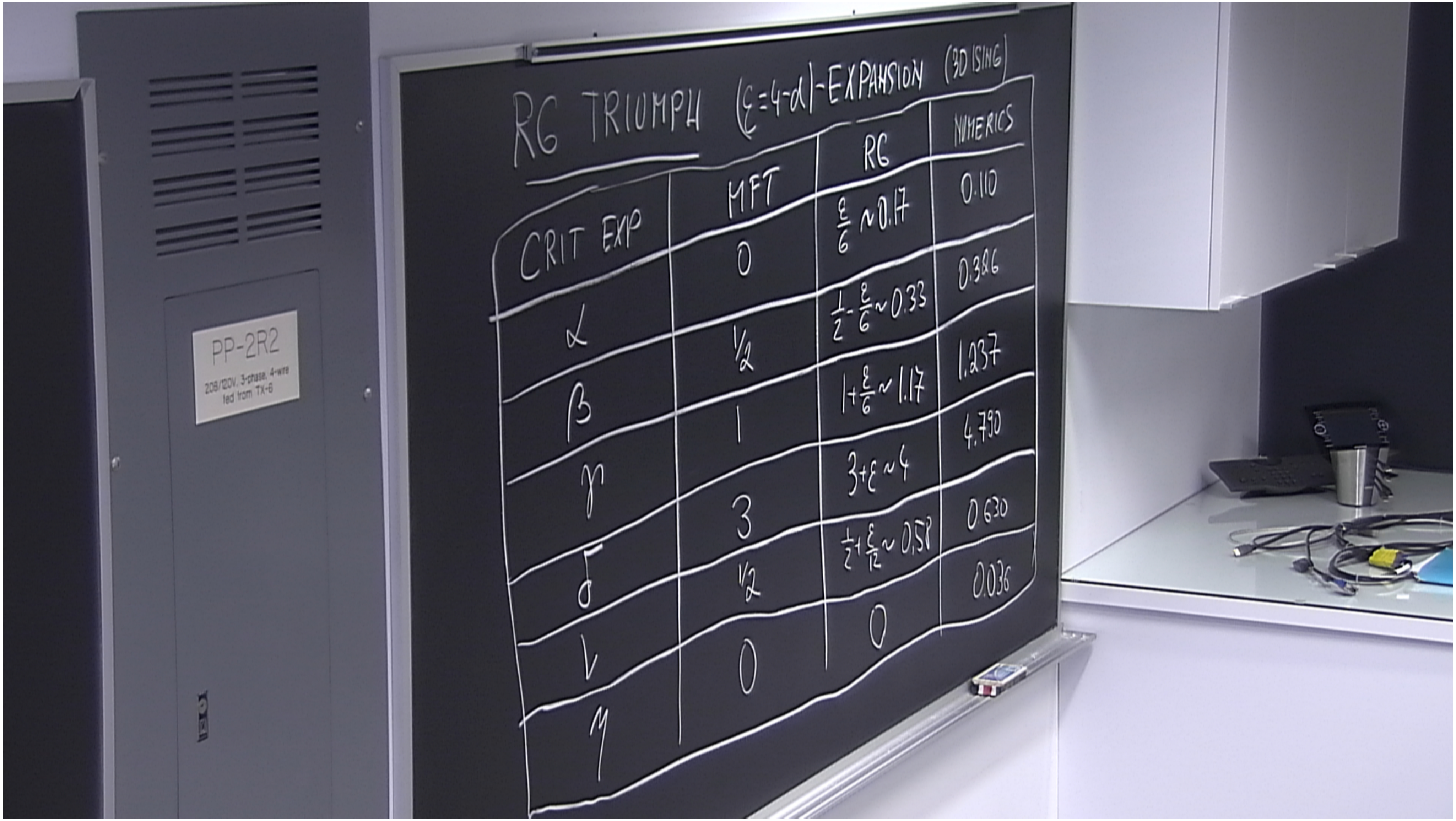
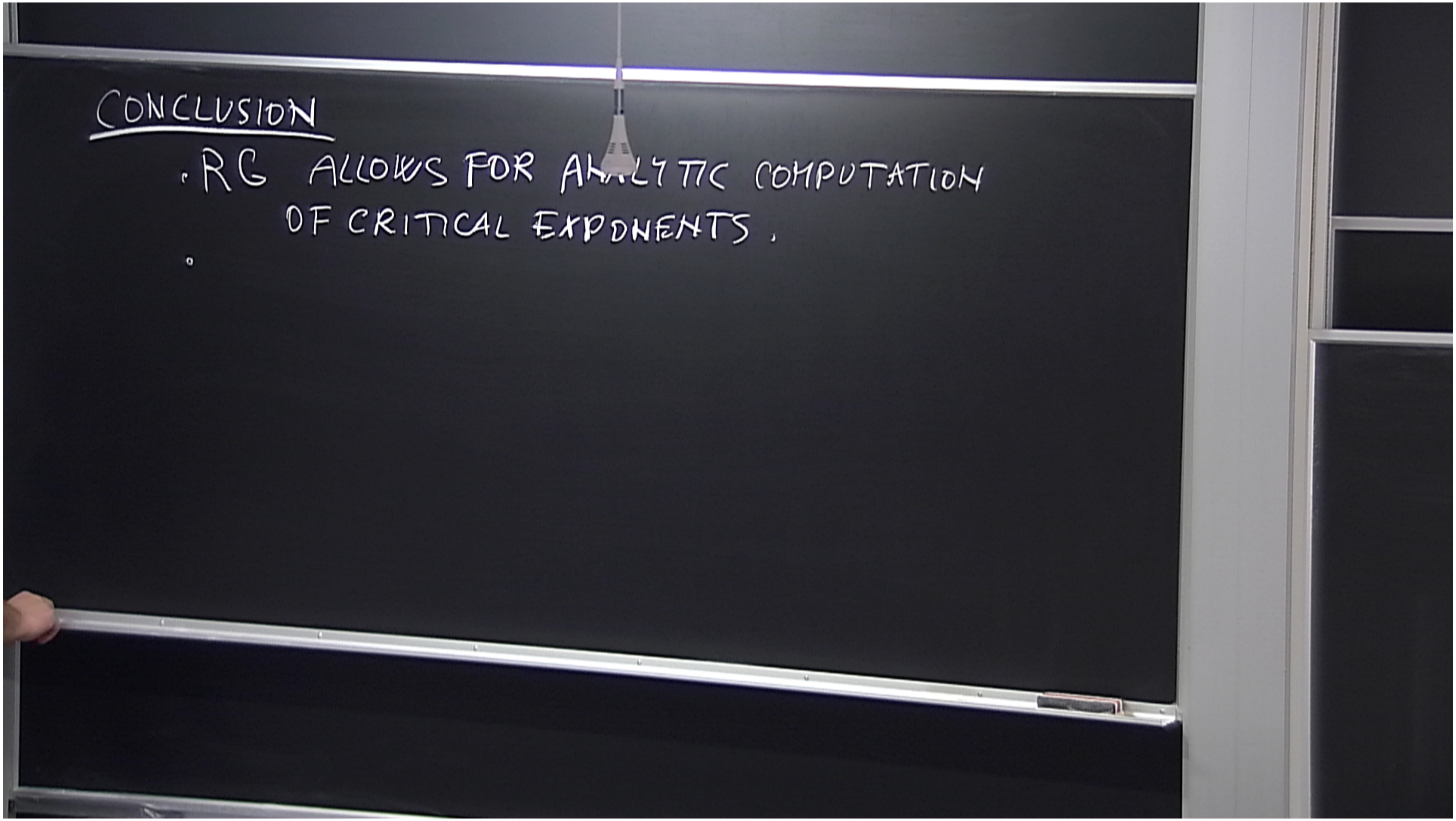
Title: PSI 2016/2017 Statistical Mechanics - Lecture 10

Date: Oct 24, 2016 10:45 AM

URL: http://pirsa.org/16100024

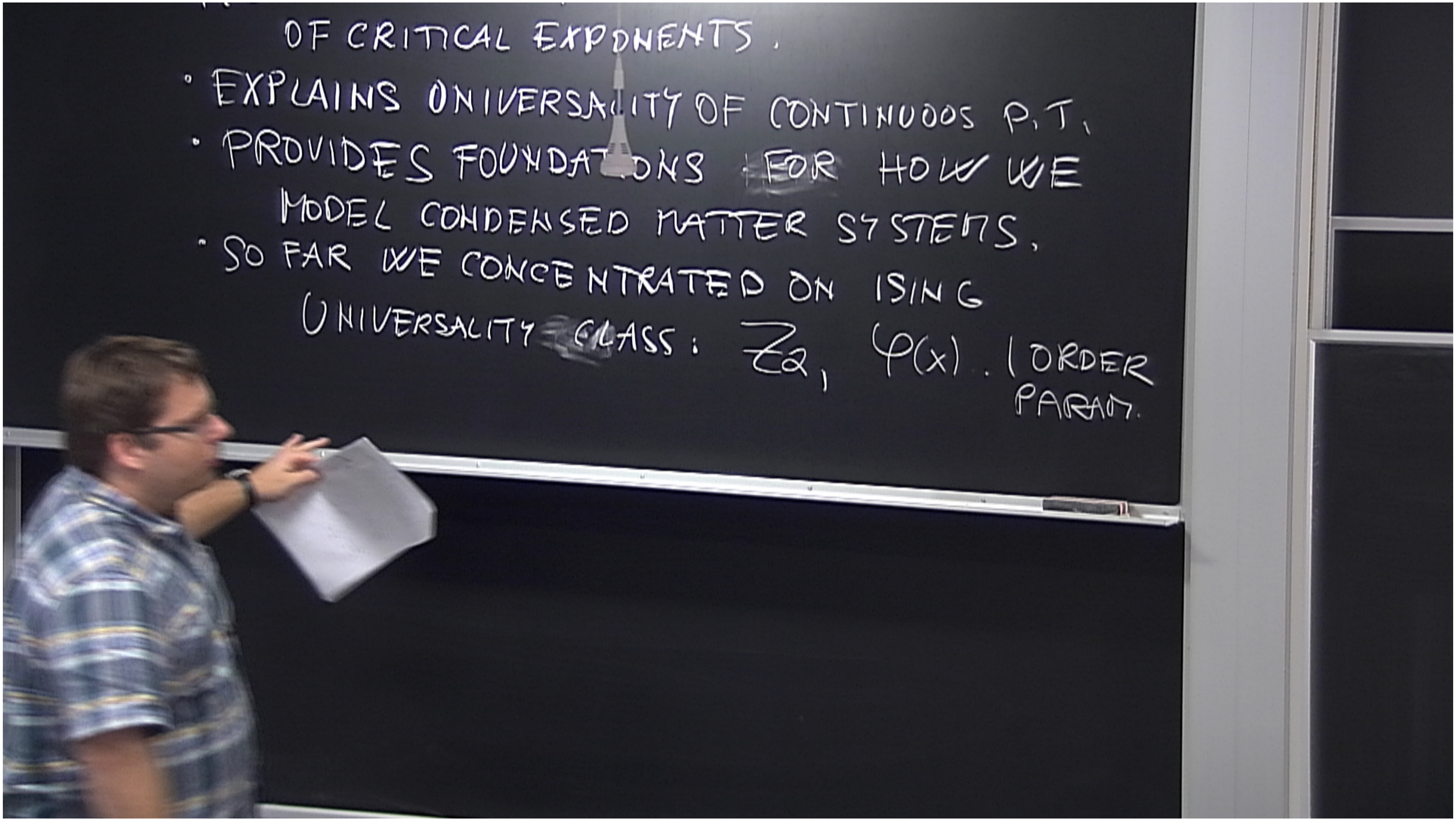
Abstract:

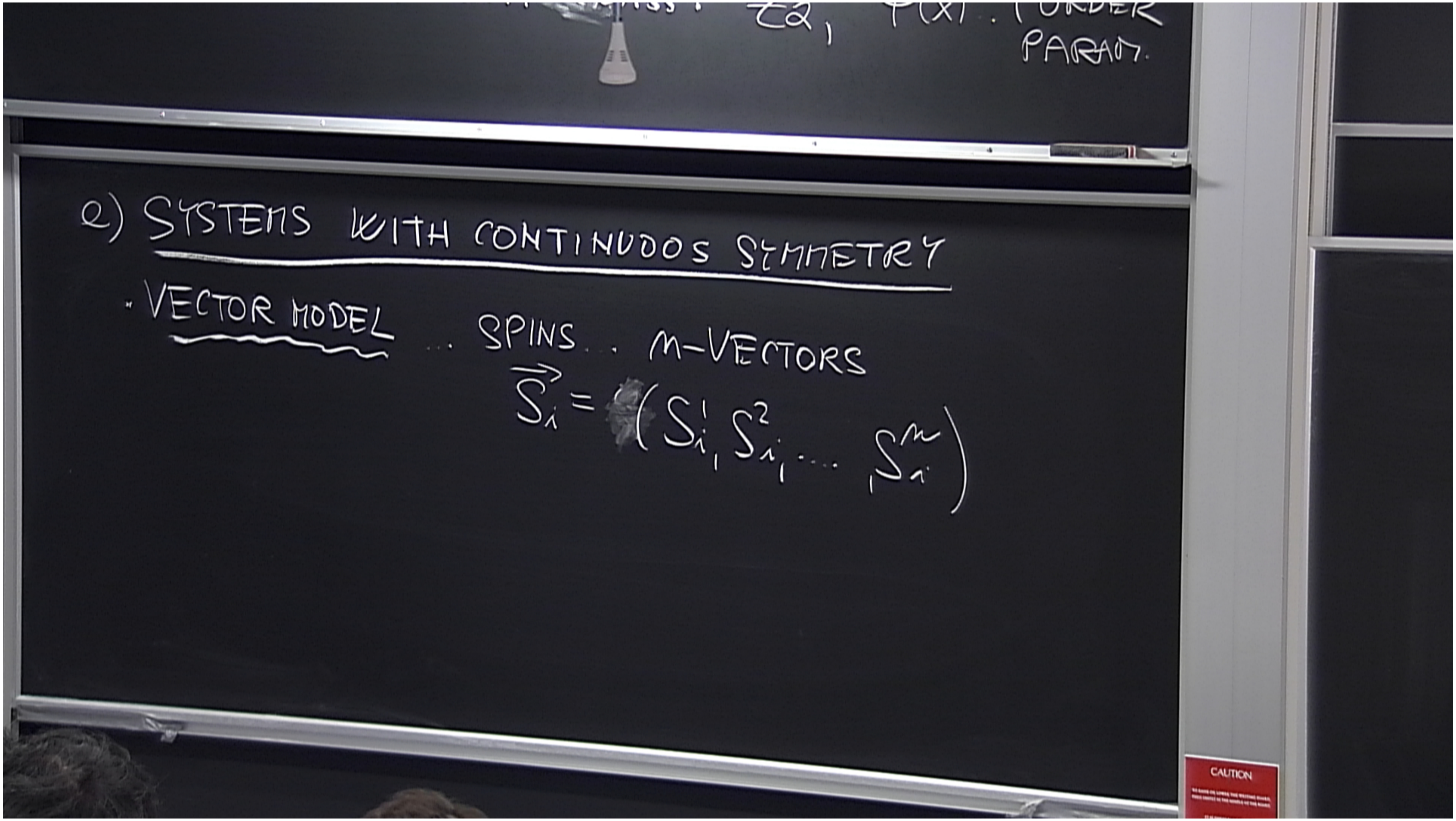

PP-2R2
208/120V, 3-phase, 4-wire
fed from TX-6
RG TRIUMPH (ε=4−d)-EXPANSION (3D ISING)
CRIT EXP MFT RG NUMERICS
α 0 ε/6 ~ 0.17 0.110
β 1/2 1/2 − ε/6 ~ 0.33 0.326
γ 1 1+ε/6 ~ 1.17 1.237
δ 3 3+ε ~ 4 4.790
ν 1/2 1/2+ε/12 ~ 0.58 0.630
η 0 0 0.036


CONCLUSION
• RG ALLOWS FOR ANALYTIC COMPUTATION
OF CRITICAL EXPONENTS.
•

CONCLUSION

- RG ALLOWS FOR ANALYTIC COMPUTATION OF CRITICAL EXPONENTS.
- EXPLAINS UNIVERSALITY OF CONTINUOUS P.T.


OF CRITICAL EXPONENTS.
· EXPLAINS UNIVERSALITY OF CONTINUOUS P.T.
· PROVIDES FOUNDATIONS FOR HOW WE MODEL CONDENSED MATTER SYSTEMS.
· SO FAR WE CONCENTRATED ON ISING UNIVERSALITY CLASS: Z2, φ(x). (ORDER PARAM.


PARAM.
2) SYSTEMS WITH CONTINUOUS SYMMETRY
· VECTOR MODEL ... SPINS ... n-VECTORS
$\vec{S}_i = (S_i^1, S_i^2, \ldots, S_i^n)$

$\mathbb{Z}_2$, $\phi(x)$: (ORDER PARAM.

2) SYSTEMS WITH CONTINUOUS SYMMETRY

- VECTOR MODEL ... SPINS ... n-VECTORS

$$\vec{S}_i = (S_i^1, S_i^2, \dots, S_i^n)$$

$$\boxed{H = -\frac{1}{2} \sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j}$$

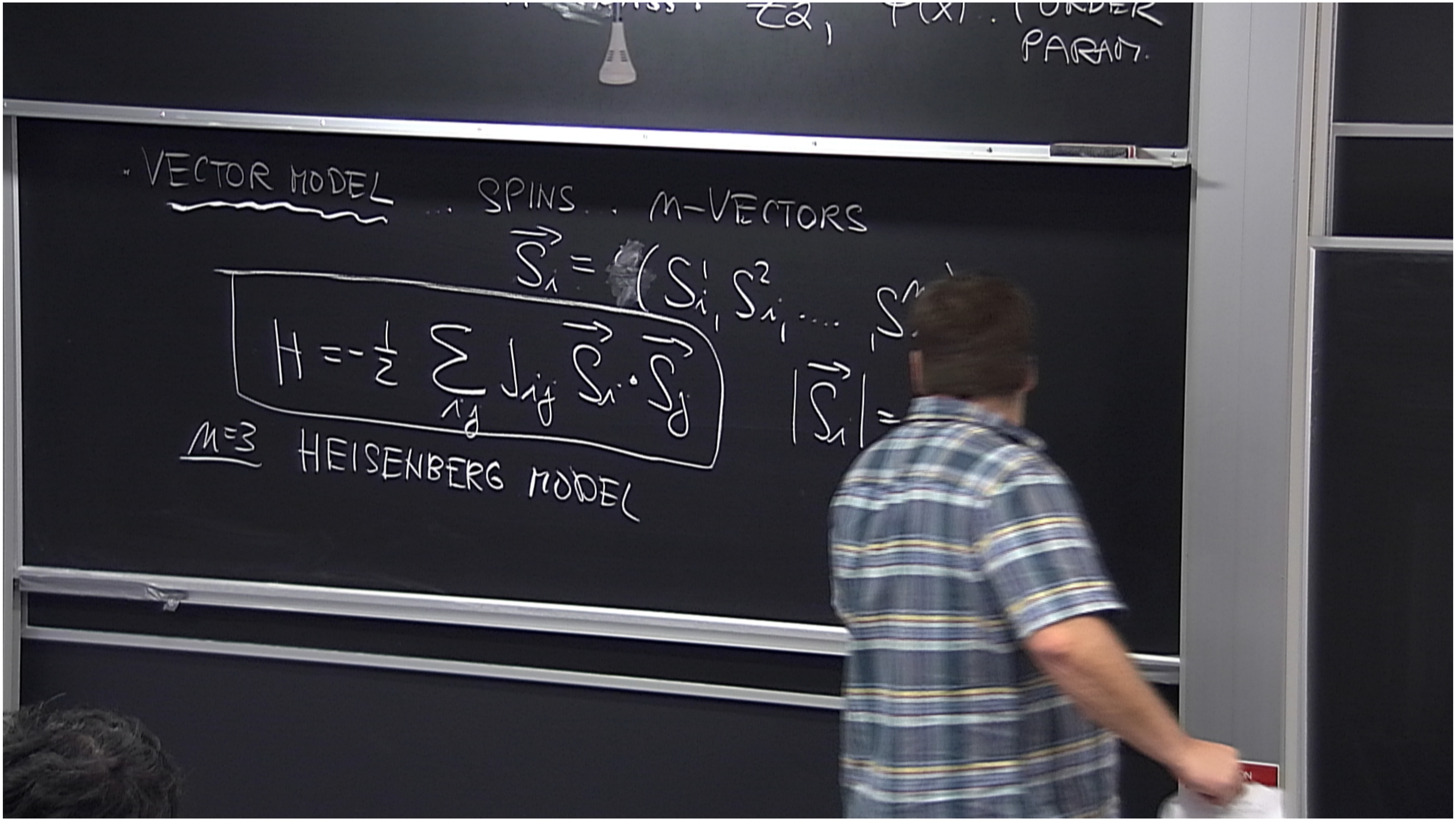
… : $\mathbb{Z}_2$, $\varphi(x)$: ORDER PARAM.

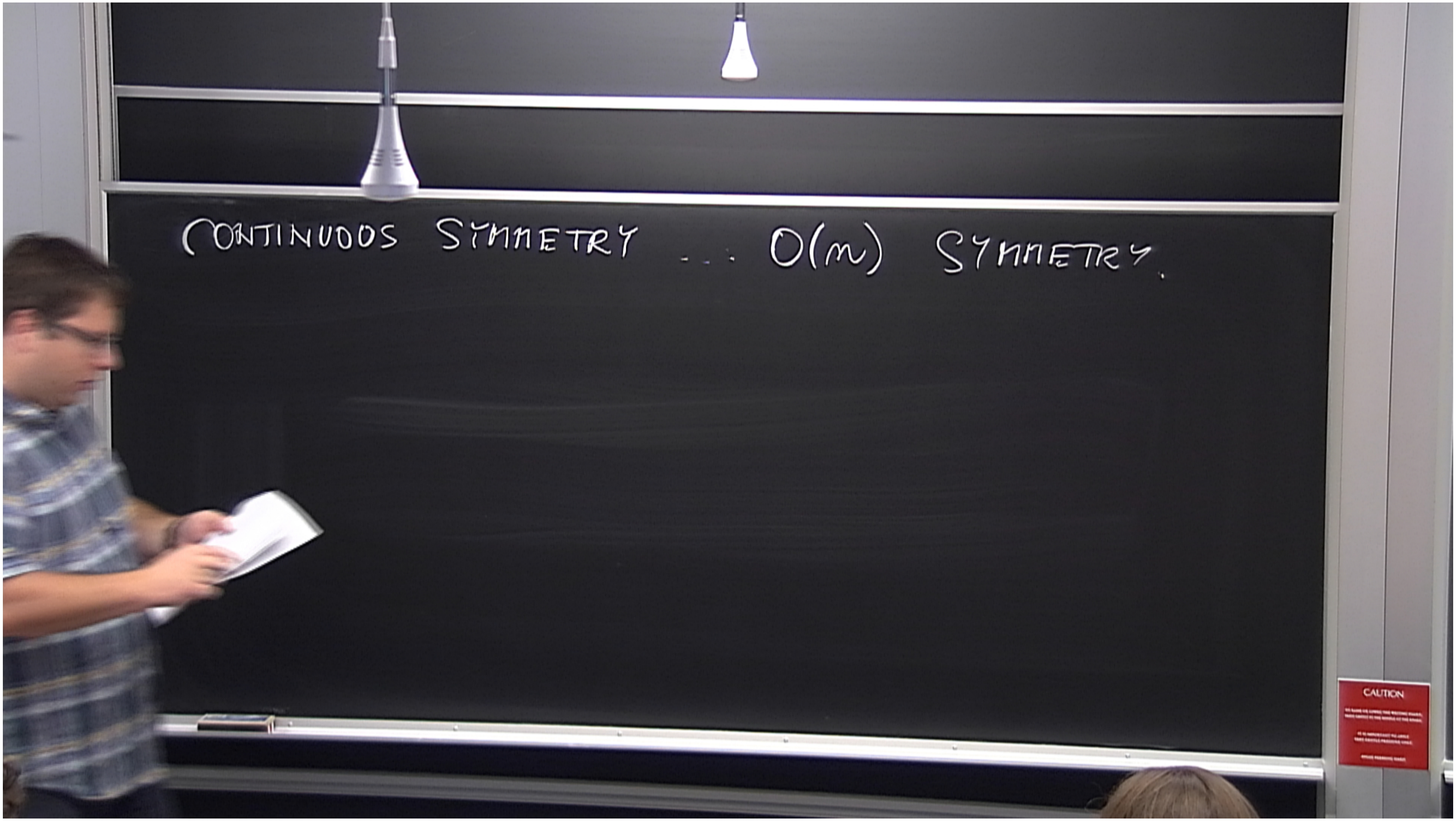
2) SYSTEMS WITH CONTINUOUS SYMMETRY

• VECTOR MODEL … SPINS … n-VECTORS

$$\vec{S}_i = (S_i^1, S_i^2, \dots, S_i^n)$$

$$\boxed{H = -\frac{1}{2} \sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j} \qquad |\vec{S}_i| = 1$$


ORDER PARAM.
VECTOR MODEL ... SPINS ... n-VECTORS
$\vec{S}_i = (S_i^1, S_i^2, \ldots, S$
$H = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$
$|\vec{S}_i| =$
n=3 HEISENBERG MODEL


CONTINUOUS SYMMETRY ... O(n) SYMMETRY.
CAUTION

CONTINUOUS SYMMETRY ... O(n) SYMMETRY.

- LANDAU-GINSBURG

$$S[\vec{\varphi}] = \int d^d x \left[\frac{1}{2} (\nabla \vec{\varphi}) \cdot (\nabla \vec{\varphi}) + \frac{r}{2} \vec{\varphi} \cdot \vec{\varphi} \right.$$

CONTINUOUS SYMMETRY ... $O(n)$ SYMMETRY.

- LANDAU-GINSBURG

$$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2} (\nabla\vec{\varphi})\cdot(\nabla\vec{\varphi}) + \frac{r}{2}\,\vec{\varphi}\cdot\vec{\varphi} + \frac{u}{4}\left(\vec{\varphi}\cdot\vec{\varphi}\right)^2 \right]$$

CONTINUOUS SYMMETRY ... O(n) SYMMETRY.

- LANDAU-GINSBURG

$$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2}(\nabla\vec{\varphi})\cdot(\nabla\vec{\varphi}) + \frac{r}{2}\vec{\varphi}\cdot\vec{\varphi} + \frac{u}{4}(\vec{\varphi}\cdot\vec{\varphi})^2 \right]$$

- CAN DO (4-d)-EXPANSION (HOMEWORK)

CRIT. EXPONENTS .. DEPEND ON $\underline{n}$.

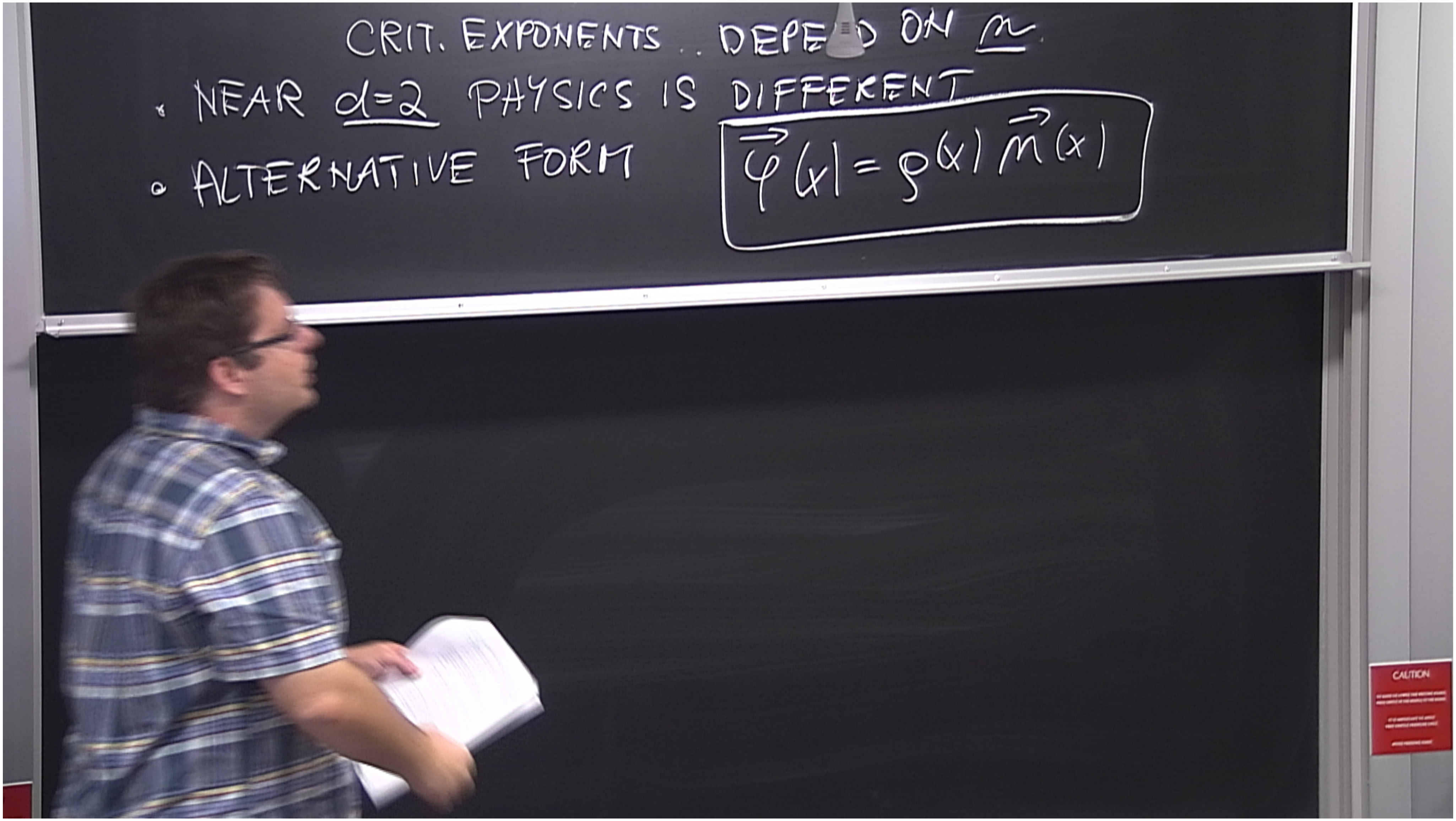
- LANDAU–GINSBURG

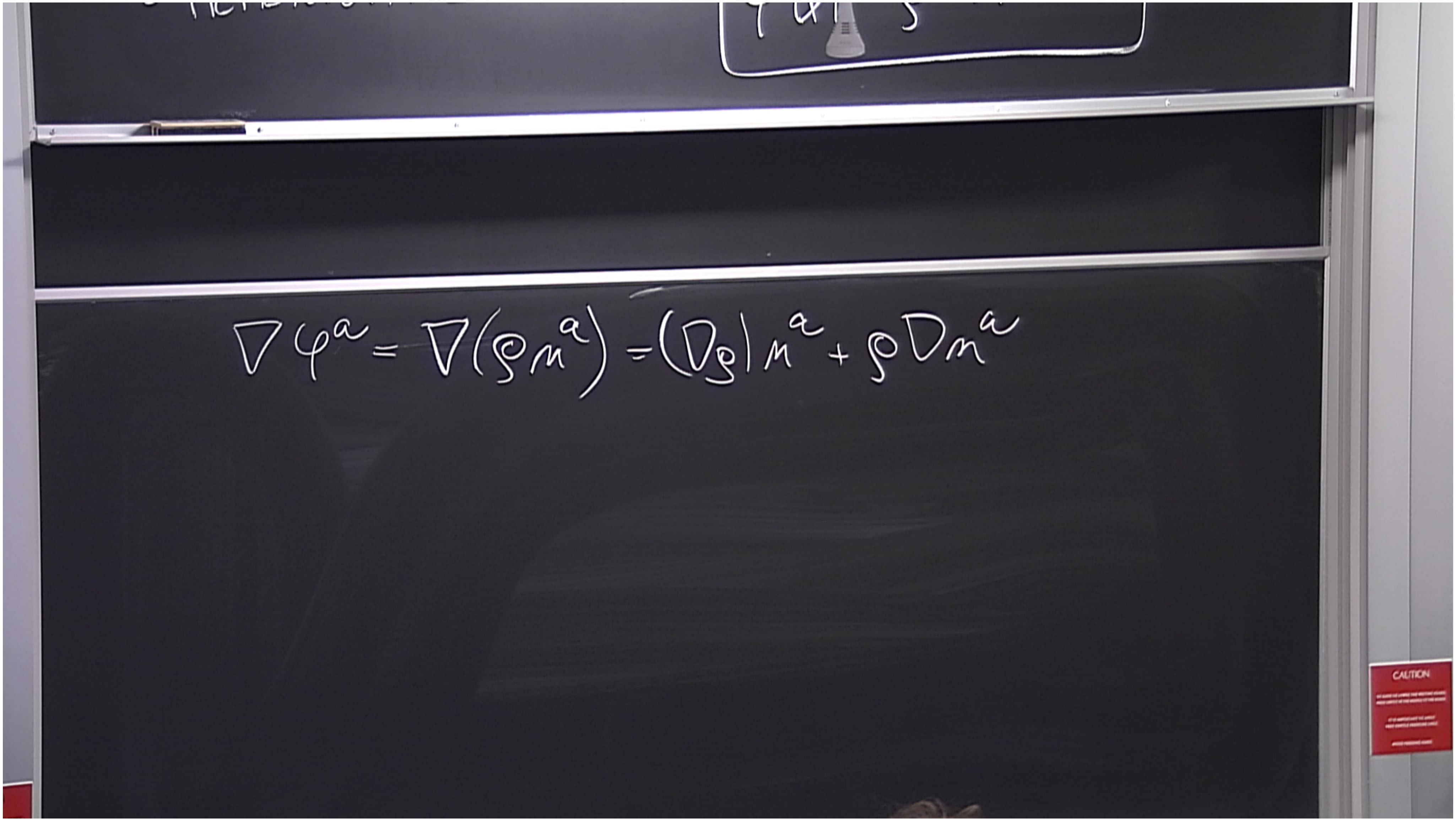
$$S[\vec{\varphi}] = \int d^d x \left[\frac{1}{2}(\nabla\vec{\varphi})\cdot(\nabla\vec{\varphi}) + \frac{r}{2}\,\vec{\varphi}\cdot\vec{\varphi} + \frac{u}{4}(\vec{\varphi}\cdot\vec{\varphi}) \right]$$

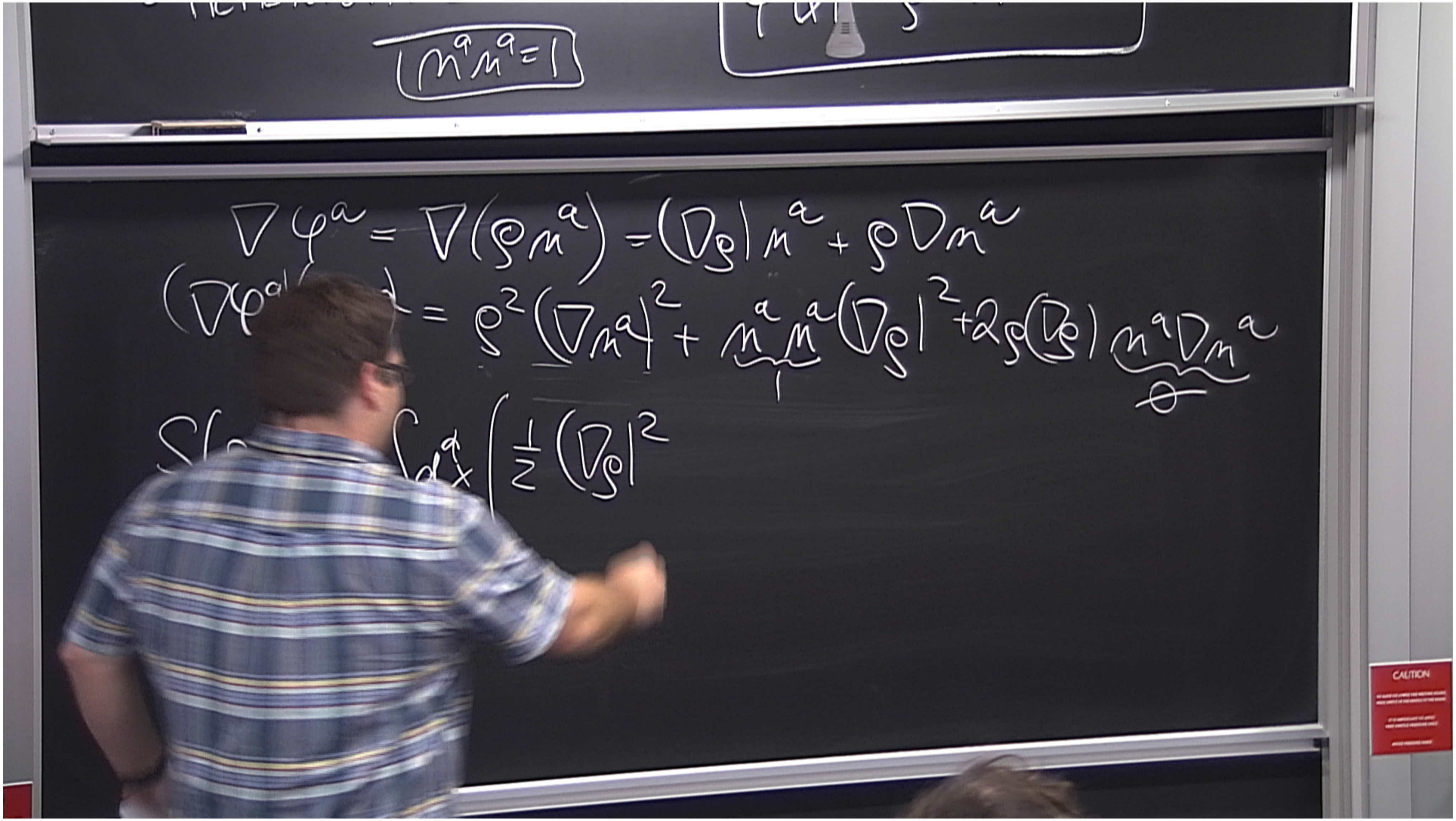
- CAN DO $(4-d)$–EXPANSION (HOMEWORK)

  CRIT. EXPONENTS .. DEPEND ON $\underline{n}$.

- NEAR $\underline{d=2}$ PHYSICS IS DIFFERENT


CRIT. EXPONENTS .. DEPE D ON n.
• NEAR d=2 PHYSICS IS DIFFERENT
• ALTERNATIVE FORM
$\vec{\varphi}(x) = \rho(x)\,\vec{m}(x)$

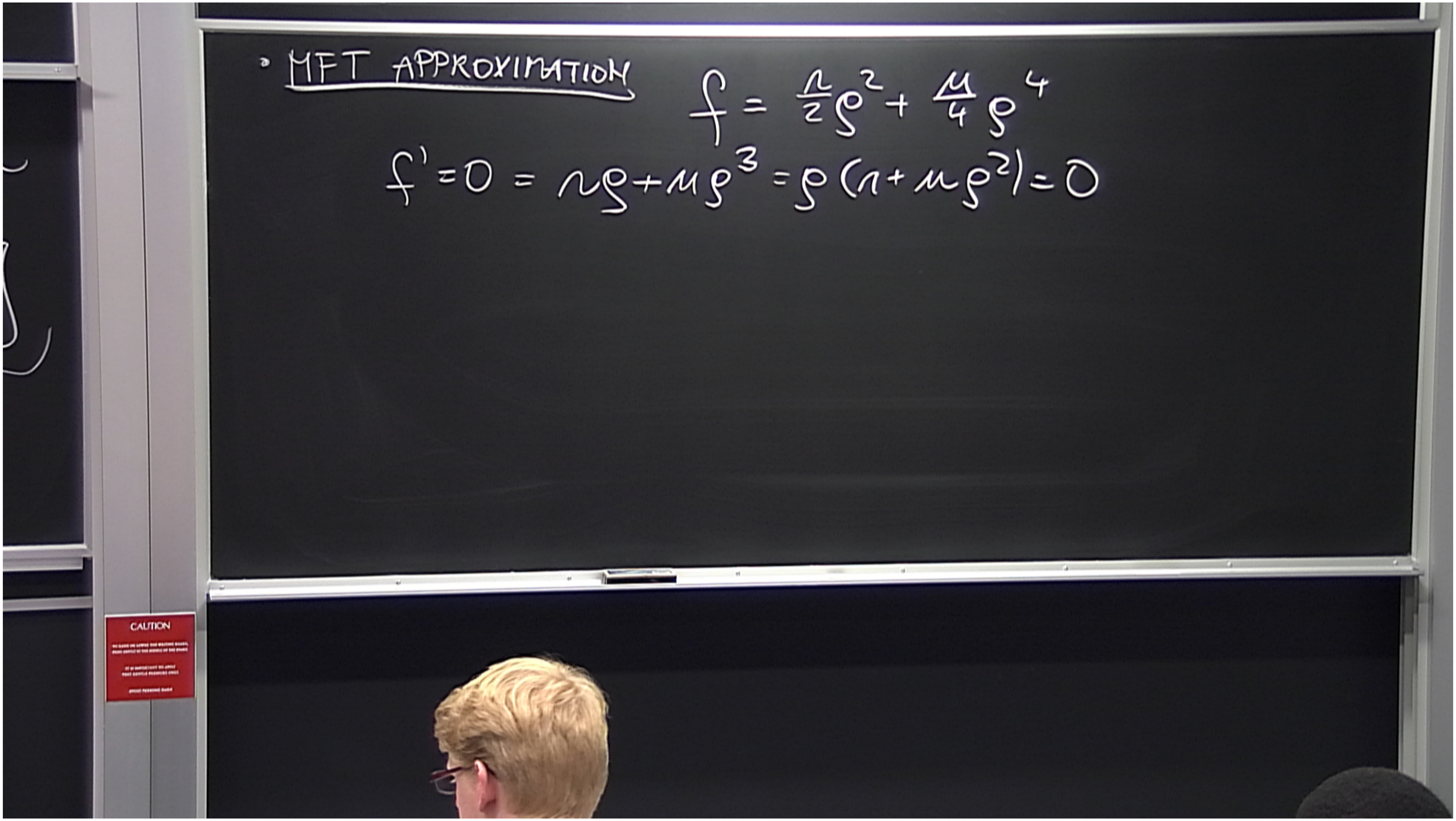

$\nabla \varphi^a = \nabla(\rho n^a) = (\nabla \rho) n^a + \rho \nabla n^a$


$m^a m^a = 1$
$\nabla \varphi^a = \nabla(\rho n^a) = (\nabla\rho) n^a + \rho \nabla n^a$
$= \rho^2 (\nabla n^a)^2 + \underbrace{n^a n^a}_{1} (\nabla\rho)^2 + 2\rho(\nabla\rho) \underbrace{n^a \nabla n^a}_{0}$
$\int d^dx \left(\frac{1}{2} (\nabla\rho)^2\right.$

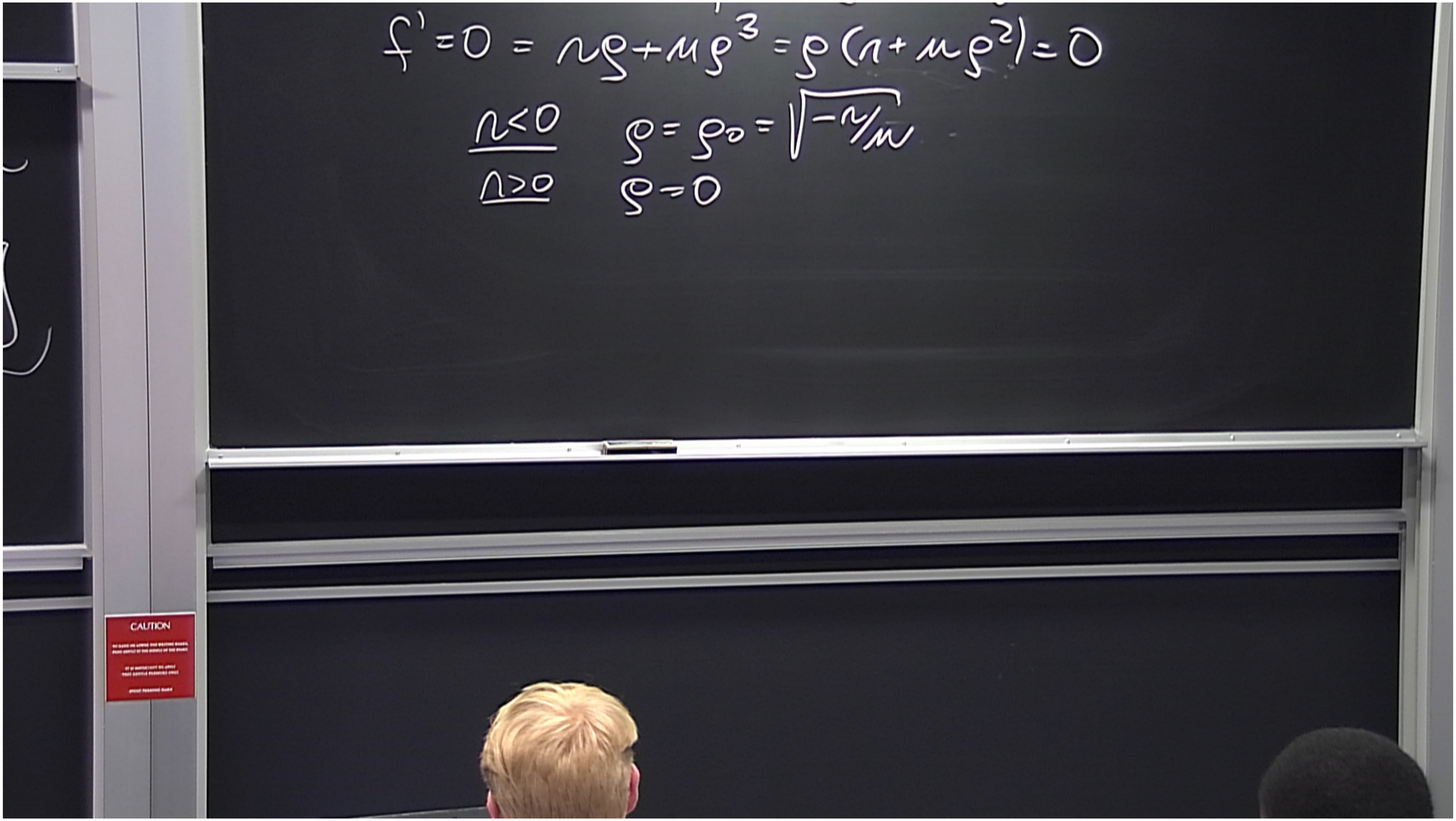
$$\boxed{n^a n^a = 1}$$

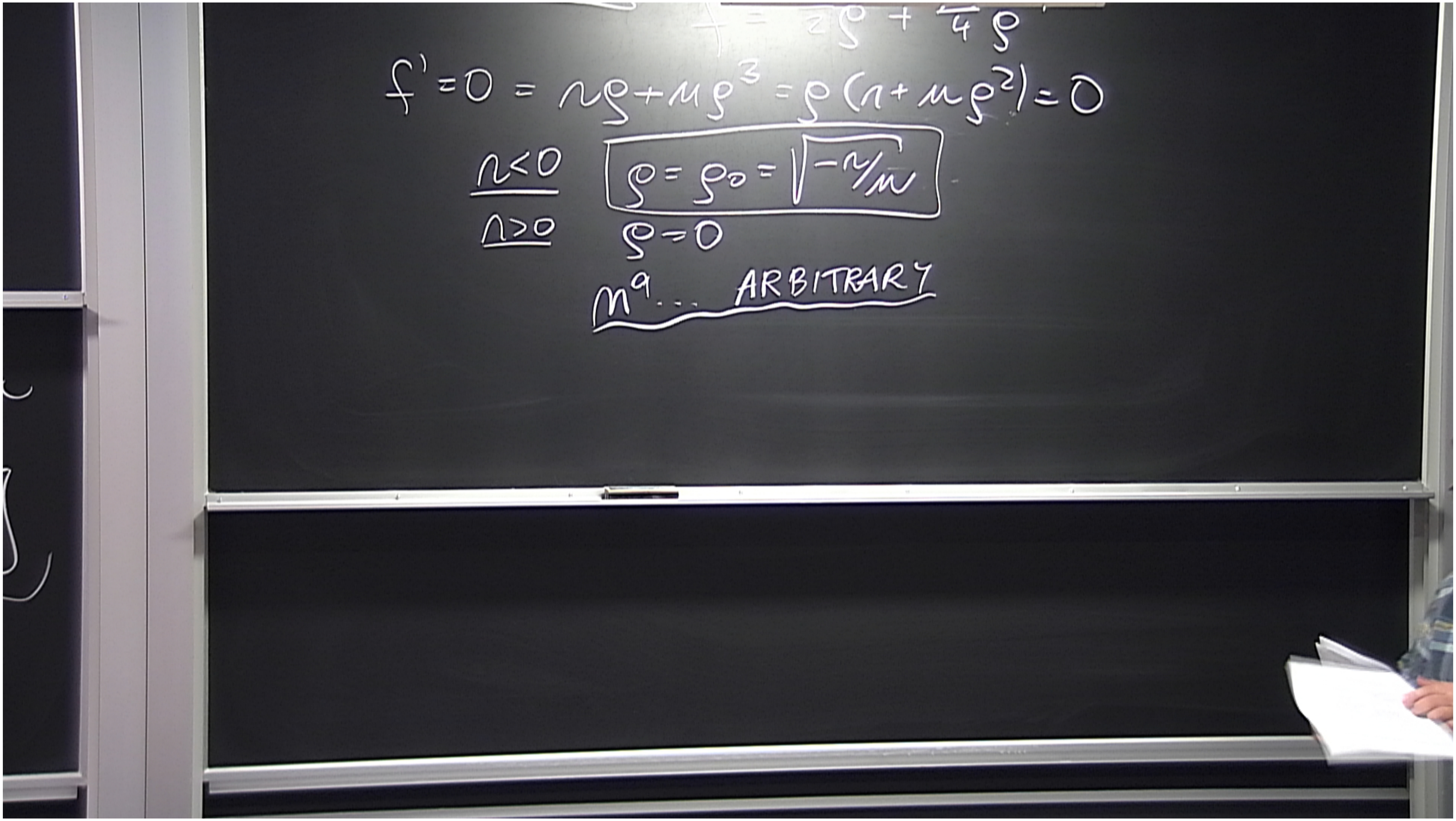
$$(\nabla\varphi^a)(\nabla\varphi^a) = \rho^2(\nabla n^a)^2 + \underbrace{n^a n^a}_{1}(\nabla\rho)^2 + 2\rho(\nabla\rho)\underbrace{n^a\nabla n^a}_{0}$$

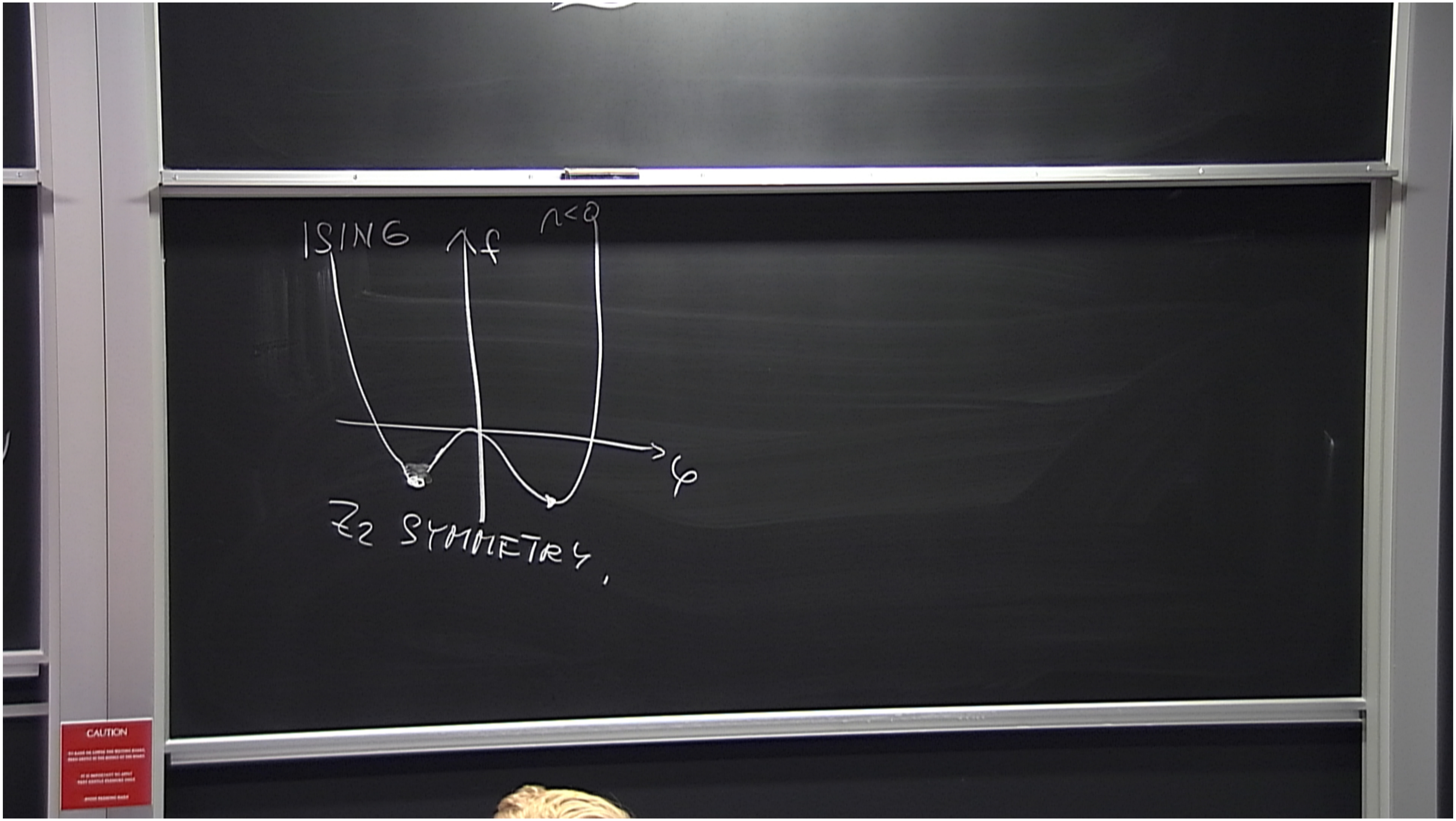
$$S(\rho,\vec{n}) = \int d^d x \left(\frac{1}{2}(\nabla\rho)^2 + \frac{1}{2}\rho^2(\nabla n^a)(\nabla n^a) + \frac{r}{2}\rho^2 + \frac{u}{4}\rho^4 \right]$$

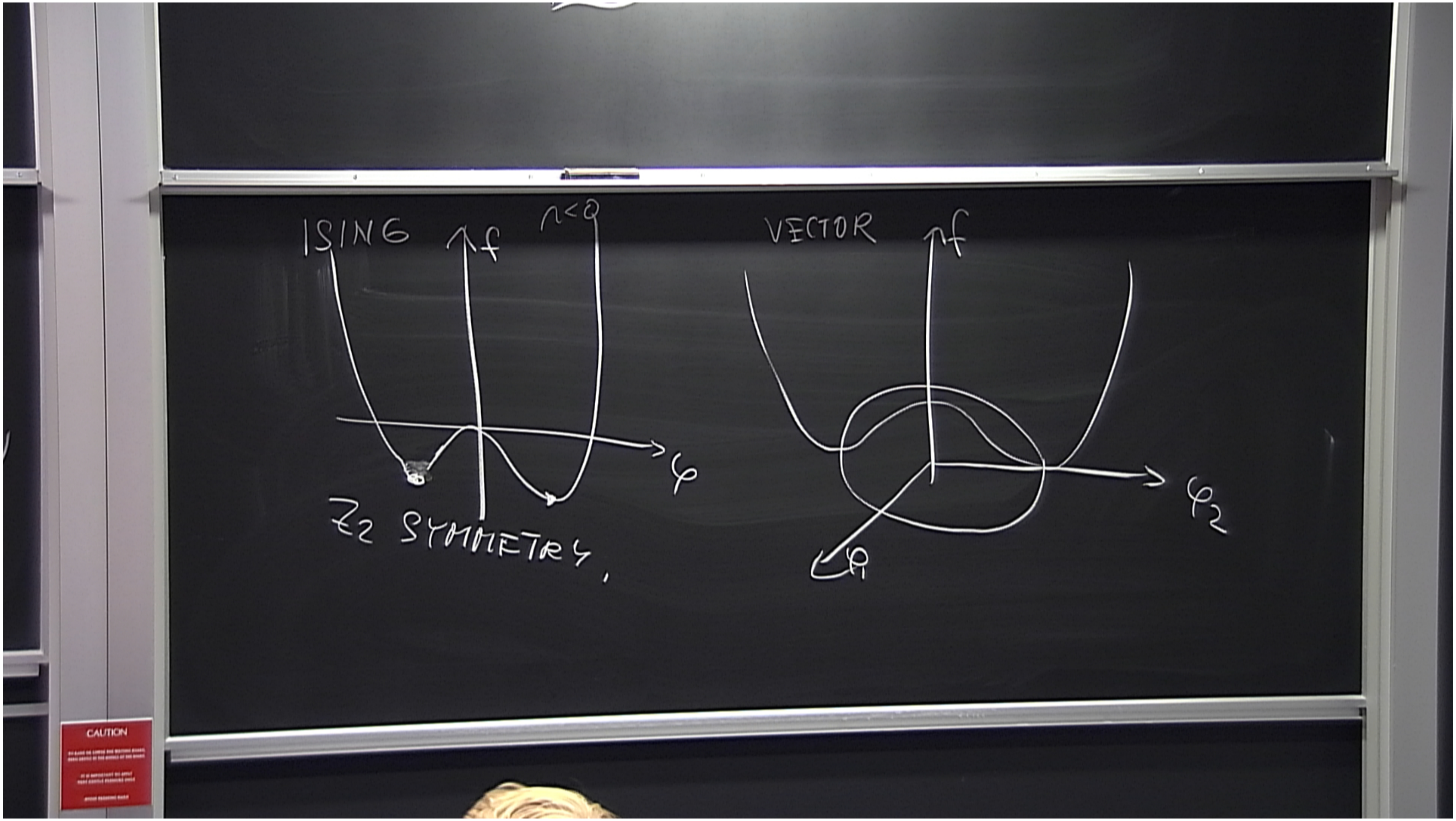


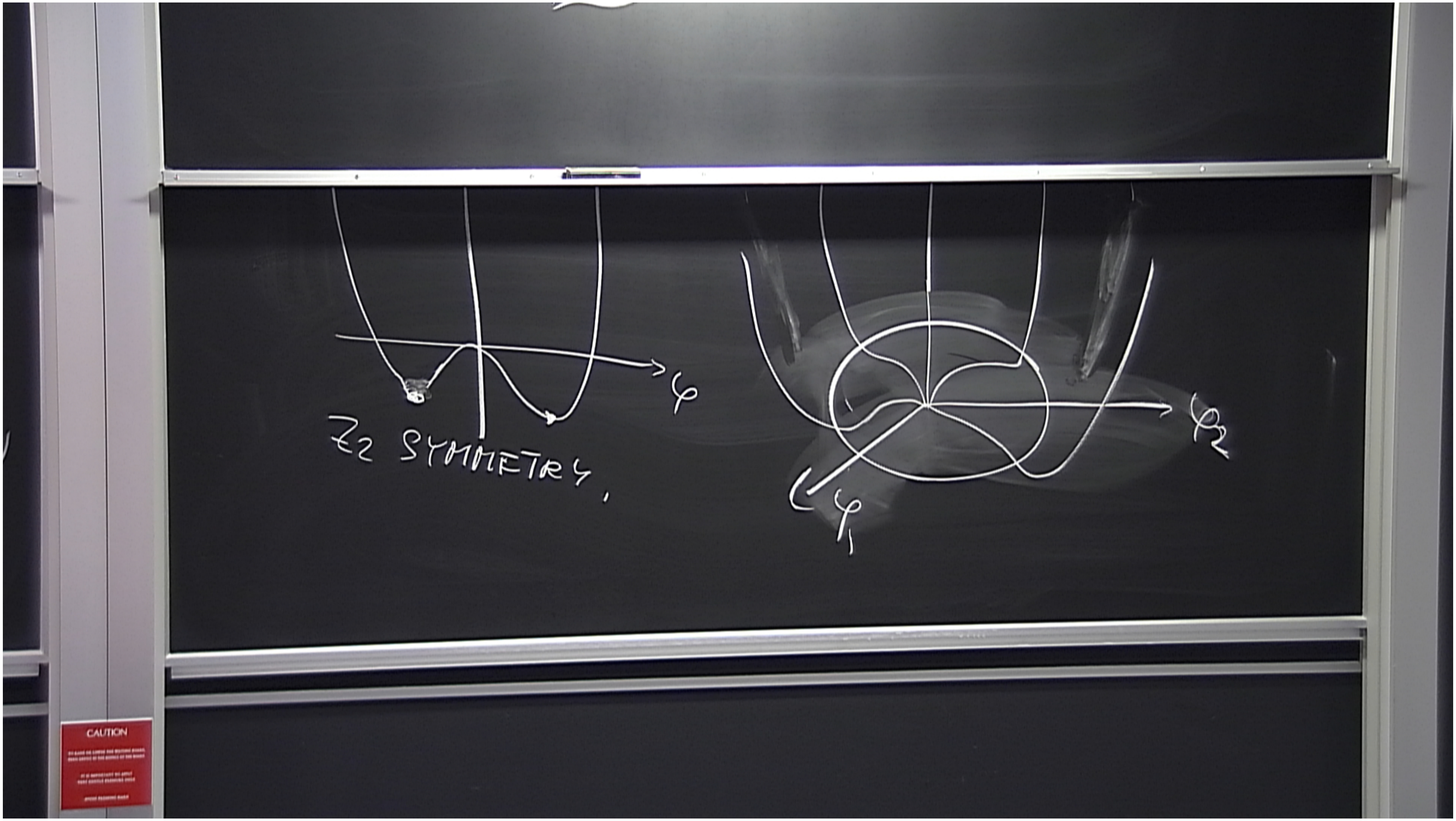
• MFT APPROXIMATION
$f = \frac{r}{2}\rho^2 + \frac{u}{4}\rho^4$
$f' = 0 = r\rho + u\rho^3 = \rho(r + u\rho^2) = 0$
CAUTION

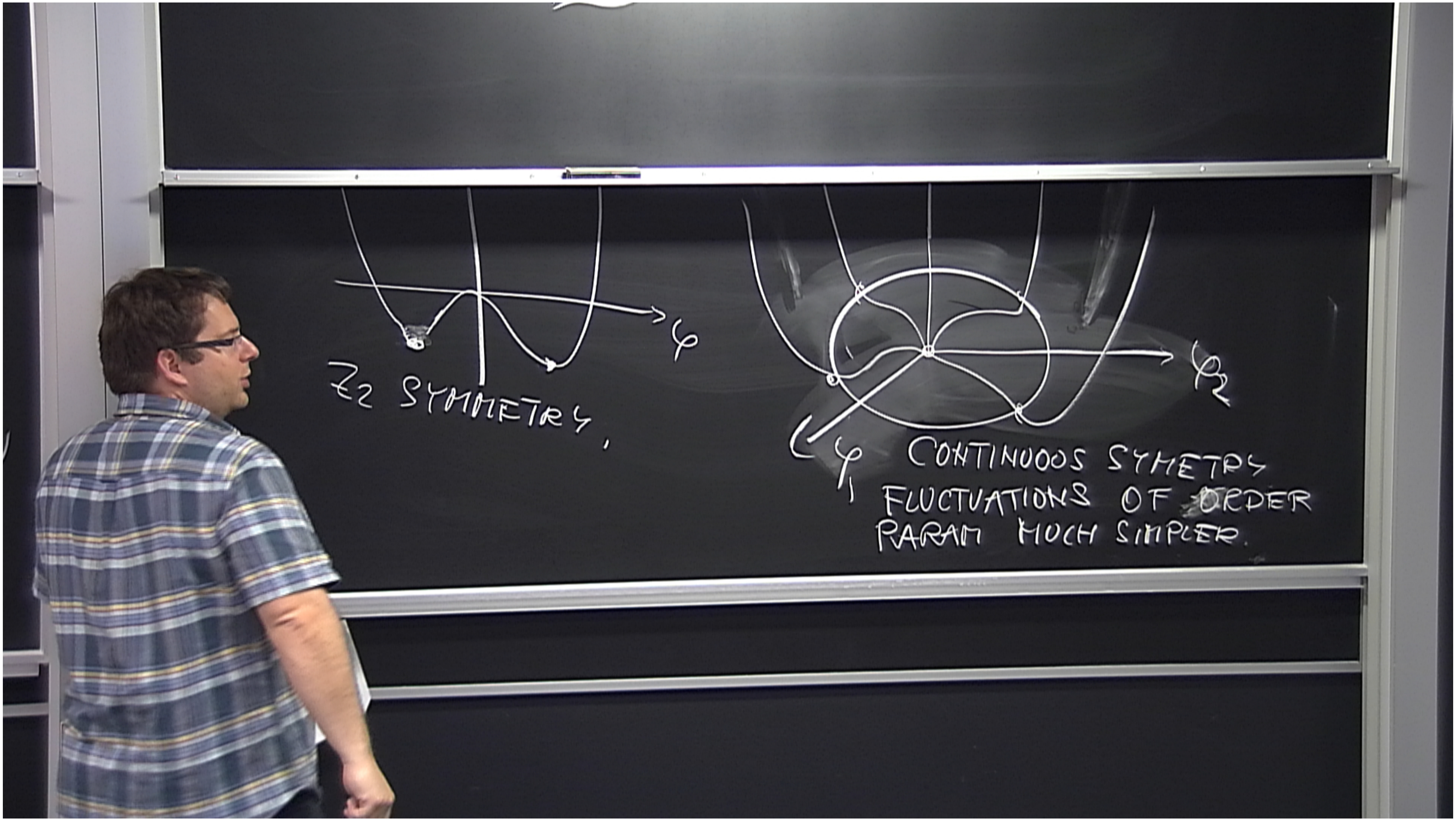

$f' = 0 = r\rho + u\rho^3 = \rho(r + u\rho^2) = 0$
$\underline{r<0}$ $\rho = \rho_0 = \sqrt{-r/u}$
$\underline{r>0}$ $\rho = 0$
CAUTION

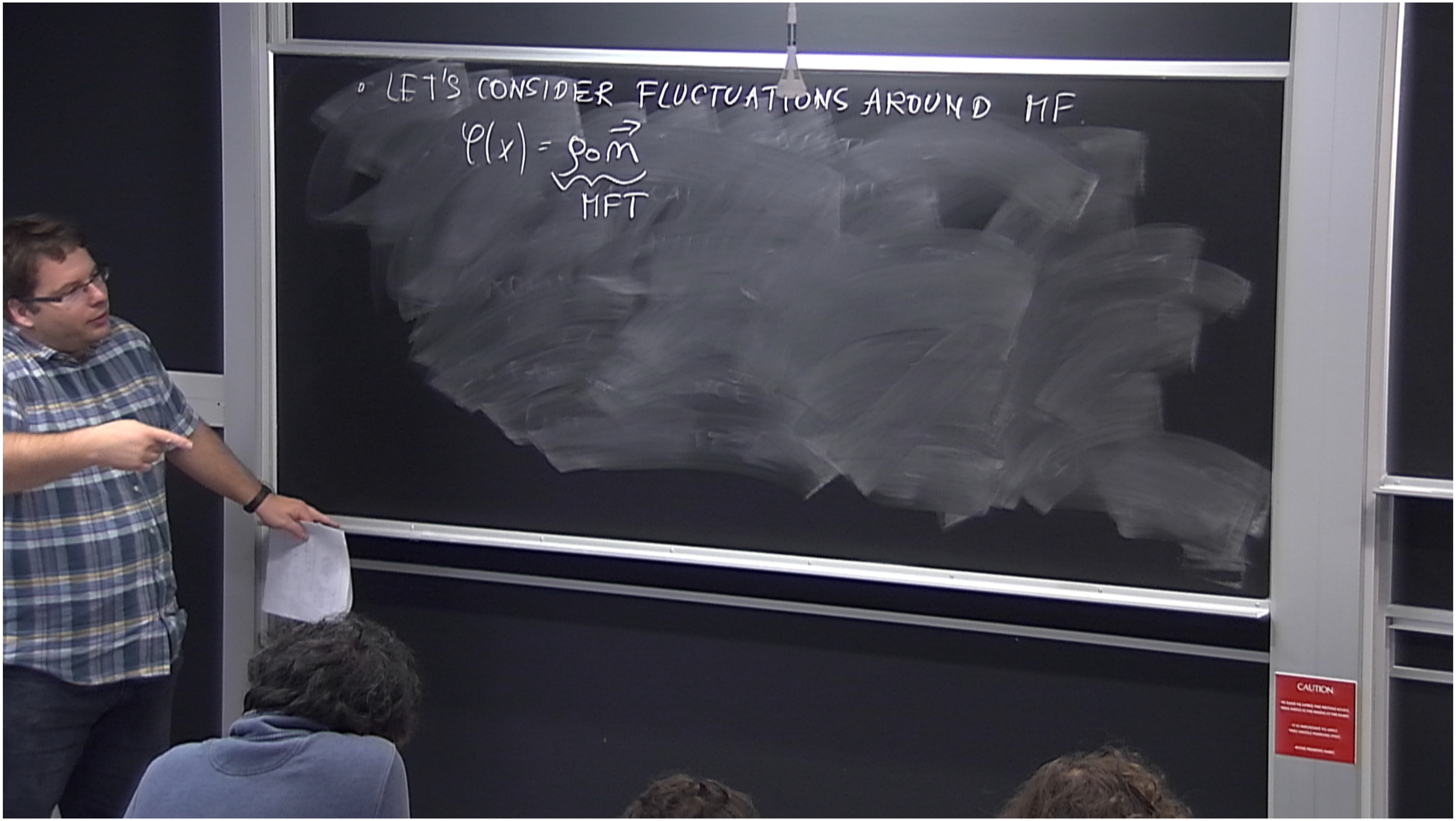

$f' = 0 = r\rho + u\rho^3 = \rho(r + u\rho^2) = 0$
$r<0$ $\boxed{\rho = \rho_0 = \sqrt{-r/u}}$
$r>0$ $\rho = 0$
$m^a \ldots$ ARBITRARY

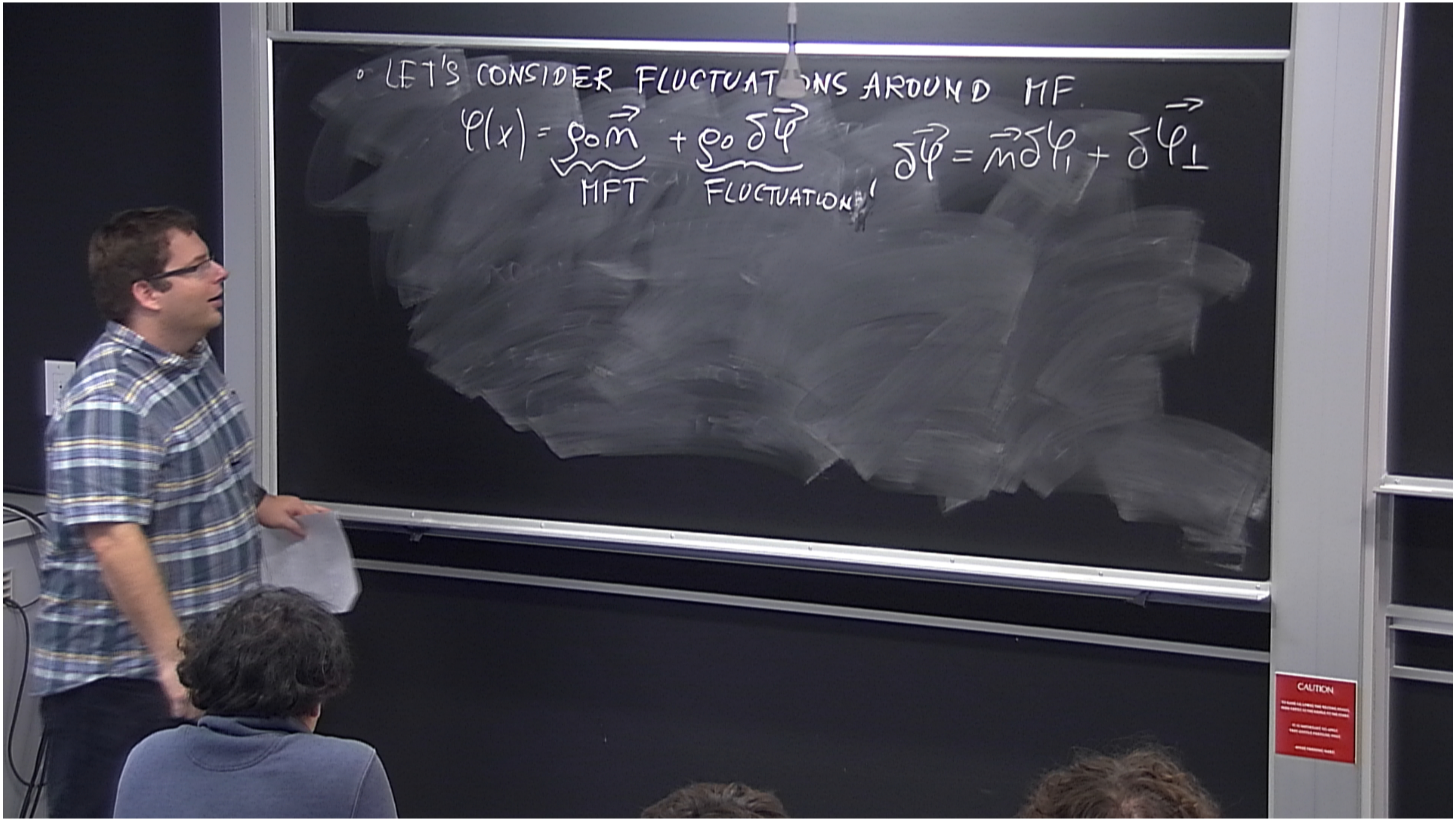

ISING
f
r<0
φ
Z2 SYMMETRY.
CAUTION

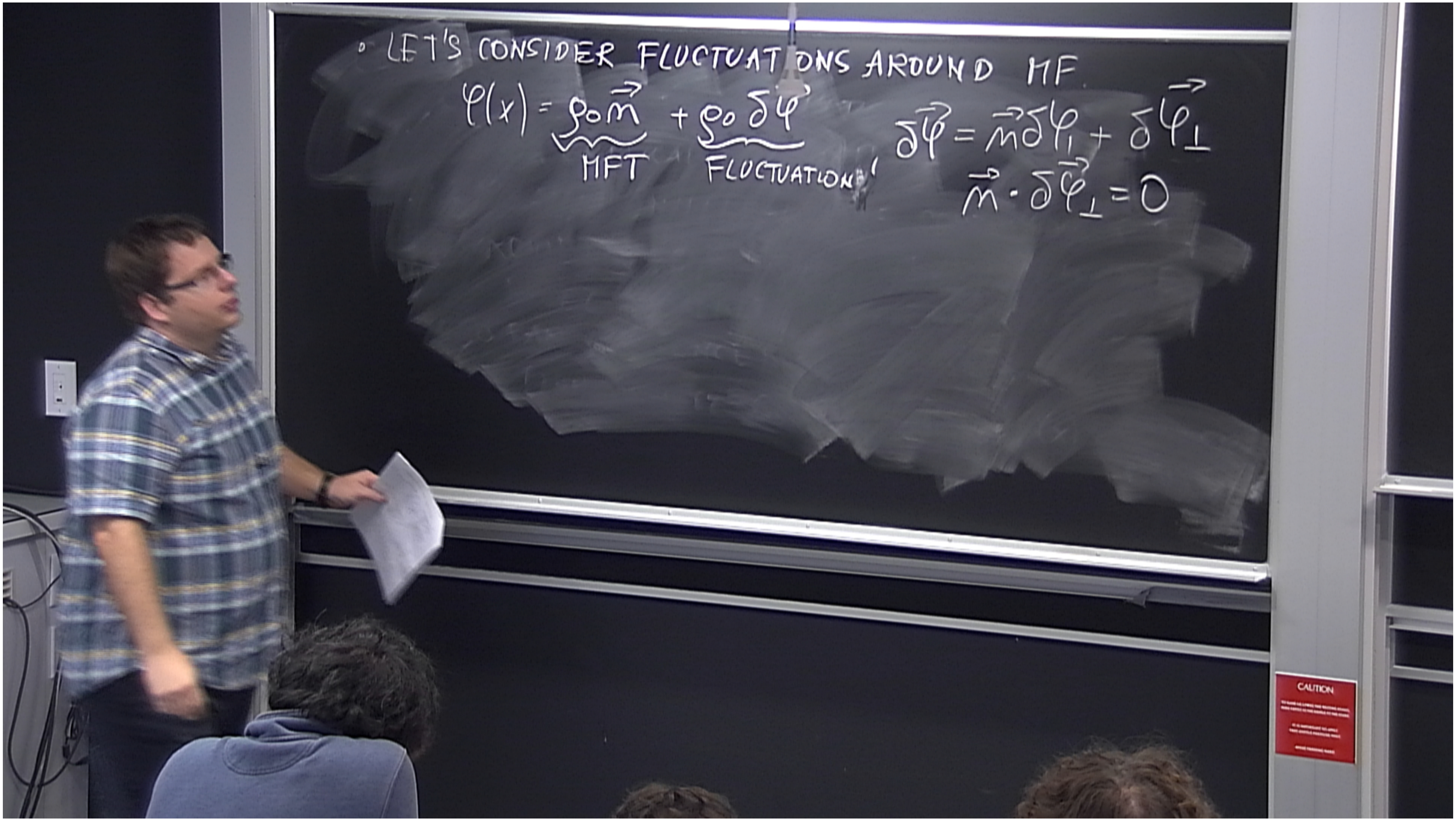

ISING
f
r<0
φ
Z2 SYMMETRY.
VECTOR
f
φ2
φ1

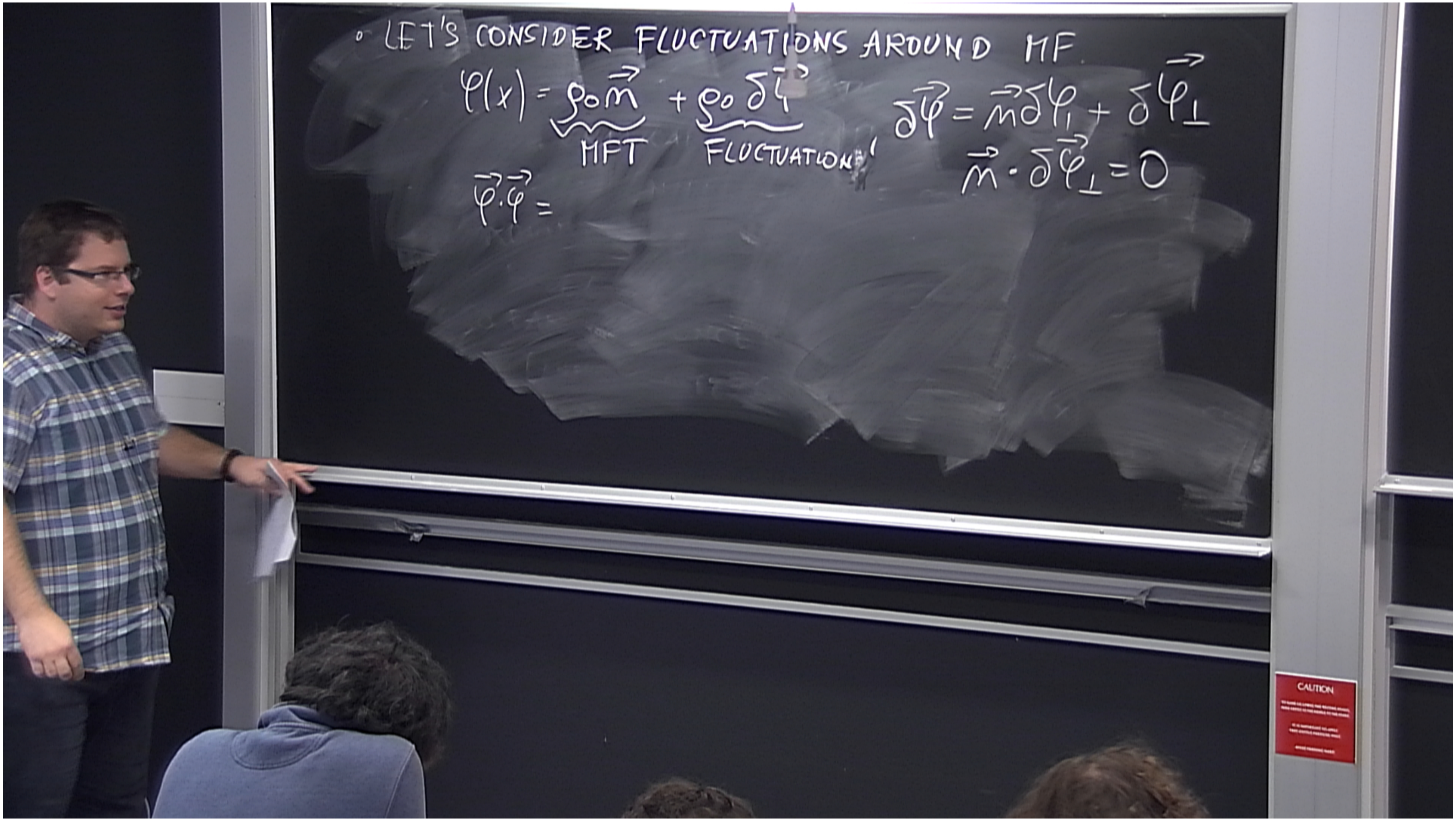

φ
Z2 SYMMETRY.
φ2
φ1

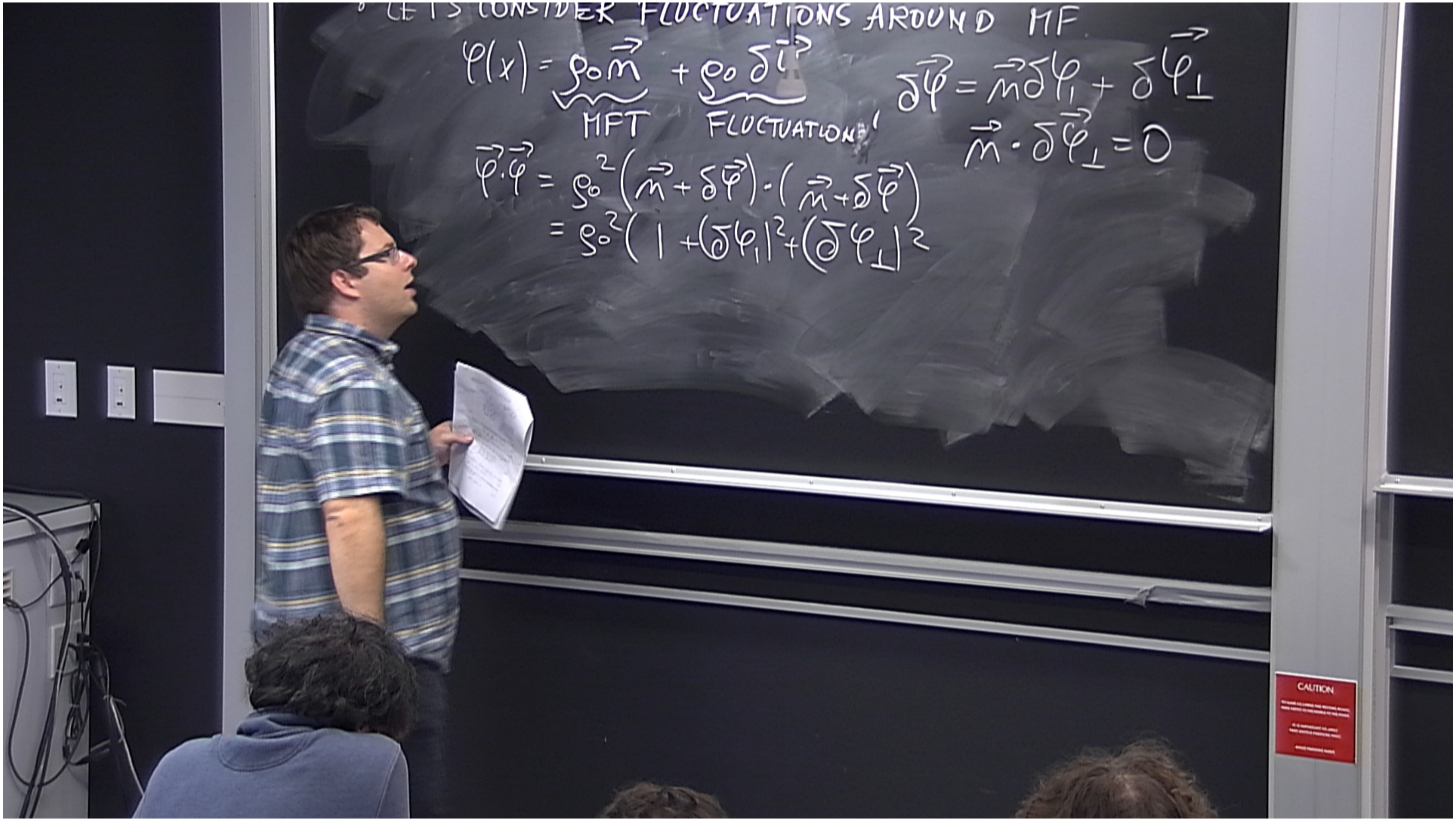

φ
Z_2 SYMMETRY.
φ_2
φ_1
CONTINUOUS SYMETRY
FLUCTUATIONS OF ORDER
PARAM MUCH SIMPLER.

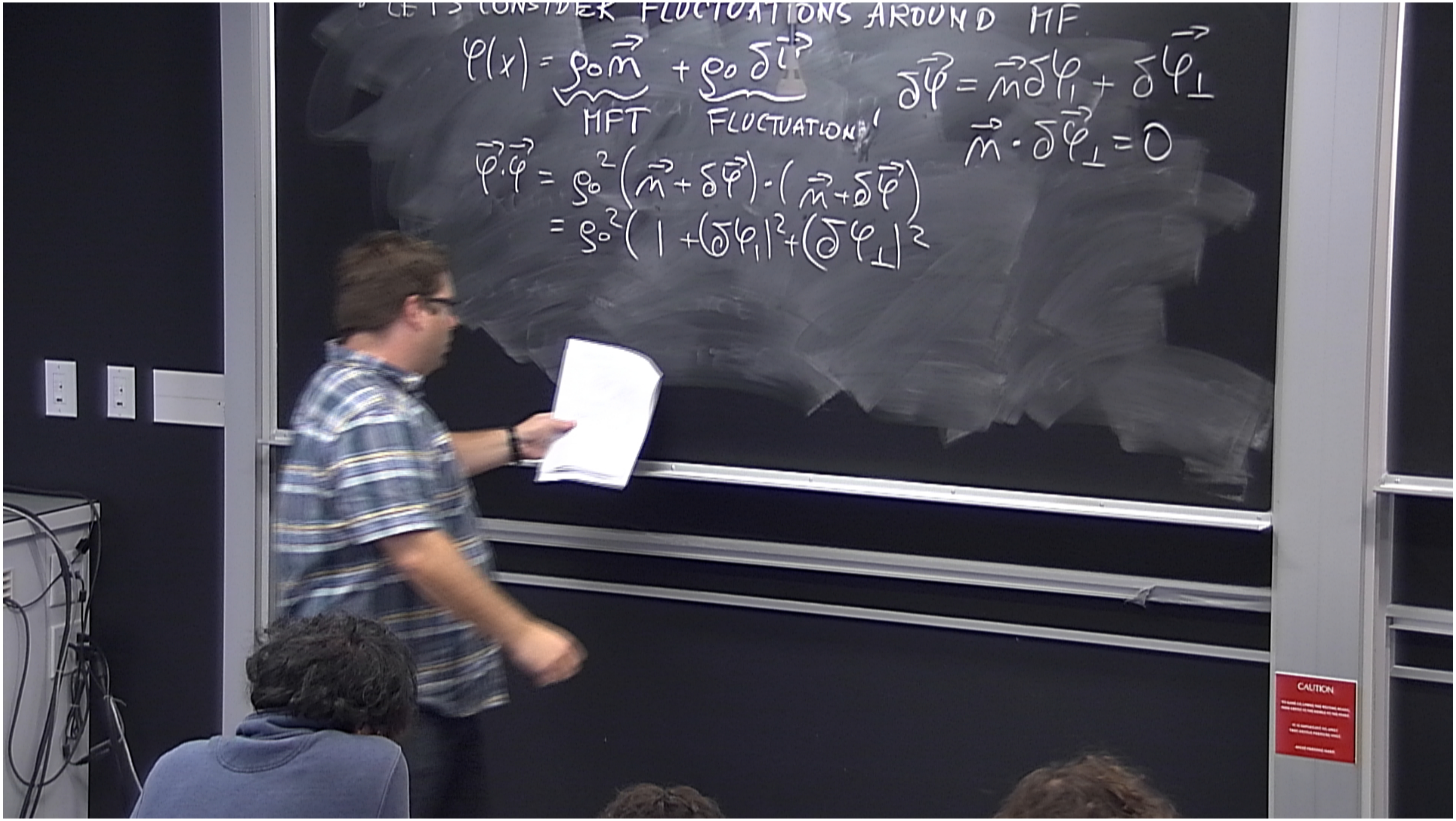

LET'S CONSIDER FLUCTUATIONS AROUND MF.
$\varphi(x) = \underbrace{\rho_0 \vec{m}}_{MFT}$

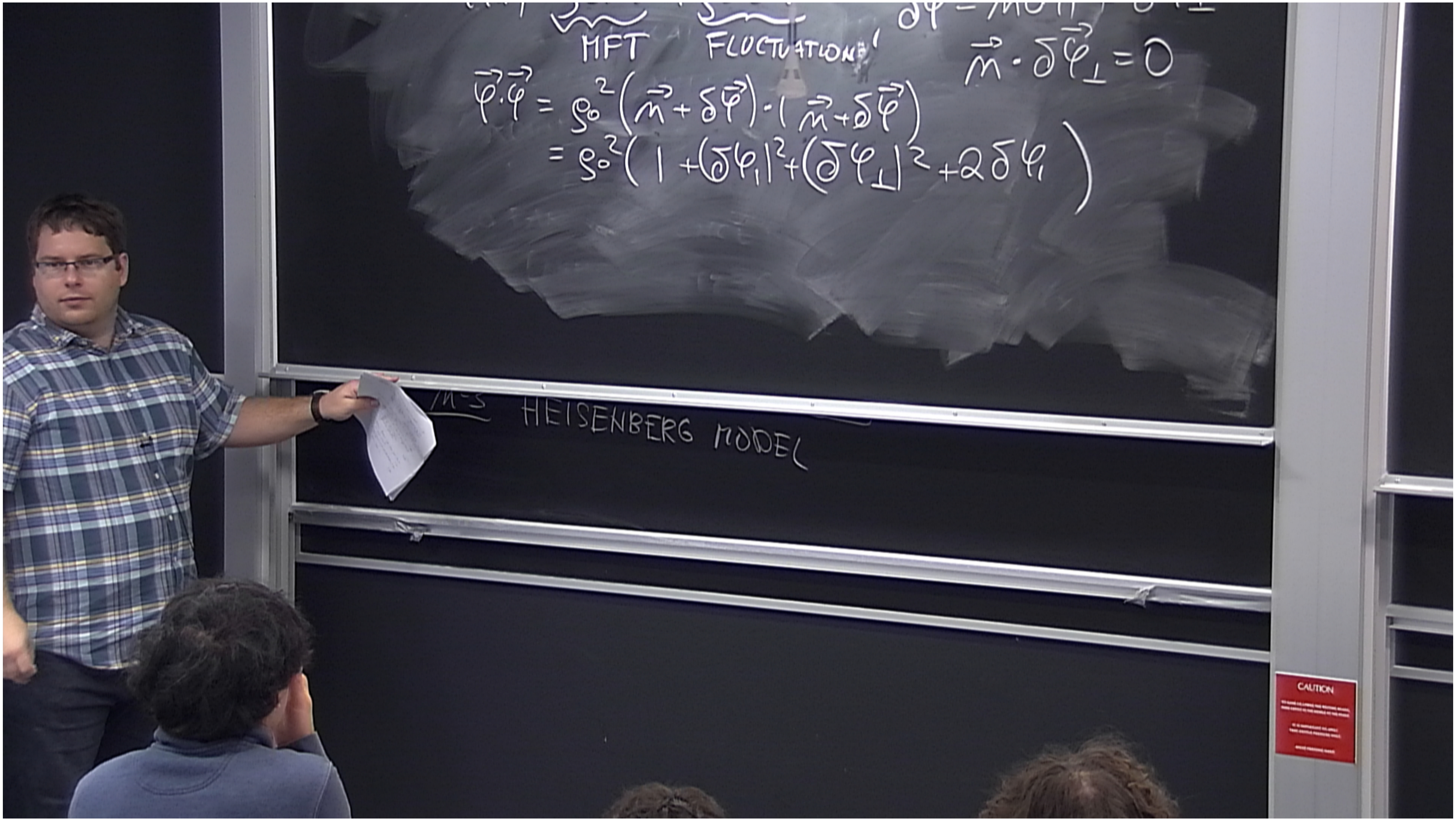

LET'S CONSIDER FLUCTUATIONS AROUND MF.
$\varphi(x) = \underbrace{\rho_0 \vec{m}}_{\text{MFT}} + \underbrace{\rho_0 \delta\vec{\varphi}}_{\text{FLUCTUATIONS}}$
$\delta\vec{\varphi} = \vec{m}\,\delta\varphi_1 + \delta\vec{\varphi}_\perp$

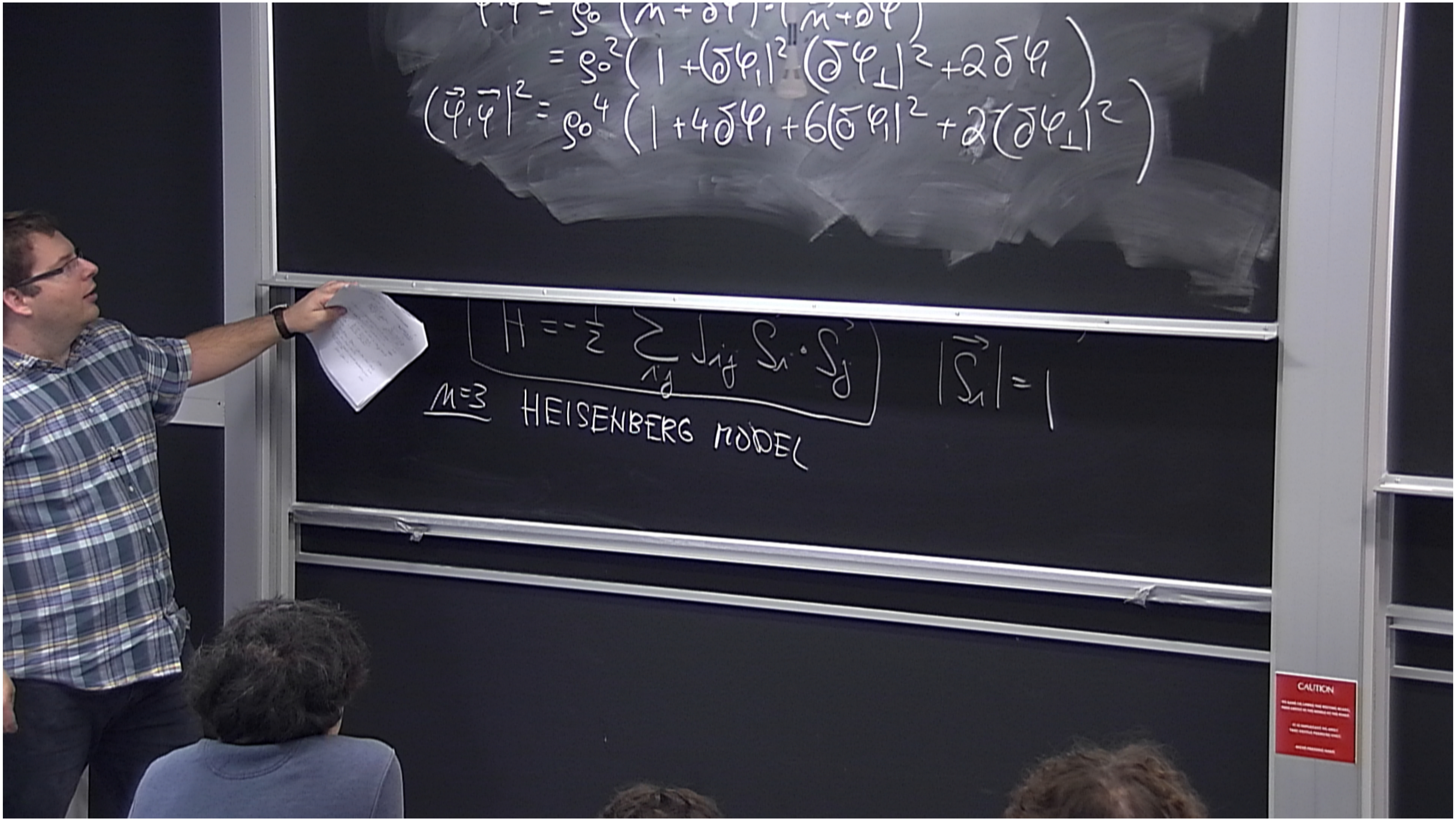

LET'S CONSIDER FLUCTUATIONS AROUND MF
$\varphi(x) = \underbrace{\rho_0 \vec{m}}_{MFT} + \underbrace{\rho_0 \delta \vec{\varphi}}_{FLUCTUATIONS}$
$\delta\vec{\varphi} = \vec{m}\,\delta\varphi_1 + \delta\vec{\varphi}_\perp$
$\vec{m} \cdot \delta\vec{\varphi}_\perp = 0$
CAUTION

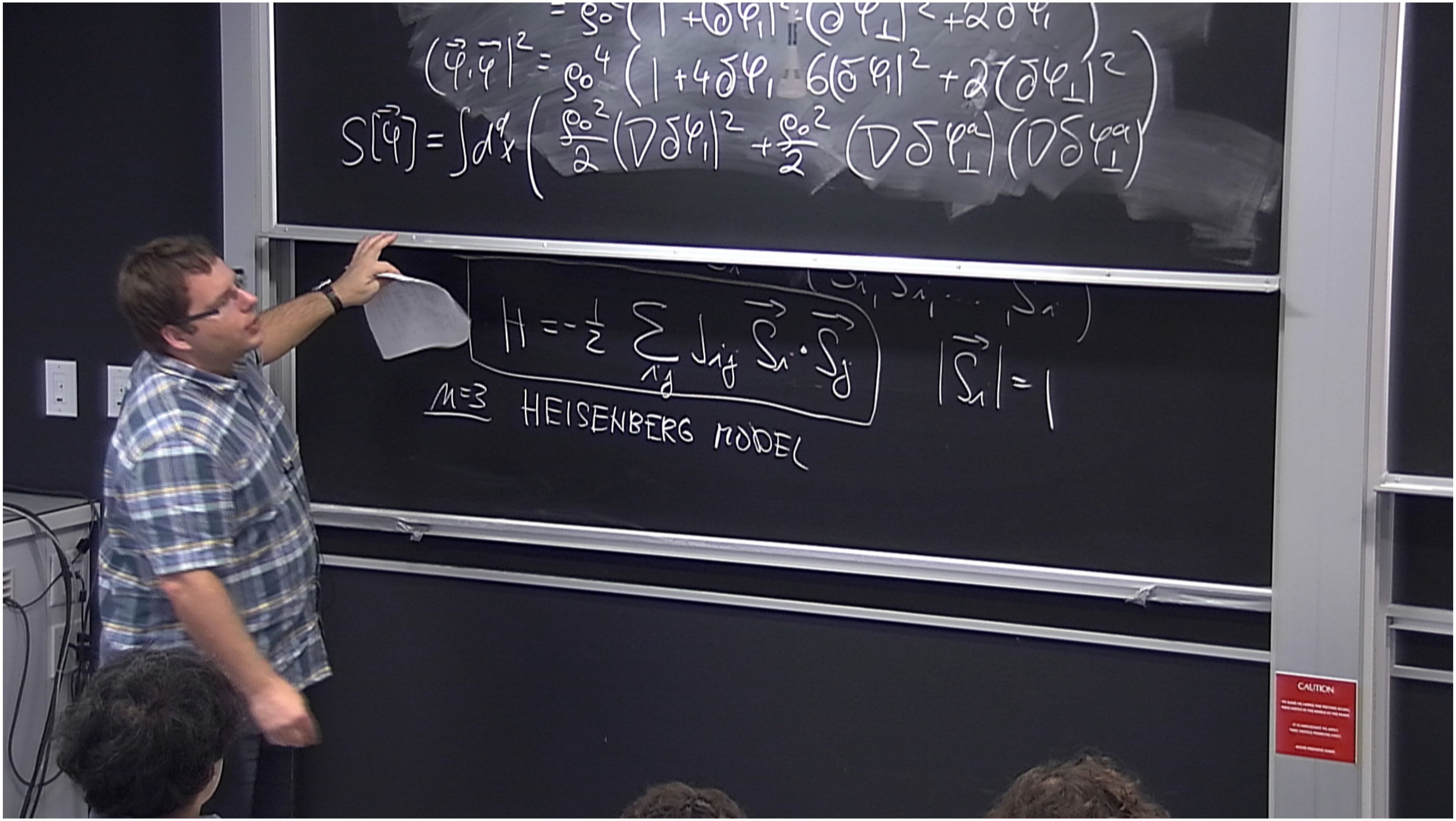

LET'S CONSIDER FLUCTUATIONS AROUND MF.
φ(x) = ρ₀m + ρ₀δφ
MFT
FLUCTUATIONS
δφ = m δφ1 + δφ⊥
m · δφ⊥ = 0
φ·φ =
CAUTION


LETS CONSIDER FLUCTUATIONS AROUND MF.
$\varphi(x) = \underbrace{\rho_0 \vec{m}}_{MFT} + \underbrace{\rho_0 \delta\vec{\varphi}}_{FLUCTUATIONS}$
$\delta\vec{\varphi} = \vec{m}\,\delta\varphi_1 + \delta\vec{\varphi}_\perp$
$\vec{m}\cdot\delta\vec{\varphi}_\perp = 0$
$\vec{\varphi}\cdot\vec{\varphi} = \rho_0^2(\vec{m}+\delta\vec{\varphi})\cdot(\vec{m}+\delta\vec{\varphi})$
$= \rho_0^2(1 + (\delta\varphi_1)^2 + (\delta\varphi_\perp)^2$
CAUTION

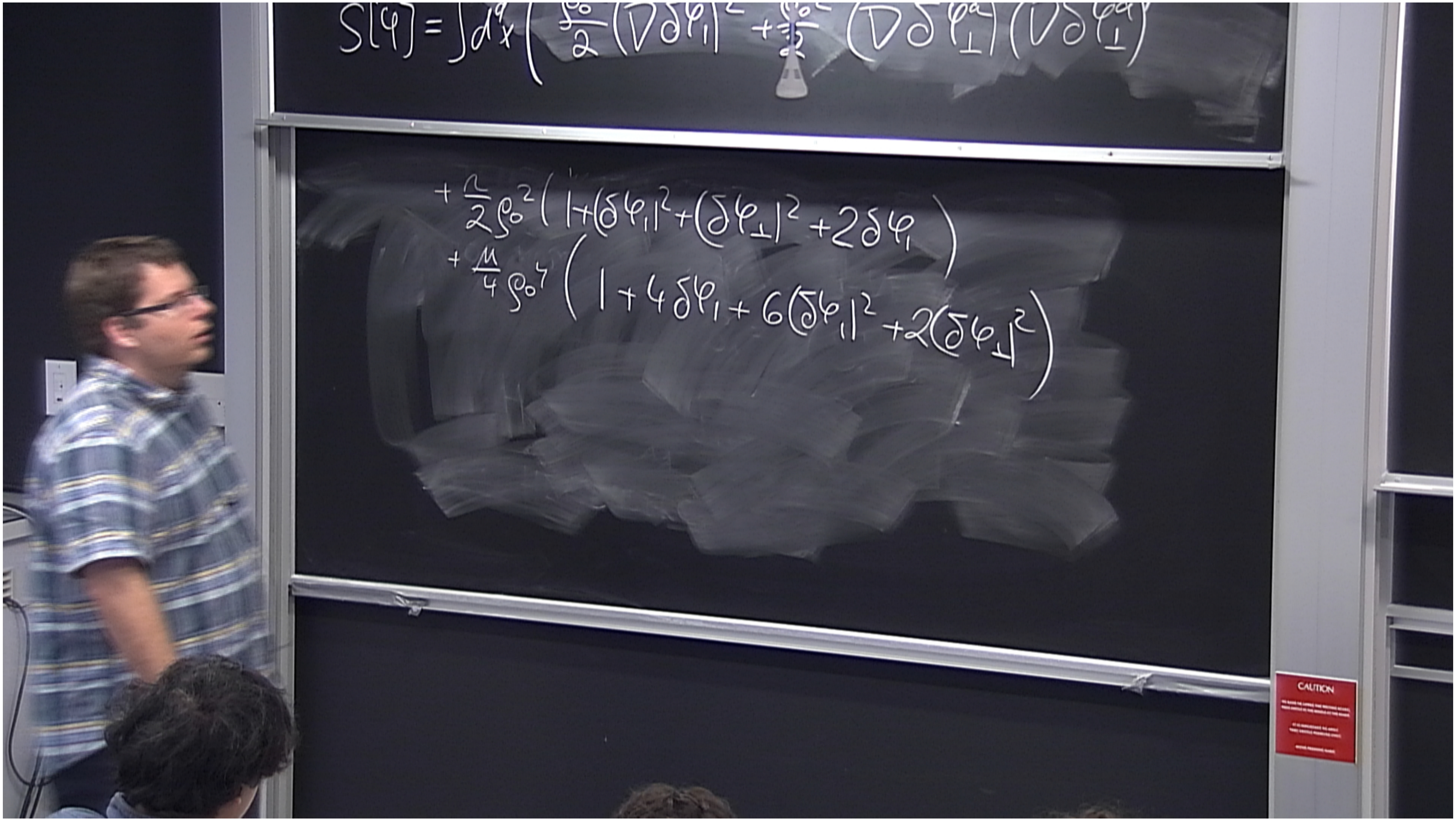

FLUCTUATIONS AROUND MF
$\varphi(x) = \underbrace{\rho_0 \vec{m}}_{MFT} + \underbrace{\rho_0 \delta\vec{\varphi}}_{FLUCTUATIONS}$
$\delta\vec{\varphi} = \vec{m}\,\delta\varphi_1 + \delta\vec{\varphi}_\perp$
$\vec{m}\cdot\delta\vec{\varphi}_\perp = 0$
$\vec{\varphi}\cdot\vec{\varphi} = \rho_0^2(\vec{m}+\delta\vec{\varphi})\cdot(\vec{m}+\delta\vec{\varphi})$
$= \rho_0^2(1 + (\delta\varphi_1)^2 + (\delta\varphi_\perp)^2$
CAUTION

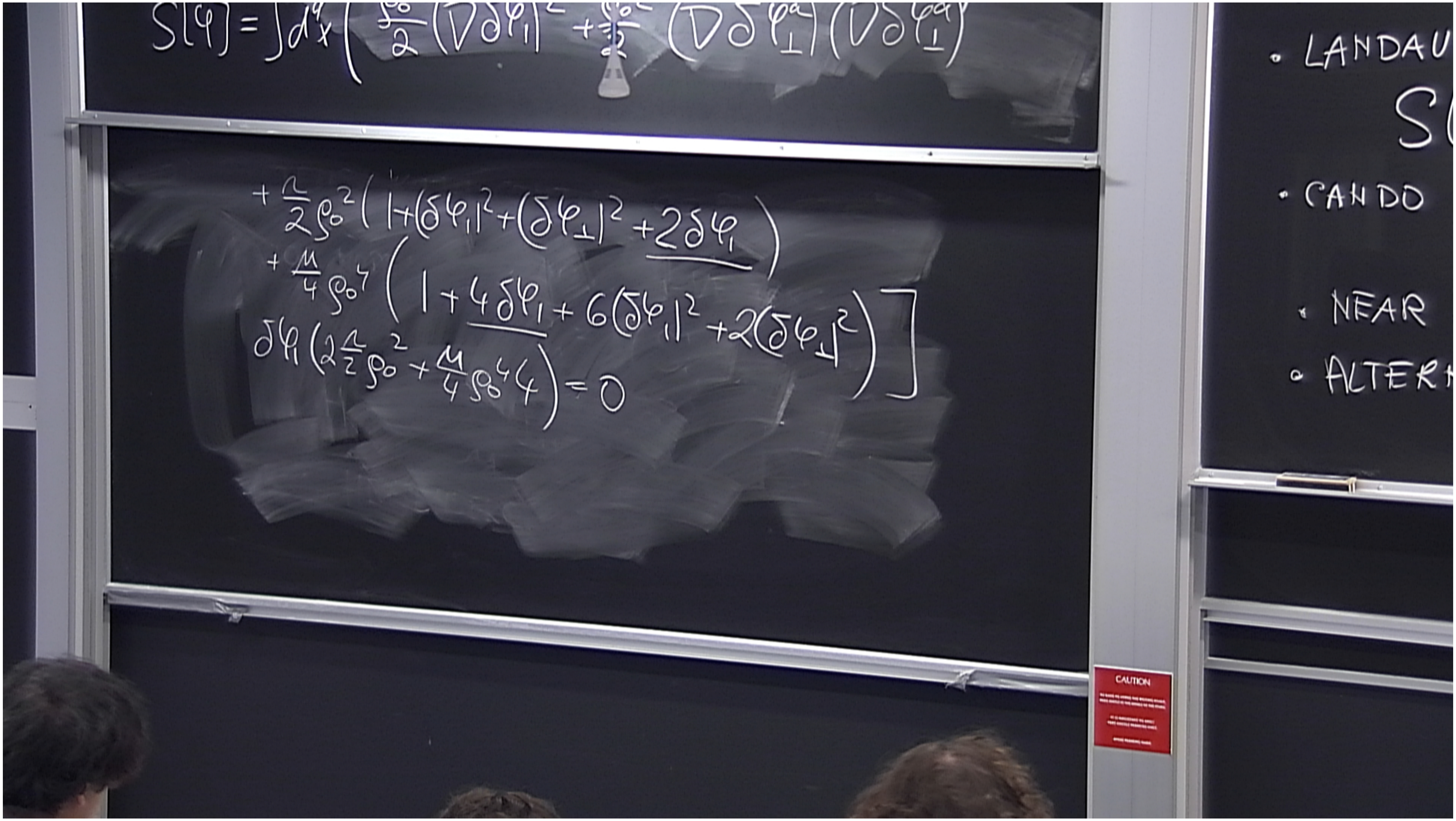

MFT
FLUCTUATIONS
$\vec{m}\cdot\delta\vec{\varphi}_\perp = 0$
$\vec{\varphi}\cdot\vec{\varphi} = \rho_0^2(\vec{m}+\delta\vec{\varphi})\cdot(\vec{m}+\delta\vec{\varphi})$
$= \rho_0^2(1+(\delta\varphi_1)^2+(\delta\varphi_\perp)^2+2\delta\varphi_1)$
HEISENBERG MODEL


$= \rho_0^2 \left(1 + (\delta\varphi_\parallel)^2 (\delta\varphi_\perp)^2 + 2\delta\varphi_\parallel \right)$
$(\vec{\varphi}\cdot\vec{\varphi})^2 = \rho_0^4 \left(1 + 4\delta\varphi_\parallel + 6(\delta\varphi_\parallel)^2 + 2(\delta\varphi_\perp)^2 \right)$
$H = -\frac{1}{2} \sum_{ij} J_{ij} S_i \cdot S_j$
$|\vec{S}_i| = 1$
n=3 HEISENBERG MODEL
CAUTION



$S[\vec{\varphi}] = \int d^dx \left(\frac{\rho_0^2}{2}(\nabla\delta\varphi_1)^2 + \frac{\rho_0^2}{2}(\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a) \right.$
$H = -\frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i\cdot\vec{S}_j$
$|\vec{S}_i| = 1$
n=3 HEISENBERG MODEL
CAUTION


$S[\varphi] = \int d^dx \Big(\frac{\rho_0}{2} (\nabla \delta\varphi_\parallel)^2 + \frac{\rho_0^2}{2} (\nabla \delta \varphi_\perp^a)(\nabla \delta \varphi_\perp^a)$
$+ \frac{r}{2} \rho_0^2 \big(1 + (\delta\varphi_\parallel)^2 + (\delta\varphi_\perp)^2 + 2\delta\varphi_\parallel \big)$
$+ \frac{u}{4} \rho_0^4 \big(1 + 4\delta\varphi_\parallel + 6(\delta\varphi_\parallel)^2 + 2(\delta\varphi_\perp)^2 \big)$
CAUTION


$S[\varphi] = \int d^dx \Big(\frac{\rho_0}{2}(\nabla\delta\varphi_1)^2 + \frac{\rho_0^2}{2}(\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a)$
$+ \frac{r}{2}\rho_0^2\big(1 + (\delta\varphi_1)^2 + (\delta\varphi_\perp)^2 + \underline{2\delta\varphi_1}\big)$
$+ \frac{u}{4}\rho_0^4\big(1 + \underline{4\delta\varphi_1} + 6(\delta\varphi_1)^2 + 2(\delta\varphi_\perp)^2\big)\Big]$
$\delta\varphi_1\big(2\frac{r}{2}\rho_0^2 + \frac{u}{4}\rho_0^4 4\big) = 0$
• LANDAU
S
• CANDO
• NEAR
• ALTER
CAUTION

$$S[\varphi] = \int d^d x \left(\frac{\rho_0^2}{2} (\nabla \delta\varphi_1)^2 + \frac{\rho_0^2}{2} (\nabla \delta\varphi_\perp^a)(\nabla \delta\varphi_\perp^a) \right.$$

$$+ \frac{r}{2}\rho_0^2 \left(1 + (\delta\varphi_1)^2 + \boxed{(\delta\varphi_\perp)^2} + \underline{2\delta\varphi_1} \right)$$

$$\left. + \frac{u}{4}\rho_0^4 \left(1 + \underline{4\delta\varphi_1} + 6(\delta\varphi_1)^2 + \boxed{2(\delta\varphi_\perp)^2} \right) \right]$$

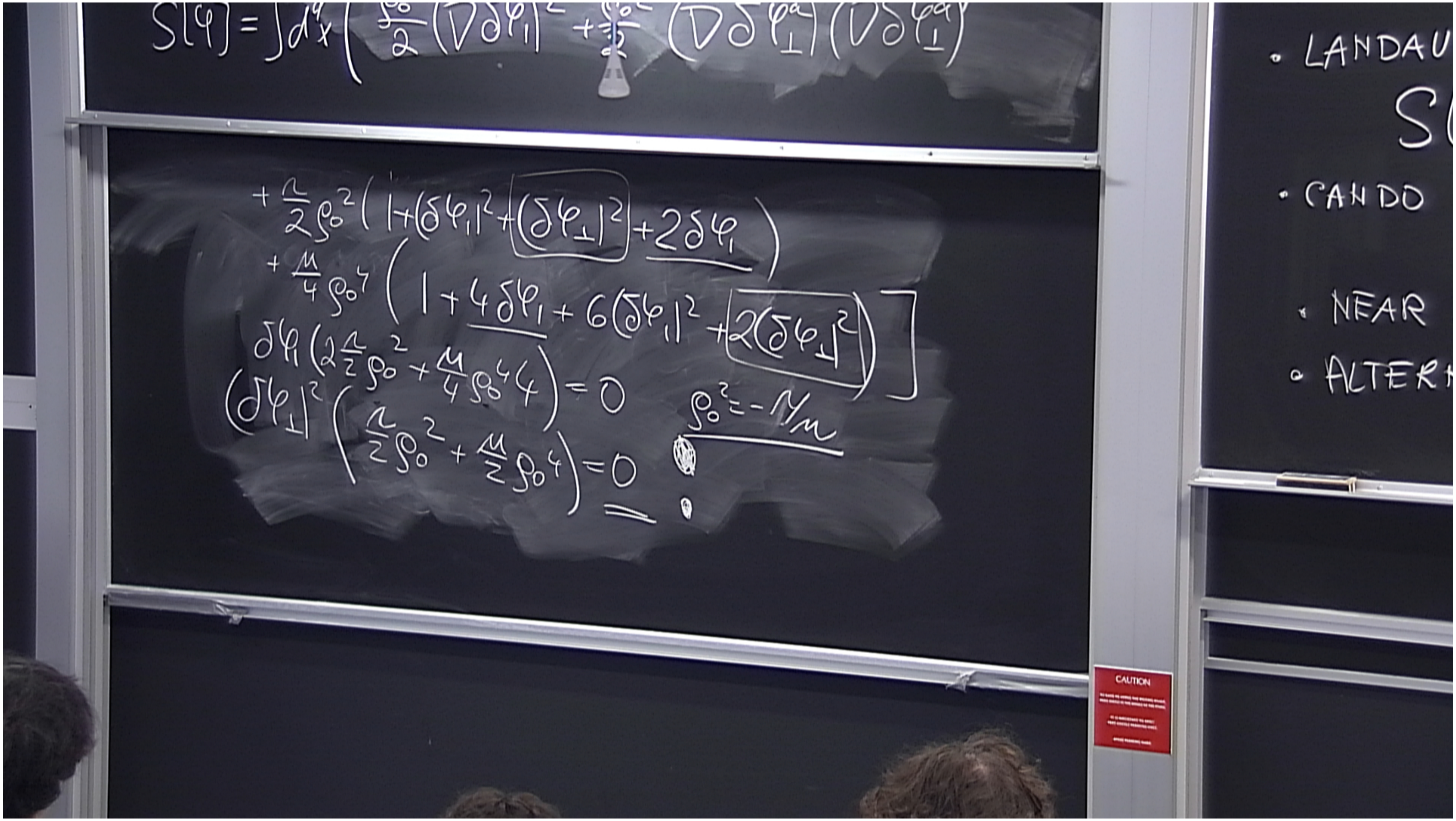
$$\delta\varphi_1 \left(2\frac{r}{2}\rho_0^2 + \frac{u}{4}\rho_0^4 4 \right) = 0 \qquad \underline{\rho_0^2 = -r/u}$$

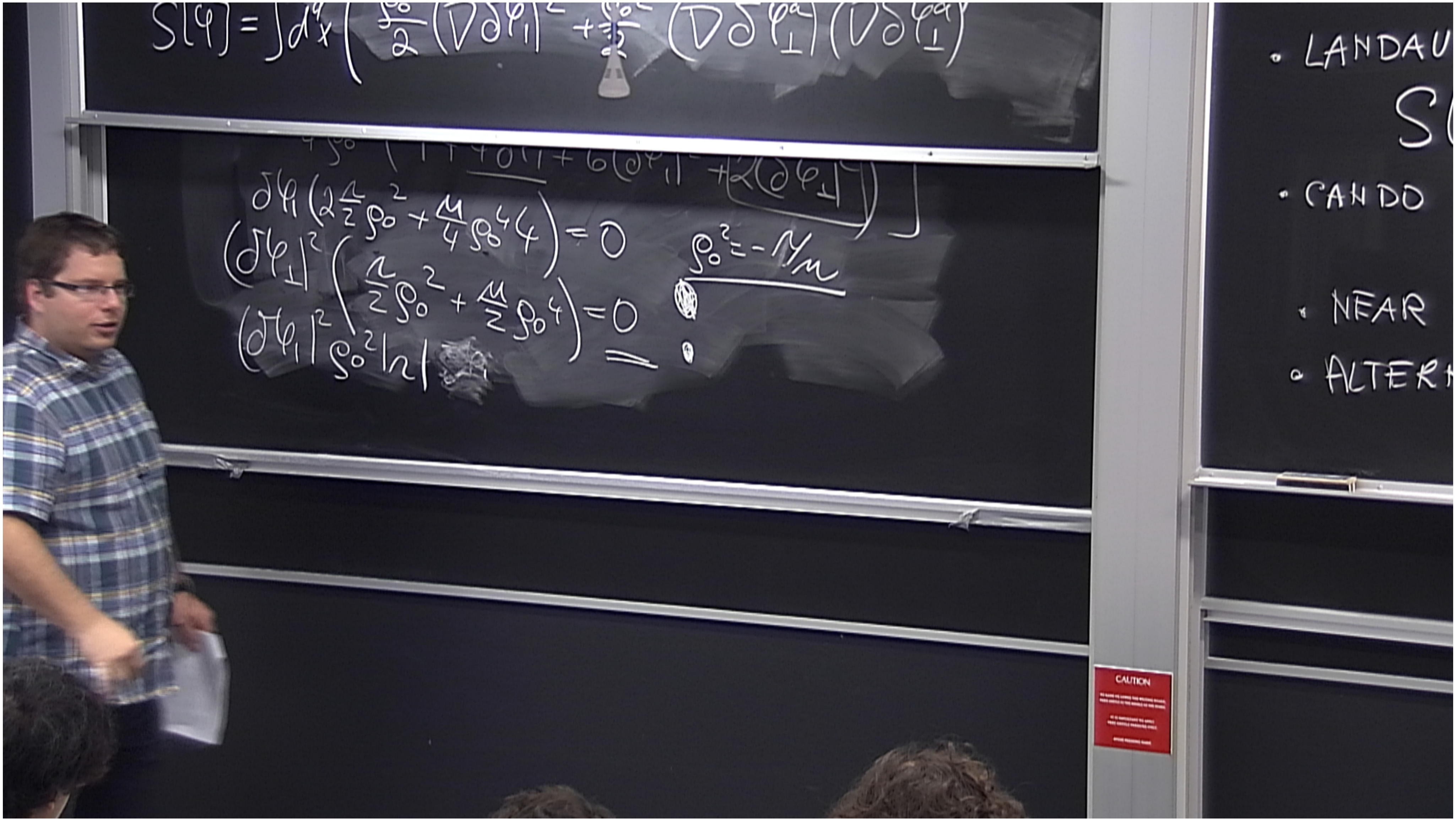
$$(\delta\varphi_\perp)^2 \left(\frac{r}{2}\rho_0^2 + \frac{u}{2}\rho_0^4 \right)$$

- LANDAU

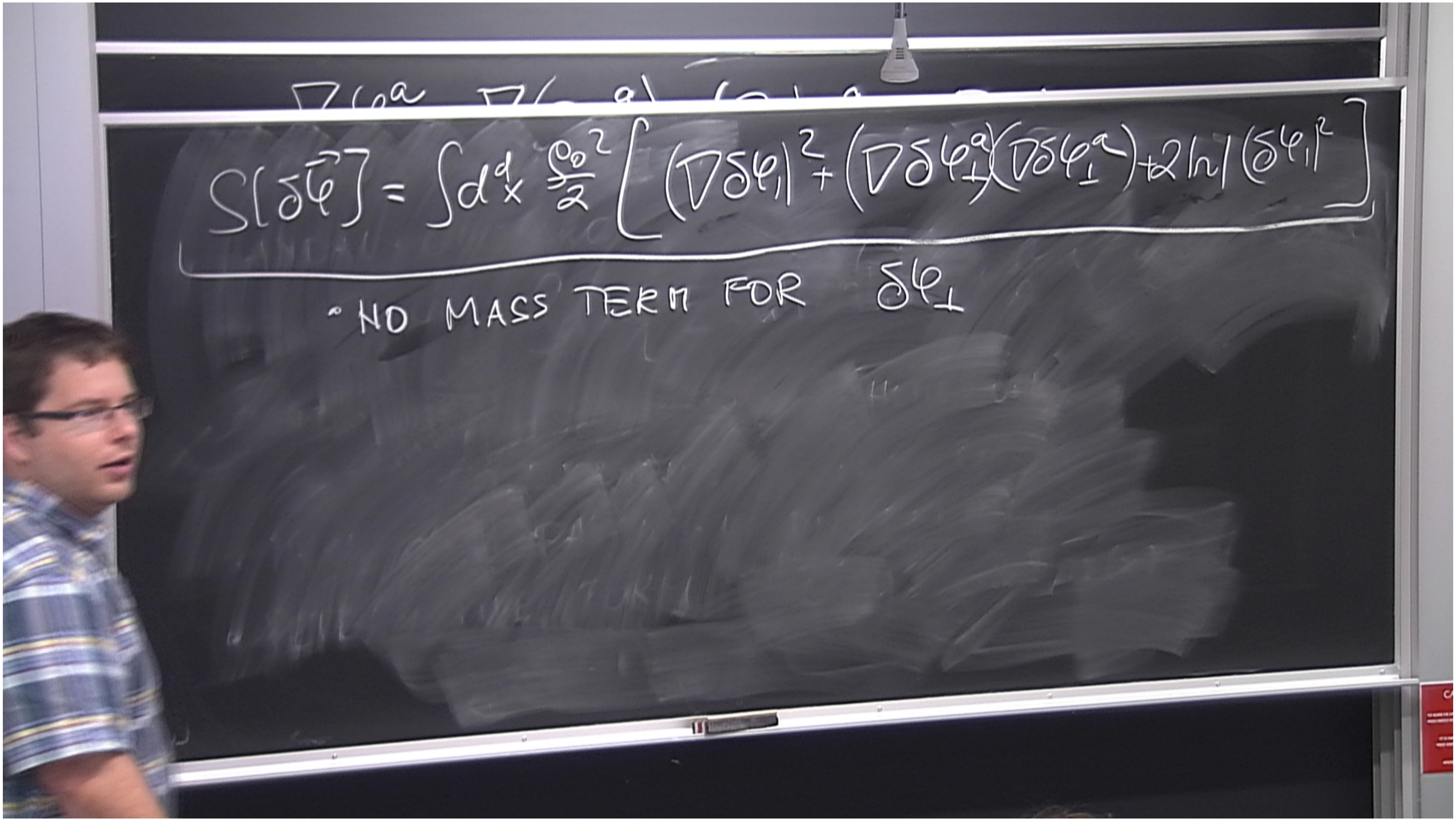
  S

- CANDO
- NEAR
- ALTER


$S[\varphi] = \int d^d x \Big(\frac{\rho_0}{2} (\nabla\delta\varphi_1)^2 + \frac{\rho_0^2}{2} (\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a)$
$+ \frac{r}{2}\rho_0^2 \left(1 + (\delta\varphi_1)^2 + \boxed{(\delta\varphi_\perp)^2} + \underline{2\delta\varphi_1} \right)$
$+ \frac{u}{4}\rho_0^4 \left(1 + \underline{4\delta\varphi_1} + 6(\delta\varphi_1)^2 + \boxed{2(\delta\varphi_\perp)^2} \right) \Big]$
$\delta\varphi_1 \left(2\frac{r}{2}\rho_0^2 + \frac{u}{4}\rho_0^4 \, 4 \right) = 0$
$(\delta\varphi_\perp)^2 \left(\frac{r}{2}\rho_0^2 + \frac{u}{2}\rho_0^4 \right) = 0$
$\rho_0^2 = -r/u$
• LANDAU
S
• CANDO
• NEAR
• ALTER
CAUTION


• LANDAU
• CANDO
• NEAR
• ALTER
CAUTION

$$S[\delta\vec{\varphi}] = \int d^d x \, \frac{\rho_0^2}{2}\left[(\nabla\delta\varphi_1)^2 + (\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a) + 2|h|(\delta\varphi_1)^2 \right]$$


$S[\delta\vec{\varphi}] = \int d^dx \, \frac{\rho_0^2}{2}\left[(\nabla\delta\varphi_\parallel)^2 + (\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a) + 2|h|(\delta\varphi_\parallel)^2\right]$
• NO MASS TERM FOR $\delta\varphi_\perp$

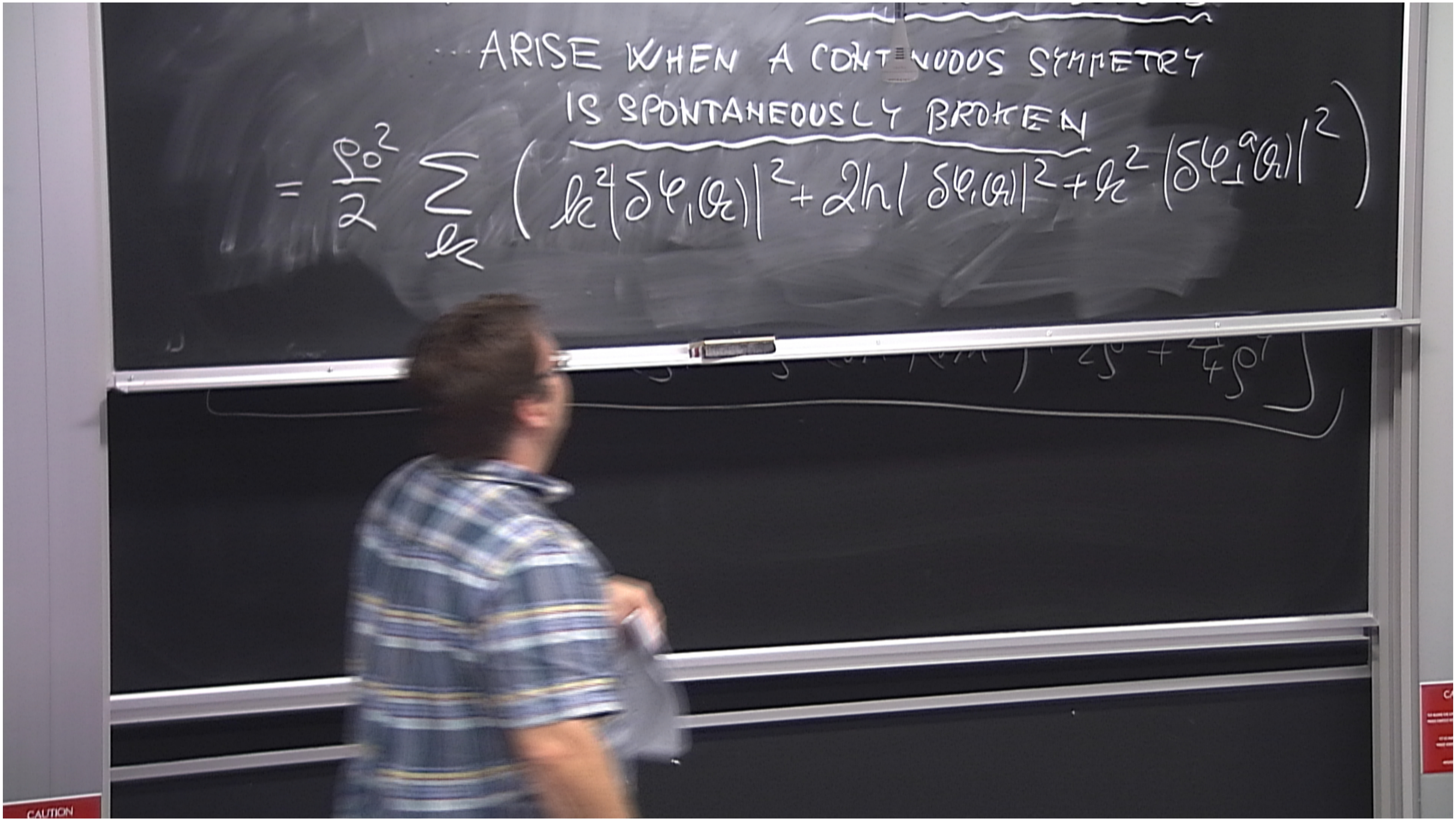
$$S[\delta\vec{\varphi}] = \int d^dx \, \frac{\rho_0^2}{2} \left[(\nabla\delta\varphi_\parallel)^2 + (\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a) + 2|h|(\delta\varphi_\parallel)^2 \right]$$

- NO MASS TERM FOR $\delta\varphi_\perp$ BEHAVE AS MASSLESS MODES. GOLDSTONE BOSONS

$$S[\delta\vec{\varphi}] = \int d^dx \, \frac{\rho_0^2}{2}\left[(\nabla\delta\varphi_1)^2 + (\nabla\delta\varphi_\perp^a)(\nabla\delta\varphi_\perp^a) + 2|h|(\delta\varphi_1)^2\right]$$

- NO MASS TERM FOR $\delta\varphi_\perp$ ... BEHAVE AS MASSLESS MODES . GOLDSTONE BOSONS

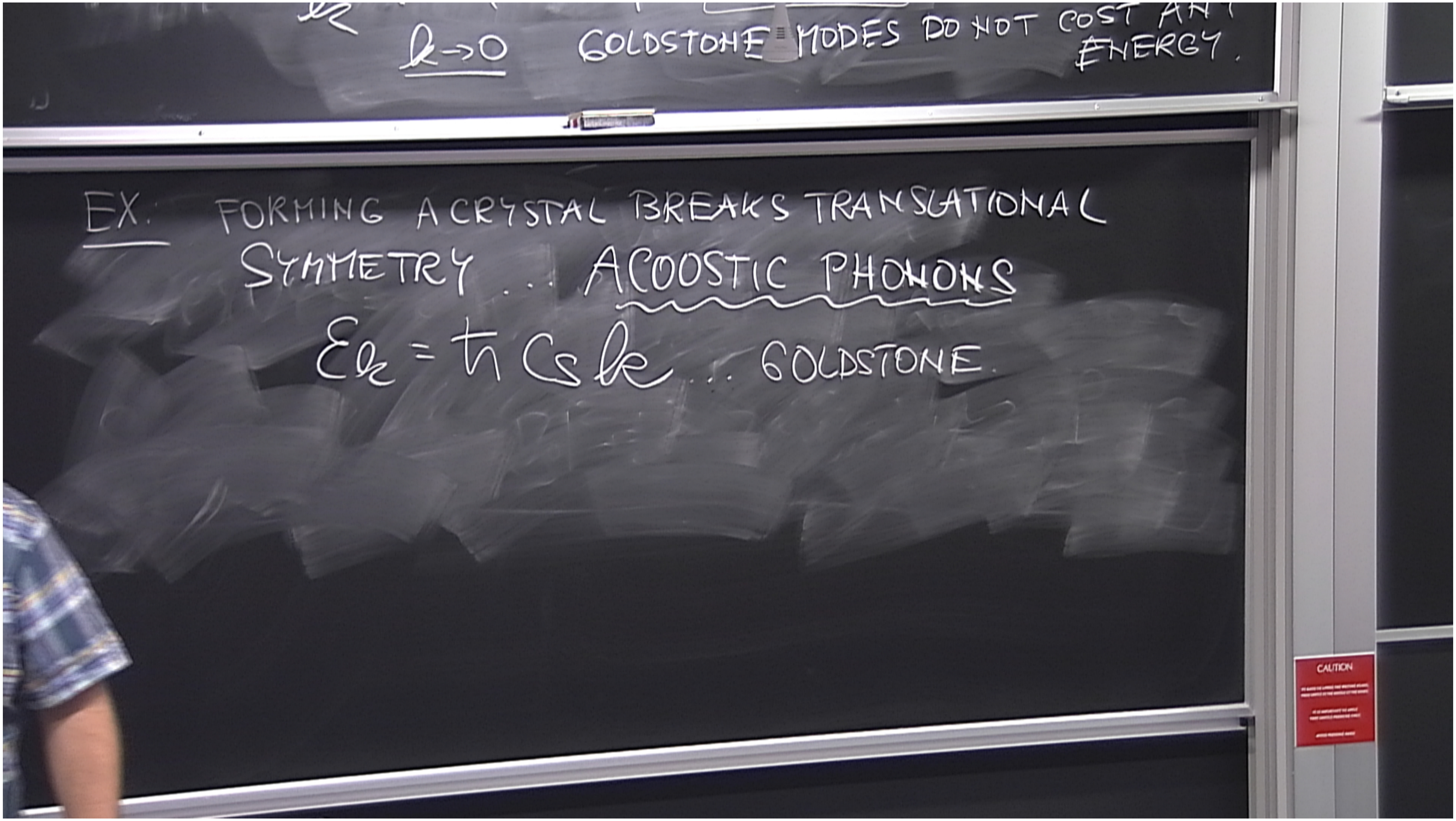
... ARISE WHEN A CONTINUOUS SYMMETRY IS SPONTANEOUSLY BROKEN

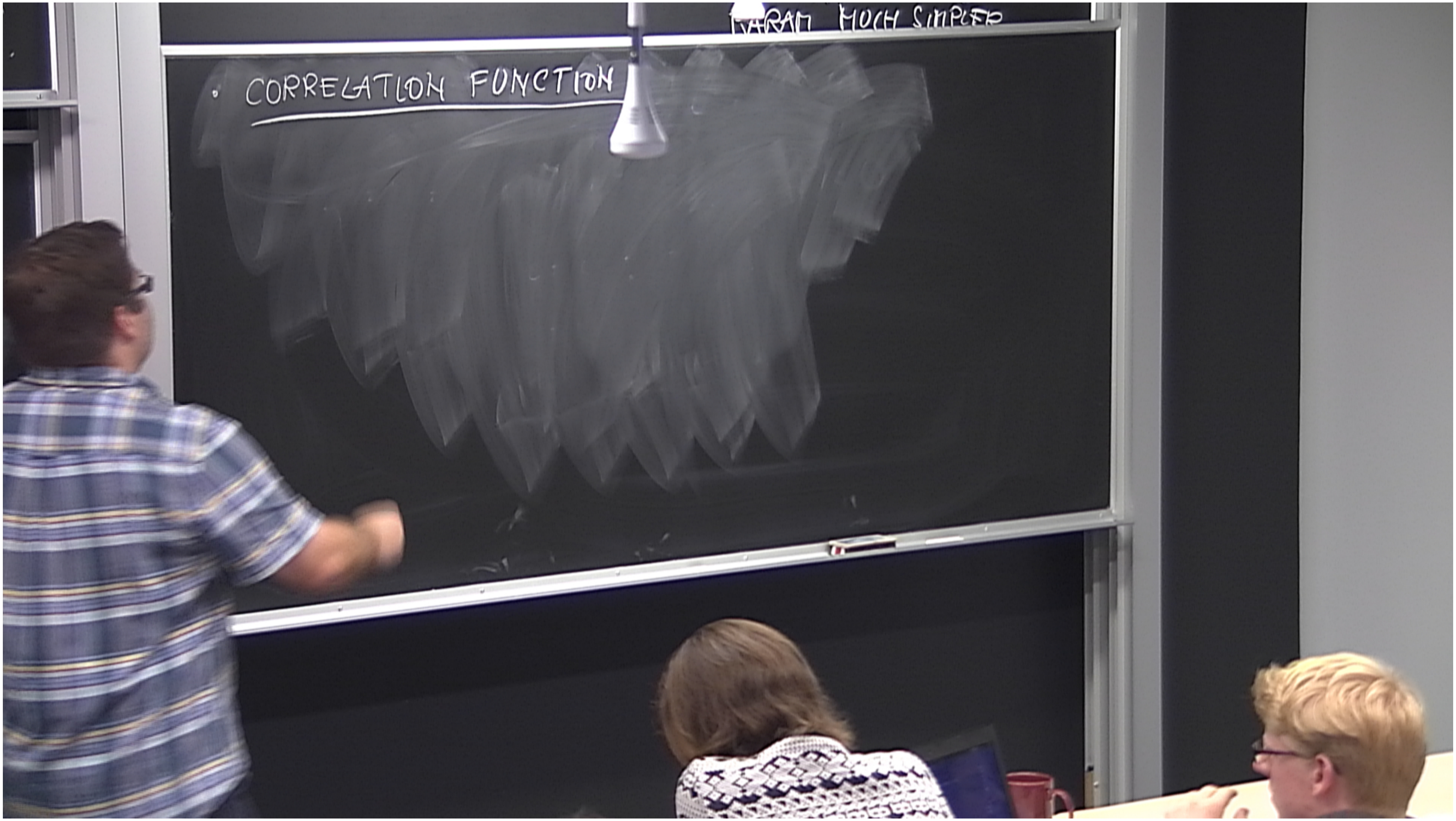

ARISE WHEN A CONTINUOUS SYMMETRY
IS SPONTANEOUSLY BROKEN
$= \frac{\rho_0^2}{2} \sum_k \left(k^2 |\delta\varphi_1(k)|^2 + 2h |\delta\varphi_1(k)|^2 + k^2 |\delta\varphi_\perp^a(k)|^2 \right)$
CAUTION

...ARISE WHEN A CONTINUOUS SYMMETRY IS SPONTANEOUSLY BROKEN

$$= \frac{\rho_0^2}{2} \sum_k \left(k^2 |\delta\varphi_\parallel(k)|^2 + 2h\, |\delta\varphi_\parallel(k)|^2 + k^2 |\delta\varphi_\perp^a(k)|^2 \right)$$

$\underline{k \to 0}$ GOLDSTONE MODES DO NOT COST ANY ENERGY.


$k \to 0$ GOLDSTONE MODES DO NOT COST ANY ENERGY.
EX: FORMING A CRYSTAL BREAKS TRANSLATIONAL SYMMETRY ... ACOUSTIC PHONONS
$E_k = \hbar c_s k$ ... GOLDSTONE.


MUCH SIMPLER
CORRELATION FUNCTION

...METRY,

$\angle\varphi$ CONTINUOUS SYM...

- CORRELATION FUNCTION

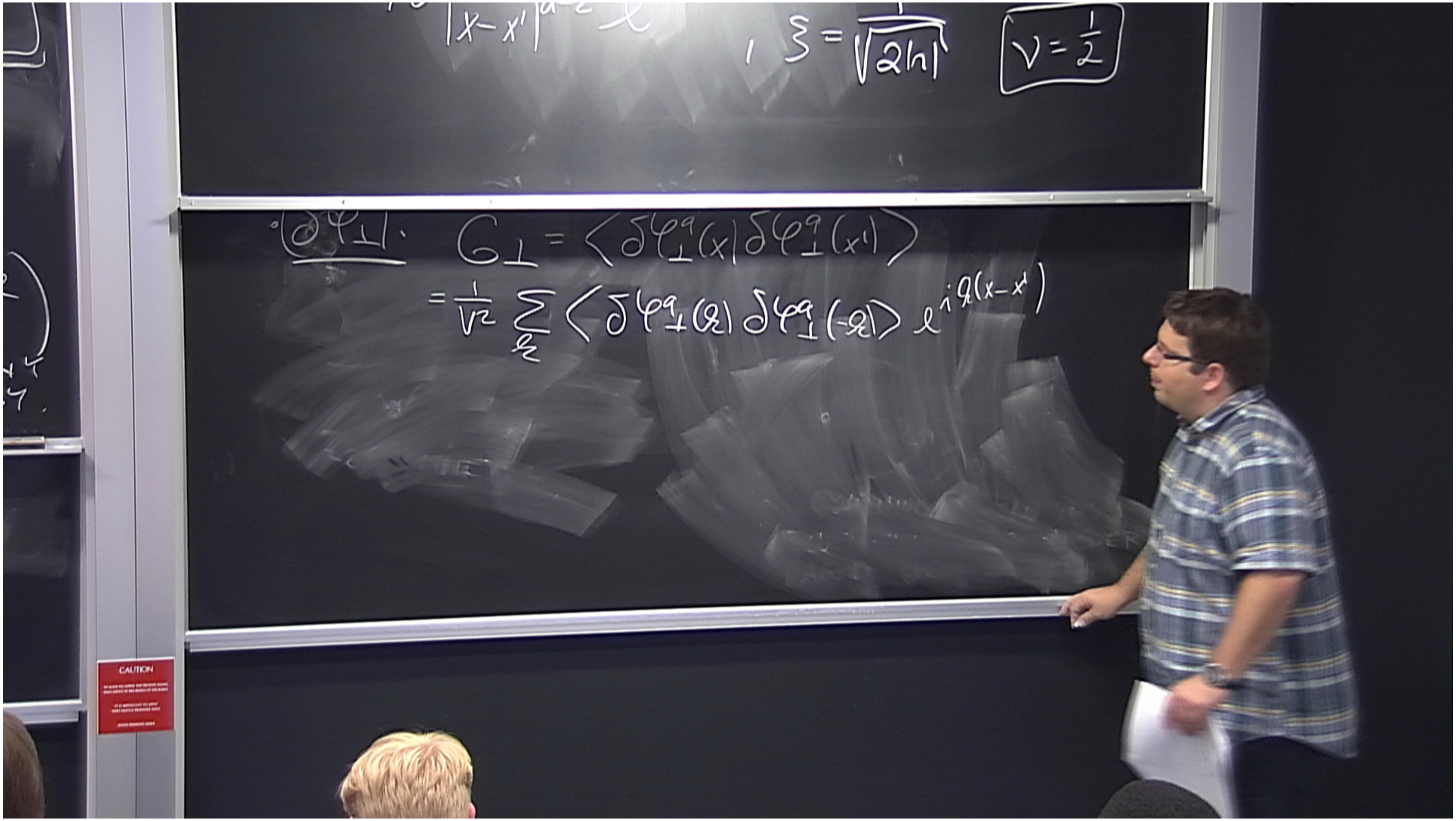
$\underline{\delta\varphi_1}$: $\langle \varphi \rangle = \vec{m}\rho_0$, $\langle \delta\varphi_1 \rangle = 0$

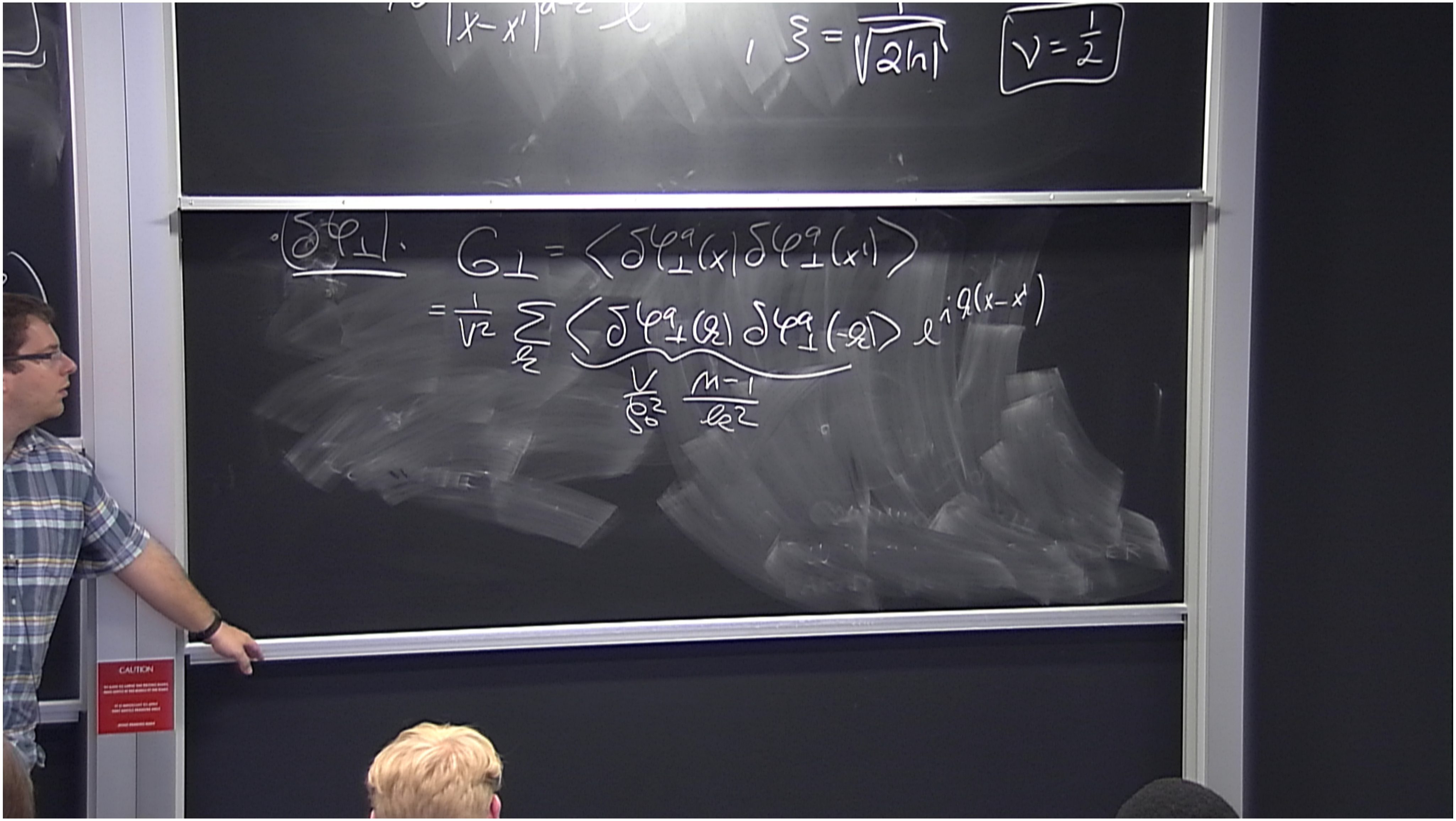
$$G_1(x-x') = \langle \delta\varphi_1(x)\,\delta\varphi_1(x') \rangle = \frac{1}{V^2}\sum_k \langle \delta\varphi_1(k)\,\delta\varphi_1(-k) \rangle\, e^{ik(x-x')}$$

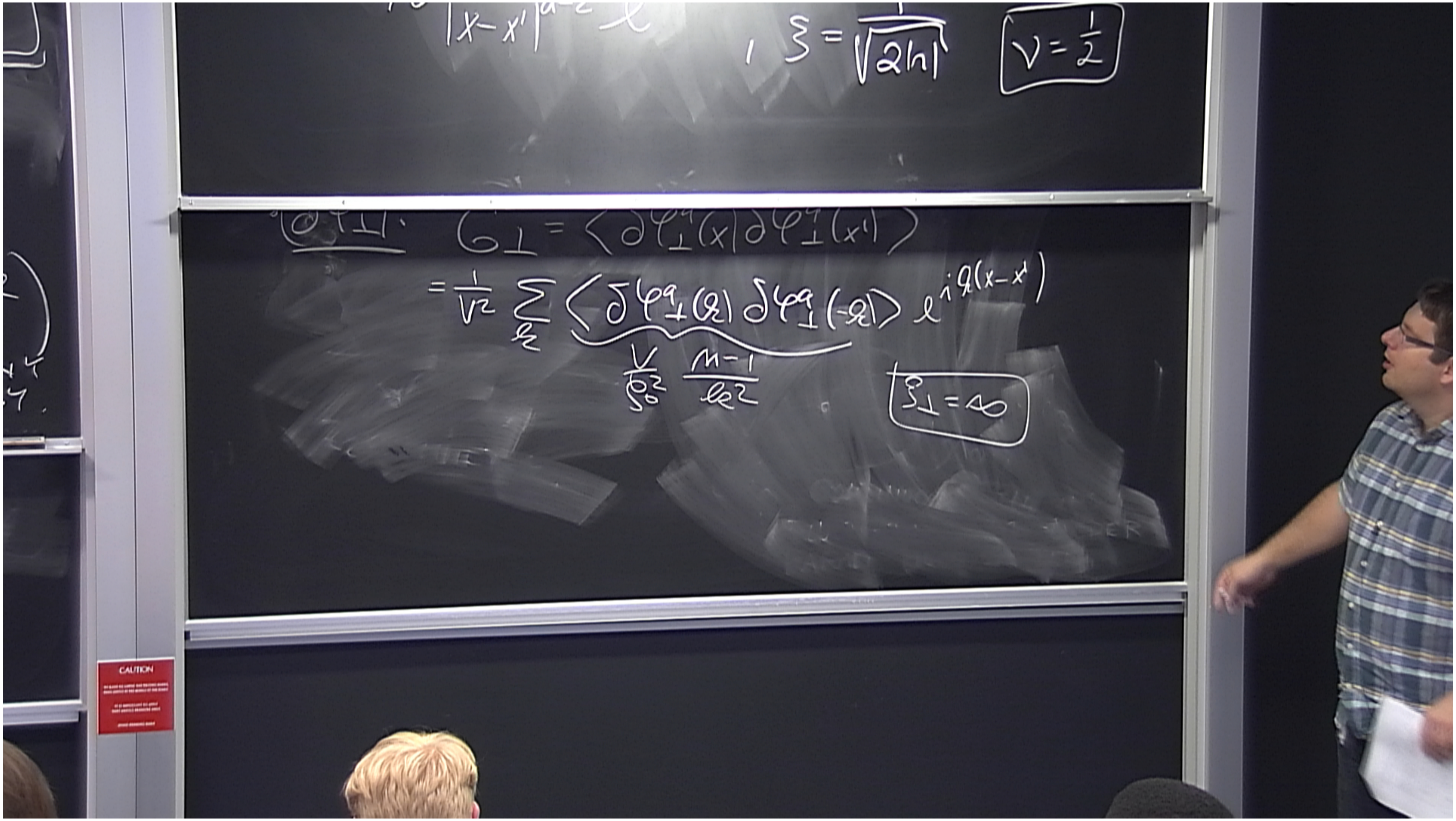
$\underline{\delta\varphi_1}$: $\langle \varphi \rangle = \vec{m}\rho_0$, $\langle \delta\varphi_1 \rangle = 0$

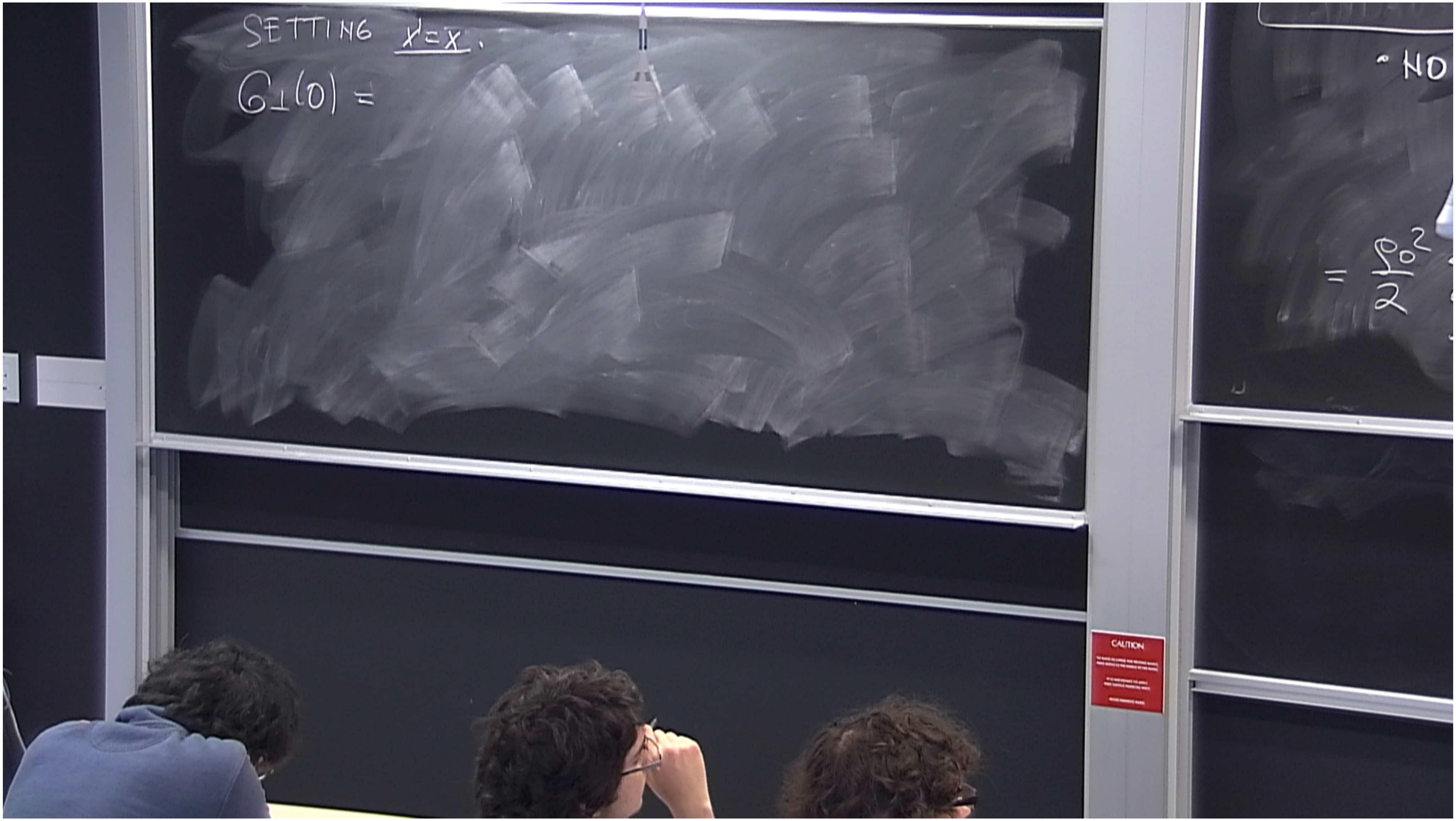
$$G_1(x-x') = \langle \delta\varphi_1(x)\,\delta\varphi_1(x') \rangle = \frac{1}{V^2}\sum_k \underbrace{\langle \delta\varphi_1(k)\,\delta\varphi_1(-k) \rangle}_{\frac{V}{\rho_0^2}\frac{1}{k^2+2|h|}} e^{ik(x-x')}$$

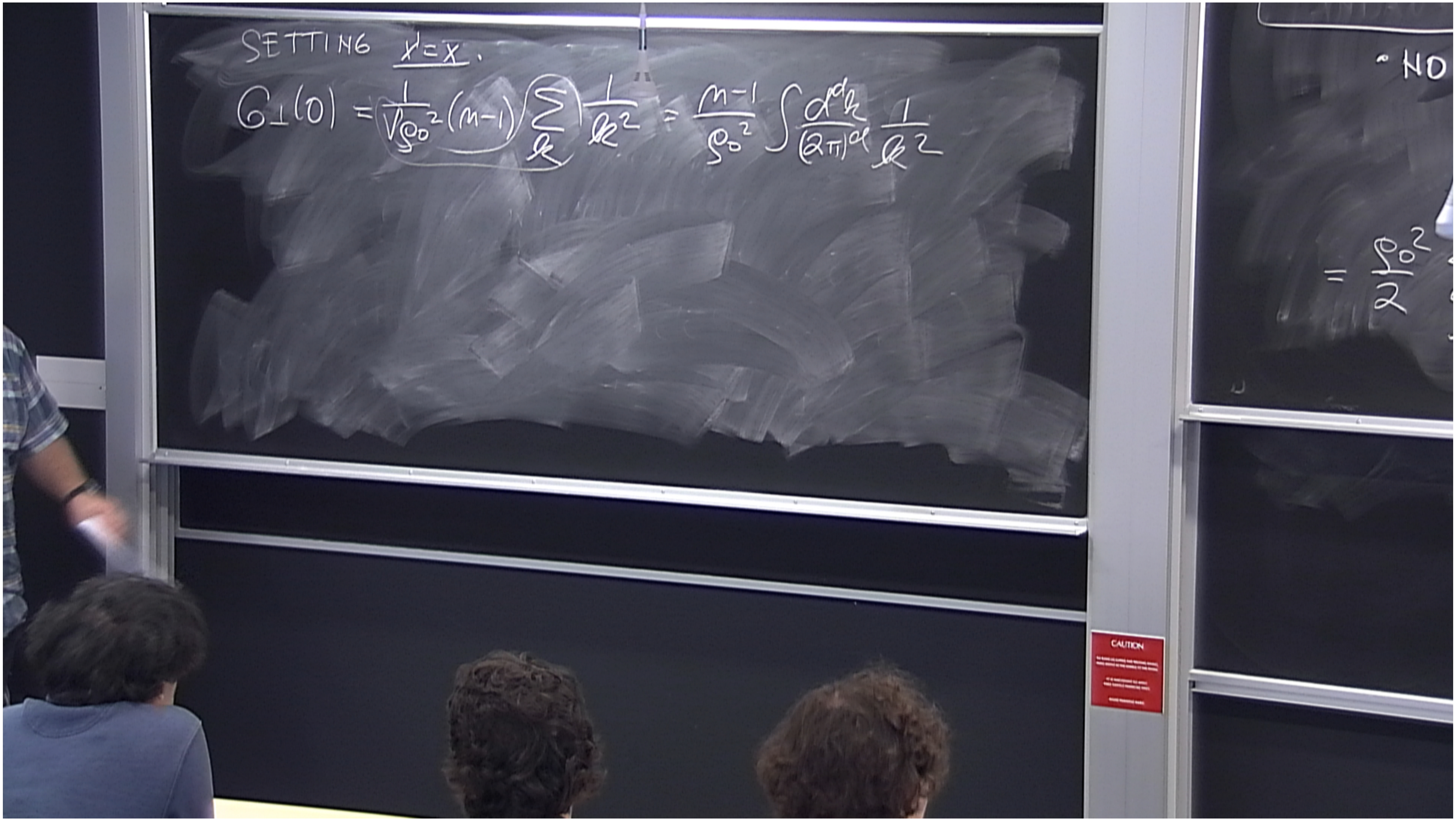
$$\sim \frac{1}{|x-x'|^{d-2}}\, e^{-\frac{|x-x'|}{\xi}} \quad , \quad \xi = \frac{1}{\sqrt{2|h|}}$$

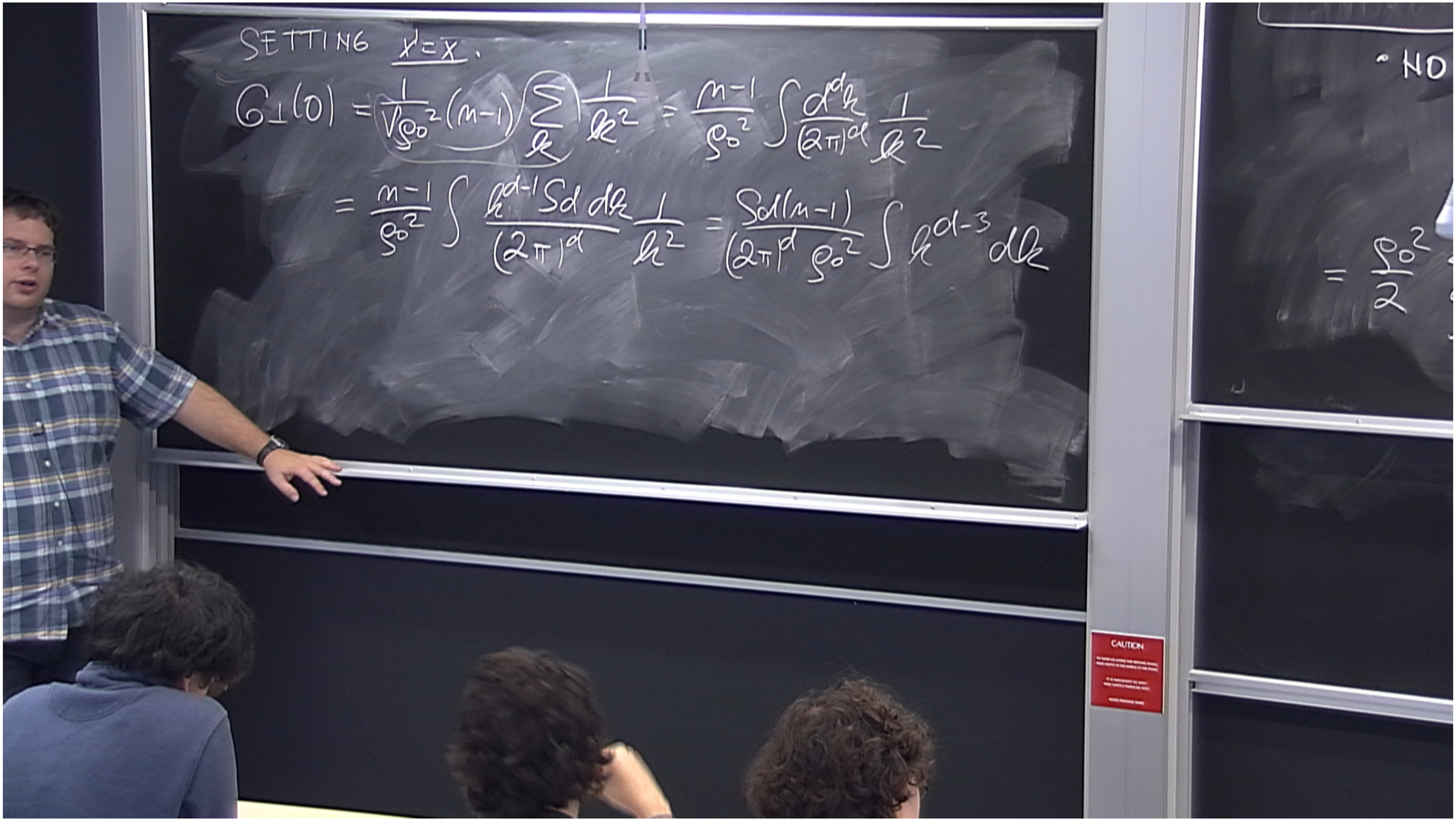

$\xi = \frac{1}{\sqrt{2|h|}}$
$\nu = \frac{1}{2}$
$\partial \varphi_\perp$
$G_\perp = \langle \delta\varphi^a_\perp(x)\, \delta\varphi^a_\perp(x') \rangle$
$= \frac{1}{V^2} \sum_q \langle \delta\varphi^a_\perp(q)\, \delta\varphi^a_\perp(-q) \rangle e^{iq(x-x')}$
CAUTION

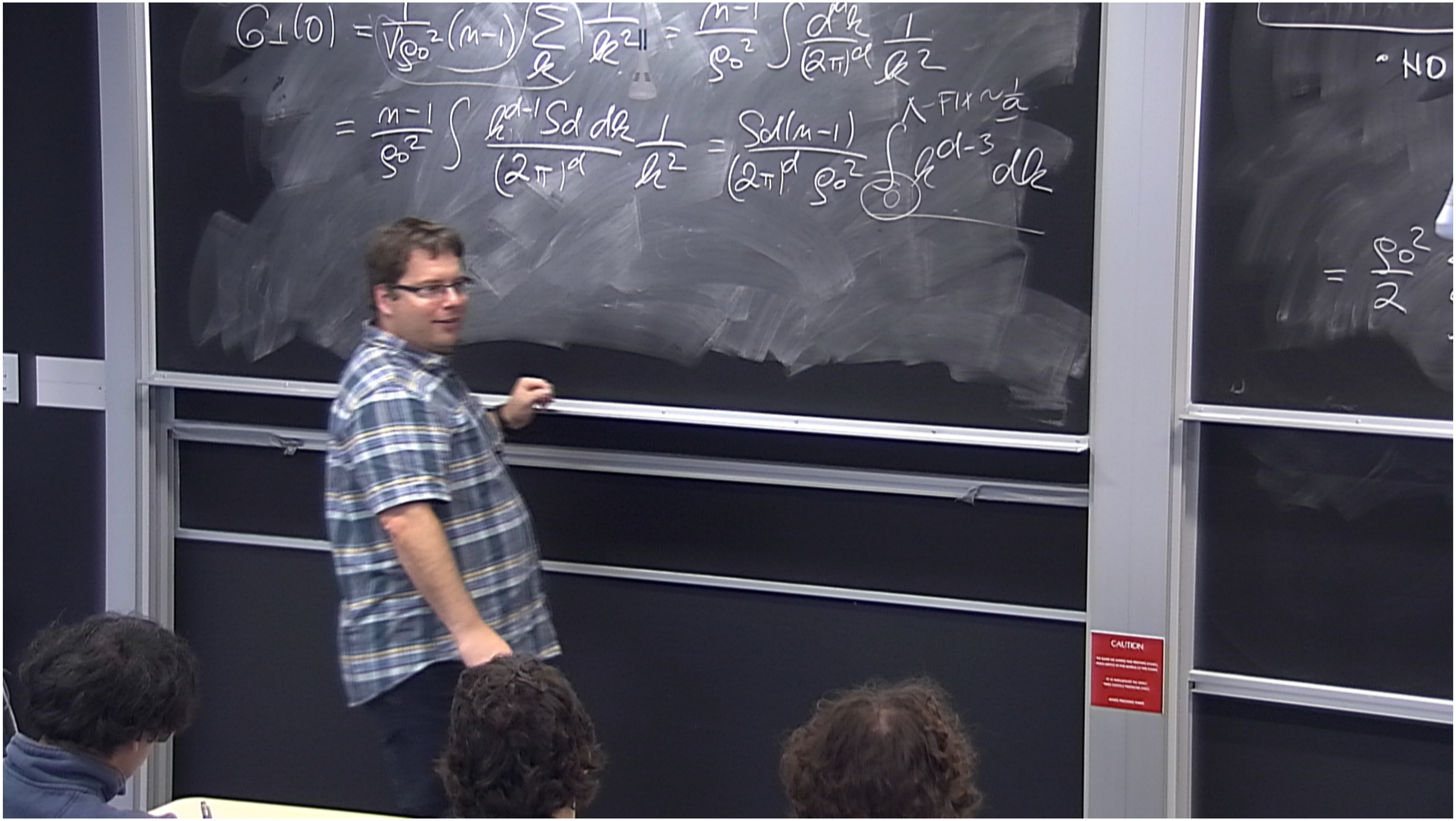

$\xi = \frac{1}{\sqrt{2|h|}}$
$\boxed{\nu = \frac{1}{2}}$
$\delta\varphi_\perp$.
$G_\perp = \langle \delta\varphi^a_\perp(x)\,\delta\varphi^a_\perp(x')\rangle$
$= \frac{1}{V^2}\sum_q \underbrace{\langle \delta\varphi^a_\perp(q)\,\delta\varphi^a_\perp(-q)\rangle}_{\frac{V}{\xi_0^2}\frac{n-1}{q^2}} e^{iq(x-x')}$
CAUTION

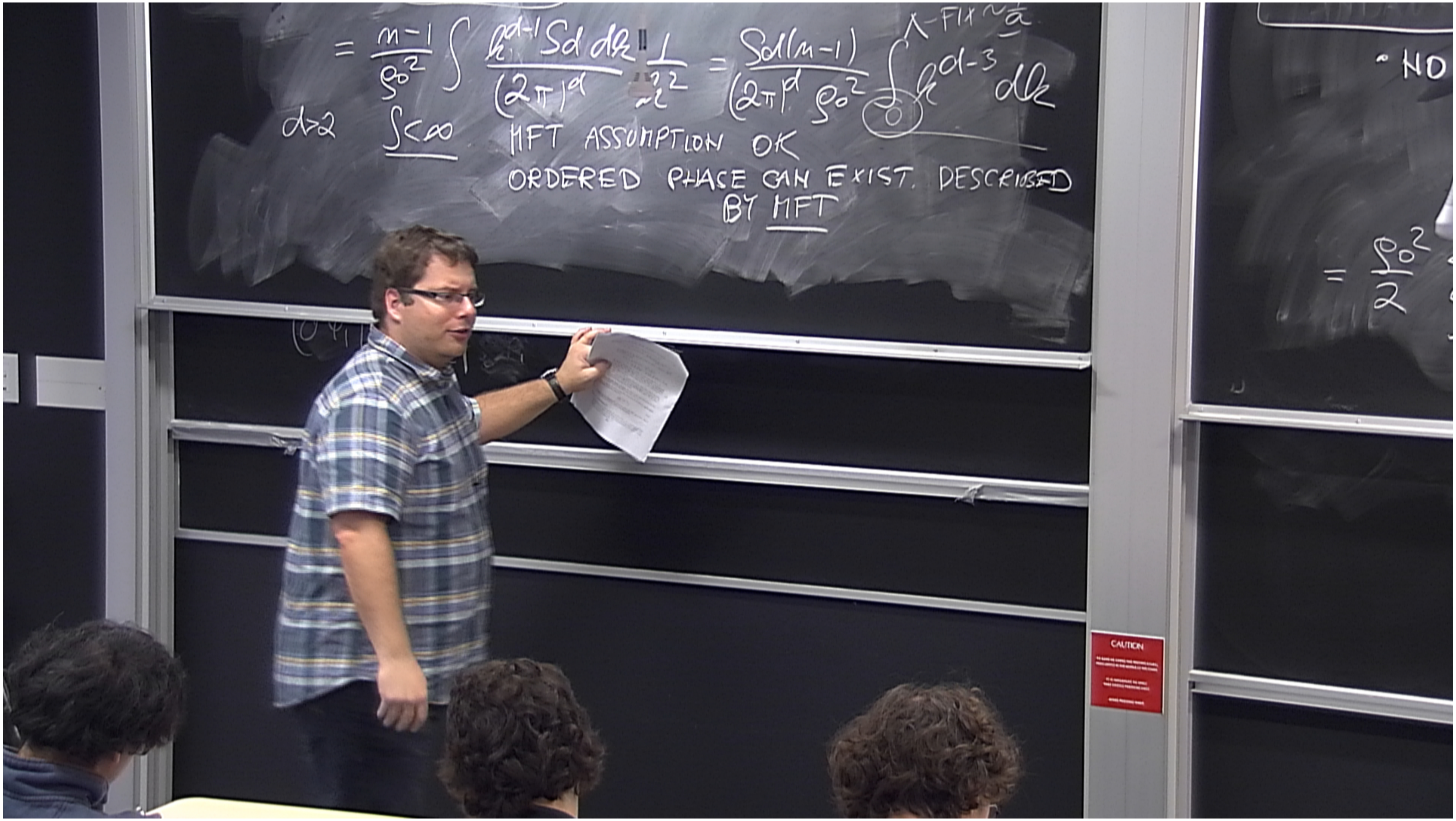

$\xi = \frac{1}{\sqrt{2|h|}}$
$\nu = \frac{1}{2}$
$G_\perp = \langle \partial\varphi_\perp^a(x)\,\partial\varphi_\perp^a(x')\rangle$
$= \frac{1}{V^2}\sum_q \langle \partial\varphi^a_\perp(q)\,\partial\varphi^a_\perp(-q)\rangle\, e^{iq(x-x')}$
$\frac{V}{\xi_0^2}\,\frac{n-1}{q^2}$
$\xi_\perp = \infty$
CAUTION

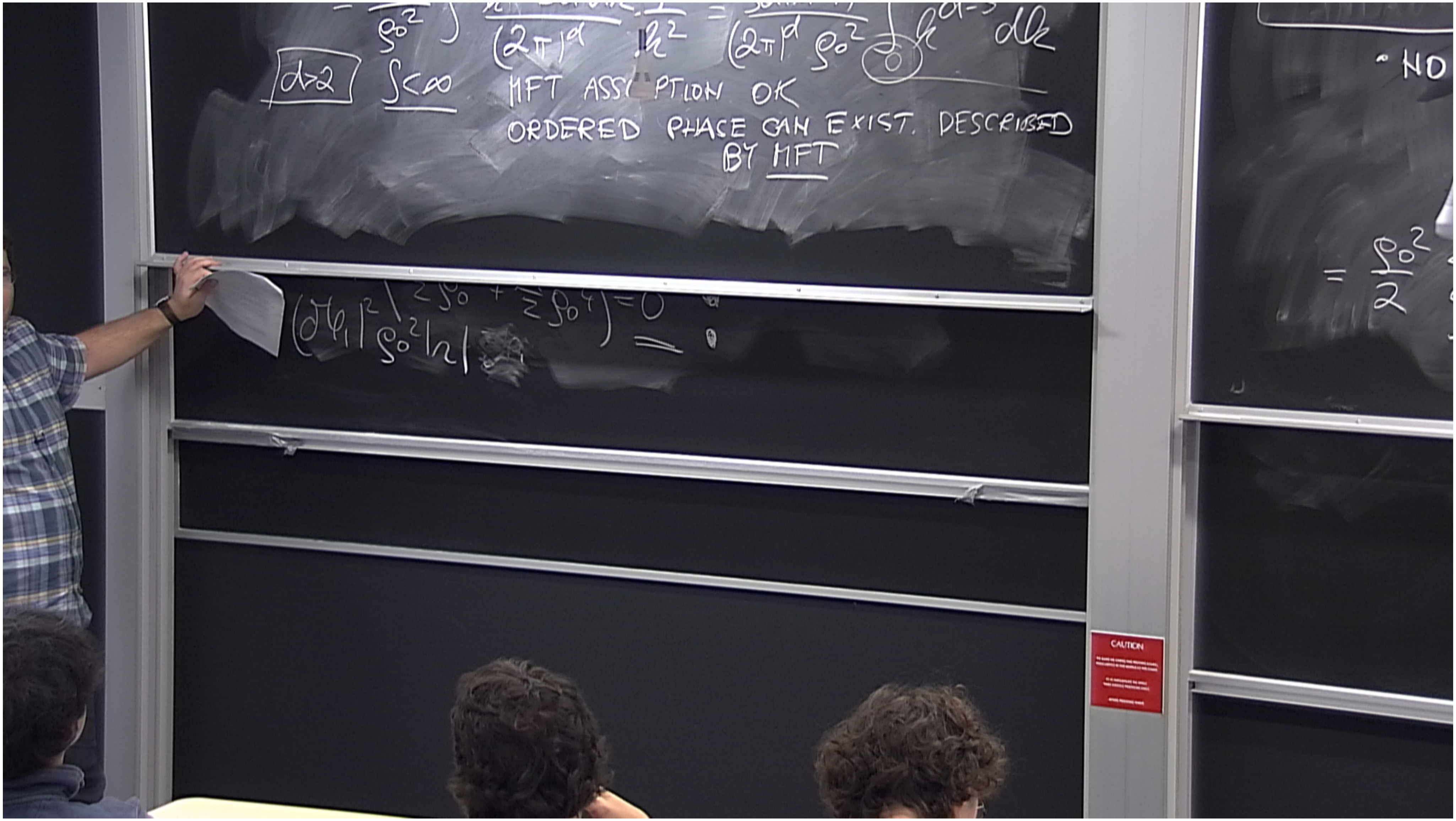

SETTING $x' = x$.
$G_\perp(0) =$
• NO
$= \frac{\rho_0^2}{2}$
CAUTION

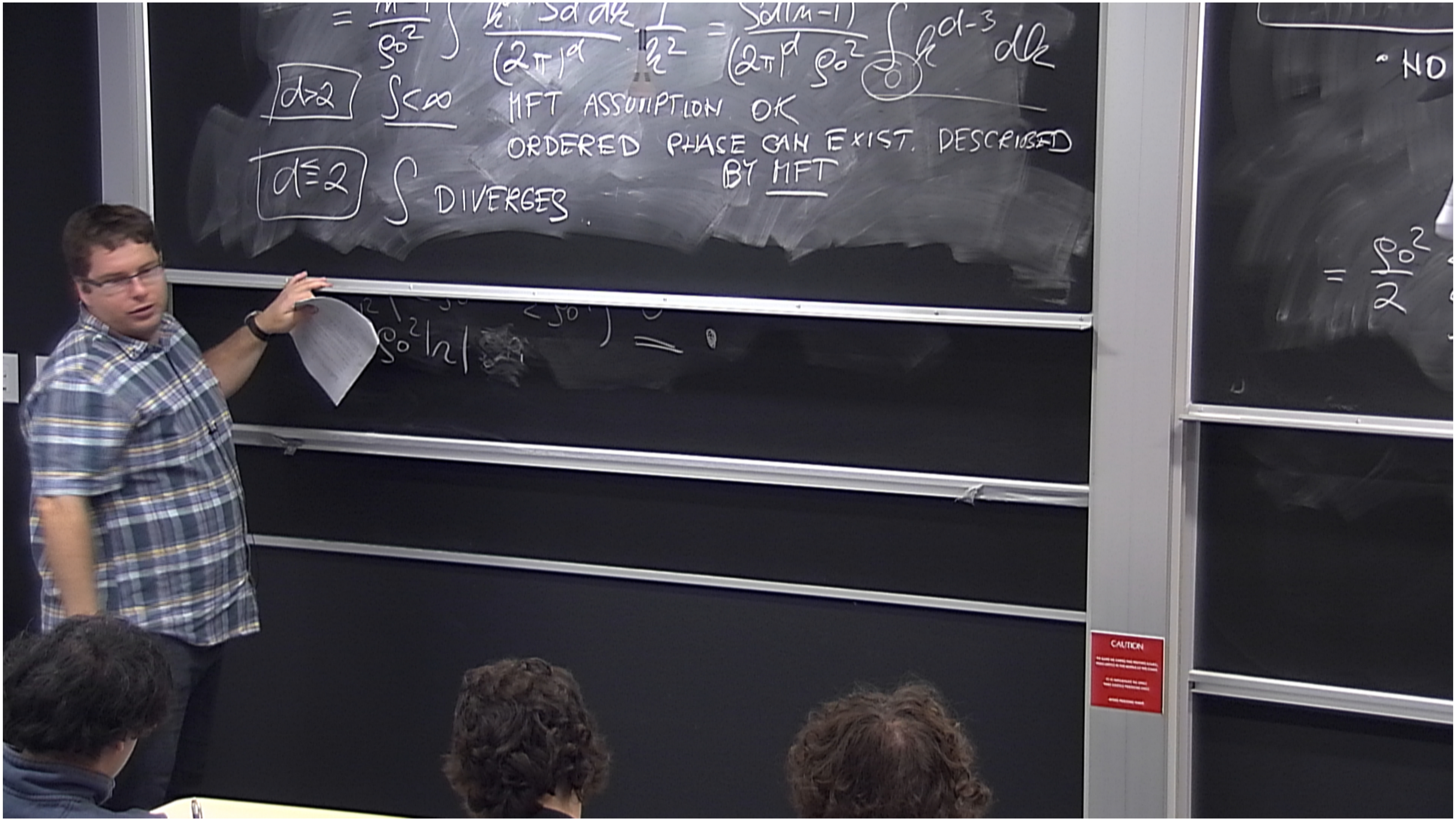

SETTING x'=x.
$G_\perp(0) = \frac{1}{V\rho_0^2}(n-1)\sum_k \frac{1}{k^2} = \frac{n-1}{\rho_0^2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2}$
• NO
$= \frac{\rho_0^2}{2}$

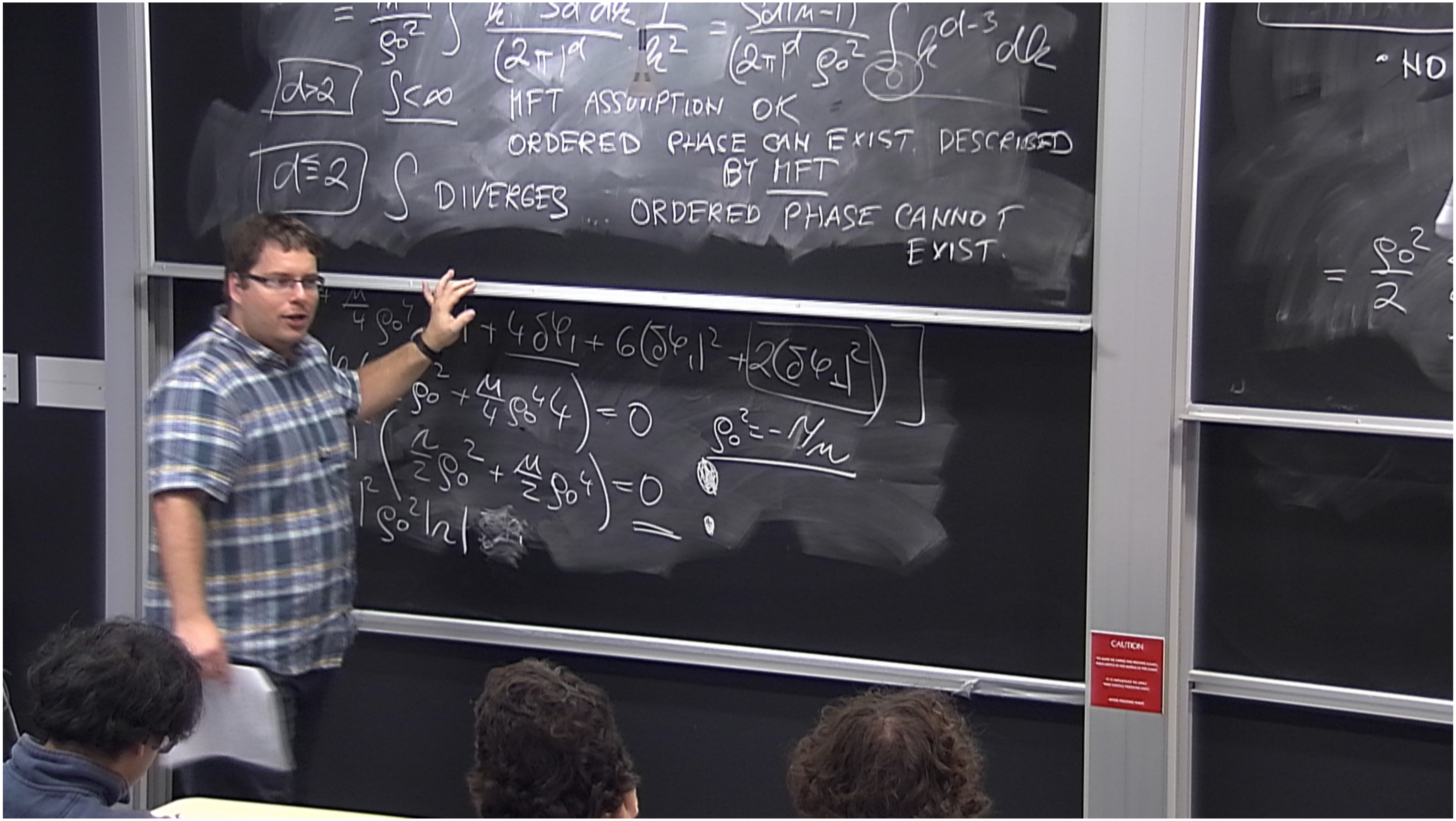

SETTING $x' = x$.
$G_\perp(0) = \frac{1}{V\rho_0^2}(n-1)\sum_k \frac{1}{k^2} = \frac{n-1}{\rho_0^2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}$
$= \frac{n-1}{\rho_0^2}\int \frac{k^{d-1} S_d \, dk}{(2\pi)^d}\frac{1}{k^2} = \frac{S_d(n-1)}{(2\pi)^d \rho_0^2}\int k^{d-3}\, dk$

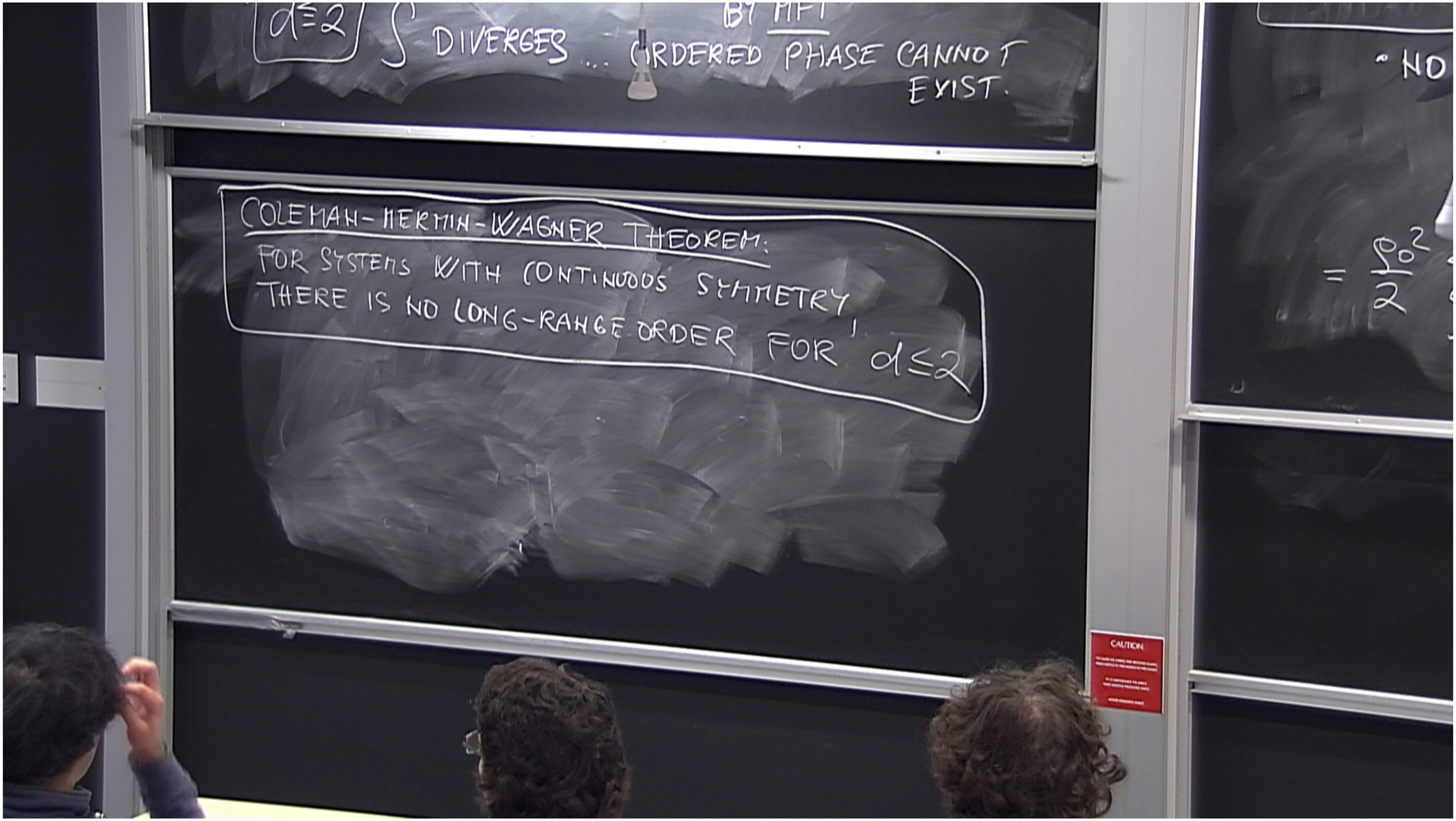

$G_\perp(0) = \frac{1}{V\rho_0^2(n-1)} \sum_k \frac{1}{k^2} = \frac{n-1}{\rho_0^2} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2}$
$= \frac{n-1}{\rho_0^2} \int \frac{k^{d-1} S_d\, dk}{(2\pi)^d} \frac{1}{k^2} = \frac{S_d(n-1)}{(2\pi)^d \rho_0^2} \int_0^{\Lambda} k^{d-3}\, dk$
Λ - FIX $\sim \frac{1}{a}$
NO
$= \frac{\rho_0^2}{2}$
CAUTION

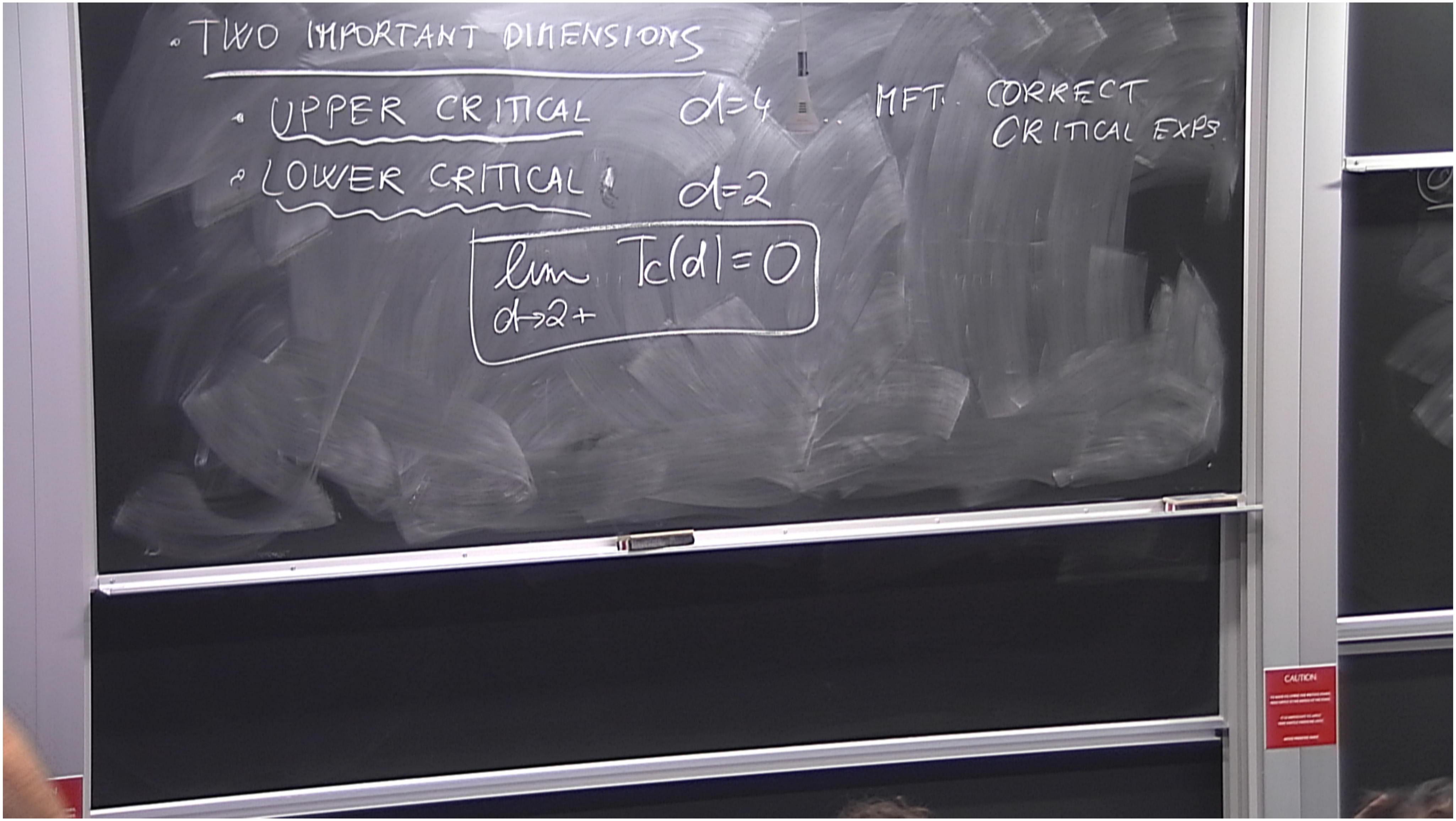

= (n-1)/ρ0² ∫ k^(d-1) Sd dk/(2π)^d 1/k² = Sd(n-1)/((2π)^d ρ0²) ∫ k^(d-3) dk
Λ-FIX ~ 1/a
d>2 ∫<∞ MFT ASSUMPTION OK
ORDERED PHASE CAN EXIST. DESCRIBED BY MFT
• NO
= ρ0²/2
CAUTION


d>2
MFT ASS PTION OK
ORDERED PHACE CAN EXIST. DESCRIBED BY MFT
HO
CAUTION

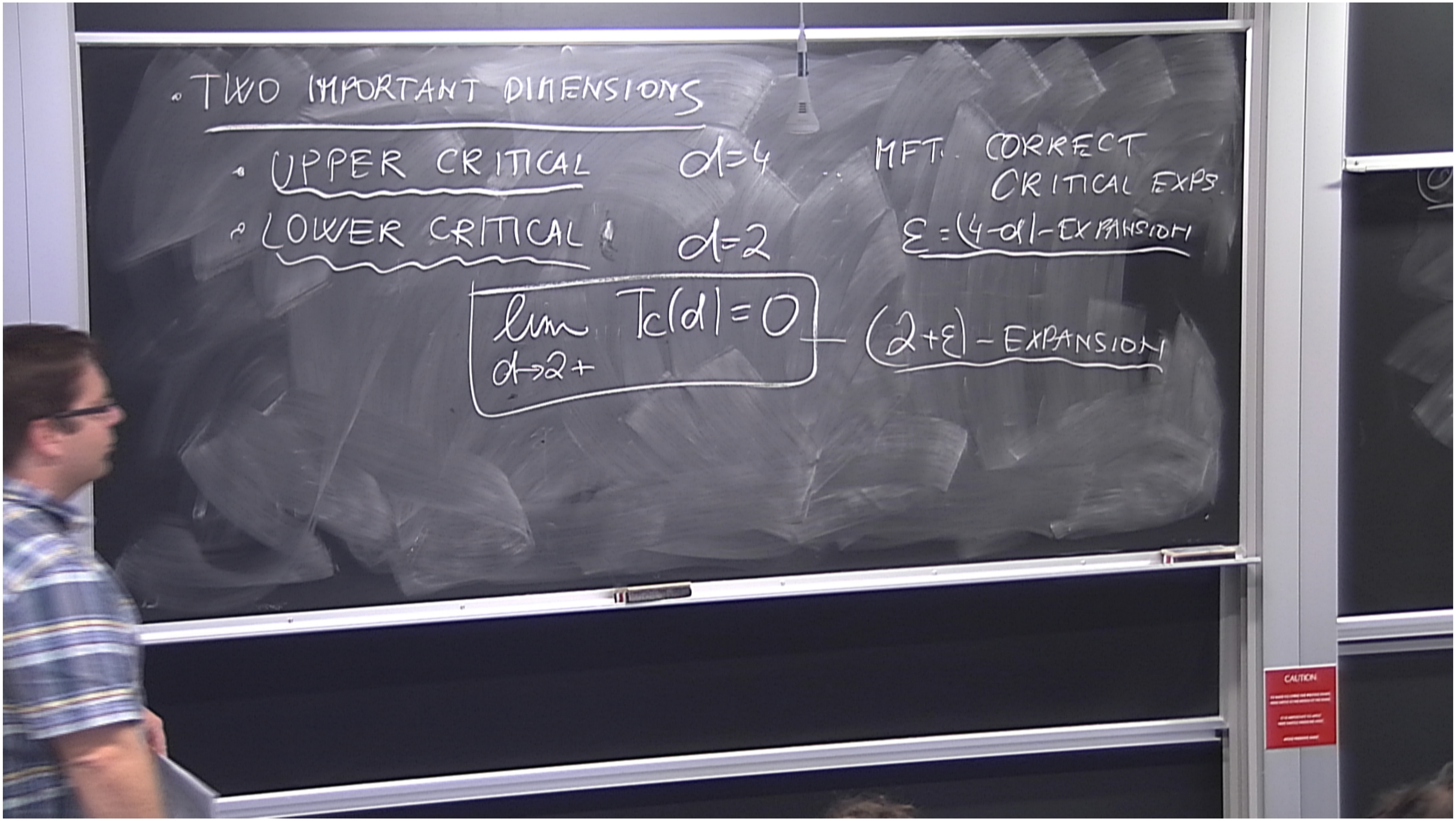

d>2 ∫<∞
MFT ASSUMPTION OK
ORDERED PHASE CAN EXIST, DESCRIBED BY MFT
d≤2 ∫ DIVERGES
NO
CAUTION

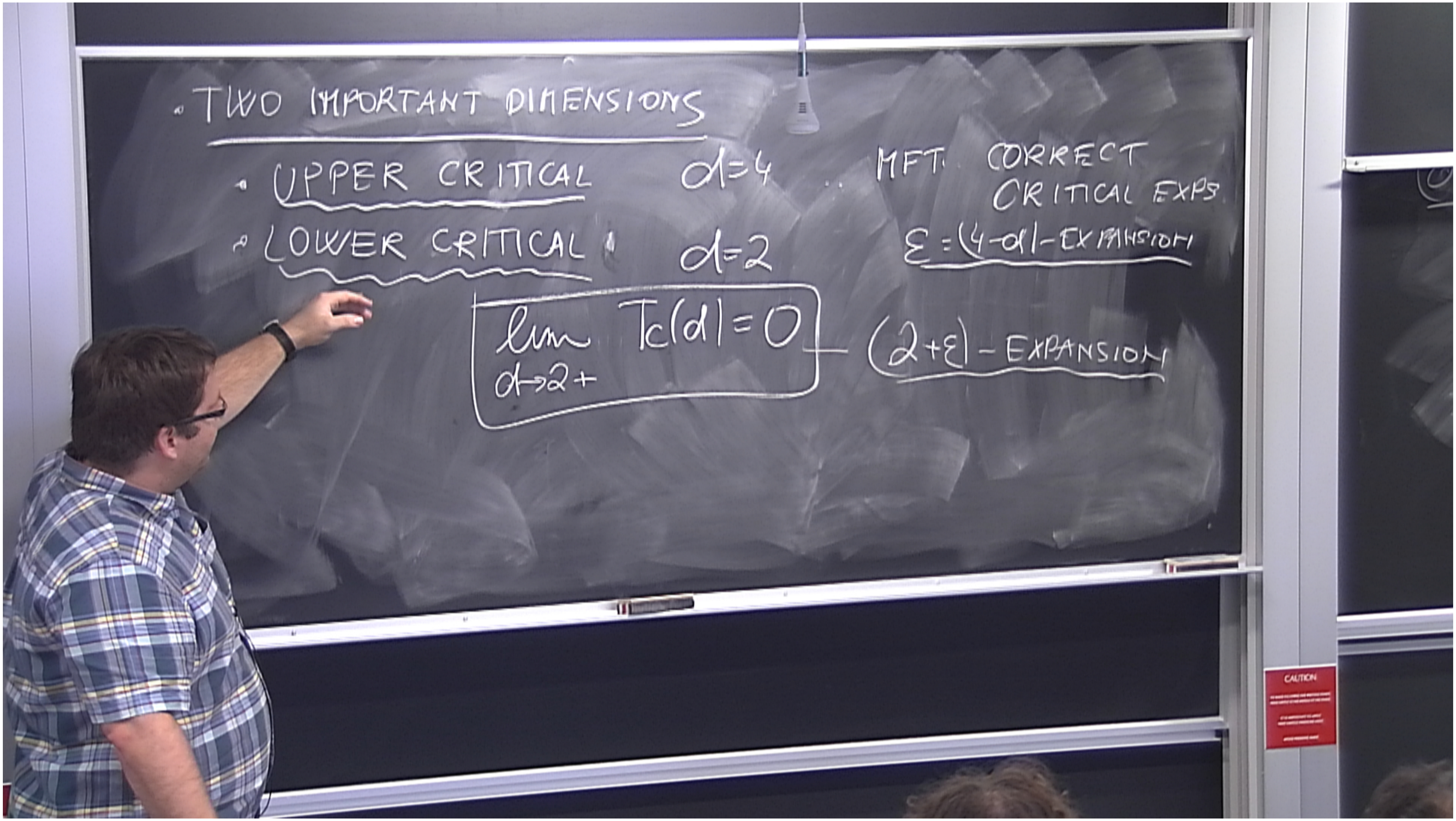

d>2
S<∞
MFT ASSUMPTION OK
ORDERED PHASE CAN EXIST, DESCRIBED BY MFT
d≤2
∫ DIVERGES ... ORDERED PHASE CANNOT EXIST.
NO


d≤2
DIVERGES ... ORDERED PHASE CANNOT EXIST.
COLEMAN-MERMIN-WAGNER THEOREM:
FOR SYSTEMS WITH CONTINUOUS SYMMETRY, THERE IS NO LONG-RANGE ORDER FOR d≤2
• NO



• TWO IMPORTANT DIMENSIONS
• UPPER CRITICAL $d=4$ .. MFT. CORRECT CRITICAL EXPS.
• LOWER CRITICAL $d=2$
$\boxed{\lim_{d \to 2+} T_c(d) = 0}$


TWO IMPORTANT DIMENSIONS
UPPER CRITICAL d=4 ... MFT: CORRECT CRITICAL EXPS.
LOWER CRITICAL d=2
ε=(4-d)-EXPANSION
lim d→2+ Tc(d)=0
(2+ε)-EXPANSION


TWO IMPORTANT DIMENSIONS
UPPER CRITICAL
d=4
MFT CORRECT CRITICAL EXPS
LOWER CRITICAL
d=2
ε=(4-d)-EXPANSION
lim Tc(d)=0
d→2+
(2+ε)-EXPANSION