Title: PSI 2016/2017 Statistical Mechanics - Lecture 5

Date: Oct 17, 2016 10:45 AM

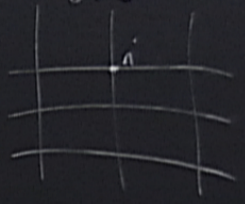
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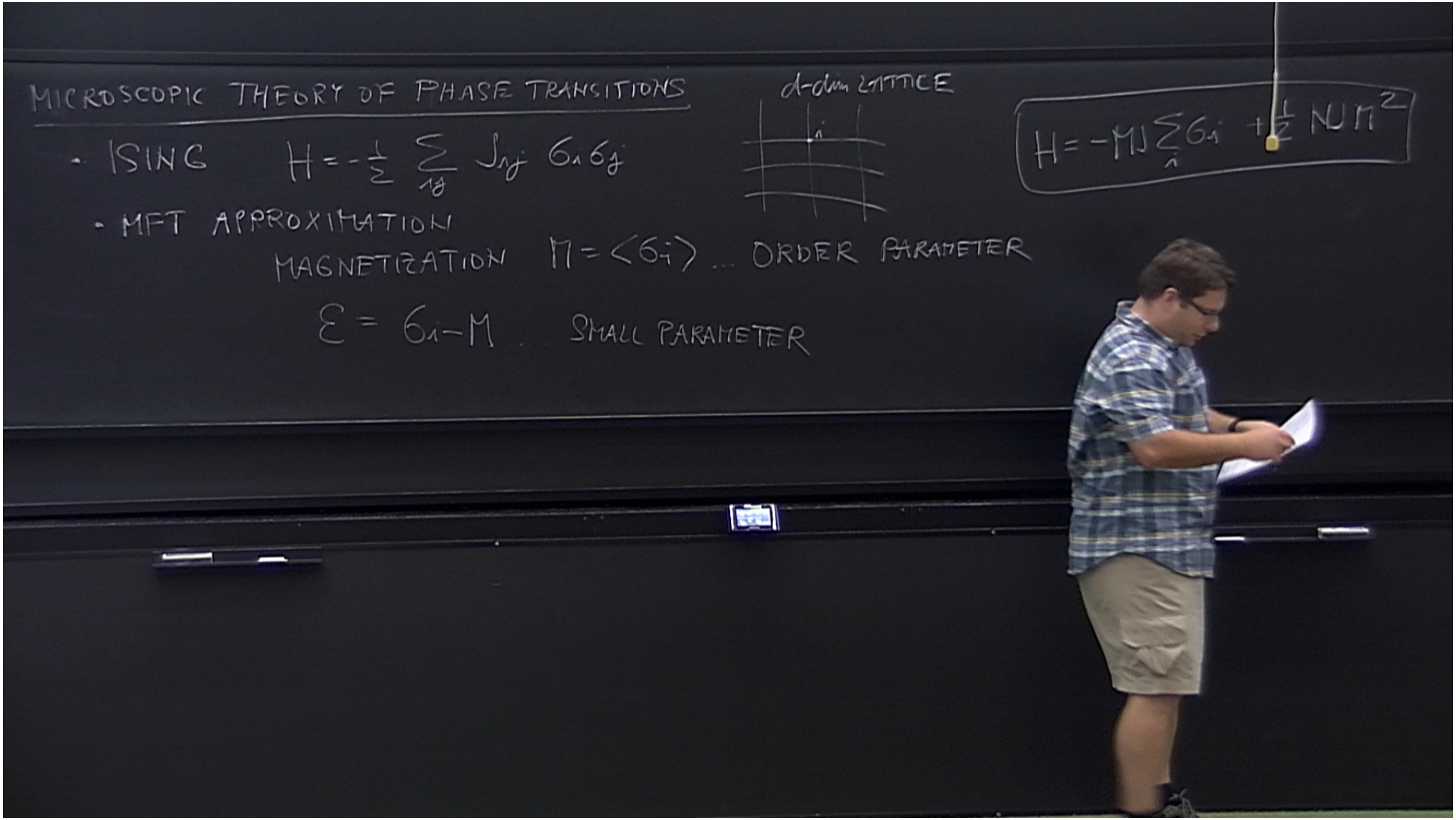
Abstract:

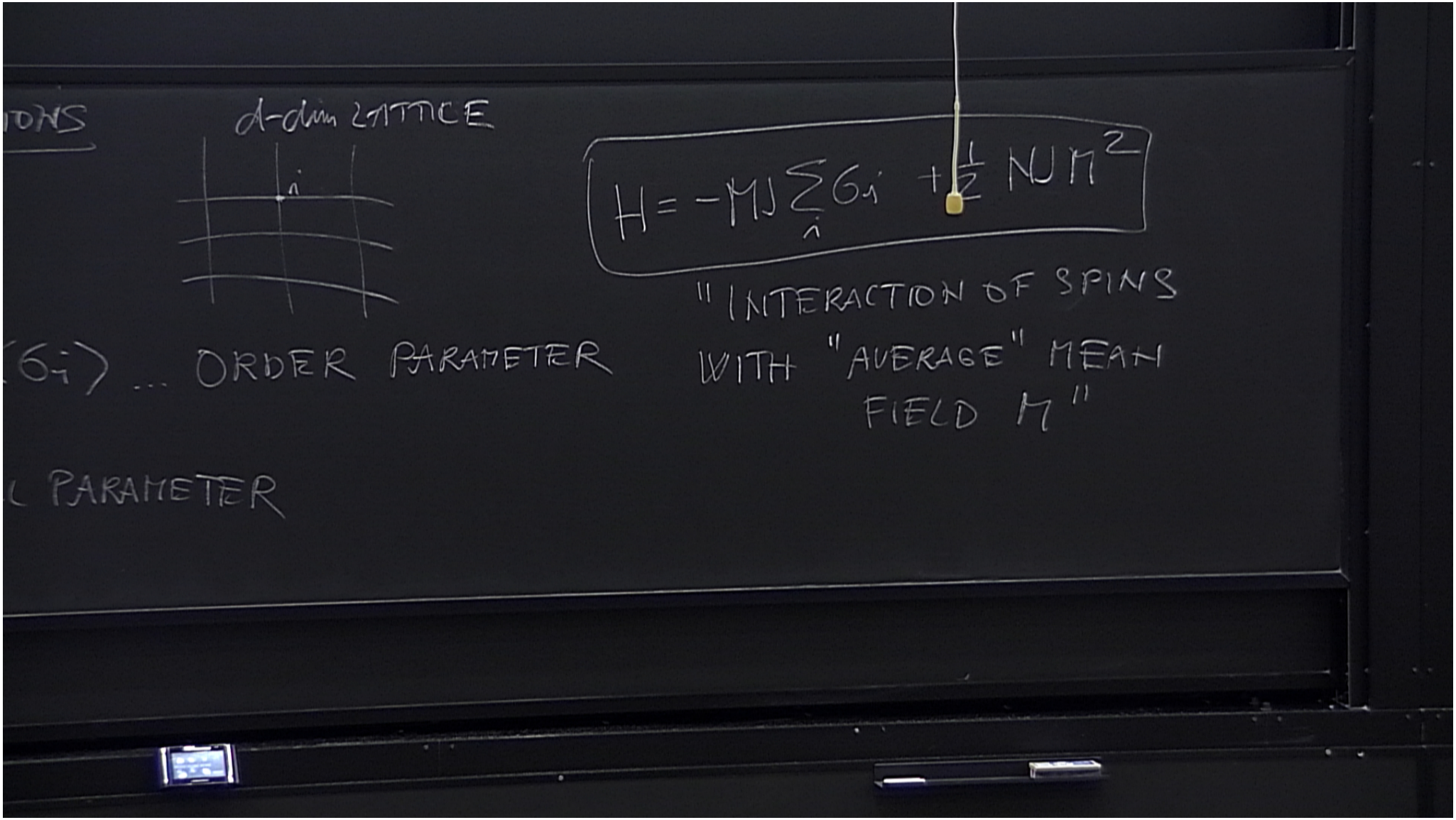
MICROSCOPIC THEORY OF PHASE TRANSITIONS

- ISING $H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$

d-dim LATTICE

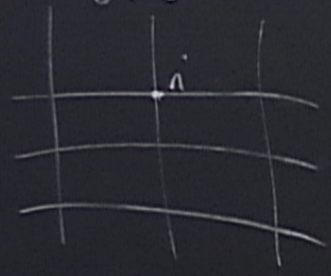



MICROSCOPIC THEORY OF PHASE TRANSITIONS
• ISING $H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j$
• MFT APPROXIMATION
MAGNETIZATION $M = \langle\sigma_i\rangle$ ... ORDER PARAMETER
$\varepsilon = \sigma_i - M$ SMALL PARAMETER
d-dim LATTICE
$H = -MJ\sum_i \sigma_i + \frac{1}{2}NJM^2$


d-dim LATTICE
i
$H = -MJ \sum_i G_i + \frac{1}{2} NJM^2$
"INTERACTION OF SPINS WITH "AVERAGE" MEAN FIELD M"
$\langle G_i \rangle$ ... ORDER PARAMETER
PARAMETER

DNS

d-dim LATTICE



$$H = -MJ \sum_i G_i + \frac{1}{2} NJM^2$$

"INTERACTION OF SPINS WITH "AVERAGE" MEAN FIELD M"

$\langle G_i \rangle$ ... ORDER PARAMETER

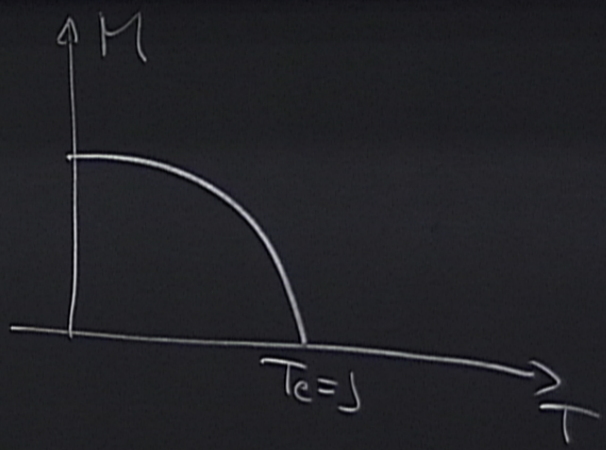
L PARAMETER

$$\Rightarrow f = \frac{F}{N} = \frac{JM^2}{2} - T \log\left[2\cosh(M/T)\right]$$

$\varepsilon = 6i - M$ ... SMALL PARAMETER $\Rightarrow f = \frac{F}{N}$

- MINIMIZE f W.R.T. M

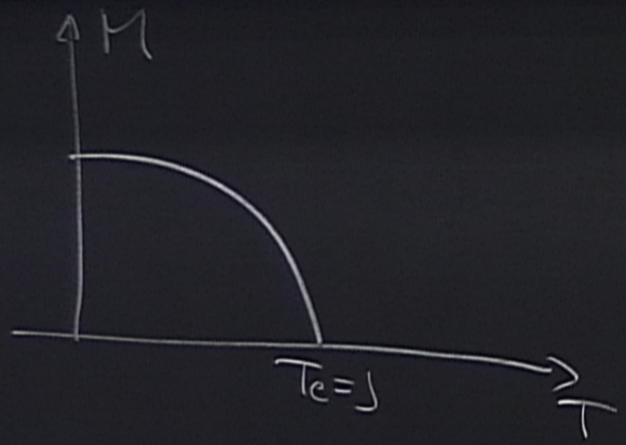
$$M = \tanh(MJ/T)$$



SMALL PARAMETER

$$\Rightarrow f = \frac{F}{N} = \frac{Jm^2}{2} - T \log\left[2\cosh(m/T)\right]$$

FIELD h

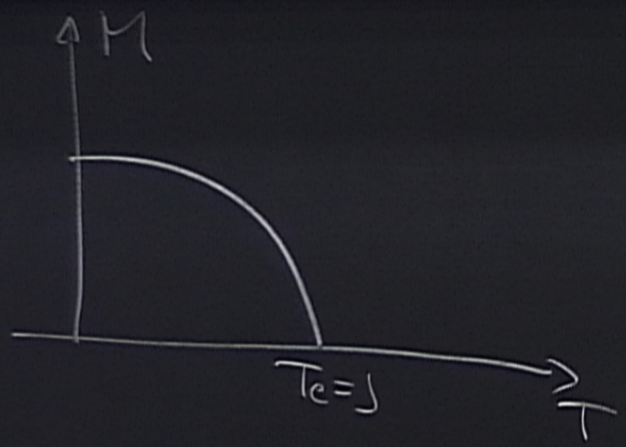


f ... NON-ANALYTIC FUNCTION OF T
(BECAUSE $M = M(T)$ IS)

SMALL PARAMETER

$$\Rightarrow f = \frac{F}{N} = \frac{J m^2}{2} - T \log\left[2\cosh(mJ/T)\right]$$

FIELD m

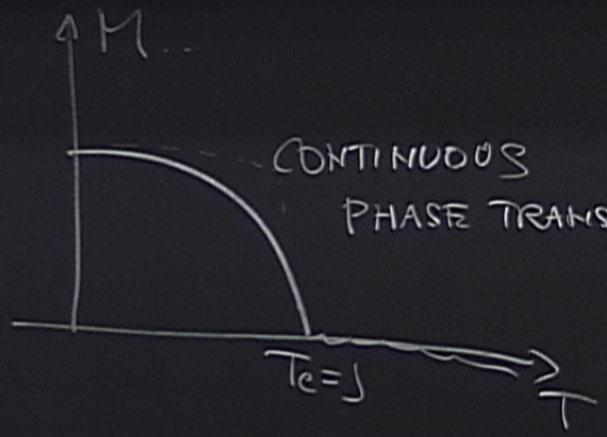


f ... NON-ANALYTIC FUNCTION OF T

(BECAUSE M=M(T) IS)

⇒ PHASE TRANSITION

Si-M ... SMALL PARAMETER $\Rightarrow f = \frac{F}{N} = \frac{JM^2}{2} - T \log\left[2\cosh(M/T)\right]$ FIELD M

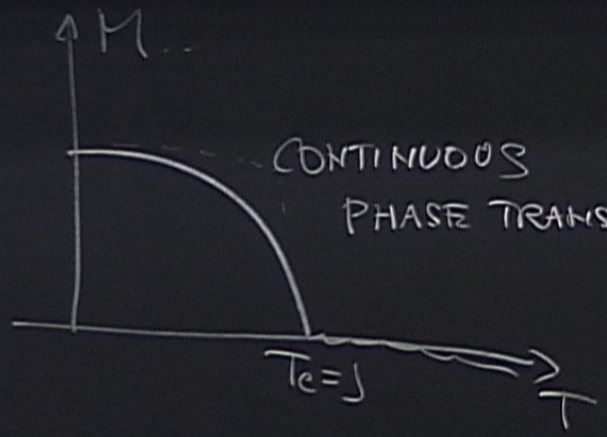


f ... NON-ANALYTIC FUNCTION OF T
(BECAUSE M=M(T) IS)
$\Rightarrow$ PHASE TRANSITION

$m_i - M$ ... SMALL PARAMETER

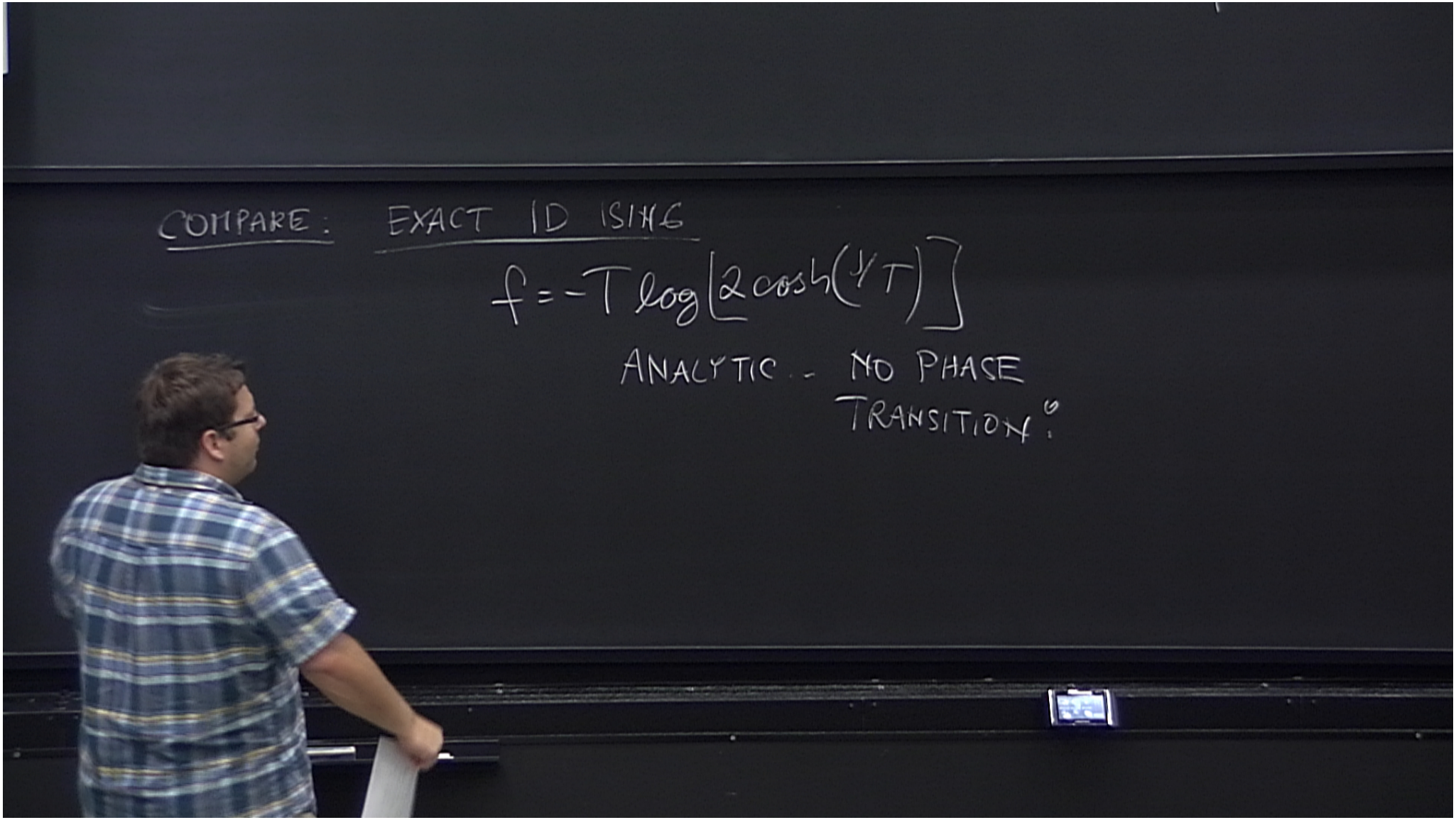
$$\Rightarrow f = \frac{F}{N} = \frac{JM^2}{2} - T \log\left[2\cosh(M/T)\right]$$

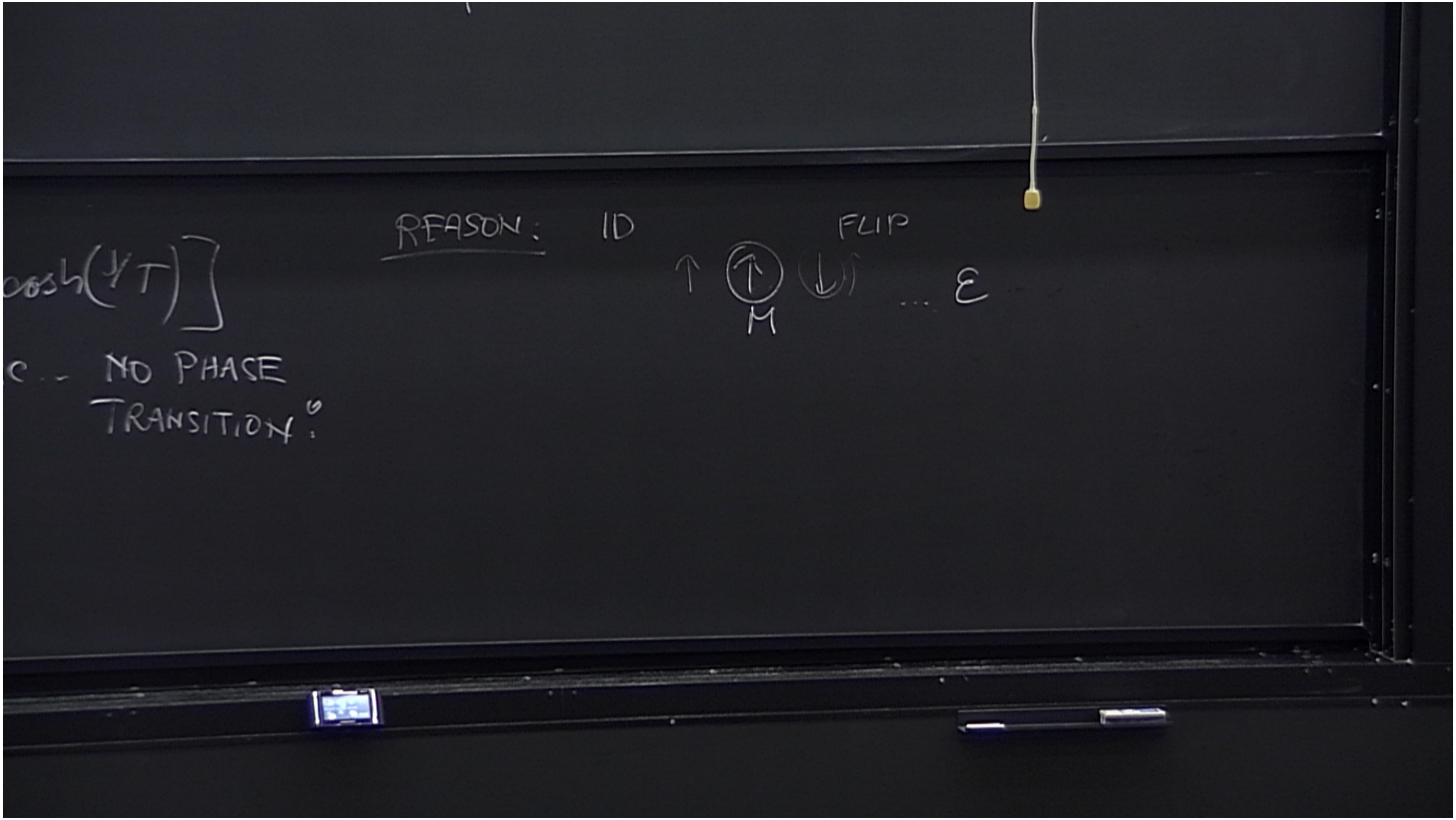
FIELD H

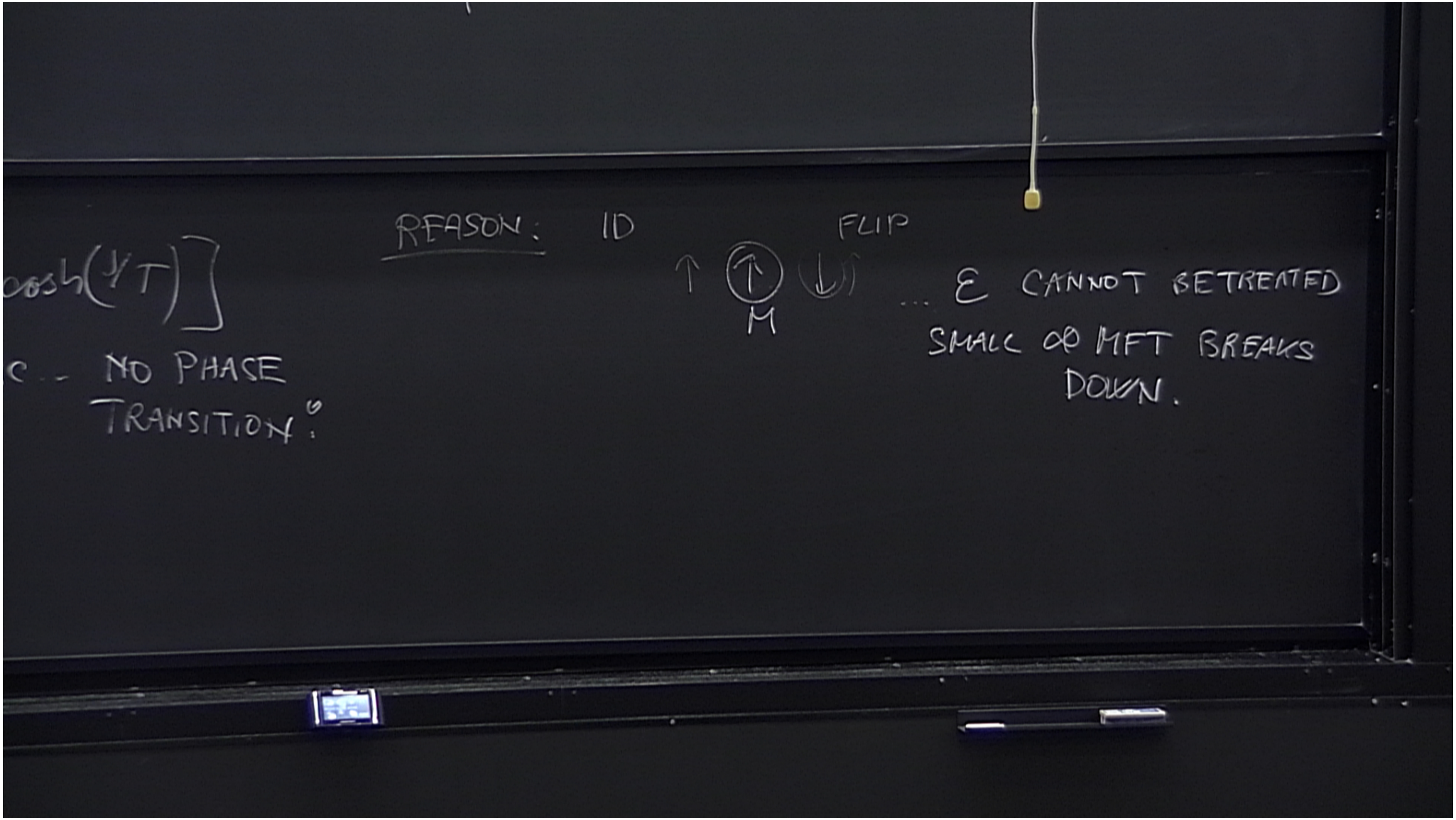


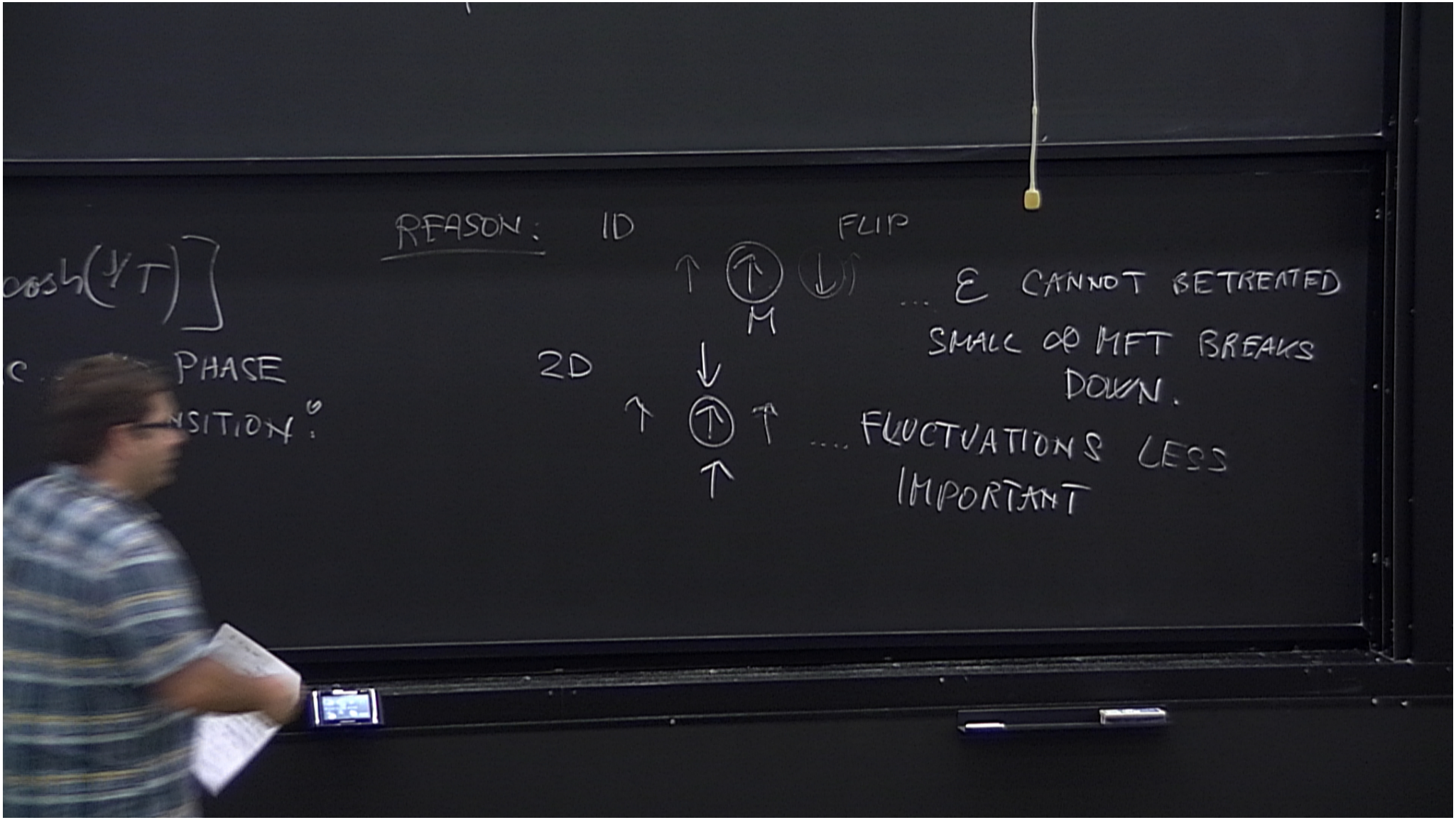
f ... NON-ANALYTIC FUNCTION OF T
(BECAUSE $M = M(T)$ IS)
$\Rightarrow$ PHASE TRANSITION

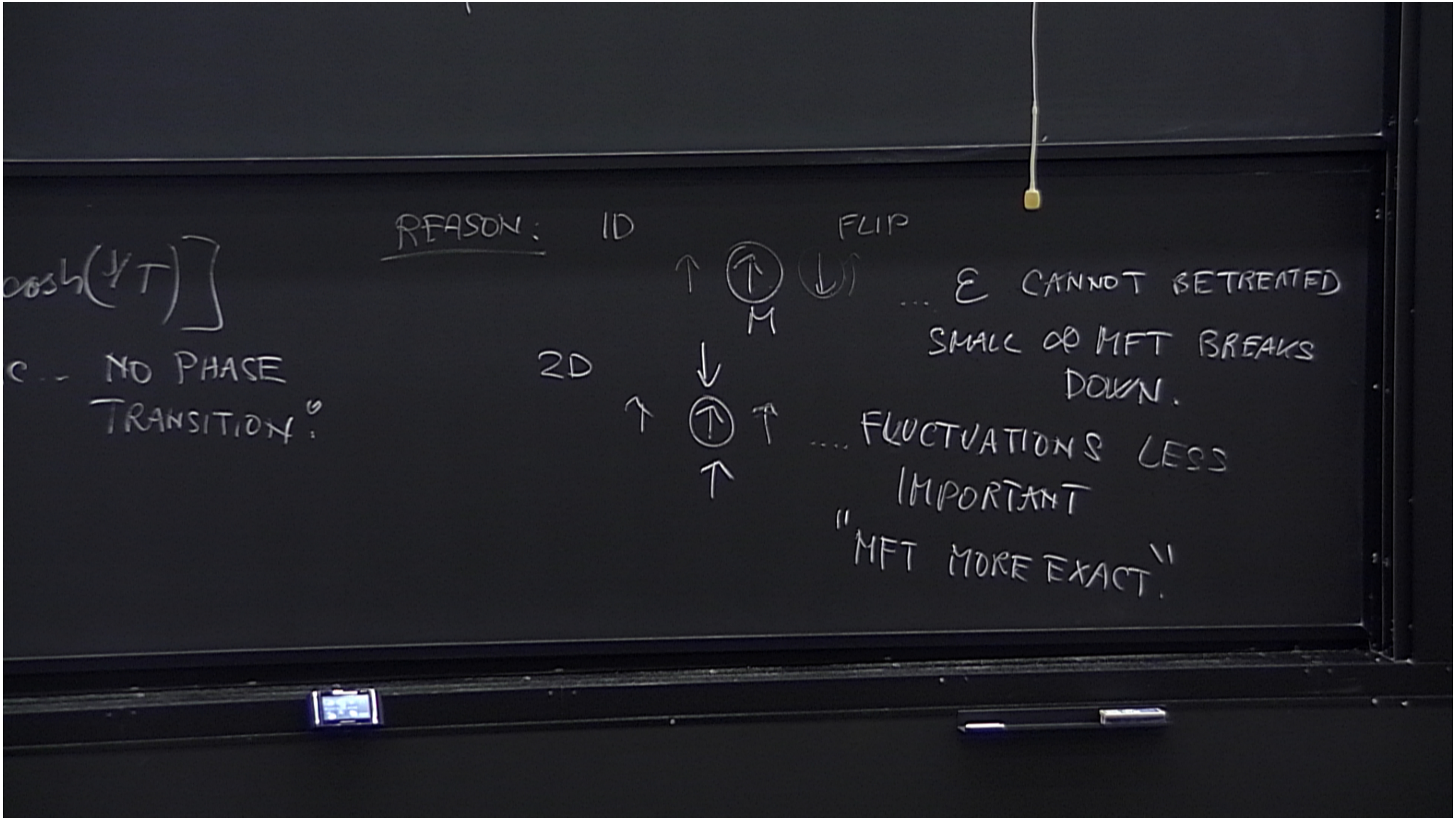
INDEPENDENT OF DIMENSION d


COMPARE: EXACT 1D ISING
$f = -T \log\left[2\cosh\left(\frac{1}{T}\right)\right]$
ANALYTIC - NO PHASE TRANSITION!


REASON: ID FLIP
M
... ε
NO PHASE TRANSITION!


REASON: ID
FLIP
M
... ε CANNOT BE TREATED
SMALL OR MFT BREAKS
DOWN.
NO PHASE
TRANSITION!


REASON: 1D
FLIP
M
... ε CANNOT BE TREATED
SMALL ⇒ MFT BREAKS DOWN.
2D
... FLUCTUATIONS LESS IMPORTANT
PHASE


REASON: 1D
FLIP
M
... ε CANNOT BE TREATED SMALL OR MFT BREAKS DOWN.
2D
... FLUCTUATIONS LESS IMPORTANT
"MFT MORE EXACT"
NO PHASE TRANSITION!

EXACT 1D ISING

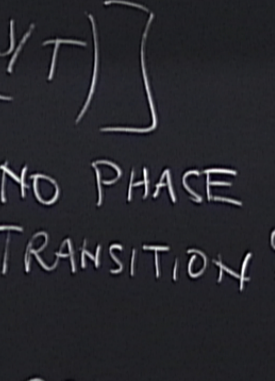
$$f = -T \log\left[2\cosh\left(\frac{J}{T}\right)\right]$$

ANALYTIC - NO PHASE TRANSITION!

$d=4$ UPPER CRITICAL DIMENSION

(MFT IS EXACT)

REASON: 1D



FLIP ... E

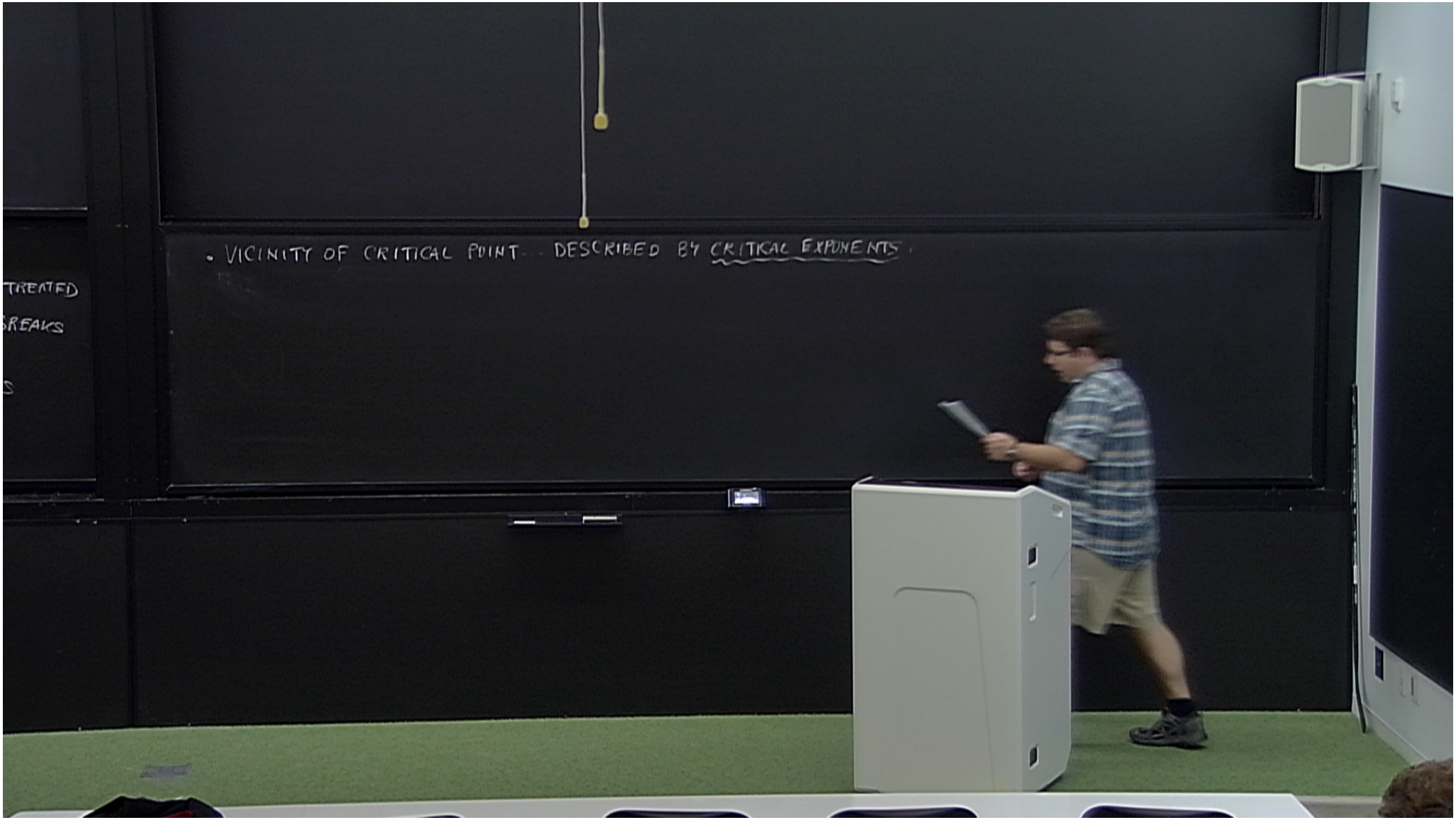
SMALL

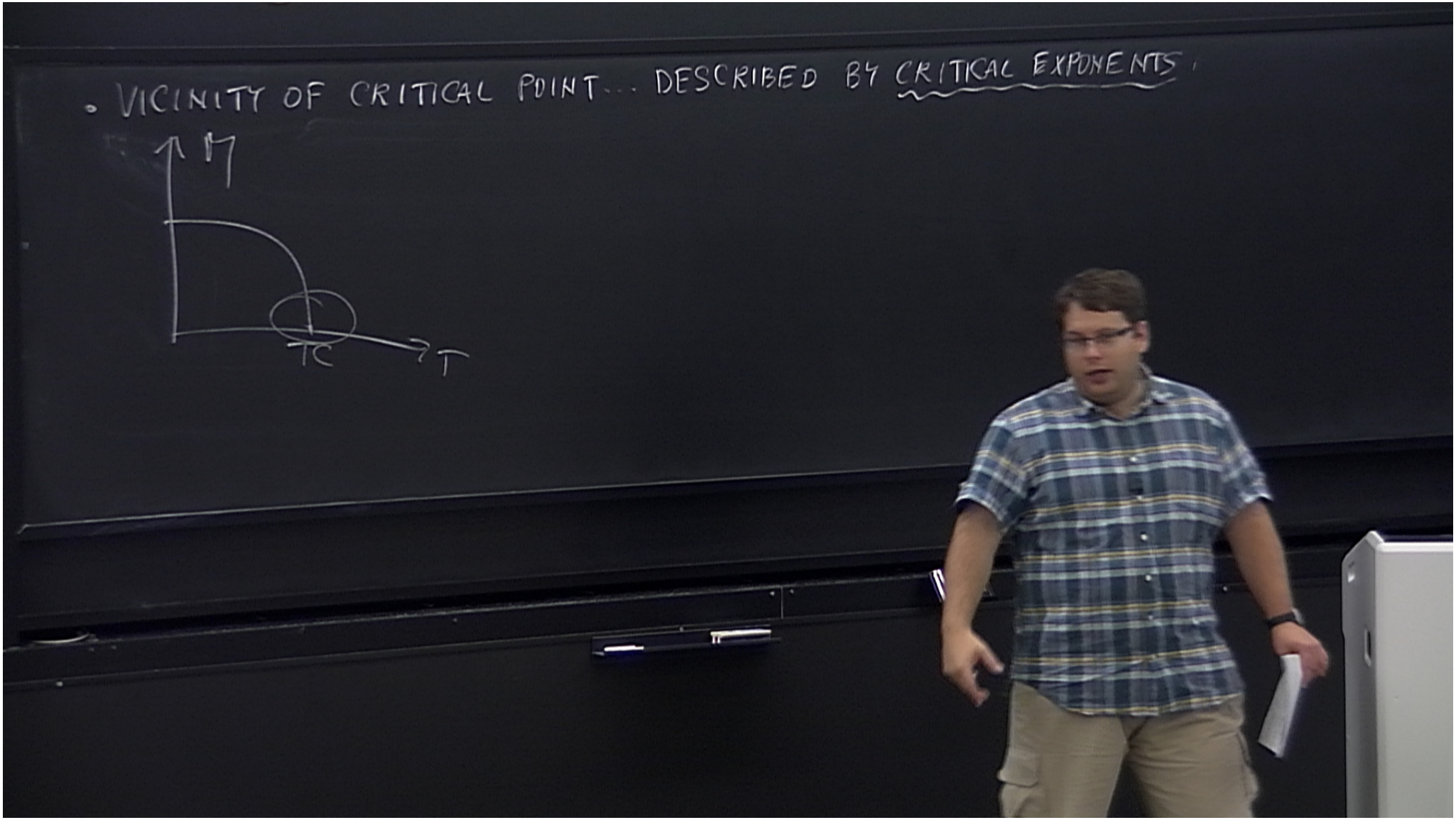
2D

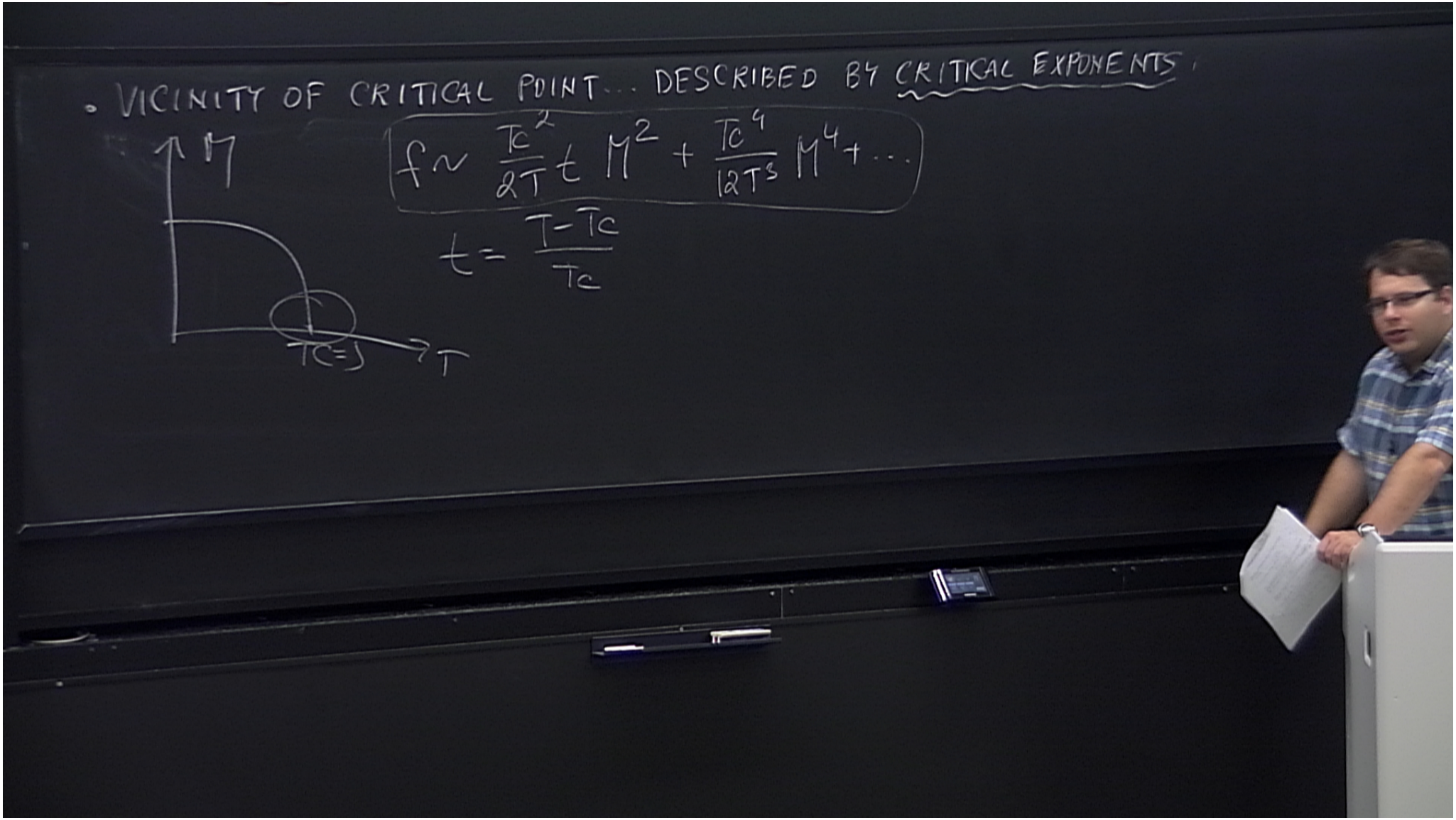
... FLUCTUATI

IMPORTA

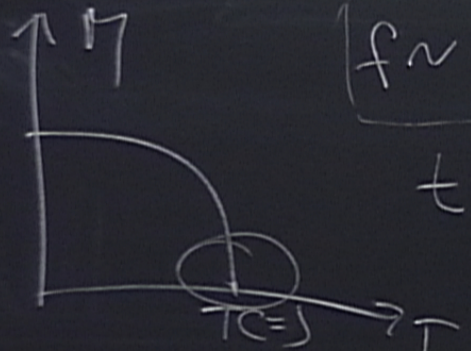
"MFT MORE


• VICINITY OF CRITICAL POINT DESCRIBED BY CRITICAL EXPONENTS
TREATED
REAKS


• VICINITY OF CRITICAL POINT... DESCRIBED BY CRITICAL EXPONENTS
M
Tc
T


• VICINITY OF CRITICAL POINT... DESCRIBED BY CRITICAL EXPONENTS:
M
$f \sim \frac{T_c^2}{2T} t M^2 + \frac{T_c^4}{12T^3} M^4 + \ldots$
$t = \frac{T - T_c}{T_c}$
$T_c = T$
T

- VICINITY OF CRITICAL POINT... DESCRIBED BY CRITICAL EXPONENTS:

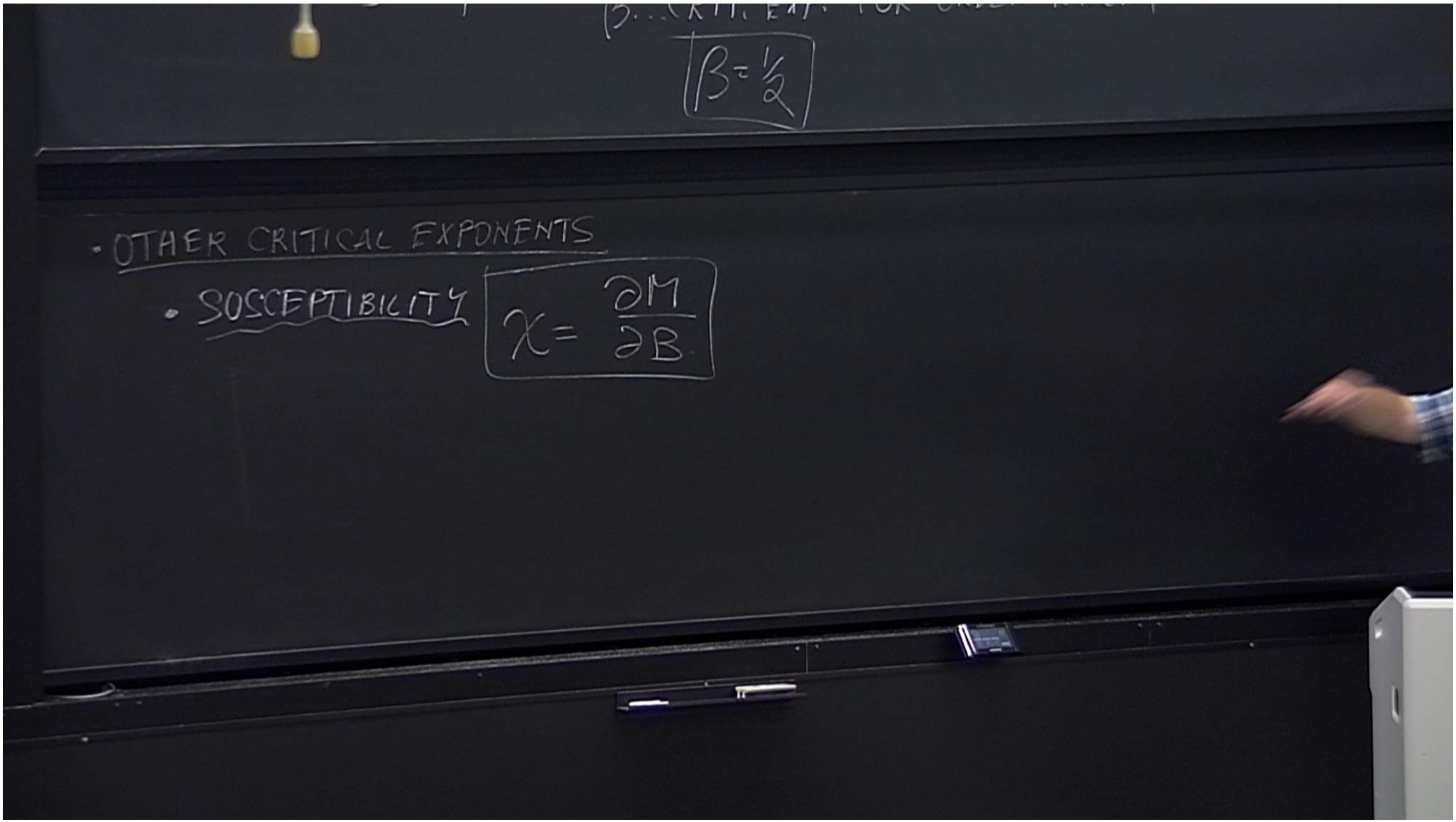


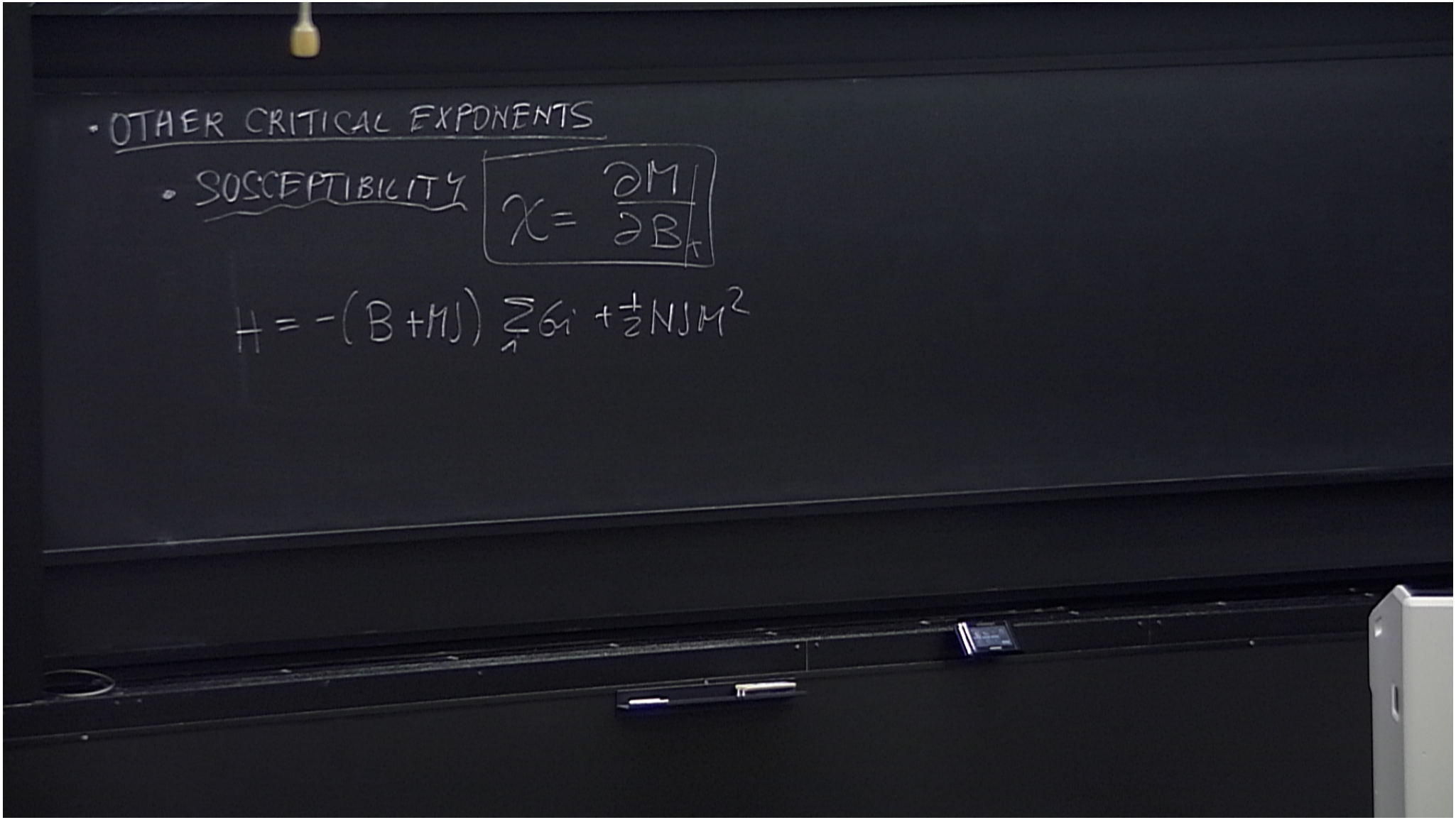
$$f \sim \frac{Tc^2}{2T} t\, M^2 + \frac{Tc^4}{12T^3} M^4 + \dots$$

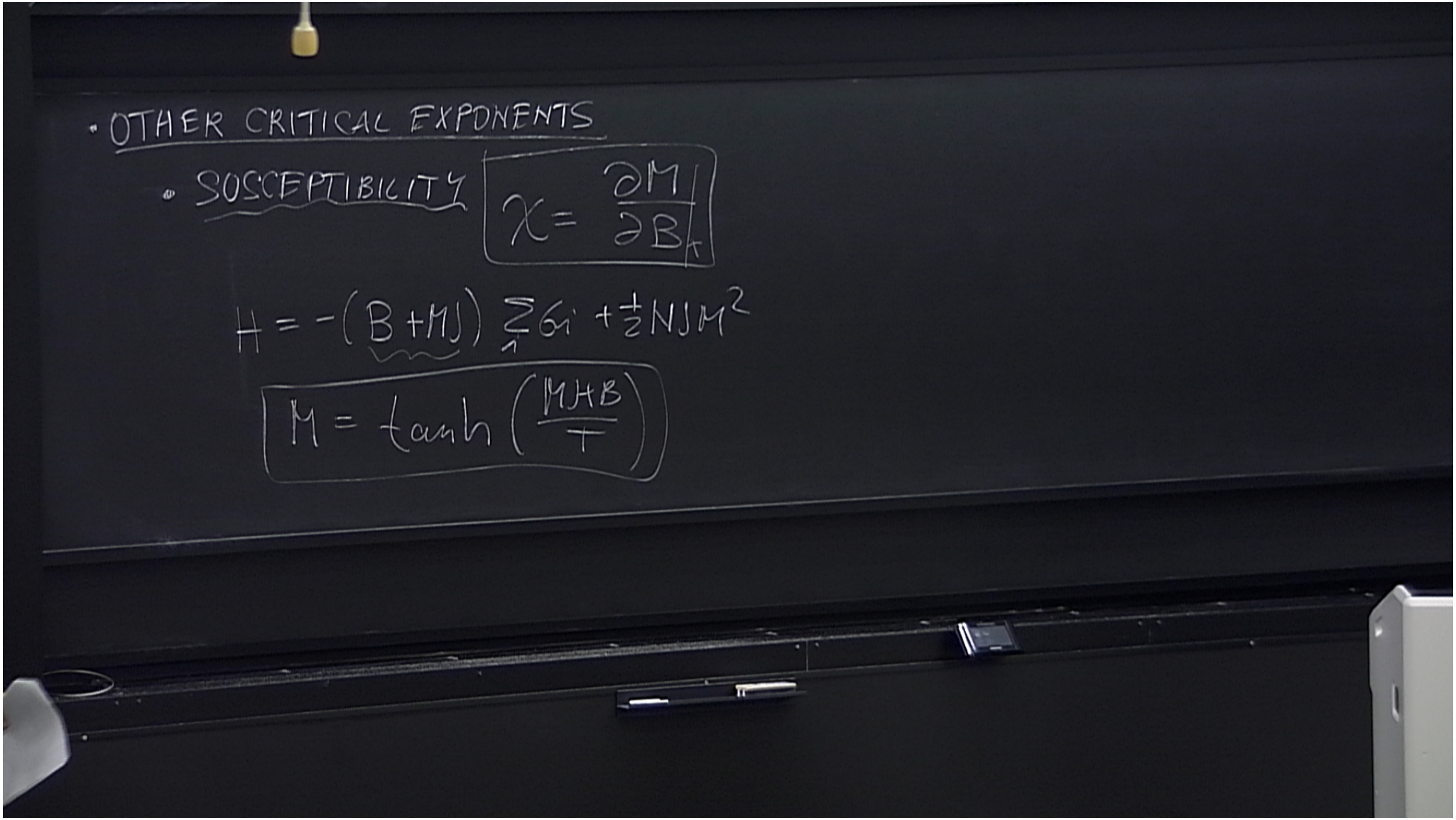
$$t = \frac{T - Tc}{Tc} \qquad M \sim \sqrt{3(1 - T/Tc)} \sim (-t)^{\beta}$$

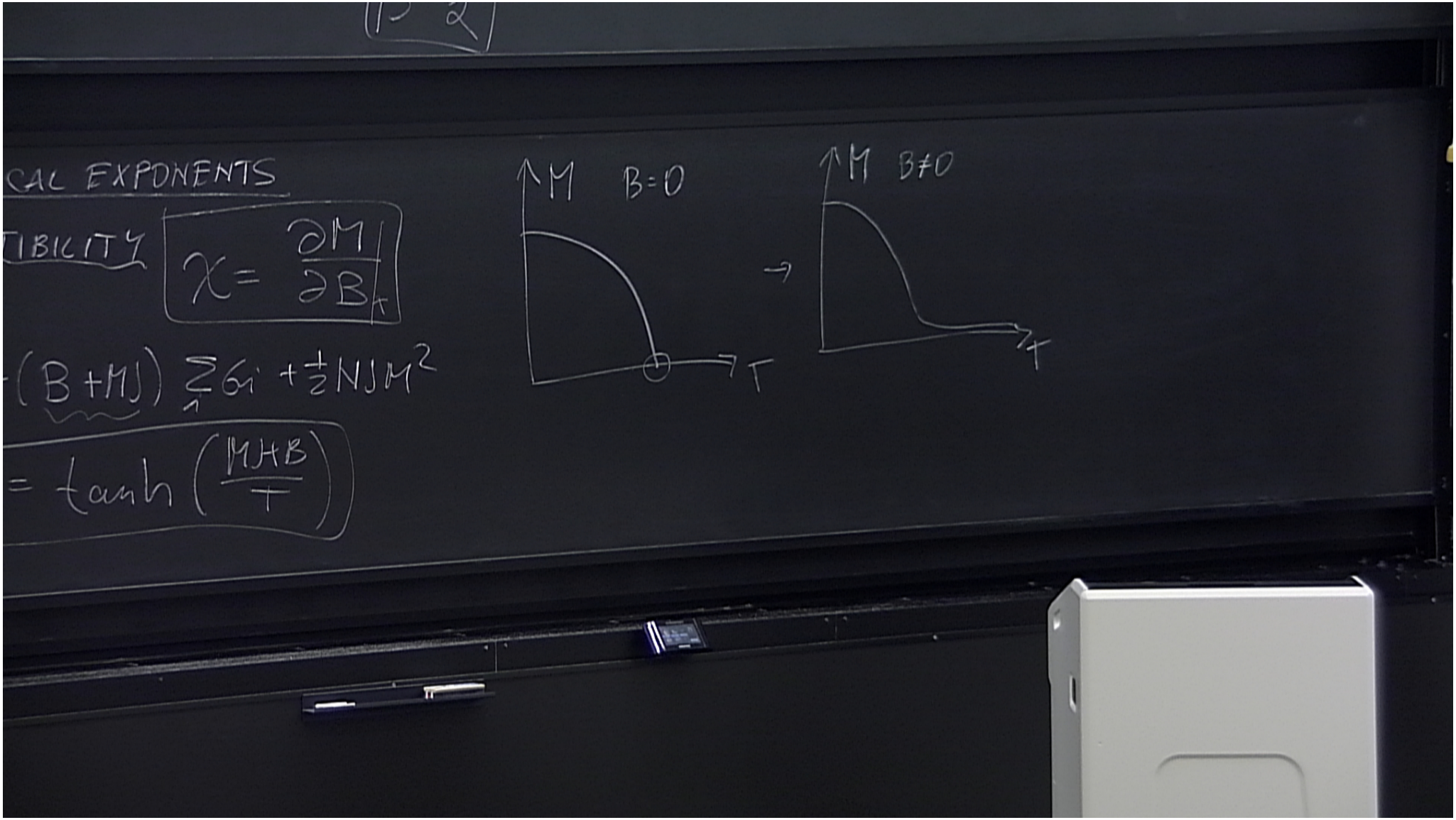
β... CRIT. EXP. FOR ORDER PARAM

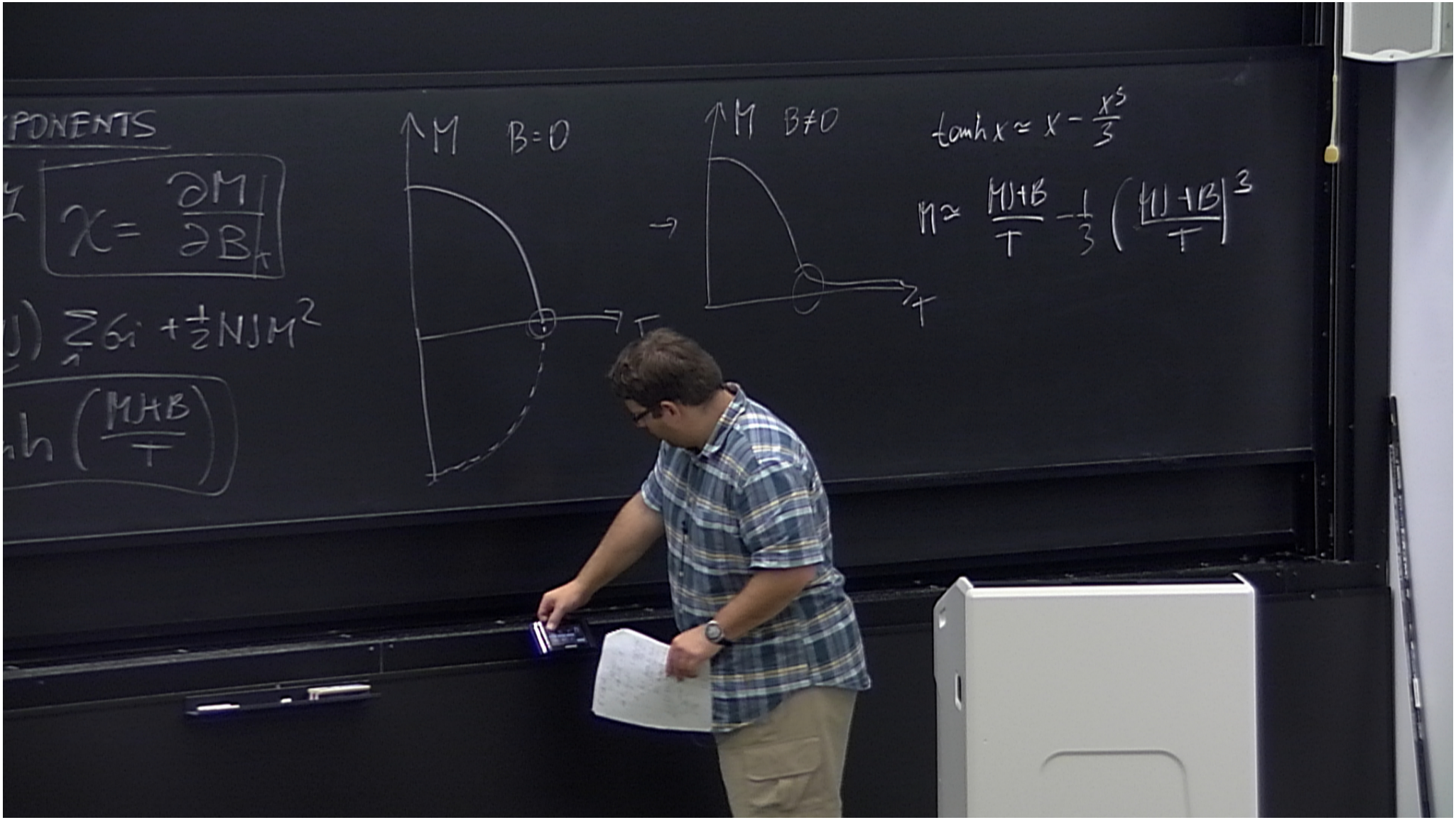
$$\beta = \tfrac{1}{2}$$

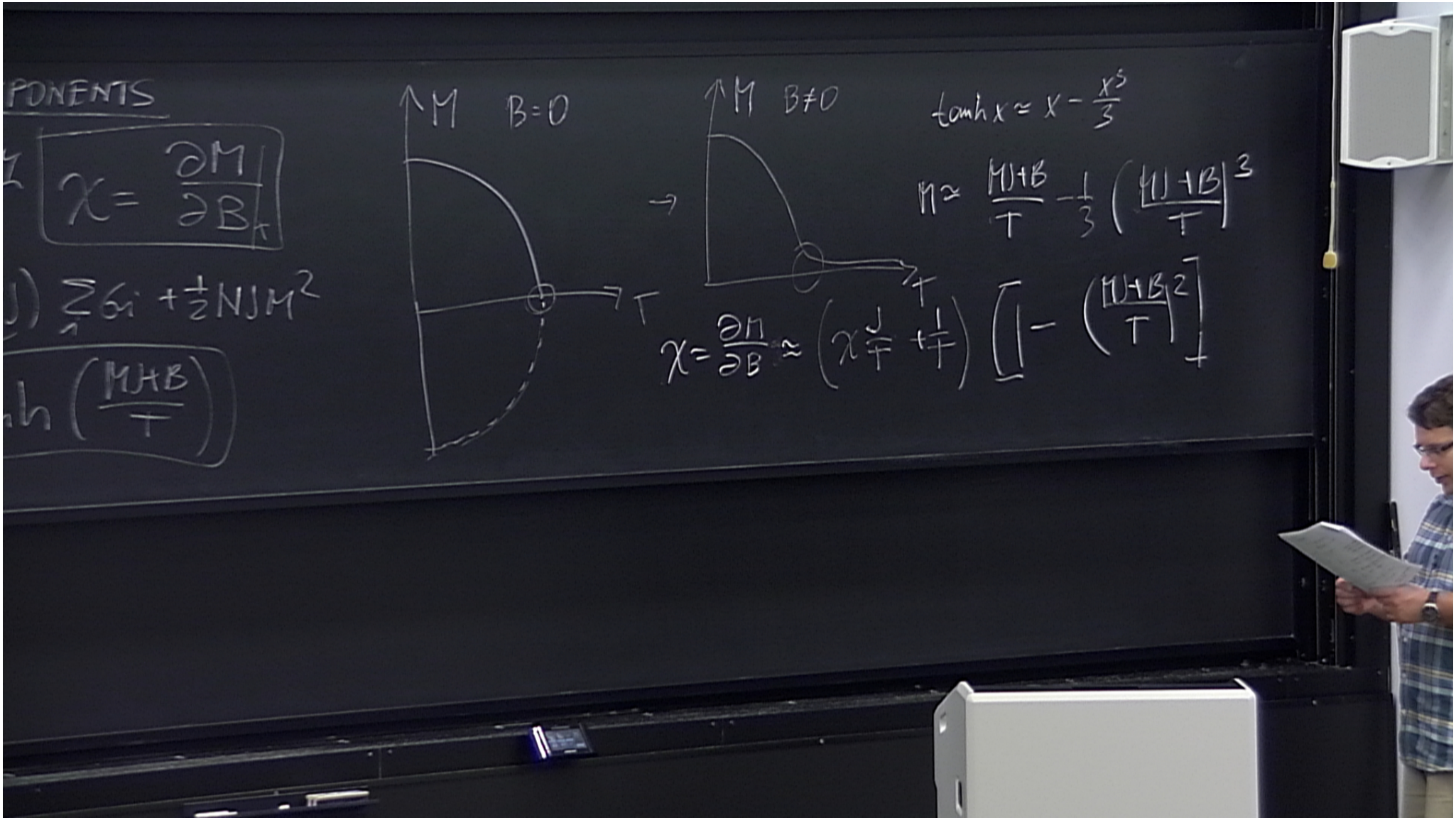

$\beta = \frac{1}{2}$
• OTHER CRITICAL EXPONENTS
• SUSCEPTIBILITY $\chi = \frac{\partial M}{\partial B}$

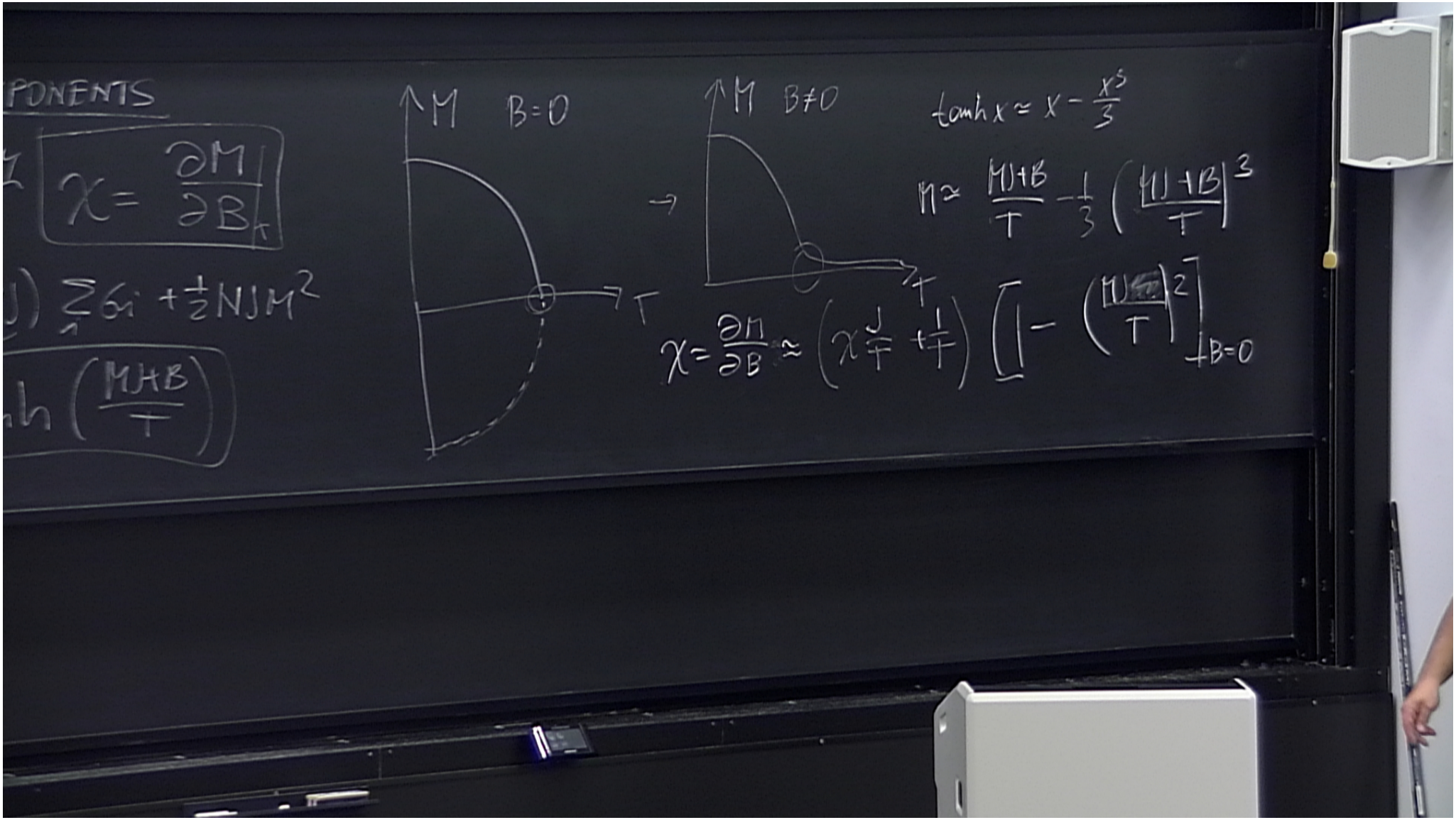

• OTHER CRITICAL EXPONENTS
• SOSCEPTIBILITY
$\chi = \frac{\partial M}{\partial B}$
$H = -(B + MJ) \sum_{\uparrow} G_i + \frac{1}{2} N J M^2$

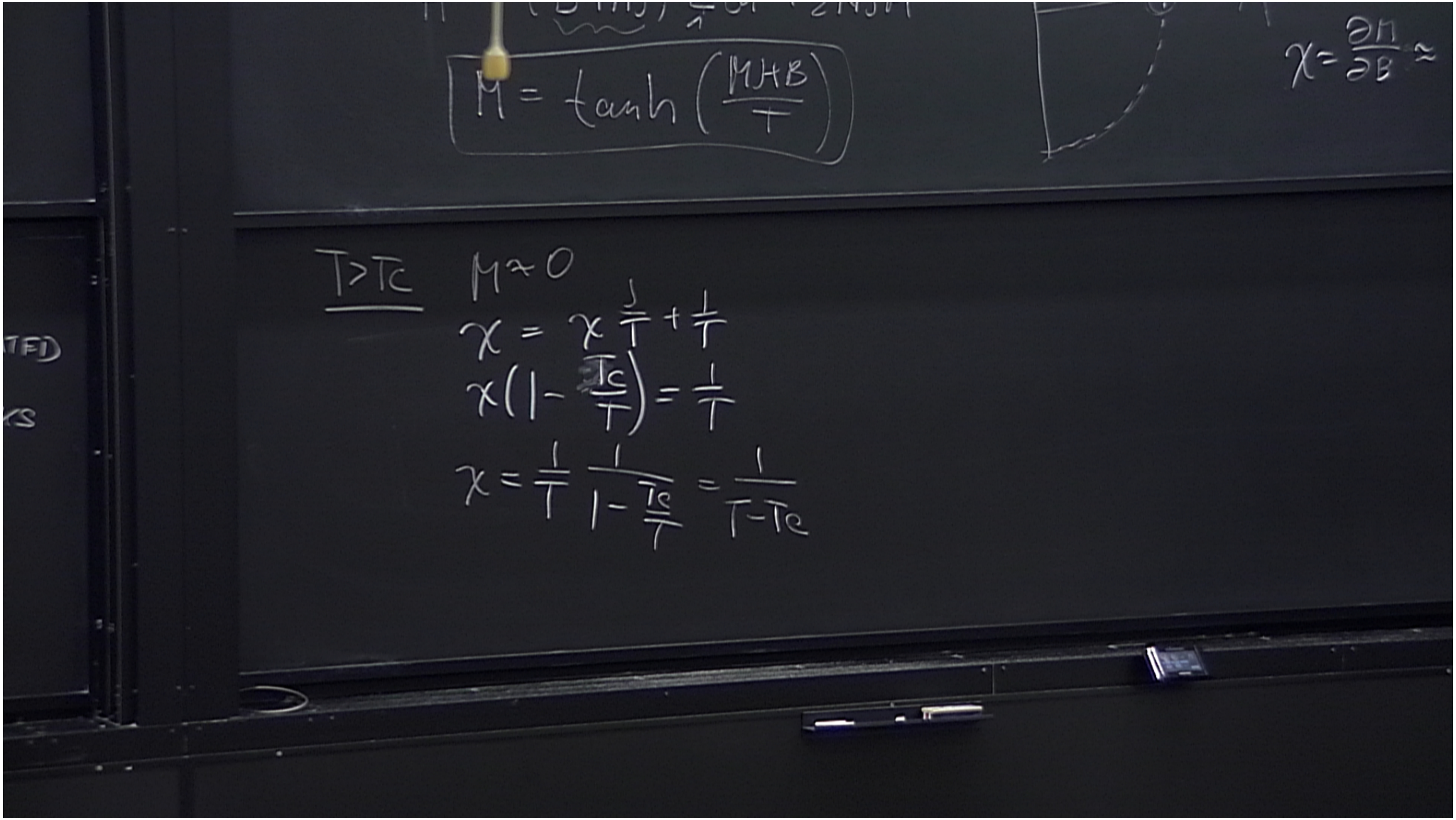

• OTHER CRITICAL EXPONENTS
• SUSCEPTIBILITY
$\chi = \frac{\partial M}{\partial B}$
$H = -(B + MJ) \sum_i \sigma_i + \frac{1}{2} N J M^2$
$M = \tanh\left(\frac{MJ + B}{T}\right)$

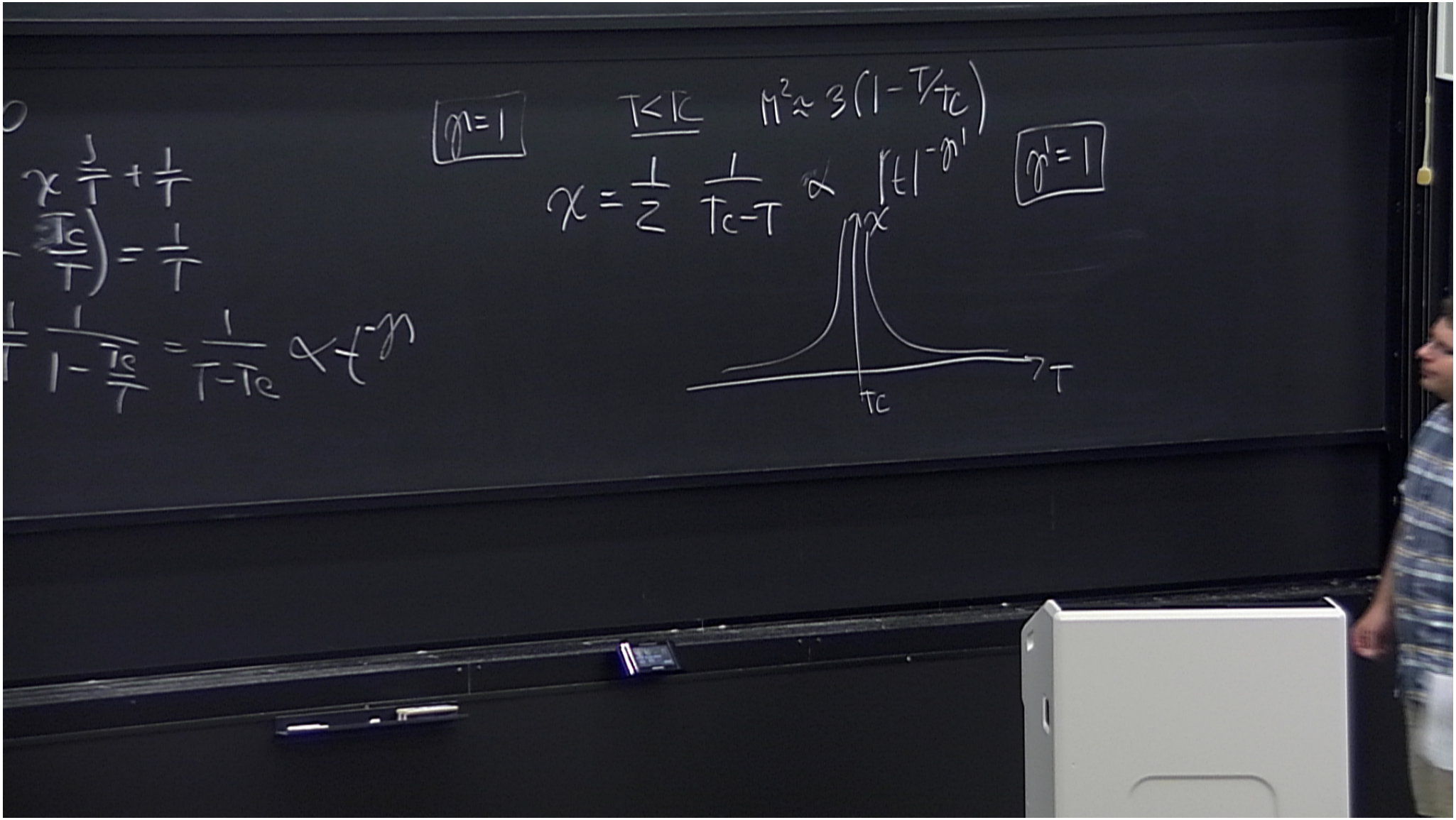

CAL EXPONENTS
TIBILITY
$\chi = \frac{\partial M}{\partial B}\Big|_T$
$(B + MJ) \sum G_i + \frac{1}{2} N J M^2$
$= \tanh\left(\frac{MJB}{T}\right)$
M B=0
T
M B≠0
T

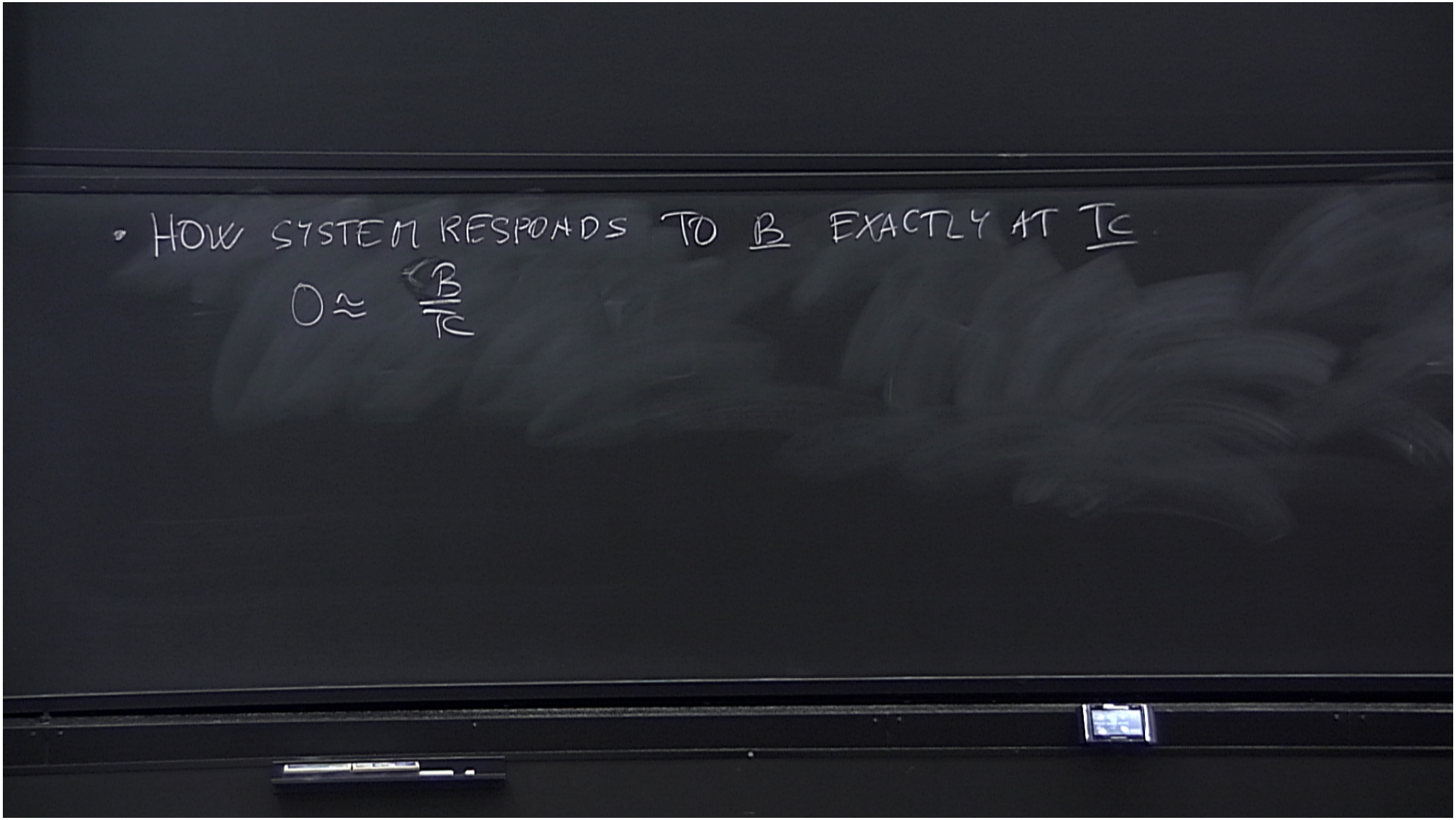

PONENTS
$\chi = \frac{\partial M}{\partial B}$
$\sum G_i + \frac{1}{2} N J M^2$
$\left(\frac{M+B}{T}\right)$
M B=0
T
M B≠0
T
$\tanh x \simeq x - \frac{x^3}{3}$
$\eta \simeq \frac{M+B}{T} - \frac{1}{3}\left(\frac{M+B}{T}\right)^3$

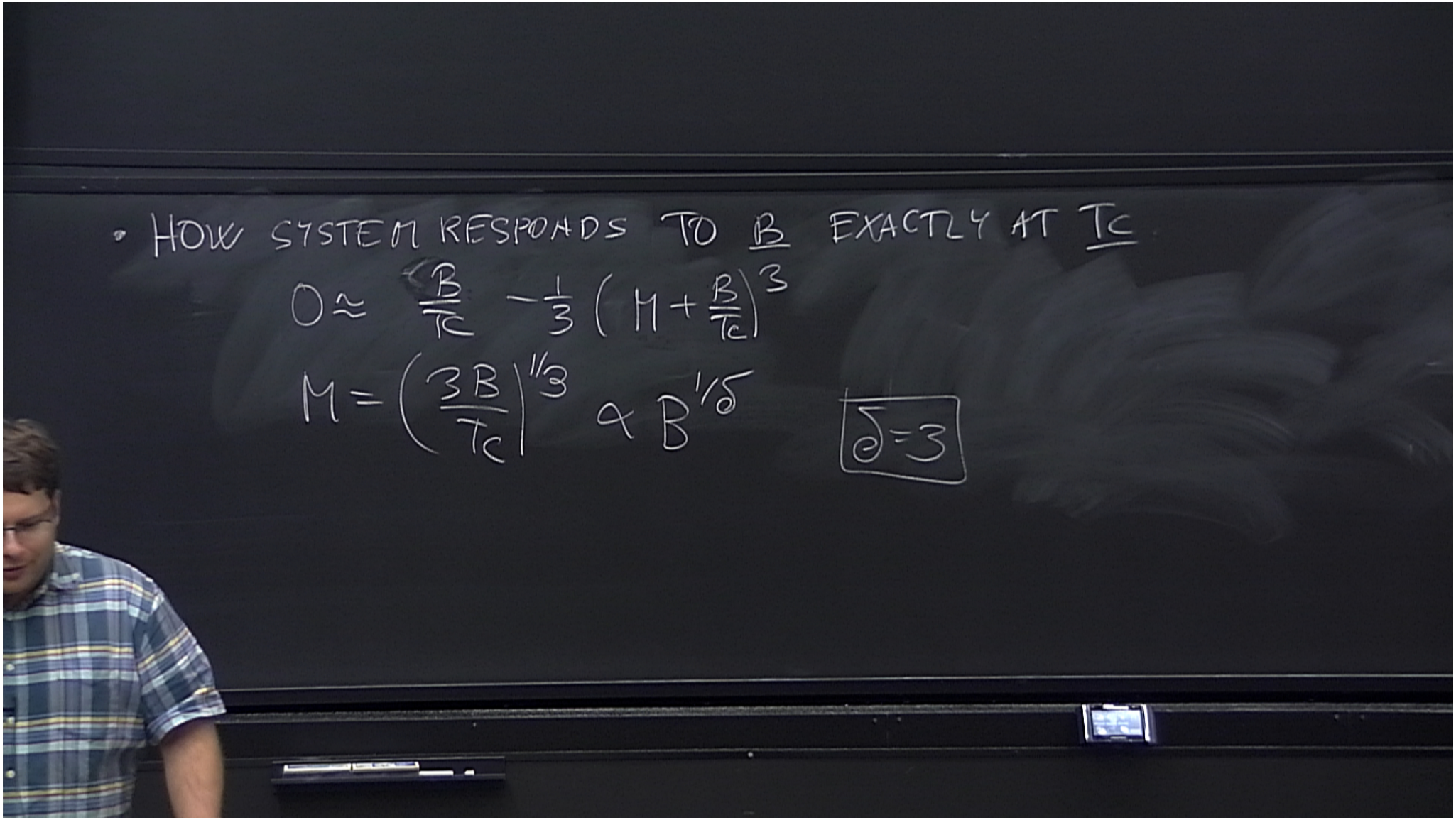

PONENTS
$\chi = \frac{\partial M}{\partial B}\Big|_T$
$\sum G_i + \frac{1}{2} N J M^2$
$\tanh\left(\frac{M+B}{T}\right)$
M B=0
M B≠0
T
$\tanh x \approx x - \frac{x^3}{3}$
$\eta \approx \frac{M+B}{T} - \frac{1}{3}\left(\frac{M+B}{T}\right)^3$
$\chi = \frac{\partial M}{\partial B} \approx \left(\chi \frac{J}{T} + \frac{1}{T}\right)\left[1 - \left(\frac{M+B}{T}\right)^2\right]$

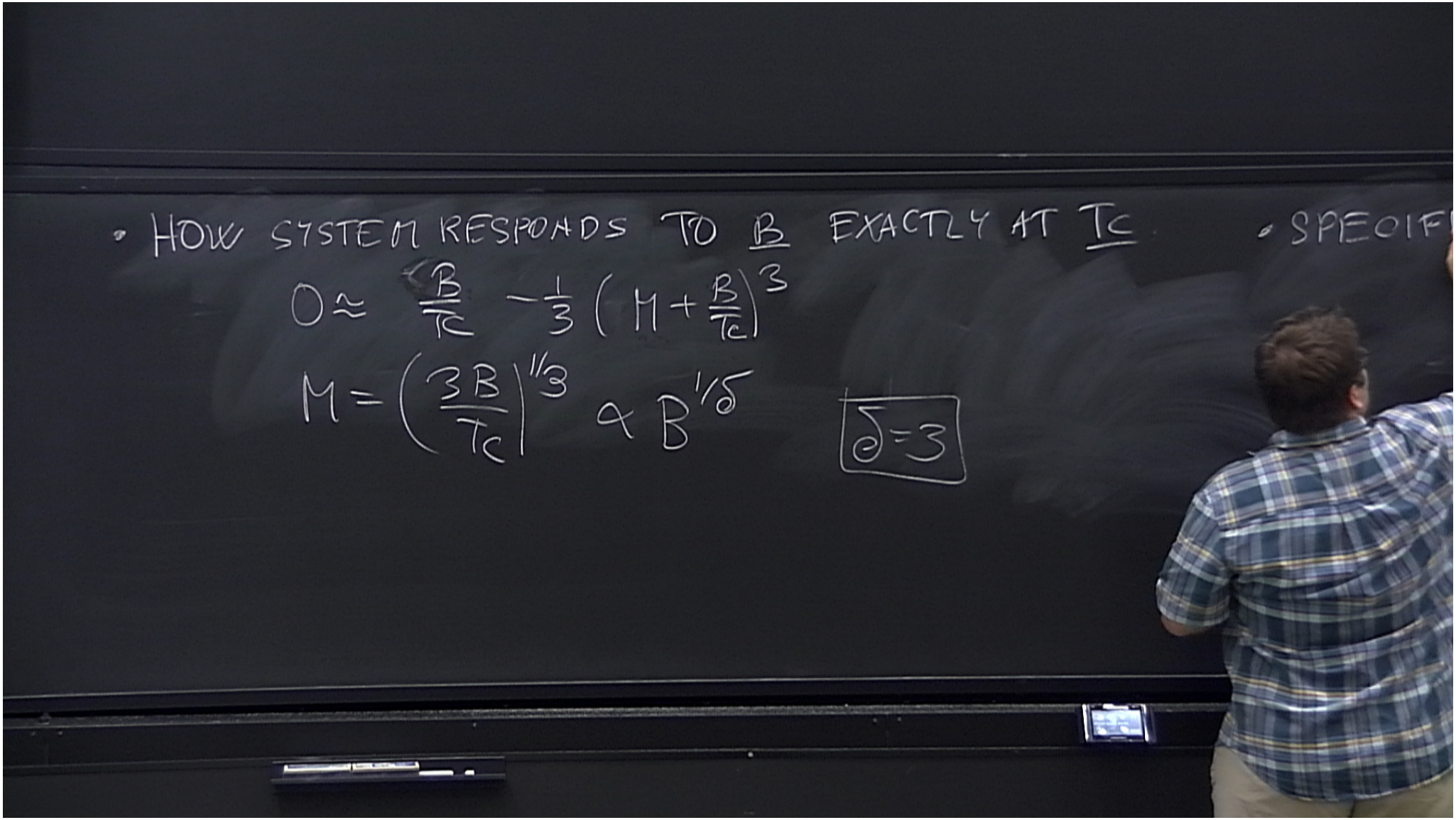

PONENTS
$\chi = \frac{\partial M}{\partial B}\Big|_T$
$\sum G_i + \frac{1}{2} N J M^2$
$\left(\frac{MHB}{T}\right)$
M B=0
M B≠0
T
$\tanh x \simeq x - \frac{x^3}{3}$
$\eta \simeq \frac{MHB}{T} - \frac{1}{3}\left(\frac{MJ+B}{T}\right)^3$
$\chi = \frac{\partial M}{\partial B} \simeq \left(\chi \frac{J}{T} + \frac{1}{T}\right)\left[1 - \left(\frac{MJ}{T}\right)^2\right]_{+B=0}$

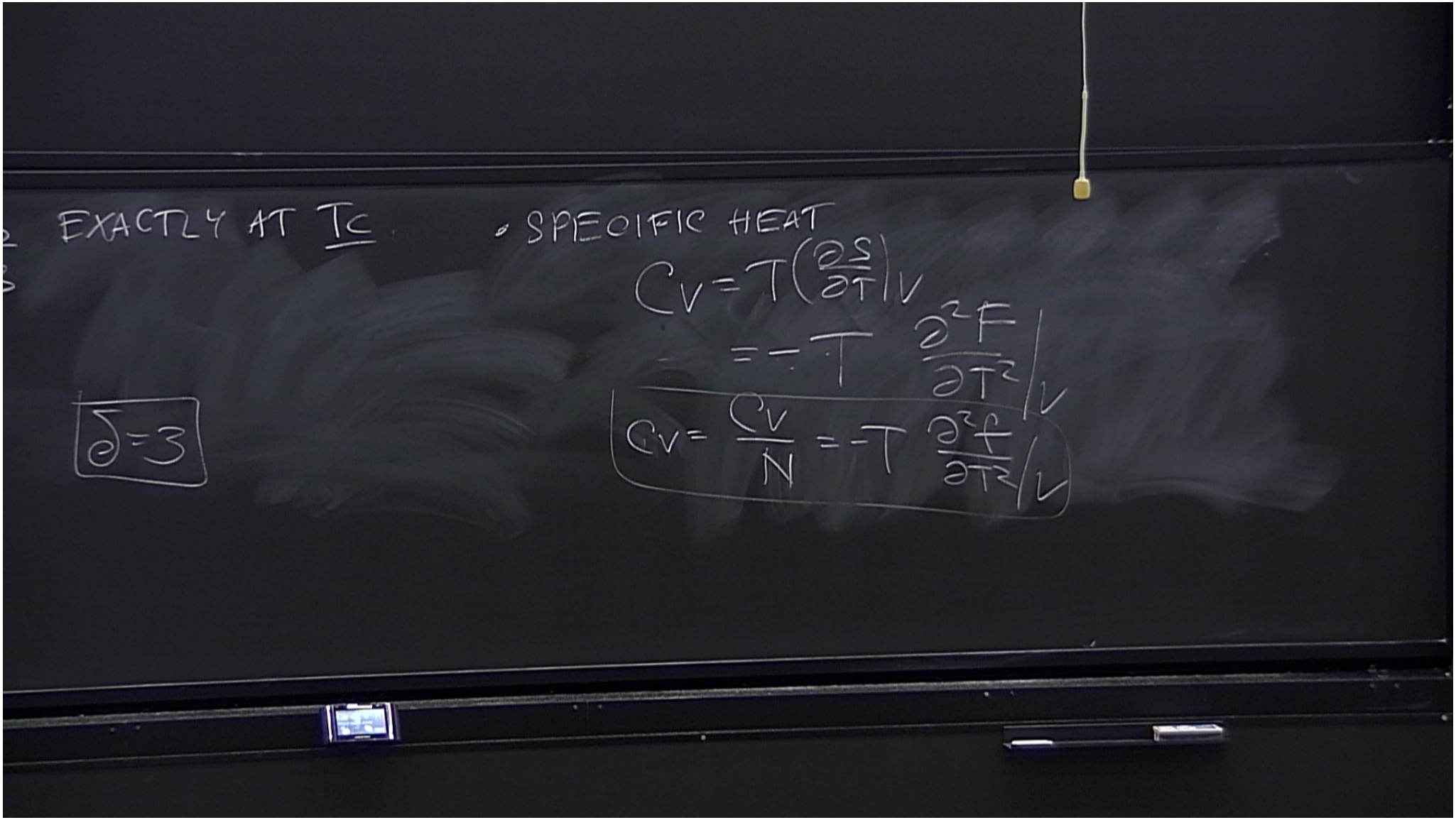

$M = \tanh\left(\frac{MJ+B}{T}\right)$
$\chi = \frac{\partial M}{\partial B} \approx$
$T > T_c$ $M \approx 0$
$\chi = \chi \frac{J}{T} + \frac{1}{T}$
$\chi\left(1 - \frac{J}{T}\right) = \frac{1}{T}$
$\chi = \frac{1}{T} \frac{1}{1 - \frac{T_c}{T}} = \frac{1}{T - T_c}$

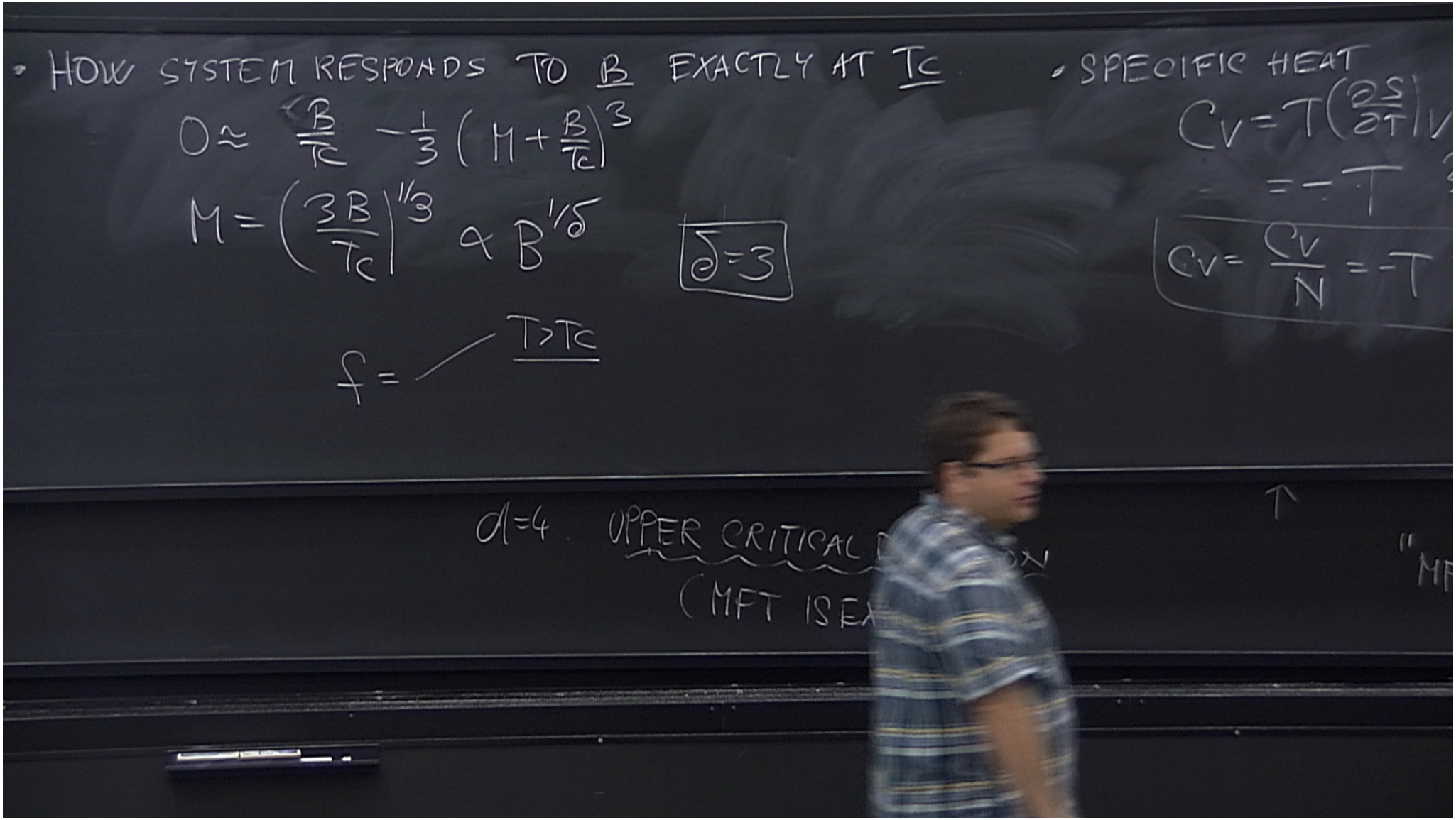

γ=1
T<Tc
M² ≈ 3(1 − T/Tc)
χ = 1/2 · 1/(Tc − T) ∝ |t|^(−γ')
γ'=1
χ
Tc
T

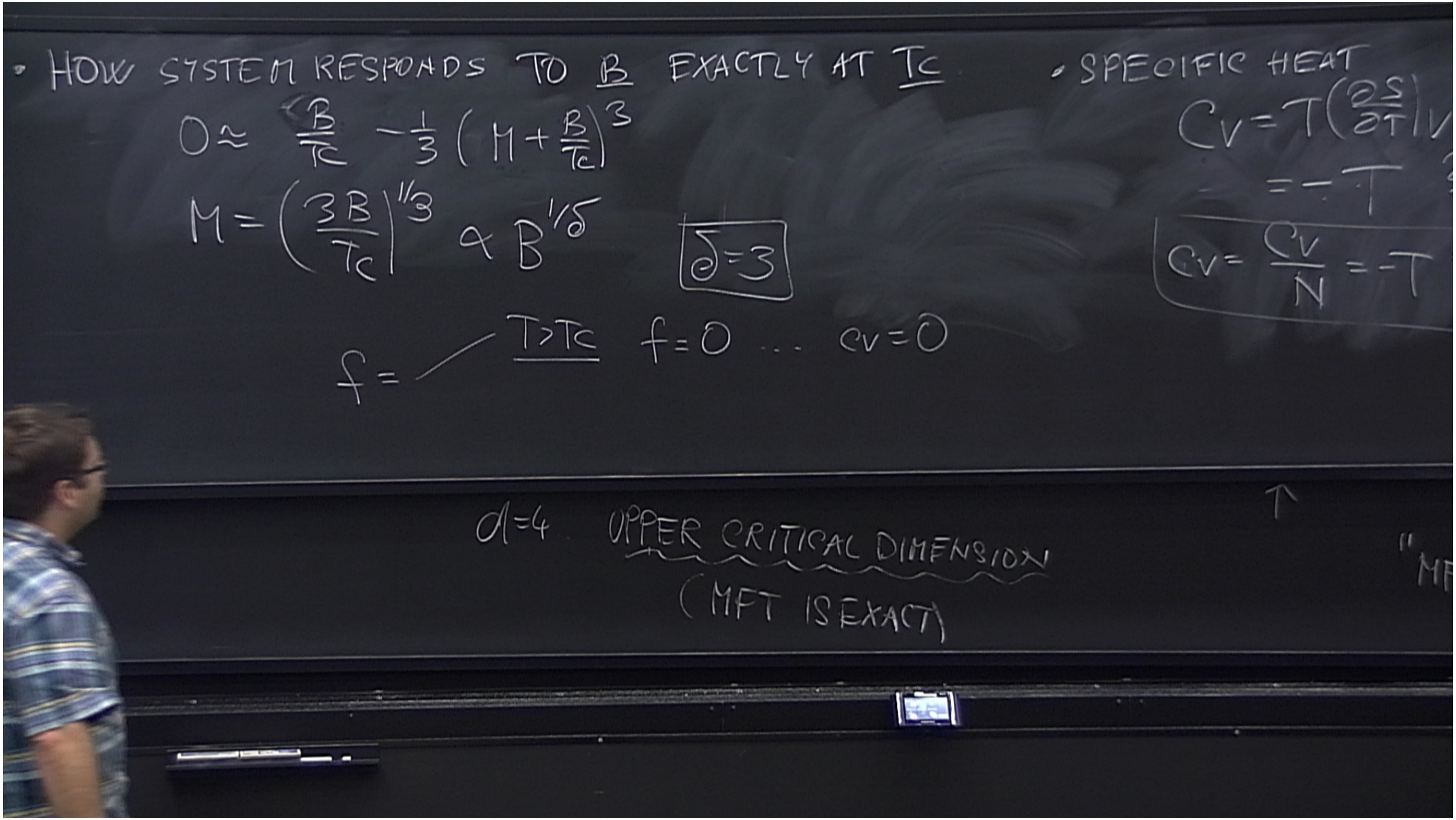

• HOW SYSTEM RESPONDS TO B EXACTLY AT Tc
0 ≃ B/Tc

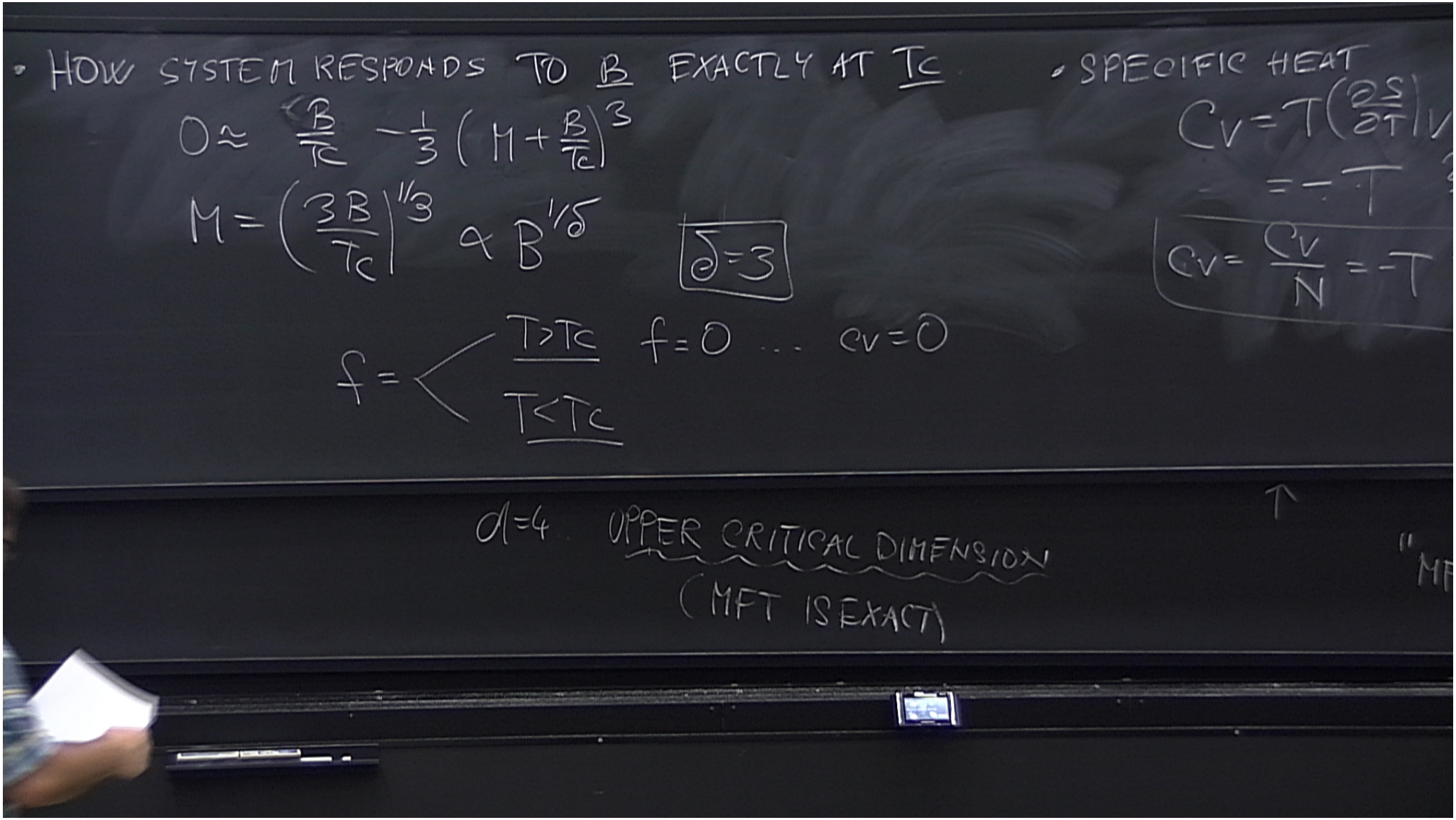

• HOW SYSTEM RESPONDS TO B EXACTLY AT Tc
$0 \approx \frac{B}{T_c} - \frac{1}{3}\left(M + \frac{B}{T_c}\right)^3$
$M = \left(\frac{3B}{T_c}\right)^{1/3} \propto B^{1/\delta}$
$\delta = 3$

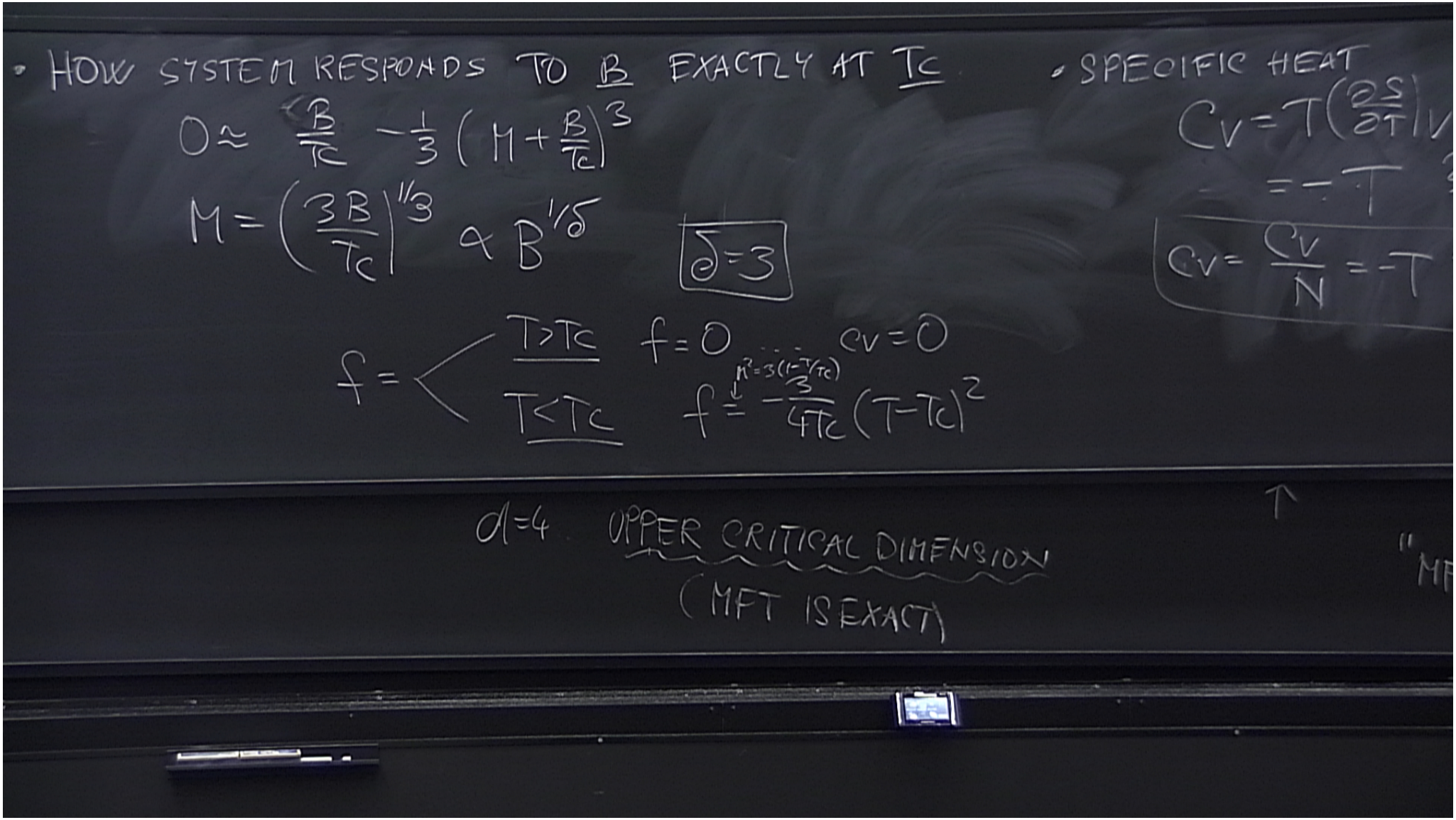

• HOW SYSTEM RESPONDS TO B EXACTLY AT Tc
• SPECIF
$0 \simeq \frac{B}{T_c} - \frac{1}{3}\left(M + \frac{B}{T_c}\right)^3$
$M = \left(\frac{3B}{T_c}\right)^{1/3} \propto B^{1/\delta}$
$\delta = 3$


EXACTLY AT Tc
• SPECIFIC HEAT
$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$
$= -T \frac{\partial^2 F}{\partial T^2}\Big|_V$
$C_V = \frac{C_V}{N} = -T \frac{\partial^2 f}{\partial T^2}\Big|_V$
$\delta = 3$


• HOW SYSTEM RESPONDS TO B EXACTLY AT Tc
$0 \simeq \frac{B}{T_c} - \frac{1}{3}\left(M + \frac{B}{T_c}\right)^3$
$M = \left(\frac{3B}{T_c}\right)^{1/3} \propto B^{1/\delta}$
$\delta = 3$
$f =$ — $T > T_c$
• SPECIFIC HEAT
$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$
$= -T$
$C_V = \frac{C_V}{N} = -T$
$d = 4$ UPPER CRITICAL
(MFT IS EX
"MF


• HOW SYSTEM RESPONDS TO B EXACTLY AT Tc
$0 \simeq \frac{B}{T_c} - \frac{1}{3}\left(M + \frac{B}{T_c}\right)^3$
$M = \left(\frac{3B}{T_c}\right)^{1/3} \propto B^{1/\delta}$
$\delta = 3$
$f = \diagup$ $T > T_c$ $f = 0$ ... $c_v = 0$
• SPECIFIC HEAT
$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$
$= -T$
$c_v = \frac{C_V}{N} = -T$
$d = 4$ UPPER CRITICAL DIMENSION
(MFT IS EXACT)


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$M^2 = 3\left(1 - \frac{T}{T_c}\right)$
$C_V = 0$
• SPECIFIC HEAT
$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$
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$C_V = \frac{C_V}{N} = -T$
$d = 4$ UPPER CRITICAL DIMENSION
(MFT IS EXACT)

M RESPONDS TO B EXACTLY AT Tc

$\frac{B}{T_c} \sim \frac{1}{3}\left(M+\frac{B}{T_c}\right)^3$

$\left(\frac{3B}{T_c}\right)^{1/3} \propto B^{1/\delta}$

$\boxed{\delta = 3}$

$f = \begin{cases} T>T_c & f=0 \quad \ldots \quad C_V=0 \\ T<T_c & f = -\frac{3}{4T_c}(T-T_c)^2 \quad M^2 = 3\left(1-\frac{T}{T_c}\right) \quad C_V = \frac{3}{2}\frac{I}{T_c} \end{cases}$

• SPECIFIC HEAT

$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = -T\frac{\partial^2 F}{\partial T^2}\Big|_V$

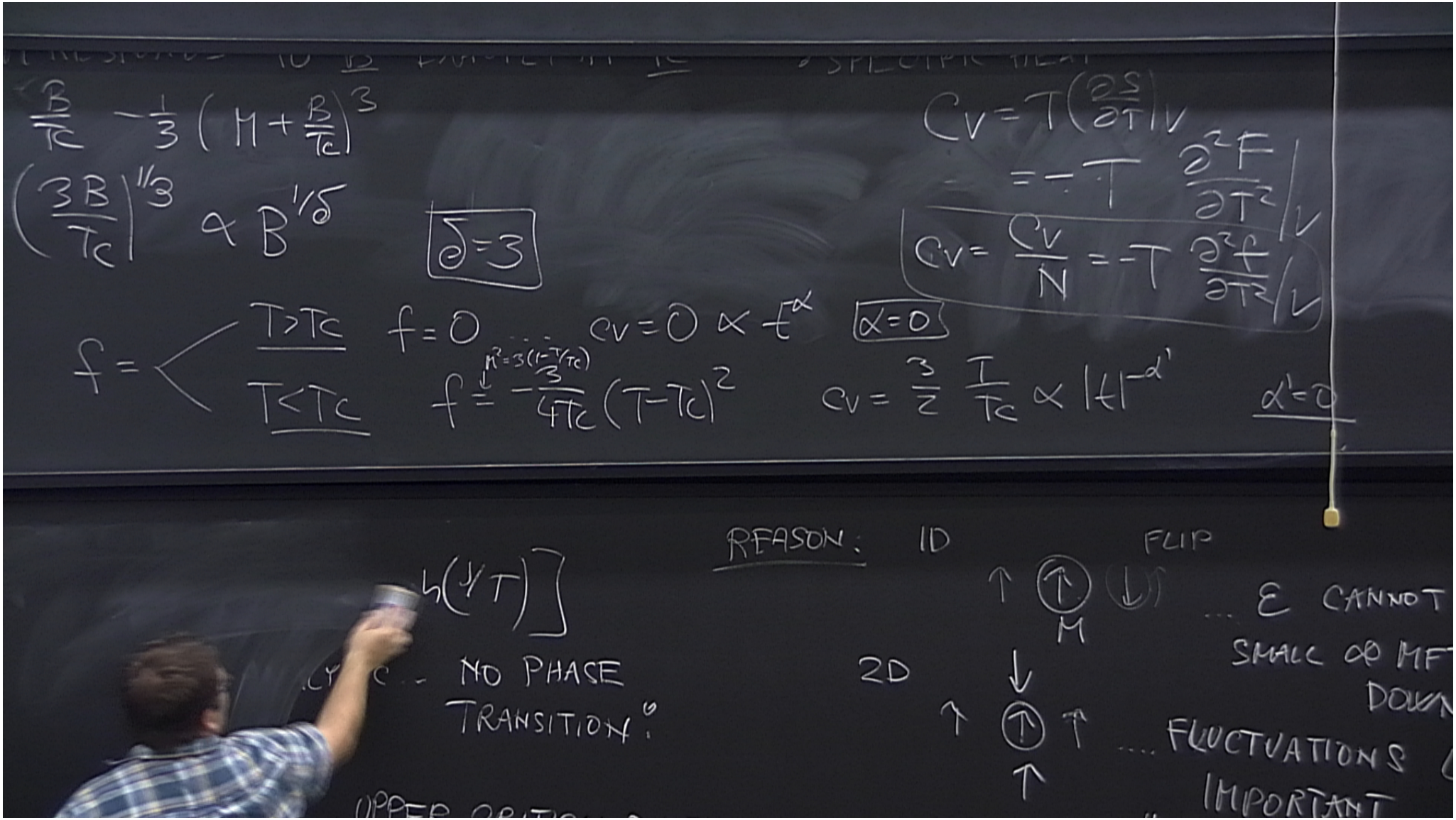
$C_V = \frac{C_V}{N} = -T\frac{\partial^2 f}{\partial T^2}\Big|_V$

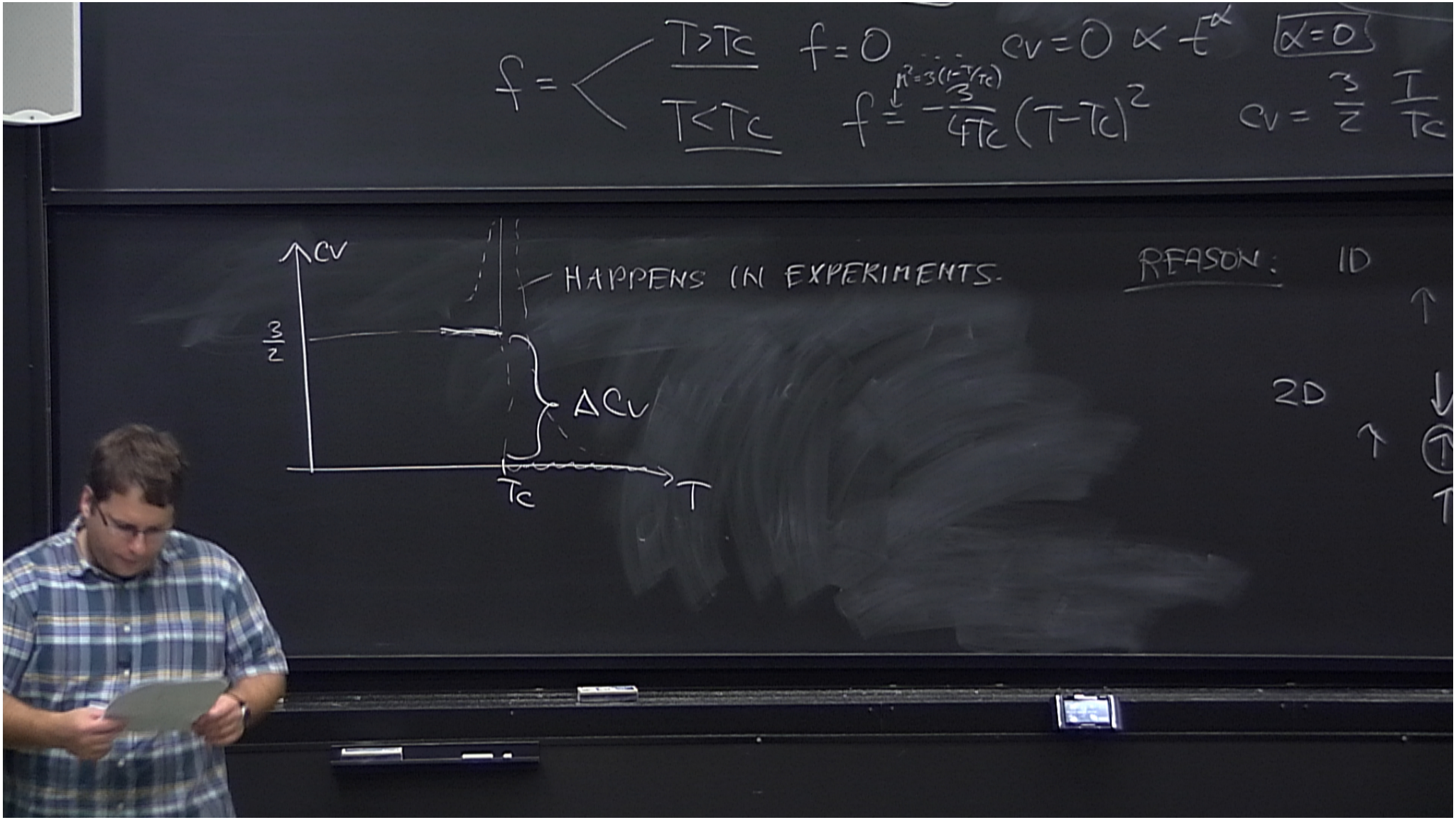
$d=4$ UPPER CRITICAL DIMENSION

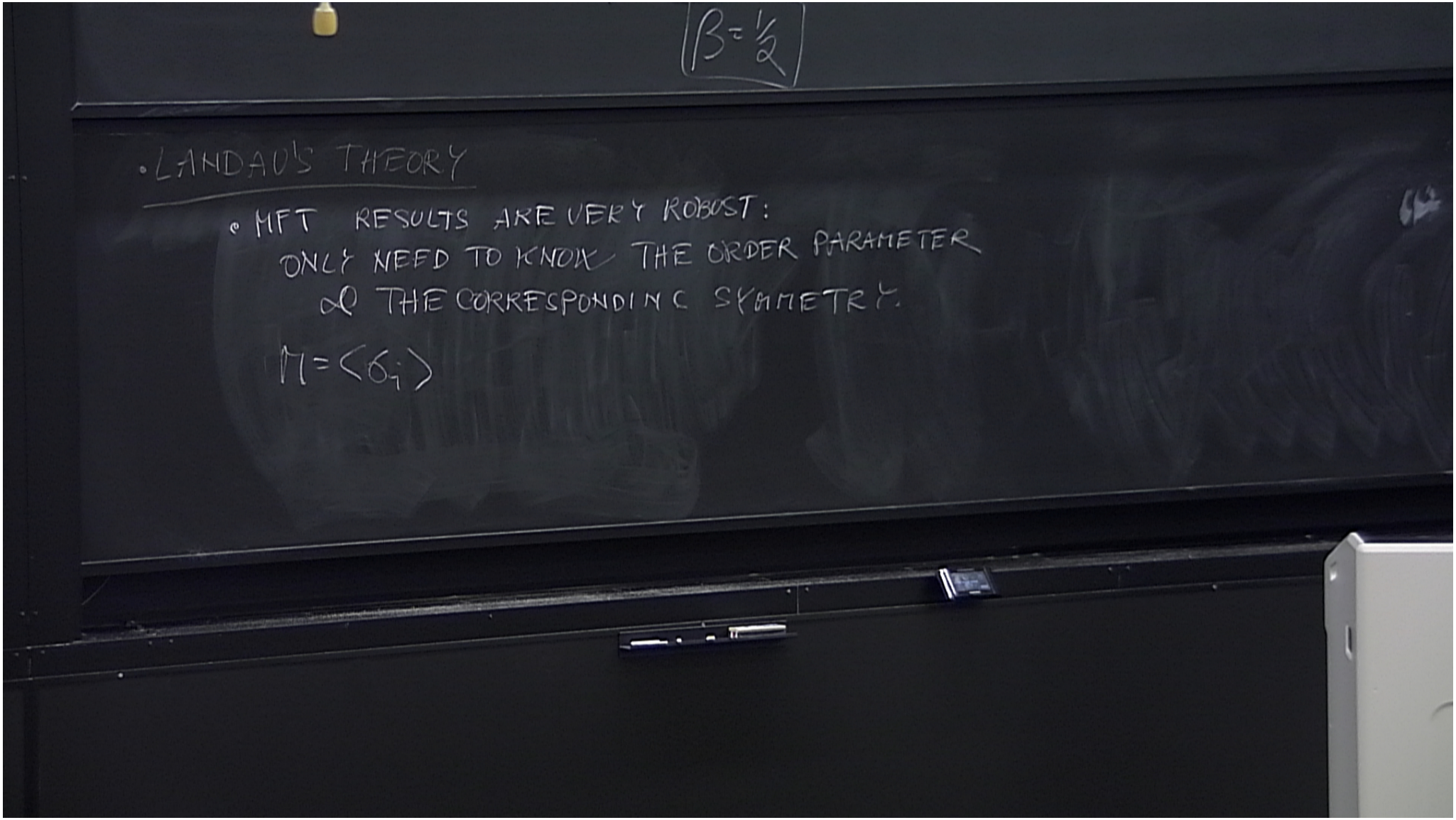
(MFT IS EXACT)

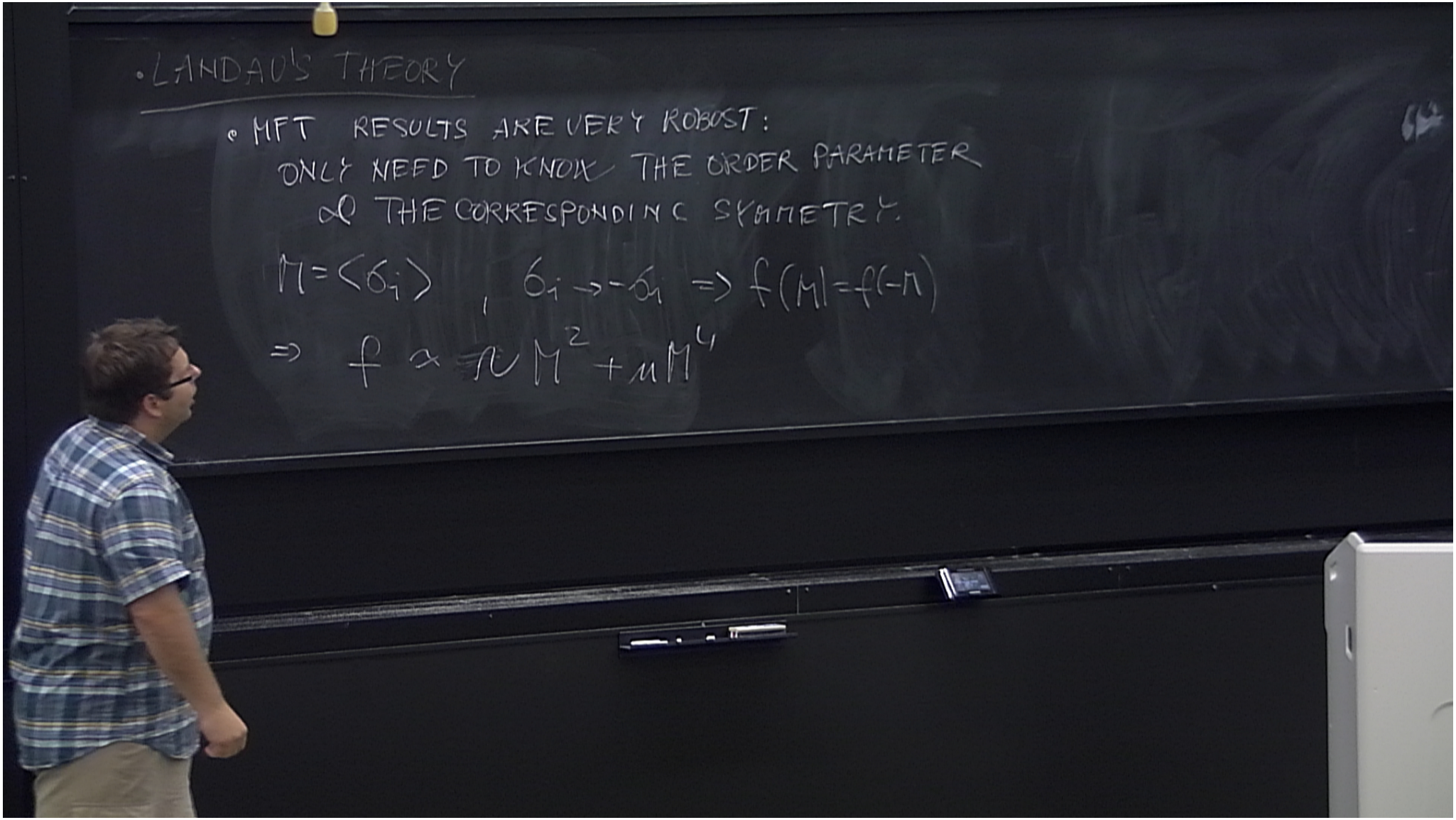
↑ IMPORTANT

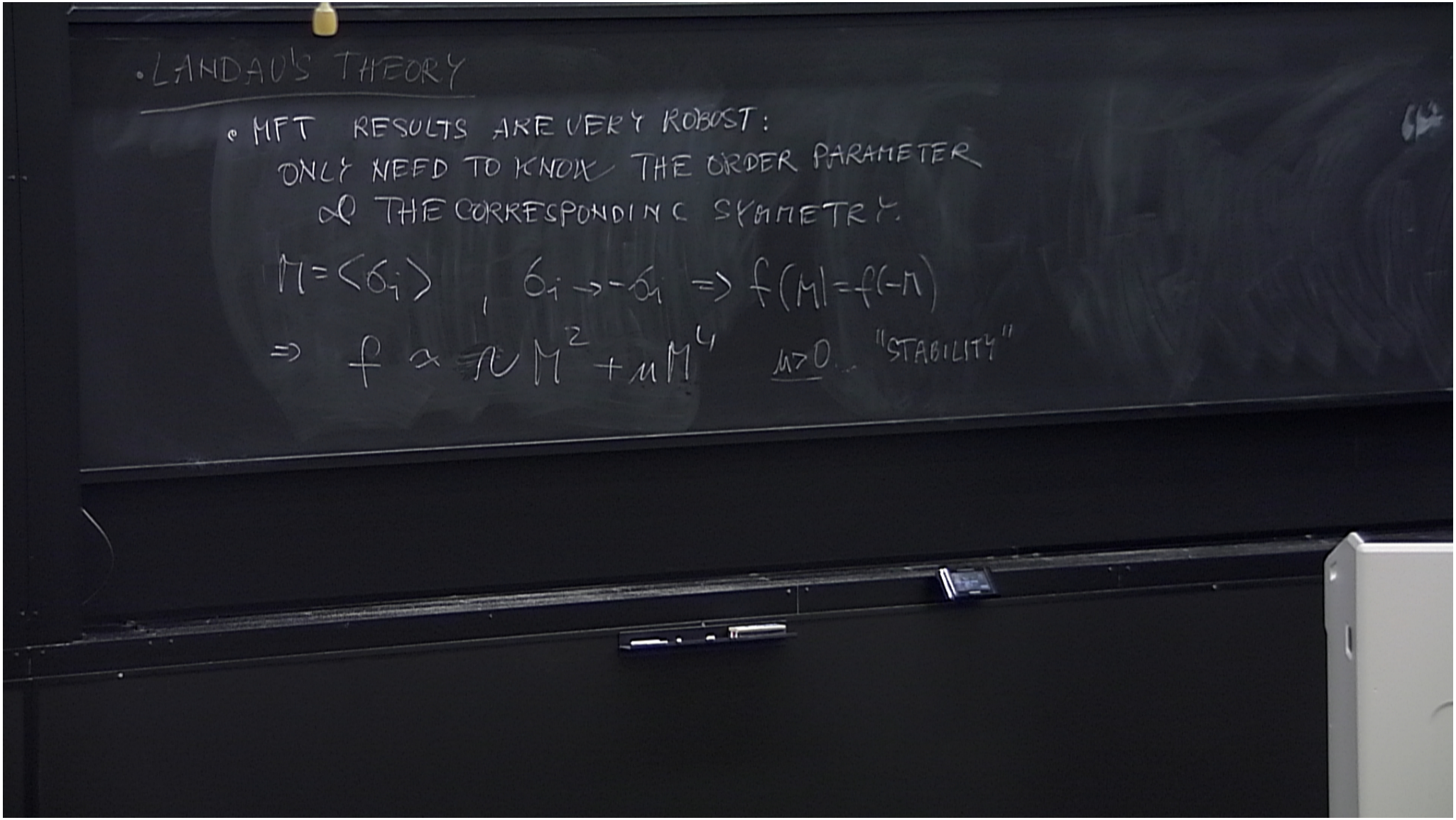
" MFT MORE EXACT

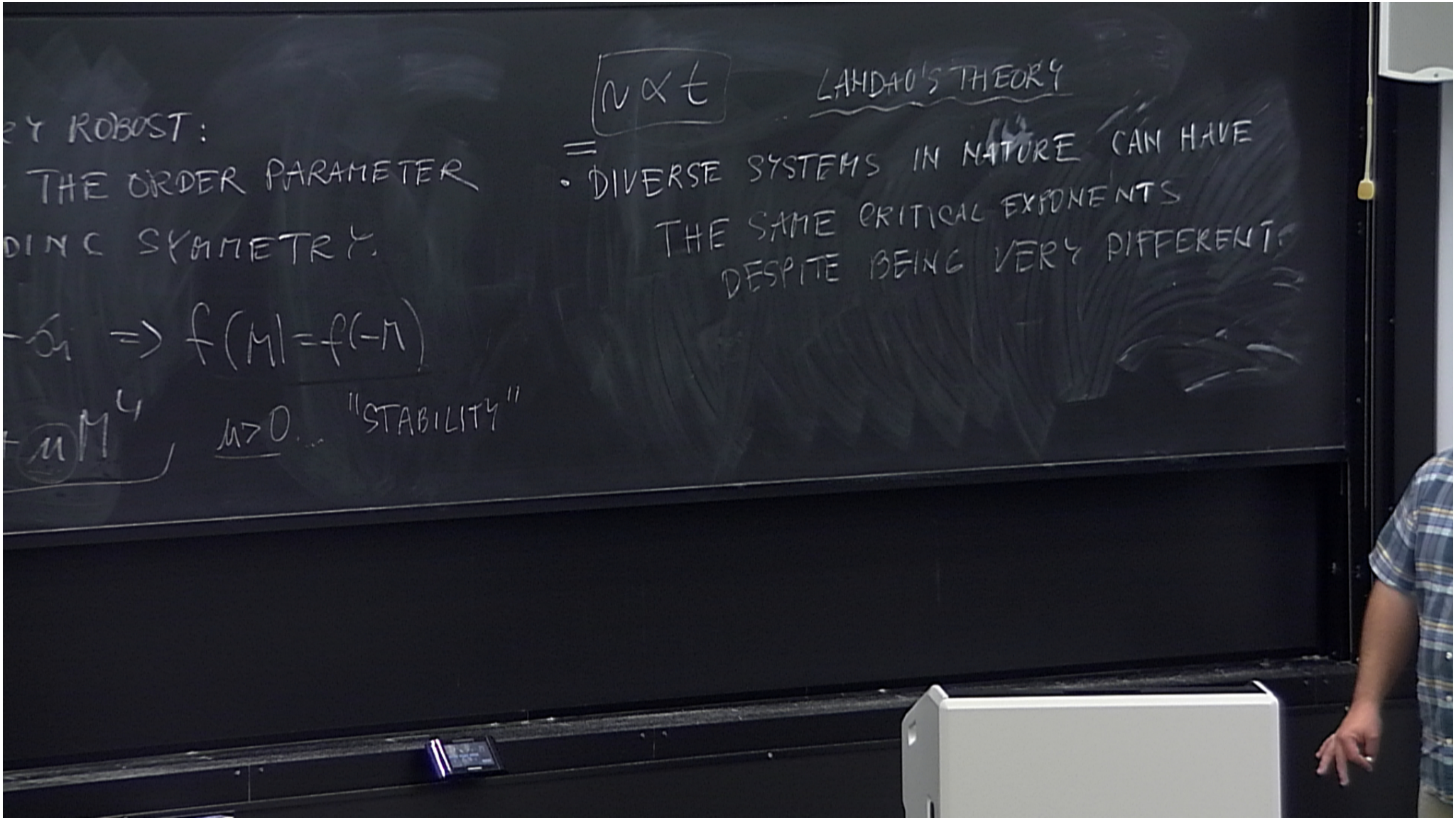

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$f = \begin{cases} T>T_c & f=0 \quad C_V = 0 \propto t^{\alpha} \quad \alpha=0 \\ T<T_c & f = -\frac{3}{4T_c}(T-T_c)^2 \quad M^2 = 3(1-T/T_c) \quad C_V = \frac{3}{2}\frac{I}{T_c} \propto |t|^{-\alpha'} \quad \alpha'=0 \end{cases}$
REASON: 1D
FLIP
M
... ε CANNOT
SMALL ⇒ MF
DOWN
... FLUCTUATIONS
IMPORTANT
2D
NO PHASE TRANSITION!

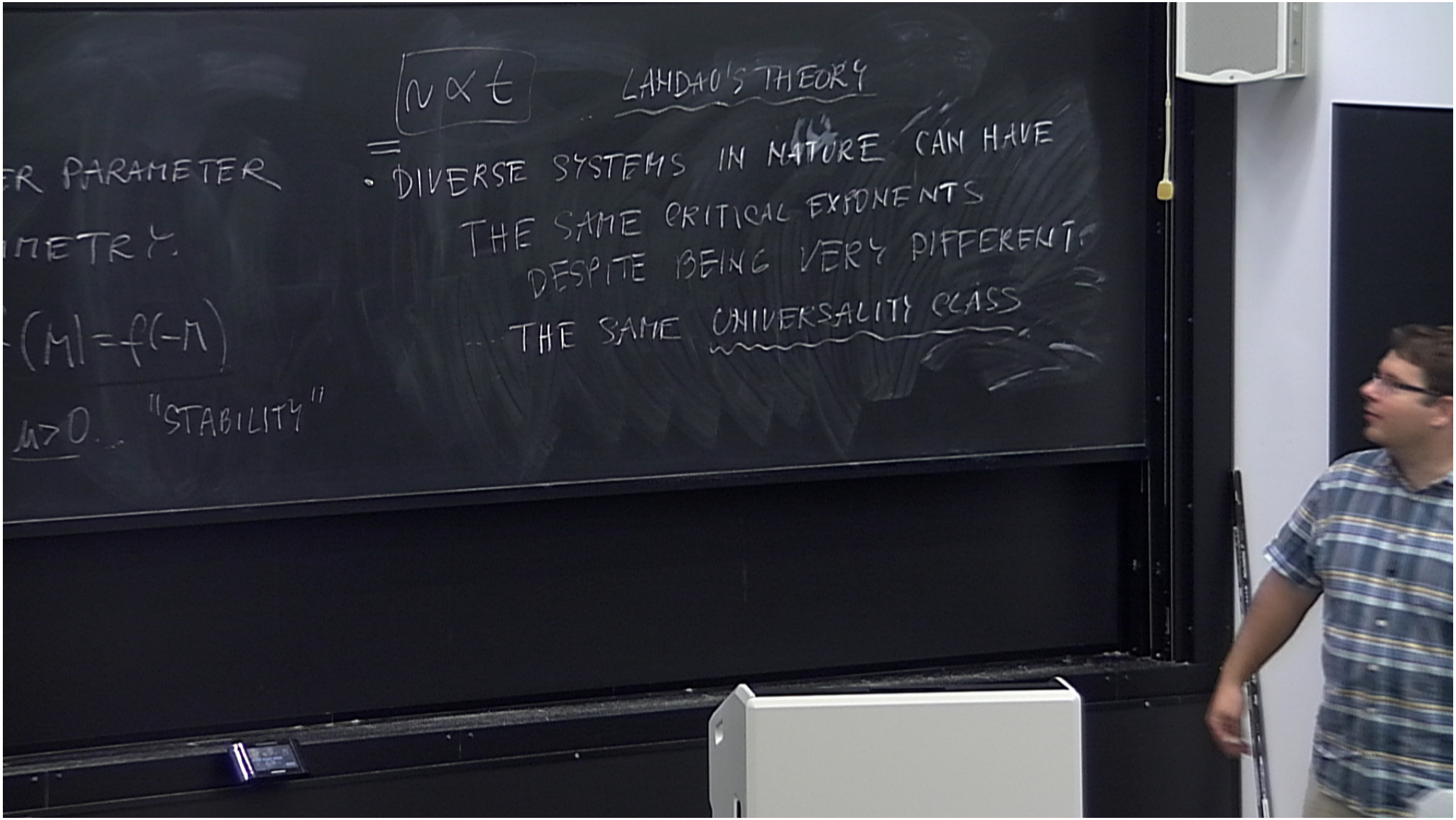

f = { T>Tc f=0 ... Cv=0 ∝ t^α α=0
T<Tc f = 1/(4Tc) (T−Tc)^2 Cv = 3/2 I/Tc
Cv
3/2
HAPPENS IN EXPERIMENTS.
ΔCv
Tc
T
REASON: 1D
2D

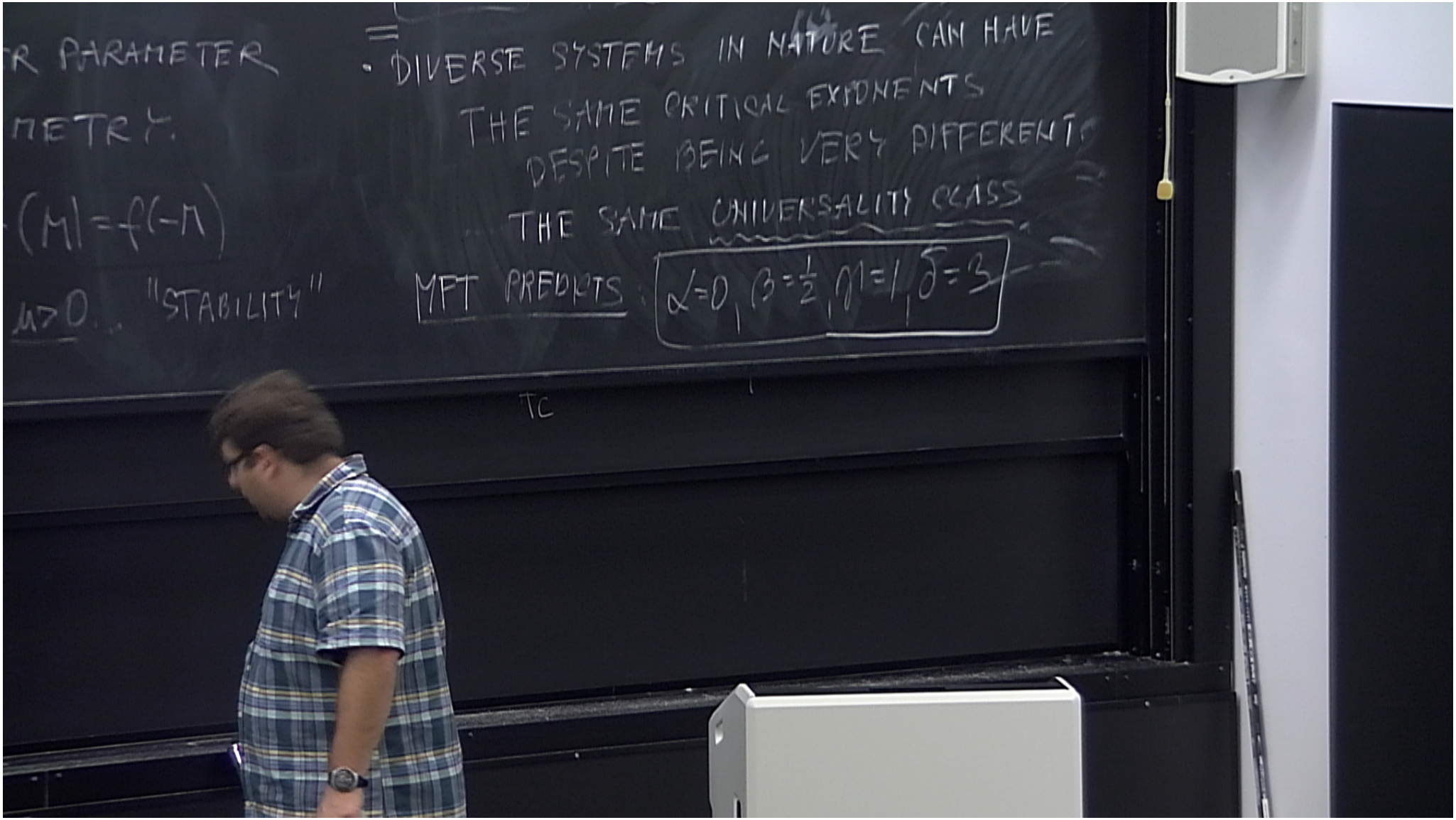

β = ½
• LANDAU'S THEORY
• MFT RESULTS ARE VERY ROBUST:
ONLY NEED TO KNOW THE ORDER PARAMETER
& THE CORRESPONDING SYMMETRY.
M = ⟨σ_i⟩

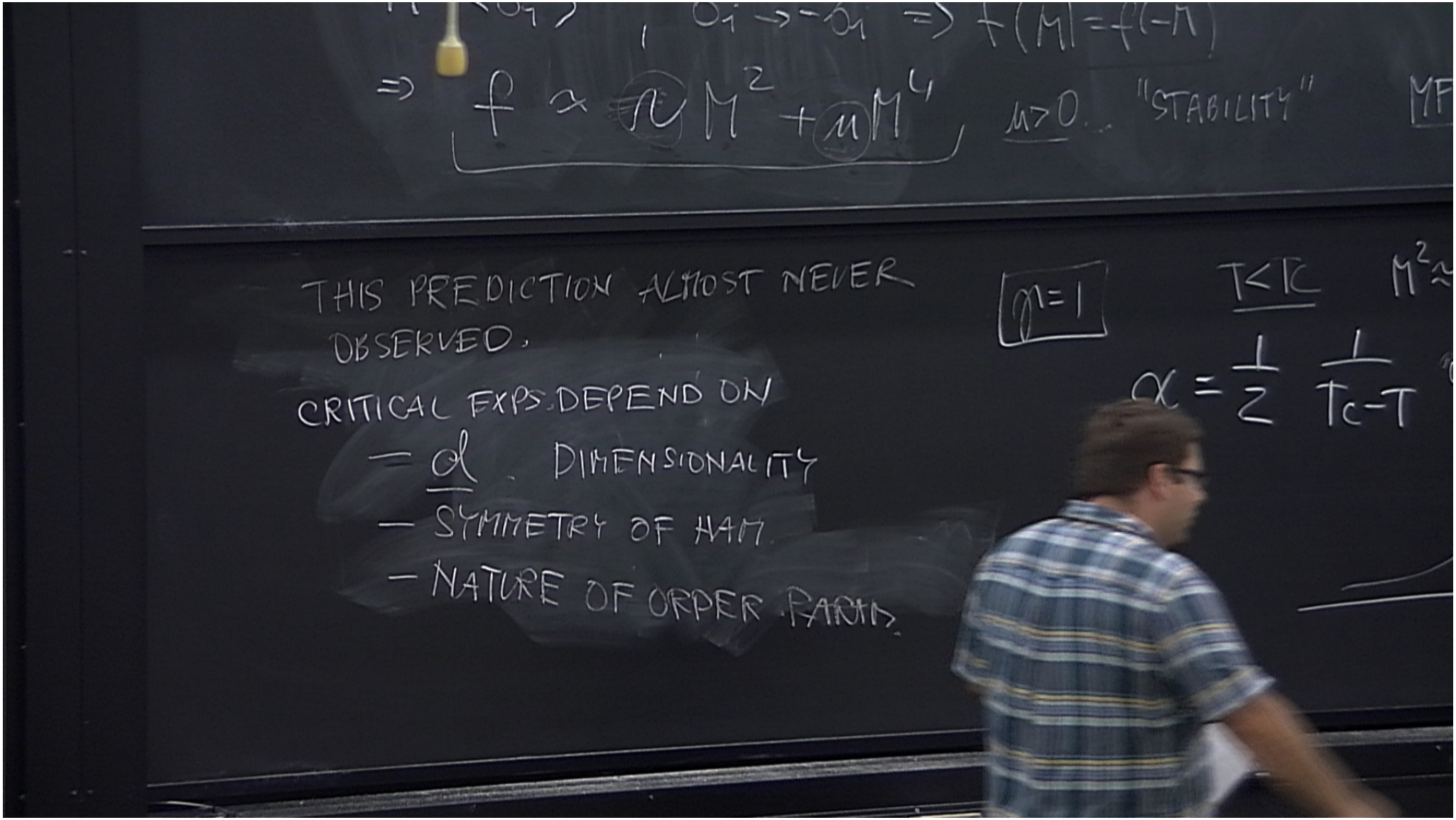

• LANDAU'S THEORY
• MFT RESULTS ARE VERY ROBUST:
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$M = \langle \sigma_i \rangle$, $\sigma_i \to -\sigma_i \Rightarrow f(M) = f(-M)$
$\Rightarrow f \propto r M^2 + u M^4$


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$\Rightarrow f \propto r M^2 + u M^4$ $u > 0$ ... "STABILITY"


ROBUST:
THE ORDER PARAMETER
DING SYMMETRY.
$\Rightarrow f(M) = f(-M)$
uM^4
$u > 0$... "STABILITY"
$n \propto t$
=
LANDAU'S THEORY
DIVERSE SYSTEMS IN NATURE CAN HAVE
THE SAME CRITICAL EXPONENTS
DESPITE BEING VERY DIFFERENT.


ER PARAMETER
METRY.
(M)=f(-M)
M>0 "STABILITY"
n∝t
LANDAU'S THEORY
=
DIVERSE SYSTEMS IN NATURE CAN HAVE
THE SAME CRITICAL EXPONENTS
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R PARAMETER
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DIVERSE SYSTEMS IN NATURE CAN HAVE
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THE SAME UNIVERSALITY CLASS
MFT PREDICTS: α=0, β=½, γ=1, δ=3
Tc


$\Rightarrow f \propto r M^2 + u M^4$
$u > 0$ ... "STABILITY"
THIS PREDICTION ALMOST NEVER OBSERVED.
CRITICAL EXPS DEPEND ON
= d DIMENSIONALITY
– SYMMETRY OF HAM
– NATURE OF ORDER PARAM.
$T < T_c$
$\alpha = \frac{1}{2}$
$\frac{1}{T_c - T}$

$\Rightarrow f(M) = f(-M)$

$\Rightarrow f \propto r M^2 + u M^4$, $u > 0$ ... "STABILITY"

MF

THIS PREDICTION ALMOST NEVER OBSERVED.

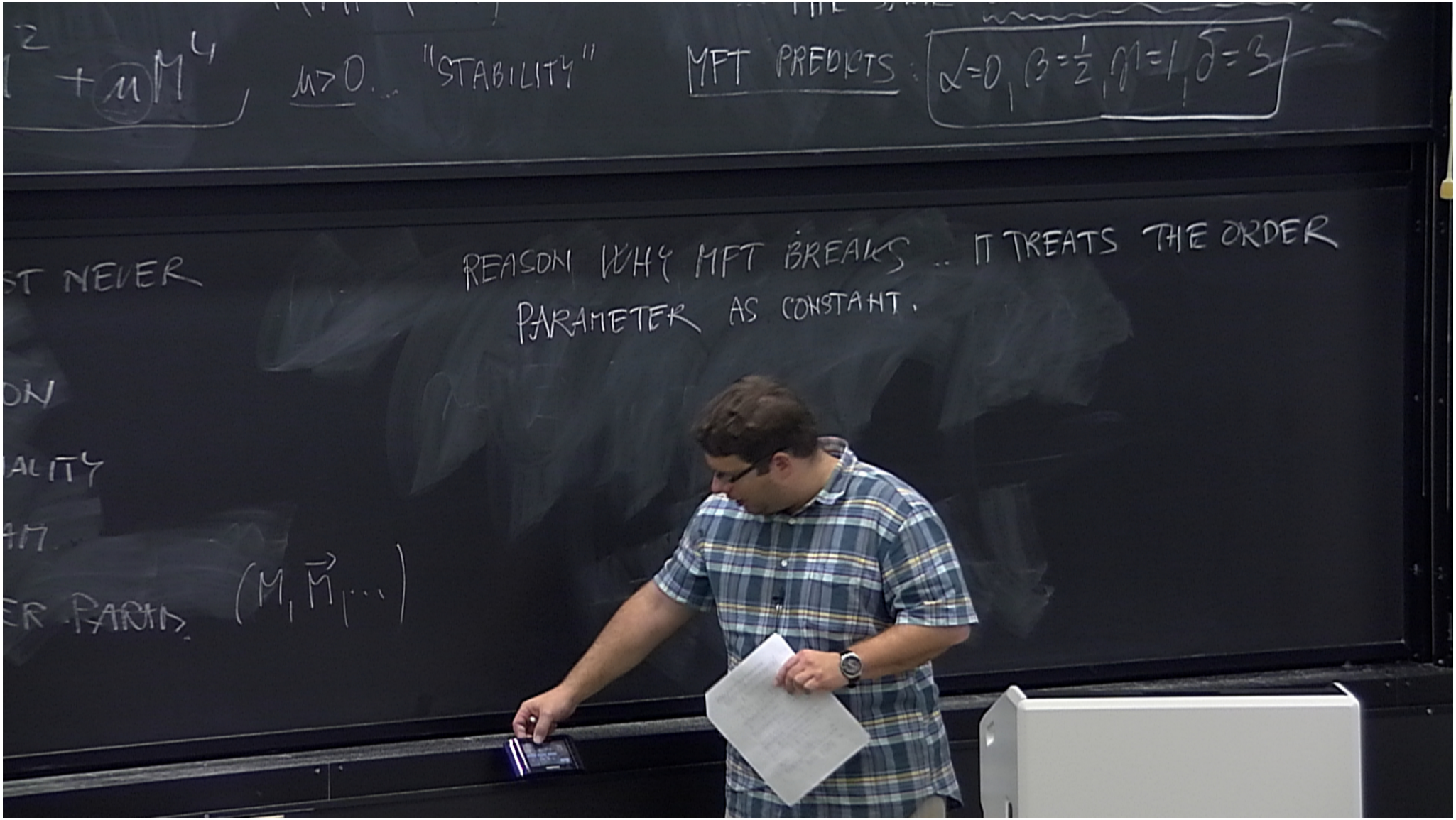
$\gamma = 1$

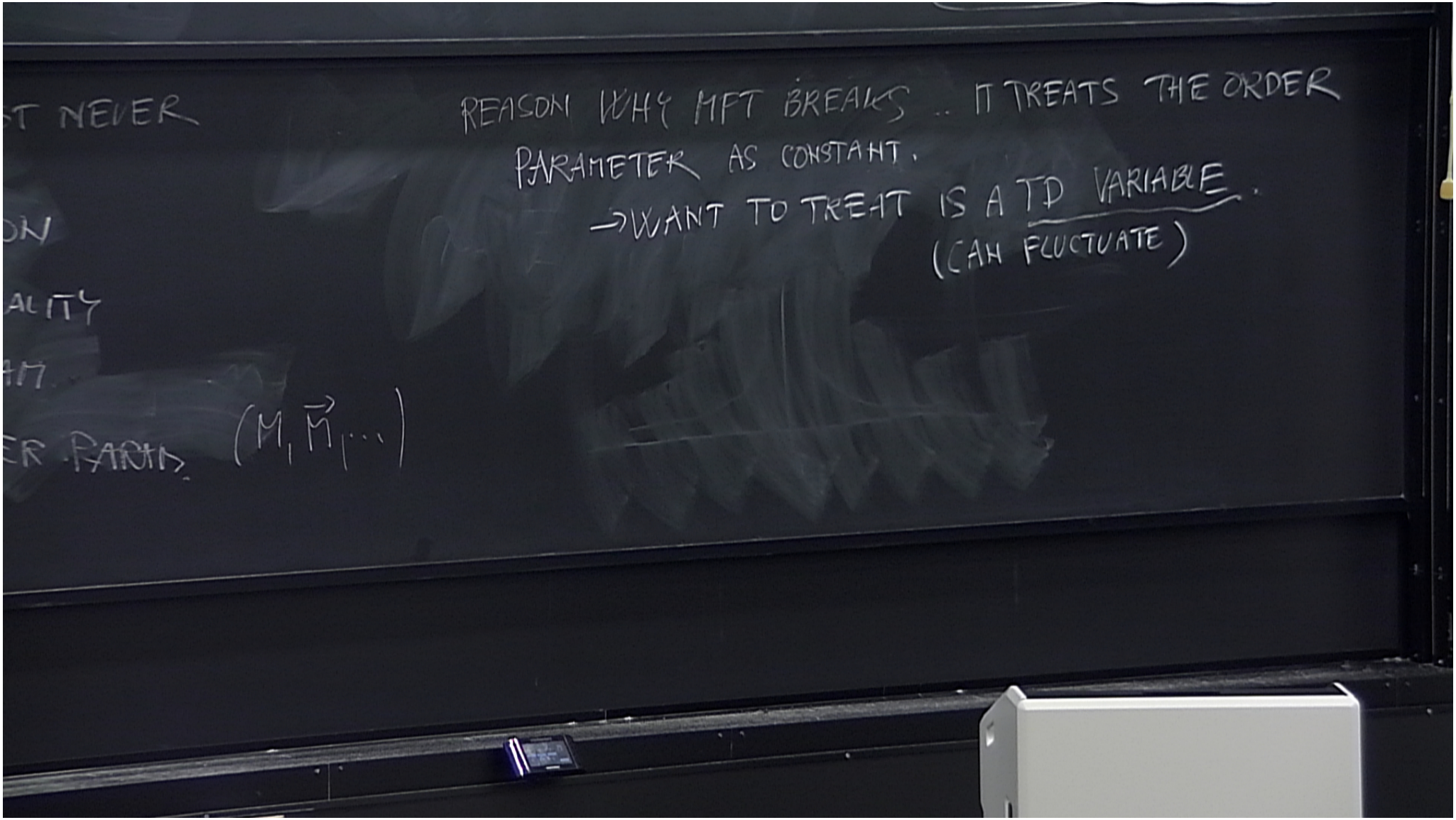
$T < T_c$ $M^2 \approx$

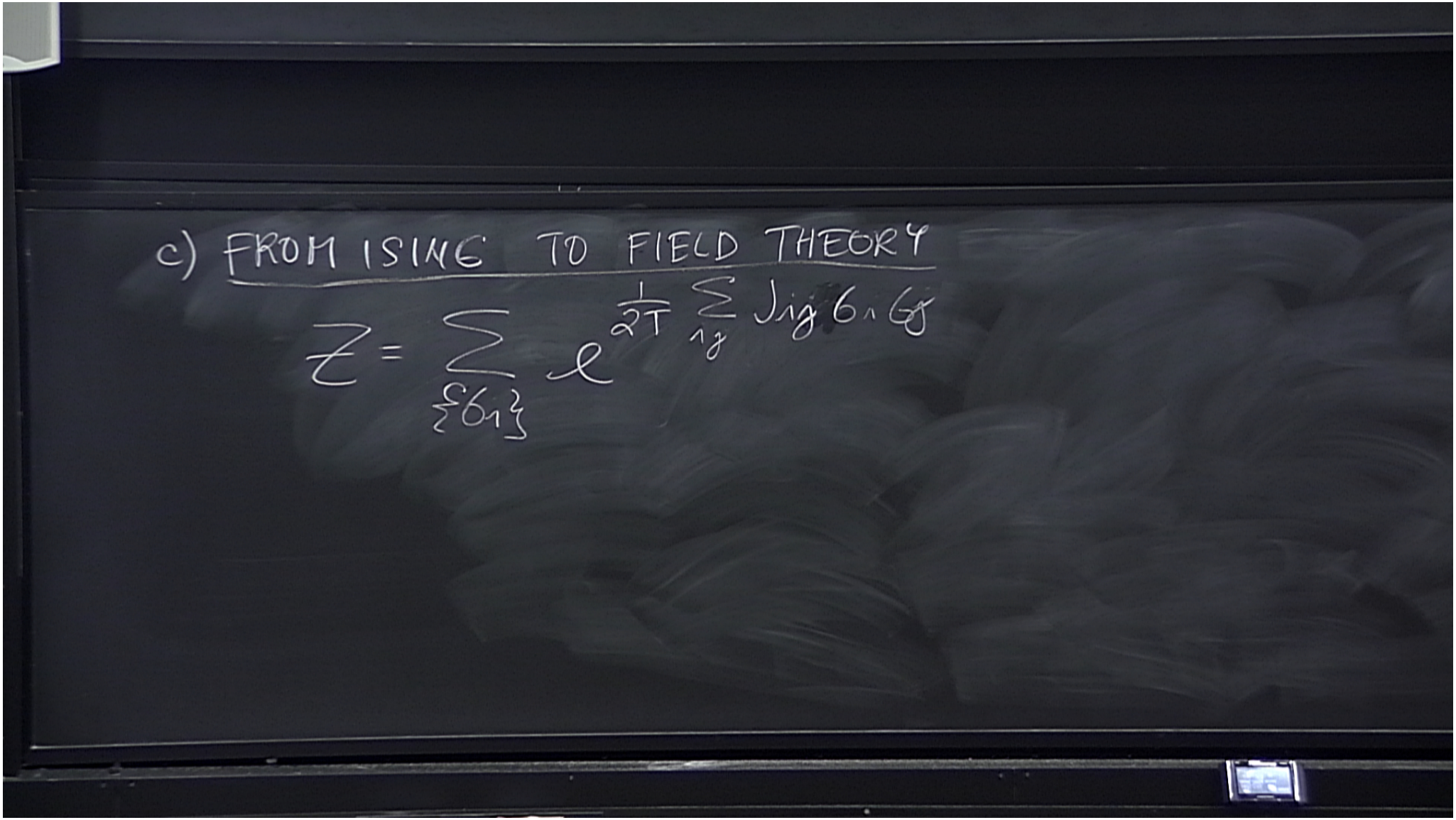
$\chi = \frac{1}{2} \frac{1}{T_c - T}$

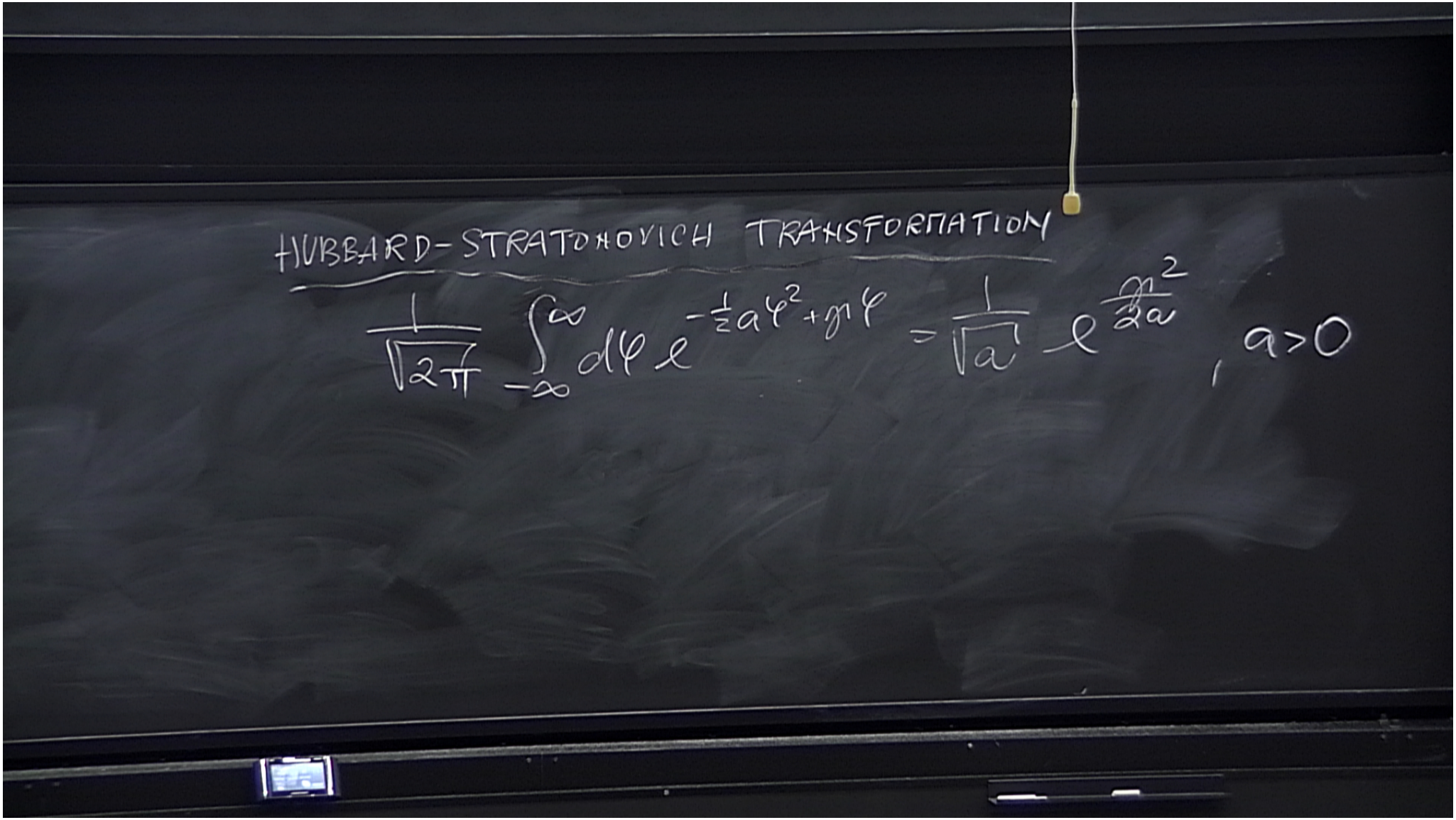
CRITICAL EXPS DEPEND ON

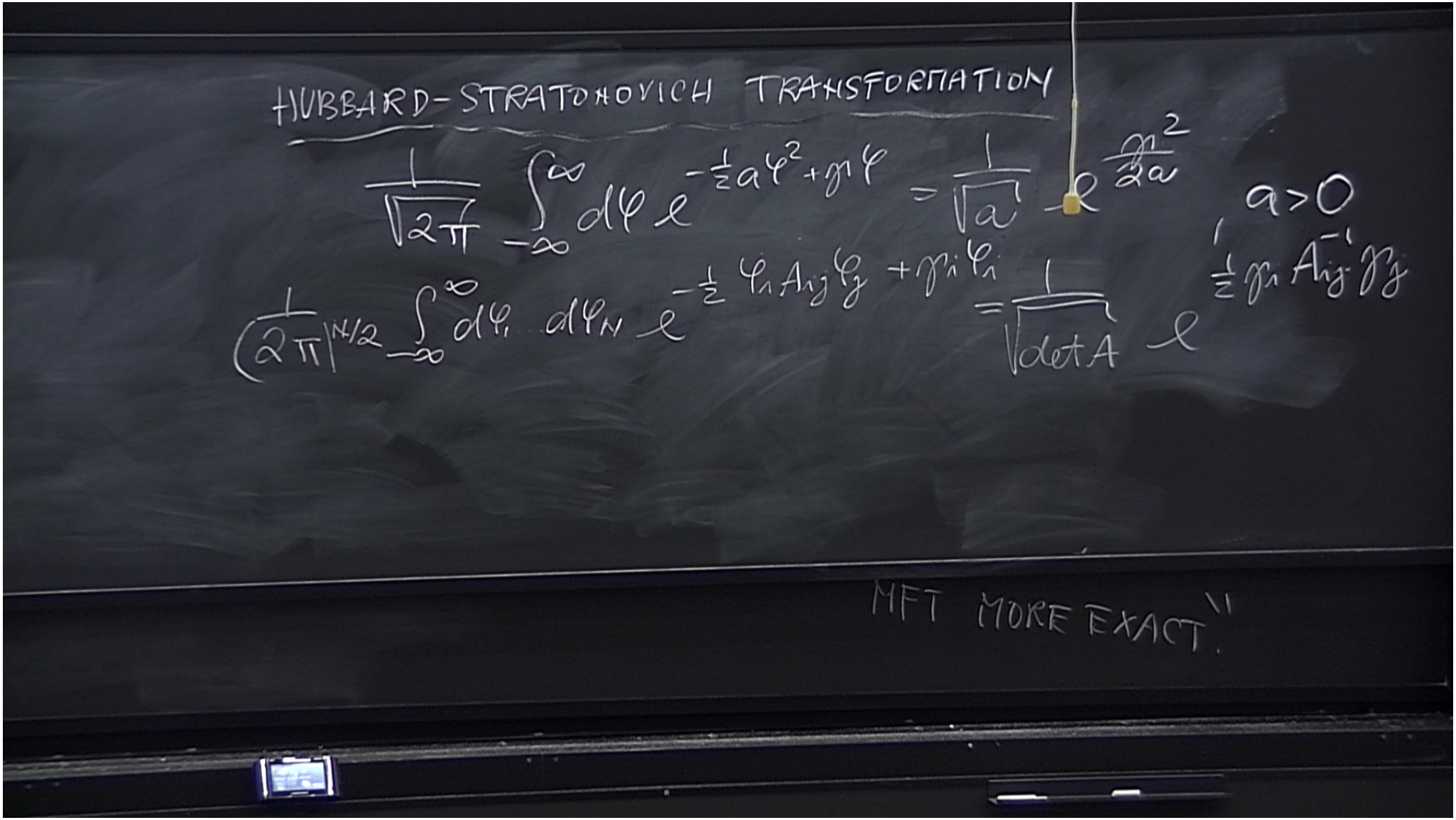
- $= \underline{d}$ DIMENSIONALITY
- SYMMETRY OF HAM
- NATURE OF ORDER PARAM. $(M, \vec{M}, \ldots)$

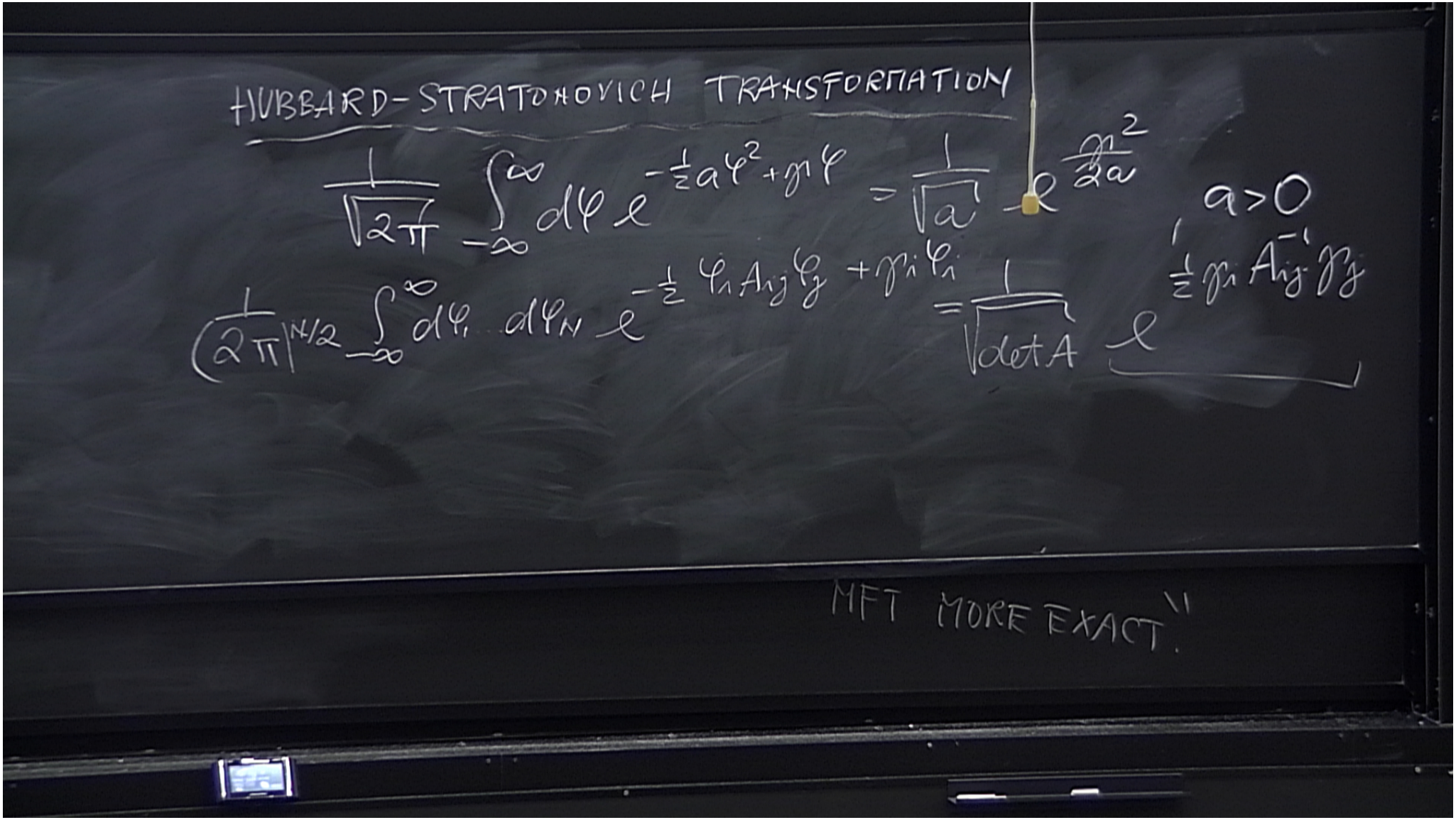

$+uM^4$
$u>0$... "STABILITY"
MFT PREDICTS: $\alpha=0, \beta=\frac{1}{2}, \gamma=1, \delta=3$
ST NEVER
REASON WHY MFT BREAKS .. IT TREATS THE ORDER
PARAMETER AS CONSTANT.
ON
ALITY
AM
ER PARAM. $(M, \vec{M}, \ldots)$

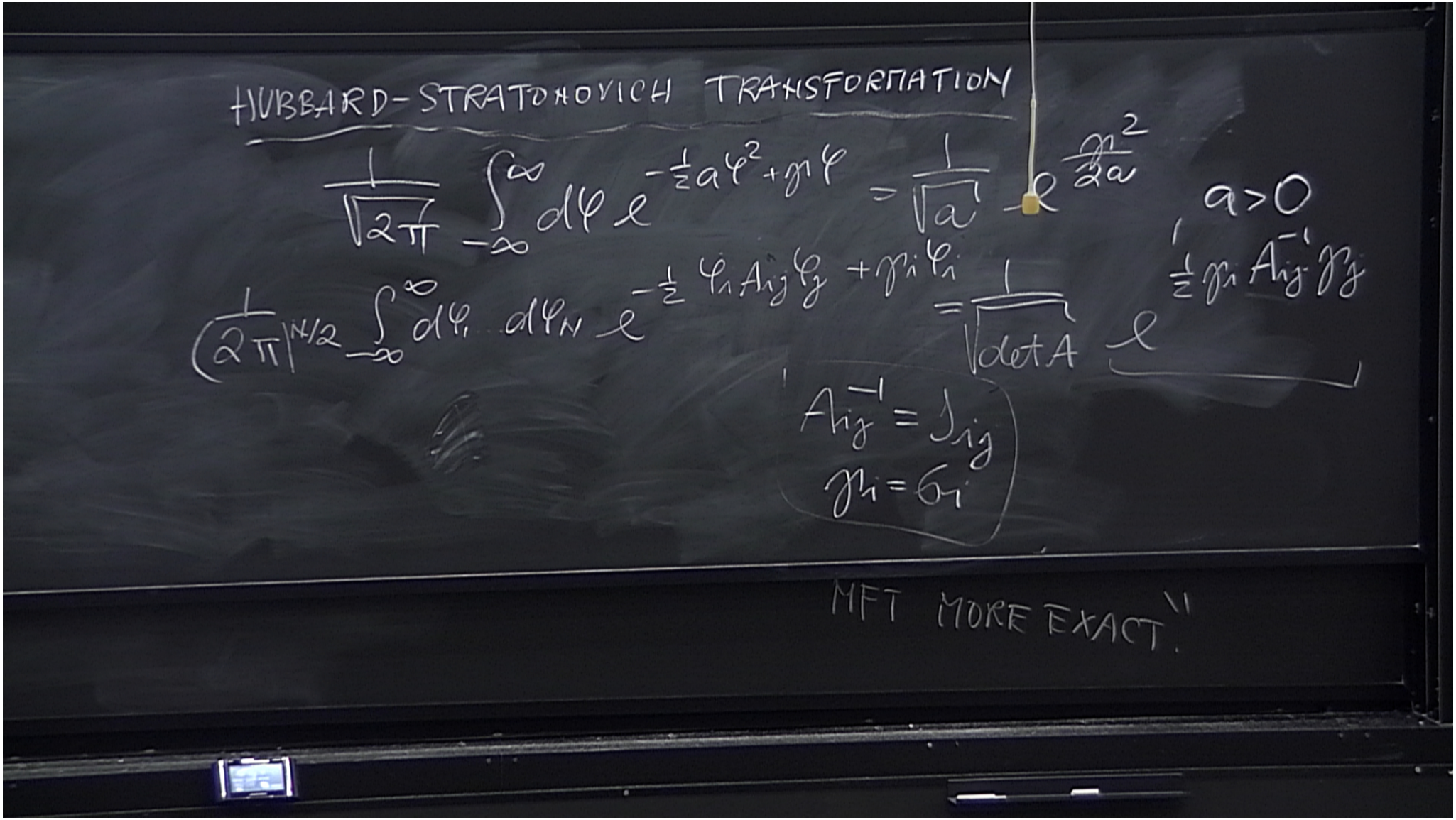

ST NEVER
ON
ALITY
AM
ER PARAM. $(M, \vec{M}, \ldots)$
REASON WHY MFT BREAKS .. IT TREATS THE ORDER
PARAMETER AS CONSTANT.
→WANT TO TREAT IS A TD VARIABLE.
(CAN FLUCTUATE)

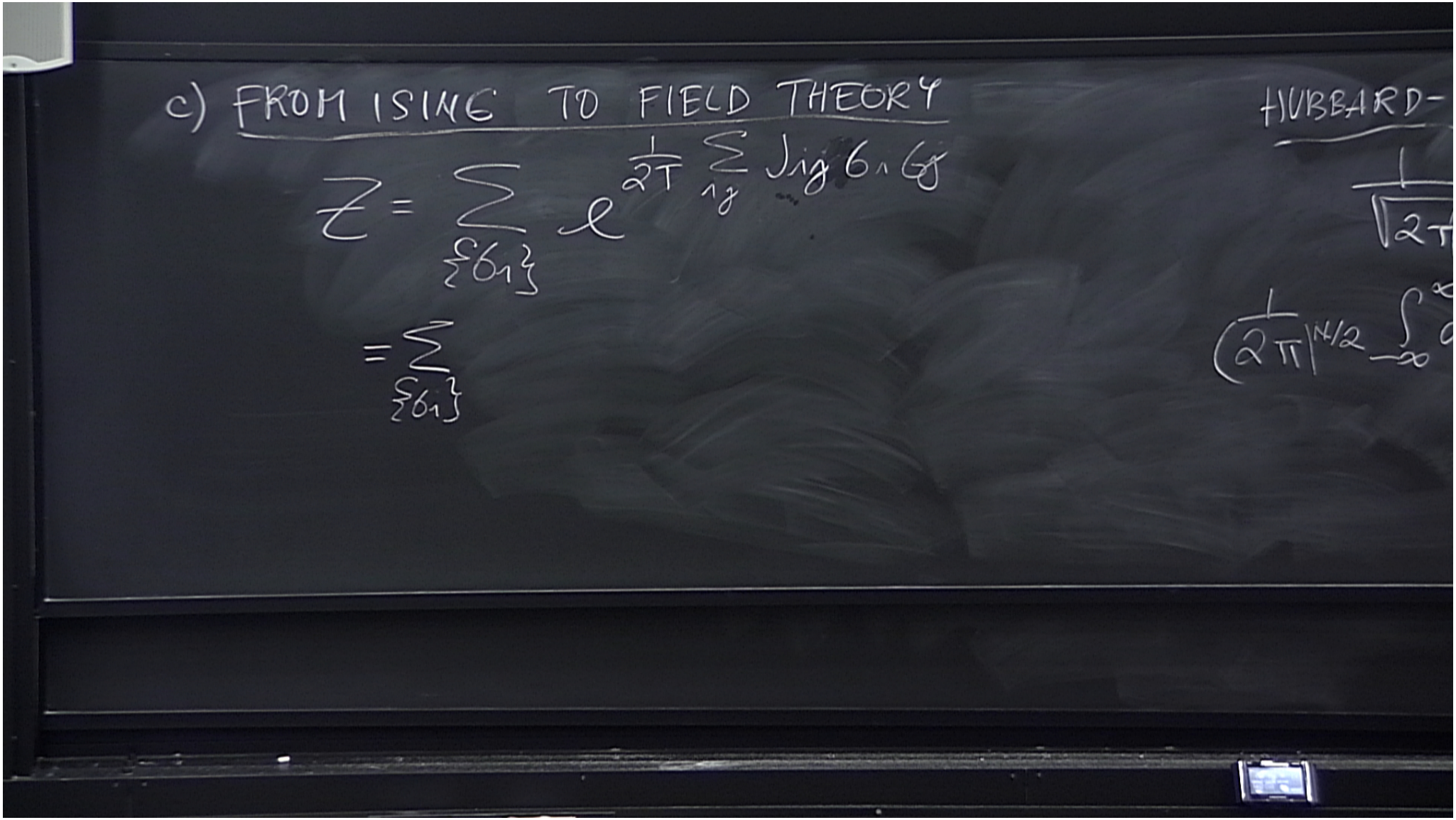

c) FROM ISING TO FIELD THEORY
$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T} \sum_{ij} J_{ij} \sigma_i \sigma_j}$

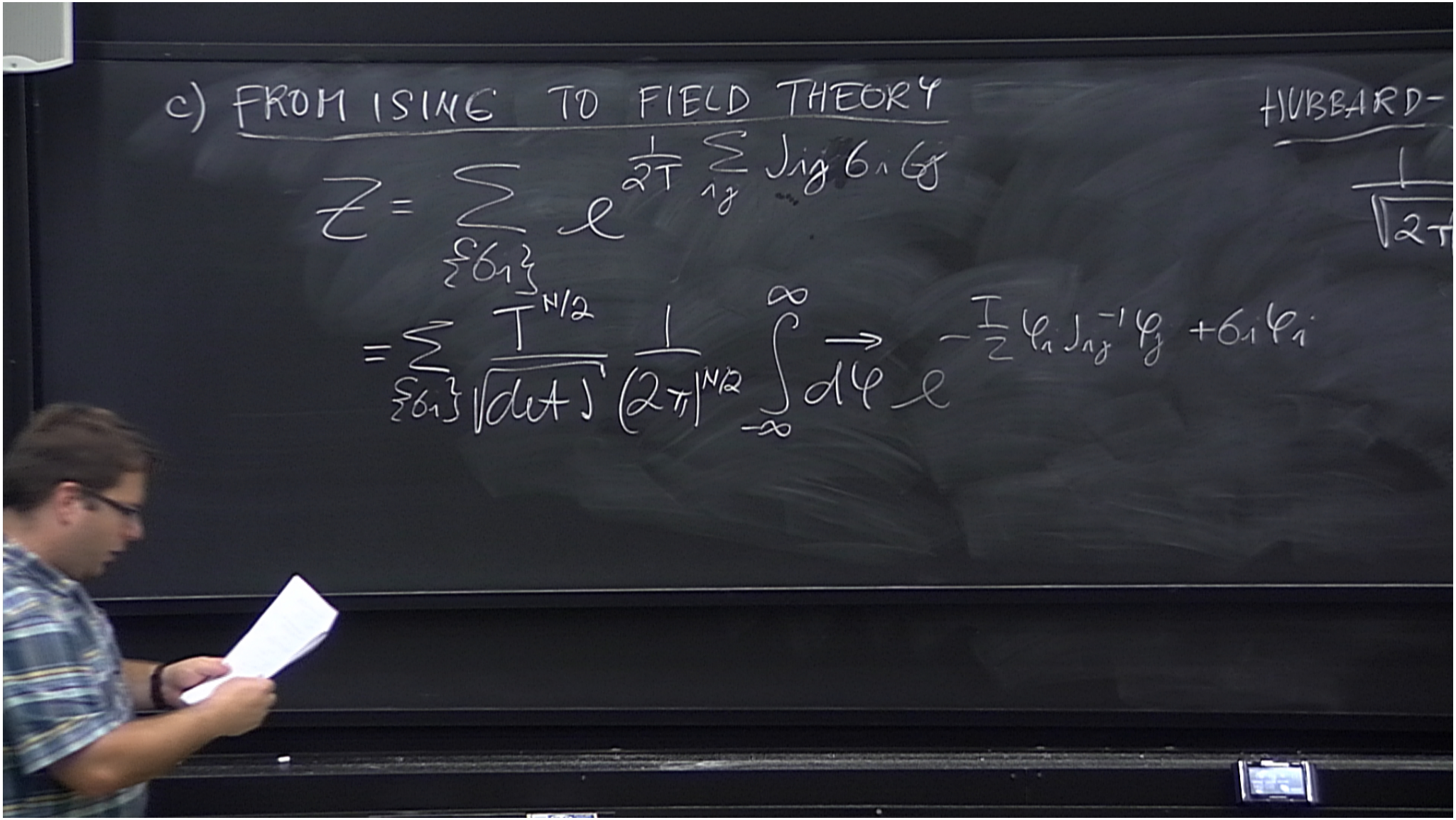

HUBBARD-STRATONOVICH TRANSFORMATION
$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\varphi\, e^{-\frac{1}{2}a\varphi^2 + g\varphi} = \frac{1}{\sqrt{a}} e^{\frac{g^2}{2a}}$, $a>0$


HUBBARD-STRATONOVICH TRANSFORMATION
$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\varphi\, e^{-\frac{1}{2}a\varphi^2 + j\varphi} = \frac{1}{\sqrt{a}}\, e^{\frac{j^2}{2a}}$, $a>0$
$\left(\frac{1}{2\pi}\right)^{N/2} \int_{-\infty}^{\infty} d\varphi_1 \ldots d\varphi_N\, e^{-\frac{1}{2}\varphi_i A_{ij} \varphi_j + j_i \varphi_i} = \frac{1}{\sqrt{\det A}}\, e^{\frac{1}{2} j_i A^{-1}_{ij} j_j}$
MFT MORE EXACT!!


HUBBARD-STRATONOVICH TRANSFORMATION
$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\varphi\, e^{-\frac{1}{2}a\varphi^2 + x\varphi} = \frac{1}{\sqrt{a}} e^{\frac{x^2}{2a}}$, $a>0$
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MFT "MORE EXACT"


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$A^{-1}_{ij} = J_{ij}$
$g_i = \sigma_i$
MFT MORE EXACT!!


c) FROM ISING TO FIELD THEORY
$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j}$
$= \sum_{\{\sigma_i\}}$
HUBBARD-
$\frac{1}{\sqrt{2\pi}}$
$\left(\frac{1}{2\pi}\right)^{N/2}\int_{-\infty}$


c) FROM ISING TO FIELD THEORY
HUBBARD-
$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j}$
$= \sum_{\{\sigma_i\}} \frac{T^{N/2}}{\sqrt{\det J}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} \overrightarrow{d\varphi}\, e^{-\frac{T}{2}\varphi_i J_{ij}^{-1}\varphi_j + \sigma_i\varphi_i}$

FROM ISING TO FIELD THEORY

$$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j}$$

$$= \sum_{\{\sigma_i\}} \frac{T^{N/2}}{\sqrt{\det J}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} \overrightarrow{d\varphi}\, e^{-\frac{T}{2}\varphi_i J_{ij}^{-1}\varphi_j + \sigma_i\varphi_i}$$

$$\left(\varphi_i \to \frac{\varphi_i}{T}\right)$$

$$= \frac{1}{\sqrt{\det J}} \frac{1}{(2\pi T)^{N/2}} \sum_{\{\sigma_i\}} \int \overrightarrow{d\varphi}\, e^{-\frac{1}{2T}\varphi_i J_{ij}^{-1}\varphi_j + \sigma_i\frac{\varphi_i}{T}}$$

HUBBARD-STRATONO

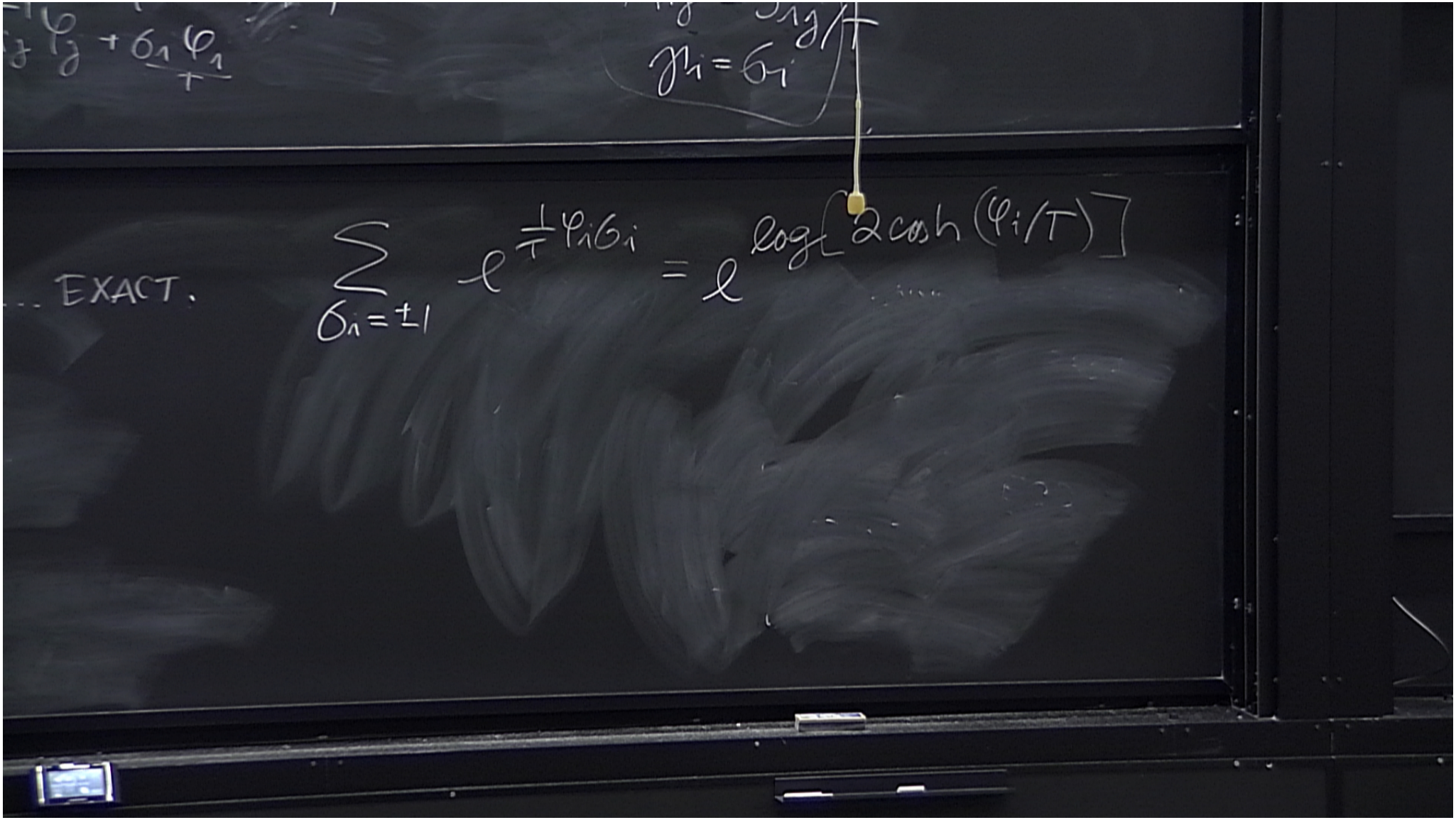
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\varphi$$

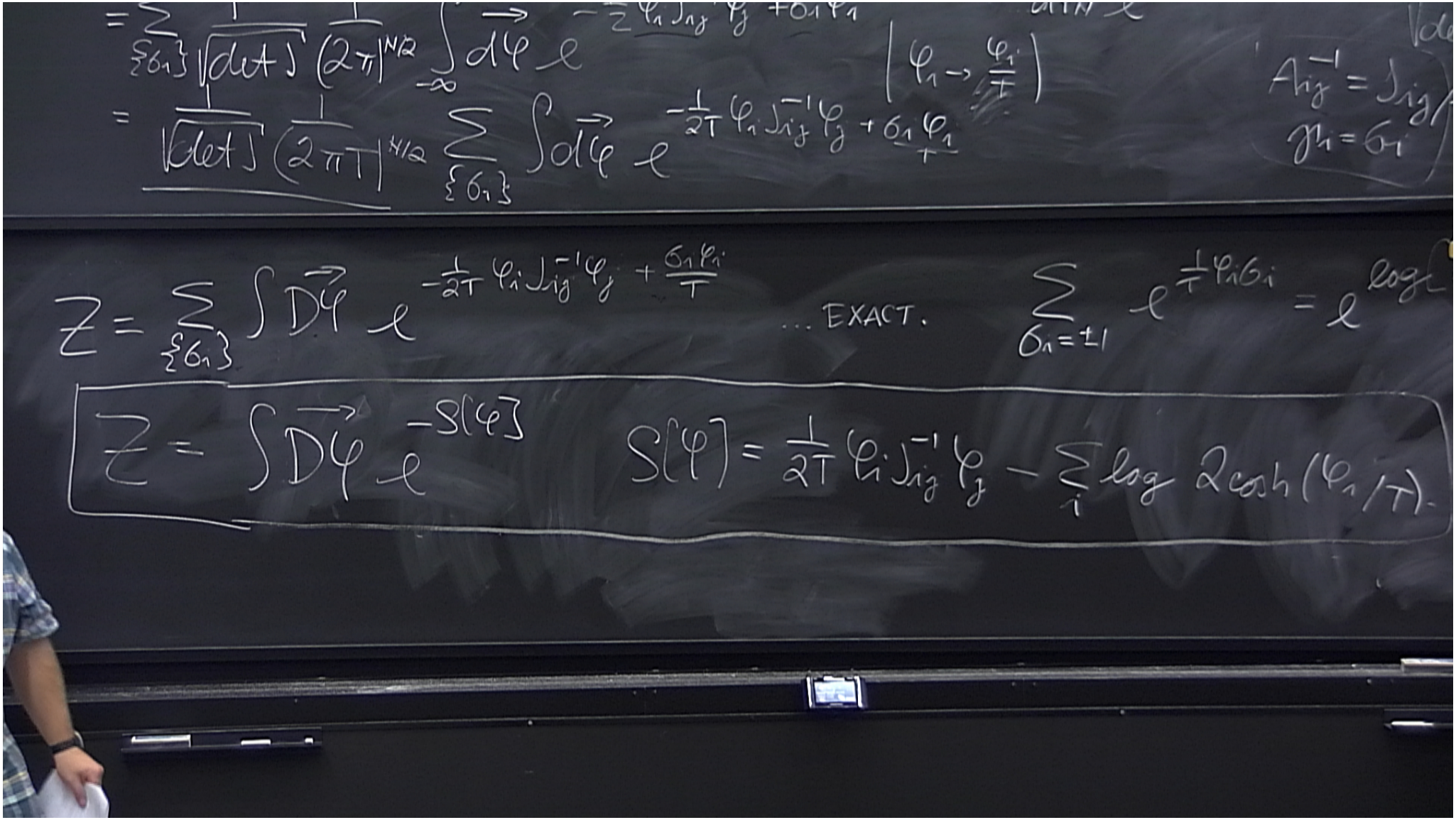
$d\varphi_N$

$$= \frac{1}{\sqrt{\det J}\,(2\pi T)^{N/2}} \sum_{\{\sigma_i\}} \int d\vec{\varphi}\; e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j + \frac{\sigma_i \varphi_i}{T}}$$

$$Z = \sum_{\{\sigma_i\}} \int D\vec{\varphi}\; e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij}\varphi_j + \frac{\sigma_i \varphi_i}{T}}$$

... EXACT.


$\varphi_j + \sigma_1 \frac{\varphi_1}{T}$
$\gamma_i = \sigma_i$
... EXACT.
$\sum_{\sigma_i = \pm 1} e^{\frac{1}{T}\varphi_i \sigma_i} = e^{\log[2\cosh(\varphi_i/T)]}$


$= \sum_{\{\sigma_i\}} \sqrt{\det J}\,(2\pi)^{N/2} \int_{-\infty}^{\infty} d\vec{\varphi}\, e^{-\frac{1}{2}\varphi_i J_{ij} \varphi_j + \sigma_i \varphi_i}$
$(\varphi_i \to \frac{\varphi_i}{T})$
$= \sqrt{\det J}\,(2\pi T)^{N/2} \sum_{\{\sigma_i\}} \int d\vec{\varphi}\, e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij} \varphi_j + \sigma_i \frac{\varphi_i}{T}}$
$A^{-1}_{ij} = J_{ij}$
$y_i = \sigma_i$
$Z = \sum_{\{\sigma_i\}} \int D\vec{\varphi}\, e^{-\frac{1}{2T}\varphi_i J^{-1}_{ij} \varphi_j + \frac{\sigma_i \varphi_i}{T}}$
... EXACT.
$\sum_{\sigma_i = \pm 1} e^{\frac{1}{T}\varphi_i \sigma_i} = e^{\log}$
$Z = \int D\vec{\varphi}\, e^{-S[\varphi]}$
$S[\varphi] = \frac{1}{2T}\varphi_i J^{-1}_{ij} \varphi_j - \sum_i \log 2\cosh(\varphi_i/T)$.

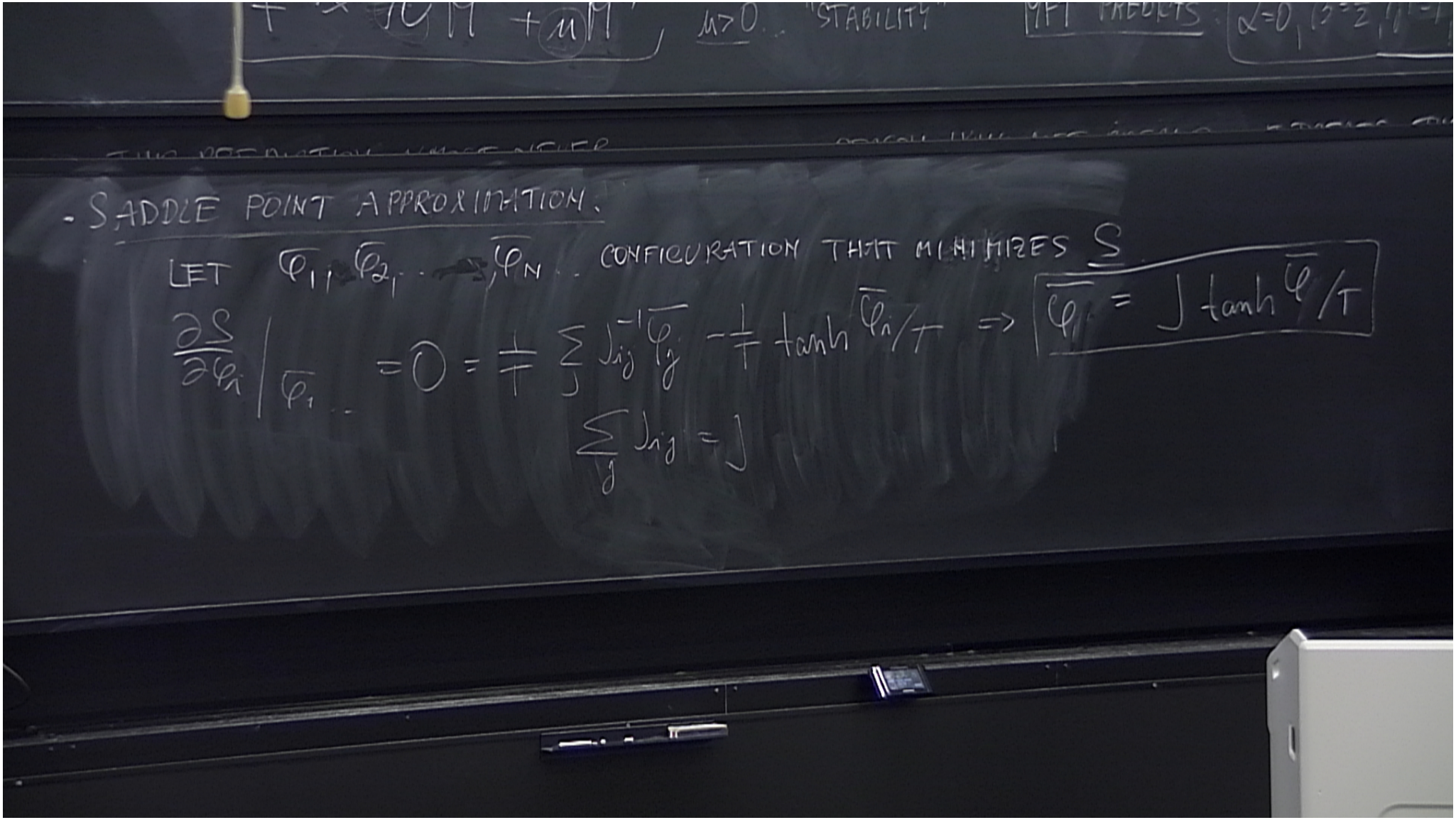
THIS PREDICTION ALMOST NEVER OBSERVED.

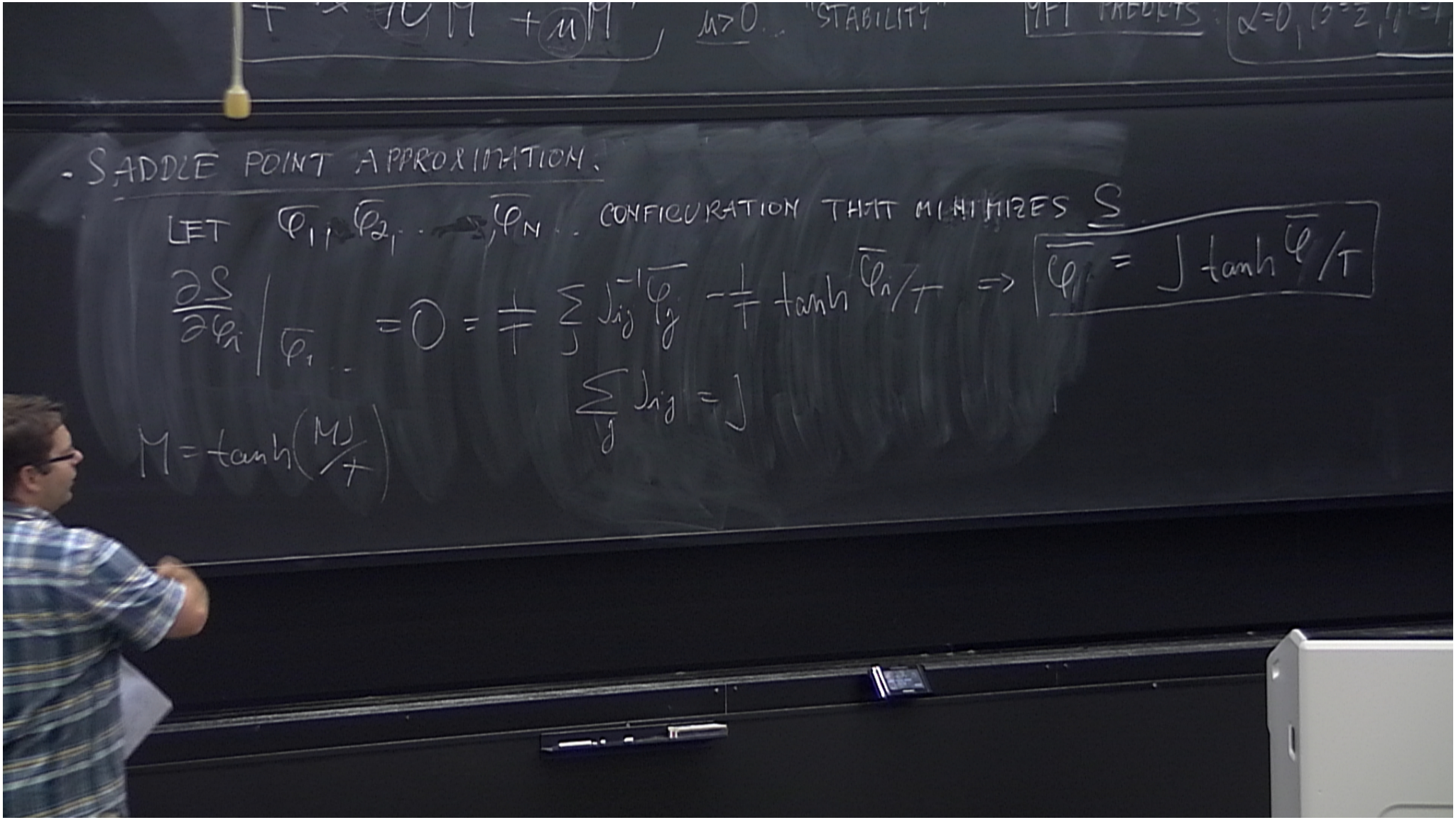
REASON WHY MFT BREAKS .. PARAMETER AS CONSTANT.

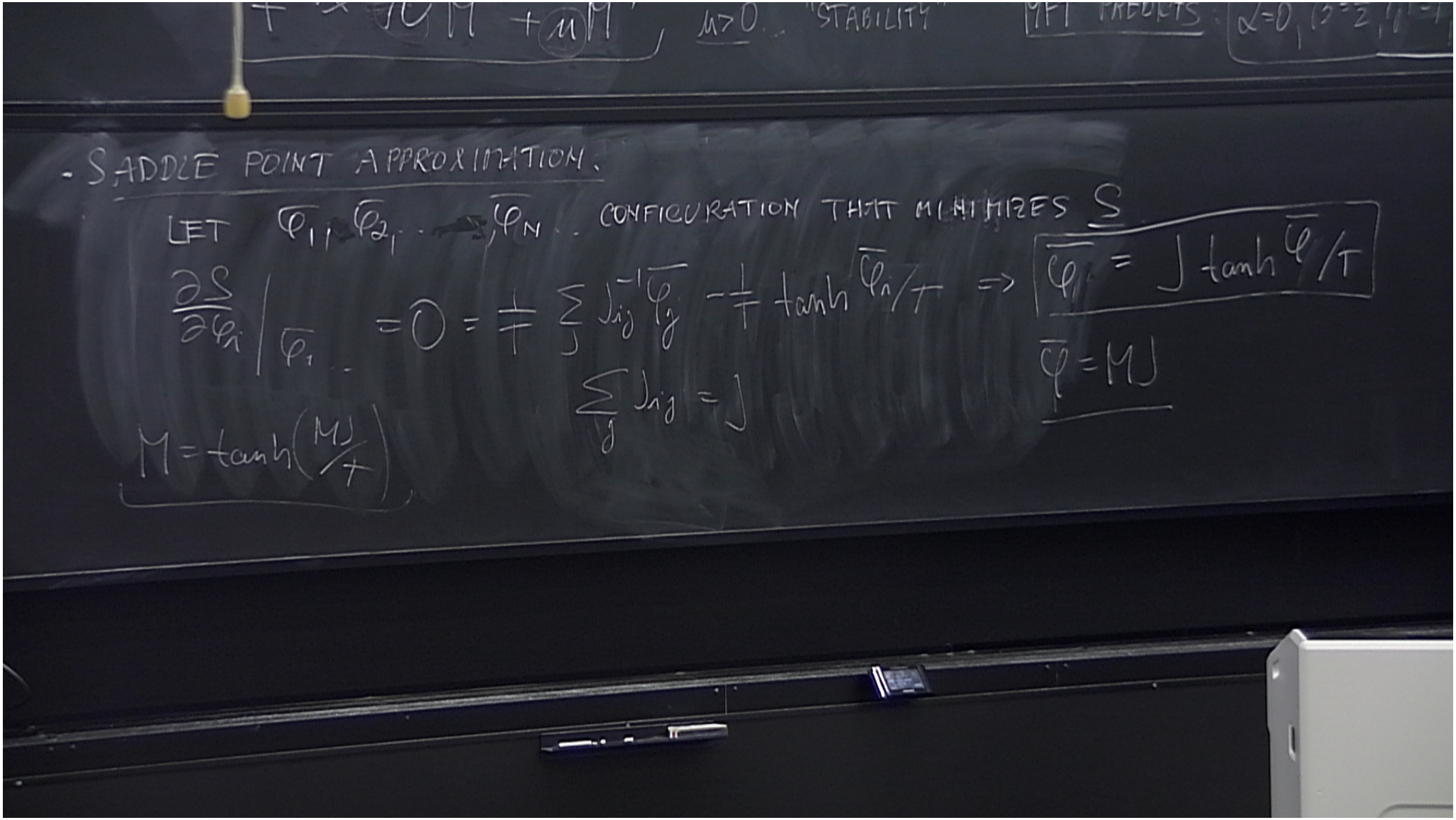
- SADDLE POINT APPROXIMATION.

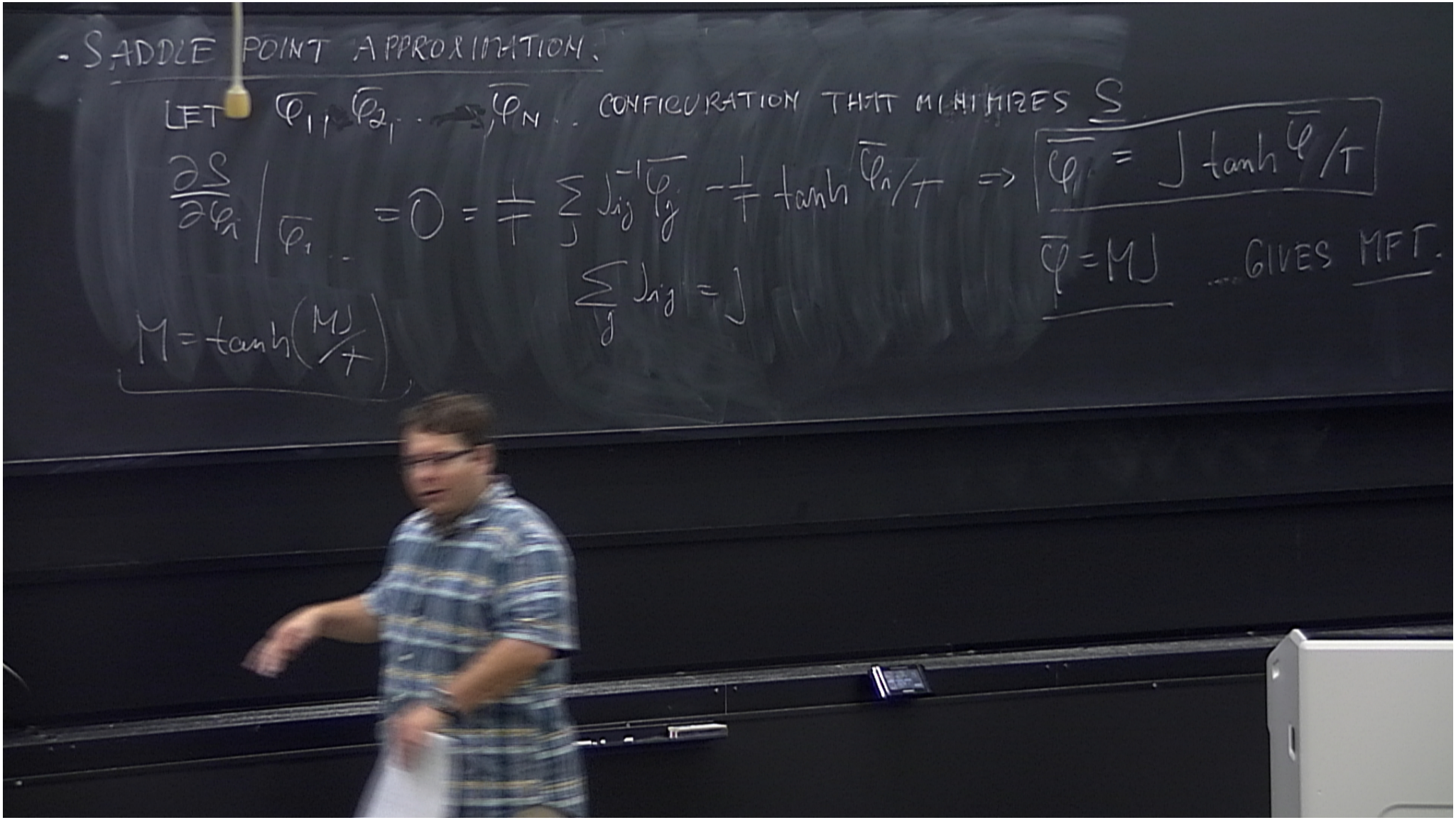
LET $\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_N$ .. CONFIGURATION THAT MINIMIZES S

$$\left.\frac{\partial S}{\partial \varphi_i}\right|_{\bar{\varphi}_1 \ldots} = 0$$


$+\frac{1}{2}\sum_{ij} J_{ij}\varphi_i\varphi_j + \mu M$, $\mu > 0$ .. "STABILITY"
MFT PREDICTS
$\alpha=0$,
- SADDLE POINT APPROXIMATION.
LET $\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_N$ .. CONFIGURATION THAT MINIMIZES S
$\frac{\partial S}{\partial \varphi_i}\Big|_{\bar{\varphi}_1 \ldots} = 0 = \mp \sum_j J_{ij}^{-1}\bar{\varphi}_j - \frac{1}{T}\tanh \bar{\varphi}_i/T \Rightarrow \bar{\varphi}_i = J \tanh \bar{\varphi}/T$
$\sum_j J_{ij} = J$


+ μM
μ>0 "STABILITY"
MFT PREDICTS
- SADDLE POINT APPROXIMATION.
LET $\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_N$ CONFIGURATION THAT MINIMIZES S
$\left.\frac{\partial S}{\partial \varphi_i}\right|_{\bar{\varphi}_1 \ldots} = 0 = \frac{1}{T}\sum_j J_{ij}^{-1}\bar{\varphi}_j - \frac{1}{T}\tanh \bar{\varphi}_i/T \Rightarrow \bar{\varphi}_i = J \tanh \bar{\varphi}/T$
$\sum_j J_{ij} = J$
$M = \tanh\left(\frac{MJ}{T}\right)$


$+\frac{1}{2}\mu M^2$
$\mu > 0$ ... "STABILITY"
MFT PREDICTS
$\alpha = 0$
- SADDLE POINT APPROXIMATION.
LET $\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_N$ .. CONFIGURATION THAT MINIMIZES S
$\frac{\partial S}{\partial \varphi_i}\Big|_{\bar{\varphi}_1 \ldots} = 0 = \frac{1}{T}\sum_j J_{ij}^{-1}\bar{\varphi}_j - \frac{1}{T}\tanh \bar{\varphi}_i/T \Rightarrow \bar{\varphi}_i = J \tanh \bar{\varphi}/T$
$\bar{\varphi} = MJ$
$M = \tanh\left(\frac{MJ}{T}\right)$
$\sum_j J_{ij} = J$


- SADDLE POINT APPROXIMATION.
LET $\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_N$ .. CONFIGURATION THAT MINIMIZES S
$\left.\frac{\partial S}{\partial \varphi_i}\right|_{\bar{\varphi}_1 \ldots} = 0 = \pm \sum_j J_{ij}^{-1} \bar{\varphi}_j - \frac{1}{T} \tanh \bar{\varphi}_i / T \Rightarrow \bar{\varphi}_i = J \tanh \bar{\varphi} / T$
$\sum_j J_{ij} = J$
$\bar{\varphi} = MJ$ .. GIVES MFT.
$M = \tanh\left(\frac{MJ}{T}\right)$