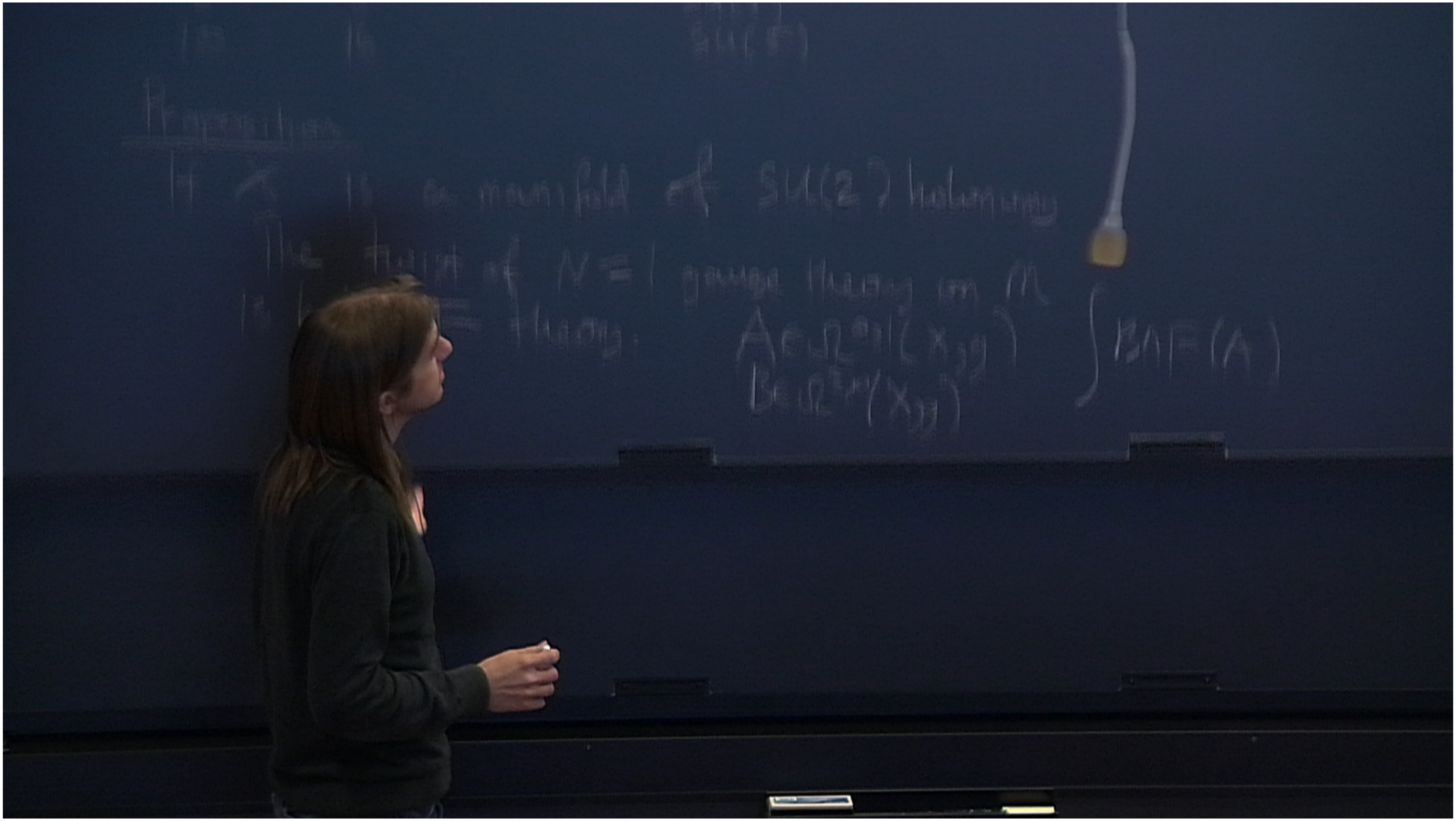
Title: Supersymmetric Field Theories for Mathematicians

Date: Oct 19, 2016 02:00 PM

URL: http://pirsa.org/16100040

Abstract:


Proposition
is a manifold of SU(2) holonomy
gauge theory on

Proposition

In 6 dimensions, on X a $CY3$, the twist of $(1,0)$ gauge th[eory]
is again hol. BF theory.

Fields $A \in \Omega^{0,1}(X, \mathfrak{g})$ $\quad S = \int B \wedge F^{0,2}(A)$

$B \in \Omega^{3,1}(X, \mathfrak{g})$ $\quad$ Extra gauge symmetry $\chi \in \Omega^{3,0}$

imensions, on X a CY_3, the twist of $(1,0)$ gauge theory

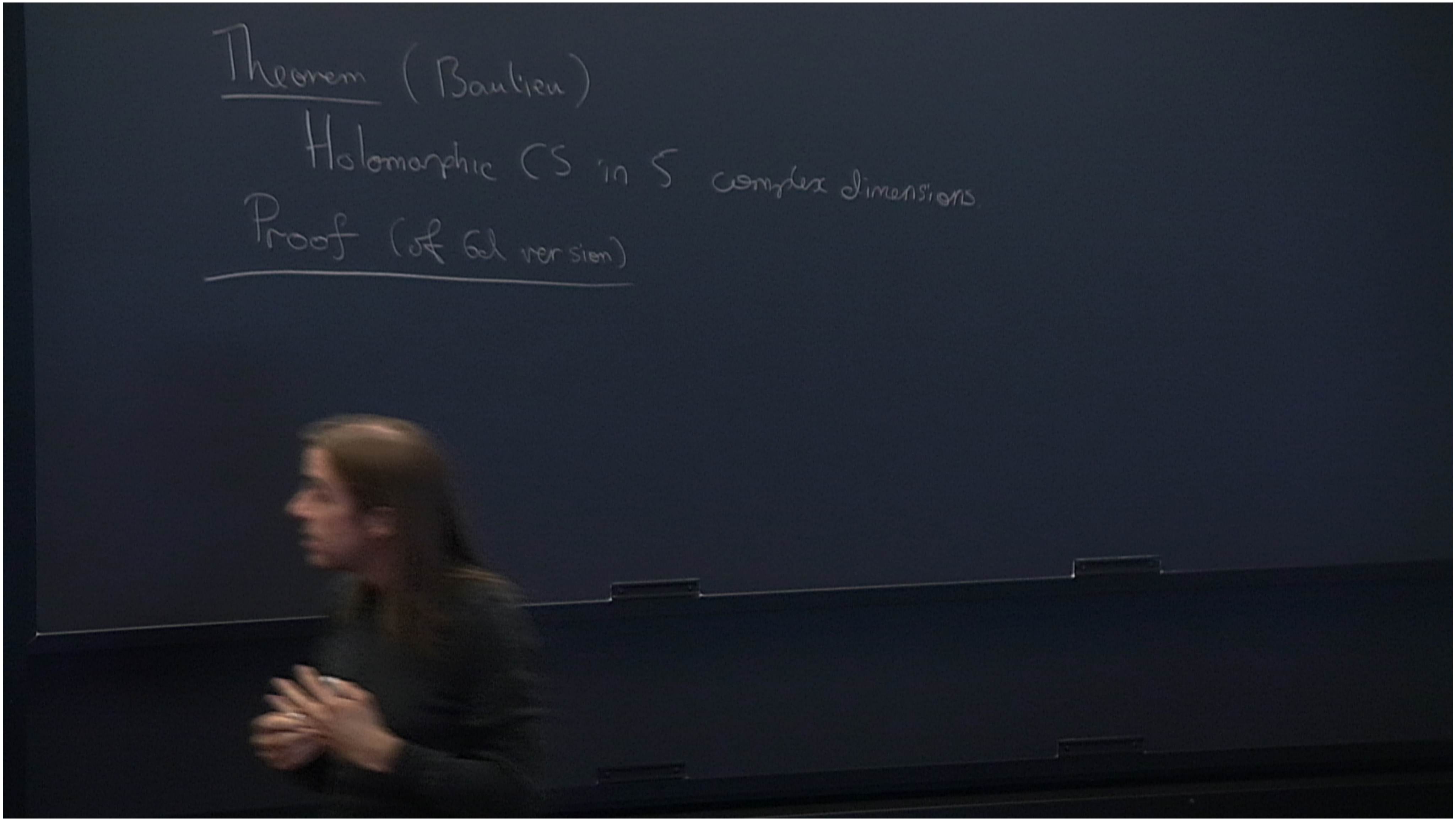
in hol. BF theory.

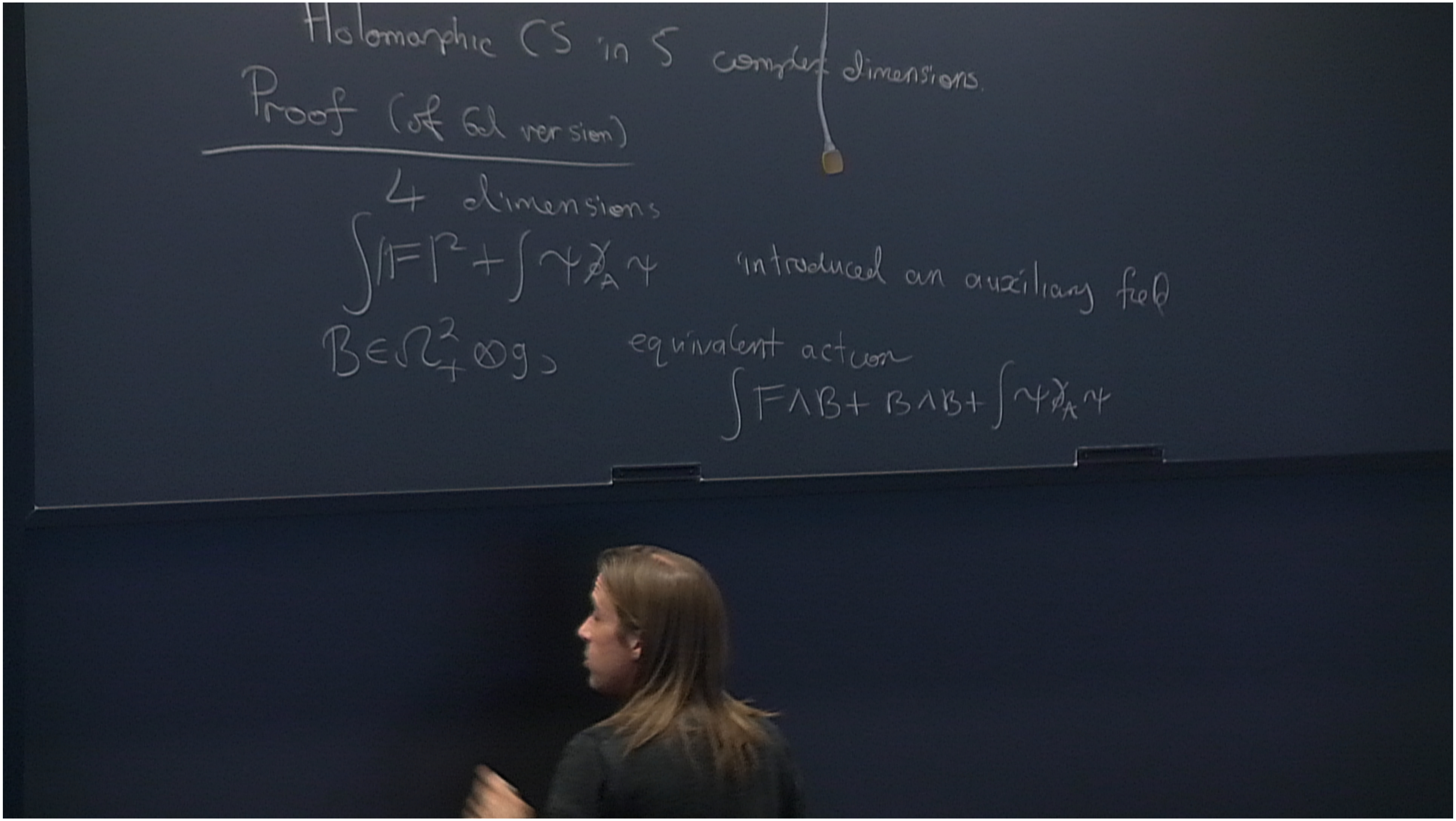
lds $A \in \Omega^{0,1}(X, \mathfrak{g})$ $\quad$ $S = \int B \wedge F^{0,2}(A)$

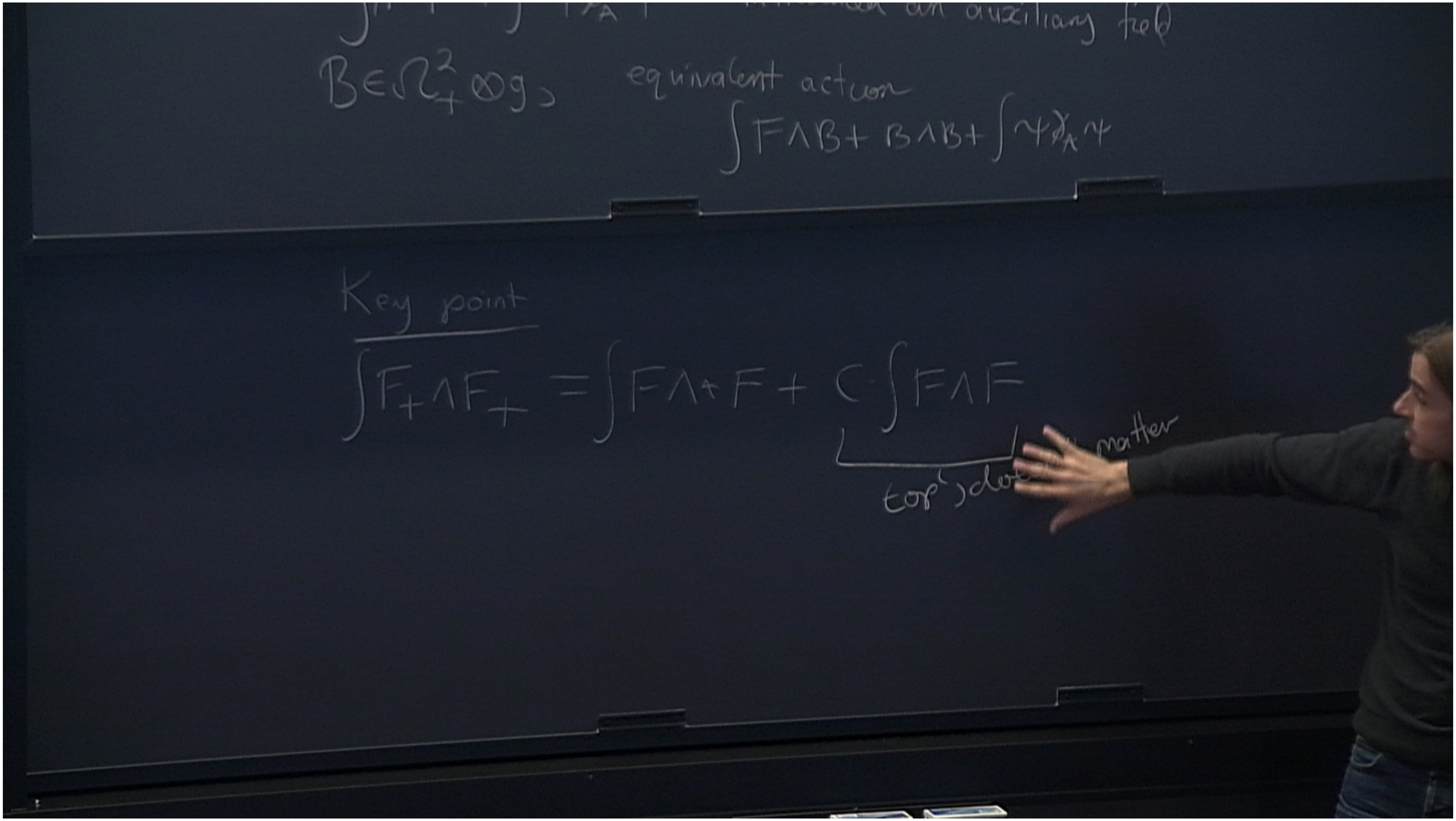
$B \in \Omega^{3,1}(X, \mathfrak{g})$

Extra gauge symmetry $\chi \in \Omega^{3,0}(X, \mathfrak{g})$

$B \longmapsto B + \partial_A \chi$


Theorem (Baulieu)
Holomorphic CS in 5 complex dimensions
Proof (of 6d version)


Holomorphic CS in 5 complex dimensions.
Proof (of 6d version)
4 dimensions
$\int |F|^2 + \int \psi \not{\partial}_A \psi$
introduced an auxiliary field
$B \in \Omega^2_+ \otimes g$,
equivalent action
$\int F \wedge B + B \wedge B + \int \psi \not{\partial}_A \psi$


an auxiliary field
$B \in \Omega^2_+ \otimes g$, equivalent action
$\int F \wedge B + B \wedge B + \int \psi \gamma_A \psi$
Key point
$\int F_+ \wedge F_+ = \int F \wedge * F + C \cdot \int F \wedge F$
matter
top'l

an auxiliary field

$B \in \Omega^2_+ \otimes \mathfrak{g}$, equivalent action

$$\int F \wedge B + B \wedge B + \int \psi \not{D}_A \psi$$

Key point

$$\int F_+ \wedge F_+ = \int F \wedge * F + c \cdot \underbrace{\int F \wedge F}_{\text{top'l doesn't matter}}$$

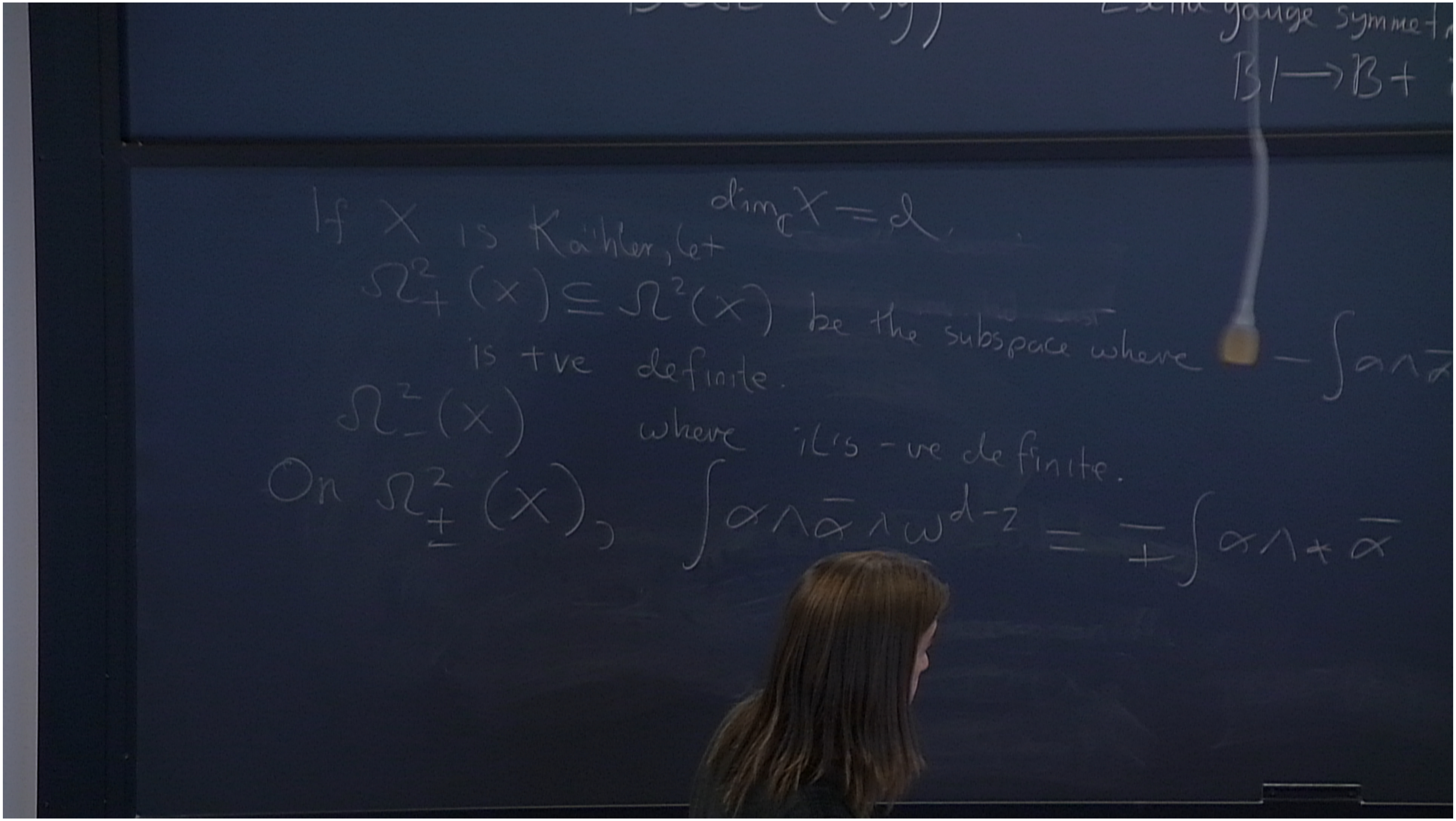
... an auxiliary field

$B\in\Omega^2_+\otimes\mathfrak{g}$, equivalent action

$$\int F\wedge B + B\wedge B + \int \psi \not{D}_A \psi$$

Key point

$$\int F_+\wedge F_+ = \int F\wedge * F + C\cdot \underbrace{\int F\wedge F}_{\text{top'l doesn't matter}}$$


$B \mapsto B +$
If X is Kähler, let $\dim_{\mathbb{C}} X = d$.
$\Omega^2_+(X) \subseteq \Omega^2(X)$ be the subspace where $-\int \alpha \wedge \bar{\alpha}$
is +ve definite.
$\Omega^2_-(X)$ where it's −ve definite.
On $\Omega^2_\pm(X)$, $\int \alpha \wedge \bar{\alpha} \wedge \omega^{d-2} = \mp \int \alpha \wedge * \bar{\alpha}$

If X is Kähler, let $\dim_{\mathbb{C}} X = d$,

$\Omega^2_+(X) \subseteq \Omega^2(X)$ be the subspace where $-\int \alpha \wedge \bar{\alpha}$ is +ve definite.

$\Omega^2_-(X)$ where it's −ve definite.

On $\Omega^2_\pm(X)$, $\int \alpha \wedge \bar{\alpha} \wedge \omega^{d-2} = \mp \int \alpha \wedge * \bar{\alpha}$

$$\Omega^2_+(X) = \Omega^{2,0}(X) \oplus \omega \cdot \Omega^{0,0}(X) \oplus \Omega^{0,2}(X)$$

$$\Omega^2_-(X) = \omega^\perp \subseteq \Omega^{1,1}(X)$$

$$\Omega^2_-(X) \cong \omega^{\perp} \subseteq \Omega^{1,1}(X)$$

In 6 dimensions, the YM action is equivalent to the action

$$\int F(A)\wedge B + \int B\wedge B$$

where $B \in \Omega^2_+(X)\otimes \mathfrak{g}$

(X is Kähler)

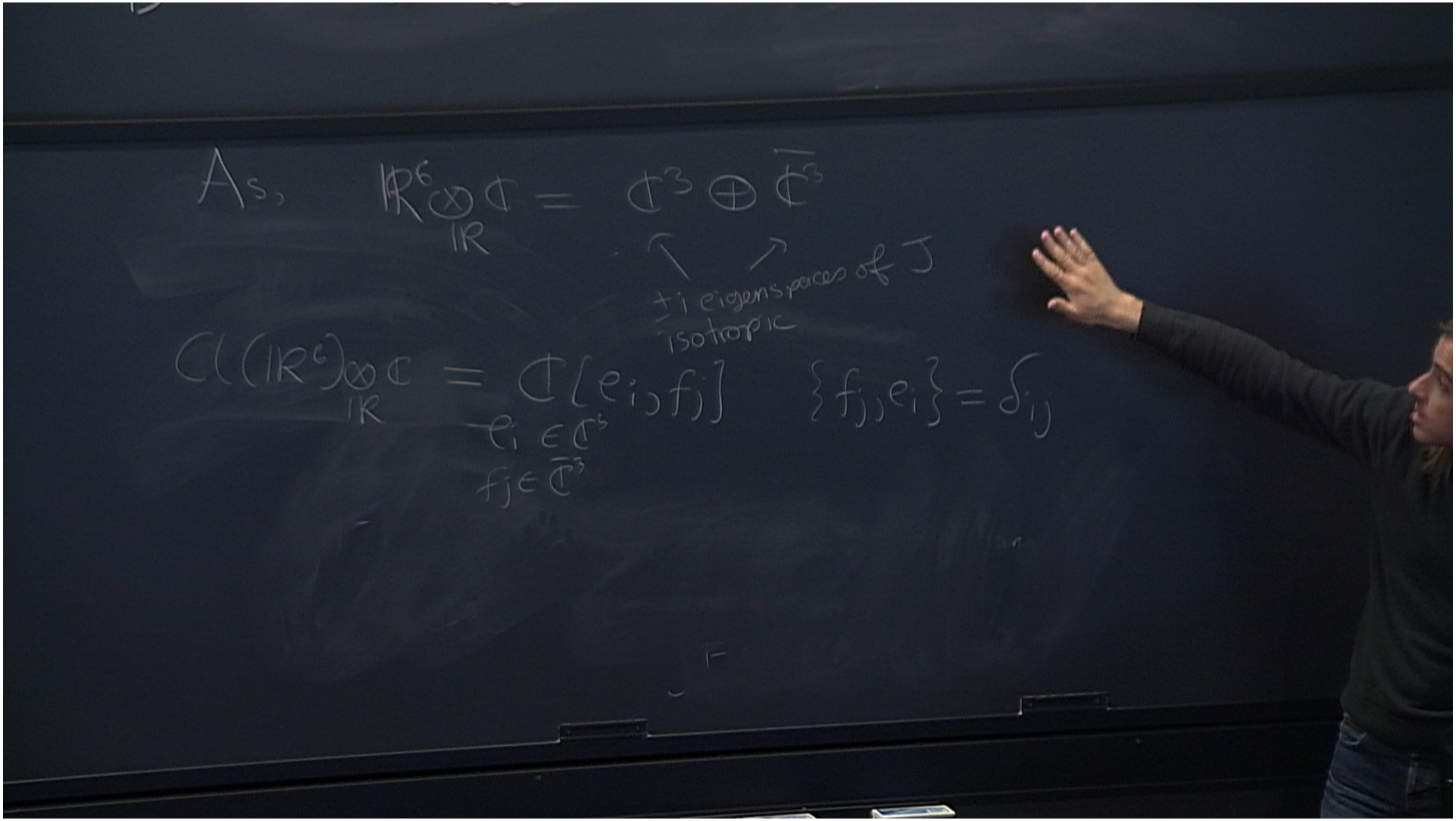
$\Omega^2_-(X) \cong \omega^\perp \subseteq \Omega^{1,1}(X)$

In 6 dimensions, the YM action is equivalent to the action

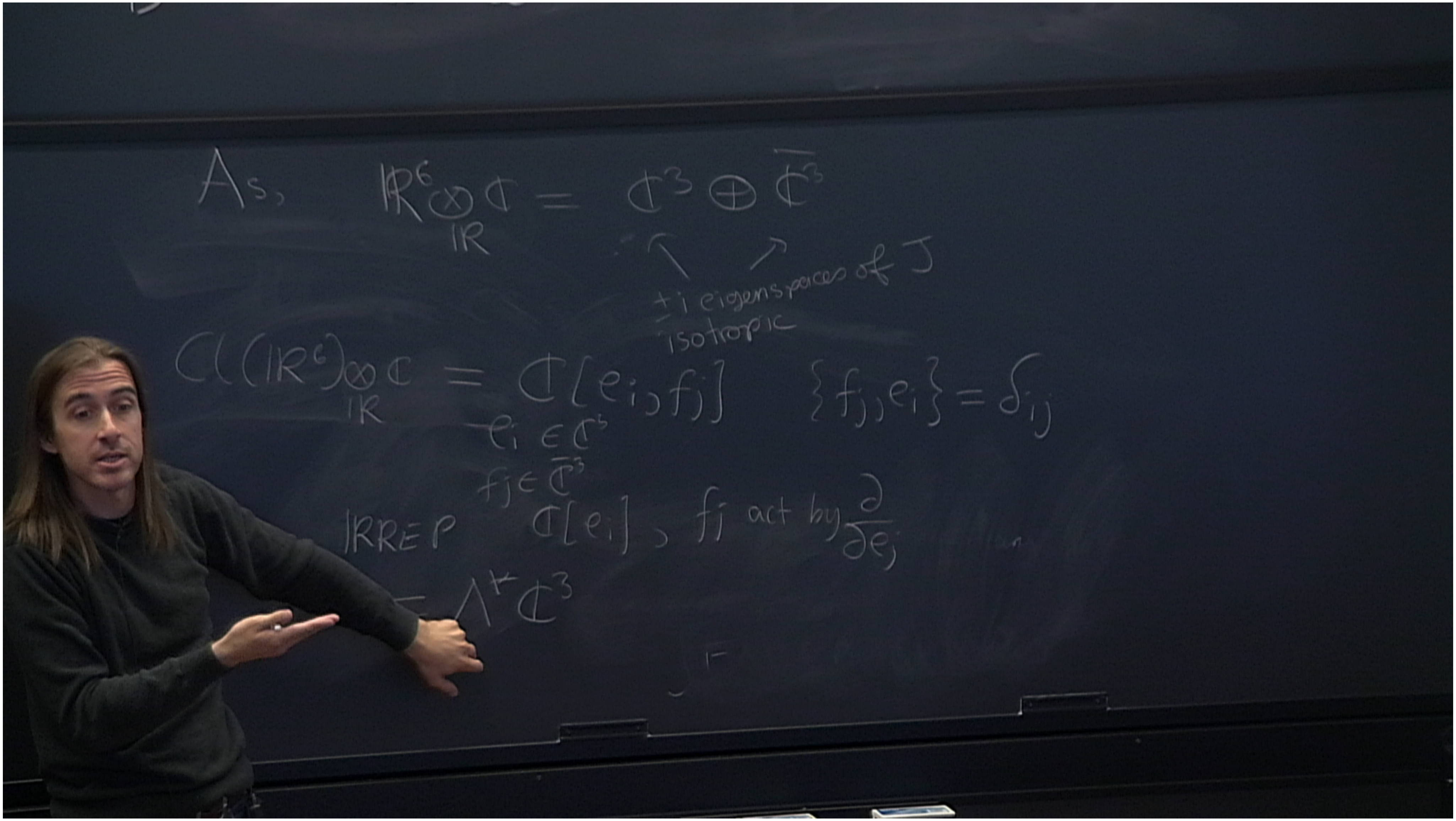
$\int F(A) \wedge B + \int B \wedge B$

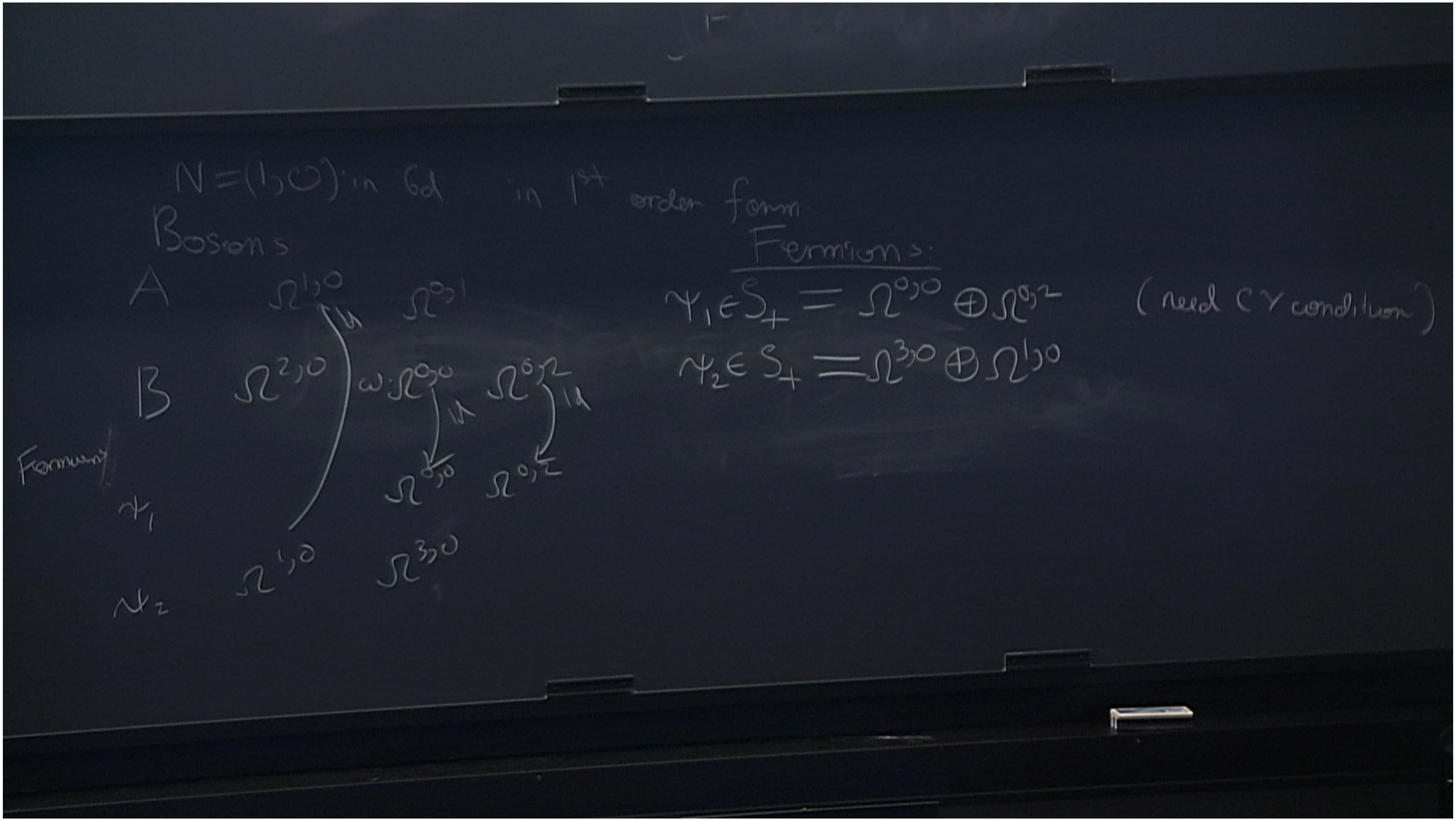
where $B \in \Omega^2_+(X) \otimes g$

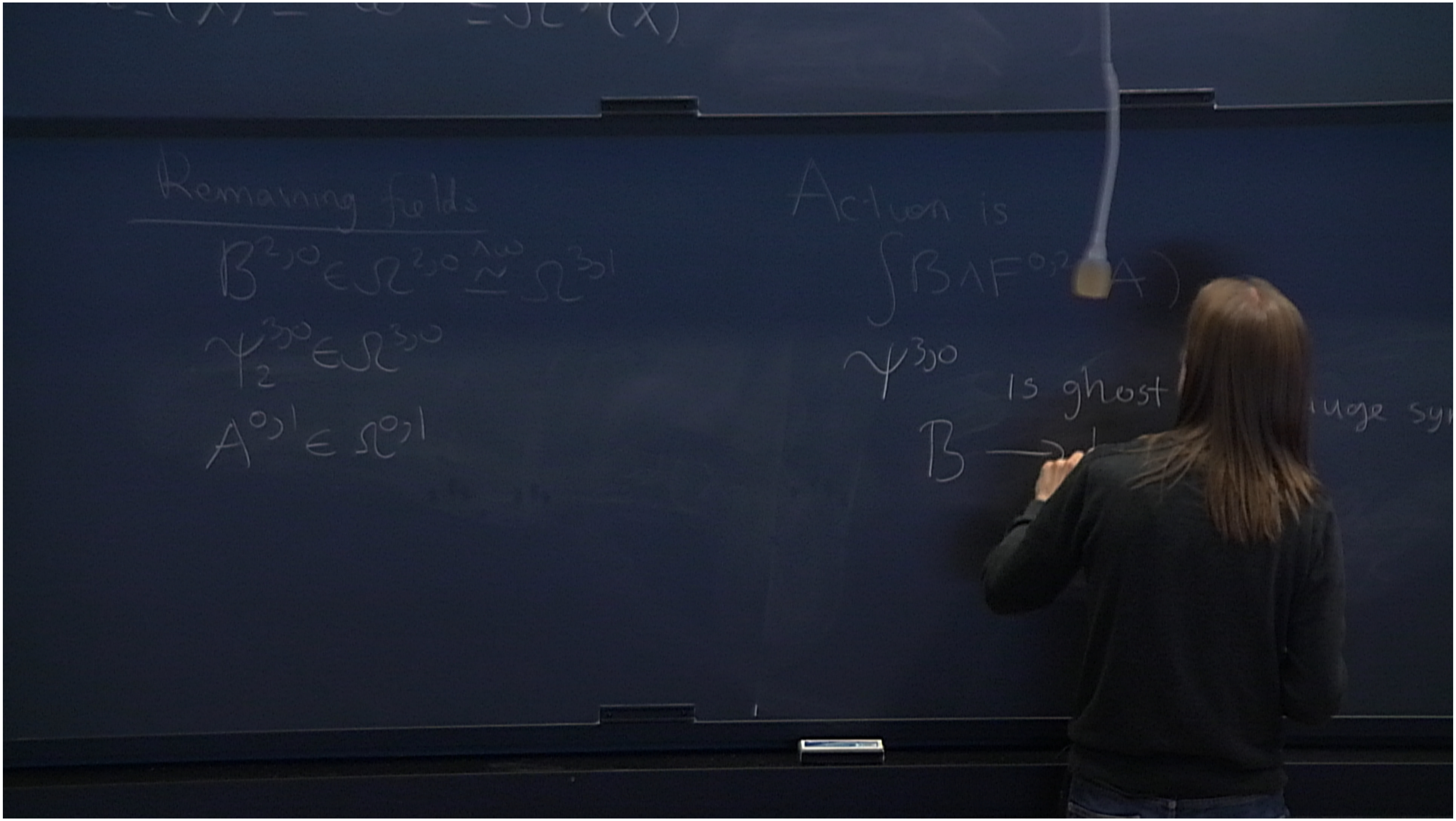
(X is Kähler)

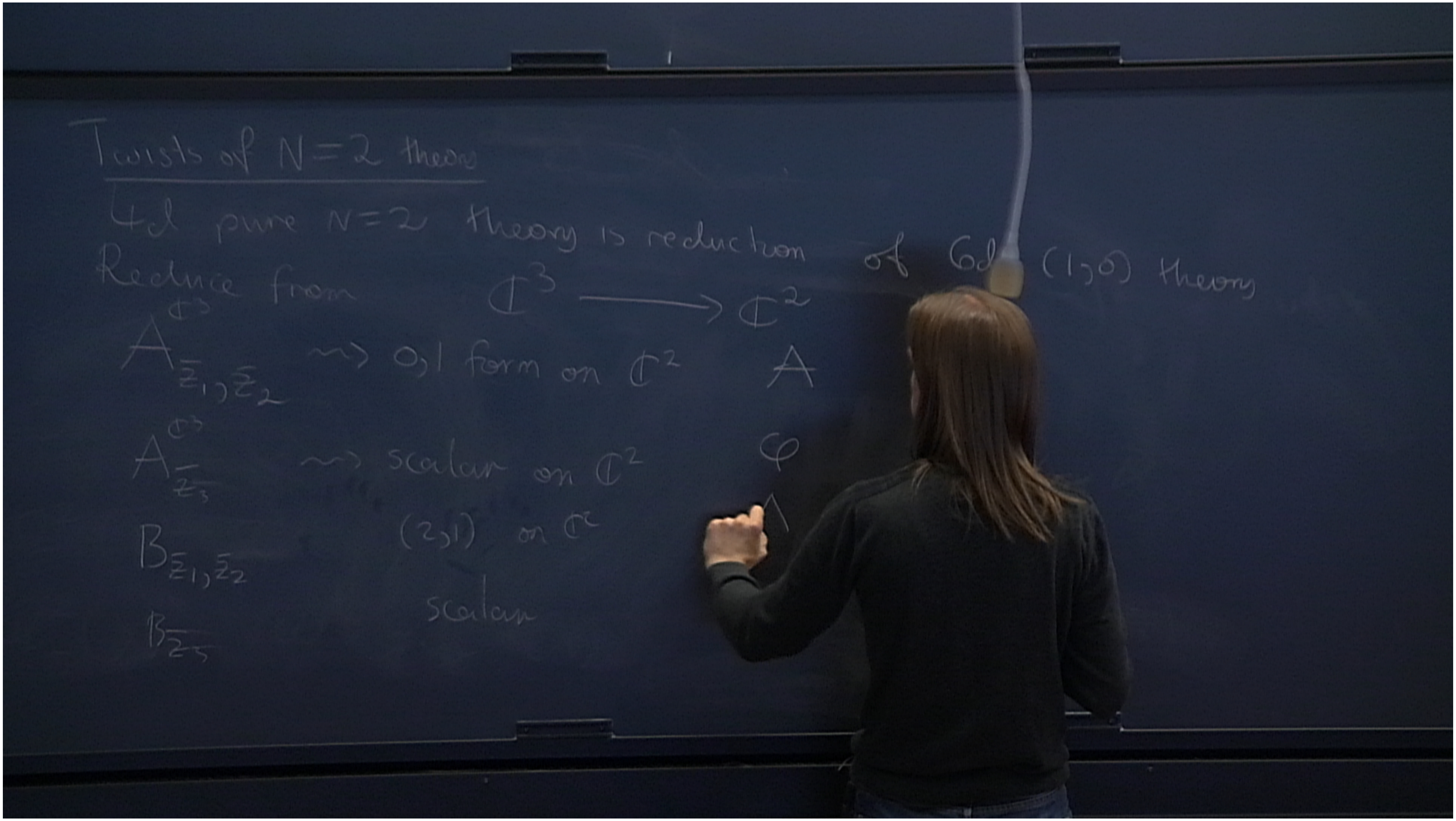


As, $\mathbb{R}^6 \otimes_{\mathbb{R}} \mathbb{C} = \mathbb{C}^3 \oplus \overline{\mathbb{C}^3}$
$\pm i$ eigenspaces of J
isotropic
$Cl(\mathbb{R}^6) \otimes_{\mathbb{R}} \mathbb{C} = \mathbb{C}[e_i, f_j]$
$\{f_j, e_i\} = \delta_{ij}$
$e_i \in \mathbb{C}^3$
$f_j \in \overline{\mathbb{C}^3}$


As, ℝ⁶ ⊗_ℝ ℂ = ℂ³ ⊕ ℂ̄³
±i eigenspaces of J
isotropic
Cl(ℝ⁶) ⊗_ℝ ℂ = ℂ[eᵢ, fⱼ]   {fⱼ, eᵢ} = δᵢⱼ
eᵢ ∈ ℂ³
fⱼ ∈ ℂ̄³
IRREP   ℂ[eᵢ], fⱼ act by ∂/∂eⱼ
Λᵏℂ³


$N=(1,0)$ in 6d   in 1st order form
Bosons
A $\Omega^{1,0}$ $\Omega^{0,1}$
B $\Omega^{2,0}$ $\omega: \Omega^{0,0}$ $\Omega^{0,2}$
Fermions
ψ_1 $\Omega^{0,0}$ $\Omega^{0,2}$
ψ_2 $\Omega^{1,0}$ $\Omega^{3,0}$
Fermions:
$\psi_1 \in S_+ = \Omega^{0,0} \oplus \Omega^{0,2}$ (need CY condition)
$\psi_2 \in S_+ = \Omega^{3,0} \oplus \Omega^{1,0}$


Remaining fields
$B^{2,0} \in \Omega^{2,0} \overset{\wedge\omega}{\simeq} \Omega^{3,1}$
$\psi_2^{3,0} \in \Omega^{3,0}$
$A^{0,1} \in \Omega^{0,1}$
Action is
$\int B \wedge F^{0,2}(A)$
$\psi^{3,0}$
is ghost
uge sy
$B \longrightarrow$


Twists of N=2 theory
4d pure N=2 theory is reduction of 6d (1,0) theory
Reduce from $\mathbb{C}^3 \longrightarrow \mathbb{C}^2$
$A^{\mathbb{C}^3}_{\bar{z}_1, \bar{z}_2}$ ⇝ 0,1 form on $\mathbb{C}^2$ A
$A^{\mathbb{C}^3}_{\bar{z}_3}$ ⇝ scalar on $\mathbb{C}^2$ φ
$B_{\bar{z}_1, \bar{z}_2}$ (2,1) on $\mathbb{C}^2$
$B_{\bar{z}_3}$ scalar

heory is reduction of 6d (1,0) theory

$\mathbb{C}^3 \longrightarrow \mathbb{C}^2$

orm on $\mathbb{C}^2$ $\quad A \quad 0,1$

$\varphi \quad 0,0$

on $\mathbb{C}^2$ $\quad \tilde{A} \quad 2,1$

on $\mathbb{C}^2$ $\quad \tilde{\varphi} \quad 2,0$

an

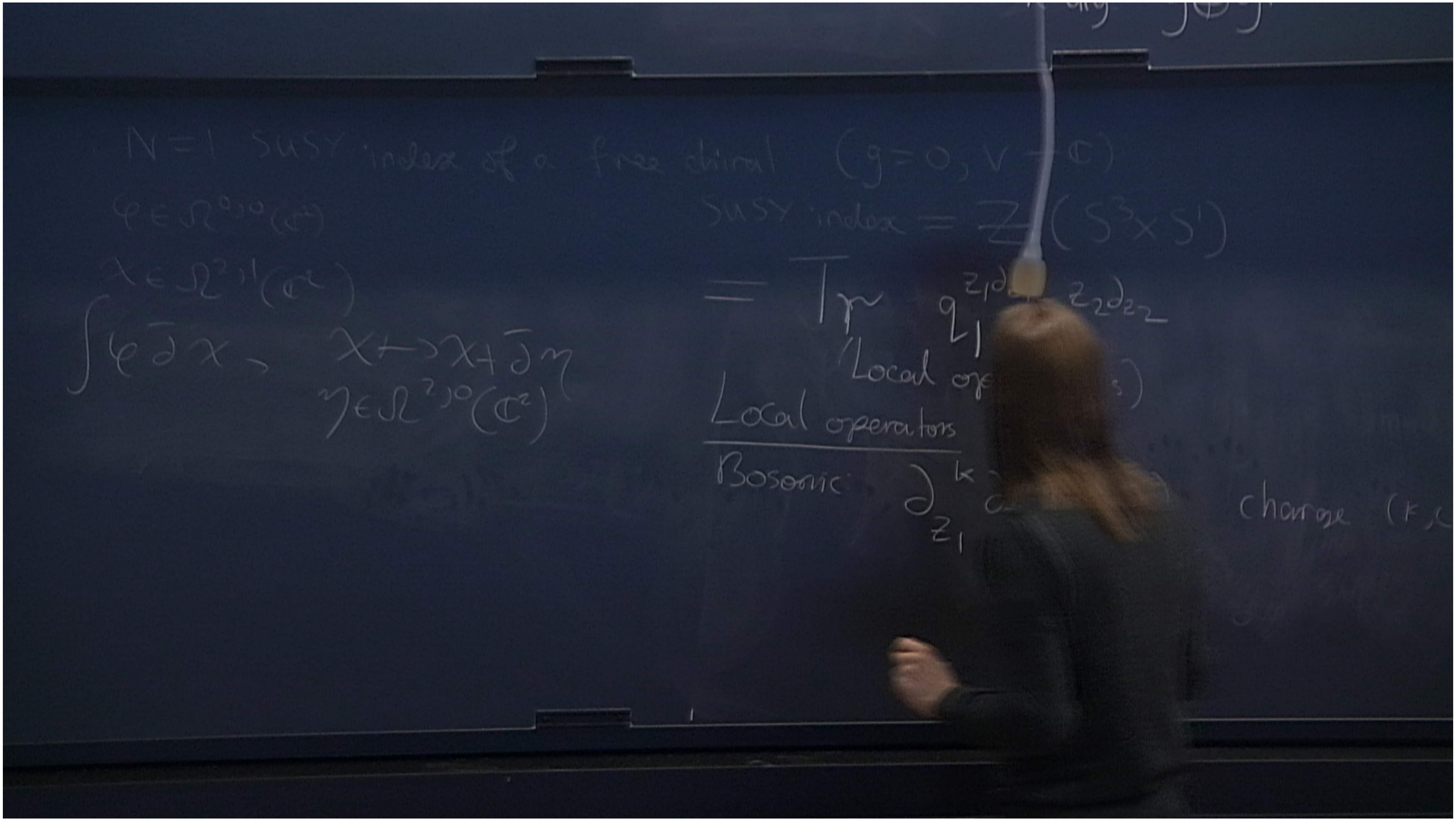
$$S = \underbrace{\int \tilde{\varphi} F^{0,2}(A)}_{\text{pure } N=1} + \underbrace{\int \varphi \bar{\partial}_A \tilde{A}}_{\text{adjoint matter for } N=1}$$

Given a rep V of $\mathfrak{g}$,
hol. twist of $N=1$ gauge theory w. matter
living in V, is hol. BF theory for
Lie algebra $\mathfrak{g} \oplus V$
Concretely, fields are

$A \in \Omega^{0,1}(X, \mathfrak{g})$
$B \in \Omega^{0,0}(X, \mathfrak{g})$
$\chi \in \Omega^{0,1}(X, V^{\vee})$
$\varphi \in \Omega^{0,0}(X, V)$

$$\int B \wedge F^{0,2}(A) \cdot \Omega^{2,0} + \int \chi \wedge \bar{\partial}_A \varphi \wedge \Omega^{2,0}$$

$$\chi \to \chi + \bar{\partial}_A \eta \qquad \eta \in \Omega^{0,0}(X, V^{\vee})$$



N=1 SUSY index of a free chiral ($g=0$, $V=\mathbb{C}$)
$\varphi \in \Omega^{0,0}(\mathbb{C}^2)$
$\chi \in \Omega^{2,1}(\mathbb{C}^2)$
$\int \varphi \bar\partial \chi$, $\chi \mapsto \chi + \bar\partial \eta$
$\eta \in \Omega^{2,0}(\mathbb{C}^2)$
SUSY index $= Z(S^3 \times S^1)$
$= \mathrm{Tr}\ q_1^{z_1 \partial} \ldots$ $z_2 \partial_{z_2}$
Local operators
Bosonic: $\partial_{z_1}^k$
charge (k,

N=1 SUSY index of a free chiral ($g=0$, $V=\mathbb{C}$)

$\varphi \in \Omega^{0,0}(\mathbb{C}^2)$

$\chi \in \Omega^{2,1}(\mathbb{C}^2)$

$\varphi \bar\partial \chi$, $\chi \mapsto \chi + \bar\partial \eta$

$\eta \in \Omega^{2,0}(\mathbb{C}^2)$

...erators build from χ

$0 \Rightarrow \chi = \partial\eta$

build operators from

SUSY index $= Z(S^3 \times S^1)$

$= \mathrm{Tr}\, q_1^{z_1\partial_{z_1}} q_2^{z_2\partial_{z_2}}$

(Local operators)

Local operators

Bosonic $\partial_{z_1}^k \partial_{z_2}^l \varphi(0)$ charge (k,l)

$\partial_{z_1}^k \partial_{z_2}^l \eta(0)$ charge $(k+1, l+1)$

Whole space of local operators is

$$S^\bullet \mathbb{C}[\partial_{z_1},\partial_{z_2}] \otimes \Lambda^\bullet\left(\partial_{z_1},\partial_{z_2}\,\mathbb{C}[\partial_{z_1},\partial_{z_2}]\right)$$

Character:

$$\prod_{k,l\geq 0} \frac{1-q_1^{k+1}q_2^{l+1}}{1-q_1^{k}q_2^{l}}$$

Whole space of local operators is

$$\ldots \; S^{\bullet}\mathbb{C}[\partial_{z_1},\partial_{z_2}] \otimes \Lambda^{\bullet}\left(\partial_{z_1}\partial_{z_2}\mathbb{C}[\partial_{z_1},\partial_{z_2}]\right)$$

Character:

$$\left.\prod_{k,l\geq 0} \frac{1-q_1^{k+1}q_2^{l+1}u^{-1}}{1-q_1^{k}q_2^{l}u}\right\} \text{correct formula!}$$