Title: Causality in the quantum world

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Abstract: <p>Einstein's causality is one of the fundamental principles underlying modern physical theories. Whereas it is readily implemented in classical physics founded on Lorentzian geometry, its status in quantum theory has long been controversial. It is usually believed that the quantum nature of spacetime at small scales induces the breakdown of causality, although there is no empirical evidence that would support such a view. In my talk, I will argue that one can have a sound notion of causality even in a `quantum spacetime' -- understood as the space of states of an abstract algebra of observables. To this end I will draw from the mathematical richness of noncommutative geometry a la Connes. I will illustrate the general concept with an `almost commutative' toy-model and discuss the potential emprical consequences.</p>

Einstein's causality – a dual viewpoint

Causality for mathematicians: a partial order relation on spacetime $\mathcal{M}$ induced by the Lorentzian metric.

$$p \preceq q \iff \exists \text{ future-directed causal curve } \gamma \text{ from } p \text{ to } q \text{ (or } p = q).$$

Causal function – $a \in C(\mathcal{M}, \mathbb{R})$ non-decreasing on every future-directed causal curve, i.e. $p \preceq q \Rightarrow a(p) \leq a(q)$.

Compare: time function – strictly increasing on fut.-dir. causal curves.

Dual viewpoint [Besnard 2009; Minguzzi 2010; Franco, M.E. 2013]

Let $\mathcal{M}$ be a causally simple spacetime, then

$$p \preceq q \Leftrightarrow \forall\, a - \text{bounded causal function } a(p) \leq a(q).$$

Causality for probability measures

Recall the Gelfand duality: $\mathcal{P}(C_0(X)) \simeq X$

- What are the elements of $S(C_0(X))$?

Riesz–Markov–Kakutani representation theorem

mixed states on $C_0(X) \overset{1:1}{\longleftrightarrow}$ Borel probability measures on X

Definition [M.E., Miller 2015]

Let $\mathcal{M}$ be a causally simple spacetime. For $\mu, \nu \in \mathfrak{P}(\mathcal{M})$ we define

$$\mu \preceq \nu \underset{\text{def}}{\Leftrightarrow} \forall\, a - \text{bounded causal function} \quad \int_{\mathcal{M}} a d\mu \leq \int_{\mathcal{M}} a d\nu.$$

The causal order on spacetime $\mathcal{M}$ extends naturally to $\mathfrak{P}(\mathcal{M})$.

Characterisation of causality 2

Theorem [M.E., Miller 2015]

Let $\mathcal{M}$ be a causally simple spacetime. For $\mu, \nu \in \mathfrak{P}(\mathcal{M})$,

$$\mu \preceq \nu \iff \exists\, \omega \in \mathfrak{P}(\mathcal{M}^2) \text{ such that:}$$

- $\forall_{A \in \mathfrak{B}(\mathcal{M})}\ \omega(A \times \mathcal{M}) = \mu(A), \omega(M \times A) = \nu(A)$,
- $\omega(J^+) = 1, \quad J^+ = \{(p,q) \in \mathcal{M}^2 \,|\, p \preceq q\}$.

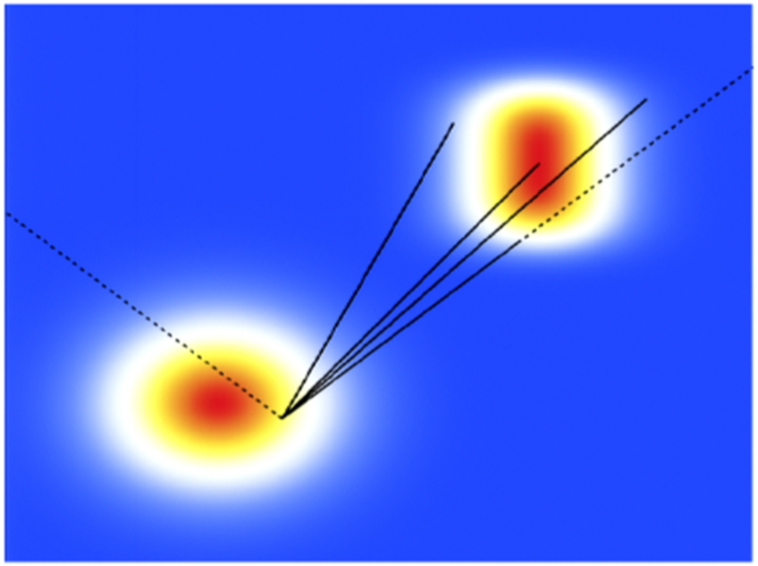
- Optimal Transport Theory $\rightsquigarrow$ How to move a pile of sand *causally*?
- ω deserves the name of a causal transference plan.
- For $\mu = \delta_p$, $\nu = \delta_q$, $\omega = \delta_{(p,q)}$ and $\delta_p \preceq \delta_q$ iff $p \preceq q$.
- Actually, ω makes sense on *any* spacetime!

Formalising intuitions

Each infinitesimal part of the probability density should travel along a future-directed causal curve.

$\omega \in \mathfrak{P}(\mathcal{M}^2)$
$\Rightarrow \exists \, \{\gamma_{p,q}\}_{(p,q)\in\mathcal{M}\times\mathcal{M}}$

$\omega(J^+) = 1$
$\Rightarrow \gamma_{p,q}$ – causal fut.-dir.



Examples of spectral triples

Connes' Reconstruction Theorem [2008]

For every *commutative* spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ there exists a smooth compact spin Riemannian manifold $\mathcal{M}$ such that:

$$\mathcal{A} = C^\infty(\mathcal{M}), \qquad \mathcal{H} = L^2(S(\mathcal{M})), \qquad \mathcal{D} = \not{D} = -\mathrm{i}\,\gamma^\mu \nabla^S_\mu.$$

Examples of spectral triples:

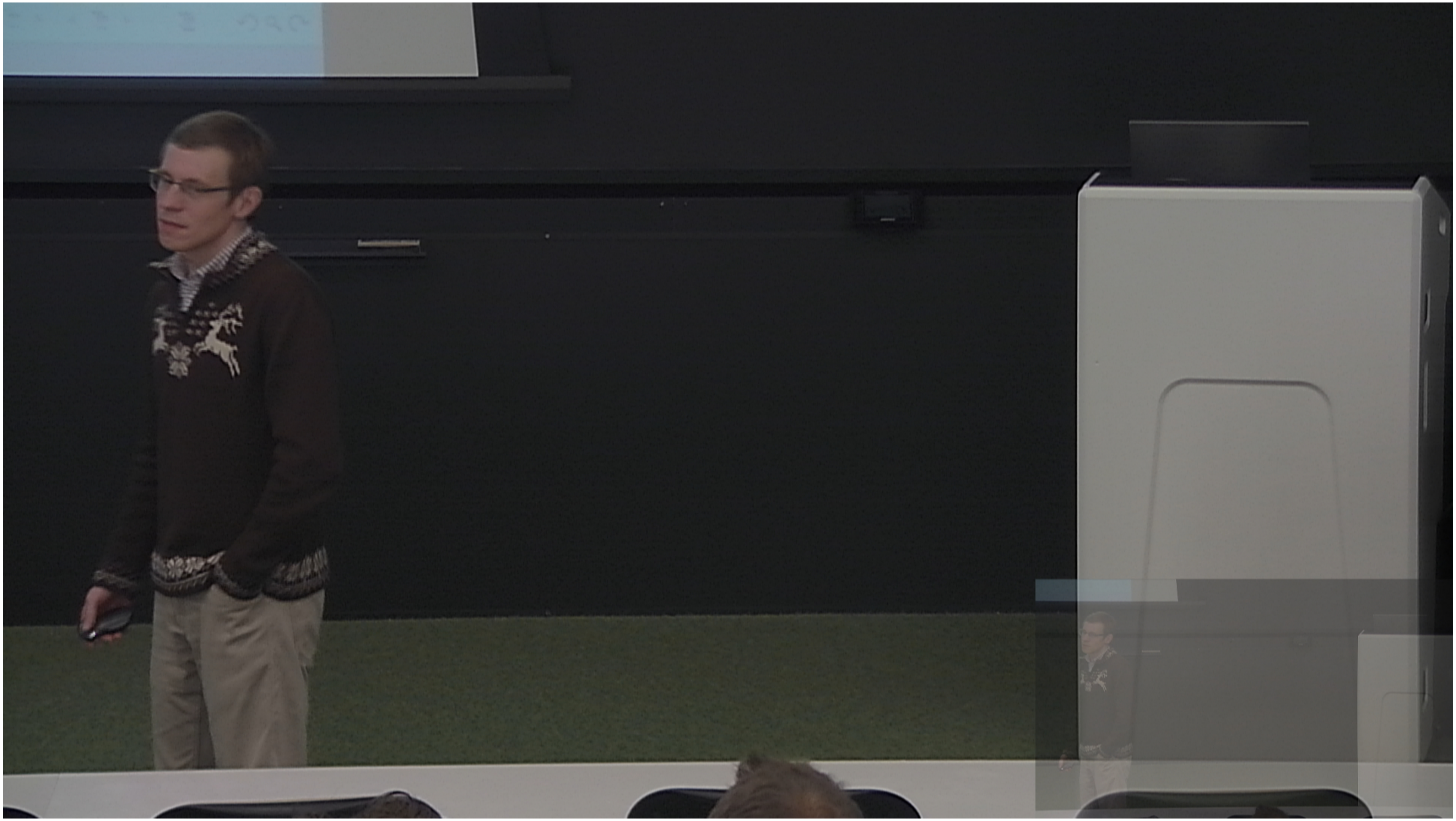
- Finite ST: $\mathcal{A}_\mathcal{F} = M_n(\mathbb{C})$, $\mathcal{H}_\mathcal{F} = \mathbb{C}^N$, $\mathcal{D}_\mathcal{F} = \mathcal{D}_\mathcal{F}^\dagger \in M_N(\mathbb{C})$.
- Direct sums & tensor products of spectral triples are spectral triples.
- $\mathcal{A} = C^\infty(\mathcal{M}) \otimes \mathcal{A}_F$ – almost commutative geometry.
- Fractals, deformations, quantum groups, quantum spaces, ...

Noncommutative geometry à la Connes

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ – spectral triple

- $\mathcal{A}$ – (dense $*$-subalgebra of) a C^*-algebra
- $\mathcal{H}$ – Hilbert space with a faithful representation $\pi(\mathcal{A}) \subset \mathcal{B}(\mathcal{H})$
- $\mathcal{D}$ – a Dirac operator – selfadjoint with $[\mathcal{D}, \pi(a)] \in \mathcal{B}(\mathcal{H}) \; \forall \, a \in \mathcal{A}$.
- $+$ technical assumptions
- $+$ additional structure – modules, chirality, ...

Heisenberg	NC algebra $\mathcal{A}$	Dirac operator
Schrödinger	NC space(time)	geometry



The noncommutative causal structure

- Choose a suitable *unitisation* of $\mathcal{A}$, $\widetilde{\mathcal{A}}$ (technicality).
- Take a *cone* $\mathcal{C} \subset \widetilde{\mathcal{A}}$ of elements satisfying $\forall_{\phi \in \mathcal{K}} \quad (\phi, [\mathcal{D}, a]\phi) \leq 0$.
- If $\overline{\mathrm{span}_{\mathbb{C}}(\mathcal{C})} = \overline{\widetilde{\mathcal{A}}}$, then $\mathcal{C}$ is a causal cone.

Proposition [N. Franco, M.E. (2013)]

Let $\mathcal{C} \subset \widetilde{\mathcal{A}}$ be a causal cone, then for every two states $\chi, \xi \in S(\widetilde{\mathcal{A}})$ define

$$\chi \preceq \xi \quad \text{iff} \quad \forall\, a \in \mathcal{C} \quad \chi(a) \leq \xi(a).$$

The relation $\preceq$ is a partial order on $S(\widetilde{\mathcal{A}})$ (and thus on $P(\mathcal{A}), S(\mathcal{A})$).

Almost-commutative spacetimes

Take $(\mathcal{A}_{\mathcal{M}}, \mathcal{K}_{\mathcal{M}}, \mathcal{D}_{\mathcal{M}})$ even ($\mathcal{K}$ is $\mathbb{Z}_2$-graded) and $(\mathcal{A}_F, \mathcal{H}_F, \mathcal{D}_F)$ finite, then $(\mathcal{A}_{\mathcal{M}}, \mathcal{K}_{\mathcal{M}}, \mathcal{D}_{\mathcal{M}}) \otimes (\mathcal{A}_F, \mathcal{H}_F, \mathcal{D}_F)$ is an almost commutative pseudo-Riemannian spectral triple.

Almost-commutative spacetime

$$\mathcal{P}(\mathcal{A}) = \mathcal{M} \times \mathcal{P}(\mathcal{A}_{\mathcal{F}})$$

Theorem (No Einstein causality violation) [M.E., Franco (2014b)]

Let $(\mathcal{A}, \mathcal{K}, \mathcal{D})$ be an almost commutative pseudo-Riemannian spectral triple, such that the causal cone $\mathcal{C}$ exists. If $\omega_{p,\xi}, \omega_{q,\chi} \in \mathcal{P}(\mathcal{A})$ are such that $\omega_{p,\xi} \preceq \omega_{q,\chi}$, then $p \preceq q$ in $\mathcal{M}$.

Two-sheeted space-time

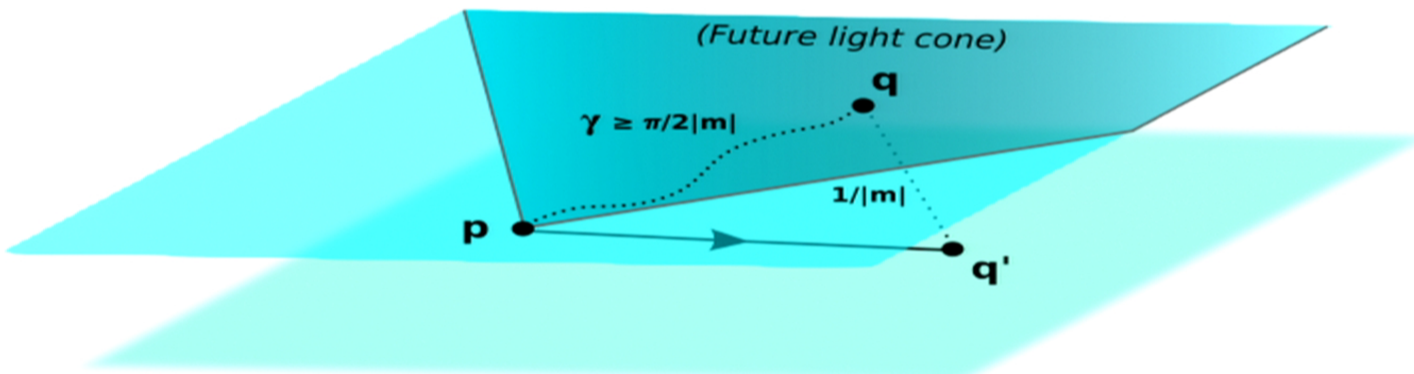
- $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{C}$,
- $\mathcal{H}_F = \mathbb{C}^2$,
- $\mathcal{D}_F = \begin{pmatrix} 0 & m \\ m^* & 0 \end{pmatrix}$, $m \in \mathbb{C}^*$.
- $P(\mathcal{A}) = M \times \{0, 1\} = M \sqcup M$.

Theorem [*N. Franco, M.E. 2015b*]

$(p, 0) \preceq (q, 1)$ iff $p \preceq q$ in M and

$$\tau(\gamma) \geq \frac{\pi}{2\,|m|},$$

where $\tau(\gamma)$ – proper time along γ.



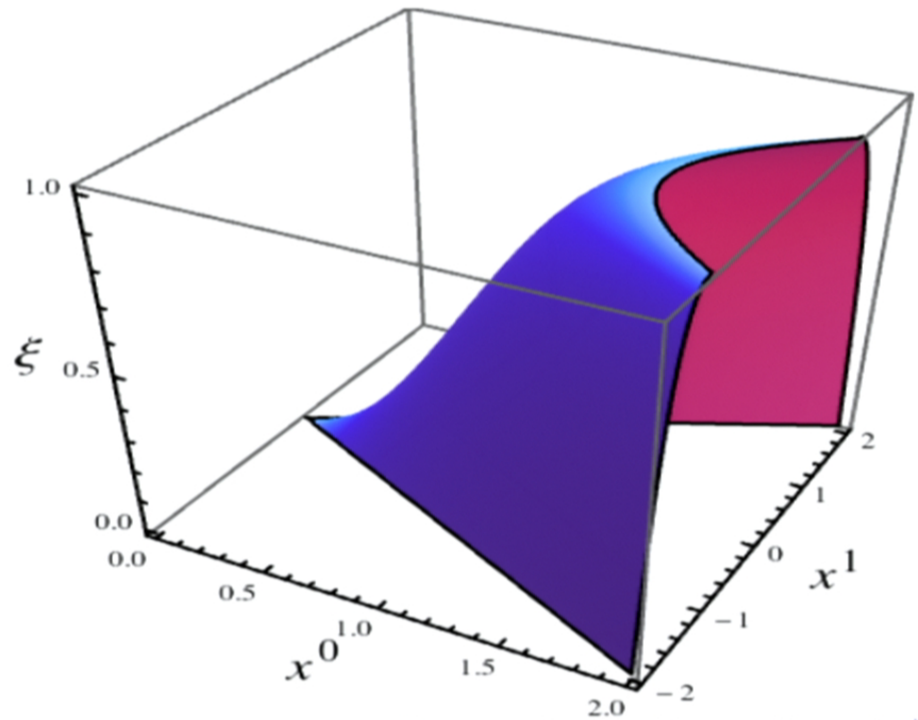
Two-sheeted space-time

- 'Internal' mixed states: $M \times [0,1] \subset S(\mathcal{A})$.

Thm [*N. Franco, M.E. 2015b*]

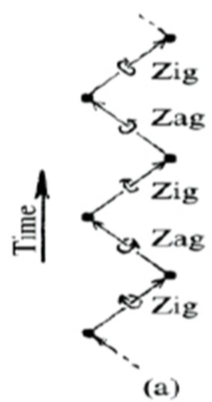
$(p,\varphi) \preceq (q,\xi)$ iff $p \preceq q$ on M and

$$\tau(\gamma) \geq \frac{\left|\arcsin\sqrt{\varphi} - \arcsin\sqrt{\xi}\right|}{|m|}.$$

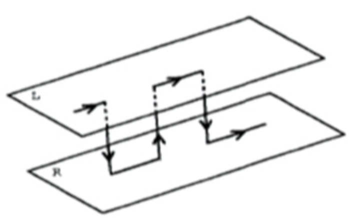


Zitterbewegung

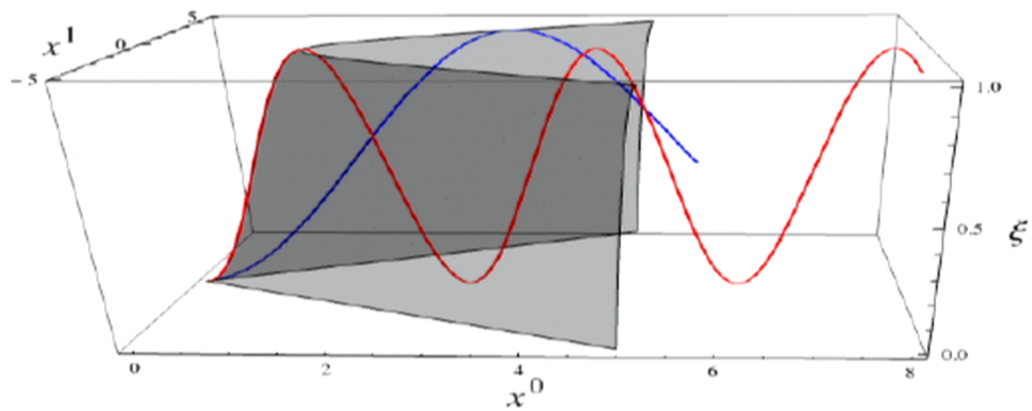
- Zitterbewegung – the 'trembling motion of the electron'.
- Free Dirac equation $i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$.
- Chirality $\gamma^5\psi_{L,R} = \pm\psi_{L,R}$ eigenstates: $i\hbar\gamma^\mu\partial_\mu\psi_{L,R} = mc\,\psi_{R,L}$.
- In rest frame: $T_{\mathrm{ZB}} = \frac{\pi\hbar}{mc^2}$.



[Penrose (2004)]



[Brout (2001)]



[M.E., N. Franco, T. Miller (2016)]

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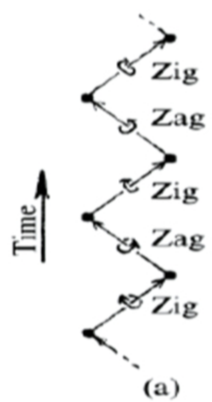

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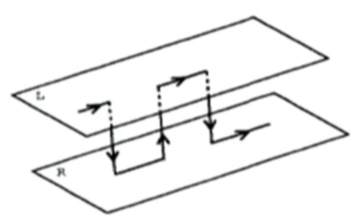

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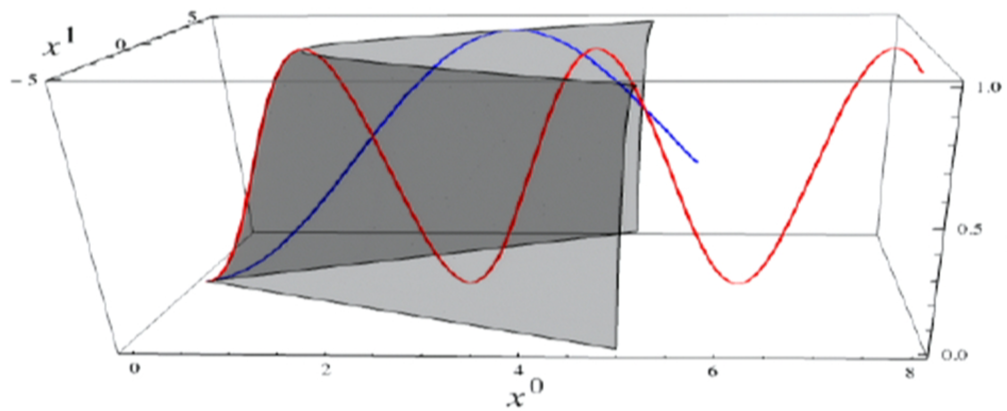
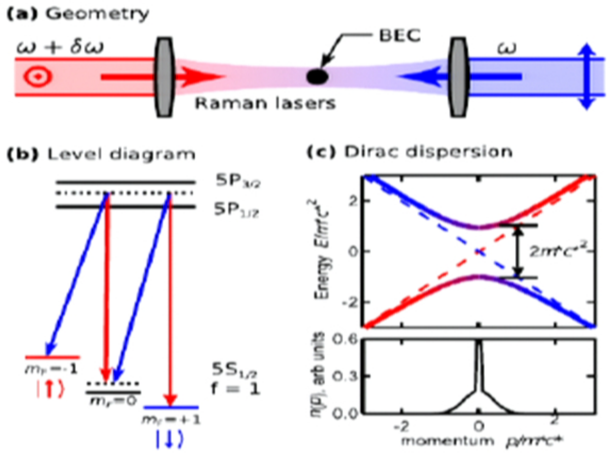


[M.E., N. Franco, T. Miller (2016)]

Quantum simulation of *Zitterbewegung*

- For a free electron $T_{\text{ZB}} \approx 10^{-21} s$.
- The true electrons require a QFT description.
- Quantum simulation! (trapped ions, BEC, photonics, ...)



L.J. LeBlanc et al., New J. of Phys. **15** 073011 (2013)

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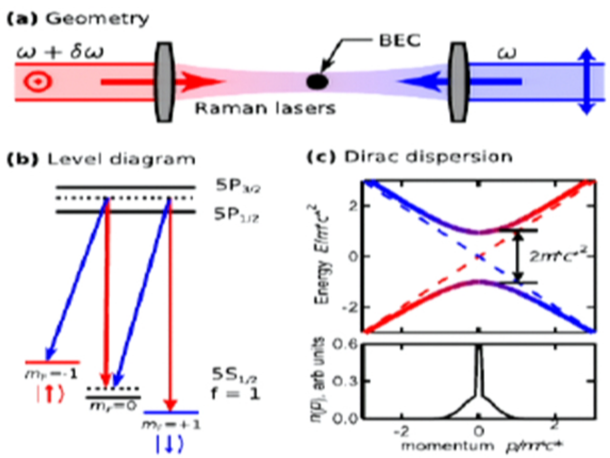

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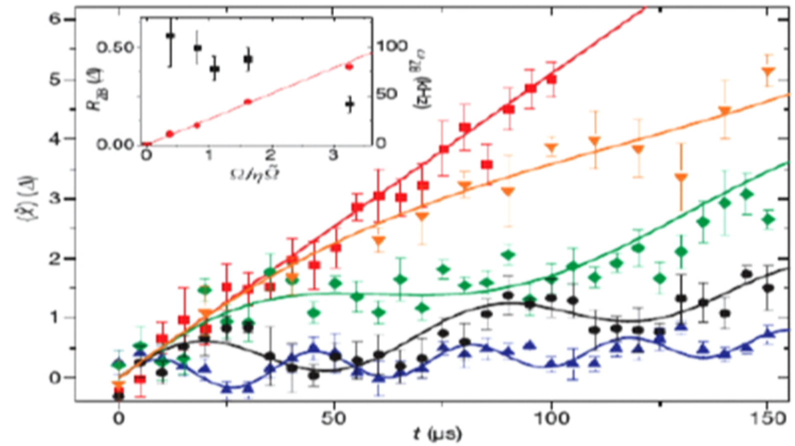
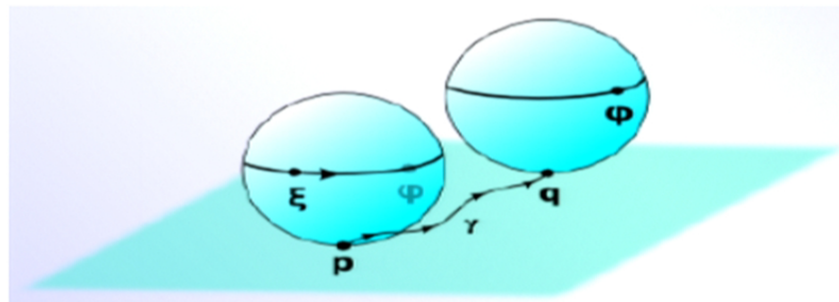


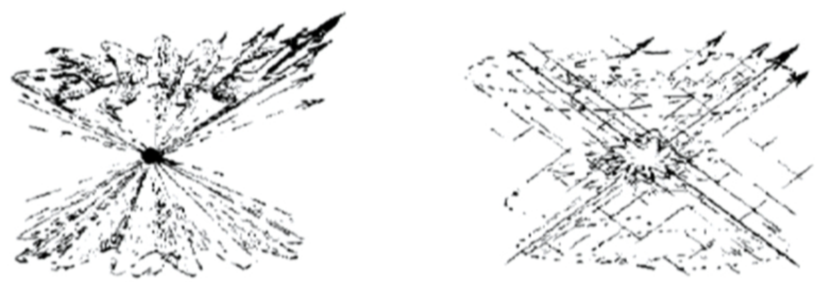
R. Gerritsma et al., Nature **463** 68 (2010)

Quantum spacetime

- Other gauge models available!
- $\mathcal{A}_F = M_2(\mathbb{C})$, $\mathcal{D}_F = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$.
- arXiv:1310.8225
- Neutrino, quark mixing, ...



- $[\hat{x}_\mu, \hat{x}_\nu] = \Theta_{\mu\nu}$
- Coherent states
  – Θ smeared events
- Rigid causal structure!
- arXiv:1507.06559



[R. Penrose, *Road to Reality*, 2004]

Outlook

Rethinking quantum field theory ...

- Idea: Establish QFT over $\mathcal{P}(\mathcal{A}) \rightsquigarrow \mathcal{A}(\mathcal{O})$ for $\mathcal{O} \subset \mathcal{P}(\mathcal{A})$.
- Example: 2 QF over $\mathcal{M}$ = QF over $\mathcal{M} \sqcup \mathcal{M}$.
- Consequence: Microcausality needs adjustment.
- Consequence: The algebras of observables are modified.

Take-home messages

- Operational viewpoint on quantum spacetime.
- Causality is possible in the quantum world!
- Matter does matter!