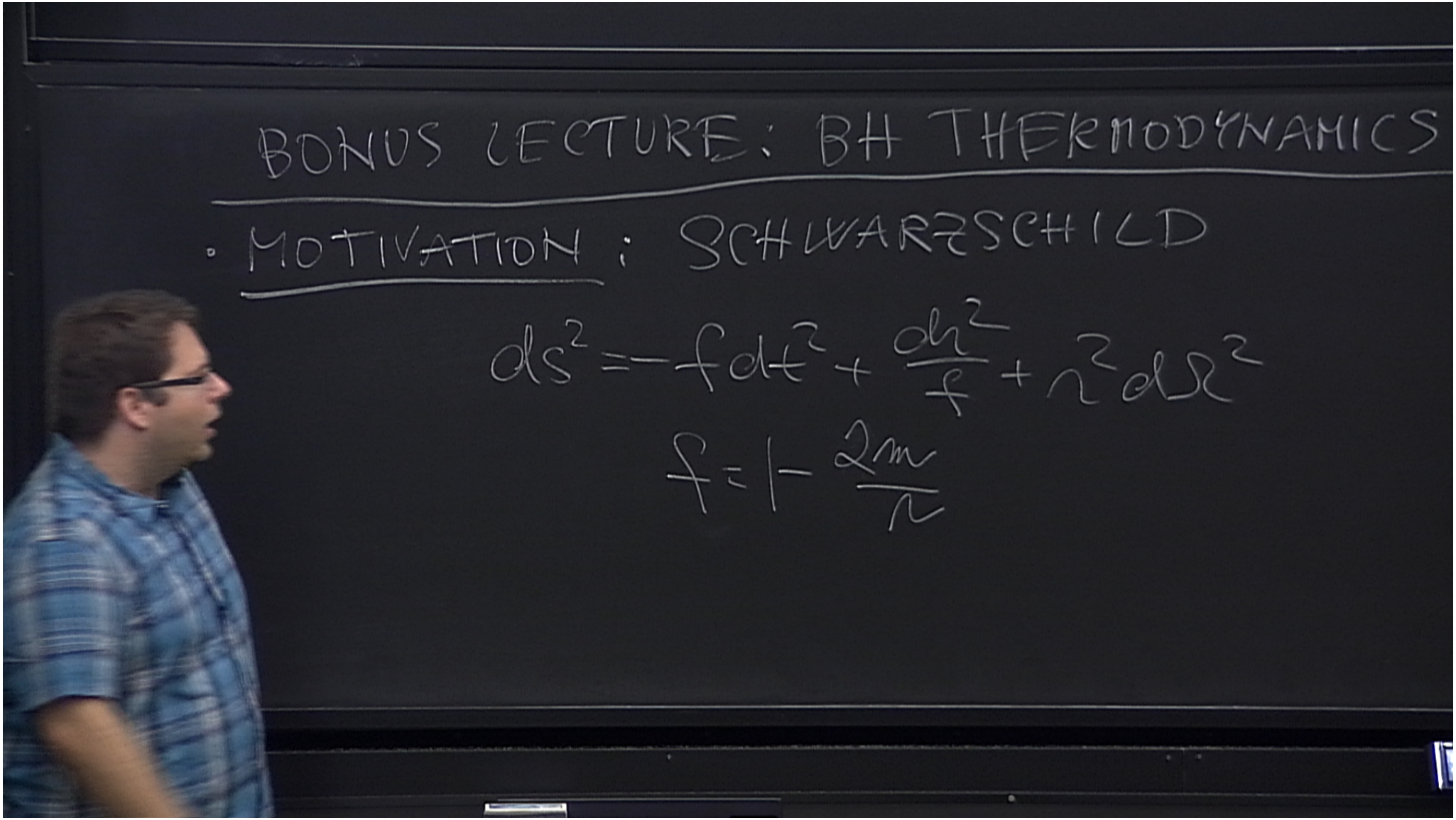
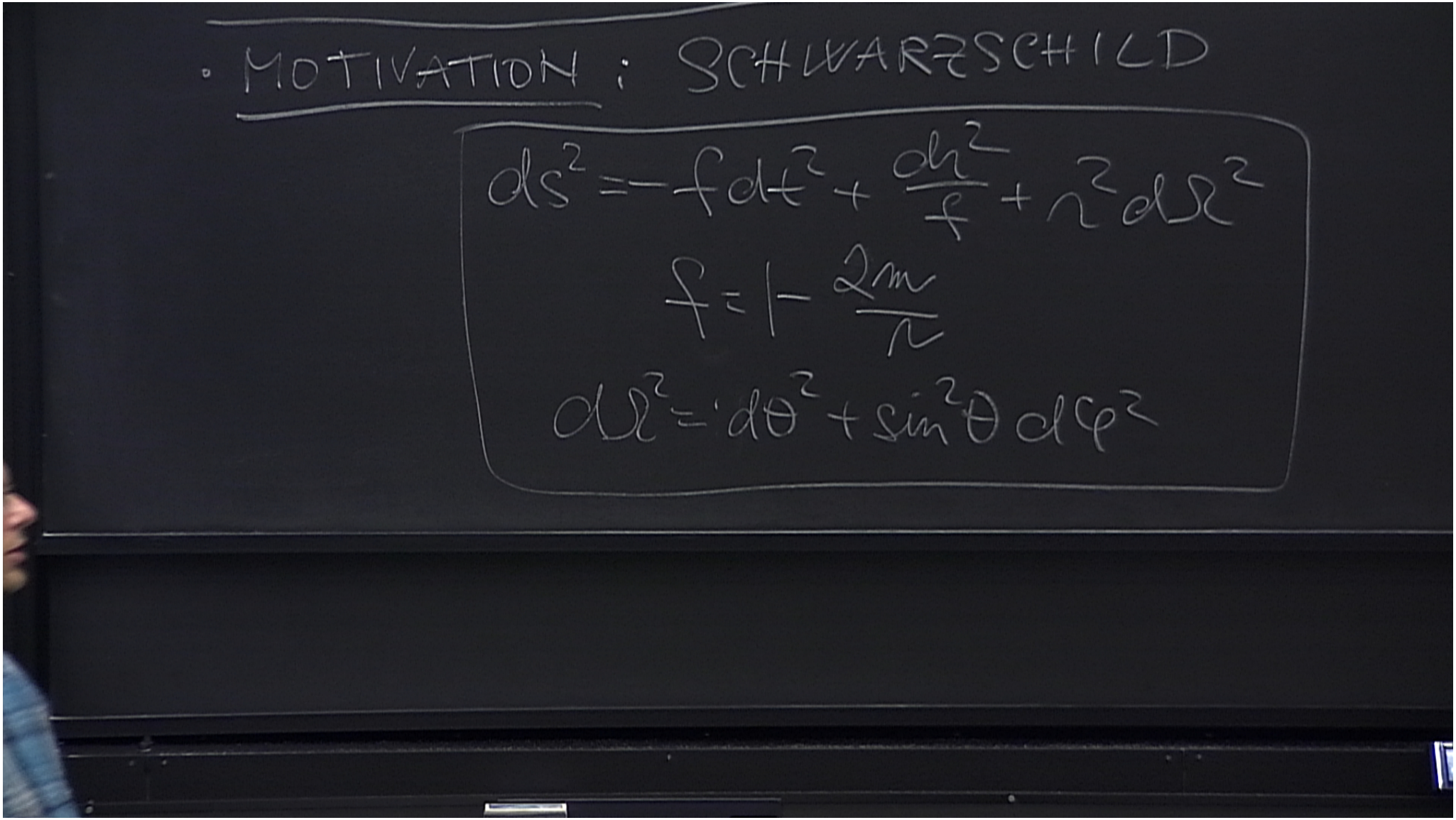
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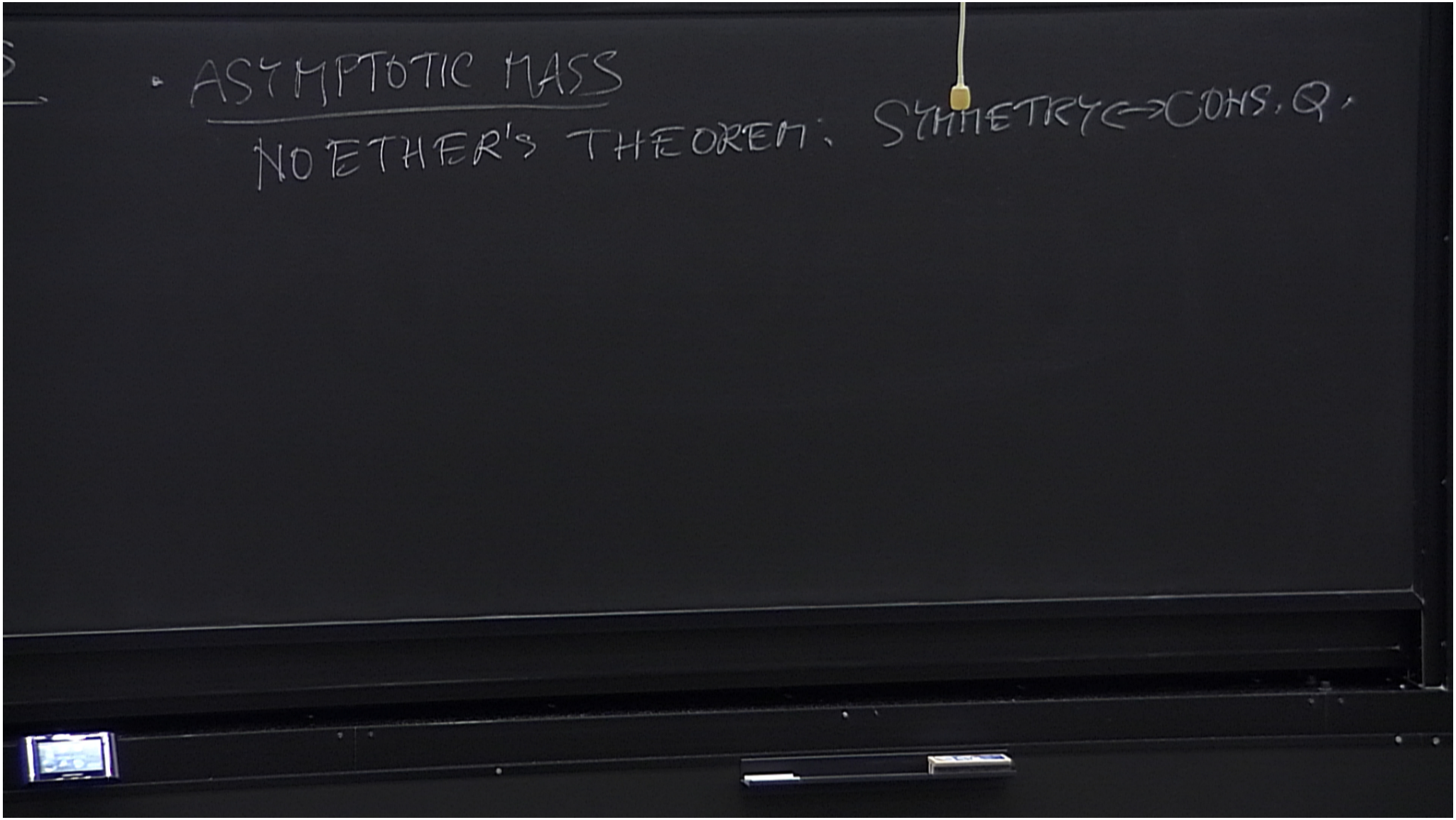
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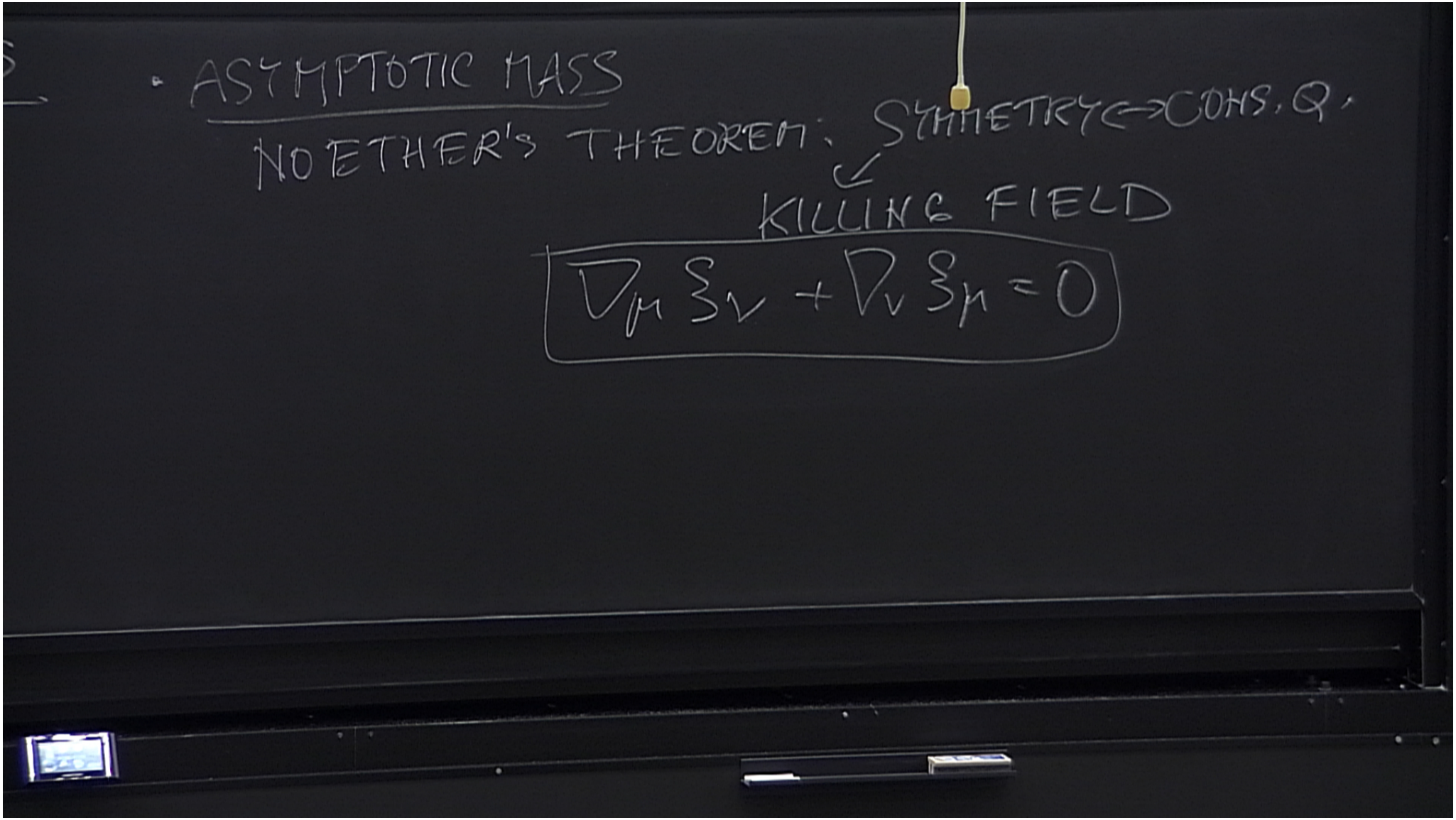
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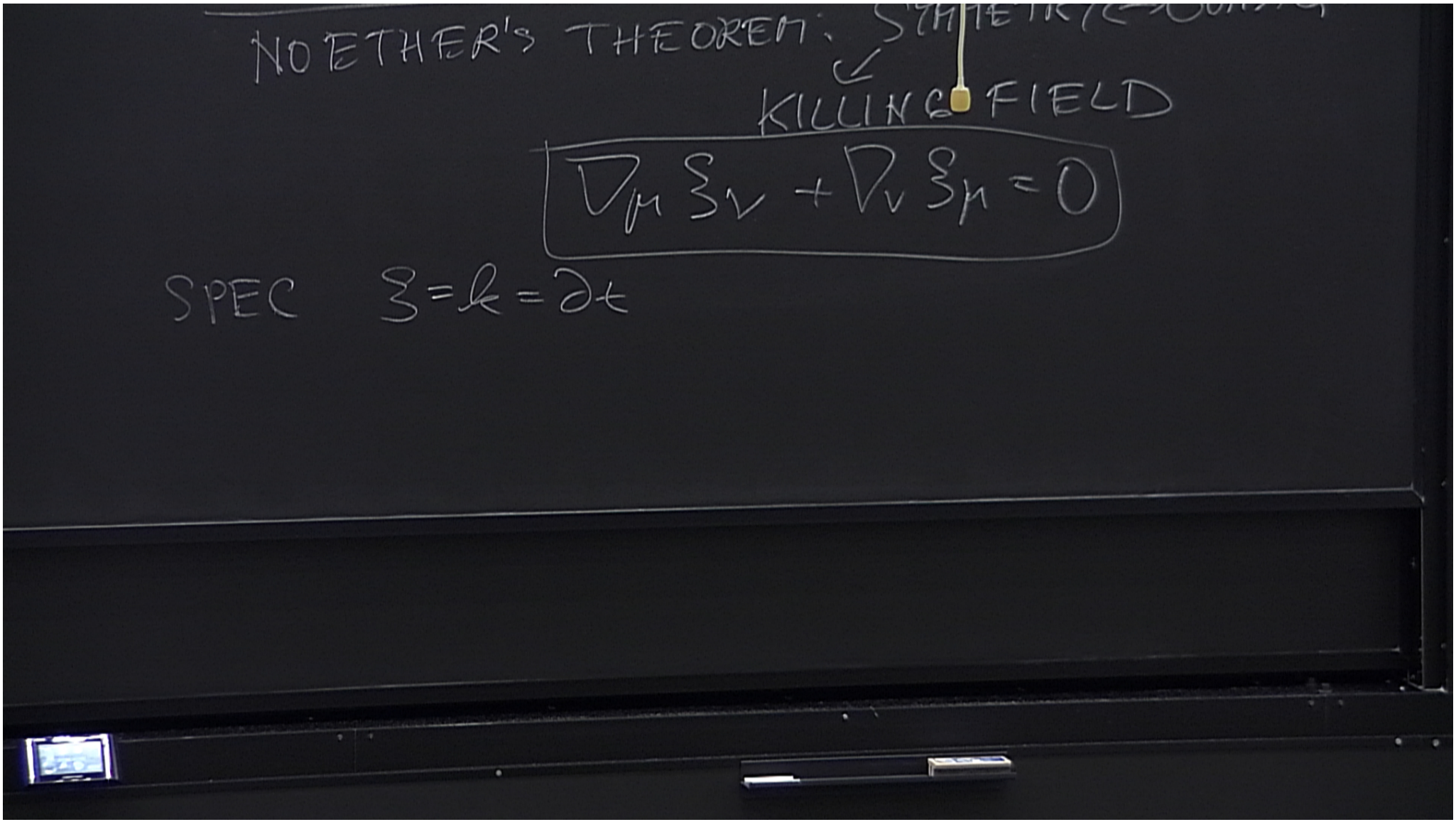
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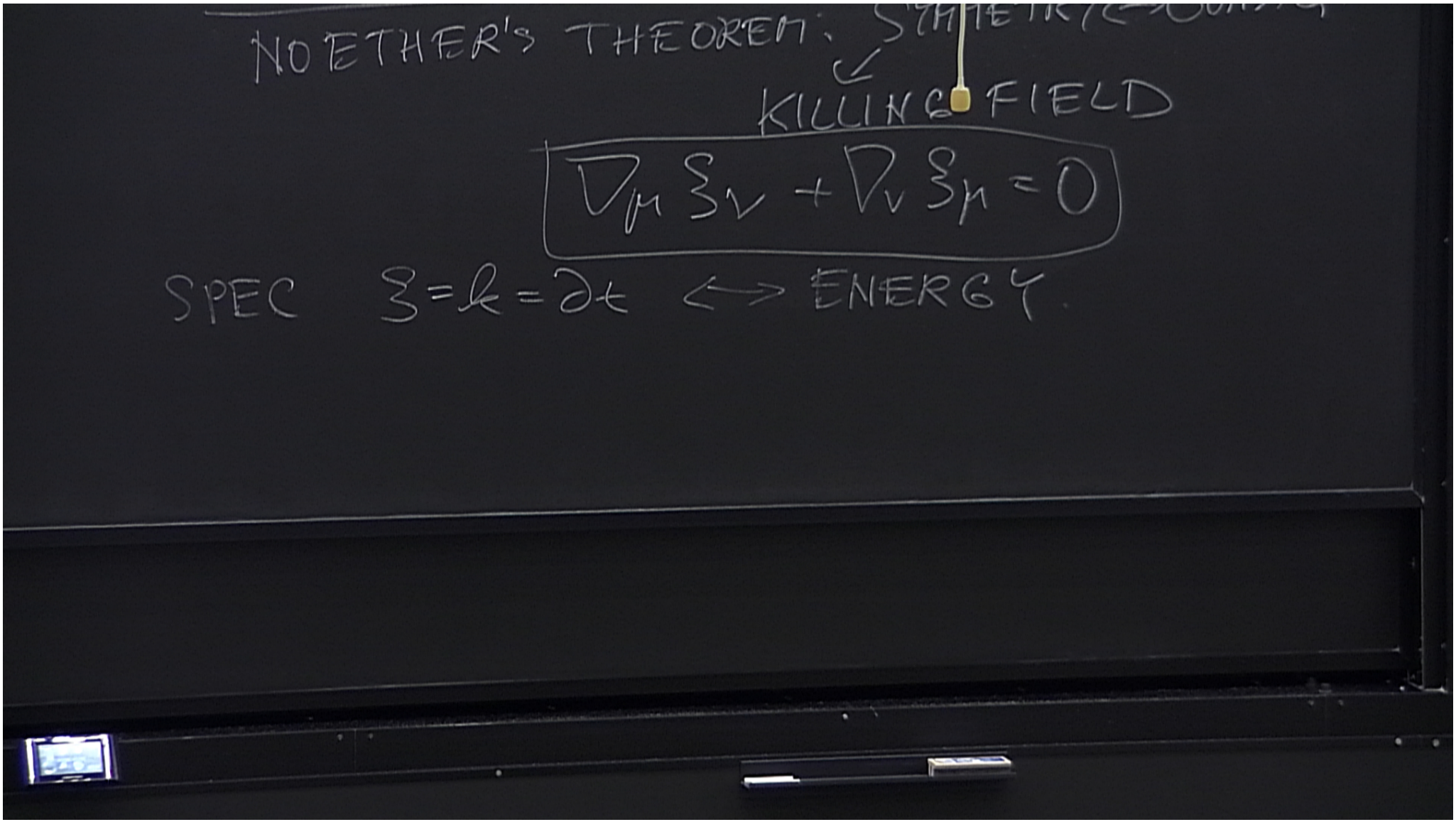

BONUS LECTURE: BH THERMODYNAMICS
• MOTIVATION: SCHWARZSCHILD
$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$
$f = 1 - \frac{2m}{r}$

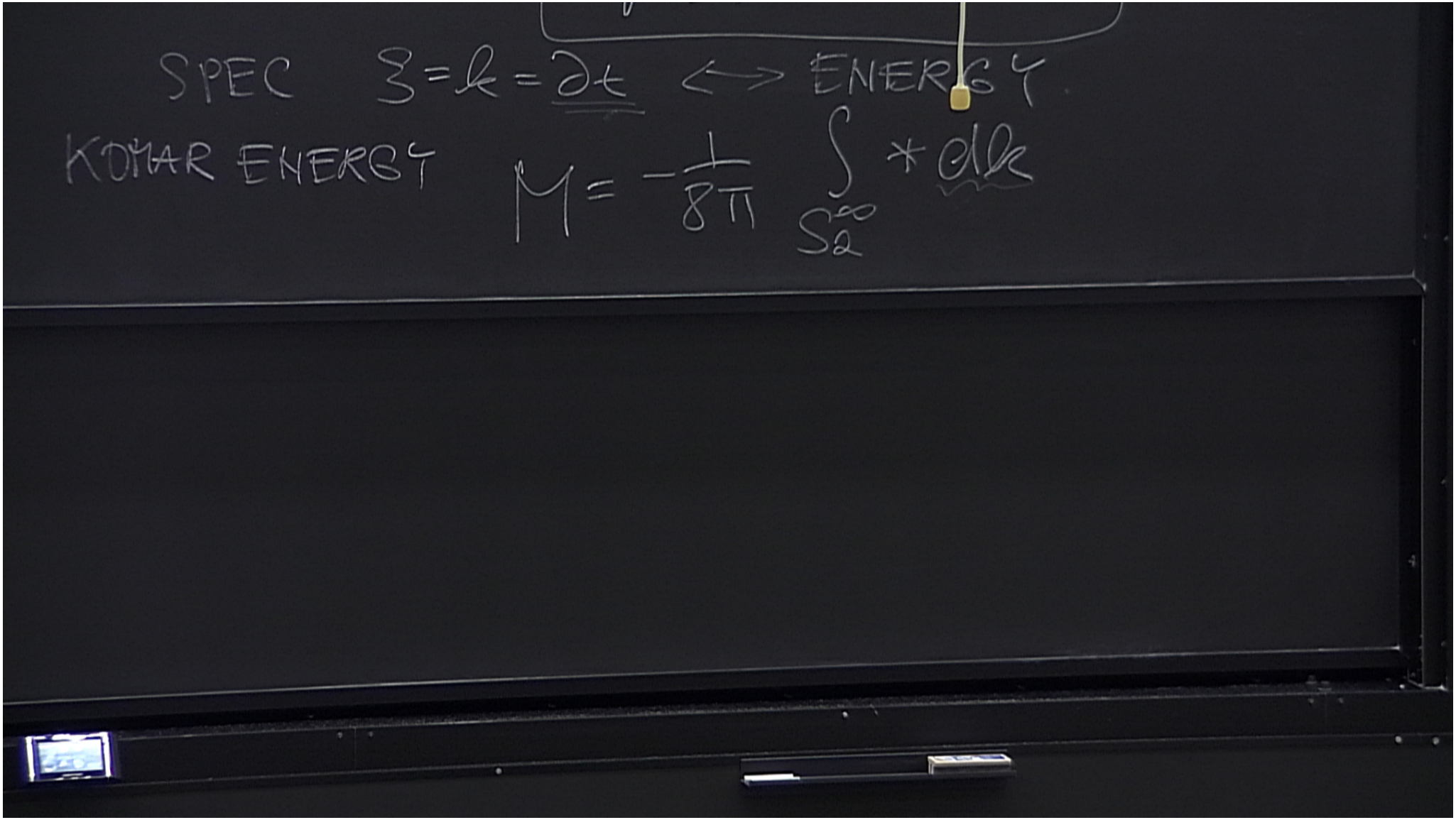

• MOTIVATION : SCHWARZSCHILD
$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$
$f = 1 - \frac{2m}{r}$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

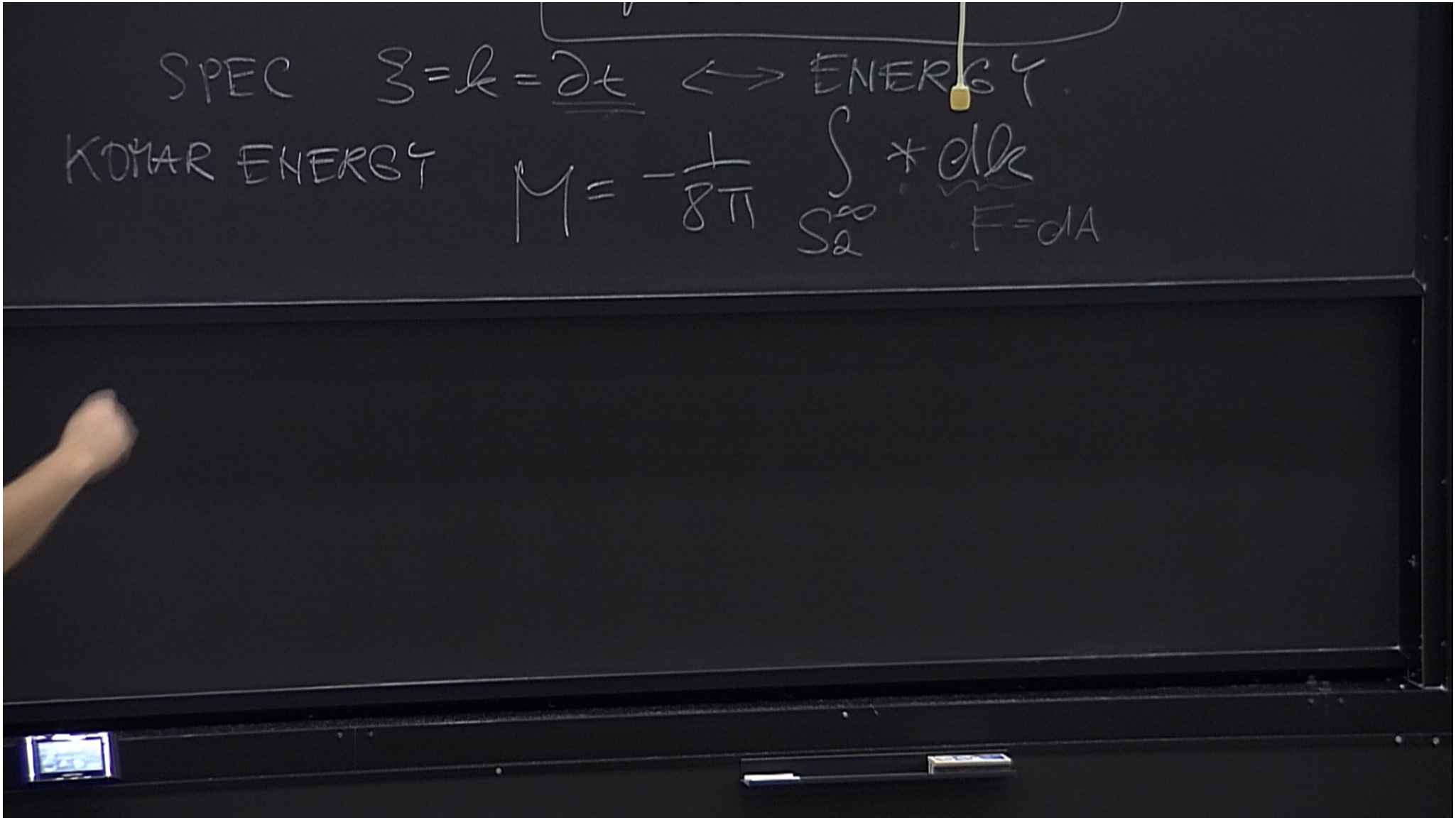

· ASYMPTOTIC MASS
NOETHER'S THEOREM: SYMMETRY ⟷ CONS. Q.

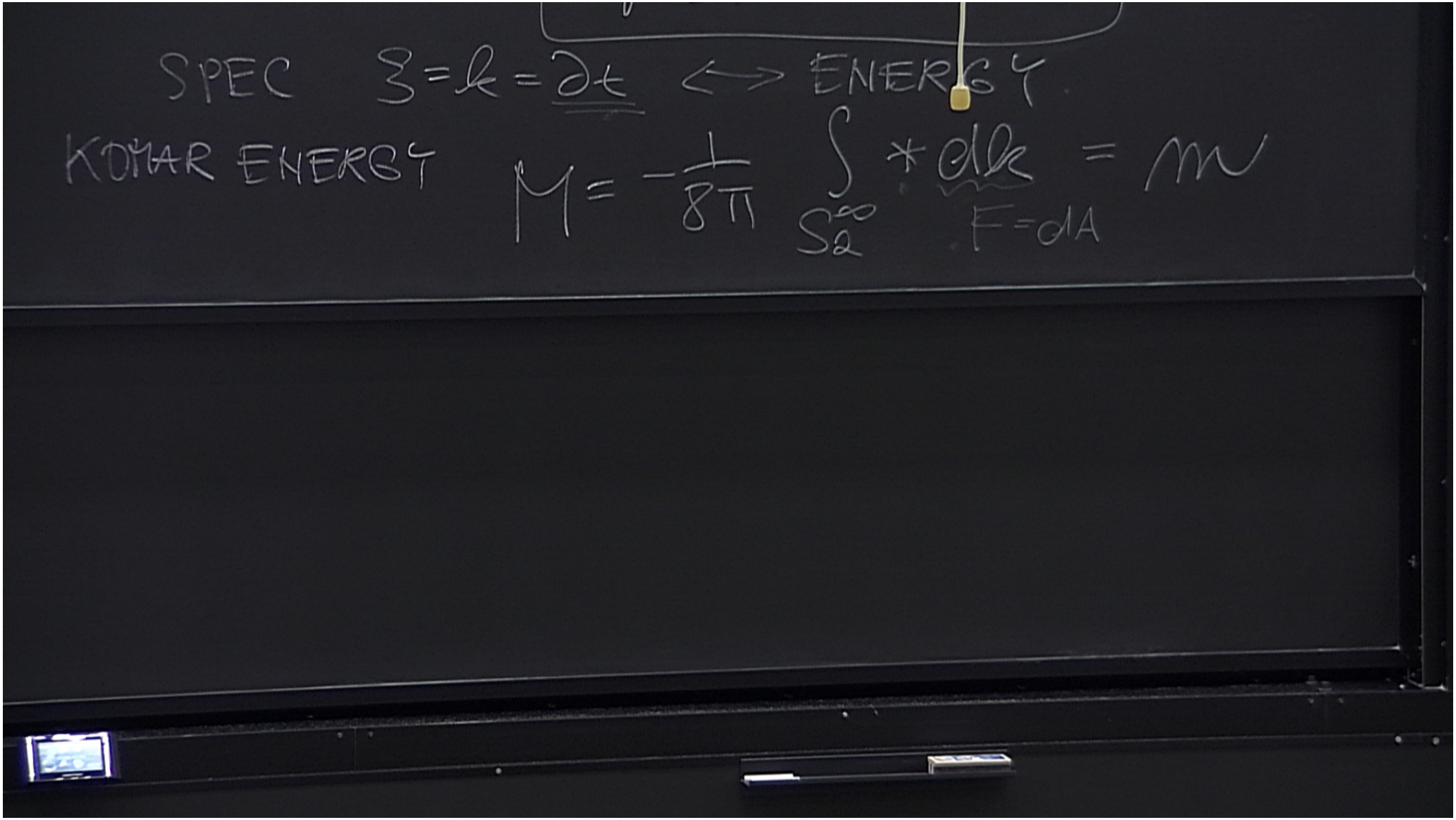

ASYMPTOTIC MASS
NOETHER'S THEOREM: SYMETRY ⟷ CONS. Q.
KILLING FIELD
$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$


NOETHER'S THEOREM:
KILLING FIELD
$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$
SPEC $\xi = k = \partial_t$


NOETHER'S THEOREM:
KILLING FIELD
$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$
SPEC $\xi = k = \partial_t$ ENERGY

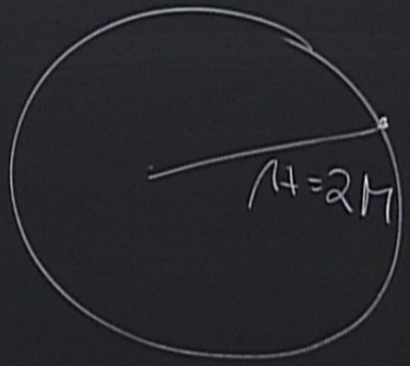

SPEC $\xi = k = \partial t$ $\leftrightarrow$ ENERGY
KOMAR ENERGY $M = -\frac{1}{8\pi} \int_{S^2_\infty} * dk$

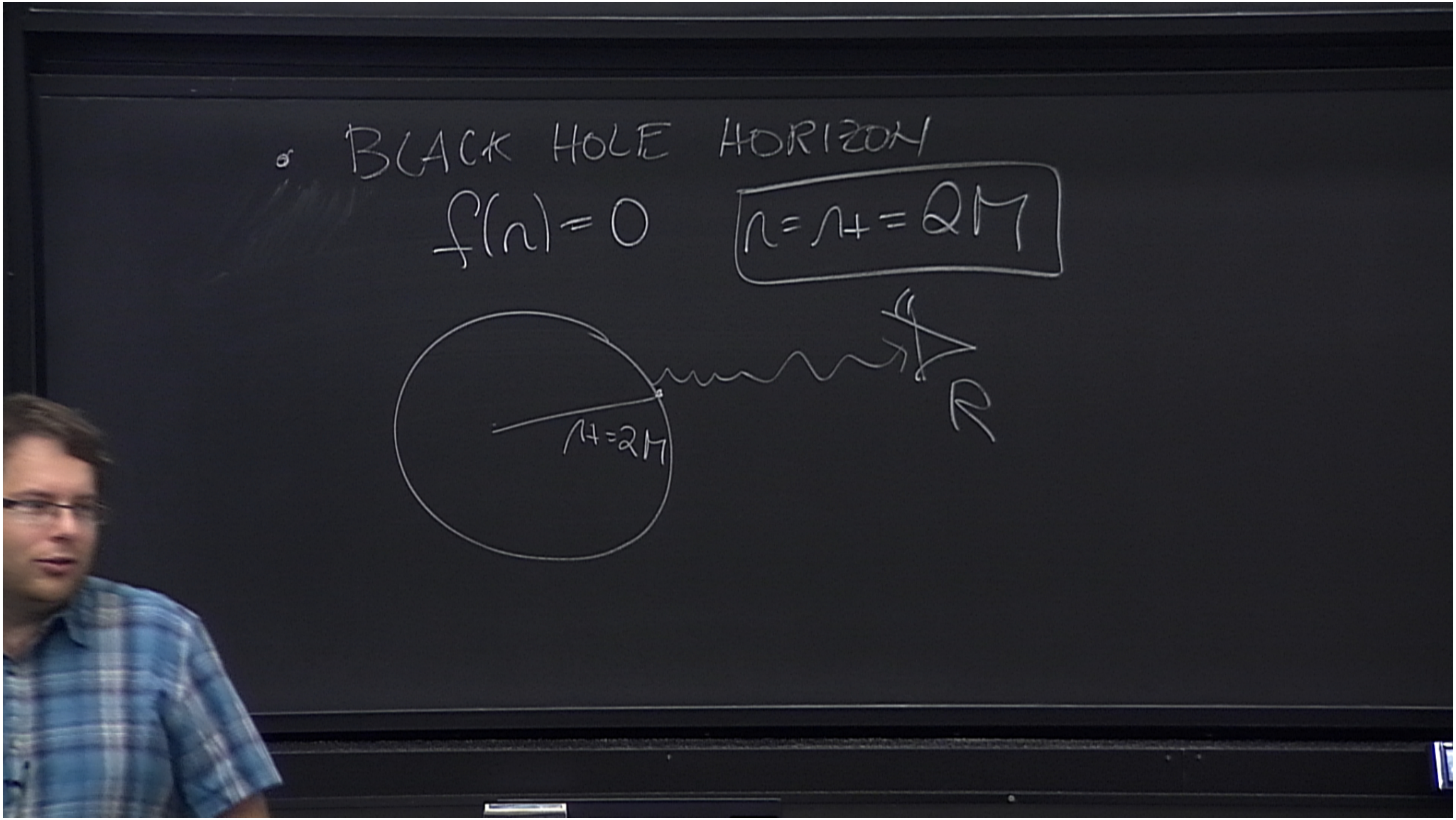

SPEC $\xi = k = \partial t$ $\leftrightarrow$ ENERGY
KOMAR ENERGY $M = -\frac{1}{8\pi} \int_{S^2_\infty} * dk$
$F = dA$

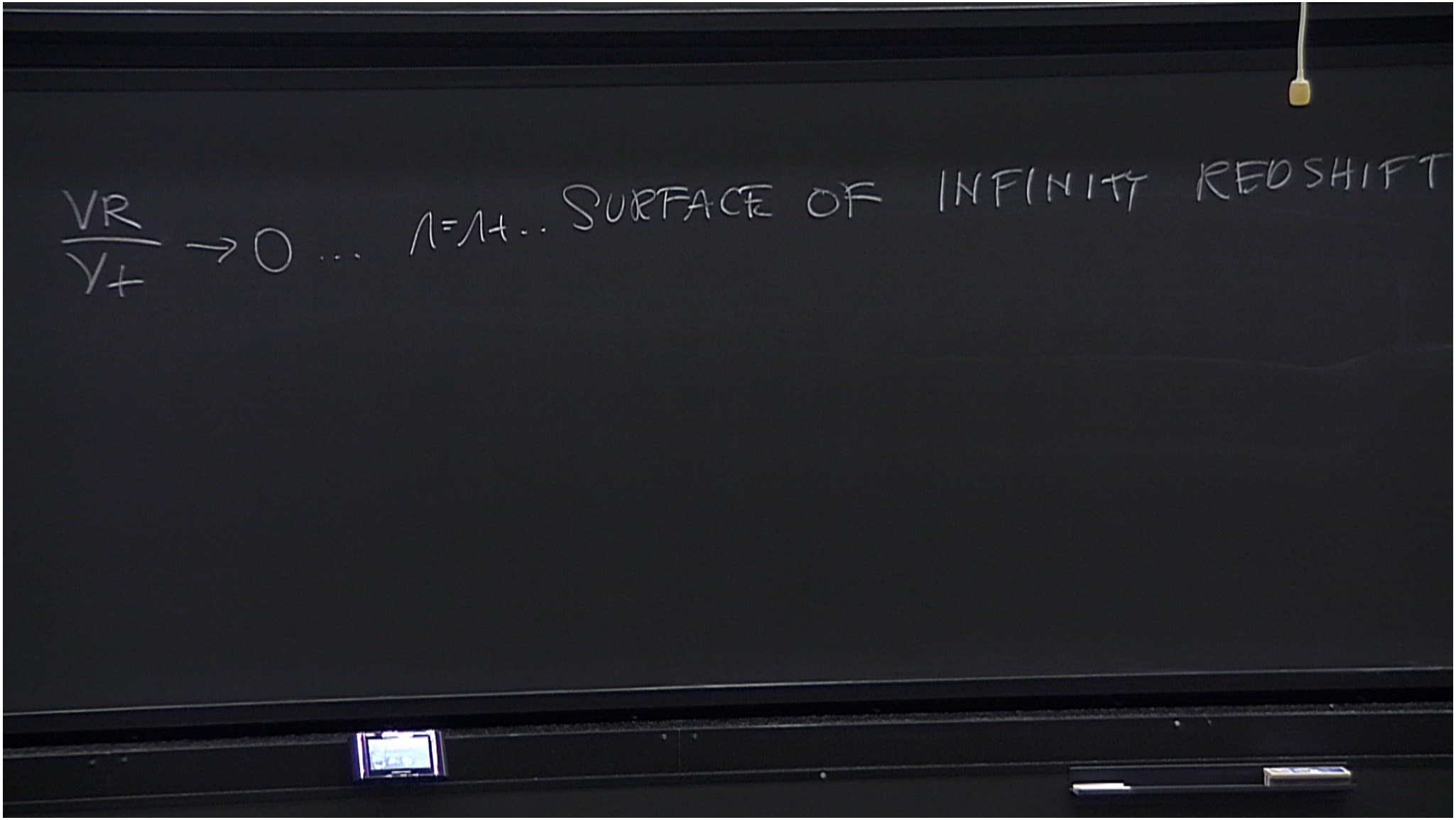

SPEC $\xi = k = \partial t$ $\leftrightarrow$ ENERGY
KOMAR ENERGY $M = -\frac{1}{8\pi} \int_{S^2_\infty} * dk = m$
$F = dA$

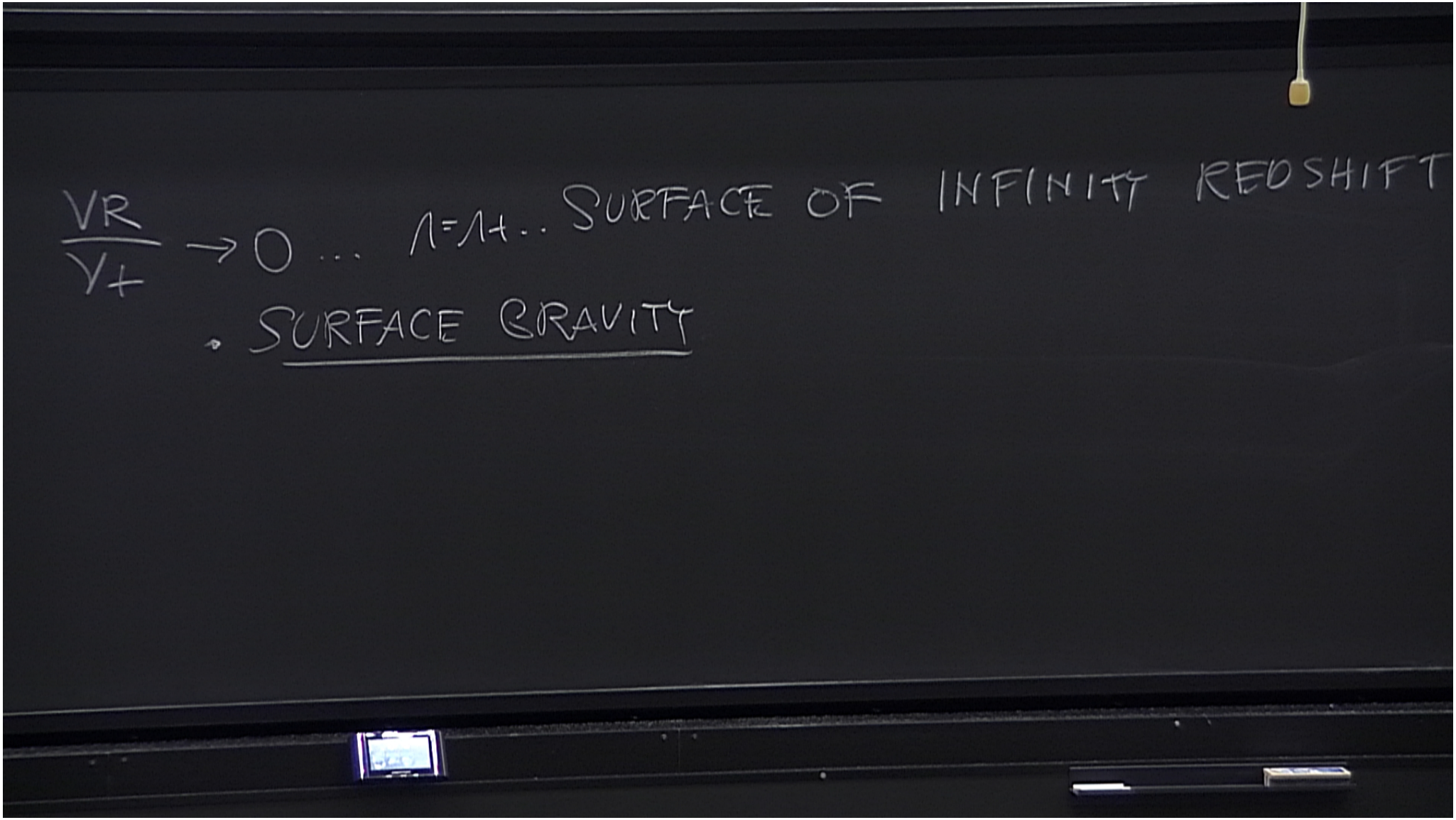
- BLACK HOLE HORIZON

$f(r) = 0$ $\boxed{r = r_+ = 2M}$



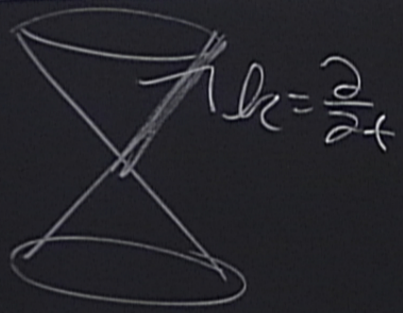

BLACK HOLE HORIZON
f(r) = 0
r = r+ = 2M
r+ = 2M
R


VR/Y+ → 0 ... Λ=Λ+.. SURFACE OF INFINITY REDSHIFT


$\frac{V_R}{V_+} \to 0$ ... $\Lambda = \Lambda_+$ .. SURFACE OF INFINITY REDSHIFT
. SURFACE GRAVITY

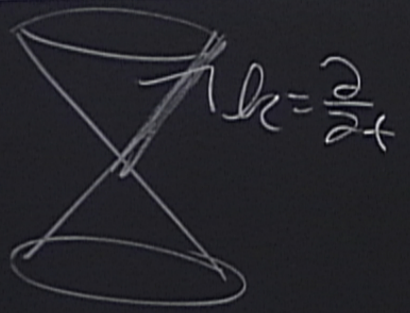
$\Lambda = \Lambda_+$ .. SURFACE OF INFINITY REDSHIFT.

RFACE GRAVITY BH HORIZON $\simeq$ KILLING HORIZON



$\Lambda = \Lambda_+$ .. SURFACE OF INFINITY REDSHIFT.

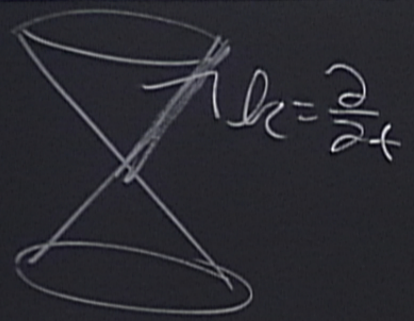
RFACE GRAVITY



BH HORIZON ≃ KILLING HORIZON

NULL SURFACE GENERATED BY KILLING FIEL $k = \frac{2}{2t}$

RFACE GRAVITY

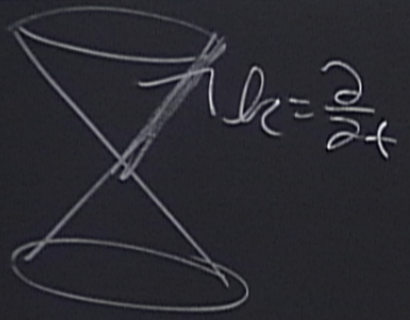


BH HORIZON ≅ KILLING HORIZON

NULL SURFACE GENERATED BY KILLING FIEL $k=\frac{2}{2t}$

$k^\mu \nabla_\mu k^\nu = \varkappa k^\nu$

RFACE GRAVITY

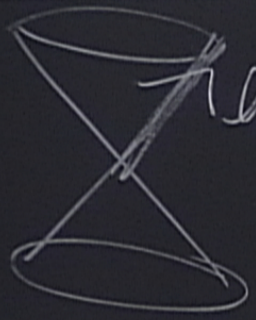


BH HORIZON ≈ KILLING HORIZON

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RFACE GRAVITY



BH HORIZON ≃ KILLING HORIZON

NULL SURFACE GENERATED BY KILLING FIEL $k = \frac{\partial}{\partial t}$

$k^\mu \nabla_\mu k^\nu = \kappa k^\nu$ SURFACE GRAVITY

CAN BE SHOWN:

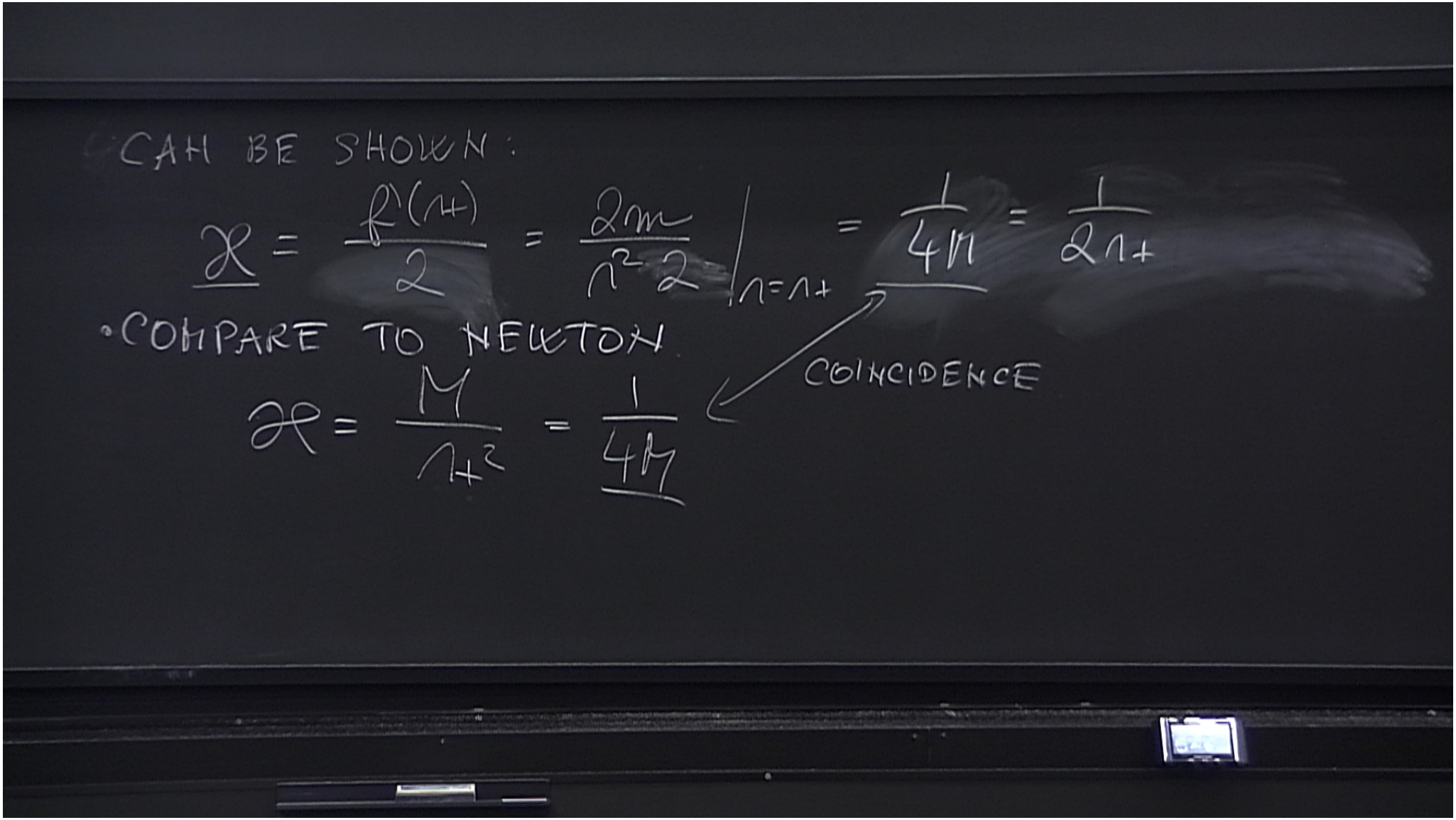
$$\chi = \frac{f'(n_+)}{2} = \frac{2m}{n^2 2}\Big|_{n=n_+} = \frac{1}{4n} = \frac{1}{2n_+}$$

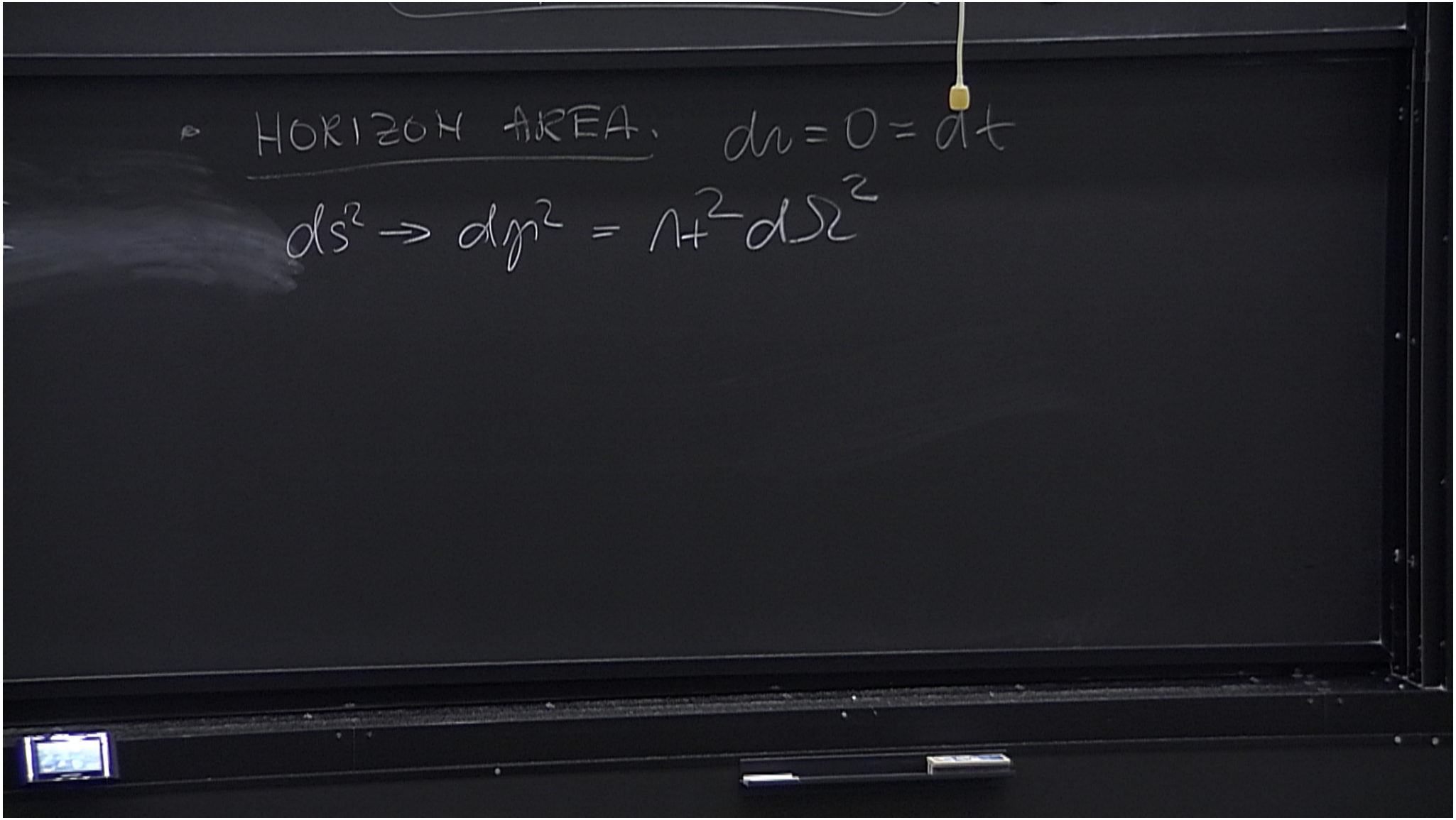
CAN BE SHOWN:

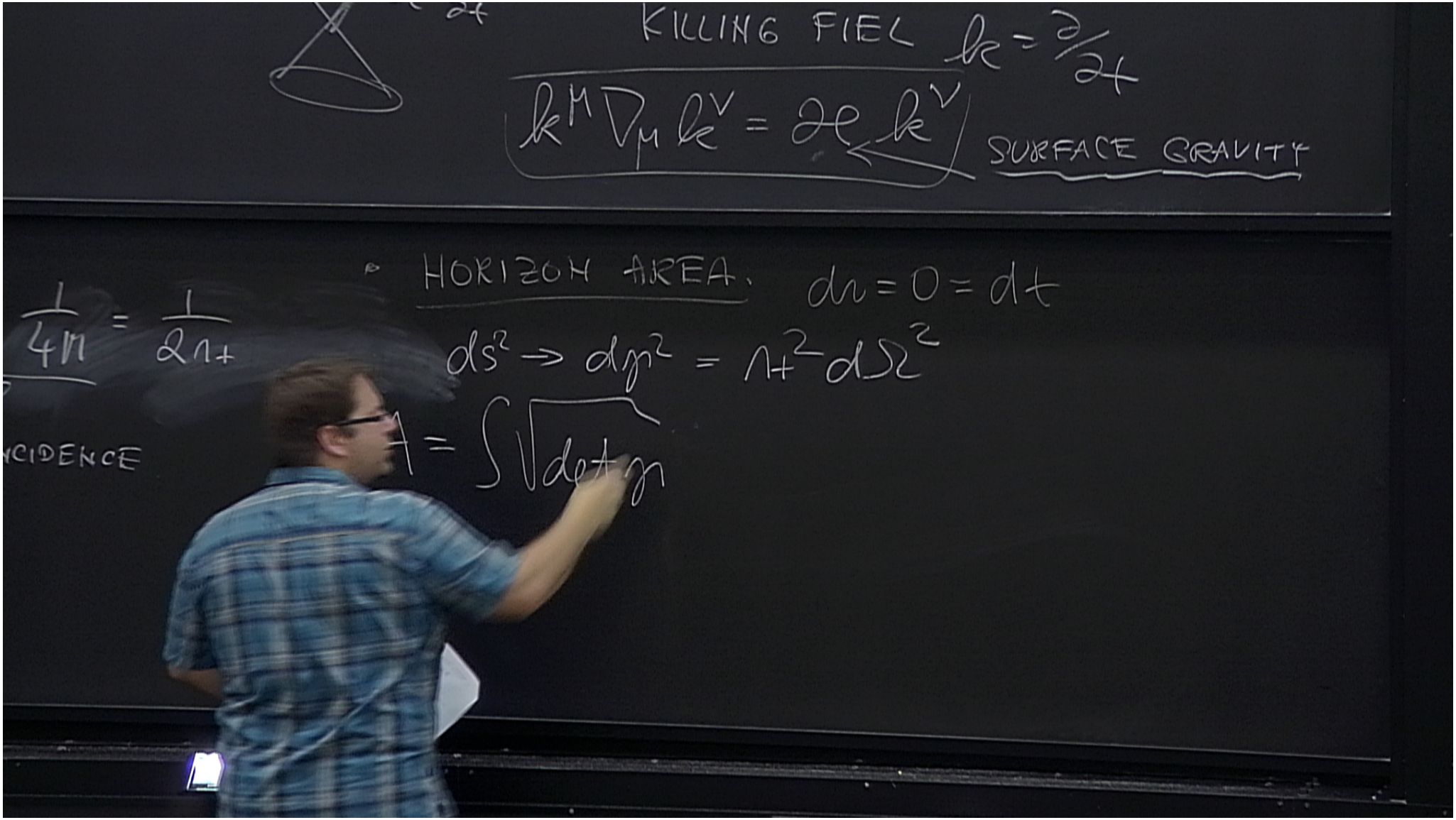
$$\kappa = \frac{f'(r_+)}{2} = \frac{2m}{r^2 2}\Big|_{r=r_+} = \frac{1}{4M} = \frac{1}{2r_+}$$

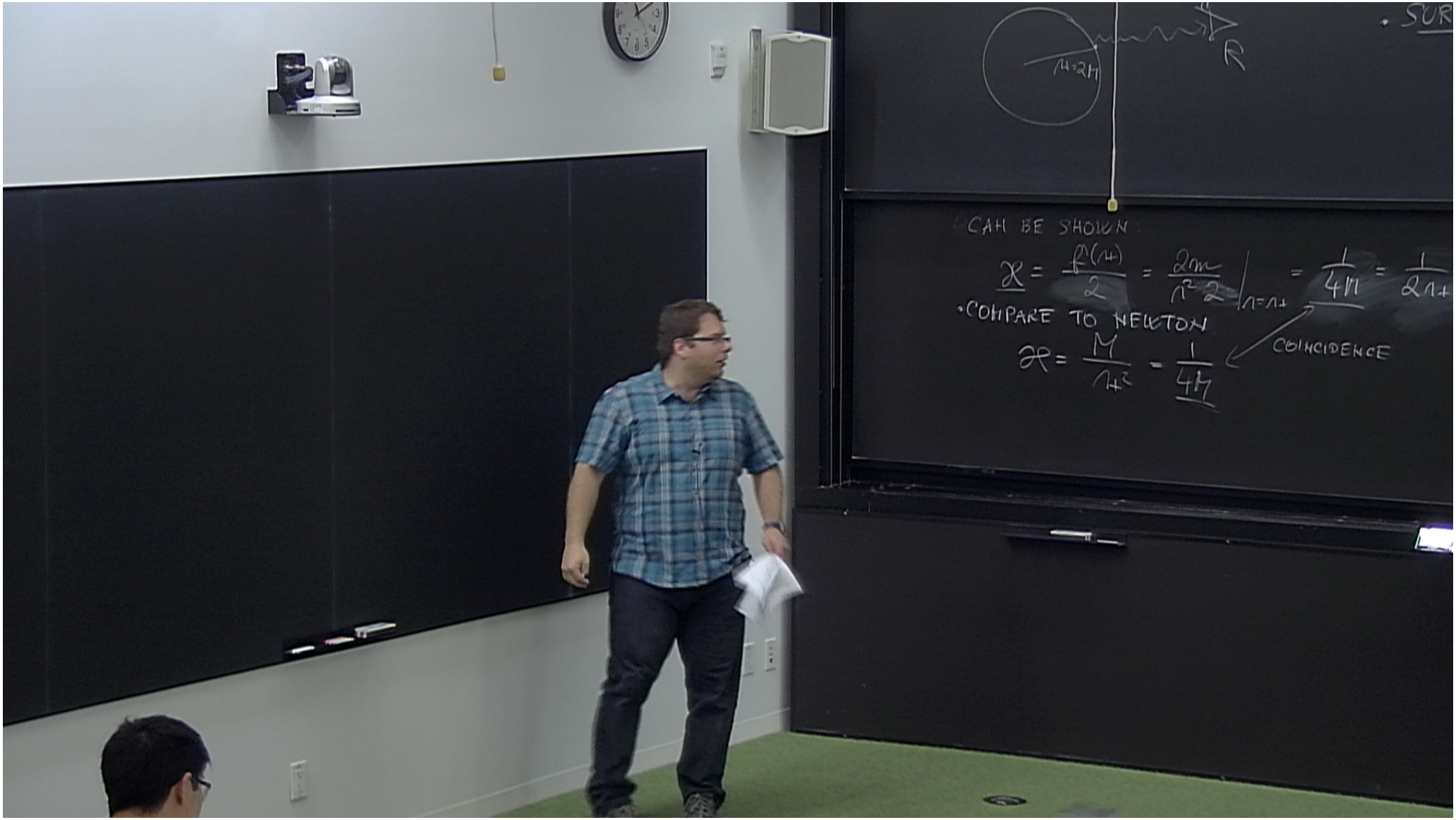
- COMPARE TO NEWTON

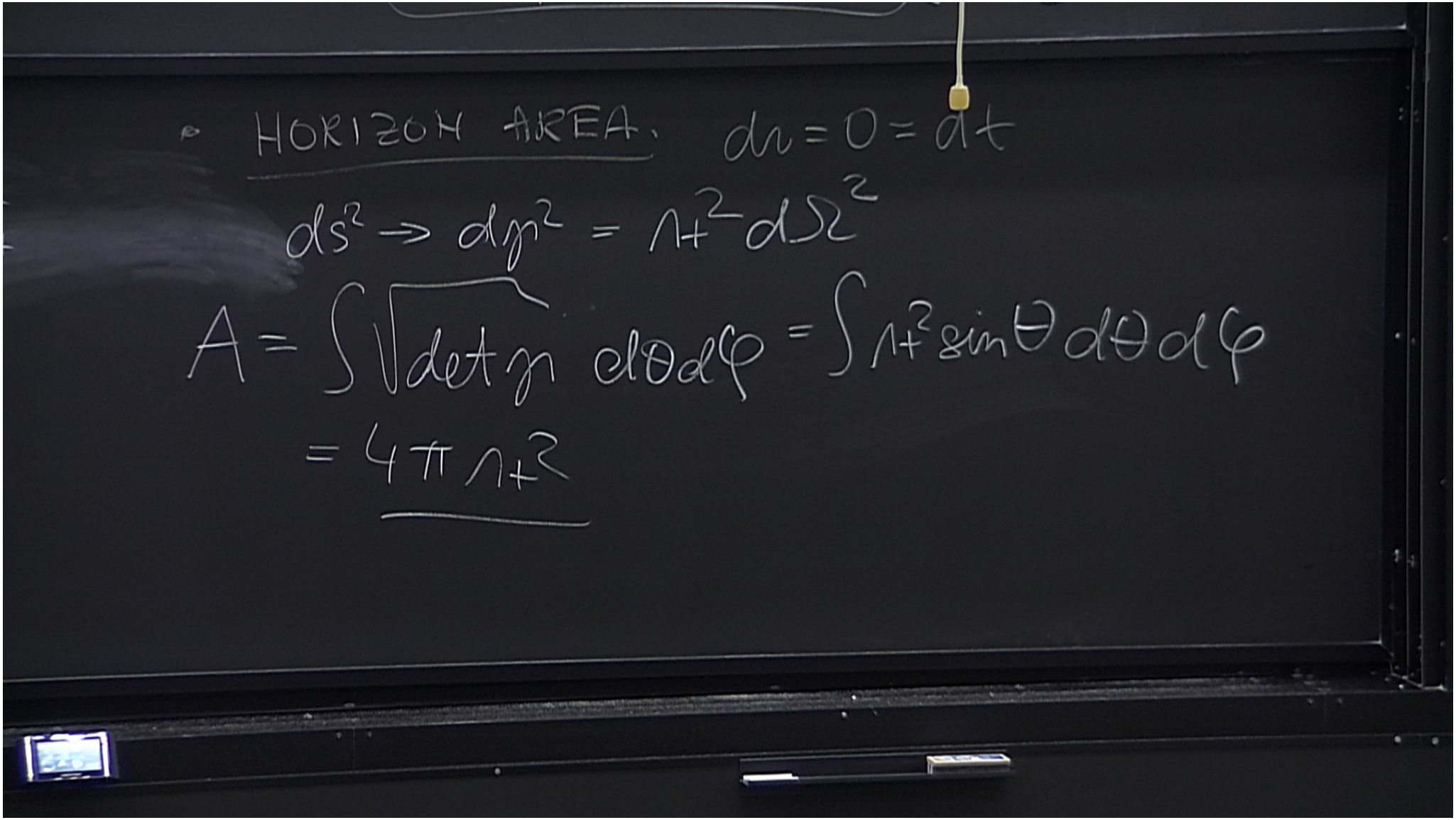
$$\kappa = \frac{M}{r_+^2} = \frac{1}{4M}$$

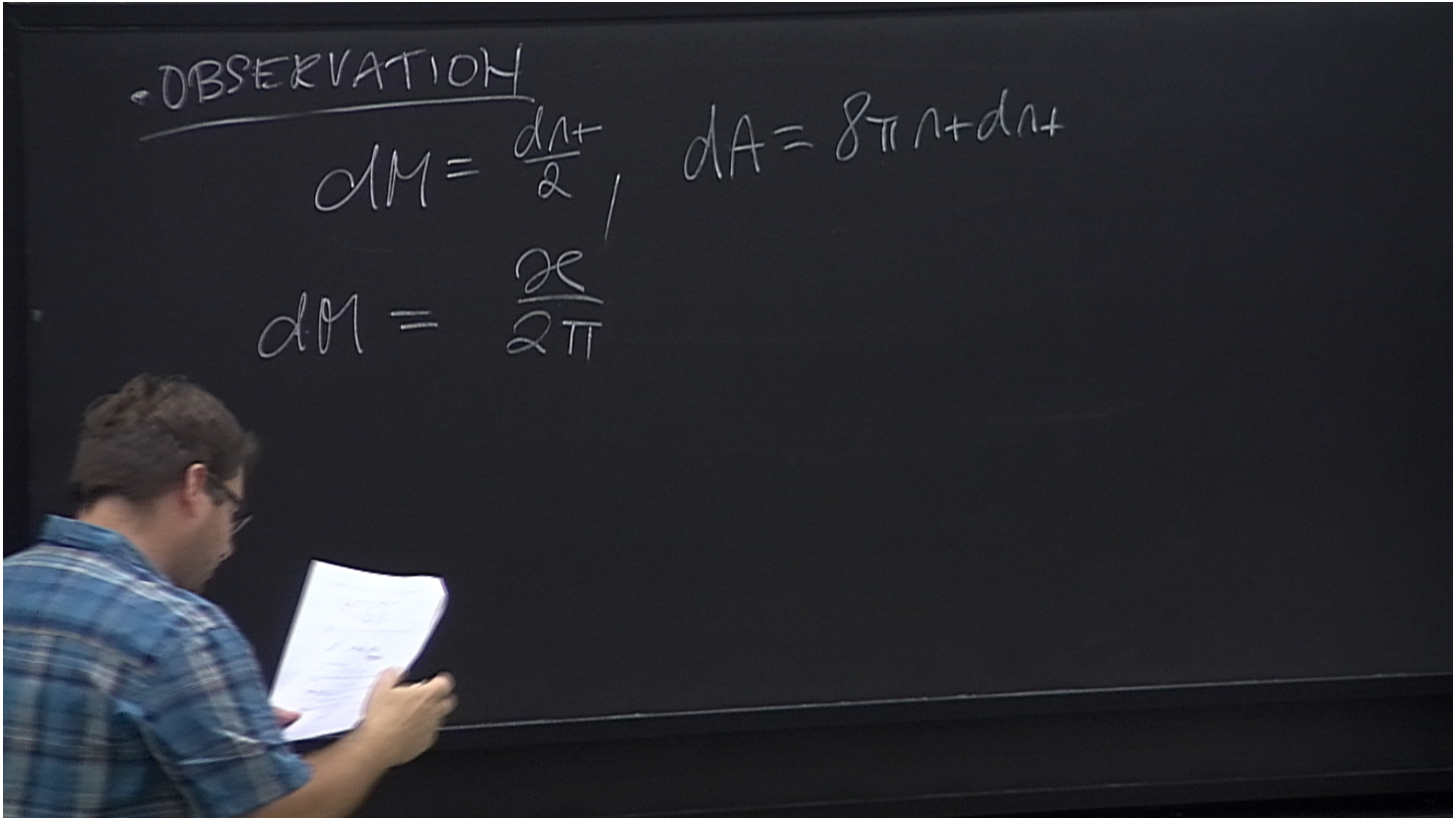

CAN BE SHOWN:
$\kappa = \frac{f'(r_+)}{2} = \frac{2m}{r^2 2}\Big|_{r=r_+} = \frac{1}{4M} = \frac{1}{2r_+}$
• COMPARE TO NEWTON
$\kappa = \frac{M}{r_+^2} = \frac{1}{4M}$
COINCIDENCE


HORIZON AREA. $dr = 0 = dt$
$ds^2 \Rightarrow dy^2 = r_+^2 d\Omega^2$


KILLING FIEL
$k = \partial/\partial t$
$k^\mu \nabla_\mu k^\nu = \kappa k^\nu$
SURFACE GRAVITY
HORIZON AREA.
$dr = 0 = dt$
$ds^2 \to dy^2 = r_+^2 d\Omega^2$
$A = \int \sqrt{\det g}$
$\frac{1}{4\pi} = \frac{1}{2r_+}$
NCIDENCE


R
CAN BE SHOWN
COMPARE TO NEWTON
COINCIDENCE


HORIZON AREA. $dr = 0 = dt$
$ds^2 \to dy^2 = r_+^2 d\Omega^2$
$A = \int \sqrt{\det \gamma}\, d\theta d\varphi = \int r_+^2 \sin\theta\, d\theta d\varphi$
$= 4\pi r_+^2$


OBSERVATION
dM = dΛ+ / 2
dA = 8π Λ+ dΛ+
dM = dE / 2π

$$dM = \frac{d\Lambda_+}{2}, \quad dA = 8\pi \Lambda_+ d\Lambda_+$$

$$\boxed{dM = \frac{\partial \varepsilon}{2\pi} \frac{dA}{4}}$$

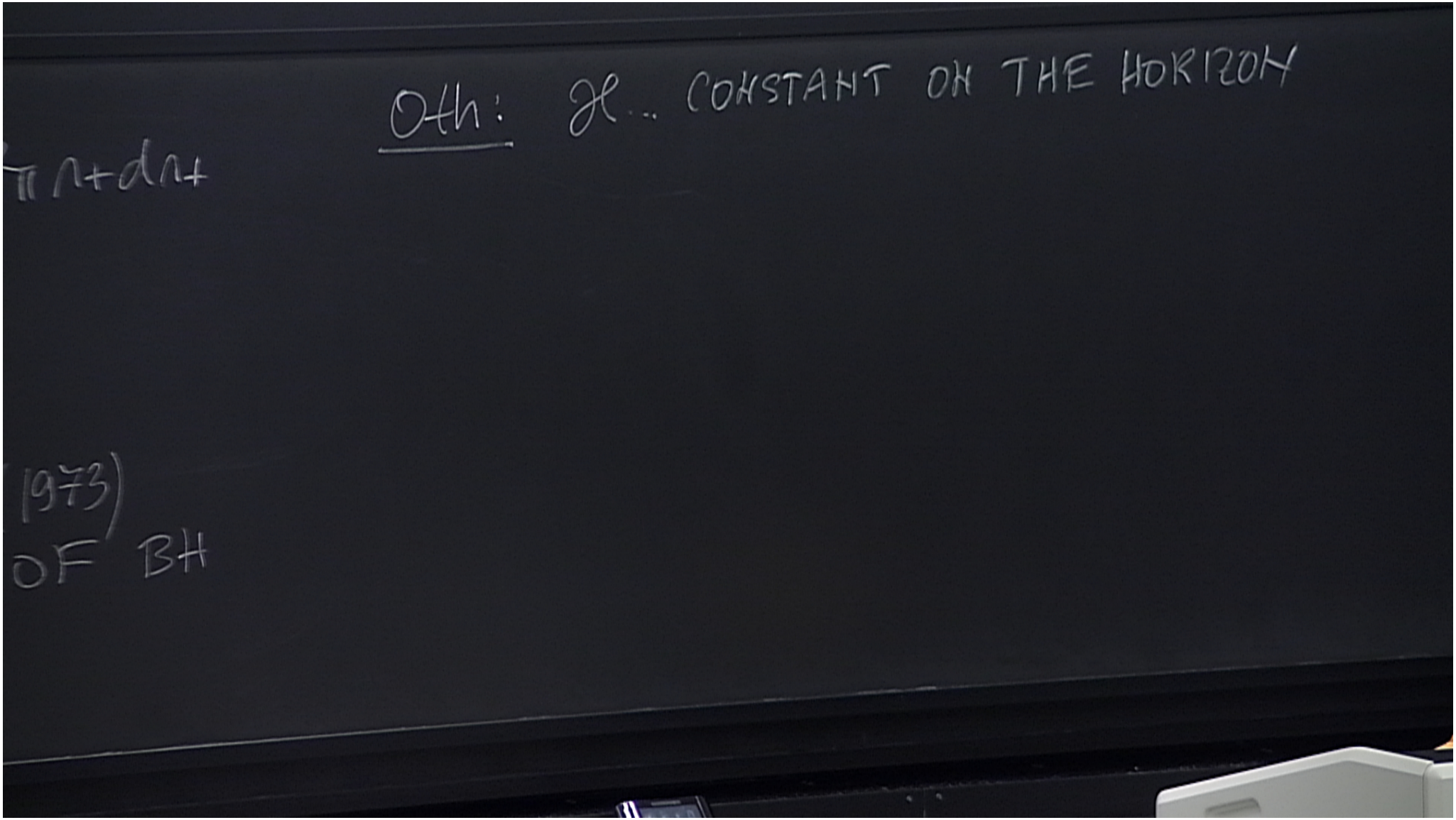
$$dM = \frac{\partial \varepsilon}{2\pi} \frac{dA}{4}$$

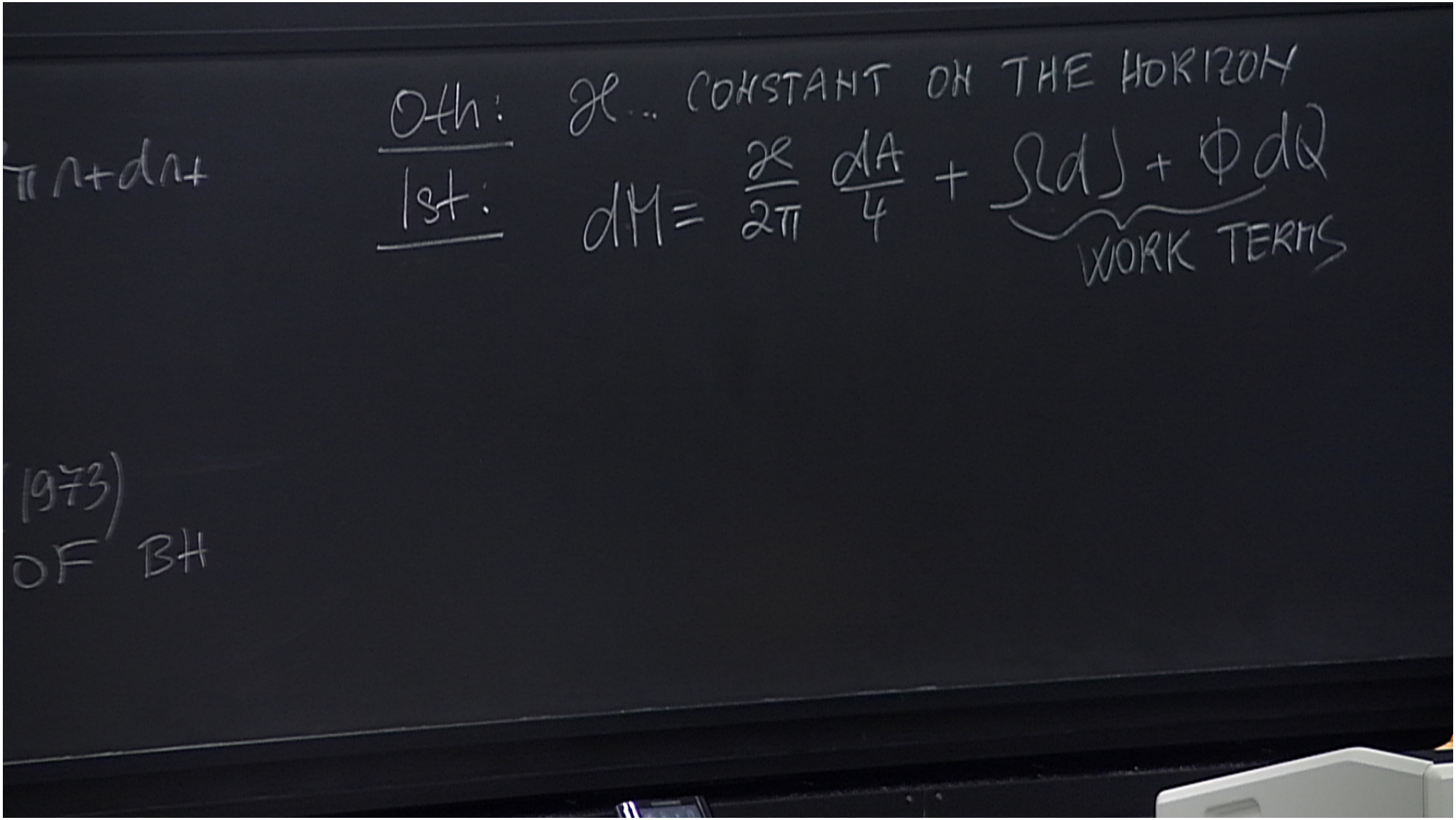
ii) BLACK HOLE MECHANICS

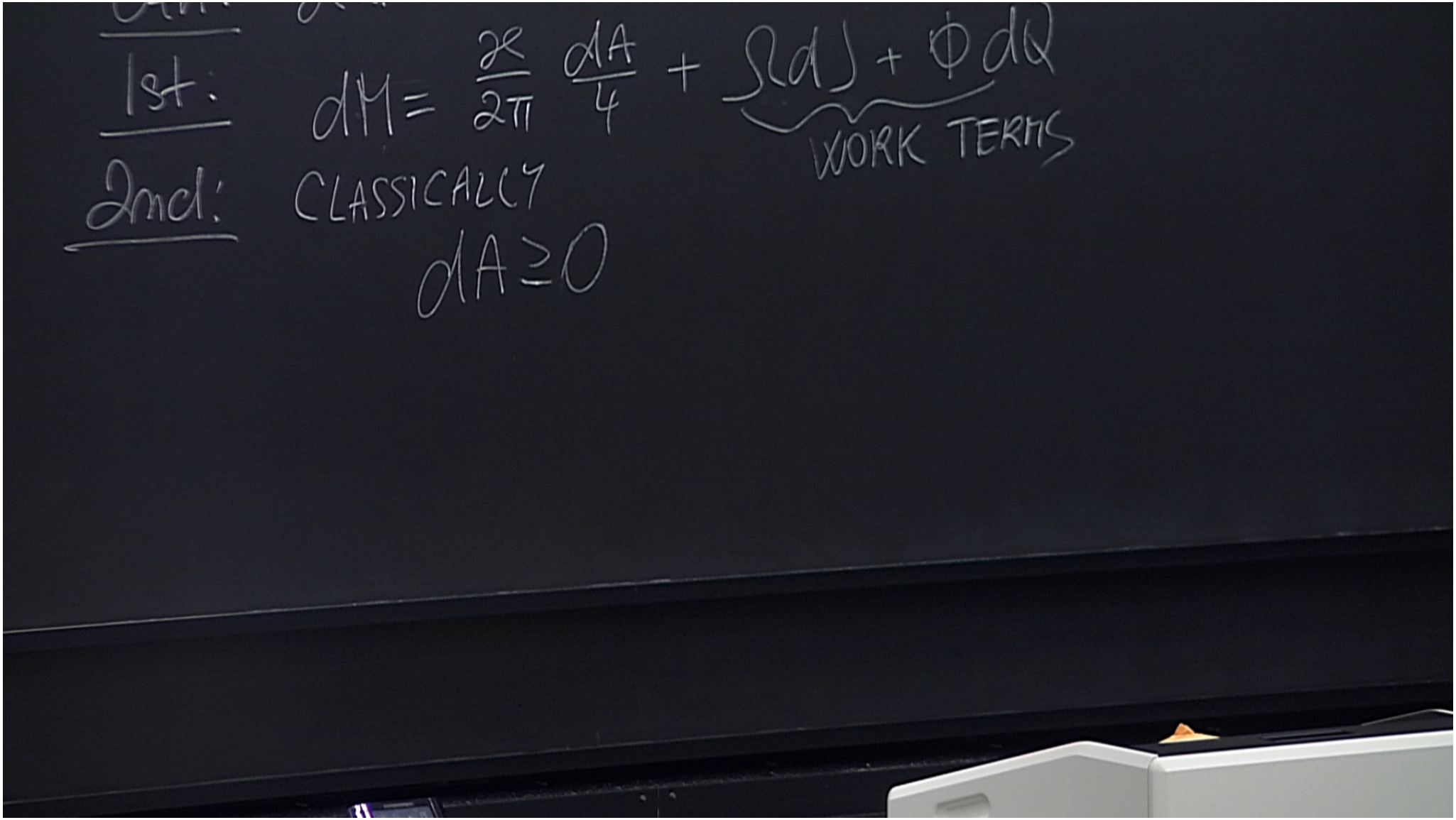
$\alpha\nu = \frac{2\pi}{4}$

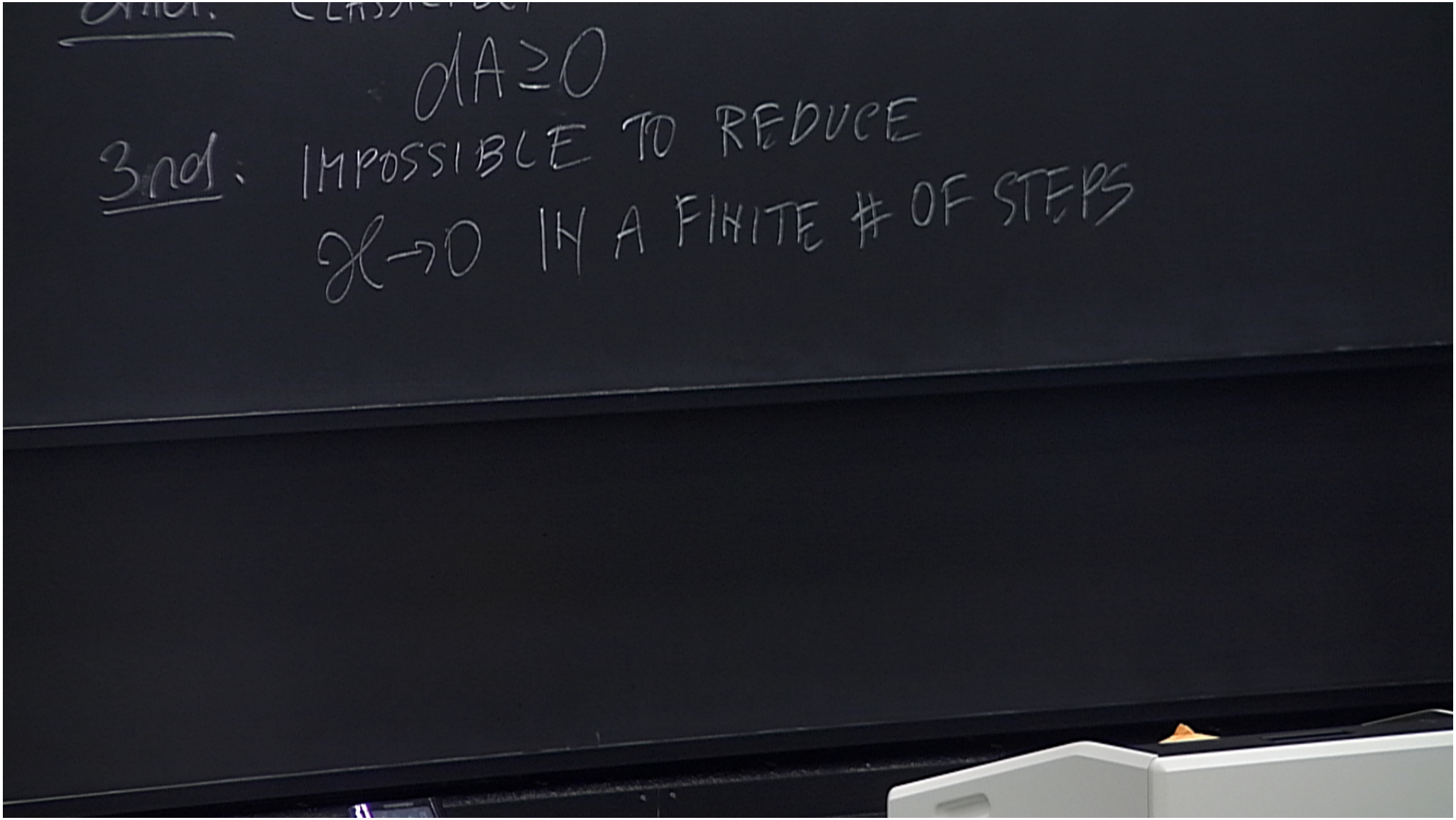
ii) BLACK HOLE MECHANICS (1973)

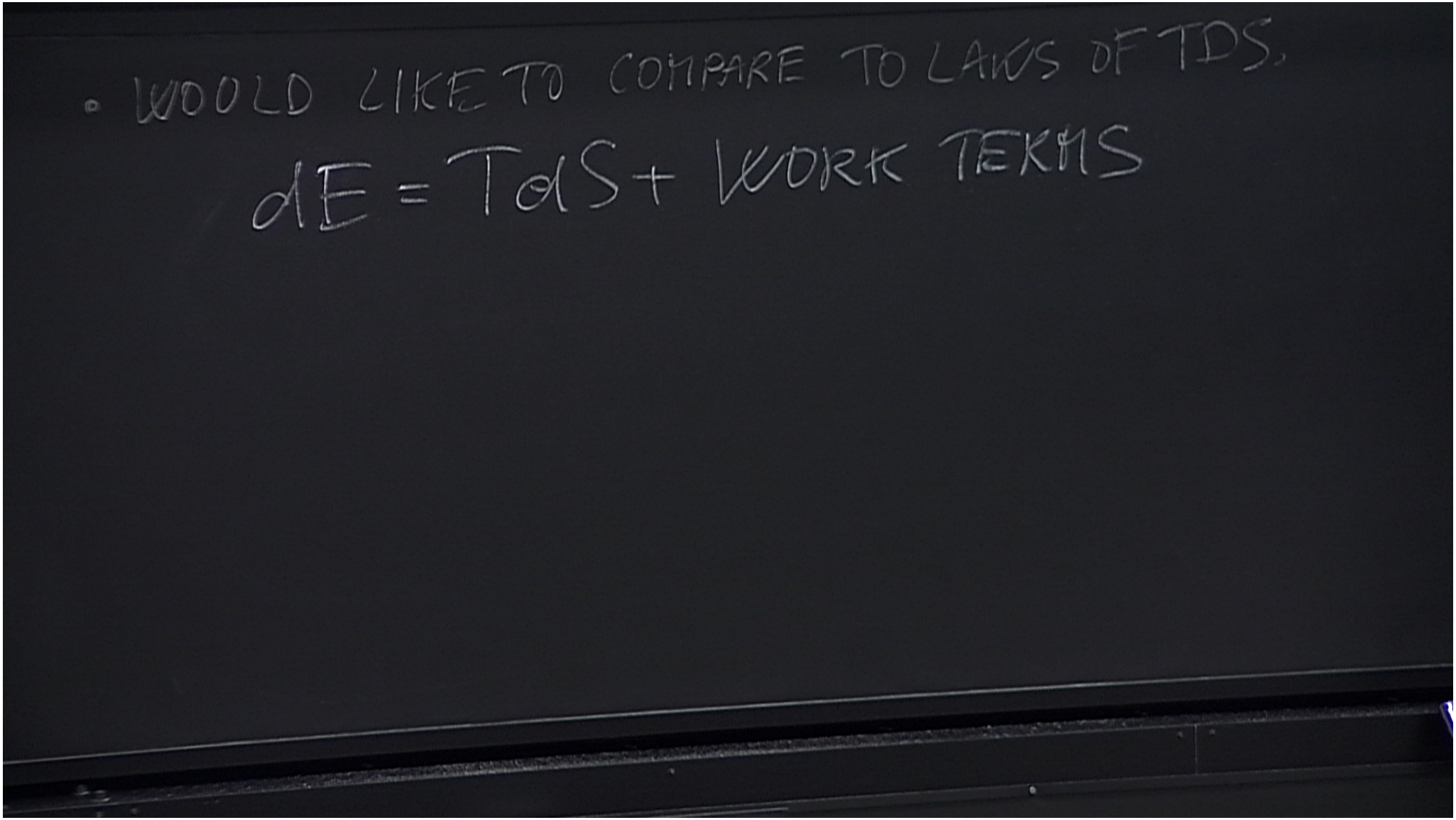
FORMULATED 4 LAWS OF BH MECHANICS.

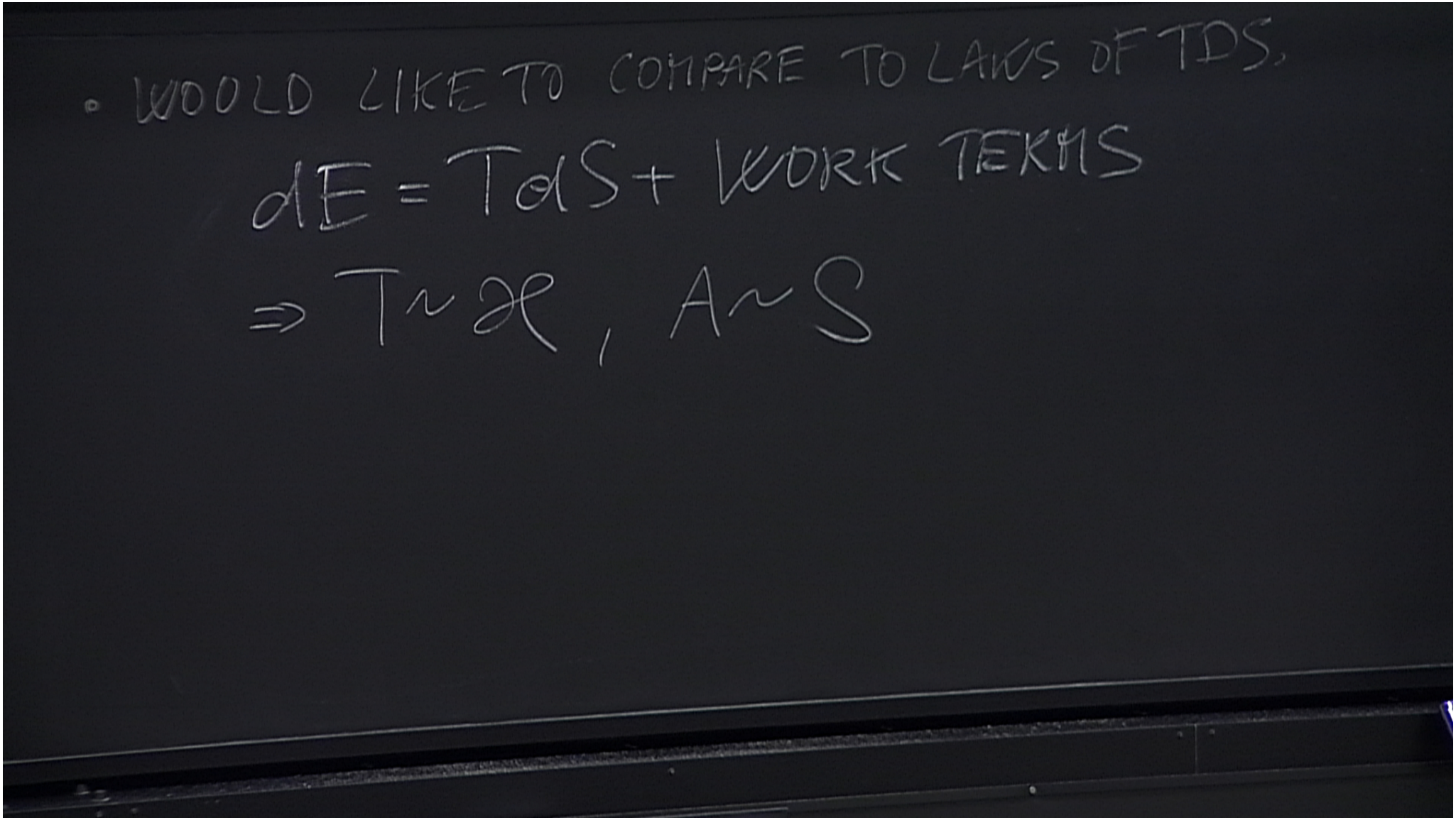

0th: $\mathcal{H}$ .. CONSTANT ON THE HORIZON
$\pi\Lambda + d\Lambda$
1973)
OF BH

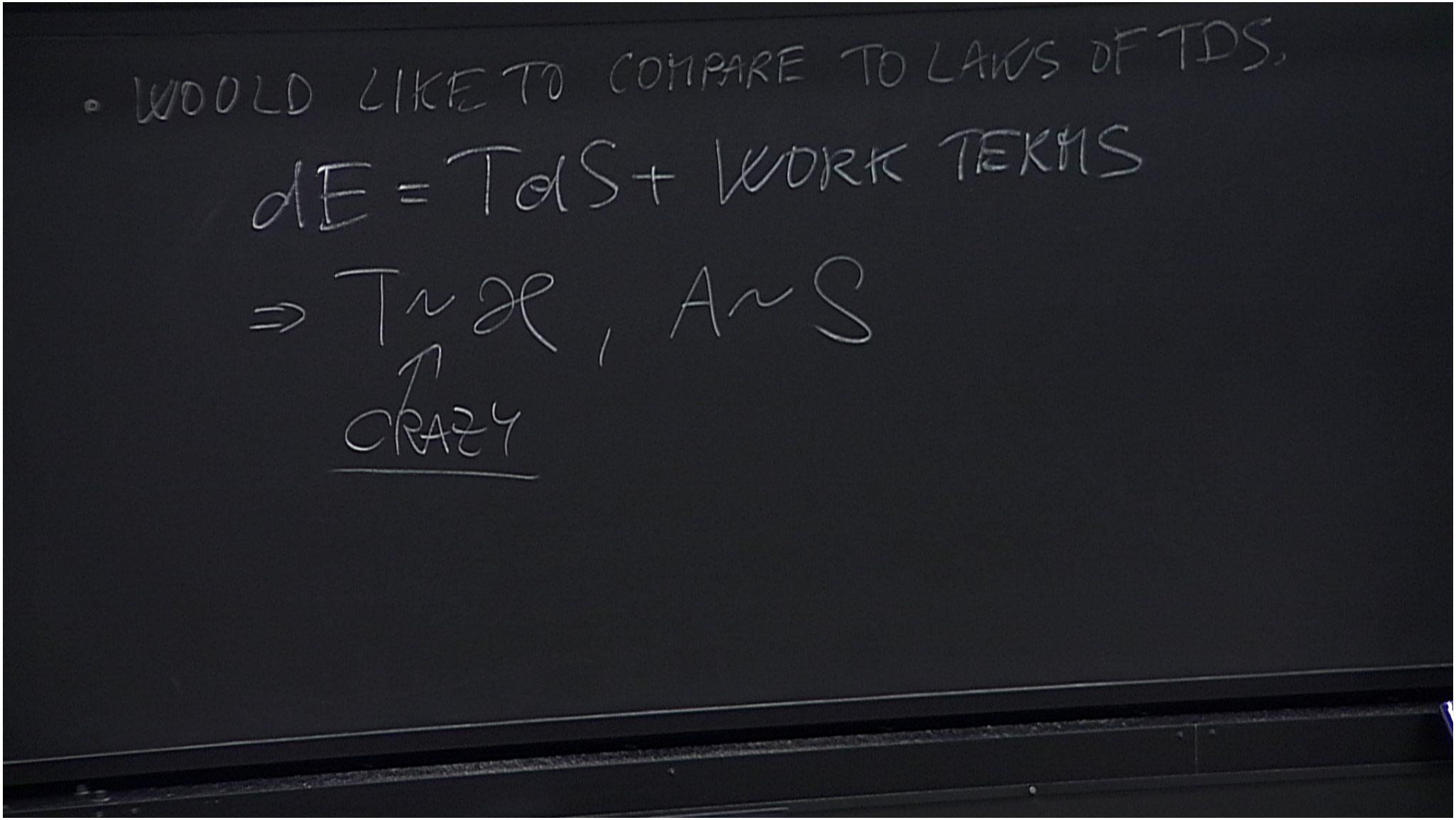

0th: $\mathcal{K}$ .. CONSTANT ON THE HORIZON
1st: $dM = \frac{\mathcal{K}}{2\pi}\frac{dA}{4} + \Omega dJ + \Phi dQ$
WORK TERMS
$\pi\Lambda + d\Lambda$
(973)
OF BH

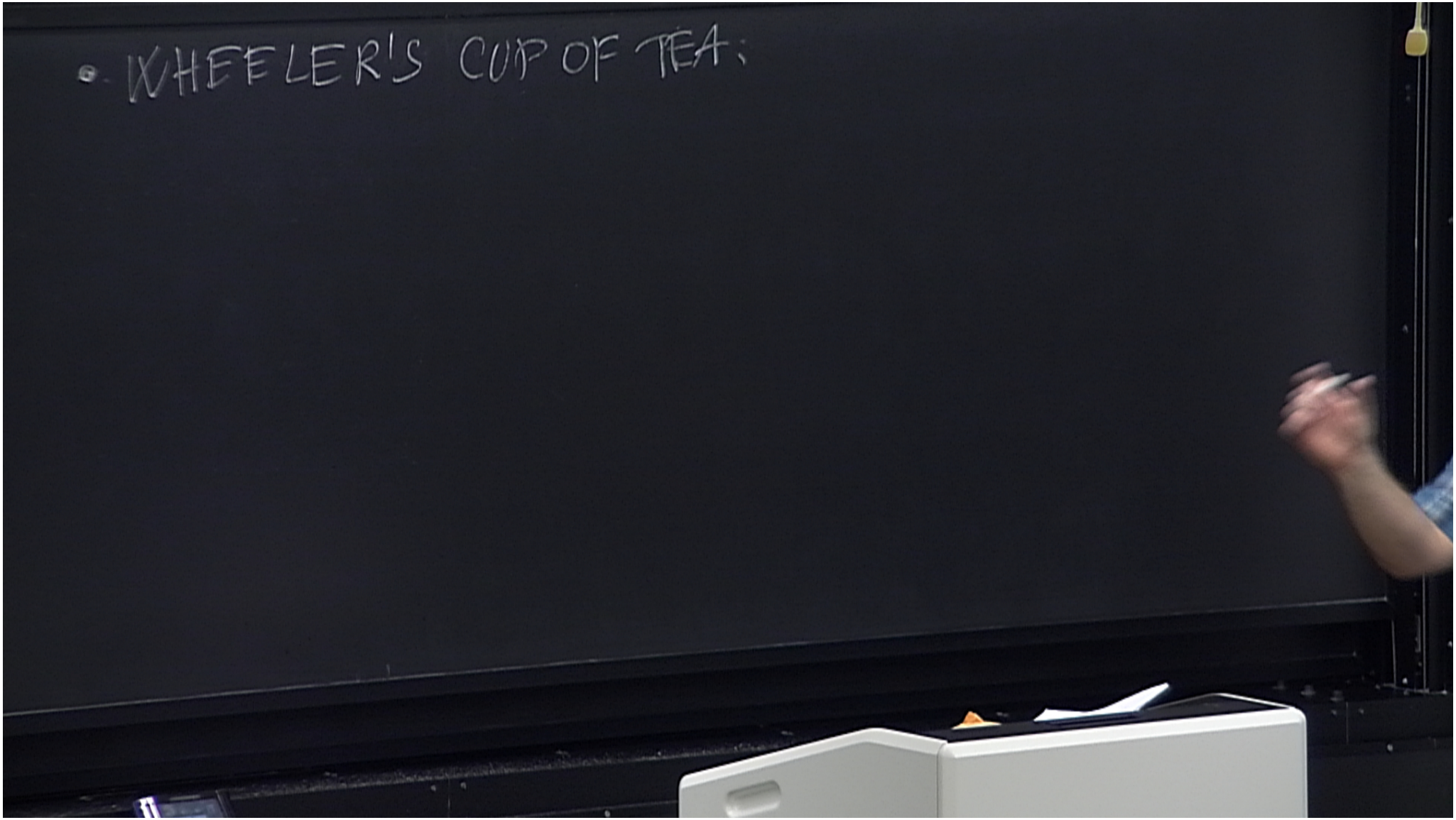

1st:
$dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \Omega dJ + \Phi dQ$
WORK TERMS
2nd:
CLASSICALLY
$dA \geq 0$

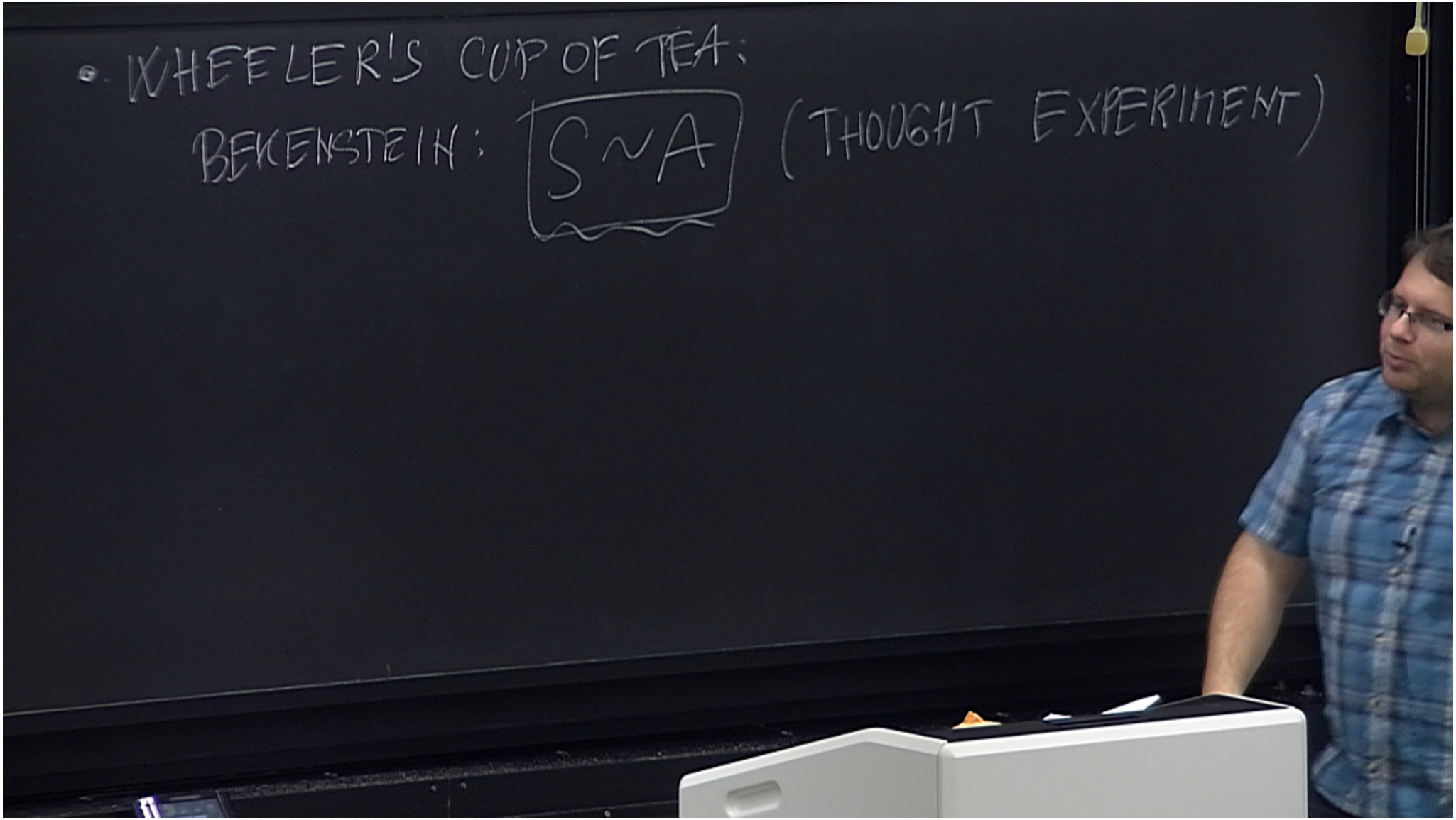

dA ≥ 0
3rd: IMPOSSIBLE TO REDUCE
$\mathcal{A} \to 0$ IN A FINITE # OF STEPS

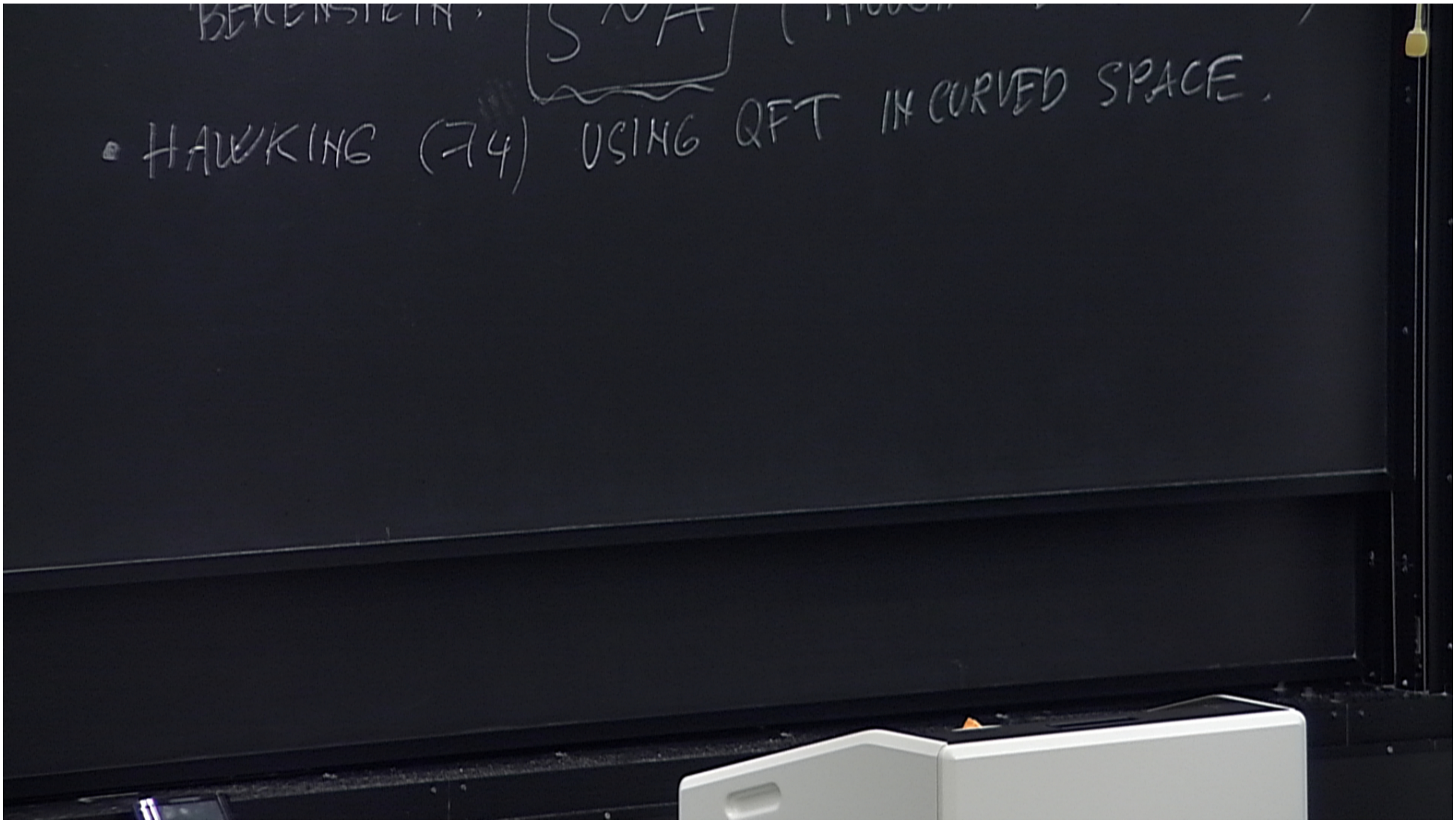

• WOULD LIKE TO COMPARE TO LAWS OF TDS.
dE = TdS + WORK TERMS

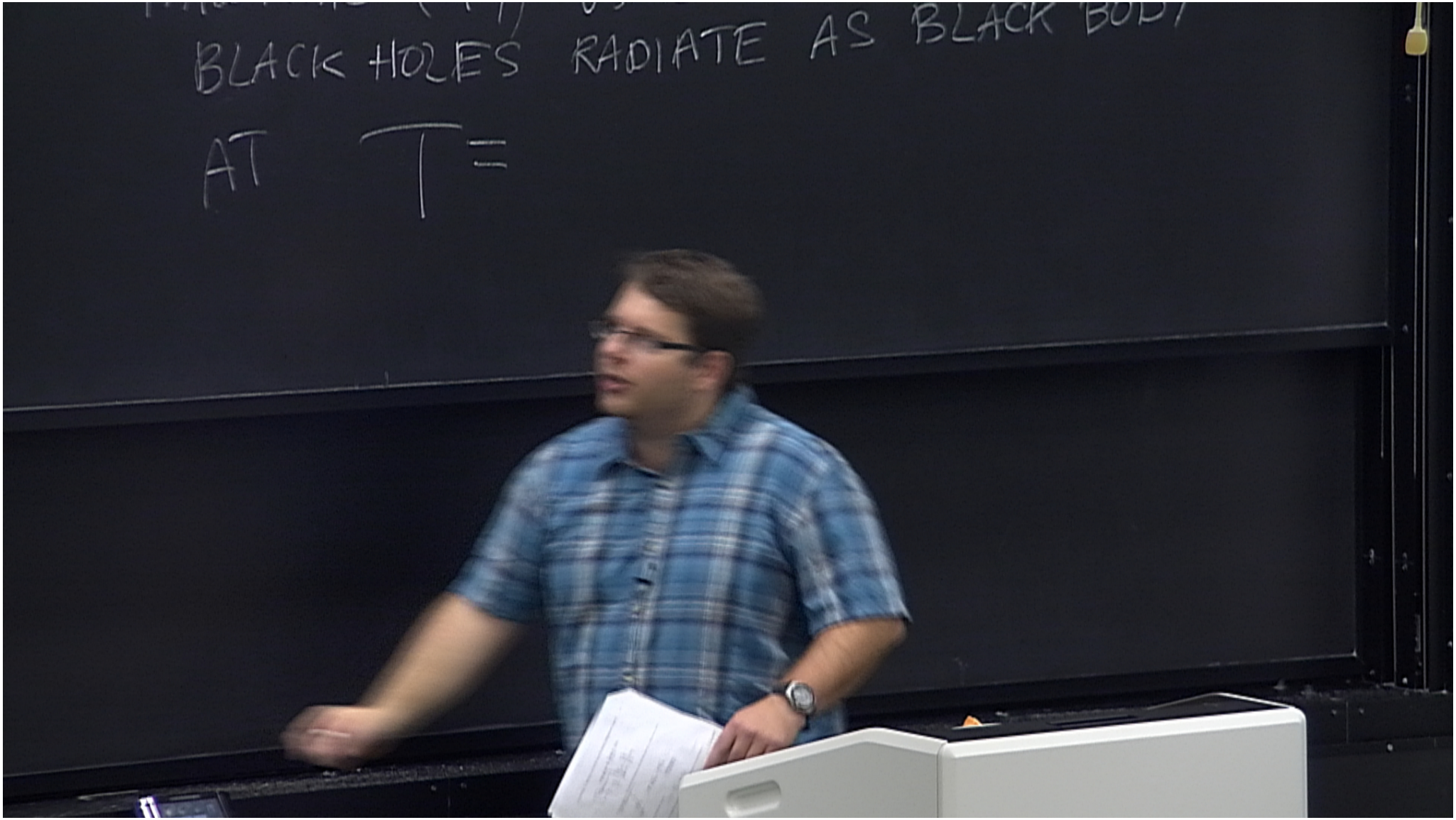

• WOULD LIKE TO COMPARE TO LAWS OF TDS.
dE = TdS + WORK TERMS
⇒ T ~ ∂ℛ, A ~ S

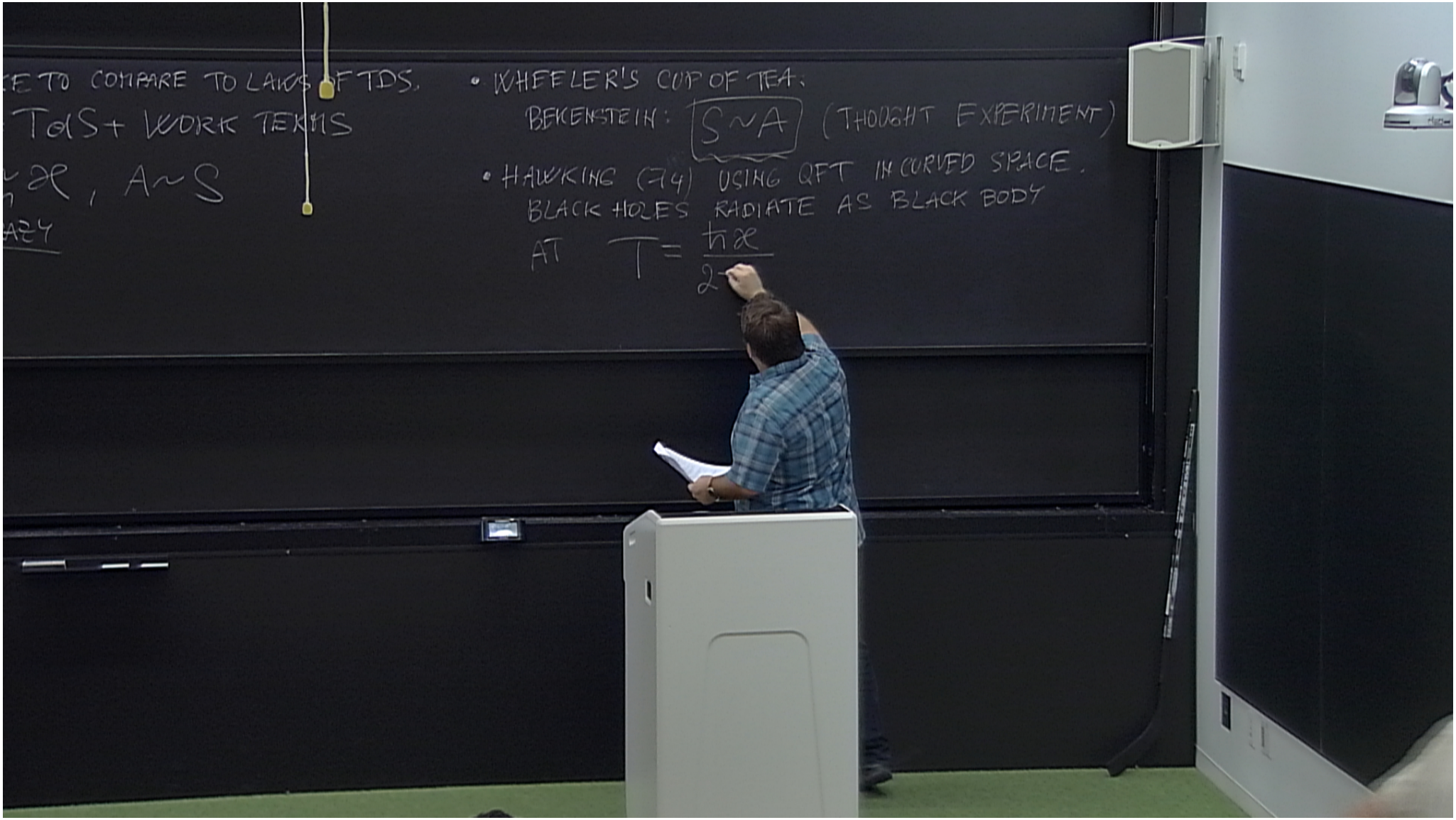

• WOULD LIKE TO COMPARE TO LAWS OF TDS.
dE = TdS + WORK TERMS
⇒ T ~ ∂E , A ~ S
CRAZY

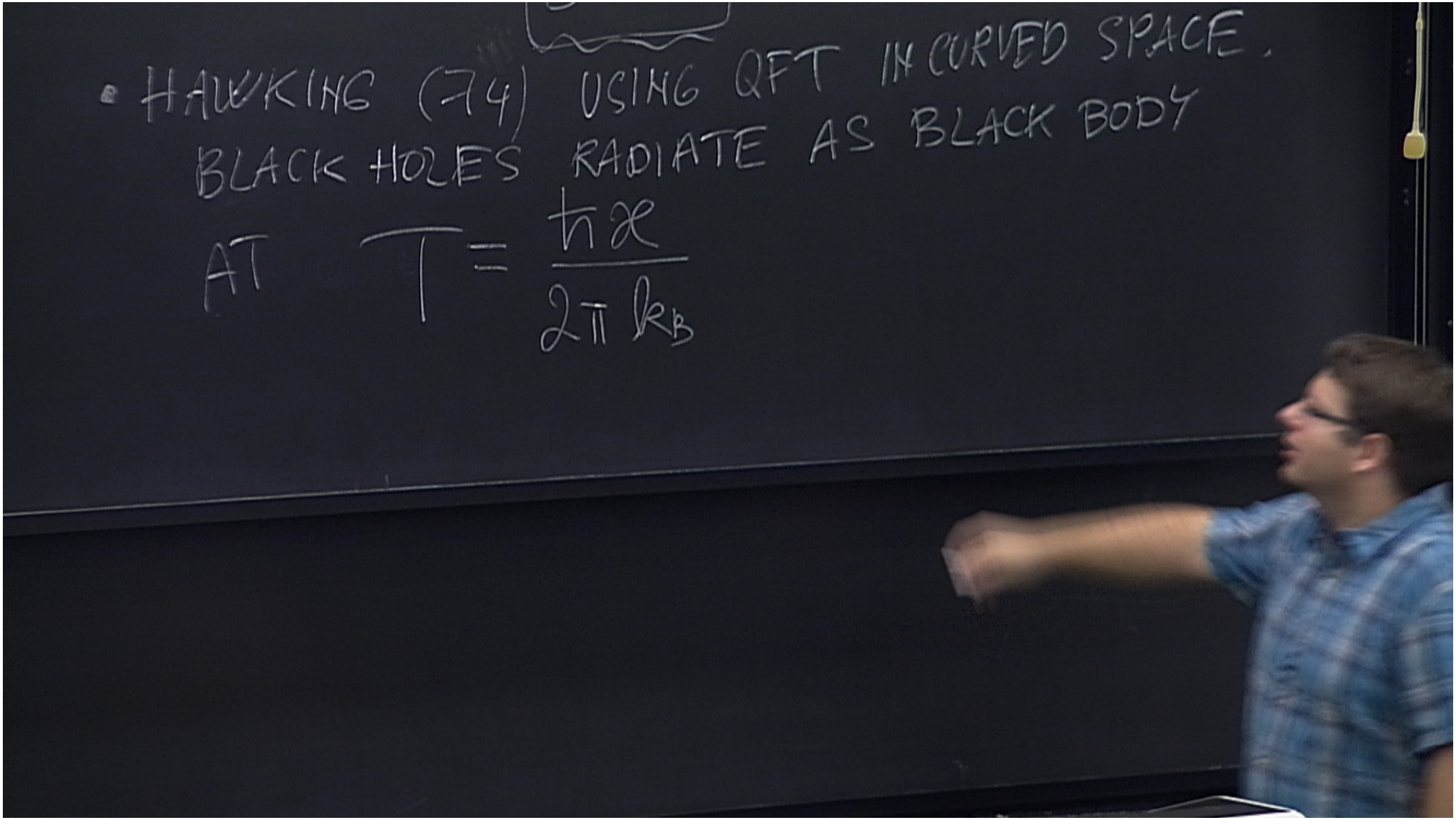

WHEELER'S CUP OF TEA;

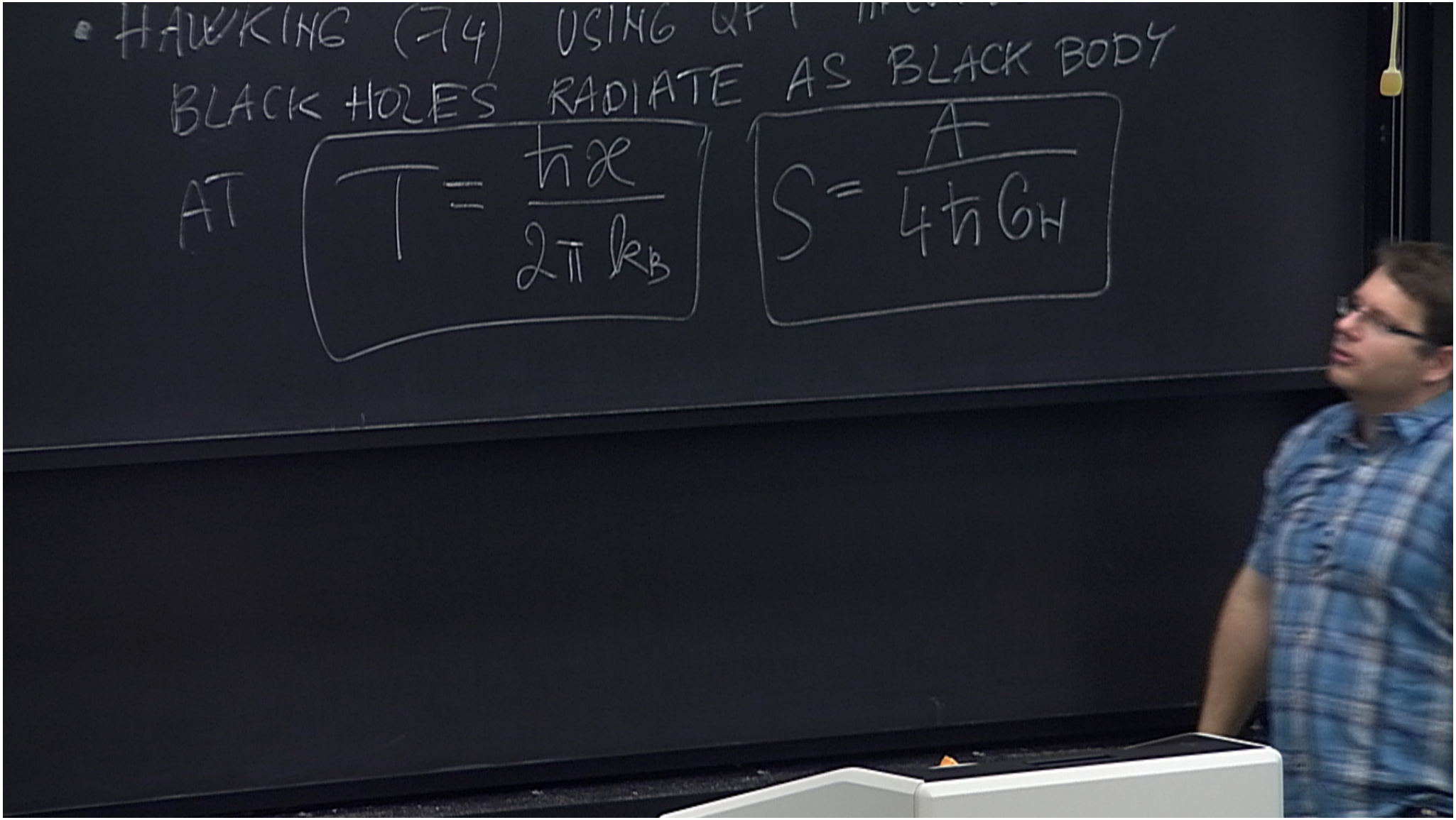

WHEELER'S CUP OF TEA:
BEKENSTEIN: S ~ A
(THOUGHT EXPERIMENT)

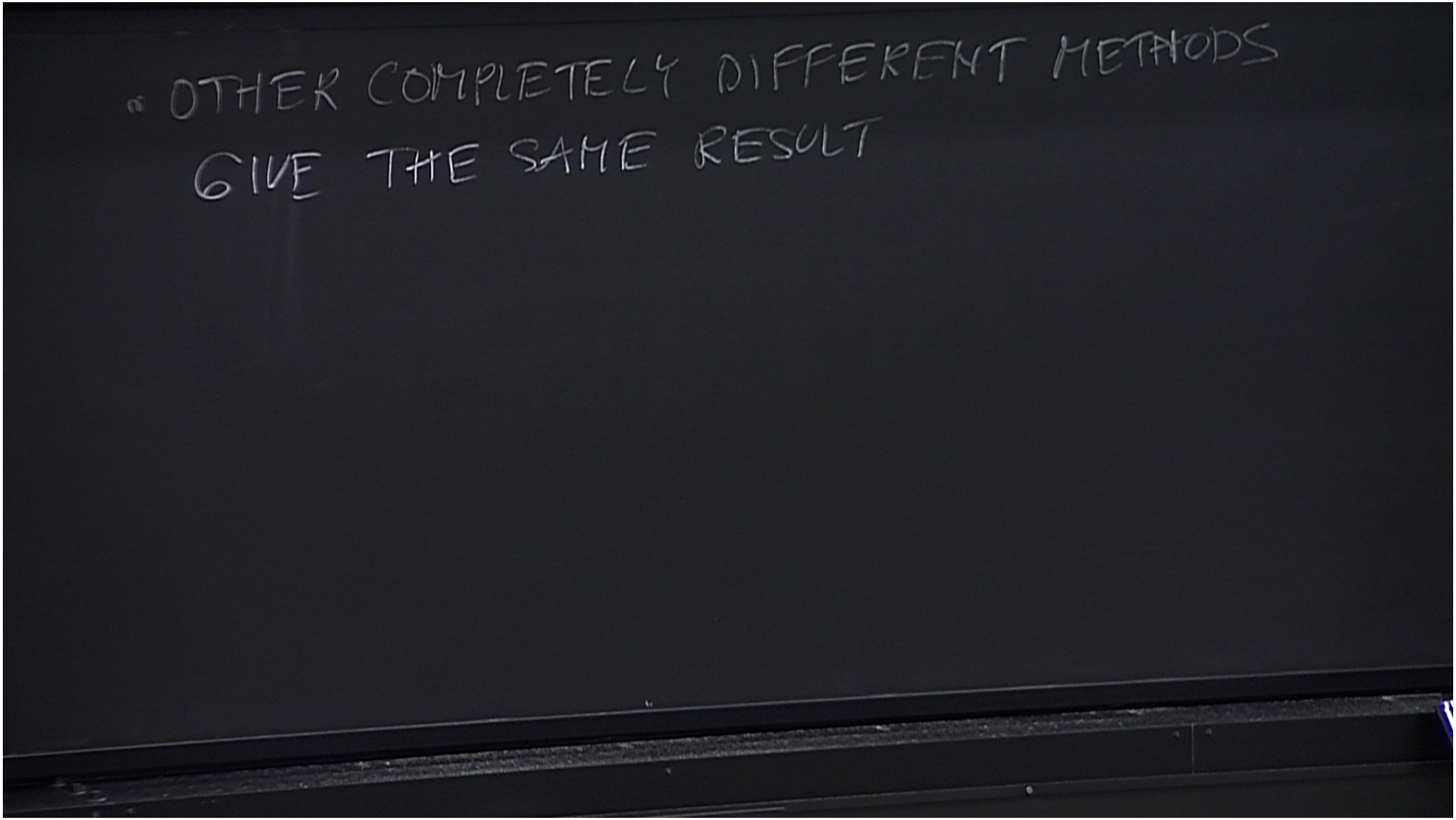

BEKENSTEIN: S ∝ A
• HAWKING (74) USING QFT IN CURVED SPACE.

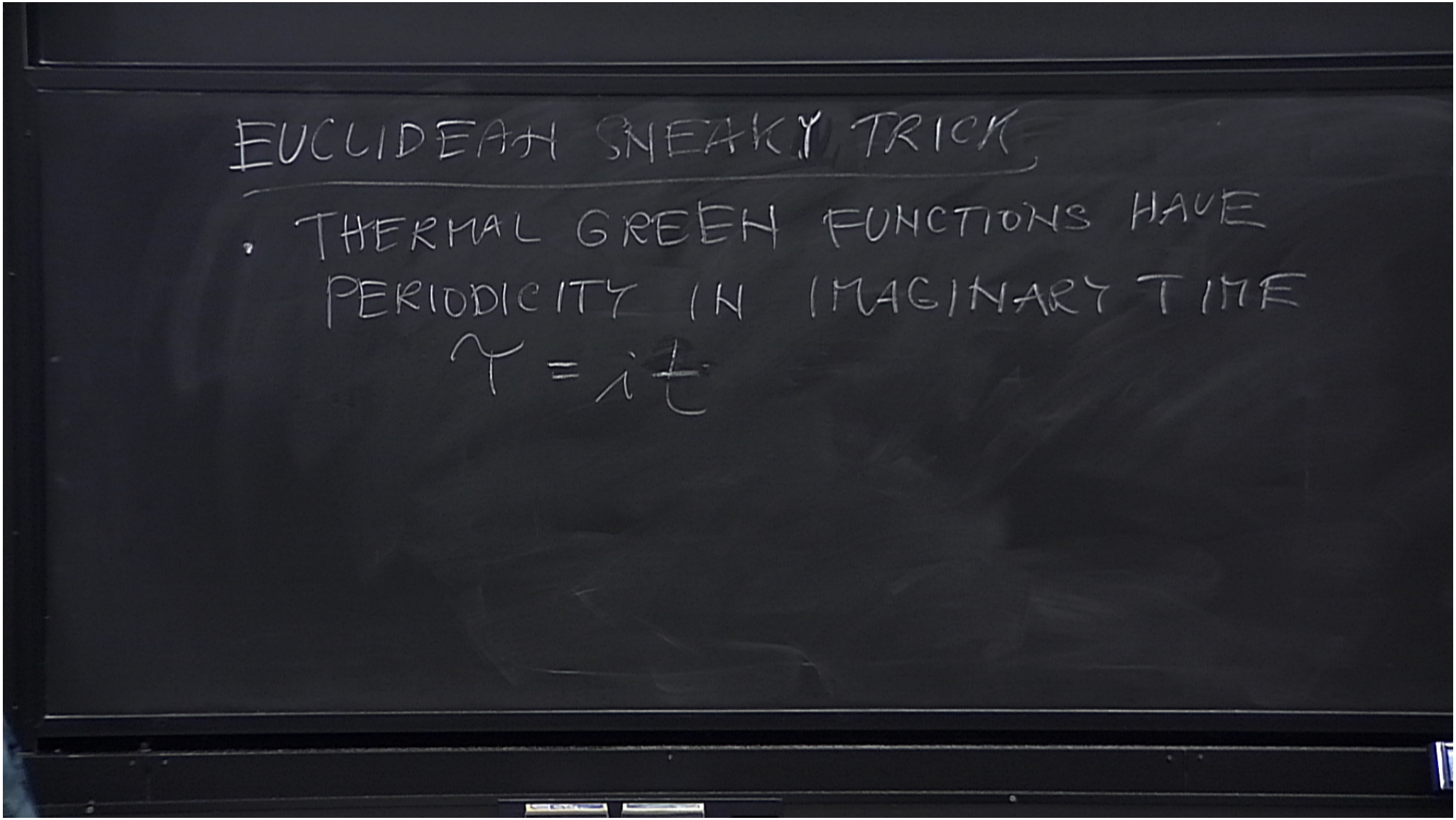

BLACK HOLES RADIATE AS BLACK BODY
AT T=


E TO COMPARE TO LAWS OF TDS.
TdS + WORK TERMS
, A~S
• WHEELER'S CUP OF TEA:
BEKENSTEIN: S~A (THOUGHT EXPERIMENT)
• HAWKING (74) USING QFT IN CURVED SPACE.
BLACK HOLES RADIATE AS BLACK BODY
AT T = ħκ/2


• HAWKING (74) USING QFT IN CURVED SPACE.
BLACK HOLES RADIATE AS BLACK BODY
AT $T = \frac{\hbar \kappa}{2\pi k_B}$


HAWKING (74) USING QFT
BLACK HOLES RADIATE AS BLACK BODY
AT $T = \frac{\hbar \varkappa}{2\pi k_B}$
$S = \frac{A}{4\hbar G_H}$


OTHER COMPLETELY DIFFERENT METHODS
GIVE THE SAME RESULT

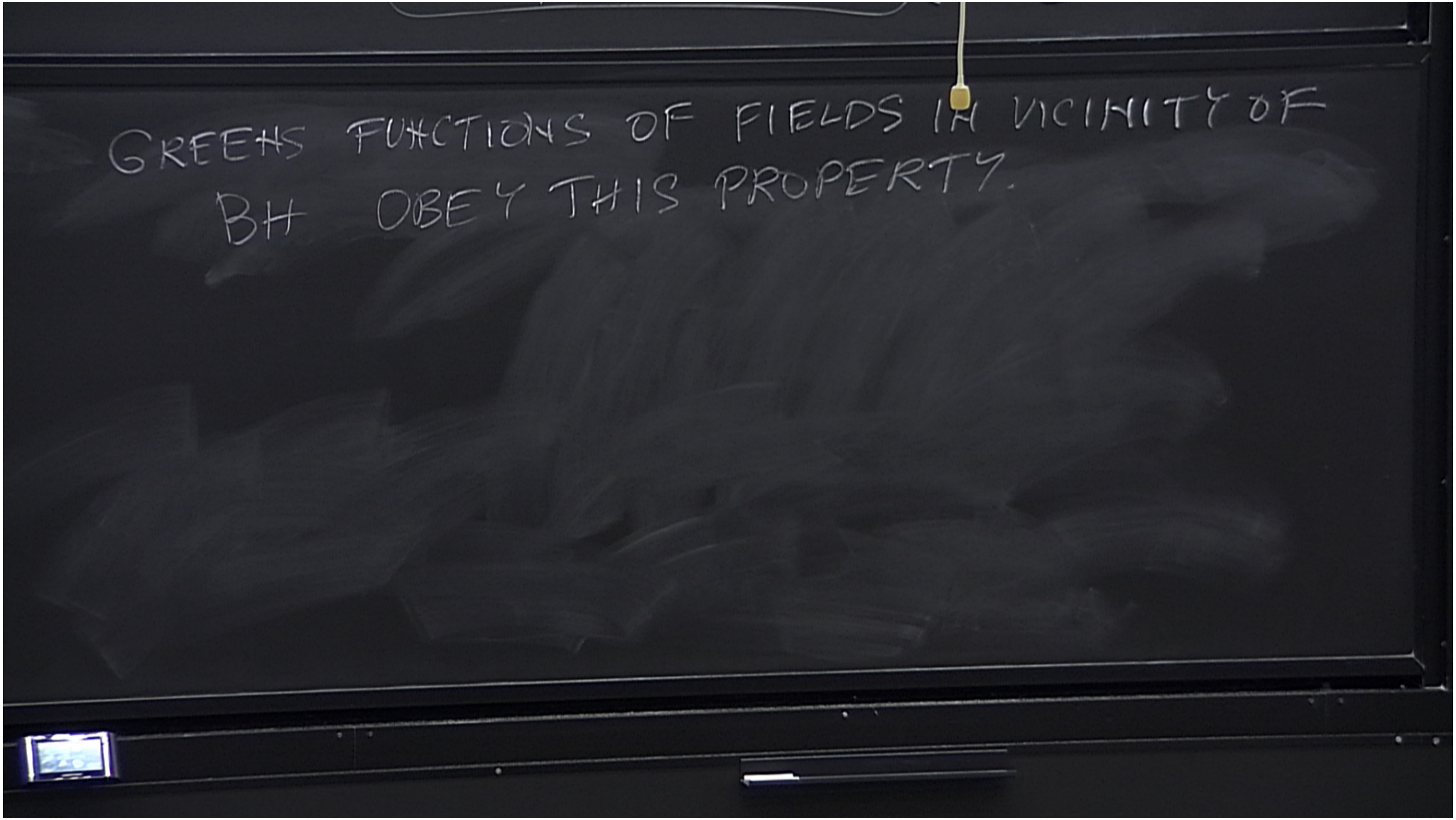

EUCLIDEAN SNEAKY TRICK
• THERMAL GREEN FUNCTIONS HAVE PERIODICITY IN IMAGINARY TIME
$\tau = i t$

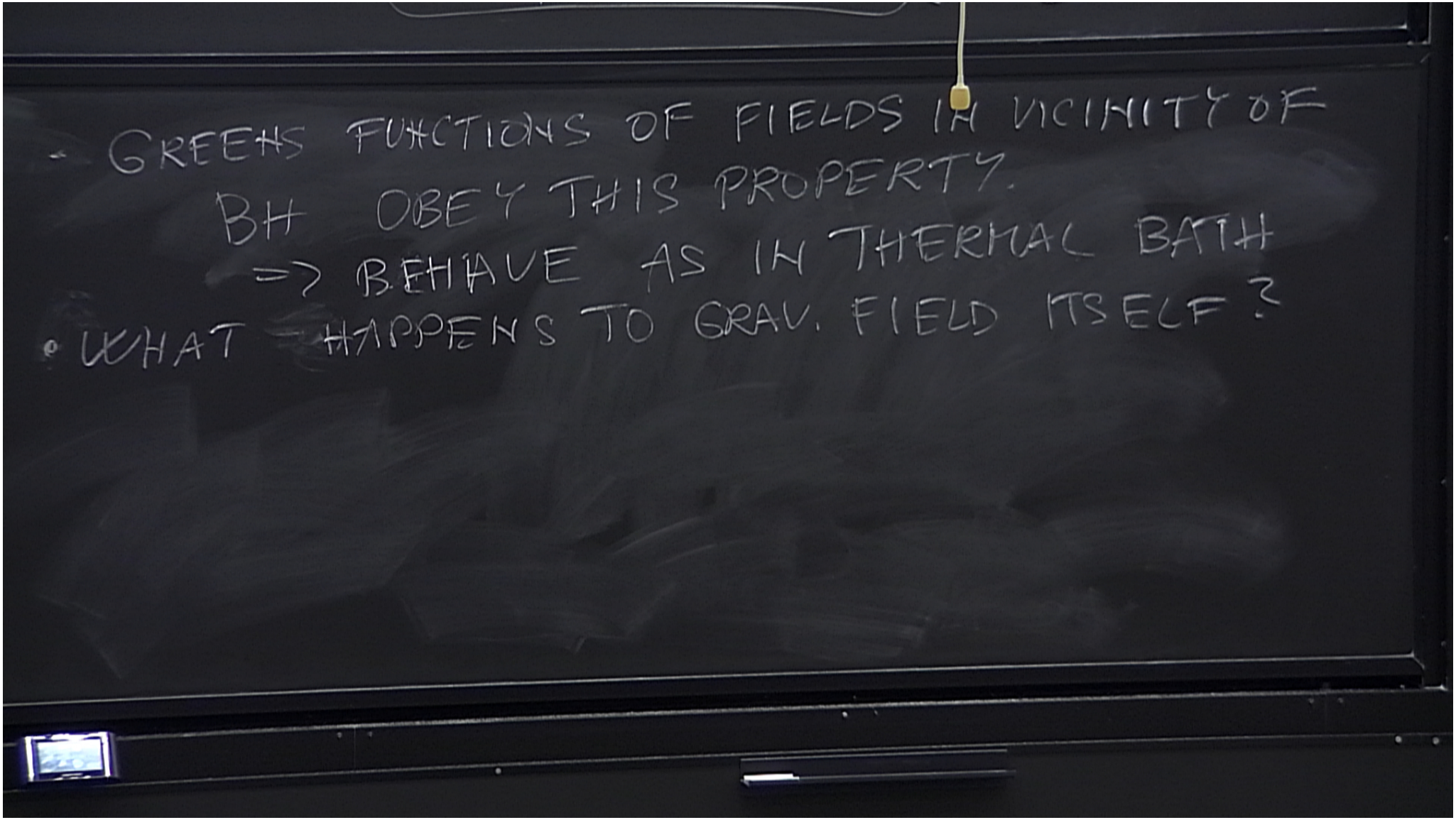
EUCLIDEAN SINGULARITIES

- THERMAL GREEN FUNCTIONS HAVE PERIODICITY IN IMAGINARY TIME

$$\tau = it$$

$$G(\tau) = G(\tau + \beta), \quad \beta = 1/T$$


GREENS FUNCTIONS OF FIELDS IN VICINITY OF
BH OBEY THIS PROPERTY.

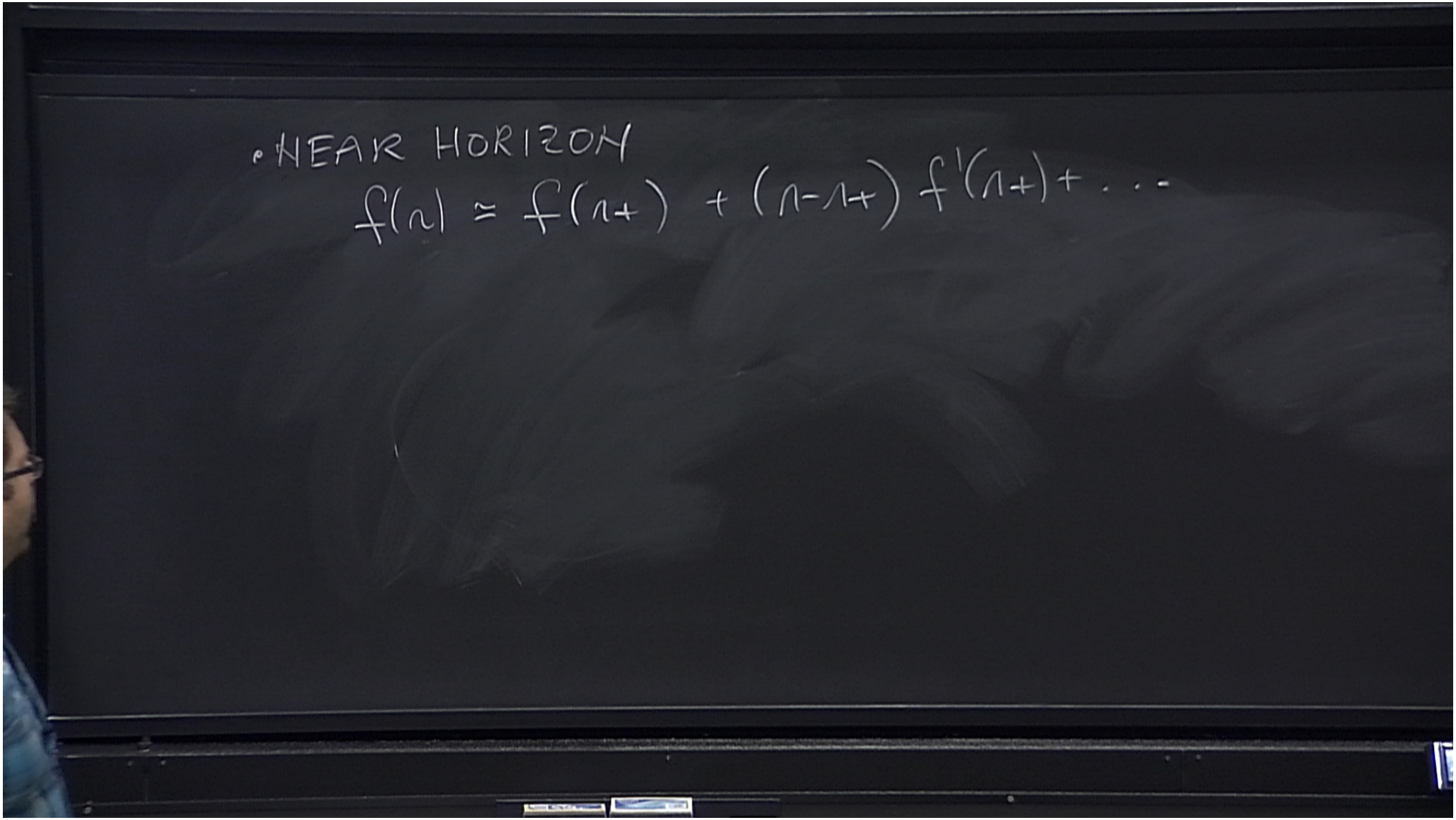

- GREENS FUNCTIONS OF FIELDS IN VICINITY OF
BH OBEY THIS PROPERTY.
=> BEHAVE AS IN THERMAL BATH
• WHAT HAPPENS TO GRAV. FIELD ITSELF?

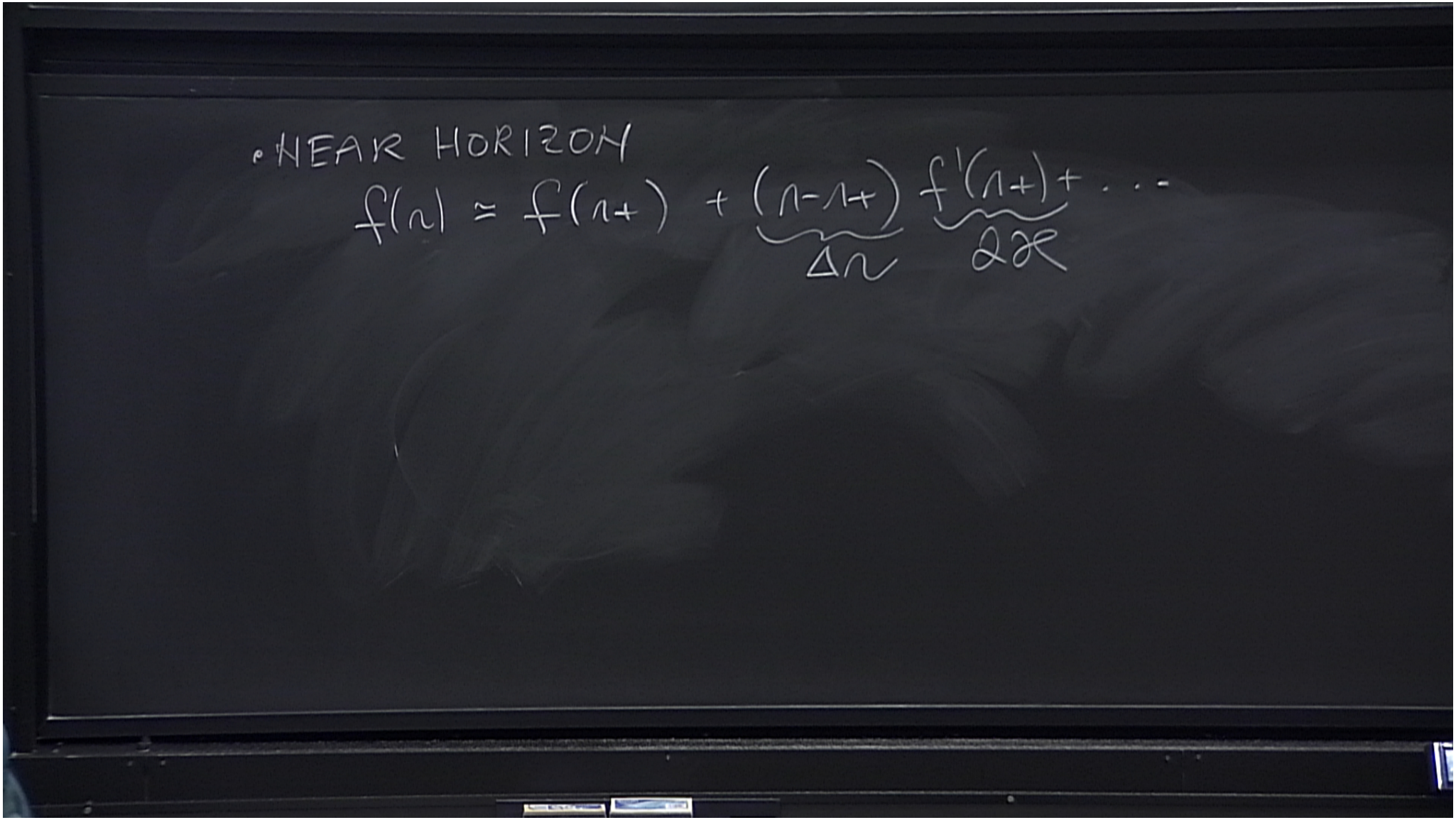
- GREENS FUNCTIONS OF FIELDS IN VICINITY OF BH OBEY THIS PROPERTY.
  => BEHAVE AS IN THERMAL BATH
- WHAT HAPPENS TO GRAV. FIELD ITSELF?

CONSIDER SCHW → EUCLIDEANIZED SCHW.

$$ds_E^2 = f d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$= 4\pi r_+^2$$


• NEAR HORIZON
$f(r) \simeq f(r_+) + (r - r_+) f'(r_+) + \ldots$


• NEAR HORIZON
$f(r) \simeq f(r_+) + \underbrace{(r - r_+)}_{\Delta r} \underbrace{f'(r_+)}_{2\kappa} + \cdots$

• NEAR HORIZON

$$f(r) \simeq \underbrace{f(r_+)}_{0} + \underbrace{(r - r_+)}_{\Delta r} \underbrace{f'(r_+)}_{2\kappa} + \cdots = 2\kappa \Delta r$$

• NEAR HORIZON

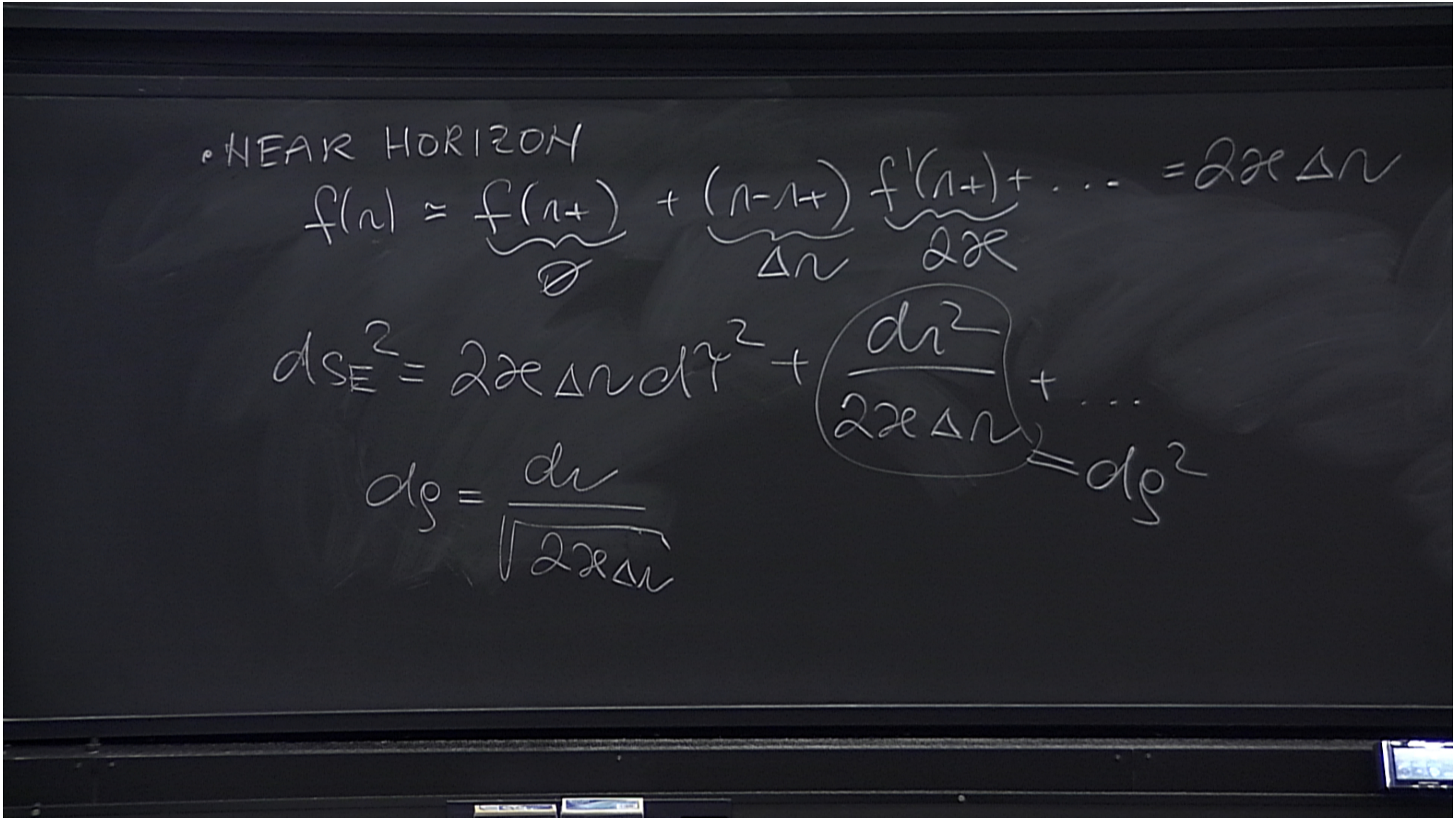
$$f(r) \simeq \underbrace{f(r_+)}_{0} + \underbrace{(r - r_+)}_{\Delta r} \underbrace{f'(r_+)}_{2\kappa} + \dots = 2\kappa \Delta r$$

$$ds_E^2 = 2\kappa \Delta r \, d\tau^2 + \frac{dr^2}{2\kappa \Delta r} + \dots$$

• NEAR HORIZON

$$f(r) \simeq \underbrace{f(r_+)}_{0} + \underbrace{(r-r_+)}_{\Delta r}\underbrace{f'(r_+)}_{2\kappa} + \dots = 2\kappa\,\Delta r$$

$$ds_E^2 = 2\kappa\,\Delta r\, d\tau^2 + \frac{dr^2}{2\kappa\,\Delta r} + \dots$$

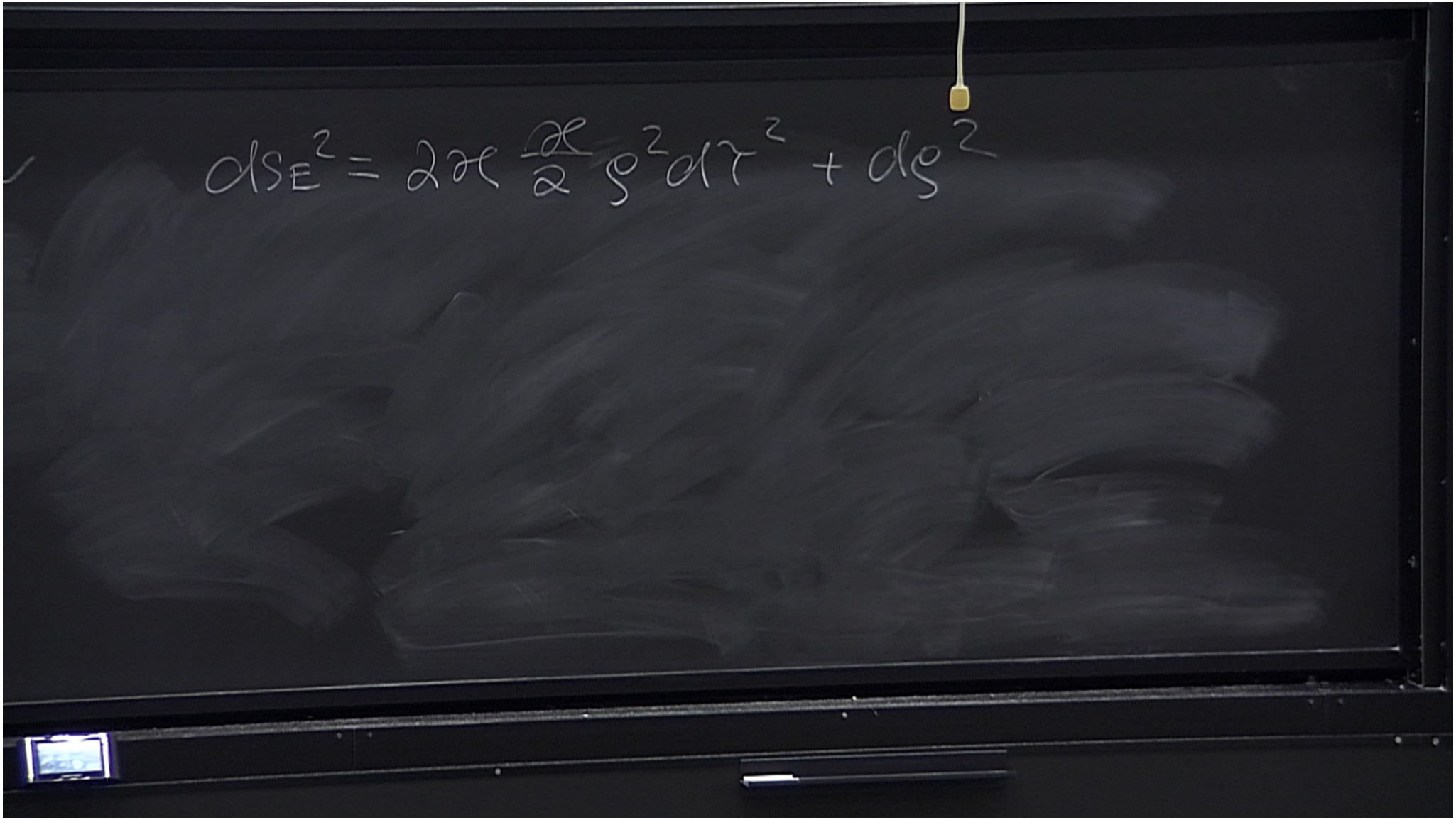

• NEAR HORIZON
$f(r) \simeq \underbrace{f(r_+)}_{0} + \underbrace{(r-r_+)}_{\Delta r} \underbrace{f'(r_+)}_{2\kappa} + \dots = 2\kappa\Delta r$
$ds_E^2 = 2\kappa\Delta r\, d\tau^2 + \frac{dr^2}{2\kappa\Delta r} + \dots = d\rho^2$
$d\rho = \frac{dr}{\sqrt{2\kappa\Delta r}}$

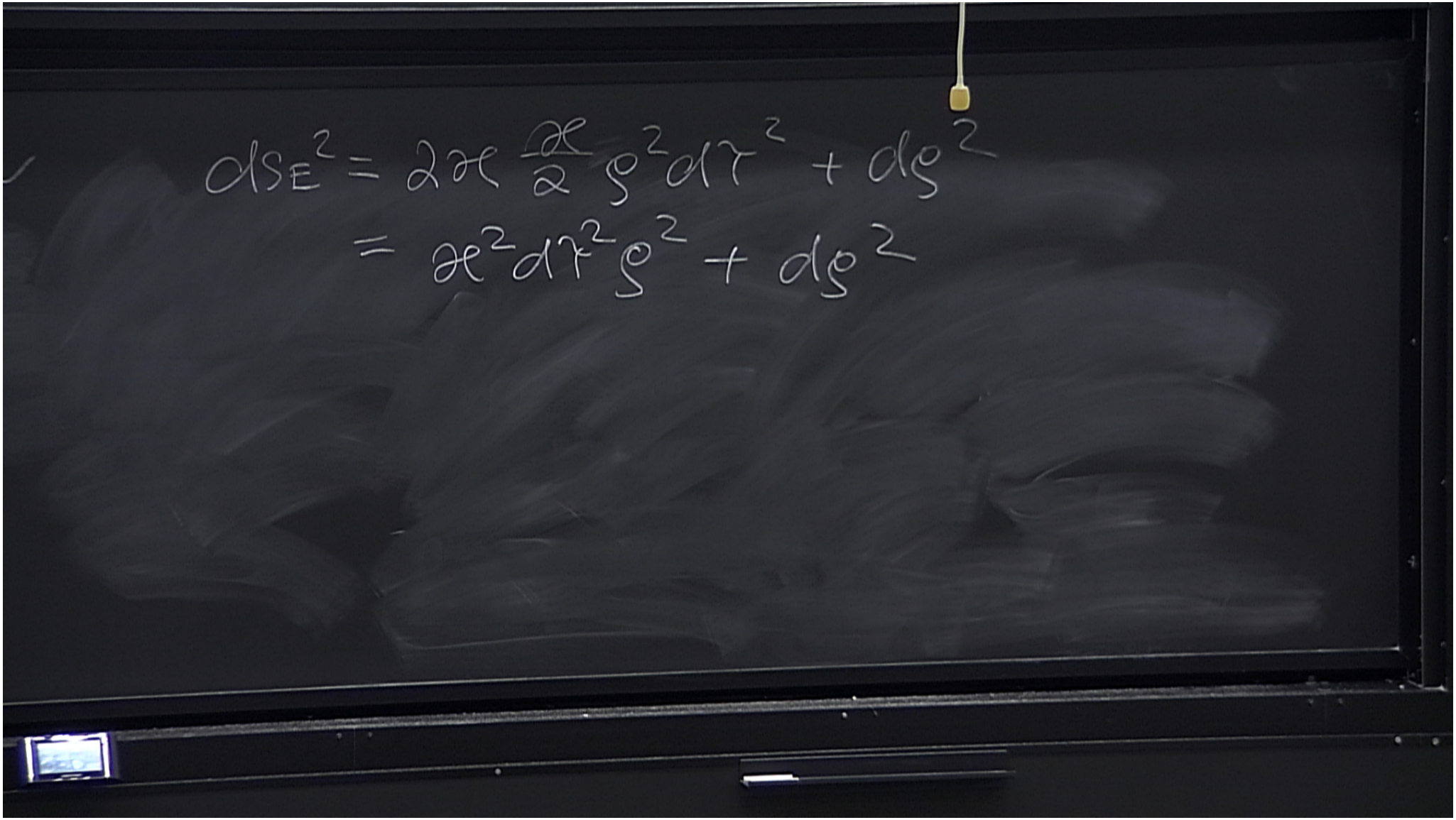
• NEAR HORIZON

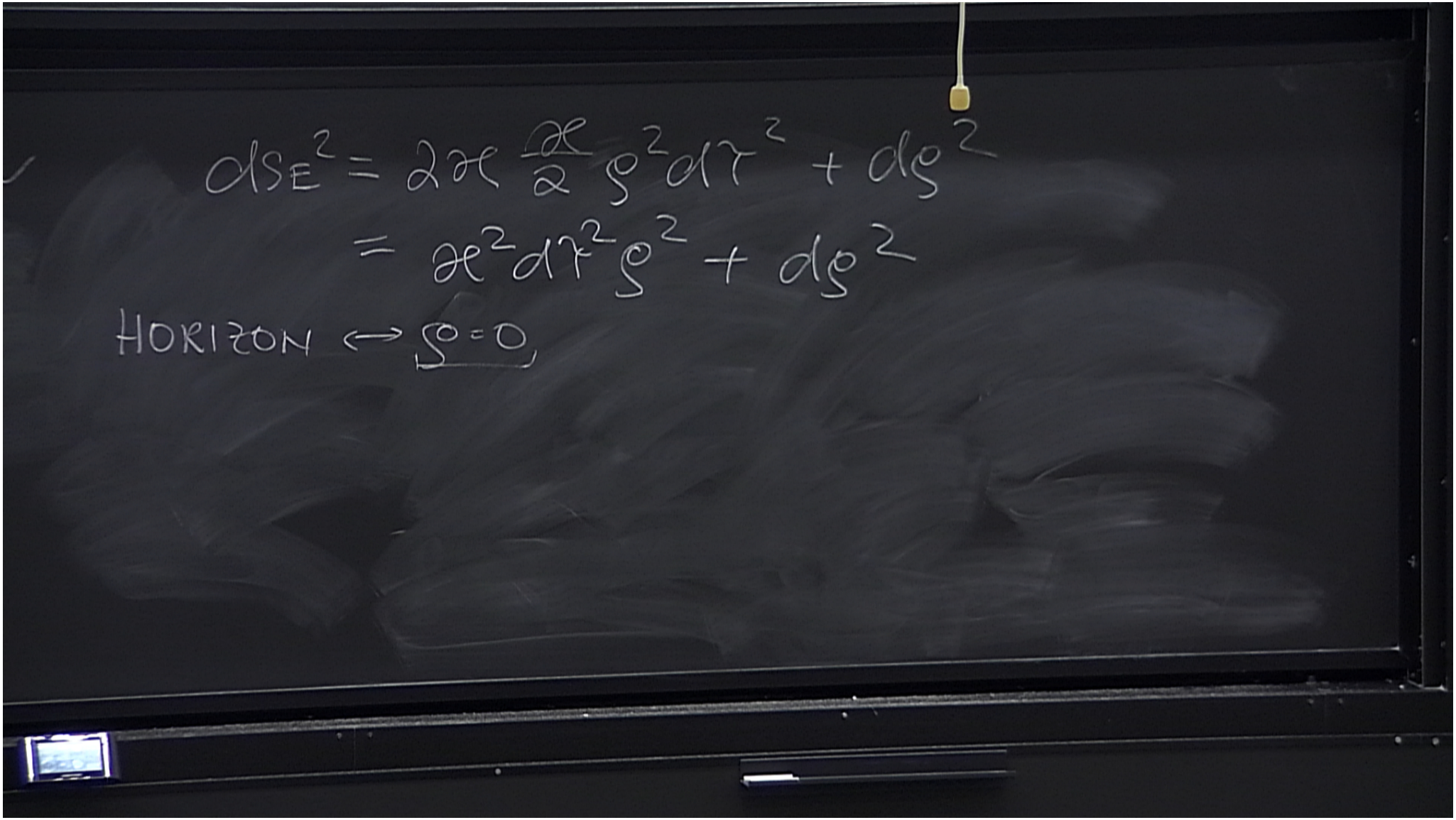
$$f(r) \simeq \underbrace{f(r_+)}_{0} + \underbrace{(r-r_+)}_{\Delta r} \underbrace{f'(r_+)}_{2\kappa} + \dots = 2\kappa\,\Delta r$$

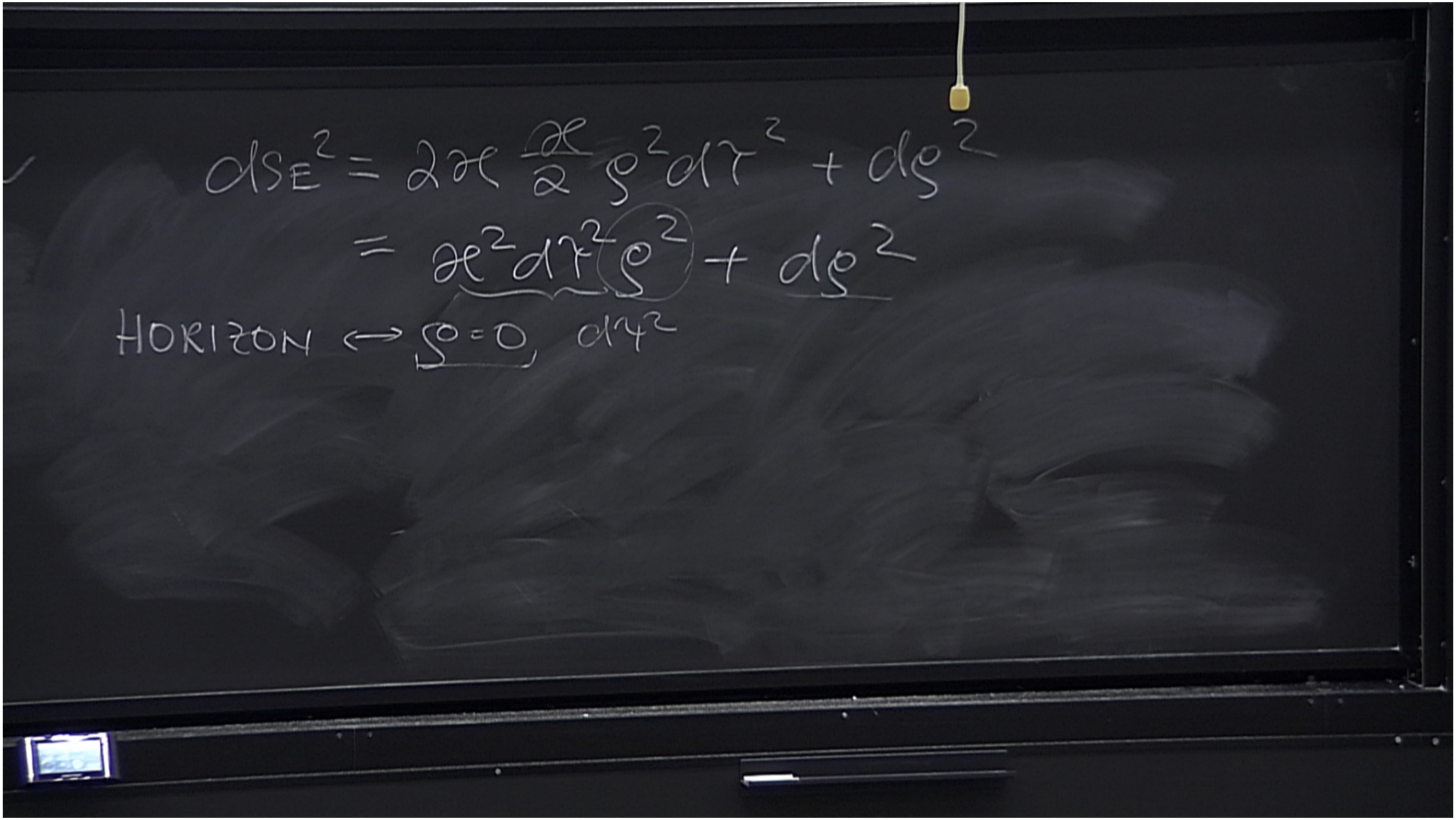
$$ds_E^2 = 2\kappa\,\Delta r\, d\tau^2 + \frac{dr^2}{2\kappa\,\Delta r} + \dots = d\rho^2$$

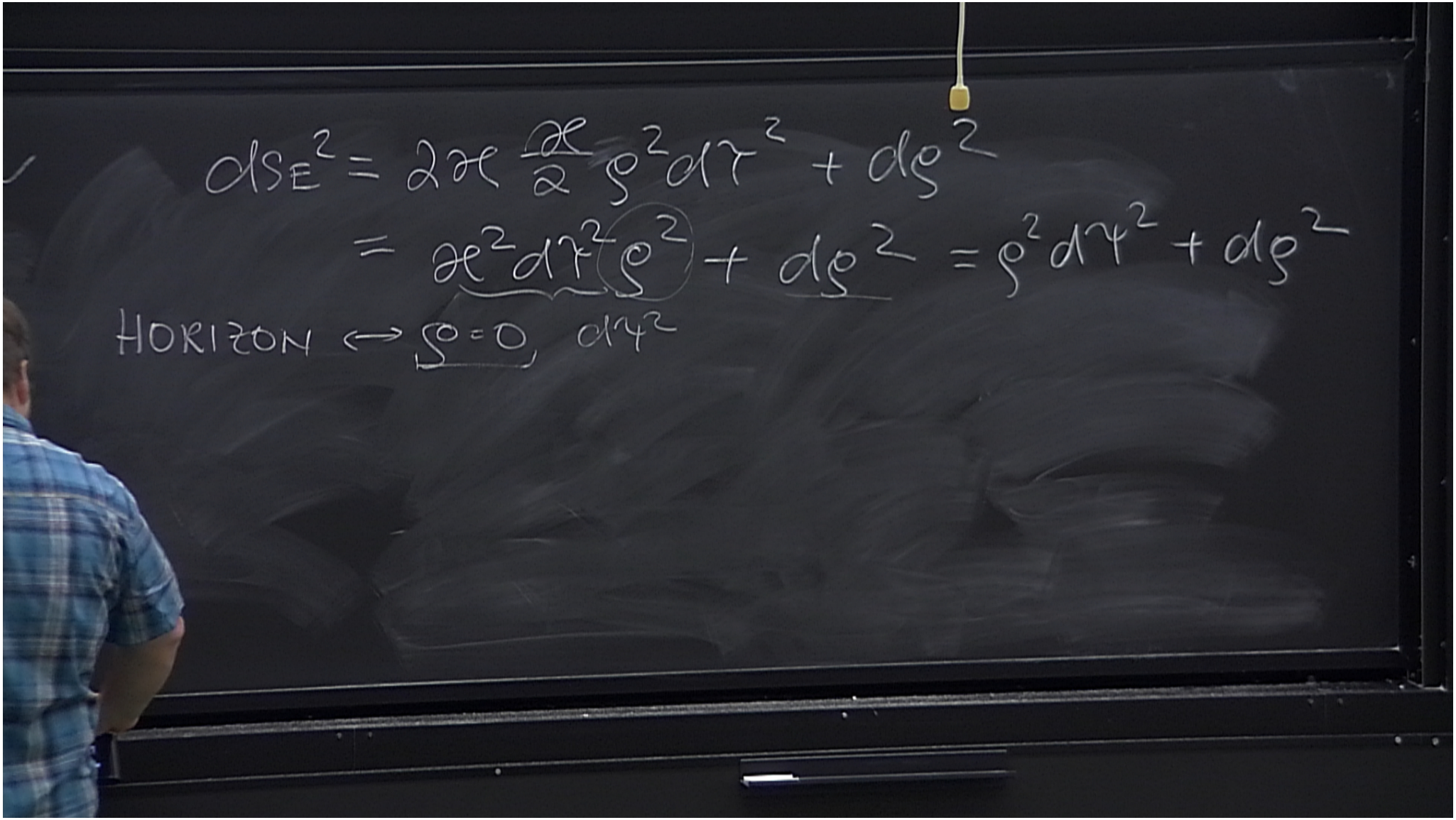
$$d\rho = \frac{dr}{\sqrt{2\kappa\,\Delta r}} \iff \boxed{\Delta r = \frac{\kappa}{2}\rho^2}$$

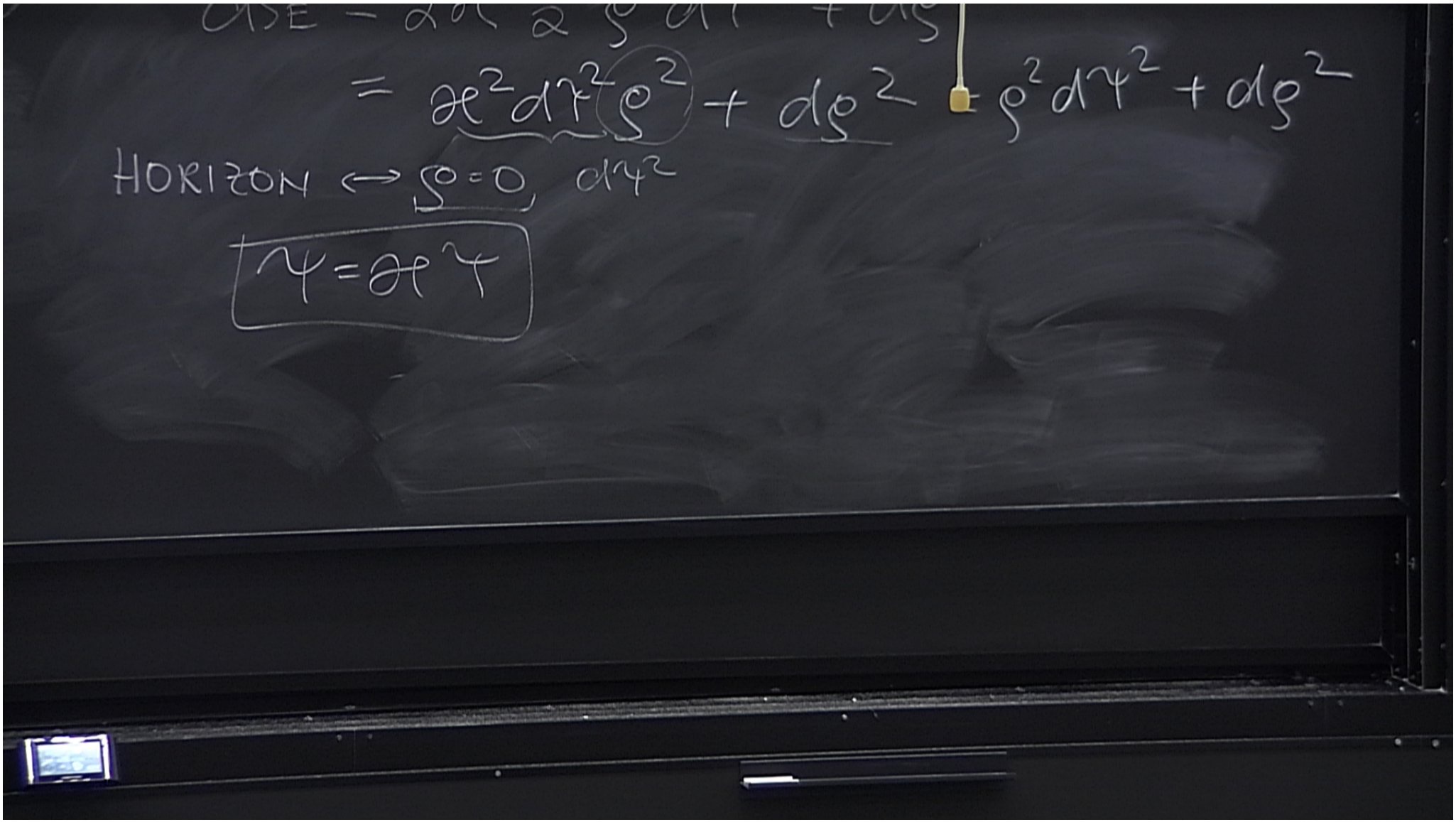

$ds_E^2 = 2r\frac{dr}{2}g^2 d\tau^2 + dg^2$

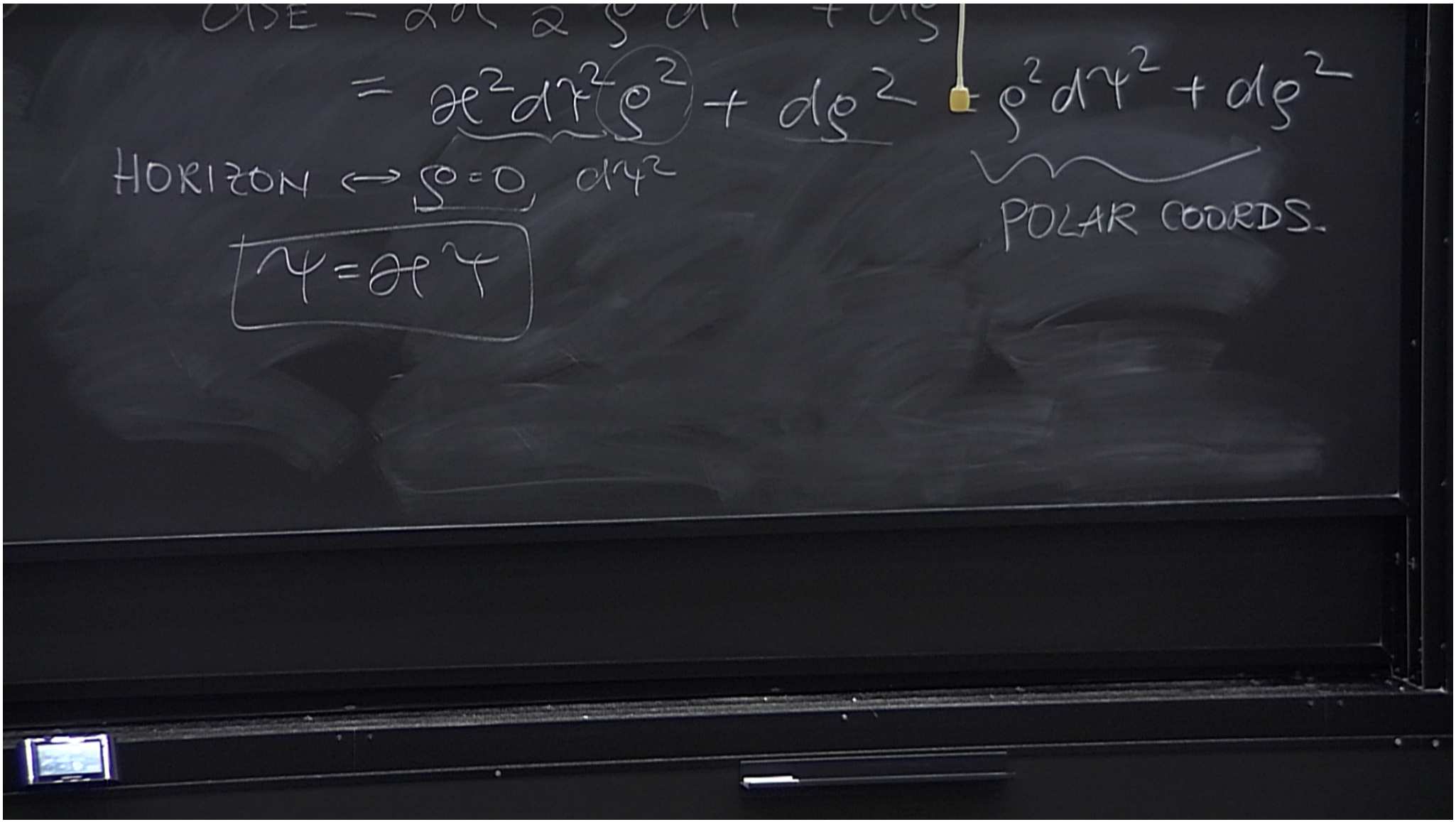

$ds_E^2 = 2r\frac{dr}{2}g^2 d\tau^2 + dg^2$
$= r^2 d\tau^2 g^2 + dg^2$

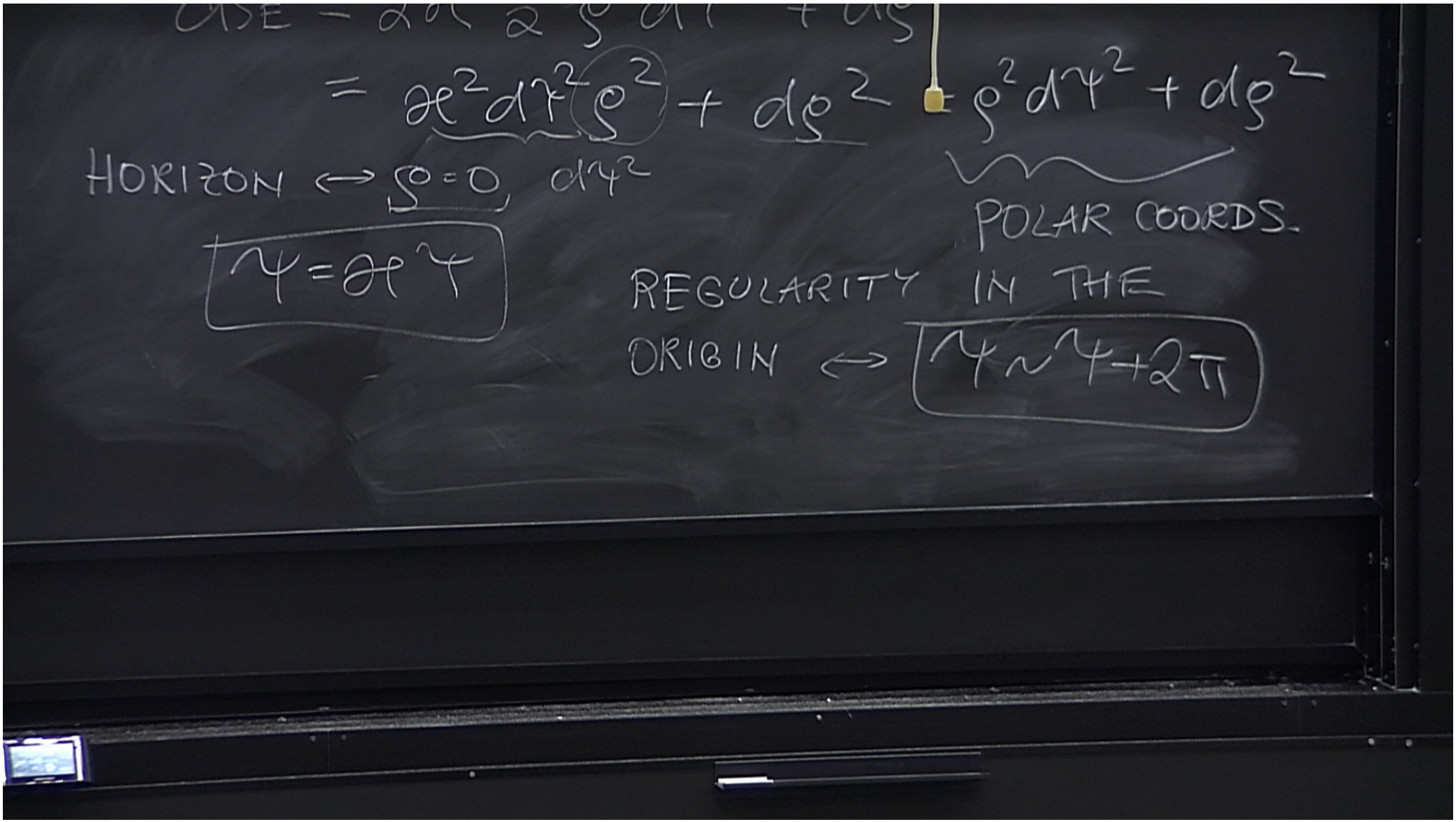

$dsE^2 = 2r\frac{dr}{2}\rho^2 d\tau^2 + d\rho^2$
$= r^2 d\tau^2 \rho^2 + d\rho^2$
HORIZON $\leftrightarrow \rho = 0$

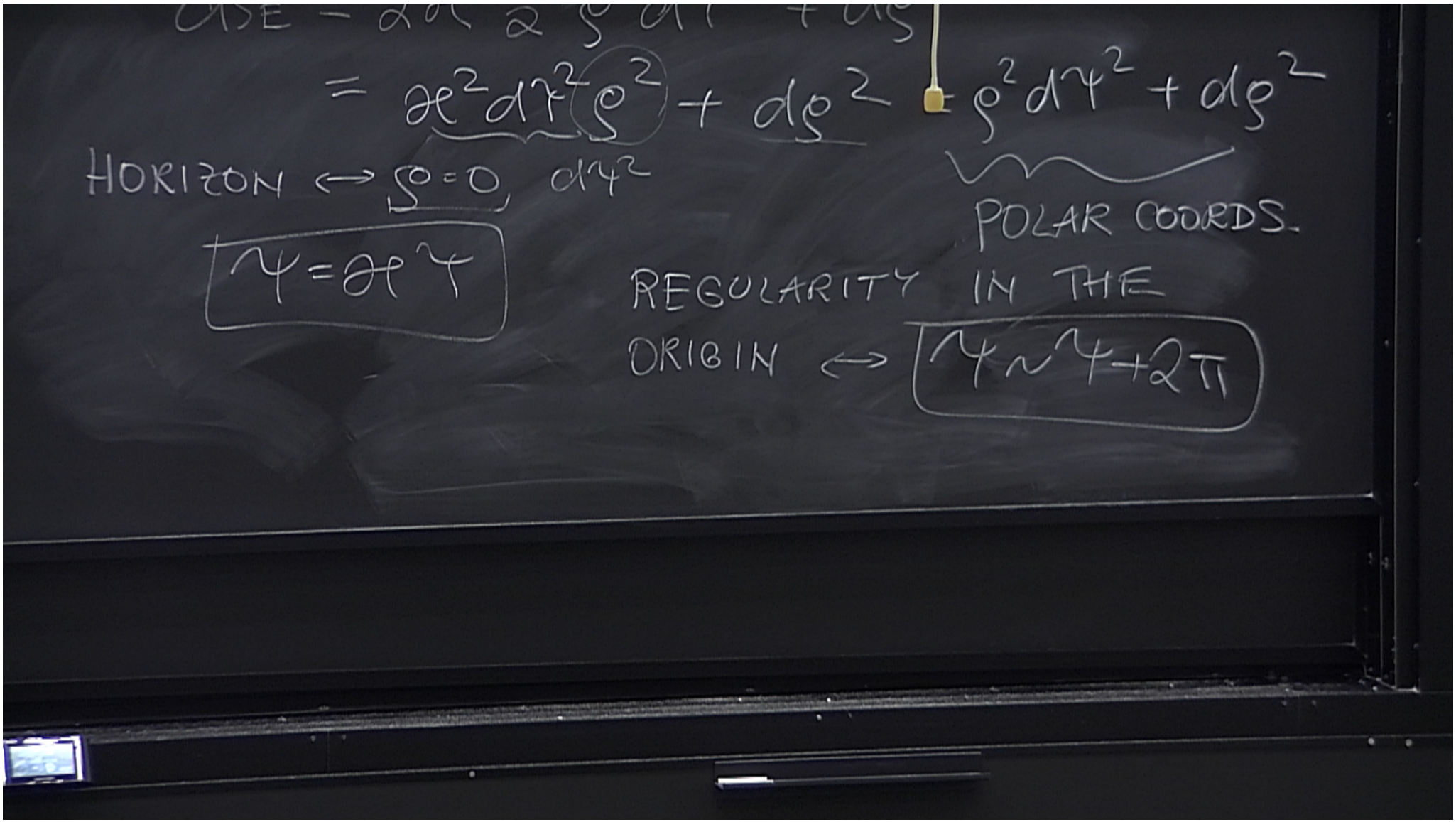

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$= r^2 d\tau^2 \rho^2 + d\rho^2$
HORIZON $\leftrightarrow \rho = 0$, $d\tau^2$

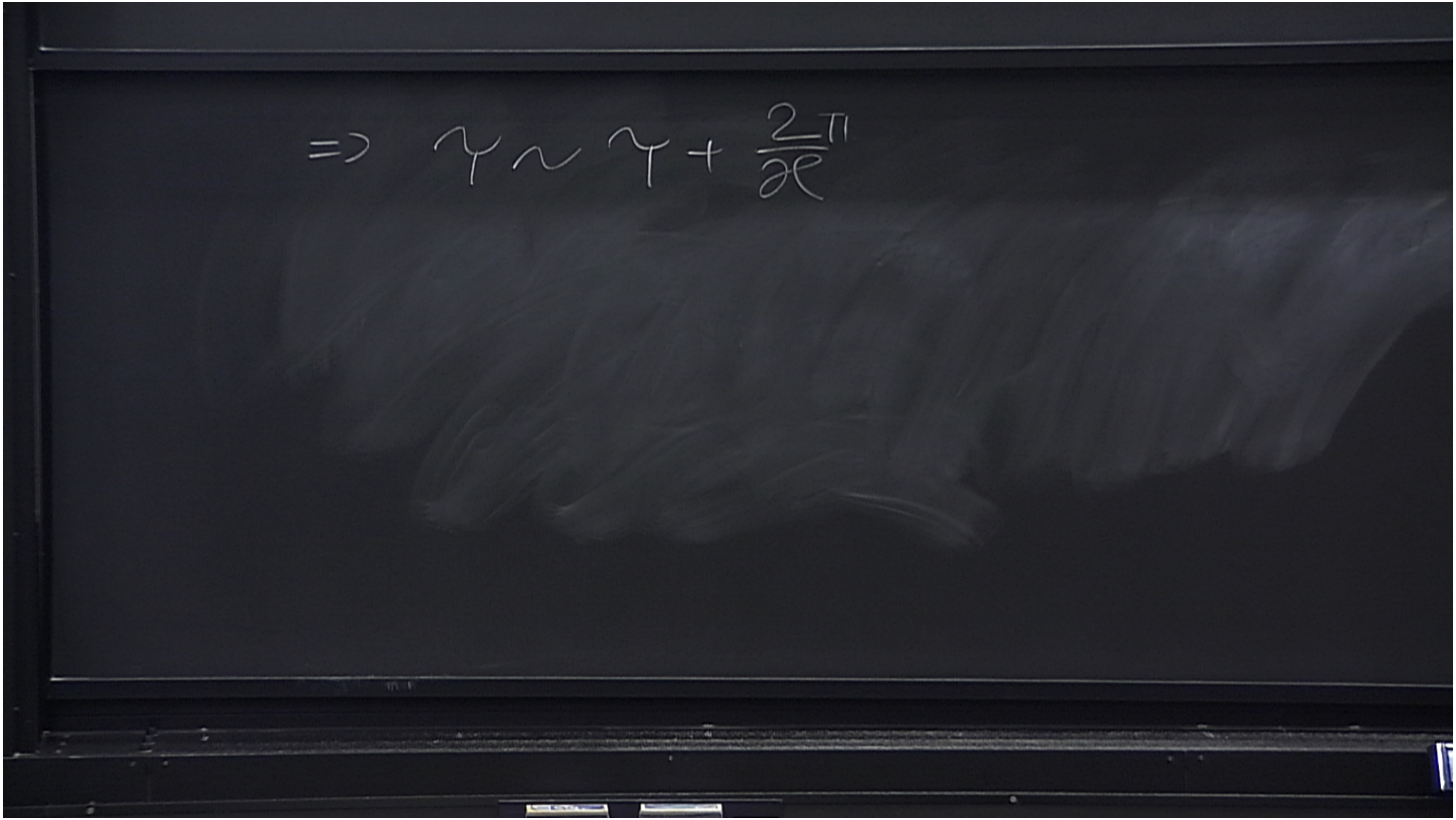

$ds_E^2 = 2r\frac{dr}{2}\rho^2 d\tau^2 + d\rho^2$
$= r^2 d\tau^2 \rho^2 + d\rho^2 = \rho^2 d\tau^2 + d\rho^2$
HORIZON $\leftrightarrow \rho = 0$, $d\tau^2$

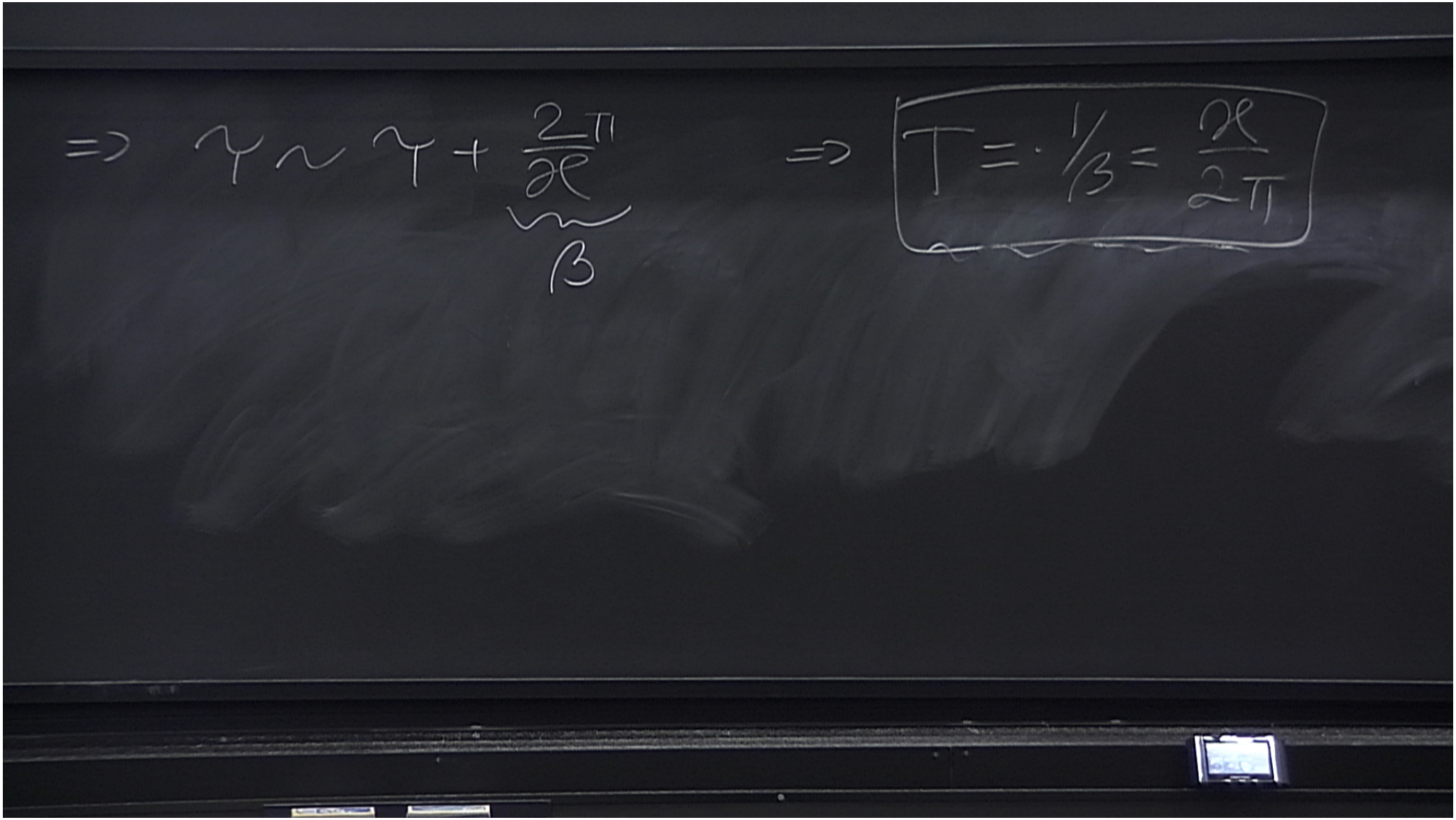

$= \rho^2 d\tau^2 \varsigma^2 + d\varsigma^2 = \varsigma^2 d\tau^2 + d\varsigma^2$
HORIZON $\leftrightarrow \varsigma = 0$, $d\tau^2$
$\tau = \rho\tau$

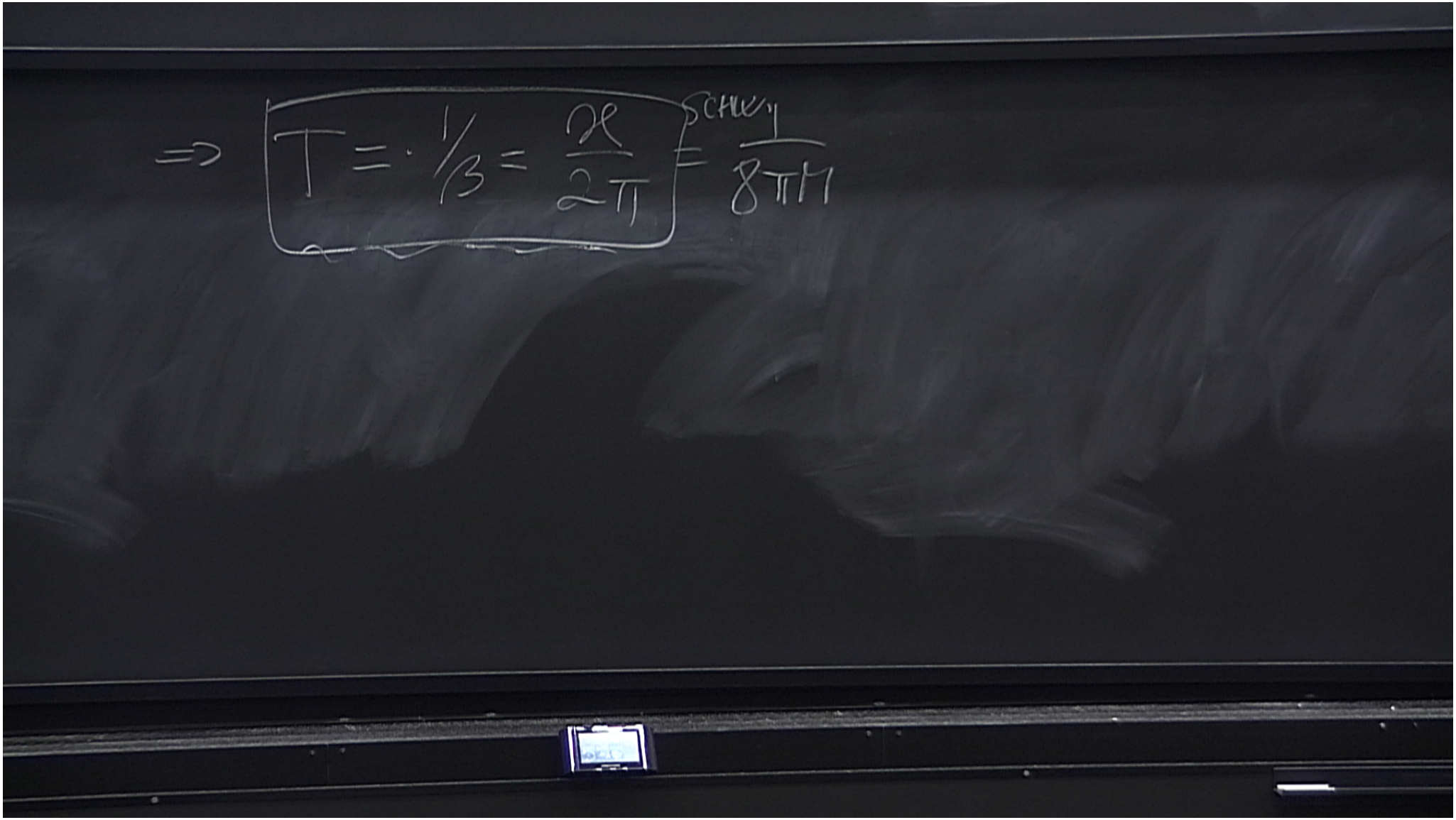

$= \alpha^2 d\gamma^2 \rho^2 + d\rho^2 = \rho^2 d\gamma^2 + d\rho^2$
HORIZON $\leftrightarrow \rho = 0$, $d\gamma^2$
POLAR COORDS.
$\gamma = \alpha\gamma$


$= \varrho^2 d\gamma^2 + d\varrho^2 = \varrho^2 d\gamma^2 + d\varrho^2$
HORIZON $\leftrightarrow \varrho = 0$, $d\gamma^2$
$\gamma = \varrho^2 \gamma$
POLAR COORDS.
REGULARITY IN THE ORIGIN $\leftrightarrow$ $\gamma \sim \gamma + 2\pi$


$= \varrho^2 d\gamma^2 + d\varrho^2 = \varrho^2 d\gamma^2 + d\varrho^2$
HORIZON $\leftrightarrow \varrho = 0$, $d\gamma^2$
$\gamma = \varrho\gamma$
POLAR COORDS.
REGULARITY IN THE ORIGIN $\leftrightarrow$ $\gamma \sim \gamma + 2\pi$

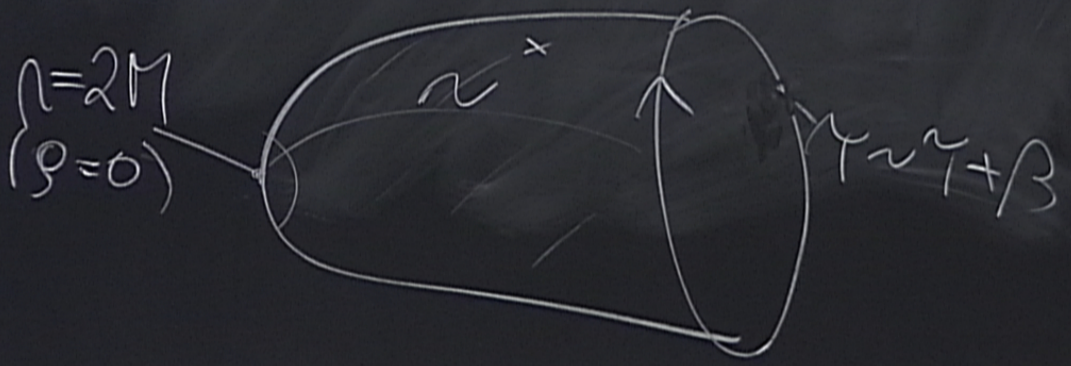

$\Rightarrow \gamma \sim \gamma + \frac{2\pi}{\partial e}$


$\Rightarrow \tau \sim \tau + \frac{2\pi}{\beta}$
β
$\Rightarrow$
$T = \frac{1}{\beta} = \frac{\beta}{2\pi}$


$\Rightarrow \boxed{T = \cdot \frac{1}{\beta} = \frac{\kappa}{2\pi}} = \frac{1}{8\pi M}$
SCHWA

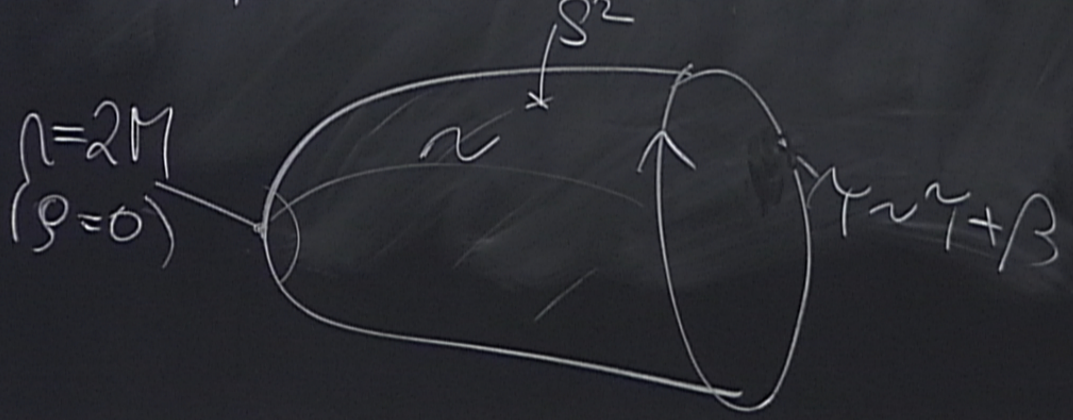
$\Rightarrow \tau \sim \tau + \underbrace{\frac{2\pi}{\partial \varepsilon}}_{\beta} \Rightarrow \boxed{T = 1/\beta = \ldots}$

EUCL. SCHW.



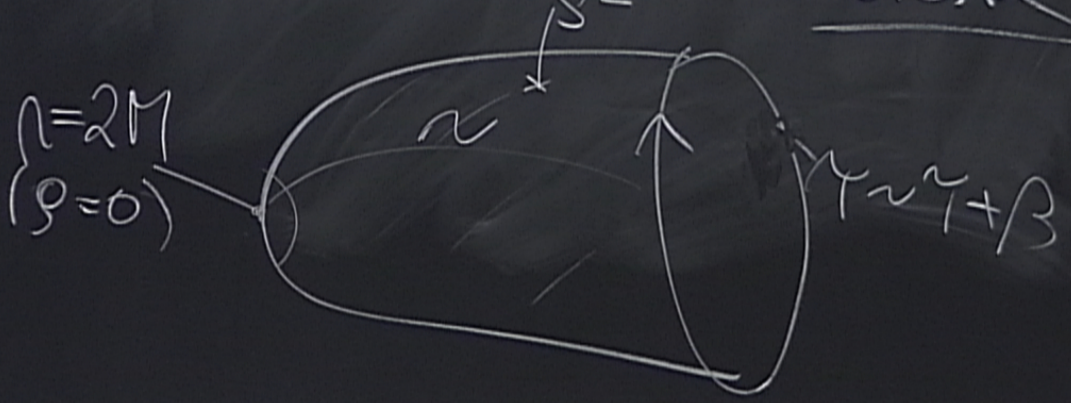
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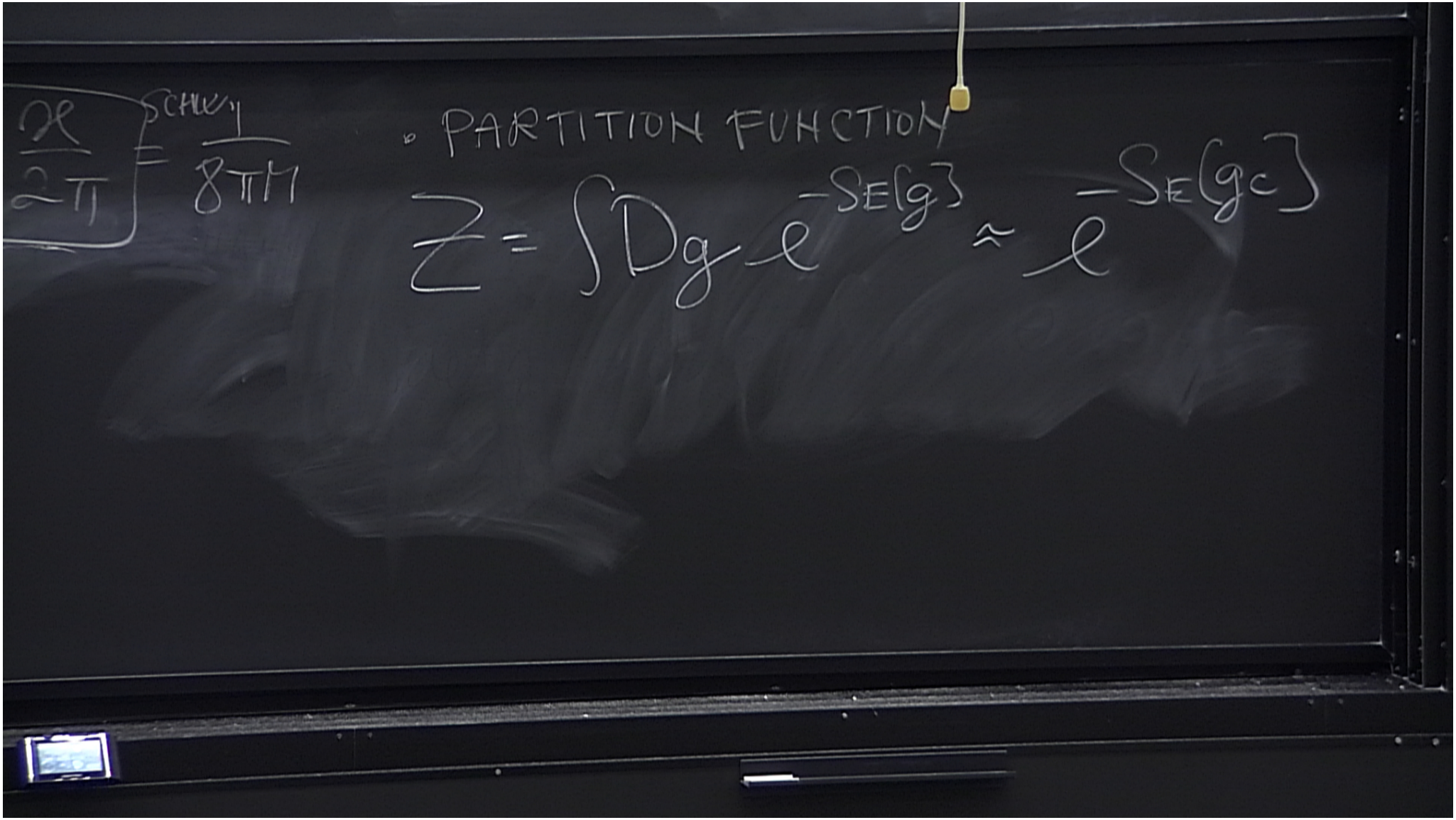
EUCL. SCHW.

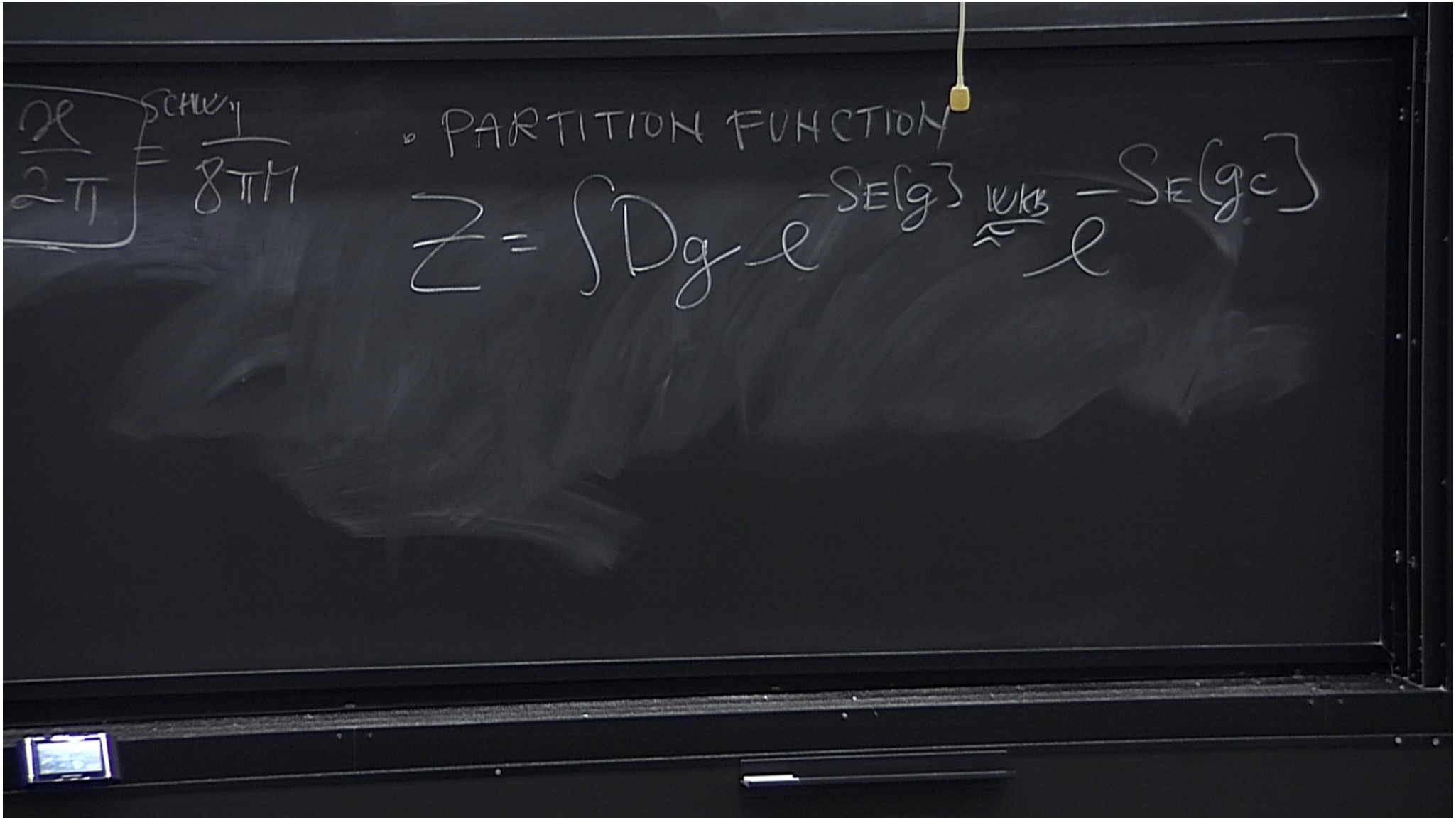


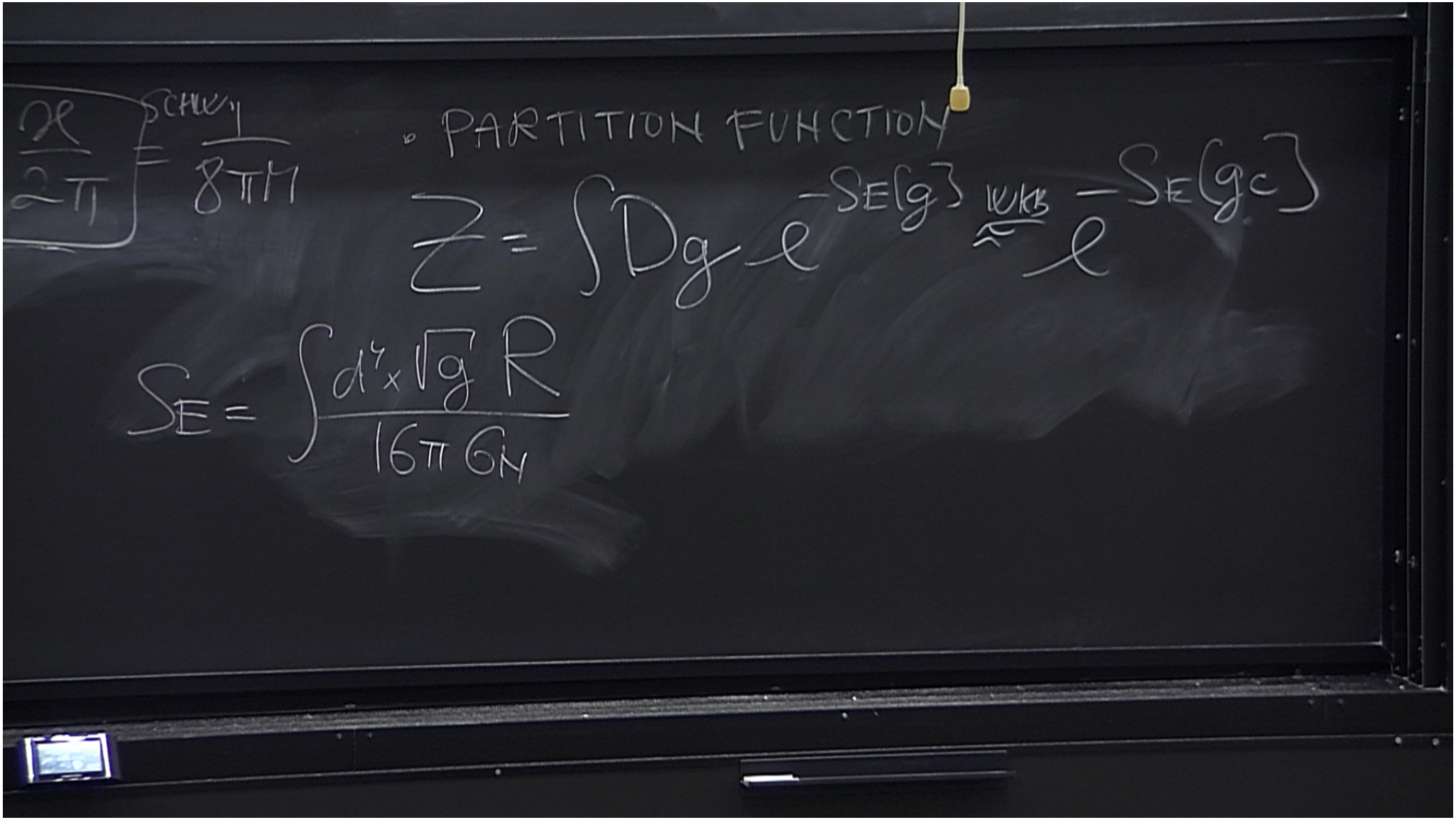
$\Rightarrow \tau \sim \tau + \underbrace{\frac{2\pi}{\partial \mathcal{E}}}_{\beta}$ $\Rightarrow$ $T = 1/\beta =$

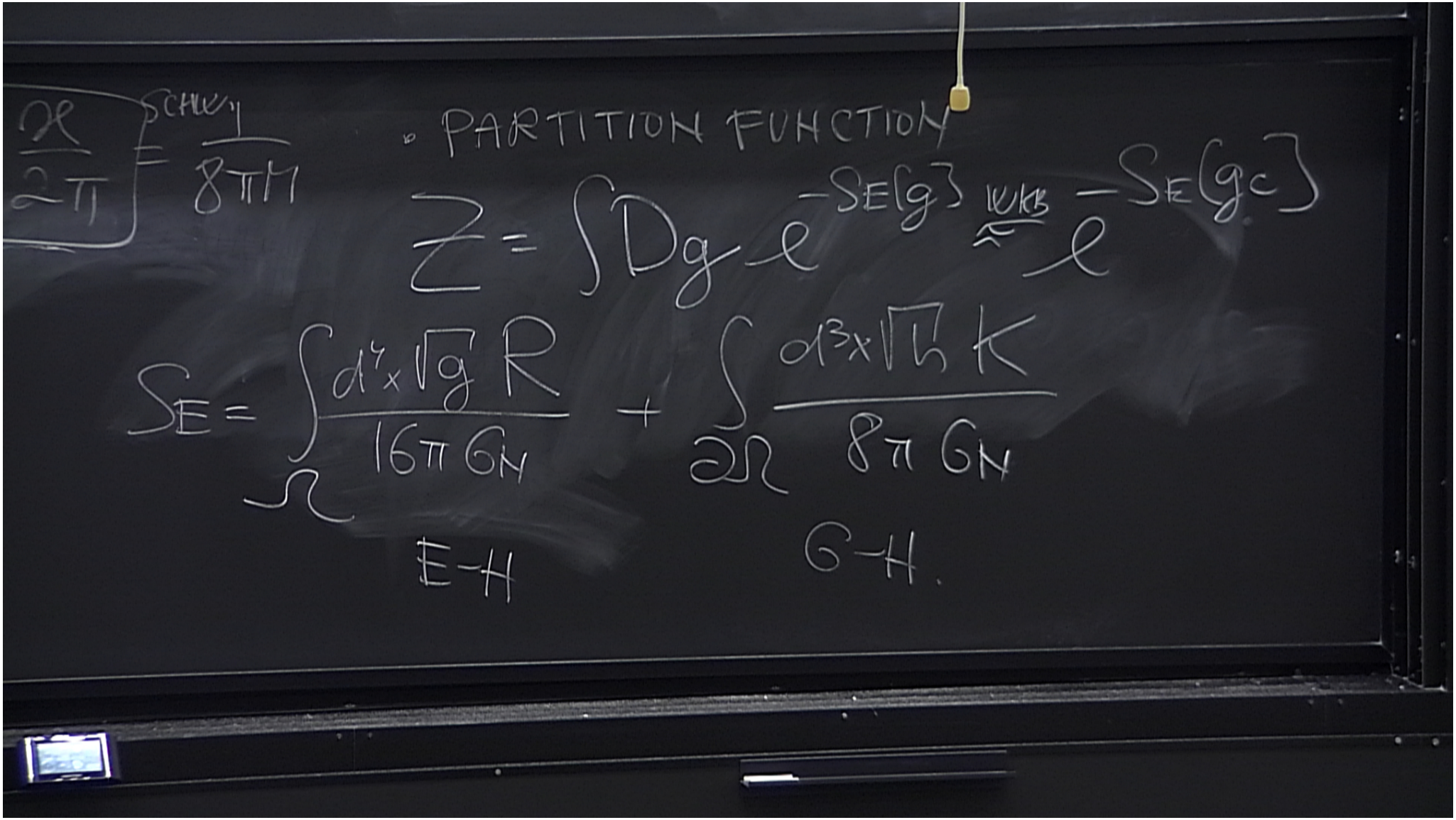
- EUCL. SCHW. $\simeq$ CIGAR

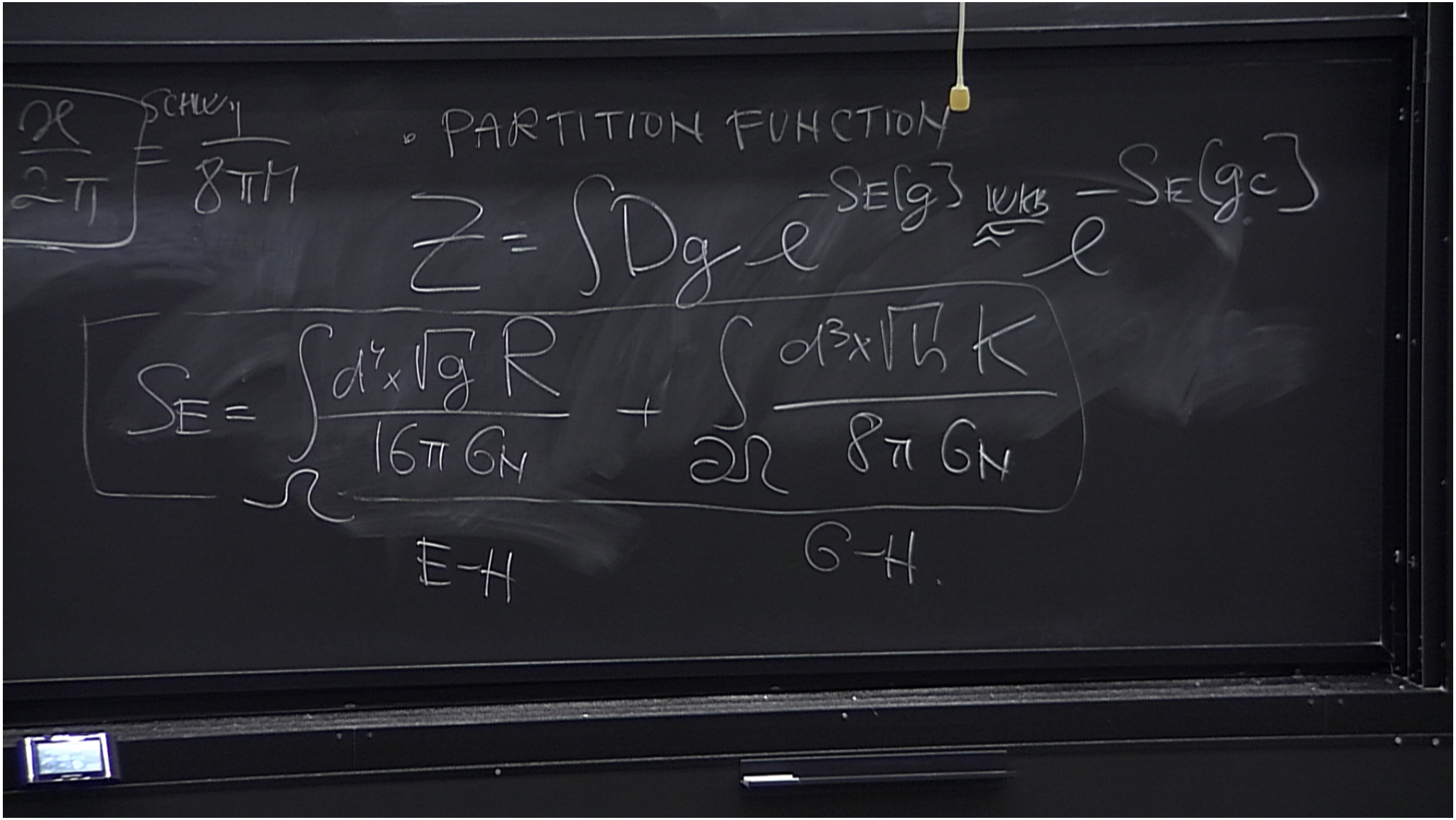


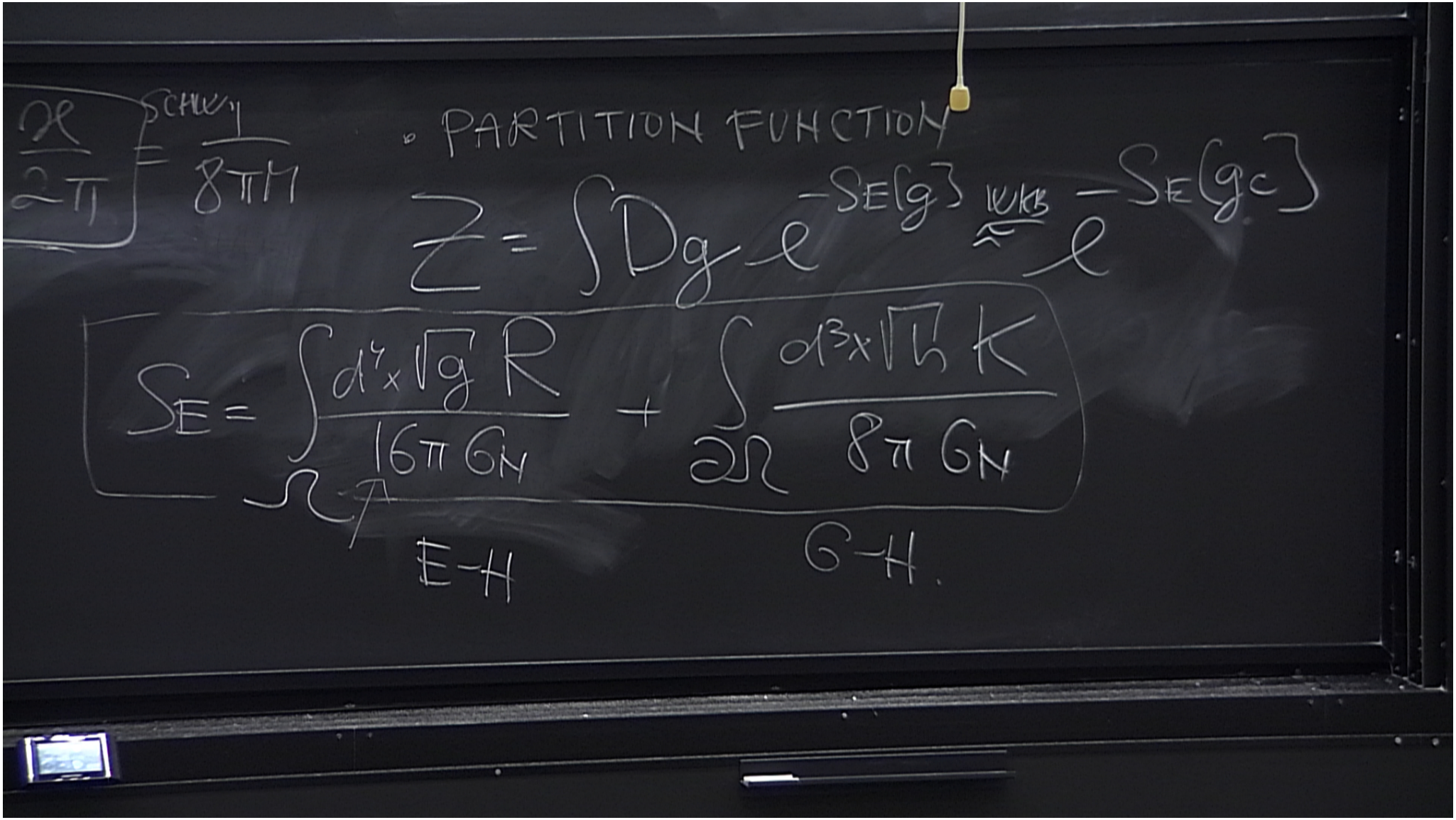

SCHW.
PARTITION FUNCTION
$Z = \int Dg\, e^{-S_E[g]} \approx e^{-S_E[g_c]}$


SCHW.
PARTITION FUNCTION
WKB


SCHWARZ
PARTITION FUNCTION
WKB


PARTITION FUNCTION
$\frac{\alpha}{2\pi}$ SCHW. $\frac{1}{8\pi G_N}$
$Z = \int Dg\, e^{-S_E[g]} \overset{WKB}{\approx} e^{-S_E[g_c]}$
$S_E = \int_{\Sigma} \frac{d^2x\sqrt{g}R}{16\pi G_N} + \int_{\partial\Sigma} \frac{d^3x\sqrt{h}K}{8\pi G_N}$
E-H
G-H.

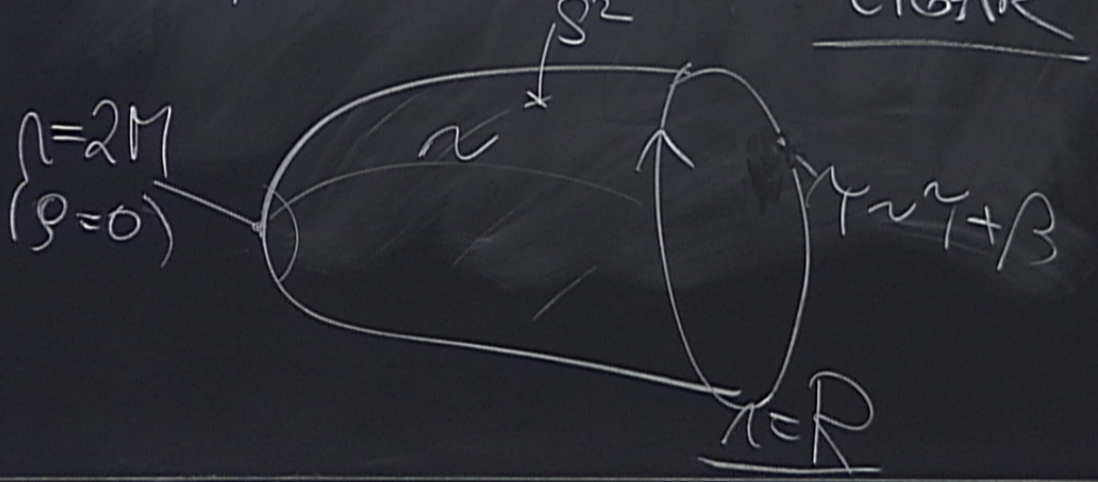

SCHW.
PARTITION FUNCTION
WKB
E-H
G-H


SCHW.
PARTITION FUNCTION
WKB
E-H
G-H.

- FOR VACUUM SOLUTION $R=0$ $\Rightarrow$ ONLY THE SECOND TERM CONTRIBUTES.

$\Rightarrow \tau \sim \tau + \underbrace{\frac{2\pi}{\partial \mathcal{E}}}_{\beta} \Rightarrow \boxed{T = 1/\beta = \ldots}$

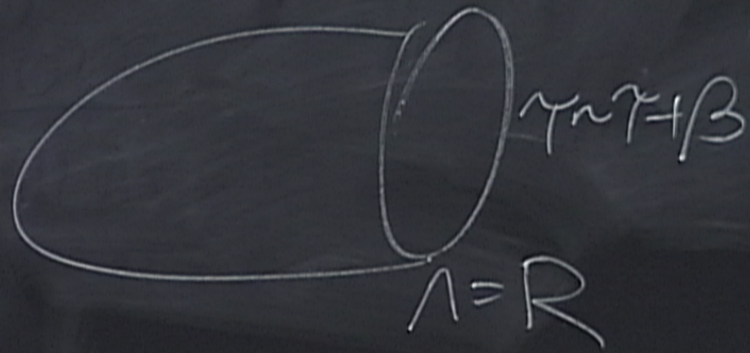
- EUCL. SCHW. $\simeq$ CIGAR



- FOR VACUUM SOLUTION $R=0$ => ONLY THE SECOND TERM CONTRIBUTES.
- ... IMPOSE CUTOFF $\Lambda = R$, $R \to \infty$

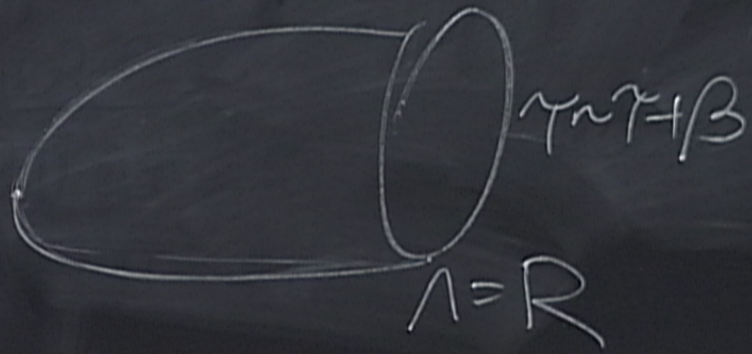
RENORMALIZATION REQUIRED

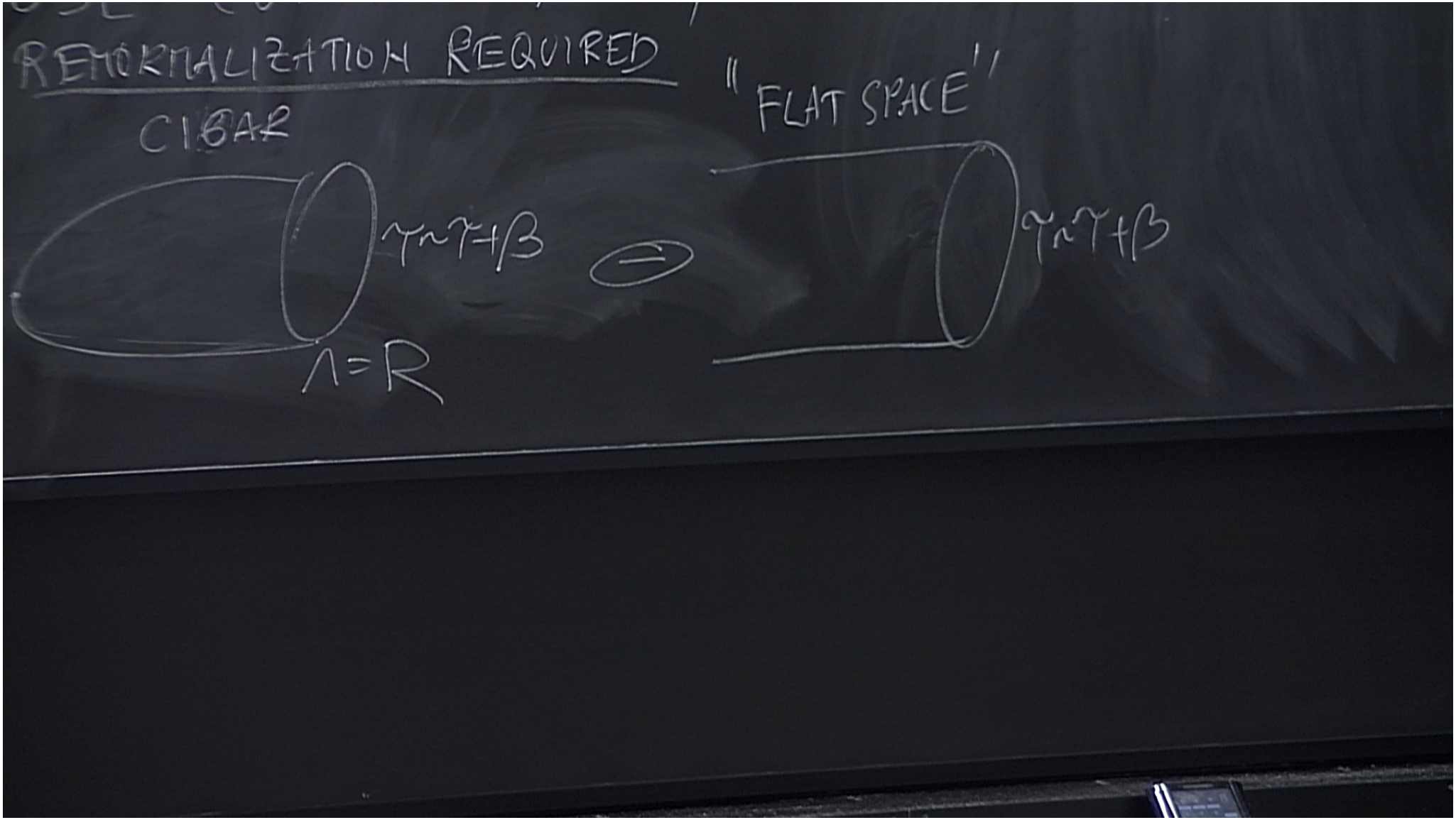
CLEAR

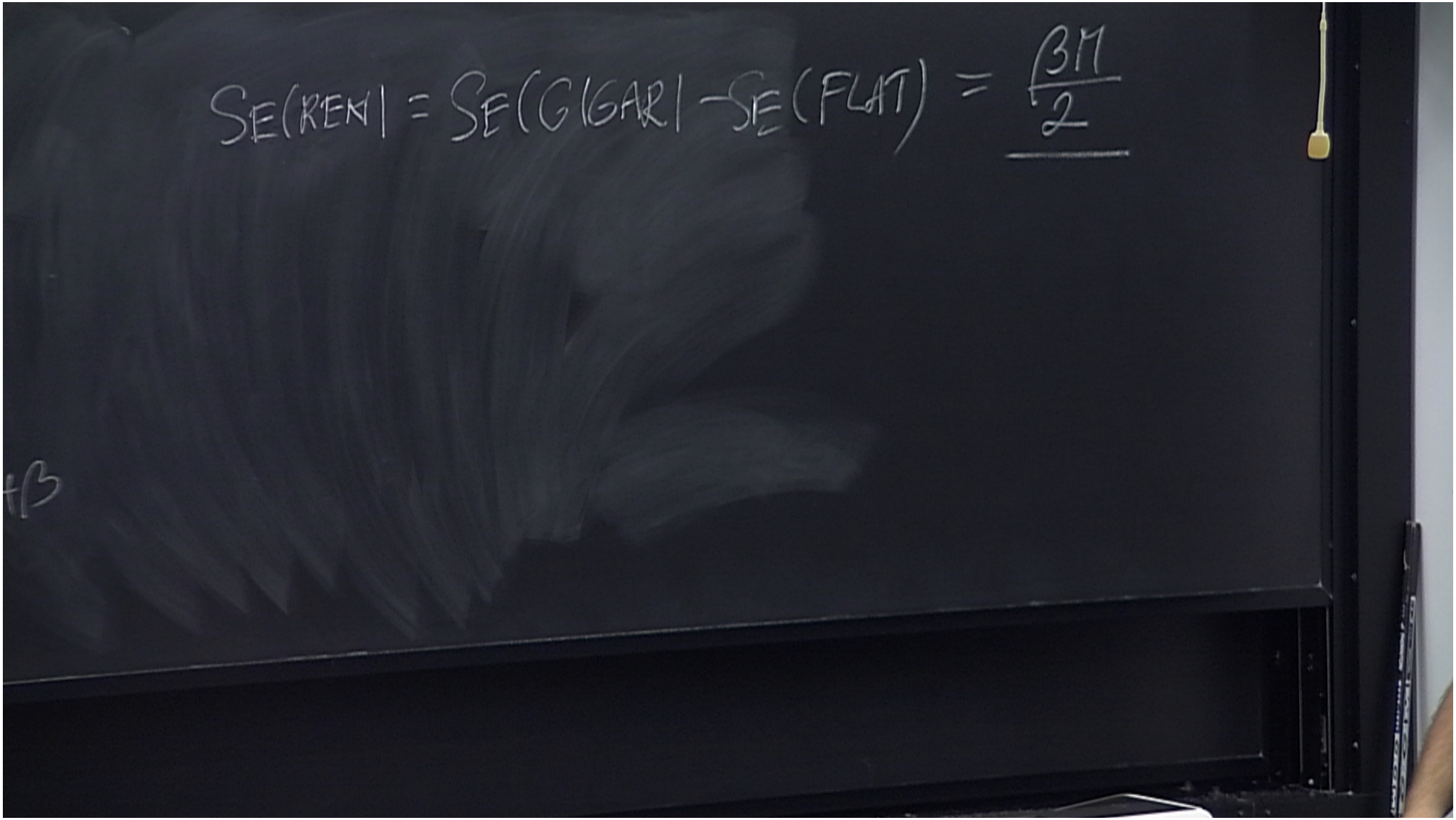


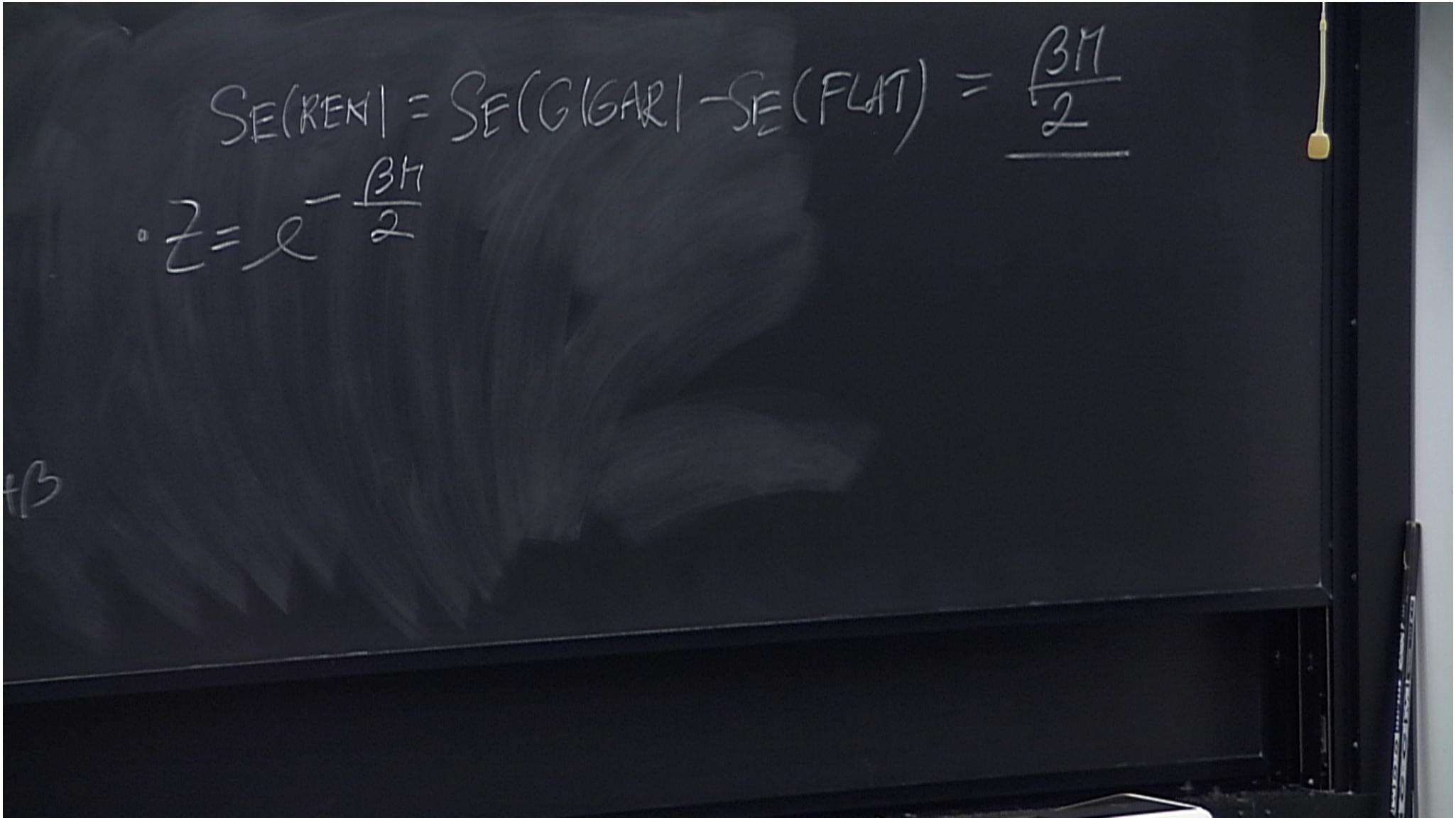
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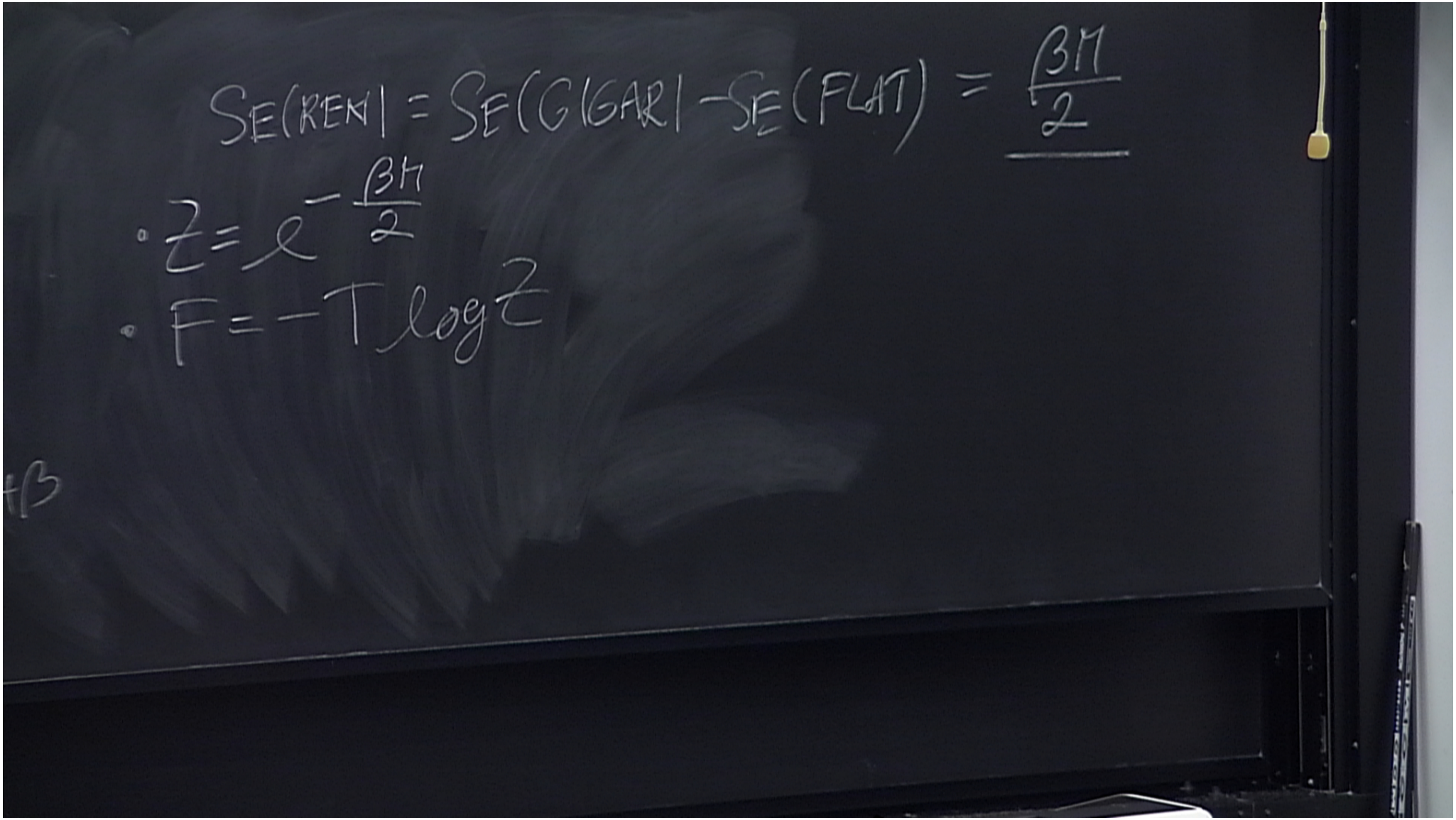
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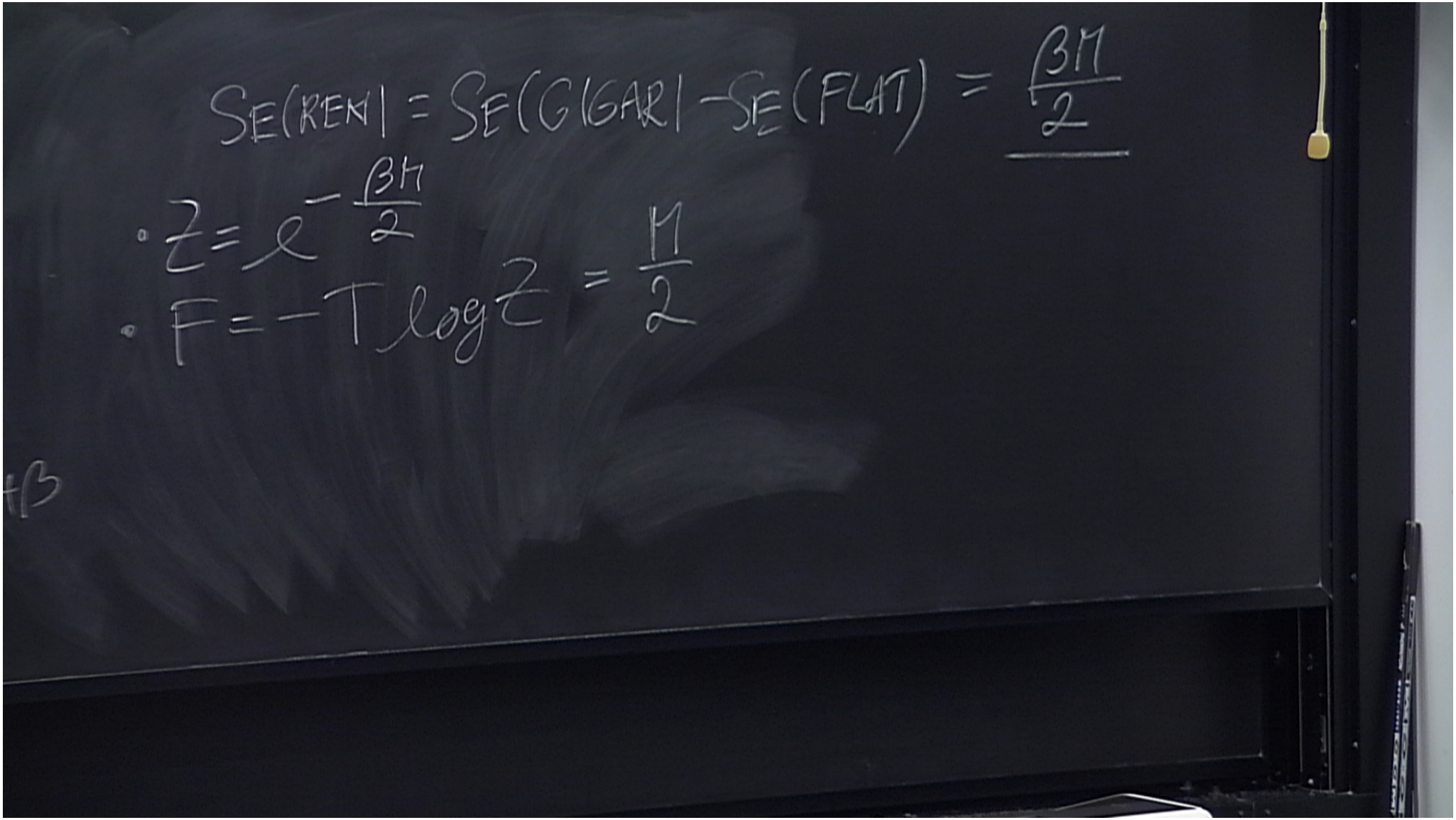


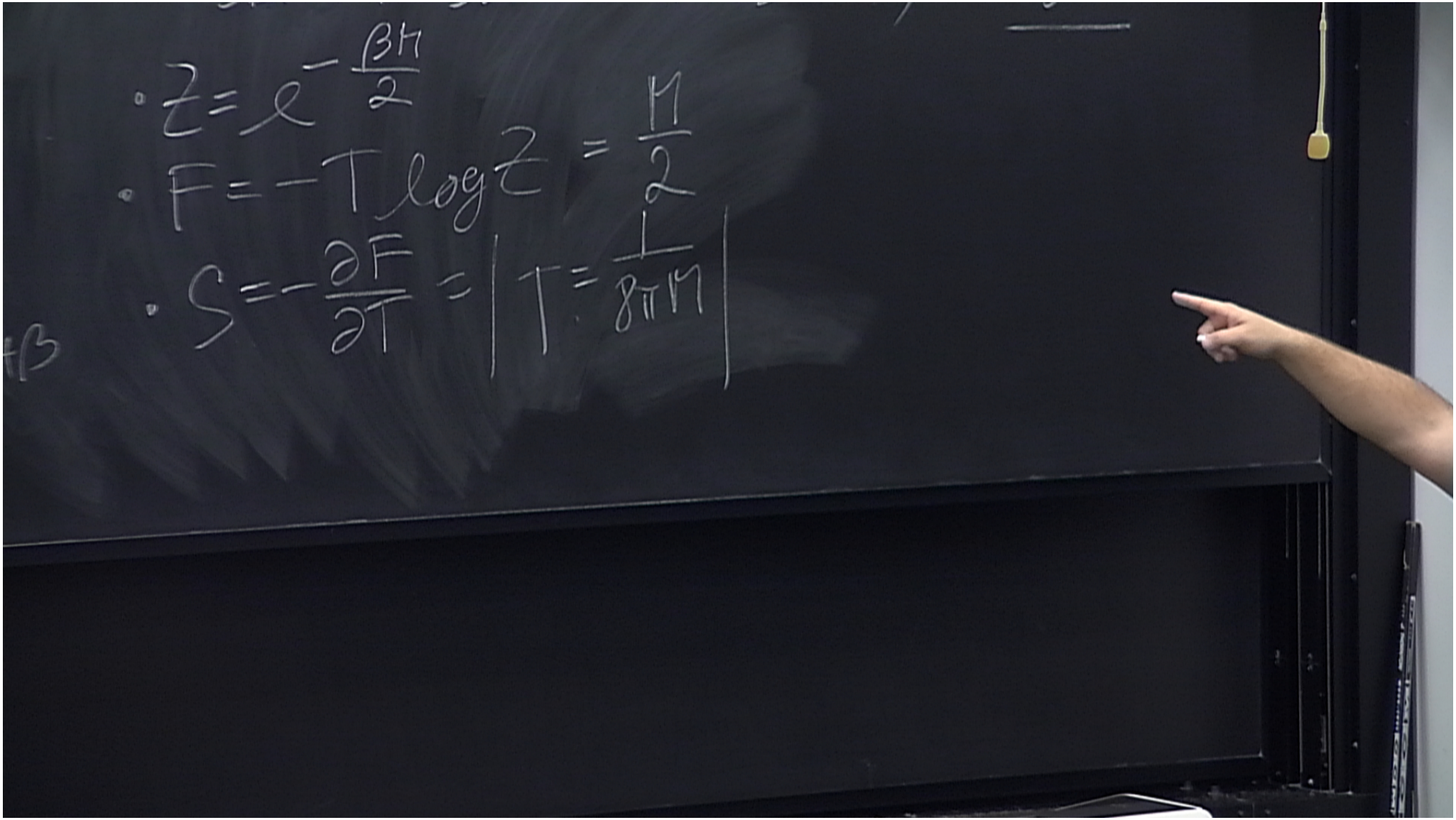

RENORMALIZATION REQUIRED
CIGAR
τ~τ+β
Λ=R
"FLAT SPACE"
τ~τ+β

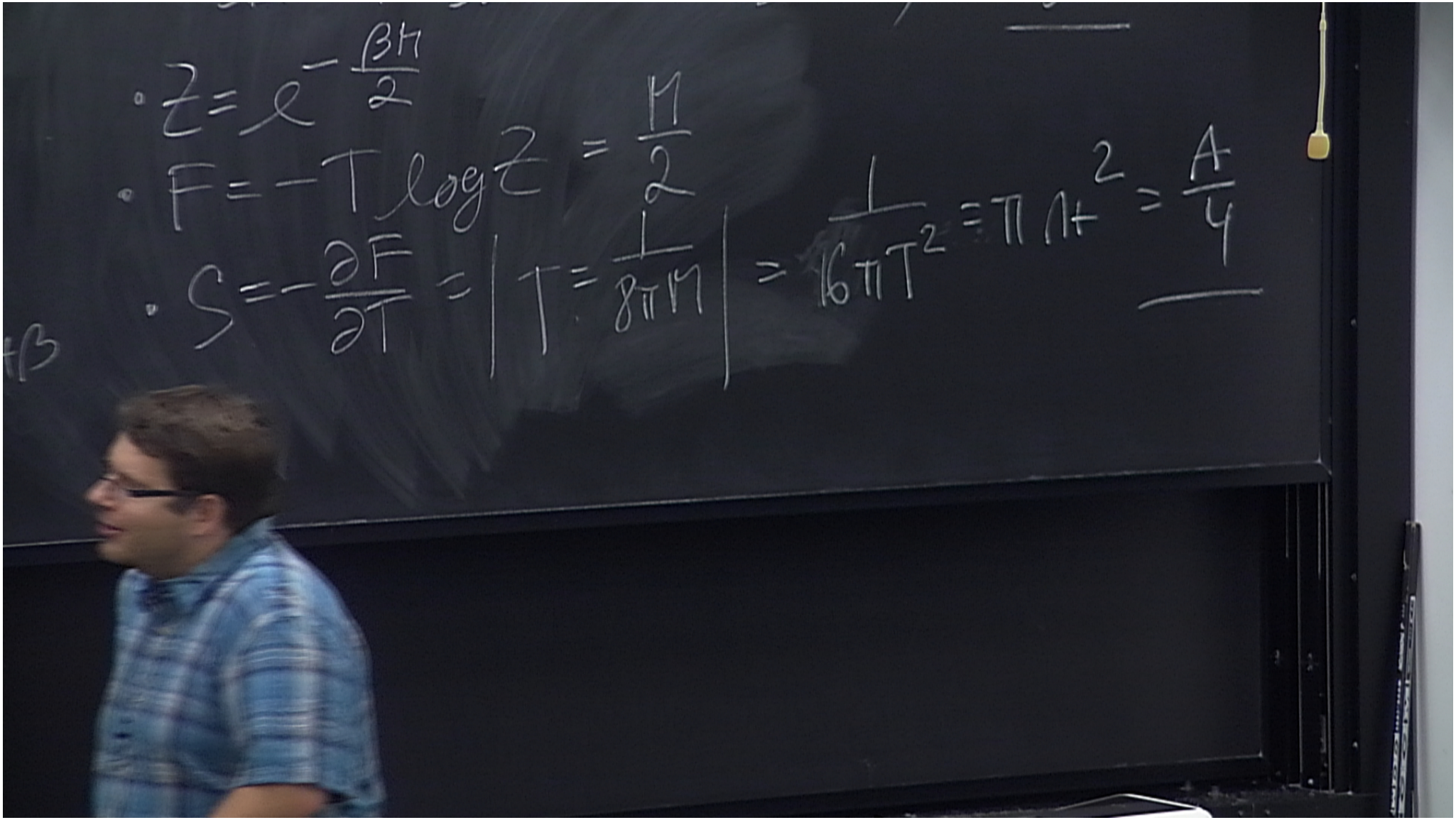

$S_E(REM) = S_E(GIGAR) - S_E(FLAT) = \frac{BM}{2}$

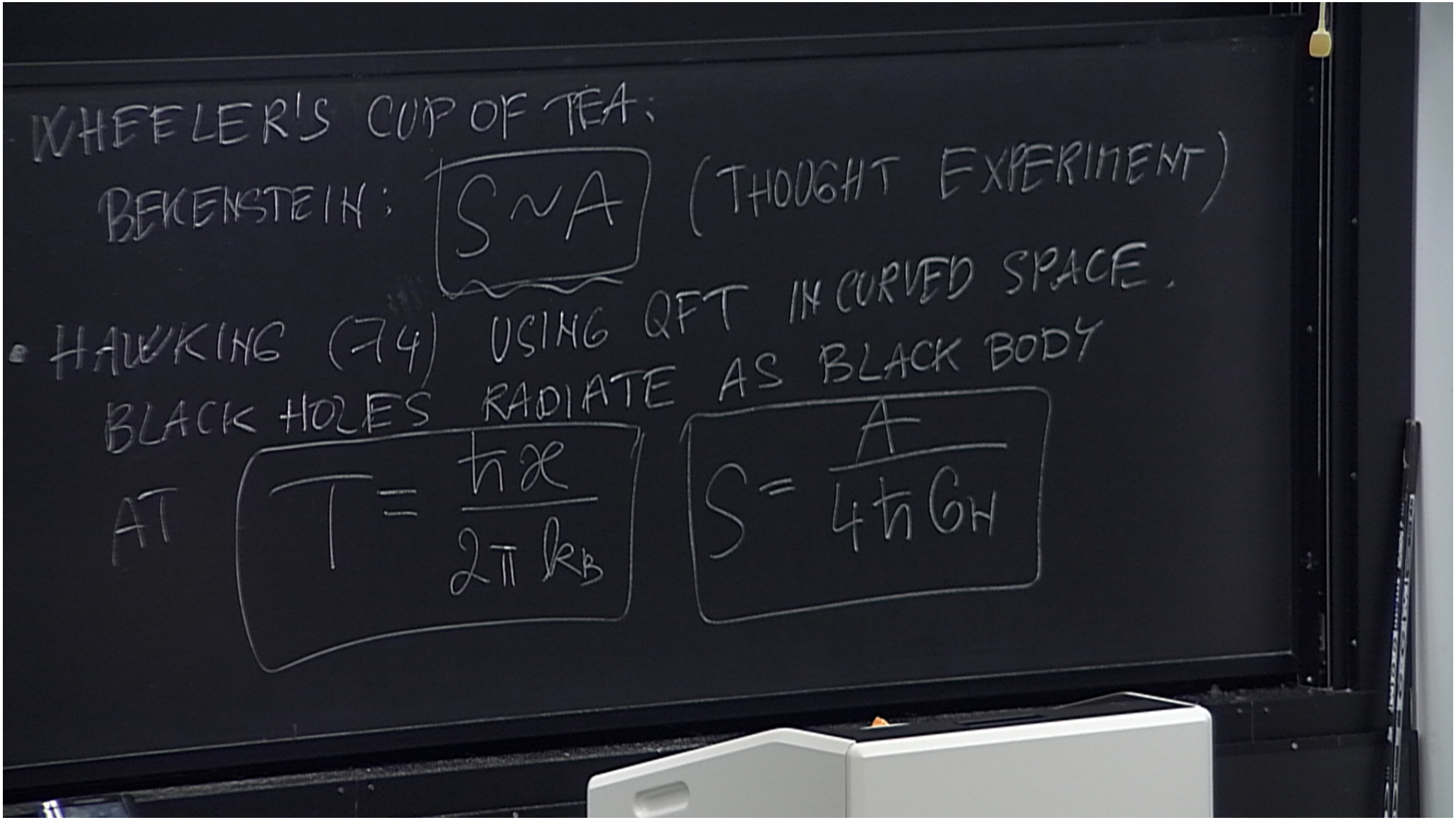

$S_E(REM) = S_E(GIGAR) - S_E(FLAT) = \frac{\beta M}{2}$
$Z = e^{-\frac{\beta M}{2}}$

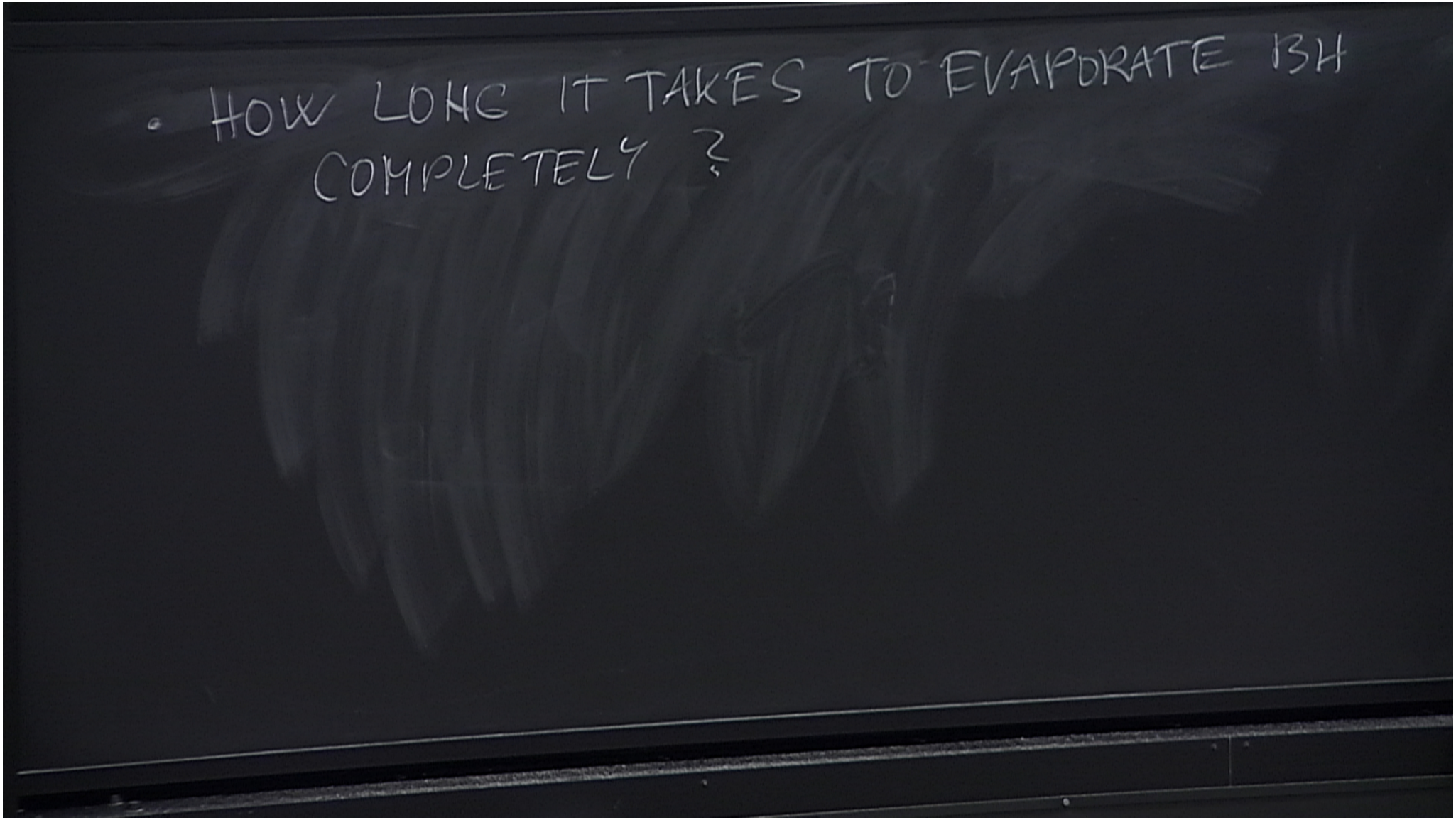

$S_E(REM) = S_E(GIGAR) - S_E(FLAT) = \frac{\beta M}{2}$
$Z = e^{-\frac{\beta M}{2}}$
$F = -T \log Z$


$S_E(REM) = S_E(GIGAR) - S_E(FLAT) = \frac{\beta H}{2}$
$Z = e^{-\frac{\beta H}{2}}$
$F = -T \log Z = \frac{H}{2}$


$Z = e^{-\frac{\beta H}{2}}$
$F = -T \log Z = \frac{H}{2}$
$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right|$


$Z = e^{-\frac{\beta H}{2}}$
$F = -T \log Z = \frac{H}{2}$
$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi r_+^2 = \frac{A}{4}$


WHEELER'S CUP OF TEA:
BEKENSTEIN: $S \sim A$ (THOUGHT EXPERIMENT)
HAWKING (-74) USING QFT IN CURVED SPACE.
BLACK HOLES RADIATE AS BLACK BODY
AT $T = \frac{\hbar \varkappa}{2\pi k_B}$
$S = \frac{A}{4\hbar G_H}$

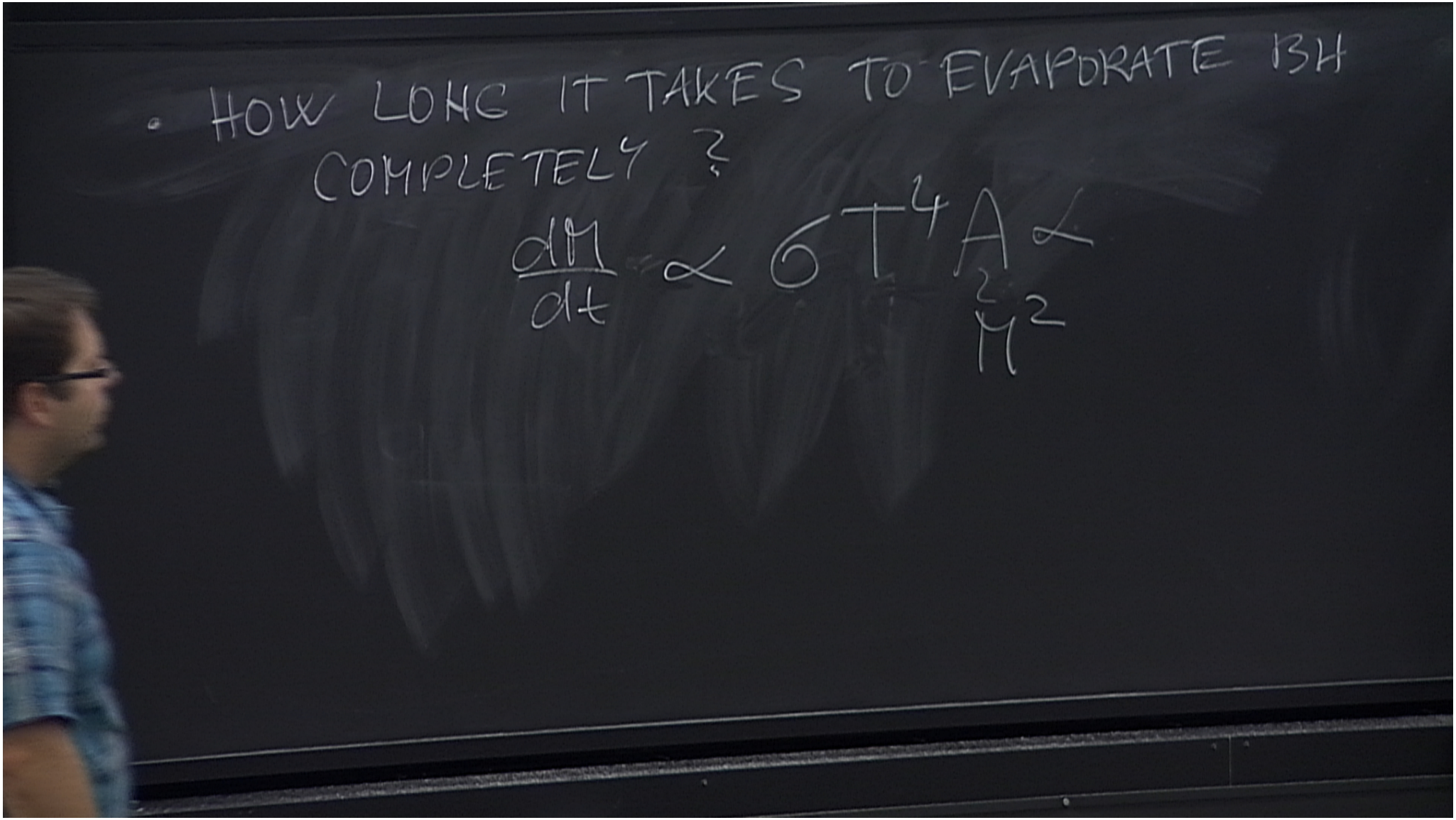

• HOW LONG IT TAKES TO EVAPORATE BH
COMPLETELY ?

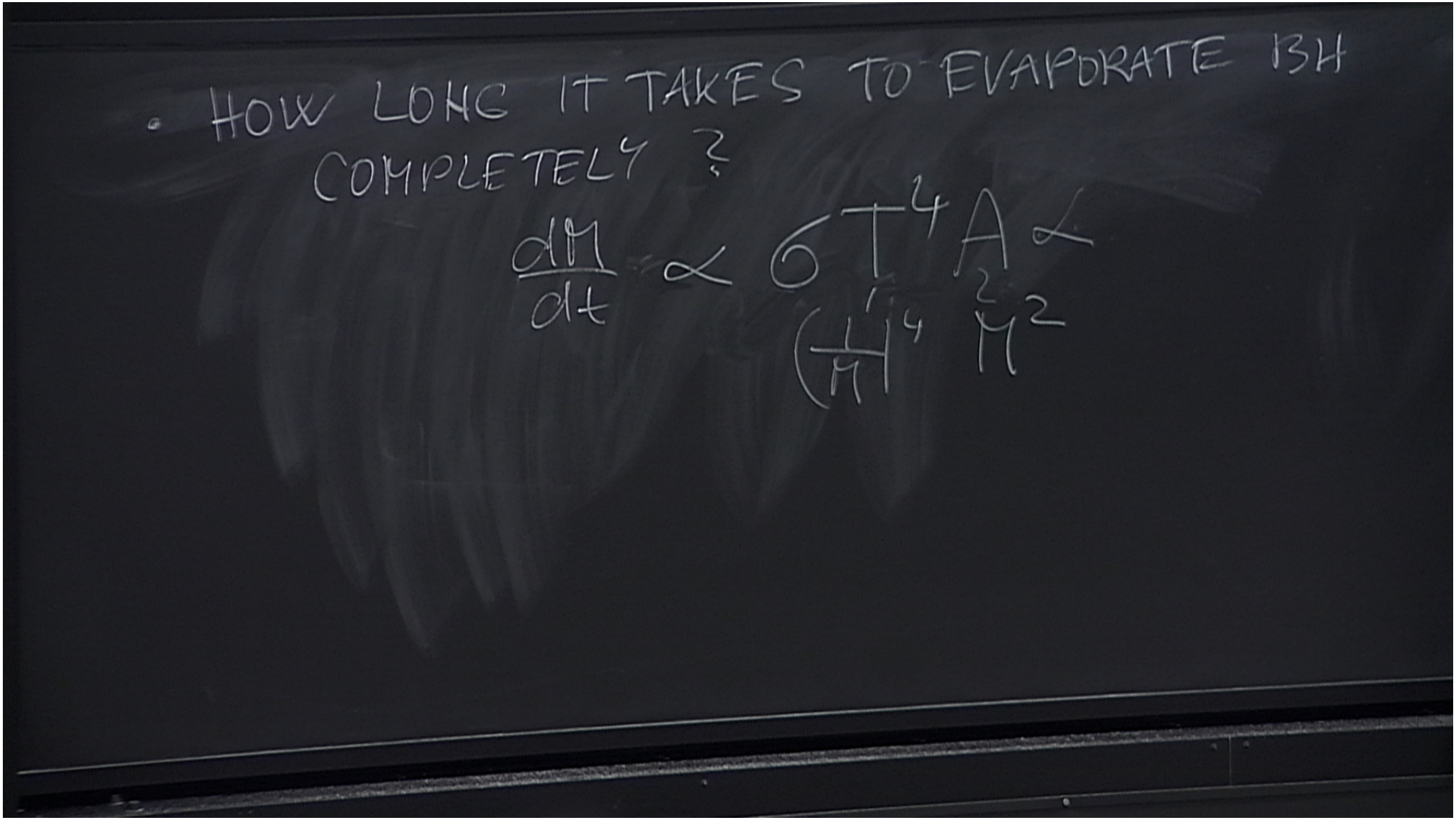
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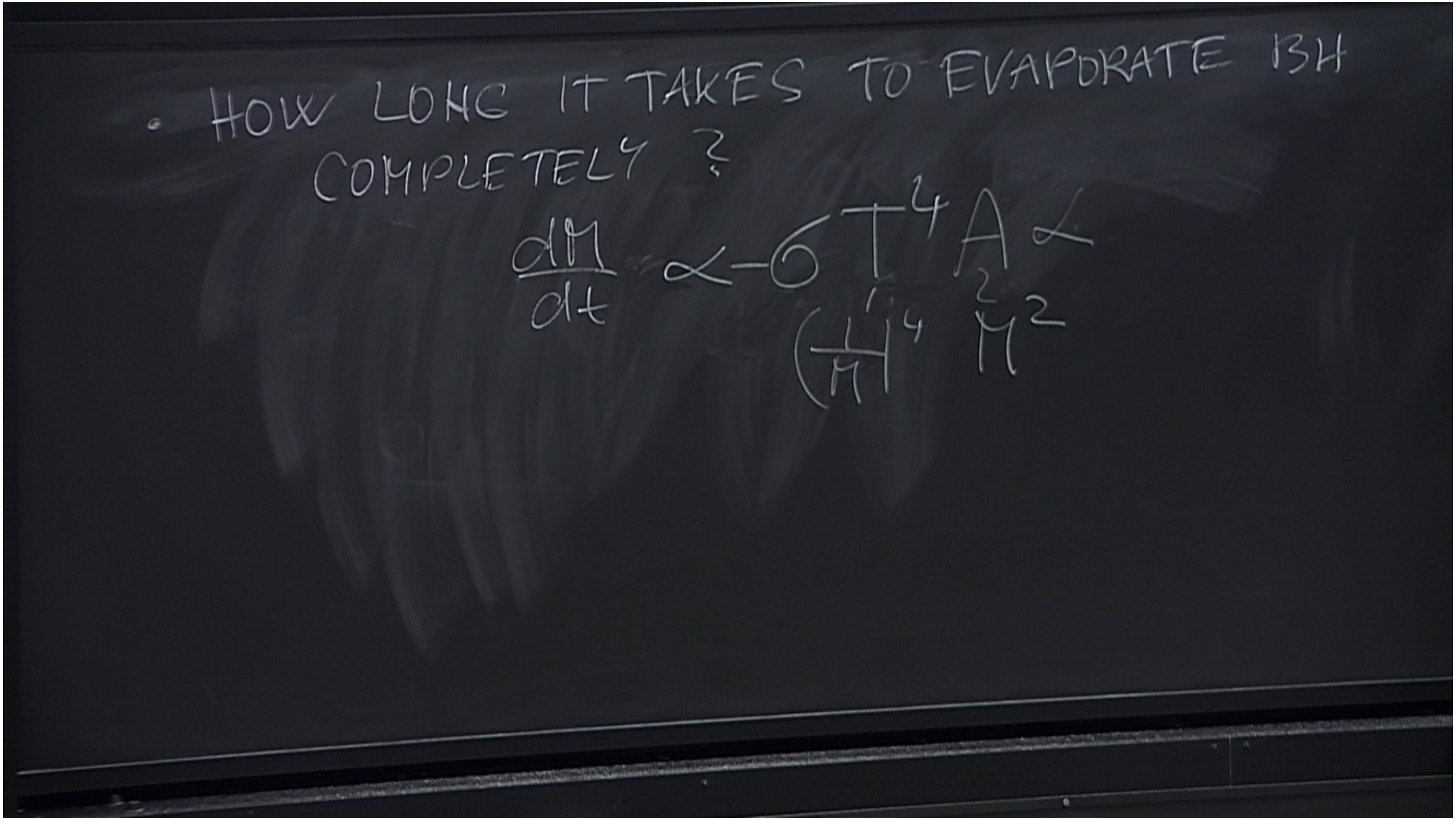
$$\frac{dM}{dt} \propto \sigma T^4$$

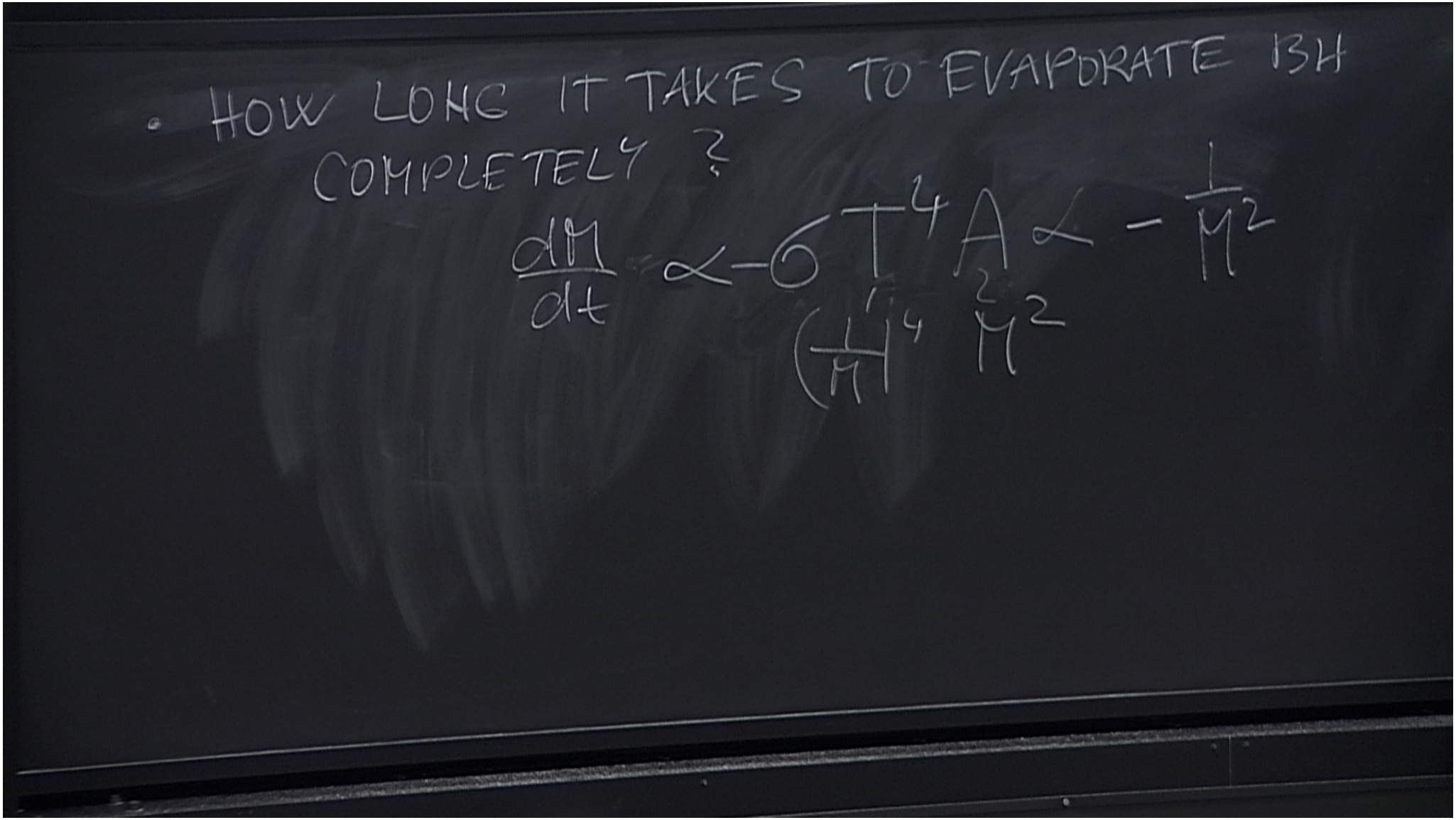
- HOW LONG IT TAKES TO EVAPORATE BH COMPLETELY ?

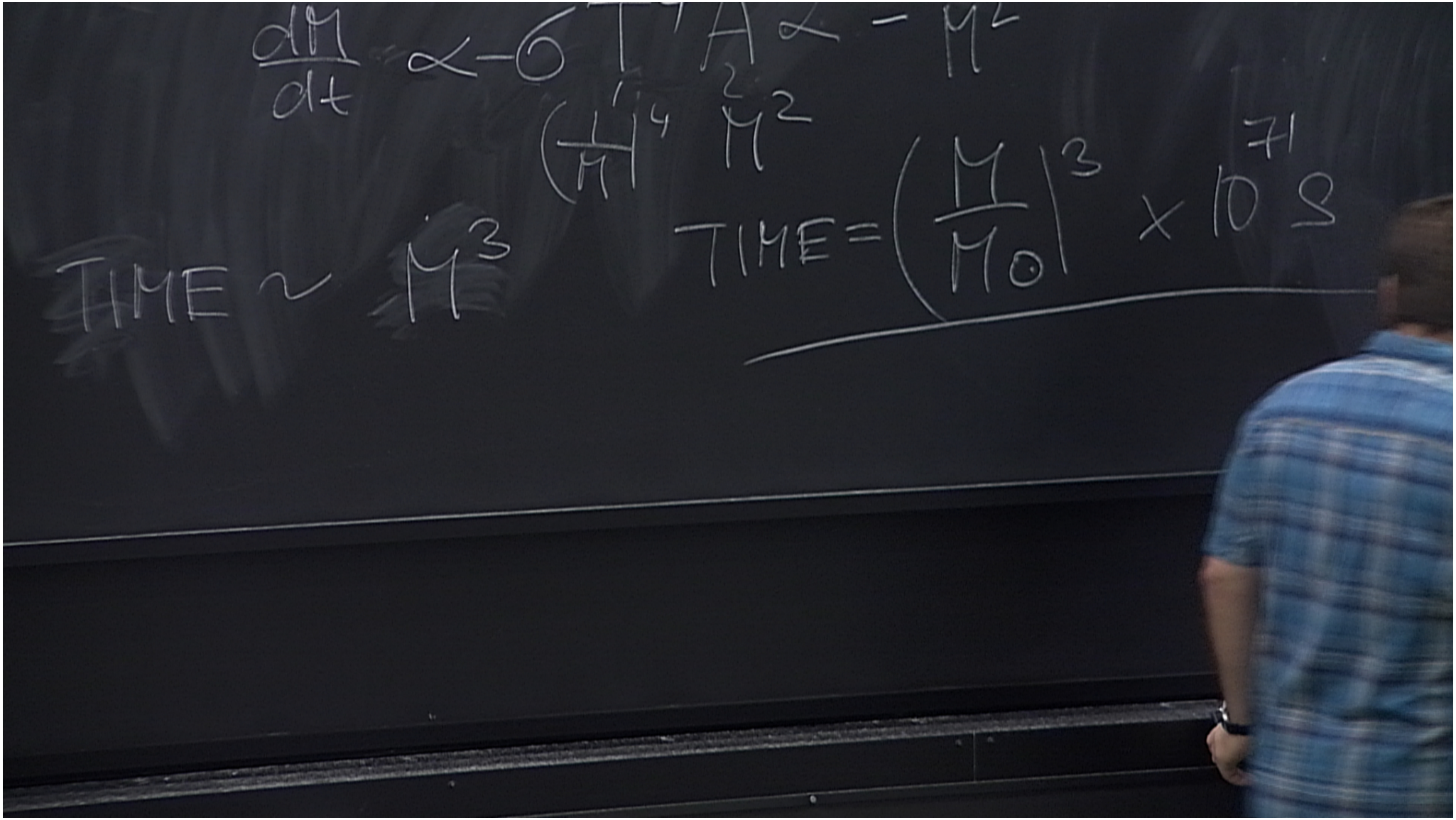
$$\frac{dM}{dt} \propto \sigma T^4 A$$

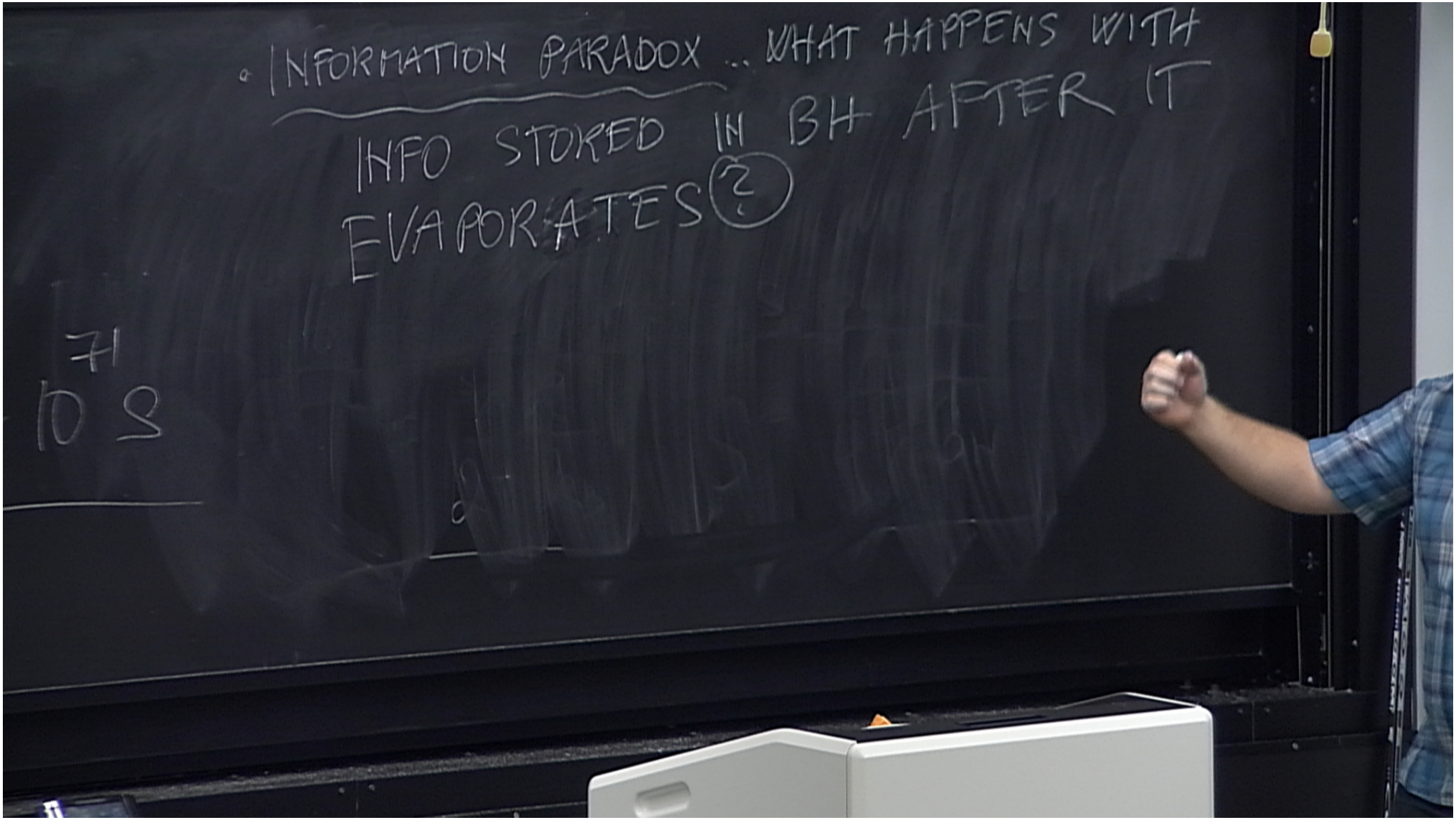

• HOW LONG IT TAKES TO EVAPORATE BH
COMPLETELY ?
$\frac{dM}{dt} \propto \sigma T^4 A \propto \frac{1}{M^2}$


• HOW LONG IT TAKES TO EVAPORATE BH
COMPLETELY ?
$\frac{dM}{dt} \propto \sigma T^4 A \propto$
$\left(\frac{1}{M}\right)^4 M^2$


• HOW LONG IT TAKES TO EVAPORATE BH
COMPLETELY ?
$\frac{dM}{dt} \propto -\sigma T^4 A \propto \left(\frac{1}{M}\right)^4 M^2$


• HOW LONG IT TAKES TO EVAPORATE BH
COMPLETELY ?
$\frac{dM}{dt} \propto -\sigma T^4 A \propto -\frac{1}{M^2}$
$\left(\frac{1}{M}\right)^4 M^2$


$\frac{dM}{dt} \propto -6$
$\left(\frac{1}{M}\right)^4 M^2$
$TIME \sim M^3$
$TIME = \left(\frac{M}{M_0}\right)^3 \times 10^{71} S$


INFORMATION PARADOX .. WHAT HAPPENS WITH
INFO STORED IN BH AFTER IT
EVAPORATES ?