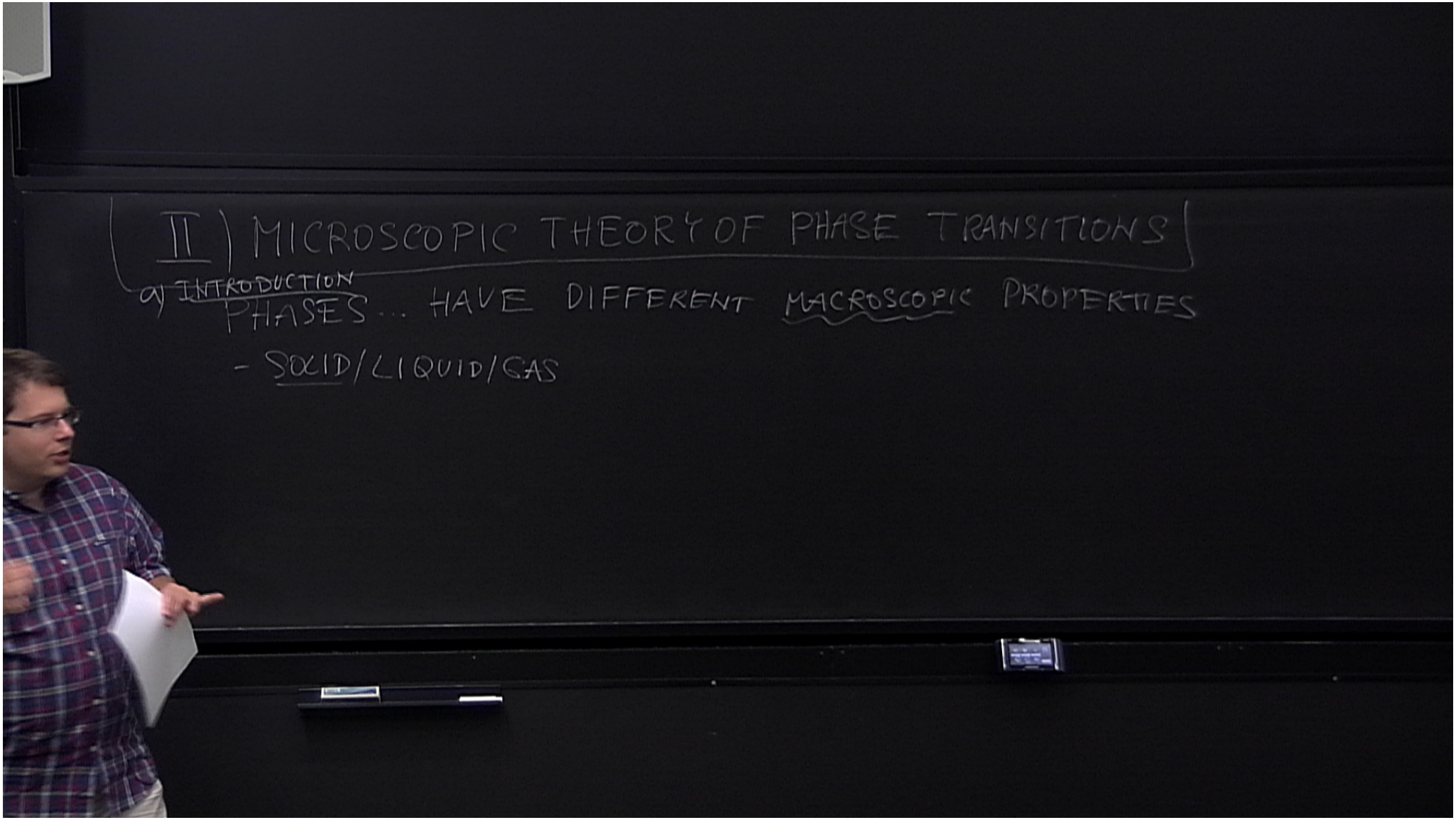
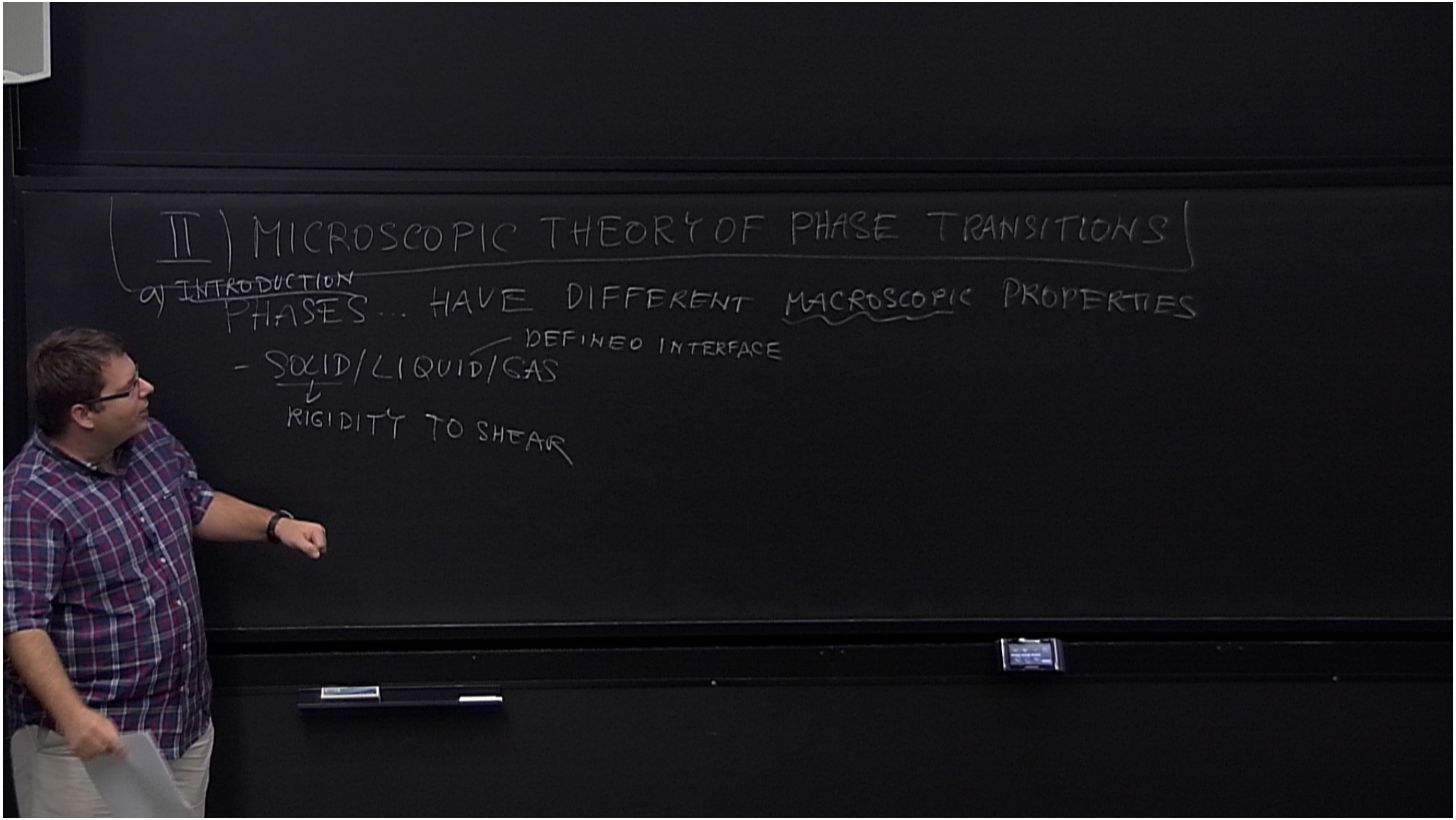
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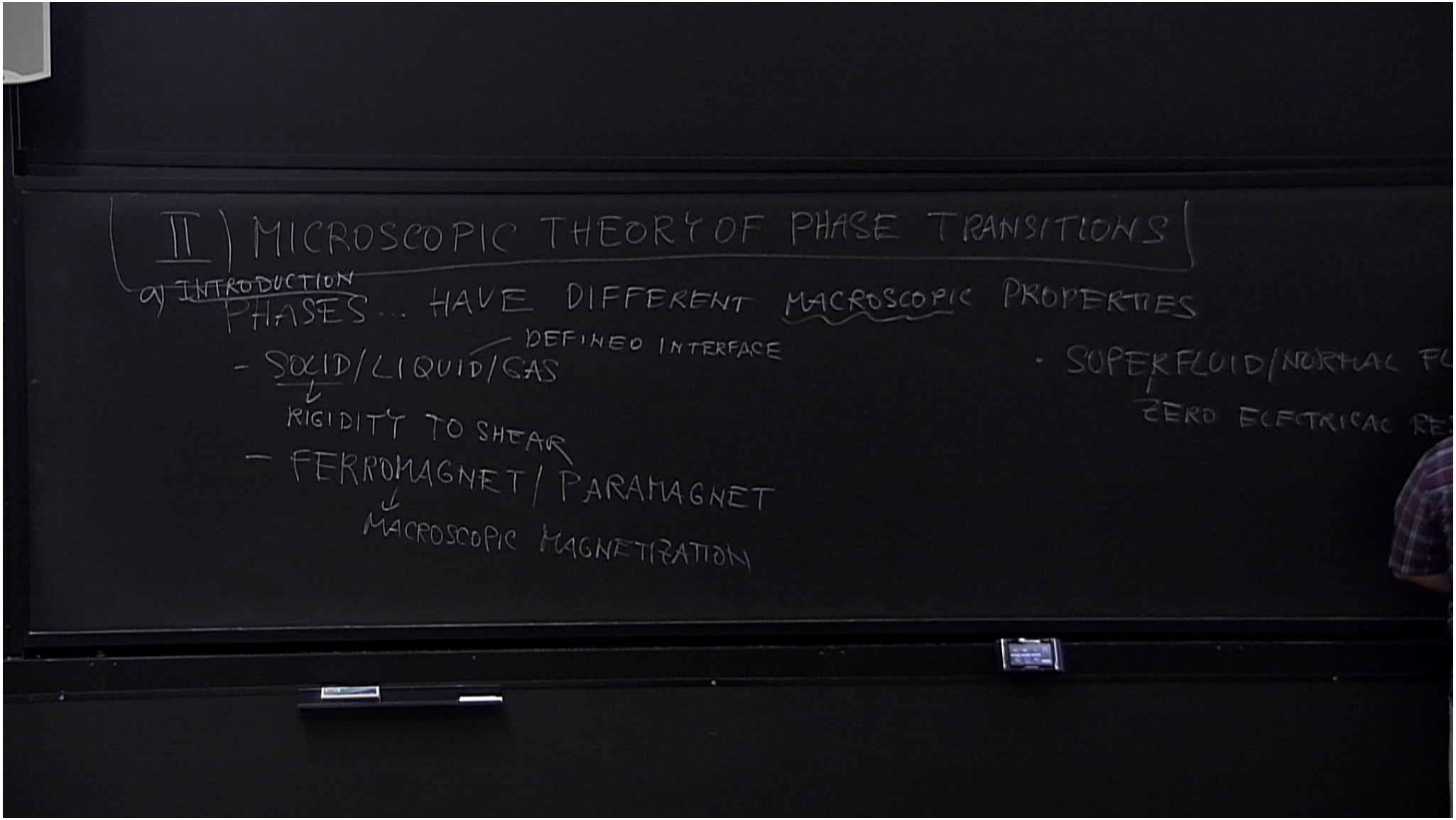
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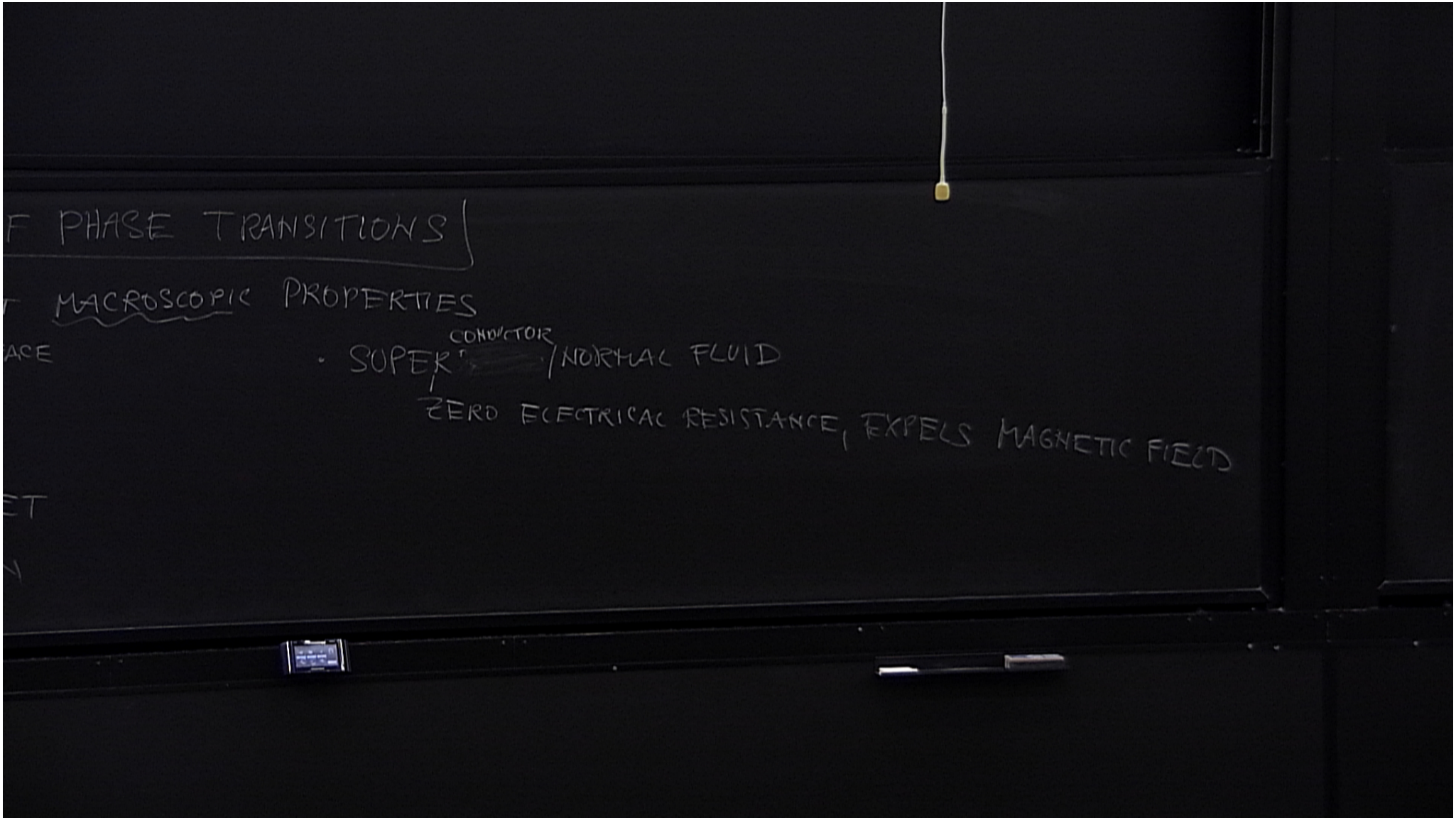
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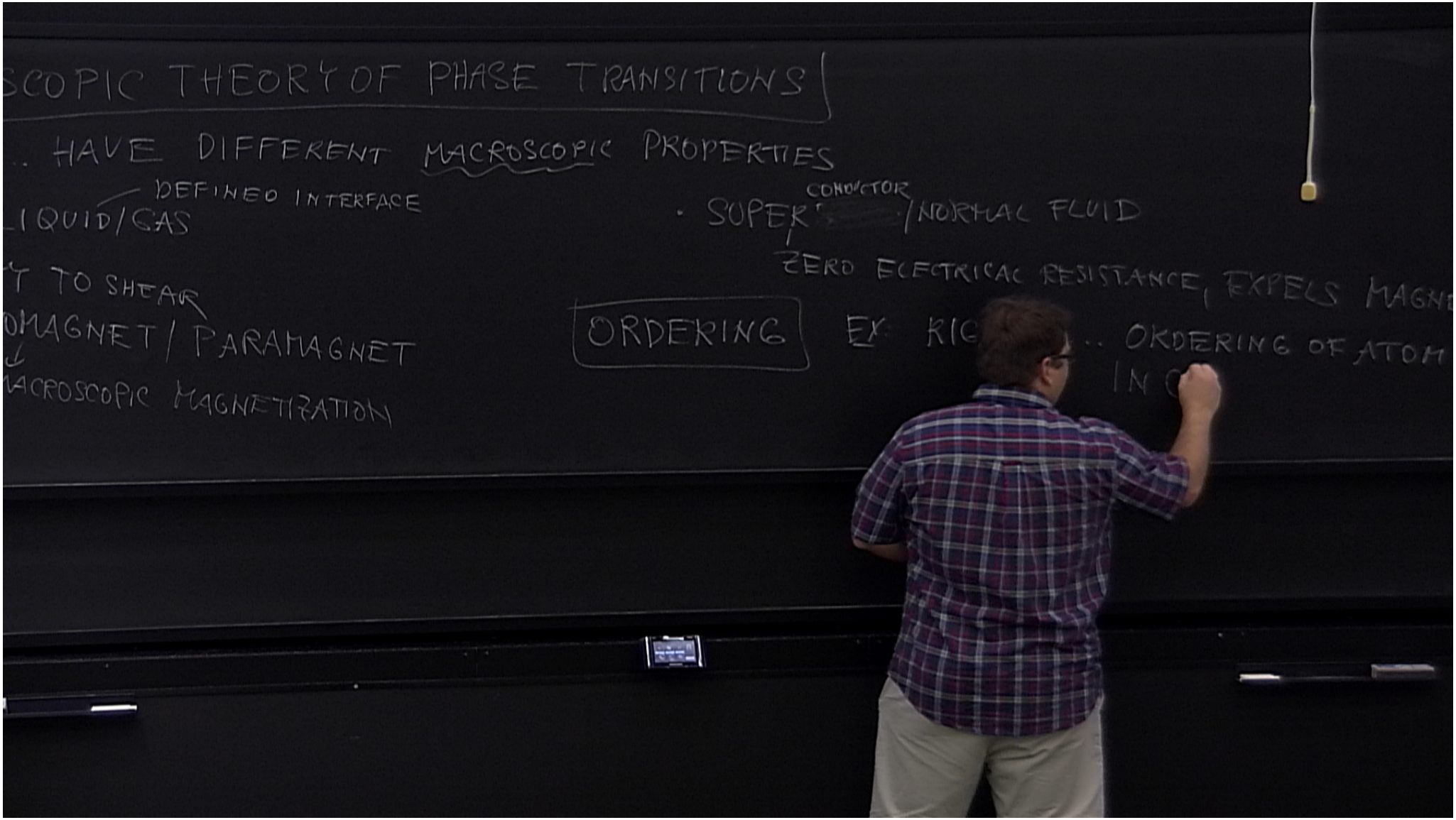
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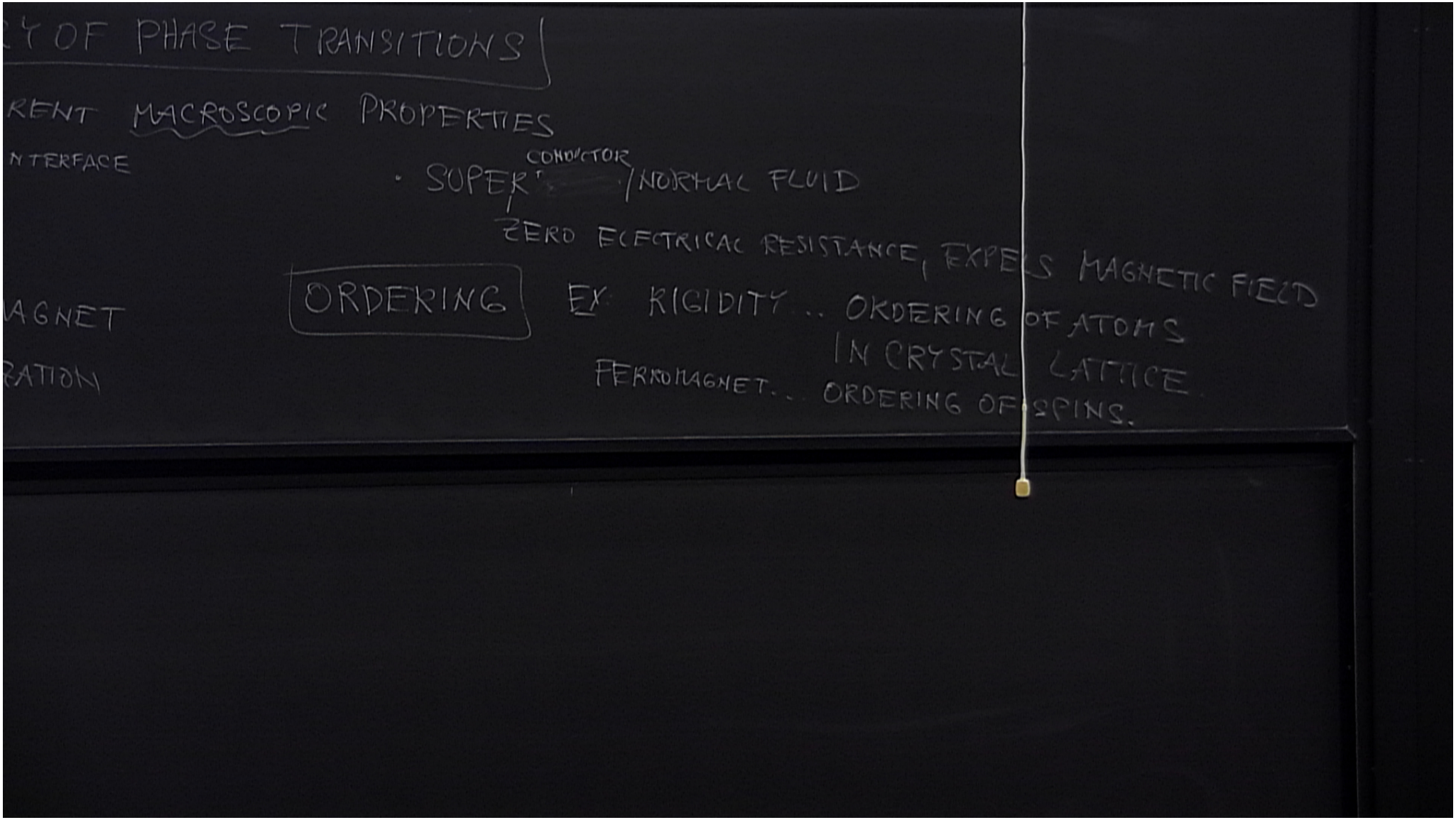

II) MICROSCOPIC THEORY OF PHASE TRANSITIONS
a) INTRODUCTION
PHASES ... HAVE DIFFERENT MACROSCOPIC PROPERTIES
- SOLID/LIQUID/GAS

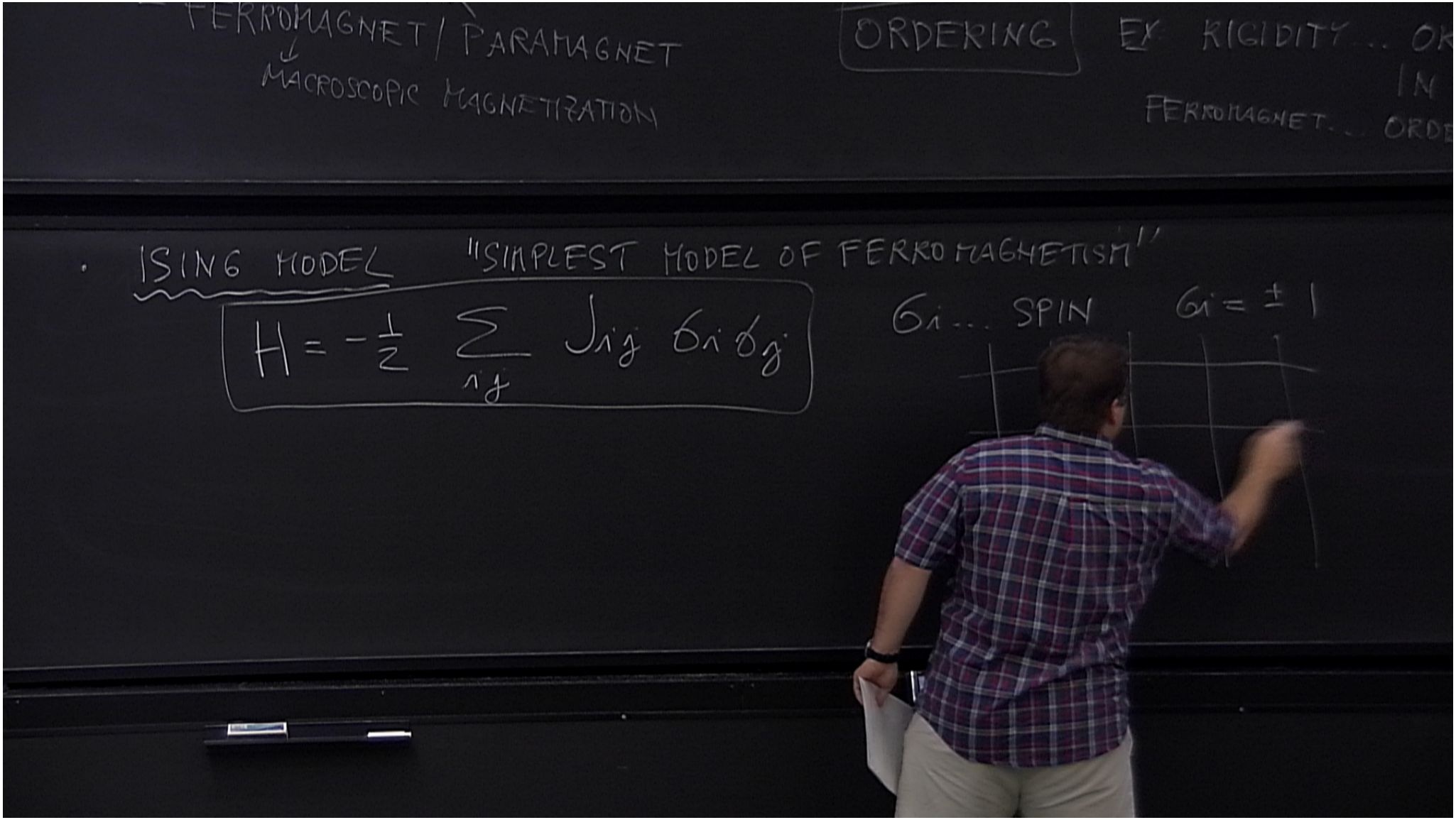

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ZERO ELECTRICAL RE


F PHASE TRANSITIONS
MACROSCOPIC PROPERTIES
SUPER CONDUCTOR / NORMAL FLUID
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SCOPIC THEORY OF PHASE TRANSITIONS
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AGNET
ORDERING
EX RIGIDITY .. ORDERING OF ATOMS IN CRYSTAL LATTICE
RATION
FERROMAGNET.. ORDERING OF SPINS.


FERROMAGNET / PARAMAGNET
MACROSCOPIC MAGNETIZATION
ORDERING
EX RIGIDITY .. OR
IN
FERROMAGNET.. ORDE
ISING MODEL "SIMPLEST MODEL OF FERROMAGNETISM"
$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$
σ_i ... SPIN
$\sigma_i = \pm 1$

FERROMAGNET / PARAMAGNET

↓

MACROSCOPIC MAGNETIZATION

ORDERING

EX RIGIDITY .. OR

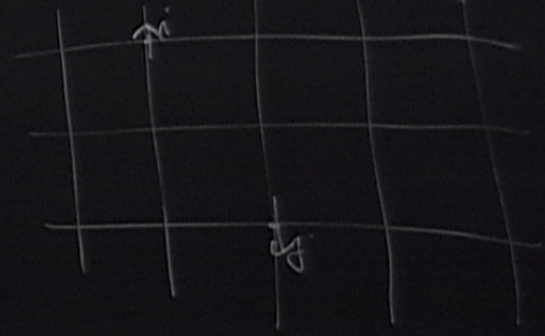
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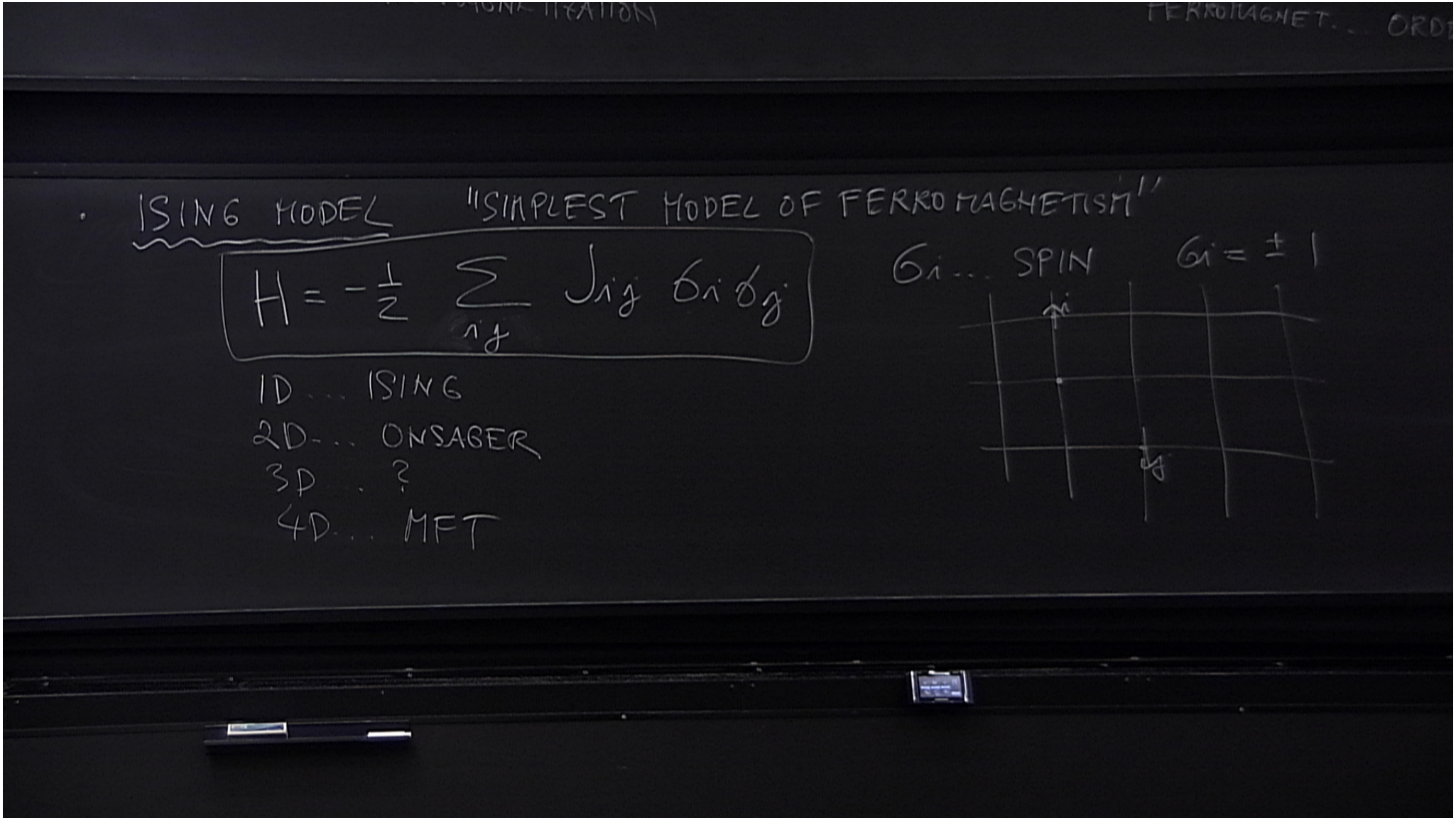
FERROMAGNET .. ORDE

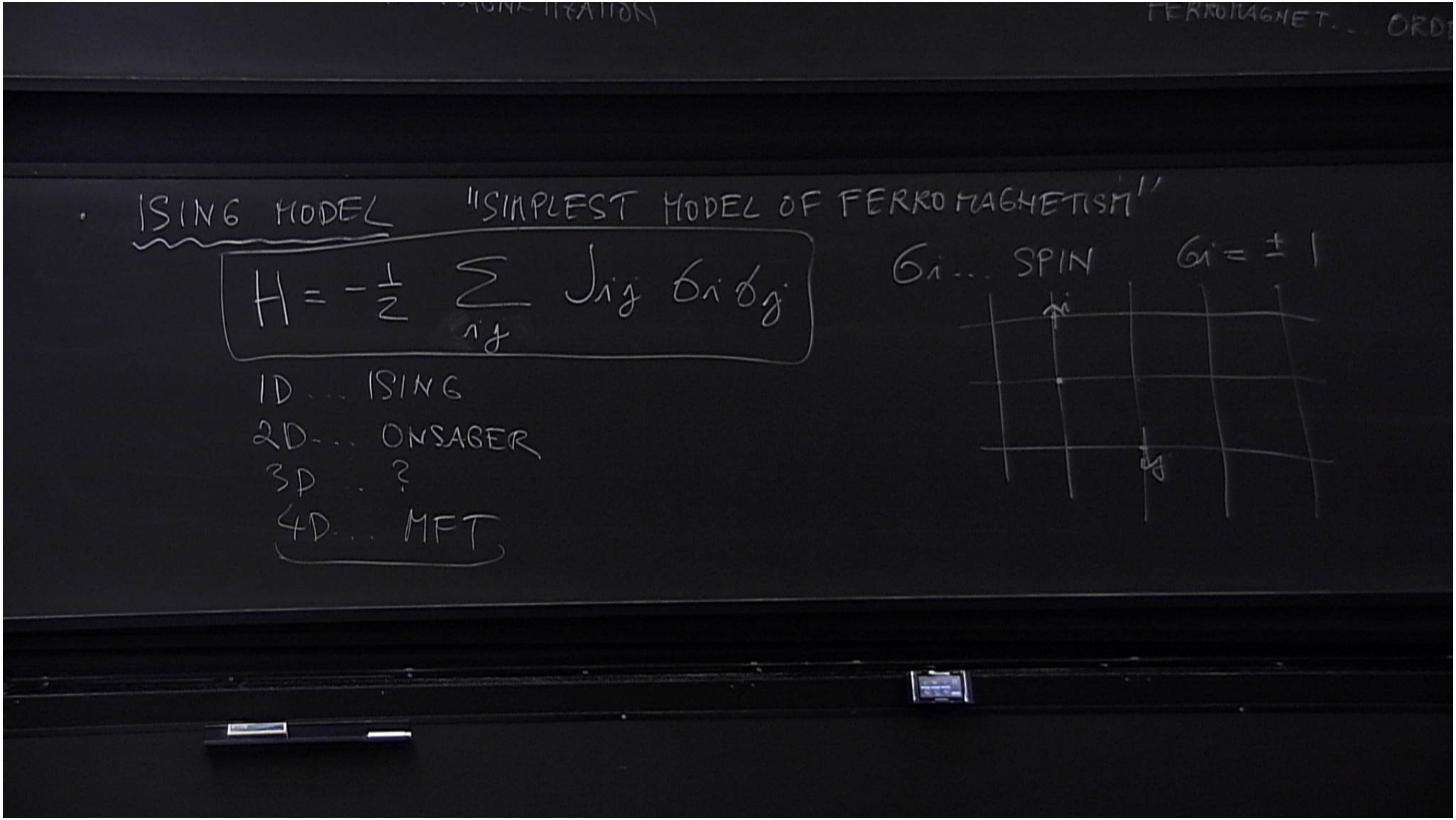
- ISING MODEL "SIMPLEST MODEL OF FERROMAGNETISM"

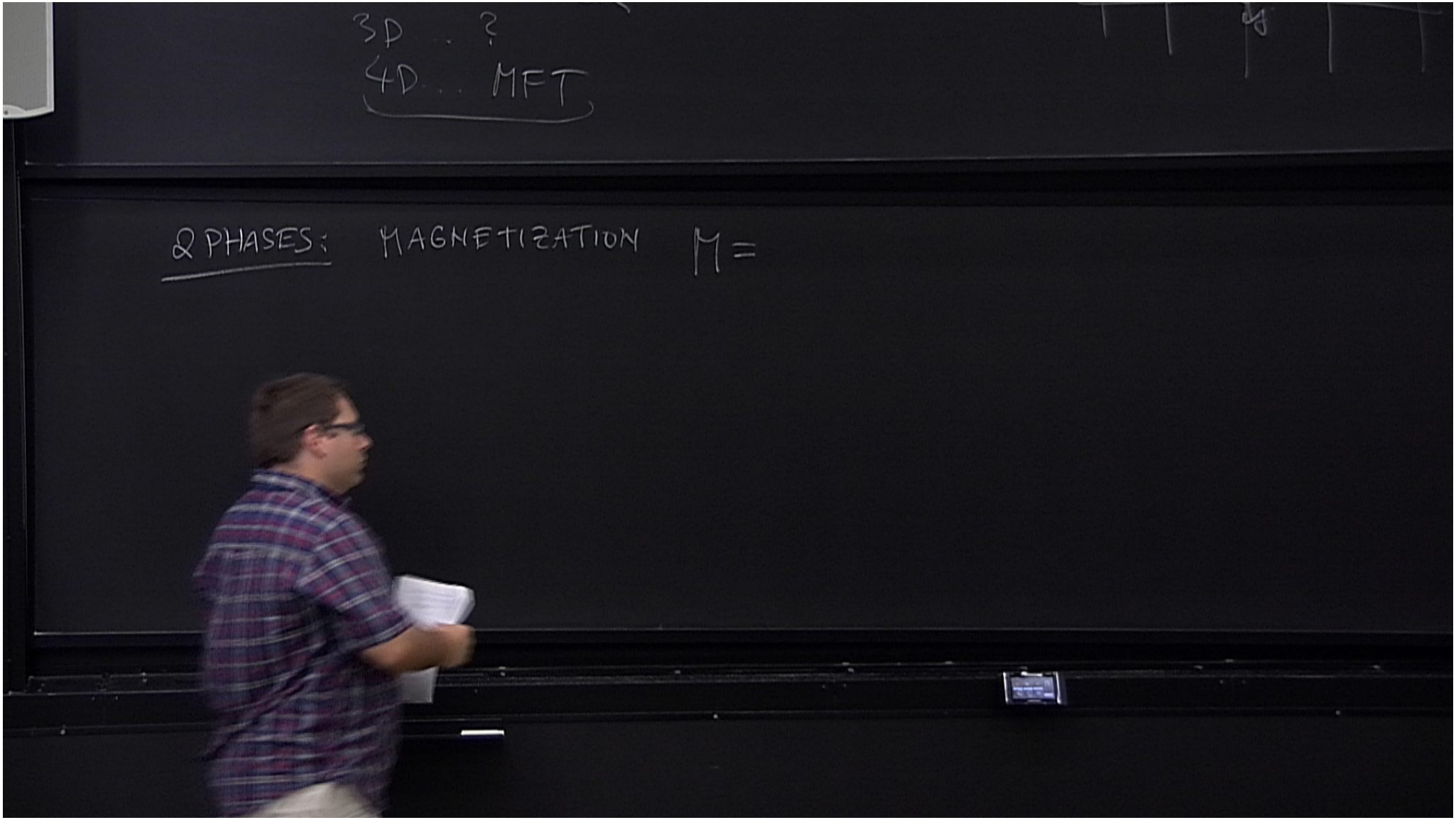
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1D ... ISING
2D ... ONSAGER
3D ... ?
4D ... MFT

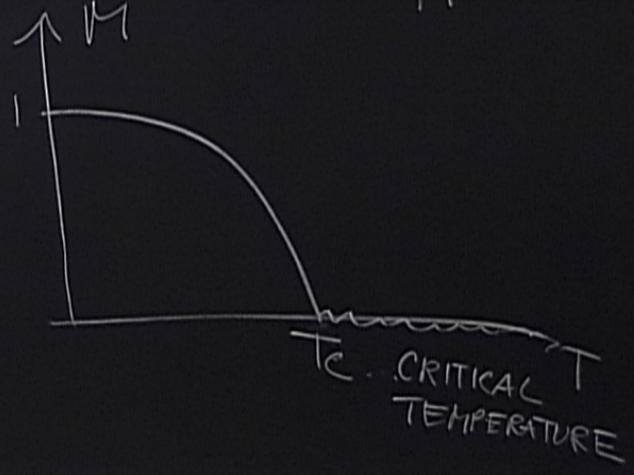

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3D . ?
4D . . MFT
2 PHASES: MAGNETIZATION M =

3D . ?

4D . . MFT

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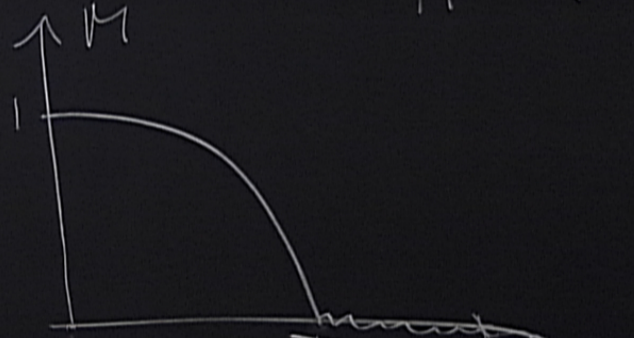


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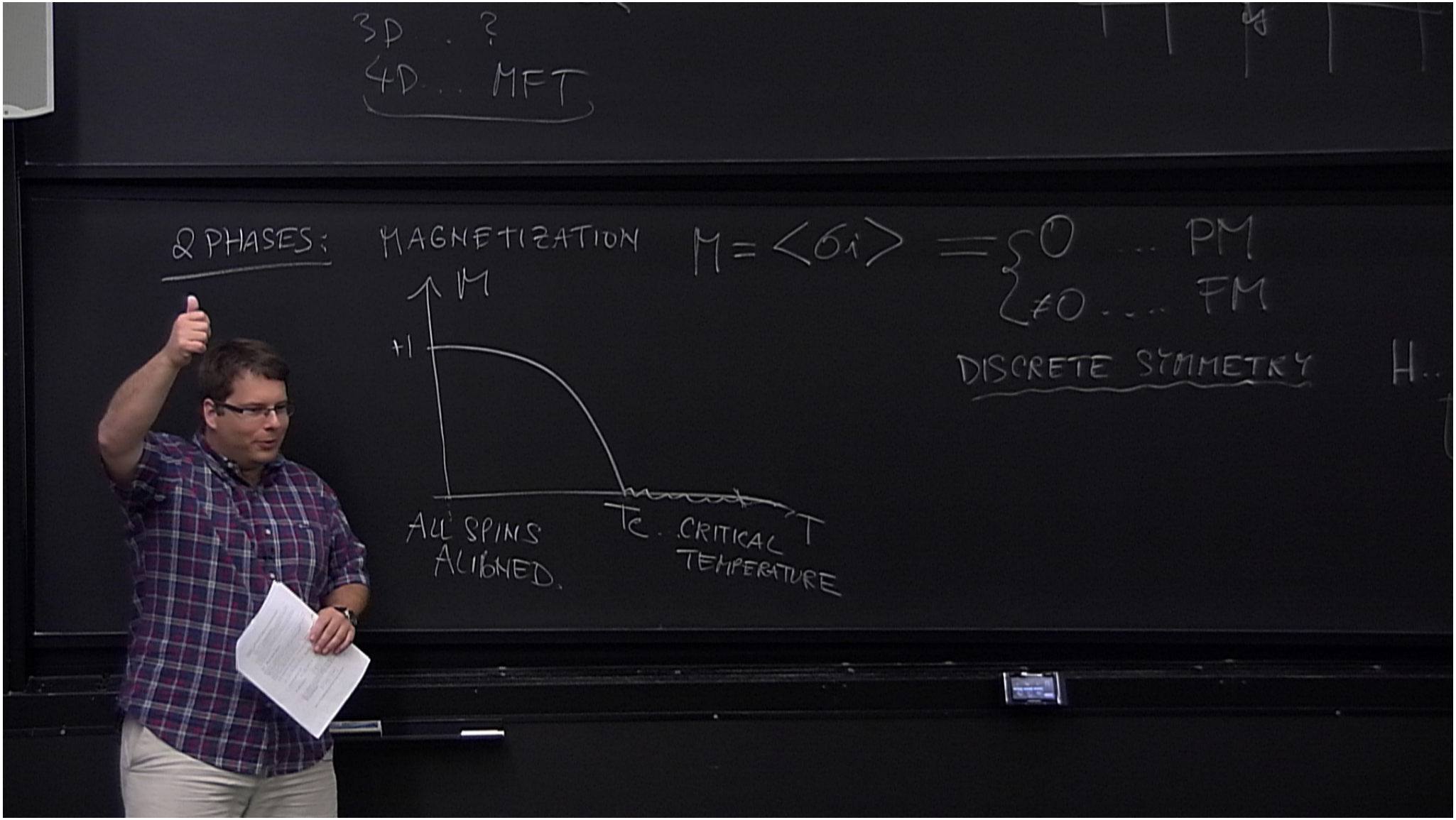
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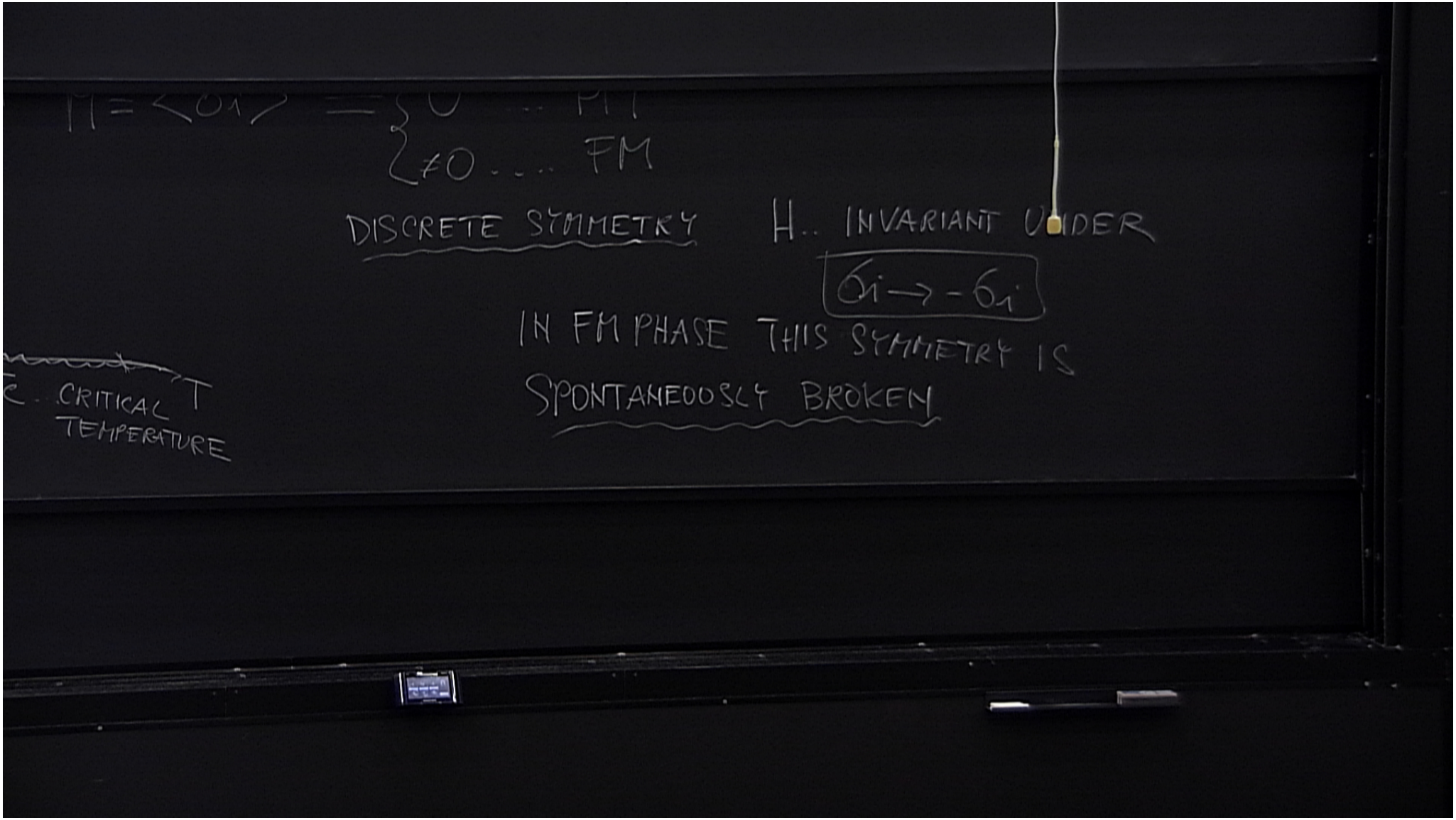
DISCRETE SYMMETRY H..

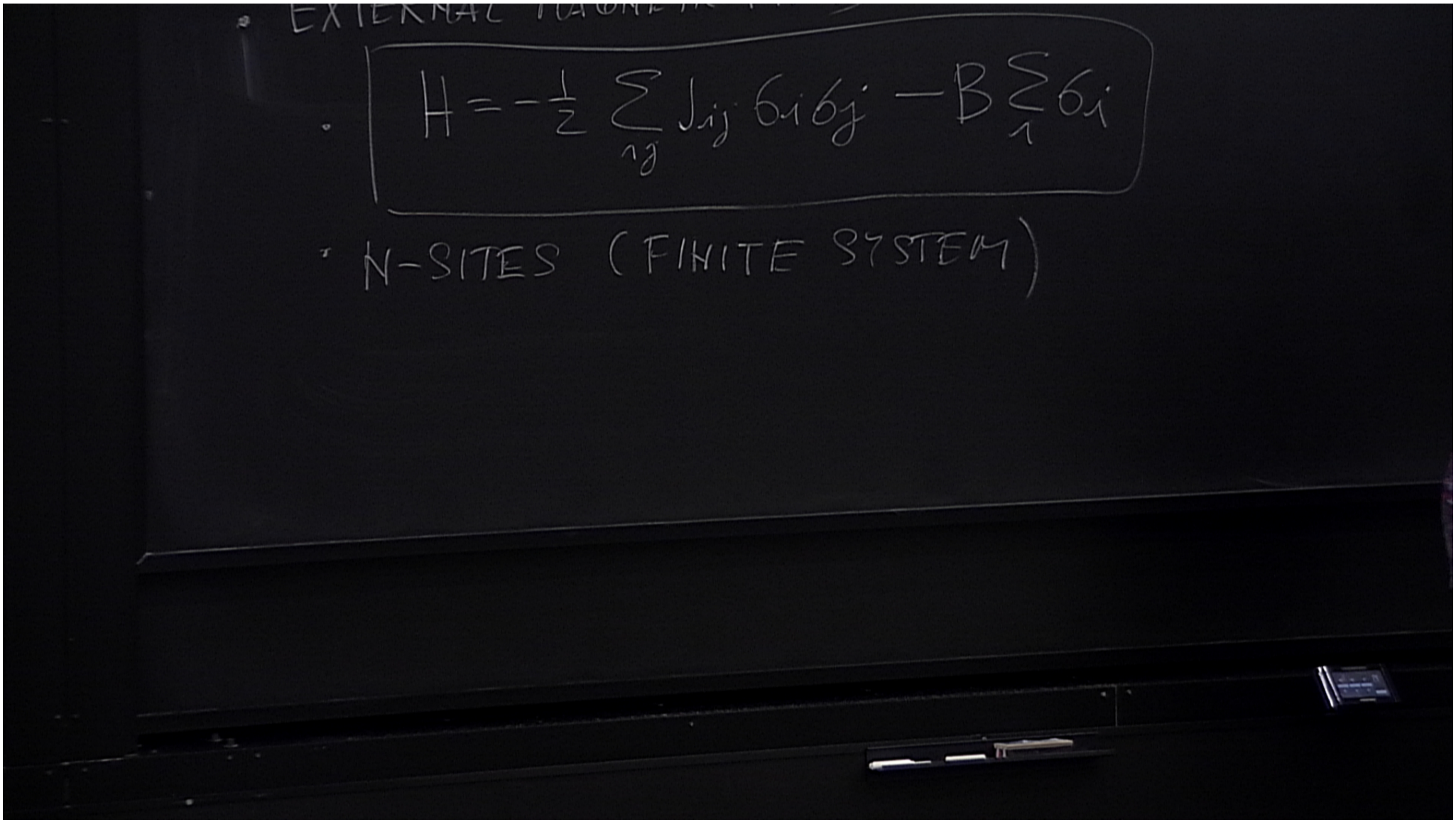


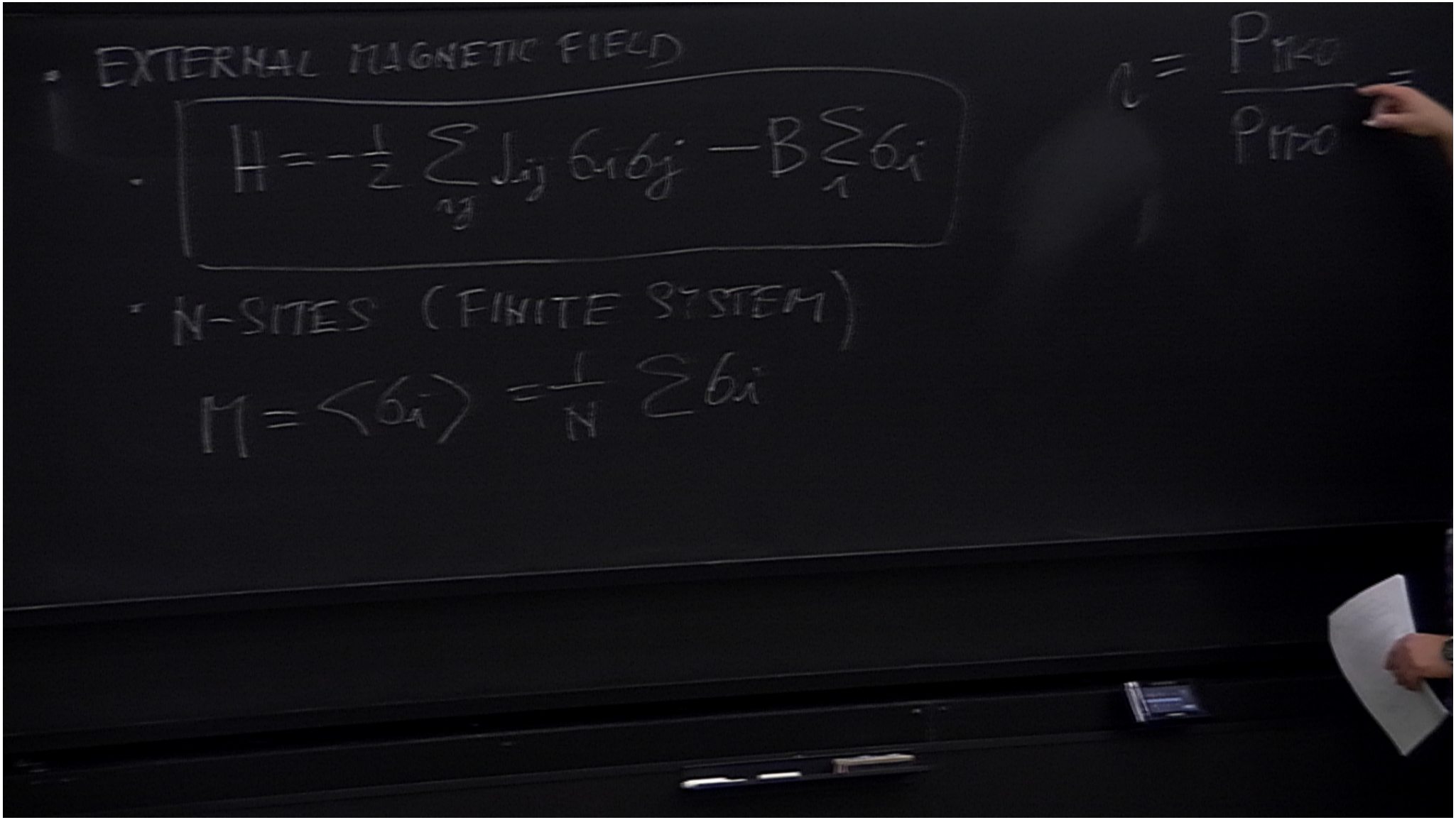
ALL SPINS ALIGNED.

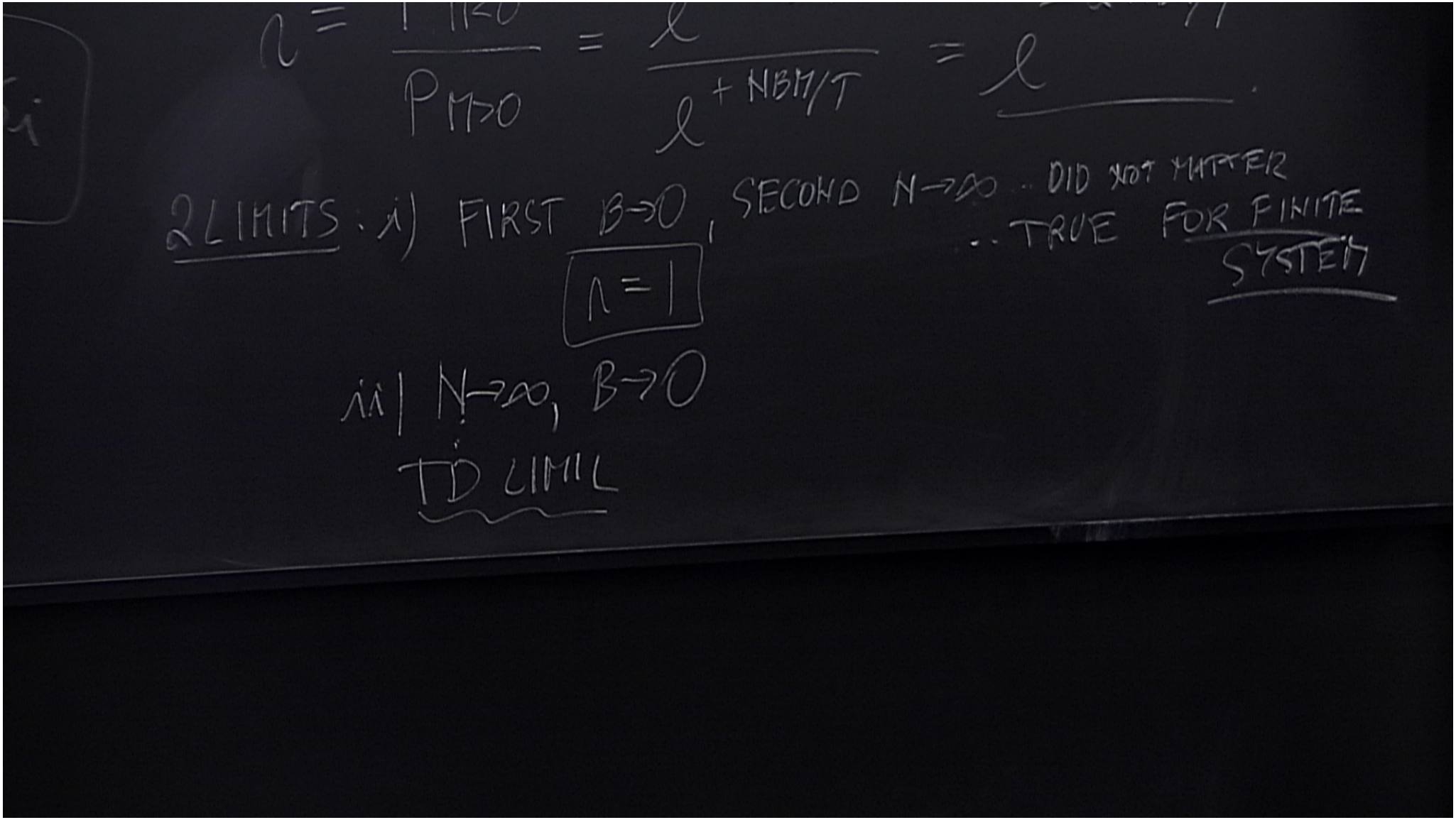
T_c . CRITICAL TEMPERATURE


3D . ?
4D . . MFT
2 PHASES:
MAGNETIZATION
M = <σi>
= { 0 . . . PM
≠0 . . . . FM
DISCRETE SYMMETRY
H..
M
+1
ALL SPINS ALIGNED.
Tc . CRITICAL TEMPERATURE
T


DISCRETE SYMMETRY
H.. INVARIANT UNDER
$\sigma_i \rightarrow -\sigma_i$
IN FM PHASE THIS SYMMETRY IS
SPONTANEOUSLY BROKEN
CRITICAL
TEMPERATURE


EXTERNAL
$H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j - B\sum_i \sigma_i$
N-SITES (FINITE SYSTEM)


• EXTERNAL MAGNETIC FIELD
$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$
• N-SITES (FINITE SYSTEM)
$M = \langle \sigma_i \rangle = \frac{1}{N}\sum \sigma_i$
$C = \frac{P_{T1K0}}{P_{T1K0}}$

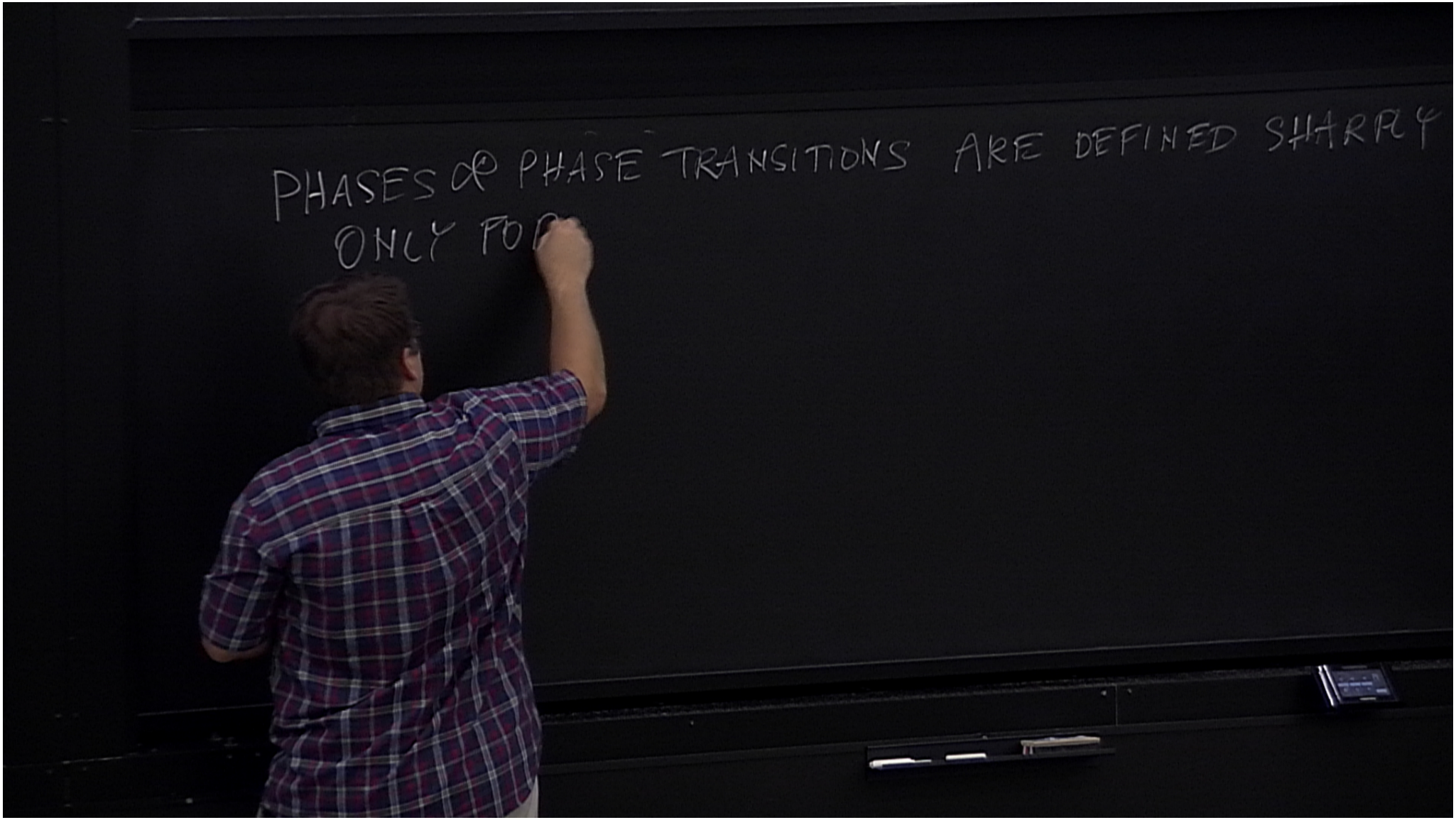

$\Lambda = \frac{1}{P_{M>0}} = \frac{\ell}{\ell^{+NBM/T}} = \ell$
2 LIMITS : i) FIRST $B \to 0$, SECOND $N \to \infty$ .. DID NOT MATTER
$\Lambda = 1$
.. TRUE FOR FINITE SYSTEM
ii) $N \to \infty$, $B \to 0$
TD LIMIT

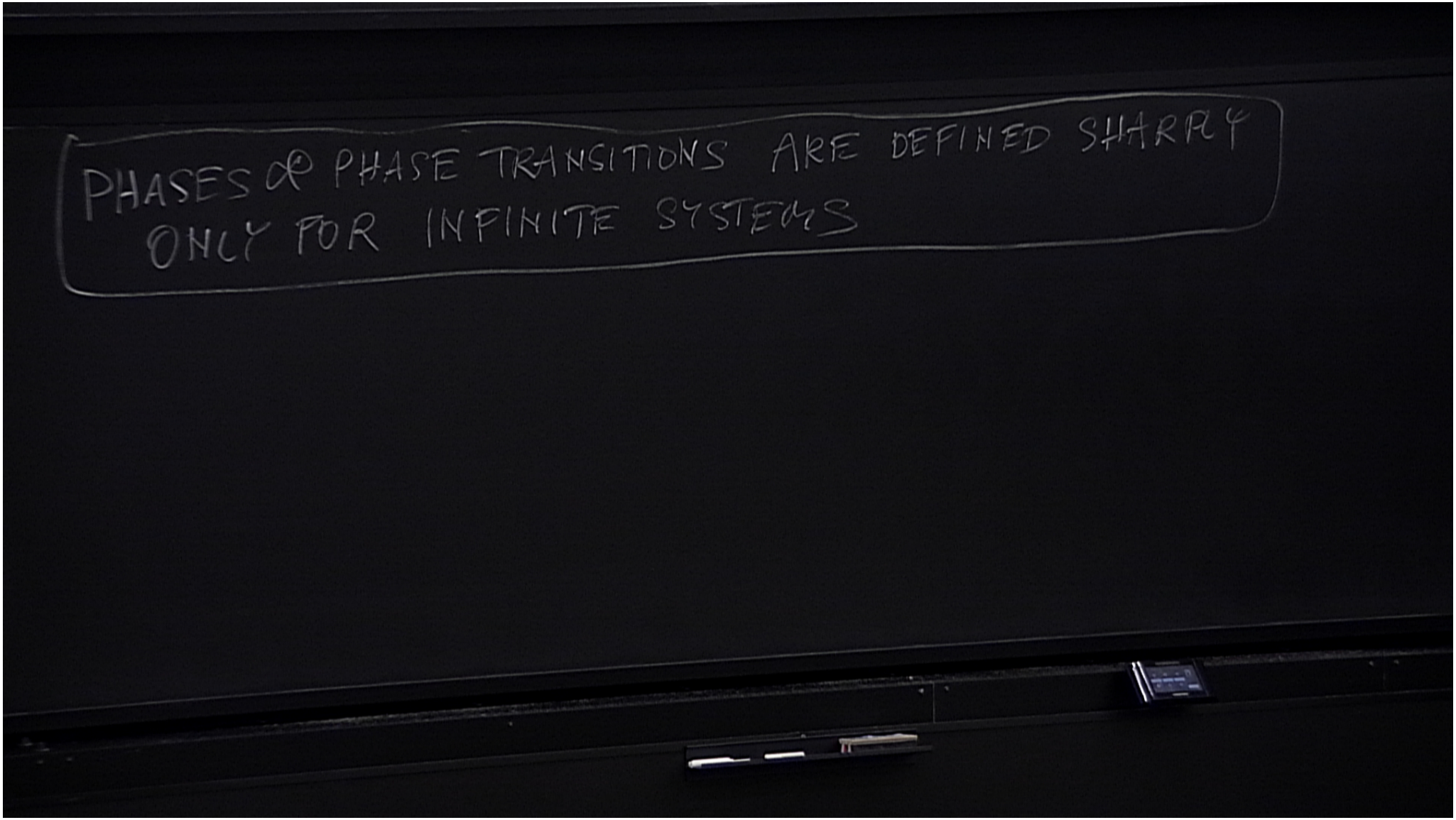
$$\Lambda = \frac{P_{M<0}}{P_{M>0}} = \frac{e^{-NBM/T}}{e^{+NBM/T}} = e^{-2NBM/T}.$$

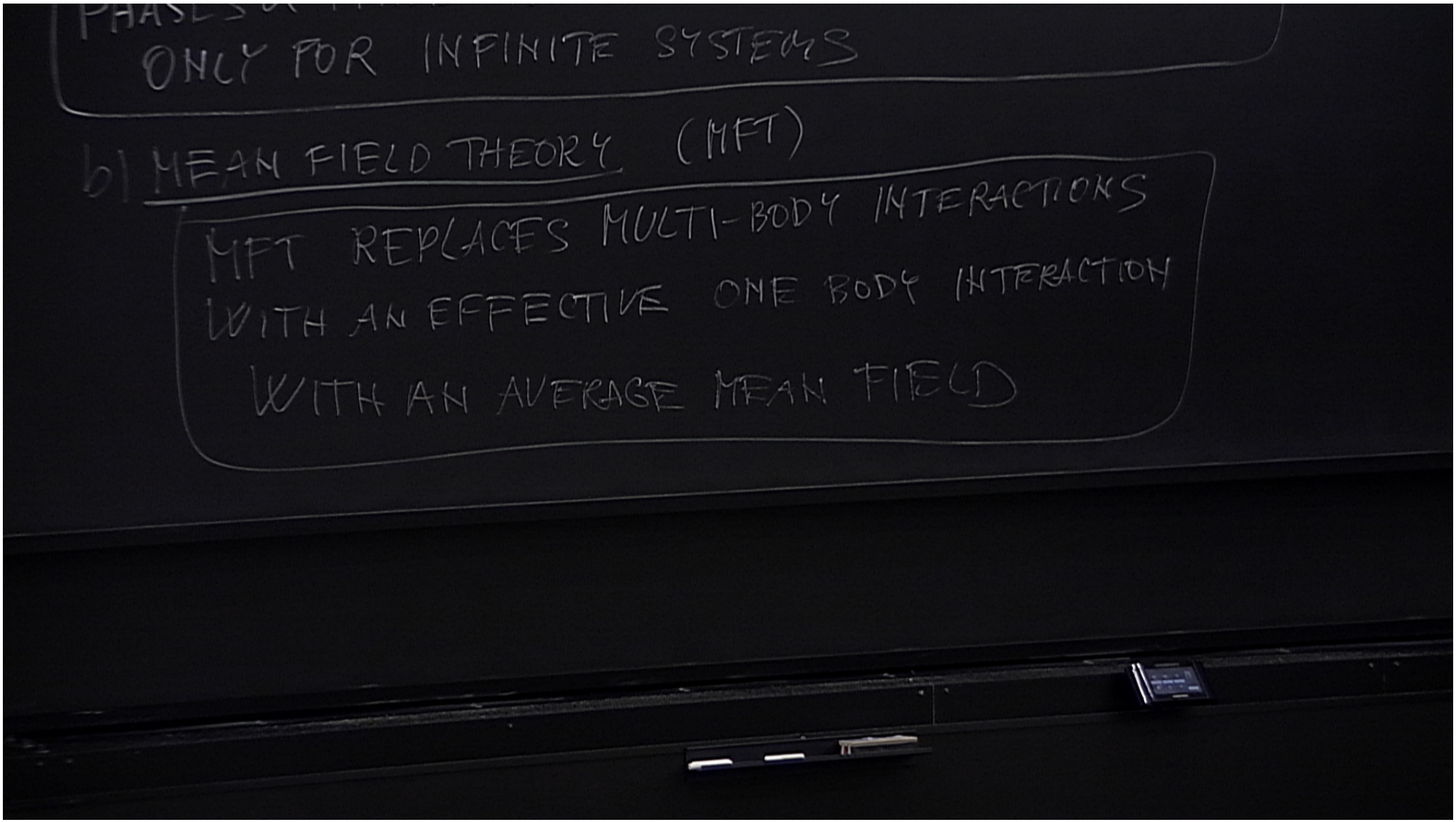
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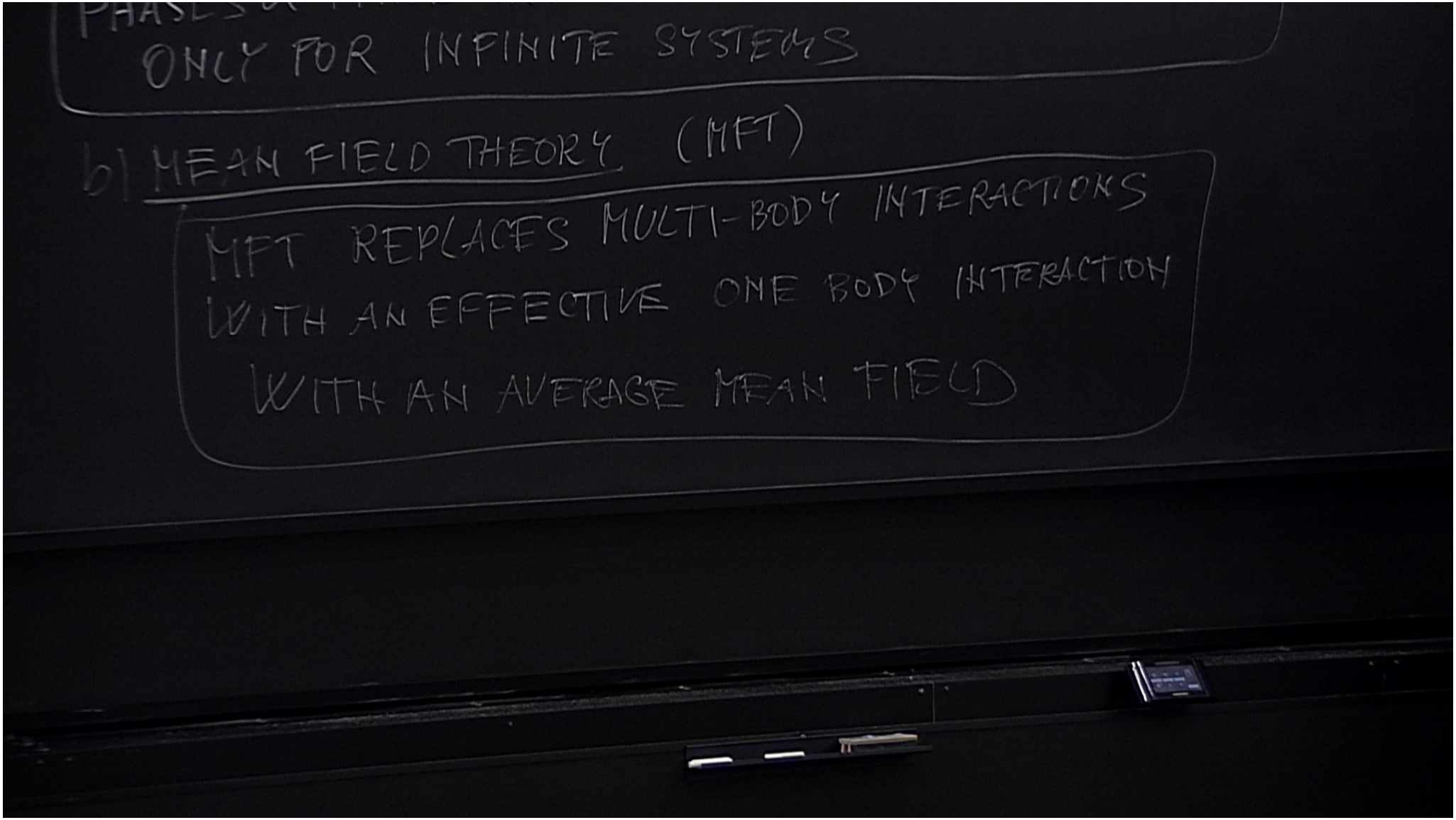
$\boxed{\Lambda = 1}$

ii) $N \to \infty$, $B \to 0$ TD LIMIT $\boxed{\Lambda = 0}$ .. SYSTEM REMAINS IN THE STATE DEFINED BY B.


PHASES & PHASE TRANSITIONS ARE DEFINED SHARPLY
ONLY FOR


PHASES & PHASE TRANSITIONS ARE DEFINED SHARPLY
ONLY FOR INFINITE SYSTEMS


ONLY FOR INFINITE SYSTEMS
b) MEAN FIELD THEORY (MFT)
MFT REPLACES MULTI-BODY INTERACTIONS
WITH AN EFFECTIVE ONE BODY INTERACTION
WITH AN AVERAGE MEAN FIELD


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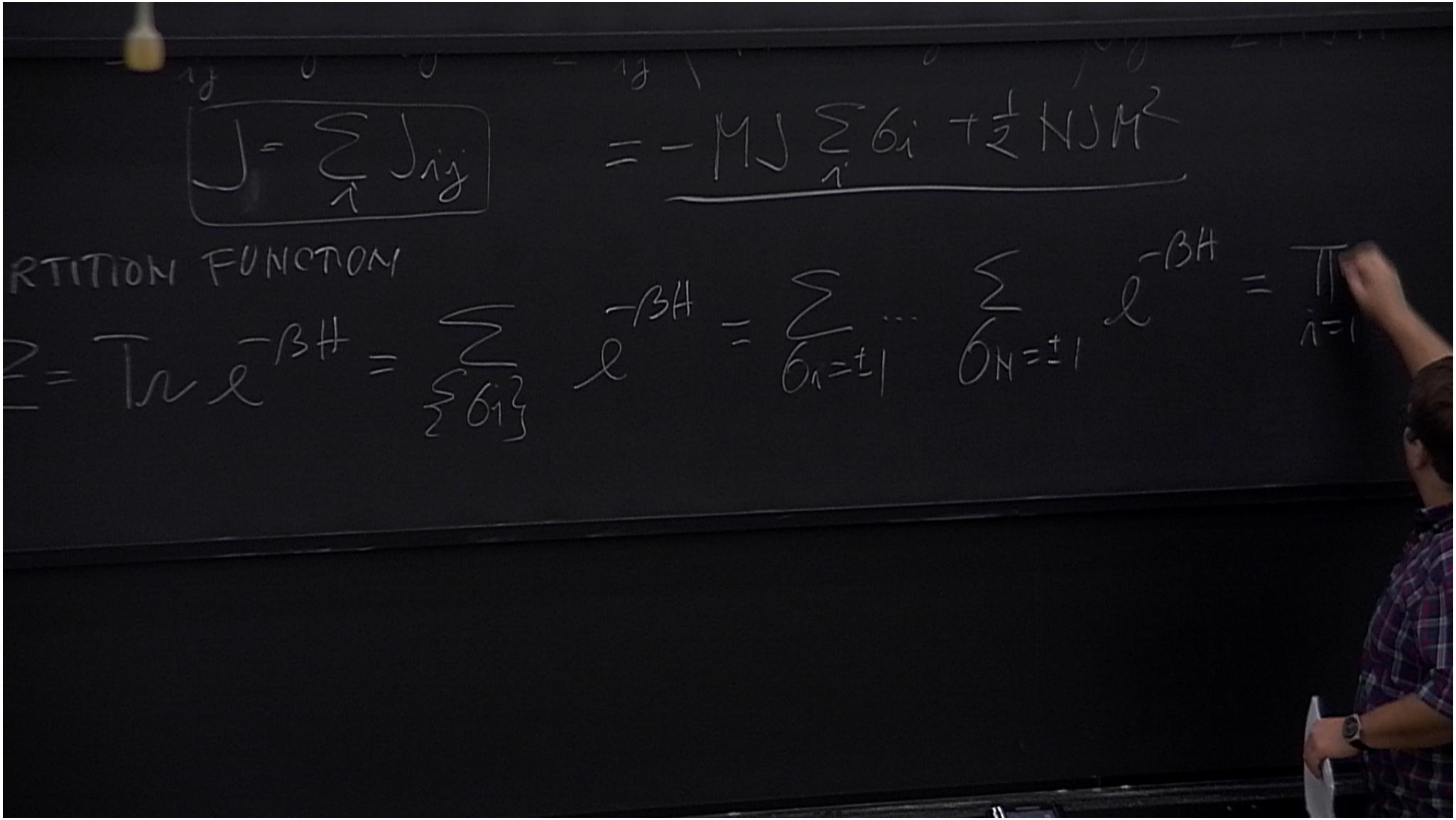
WITH AN AVERAGE MEAN FIELD

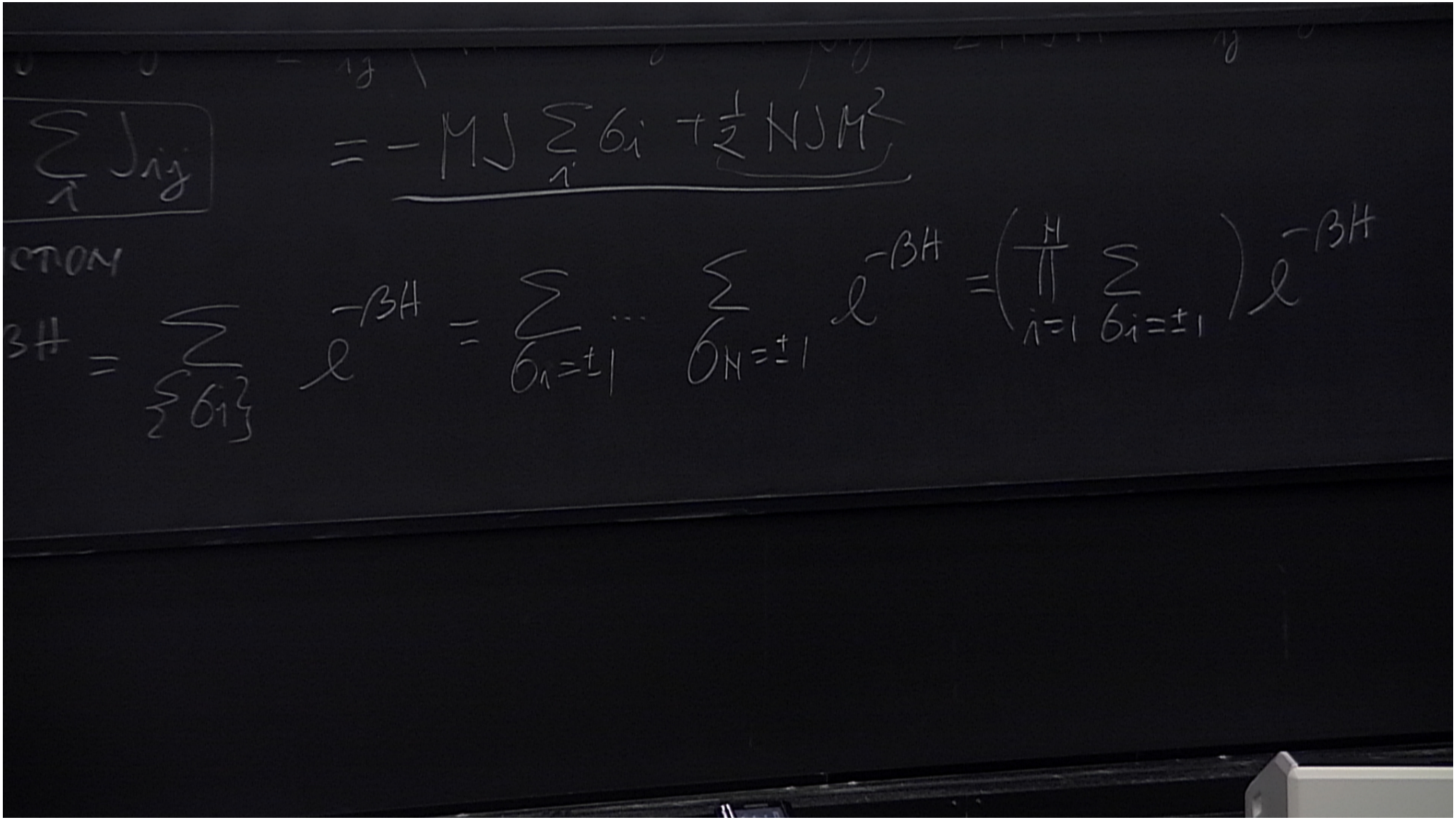
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j = -\frac{1}{2}\sum_{ij}\left(M\sigma_i + M\sigma_j - M^2\right)J_{ij}$$

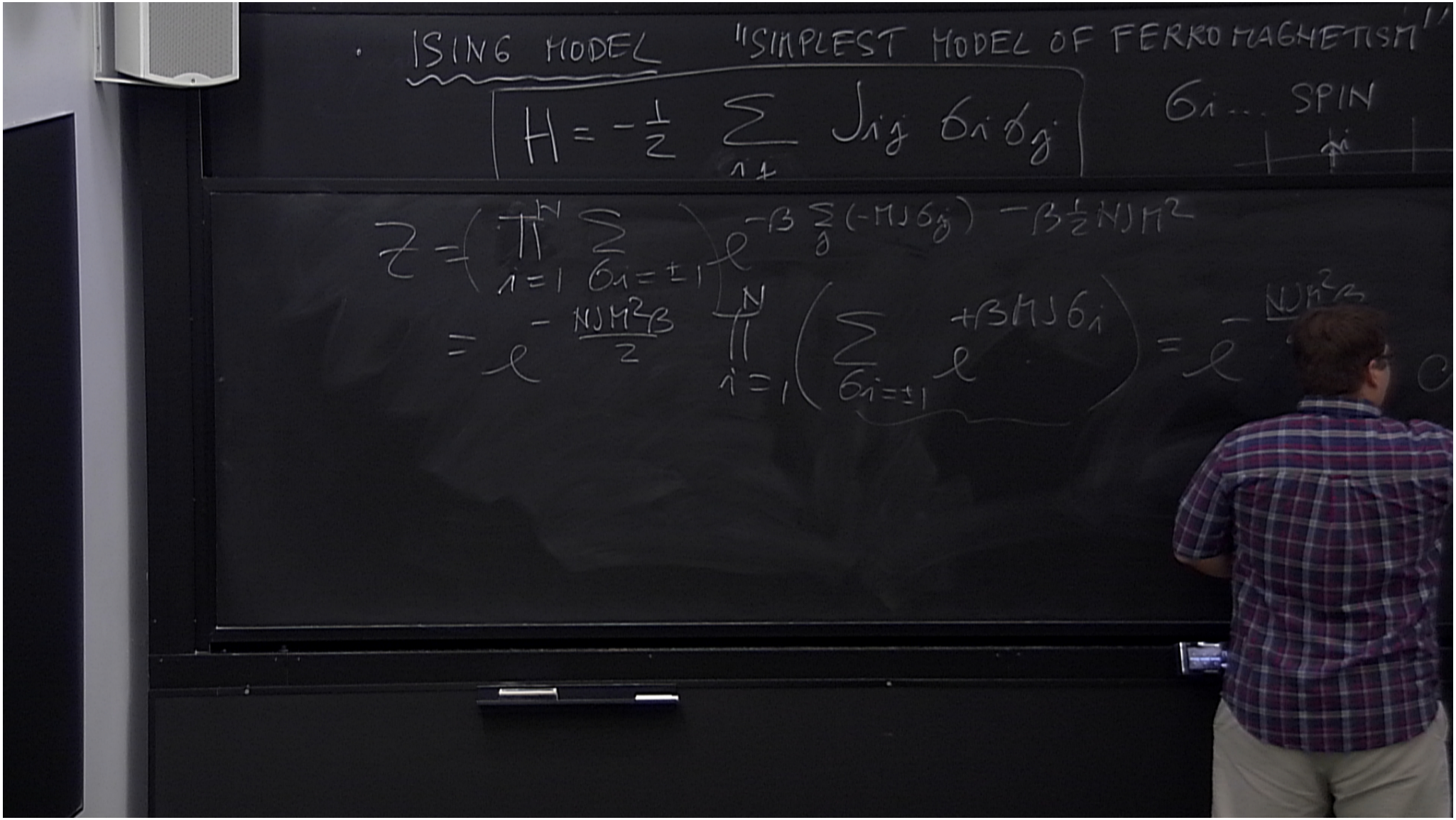
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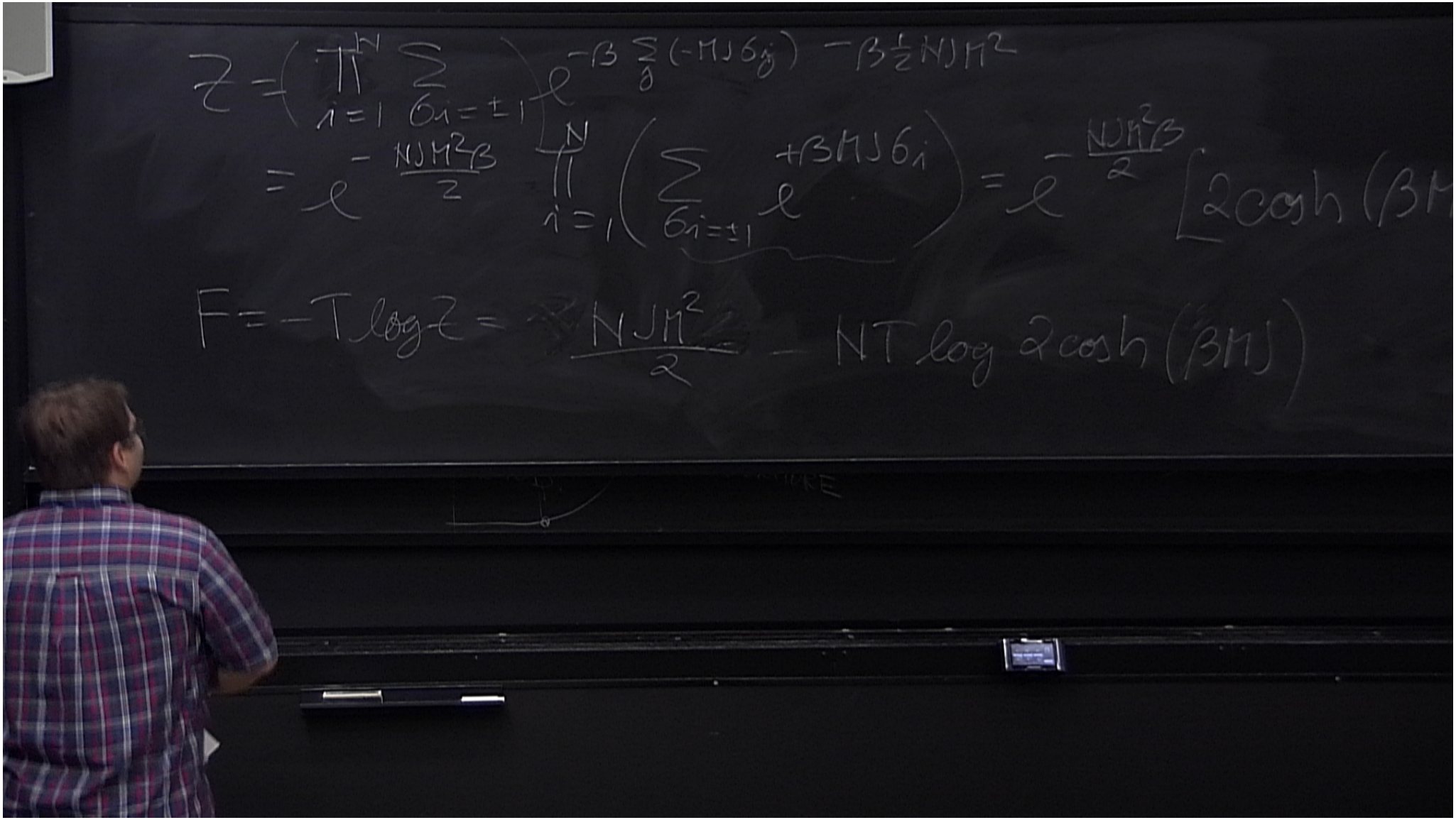
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j = -\frac{1}{2}\sum_{ij}\left(M\sigma_i + M\sigma_j - M^2\right)J_{ij} = +\frac{1}{2}$$

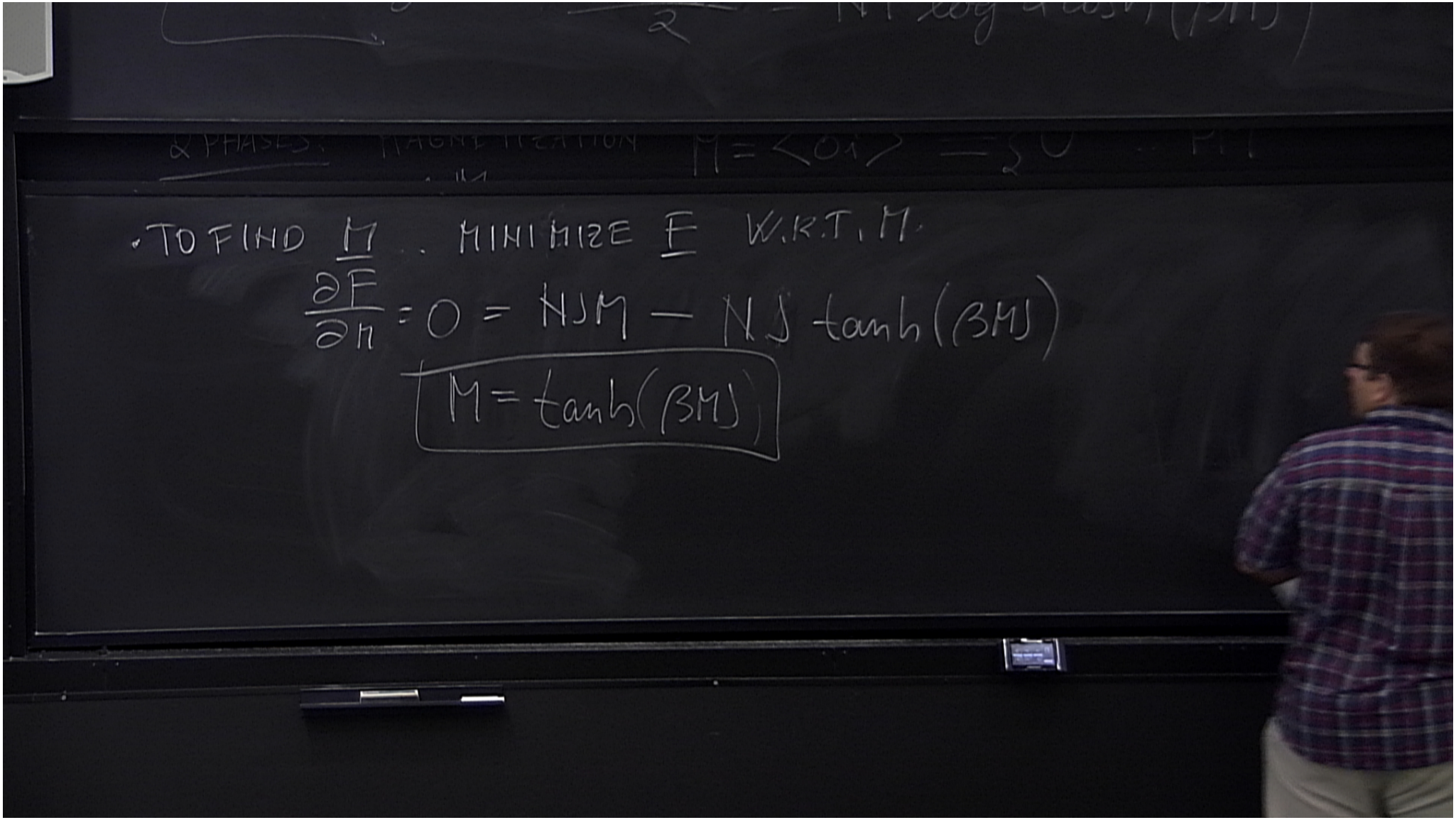
$$J = \sum_{i} J_{ij}$$

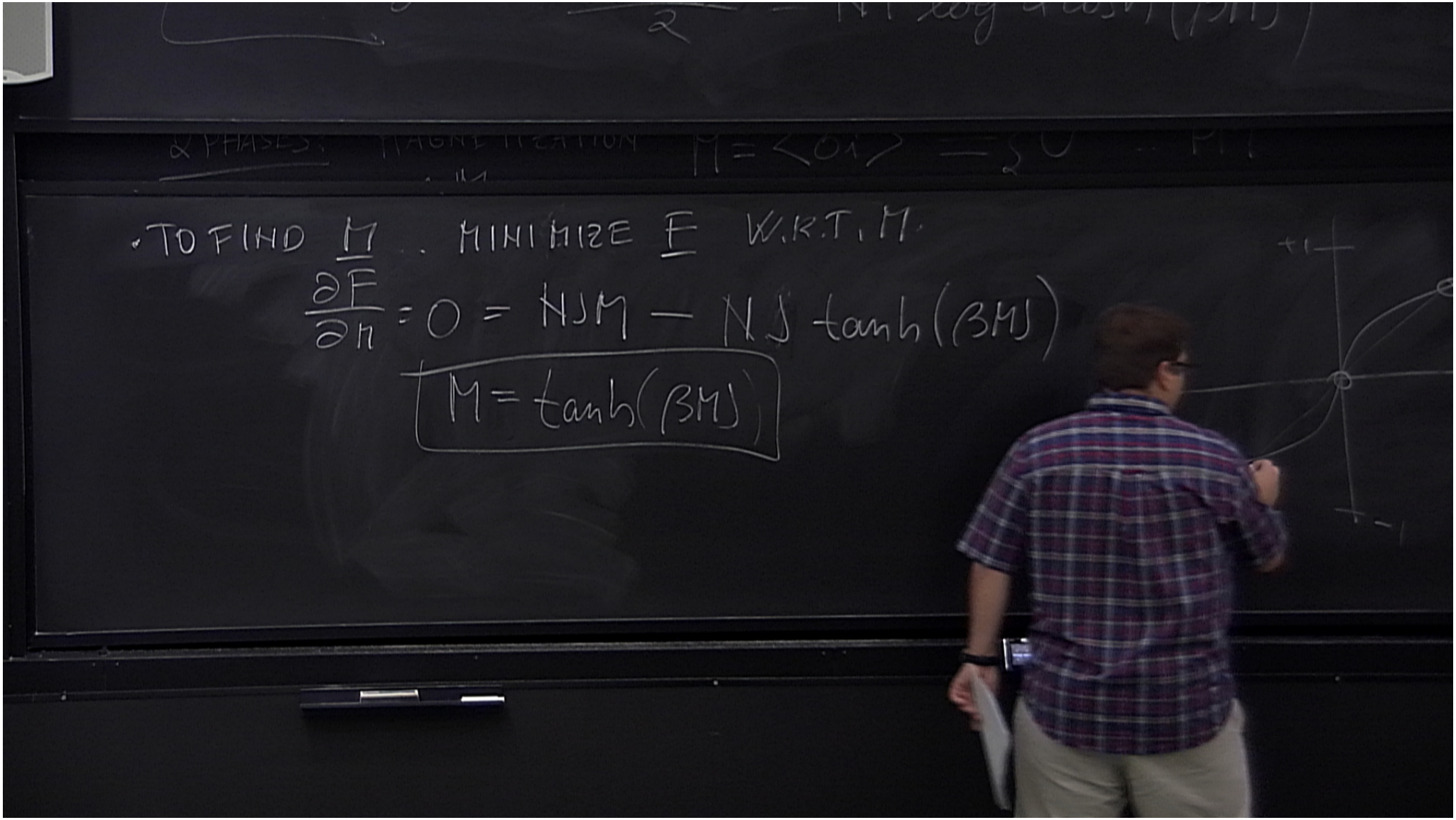

$J = \sum_{\hat{1}} J_{1j}$
$= -MJ \sum_{\hat{1}} \sigma_i + \frac{1}{2} NJM^2$
RTITION FUNCTION
$Z = Tr\, e^{-\beta H} = \sum_{\{\sigma_i\}} e^{-\beta H} = \sum_{\sigma_1 = \pm 1} \cdots \sum_{\sigma_N = \pm 1} e^{-\beta H} = \prod_{i=1}$

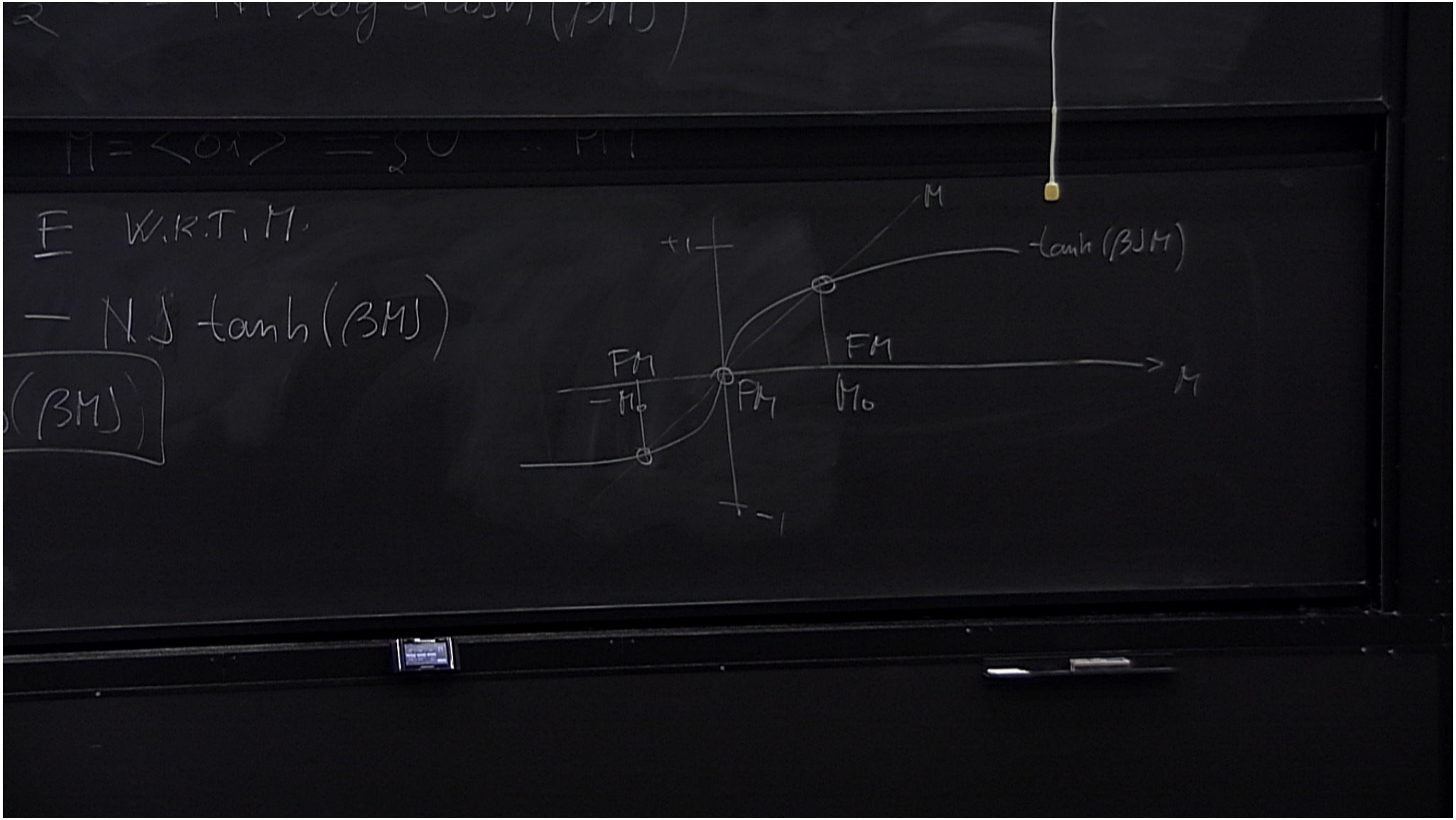

$\sum_i J_{ij}$
$= -MJ \sum_i \sigma_i + \frac{1}{2} NJM^2$
CTION
$BH = \sum_{\{\sigma_i\}}$
$e^{-\beta H} = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1} \right) e^{-\beta H}$


ISING MODEL
"SIMPLEST MODEL OF FERROMAGNETISM"
$H = -\frac{1}{2} \sum_{i,j} J_{ij} \sigma_i \sigma_j$
σ_i ... SPIN
$Z = \left(\prod_{i=1}^{N} \sum_{\sigma_i = \pm 1} \right) e^{-\beta \sum_j (-MJ\sigma_j)} - \beta \frac{1}{2} NJM^2$
$= e^{-\frac{NJM^2\beta}{2}} \prod_{i=1}^{N} \left(\sum_{\sigma_i = \pm 1} e^{+\beta MJ \sigma_i} \right) = e^{-\frac{NJM^2\beta}{2}}$


$Z = \left(\prod_{i=1}^{N} \sum_{\sigma_i = \pm 1} \right) e^{-\beta \sum_j (-MJ\sigma_j)} - \beta \frac{1}{2} NJM^2$
$= e^{-\frac{NJM^2\beta}{2}} \prod_{i=1}^{N} \left(\sum_{\sigma_i = \pm 1} e^{+\beta MJ\sigma_i} \right) = e^{-\frac{NJM^2\beta}{2}} [2\cosh(\beta M$
$F = -T \log Z = \frac{NJM^2}{2} - NT \log 2\cosh(\beta MJ)$


2 PHASES: MAGNETIZATION
M = <σ> = 2U
• TO FIND M . MINIMIZE F W.R.T. M.
∂F/∂M = 0 = NJM − NJ tanh(βMJ)
M = tanh(βMJ)

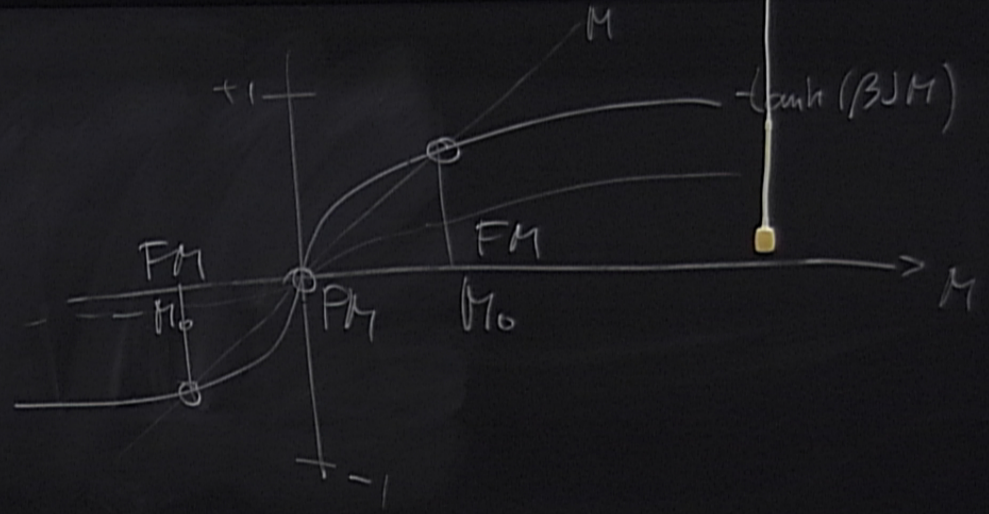

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M = tanh(βMJ)
+1
−1

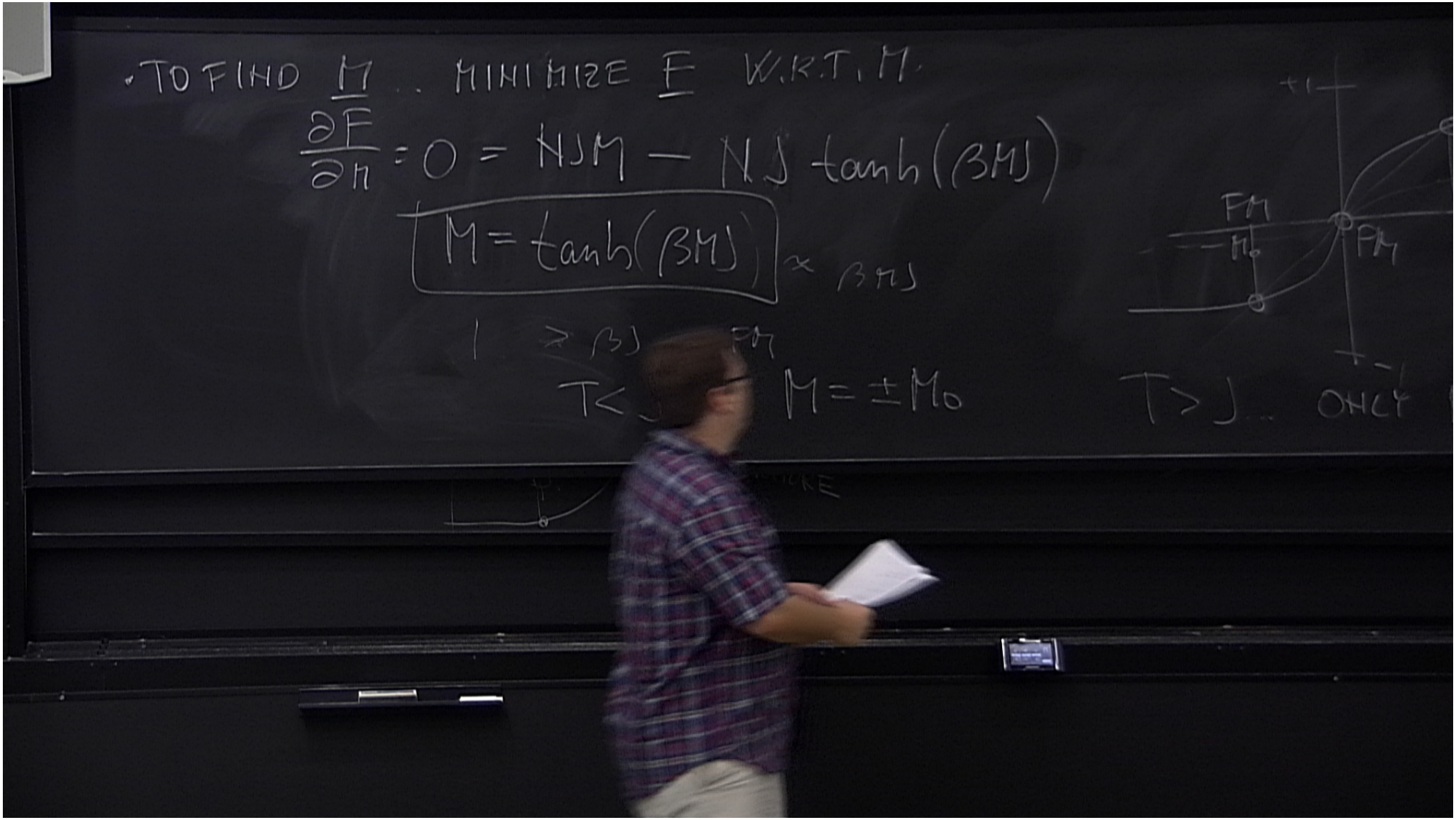

F W.R.T. M.
− NJ tanh(βMJ)
M
+1
tanh(βJM)
FM
FM
−M₀
M₀
M
−1

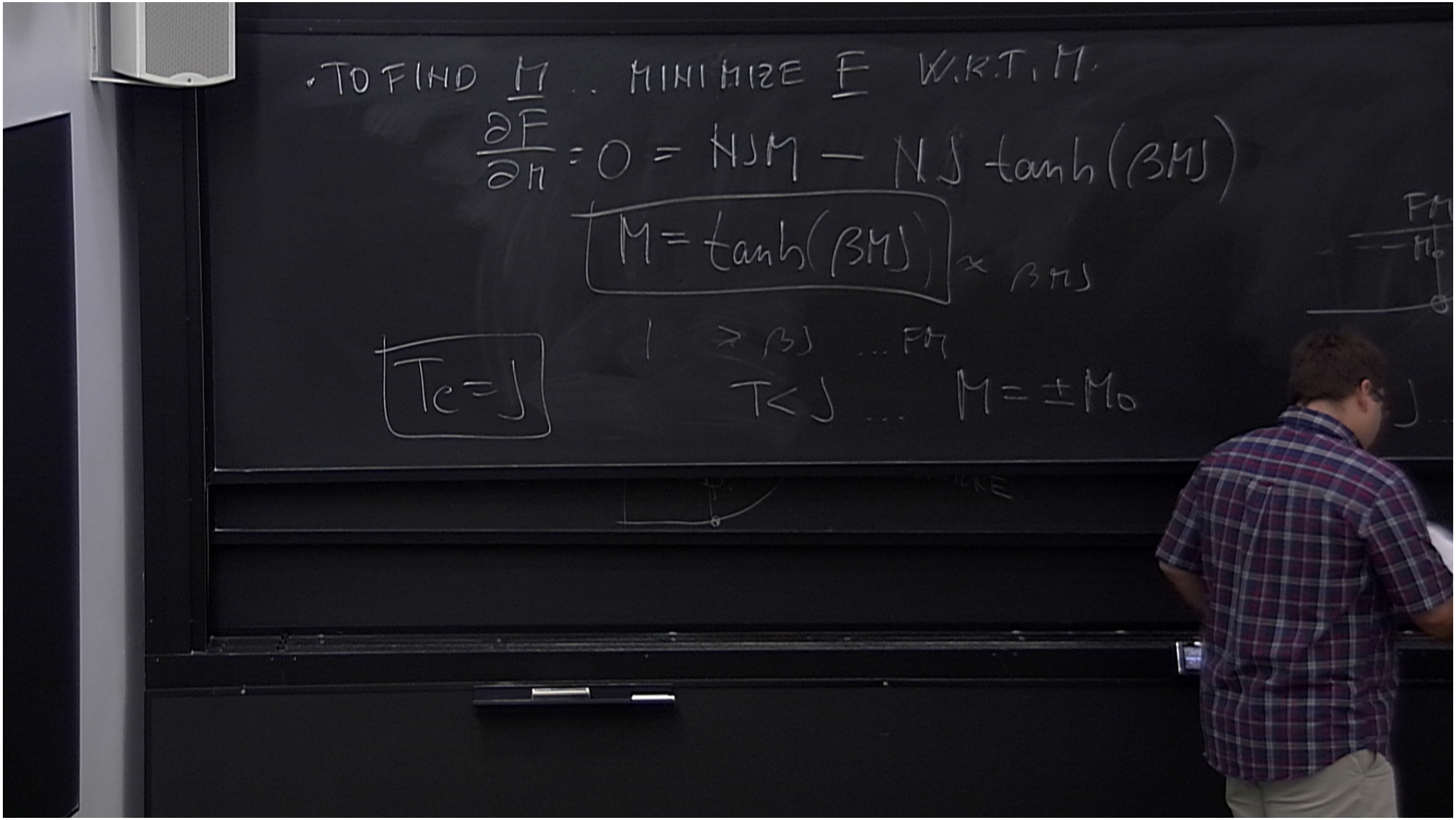
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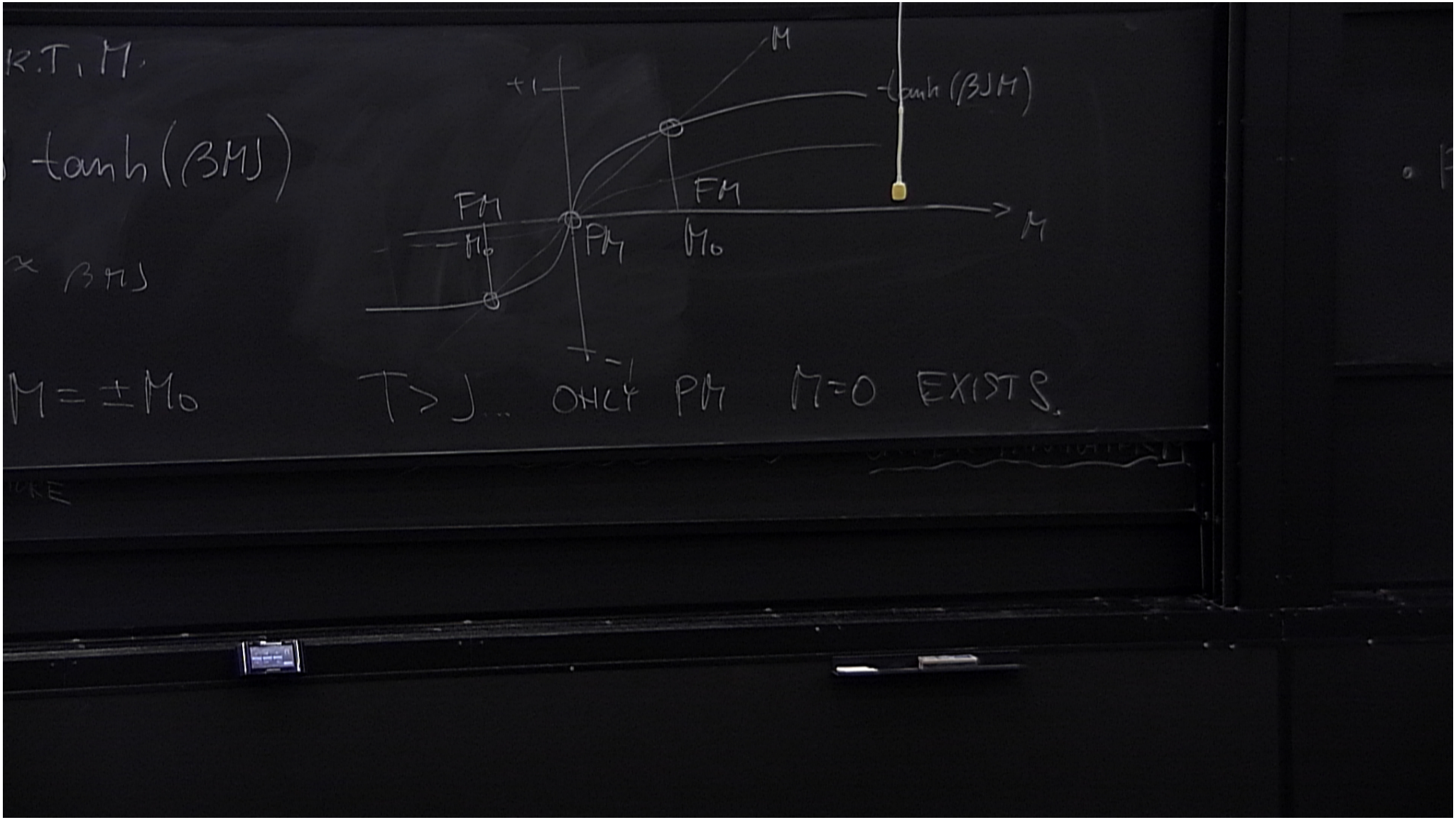
$-NJ\tanh(\beta MJ)$

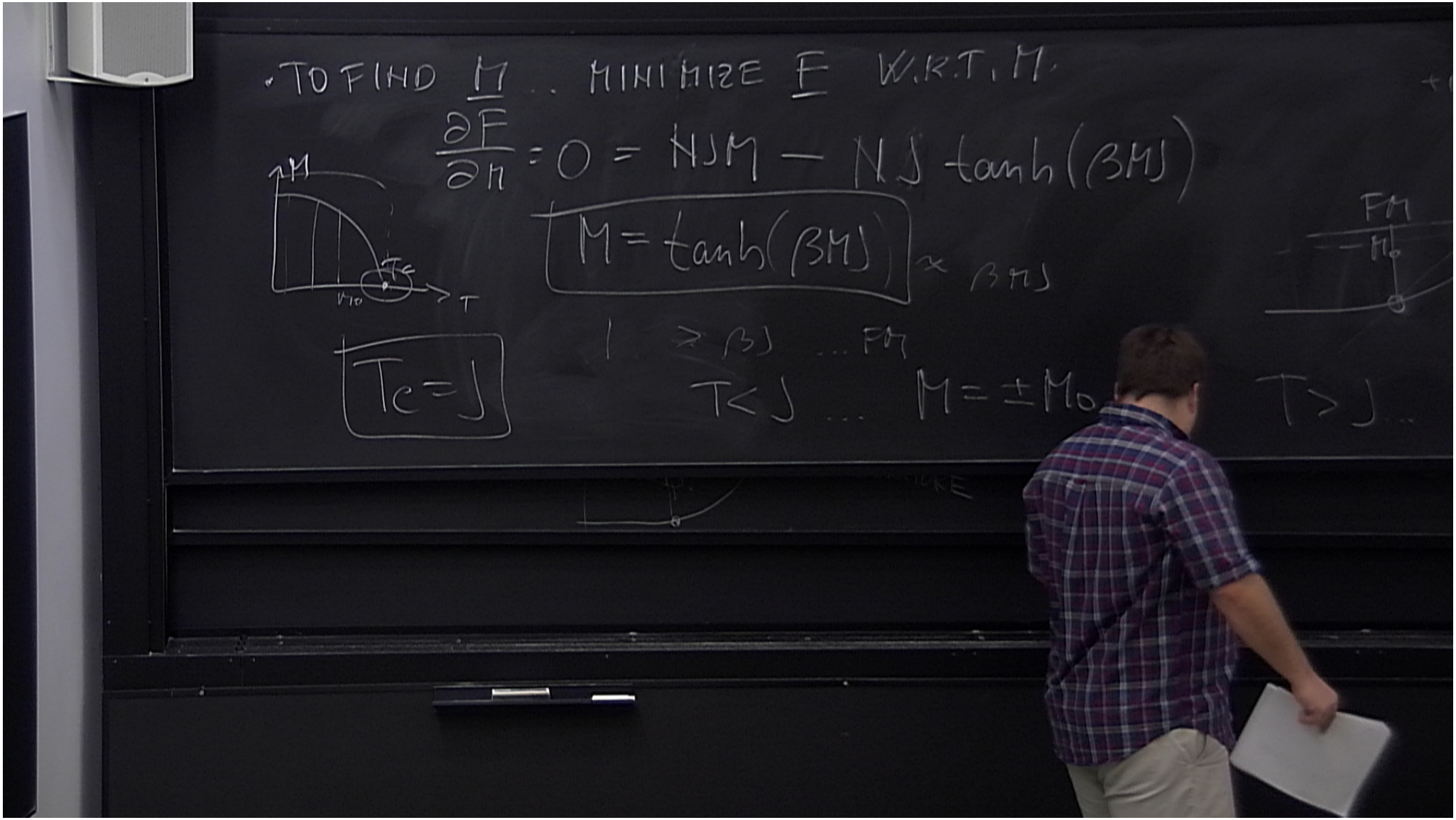
$(\beta MJ) \propto \beta MJ$

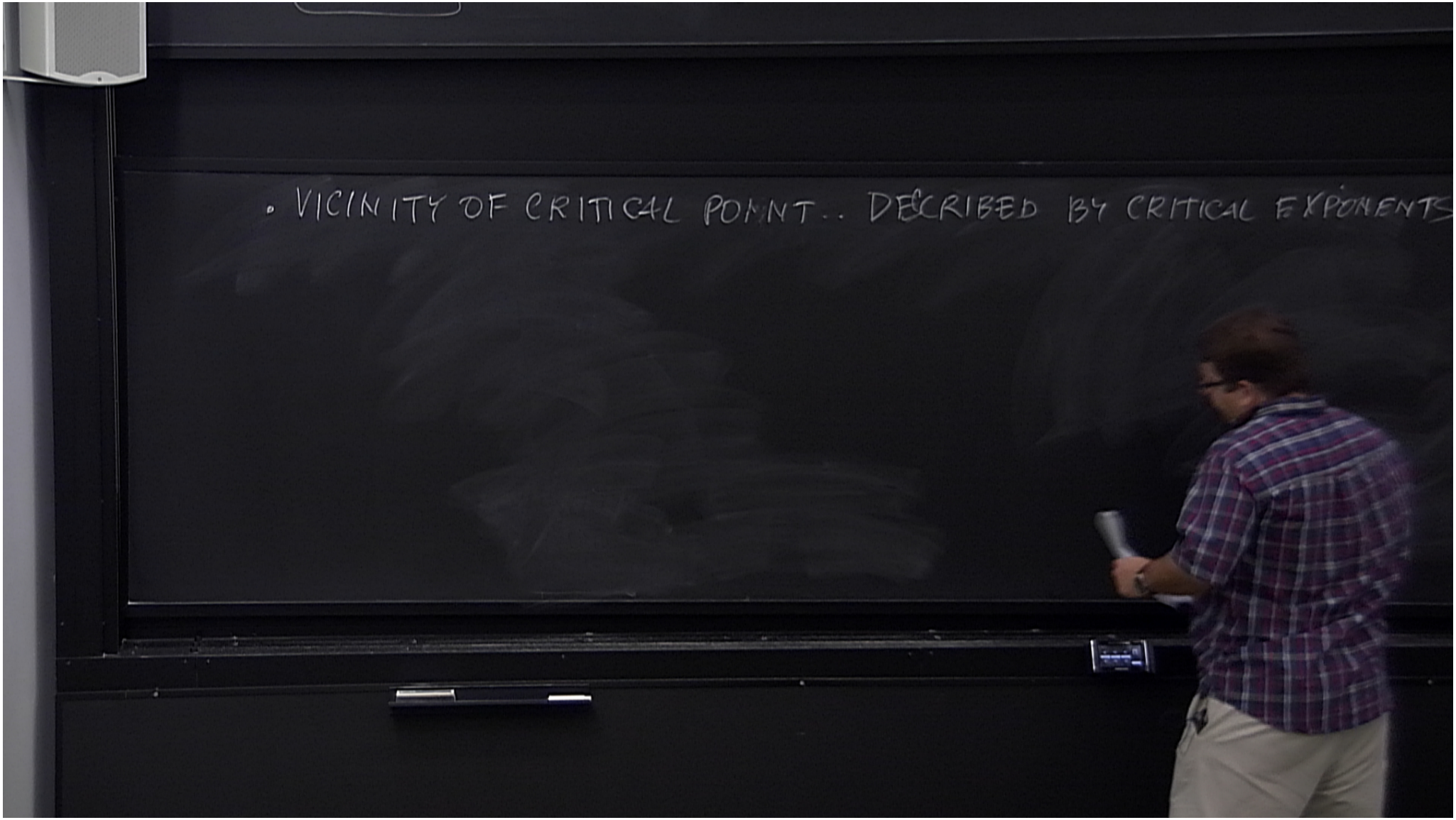


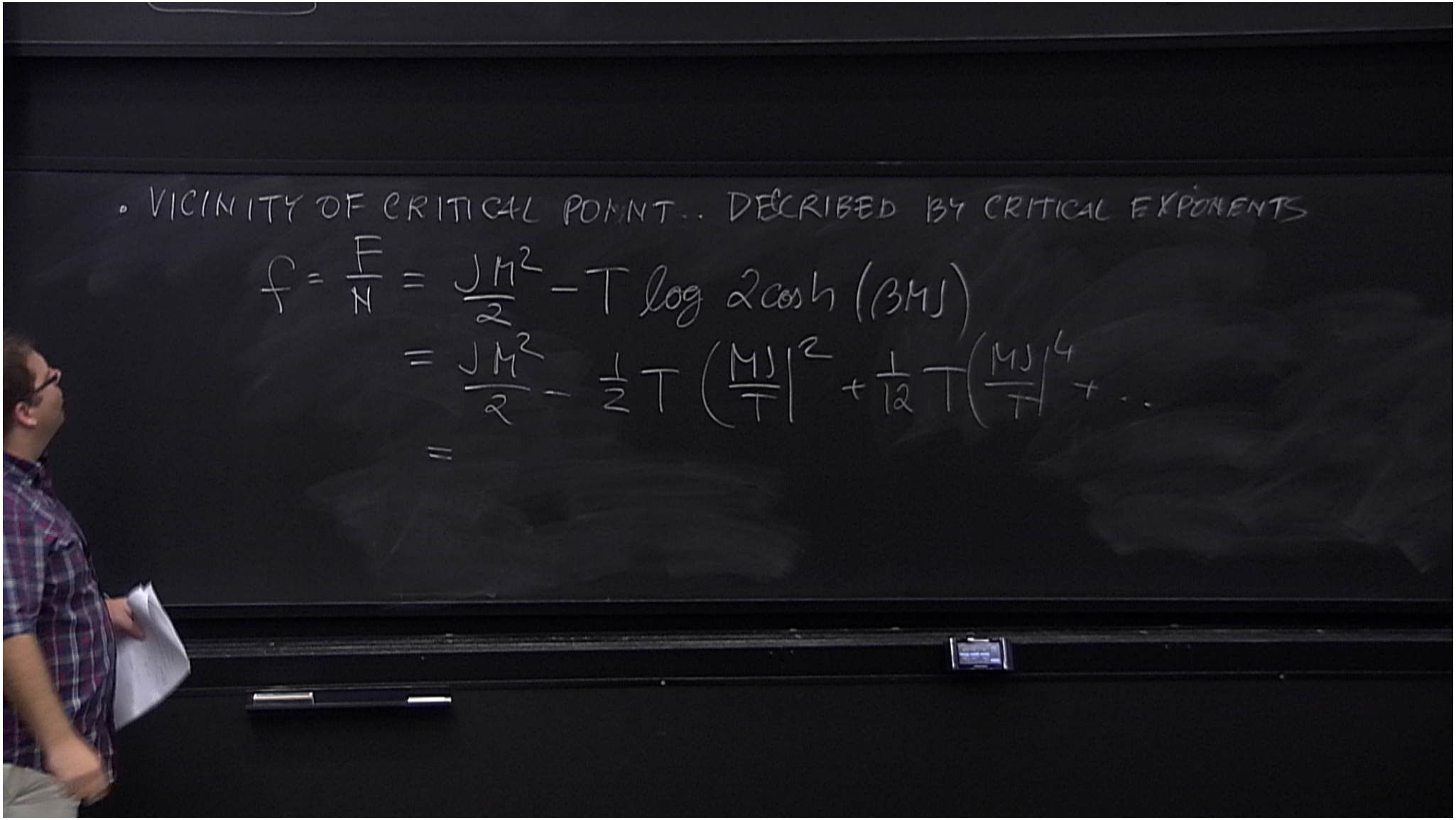

• TO FIND M . . MINIMIZE F W.R.T. M.
$\frac{\partial F}{\partial m} = 0 = NJM - NJ\tanh(\beta MJ)$
$M = \tanh(\beta MJ)$
$\propto \beta MJ$
$1 \geq \beta J$
FM
$T < J$
$M = \pm M_0$
$T > J$ . . . OHCP
+1
FM
$-M_0$
PM
-1

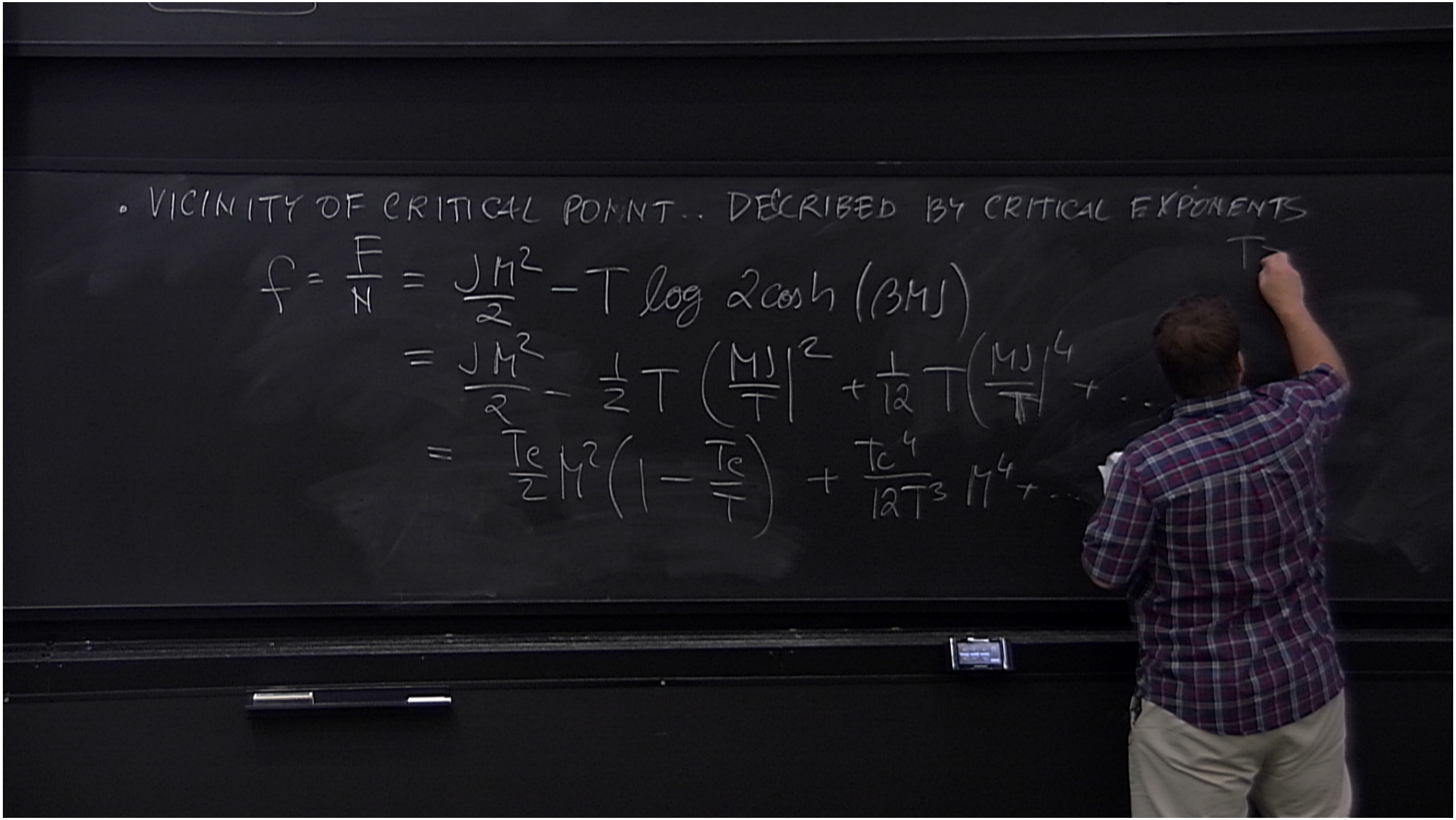

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$M = \tanh(\beta MJ)$
$\approx \beta MJ$
$T_c = J$
$1 \geq \beta J$ ... FM
$T < J$ ... $M = \pm M_0$


R.T.M.
tanh(βMJ)
x βMJ
M = ±M₀
T > J ... ONLY PM M=0 EXISTS.
+1
-1
M
tanh(βJM)
FM
-M₀
PM
FM
M₀
M


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FM
M_0
T_c
M_0
T


• VICINITY OF CRITICAL POINT.. DESCRIBED BY CRITICAL EXPONENTS


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=

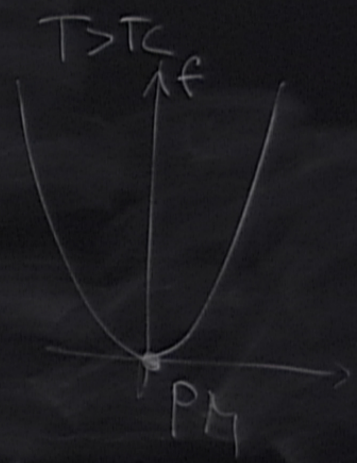

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CINITY OF CRITICAL POINT.. DESCRIBED BY CRITICAL EXPONENTS

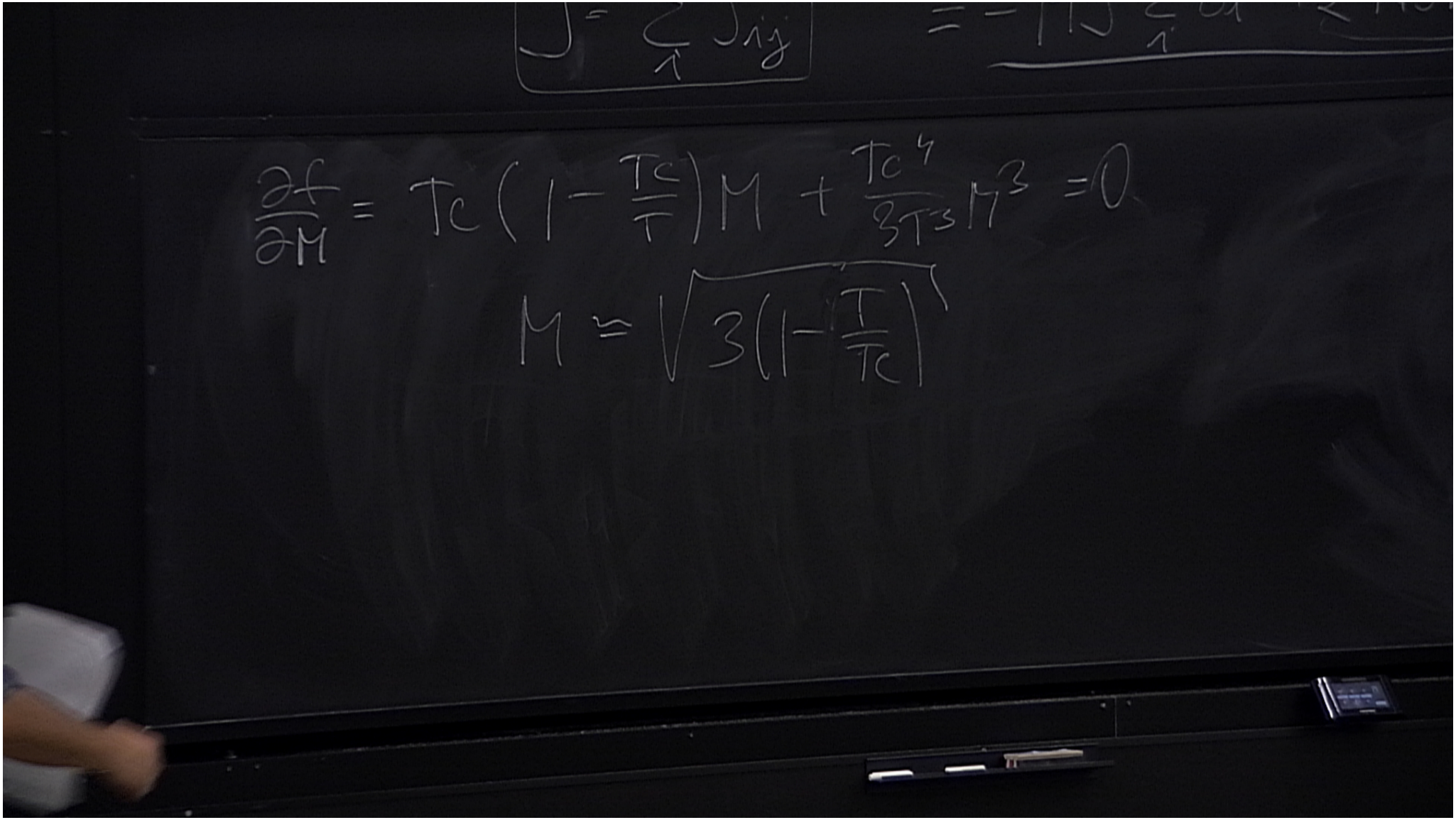
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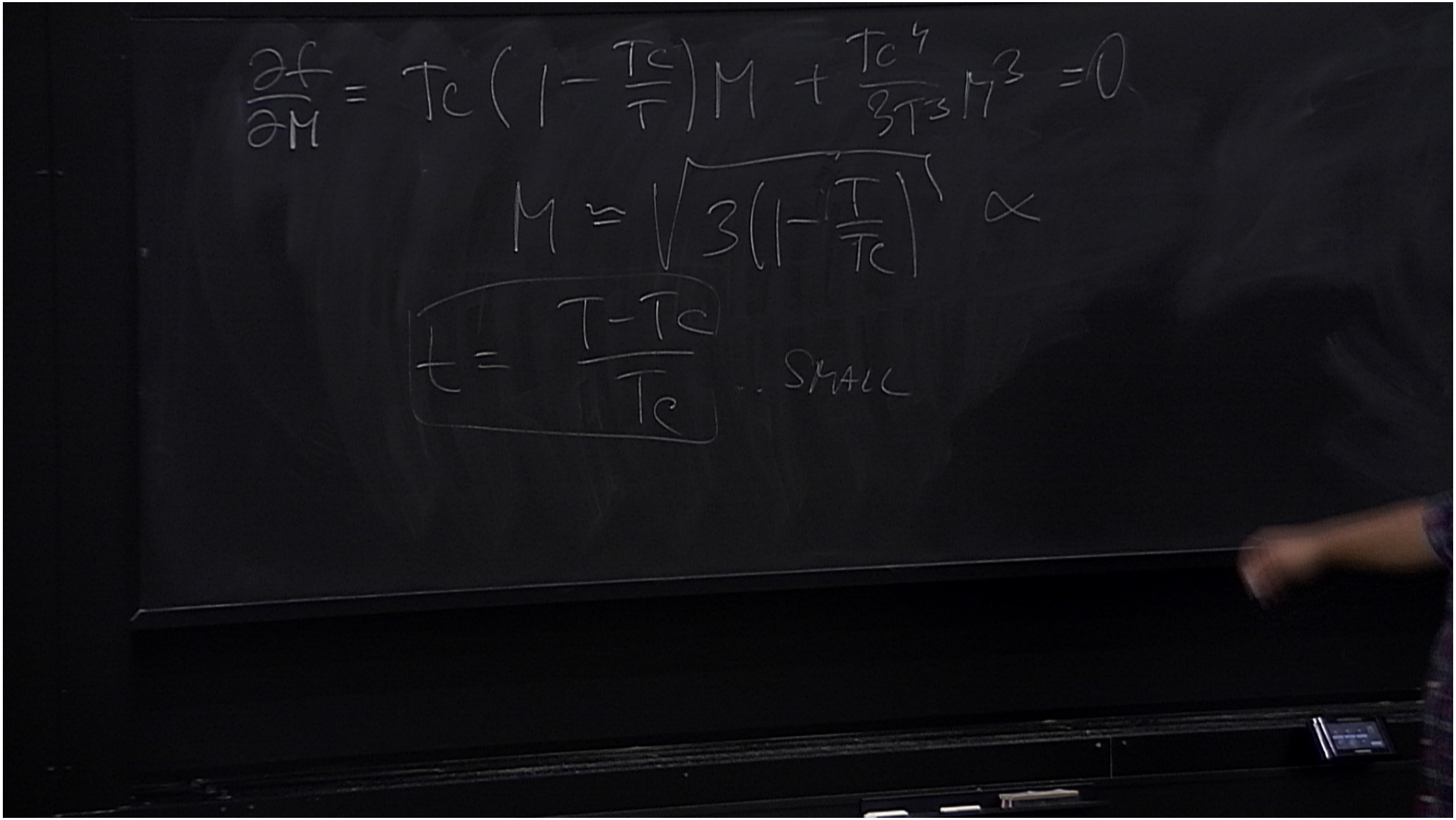
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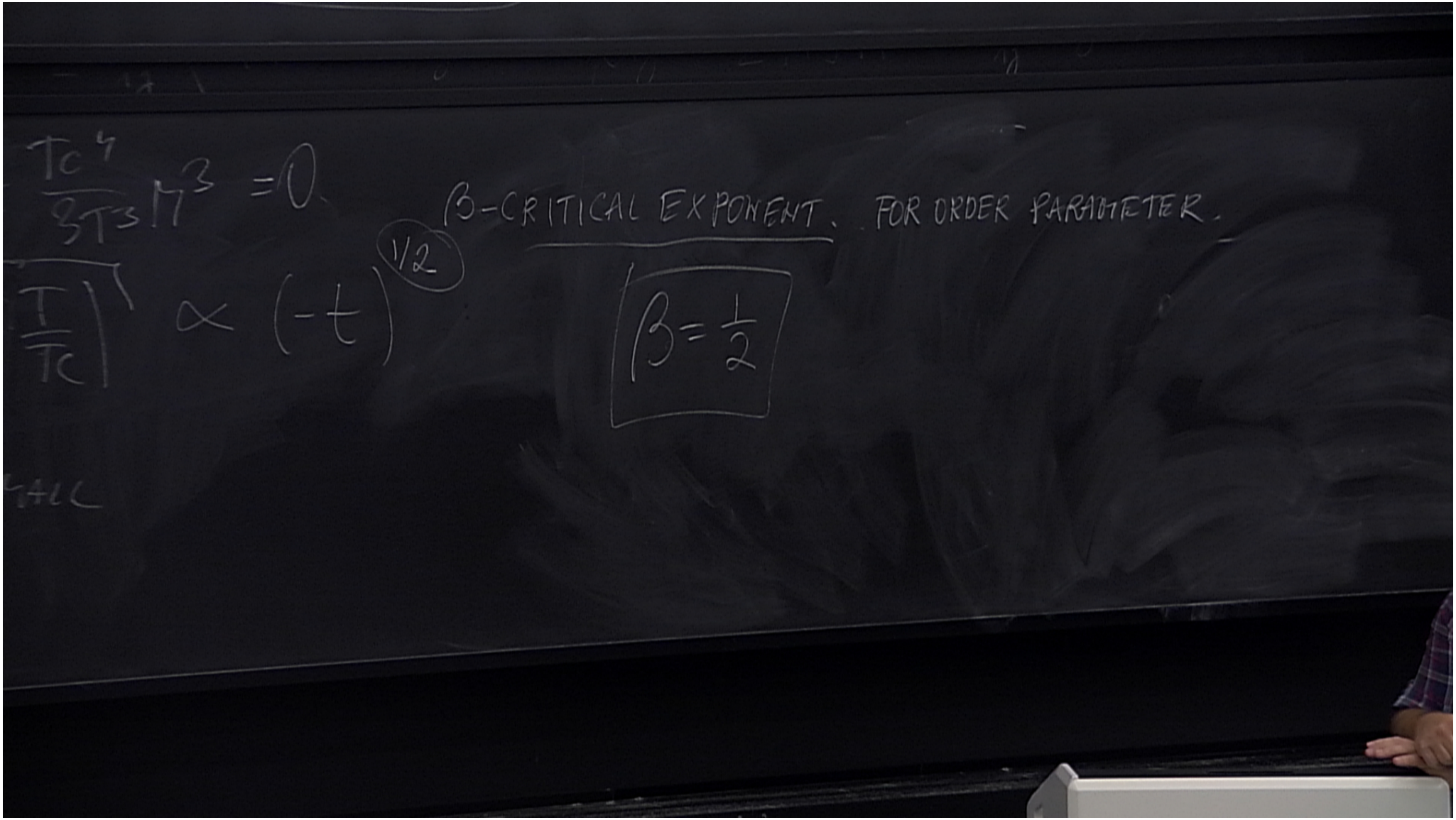
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$t = \frac{T - T_c}{T_c}$ .. SMALL


T_c^4
$3T^3 M^3 = 0$
β-CRITICAL EXPONENT. FOR ORDER PARAMETER.
$\left|\frac{T}{T_c}\right| \propto (-t)^{1/2}$
$\beta = \frac{1}{2}$