Title: PSI 2016/2017 Statistical Mechanics - Lecture 9

Date: Oct 21, 2016 10:45 AM

URL: http://pirsa.org/16100023

Abstract:

RENORMALIZATION

$$Z = \int D\varphi_< D\varphi_> e^{-S[\varphi_<,\varphi_>]} \equiv \int D\varphi_< e^{-\tilde{S}[\varphi_<]} \qquad (1)$$

STEP A FIND $\tilde{S}[\varphi_<]$ EXPLICITELY

$$\tilde{S}[\varphi_<] = \frac{1}{2}\int_q^{\Lambda_b} \underbrace{\left[\tilde{\Lambda}_2 q^2 + \tilde{\Lambda}_0\right]|\varphi_<(q)|^2}_{\text{2 EXTERNAL SLOW LEGS}} + \frac{\tilde{\lambda}_4}{4!}\int_{q_1\ldots q_4}^{\Lambda_b} \underbrace{\varphi_<(q_1)\varphi_<(q_2)\varphi_<(q_3)\varphi_<(q_4)}_{\text{4 LEGS}} (2\pi)^d \delta(\Sigma_{q_i})$$

- FROM (1)

$$\tilde{S}[\varphi_<] = \underbrace{S_0[\varphi_<]}_{\text{0th-ORDER}} + \underbrace{\langle S_{int}\rangle_{FAST}}_{\text{1ST}} \underbrace{-\frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right]}_{\text{2ND}} + \ldots$$

2 EXTERNAL SLOW LEGS

- FROM (1) $\tilde{S}[\varphi_<] = \underbrace{S_0[\varphi_<]}_{\text{0th-ORDER}} + \underbrace{\langle S_{int} \rangle_{FAST}}_{\text{1ST}} \underbrace{- \frac{1}{2}\left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right]}_{\text{2ND}} + \dots$

0-th ORDER: $S_0[\varphi_<] = \frac{1}{2} \int (k^2 + r) |\varphi_<(k)|^2$

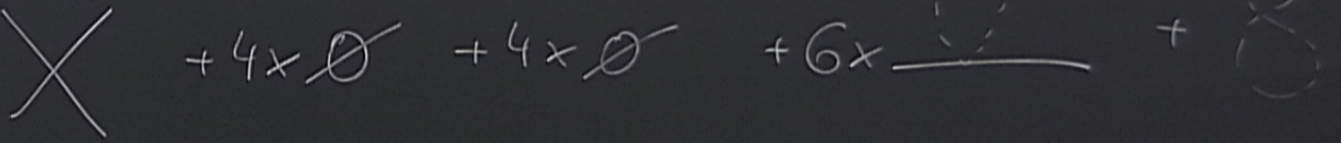
1-ST-ORDER: $S_{INT} =$  $+ \dots$

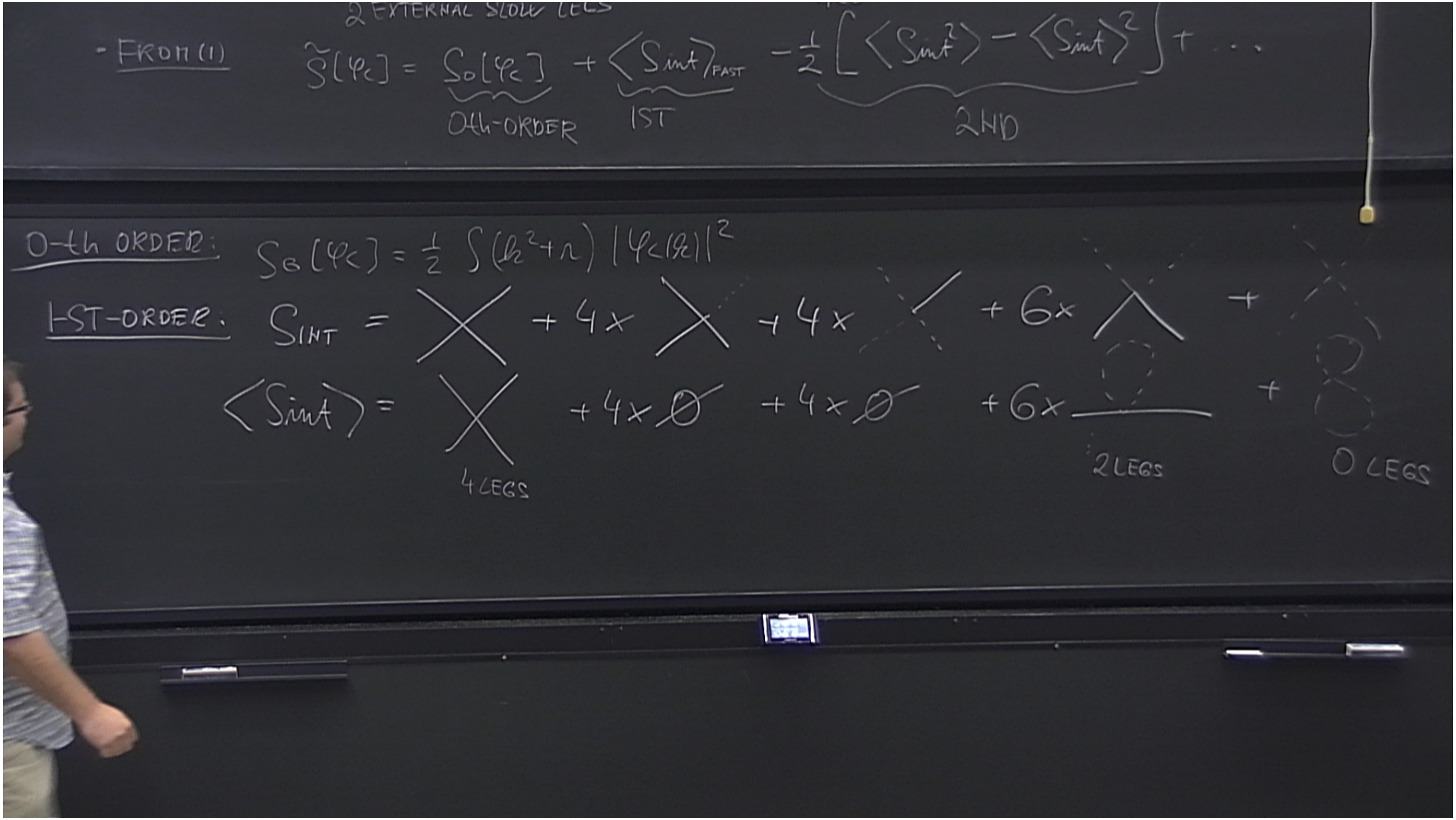
2 EXTERNAL SLOW LEGS

- FROM (1) $\tilde{S}[\varphi_<] = \underbrace{S_0[\varphi_<]}_{0\text{th-ORDER}} + \underbrace{\langle S_{int}\rangle_{FAST}}_{1ST} - \underbrace{\frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right]}_{2ND} + \dots$

0-th ORDER: $S_0[\varphi_<] = \frac{1}{2}\int (k^2+r)\,|\varphi_<(p_<)|^2$

1-ST-ORDER: $S_{INT} =$  + 4× + 4× + 6× +

$\langle S_{int}\rangle =$  + 4× + 4× + 6× +

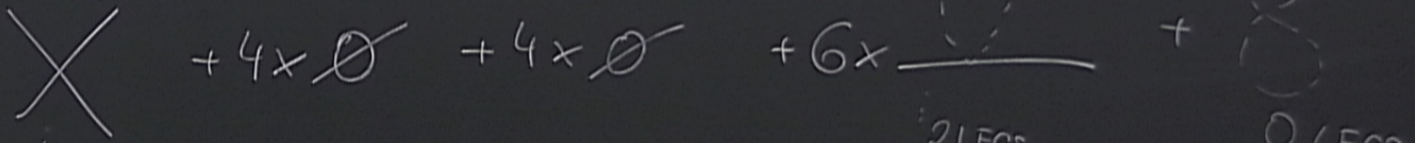

2 EXTERNAL SLOW LEGS
- FROM (1)
$\tilde{S}[\varphi_<] = S_0[\varphi_<] + \langle S_{int} \rangle_{FAST} - \frac{1}{2}\left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2\right] + \ldots$
0th-ORDER
1ST
2ND
0-th ORDER: $S_0[\varphi_<] = \frac{1}{2}\int (k^2+1)\,|\varphi_<(k)|^2$
1-ST-ORDER: $S_{INT} =$ + 4× + 4× + 6× +
$\langle S_{int} \rangle =$ + 4×0 + 4×0 + 6× +
4 LEGS
2 LEGS
0 LEGS

2 EXTERNAL SLOW LEGS

- FROM (1) $\tilde{S}[\varphi_c] = \underbrace{S_0[\varphi_c]}_{0\text{th-ORDER}} + \underbrace{\langle S_{int} \rangle_{FAST}}_{1ST} \underbrace{- \frac{1}{2}\left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right]}_{2ND} + \dots$

0-th ORDER: $S_0[\varphi_c] = \frac{1}{2} \int (k^2 + \Omega) |\varphi_c(k)|^2$

1-ST-ORDER: $S_{INT} =$  + 4× + 4× + 6× +

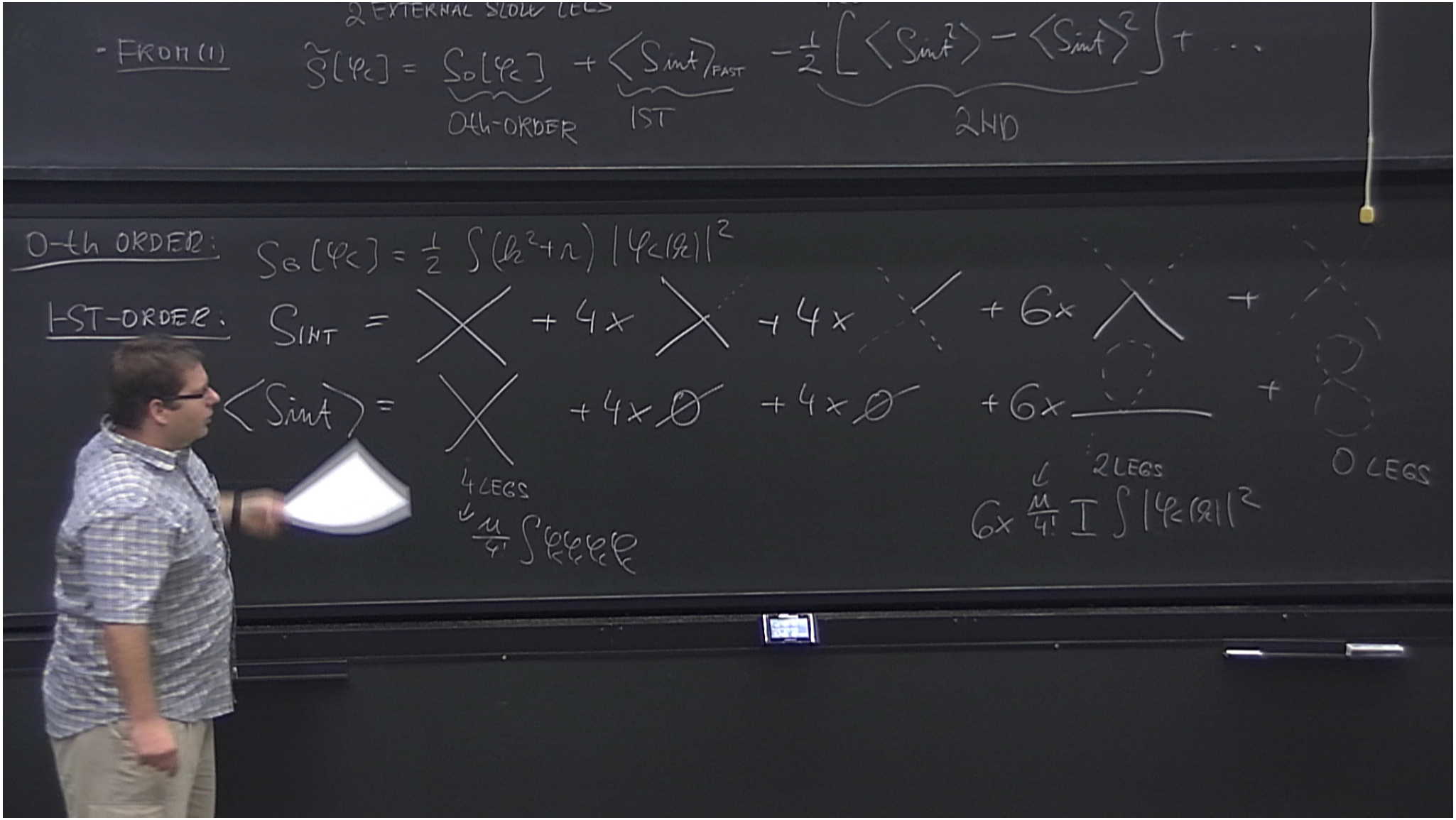
$\langle S_{int} \rangle =$  + 4×0 + 4×0 + 6×

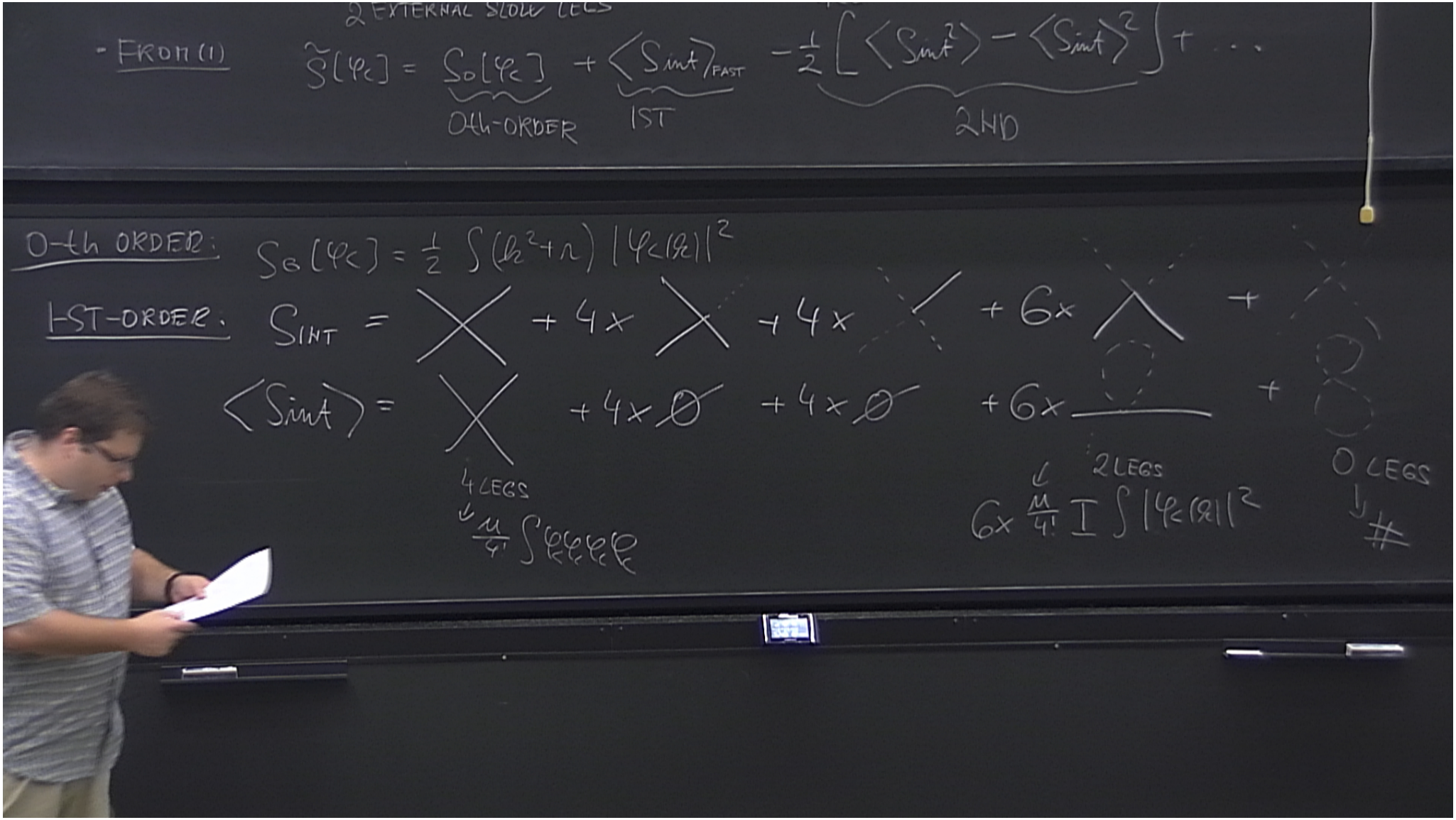
4 LEGS

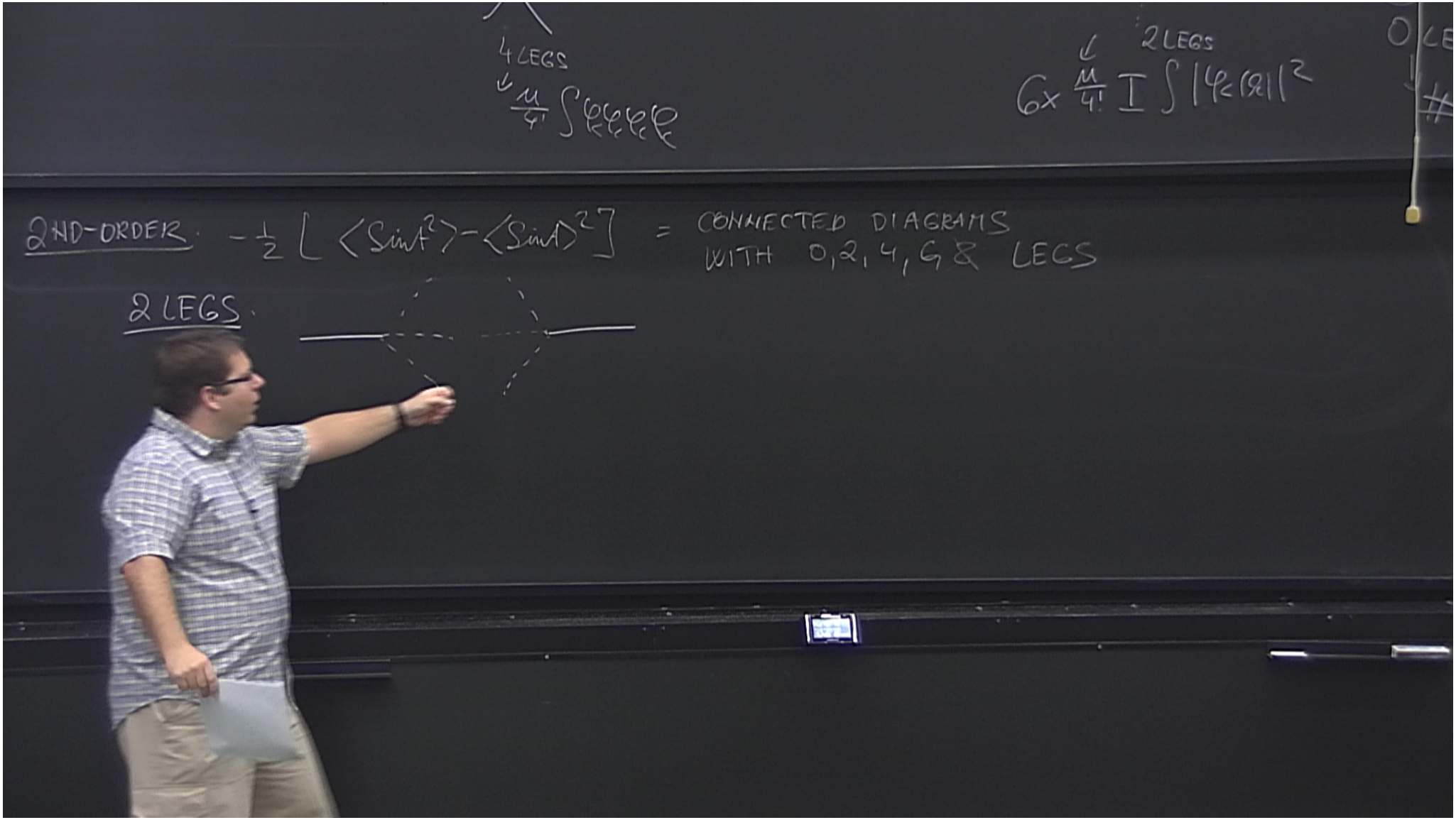
$\downarrow \frac{u}{4} \int \varphi_c \varphi_c \varphi_c \varphi_c$

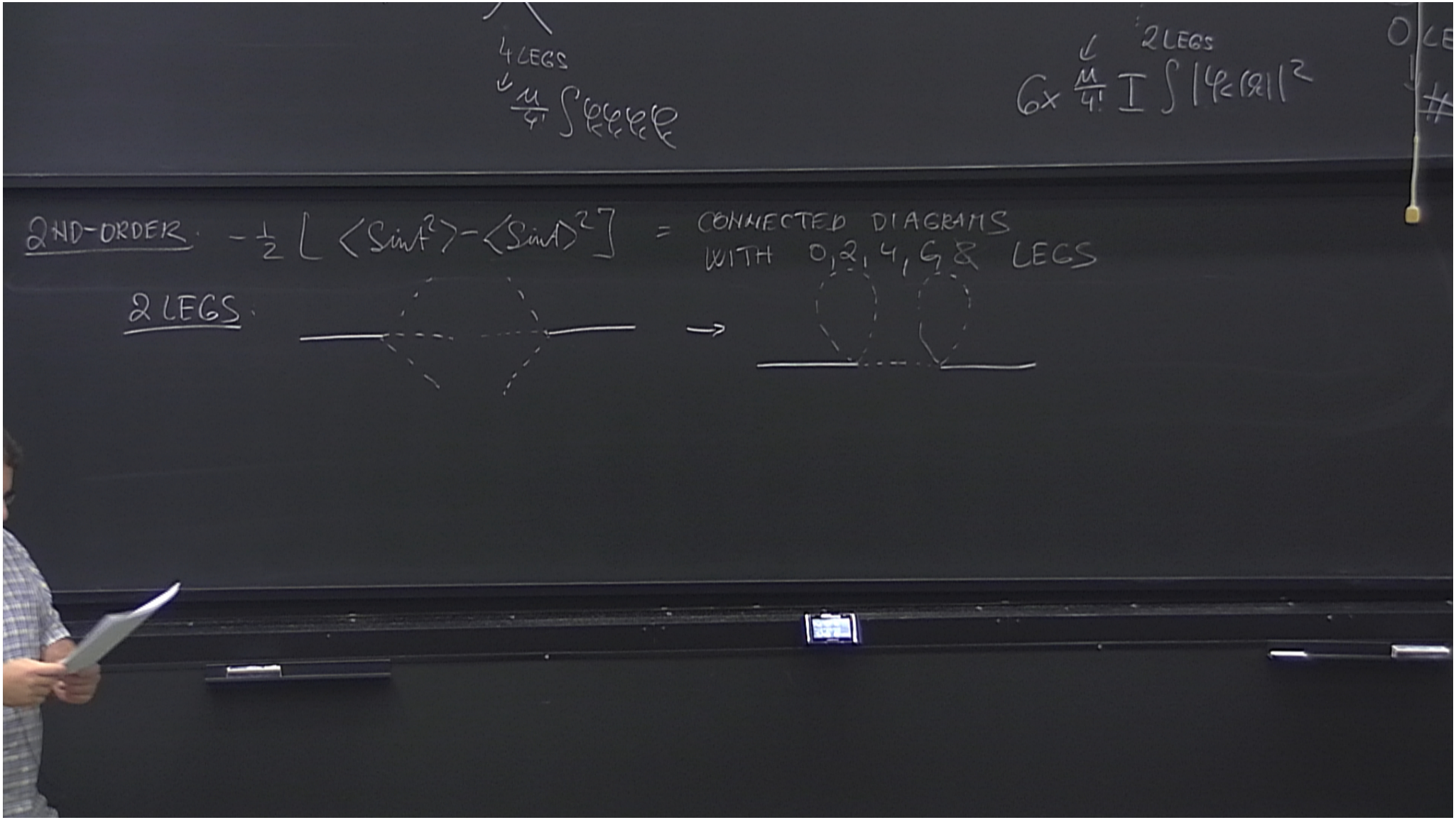
2 LEGS

0 LEGS


2 EXTERNAL SLOW LEGS
- FROM (1)
$\tilde{S}[\varphi_<] = S_0[\varphi_<] + \langle S_{int}\rangle_{FAST} - \frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right] + \dots$
0th-ORDER
1ST
2ND
0-th ORDER: $S_0[\varphi_<] = \frac{1}{2}\int (k^2+n)\,|\varphi_<(k)|^2$
1-ST-ORDER: $S_{INT} =$ + 4× + 4× + 6× +
$\langle S_{int}\rangle =$ + 4×0 + 4×0 + 6× +
4 LEGS
$\downarrow \frac{u}{4!}\int \varphi_< \varphi_< \varphi_< \varphi_<$
2 LEGS
0 LEGS
$6\times \downarrow \frac{u}{4!} I \int |\varphi_<(k)|^2$


2 EXTERNAL SLOW LEGS
- FROM (1)
$\tilde{S}[\varphi_<] = S_0[\varphi_<] + \langle S_{int} \rangle_{FAST} - \frac{1}{2}\left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2\right] + \ldots$
0th-ORDER
1ST
2ND
0-th ORDER: $S_0[\varphi_<] = \frac{1}{2}\int (k^2+1)\,|\varphi_<(k)|^2$
1-ST-ORDER: $S_{INT} =$ + 4× + 4× + 6× +
$\langle S_{int} \rangle =$ + 4×0 + 4×0 + 6× +
4 LEGS
$\frac{u}{4!}\int \varphi_< \varphi_< \varphi_< \varphi_<$
2 LEGS
$6\times \frac{u}{4!} I \int |\varphi_<(k)|^2$
0 LEGS
#


4 LEGS
$\frac{u}{4!}\int$
6× $\frac{u}{4!}$ I $\int |\psi_c|^2$
2 LEGS
2ND-ORDER $-\frac{1}{2}[\langle \sin^2 \rangle - \langle \sin \rangle^2]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
2 LEGS


4 LEGS
$\frac{u}{4!}\int \varphi\varphi\varphi\varphi$
2 LEGS
$6\times\frac{u}{4!} I \int |\varphi_k|^2$
2ND-ORDER $-\frac{1}{2}\left[\langle \sin^2\rangle - \langle \sin\rangle^2\right]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
2 LEGS

4 LEGS

$\downarrow \frac{u}{4!} \int \varphi\varphi\varphi\varphi$

$6 \times \frac{u}{4!} I \int |\varphi_k(R)|^2$

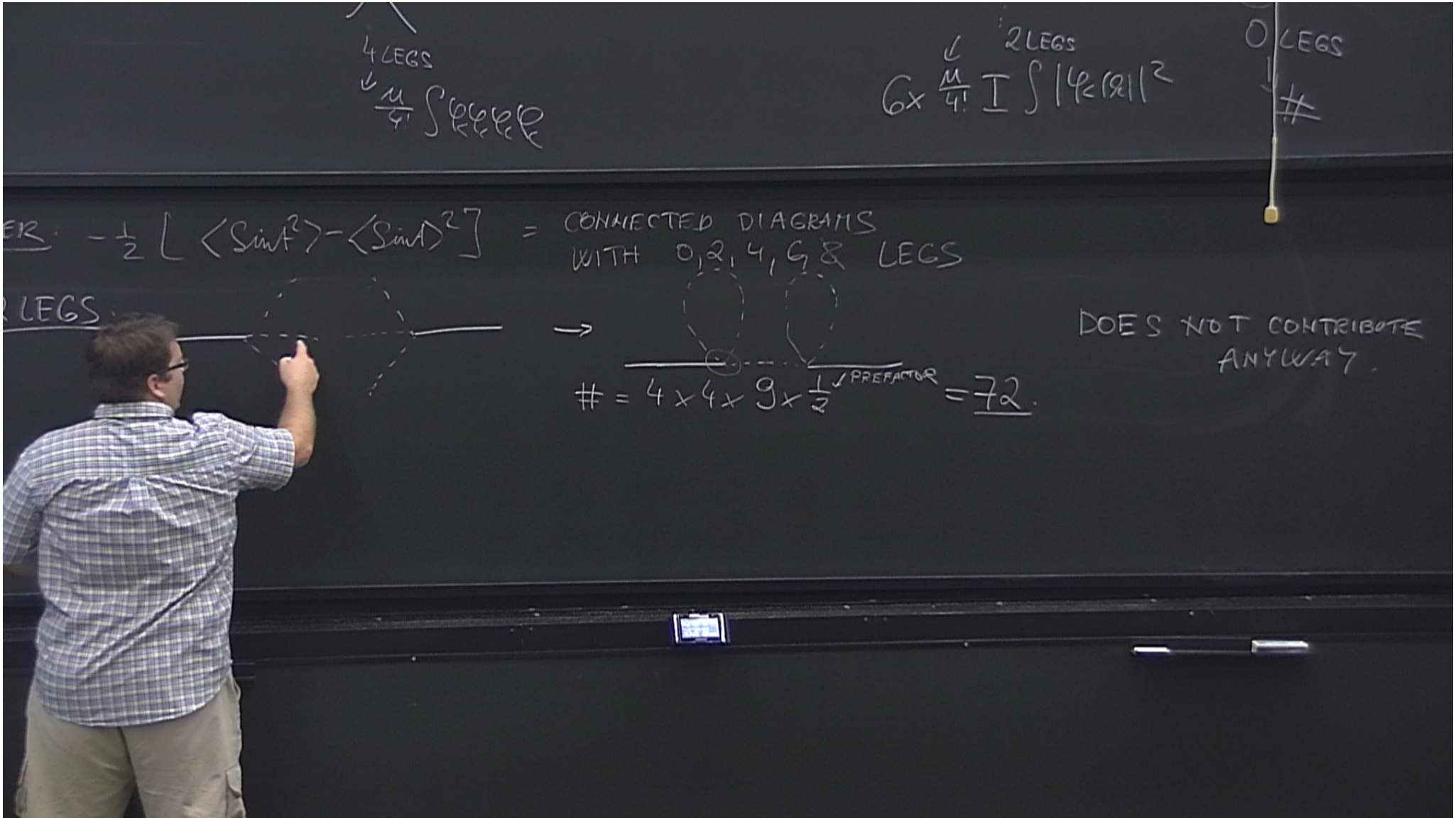
2 LEGS

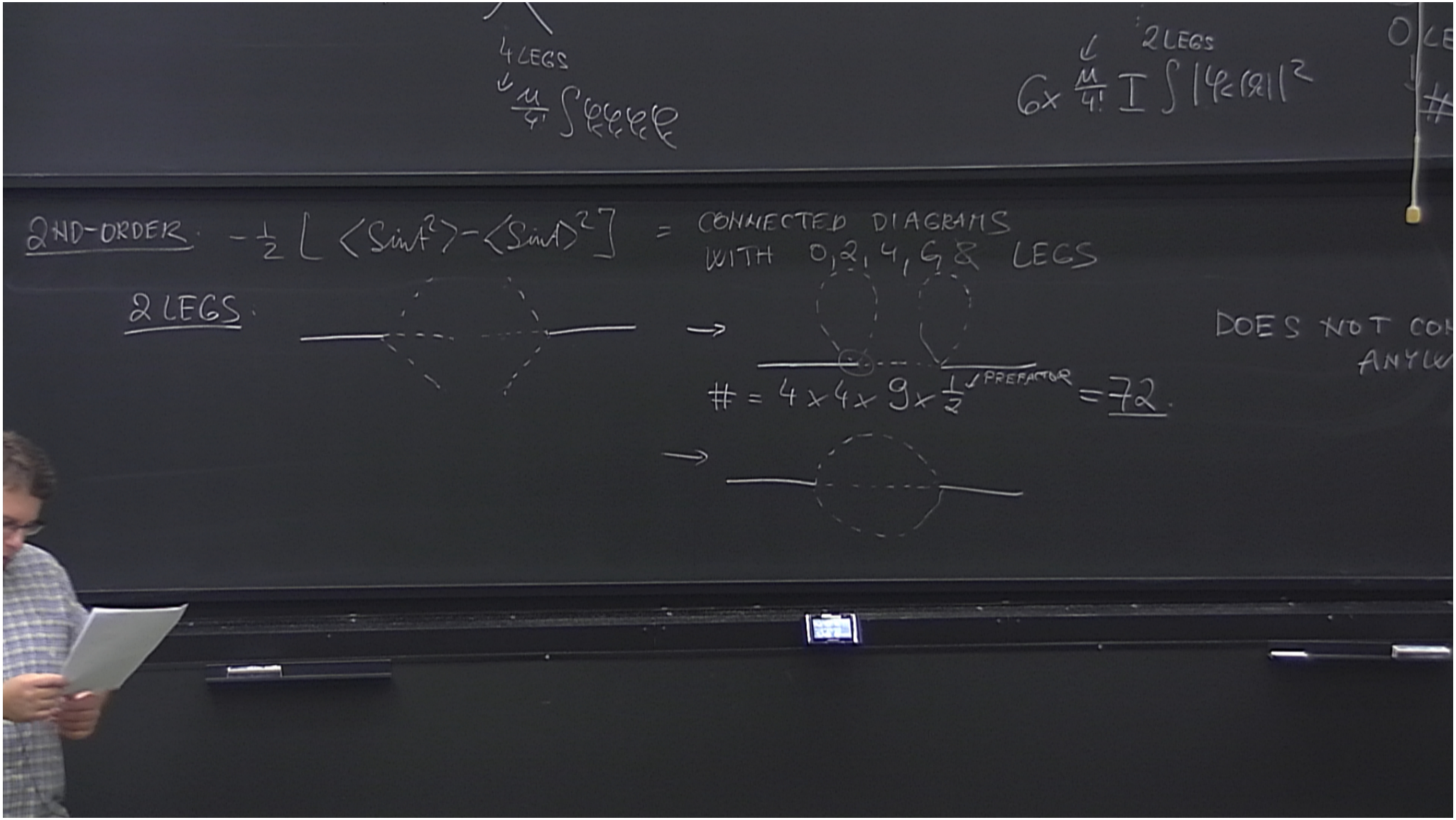
2ND-ORDER $-\frac{1}{2}\left[\langle \sin^2 \rangle - \langle \sin \rangle^2\right]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS

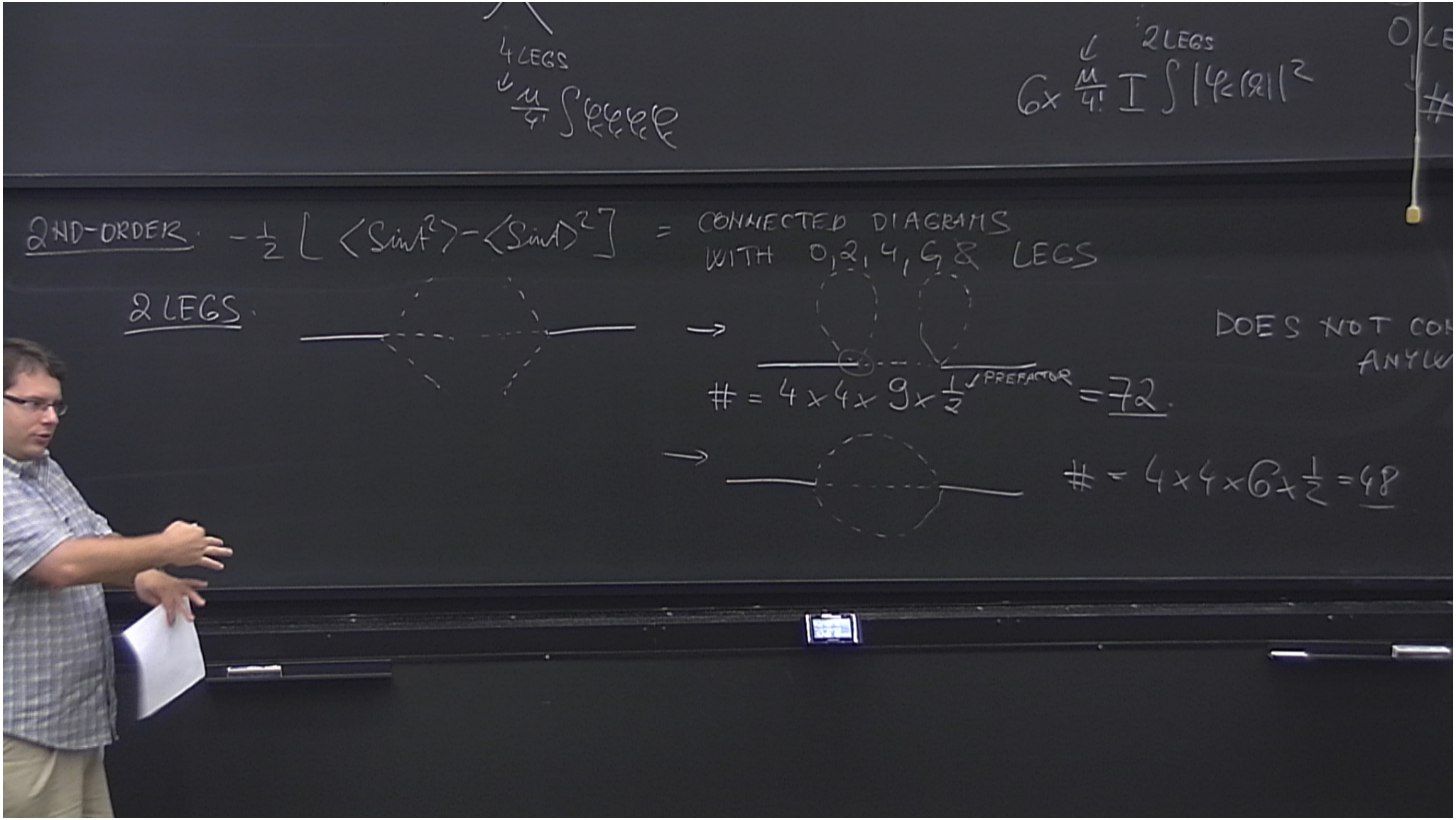
2 LEGS



$\#$ = $4 \times 4 \times 9 \times \frac{1}{2}$ ↓ PREFACTOR $= 72$.


4 LEGS
$\frac{u}{4!}\int \phi\phi\phi\phi$
2 LEGS
$6\times\frac{u}{4!} I \int |\phi_k(x)|^2$
0 LEGS
#
$-\frac{1}{2}[\langle \sin t^2\rangle - \langle \sin t\rangle^2]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
LEGS
DOES NOT CONTRIBUTE ANYWAY.
= 4 × 4 × 9 × $\frac{1}{2}$ ← PREFACTOR = 72.


4 LEGS
$\frac{\mu}{4!}\int \phi\phi\phi\phi$
2 LEGS
$6\times\frac{\mu}{4!} I \int |\phi(k)|^2$
2ND-ORDER $-\frac{1}{2}[\langle \sin^2 \rangle - \langle \sin \rangle^2]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
2 LEGS
DOES NOT CO
ANYW
$\# = 4\times 4\times 9\times \frac{1}{2}$ ↓ PREFACTOR $= 72$


4 LEGS
$\frac{u}{4!}\int \phi\phi\phi\phi$
2 LEGS
$6\times\frac{u}{4!} I \int |\phi_k(R)|^2$
2ND-ORDER $-\frac{1}{2}\left[\langle \sin t^2\rangle - \langle \sin t\rangle^2\right]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
2 LEGS
DOES NOT CO
ANYW
$\# = 4\times 4\times 9\times\frac{1}{2}$ ↓ PREFACTOR $= 72$
$\# = 4\times 4\times 6\times\frac{1}{2} = 48$

4 LEGS

$\frac{u}{4!}\int \varepsilon\varepsilon\varepsilon\varepsilon$

$6\times \frac{u}{4!} I \int |\varepsilon(R)|^2$ (2 LEGS)

2ND-ORDER $-\frac{1}{2}\left[\langle \sin t^2\rangle - \langle \sin t\rangle^2\right]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS

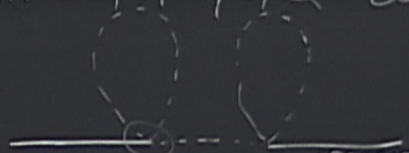
2 LEGS

↓

CAN CONTRIBUTE TO $\tilde{\lambda}_2, \tilde{\lambda}_0$



→



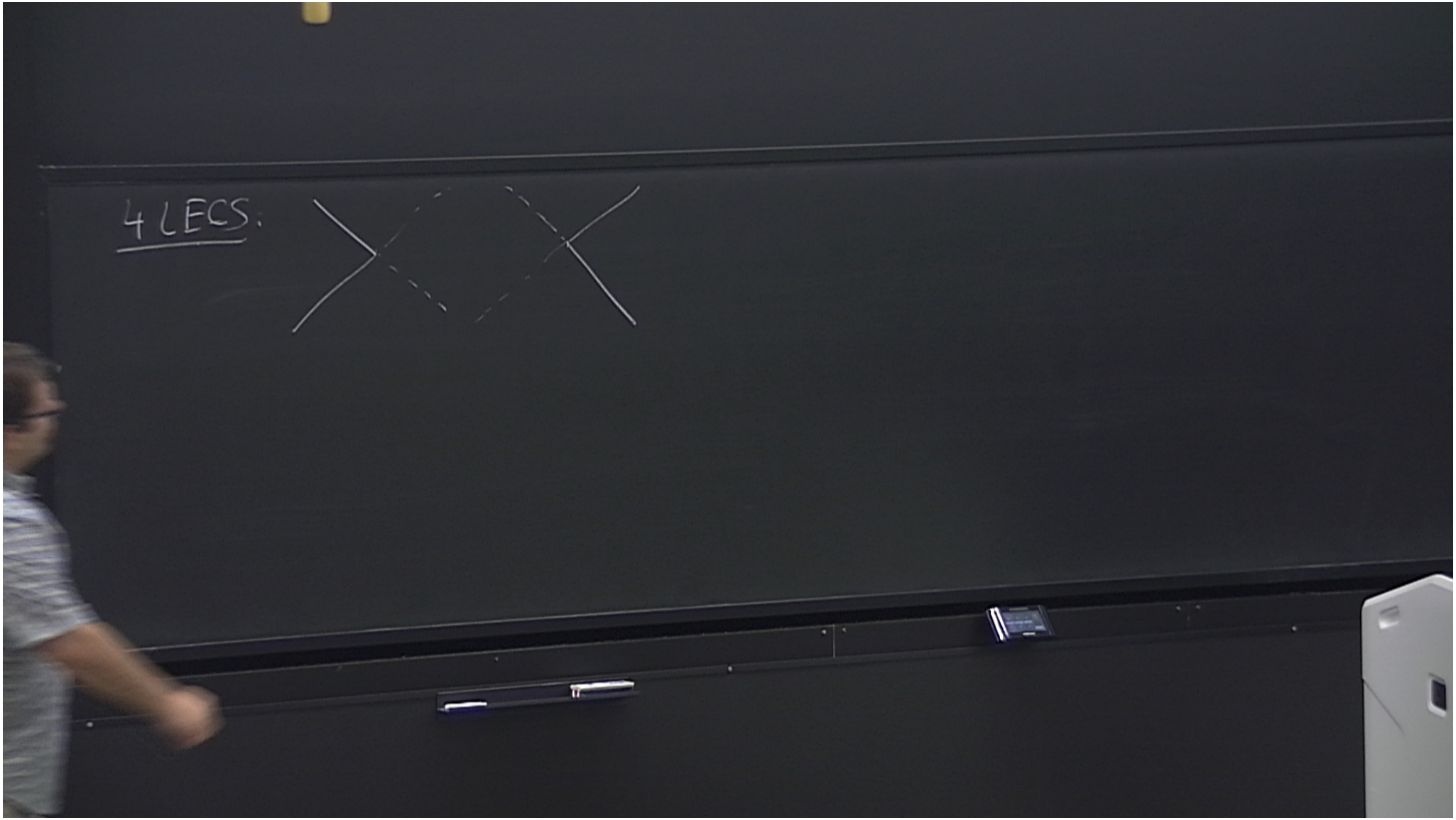
$\# = 4\times 4\times 9\times \frac{1}{2}$ (↓ PREFACTOR) $= 72$

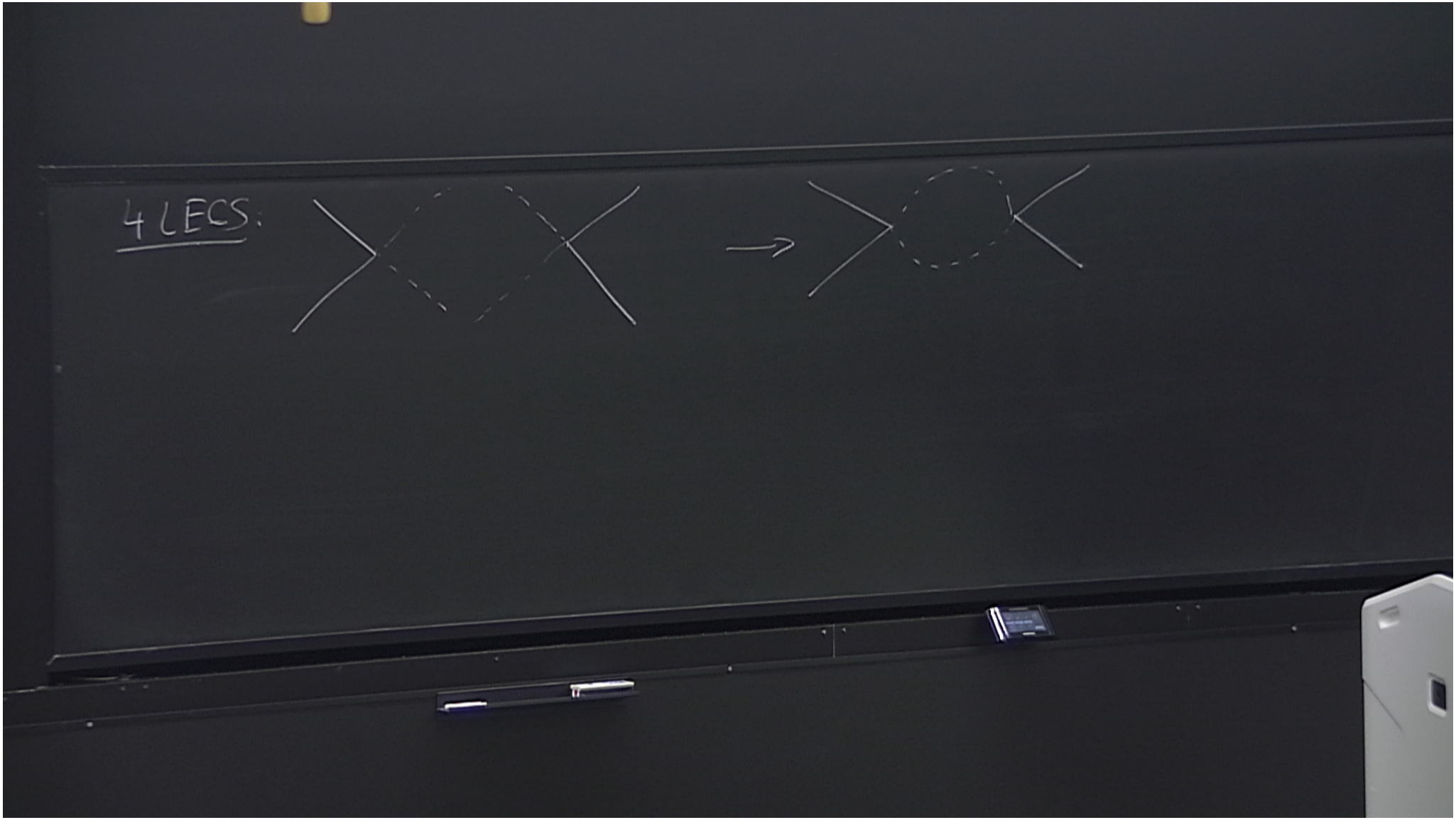
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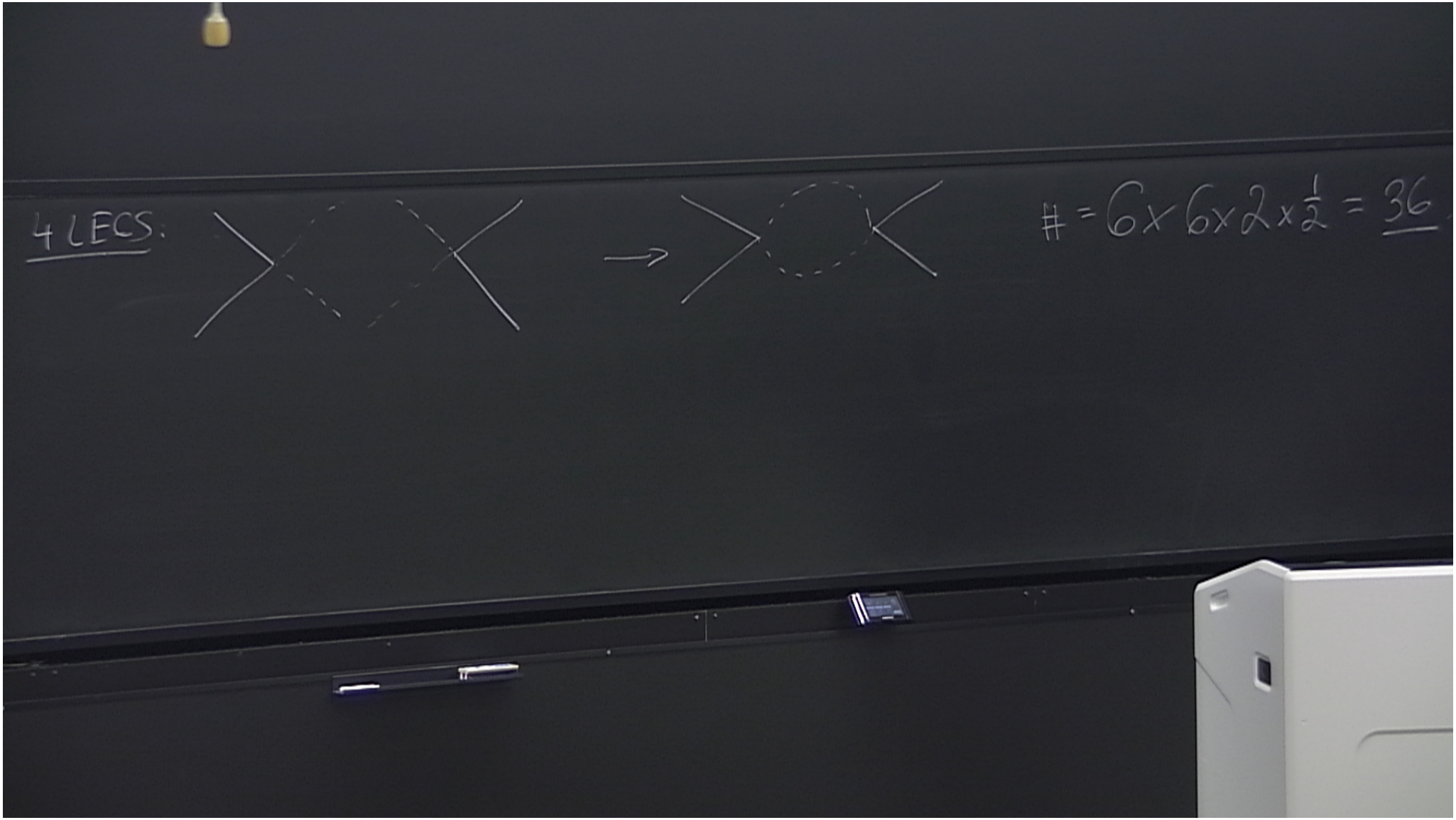


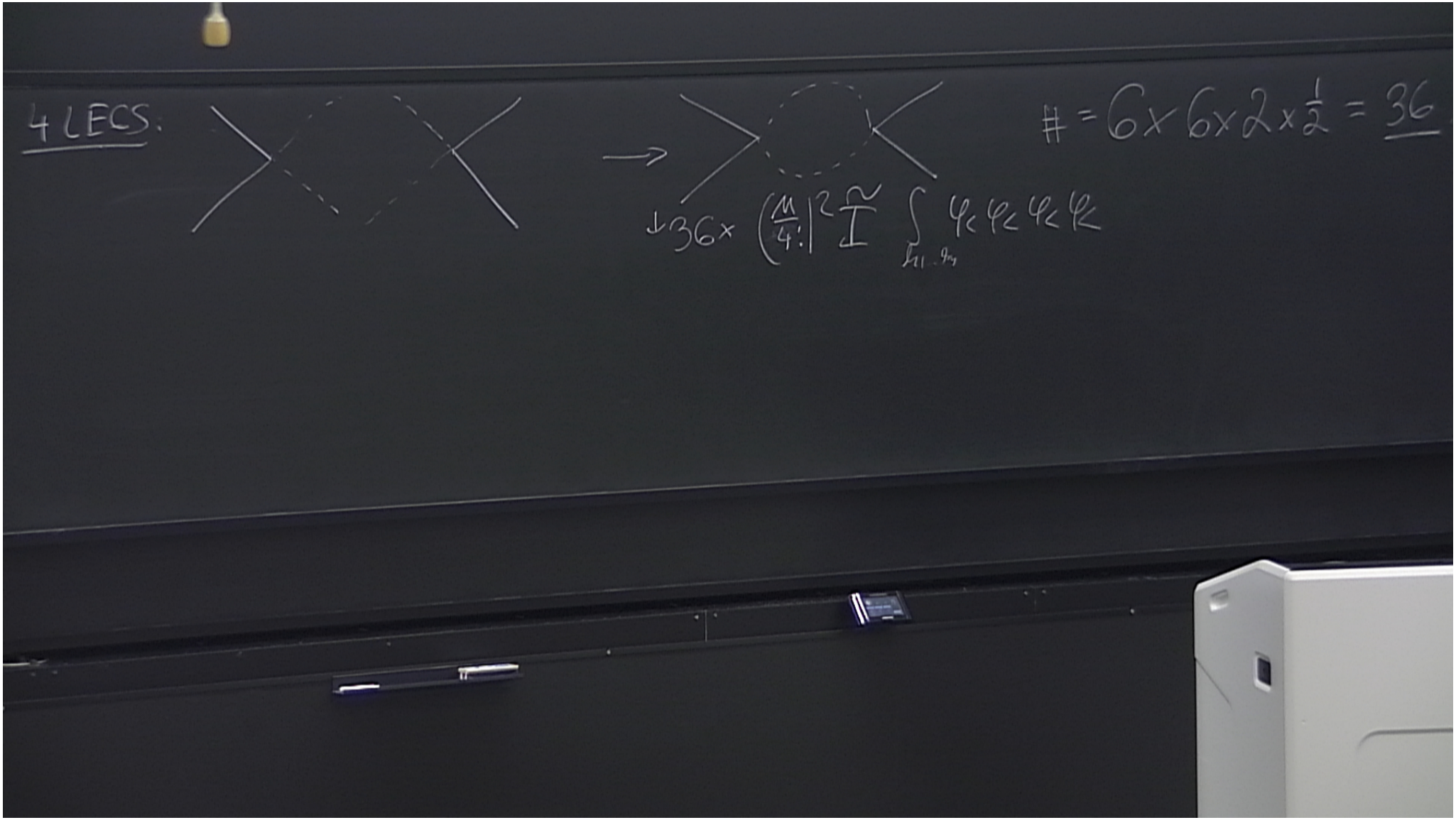
$\# = 4\times 4\times 6\times \frac{1}{2} = 48$

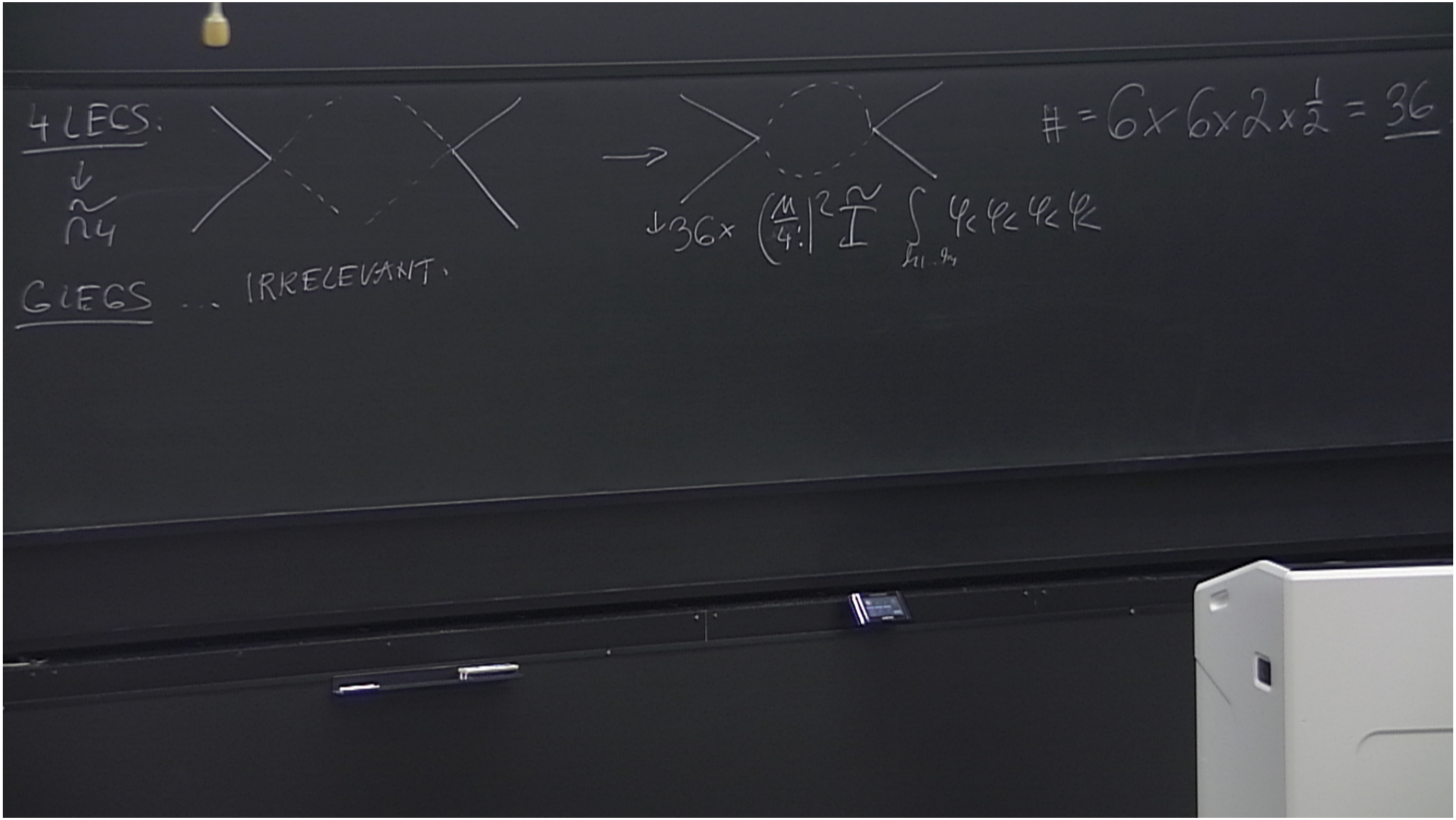
DOES NOT CO
ANYW

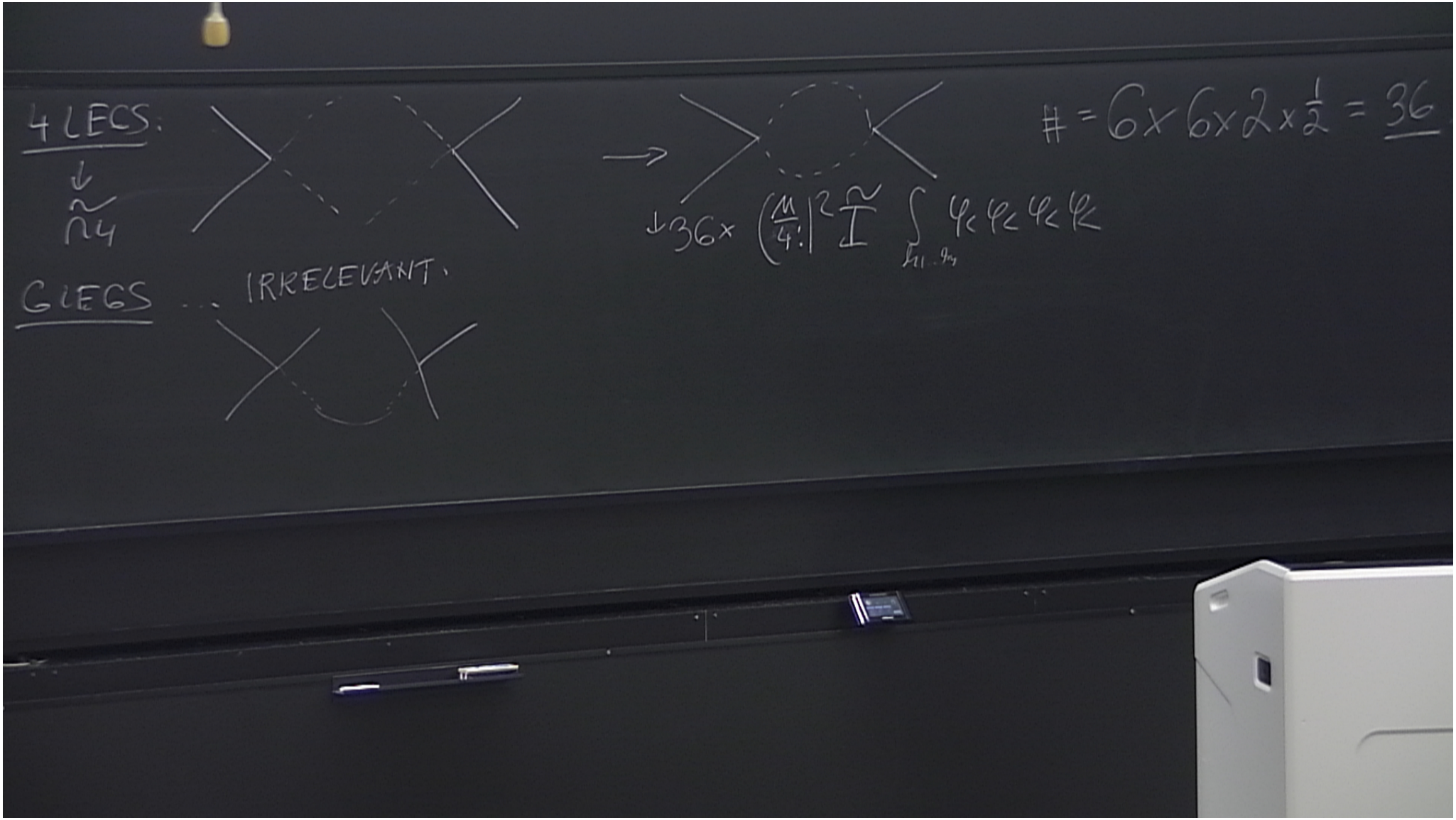

4 LECS:

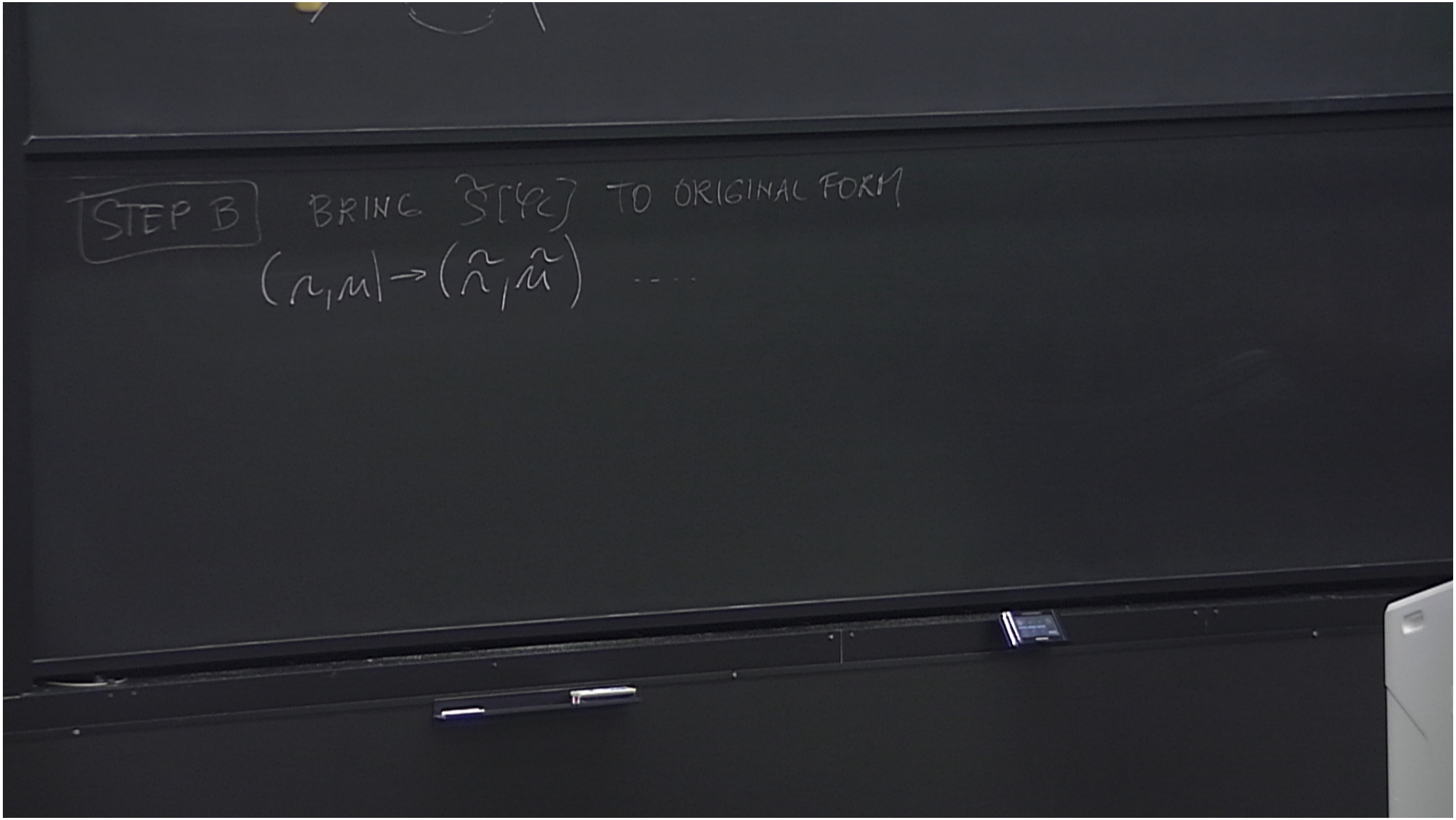

4 LECS:

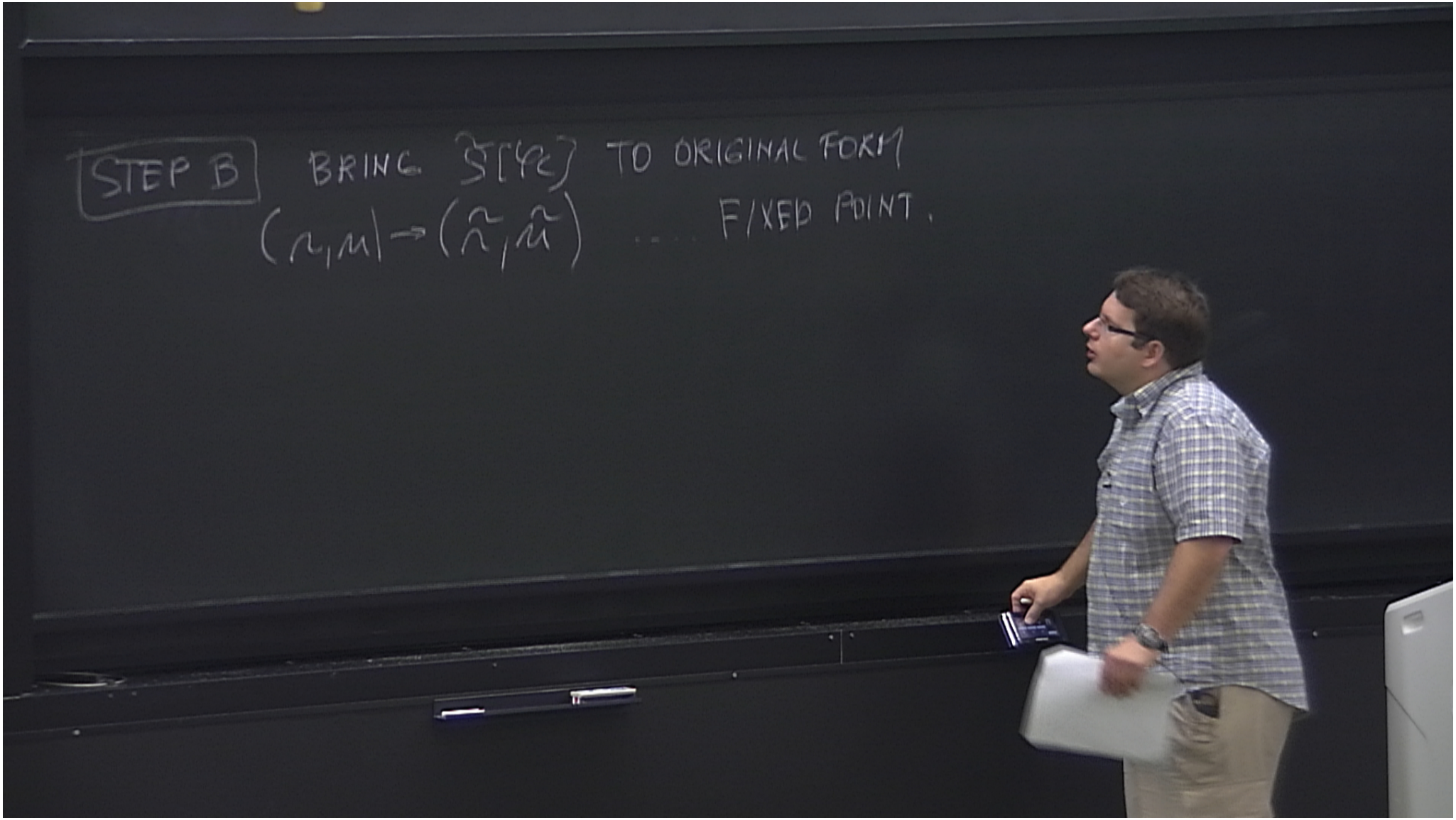

4 LECS:
= 6 × 6 × 2 × 1/2 = 36

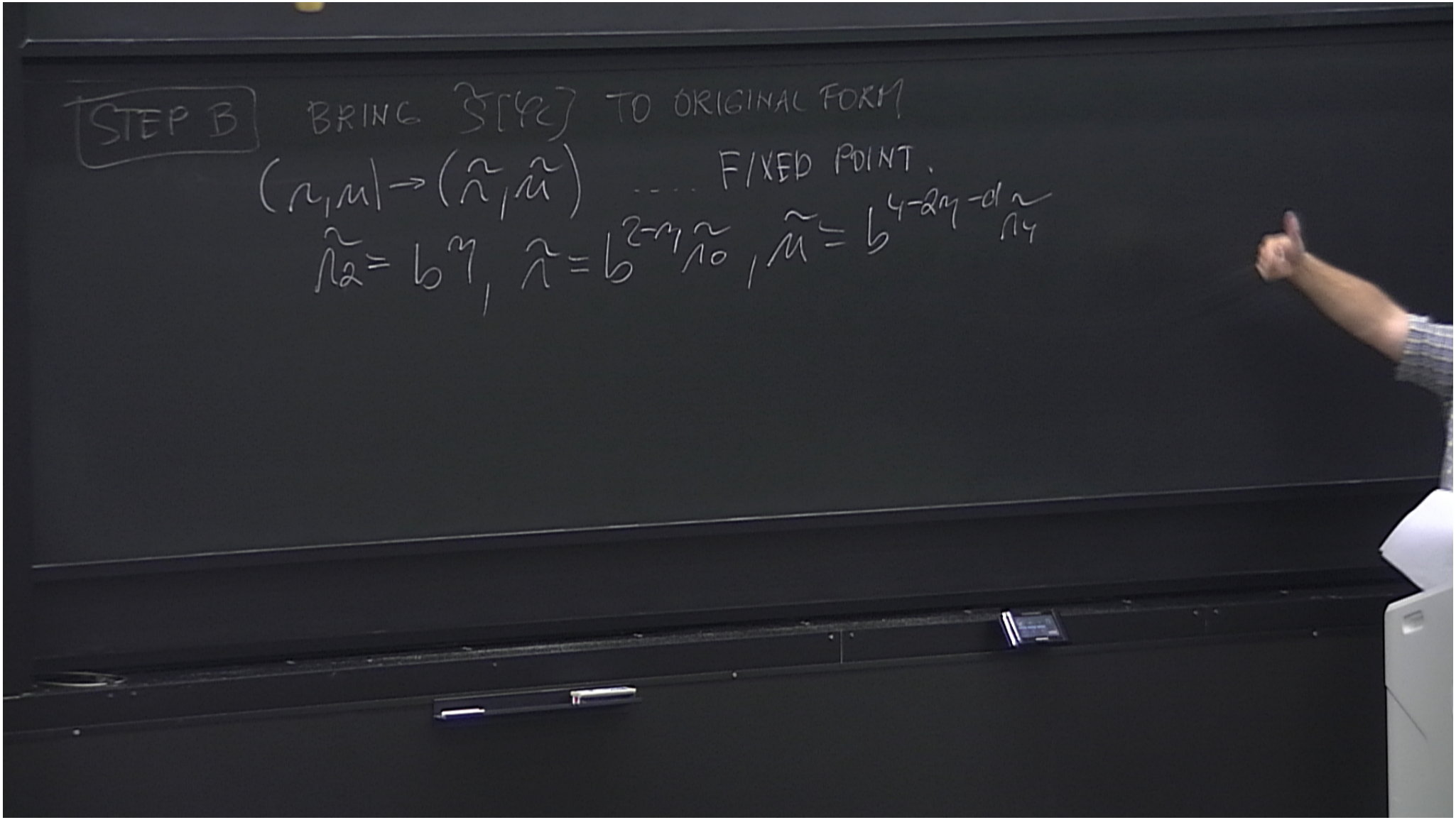

4 LECS:
= 6 × 6 × 2 × 1/2 = 36
↓36 × (M/4!)² Ĩ ∫ dq₁ dq₂ dq₃ dq₄

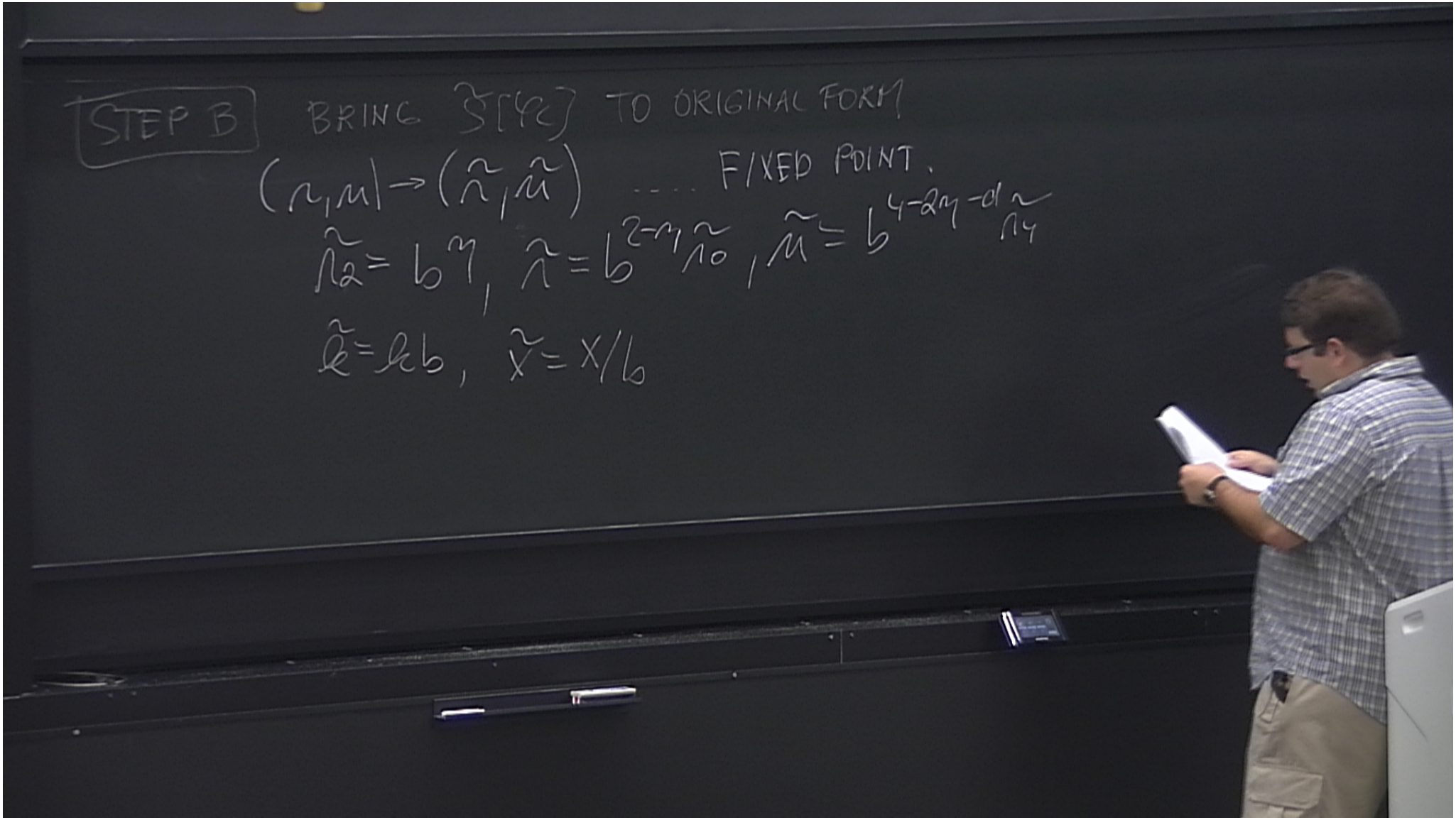

4 LEGS:
$\sim \mathcal{N}4$
GLEGS ... IRRELEVANT.
$\# = 6 \times 6 \times 2 \times \frac{1}{2} = 36$
$\downarrow 36 \times \left(\frac{M}{4!}\right)^2 \tilde{I} \int_{q_1 \ldots q_4} \varphi_{k_1} \varphi_{k_2} \varphi_{k_3} \varphi_{k_4}$


4 LEGS:
$\sim \mathcal{N}_4$
GLEGS ... IRRELEVANT.
$\# = 6 \times 6 \times 2 \times \frac{1}{2} = 36$
$\downarrow 36 \times \left(\frac{M}{4!}\right)^2 \tilde{I} \int_{q_1 \ldots q_4} \varphi_k \varphi_k \varphi_k \varphi_k$


STEP B
BRING $\tilde{S}[\psi]$ TO ORIGINAL FORM
$(n,m) \to (\tilde{n},\tilde{m})$ ....


STEP B
BRING $\tilde{S}[\tilde{\rho}]$ TO ORIGINAL FORM
$(\rho, u) \to (\tilde{\rho}, \tilde{u})$ .... FIXED POINT.


STEP B
BRING $\mathcal{S}[\psi]$ TO ORIGINAL FORM
$(\lambda, \mu) \to (\tilde{\lambda}, \tilde{\mu})$ .... FIXED POINT.
$\tilde{\lambda}_2 = b^n$, $\tilde{\lambda} = b^{2-n}\tilde{\lambda}_0$, $\tilde{\mu} = b^{4-2n-d}\tilde{\lambda}_4$

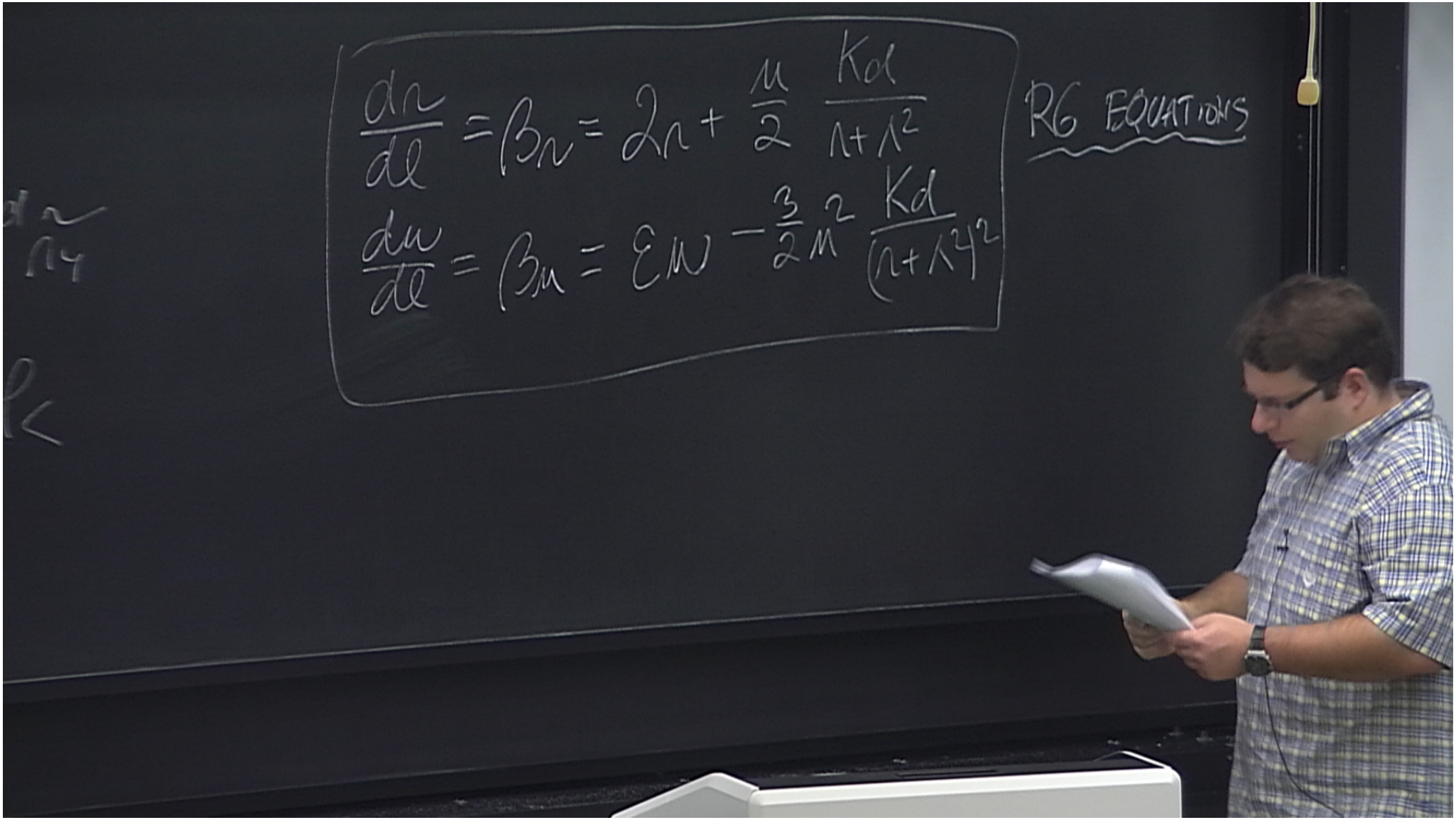

STEP B
BRING $\mathcal{S}[\varphi]$ TO ORIGINAL FORM
$(r,u) \to (\tilde{r},\tilde{u})$ .... FIXED POINT.
$\tilde{r}_2 = b^2 r_2$, $\tilde{r} = b^{2-d_\phi} r_0$, $\tilde{u} = b^{4-2d_\phi-d} r_4$
$\tilde{k} = kb$, $\tilde{x} = x/b$

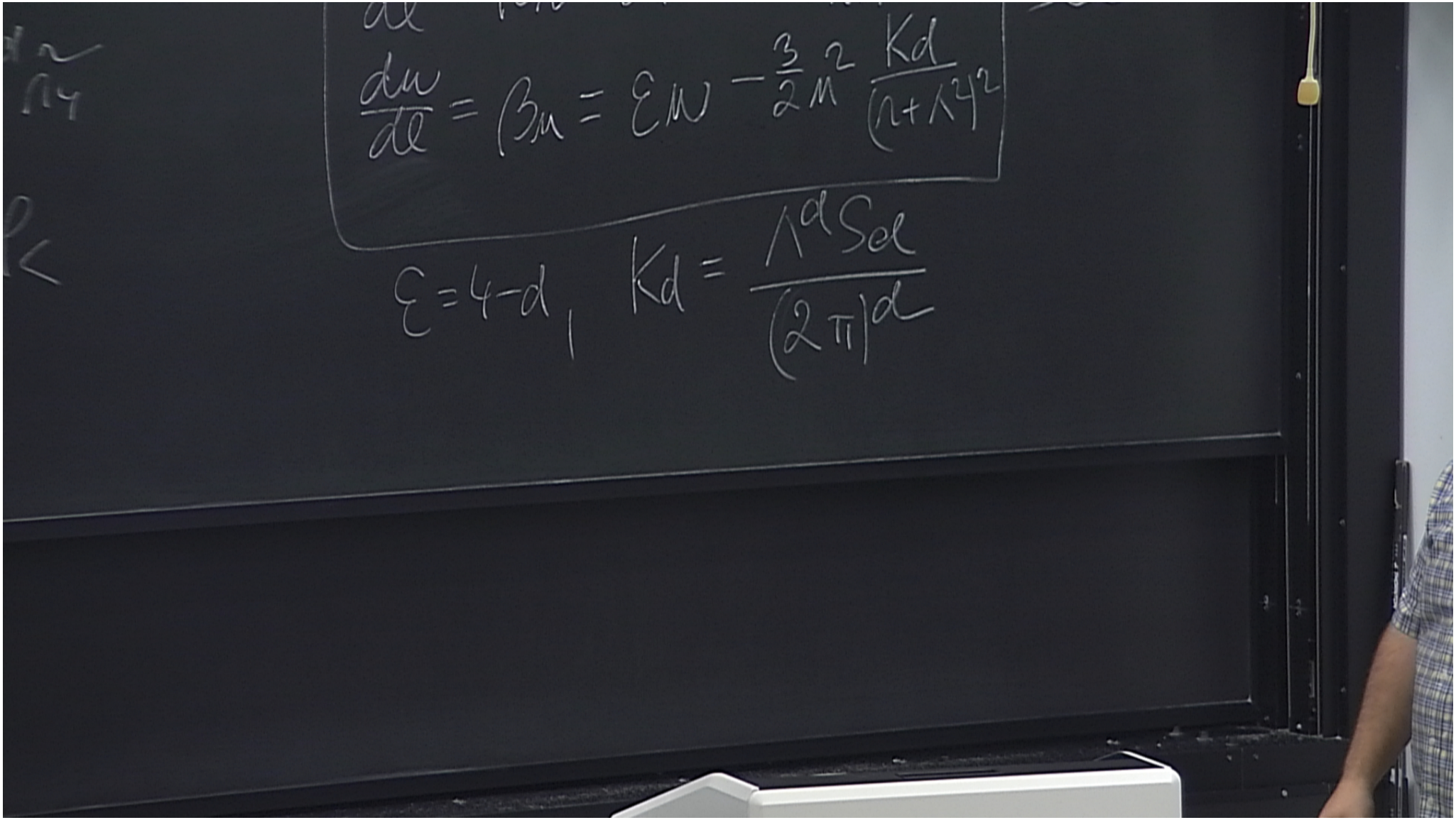
STEP B BRING $\tilde{S}[\varphi_<]$ TO ORIGINAL FORM

$(r, u) \to (\tilde{r}, \tilde{u})$ .... FIXED POINT.

$\tilde{r}_2 = b^2 r_2$, $\tilde{r} = b^{2-u} r_{10}$, $\tilde{u} = b^{4-2u-d} r_4$

$\tilde{k} = kb$, $\tilde{x} = x/b$ $\tilde{\varphi} = b^{-\frac{d+2-u}{2}} \varphi_<$

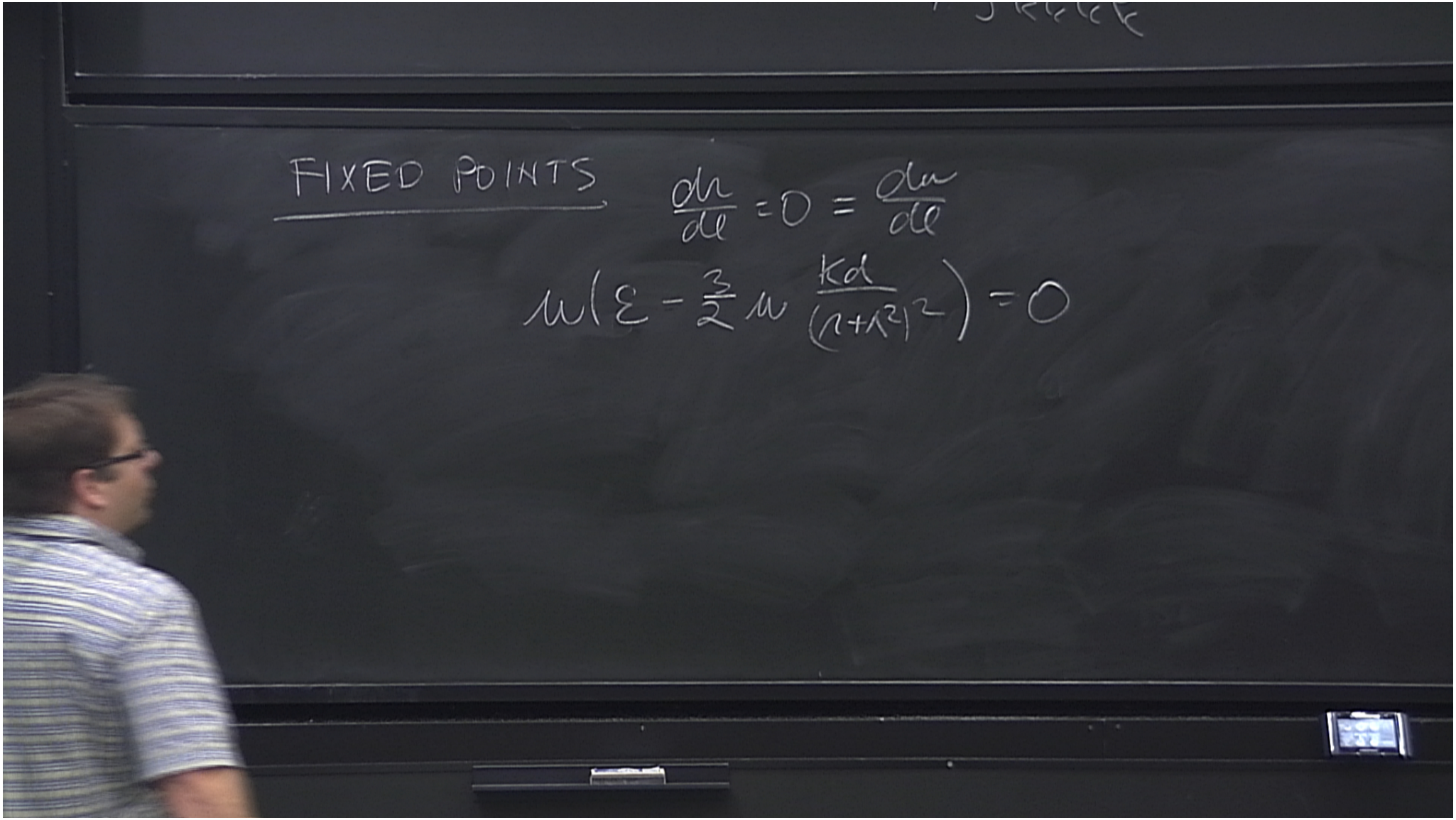

$\frac{dr}{dl} = \beta_r = 2r + \frac{u}{2}\frac{Kd}{1+r^2}$
$\frac{du}{dl} = \beta_u = \varepsilon u - \frac{3}{2}u^2\frac{Kd}{(1+r^2)^2}$
RG EQUATIONS

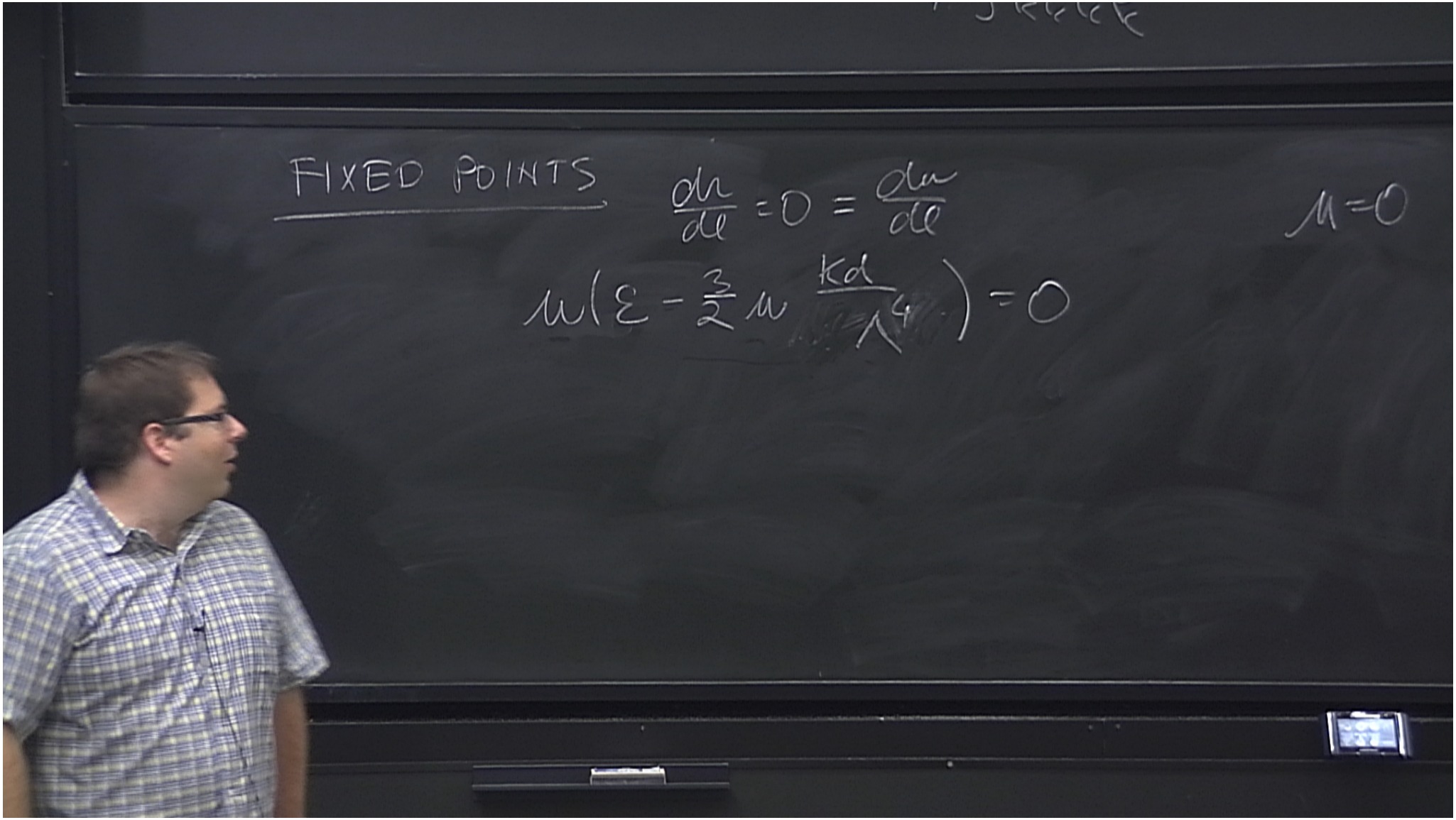

$\frac{dw}{dl} = \beta_w = \varepsilon w - \frac{3}{2} w^2 \frac{Kd}{(1+\Lambda^2)^2}$
$\varepsilon = 4-d, \quad Kd = \frac{\Lambda^d S_{el}}{(2\pi)^d}$

$\tilde{\Lambda}_2 = b^{\eta}$, $\tilde{\lambda} = b^{2-\eta}\tilde{\lambda}_0$, $\tilde{u} = b^{4-2\eta-d}\tilde{u}_4$

$\tilde{k} = kb$, $\tilde{x} = x/b$ $\tilde{\varphi} = b^{-\frac{d+2-\eta}{2}}\varphi_<$

STEPC .. ANALYZE FIXED POINTS --> PHYSICS


FIXED POINTS
$\frac{dh}{dl} = 0 = \frac{dw}{dl}$
$w\left(\varepsilon - \frac{3}{2} w \frac{kd}{(1+\lambda^2)^2}\right) = 0$


FIXED POINTS
$\frac{dh}{dl} = 0 = \frac{du}{dl}$
$u = 0$
$u\left(\varepsilon - \frac{3}{2} u \frac{Kd}{\Lambda^4}\right) = 0$

FIXED POINTS $\frac{dk}{dt} = 0 = \frac{du}{dt}$

$u = 0$

$u\left(\varepsilon - \frac{3}{2} u \frac{kd}{\Lambda^4}\right) = 0$

$u = \frac{2}{3} \varepsilon \frac{\Lambda^4}{kd}$

INTS $\frac{dh}{dl} = 0 = \frac{dw}{dl}$

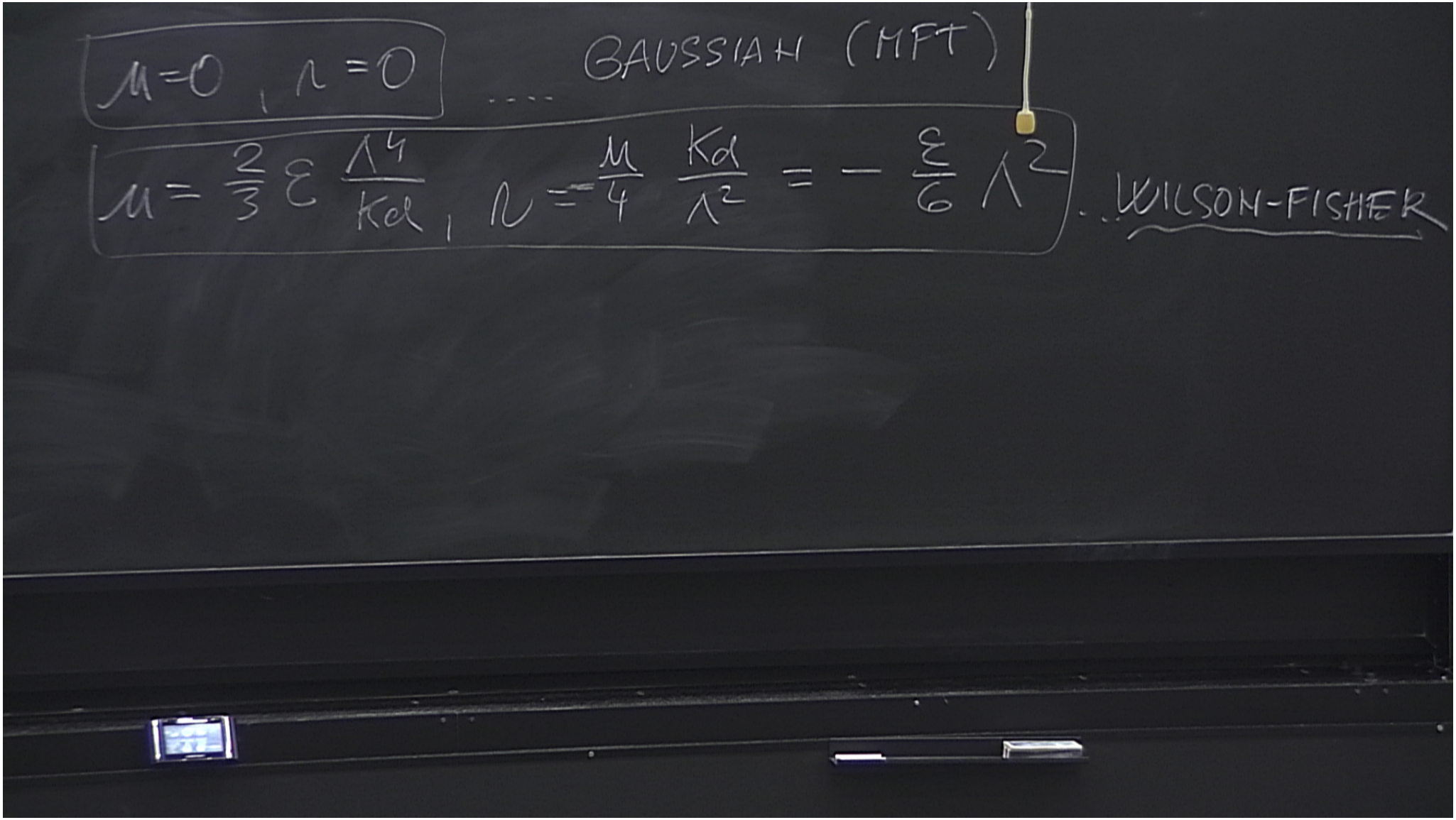
$w\left(\varepsilon - \frac{3}{2} w \frac{Kd}{\Lambda^4}\right) = 0$

$w = 0$, $h = 0$

$w = \frac{2}{3}\varepsilon \frac{\Lambda^4}{Kd}$, $h = \frac{w}{4}\frac{Kd}{\Lambda^2}$

$$M = 0 \,, \quad \Lambda = 0$$

$$\mu = \frac{2}{3}\varepsilon \frac{\Lambda^4}{Kd} \,, \quad \nu = -\frac{M}{4}\frac{Kd}{\Lambda^2} = -\frac{\varepsilon}{6}\Lambda^2$$


$u=0, \lambda=0$ .... GAUSSIAN (MFT)
$u=\frac{2}{3}\varepsilon\frac{\Lambda^4}{Kd}, \lambda=\frac{u}{4}\frac{Kd}{\Lambda^2}=-\frac{\varepsilon}{6}\Lambda^2$ .. WILSON-FISHER

$u = 0 \;,\; \lambda = 0$ .... GAUSSIAN (MFT)

$u = \frac{2}{3}\varepsilon \frac{\Lambda^4}{K_d} \;,\; \lambda = \frac{u}{4}\frac{K_d}{\Lambda^2} = -\frac{\varepsilon}{6}\Lambda^2$ .. WILSON-FISHER

SMALL ε EXPANSION.

FIXED POINTS $\frac{dn}{dt} = 0 = \frac{du}{dt}$

$u = 0$

$u\left(\varepsilon - \frac{3}{2} u \frac{Kd}{\Lambda^4}\right) = 0$

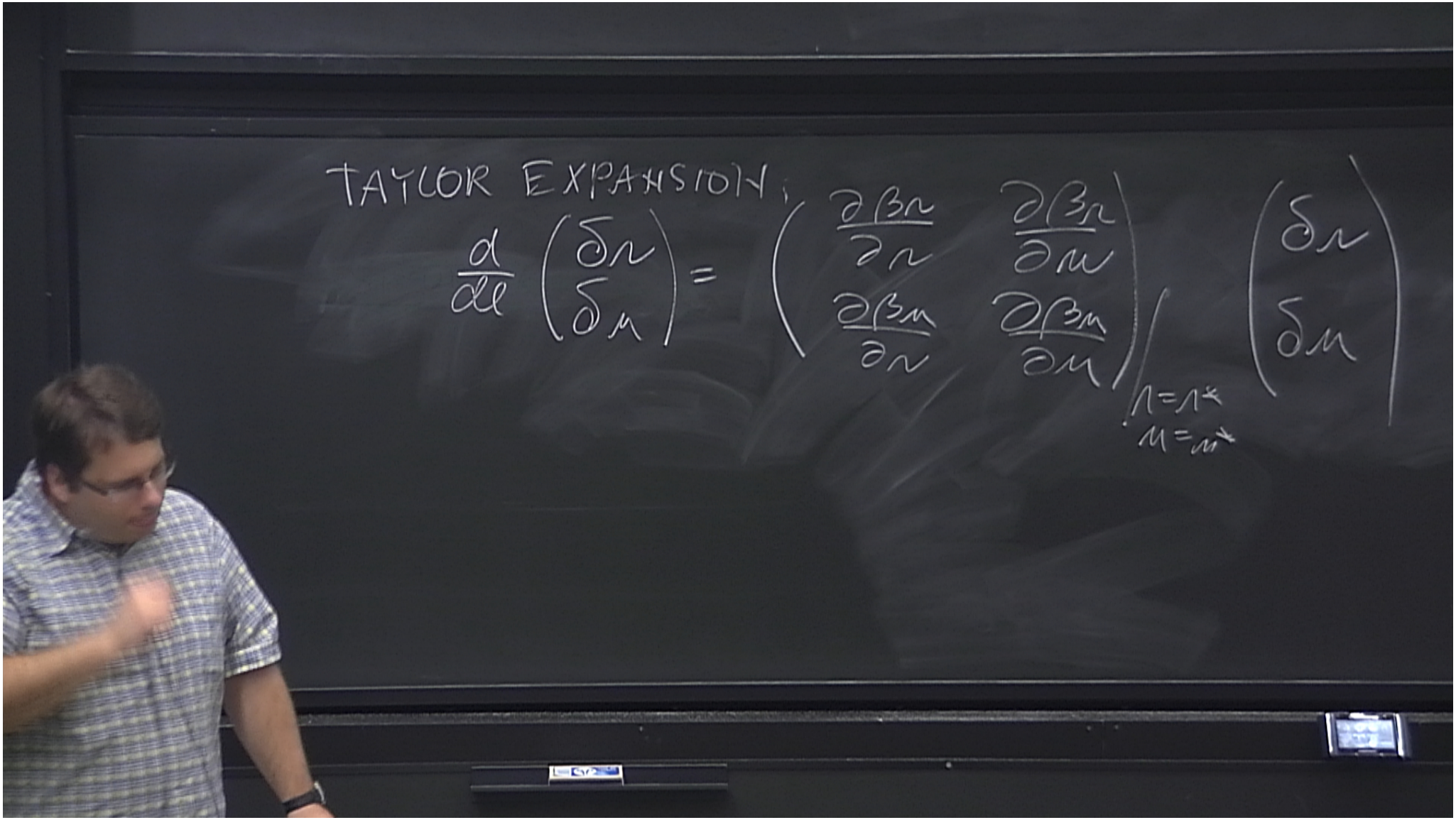
$u = \frac{2}{3}$

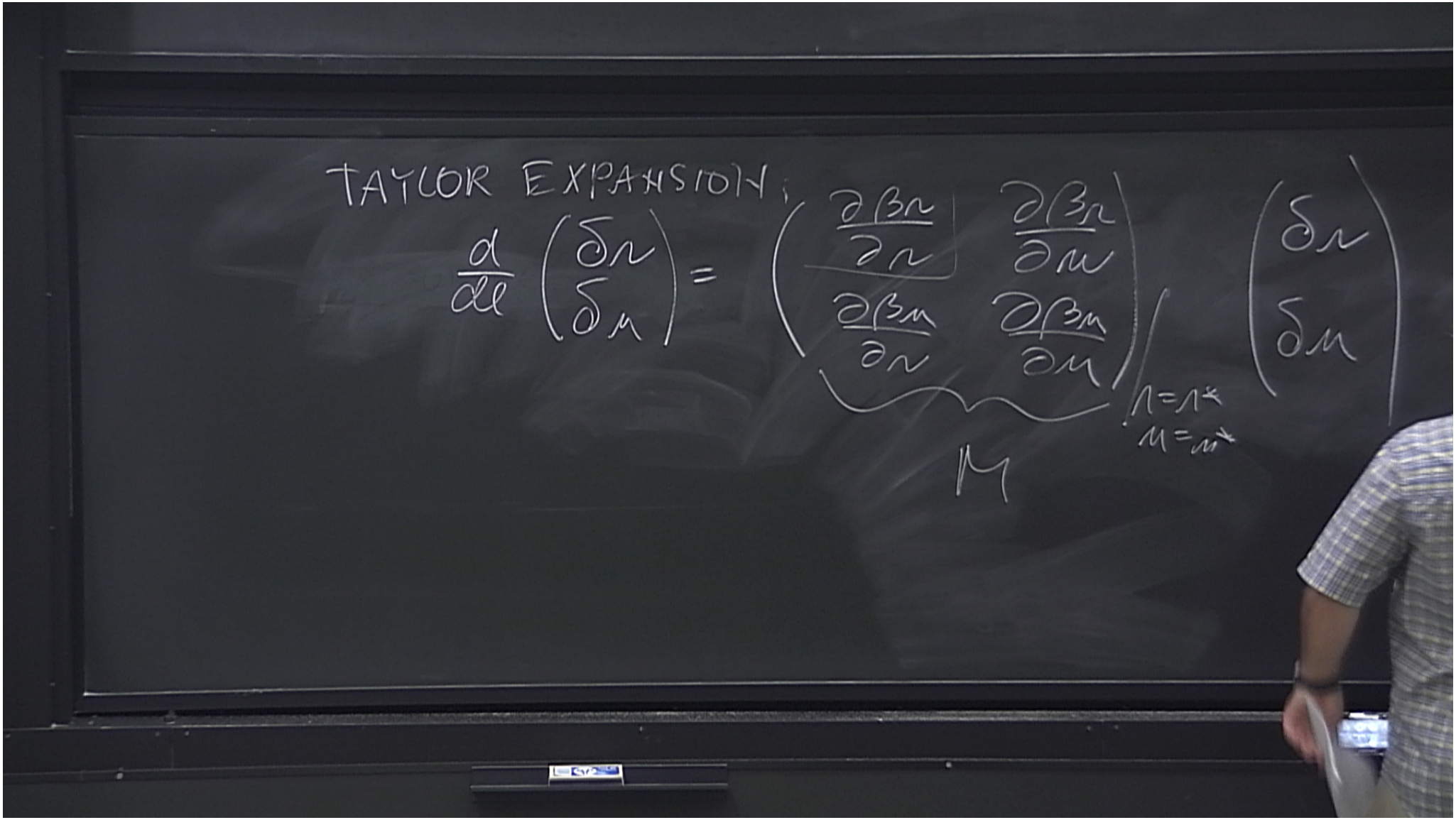
A) GAUSSIAN $(u^*, n^*) = (0, 0)$

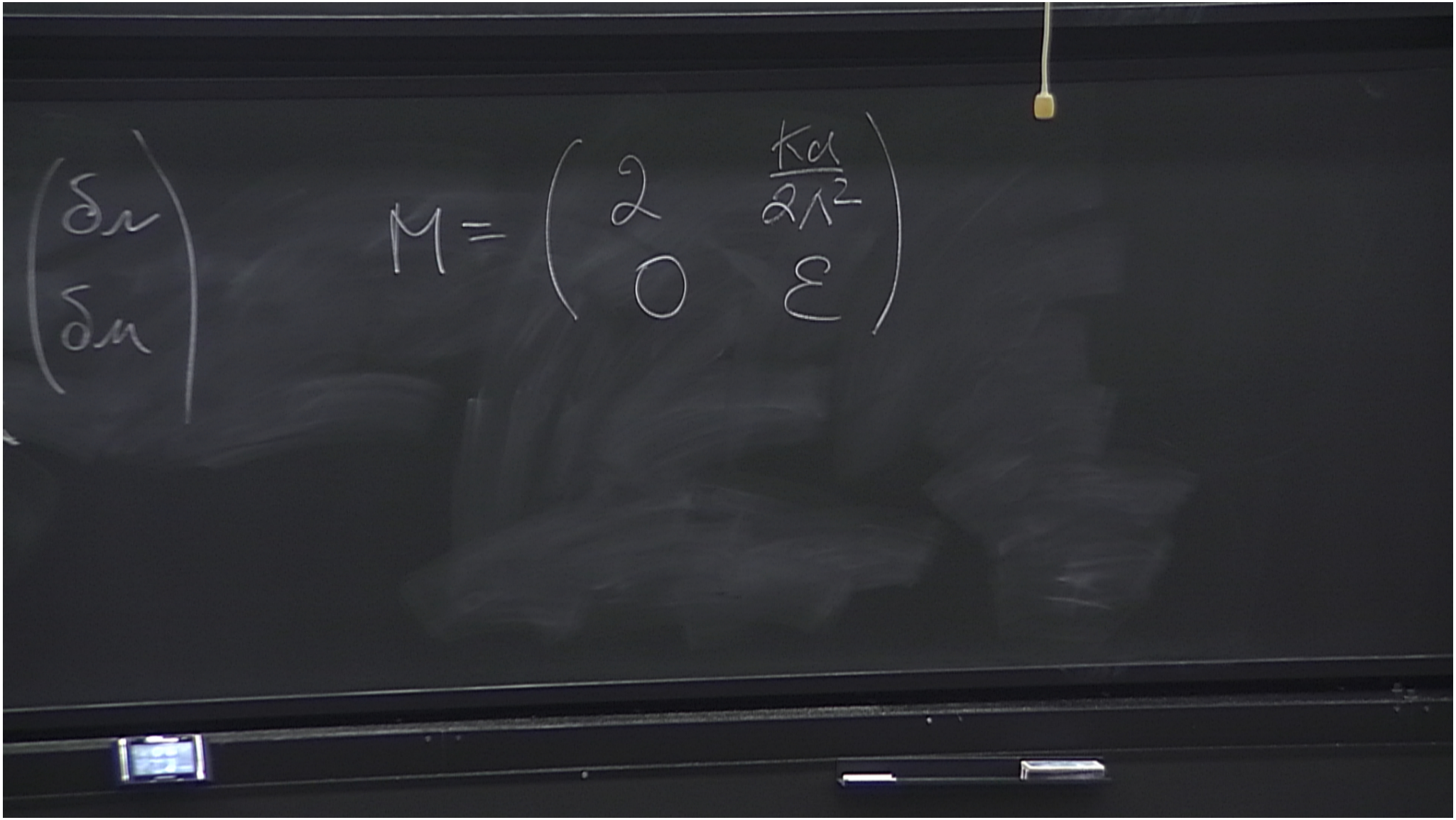
LINEARIZE RG EQS.

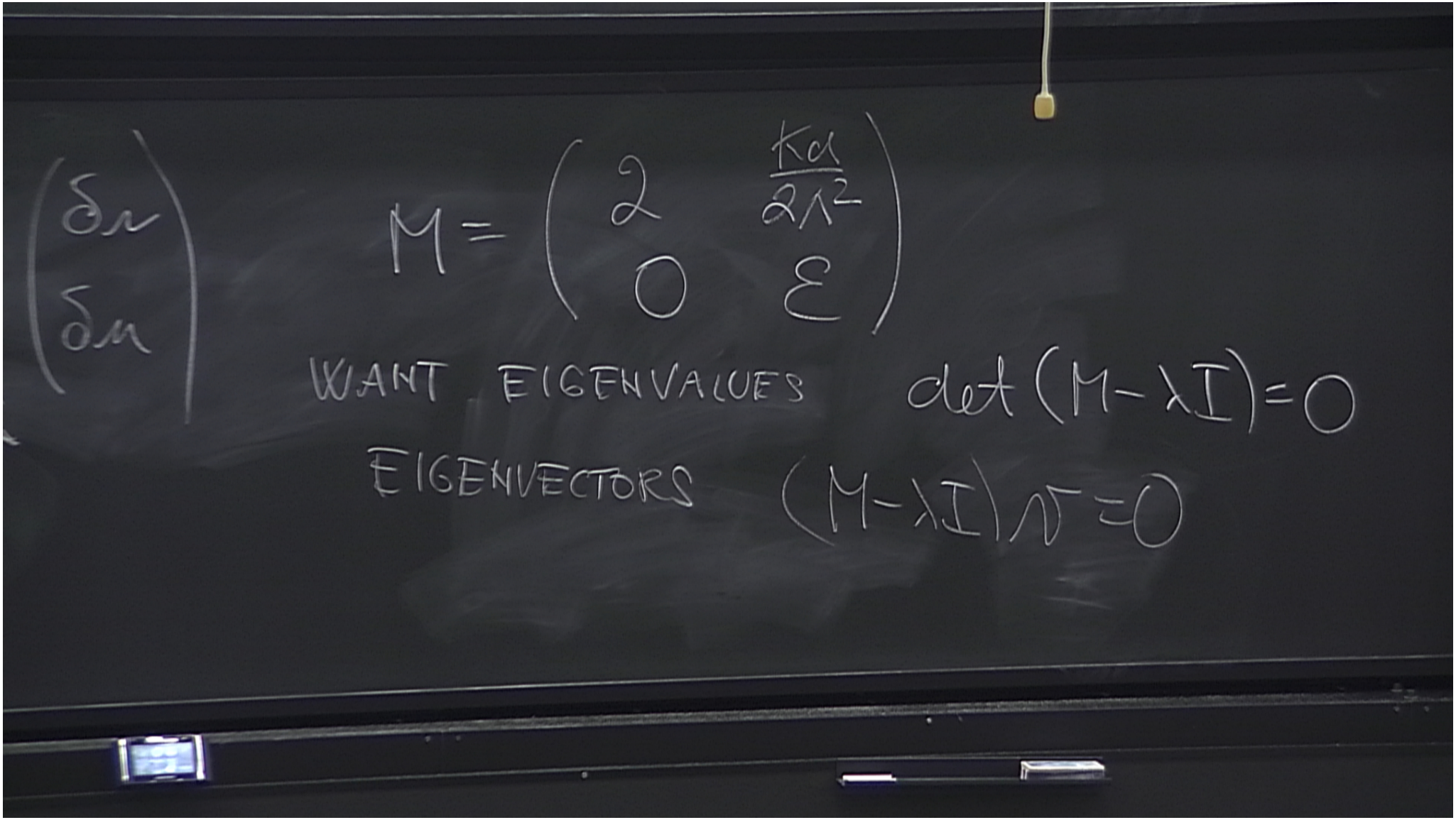
$\delta n = n - n^*$, $\delta u = u - u^*$

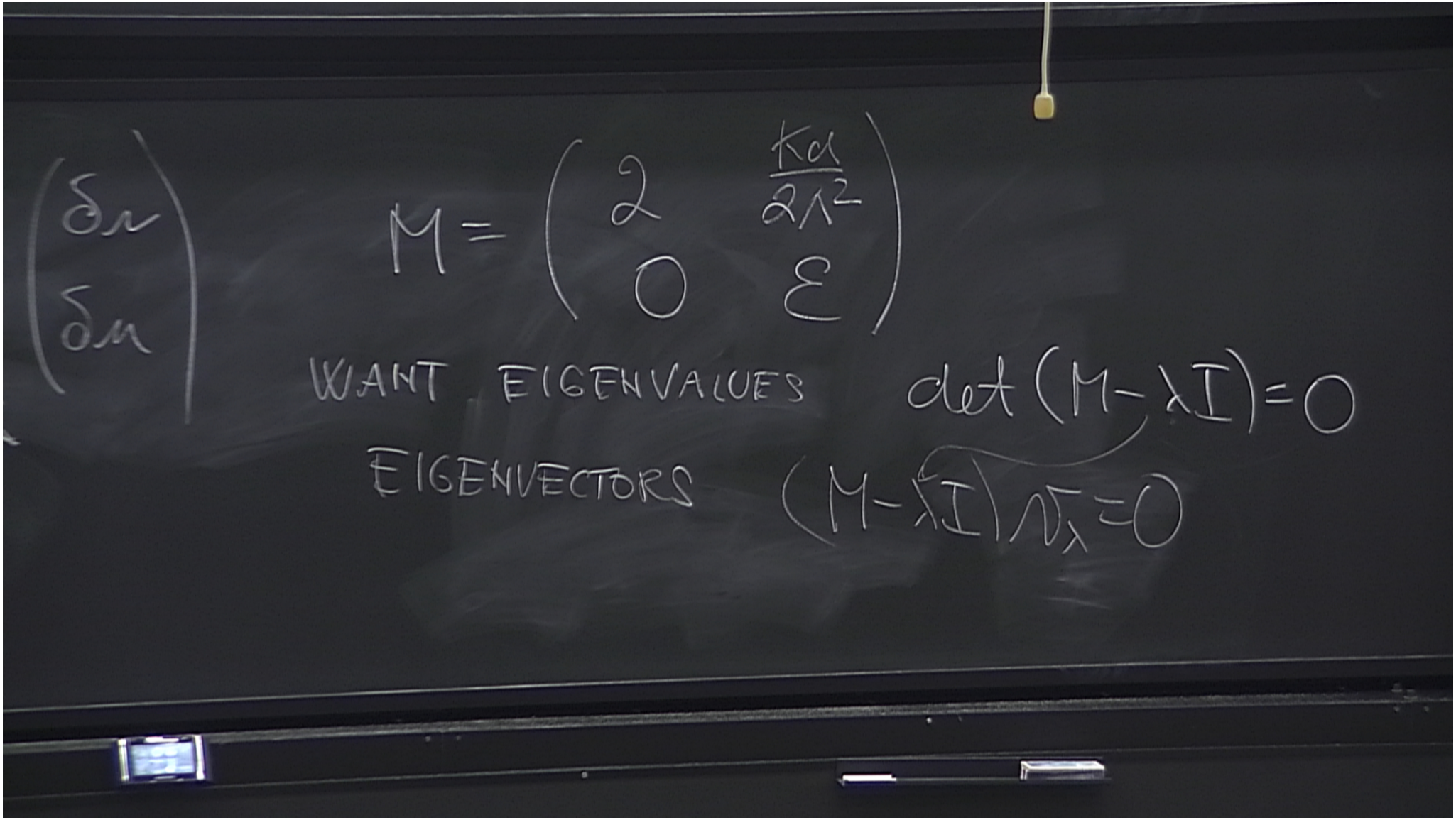
0-th ORDER: $S_G[\varphi_<] = \frac{1}{2} \int (k^2 + n) |\varphi_<(k)|^2$

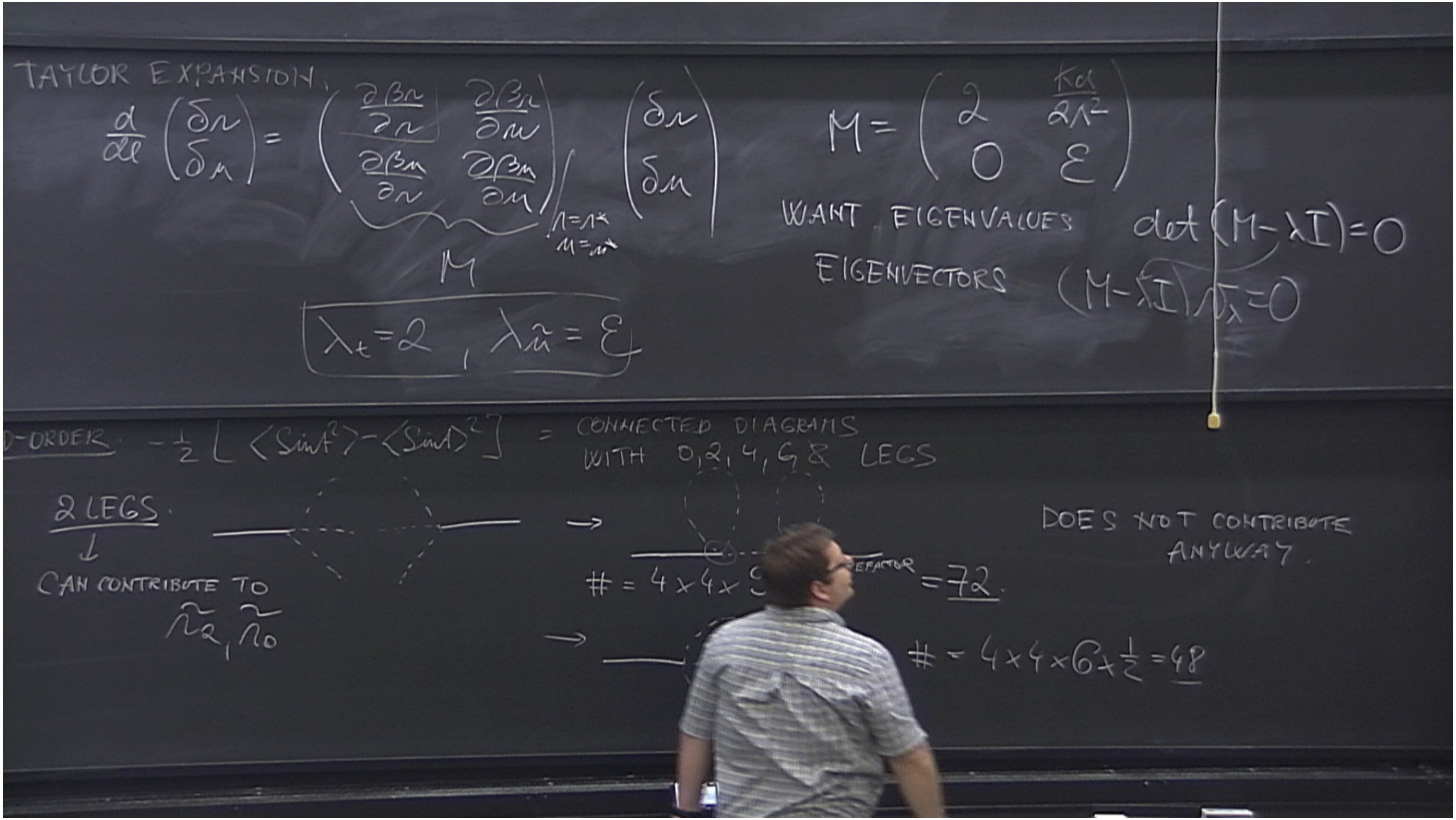

TAYLOR EXPANSION:
$\frac{d}{dl}\begin{pmatrix}\delta_n\\ \delta_m\end{pmatrix} = \begin{pmatrix}\frac{\partial\beta_n}{\partial n} & \frac{\partial\beta_n}{\partial m}\\ \frac{\partial\beta_m}{\partial n} & \frac{\partial\beta_m}{\partial m}\end{pmatrix}\Big|_{\substack{n=n^*\\ m=m^*}} \begin{pmatrix}\delta_n\\ \delta_m\end{pmatrix}$

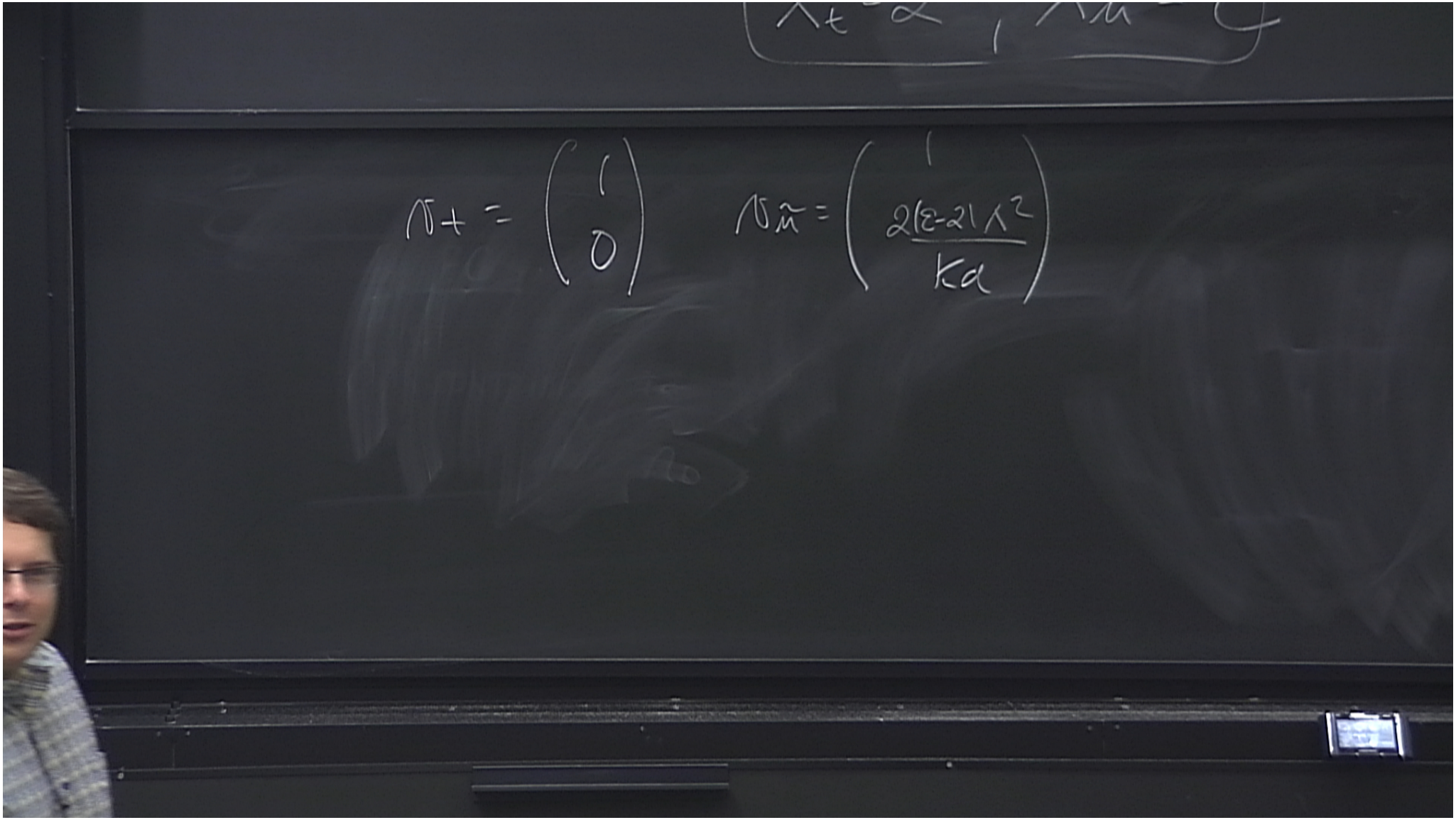

TAYLOR EXPANSION:
$\frac{d}{dt}\begin{pmatrix}\delta\lambda\\ \delta\mu\end{pmatrix} = \begin{pmatrix}\frac{\partial\beta_\lambda}{\partial\lambda} & \frac{\partial\beta_\lambda}{\partial\mu}\\ \frac{\partial\beta_\mu}{\partial\lambda} & \frac{\partial\beta_\mu}{\partial\mu}\end{pmatrix}\Big|_{\substack{\lambda=\lambda^*\\ \mu=\mu^*}} \begin{pmatrix}\delta\lambda\\ \delta\mu\end{pmatrix}$
M

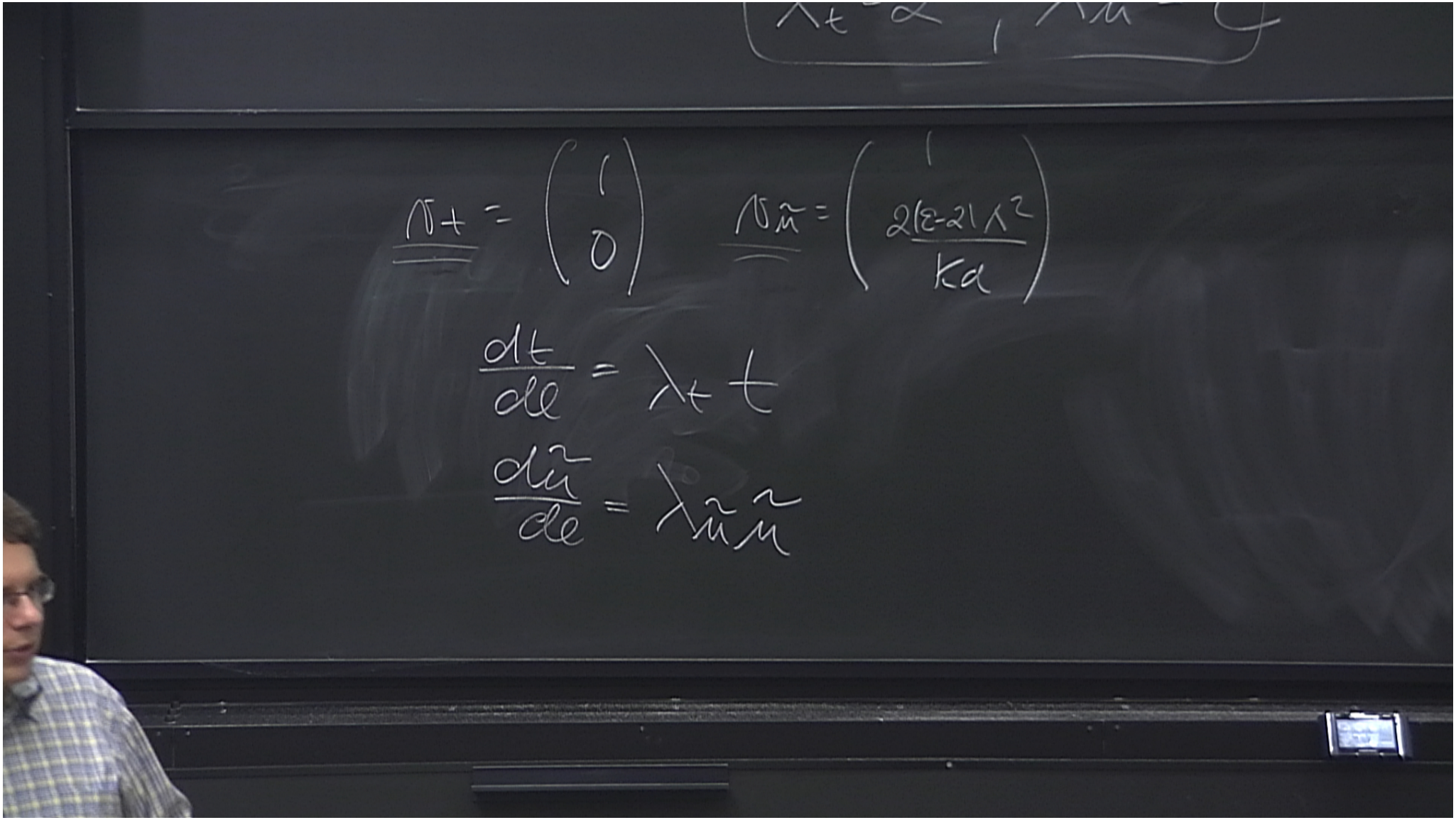

$\begin{pmatrix} \delta_n \\ \delta_m \end{pmatrix}$
$M = \begin{pmatrix} 2 & \frac{k\alpha}{2\lambda^2} \\ 0 & \varepsilon \end{pmatrix}$


$\begin{pmatrix} \delta_n \\ \delta_m \end{pmatrix}$
$M = \begin{pmatrix} 2 & \frac{k\alpha}{2\lambda^2} \\ 0 & \varepsilon \end{pmatrix}$
WANT EIGENVALUES $\det(M - \lambda I) = 0$
EIGENVECTORS $(M - \lambda I)\vec{v} = 0$


$\begin{pmatrix} \delta_n \\ \delta_m \end{pmatrix}$
$M = \begin{pmatrix} 2 & \frac{ka}{2\lambda^2} \\ 0 & \varepsilon \end{pmatrix}$
WANT EIGENVALUES $\det(M - \lambda I) = 0$
EIGENVECTORS $(M - \lambda I)\vec{v}_\lambda = 0$


TAYLOR EXPANSION:
$\frac{d}{dl}\begin{pmatrix}\delta\nu\\ \delta u\end{pmatrix} = \begin{pmatrix}\frac{\partial\beta_\nu}{\partial\nu} & \frac{\partial\beta_\nu}{\partial u}\\ \frac{\partial\beta_u}{\partial\nu} & \frac{\partial\beta_u}{\partial u}\end{pmatrix}\Big|_{\substack{\nu=\nu^*\\ u=u^*}} \begin{pmatrix}\delta\nu\\ \delta u\end{pmatrix}$
M
$M = \begin{pmatrix}2 & \frac{\kappa\alpha}{2\lambda^2}\\ 0 & \varepsilon\end{pmatrix}$
WANT EIGENVALUES $\det(M-\lambda I)=0$
EIGENVECTORS $(M-\lambda I)\vec{v}_\lambda = 0$
$\lambda_t = 2\,,\ \lambda_{\tilde{u}} = \varepsilon$
D-ORDER $-\frac{1}{2}\left[\langle \sin t^2\rangle - \langle \sin t\rangle^2\right]$ = CONNECTED DIAGRAMS WITH 0, 2, 4, 6 & LEGS
2 LEGS
CAN CONTRIBUTE TO
$\tilde{\nu}_2, \tilde{\nu}_0$
DOES NOT CONTRIBUTE ANYWAY.
$\# = 4\times 4\times$ [illegible] $= 72$.
$\# = 4\times 4\times 6\times\frac{1}{2} = 48$


$\Lambda_t = \alpha$
$\Lambda_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\Lambda_{\tilde{\pi}} = \begin{pmatrix} 1 \\ \frac{2(\varepsilon - 2)\Lambda^2}{ka} \end{pmatrix}$


$\lambda_t = \alpha$, $\lambda_{\tilde{u}} =$
$\underline{\underline{n}}_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\underline{\underline{n}}_{\tilde{u}} = \begin{pmatrix} 1 \\ \frac{2(\varepsilon - 2)\Lambda^2}{ka} \end{pmatrix}$
$\frac{dt}{d\ell} = \lambda_t t$
$\frac{d\tilde{u}}{d\ell} = \lambda_{\tilde{u}} \tilde{u}$

$\lambda_t = \alpha$, $\lambda_{\tilde{u}} = \gamma$

$$\underline{t} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \underline{\tilde{u}} = \begin{pmatrix} 1 \\ \frac{2(\varepsilon - 2|\Lambda^2}{ka} \end{pmatrix}$$

$$\frac{dt}{d\ell} = \lambda_t t \quad \Rightarrow \quad t = t_0 e^{\lambda_t \ell}$$

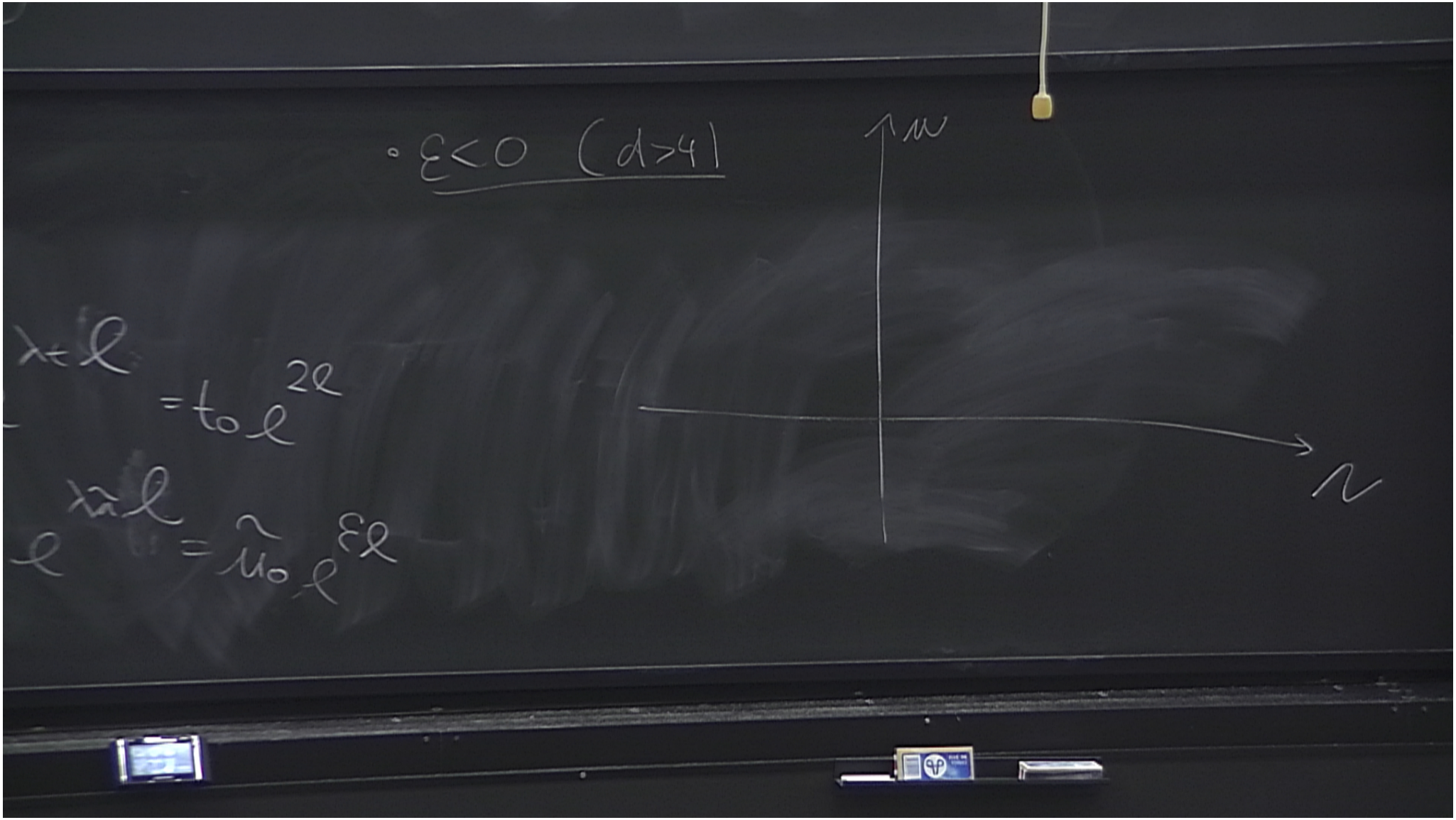
$$\frac{d\tilde{u}}{d\ell} = \lambda_{\tilde{u}} \tilde{u} \qquad \tilde{u} = \tilde{u}_0 e^{\lambda_{\tilde{u}} \ell}$$

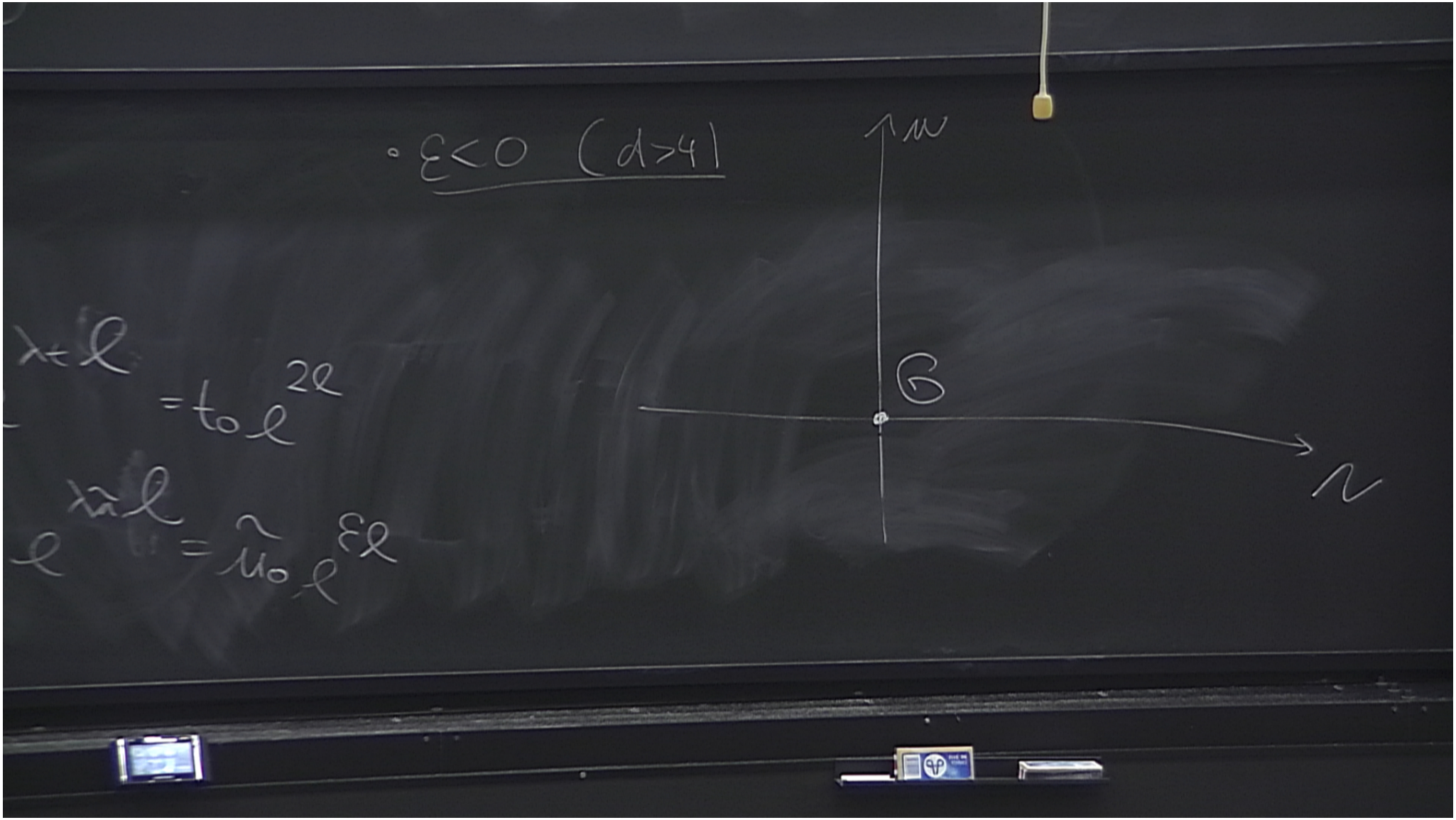
$$\lambda_t = 2, \quad \lambda_{\tilde{u}} = 0$$

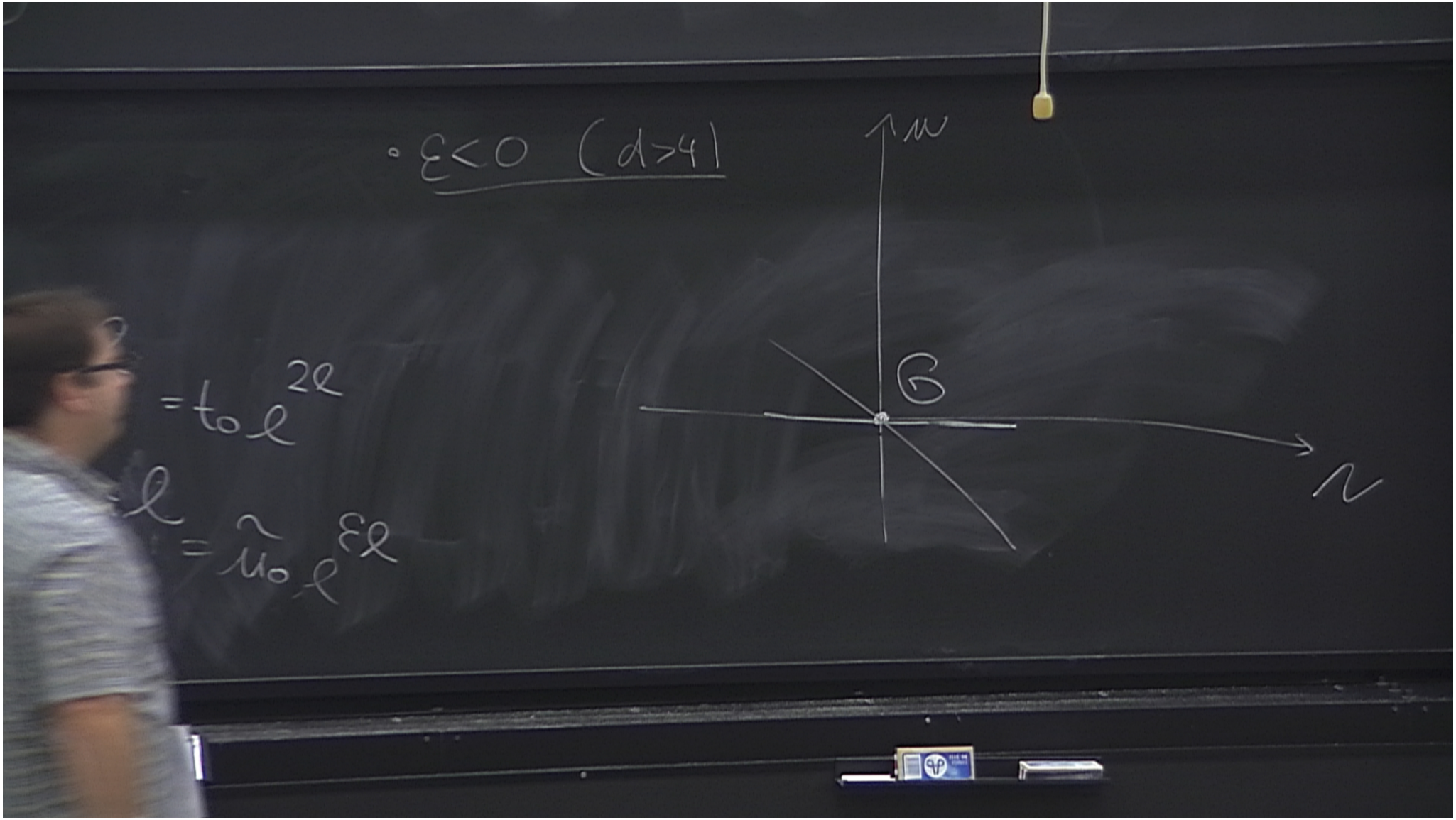
$$\underline{\underline{t}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \underline{\underline{\Lambda}}\,\tilde{u} = \begin{pmatrix} 1 \\ \frac{2(\varepsilon - 2)\Lambda^2}{ka} \end{pmatrix}$$

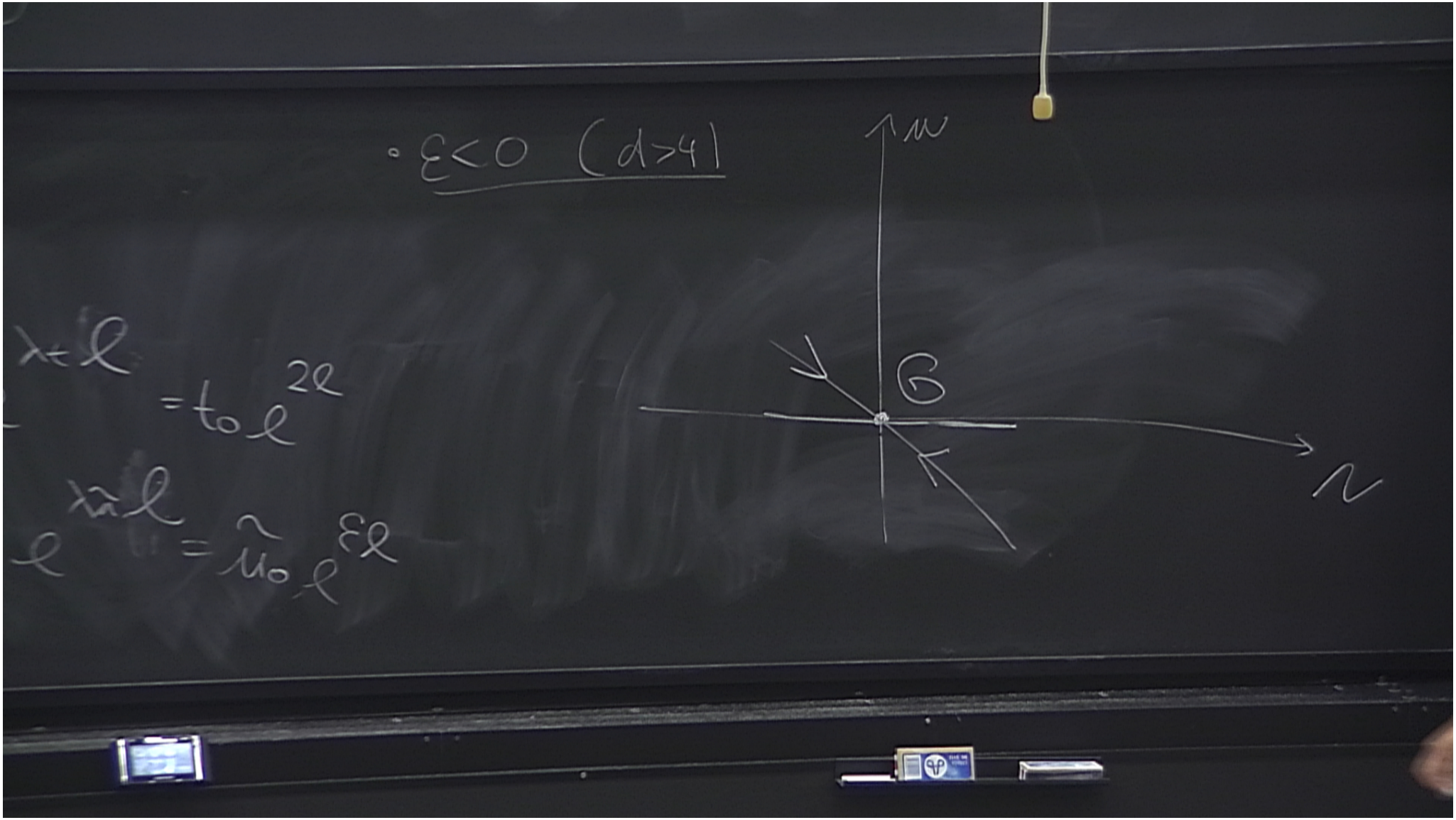
$$\frac{dt}{d\ell} = \lambda_t t \quad \Rightarrow \quad t = t_0 e^{\lambda_t \ell} = t_0 e^{2\ell}$$

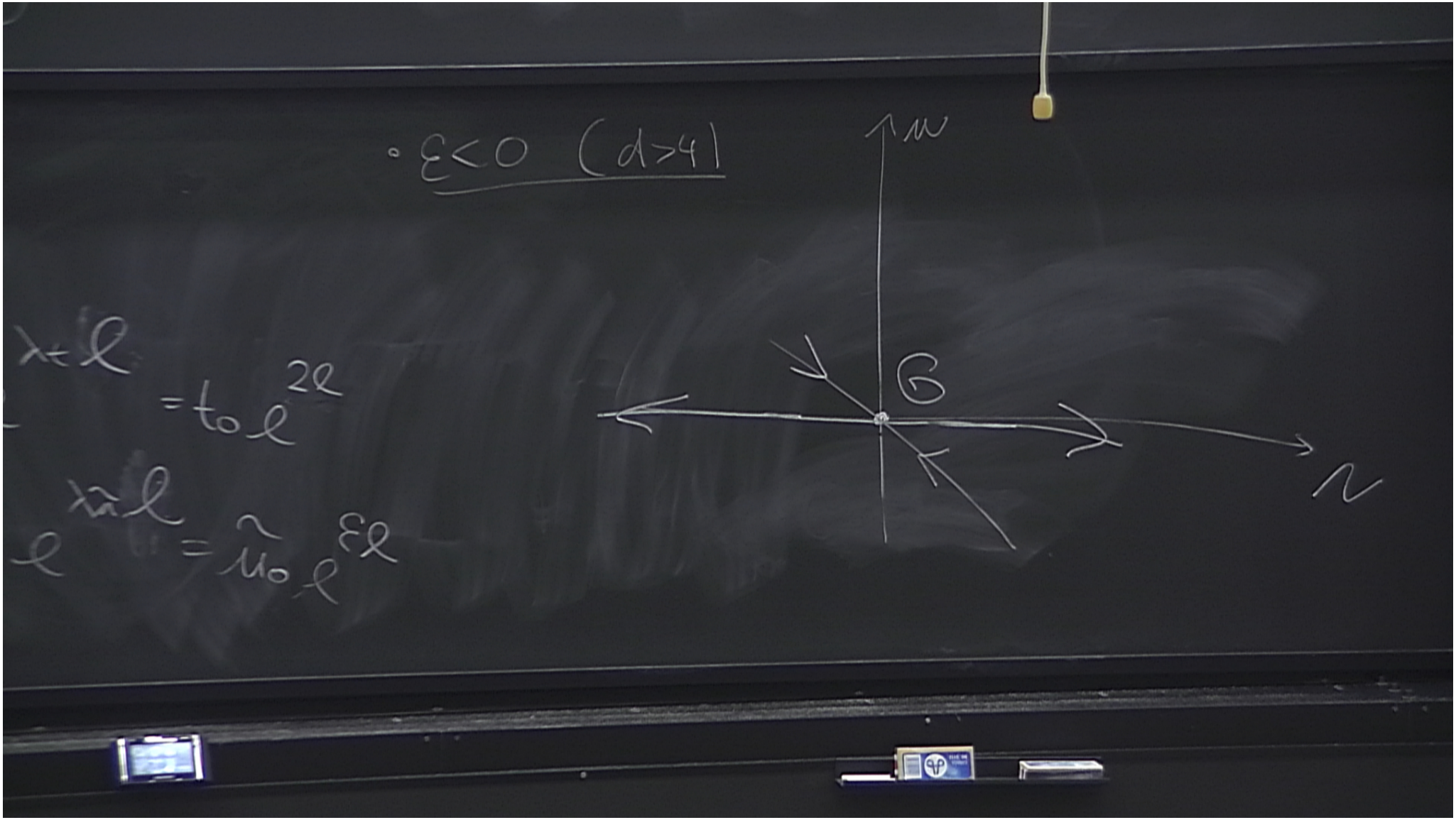
$$\frac{d\tilde{u}}{d\ell} = \lambda_{\tilde{u}} \tilde{u} \qquad \tilde{u} = \tilde{u}_0 e^{\lambda_{\tilde{u}} \ell} = \tilde{u}_0 e^{0 \cdot \ell}$$

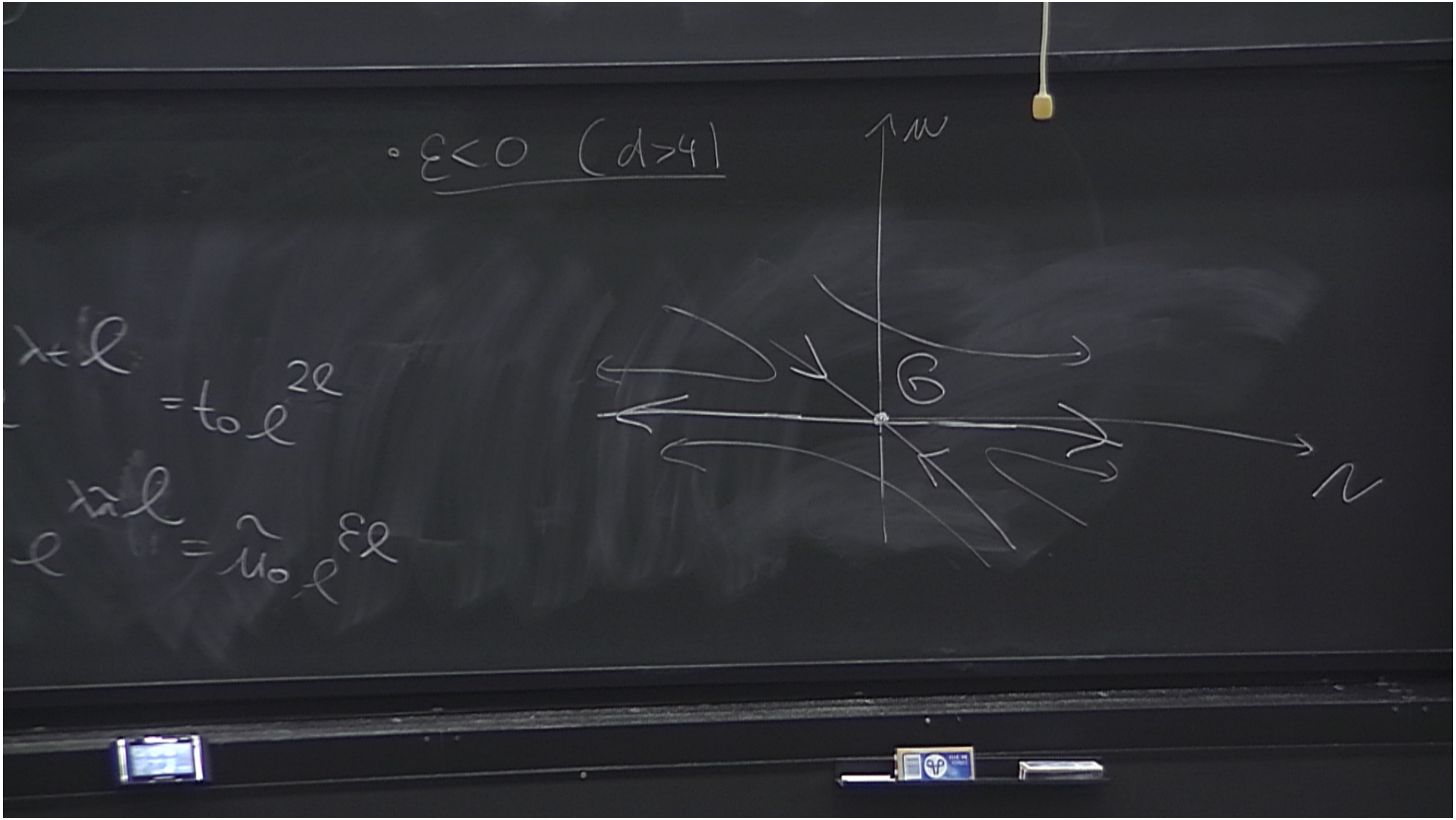

• ε<0 (d>4)
N
N

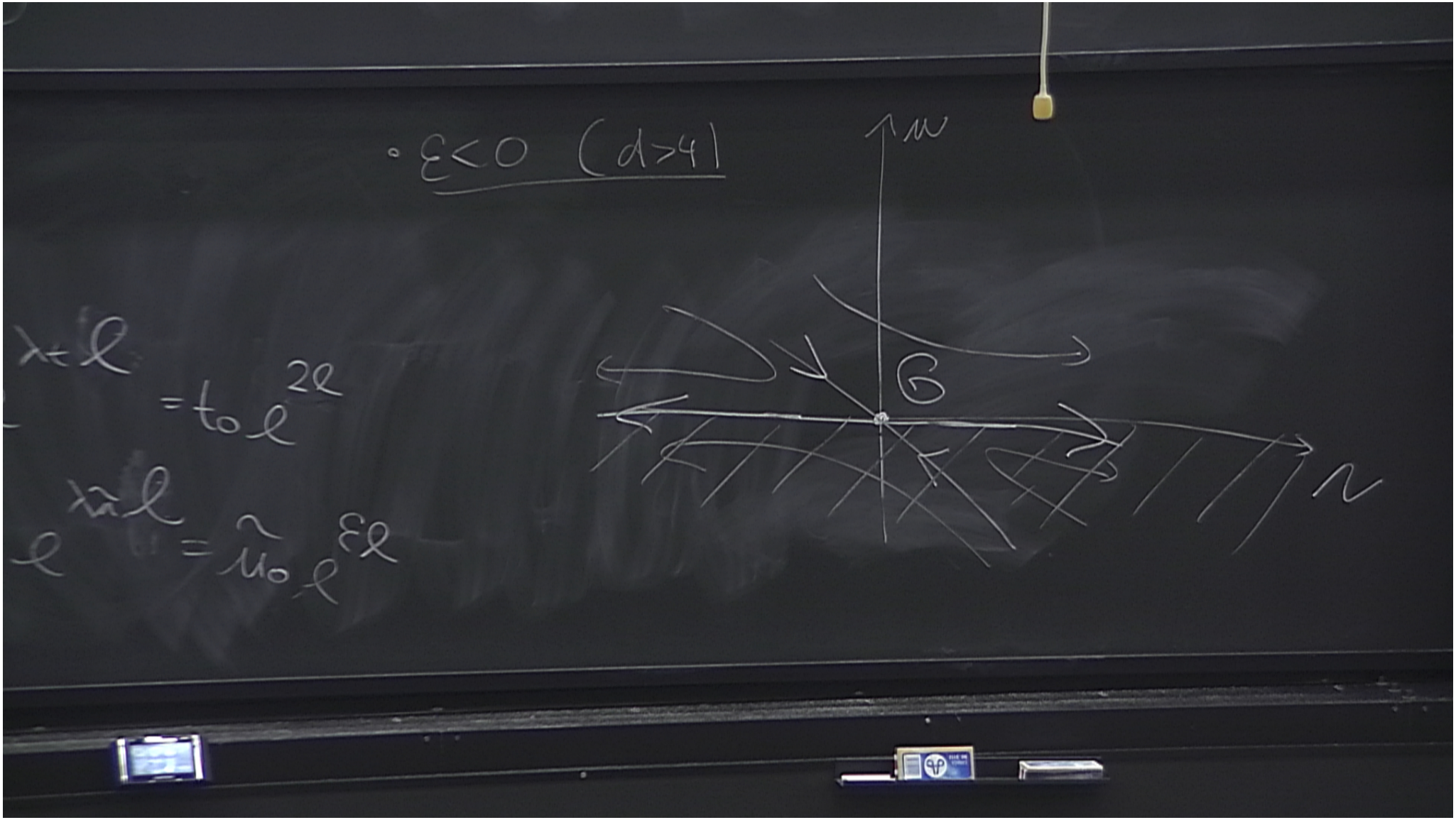

$\cdot\ \underline{\mathcal{E}<0 \quad (d>4)}$
$\chi \sim \ell$
$= t_0 \ell^{2z}$
$\tilde{\omega} \sim \ell$
$\ell^{\quad} = \tilde{\omega}_0 \ell^{\mathcal{E}z}$
ω
G
N

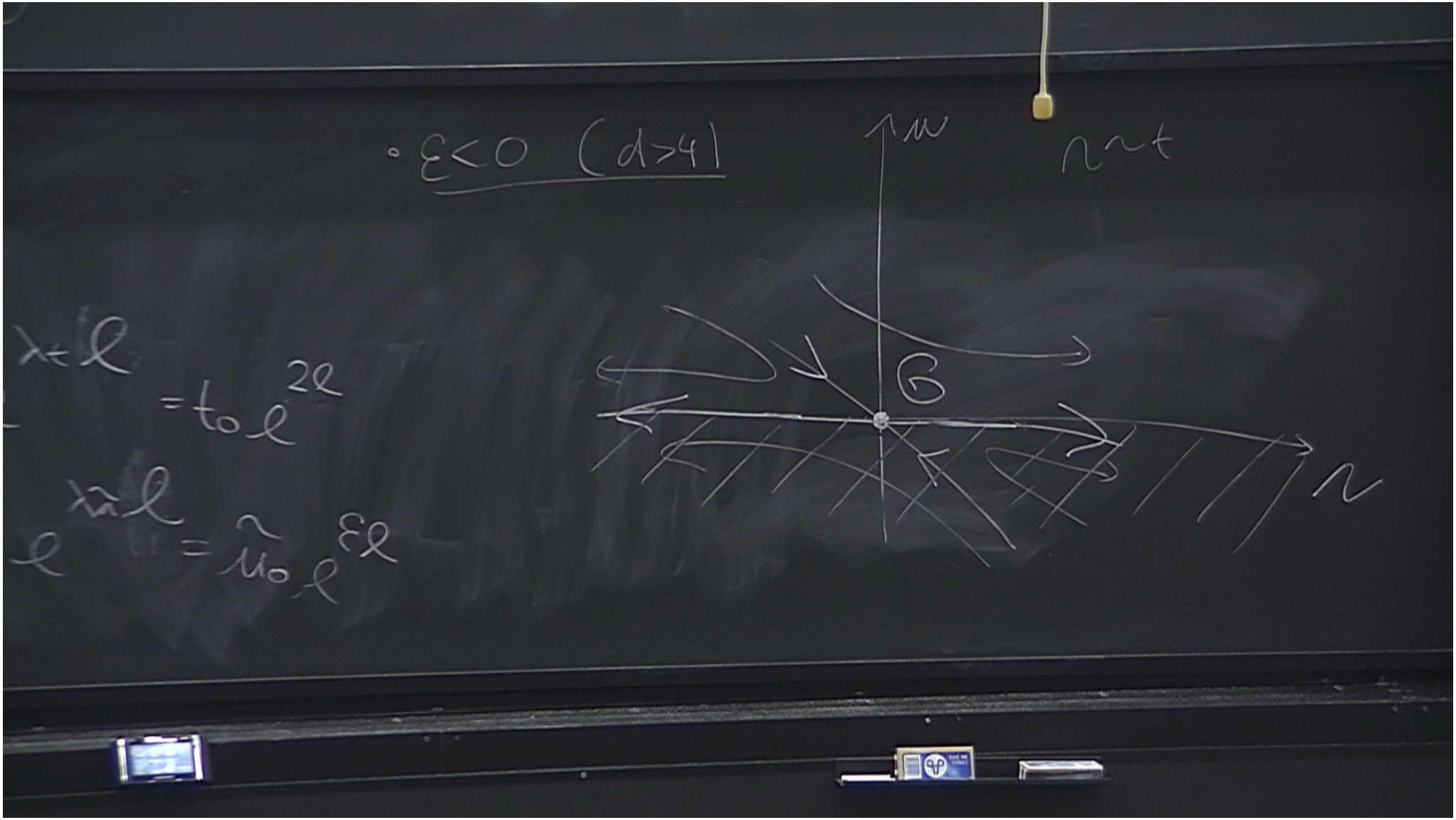

$\cdot \varepsilon < 0 \quad (d > 4)$
$= t_0 \ell^{2\ell}$
$= \tilde{\mu}_0 \ell^{\varepsilon\ell}$
w
G
N

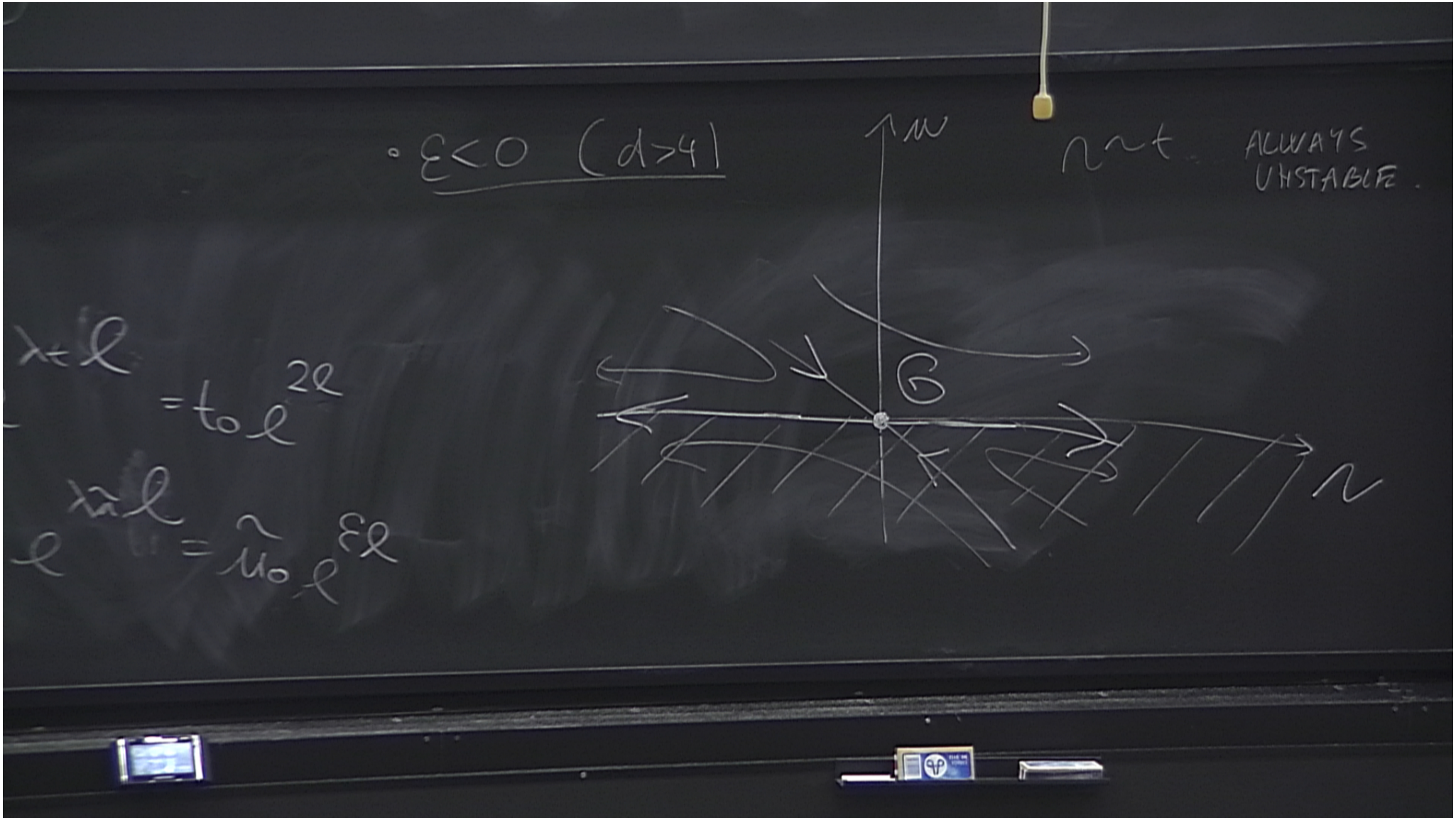

• ε<0 (d>4)
λ=ℓ
=t₀ℓ^{2ℓ}
ℓ^{λ̃ℓ}
=μ̃₀ℓ^{εℓ}
↑N
G
N

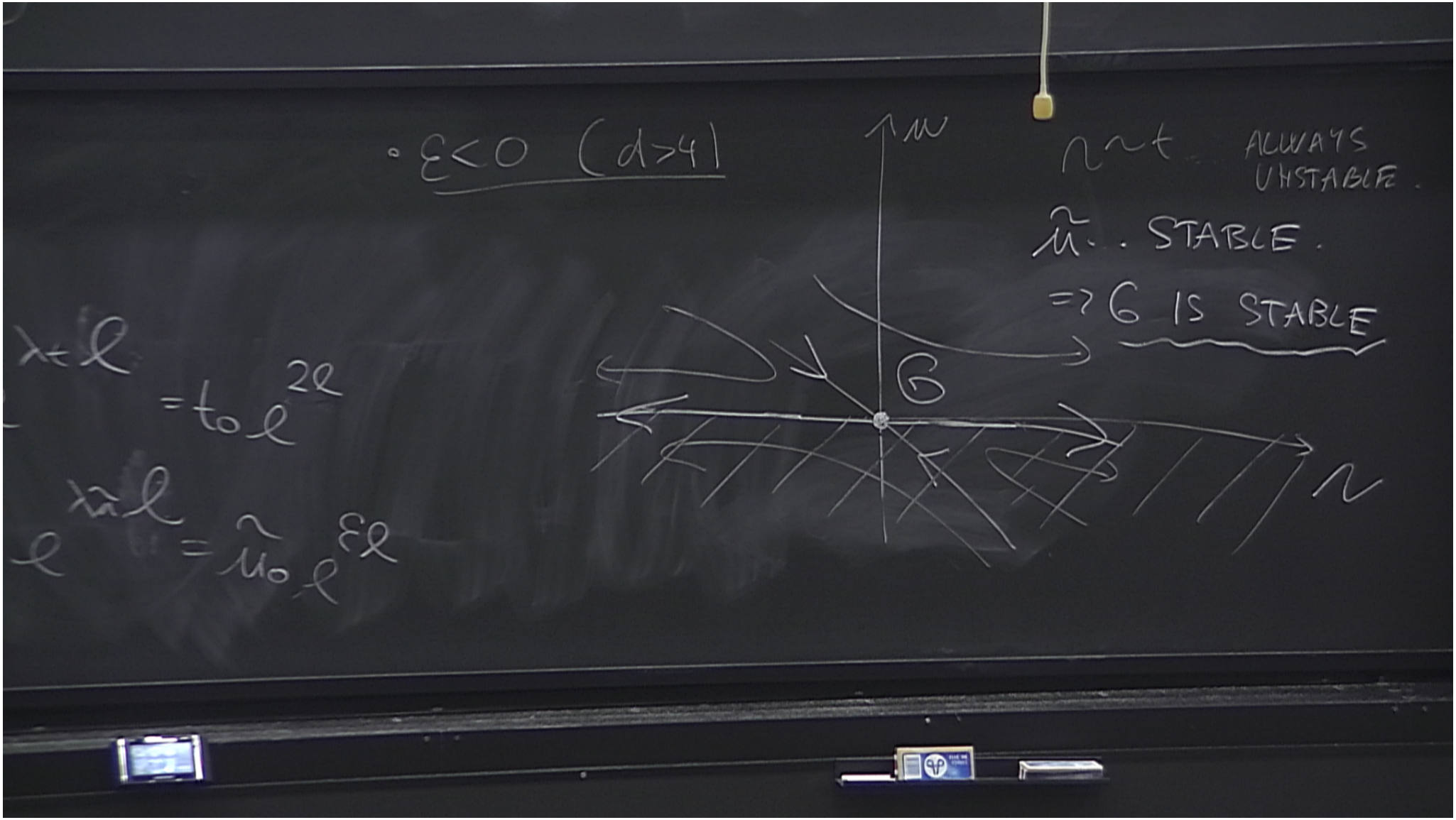

• ε<0 (d>4)
λ=ℓ
=t₀ℓ^{2ℓ}
ℓ^{λ=ℓ}
=μ̂₀ℓ^{εℓ}
M
G
N

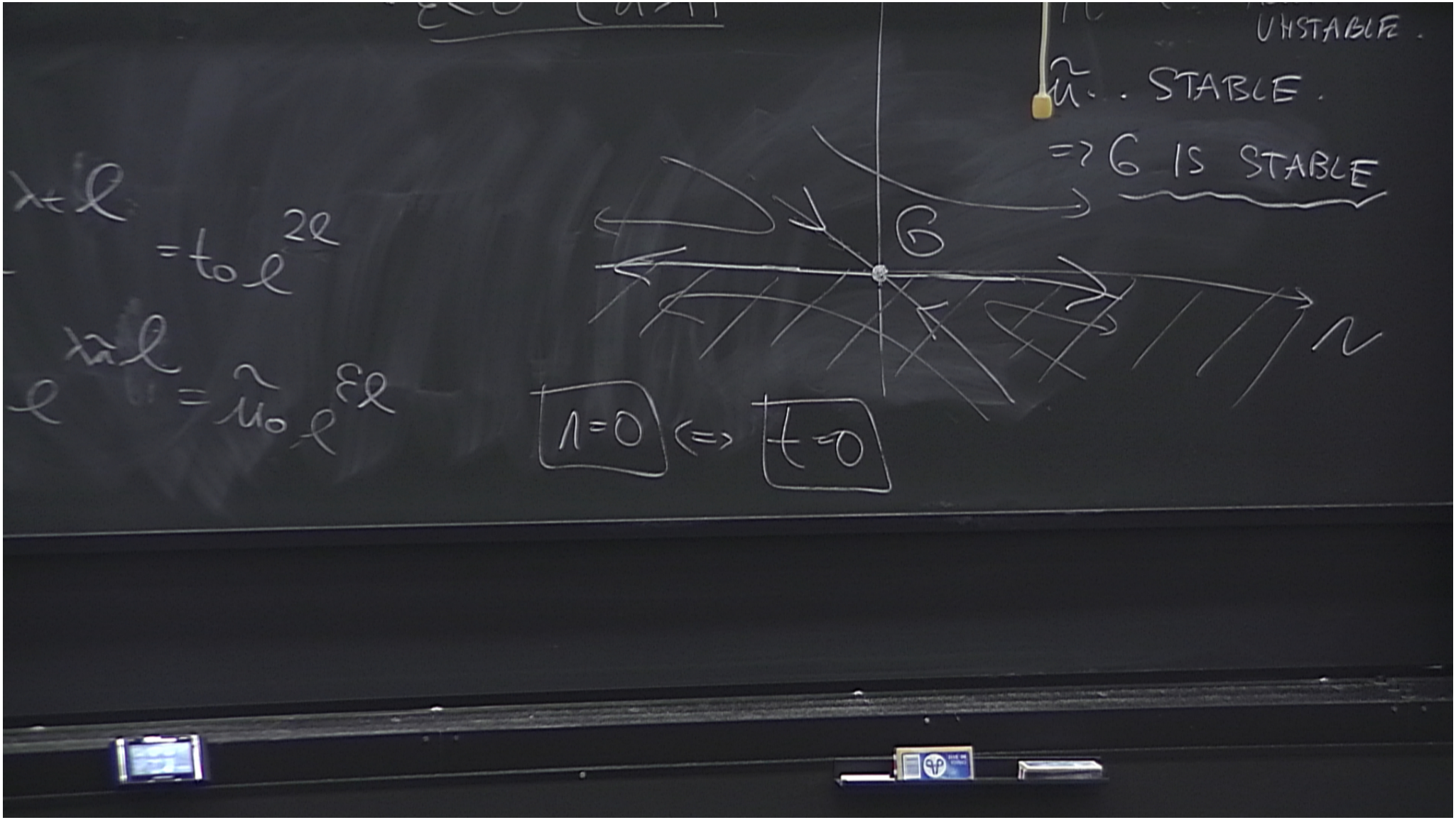

$\varepsilon < 0 \quad (d > 4)$
$\chi = \ell$
$= t_0 \ell^{2\ell}$
$\tilde{\chi} = \ell$
$= \tilde{\mu}_0 \ell^{\varepsilon \ell}$
μ
G
N

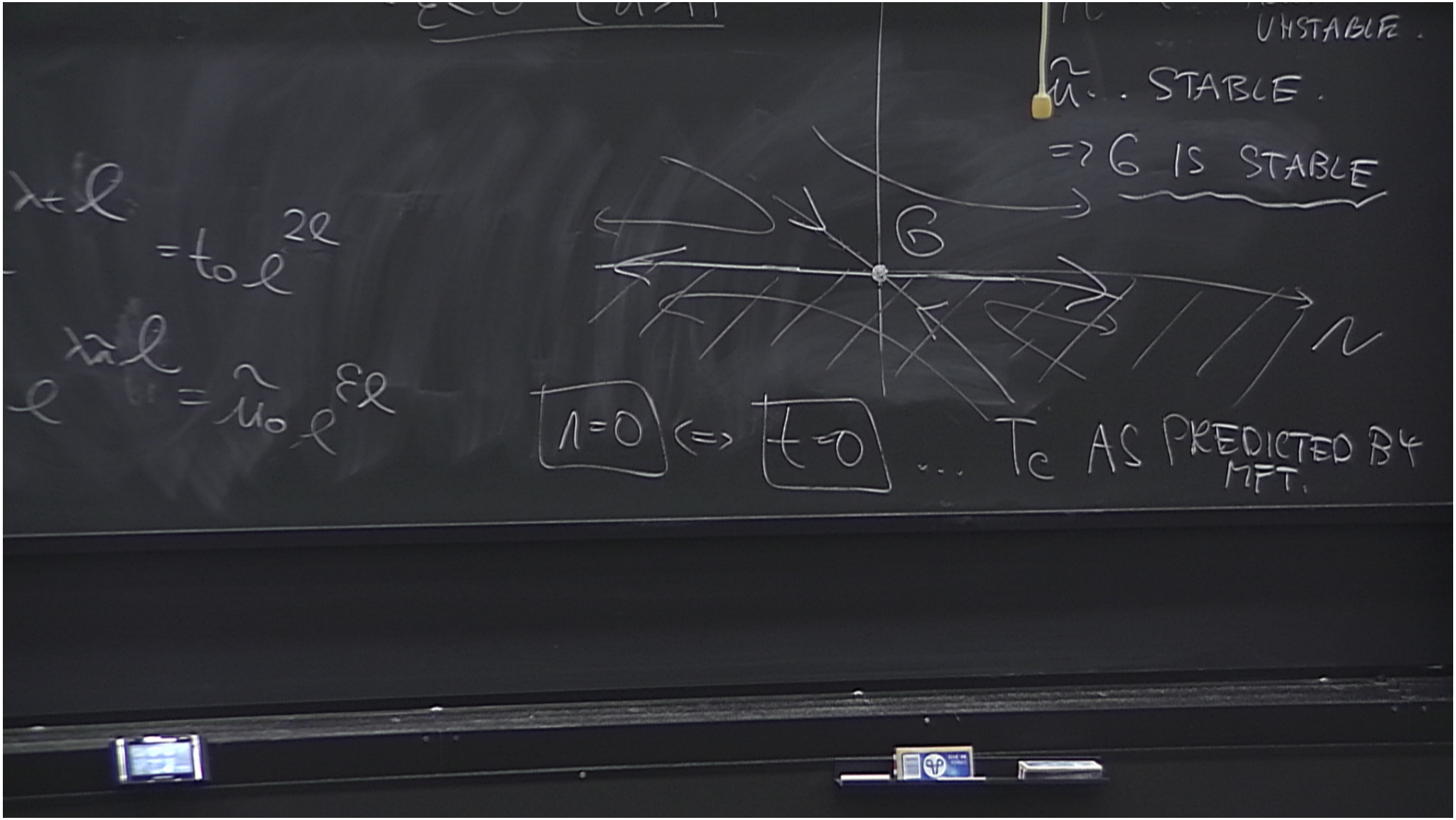

$\varepsilon < 0 \quad (d > 4)$
$\chi = \ell$
$= t_0 \ell^{2\ell}$
$\ell^{\chi = \ell} = \hat{\mu}_0 \ell^{\varepsilon \ell}$
μ
G
N


$\varepsilon < 0 \quad (d > 4)$
$\uparrow N$
$r \sim t$
G
N
$\chi = \ell$
$= t_0 \ell^{2\ell}$
$\ell^{\chi = \ell} = \hat{\mu}_0 \ell^{\varepsilon \ell}$

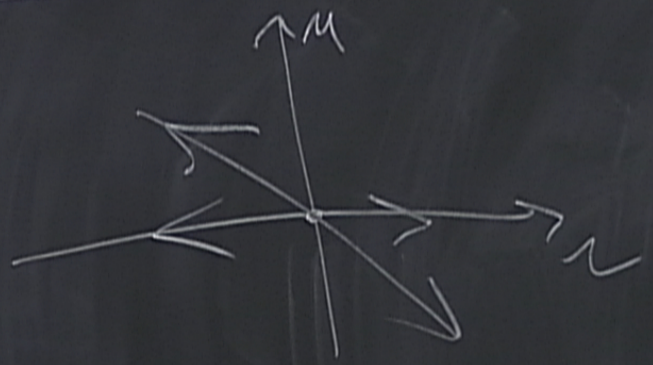

• ε<0 (d>4)
N
n~t
ALWAYS UNSTABLE.
G
N


• ε<0 (d>4)
n~t ALWAYS UNSTABLE.
û.. STABLE.
⇒ G IS STABLE
G
N

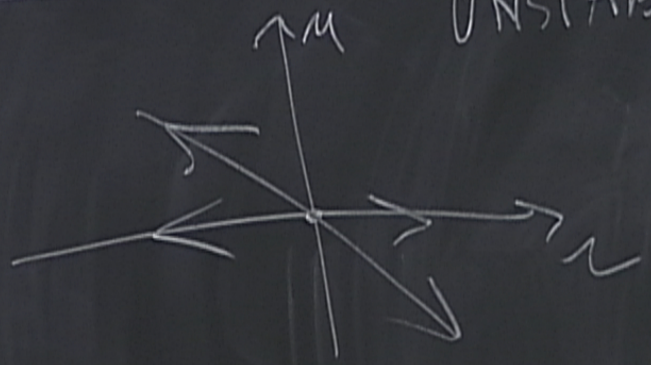

UNSTABLE.
STABLE.
⇒ G IS STABLE
G
N
λ=0 ⇔ t=0


UNSTABLE.
$\tilde{u}$ .. STABLE.
$\Rightarrow$ G IS STABLE
G
N
$\lambda = l$
$= t_0 l^{2\beta}$
$\lambda = l$
$= \tilde{u}_0 l^{\delta\beta}$
$\lambda = 0 \Leftrightarrow t = 0$ .. T_c AS PREDICTED BY MFT.

$\varepsilon > 0 \quad (d < 4)$

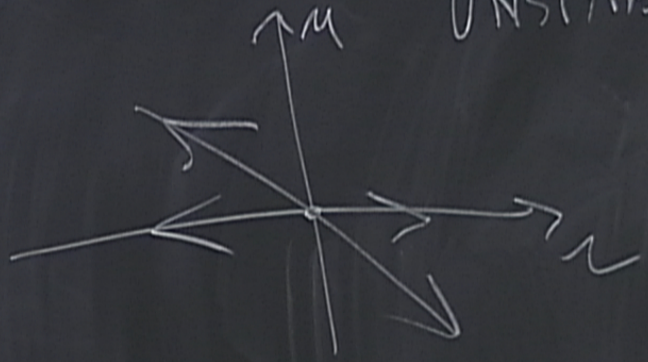


$\varepsilon > 0 \quad (d < 4)$



UNSTABLE .. CANNOT BE REACHED.

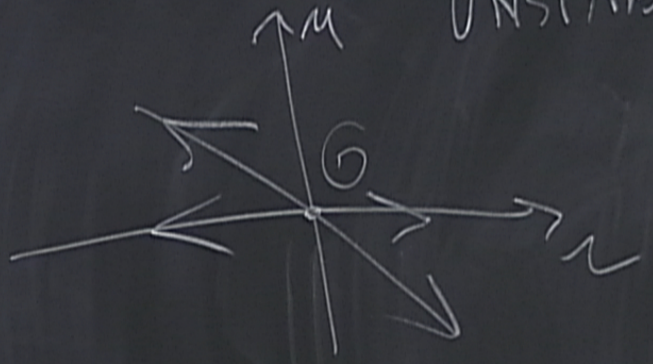
$\varepsilon > 0$ $(d < 4)$



UNSTABLE .. CANNOT BE REACHED,

... MFT ... FAILS BELOW $d = 4$.

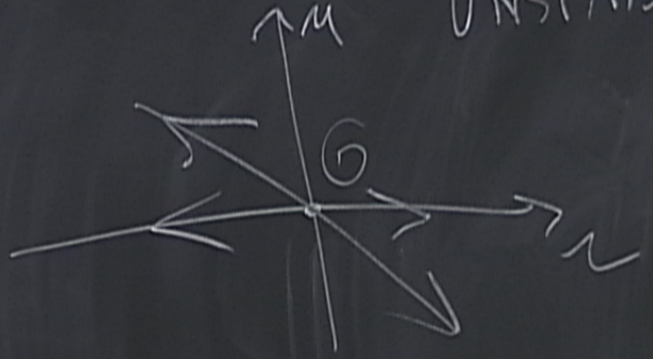
$\varepsilon > 0$ $(d < 4)$



UNSTABLE .. CANNOT BE REACHED,

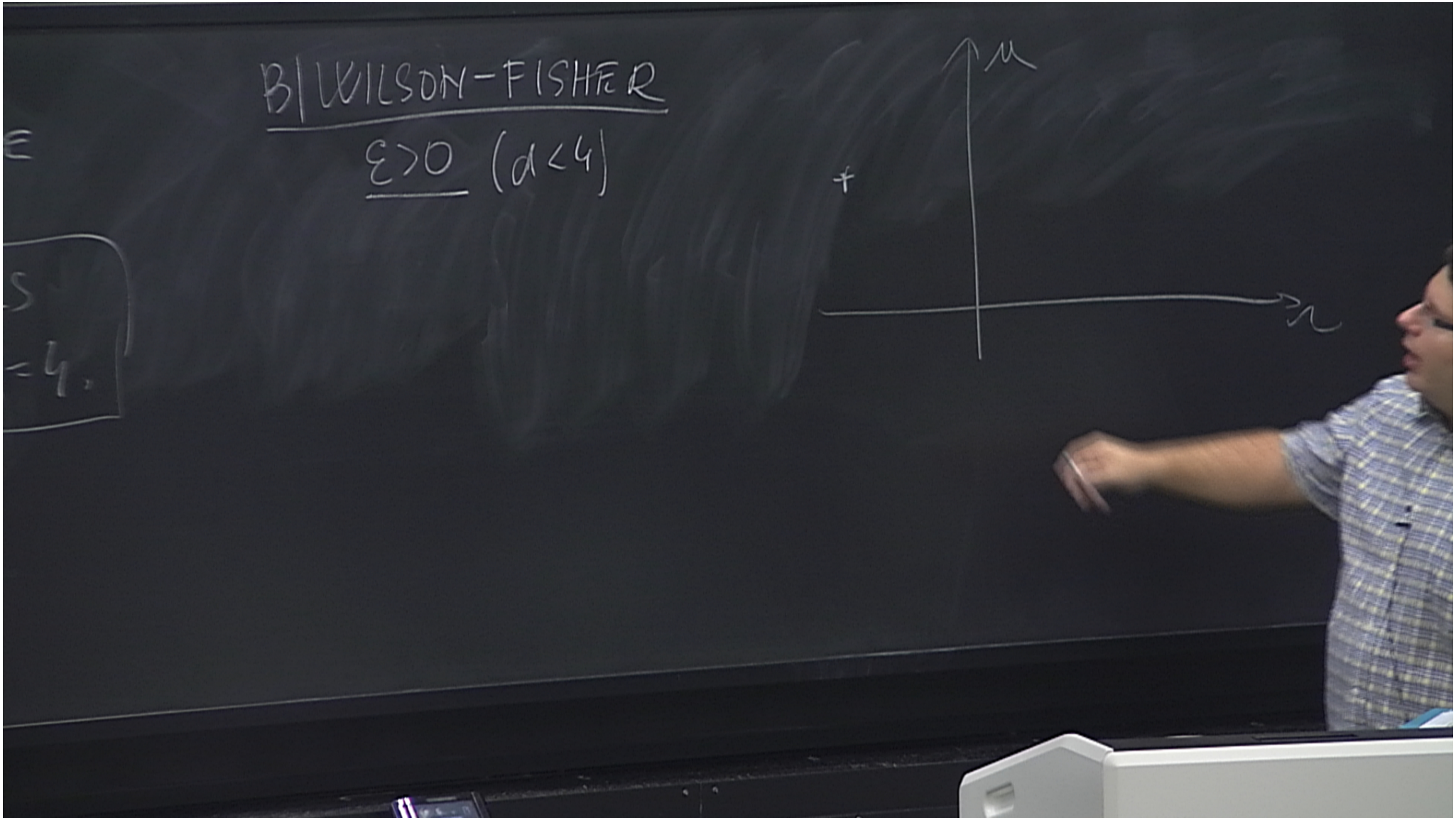
... MFT ... FAILS BELOW $d = 4$.

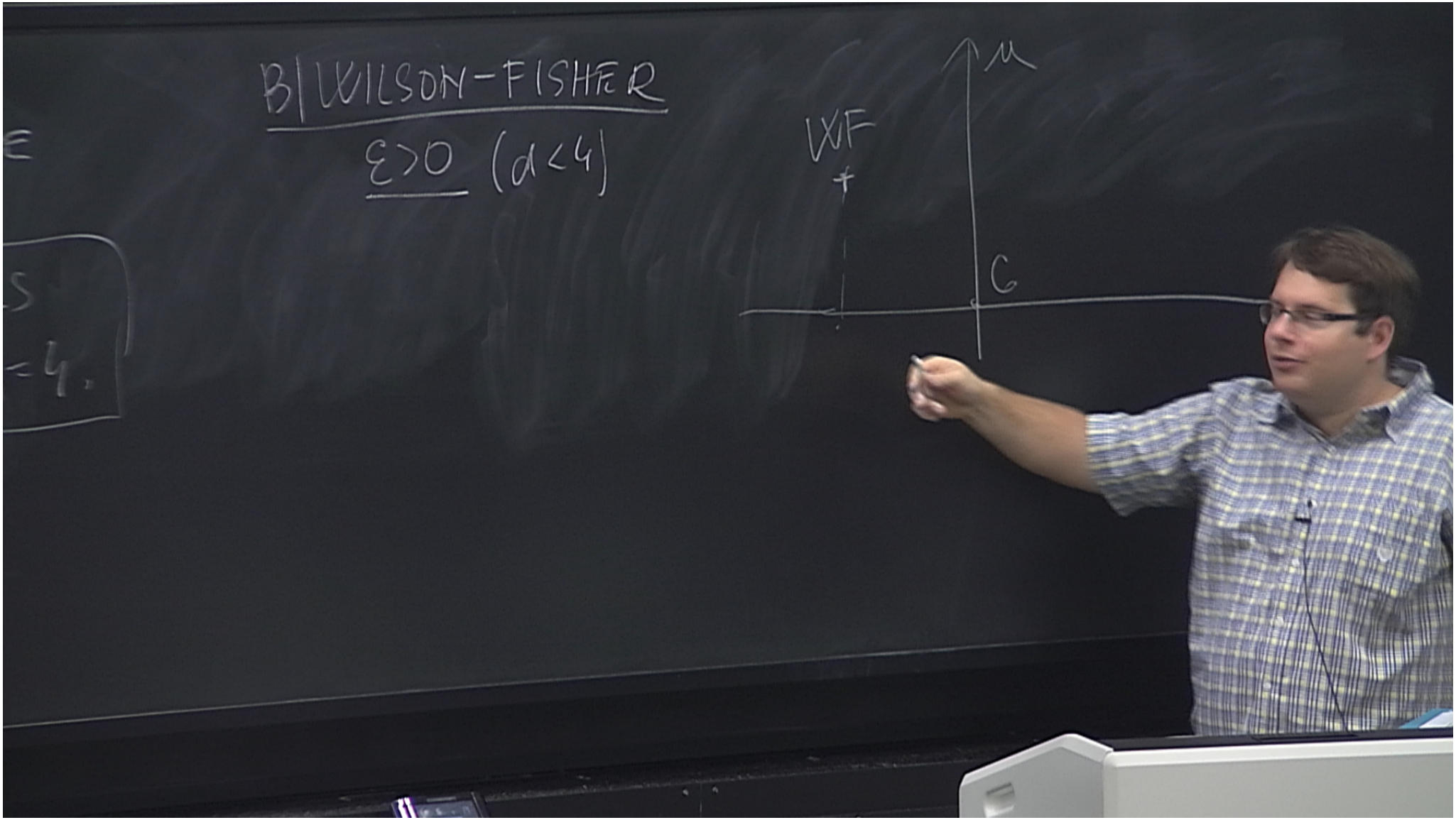
$\varepsilon > 0$ $(d < 4)$

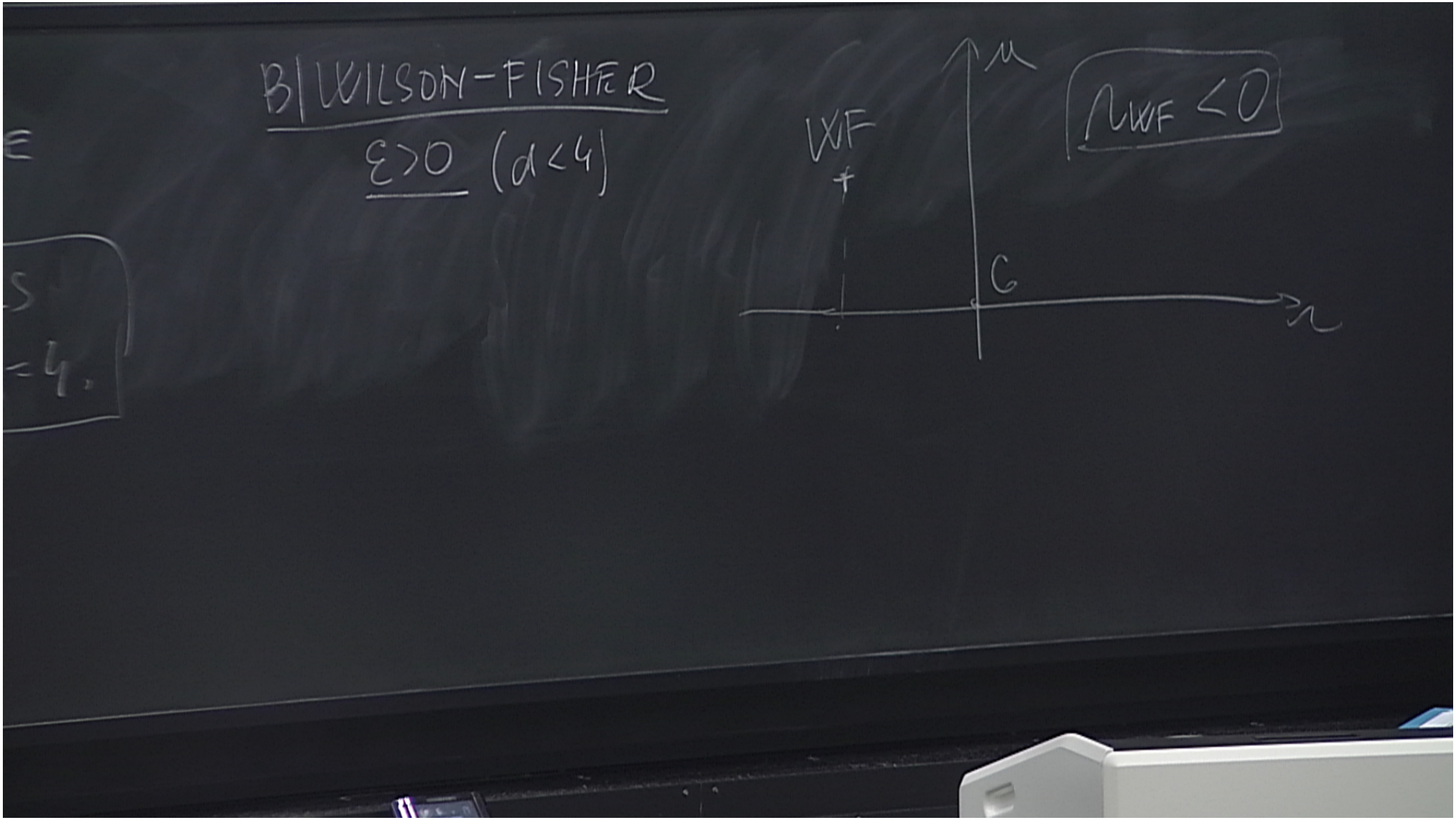


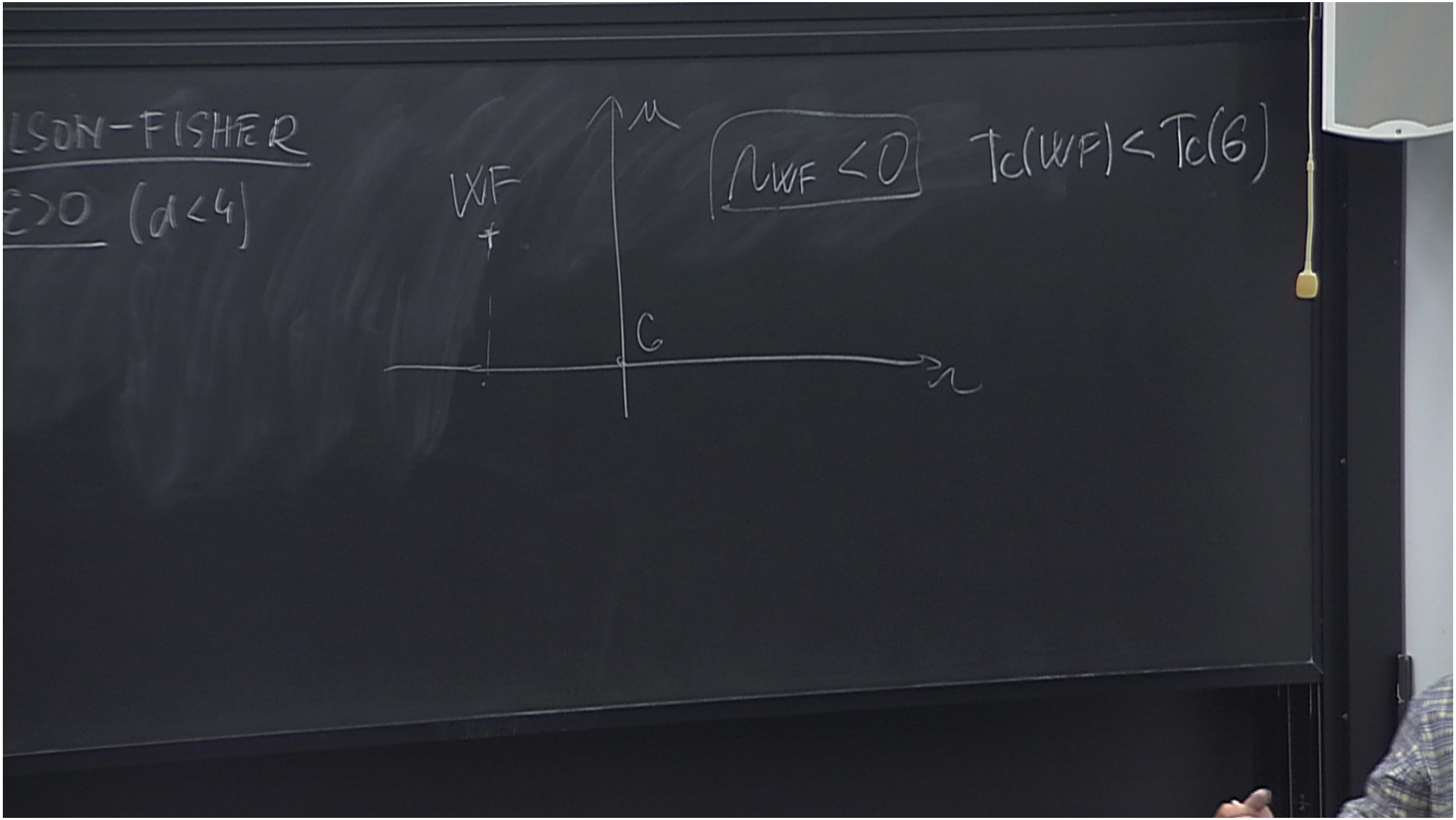
UNSTABLE .. CANNOT BE REACHED.

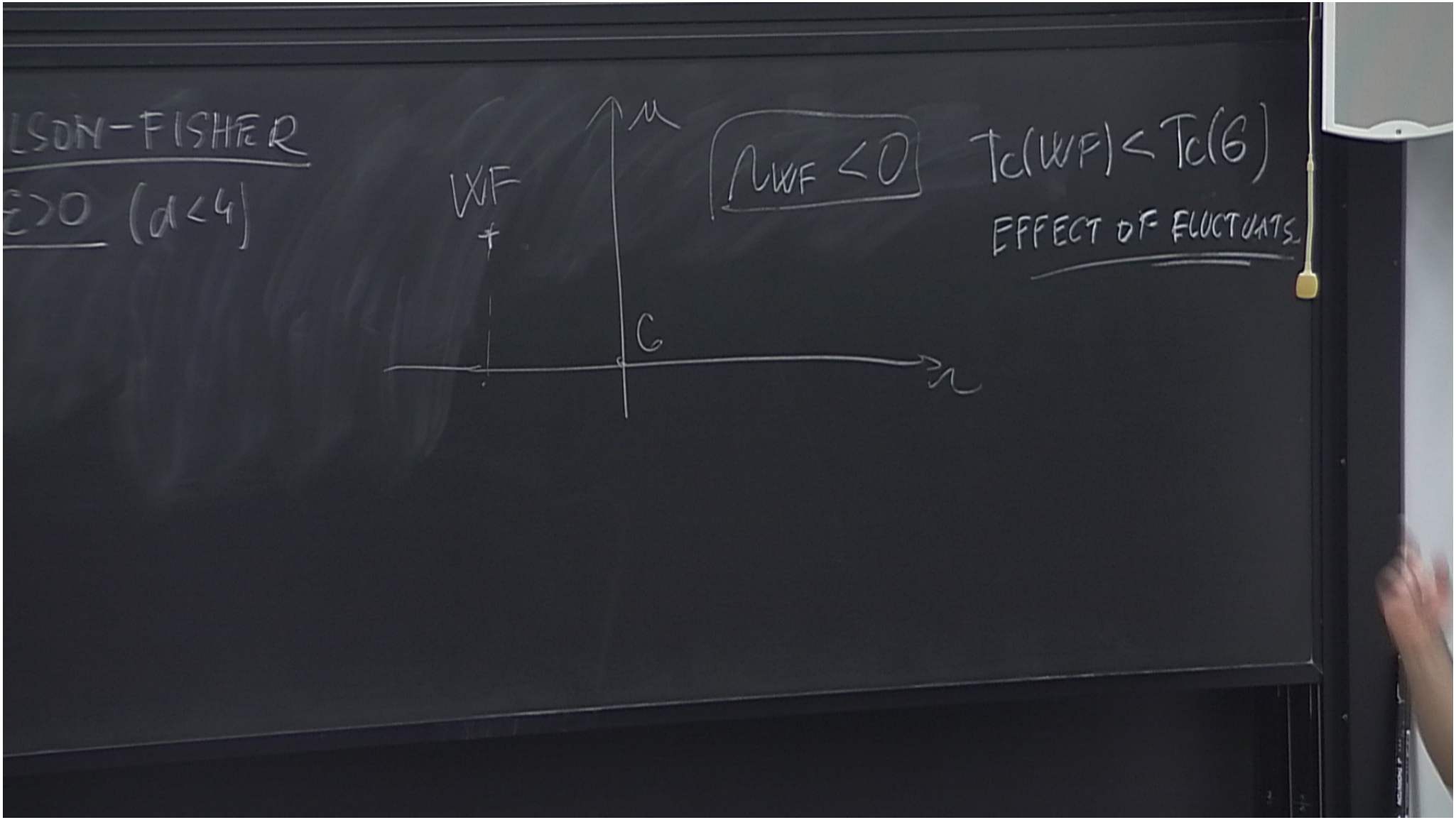
MFT ... FAILS BELOW $d = 4$.

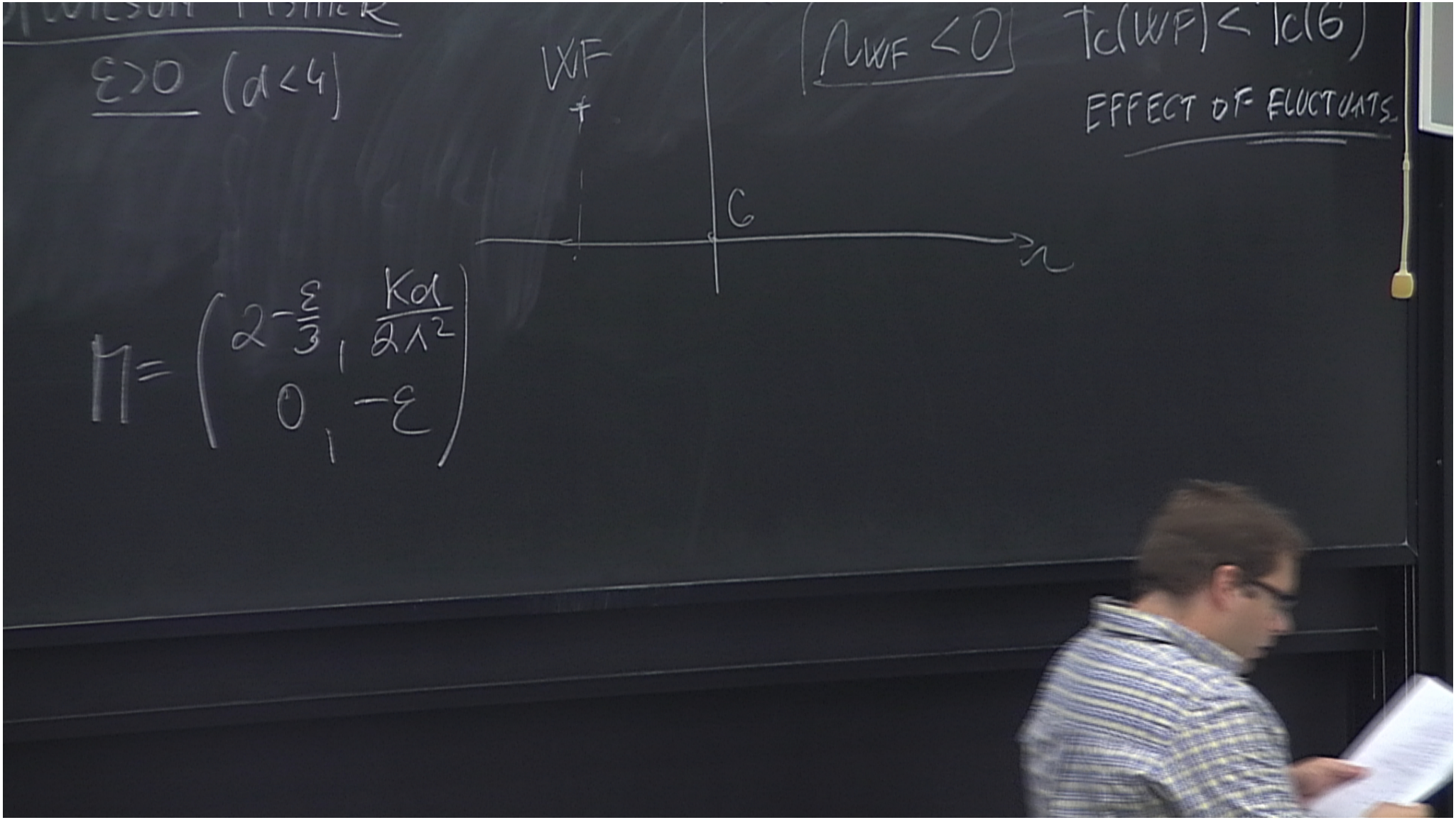

B| WILSON-FISHER
ε>0 (d<4)
+
u
r

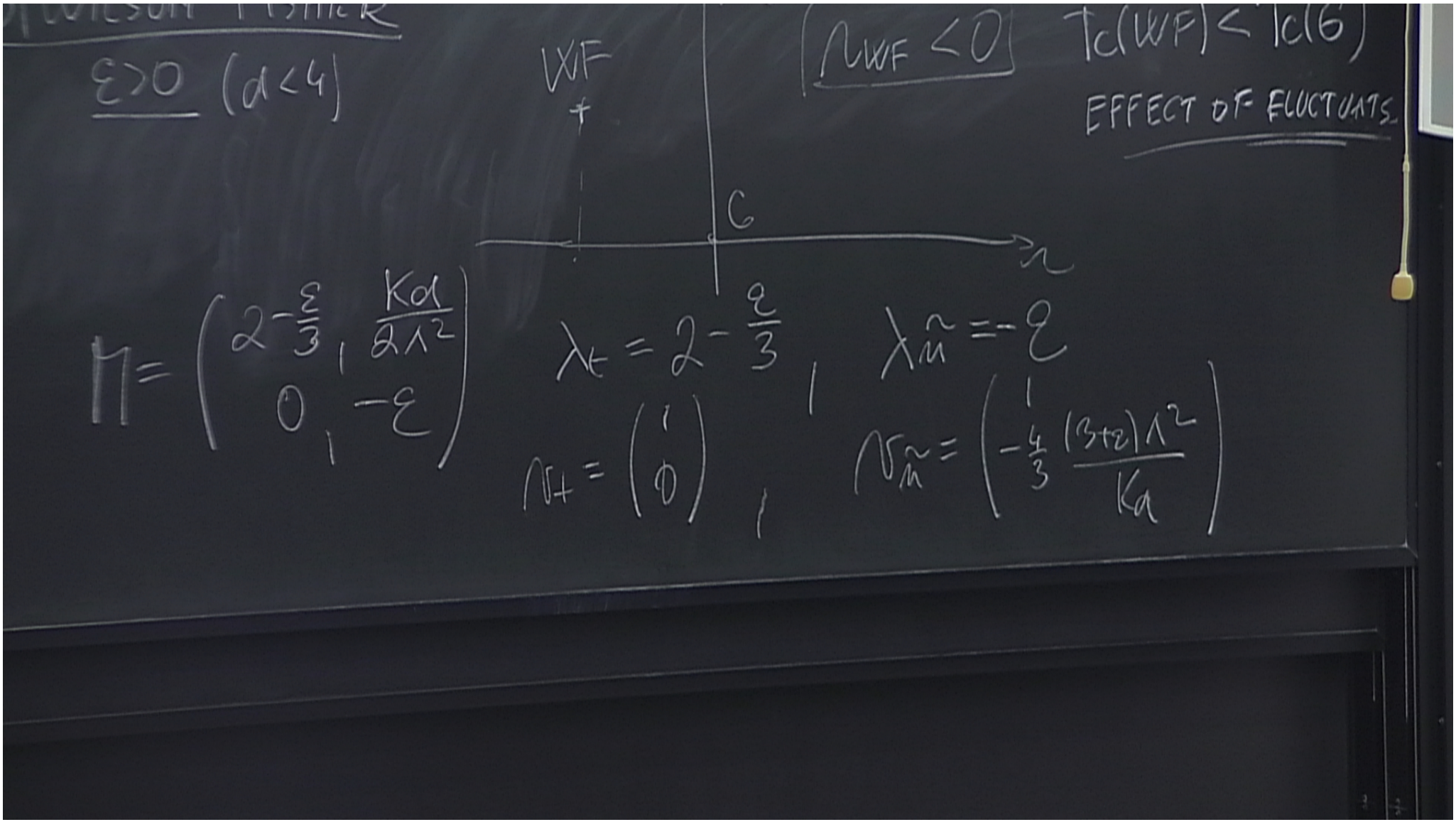

B| WILSON-FISHER
ε>0 (d<4)
WF
C

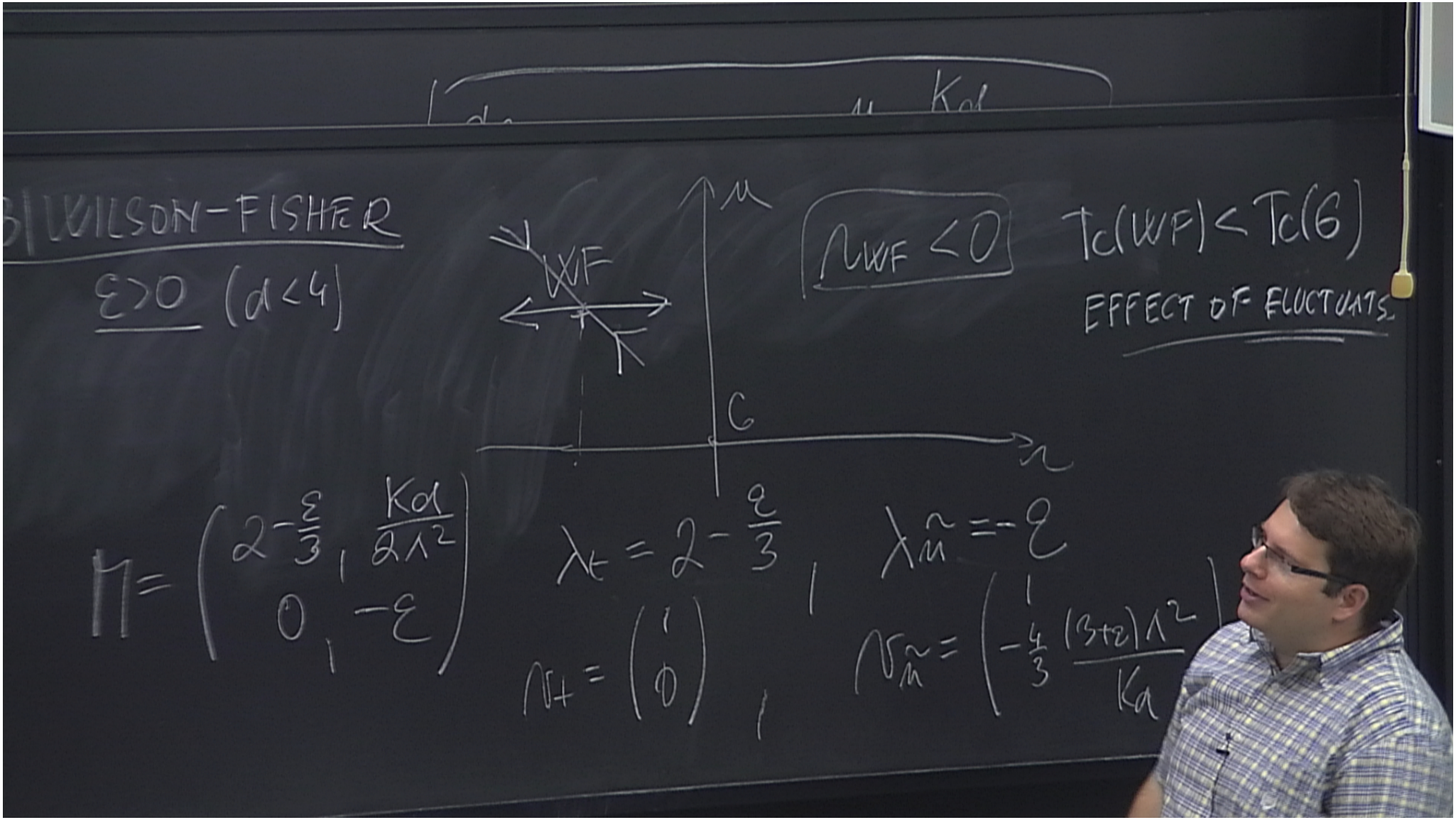

B| WILSON-FISHER
ε>0 (d<4)
WF
u
C
r
ΛWF <0

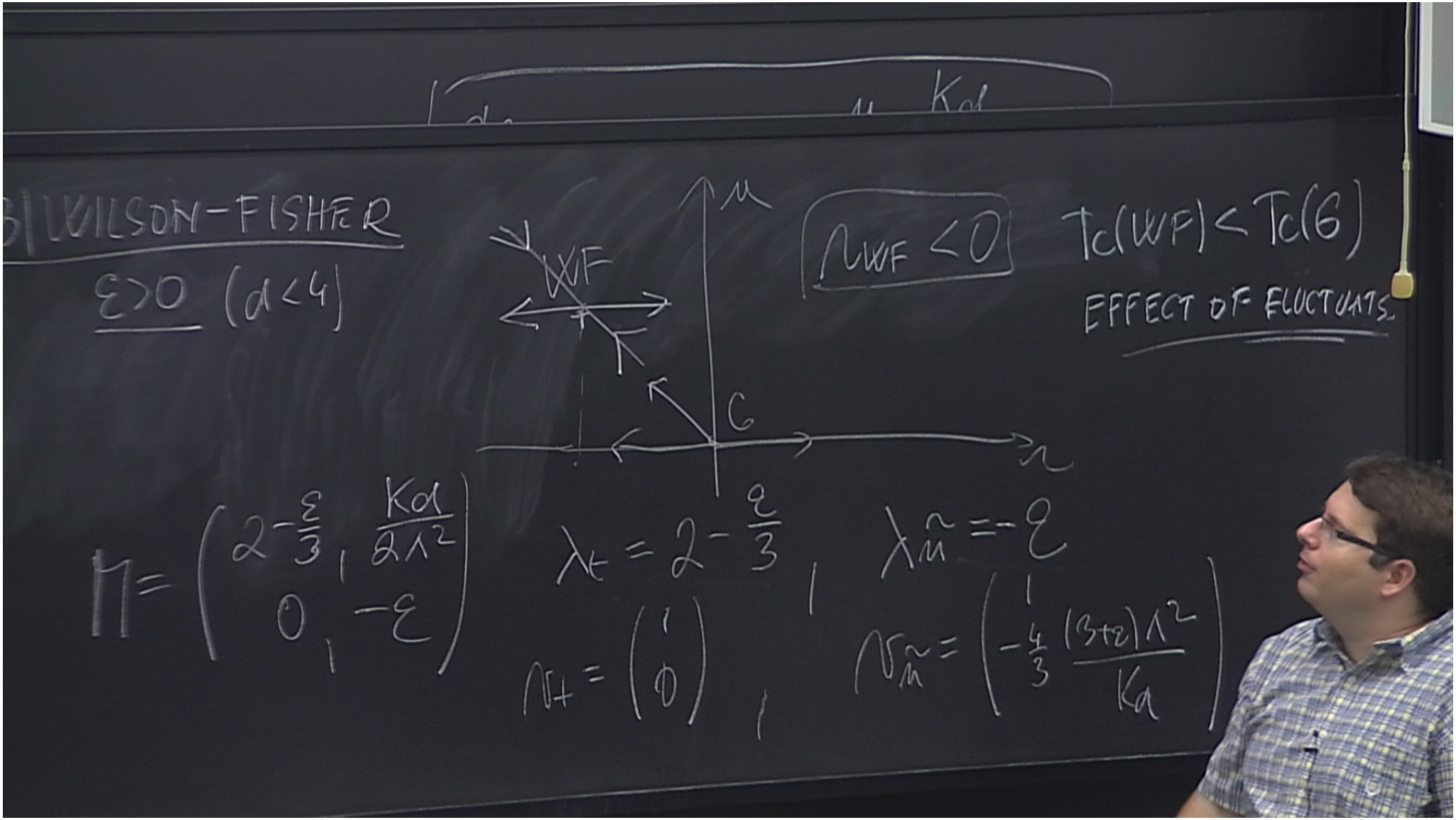

LSON-FISHER
ε>0 (d<4)
WF
u
C
r
$r_{WF} < 0$
$T_c(WF) < T_c(G)$

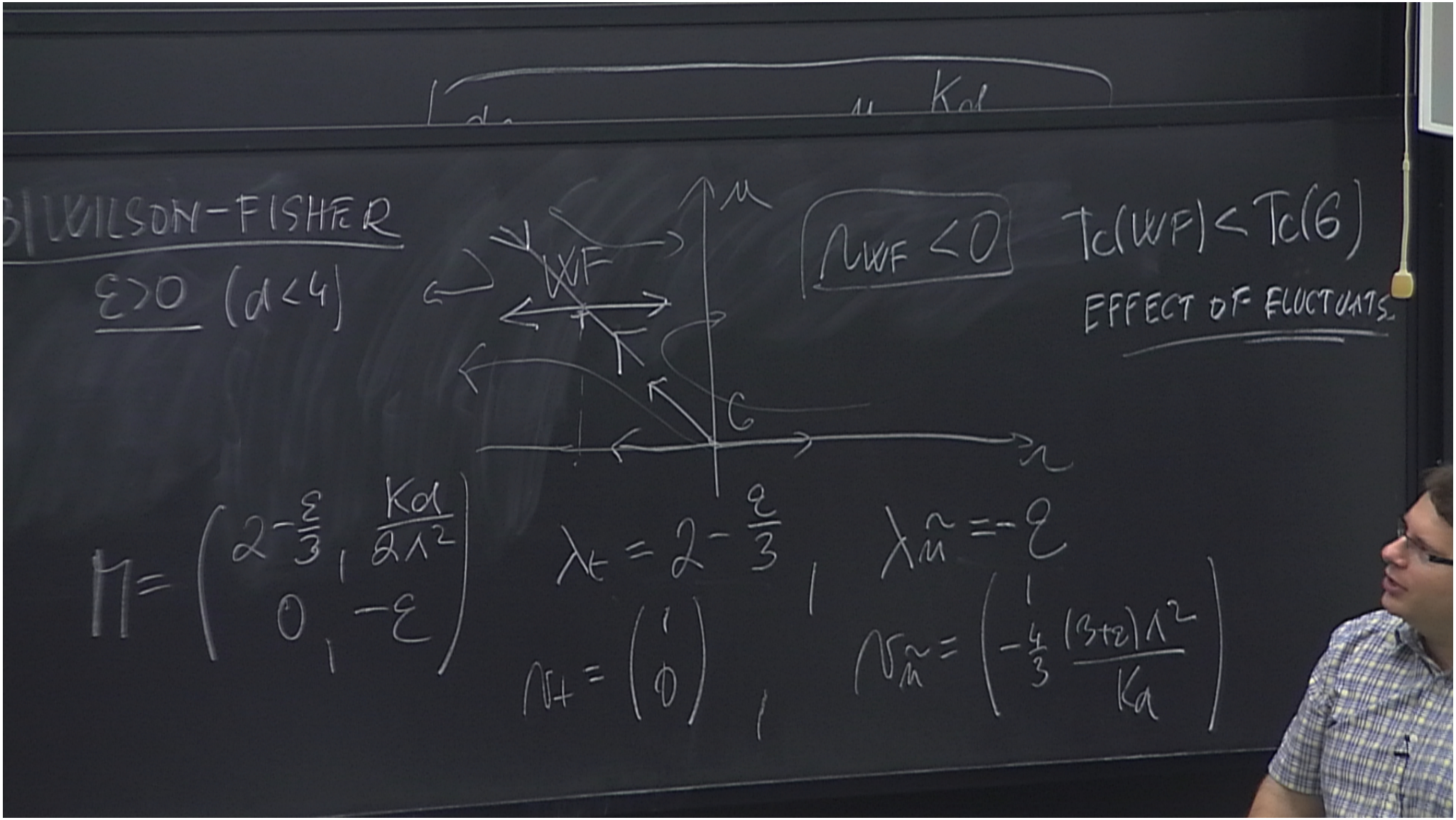

LSON-FISHER
ε>0 (d<4)
WF
u
C
r
$\Gamma_{WF} < 0$
$T_c(WF) < T_c(G)$
EFFECT OF FLUCTUATS.

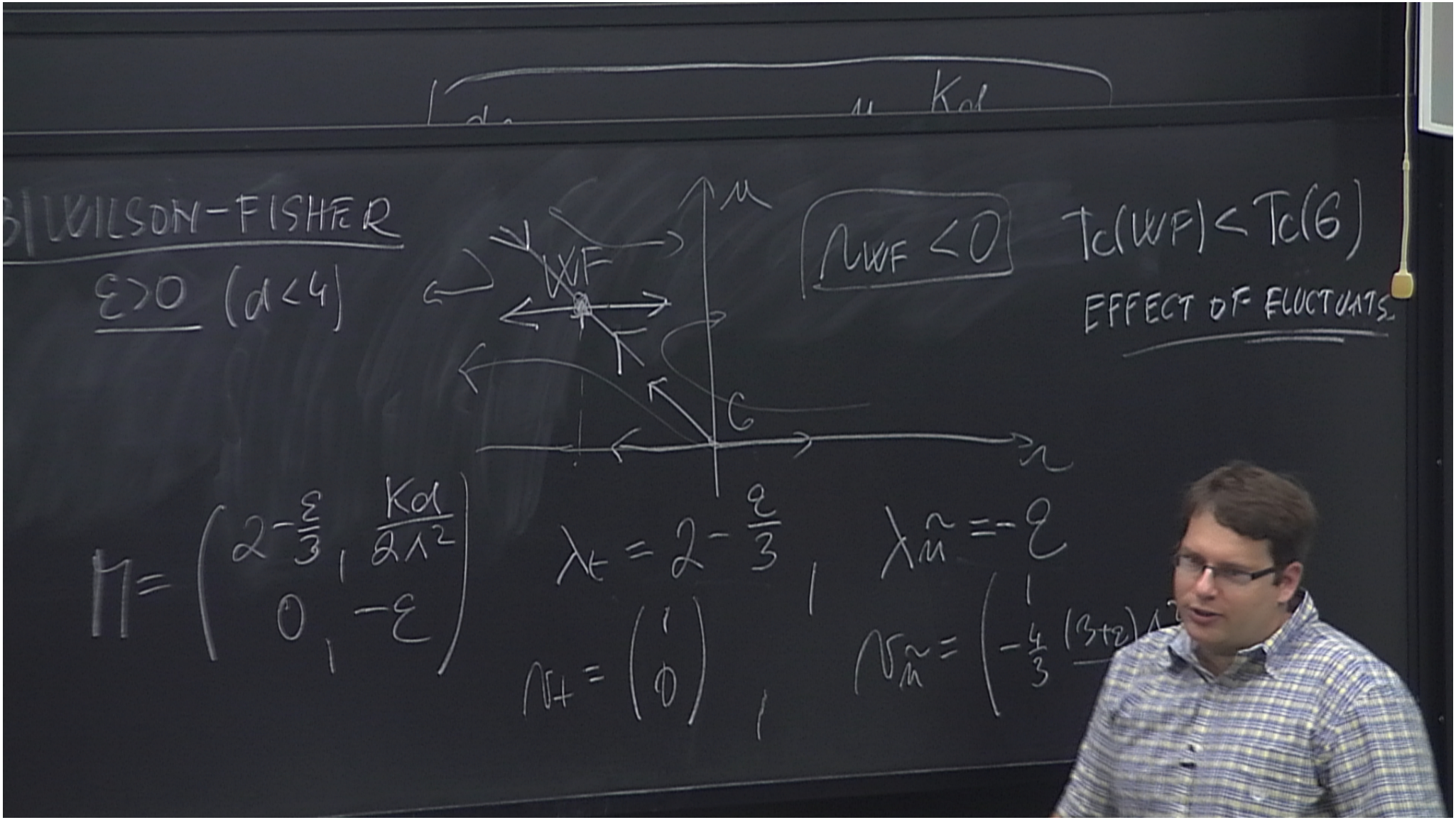

ε>0 (d<4)
WF
C
Λ_WF <0
T_c(WF) < T_c(G)
EFFECT OF FLUCTUATS.
η = (2 − ε/3, Kd/(2Λ²); 0, −ε)


WF
G
$\Lambda_{WF} < 0$
$T_c(WF) < T_c(G)$
EFFECT OF FLUCTUATS.
$\varepsilon > 0 \quad (d < 4)$
$M = \begin{pmatrix} 2 - \frac{\varepsilon}{3} & \frac{Kd}{2\Lambda^2} \\ 0 & -\varepsilon \end{pmatrix}$
$\lambda_t = 2 - \frac{\varepsilon}{3}$
$\lambda_{\tilde{m}} = -\varepsilon$
$v_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$v_{\tilde{m}} = \begin{pmatrix} 1 \\ -\frac{4}{3}\frac{(3+2)\Lambda^2}{Kd} \end{pmatrix}$


3| WILSON-FISHER
ε>0 (d<4)
WF
G
u
r
Λ_WF < 0
Tc(WF) < Tc(G)
EFFECT OF FLUCTUATS.


Kd
3| WILSON-FISHER
ε>0 (d<4)
u
WF
G
r
Λ_WF < 0
Tc(WF) < Tc(G)
EFFECT OF FLUCTUATS.
M = (2 − ε/3 , Kd/2Λ² ; 0 , −ε)
λ_t = 2 − ε/3 , λ_ũ = −ε
V_t = (1 ; 0) , V_ũ = (1 ; −4/3 (3+2)Λ²/Kd)


Kd
3| WILSON-FISHER
ε>0 (d<4)
WF
G
u
r
Λ_WF < 0
Tc(WF) < Tc(G)
EFFECT OF FLUCTUATS.
Π = (2 − ε/3, Kd/2Λ²; 0, −ε)
λ_t = 2 − ε/3 , λ_û = −ε
v_t = (1; 0) , v_û = (1; −4/3 (3+2)Λ²/Kd)

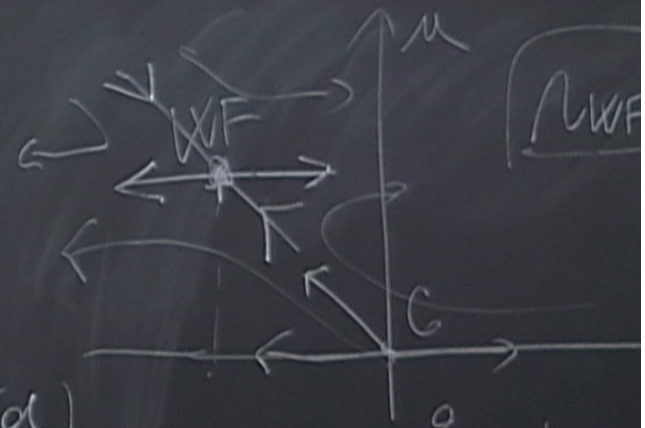

3| WILSON-FISHER
ε>0 (d<4)
WF
G
u
r
Λ_WF < 0
Tc(WF) < Tc(G)
EFFECT OF FLUCTUATS.

NSTABLE .. CANNOT BE REACHED.

MFT .. FAILS BELOW $d=4$.

B| WILSON-FISHER

$\varepsilon > 0$ $(d<4)$



Λ_{WF}

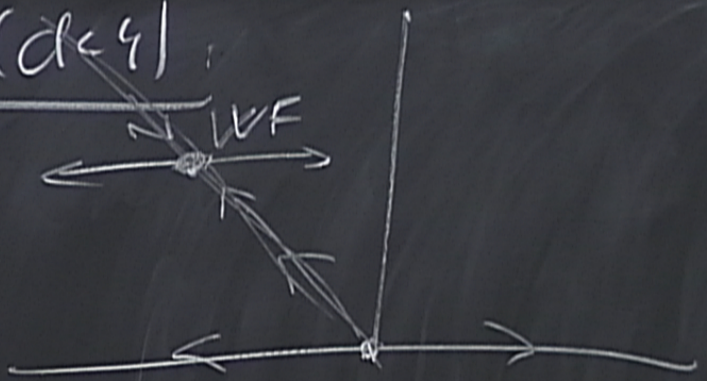
$$M = \begin{pmatrix} 2-\frac{\varepsilon}{3} & \frac{K_d}{2\Lambda^2} \\ 0 & -\varepsilon \end{pmatrix}$$

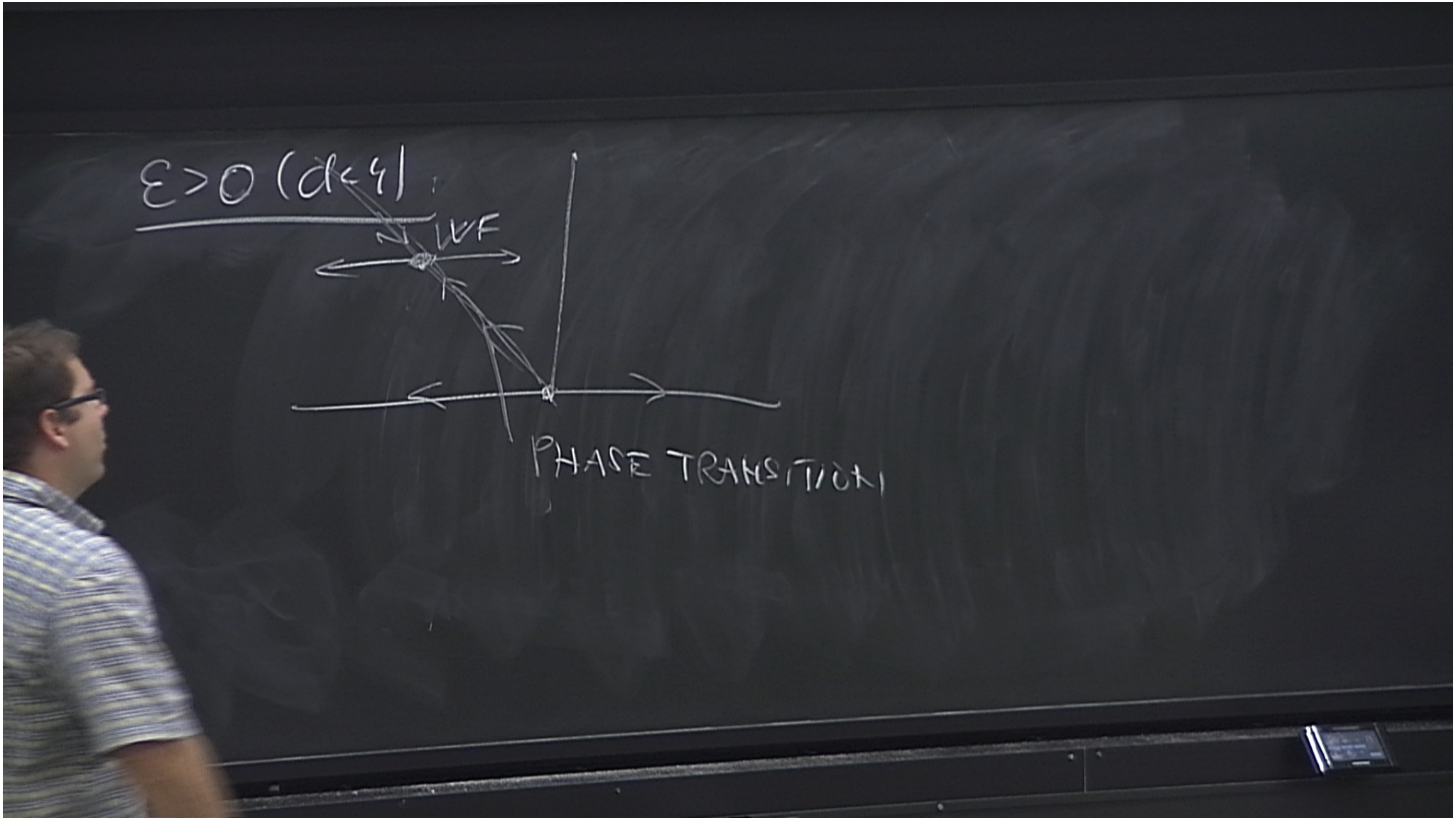
$\lambda_t = 2 - \frac{\varepsilon}{3}$, λ

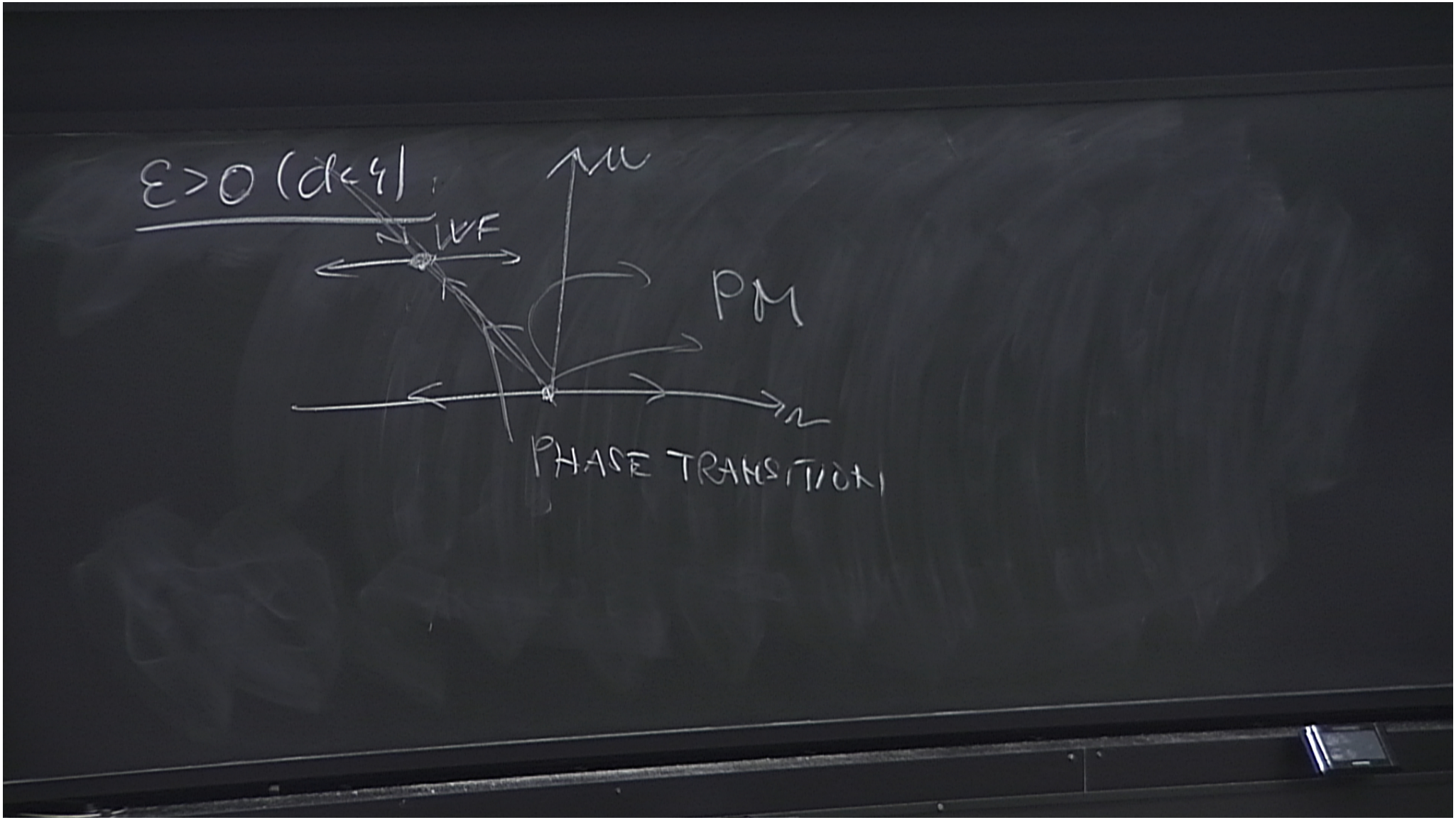
$\Lambda_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, Λ

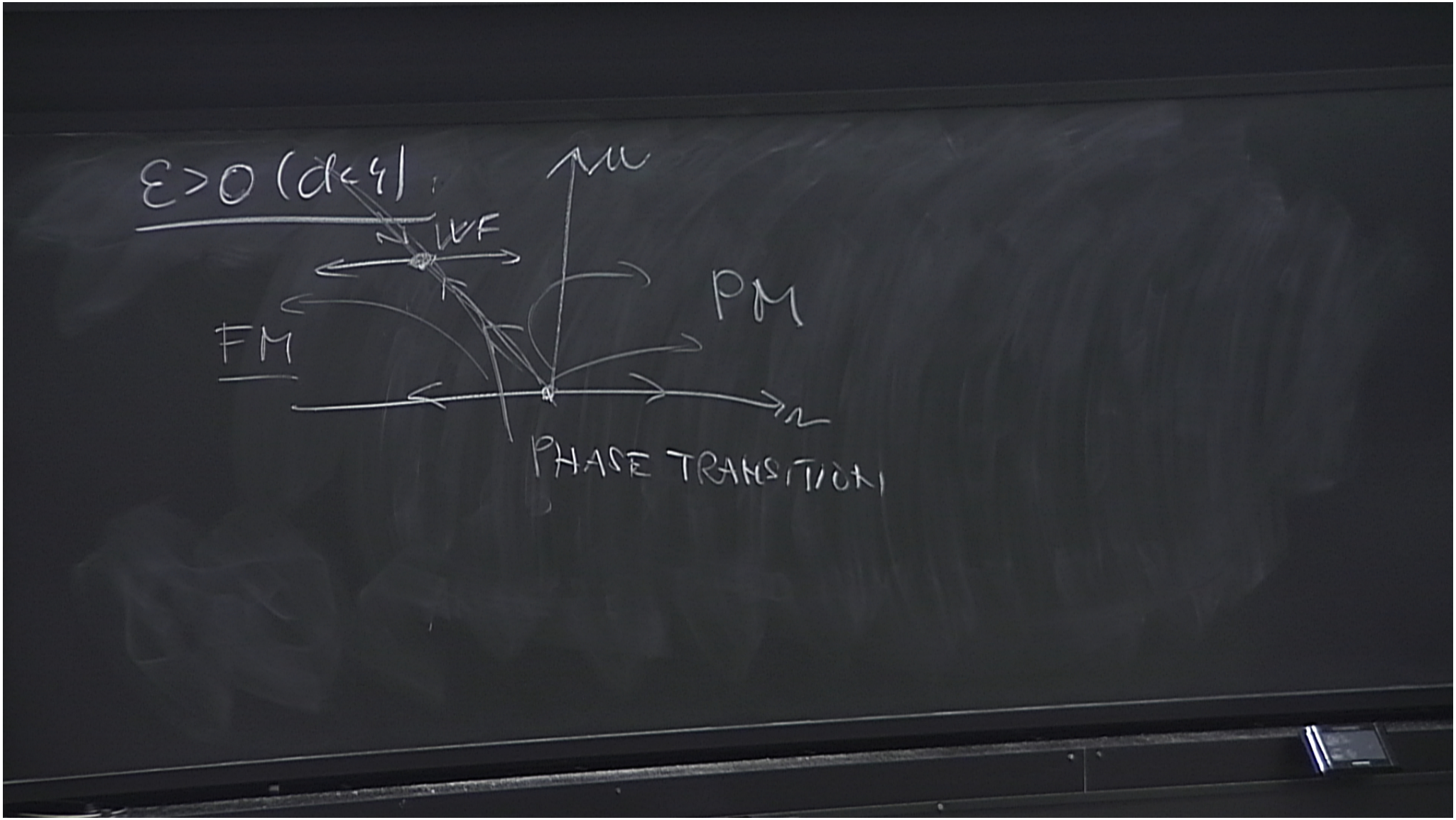
$\varepsilon < 0$ .. WF UNPHYSICAL ($u^* < 0$)

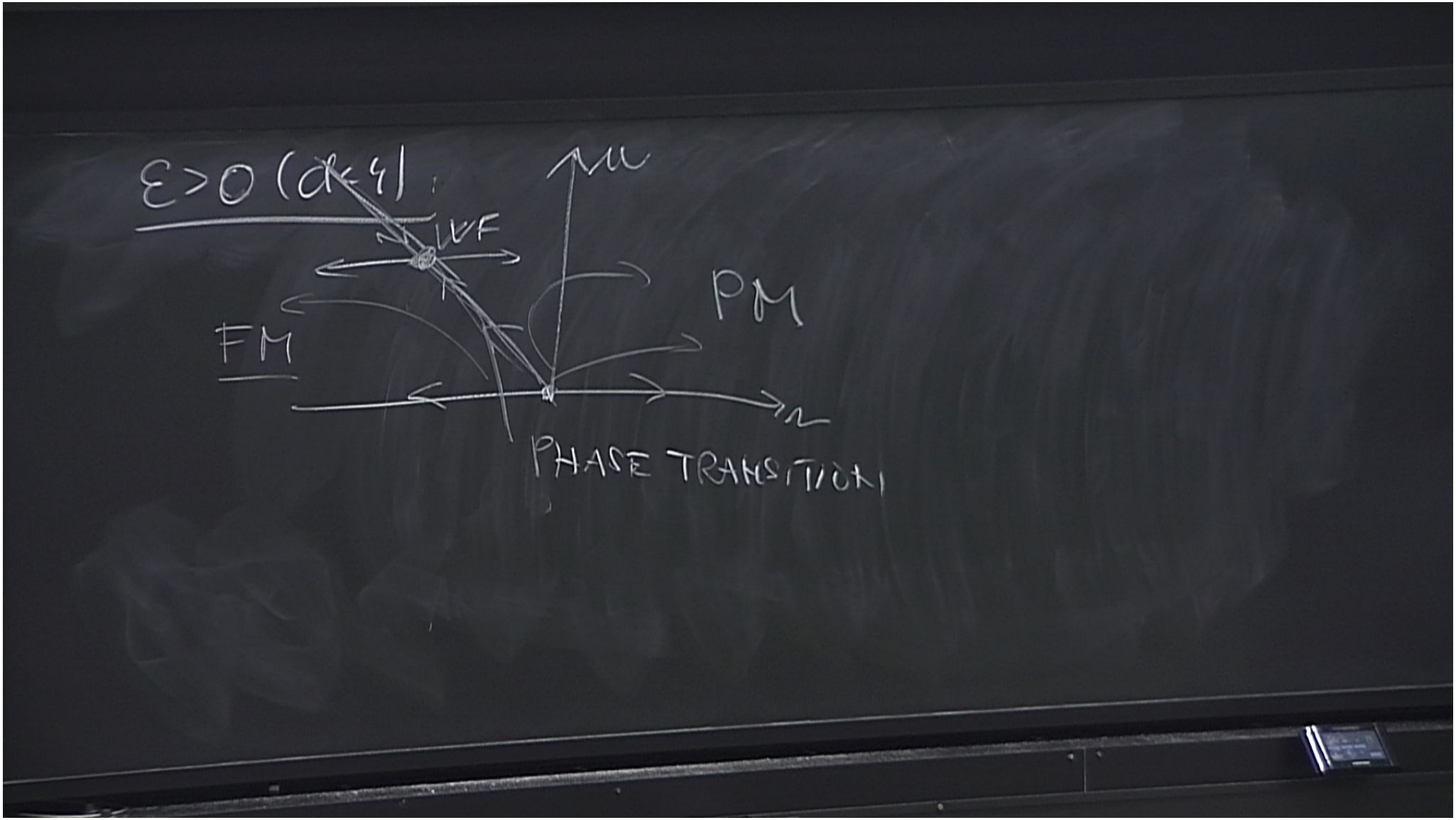
$\varepsilon > 0$ $(d < 4)$:

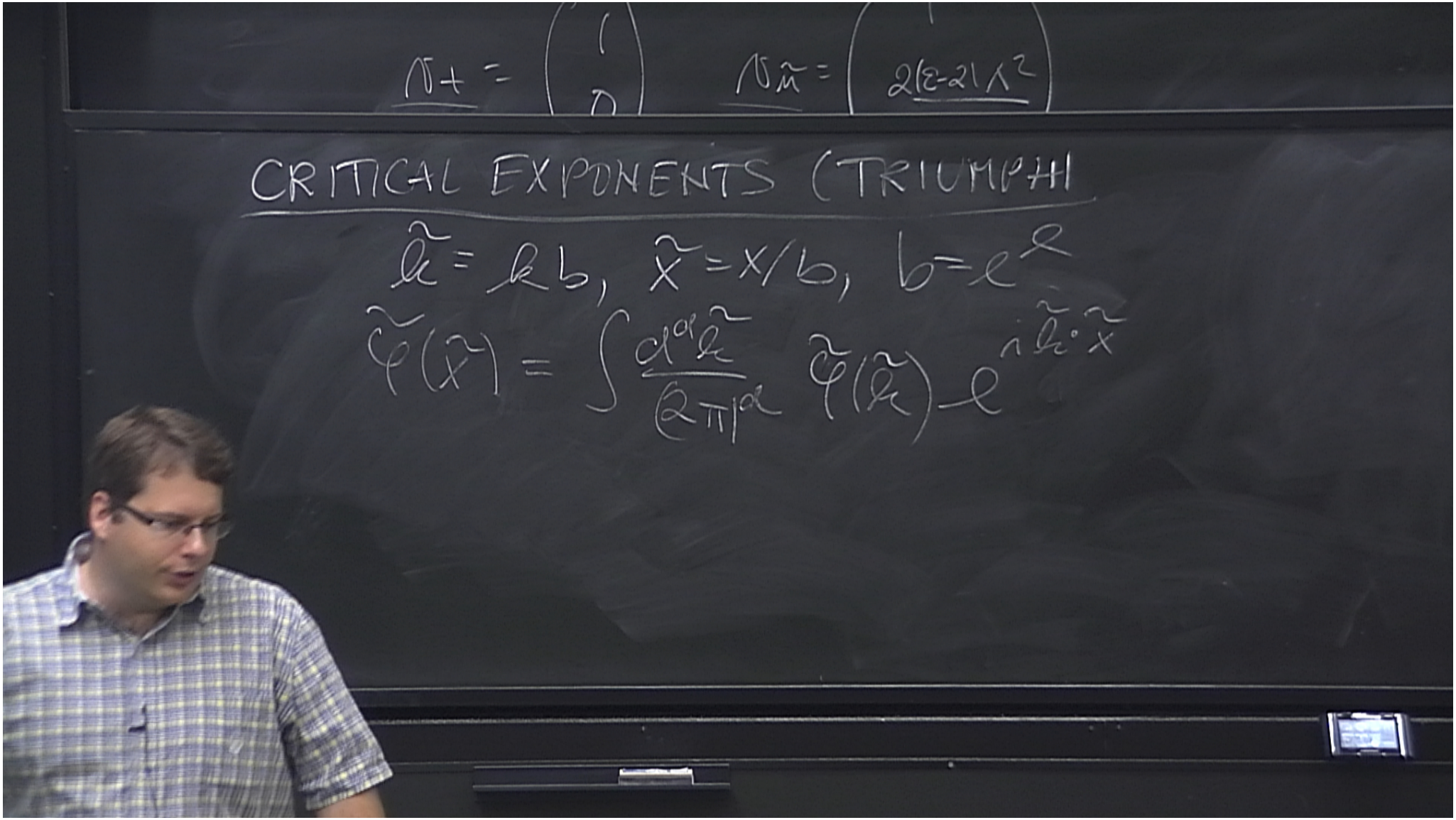



ε>0 (d<4)
IVF
PHASE TRANSITION


ε>0 (d<4)
IVF
PM
PHASE TRANSITION


ε>0 (d<4)
LVF
PM
FM
PHASE TRANSITION


ε>0 (d=4)
LVF
PM
FM
PHASE TRANSITION

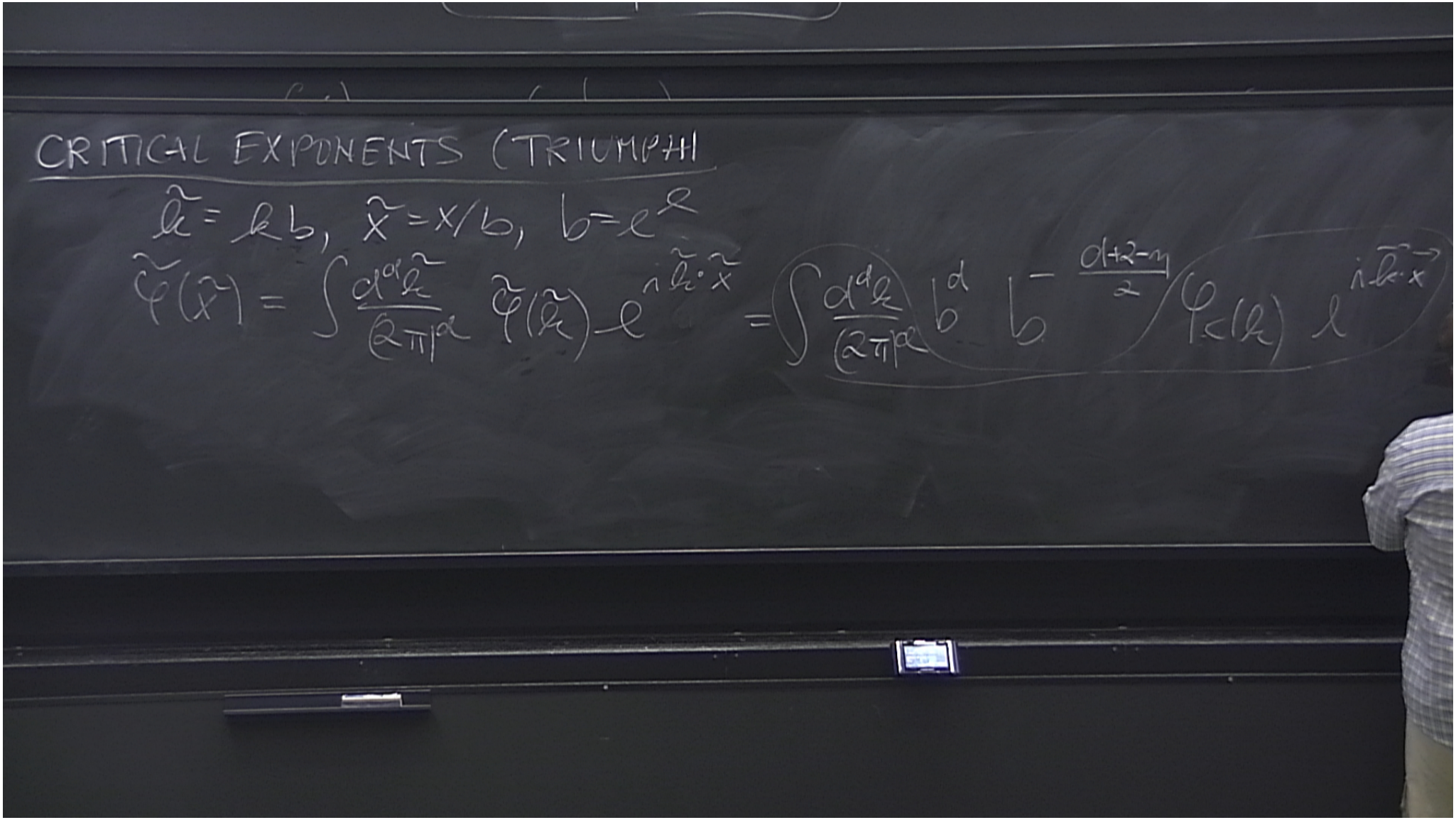

$\Lambda_+ = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$
$\Lambda_{\tilde{M}} = \begin{pmatrix} 1 & \\ & 2(\varepsilon-2)\Lambda^2 \end{pmatrix}$
CRITICAL EXPONENTS (TRIUMPH!
$\tilde{k} = kb, \quad \tilde{x} = x/b, \quad b = e^{\ell}$
$\tilde{\varphi}(\tilde{x}) = \int \frac{d^d\tilde{k}}{(2\pi)^d} \tilde{\varphi}(\tilde{k}) e^{i\tilde{k}\cdot\tilde{x}}$

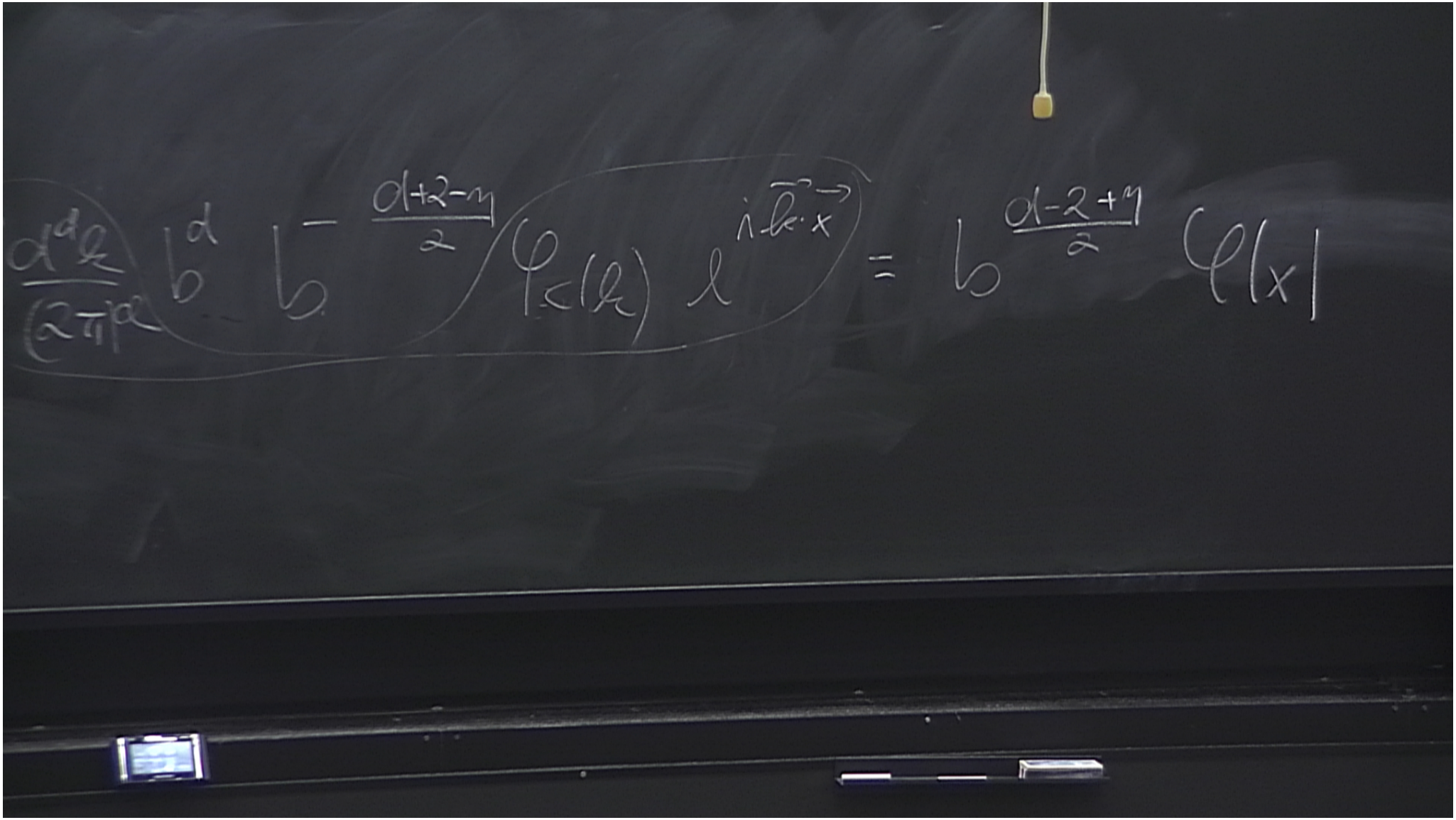
$\Lambda_+ = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$ $\Lambda_{\tilde{\pi}} = \begin{pmatrix} 1 & \\ & 2(\varepsilon-2)\Lambda^2 \end{pmatrix}$

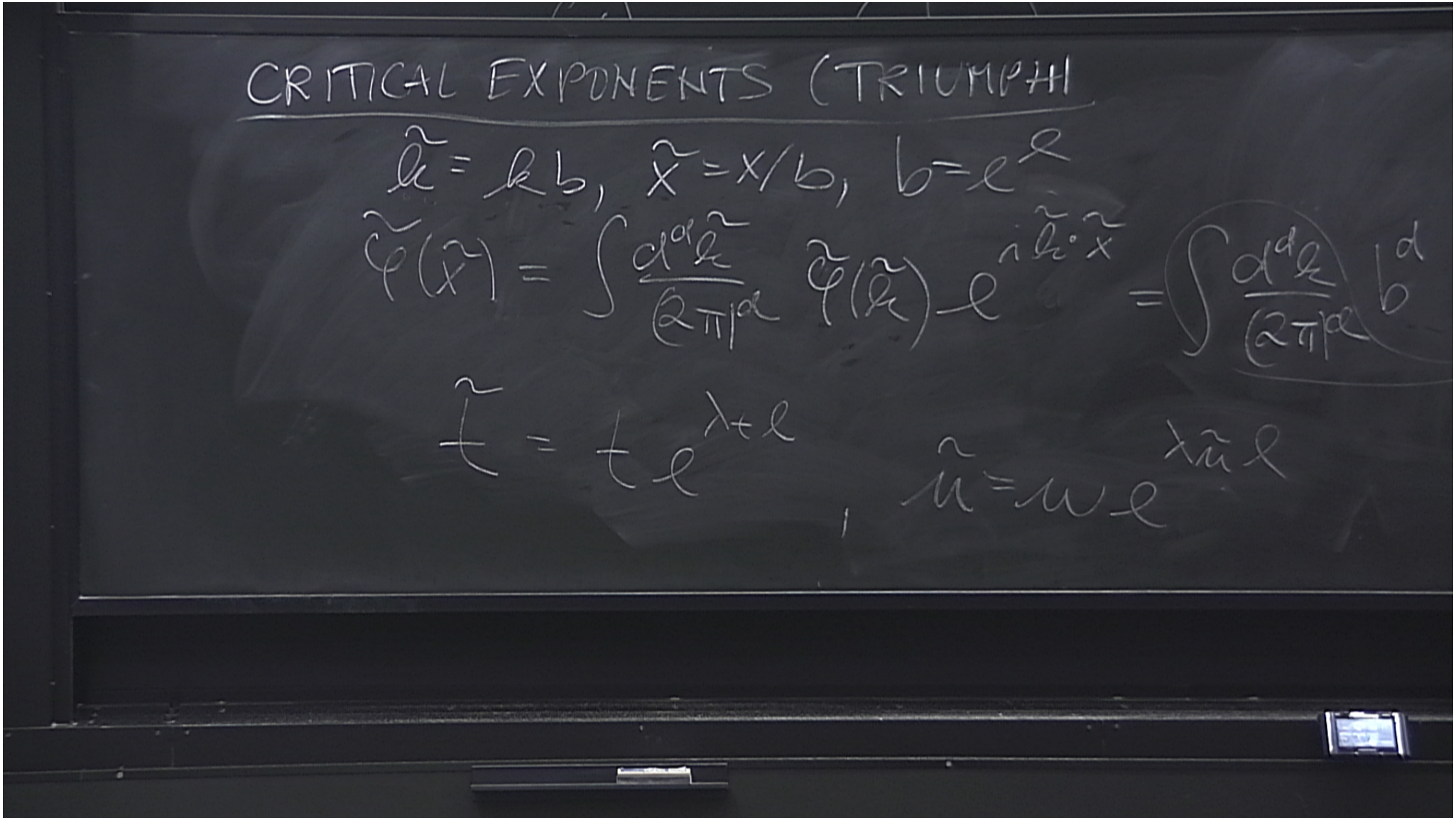
CRITICAL EXPONENTS (TRIUMPH!)

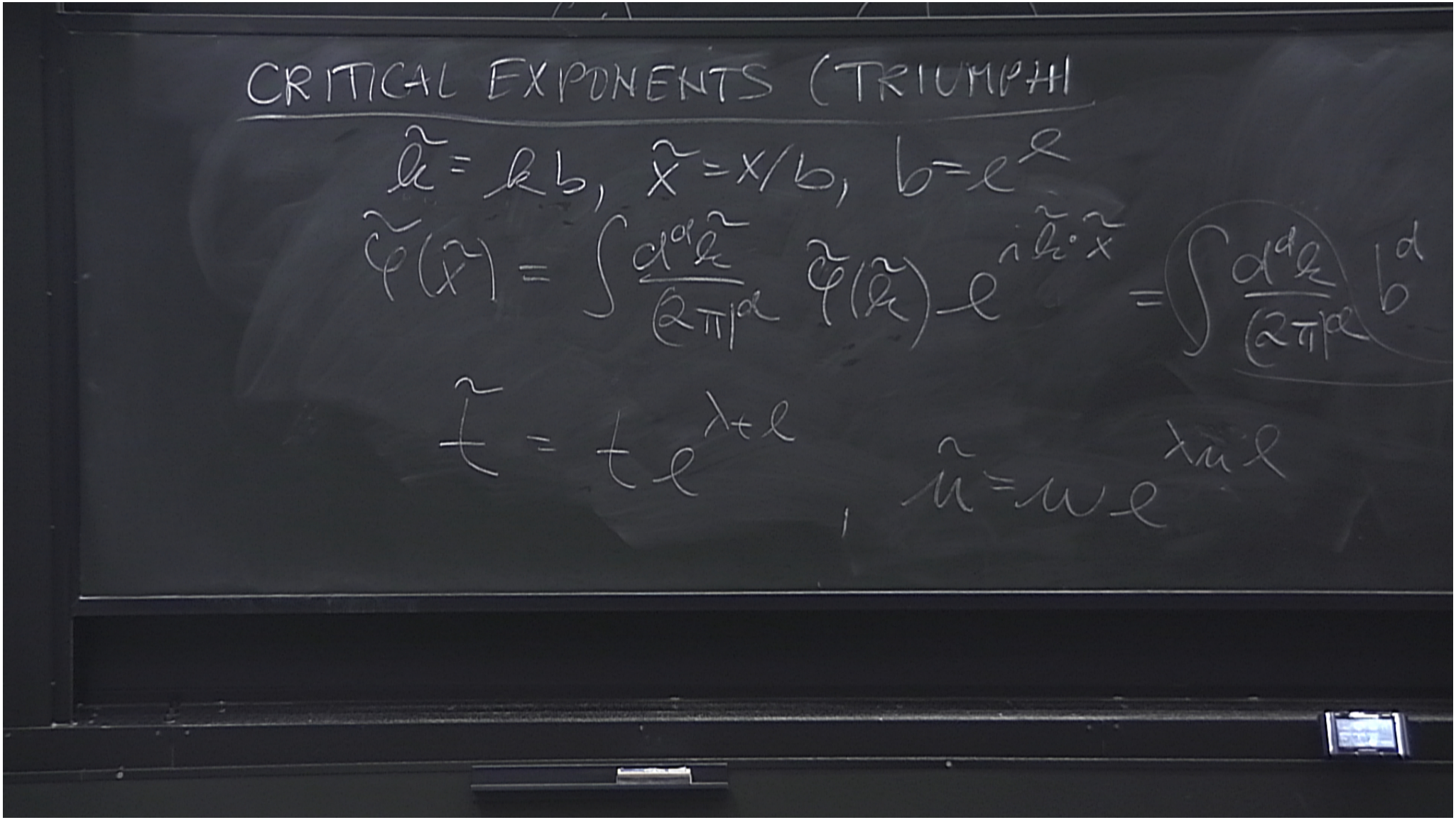
$\tilde{k} = kb, \quad \tilde{x} = x/b, \quad b = e^{\ell}$

$$\tilde{\varphi}(\tilde{x}) = \int \frac{d^d\tilde{k}}{(2\pi)^d} \tilde{\varphi}(\tilde{k}) e^{i\tilde{k}\cdot\tilde{x}} = \int \frac{d^d k}{(2\pi)^d} b^d$$


CRITICAL EXPONENTS (TRIUMPH!)
$\tilde{k} = kb, \quad \tilde{x} = x/b, \quad b = e^{\ell}$
$\tilde{\varphi}(\tilde{x}) = \int \frac{d^d\tilde{k}}{(2\pi)^d} \tilde{\varphi}(\tilde{k}) e^{i\tilde{k}\cdot\tilde{x}} = \int \frac{d^dk}{(2\pi)^d} b^d \, b^{-\frac{d+2-\eta}{2}} \varphi_{<}(k) \, e^{i\vec{k}\cdot\vec{x}}$

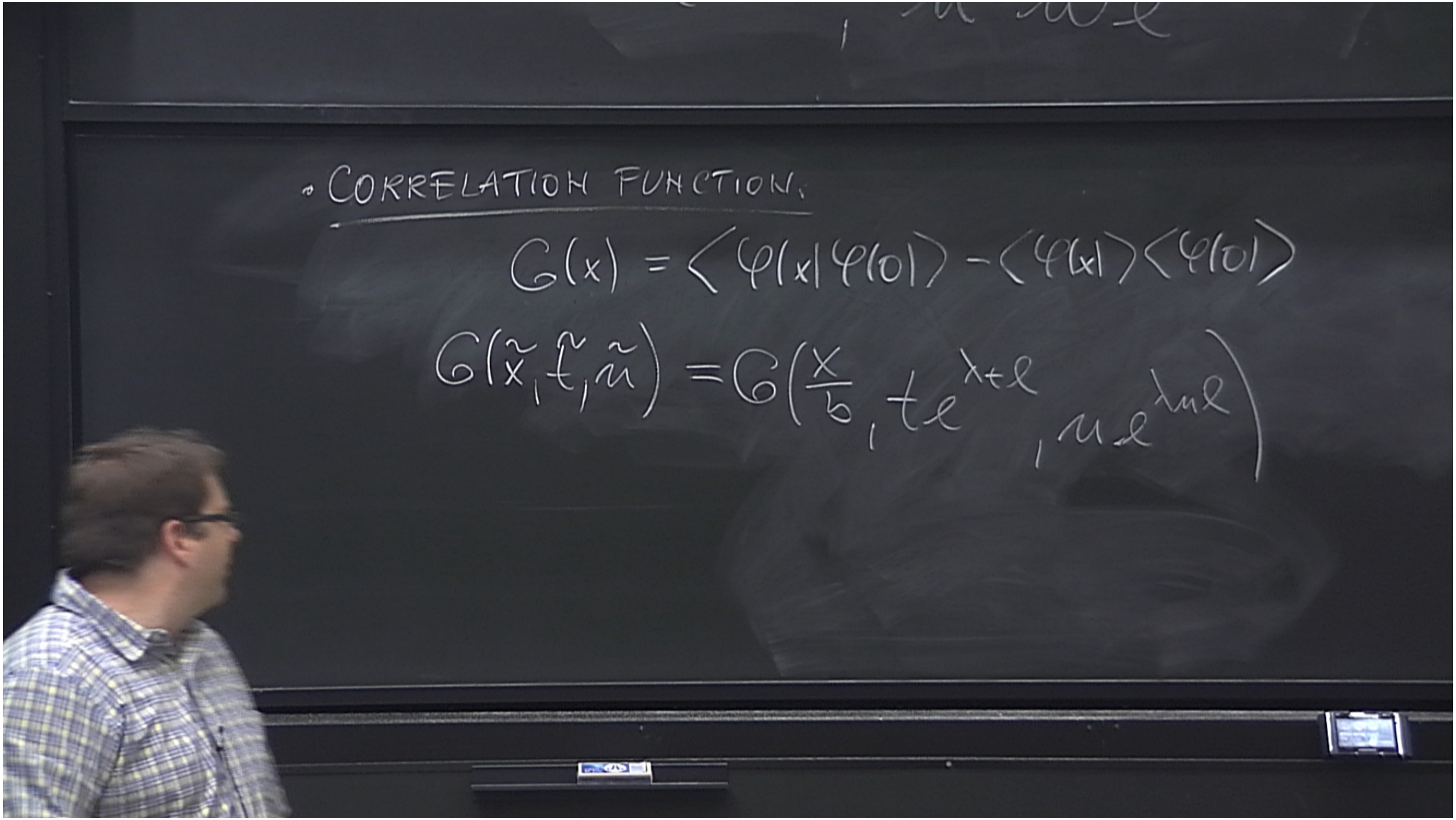

$\frac{d^d k}{(2\pi)^d} b^d b^{-\frac{d+2-\eta}{2}} \varphi_{<}(k)\, e^{i\vec{k}\cdot\vec{x}} = b^{\frac{d-2+\eta}{2}} \varphi(x)$

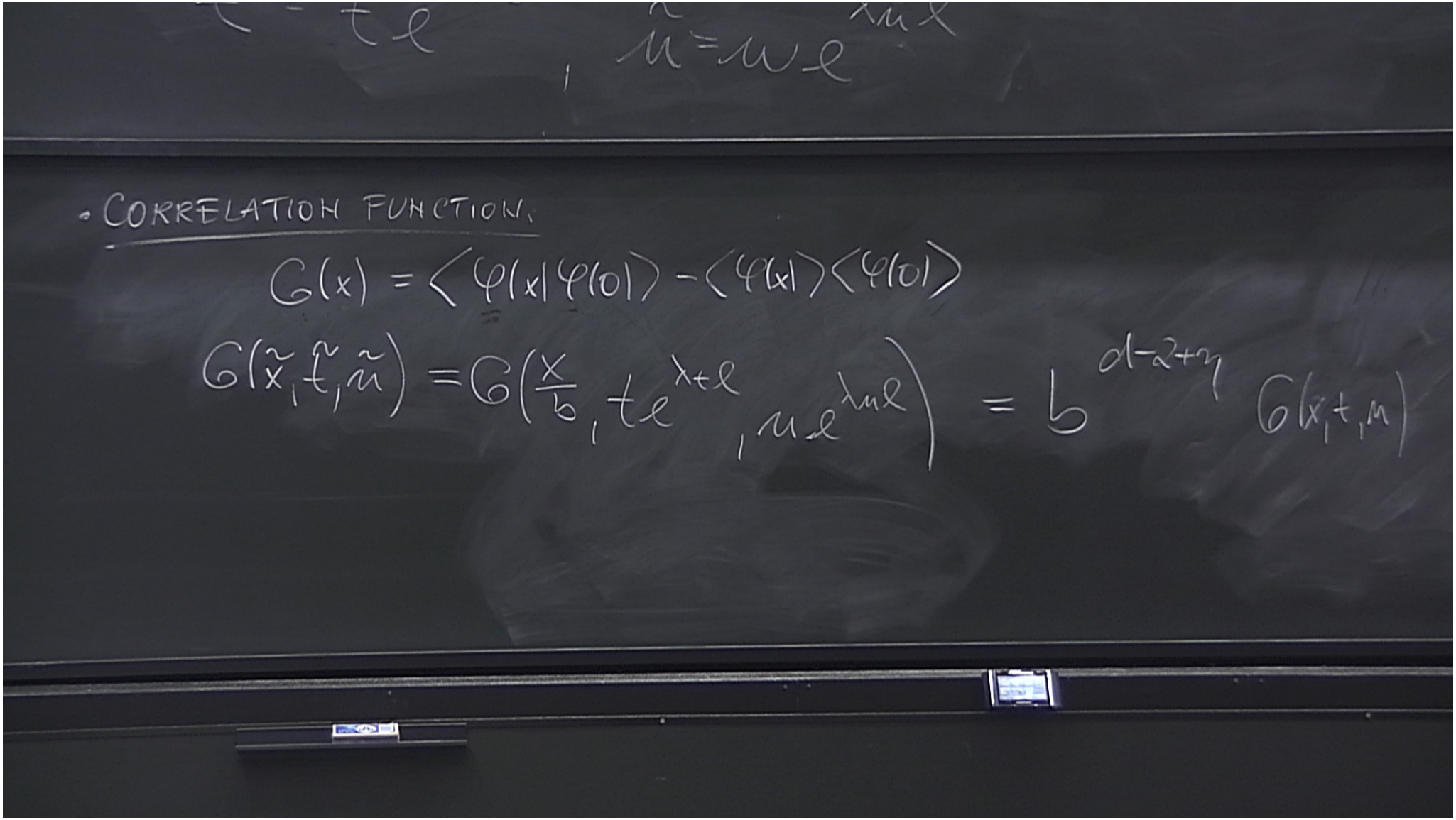

CRITICAL EXPONENTS (TRIUMPH!
$\tilde{k} = kb$, $\tilde{x} = x/b$, $b = e^{\ell}$
$\tilde{\varphi}(\tilde{x}) = \int \frac{d^d\tilde{k}}{(2\pi)^d} \tilde{\varphi}(\tilde{k}) e^{i\tilde{k}\cdot\tilde{x}} = \int \frac{d^dk}{(2\pi)^d} b^d$
$\tilde{t} = t e^{\lambda_t \ell}$, $\tilde{u} = u e^{\lambda_u \ell}$

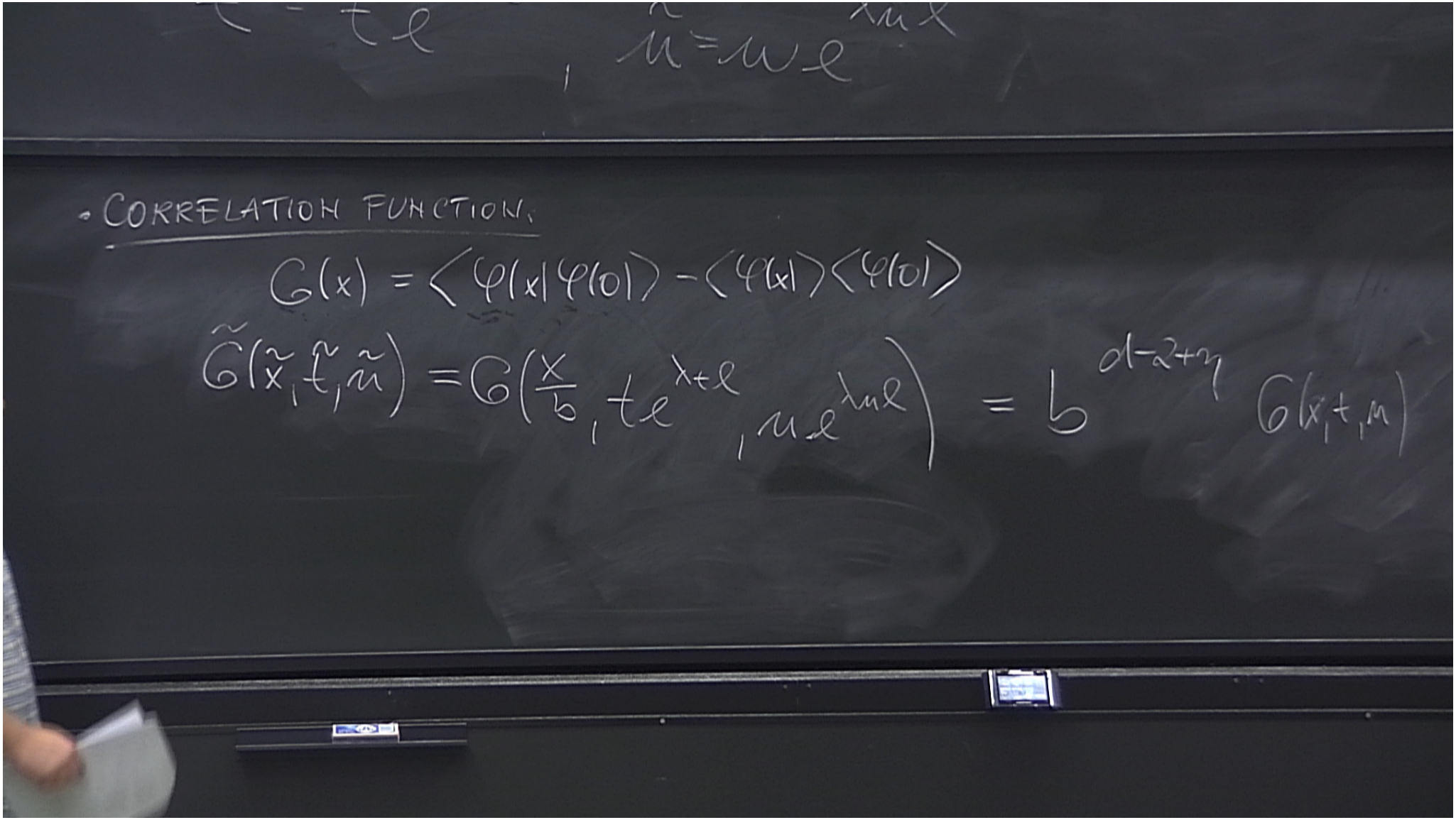

CRITICAL EXPONENTS (TRIUMPH!
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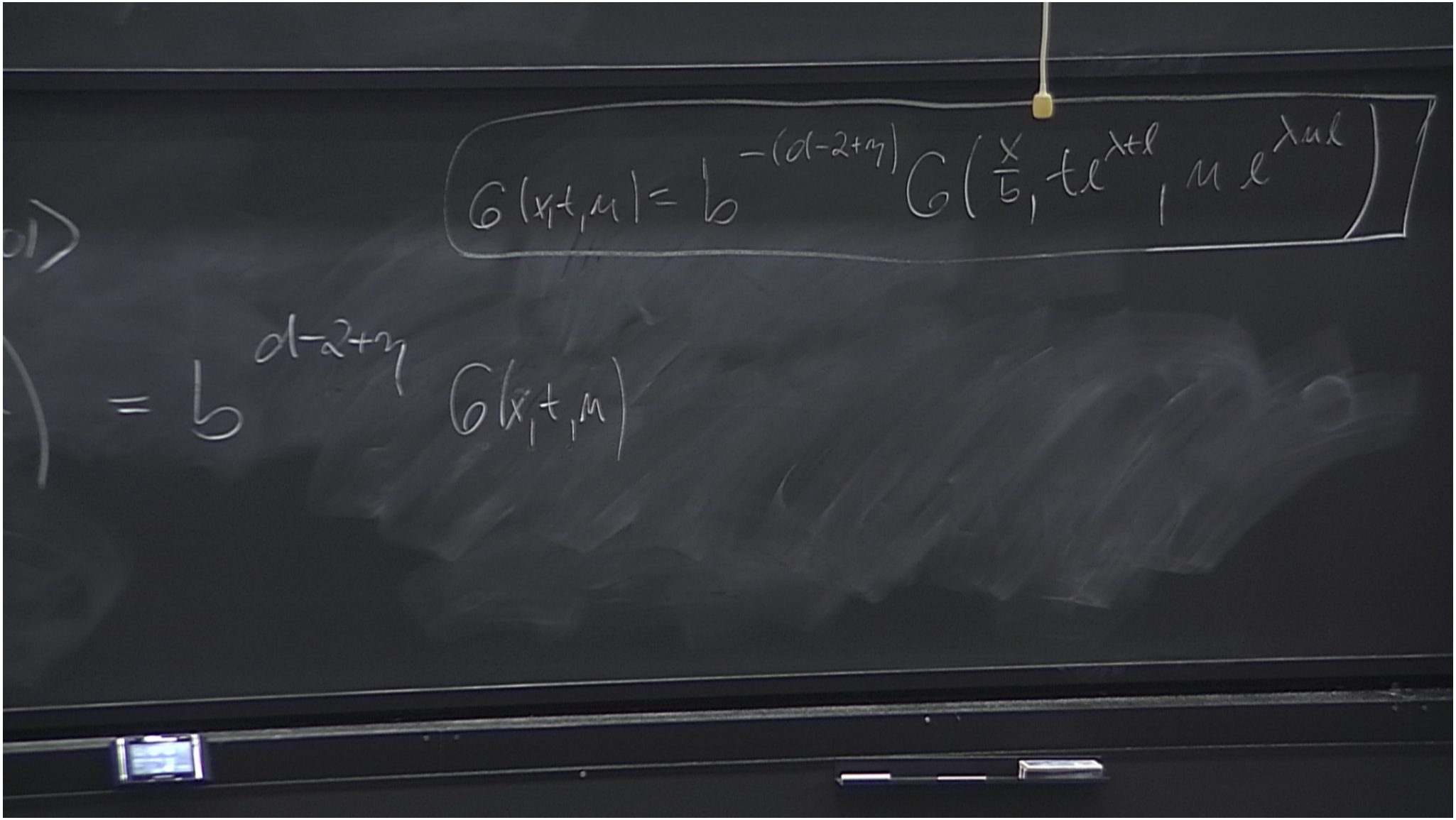
- CORRELATION FUNCTION:

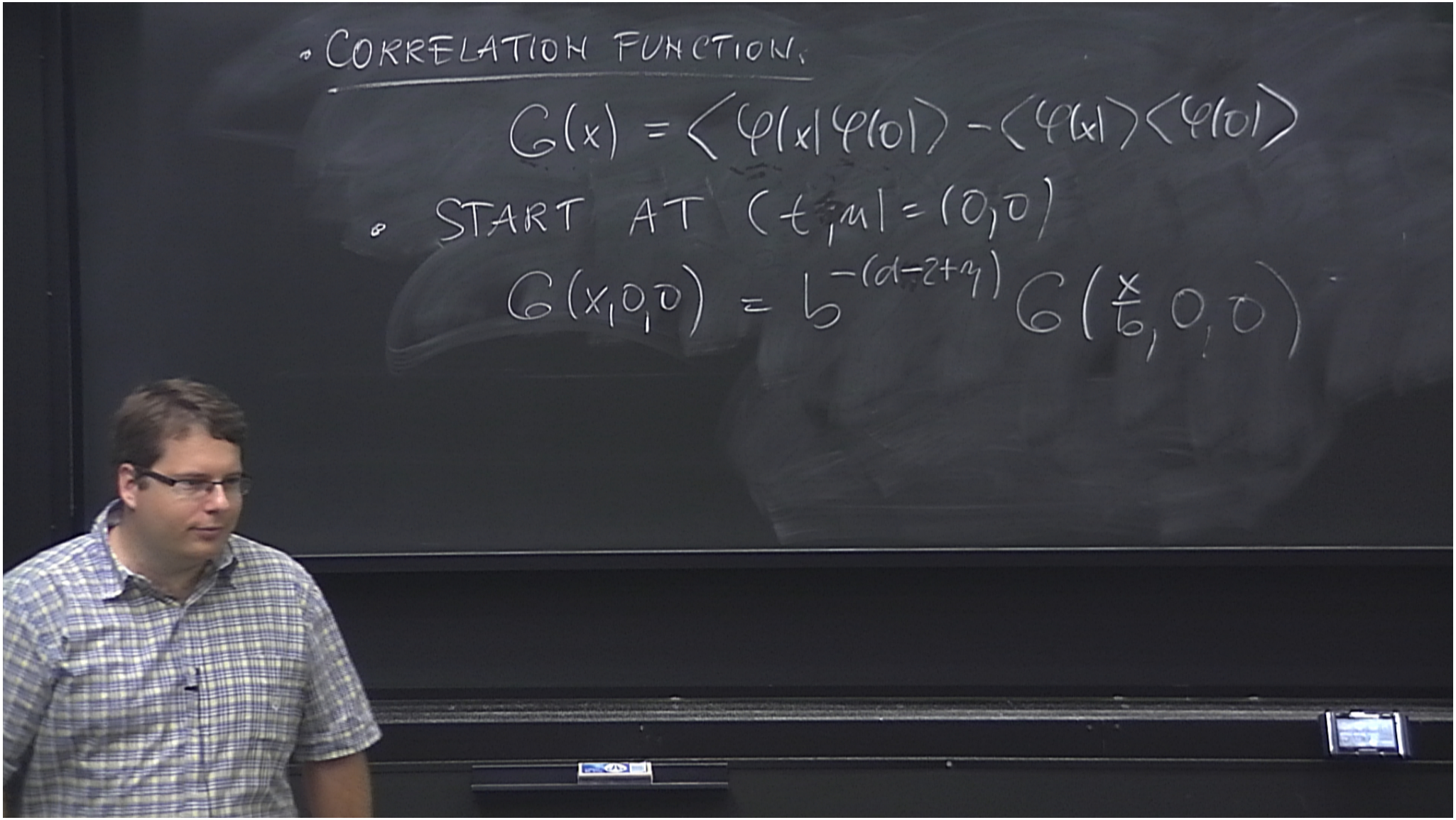
$$G(x) = \langle \varphi(x)\varphi(0) \rangle - \langle \varphi(x) \rangle \langle \varphi(0) \rangle$$

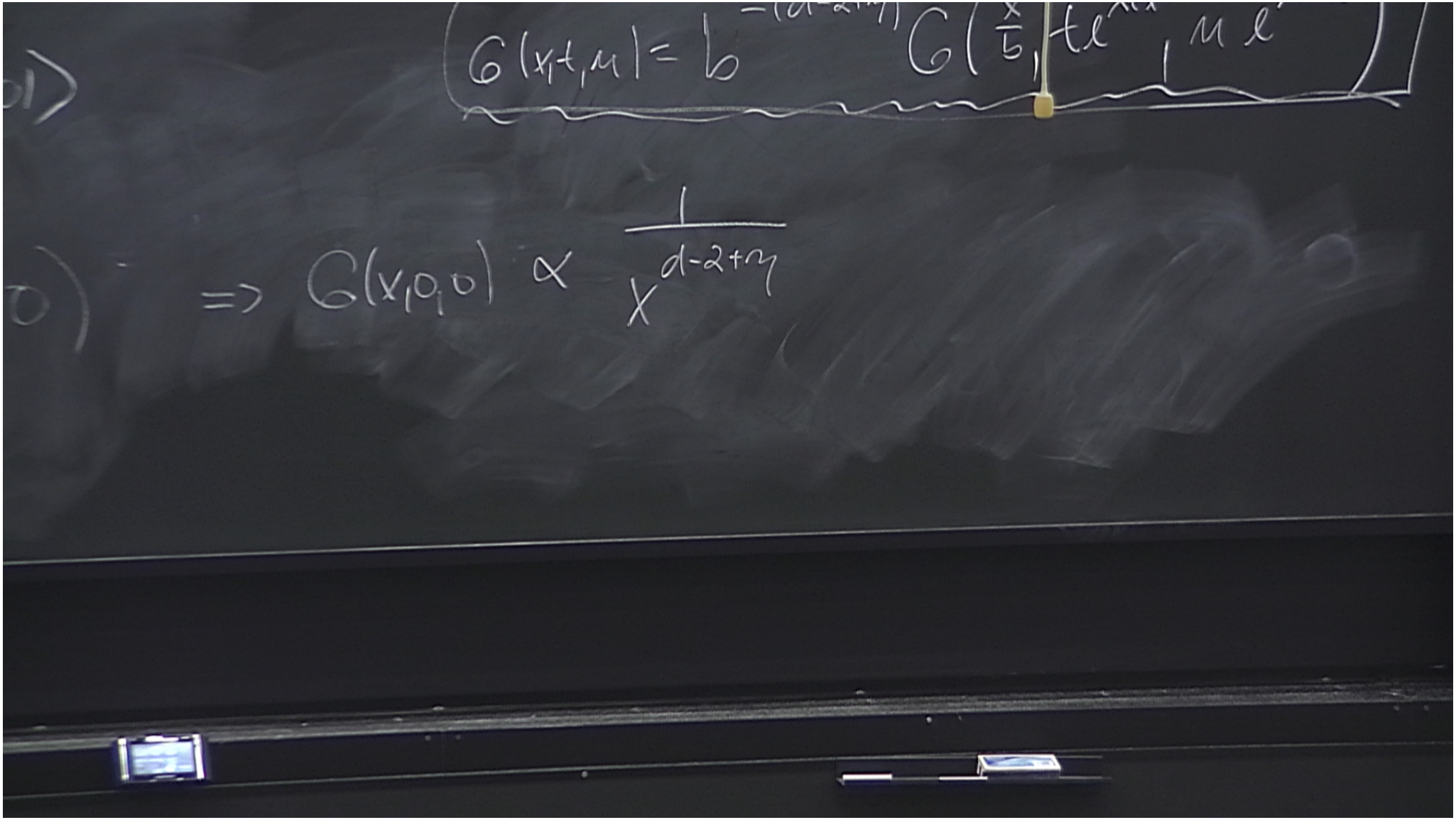

• CORRELATION FUNCTION:
$G(x) = \langle \varphi(x)\varphi(0) \rangle - \langle \varphi(x) \rangle \langle \varphi(0) \rangle$
$G(\tilde{x}, \tilde{t}, \tilde{u}) = G\left(\frac{x}{b}, t e^{\lambda + \ell}, u e^{\lambda u \ell}\right)$

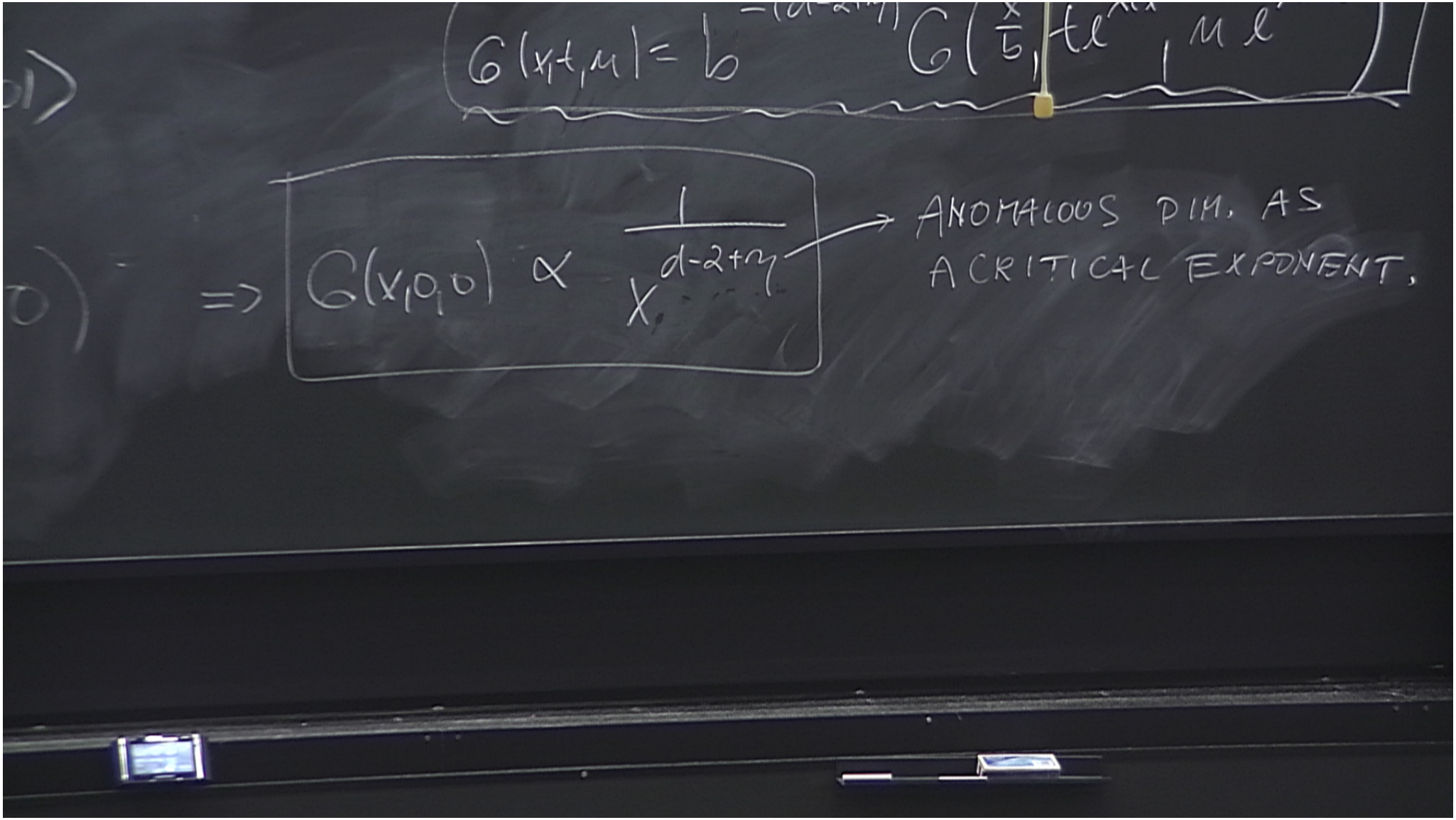

$\tilde{u} = u e^{\lambda u \ell}$
• CORRELATION FUNCTION.
$G(x) = \langle \varphi(x) \varphi(0) \rangle - \langle \varphi(x) \rangle \langle \varphi(0) \rangle$
$G(\tilde{x}, \tilde{t}, \tilde{u}) = G\left(\frac{x}{b}, t e^{\lambda_t \ell}, u e^{\lambda_u \ell}\right) = b^{d-2+\eta} G(x, t, u)$

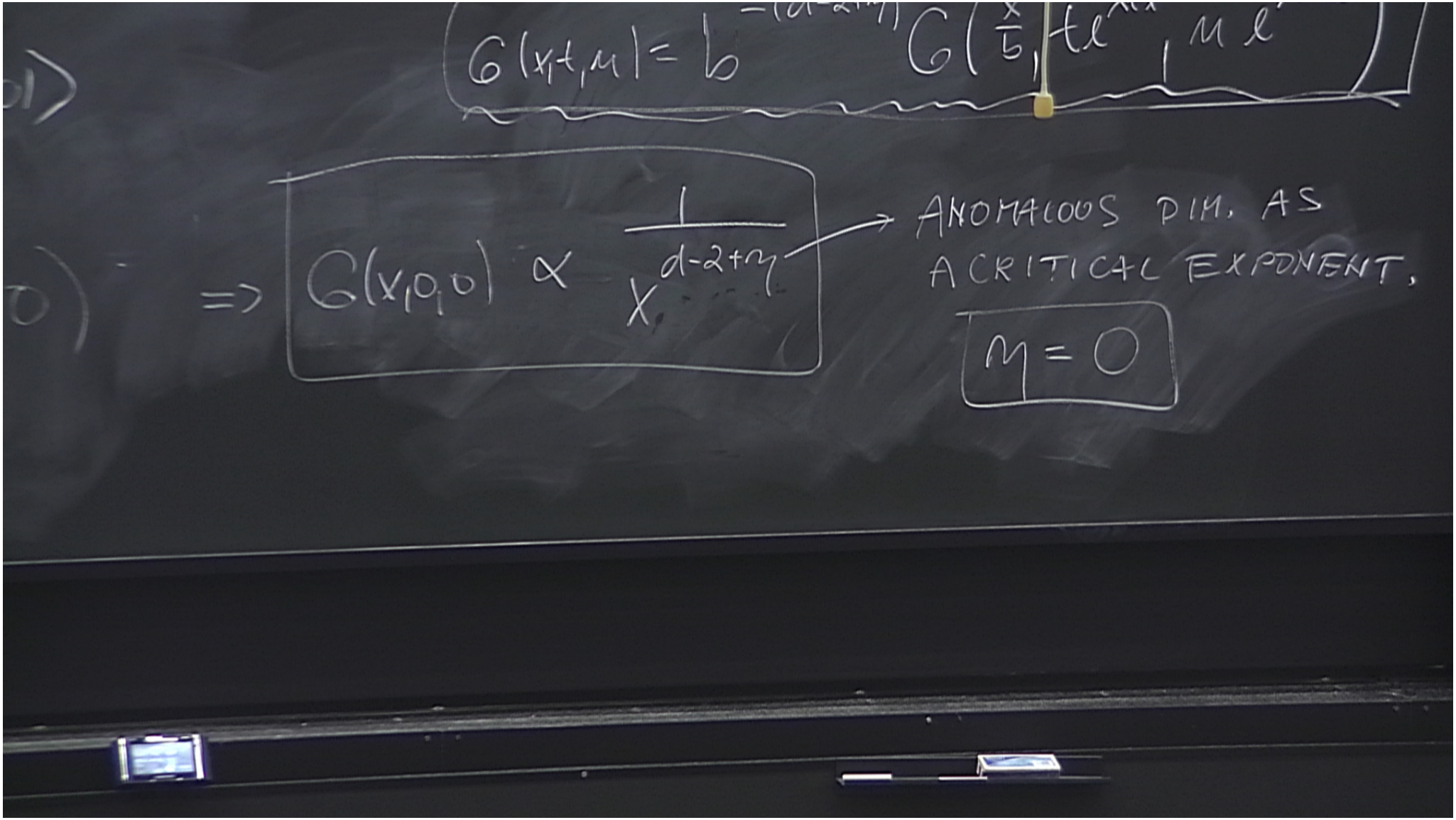

$\tilde{u} = u\, e^{\lambda u \ell}$
• CORRELATION FUNCTION.
$G(x) = \langle \varphi(x)\varphi(0)\rangle - \langle\varphi(x)\rangle\langle\varphi(0)\rangle$
$\tilde{G}(\tilde{x},\tilde{t},\tilde{u}) = G\left(\frac{x}{b}, t e^{\lambda_t \ell}, u e^{\lambda_u \ell}\right) = b^{d-2+\eta} G(x,t,u)$


$G(x,t,\mu) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t\ell^{\lambda_t \ell}, \mu \ell^{\lambda_\mu \ell}\right)$
$= b^{d-2+\eta} G(x,t,\mu)$


• CORRELATION FUNCTION:
$G(x) = \langle \varphi(x)\varphi(0) \rangle - \langle \varphi(x) \rangle \langle \varphi(0) \rangle$
• START AT $(t, m) = (0, 0)$
$G(x, 0, 0) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, 0, 0\right)$


$G(x,t,\mu) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t b^{\frac{1}{\nu}}, \mu b^{\cdot}\right)$
$\Rightarrow G(x,0,0) \propto \frac{1}{x^{d-2+\eta}}$


$G(x,t,\mu) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t b^{x_t}, \mu b^{\cdot}\right)$
$\Rightarrow G(x,0,0) \propto \frac{1}{x^{d-2+\eta}}$
ANOMALOUS DIM. AS A CRITICAL EXPONENT,


$G(x,t,\mu) = b^{-(d-2+\eta)} G(\frac{x}{b}, t\ell^{x_t}, \mu\ell^{x_\mu})$
$\Rightarrow G(x,0,0) \propto \frac{1}{x^{d-2+\eta}}$
ANOMALOUS DIM. AS A CRITICAL EXPONENT,
$\mu = 0$

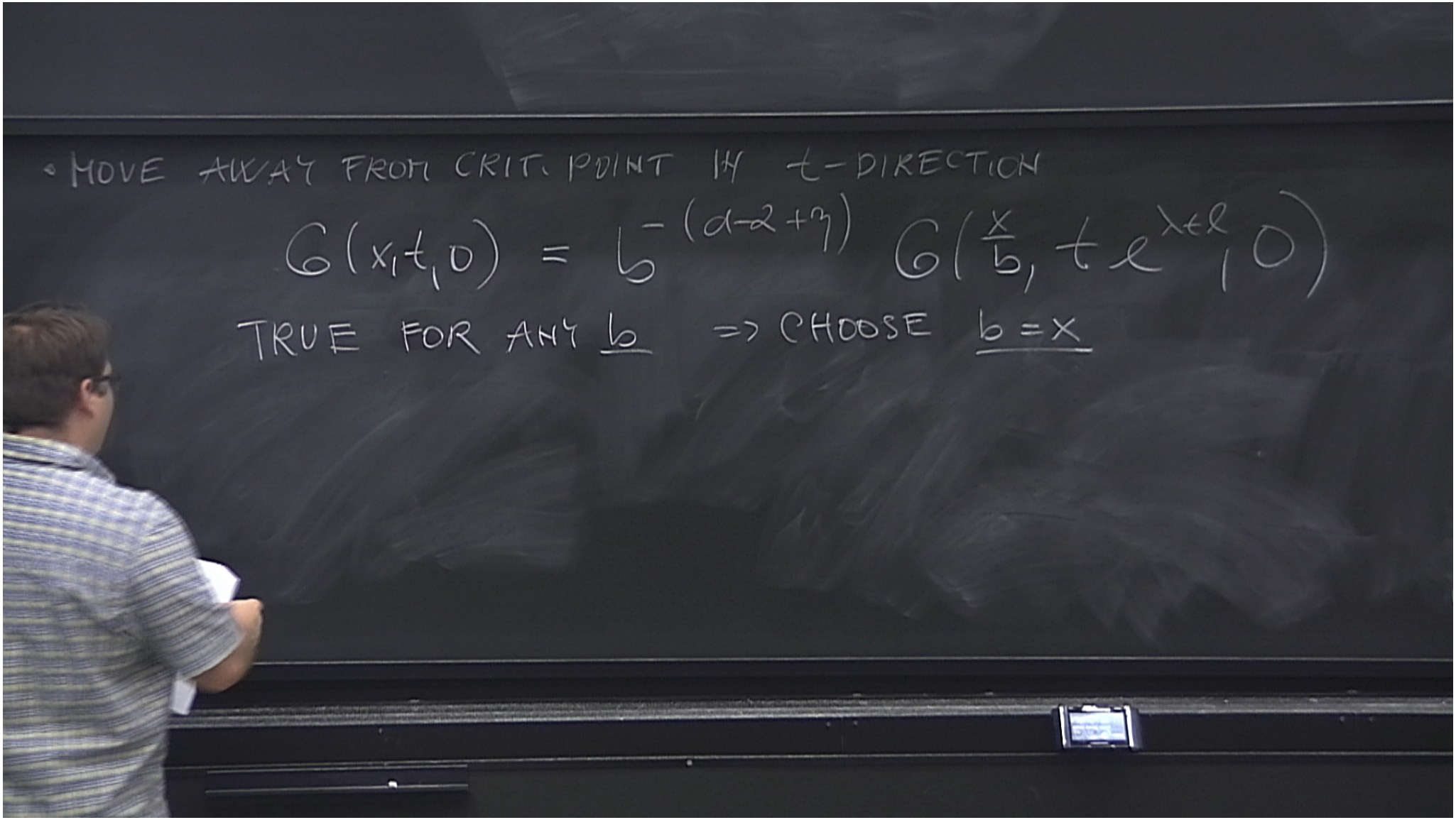
- MOVE AWAY FROM CRIT. POINT IN t-DIRECTION

$$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t e^{\lambda_t \ell}, 0\right)$$

- MOVE AWAY FROM CRIT. POINT IN t-DIRECTION

$$G(x,t,0) = b^{-(\alpha-2+\eta)}\, G\left(\frac{x}{b}, t e^{\lambda \ell}, 0\right)$$

TRUE FOR ANY b


• MOVE AWAY FROM CRIT. POINT IN t-DIRECTION
$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, te^{\lambda_t \ell}, 0\right)$
TRUE FOR ANY b => CHOOSE $b = x$

- MOVE AWAY FROM CRIT. POINT IN t-DIRECTION

$$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t e^{\lambda_t \ell}, 0\right)$$

TRUE FOR ANY b $\Rightarrow$ CHOOSE $b = x$

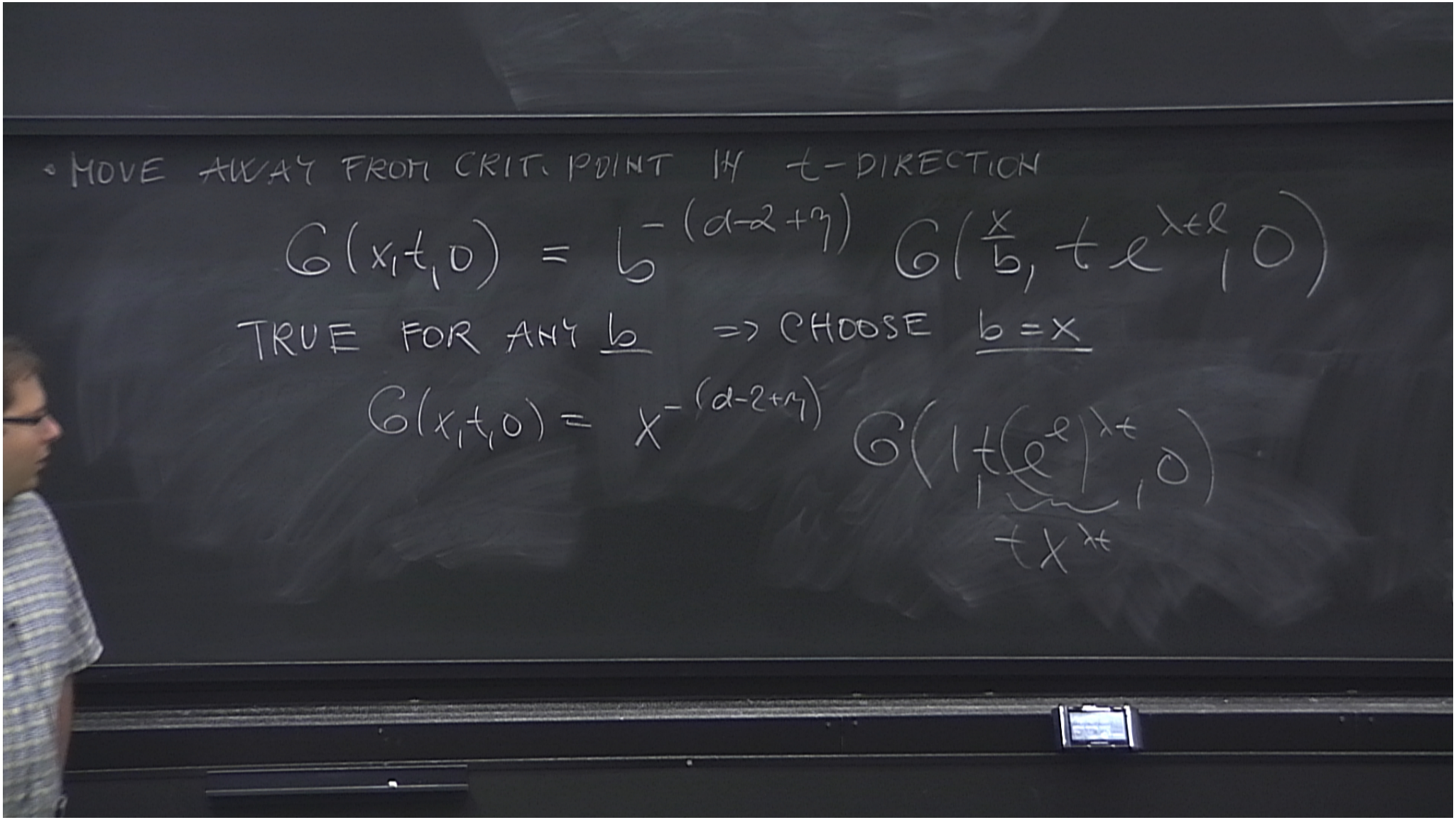
$$G(x,t,0) = x^{-(d-2+\eta)}\, G\left(1, (e^{\ell})^{\lambda_t}, 0\right)$$

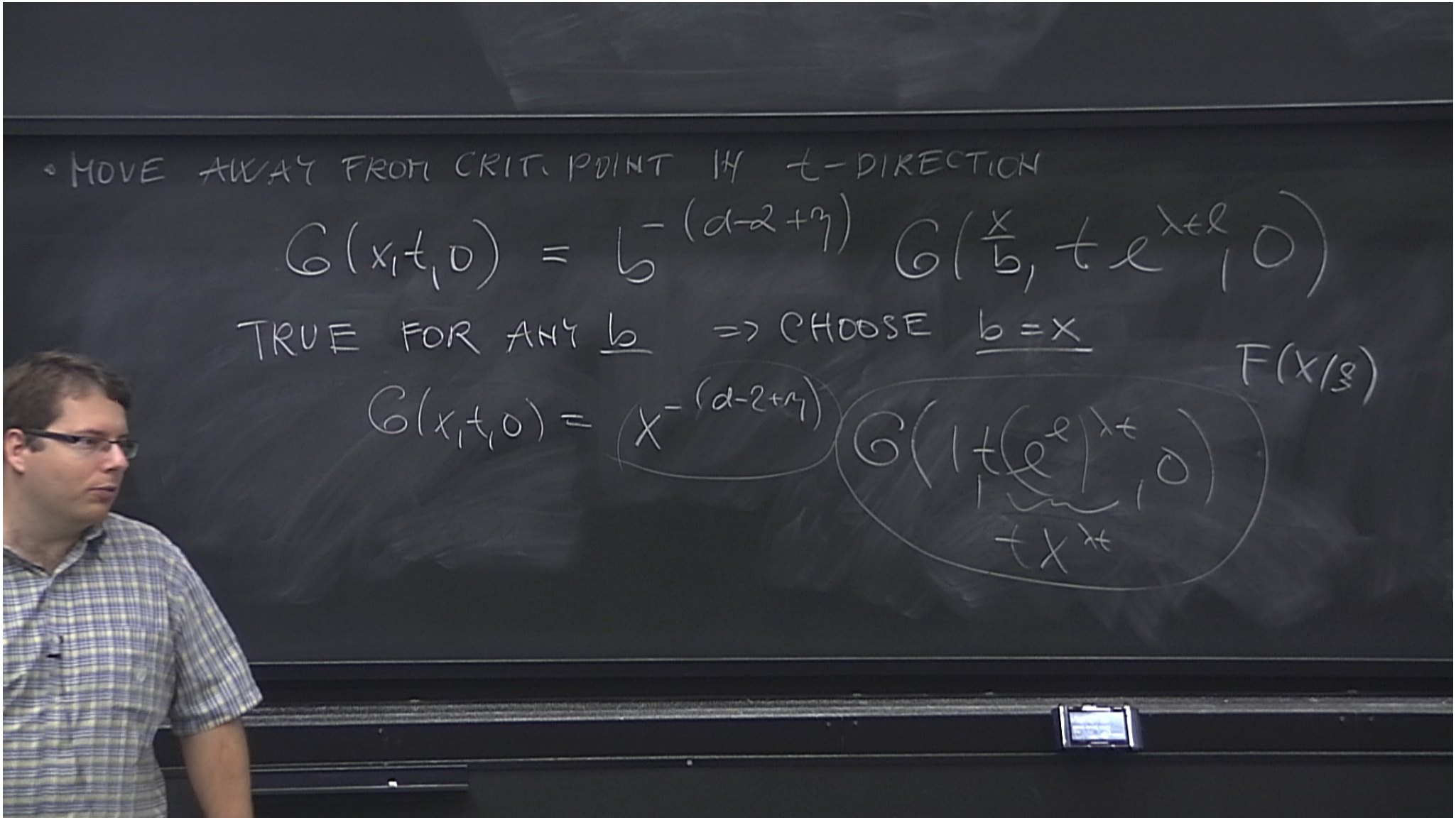
- MOVE AWAY FROM CRIT. POINT IN t-DIRECTION

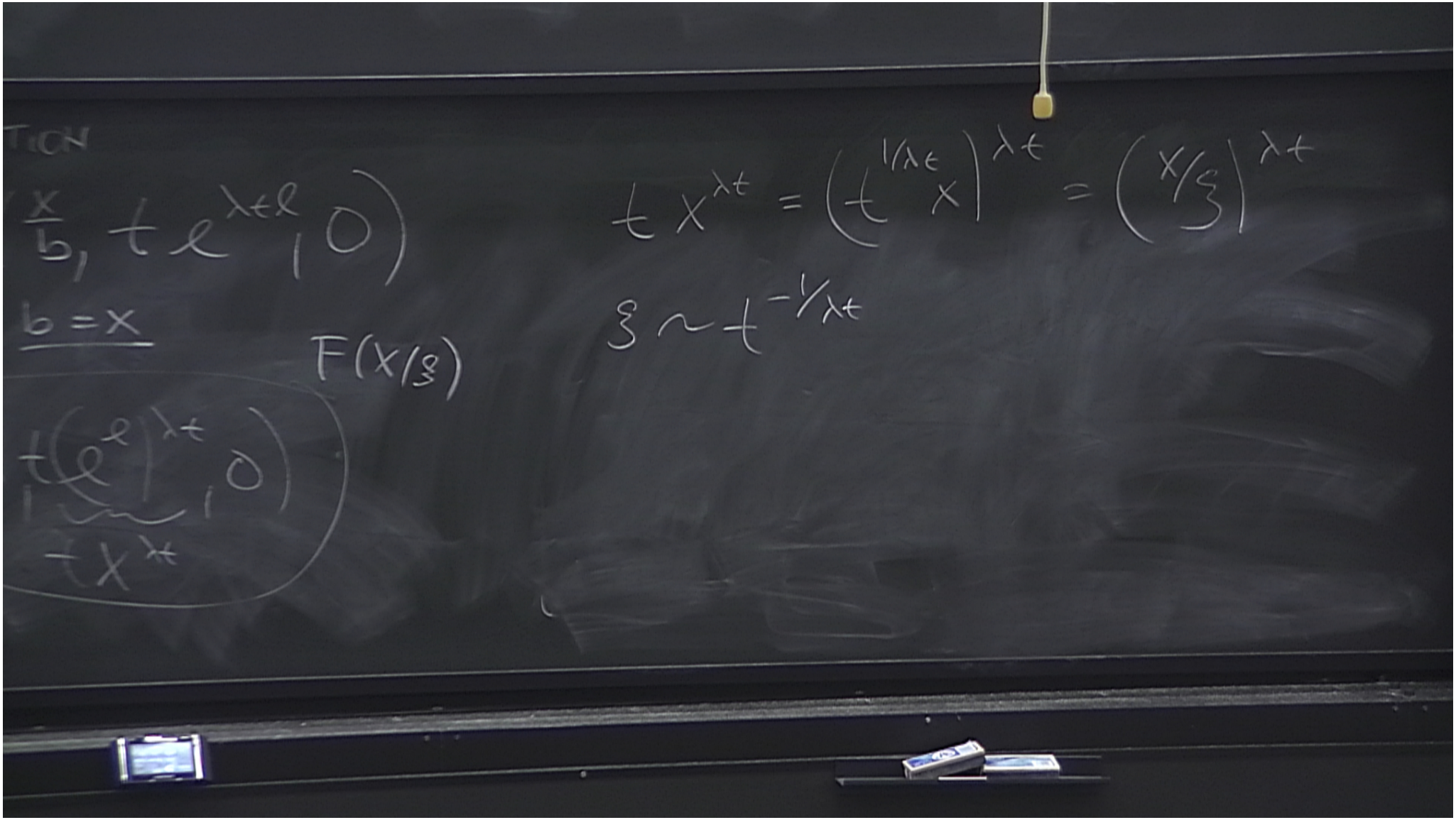
$$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t\,e^{\lambda_t \ell}, 0\right)$$

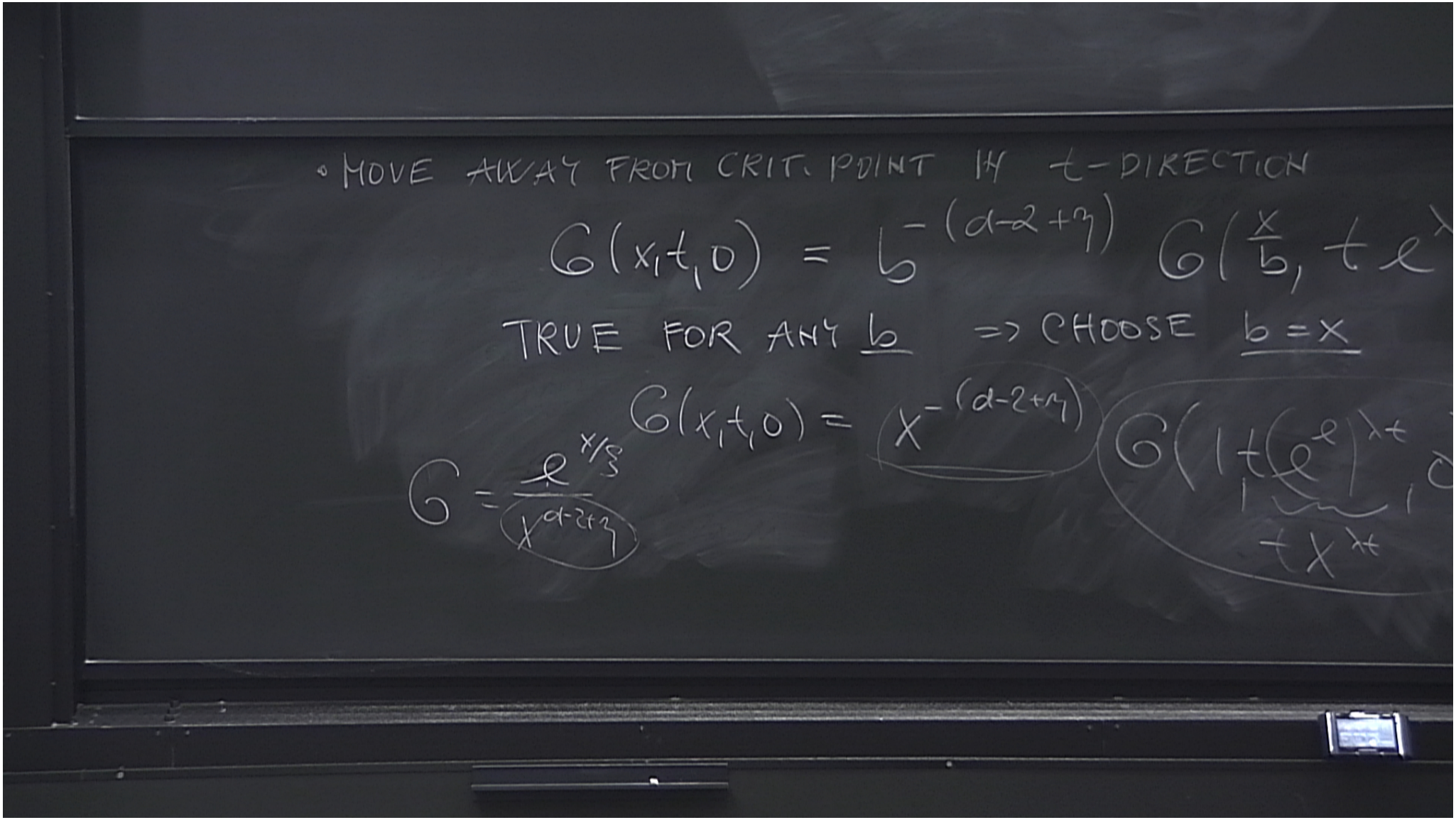
TRUE FOR ANY b $\Rightarrow$ CHOOSE $b = x$

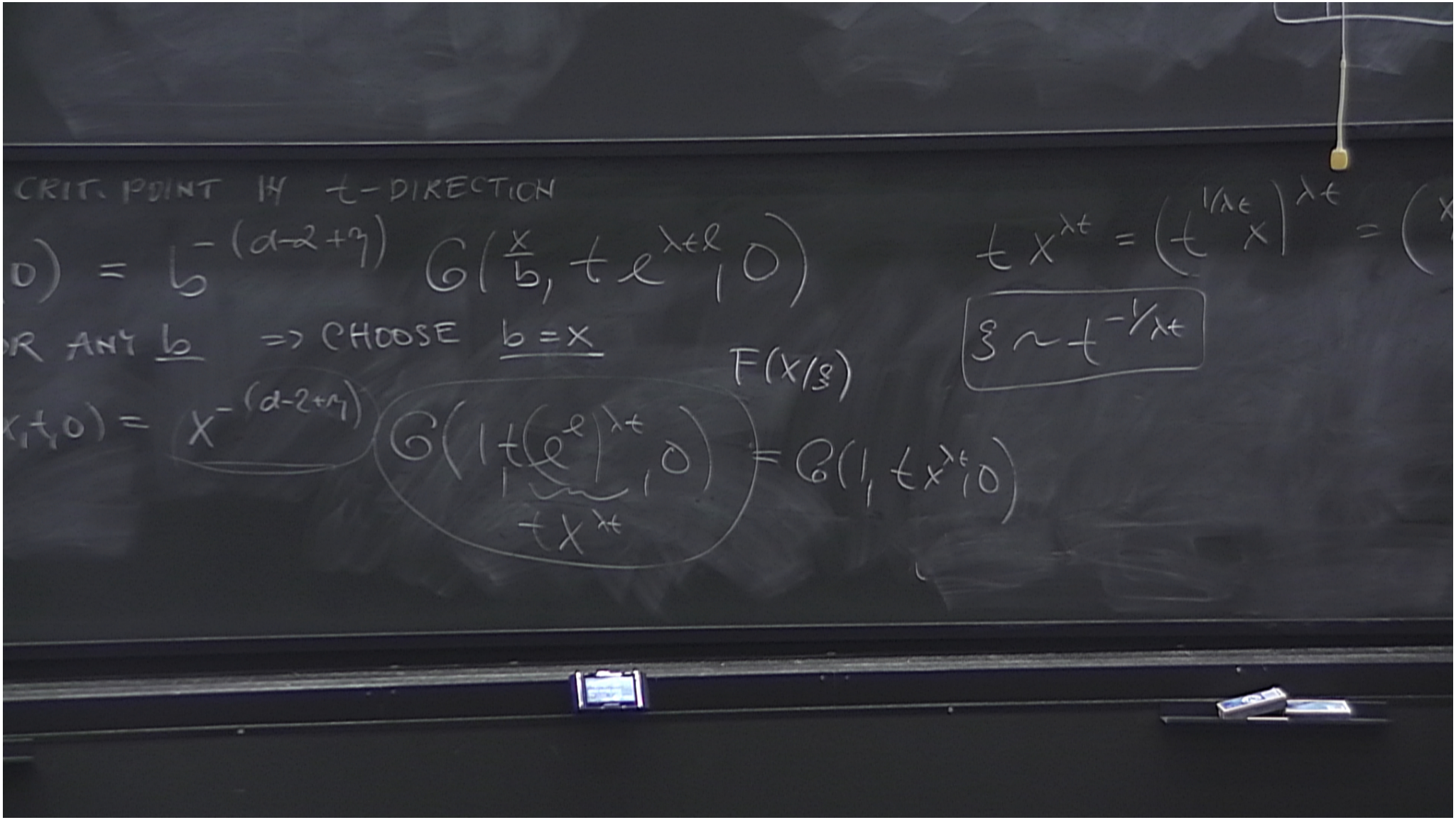
$$G(x,t,0) = x^{-(d-2+\eta)}\, G\left(1, \underbrace{(e^{\ell})^{\lambda_t}}_{x^{\lambda_t}}, 0\right)$$

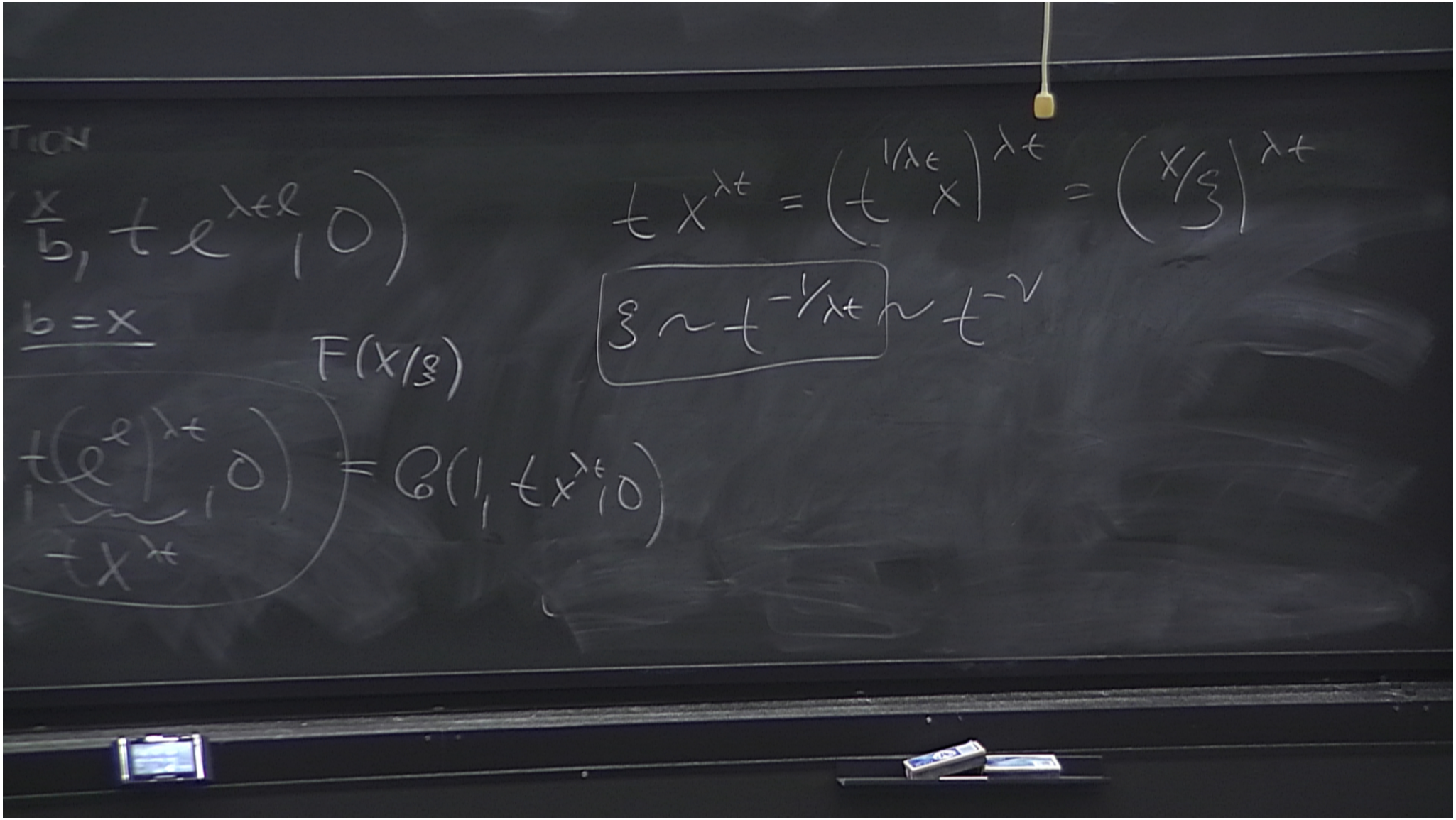

• MOVE AWAY FROM CRIT. POINT IN t-DIRECTION
$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t e^{\lambda_t \ell}, 0\right)$
TRUE FOR ANY b => CHOOSE b = x
$G(x,t,0) = x^{-(d-2+\eta)}\, G\left(1, t\underbrace{(e^{\ell})^{\lambda_t}}_{t x^{\lambda_t}}, 0\right)$

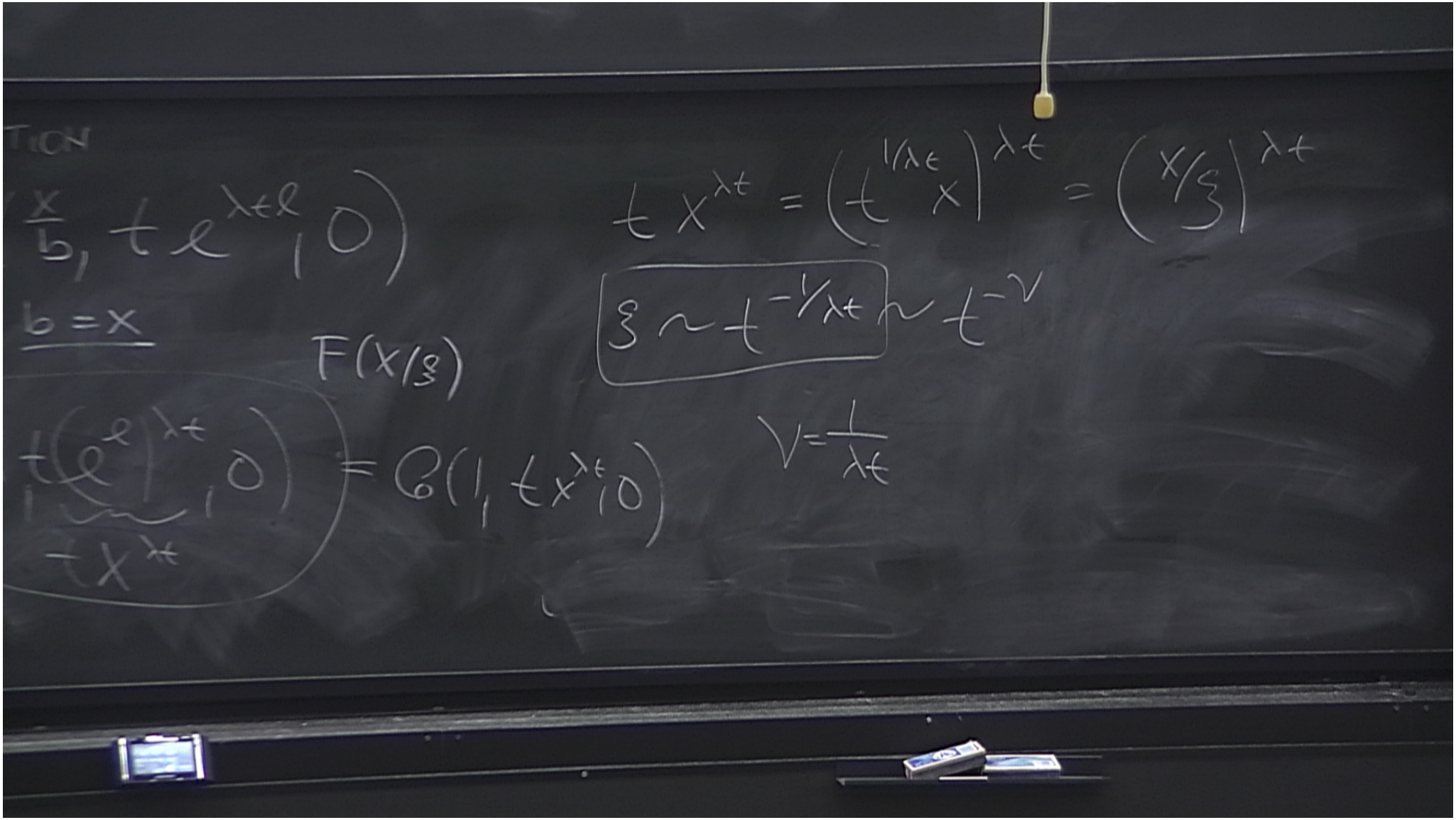

• MOVE AWAY FROM CRIT. POINT IN t-DIRECTION
$G(x,t,0) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, te^{\lambda t}, 0\right)$
TRUE FOR ANY b => CHOOSE b = x
$G(x,t,0) = x^{-(d-2+\eta)} G\left(1, te^{\lambda t}, 0\right)$
$F(x/\xi)$
$\xi \sim t^{-\nu}$

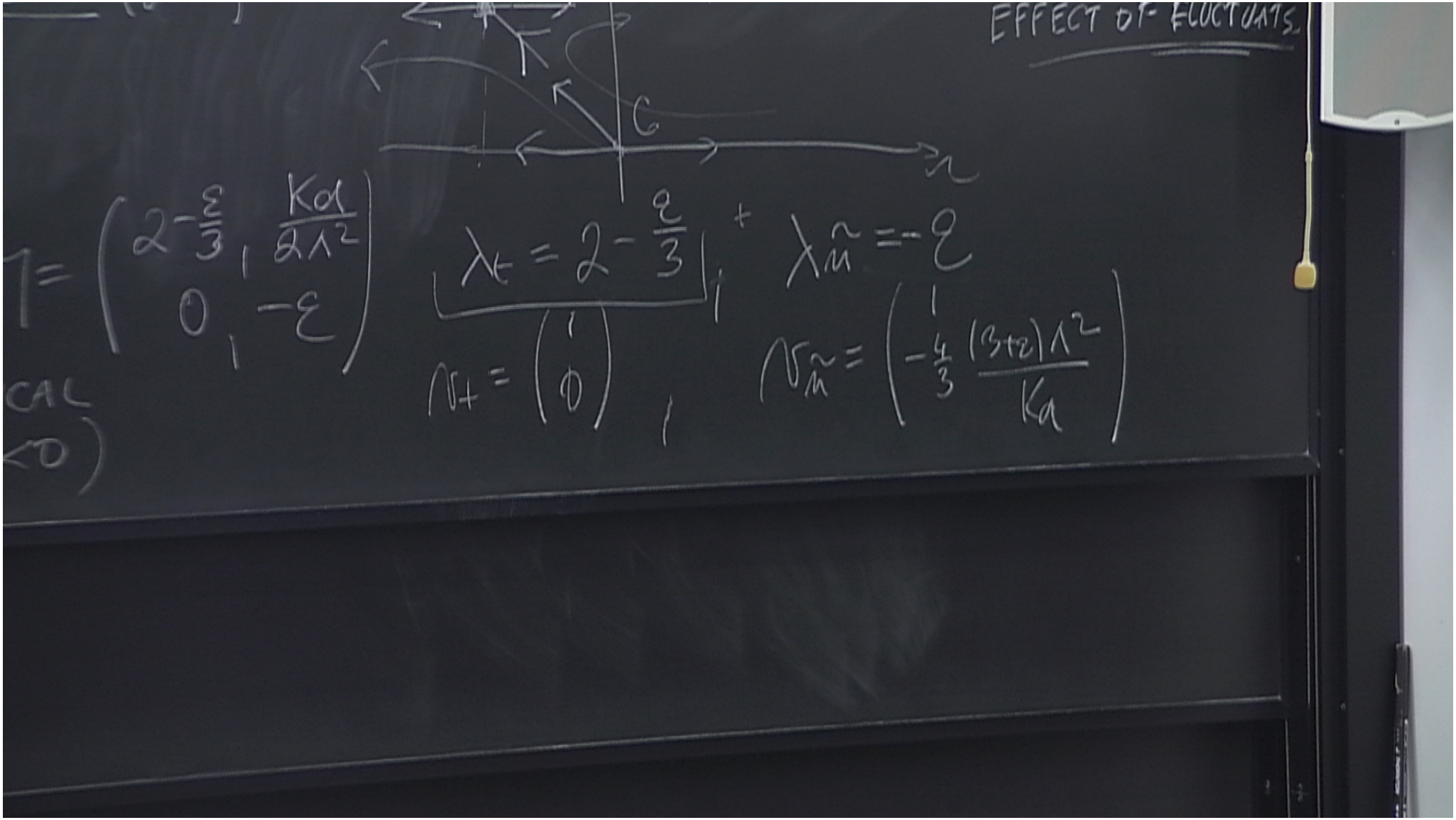

TION
$\left(\frac{x}{b}, te^{\lambda t}, 0\right)$
$b = x$
$F(x/\xi)$
$t\left(e^{\lambda t}, 0\right)$
$= x^{\lambda t}$
$tx^{\lambda t} = \left(t^{1/\lambda t} x\right)^{\lambda t} = \left(x/\xi\right)^{\lambda t}$
$\xi \sim t^{-1/\lambda t}$

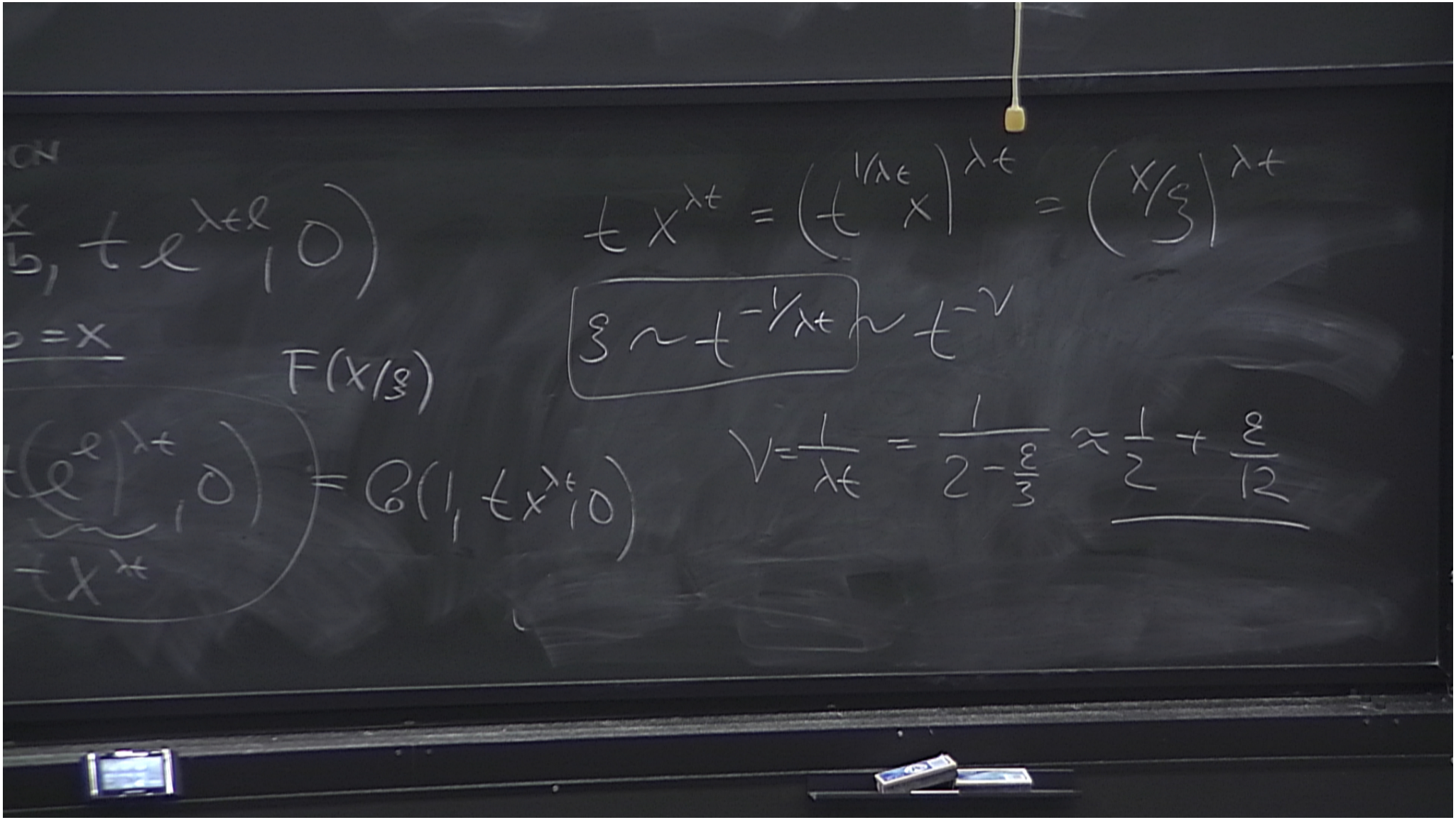

• MOVE AWAY FROM CRIT. POINT IN t-DIRECTION
$G(x,t,0) = b^{-(d-2+\eta)} G(\frac{x}{b}, te^{\lambda$
TRUE FOR ANY b => CHOOSE b = x
$G(x,t,0) = x^{-(d-2+\eta)}$
$G = \frac{e^{x/\xi}}{x^{d-2+\eta}}$
$G(1, te^{\lambda t}, ce^{\lambda t}, \ldots)$
$+ x^{\lambda t}$

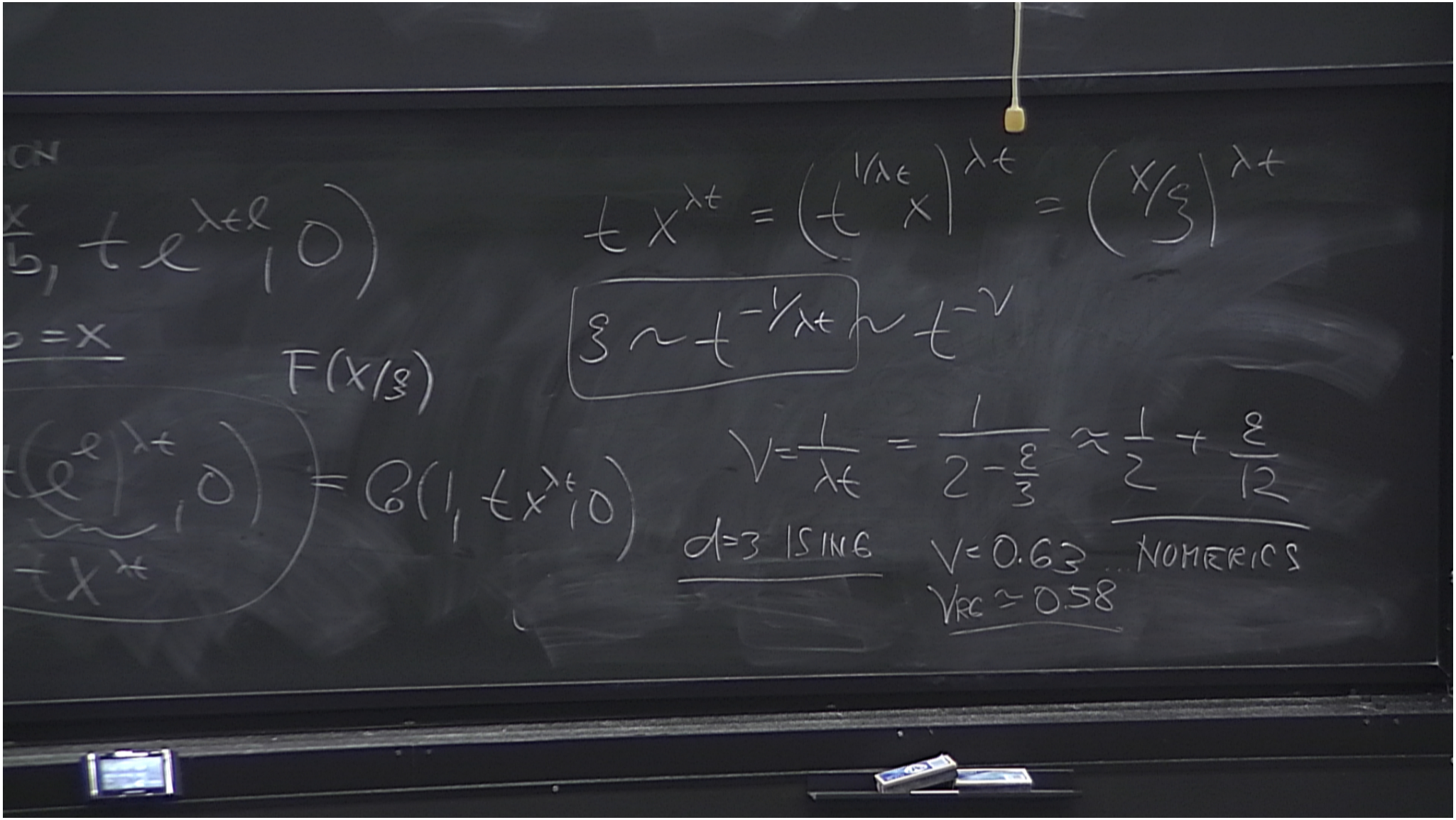

CRIT. POINT IN t-DIRECTION
$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t\,e^{\lambda_t \ell}, 0\right)$
$t\,x^{\lambda_t} = \left(t^{1/\lambda_t} x\right)^{\lambda_t}$
$\xi \sim t^{-1/\lambda_t}$
FOR ANY b => CHOOSE $b = x$
$G(x,t,0) = x^{-(d-2+\eta)}\, G\left(1, t\,(e^{\ell})^{\lambda_t}, 0\right) = G(1, t\,x^{\lambda_t}, 0)$
$F(x/\xi)$
$\sim t\,x^{\lambda_t}$


TION
$\frac{x}{b}, te^{\lambda t}, 0)$
$tx^{\lambda t} = \left(t^{1/\lambda t} x\right)^{\lambda t} = \left(x/\xi\right)^{\lambda t}$
$\xi \sim t^{-1/\lambda t} \sim t^{-\nu}$
$b = x$
$F(x/\xi)$
$t(e^{\lambda t}, 0) = G(1, tx^{\lambda t}, 0)$
$\sim$
$-x^{\lambda t}$


TION
$\left(\frac{x}{b}, t e^{\lambda t \ell}, 0\right)$
$b = x$
$F(x/\xi)$
$t\left(e^{\ell}\right)^{\lambda t}$
$= G(1, t x^{\lambda t}, 0)$
$t x^{\lambda t} = \left(t^{1/\lambda t} x\right)^{\lambda t} = \left(x/\xi\right)^{\lambda t}$
$\xi \sim t^{-1/\lambda t} \sim t^{-\nu}$
$\nu = \frac{1}{\lambda t}$


EFFECT OF FLUCTUATIONS
C
$\lambda_t = 2 - \frac{\varepsilon}{3}$
$\lambda_{\tilde{u}} = -\varepsilon$
$v_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$v_{\tilde{u}} = \begin{pmatrix} 1 \\ -\frac{4}{3}\frac{(3+2)\Lambda^2}{K_0} \end{pmatrix}$


$t x^{\lambda t} = \left(t^{1/\lambda t} x\right)^{\lambda t} = \left(x/\xi\right)^{\lambda t}$
$\xi \sim t^{-1/\lambda t} \sim t^{-\nu}$
$F(x/\xi)$
$= G(1, t x^{\lambda t}, 0)$
$\nu = \frac{1}{\lambda t} = \frac{1}{2 - \frac{\varepsilon}{3}} \approx \frac{1}{2} + \frac{\varepsilon}{12}$


$t\,x^{\lambda t} = \left(t^{1/\lambda t}\, x\right)^{\lambda t} = \left(x/\xi\right)^{\lambda t}$
$\xi \sim t^{-1/\lambda t} \sim t^{-\nu}$
$F(x/\xi)$
$= G(1, t x^{\lambda t}, 0)$
$\nu = \frac{1}{\lambda t} = \frac{1}{2 - \frac{\varepsilon}{3}} \approx \frac{1}{2} + \frac{\varepsilon}{12}$
$d=3$ ISING
$\nu = 0.63$
$\nu_{RG} \simeq 0.58$
NUMERICS