Title: PSI 2016/2017 Statistical Mechanics - Lecture 11

Date: Oct 25, 2016 10:45 AM

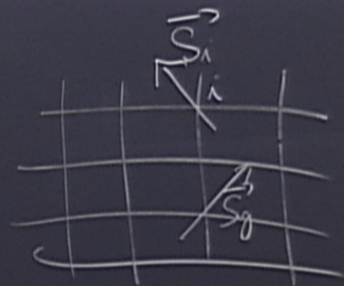
URL: http://pirsa.org/16100025

Abstract:

YESTERDAY

n-VECTOR MODEL

(d-DIM)



$\vec{S}_i$ ... n-VECTOR

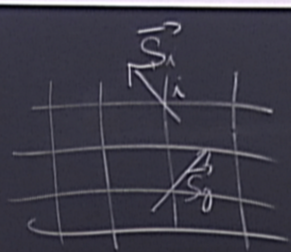
$$H = -\frac{1}{2} \sum_{ij} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

O(n) CONTINUOUS SYMMETRY

...Y

...R MODEL

(...M)



$\vec{S}_i$ ... n-VECTOR

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

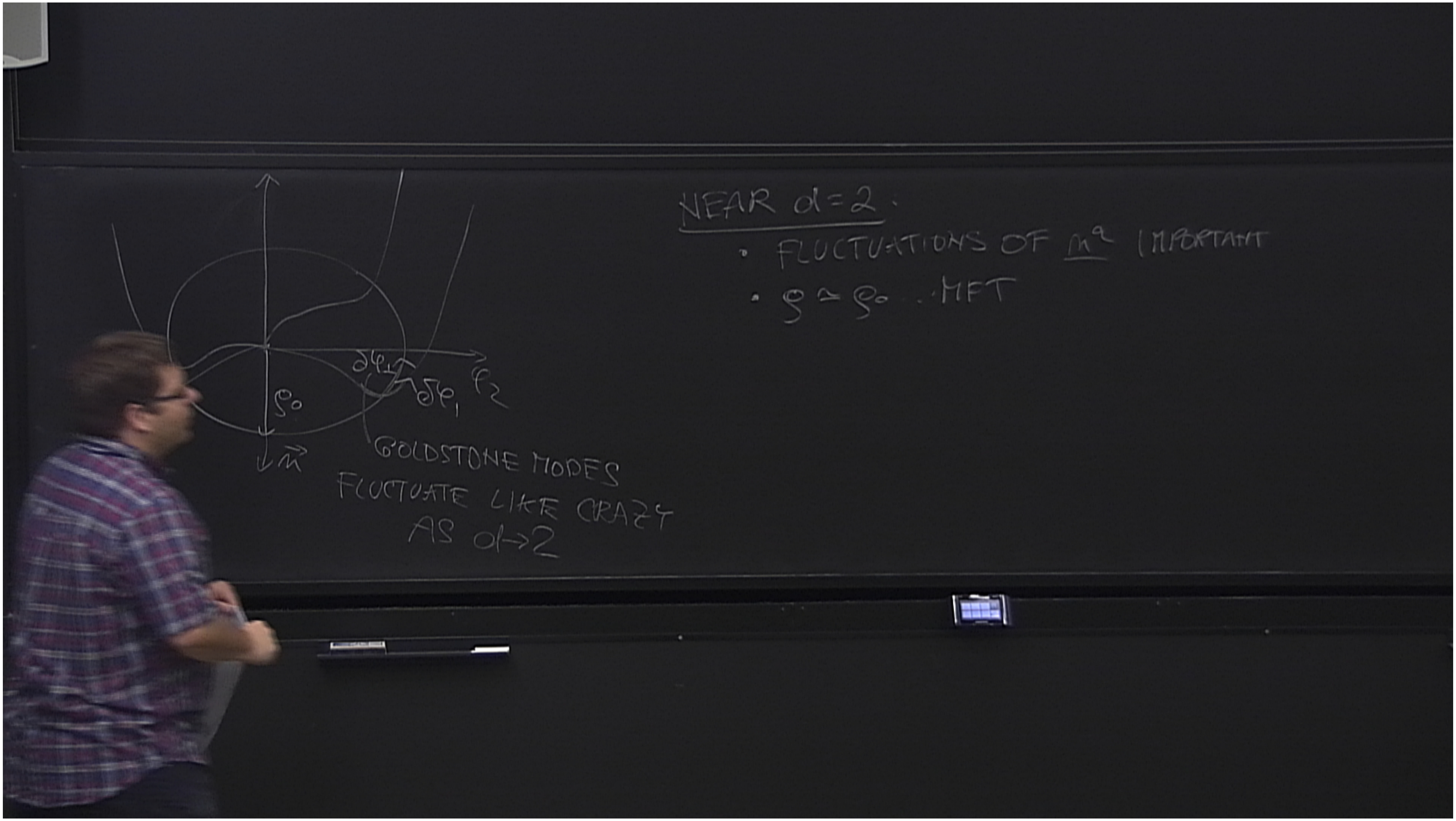
O(n) CONTINUOUS SYMMETRY

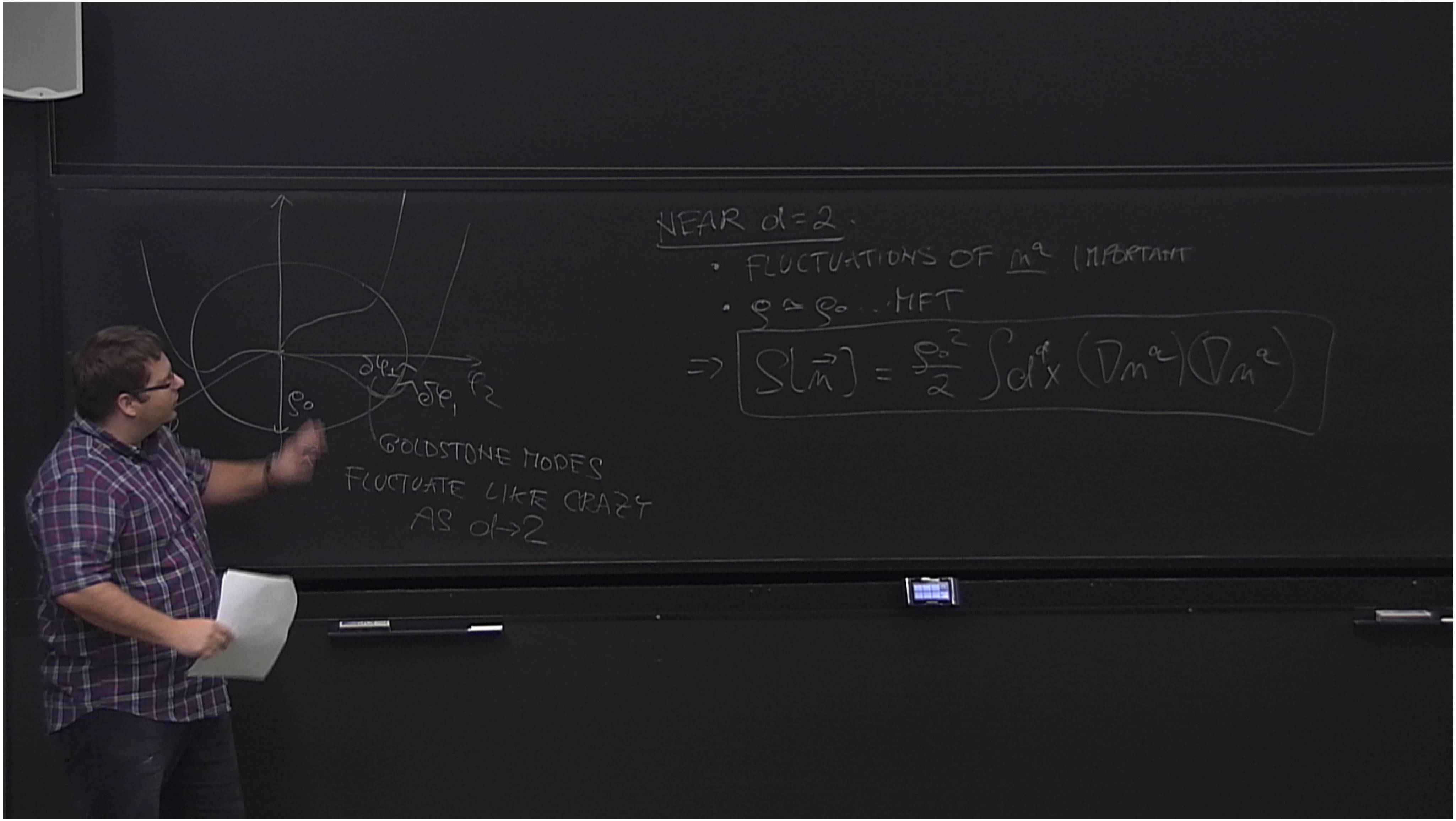
L-6

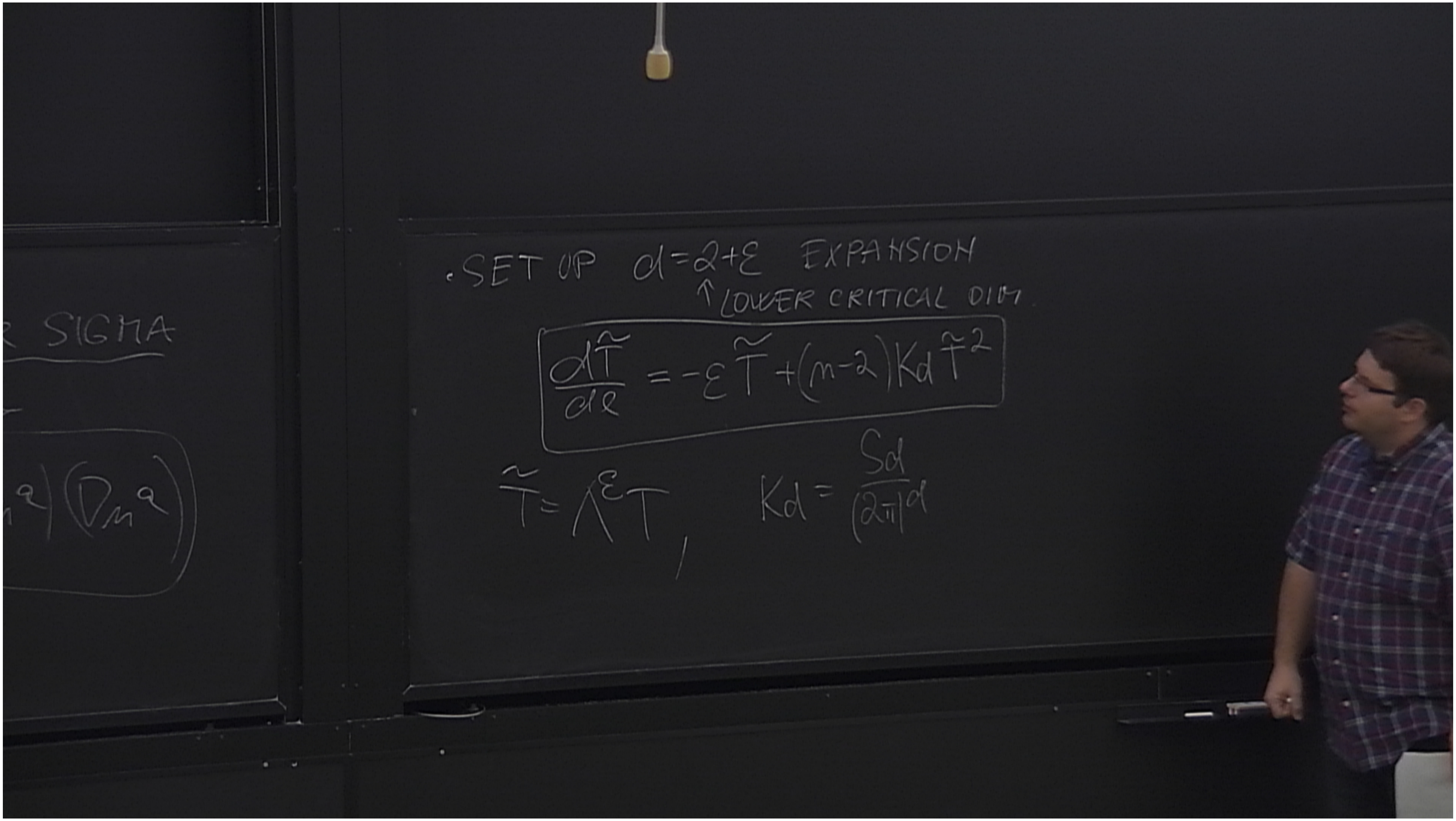
$$\varphi^a = \rho n^a \qquad a = 1, \dots, n$$

$$|\vec{n}| = 1$$

$$S[\rho, \vec{n}] = \int d^dx \left(\frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} \rho^2 (\nabla n^a)(\nabla n^a) + \frac{r}{2} \rho^2 + \frac{u}{4} \rho^4 \right)$$


NEAR d=2.
• FLUCTUATIONS OF n^a IMPORTANT
• ρ ≃ ρ0 .. MFT
ρ0
δφ1
φ2
GOLDSTONE MODES
FLUCTUATE LIKE CRAZY
AS d→2


NEAR $d=2$:
• FLUCTUATIONS OF $\underline{n}^a$ IMPORTANT
• $\rho \simeq \rho_0$ .. MFT
$\Rightarrow \boxed{S[\vec{n}] = \frac{\rho_0^2}{2} \int d^dx \, (\nabla n^a)(\nabla n^a)}$
ρ_0
$\delta\varphi_2$
$\delta\varphi_1$
φ_2
GOLDSTONE MODES
FLUCTUATE LIKE CRAZY
AS $d \to 2$


SIGMA
• SET UP d=2+ε EXPANSION
↑ LOWER CRITICAL DIM
dT̃/dl = −εT̃ + (n−2) Kd T̃²
T̃ = Λ^ε T,
Kd = Sd/(2π)^d

• SET UP $d=2+\varepsilon$ EXPANSION

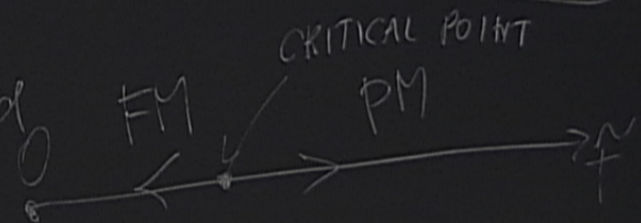
↑ LOWER CRITICAL DIM.

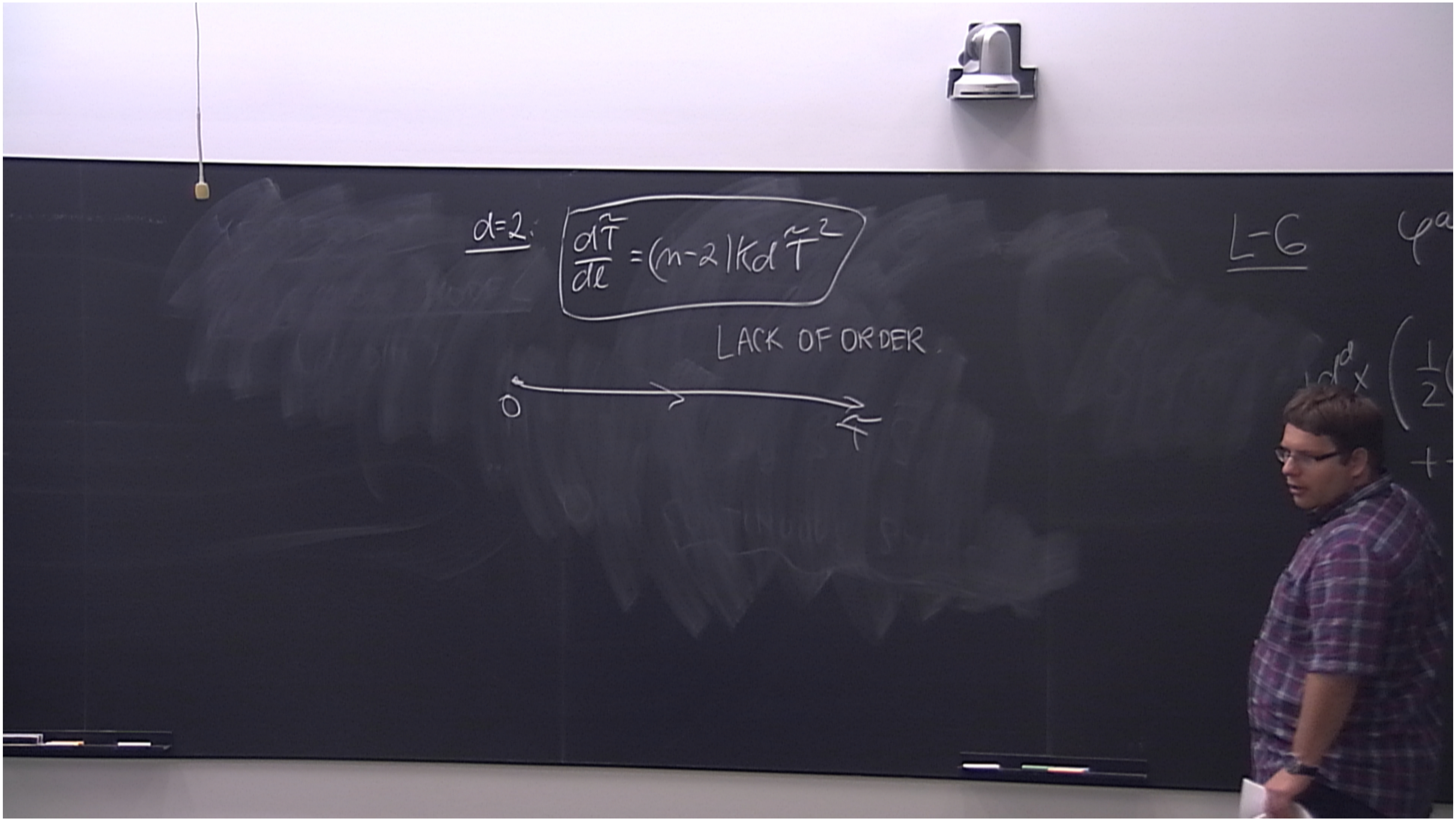
$$\boxed{\frac{d\tilde{T}}{dl} = -\varepsilon \tilde{T} + (n-2) K_d \tilde{T}^2}$$

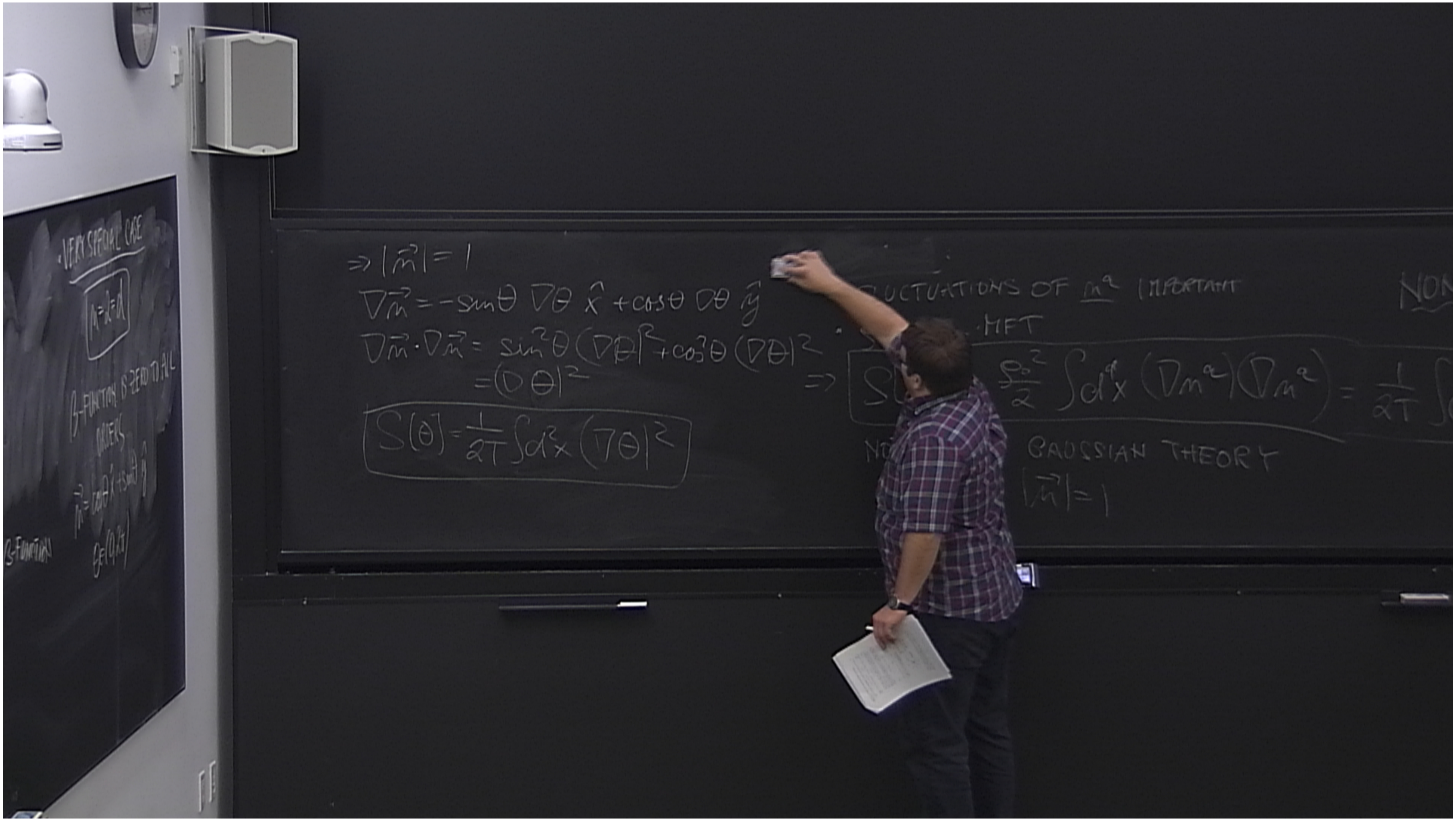
2 FIXED POINTS

$$\boxed{\begin{aligned} T^* &= 0 \\ T^* &= \frac{\varepsilon}{(n-2)K_d} \end{aligned}}$$

$\tilde{T} = \Lambda^{\varepsilon} T$, $\quad K_d = \dfrac{S_d}{(2\pi)^d}$




d=2:
$\frac{d\tilde{T}}{dl} = (n-2)K_d \tilde{T}^2$
LACK OF ORDER
L-G


VERY SPECIAL CASE
β-FUNCTION IS ZERO TO ALL ORDERS
$\Rightarrow |\vec{m}| = 1$
$\nabla\vec{m} = -\sin\theta\,\nabla\theta\,\hat{x} + \cos\theta\,\nabla\theta\,\hat{y}$
$\nabla\vec{m}\cdot\nabla\vec{m} = \sin^2\theta(\nabla\theta)^2 + \cos^2\theta(\nabla\theta)^2$
$= (\nabla\theta)^2$
$S(\theta) = \frac{1}{2T}\int d^2x\,(\nabla\theta)^2$
UCTUATIONS OF m^a IMPORTANT
MFT
GAUSSIAN THEORY
$|\vec{m}| = 1$

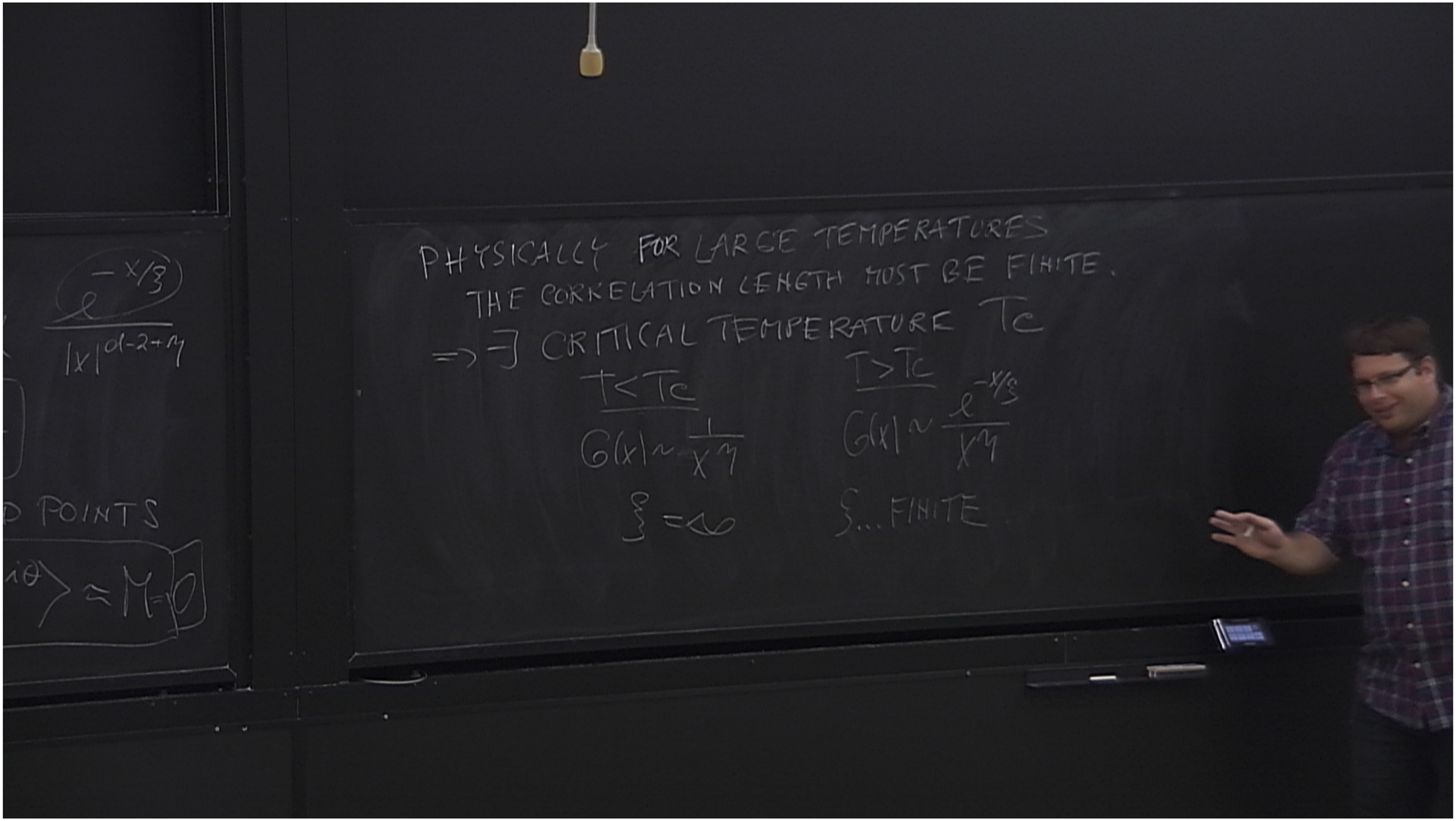
• LET'S PRETEND THAT θ IS NOT COMPACTIFIED!

$$G(\vec{x}) = \left\langle e^{i\theta(x)}\, e^{-i\theta(0)} \right\rangle = = \dots = \left(\frac{a}{x}\right)^{\frac{T}{2\pi}} \propto \frac{e^{-x/\xi}}{|x|^{d-2+\eta}}$$

$$\boxed{\frac{dT}{dl} = 0} \qquad \boxed{\xi \to \infty}\,,\ \boxed{\eta = \frac{T}{2\pi}}$$

SEEMS LIKE A LINE OF FIXED POINTS
(CRITICAL POINTS)

BUT THERE IS NO LONG-RANGE ORDER $\boxed{\langle e^{i\theta} \rangle \simeq M = 0}$


PHYSICALLY FOR LARGE TEMPERATURES
THE CORRELATION LENGTH MUST BE FINITE.
$\Rightarrow \exists$ CRITICAL TEMPERATURE T_c
$T<T_c$
$G(x) \sim \frac{1}{x^\eta}$
$\xi = \infty$
$T>T_c$
$G(x) \sim \frac{e^{-x/\xi}}{x^\eta}$
ξ... FINITE.
$\frac{e^{-x/\xi}}{|x|^{d-2+\eta}}$
POINTS
$\simeq M = 0$

$$\frac{e^{-x/\xi}}{|x|^{d-2+\eta}}$$

...D POINTS

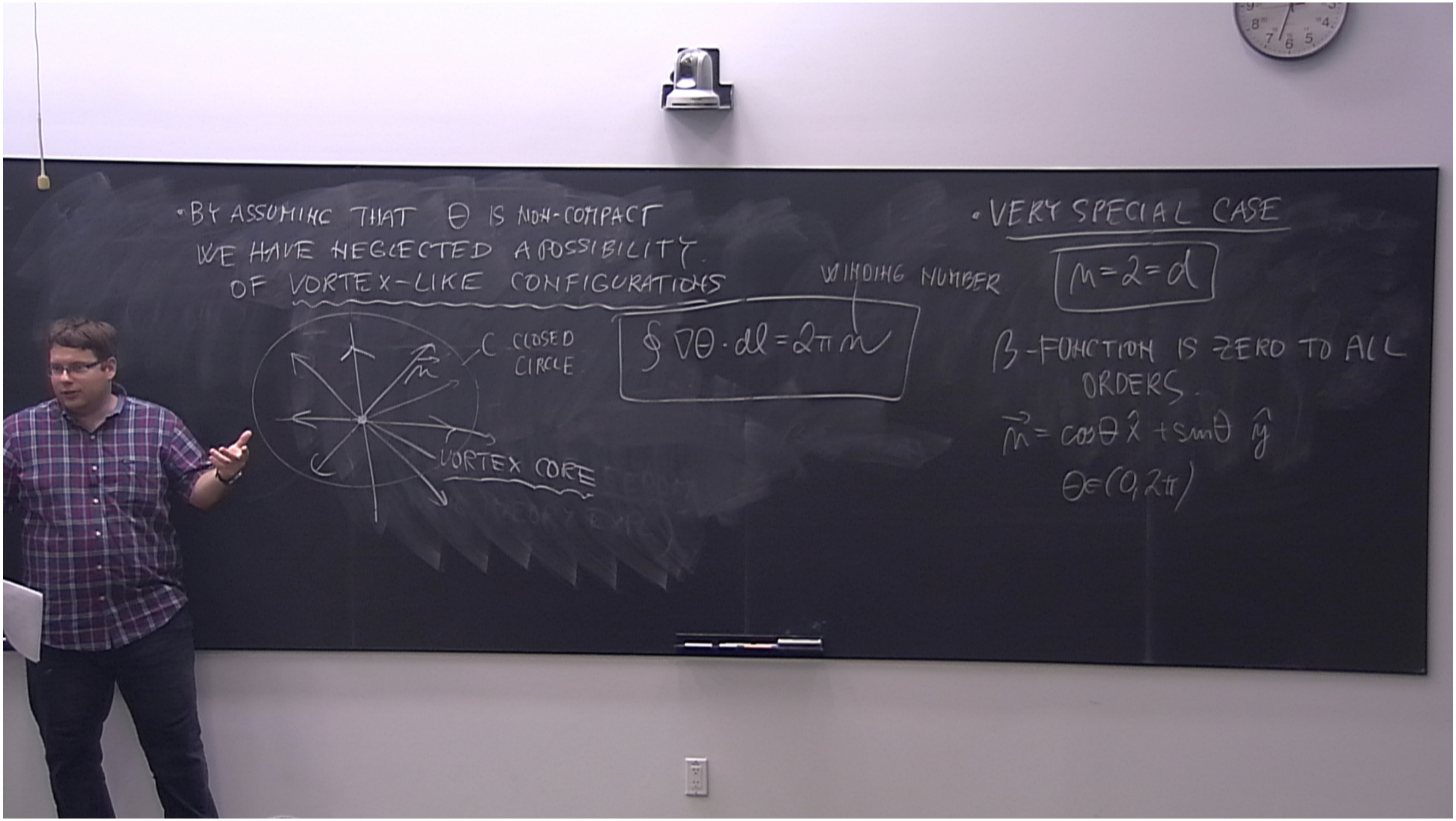
$$\ldots i\theta \rangle \simeq M = 0$$

PHYSICALLY FOR LARGE TEMPERATURES
THE CORRELATION LENGTH MUST BE FINITE.

$\Rightarrow \exists$ CRITICAL TEMPERATURE T_c

$T < T_c$		$T > T_c$
$G(x) \sim \frac{1}{x^{\eta}}$	$\longleftrightarrow$	$G(x) \sim \frac{e^{-x/\xi}}{x^{\eta}}$
$\xi = \infty$		ξ... FINITE

HAS NOTHING TO DO WITH LONG-RANGE ORDER


• BY ASSUMING THAT Θ IS NON-COMPACT
WE HAVE NEGLECTED A POSSIBILITY
OF VORTEX-LIKE CONFIGURATIONS
C CLOSED CIRCLE
VORTEX CORE
WINDING NUMBER
$\oint \nabla\theta \cdot dl = 2\pi n$
• VERY SPECIAL CASE
$n = 2 = d$
β-FUNCTION IS ZERO TO ALL ORDERS.
$\vec{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$
$\theta \in (0, 2\pi)$

IS NON-COMPACT

D A POSSIBILITY

E CONFIGURATIONS

C CLOSED CIRCLE

RTEX CORE

WINDING NUMBER

$$\boxed{\oint \nabla\theta \cdot dl = 2\pi n}$$

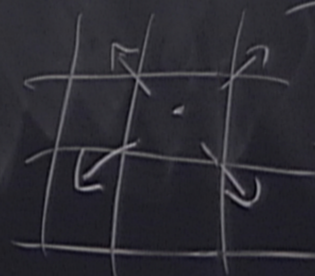
VORTEX CAN BE DETECTED FROM FAR AWAY;

COSTS INFINITE ENERGY

EXAMPLE OF A TOPOLOGICAL DEFECT

$$\boxed{\nabla\theta \sim \frac{1}{r}} \xrightarrow{r\to 0} \infty$$

CONTINUUM LIMIT BREAKS DOWN NEAR VORTEX CORE



$$|\theta_i - \theta_j| = \frac{\pi}{2}$$

• WHY CAN WE HAVE VORTICES ?

• $\underline{E}$ SINGLE VORTEX.

$$\boxed{H = \frac{1}{2} \int d^2x \, (\nabla\theta)^2}$$

WANT TO MINIMIZE
SUBJECT TO

$$\boxed{\oint_C \nabla\theta \cdot d\ell = 2\pi n}$$

…AVE VORTICES ?

VORTEX.

$$\boxed{\int d^2x\, (\nabla\theta)^2}$$

… MINIMIZE

…CT TO

$$\boxed{\oint \nabla\theta \cdot dl = 2\pi n} \checkmark$$

$$\delta H = \frac{1}{2}\int d^2x\, 2\, \nabla\delta\theta\, \nabla\theta = -\int d^2x\, (\nabla^2\theta)\, \delta\theta = 0$$

$$\boxed{\nabla^2\theta = 0} \Rightarrow \boxed{\theta = n\varphi}$$

POLAR ANGLE.

$$\Rightarrow \nabla\theta = \frac{n\hat{\varphi}}{r} \to \qquad E = \frac{1}{2}\int d^2x\, (\nabla\theta)^2 = \frac{1}{2}\int r\,dr\,d\varphi\, \frac{n^2}{r^2}$$

$$= \pi n^2 \int_a^L \frac{dr}{r} = \underline{\pi n^2 \log L/a}$$

EXCLUDE THE CORE

• ACTUALLY WE WANT TO MINIMIZE FREE ENERGY

$$F = E - TS$$

$$S \sim \log \# \text{POSSIBILITIES} \simeq \log \left(\frac{L}{a}\right)^2 = \underline{2 \log L/a}$$

$$\boxed{F = (\pi n^2 - 2T) \log L/a}$$

- ACTUALLY WE WANT TO MINIMIZE FREE ENERGY

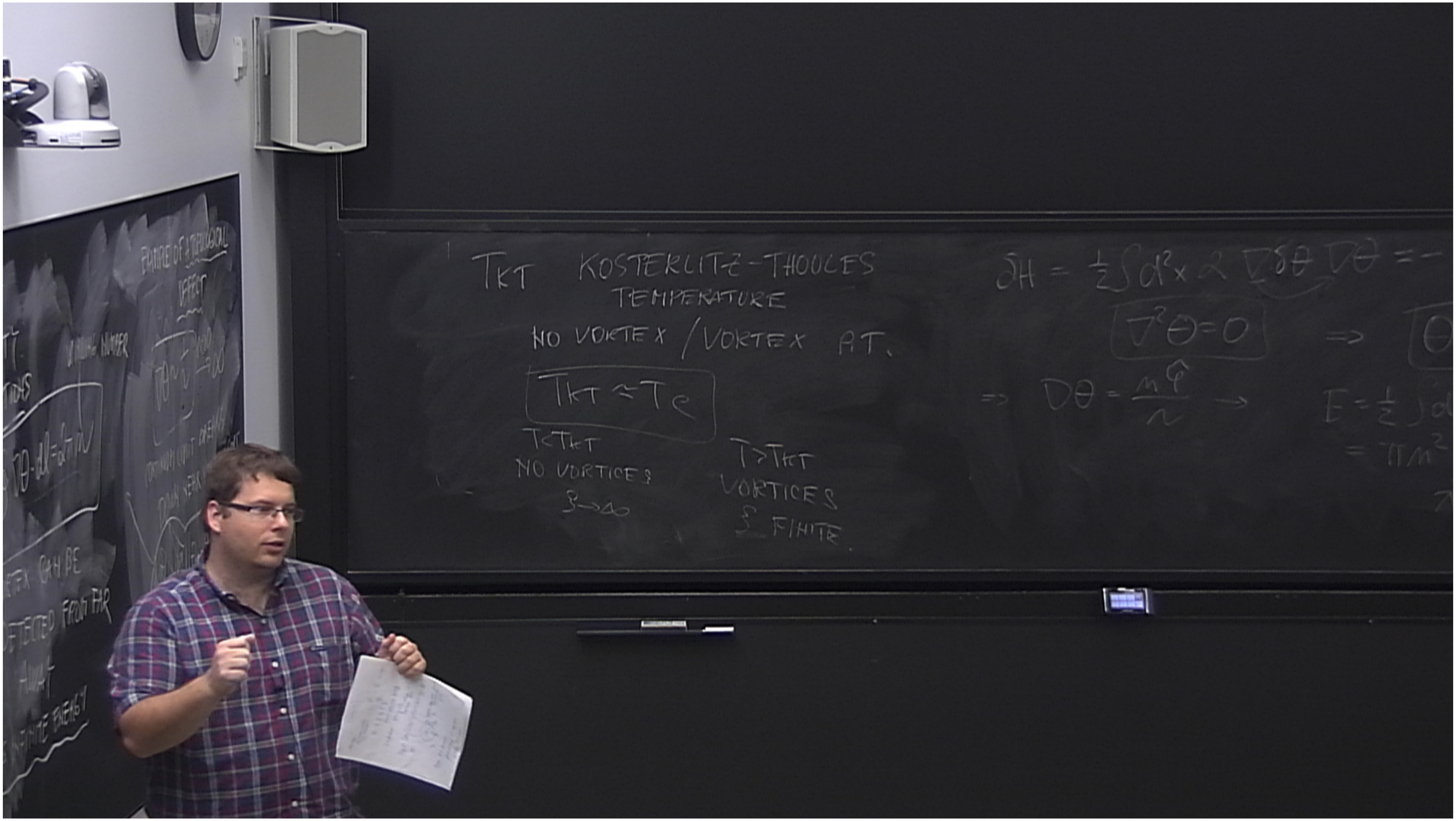
$$F = E - TS$$

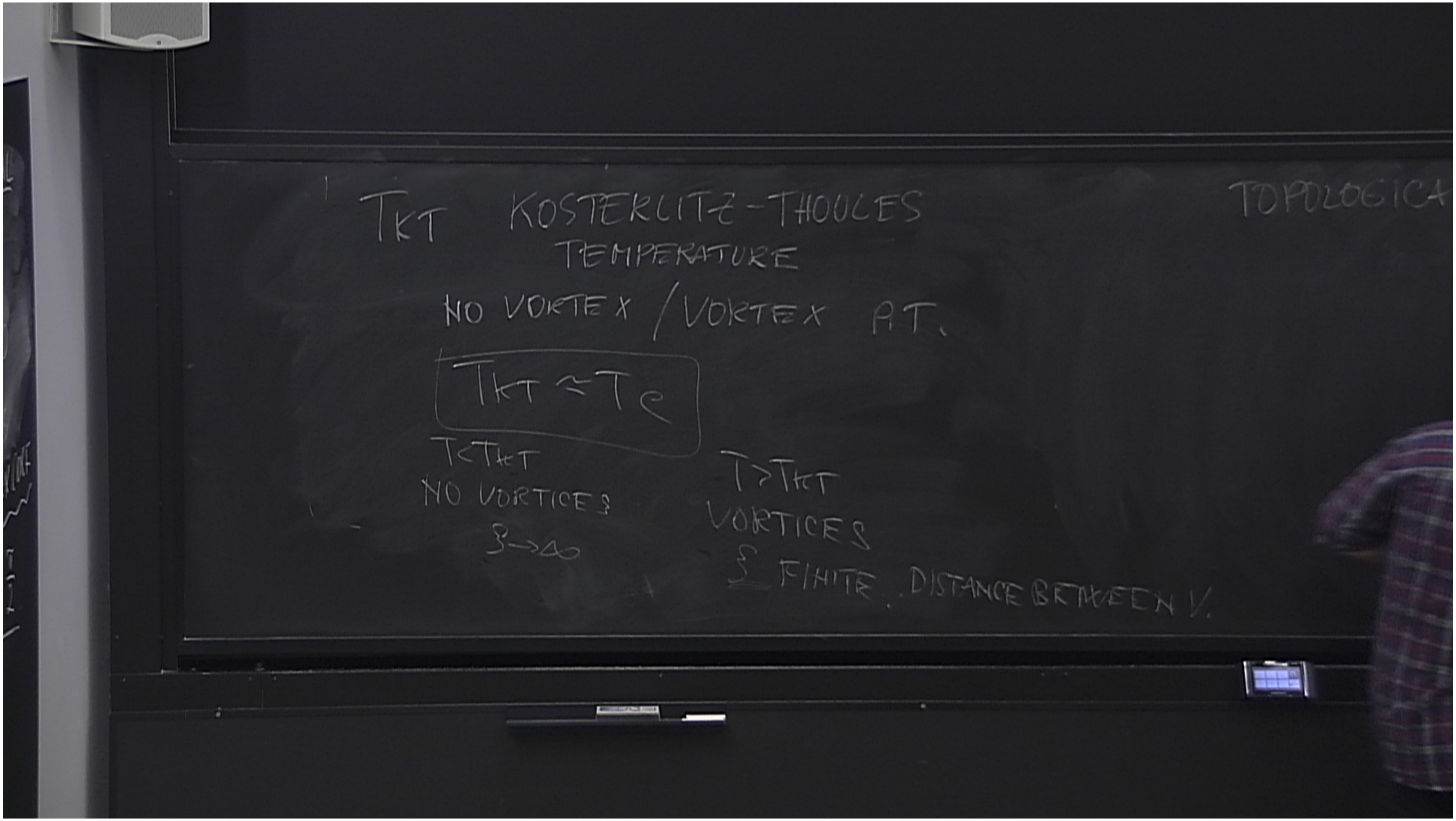
$$S \sim \log \#\text{POSSIBILITIES} \simeq \log\left(\frac{L}{a}\right)^2 = \underline{2 \log L/a}$$

$$\boxed{F = \left(\pi n^2 - 2T\right) \log L/a}$$

$F < 0$.. VORTEX IS PREFERABLE

$\boxed{n = 1}$ $\boxed{T > T_K = \pi/2}$


TKT KOSTERLITZ-THOULES
TEMPERATURE
NO VORTEX / VORTEX P.T.
TKT ≈ Tc
T<TKT
NO VORTICES
ξ→∞
T>TKT
VORTICES
ξ FINITE
∇²θ=0


TKT KOSTERLITZ-THOULES
TEMPERATURE
NO VORTEX / VORTEX P.T.
TKT ≃ Tc
T<TKT
NO VORTICES
ξ→∞
T>TKT
VORTICES
ξ FINITE . DISTANCE BETWEEN V.
TOPOLOGICA

TOPOLOGICAL PHASE TRANSITION

2016 NOBEL PRIZE

FINA REMARK $T=0$ NO VORTICES AT ALL

EVEN AT $T<T_{KT}$... CAN HAVE SOME.

STANCE BETWEEN V.

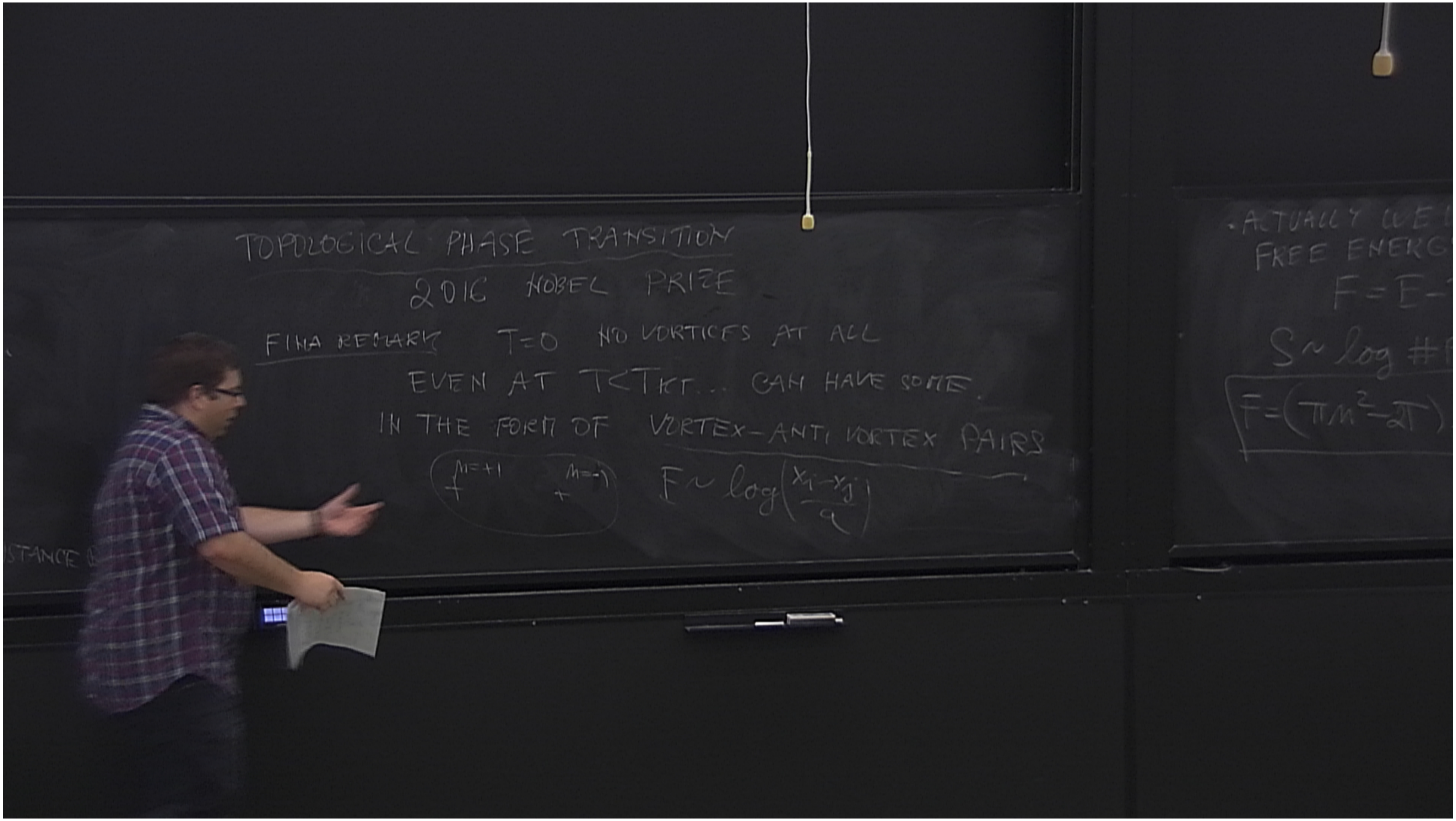
· ACTUALLY WE

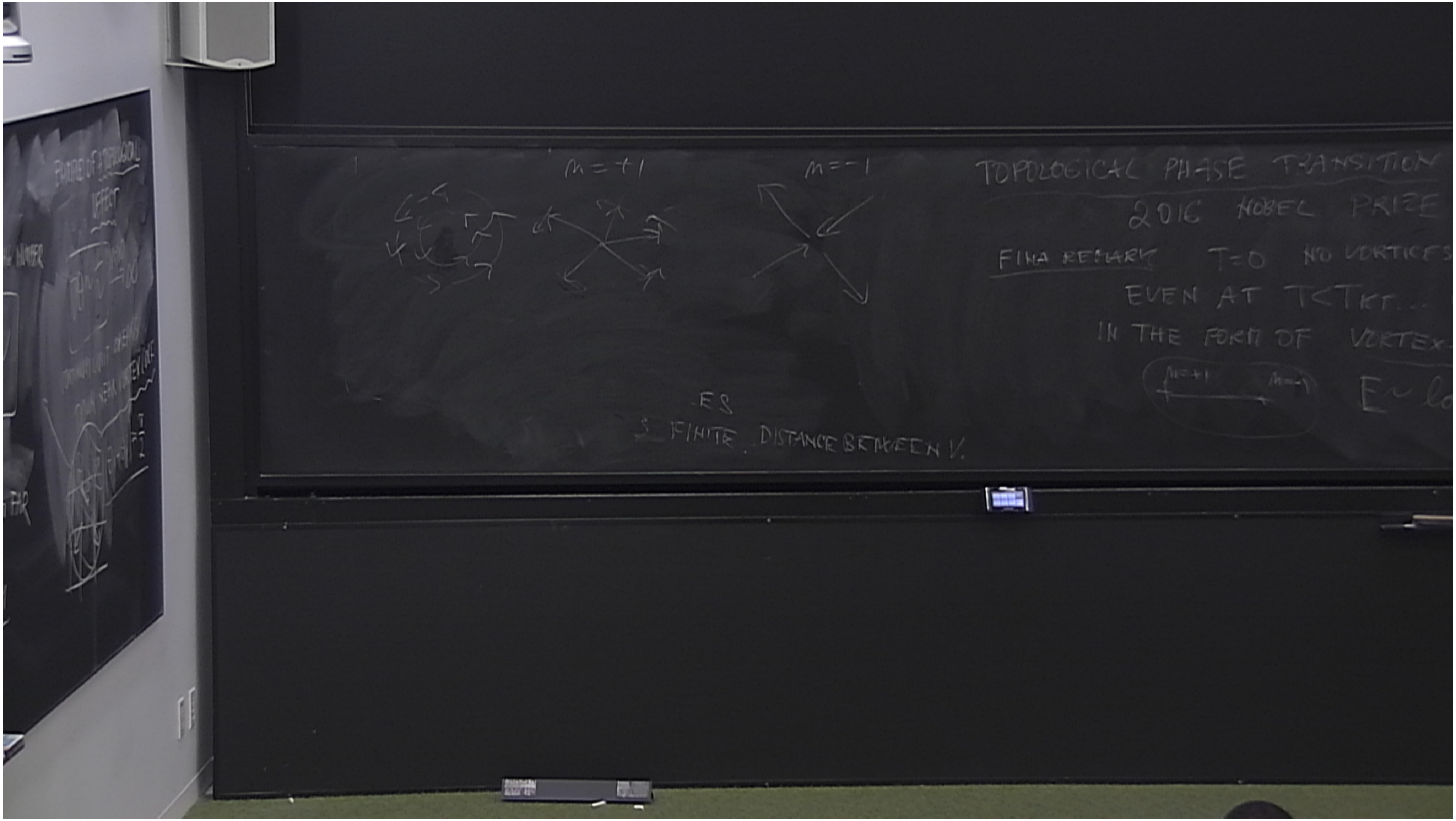
FREE ENERG

$F = E -$

$S \sim \log \#$

$F = (\pi m^2 - 2T)$


TOPOLOGICAL PHASE TRANSITION
2016 NOBEL PRIZE
FINA REMARK T=0 NO VORTICES AT ALL
EVEN AT T<TKT... CAN HAVE SOME
IN THE FORM OF VORTEX-ANTI VORTEX PAIRS
n=+1
n=-1
F ~ log((xi-xj)/a)


m = +1
m = -1
TOPOLOGICAL PHASE TRANSITION
2016 NOBEL PRIZE
FINAL REMARK T=0 NO VORTICE
EVEN AT T<TKT
IN THE FORM OF
FINITE DISTANCE BETWEEN V.