Title: PSI 2016/2017 Statistical Mechanics - Lecture 6

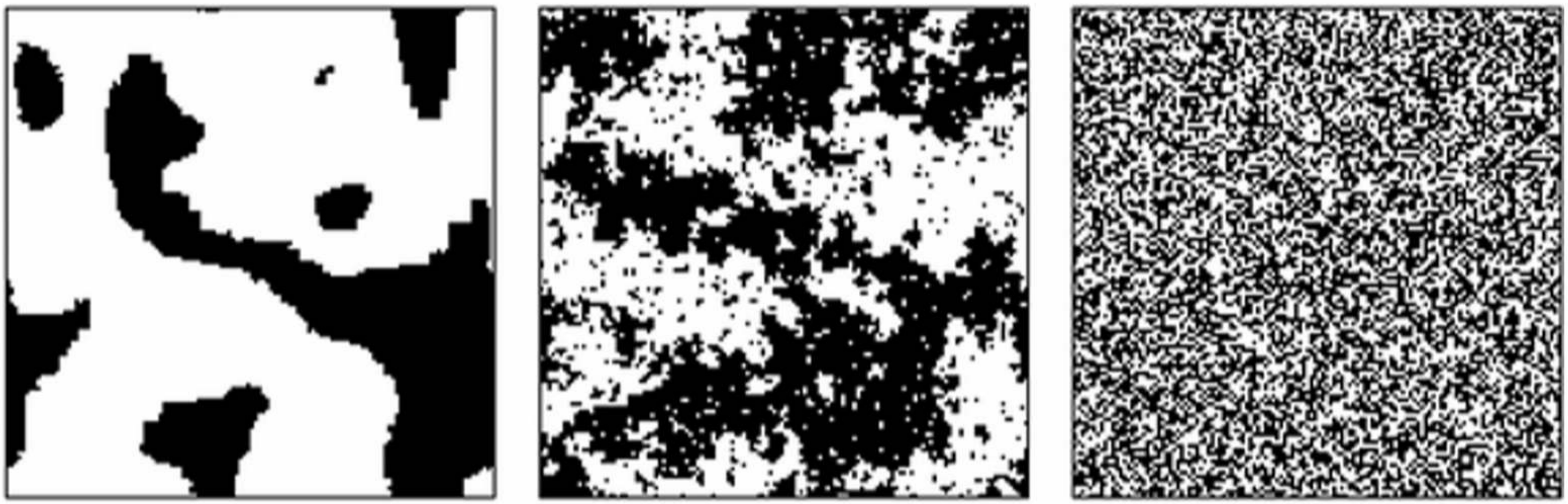
Date: Oct 18, 2016 10:45 AM

URL: http://pirsa.org/16100020

Abstract:

ISING MODEL: ordering

At hot temperature the spins are **randomly configured**, at low temperature they are close to an entirely **ordered** state, and at critical temperature they have a **fractal configuration**.



T<Tc **T=Tc** **T>Tc**

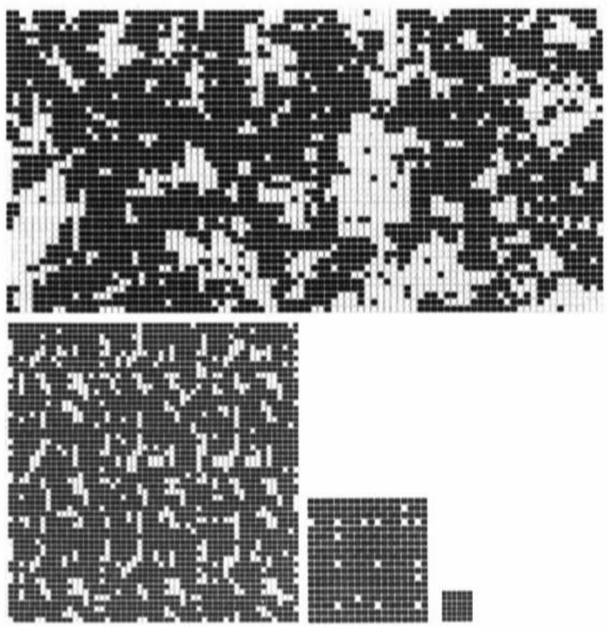
At Tc: scale invariant (fractal) structure

https://brainnetworkdynamics.wordpress.com/research/criticality/

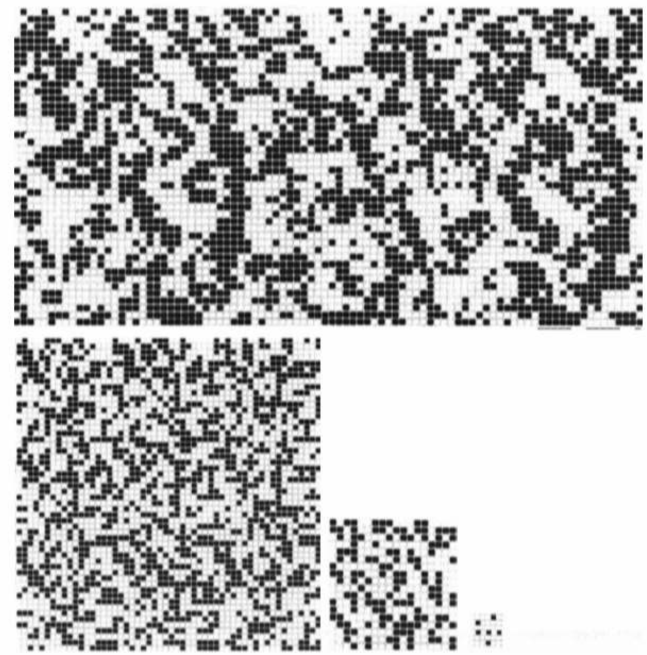
See also: https://www.youtube.com/watch?v=kjwKgpQ-I1s

Block Spin RG

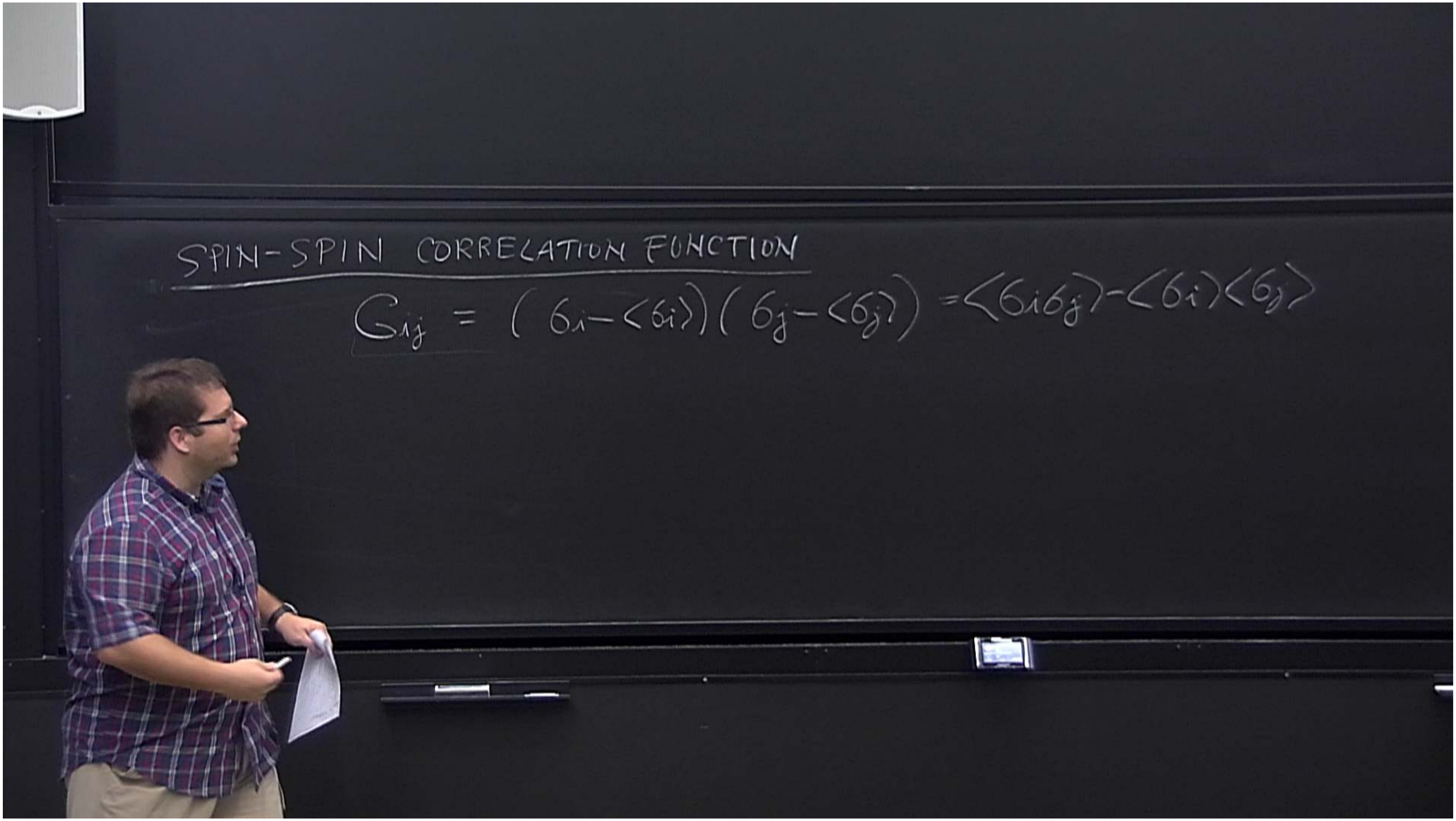
Below Tc (M=M0)

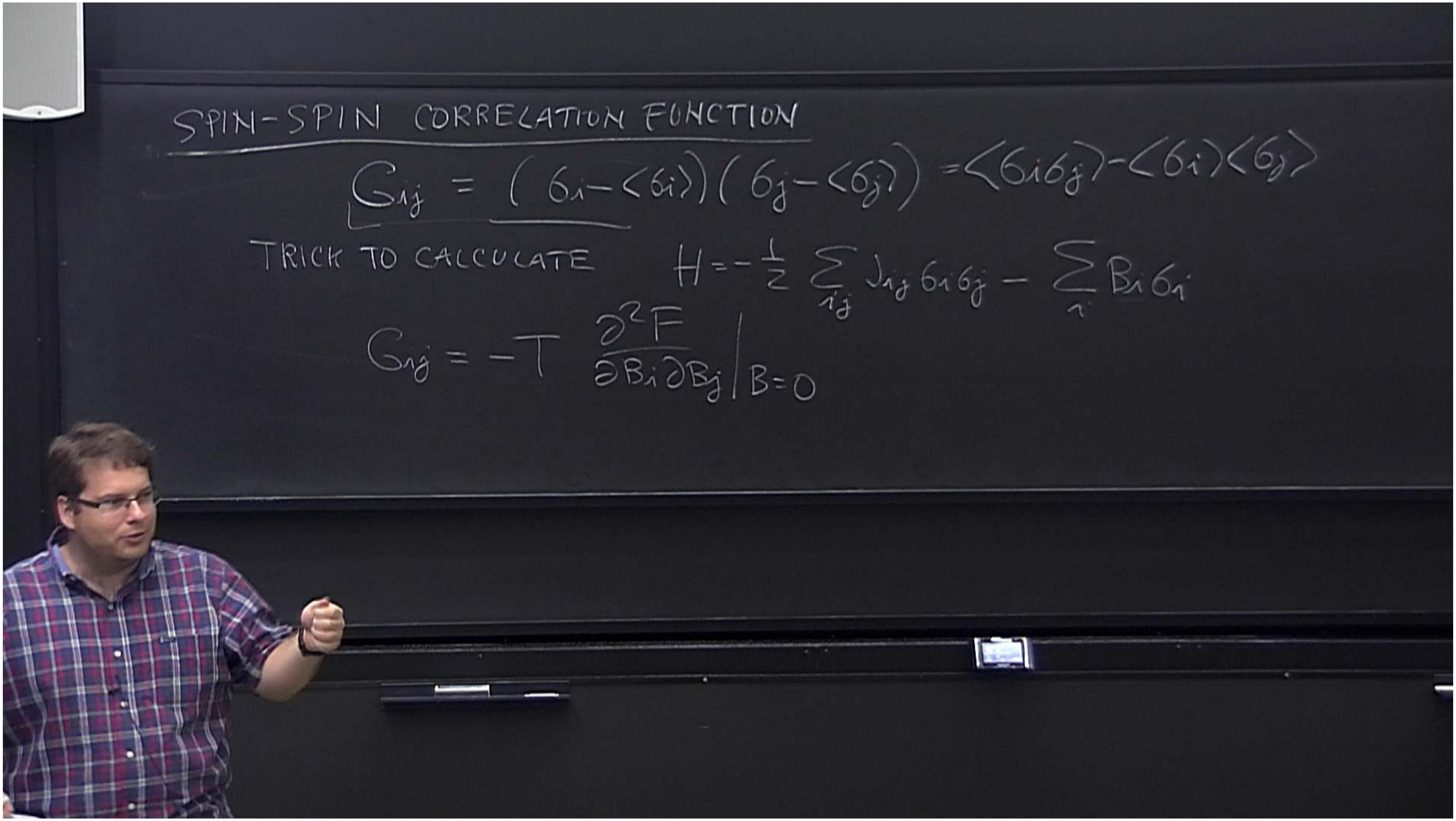


Above Tc (M=0)



https://plus.maths.org/content/going-flow-0


SPIN-SPIN CORRELATION FUNCTION
$G_{ij} = (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$

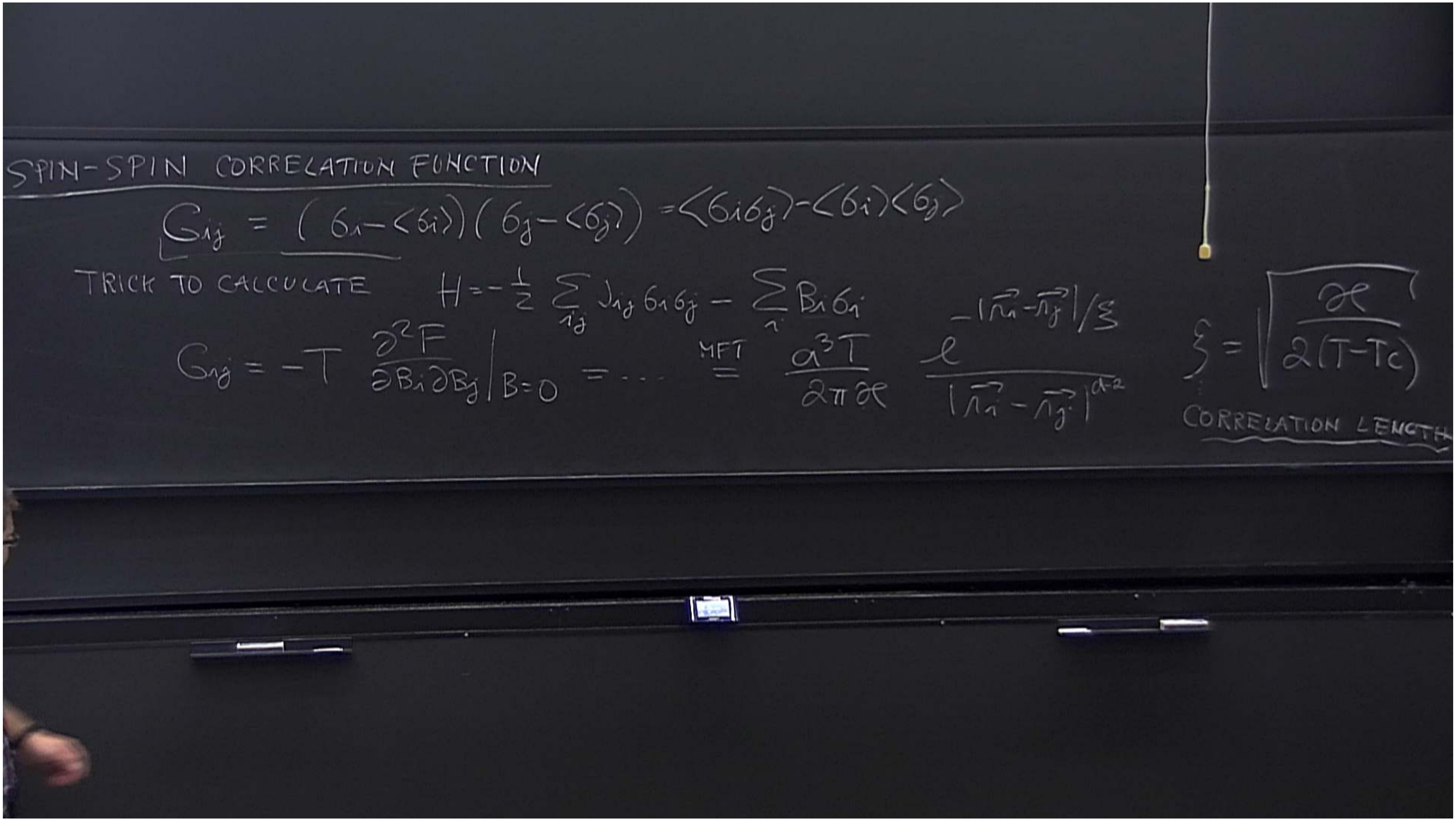

SPIN-SPIN CORRELATION FUNCTION
$G_{ij} = \overline{(\sigma_i - \langle\sigma_i\rangle)(\sigma_j - \langle\sigma_j\rangle)} = \langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$
TRICK TO CALCULATE
$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i B_i\sigma_i$
$G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0}$

SPIN-SPIN CORRELATION FUNCTION

$$G_{ij} = (\sigma_i - \langle\sigma_i\rangle)(\sigma_j - \langle\sigma_j\rangle) = \langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$$

TRICK TO CALCULATE $H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i B_i\sigma_i$

$$G_{ij} = -T\,\frac{\partial^2 F}{\partial B_i \partial B_j}\bigg|_{B=0} = \dots \overset{MFT}{=} \frac{a^3 T}{2\pi\varkappa}\,\frac{e^{-|\vec{r}_i - \vec{r}_j|/\xi}}{|\vec{r}_i - \vec{r}_j|}$$


SPIN-SPIN CORRELATION FUNCTION
$G_{ij} = (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$
TRICK TO CALCULATE
$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - \sum_i B_i \sigma_i$
$G_{ij} = -T \left. \frac{\partial^2 F}{\partial B_i \partial B_j} \right|_{B=0} = \dots \overset{MFT}{=} \frac{a^3 T}{2\pi \chi} \frac{e^{-|\vec{r}_i - \vec{r}_j|/\xi}}{|\vec{r}_i - \vec{r}_j|^{d-2}}$
$\xi = \sqrt{\frac{\chi}{2(T-T_c)}}$
CORRELATION LENGTH

CTION

$$)(\sigma_j - \langle\sigma_j\rangle) = \langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$$

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i B_i\sigma_i$$

$$B_j\Big|_{B=0} = \ldots \overset{MFT}{=} \frac{a^3 T}{2\pi \varkappa} \frac{e^{-|\vec{r}_i - \vec{r}_j|/\xi}}{|\vec{r}_i - \vec{r}_j|^{d-2}}$$

$\varkappa$: SPIN STIFFNESS

a: i to $i+1$

$$\xi = \sqrt{\frac{\varkappa}{2(T-T_c)}}$$

CORRELATION LENGTH

$$G_{ij} = -T \frac{\partial^2 \ldots}{\partial B_i \partial B_j}\Big|_{B=0} = \ldots \overset{MFT}{=} \frac{a^2 T}{2\pi \varkappa} \frac{\ell}{|\vec{r}_i - \ldots}$$

$i \xrightarrow{a} i+1$

SPIN STIFFNESS

MORE GENERALLY

$$\boxed{G(r) \propto \frac{e^{-r/\xi}}{r^{d-2+\eta}}}$$

$$\xi \propto |t|^{-\nu}$$

$$G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0} = \dots \overset{MFT}{=} \frac{a^2 T}{2\pi \varkappa} \frac{\ell}{|\vec{r}_i - \dots}$$

$i \xrightarrow{a} i+1$

$\varkappa$ ... SPIN STIFFNESS

MORE GENERALLY:

$$\boxed{G(r) \propto \frac{e^{-r/\xi}}{r^{d-2+\eta}}} \qquad \xi \propto |t|^{-\nu}$$

η ... ANOMALOUS DIMENSION CRIT. EXPONENT

ν ... CORREL. LENGTH CRIT. EXP.

$G_{ij}\big|_{B=0} = \ldots \overset{MFT}{=} \frac{a^{?}T}{2\pi\varkappa}\,\frac{1}{|\vec{r}_i - \vec{r}_j|^{d-2}}$

a: r_1 to r_{1+1}

$\varkappa$: SPIN STIFFNESS

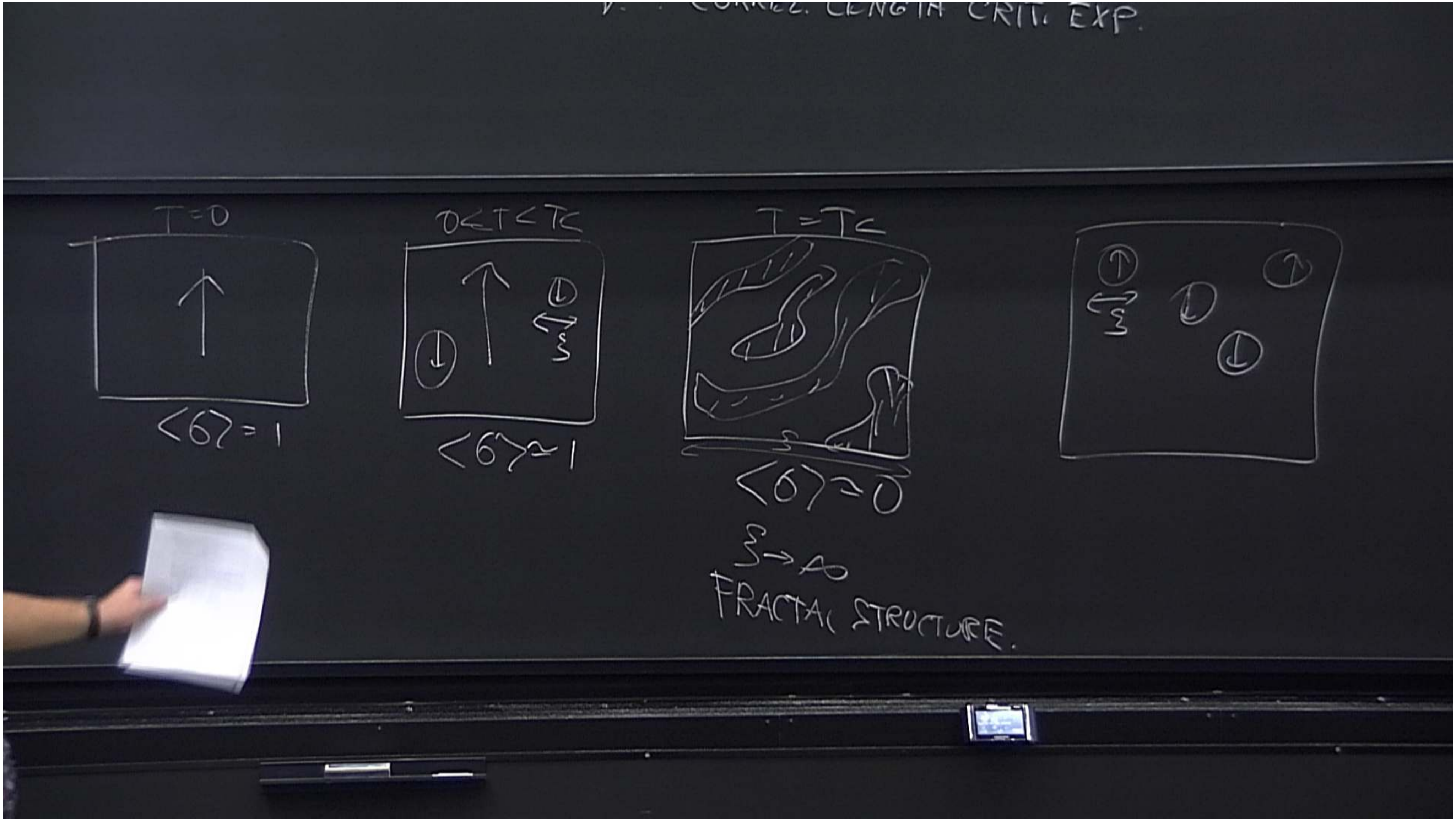
$\xi = \sqrt{2(T-T_c)}$ ...

CORRELATION LENGTH

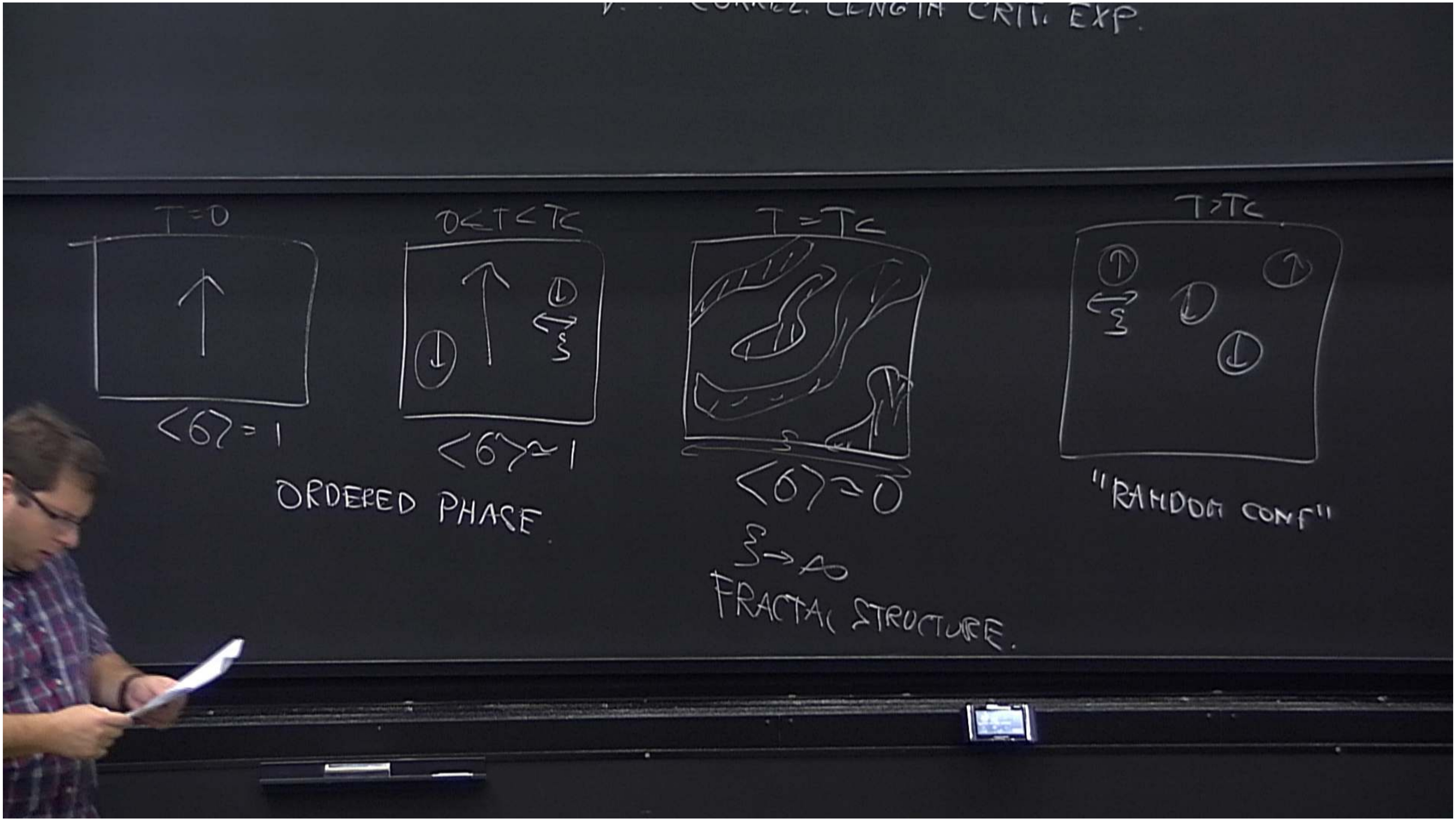
$$\propto \boxed{\frac{e^{-r/\xi}}{r^{d-2+\eta}}} \qquad \boxed{\xi \propto |t|^{-\nu}}$$

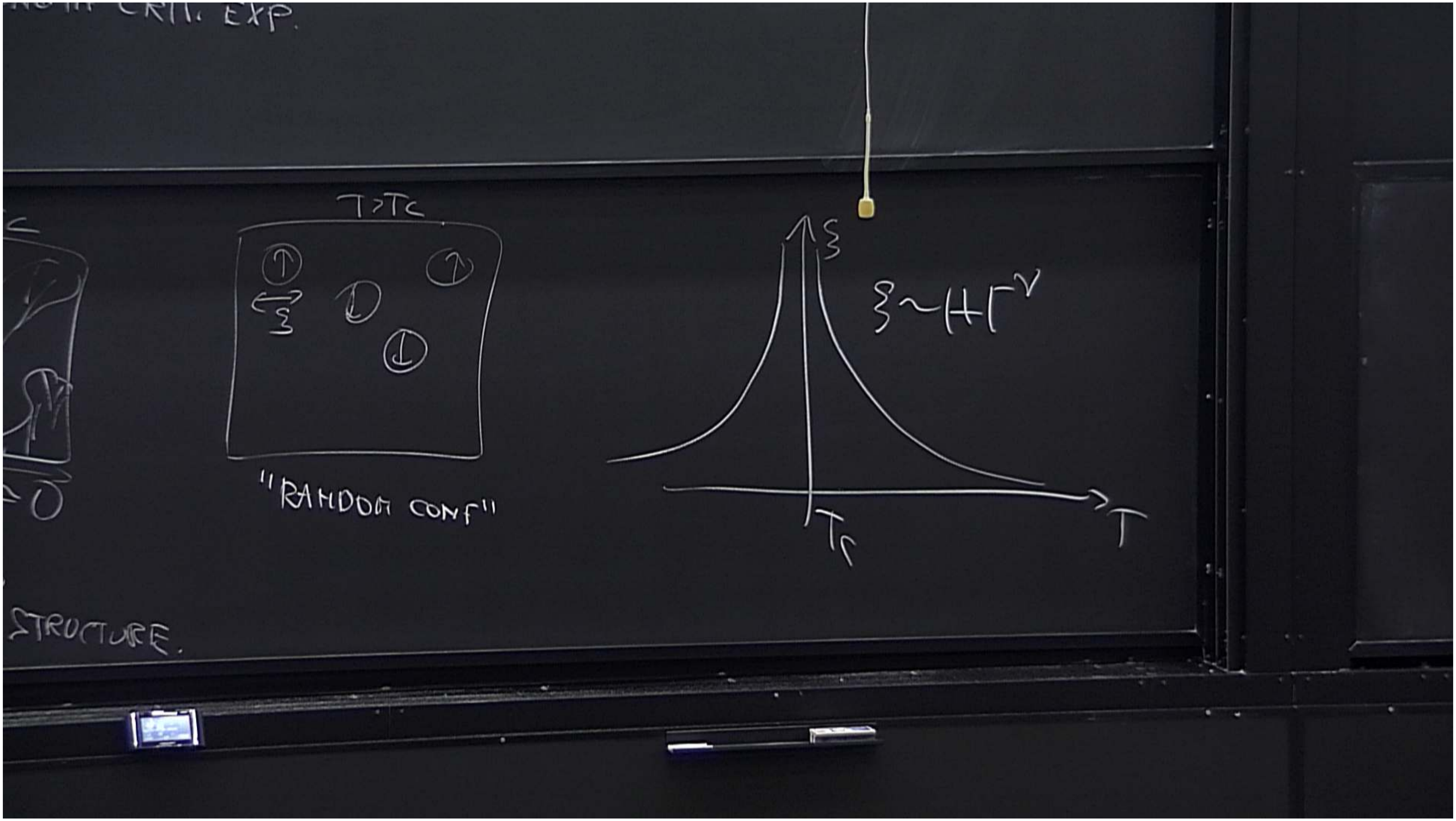
MFT: $\boxed{\begin{array}{l}\nu = \frac{1}{2} \\ \eta = 0\end{array}}$

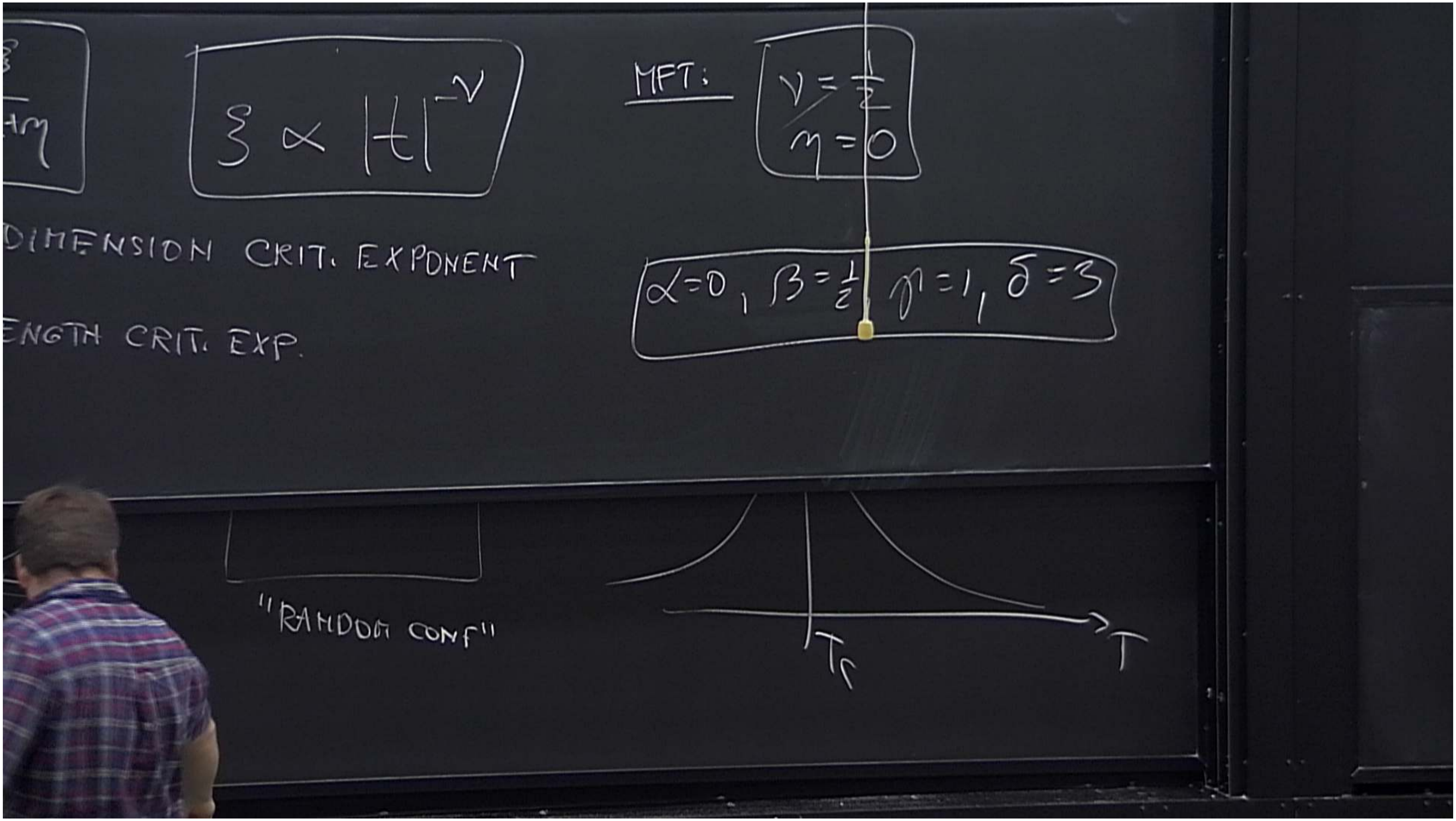
- ANOMALOUS DIMENSION CRIT. EXPONENT
- CORREL. LENGTH CRIT. EXP.

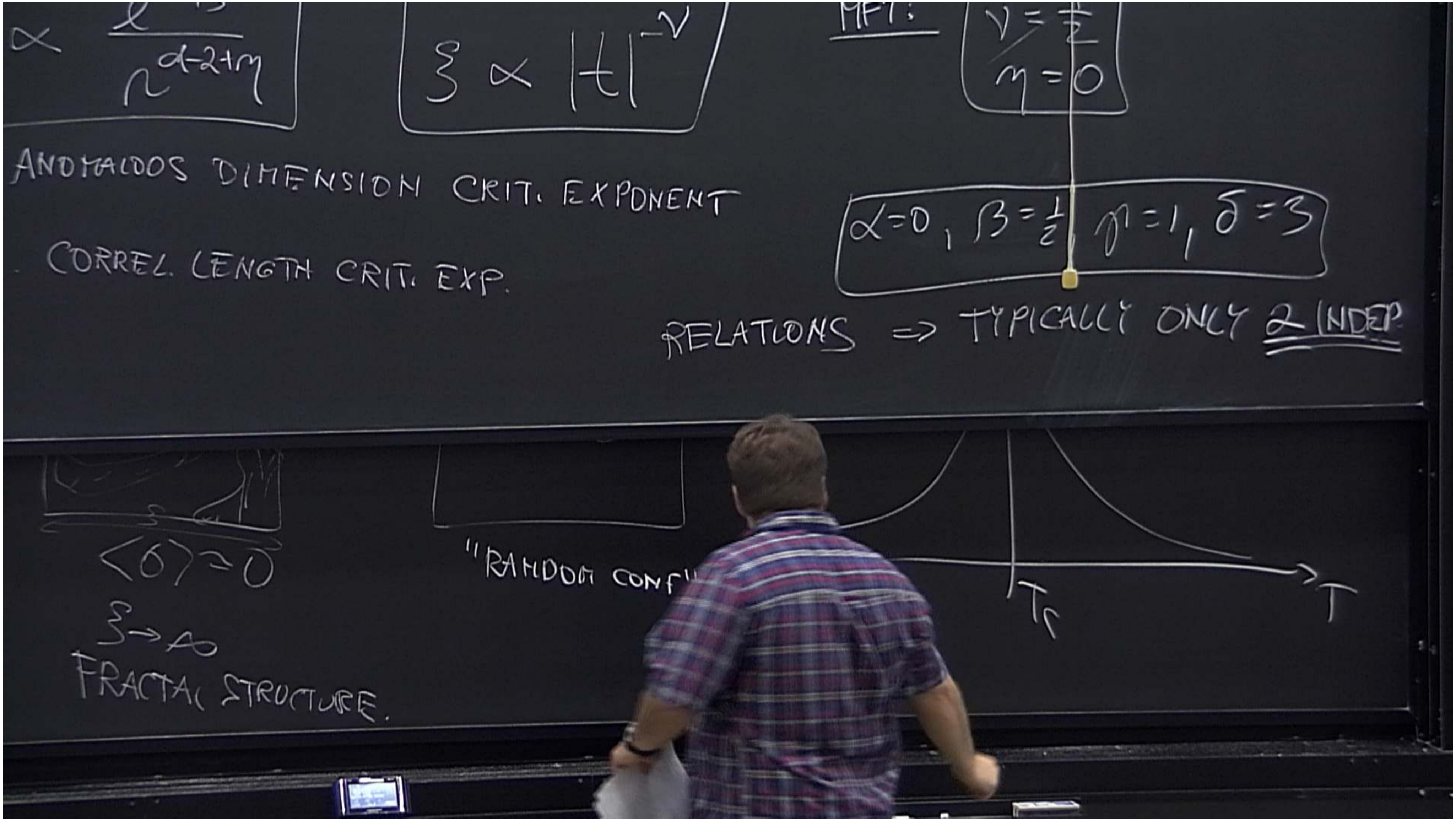


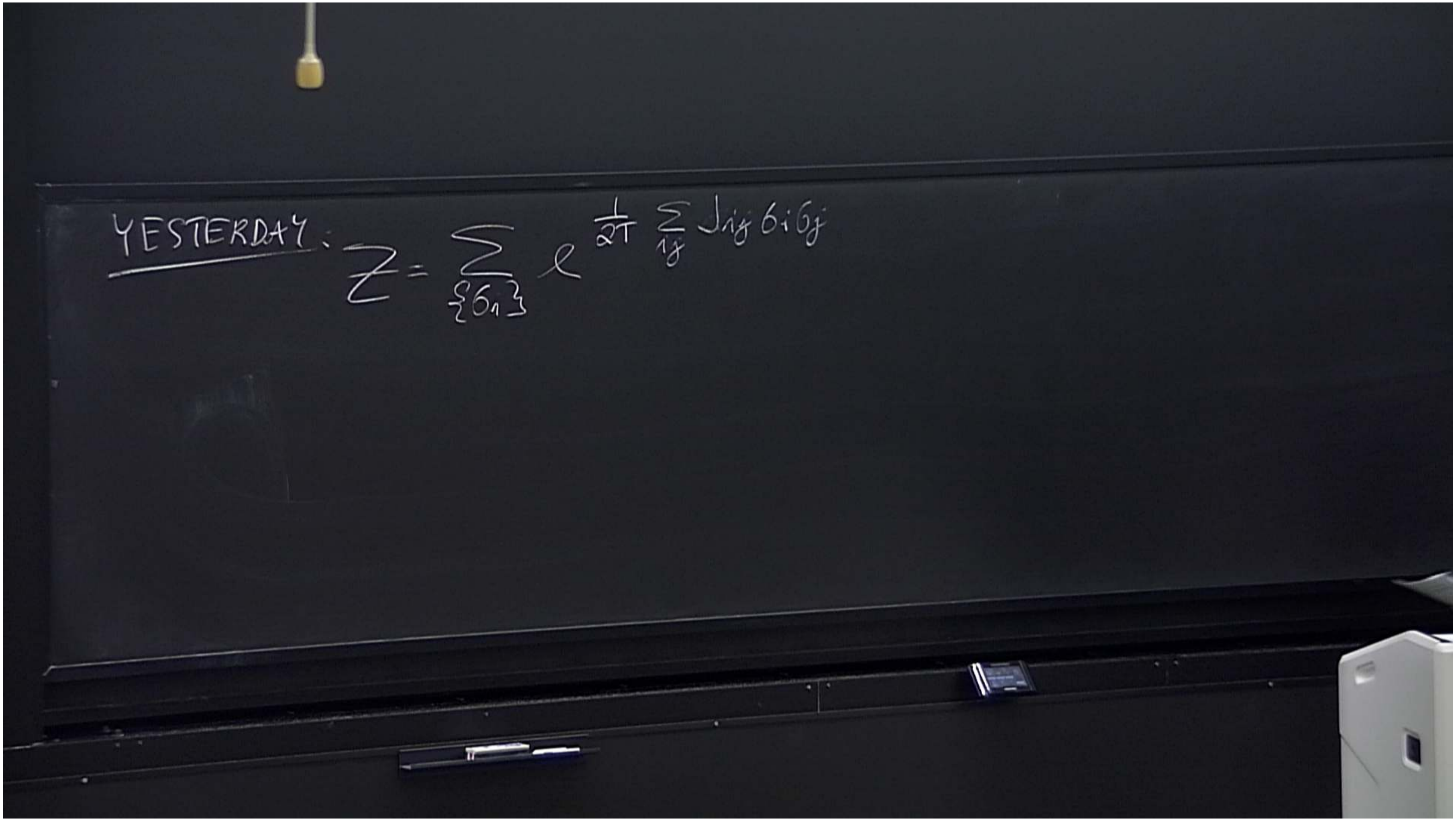
LENGTH CRIT. EXP.
T=0
<σ>=1
0<T<Tc
<σ>≃1
T=Tc
<σ>≃0
ξ→∞
FRACTAL STRUCTURE.


LENGTH CRIT. EXP.
T=0
0<T<Tc
T=Tc
T>Tc
<σ>=1
<σ>≃1
ORDERED PHASE
<σ>≃0
ξ→∞
FRACTAL STRUCTURE
"RANDOM CONF"


CRIT. EXP.
T>Tc
ξ
"RANDOM CONF"
ξ
ξ~|t|^-ν
Tc
T
STRUCTURE.


$\xi \propto |t|^{-\nu}$
MFT:
$\nu = \frac{1}{2}$
$\eta = 0$
DIMENSION CRIT. EXPONENT
ENGTH CRIT. EXP.
$\alpha = 0,\ \beta = \frac{1}{2},\ \gamma = 1,\ \delta = 3$
"RANDOM CONF"
T_c
T


$\propto \frac{\ell}{r^{d-2+\eta}}$
$\xi \propto |t|^{-\nu}$
MFT:
$\nu = \frac{1}{2}$
$\eta = 0$
ANOMALOUS DIMENSION CRIT. EXPONENT
CORREL. LENGTH CRIT. EXP.
$\alpha = 0, \beta = \frac{1}{2}, \gamma = 1, \delta = 3$
RELATIONS ⇒ TYPICALLY ONLY 2 INDEP.
$\langle \sigma \rangle = 0$
$\xi \to \infty$
FRACTAL STRUCTURE.
"RANDOM CONF
T_c
T


YESTERDAY:
$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T} \sum_{ij} J_{ij} \sigma_i \sigma_j}$

$\sigma_i \sigma_j$

$$e^{\frac{1}{2}\rho_i A_{ij} \rho_j} = \frac{1}{\sqrt{\det \hat{A}}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} d\vec{\varphi}\, e^{-\frac{1}{2}\varphi_i A^{-1}_{ij} \varphi_j + \rho_i \varphi_i}$$

$\sigma_i \sigma_j$

$$e^{\frac{1}{2} \gamma_i A_{ij} \gamma_j} = \frac{1}{\sqrt{\det \hat{A}}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} d\vec{\varphi}\, e^{-\frac{1}{2}\varphi_i A^{-1}_{ij} \varphi_j + \gamma_i \varphi_i}$$

$$A_{ij} = \frac{J_{ij}}{T}, \quad \gamma_i = \sigma_i \qquad \varphi_i \to \varphi_i / T$$

YESTERDAY:

$$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j}$$

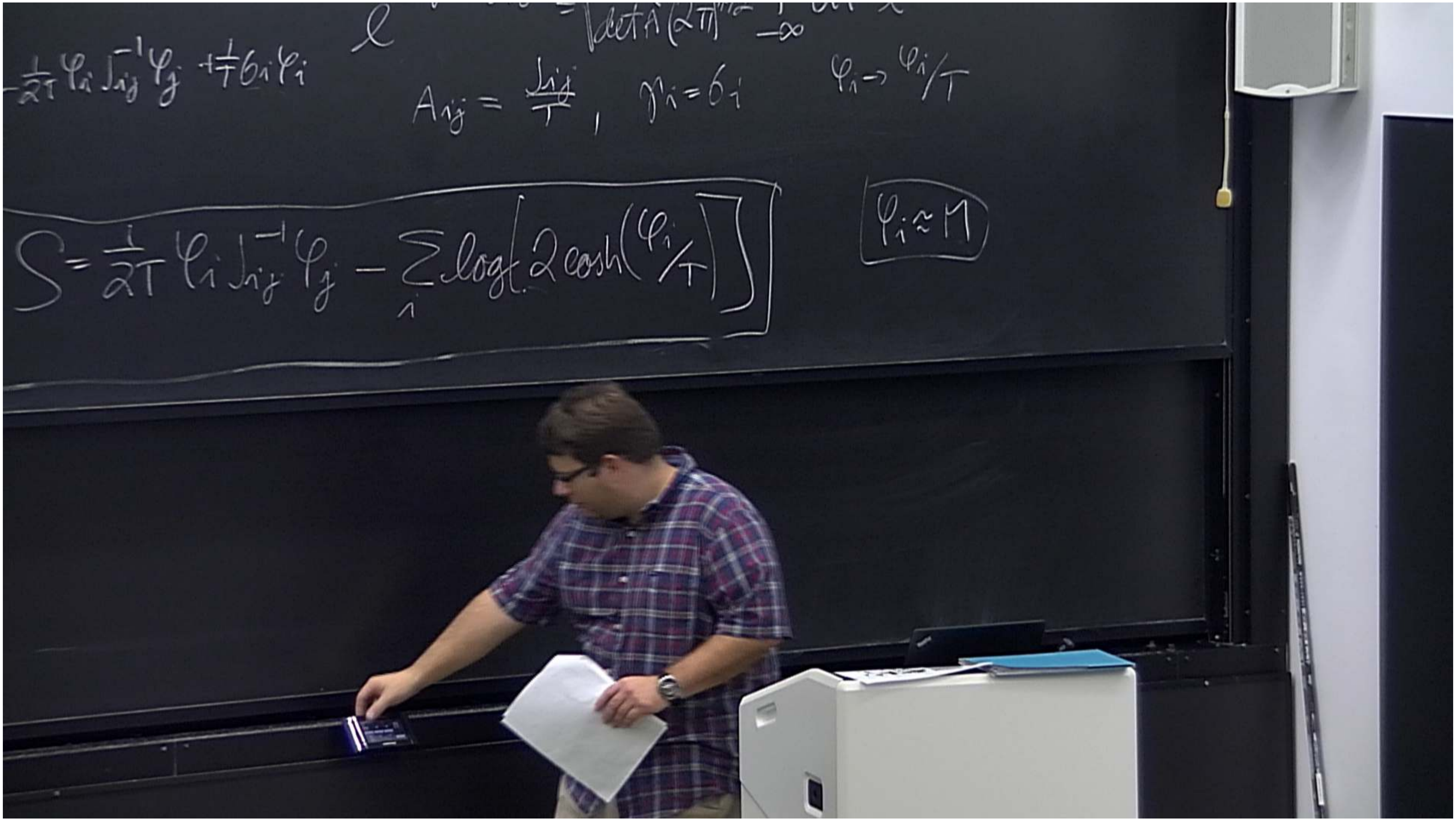
$$= \sum_{\{\sigma_i\}} \int_{-\infty}^{\infty} D\vec{\varphi}\; e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j + \frac{1}{T}\sigma_i\varphi_i}$$

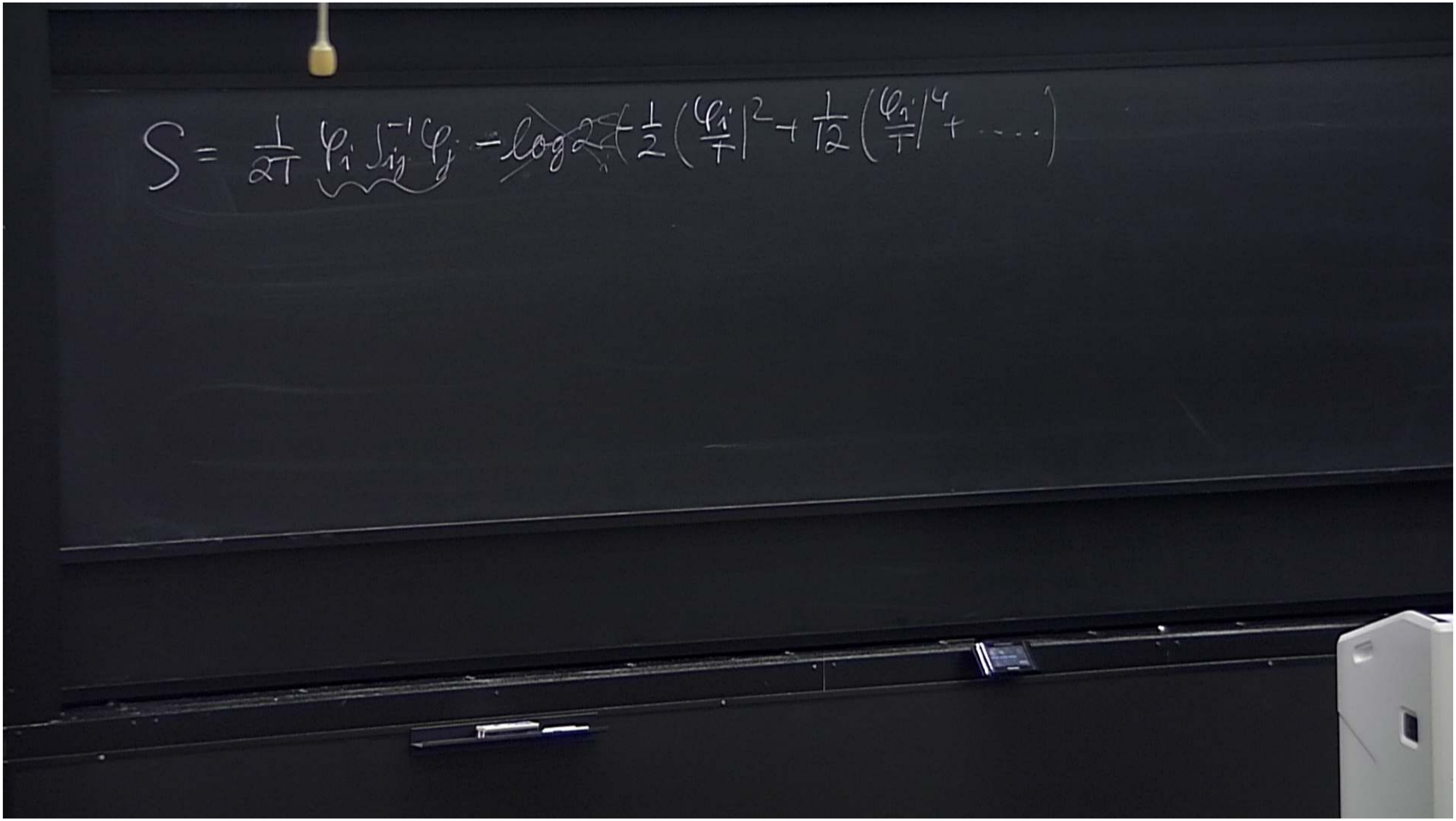
$$Z = \sum_{\{\sigma_i\}} e \quad \cdots \quad e \quad \cdots \sqrt{\det A\,(2\pi)} \quad -\infty$$

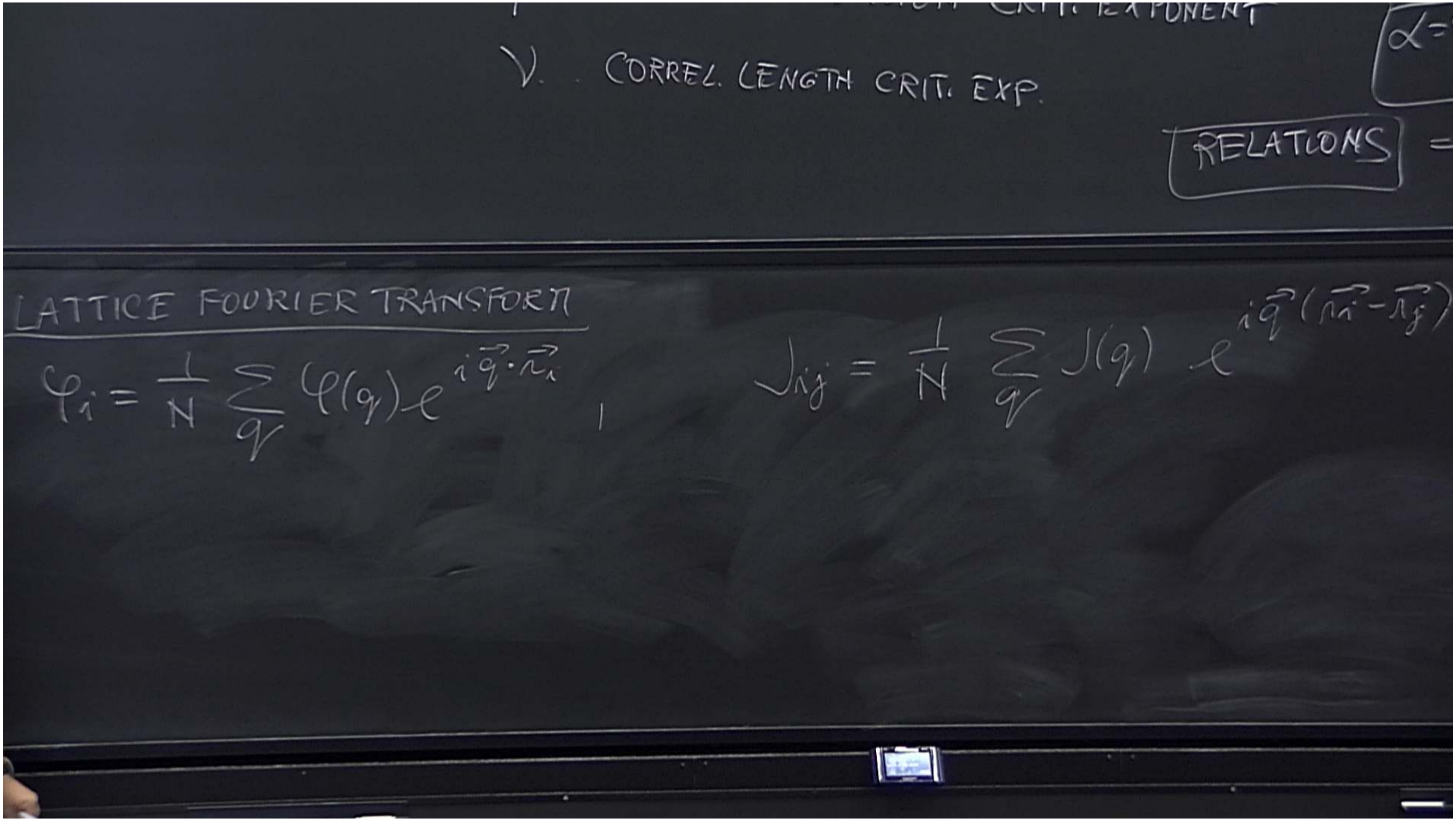
$$= \sum_{\{\sigma_i\}} \int_{-\infty}^{\infty} D\vec{\varphi}\, e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j + \frac{1}{T}\sigma_i\varphi_i}$$

$$A_{ij} = \frac{J_{ij}}{T}, \quad \gamma_i = \sigma_i$$

$$\boxed{Z = \int D\varphi\, e^{-S[\varphi]}, \quad S = \frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \log\left[2\cosh\left(\varphi_i/T\right)\right]}$$


$-\frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j + \frac{1}{T}\sigma_i\varphi_i$
$A_{ij} = \frac{J_{ij}}{T}, \quad \gamma_i = \sigma_i \qquad \varphi_i \to \varphi_i/T$
$S = \frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \log\left[2\cosh\left(\varphi_i/T\right)\right]$
$\varphi_i \approx M$


$S = \frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j - \cancel{\log 2} + \frac{1}{2}\left(\frac{\varphi_i}{T}\right)^2 + \frac{1}{12}\left(\frac{\varphi_i}{T}\right)^4 + \dots\Big)$



ν . CORREL. LENGTH CRIT. EXP.
RELATIONS
LATTICE FOURIER TRANSFORM
$\varphi_i = \frac{1}{N} \sum_q \varphi(q)\, e^{i \vec{q} \cdot \vec{r}_i}$
$J_{ij} = \frac{1}{N} \sum_q J(q)\, e^{i \vec{q}(\vec{r}_i - \vec{r}_j)}$

η .. ANOMALOUS DIMENSION CRIT. EXPONENT

ν .. CORREL. LENGTH CRIT. EXP.

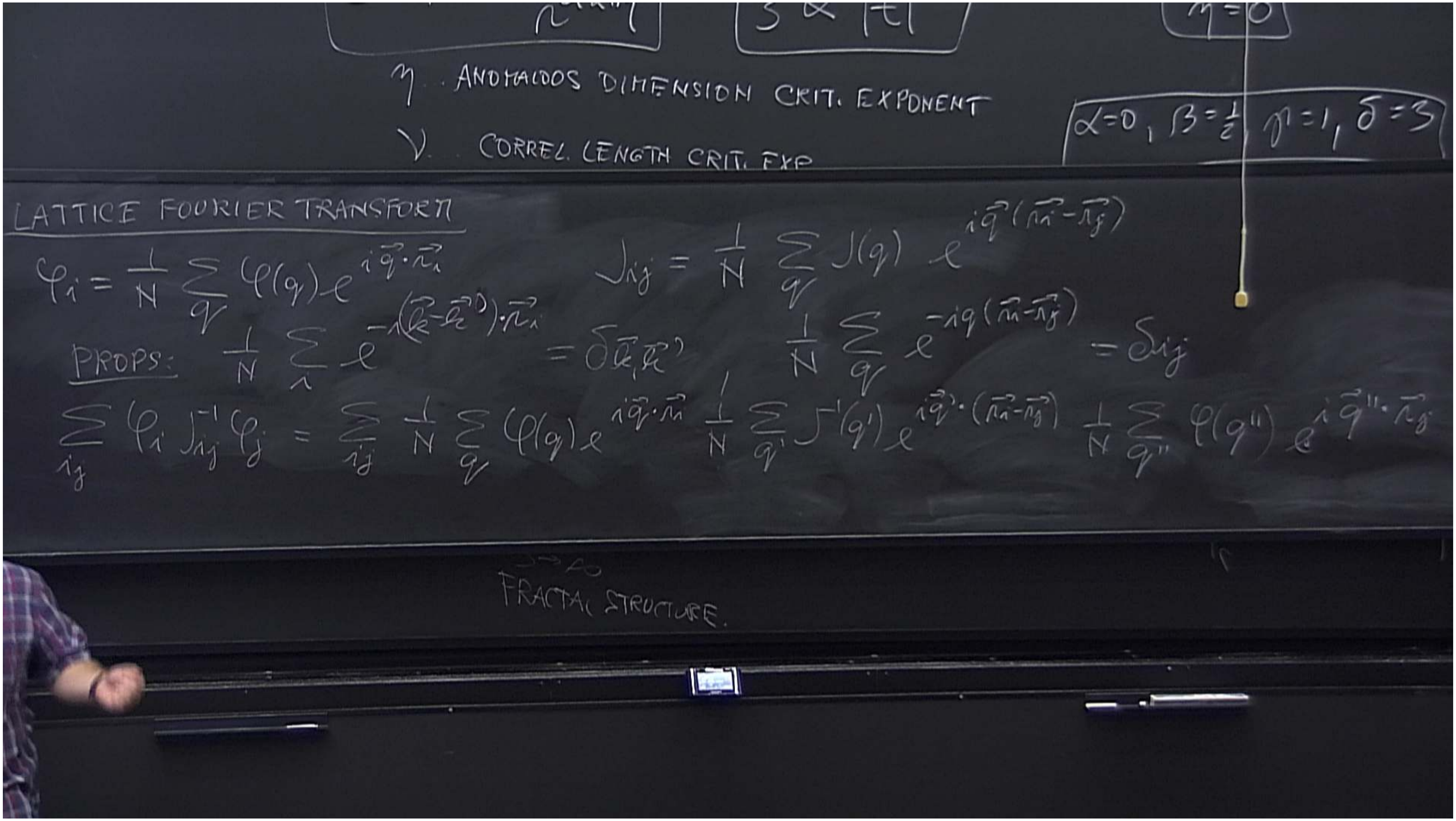
$\boxed{\alpha = 0 \,,\ \beta = \ldots}$

$\boxed{\text{RELATIONS}} \Rightarrow$ TYPIC

LATTICE FOURIER TRANSFORM

$$\varphi_i = \frac{1}{N} \sum_q \varphi(q) e^{i\vec{q}\cdot\vec{r}_i} \qquad J_{ij} = \frac{1}{N} \sum_q J(q)\, e^{i\vec{q}(\vec{r}_i - \vec{r}_j)}$$

PROPS: $\frac{1}{N} \sum_i e^{-i(\vec{k}-\vec{k}')\cdot\vec{r}_i} = \delta_{\vec{k},\vec{k}'} \,, \quad \frac{1}{N} \sum_q e^{-iq(\vec{r}_i - \vec{r}_j)} = \delta_{ij}$


η . ANOMALOUS DIMENSION CRIT. EXPONENT
ν . CORREL. LENGTH CRIT. EXP
η = 0
α = 0, β = 1/2, γ = 1, δ = 3
LATTICE FOURIER TRANSFORM
$\varphi_i = \frac{1}{N} \sum_q \varphi(q) e^{i\vec{q}\cdot\vec{r}_i}$
$J_{ij} = \frac{1}{N} \sum_q J(q) e^{i\vec{q}(\vec{r}_i - \vec{r}_j)}$
PROPS:
$\frac{1}{N} \sum_i e^{-i(\vec{k}-\vec{k}')\cdot\vec{r}_i} = \delta_{\vec{k},\vec{k}'}$
$\frac{1}{N} \sum_q e^{-iq(\vec{r}_i - \vec{r}_j)} = \delta_{ij}$
$\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j = \sum_{ij} \frac{1}{N} \sum_q \varphi(q) e^{i\vec{q}\cdot\vec{r}_i} \frac{1}{N} \sum_{q'} J^{-1}(q') e^{i\vec{q}'\cdot(\vec{r}_i - \vec{r}_j)} \frac{1}{N} \sum_{q''} \varphi(q'') e^{i\vec{q}''\cdot\vec{r}_j}$
FRACTAL STRUCTURE.

$$\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j = \sum_{ij} \frac{1}{N} \sum_{q} \varphi(q) e^{i\vec{q}\cdot\vec{r}_i} \frac{1}{N} \sum_{q'} J^{-1}(q') e^{i\vec{q}'\cdot(\vec{r}_i - \vec{r}_j)} \frac{1}{N} \sum_{q''} \varphi($$

$$\sum_i \to \delta_{q,q'} \qquad \sum_j \to \delta_{q',q''}$$

$$= \frac{1}{N} \sum_{q,q',q''} \delta_{q,q'} \delta_{q',q''} \varphi(q) J^{-1}(q') \varphi(q'')$$

PROPS: $\frac{1}{N}\sum_{i} e^{\cdots} = \delta_{k,k'}$ $\quad \frac{1}{N}\sum_{q} e^{\cdots} = \delta_{ij}$

$$\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j = \sum_{ij} \frac{1}{N}\sum_{q} \varphi(q)\, e^{i\vec{q}\cdot\vec{r}_i}\, \frac{1}{N}\sum_{q'} J^{-1}(q')\, e^{i\vec{q}'\cdot(\vec{r}_i-\vec{r}_j)}\, \frac{1}{N}\sum_{q''} \varphi(q'')$$

$$\sum_{i} \rightarrow \delta_{q,-q'} \qquad \sum_{j} \rightarrow \delta_{q',q''}$$

$$= \frac{1}{N}\sum_{q,q',q''} \delta_{q,-q'}\,\delta_{q',q''}\, \varphi(q)\, J^{-1}(q')\, \varphi(q'') = \frac{1}{N}\sum_{q} \varphi(q)\, J^{-1}(-q)\, \varphi(-q)$$

$$= \frac{1}{N}\sum_{q} \varphi(-q)\, J^{-1}(q)\, \varphi(q)$$

PROPS: $\frac{1}{N}\sum_{i} e^{\dots} = \delta_{k,k'}$ $\quad \frac{1}{N}\sum_{q} e^{\dots} = \delta_{ij}$

$$\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j = \sum_{ij} \frac{1}{N}\sum_{q} \varphi(q)\, e^{i\vec{q}\cdot\vec{r}_i}\, \frac{1}{N}\sum_{q'} J^{-1}(q')\, e^{i\vec{q}'\cdot(\vec{r}_i-\vec{r}_j)}\, \frac{1}{N}\sum_{q''} \varphi(q'')$$

$$\sum_{i} \to \delta_{q,q'} \qquad \sum_{j} \to \delta_{q',q''}$$

$$= \frac{1}{N}\sum_{q,q',q''} \delta_{q,q'}\,\delta_{q',q''}\, \varphi(q)\, J^{-1}(q')\, \varphi(q'') = \frac{1}{N}\sum_{q} \varphi(q)\, J^{-1}(-q)\, \varphi(-q)$$

$$= \frac{1}{N}\sum_{q} \varphi(-q)\, J^{-1}(q)\, \varphi(q)$$

NEED TO KNOW $J(q)$: CRITICAL POINT $\simeq$ "LARGE SCALE" $\simeq$ "SMALL MOMENTA"

$$\sum_i \to \delta_{q,-q'} \qquad \sum_j \to \delta_{q',q''}$$

$$= \frac{1}{N}\sum_{q,q',q''} \delta_{q,-q'}\,\delta_{q',q''}\,\varphi(q)\, J^{-1}(q')\,\varphi(q'') = \frac{1}{N}\sum_q \varphi(q)\, J^{-1}(-q)\,\varphi(-q)$$

$$= \frac{1}{N}\sum_q \varphi(-q)\, J^{-1}(q)\,\varphi(q)$$

NEED TO KNOW $J(q)$: CRITICAL POINT $\simeq$ "LARGE SCALE" $\simeq$ "SMALL MOMENTA"

$$J(q) = \frac{1}{N}\sum_{ij} J_{ij}\, e^{-i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)}$$

$$\simeq \frac{1}{N}\sum_{ij} J_{ij}\left(1 - i\vec{q}\cdot(\vec{r}_i-\vec{r}_j) - \frac{1}{2}\left[\vec{q}\cdot(\vec{r}_i-\vec{r}_j)\right]^2 + \dots\right)$$

$$S(q) = \frac{1}{N} \sum_{ij} J_{ij} e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

$q^2 R^2$

$$\approx \frac{1}{N} \sum_{ij} J_{ij} \left(1 - i\vec{q}\cdot(\vec{r}_i - \vec{r}_j) - \frac{1}{2}\left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2 + \dots \right]$$

$$\simeq \frac{1}{N} \sum_{ij} J_{ij} - \frac{1}{2N} \sum_{ij} J_{ij} \left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2$$

$$J(q) = \frac{1}{N}\sum_{ij} J_{ij}\, e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

SMALL MOMENTA

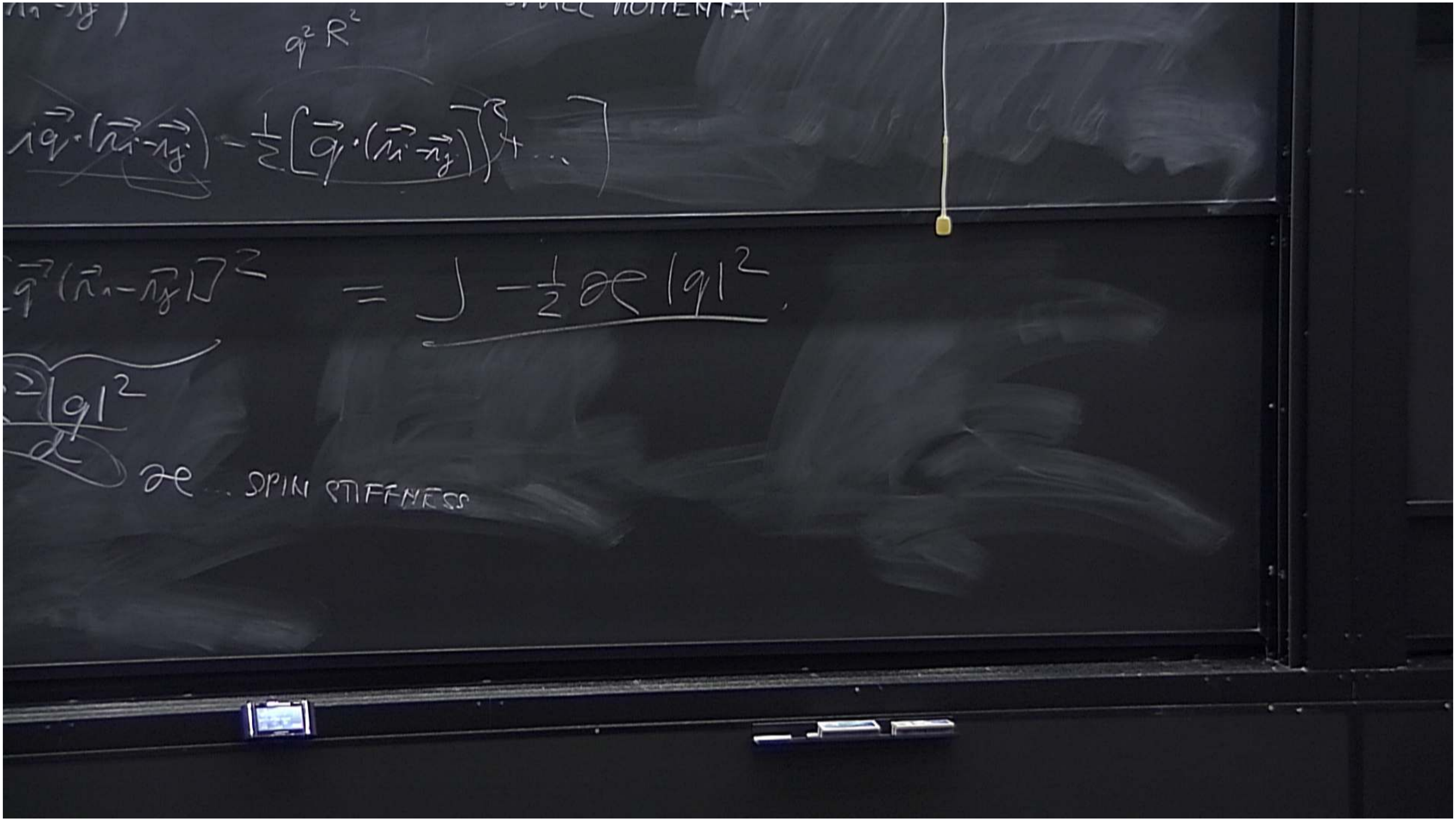
$q^2 R^2$

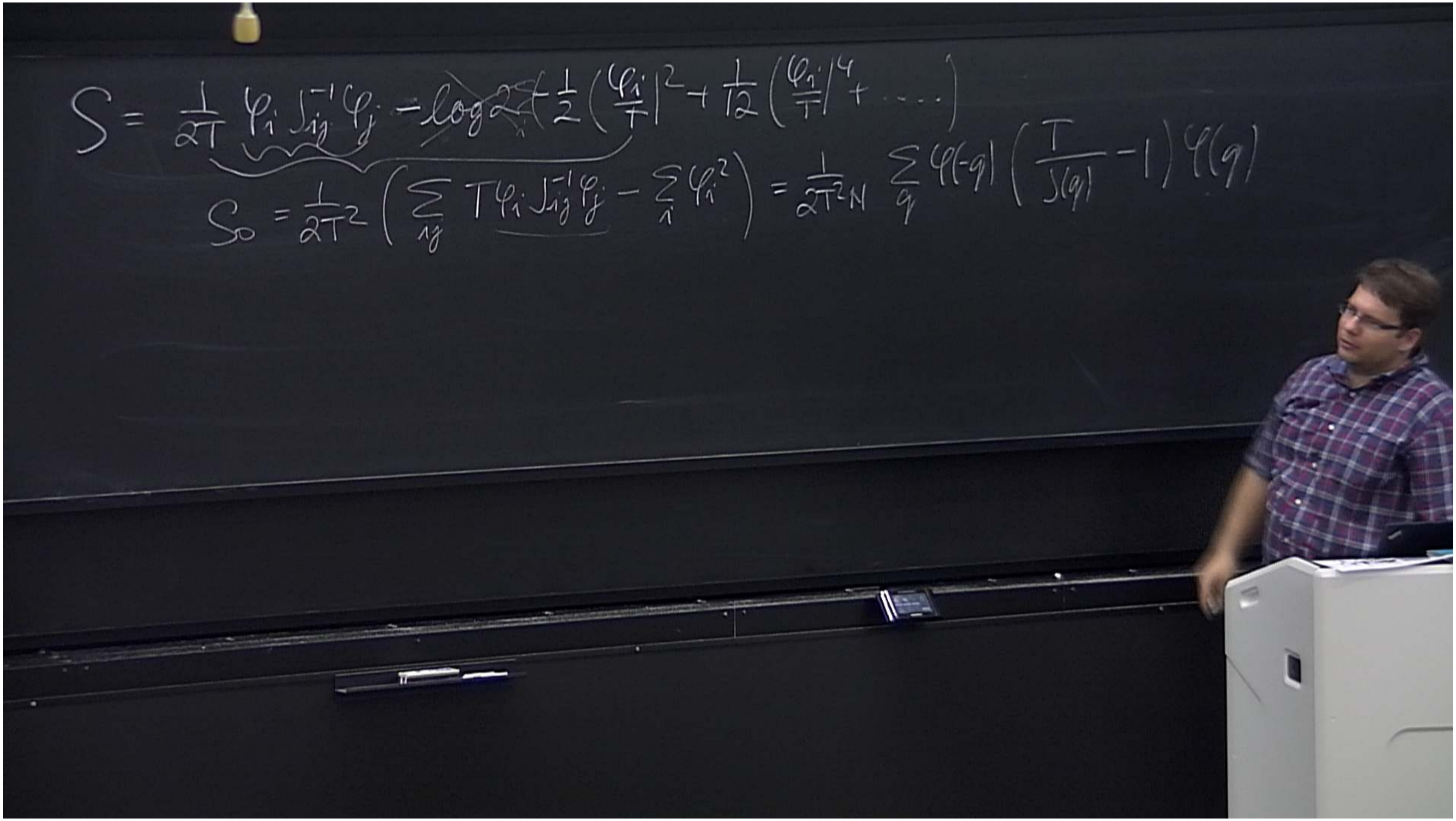
$$\approx \frac{1}{N}\sum_{ij} J_{ij}\left(1 - i\vec{q}\cdot(\vec{r}_i - \vec{r}_j) - \frac{1}{2}\left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2 + \dots\right)$$

$$\approx \underbrace{\underbrace{\frac{1}{N}\sum_{ij} J_{ij}}_{\sum_j J}}_{J} - \frac{1}{2N}\underbrace{\sum_{ij} J_{ij}\left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2}$$

$$-\frac{1}{2}\,\frac{J R^2 |q|^2}{d}$$

$\varkappa$ ... SPIN STIFFNESS


q² R²
= J − ½ æ |q|²
æ … SPIN STIFFNESS


$S = \frac{1}{2T} \varphi_i J^{-1}_{ij} \varphi_j \cancel{- \log 2} + \frac{1}{2} \left(\frac{\varphi_i}{T}\right)^2 + \frac{1}{12} \left(\frac{\varphi_i}{T}\right)^4 + \ldots$
$S_0 = \frac{1}{2T^2} \left(\sum_{ij} T \varphi_i J^{-1}_{ij} \varphi_j - \sum_i \varphi_i^2 \right) = \frac{1}{2T^2 N} \sum_q \varphi(-q) \left(\frac{T}{J(q)} - 1 \right) \varphi(q)$

$$S = \frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j - \cancel{\log 2}\left(-\frac{1}{2}\left(\frac{\varphi_i}{T}\right)^2 + \frac{1}{12}\left(\frac{\varphi_i}{T}\right)^4 + \dots\right)$$

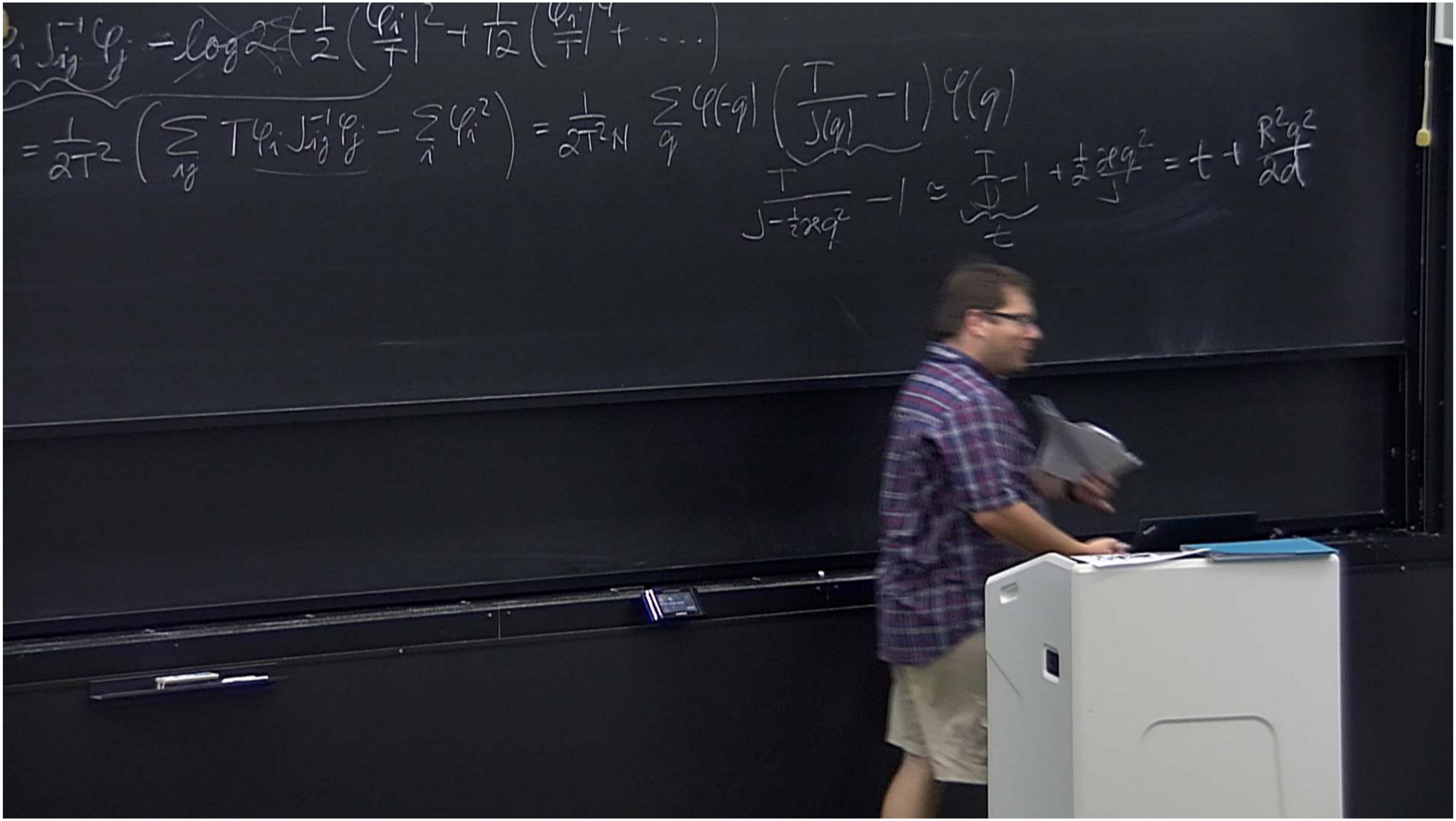
$$S_0 = \frac{1}{2T^2}\left(\sum_{ij} T\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N}\sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$$

$$\frac{T}{J - \frac{1}{2}x q^2} - 1 \simeq \frac{T}{J} - 1 + \frac{1}{2}\frac{x q^2}{J}$$

$$S = \frac{1}{2T}\,\varphi_i J^{-1}_{ij}\varphi_j - \cancel{\log 2} \left(-\frac{1}{2}\left(\frac{\varphi_i}{T}\right)^2 + \frac{1}{12}\left(\frac{\varphi_i}{T}\right)^4 + \dots \right)$$

$$S_0 = \frac{1}{2T^2}\left(\sum_{ij} T\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N}\sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$$

$$\frac{T}{J - \frac{1}{2}\kappa q^2} - 1 \simeq \underbrace{\frac{T}{J} - 1}_{t} + \frac{1}{2}\frac{\kappa q^2}{J}$$


$\varphi_i J^{-1}_{ij} \varphi_j - \log 2 + \frac{1}{2}\left(\frac{\varphi_i}{T}\right)^2 + \frac{1}{12}\left(\frac{\varphi_i}{T}\right)^4 + \dots)$
$= \frac{1}{2T^2}\left(\sum_{ij} T \varphi_i J^{-1}_{ij} \varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N} \sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$
$\frac{T}{J - \frac{1}{2}\kappa q^2} - 1 \simeq \frac{T}{J} - 1 + \frac{1}{2}\frac{\kappa q^2}{J} = t + \frac{R^2 q^2}{2d}$
t

$$S_0 = \frac{1}{2T^2}\left(\sum_{ij} T\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N}\sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$$

$$\frac{T}{J - \frac{1}{2}xq^2} - 1 \simeq \underbrace{\frac{T}{J} - 1}_{t} + \frac{1}{2}$$

$$\varphi \to \frac{T_c\sqrt{2d}}{R a^{d/2}}, \quad \boxed{r = \frac{2d\,t}{R^2}}$$

$$S_0 = \frac{1}{2}\frac{1}{Na^d}\sum_q \varphi(-q)\,(r+q^2)\,\varphi(q)$$

$$S_0 = \frac{1}{2T^2}\left(\sum_{ij} T\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N}\sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$$

$$\frac{T}{J - \frac{1}{2}x q^2} - 1 \simeq \underbrace{\frac{T}{J} - 1}_{t} + \frac{1}{2}\frac{x q^2}{J} = t + \frac{R^2 q^2}{2d}$$

$$\varphi \to \frac{T_c\sqrt{2d}}{R a^{d/2}}, \quad \boxed{r = \frac{2d\,t}{R^2}}$$

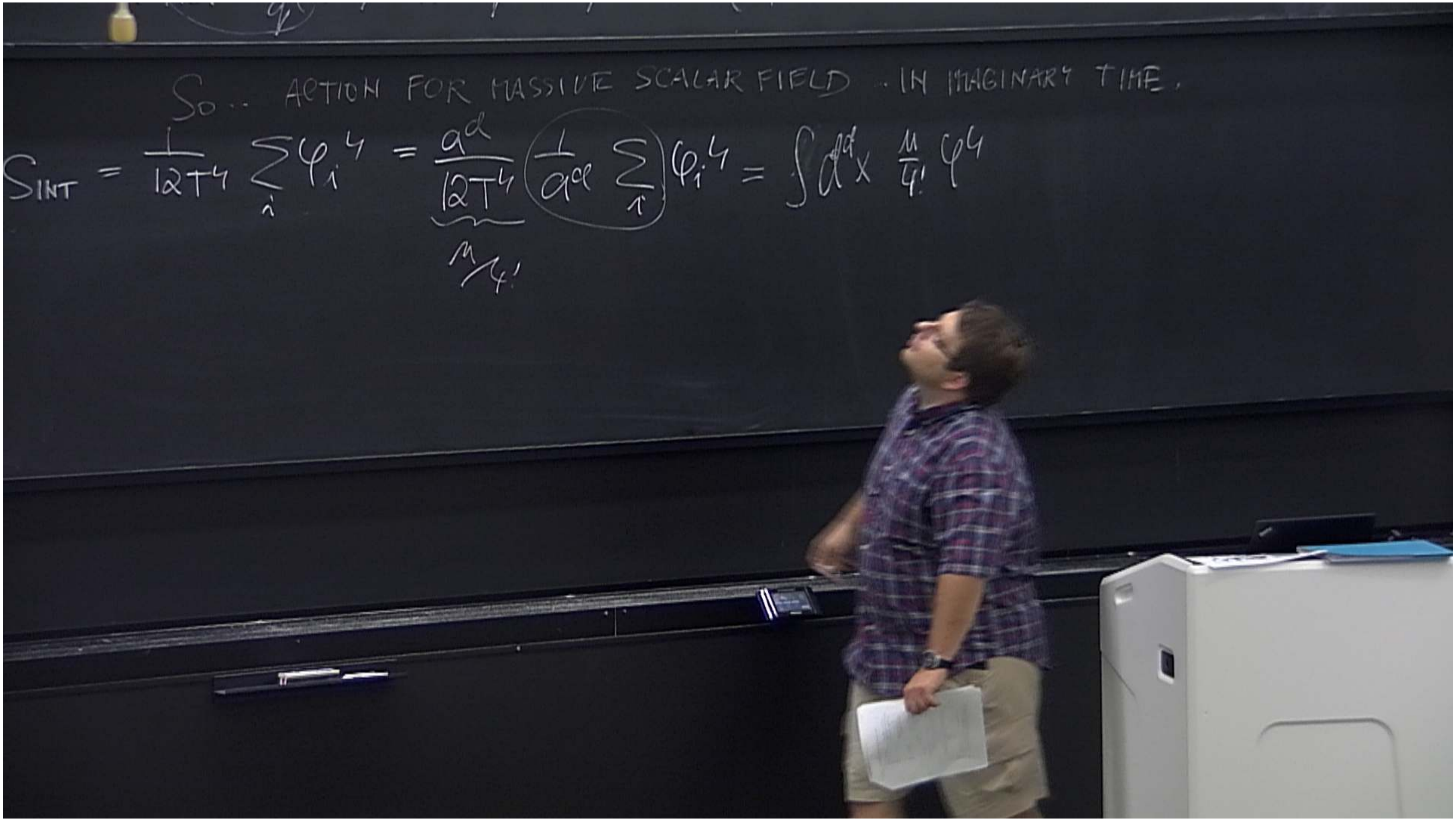
$$S_0 = \frac{1}{2}\frac{1}{N a^d}\sum_q \varphi(-q)\,(r+q^2)\,\varphi(q) = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\,\varphi(-q)\,(r+q^2)\,\varphi(q) =$$

$$S_0 = \frac{1}{2T^2}\left(\sum_{ij} T\varphi_i J^{-1}_{ij}\varphi_j - \sum_i \varphi_i^2\right) = \frac{1}{2T^2 N}\sum_q \varphi(-q)\left(\frac{T}{J(q)} - 1\right)\varphi(q)$$

$$\frac{T}{J - \frac{1}{2}\kappa q^2} - 1 \simeq \underbrace{\frac{T}{J} - 1}_{t} + \frac{1}{2}\frac{\kappa q^2}{J} = t + \frac{R^2 q^2}{2d}$$

$$\varphi \to \frac{T_c\sqrt{2d}}{R a^{d/2}}, \quad \boxed{r = \frac{2d\,t}{R^2}}$$

$$S_0 = \frac{1}{2}\frac{1}{Na^d}\sum_q \varphi(-q)\,(r+q^2)\,\varphi(q) = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\,\varphi(-q)\,(r+q^2)\,\varphi(q) = \frac{1}{2}\int d^dx\left(r\varphi^2 + (\nabla\varphi)^2\right)$$


SO... ACTION FOR MASSIVE SCALAR FIELD . IN IMAGINARY TIME.
$S_{INT} = \frac{1}{12T^4}\sum_i \varphi_i^4 = \underbrace{\frac{a^d}{12T^4}}_{\mu/4!}\left(\frac{1}{a^d}\sum_i\right)\varphi_i^4 = \int d^dx\, \frac{\mu}{4!}\varphi^4$

SO... ACTION FOR MASSIVE SCALAR FIELD. IN IMAGINARY TIME.

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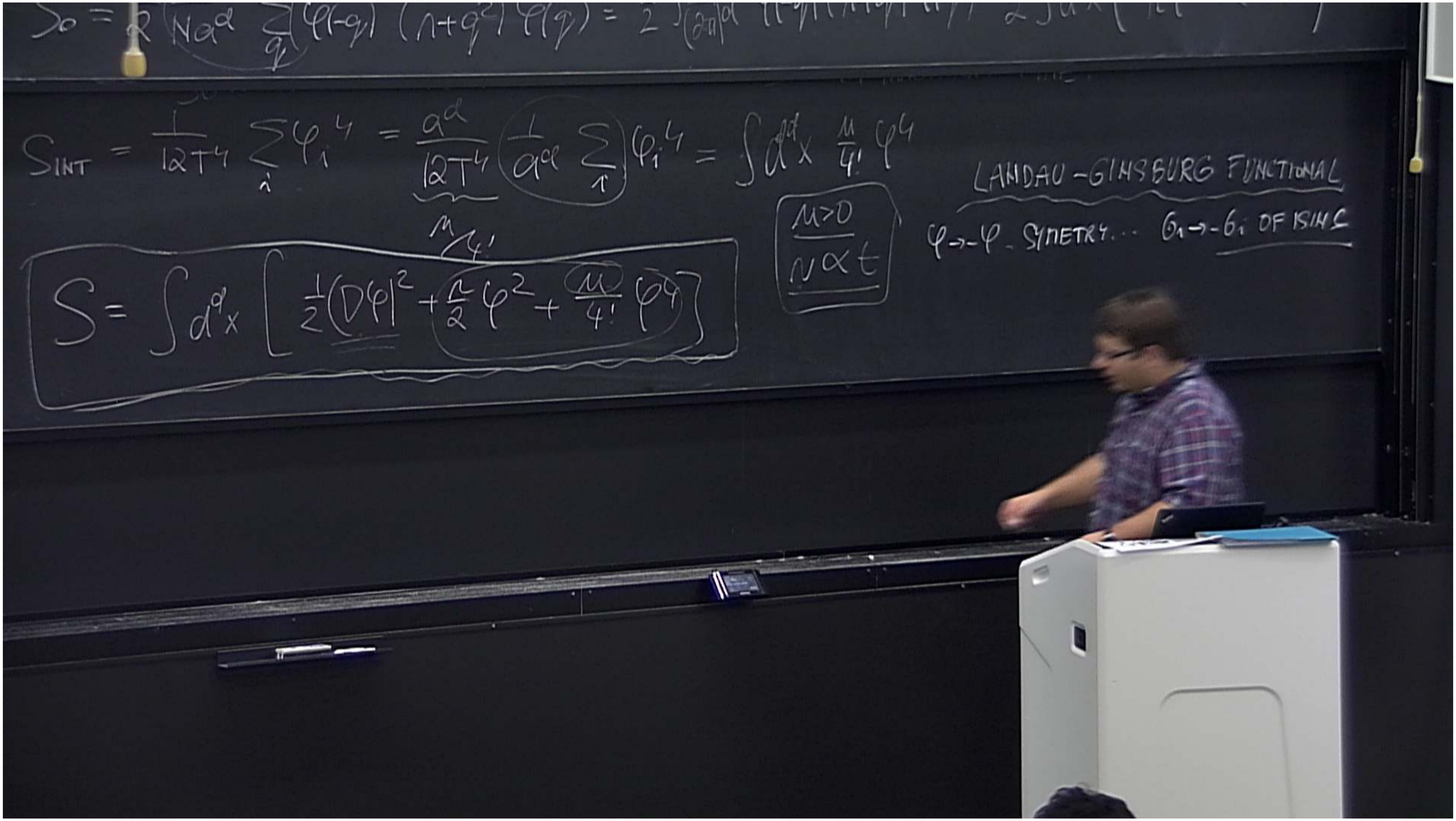
$$\boxed{S = \int d^dx \left[\frac{1}{2}(D\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{\mu}{4!}\varphi^4\right]}$$


$S_{INT} = \frac{1}{12T^4} \sum_i \varphi_i^4 = \frac{a^d}{12T^4} \frac{1}{a^d} \sum_i \varphi_i^4 = \int d^dx \frac{u}{4!} \varphi^4$
LANDAU-GINSBURG FUNCTIONAL
$u>0$
$r \propto t$
$\varphi \to -\varphi$ SYMETRY... $\sigma_i \to -\sigma_i$ OF ISING
$S = \int d^dx \left[\frac{1}{2}(\nabla\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$

"LARGE SCALE" $\propto$ "SMALL MOMENTA"

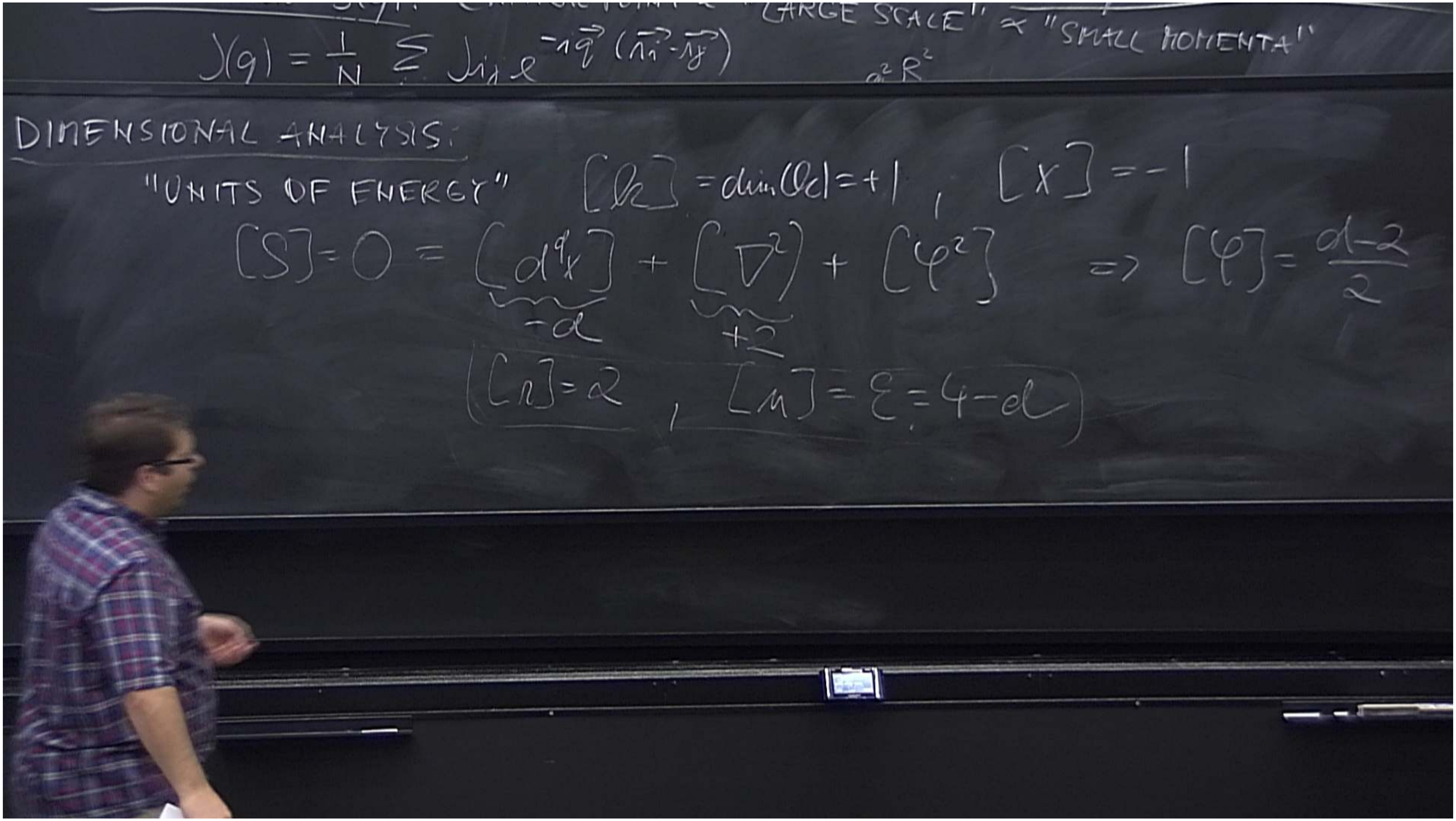
$$J(q) = \frac{1}{N}\sum_{ij} J_{ij}\, e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} \qquad q^2 R^2$$

$$\approx \frac{1}{N}\sum_{ij} J_{ij}\left(1 - i\vec{q}\cdot(\vec{r}_i - \vec{r}_j) - \frac{1}{2}\left[\vec{q}\cdot(\vec{r}_i - \vec{r}_j)\right]^2 + \dots\right]$$

DIMENSIONAL ANALYSIS:

"UNITS OF ENERGY" $\quad [k] = \dim(k) = +1, \quad [x] = -1$

$$[S] = 0 = \underbrace{[d^d x]}_{-d} + \underbrace{[\nabla^2]}_{+2} + [\varphi^2] \Rightarrow [\varphi] = \frac{d-2}{2}$$


"LARGE SCALE" ∝ "SMALL MOMENTA"
J(q) = 1/N Σ J_ij e^{-iq(r_i - r_j)}
a²R²
DIMENSIONAL ANALYSIS:
"UNITS OF ENERGY"
[k] = dim(k) = +1 , [x] = -1
[S] = 0 = [d^d x] + [∇²] + [φ²] => [φ] = (d-2)/2
-d
+2
[r] = 2 , [u] = ε = 4-d

$$[\lambda] = 2 \quad , \quad [u] = \varepsilon = 4-d$$

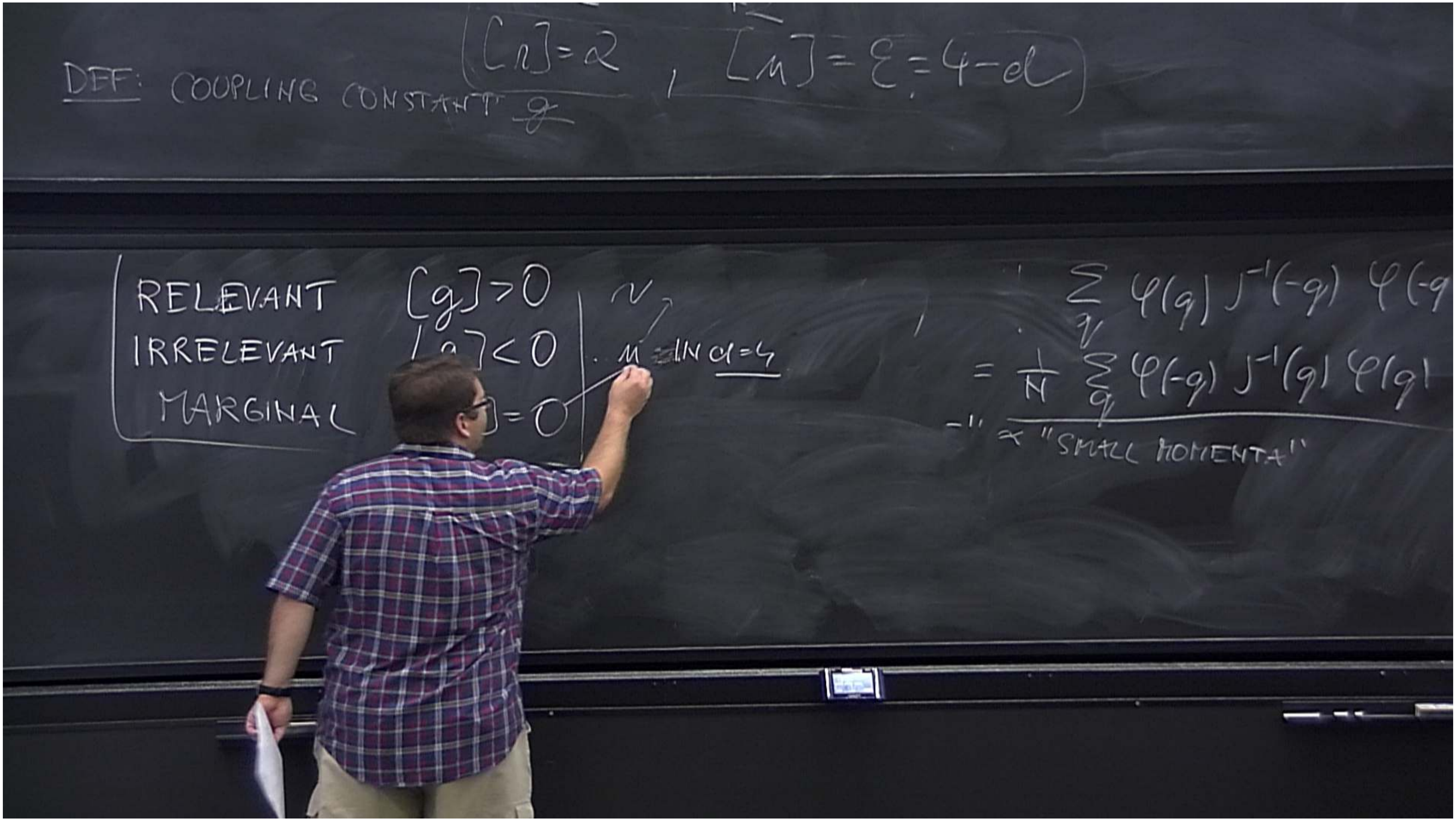
DEF: COUPLING CONSTANT g

RELEVANT	$[g] > 0$
IRRELEVANT	$[g] < 0$
MARGINAL	$[g] = 0$

$$\sum_q \varphi(q)\, J^{-1}(-q)\, \varphi(-q$$

$$= \frac{1}{N} \sum_q \varphi(-q)\, J^{-1}(q)\, \varphi(q)$$

$\approx$ "SMALL MOMENTA"


DEF: COUPLING CONSTANT g
RELEVANT [g] > 0
IRRELEVANT [g] < 0
MARGINAL
"SMALL MOMENTA"

$$S_0 = \frac{1}{2} \frac{1}{Na^d} \sum_q \varphi(-q)\,(r+q^2)\,\varphi(q) = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d} \varphi(-q)(r+q^2)\varphi(q) \ldots$$

$$S_{INT} = \frac{1}{12T^4}\sum_i \varphi_i^4 = \underbrace{\frac{a^d}{12T^4}}_{u/4!}\frac{1}{a^d}\sum_i \varphi_i^4 = \int d^dx\, \frac{u}{4!}\varphi^4$$

LANDAU-GINSBURG FUNCTIONAL

$\varphi \to -\varphi$ SYMETRY... $\sigma_i \to -\sigma_i$ OF ISING

$$\boxed{S = \int d^dx \left[\frac{1}{2}(\nabla\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 + w\,\varphi^6 \right.}$$

$$\boxed{\begin{array}{c} u>0 \\ r \propto t \end{array}}$$