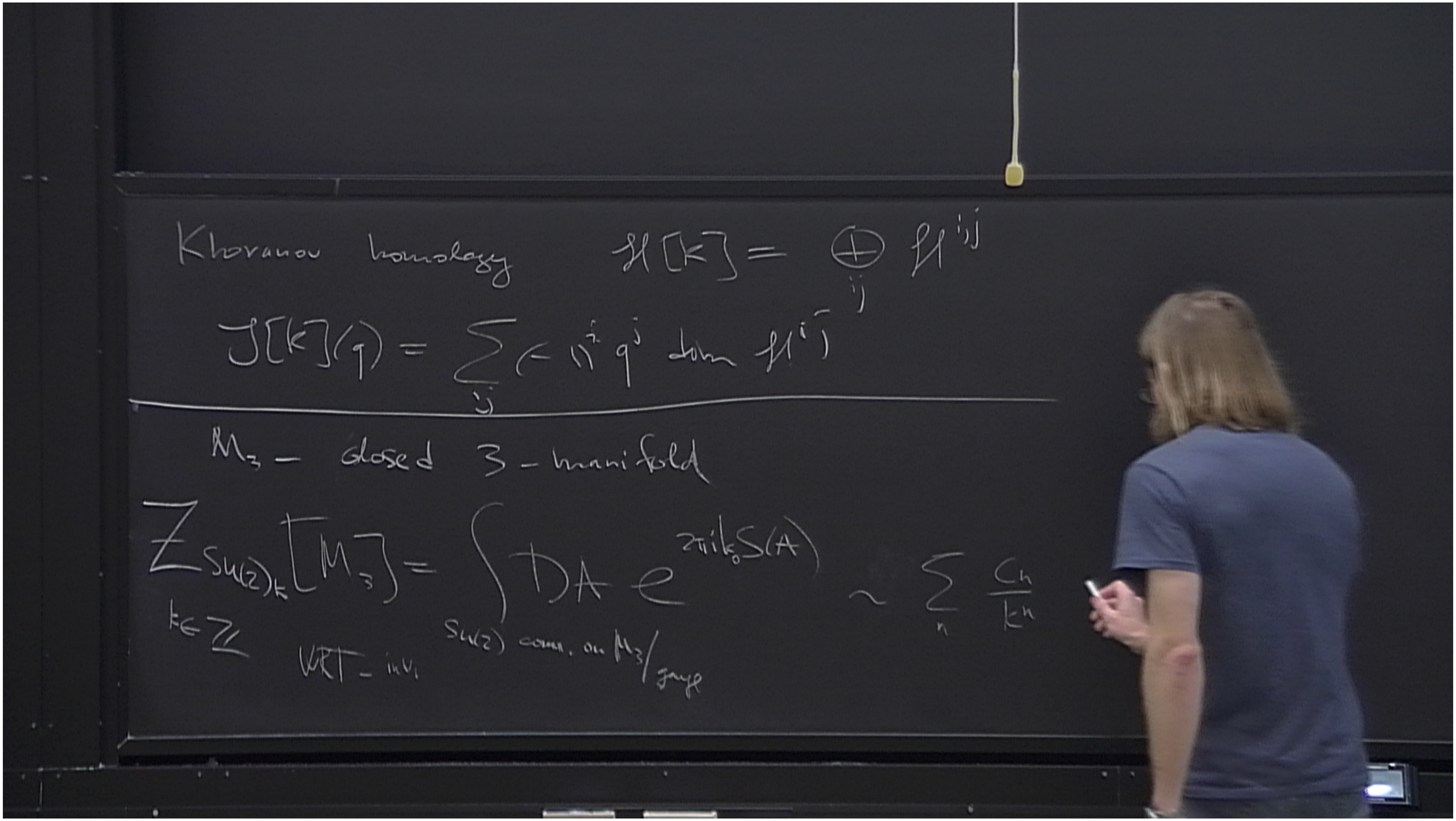
M_3 – closed 3-manifold

$$Z_{SU(2)_k}[M_3] = \int_{SU(2) \text{ conn. on } M_3/\text{gauge}} DA\, e^{2\pi i k_0 S(A)}$$

$k \in \mathbb{Z}$


Khovanov homology $\mathcal{H}[K] = \bigoplus_{i,j} \mathcal{H}^{i,j}$
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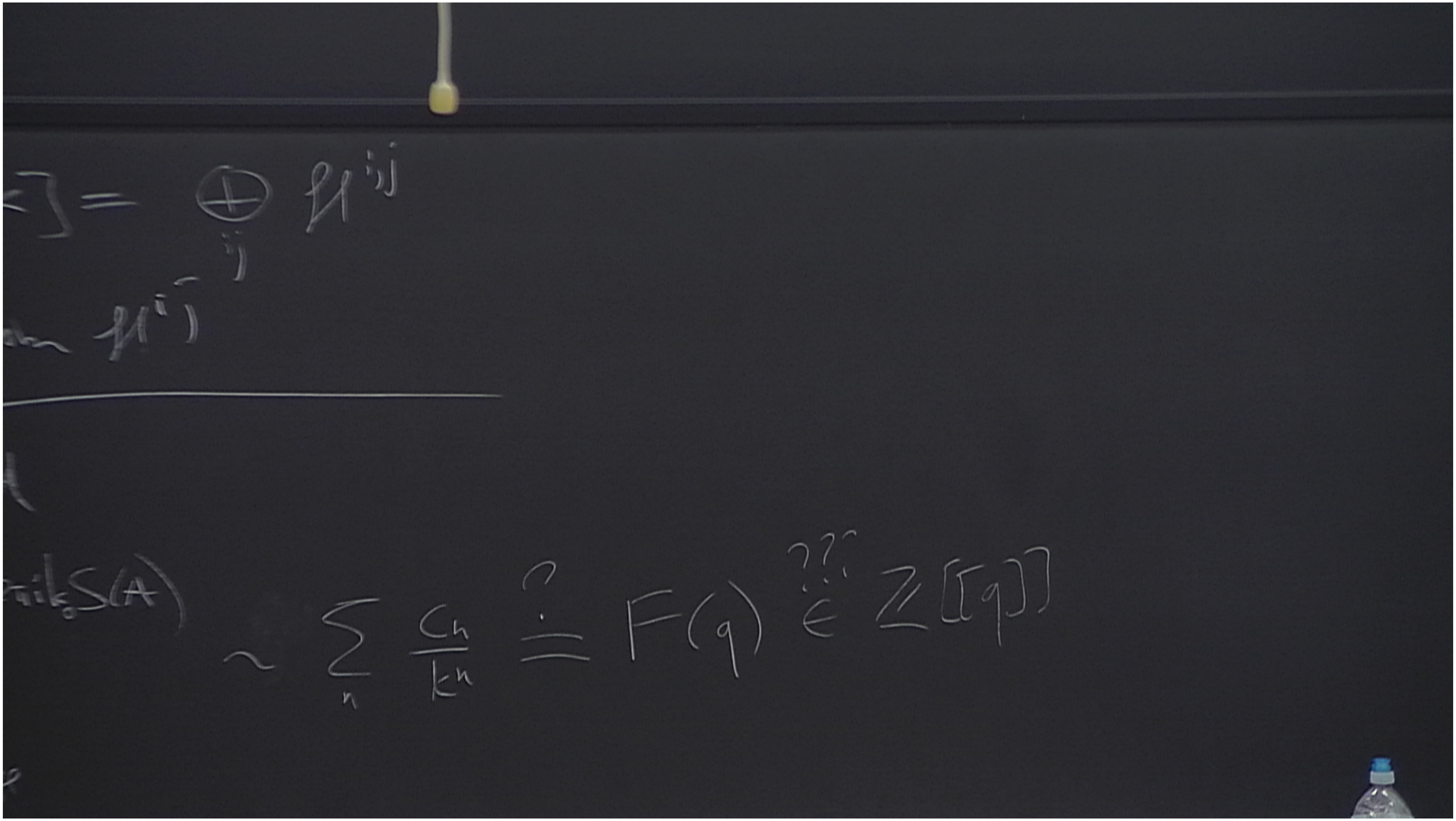
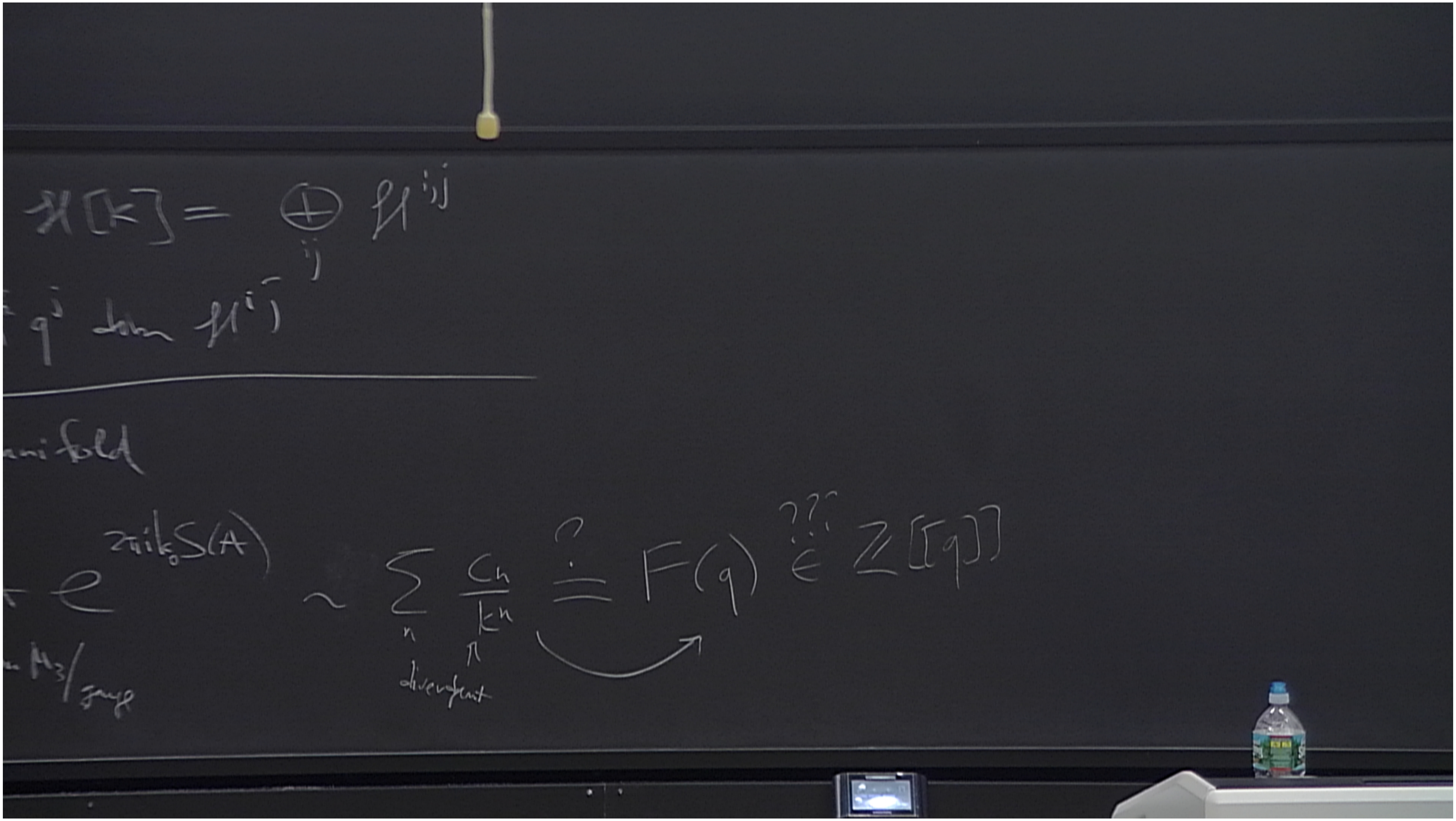
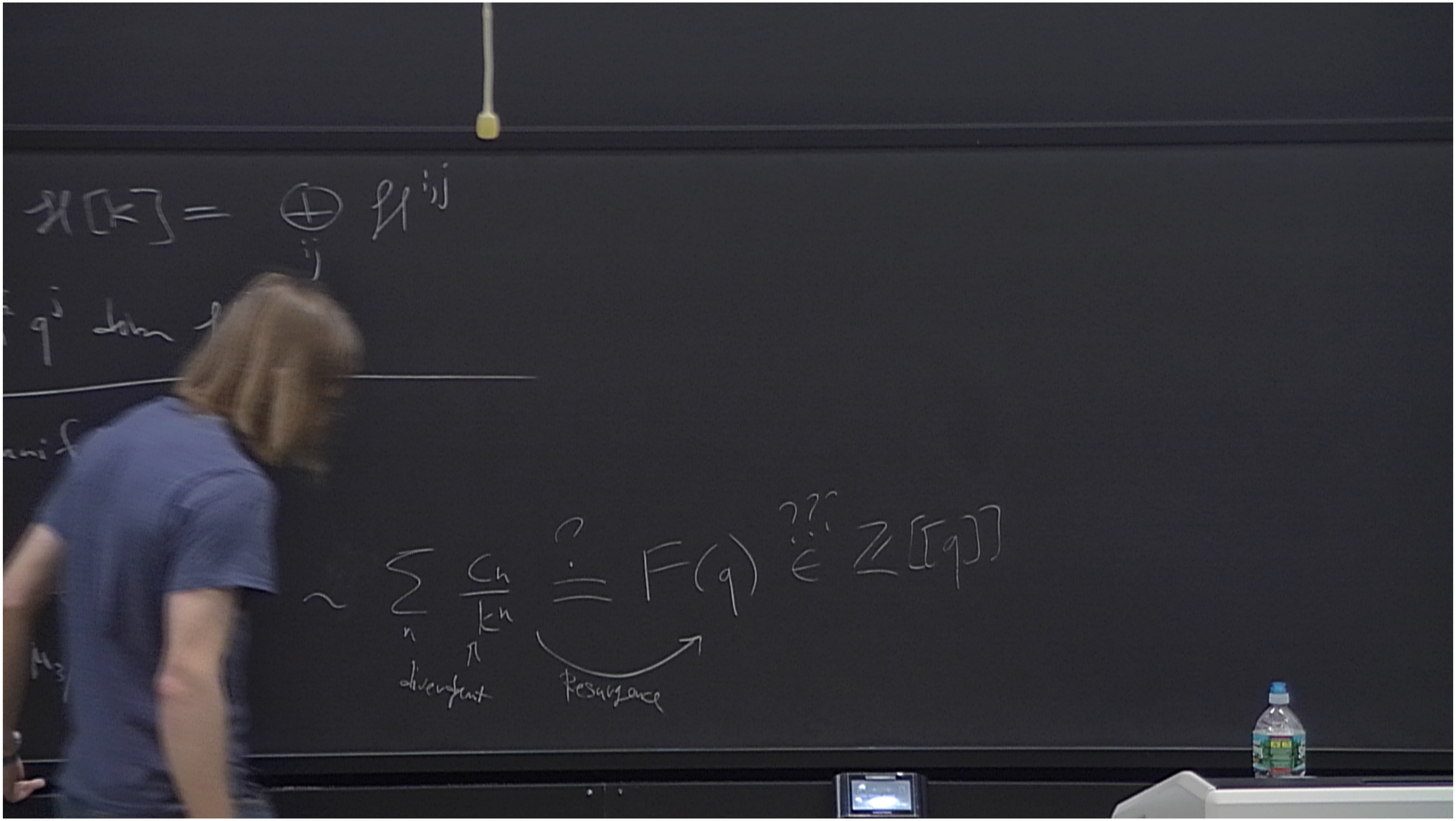
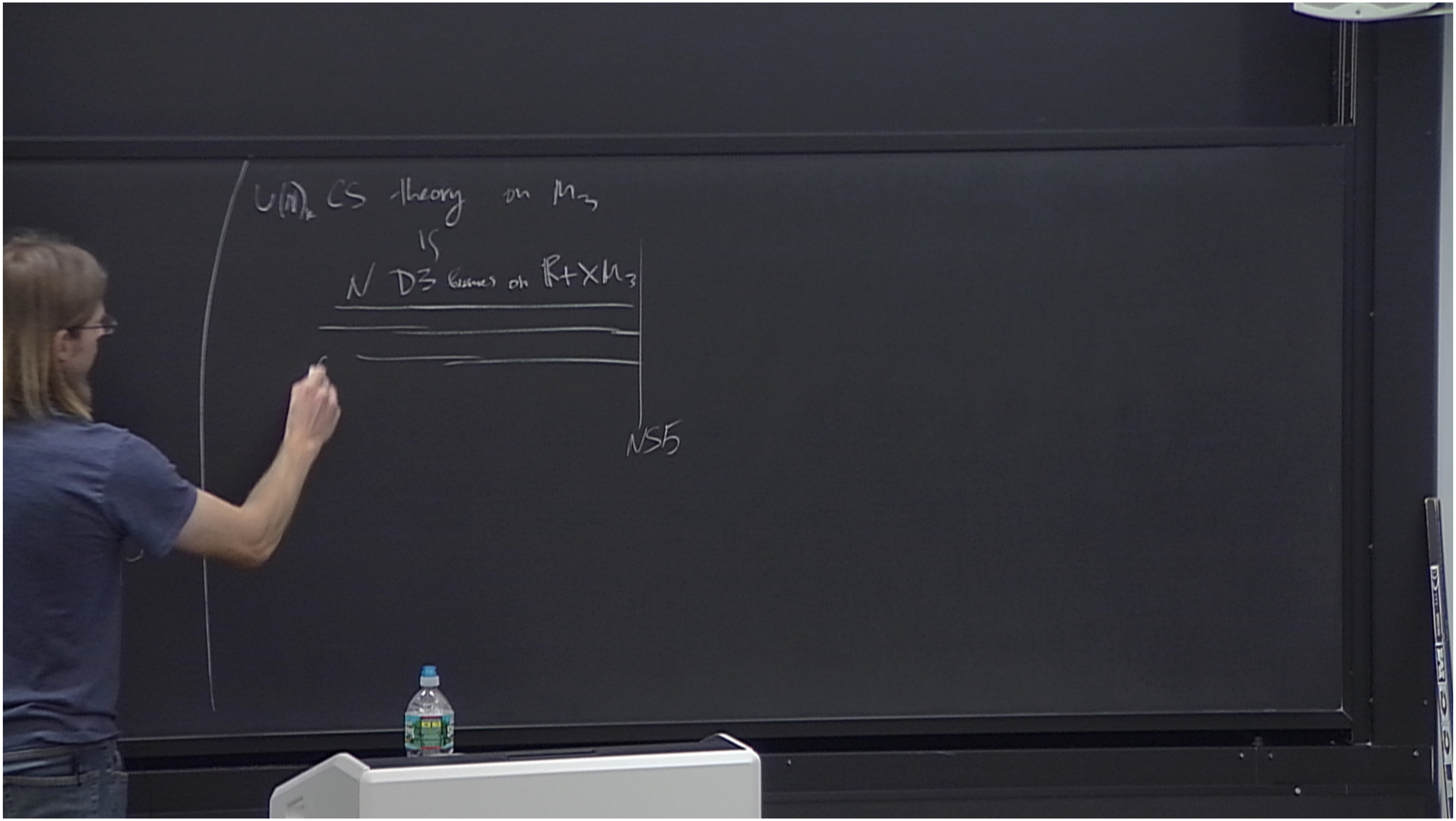
M_3 – closed 3-manifold
$Z_{SU(2)_k}[M_3] = \int DA\, e^{2\pi i k S(A)} \sim \sum_n \frac{c_n}{k^n}$
$k \in \mathbb{Z}$
WRT-inv.
SU(2) conn. on M_3/gauge


$\sim \sum_n \frac{c_n}{k^n} \overset{?}{=} F(q) \overset{???}{\in} \mathbb{Z}[[q]]$


$\mathcal{H}[k] = \bigoplus_j \mathcal{H}^{i,j}$
$q^j \dim \mathcal{H}^{i,j}$
manifold
$e^{2\pi i k_0 S(A)} \sim \sum_n \frac{c_n}{k^n} \overset{?}{=} F(q) \overset{???}{\in} \mathbb{Z}[[q]]$
divergent
M_3/gauge



H[K] = ⊕_{i,j} H^{i,j}
~ Σ_n c_n / k^n ≟ F(q) ∈??? Z[[q]]
divergent
Resurgence


U(N)_k CS theory on M_3
is
N D3 branes on R+ × M_3
NS5

$U(N)_k$ CS theory on M_3

is

N D3 branes on $\mathbb{R}_+ \times M_3$

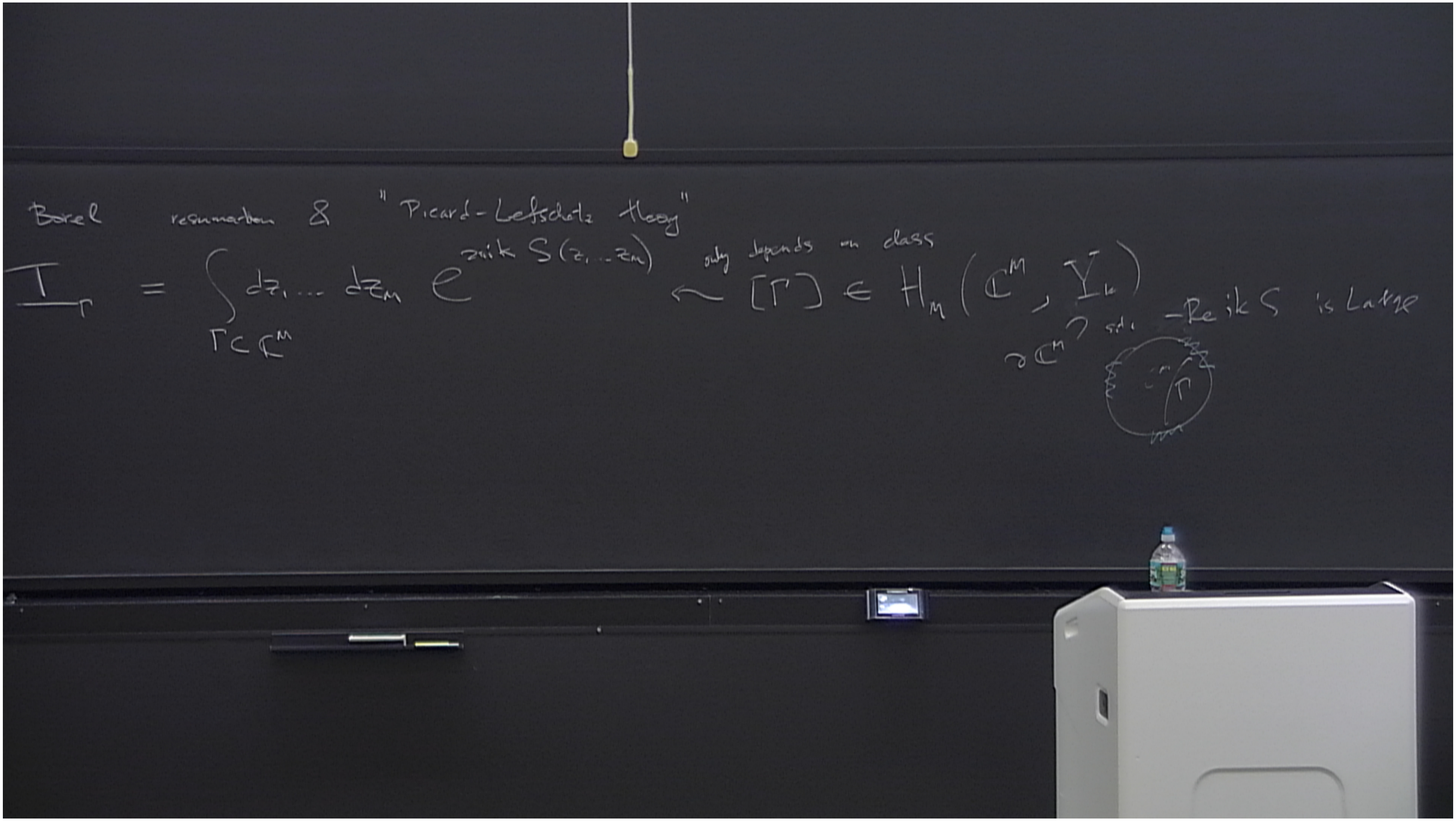
$\Sigma = k$

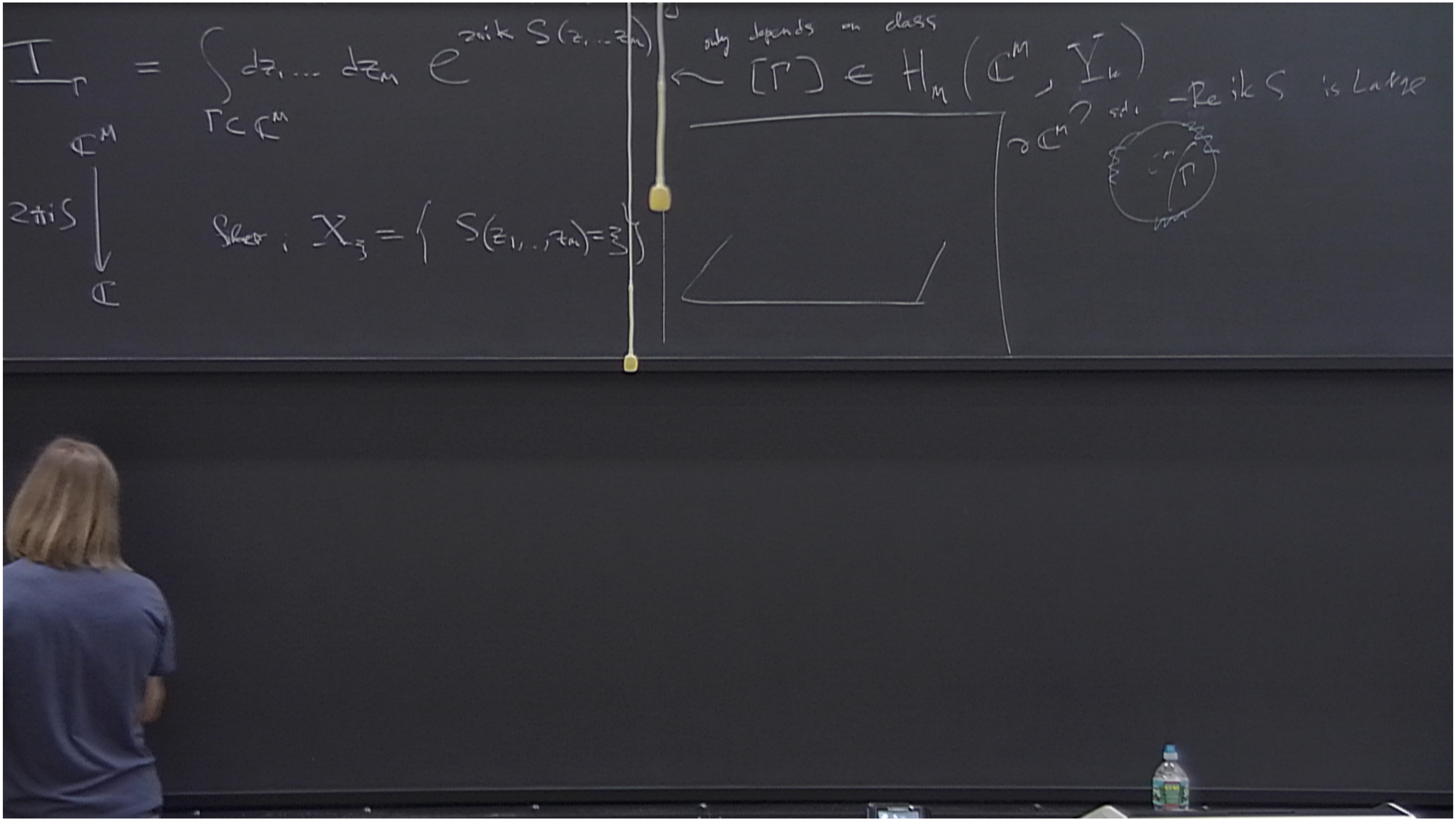
$a \leftarrow$ flat connection on M_3

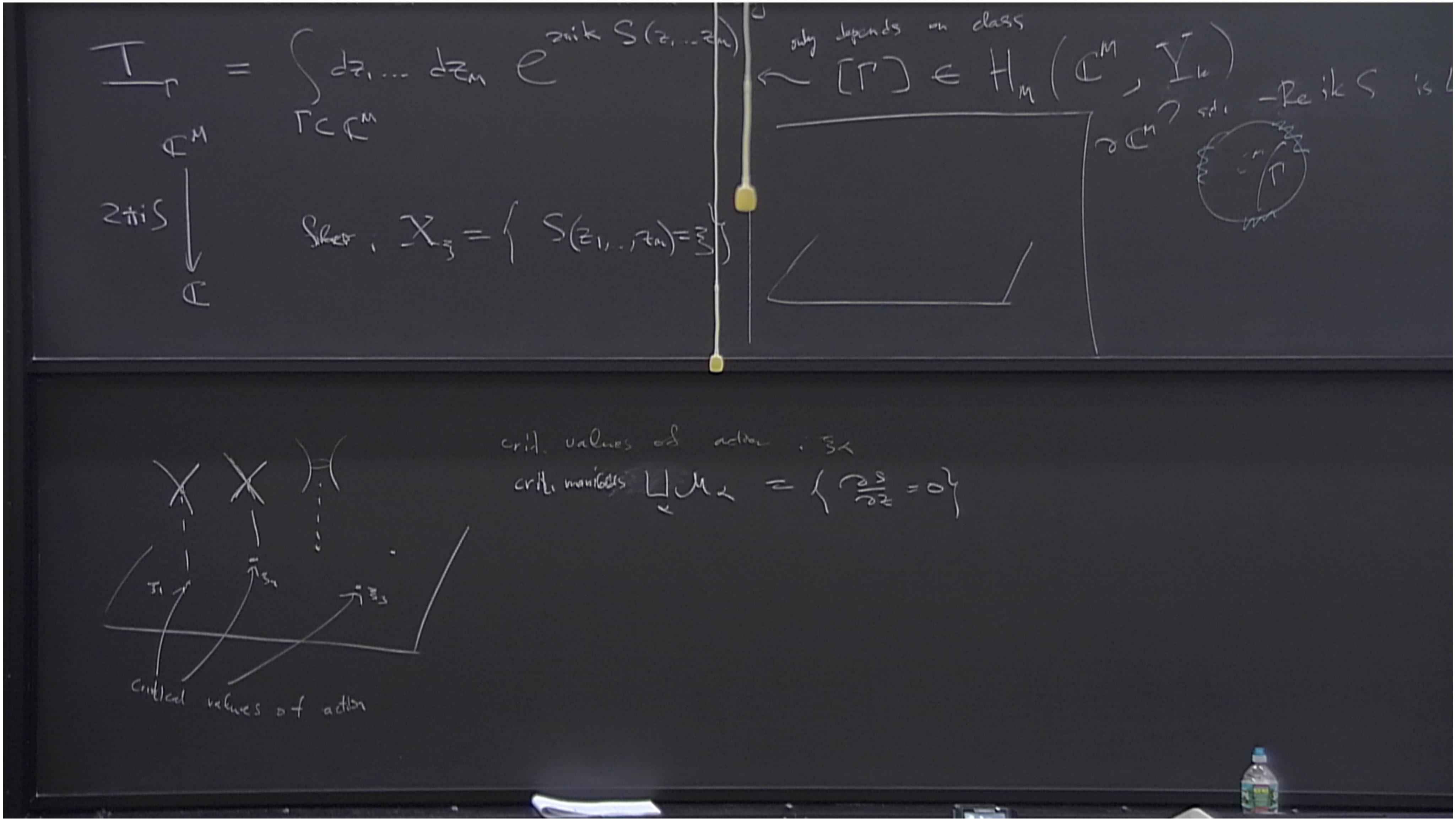
NS5

$\underset{S,T}{\simeq}$ D4 D6 $=$ N M5 branes on $M_3 \times D^2 \times S^1$, L_0

$$\mathbb{Z}[[q]] \qquad Z_a^{U(N)}[M_3](k) \overset{???}{=} \sum_b S_{ab} \, \mathrm{Tr}_{\mathcal{H}_b} (-1)^F q^{L_0} \in \mathbb{Z}[[q]]$$


Borel resummation & "Picard-Lefschetz theory"
$I_\Gamma = \int_{\Gamma \subset \mathbb{C}^M} dz_1 \ldots dz_M \, e^{2\pi i k S(z_1 \ldots z_M)}$
only depends on class
$[\Gamma] \in H_M(\mathbb{C}^M, Y_k)$
$\supset \mathbb{C}^M$ s.t. $-\mathrm{Re}\, ikS$ is Large
$\mathbb{C}^M$
Γ


$I_\Gamma = \int d z_1 \dots d z_M \, e^{2\pi i k S(z_1 \dots z_M)}$
$\Gamma \subset \mathbb{C}^M$
only depends on class
$[\Gamma] \in H_M(\mathbb{C}^M, Y_k)$
s.t. $-\mathrm{Re}\, ikS$ is Large
$\mathbb{C}^M$
$2\pi i S$
$\mathbb{C}$
fiber: $X_\xi = \{ S(z_1, .., z_M) = \xi \}$



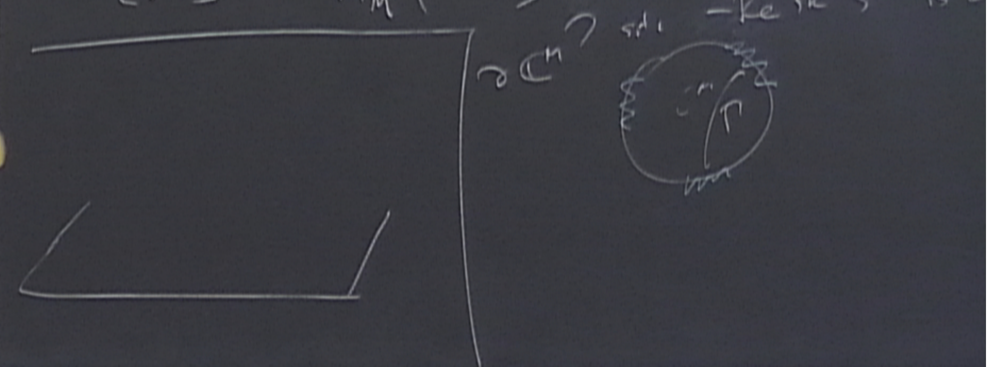
I_Γ = ∫ dz_1 ... dz_M e^{2πik S(z_1...z_M)}
Γ ⊂ C^M
only depends on class
[Γ] ∈ H_M(C^M, Y_k)
−Re ikS is
C^M
2πiS
C
fiber: X_ξ = { S(z_1,..,z_M) = ξ }
crit. values of action : ξ_α
crit. manifolds ⊔_α M_α = { ∂S/∂z = 0 }
critical values of action

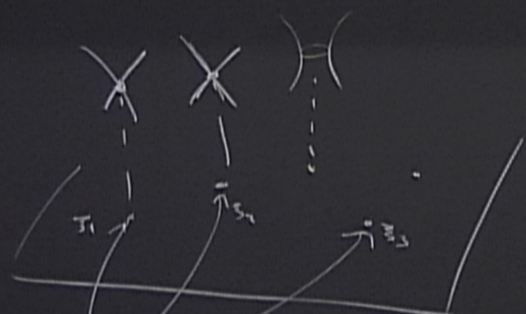
$$I_\Gamma = \int_{\Gamma \subset \mathbb{C}^M} dz_1 \dots dz_M \, e^{2\pi i k S(z_1, \dots, z_M)}$$

← only depends on class $[\Gamma] \in H_M(\mathbb{C}^M, Y_k)$

$$\mathbb{C}^M \xrightarrow{2\pi i S} \mathbb{C}$$

fiber: $X_\xi = \{ S(z_1, \dots, z_M) = \xi \}$





critical values of action

crit. values of action: ξ_α

crit. manifolds $\coprod_\alpha \mathcal{M}_\alpha = \{ \frac{\partial S}{\partial z} = 0 \}$

$\mathcal{M}_\alpha \subset X_{\xi_\alpha}$ ← sing. fibers

isolated crit. point $\mathcal{M}_\alpha = pt$

$\mathcal{M}_\alpha = T^* L_\alpha$

[illegible] comp manifold

action : ξ_α

$\mathcal{M}_\alpha = \{ \frac{\partial S}{\partial z} = 0 \}$

$\subset X_{\xi_\alpha} \leftarrow$ sing. fibers

manifold

$H_{n-1}(X_\xi)$ generated by $\{\Gamma_*^\alpha\}$

Γ_*^α at ξ_α degenerates to λ_α

Lefschetz thimbles: $\Gamma_\alpha \leftarrow$ generators of $H_n(\mathbb{C}^n, Y_L)$

$\Gamma_\alpha \overset{\text{locally}}{\simeq} \gamma_\alpha \times \Gamma_*^\alpha$

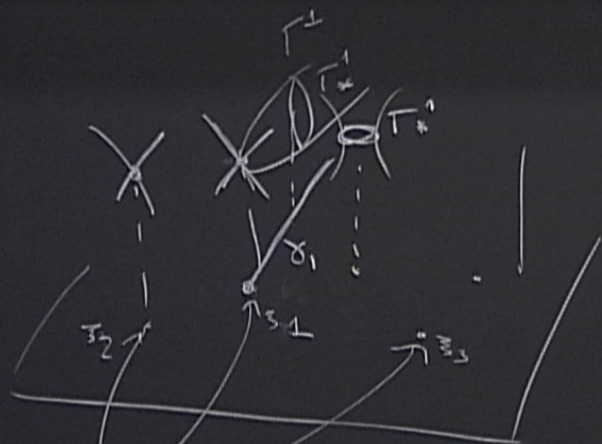
$\gamma_\alpha \subset \mathbb{C}$

$2\pi i S$

$\downarrow$

$\mathbb{C}$

fiber: $X_\xi = \{ S(z_1, .., z_n) = \xi \}$



critical values of action

crit. values of action: ξ_α

crit. manifolds $\bigsqcup_\alpha \mathcal{M}_\alpha = \left\{ \frac{\partial S}{\partial z} = 0 \right\}$

$\mathcal{M}_\alpha \subset X_{\xi_\alpha} \leftarrow$ sing. fibers

isolated crit. point

$\mathcal{M}_\alpha = pt$

$\mathcal{M}_\alpha = T^* \Lambda_\alpha$

Kähl. comp. manifold

$H_{n-1}(X_\xi,$

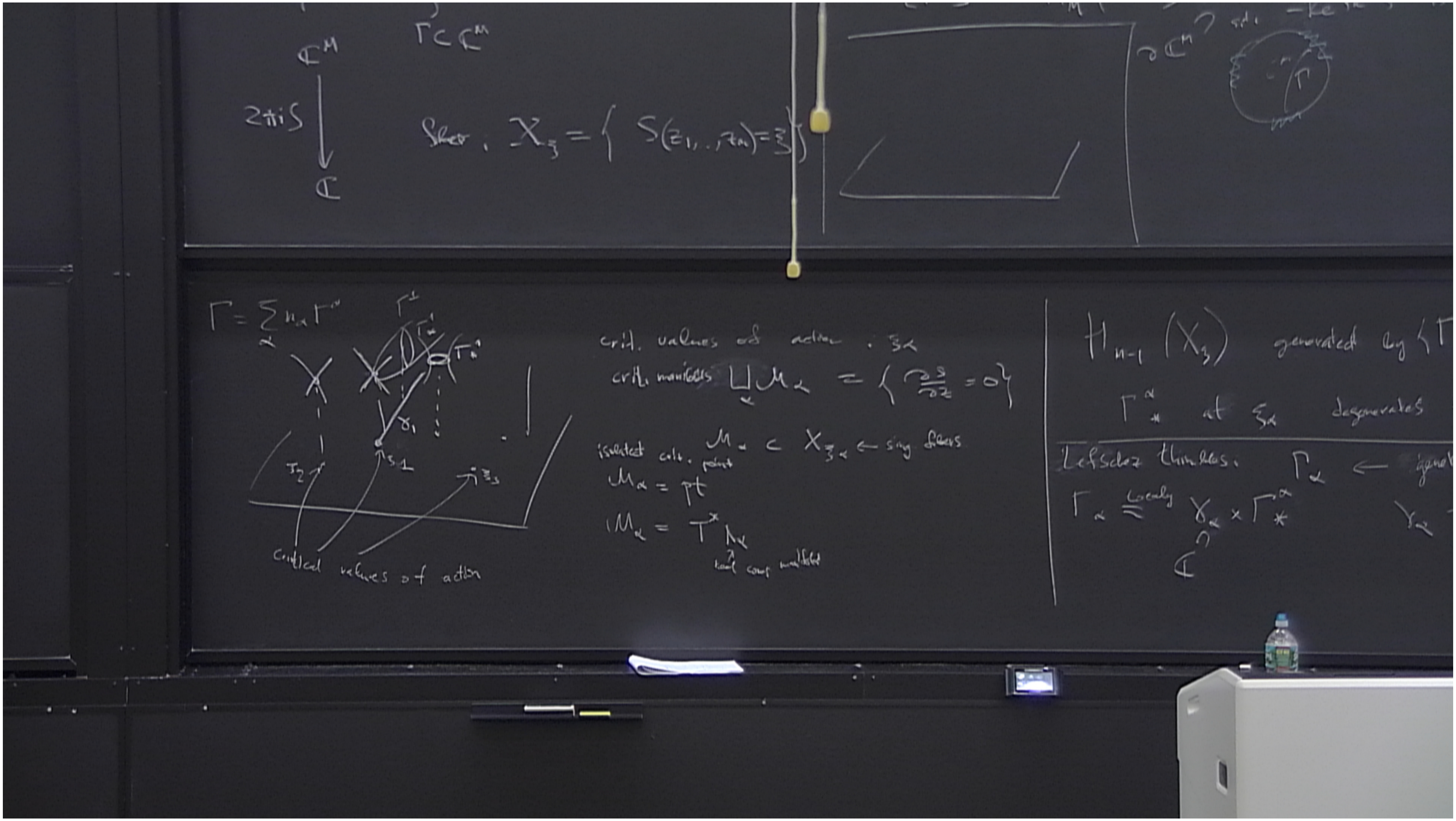
Γ^α_* at

Lefschetz thimble

$\Gamma_\alpha \overset{\text{locally}}{\cong} \gamma_\alpha$

$\cap$

$\mathbb{C}$



$\Gamma \subset \mathbb{C}^M$
$2\pi i S$
$\mathbb{C}$
fiber. $X_\xi = \{ S(z_1,..,z_M) = \xi \}$
$\Gamma = \sum_\alpha n_\alpha \Gamma^\alpha$
critical values of action
crit. values of action. ξ_α
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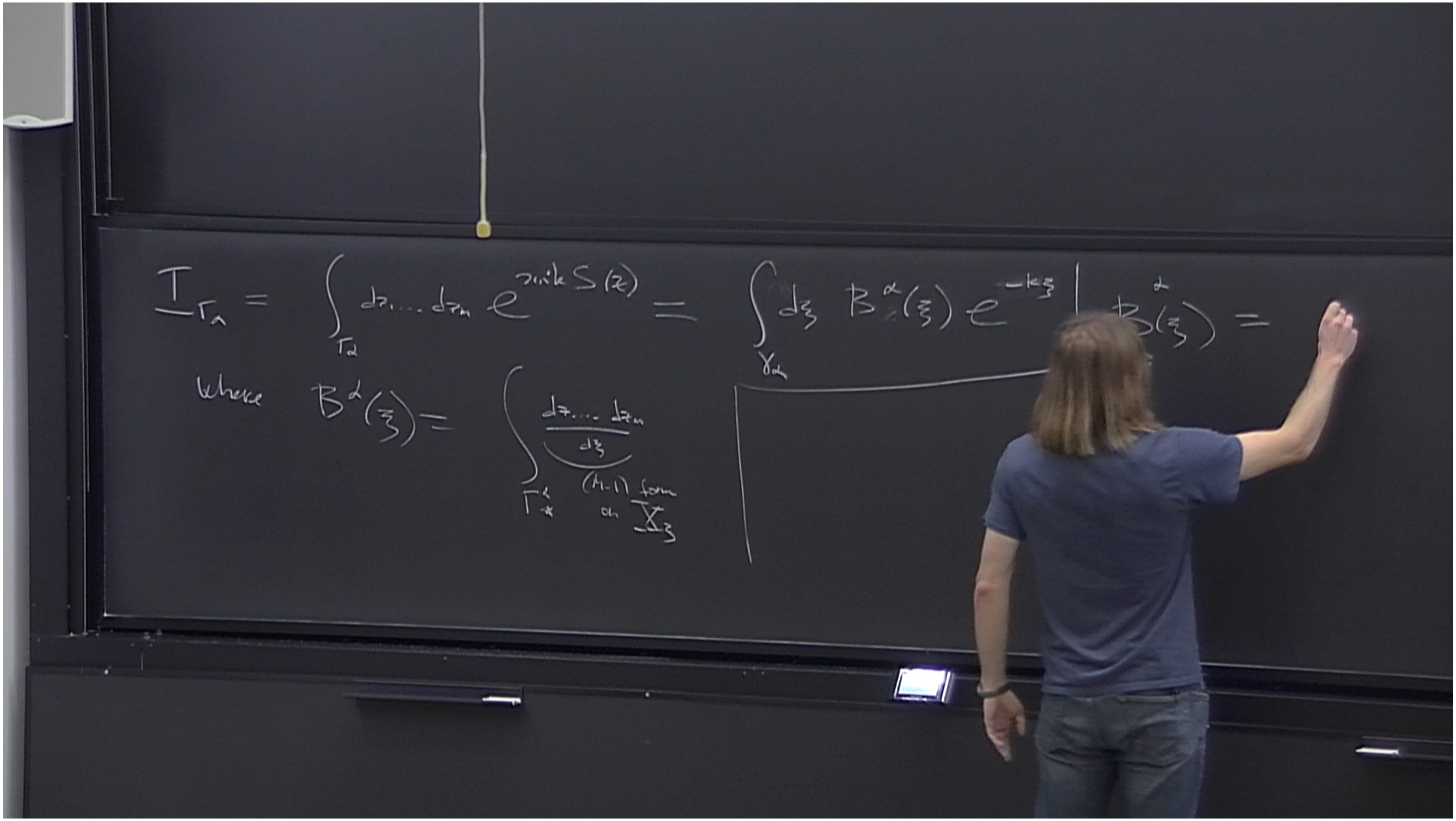
Γ^α_* at ξ_α degenerates
Lefschetz thimbles. Γ_α
$\Gamma_\alpha \overset{\text{locally}}{\approx} \gamma_\alpha \times \Gamma^\alpha_*$


$I_{\Gamma_\alpha} = \int_{\Gamma_\alpha} dz_1 \ldots dz_n \, e^{2\pi i k S(z)} = \int_{\gamma_\alpha} d\xi \; B^\alpha(\xi) \, e^{-k\xi}$
$B^\alpha(\xi) =$
where $B^\alpha(\xi) = \int_{\Gamma_\xi^\alpha} \frac{dz_1 \ldots dz_n}{d\xi}$
$(n-1)$ form on X_ξ

$$e^{2\pi i k S(z)} = \int_{\gamma_\alpha} d\xi \; B^{\alpha}(\xi)\, e^{-k\xi}$$

$$B^{\alpha}(\xi) = \sum_{n \geq 0} c_n (\xi - \xi_\alpha)^{a_\beta + n}$$

finite radius of convergence

$$a_\beta = \text{const} - \dim \Delta_\alpha / 2$$

$$\int_{\Gamma^\alpha_*} \frac{dz_1 \dots dz_n}{d\xi}$$

$(n-1)$ form on X_ξ

$$e^{2\pi i k S(z)} = \int_{\gamma_\alpha} d\xi \; B^{\alpha}(\xi)\, e^{-k\xi}$$

$$B^{\alpha}(\xi) = \sum_{n\geq 0} c_n (\xi - \xi_\alpha)^{a_\beta + n}$$

$a_\beta = \text{const} - \dim \Lambda_\alpha / 2$

$$\int_{\Gamma^\alpha_*} \underbrace{\frac{dz_1 \dots dz_n}{d\xi}}_{(n-1)\text{ form on } X_\xi}$$

Laplace transform

Borel transform

finite radius of convergence

$$I_{\Gamma_\alpha} = \sum_{n\geq 0} \frac{c_n \Gamma(n+a_\beta+1)}{k^{a_\beta+n+1}}$$

asympt. (divergent)

$$B(\xi) = \int_{\Gamma^{\alpha}_{*}} \frac{dz_1 \dots dz_n}{d\xi}$$

$(n-1)$ form on X_{ξ}

Borel transform

$$I_{\Gamma_{\alpha}} = \sum_{n \geq 0} \frac{c_n \, \Gamma(n + a_p + 1)}{k^{a_p + n + 1}} \longleftarrow \text{asympt. (divergent)}$$

ξ_p

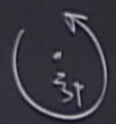
Monodromy: $[\Gamma^{\alpha}_{*}] \longrightarrow [\Gamma^{\alpha}_{*}] + m^{\alpha}_{\beta} [\Gamma^{\beta}_{*}]$

$$B^{\alpha}(\xi) \longrightarrow B^{\alpha}(\xi) + m^{\alpha}_{\beta} \, B^{\beta}(\xi)$$

$B^\alpha(\xi) = \int_{\Gamma^\alpha_*} \frac{dz_1 \ldots dz_n}{d\xi}$ ($n-1$) form on X_ξ

Borel transform

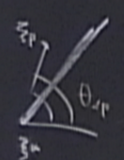
$I_{\Gamma_\alpha} \sim \sum_{n \geq 0} \frac{c_n \Gamma(n+a_\beta+1)}{k^{a_\beta+n+1}}$ ⟵ asympt. (divergent)

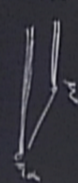


Monodromy: $[\Gamma^\alpha_*] \longrightarrow [\Gamma^\alpha_*] + m^\alpha_\beta [\Gamma^\beta_*]$

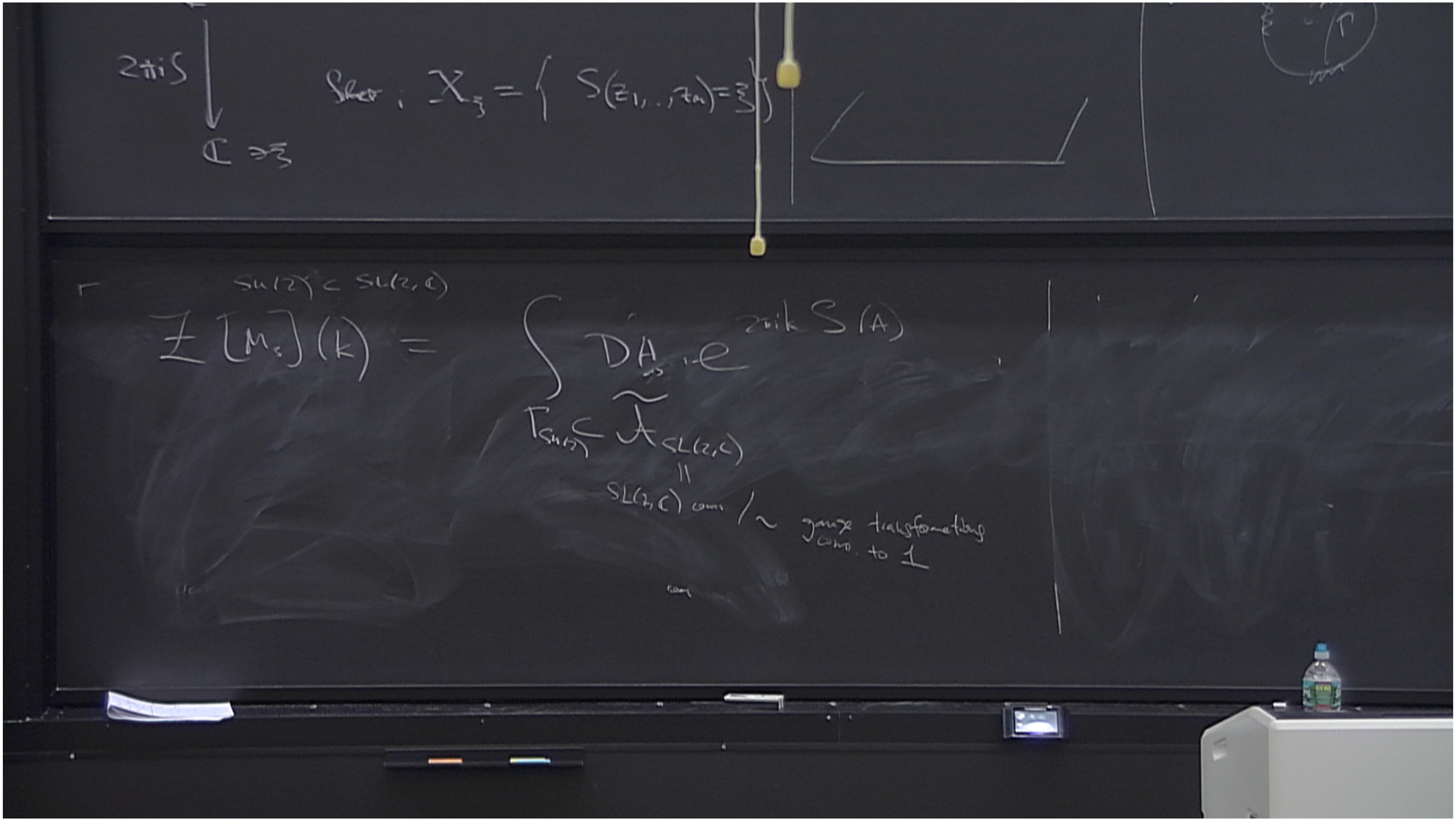
$$B^\alpha(\xi) \longrightarrow B^\alpha(\xi) + m^\alpha_\beta B^\beta(\xi)$$

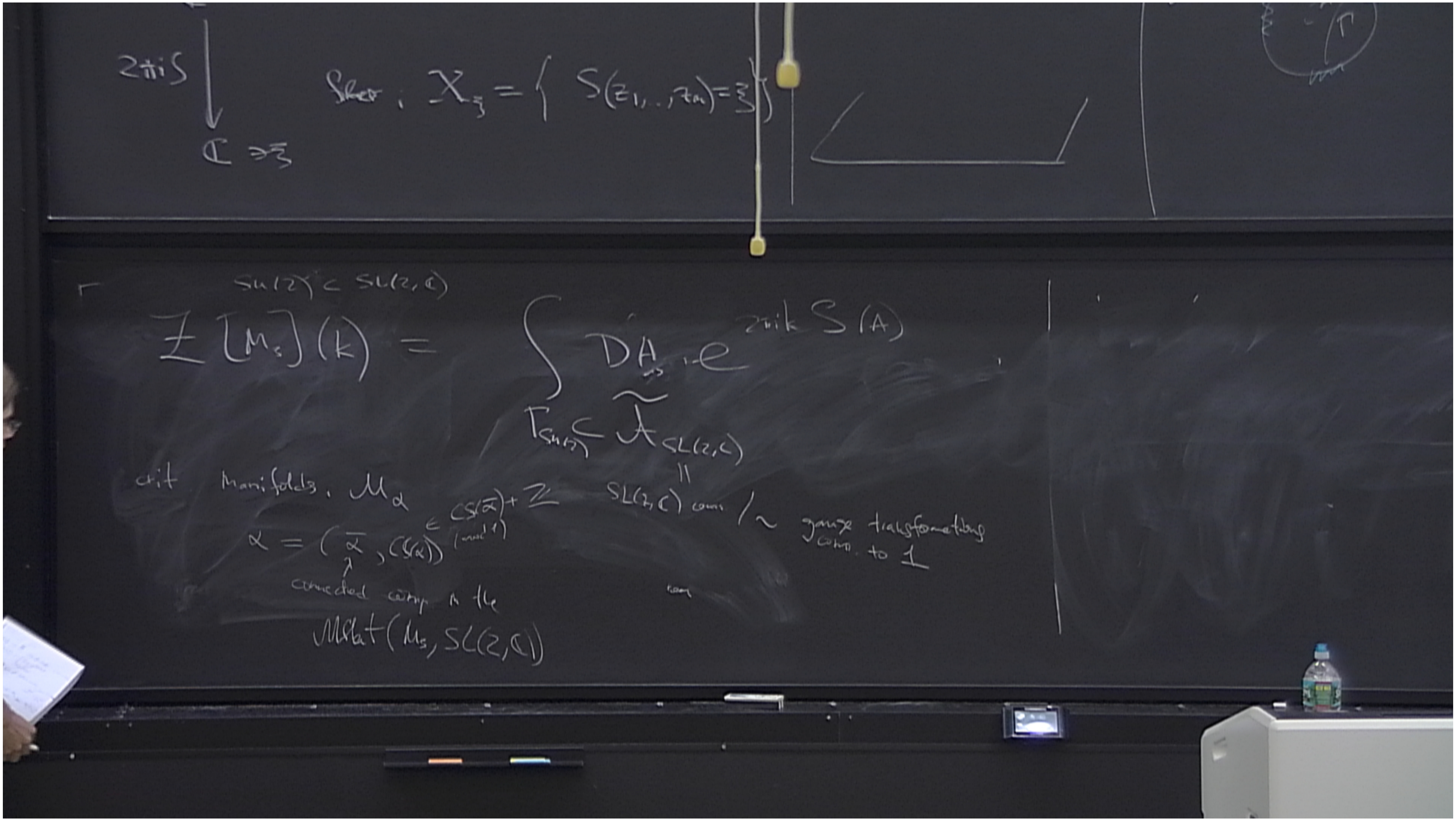
in $m^\alpha_\beta \neq 0$ $B^\alpha(\xi)$ has sing at ξ_β





$$I_{\Gamma_\alpha}\Big|_{\arg k = \theta_{\alpha\beta}-\epsilon} = \left(I_{\Gamma_\alpha} + m^\alpha_\beta I_{\Gamma_\beta}\right)\Big|_{\arg k = \theta_{\alpha\beta}+\epsilon}$$


2πiS
Sker : X_ξ = { S(z_1, .., z_n) = ξ }
ℂ ∋ ξ
SU(2) ⊂ SL(2,ℂ)
Z[M_3](k) = ∫ DA e^{2πik S(A)}
Γ_{SU(2)} ⊂ Ã_{SL(2,ℂ)}
SL(2,ℂ) conn. / ~ gauge transformations conn. to 1


2πiS
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fiber: X_ξ = { S(z_1,..,z_m) = ξ }
SU(2) ⊂ SL(2,ℂ)
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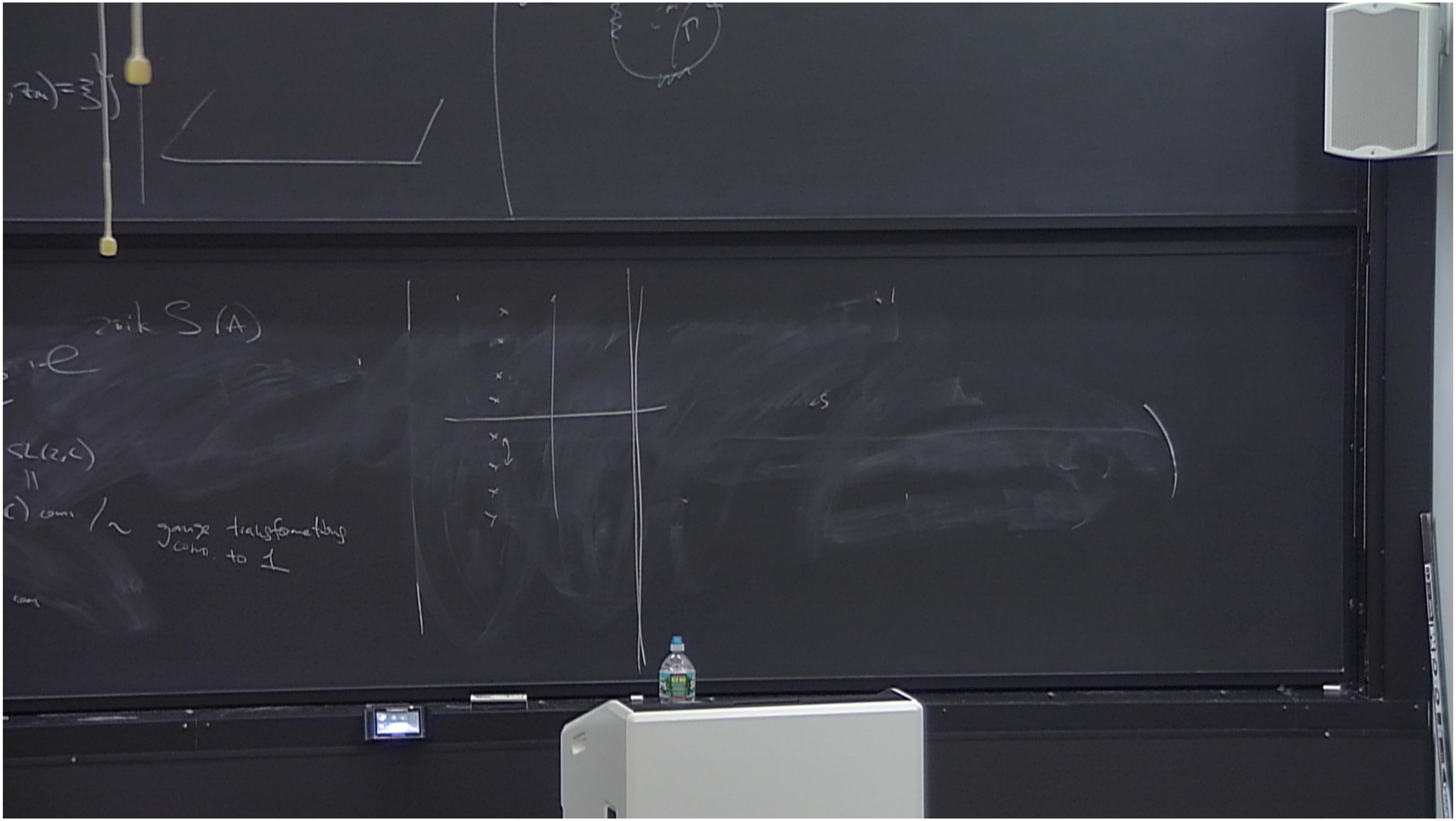
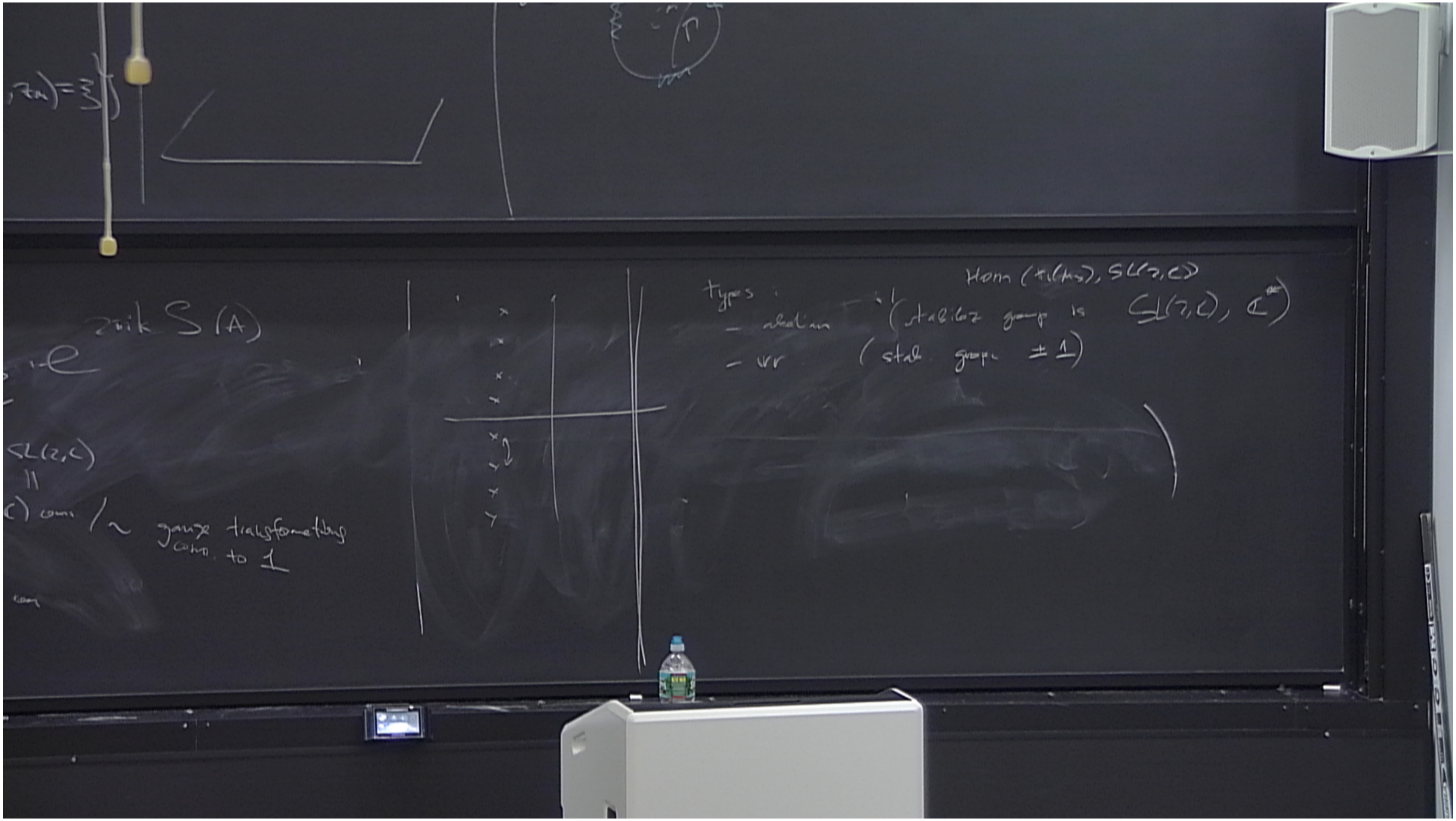
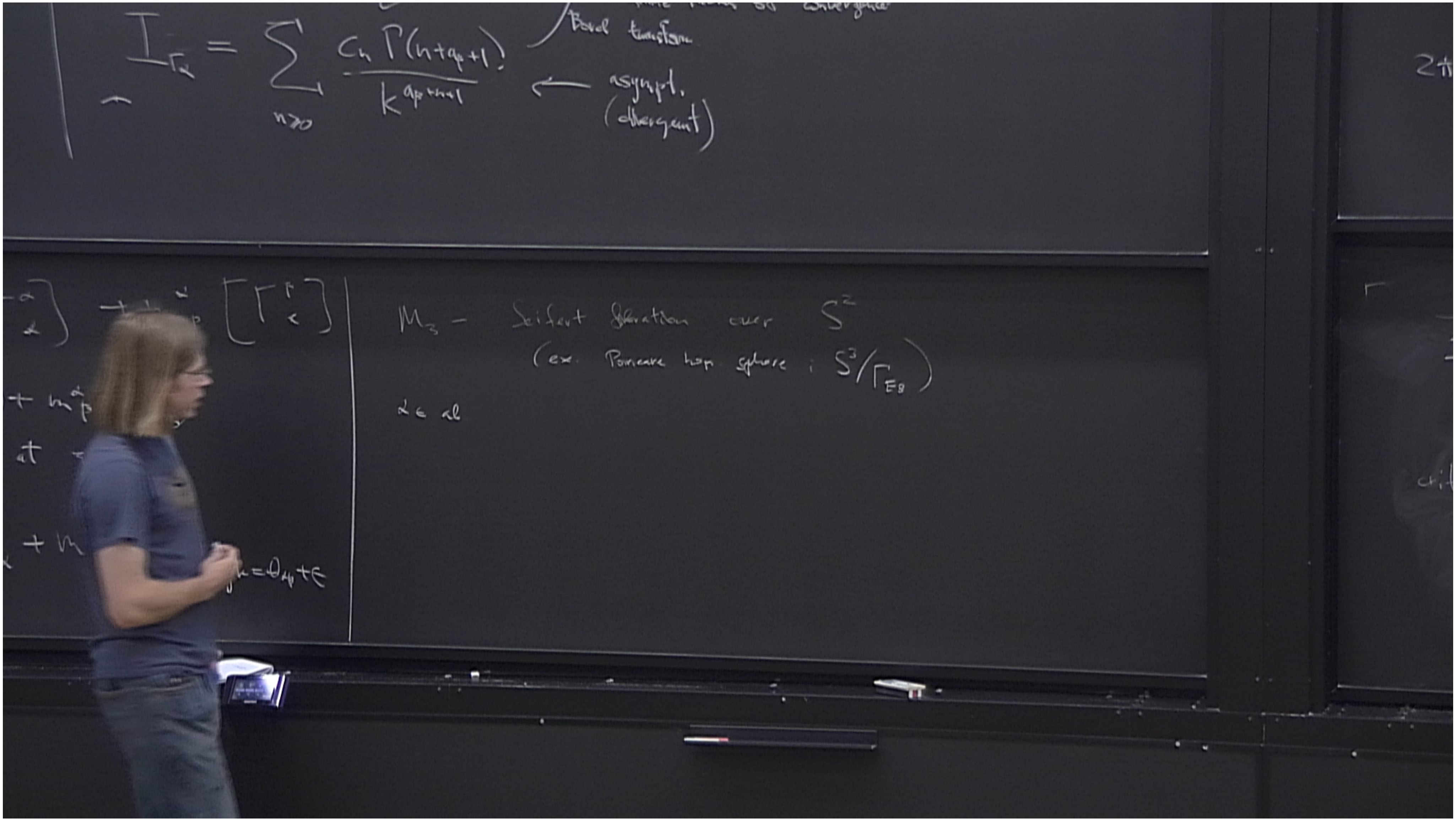
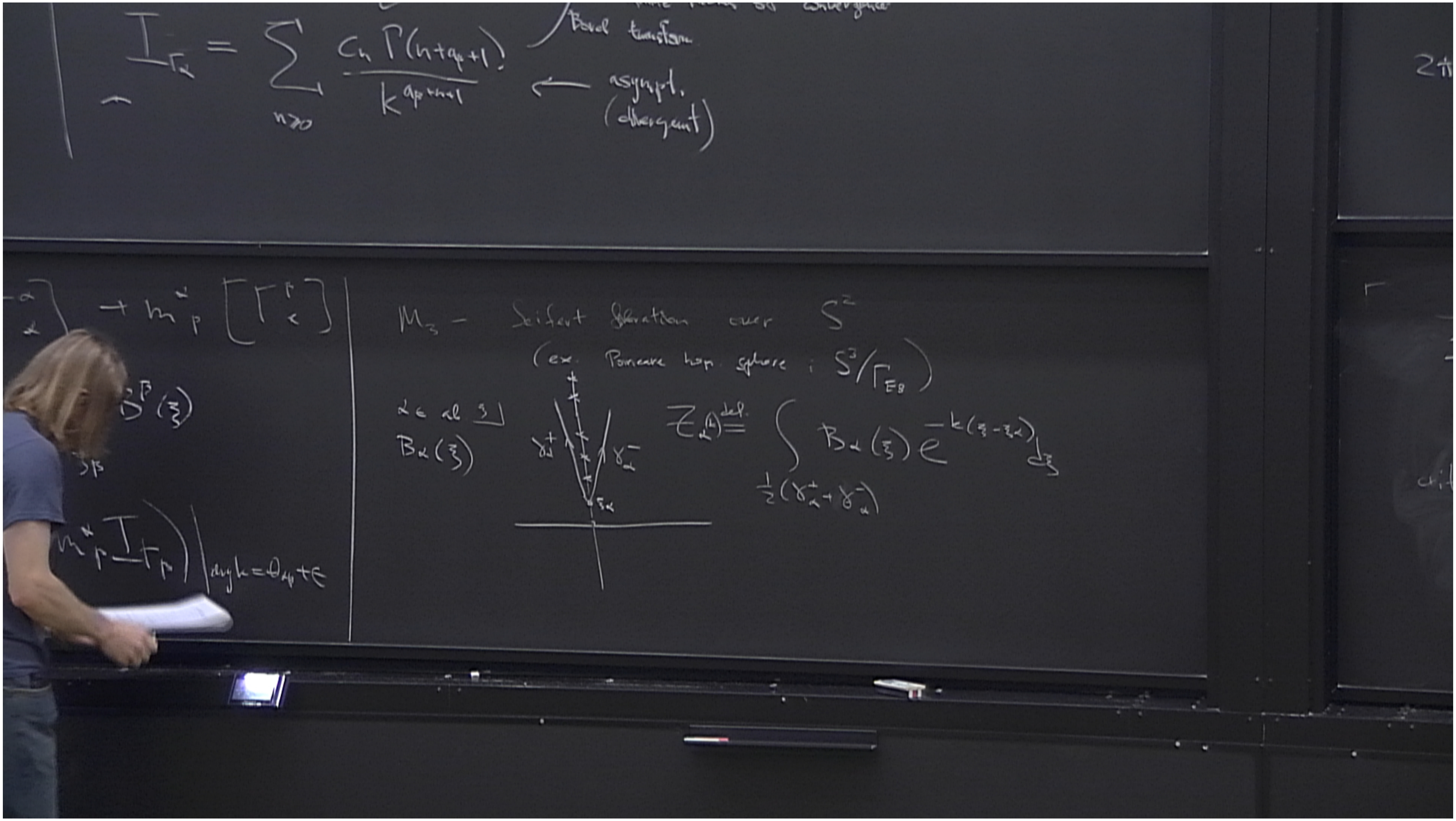
Γ_{SU(2)} ⊂ Ã_{SL(2,ℂ)}
SL(2,ℂ) conn. / ~ gauge transformations conn. to 1
crit manifolds: M_α
α = (ᾱ, CS(α)) ∈ CS(ᾱ) + ℤ (mod 1)
connected comp. in the
Mflat(M_3, SL(2,ℂ))

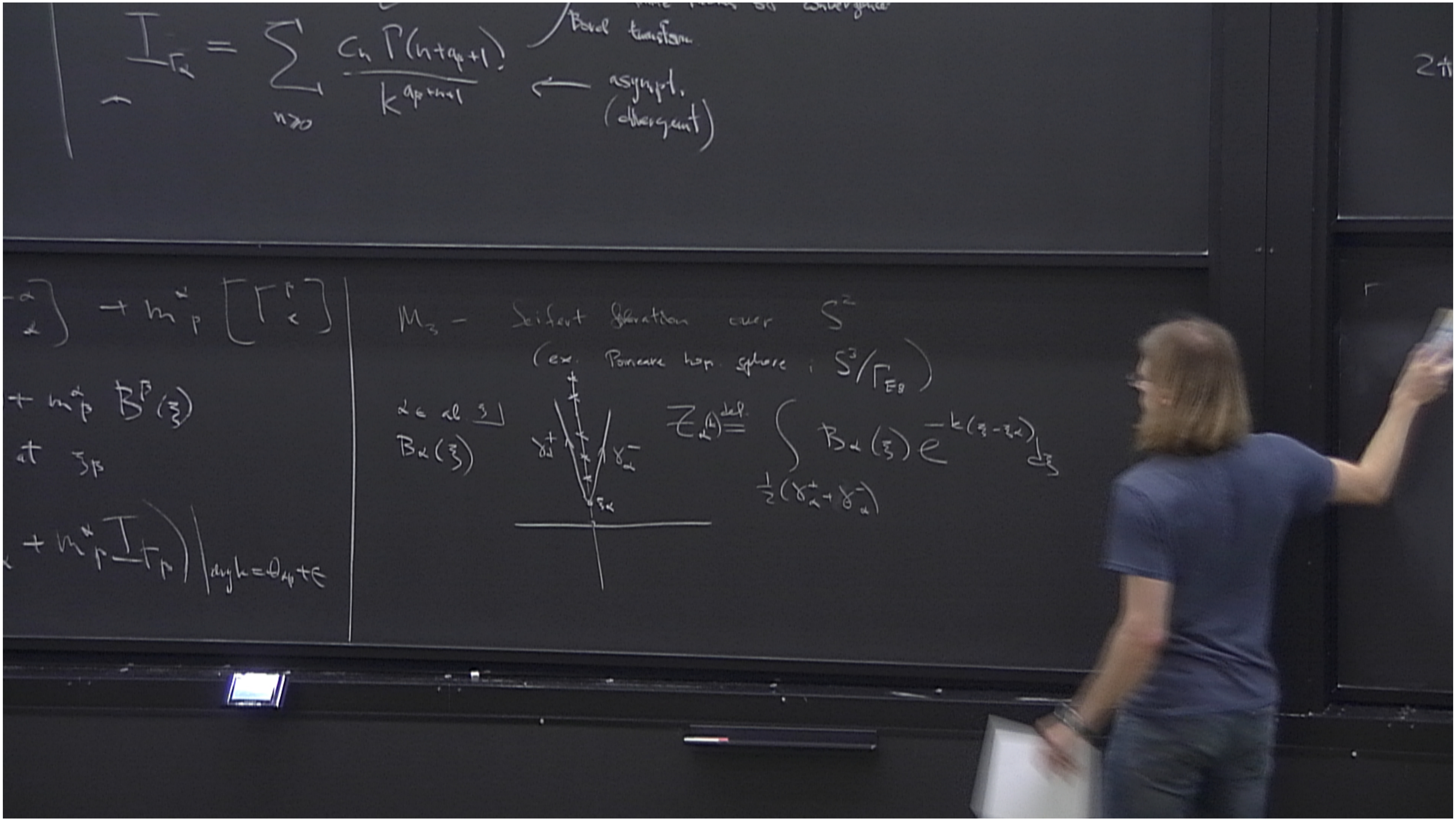

e 2πik S(A)
SL(2,C)
gauge transformations
conn. to 1


types :
- abelian
(stabilizer group is SL(2,C), C*)
- irr
(stab. group ±1)
gauge transformations
conn. to 1


Borel transform
asympt.
(divergent)
$I_{\Gamma_\alpha} = \sum_{n\geq 0} \frac{c_n \Gamma(n+a_p+1)}{k^{a_p+n+1}}$
M_3 – Seifert fibration over S^2
(ex. Poincare hom. sphere : S^3/Γ_{E_8})



Borel transform
$I_{\Gamma_\alpha} = \sum_{n\geq 0} \frac{c_n \Gamma(n+a_p+1)}{k^{a_p+n+1}}$
asympt. (divergent)
$+ m_p^\alpha [\Gamma_\alpha^\beta]$
M_3 - Seifert fibration over S^2
(ex. Poincare hom. sphere : S^3/Γ_{E_8})
$B_\alpha(\xi)$
γ_α^+ γ_α^- ξ_α
$Z_\alpha(k) \stackrel{\text{def.}}{=} \int_{\frac{1}{2}(\gamma_\alpha^+ + \gamma_\alpha^-)} B_\alpha(\xi)\, e^{-k(\xi-\xi_\alpha)}\, d\xi$
$\Big|_{\arg k = \theta_{\alpha\beta}+\epsilon}$


Borel transform
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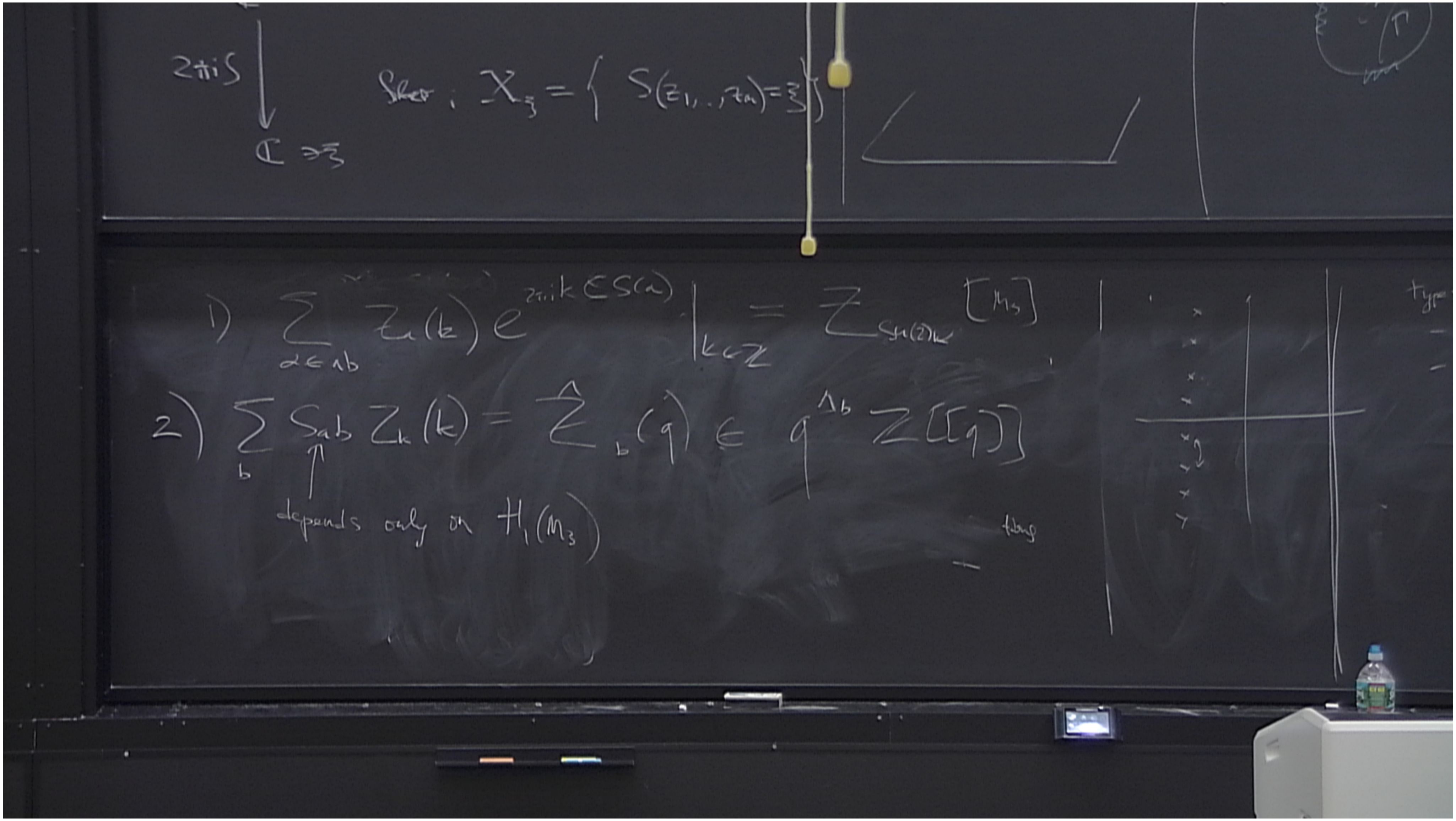
M_3 - Seifert fibration over S^2
(ex. Poincaré hom. sphere : S^3/Γ_{E_8})
$B_\alpha(\xi)$
$Z_\alpha(k) \stackrel{\text{def}}{=} \int_{\frac{1}{2}(\gamma_\alpha^+ + \gamma_\alpha^-)} B_\alpha(\xi) e^{-k(\xi-\xi_\alpha)} d\xi$
γ_α^+
γ_α^-
ξ_α
$+ m_\beta^\alpha B^\beta(\xi)$
at ξ_β
$+ m_\beta^\alpha I_{\Gamma_\beta}$
$\arg k = \theta_{\alpha\beta} + \epsilon$


2πiS
ℂ ∋ ξ
Spec : X_ξ = { S(z_1,..,z_n) = ξ }
1) ∑_{α∈Λb} Z_α(k) e^{2πik ∈ S(α)} |_{k∈Z} = Z_{SU(2)_k}[M_3]
2) ∑_b S_{ab} Z_k(k) = Ẑ_b(q) ∈ q^{Δ_b} Z[[q]]
depends only on H_1(M_3)