Title: PSI 2016/2017 Quantum Field Theory I - Lecture 13

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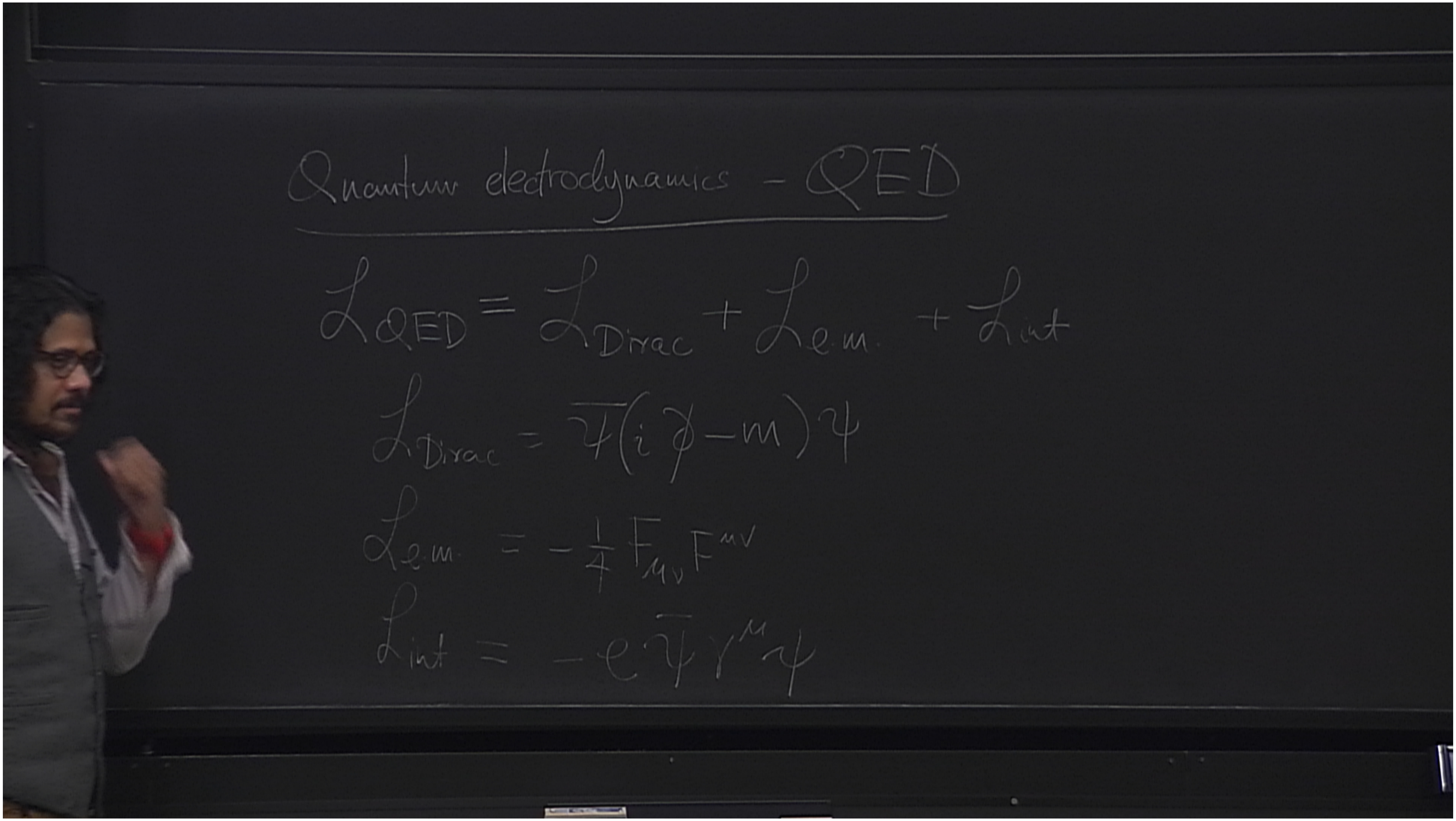
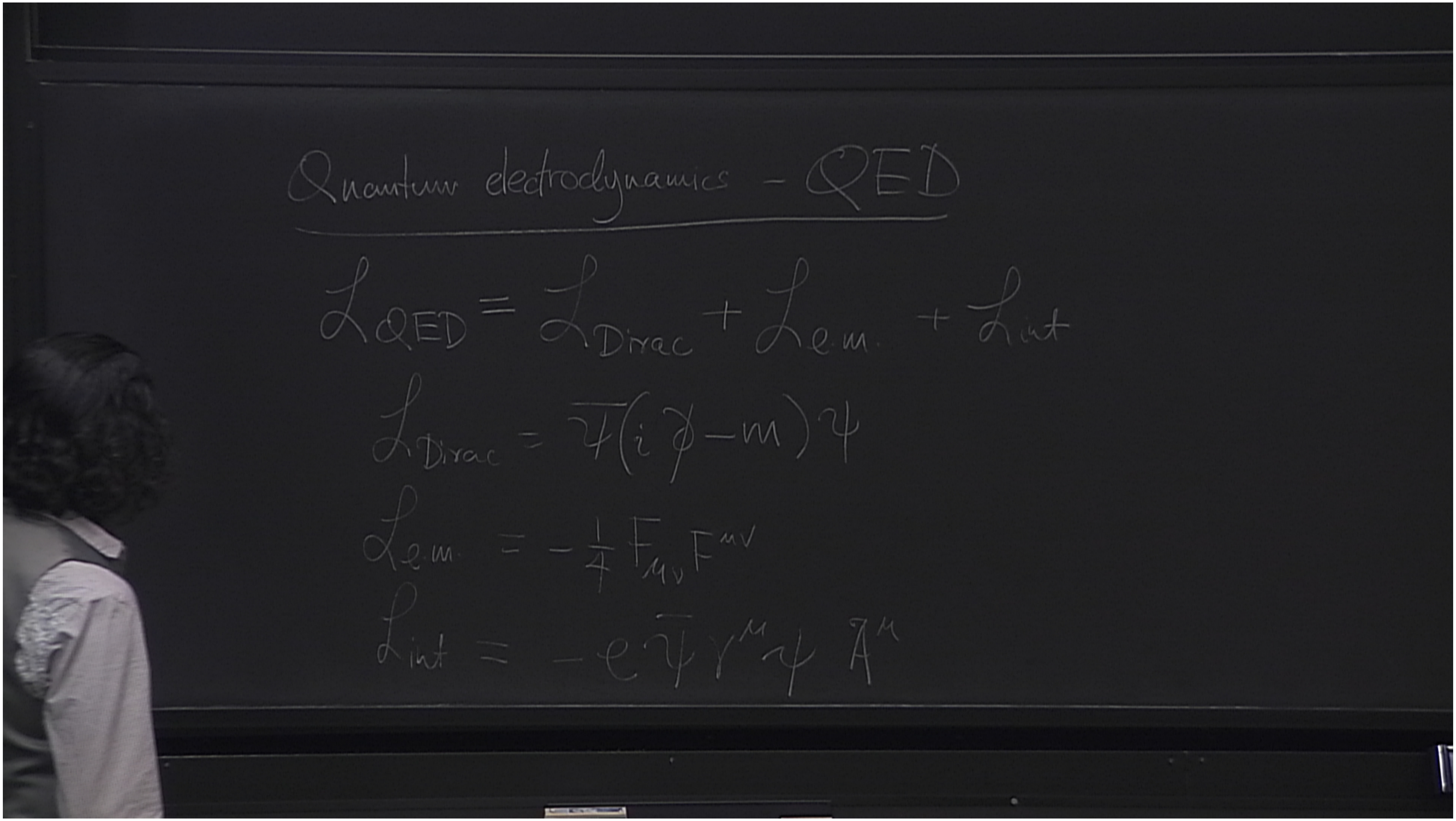
Abstract:


Quantum electrodynamics - QED
$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$
$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi$
$\mathcal{L}_{e.m.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
$\mathcal{L}_{int} = -e\bar{\psi}\gamma^{\mu}\psi$


Quantum electrodynamics - QED
$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$
$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi$
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Quantum electrodynamics - QED

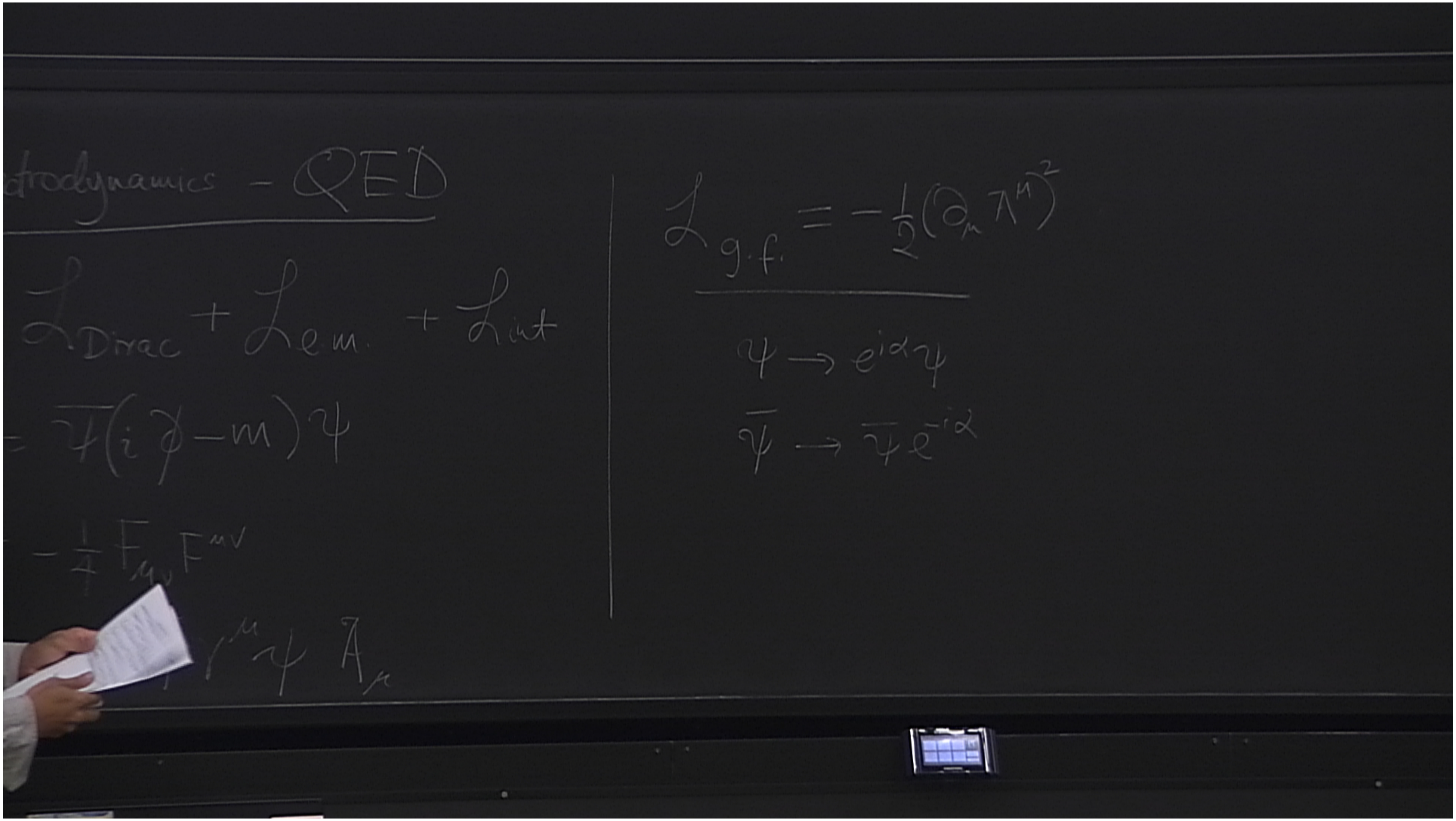
$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$$

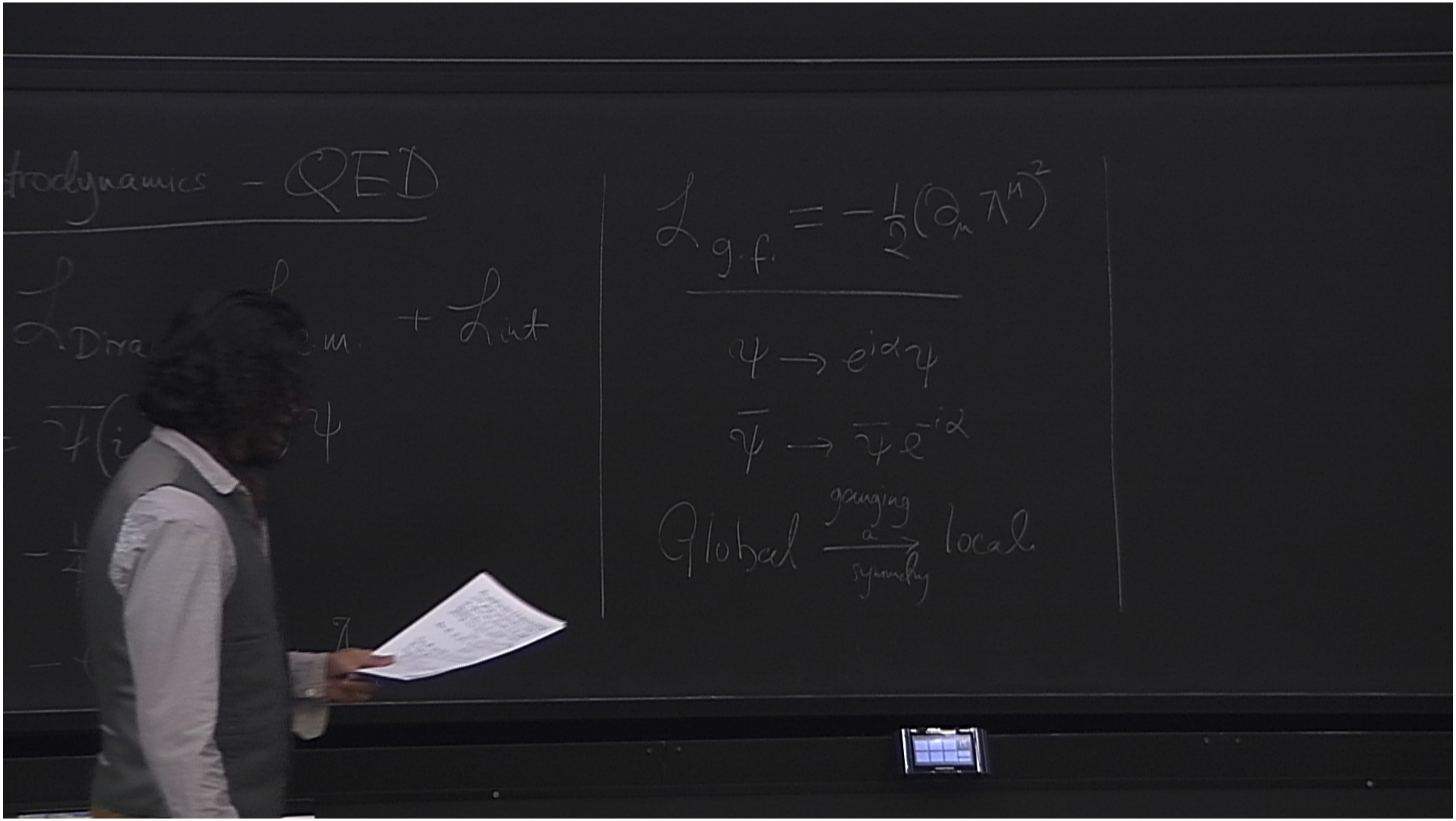
$$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi$$

$$\mathcal{L}_{e.m.} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{int} = -e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

$$\mathcal{L}_{g.f.} = -\frac{1}{2}$$


...trodynamics - QED
$\mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$
$= \bar{\psi}(i\not{\partial} - m)\psi$
$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
$\gamma^\mu\psi\, A_\mu$
$\mathcal{L}_{g.f.} = -\frac{1}{2}(\partial_\mu A^\mu)^2$
$\psi \to e^{i\alpha}\psi$
$\bar{\psi} \to \bar{\psi}e^{-i\alpha}$


trodynamics - QED
L g.f. = -1/2 (∂μ A^μ)^2
+ L int
ψ → e^iα ψ
ψ̄ → ψ̄ e^-iα
Global gauging a symmetry → local

$$\mathcal{L}_{g.f.} = -\frac{1}{2}(\partial_\mu A^\mu)^2$$

$$\psi \to e^{i\alpha}\psi$$

$$\bar{\psi} \to \bar{\psi} e^{-i\alpha}$$

$$\text{Global} \xrightarrow[\text{symmetry}]{\text{gauging } \alpha} \text{local}$$

$$\psi(x) \to \psi'(x) = e^{-ie\alpha(x)}\psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}'(x) = e^{+ie\alpha(x)}\bar{\psi}(x)$$

$$\text{f.} = -\frac{1}{2}(\partial_\mu A^\mu)^2$$

$$\psi \to e^{i\alpha}\psi$$

$$\bar{\psi} \to \bar{\psi} e^{-i\alpha}$$

al $\xrightarrow[\text{symmetry}]{\text{gauging a}}$ local

$$\psi(x) \to \psi'(x) = e^{-ie\alpha(x)}\psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}'(x) = e^{+ie\alpha(x)}\bar{\psi}(x)$$

$$\bar{\psi} i \partial\!\!\!/ \psi \to \bar{\psi}(e\, \partial\!\!\!/ \alpha)\psi + \bar{\psi} i \partial\!\!\!/ \psi$$

$$\ldots f. = -\frac{1}{2}(\partial_\mu A^\mu)^2$$

$$\psi \longrightarrow e^{i\alpha}\psi$$

$$\bar\psi \longrightarrow \bar\psi e^{-i\alpha}$$

$$\ldots al \xrightarrow[\text{symmetry}]{\text{gauging a}} \text{local}$$

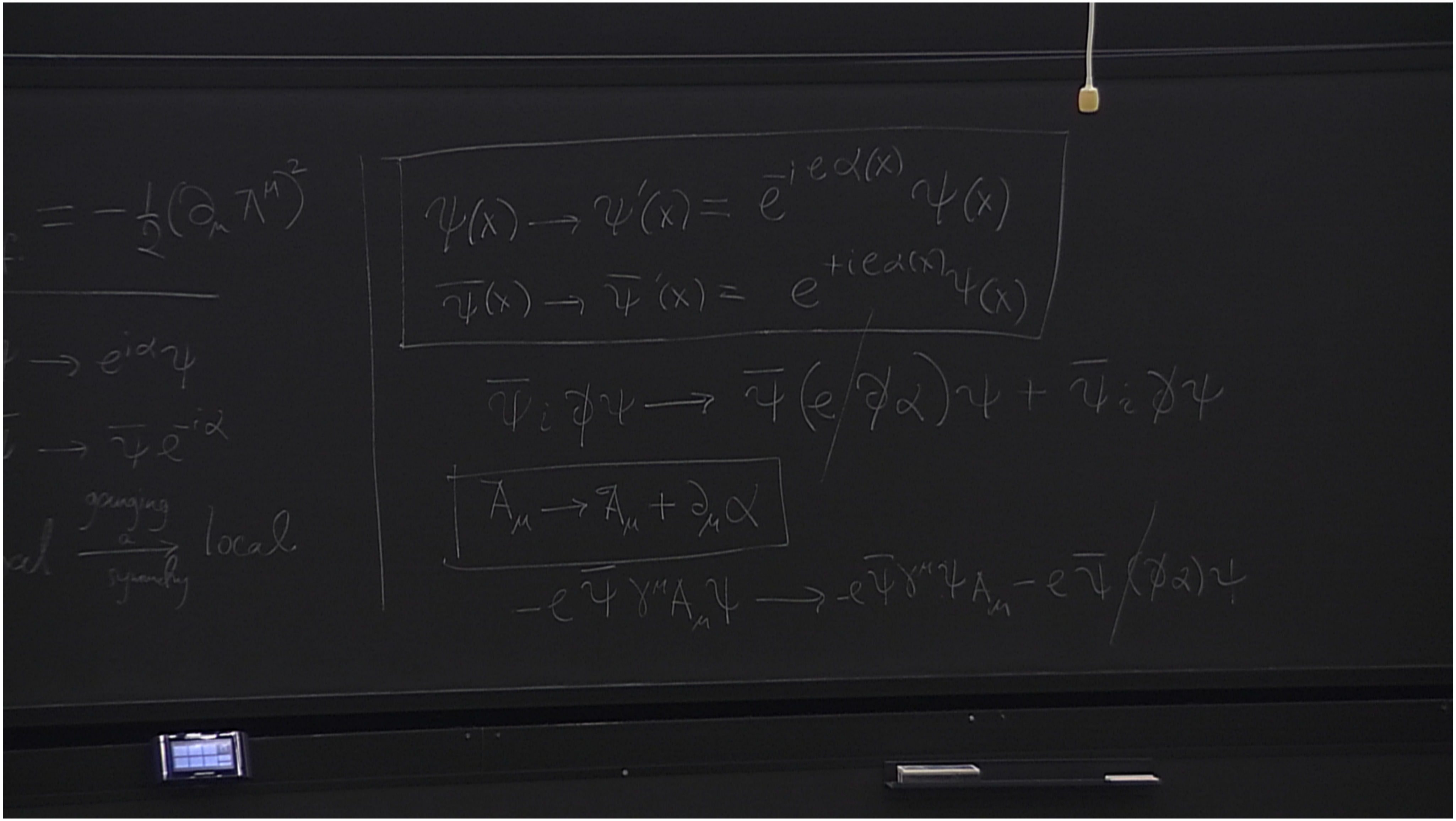
$$\psi(x) \longrightarrow \psi'(x) = e^{-ie\alpha(x)}\psi(x)$$

$$\bar\psi(x) \longrightarrow \bar\psi'(x) = e^{+ie\alpha(x)}\bar\psi(x)$$

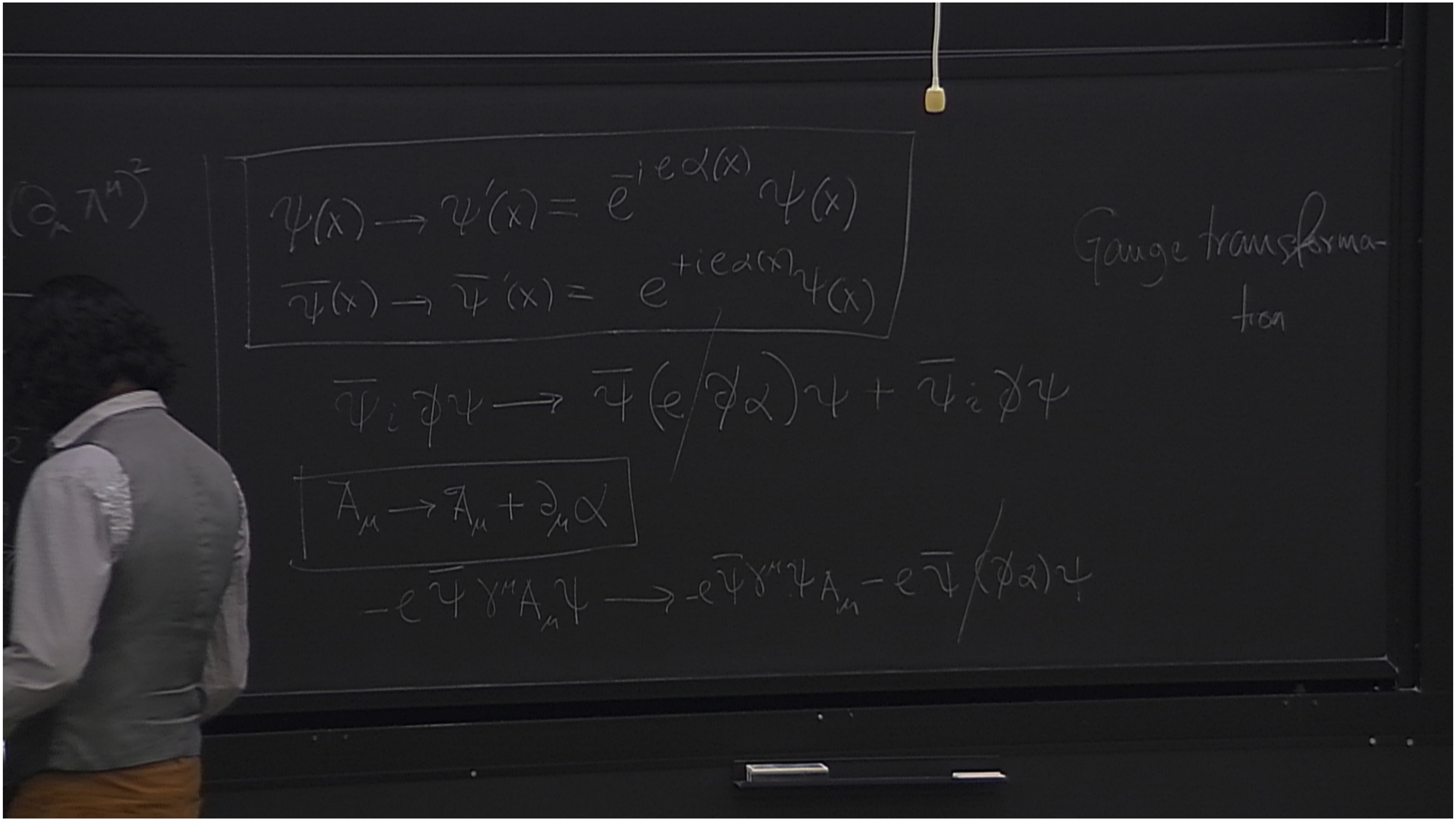
$$\bar\psi i\partial\!\!\!/\psi \longrightarrow \bar\psi(e\,\partial\!\!\!/\alpha)\psi + \bar\psi i\partial\!\!\!/\psi$$

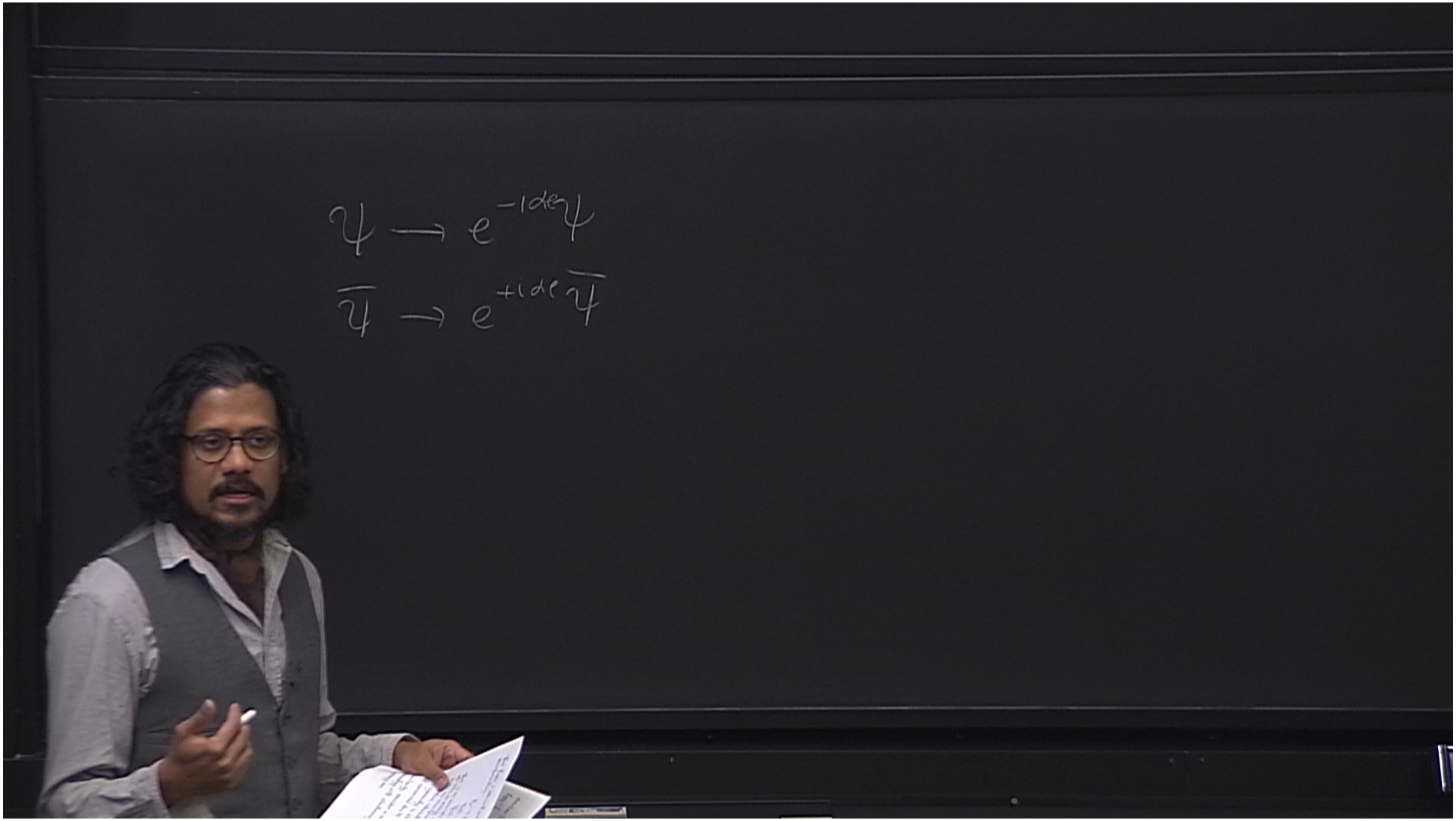
$$A_\mu \longrightarrow A_\mu + \partial_\mu\alpha$$

$$-e\bar\psi\gamma^\mu A_\mu\psi \longrightarrow -e\bar\psi\gamma^\mu\psi A_\mu - e\bar\psi(\partial\!\!\!/\alpha)\psi$$



$= -\frac{1}{2}(\partial_\mu A^\mu)^2$
$\psi \to e^{i\alpha}\psi$
$\bar\psi \to \bar\psi e^{-i\alpha}$
local $\xrightarrow[\text{symmetry}]{\text{gauging a}}$ local
$\psi(x) \to \psi'(x) = e^{-ie\alpha(x)}\psi(x)$
$\bar\psi(x) \to \bar\psi'(x) = e^{+ie\alpha(x)}\bar\psi(x)$
$\bar\psi i\not\partial\psi \to \bar\psi(e\not\partial\alpha)\psi + \bar\psi i\not\partial\psi$
$A_\mu \to A_\mu + \partial_\mu\alpha$
$-e\bar\psi\gamma^\mu A_\mu\psi \to -e\bar\psi\gamma^\mu\psi A_\mu - e\bar\psi(\not\partial\alpha)\psi$


$(\partial_\mu \lambda^\mu)^2$
$\boxed{\begin{aligned}\psi(x) &\to \psi'(x) = e^{-ie\alpha(x)}\psi(x)\\ \bar\psi(x) &\to \bar\psi'(x) = e^{+ie\alpha(x)}\bar\psi(x)\end{aligned}}$
Gauge transformation
$\bar\psi i \not\partial \psi \to \bar\psi(e\not\partial\alpha)\psi + \bar\psi i \not\partial \psi$
$\boxed{A_\mu \to A_\mu + \partial_\mu \alpha}$
$-e\bar\psi\gamma^\mu A_\mu \psi \to -e\bar\psi\gamma^\mu\psi A_\mu - e\bar\psi(\not\partial\alpha)\psi$


$\psi \to e^{-i\alpha e}\psi$
$\bar{\psi} \to e^{+i\alpha e}\bar{\psi}$

$$\psi \to e^{-i\alpha e}\psi$$

$$\bar{\psi} \to e^{+i\alpha e}\bar{\psi}$$

Scalar QED:

$$\mathcal{L}_{\substack{\text{scalar}\\ \text{QED}}} = \mathcal{L}_{\varphi} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$$

$$\mathcal{L}_{\varphi} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - m\varphi^\dagger\varphi$$

$$\mathcal{L}_{e.m.} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

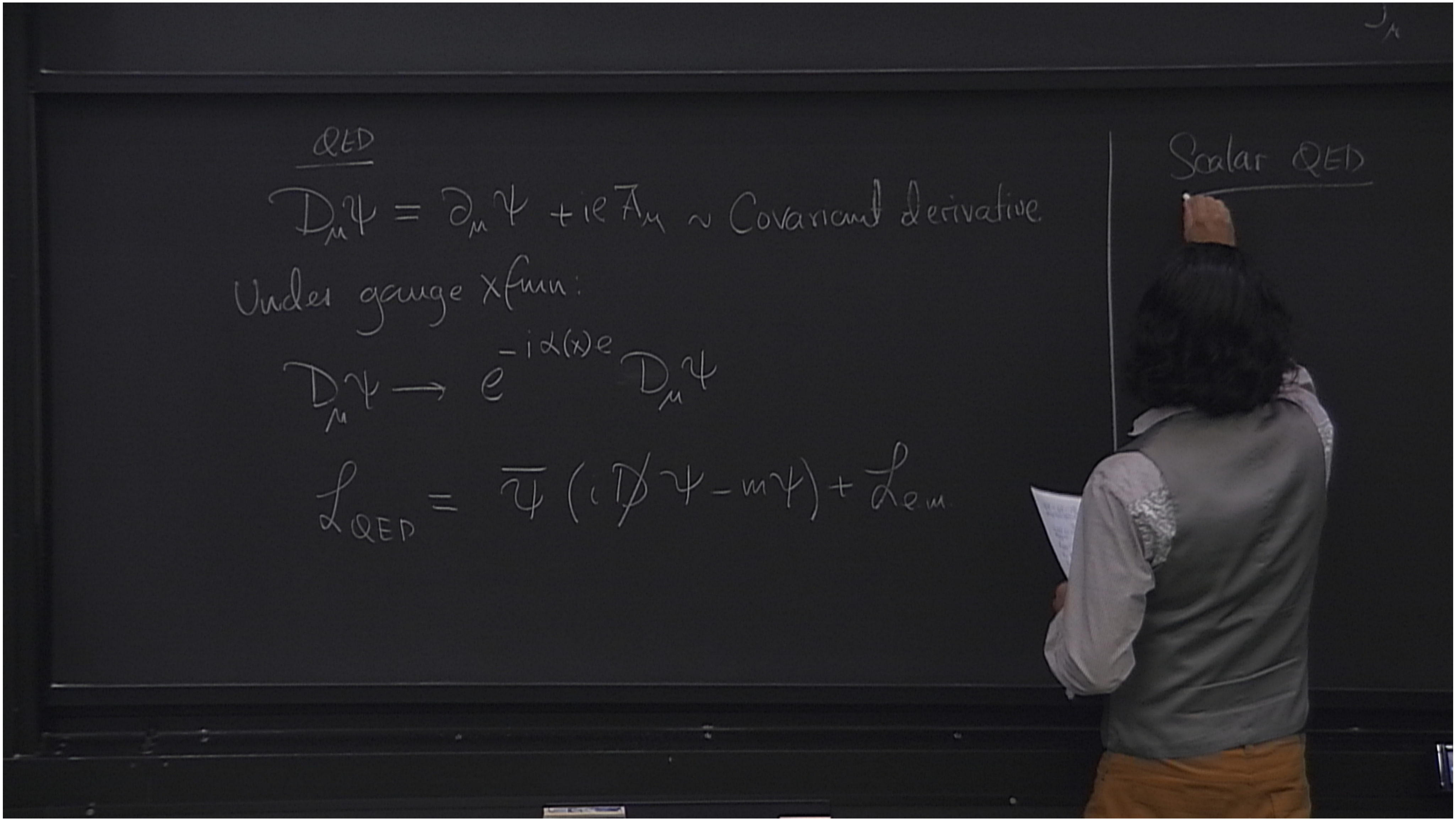
$$\mathcal{L}_{int} = -ie\underbrace{\left(\partial_\mu \varphi^\dagger \varphi - \varphi^\dagger \partial_\mu \varphi\right)}_{J_\mu} A^\mu$$

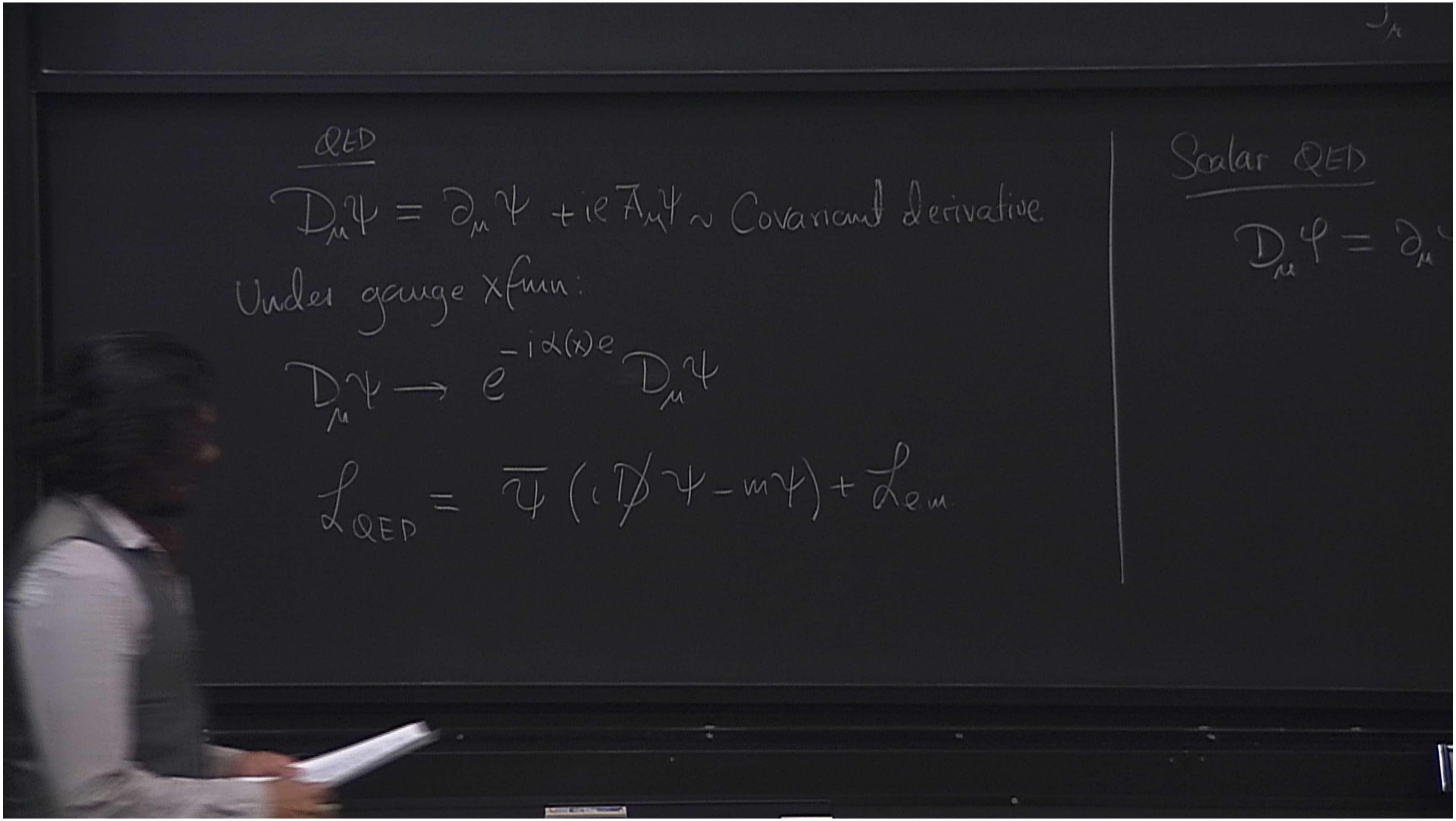
$$\varphi^\dagger \partial_\mu \varphi) A^\mu$$

$$\varphi(x) \longrightarrow e^{-ie\alpha(x)} \varphi(x)$$

$$\varphi^\dagger(x) \longrightarrow e^{+ie\alpha(x)} \varphi^\dagger(x)$$

$$A_\mu \longrightarrow A_\mu + \partial_\mu \alpha$$


QED
$D_\mu \psi = \partial_\mu \psi + ie A_\mu$ ~ Covariant derivative
Under gauge xfmn:
$D_\mu \psi \longrightarrow e^{-i\alpha(x)e} D_\mu \psi$
$\mathcal{L}_{QED} = \bar{\psi}(i \not{D} \psi - m\psi) + \mathcal{L}_{em}$
Scalar QED
J_μ


QED
$D_\mu \psi = \partial_\mu \psi + ie A_\mu \psi$ ~ Covariant derivative
Under gauge xfmn:
$D_\mu \psi \longrightarrow e^{-i\alpha(x)e} D_\mu \psi$
$\mathcal{L}_{QED} = \bar{\psi}(i \not{D} \psi - m\psi) + \mathcal{L}_{em}$
Scalar QED
$D_\mu \varphi = \partial_\mu$

J_μ

QED

$$D_\mu\psi = \partial_\mu\psi + ie A_\mu\psi \sim \text{Covariant derivative}$$

Under gauge xfmn:

$$D_\mu\psi \longrightarrow e^{-i\alpha(x)e} D_\mu\psi$$

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\not{D}\psi - m\psi\right) + \mathcal{L}_{em}$$

Scalar QED

$$D_\mu\varphi = \partial_\mu$$

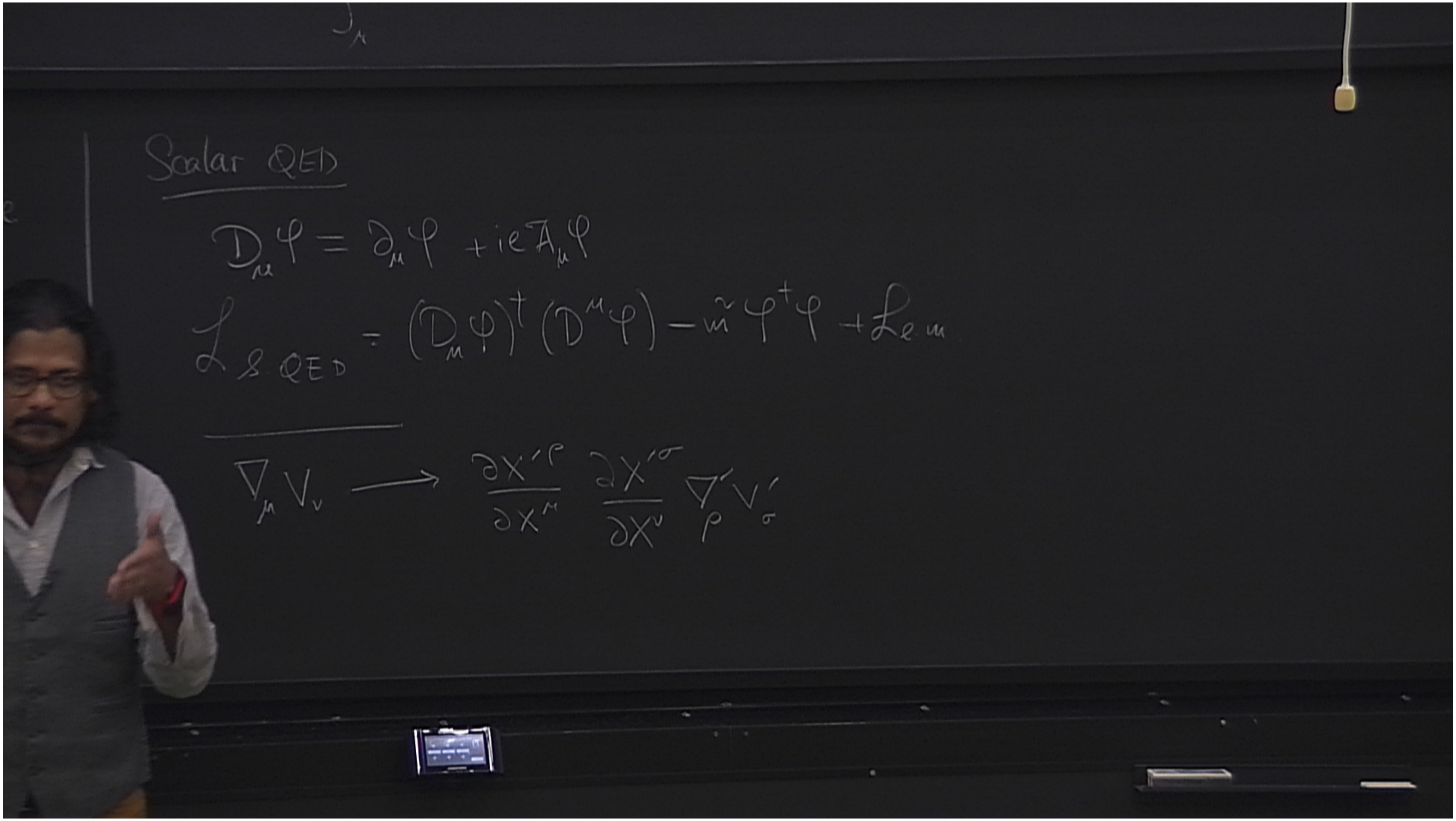
$$\mathcal{L}$$

J_μ

Scalar QED

$$D_\mu \varphi = \partial_\mu \varphi + ieA_\mu \varphi$$

$$\mathcal{L}_{s.QED} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - m^2 \varphi^\dagger \varphi + \mathcal{L}_{e.m.}$$


Scalar QED
$D_\mu \varphi = \partial_\mu \varphi + ie A_\mu \varphi$
$\mathcal{L}_{S.QED} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - m^2 \varphi^\dagger \varphi + \mathcal{L}_{e.m.}$
$\nabla_\mu V_\nu \longrightarrow \frac{\partial x'^\rho}{\partial x^\mu} \frac{\partial x'^\sigma}{\partial x^\nu} \nabla'_\rho V'_\sigma$

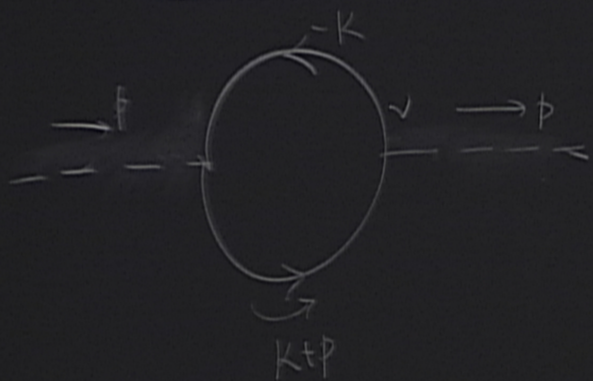
$$[D_\mu, D_\nu]\psi = F_{\mu\nu}\psi$$

$${}_\mu\varphi + ieA_\mu\varphi$$

$$(D_\mu\varphi)^\dagger(D^\mu\varphi) - m^2\varphi^\dagger\varphi + \mathcal{L}_{em}$$

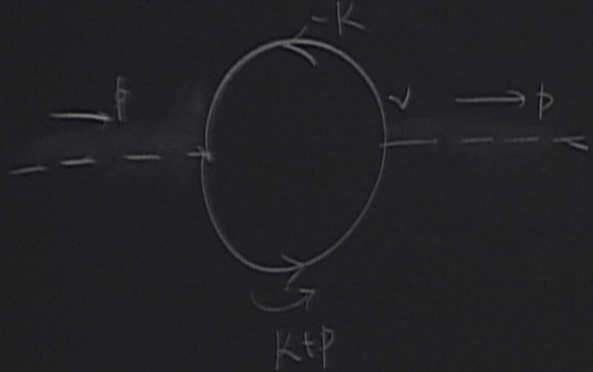
$$\longrightarrow \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu}\nabla'_\rho V'_\sigma \qquad [\nabla, \nabla]V = R\ldots V$$

Feynman Rules for QED:



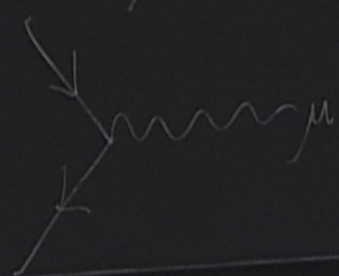
$$-\int \frac{d^4k}{(2\pi)^4}\, S_{\alpha\beta}\, S^{\nu}_{\beta\alpha}$$

Feynman Rules for QED:



$$-\int \frac{d^4k}{(2\pi)^4}\, S_{\alpha\beta}\, S_{\beta\alpha}$$

\# Interaction vertex between two (anti-)fermions & a photon



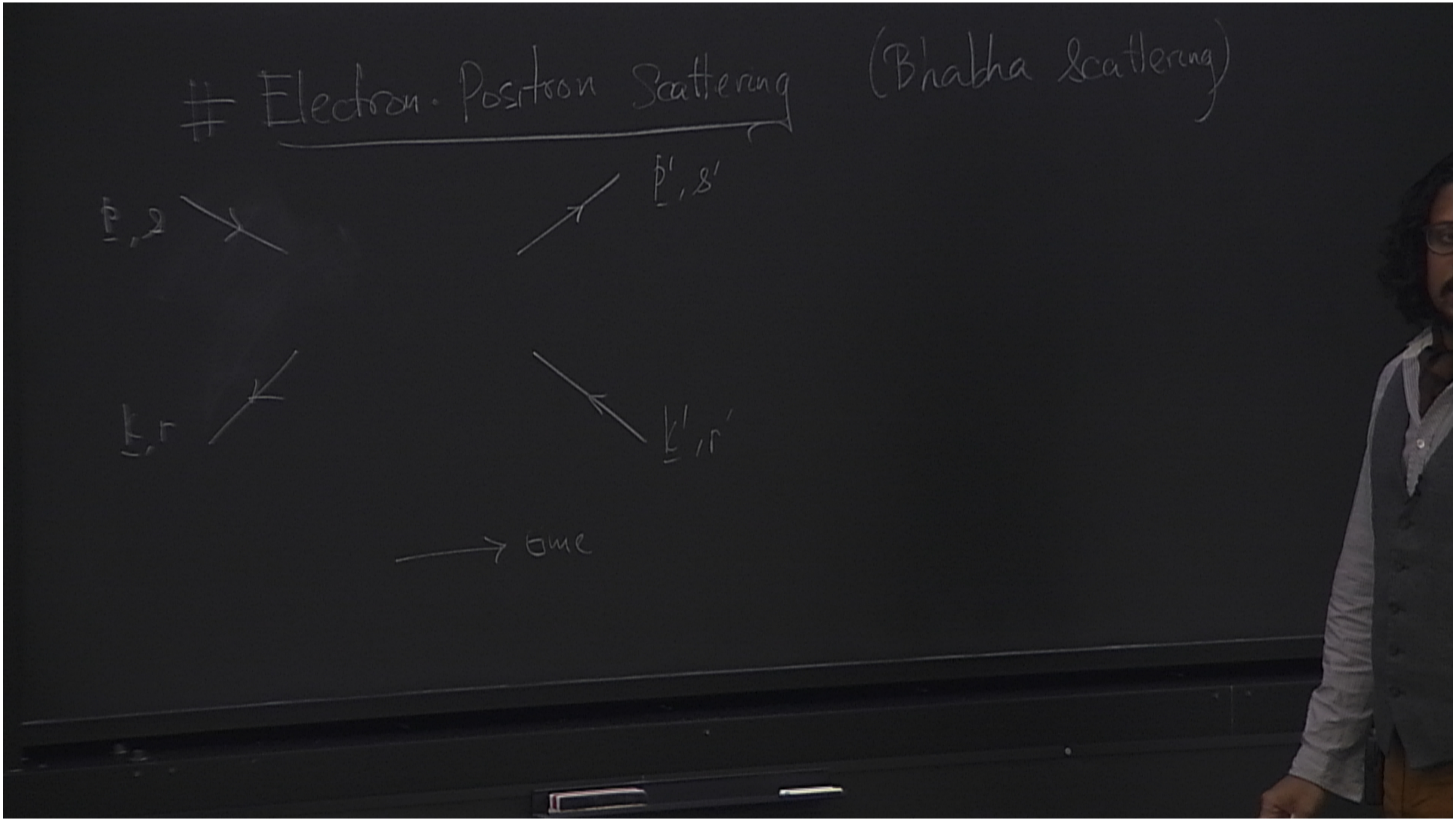
$$= -ie\gamma^{\mu}$$

Photon prop (Lorenz gauge)

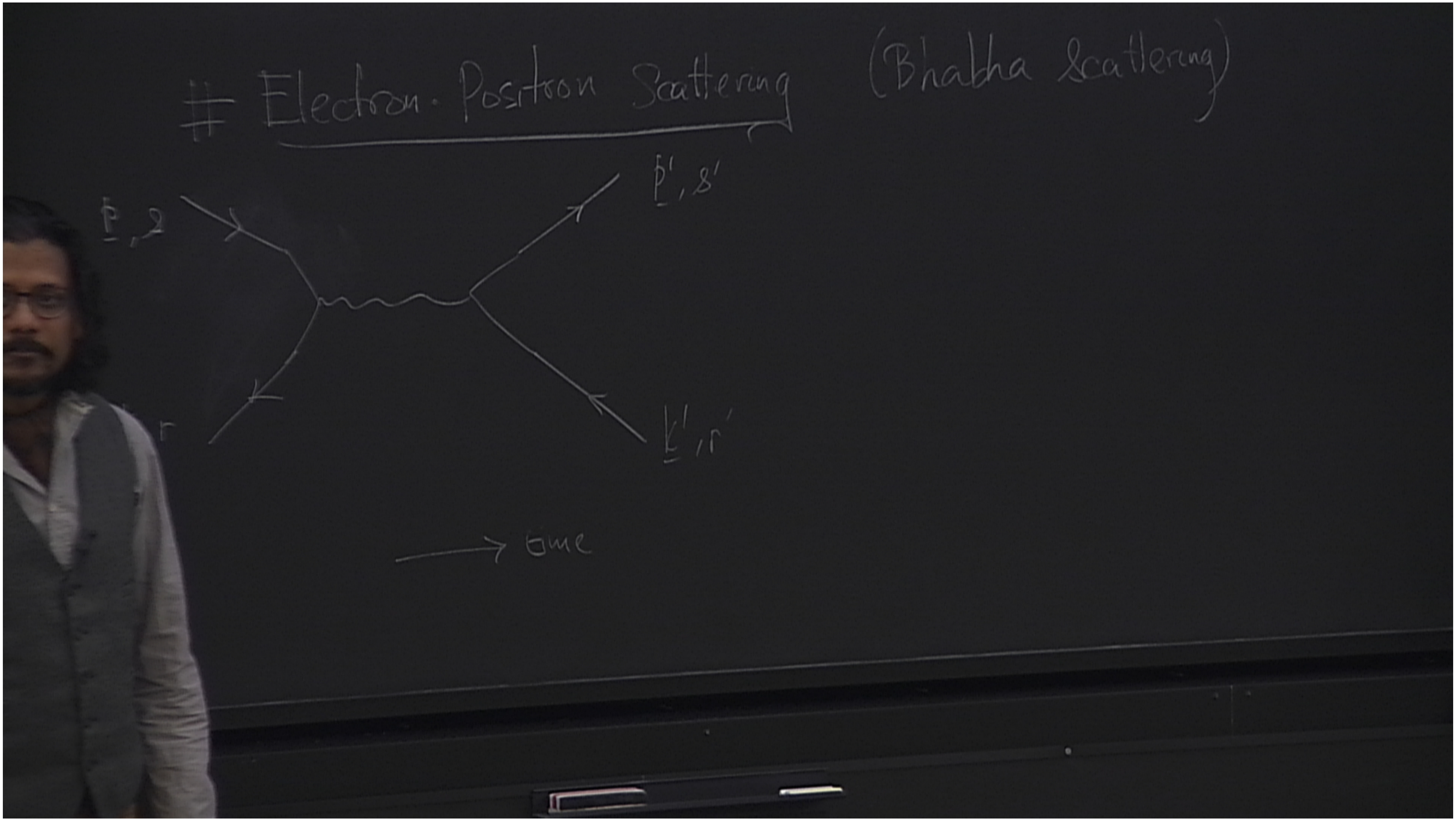
$$\mu \sim\sim\sim\sim \nu = \frac{-i\eta_{\mu\nu}}{p^2+i\epsilon}$$

Incoming photon $= \epsilon_\mu(p)$

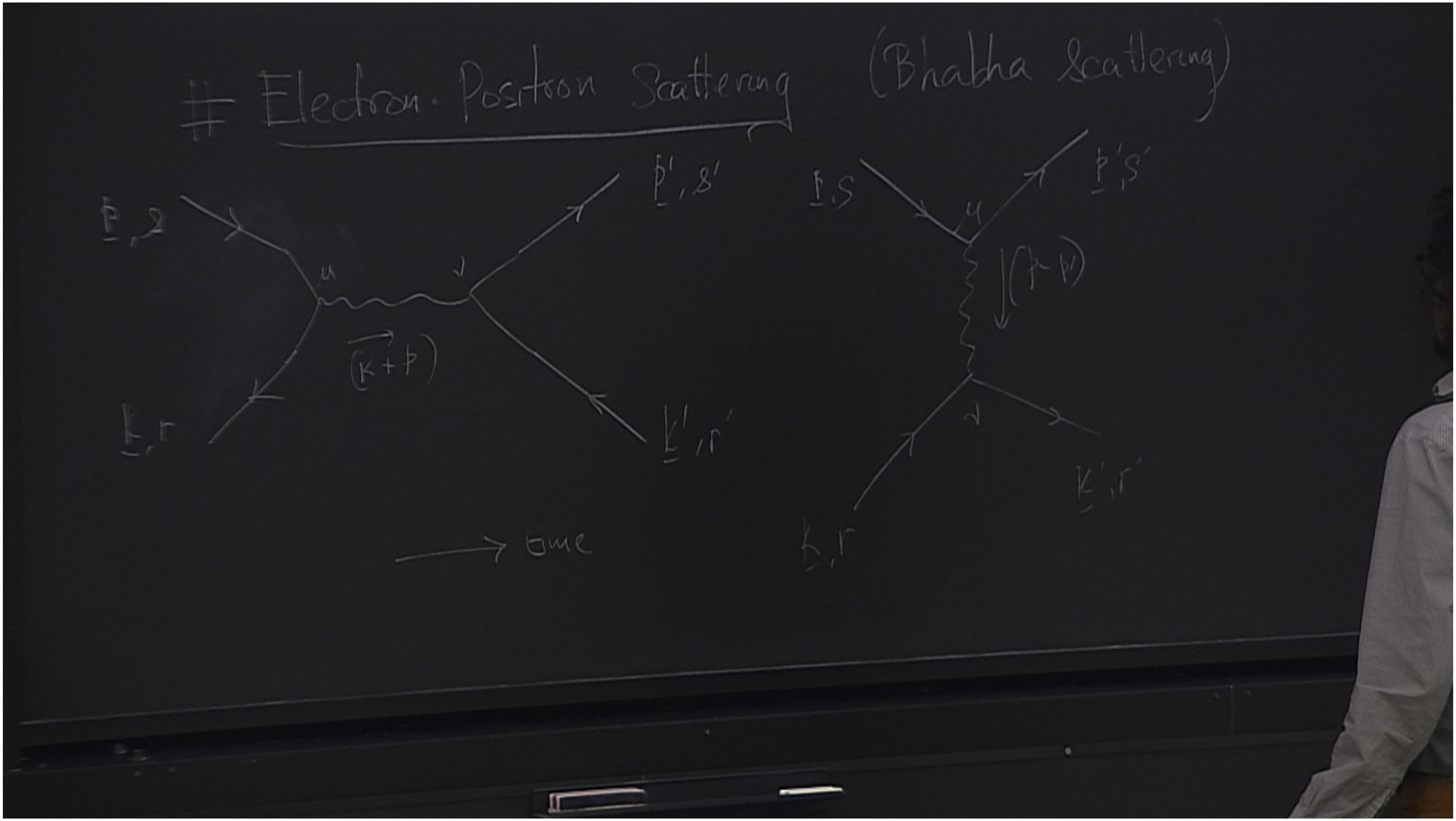
Outgoing photon $= \epsilon^*_\mu(p)$



Electron-Positron Scattering (Bhabha Scattering)
p, s
p', s'
k, r
k', r'
time



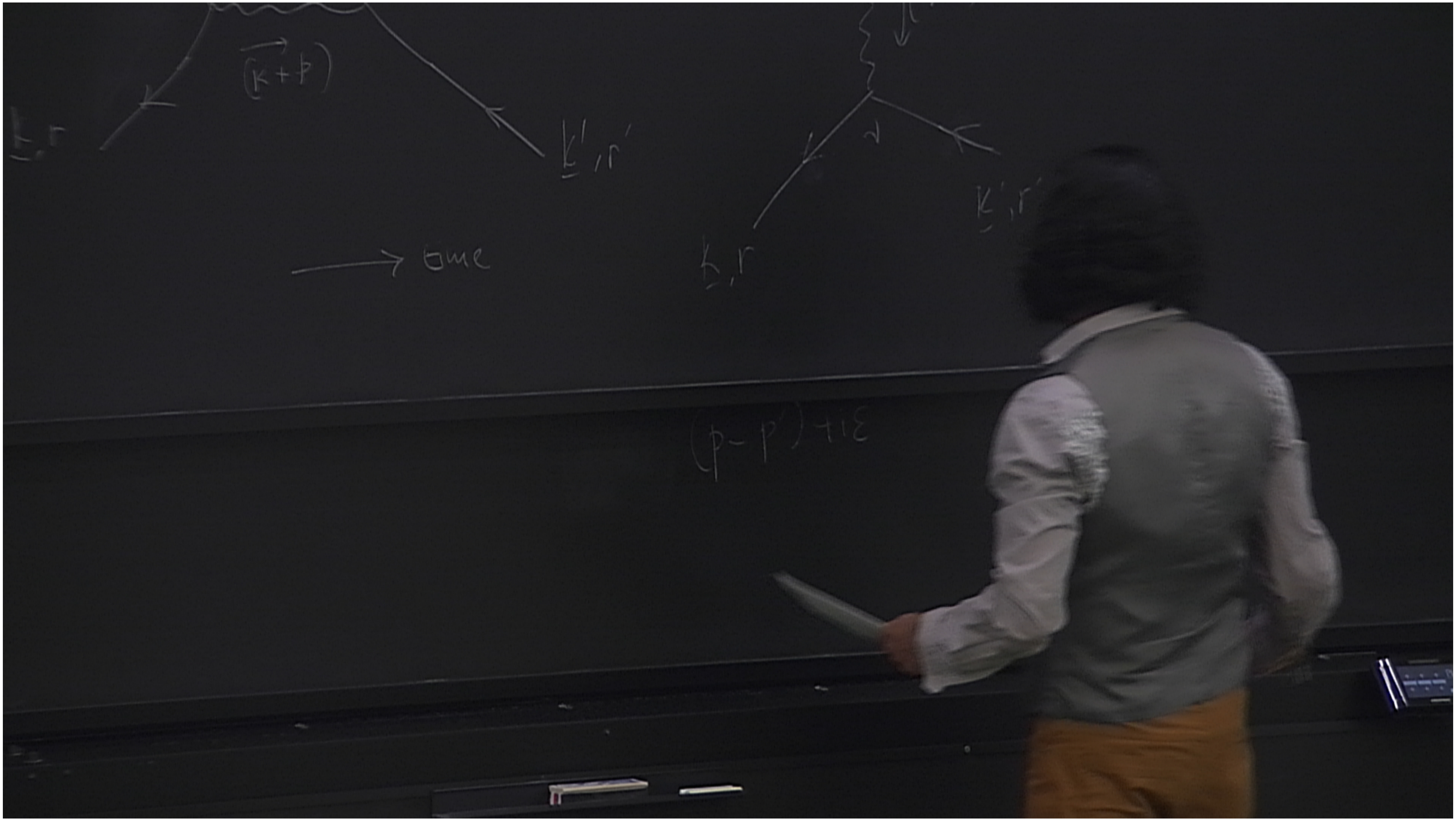
Electron-Positron Scattering (Bhabha Scattering)
p, s
p', s'
k', r'
r
time

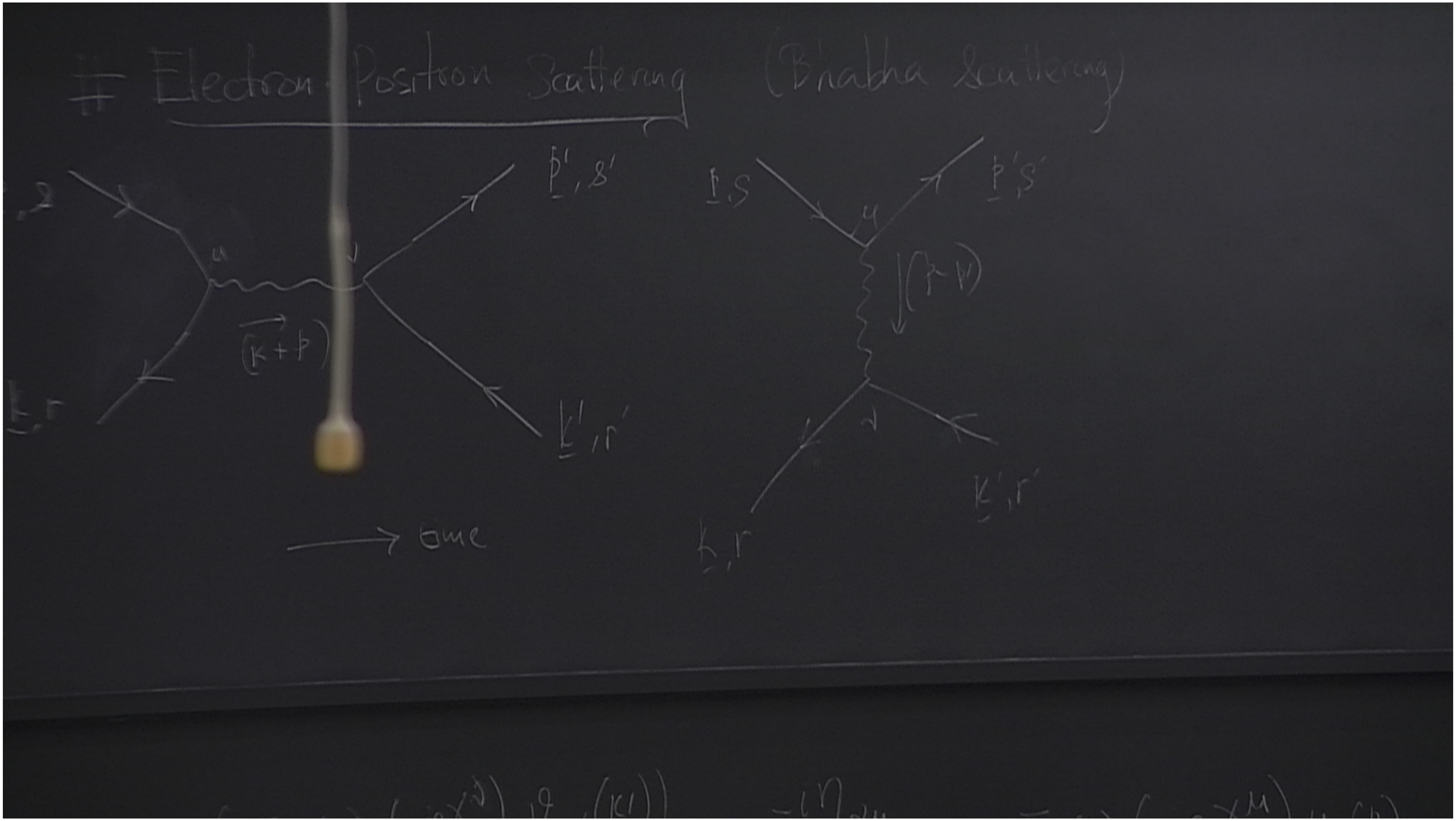


Electron-Positron Scattering
(Bhabha Scattering)
p, s
k, r
p', s'
k', r'
μ
ν
(k + p)
(p − p')
time

$$i\mathcal{M} = \left(\bar{u}_{s'}(p') \left(-ie\gamma^{\mu}\right) v_{r'}(k') \right) \quad \frac{-i\eta_{\nu\mu}}{(p+k)^2 + i\varepsilon}$$

$$i\mathcal{M} = \left(\bar{u}_{s'}(p')(-ie\gamma^{\nu})\, v_{r'}(k')\right) \frac{-i\eta_{\nu\mu}}{(p+k)^2+i\epsilon}\, \bar{v}_r(k)(-ie\gamma^{\mu})\, u_s(p)$$


(k+p)
k,r
k',r'
time
k,r
k',r'
(p-p')+iε



Electron-Positron Scattering (Bhabha Scattering)
p', s'
k', r'
(k + p)
time
p, s
p', s'
(p - p')
k', r'
k, r

$$i\mathcal{M} = \left(\bar{u}_{s'}(p') (-ie\gamma^{\nu}) v_{r'}(k') \right) \frac{-i\eta_{\nu\mu}}{(p+k)^2 + i\varepsilon} \bar{v}_r(k) (-ie\gamma^{\mu}) u_s(k)$$

$$- \bar{u}_{s'}(p') (-ie\gamma^{\mu}) u_s(p) \frac{-i\eta_{\mu\nu}}{(p-p')^2 + i\varepsilon} \bar{v}_r(k) (-ie\gamma^{\nu}) v_{r'}(k')$$

\# $e^+e^- \longrightarrow \mu^+\mu^-$

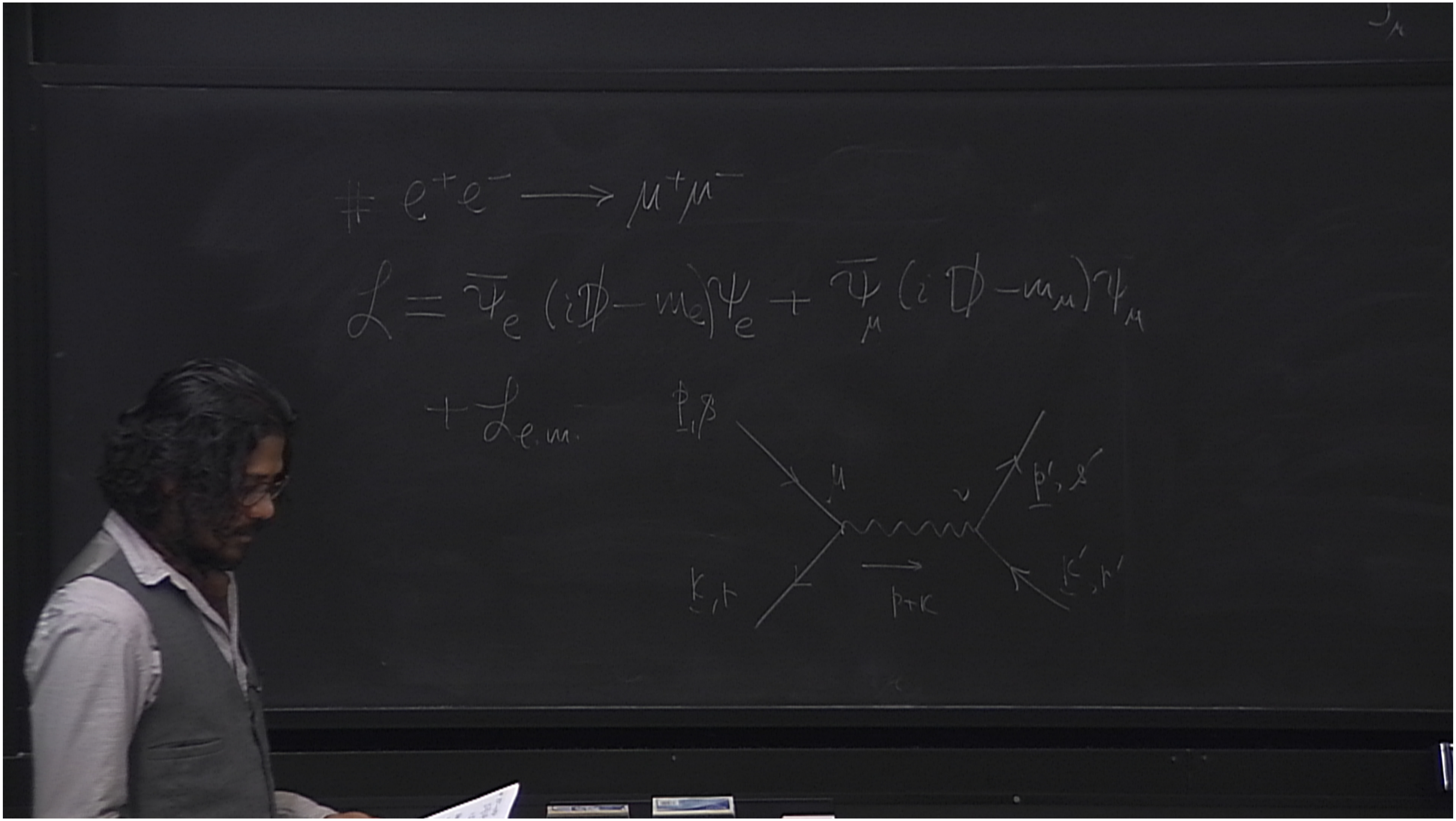
$$\mathcal{L} = \bar{\Psi}_e (i\not{D} - m_e)\Psi + \bar{\Psi}_\mu (i\not{D} - m_\mu)\Psi_\mu$$

$$+ \mathcal{L}_{e.m.}$$

$e^+e^- \longrightarrow \mu^+\mu^-$

$$\mathcal{L} = \bar{\Psi}_e (i\not{D} - m_e)\Psi_e + \bar{\Psi}_\mu (i\not{D} - m_\mu)\Psi_\mu$$

$$+ \mathcal{L}_{e.m.}$$



$e^+e^- \longrightarrow \mu^+\mu^-$
$\mathcal{L} = \bar{\psi}_e (i\not{D} - m_e)\psi_e + \bar{\psi}_\mu (i\not{D} - m_\mu)\psi_\mu$
$+ \mathcal{L}_{e.m.}$
p, s
μ
ν
p', s'
k, r
$p+k$
k', r'

$$i\mathcal{M} = \left(\bar{u}_{s'}(p') \left(-ie\gamma^{\nu}\right) v_{r'}(k') \right) \frac{-i\eta_{\nu\mu}}{(p+k)^2 + i\varepsilon} \bar{v}_r(k) \left(-ie\gamma^{\mu}\right) u_s(k)$$

$$- \bar{u}_{s'}(p') \left(-ie\gamma^{\mu}\right) u_s(p) \frac{-i\eta_{\mu\nu}}{(p-p')^2 + i\varepsilon} \bar{v}_r(k) \left(-ie\gamma^{\nu}\right) v_{r'}(k')$$

$$= \underbrace{\bar{v}_r(k)\,(-ie\gamma^\mu)\,u_s(p)}_{e^+e^-}\;\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\epsilon}\;\underbrace{\bar{u}_{s'}(p')\,(-ie\gamma^\nu)\,u_{r'}(k')}_{\mu^+\mu^-}$$

$$= \underbrace{\bar{v}_r(k)\,(-ie\gamma^\mu)\,u_s(p)}_{e^+e^-} \; \frac{-i\,\eta_{\mu\nu}}{(p+k)^2+i\varepsilon} \; \underbrace{\bar{u}_{s'}(p')\,(-ie\gamma^\nu)\,v_{r'}(k')}_{\mu^+\mu^-}$$

$$i\mathcal{M} = \underbrace{\bar{v}_r(k)\left(-ie\gamma^\mu\right)u_s(p)}_{e^+e^-}\;\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon}\;\underbrace{\bar{u}_{s'}(p')\left(-ie\gamma^\nu\right)v_{r'}(k')}_{\mu^+\mu^-}$$

$$\frac{1}{4}\sum_{\substack{s,r\\ s',r'}} |\mathcal{M}|^2 = \frac{1}{4}\sum \Big[e^4 \left(\bar{v}_r(k)\gamma^\mu u_s(p)\,\bar{u}_{s'}(p')\gamma_\mu v_{r'}(k')\right)^* \times \left(\bar{v}_r(k)\gamma^\sigma u_s(p)\,\bar{u}_{s'}(p')\gamma_\sigma v_{r'}(k')\right) \times \frac{1}{(p+k)^4}\Big]$$

$$i\mathcal{M} = \underbrace{\bar{v}_r(k)(-ie\gamma^\mu)u_s(p)}_{e^+e^-} \frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon} \underbrace{\bar{u}_{s'}(p')(-ie\gamma^\nu)v_{r'}(k')}_{\mu^+\mu^-}$$

$$\frac{1}{4}\sum_{\substack{s,r\\s',r'}} |\mathcal{M}|^2 = \frac{1}{4}\sum\Big[e^4 \big(\bar{v}_r(k)\gamma^\mu u_s(p)\,\bar{u}_{s'}(p')\gamma_\mu v_{r'}(k')\big)^* \times \big(\bar{v}_r(k)\gamma^\sigma u_s(p)\,\bar{u}_{s'}(p')\gamma_\sigma v_{r'}(k')\big) \times \frac{1}{(p+k)^4}\Big]$$

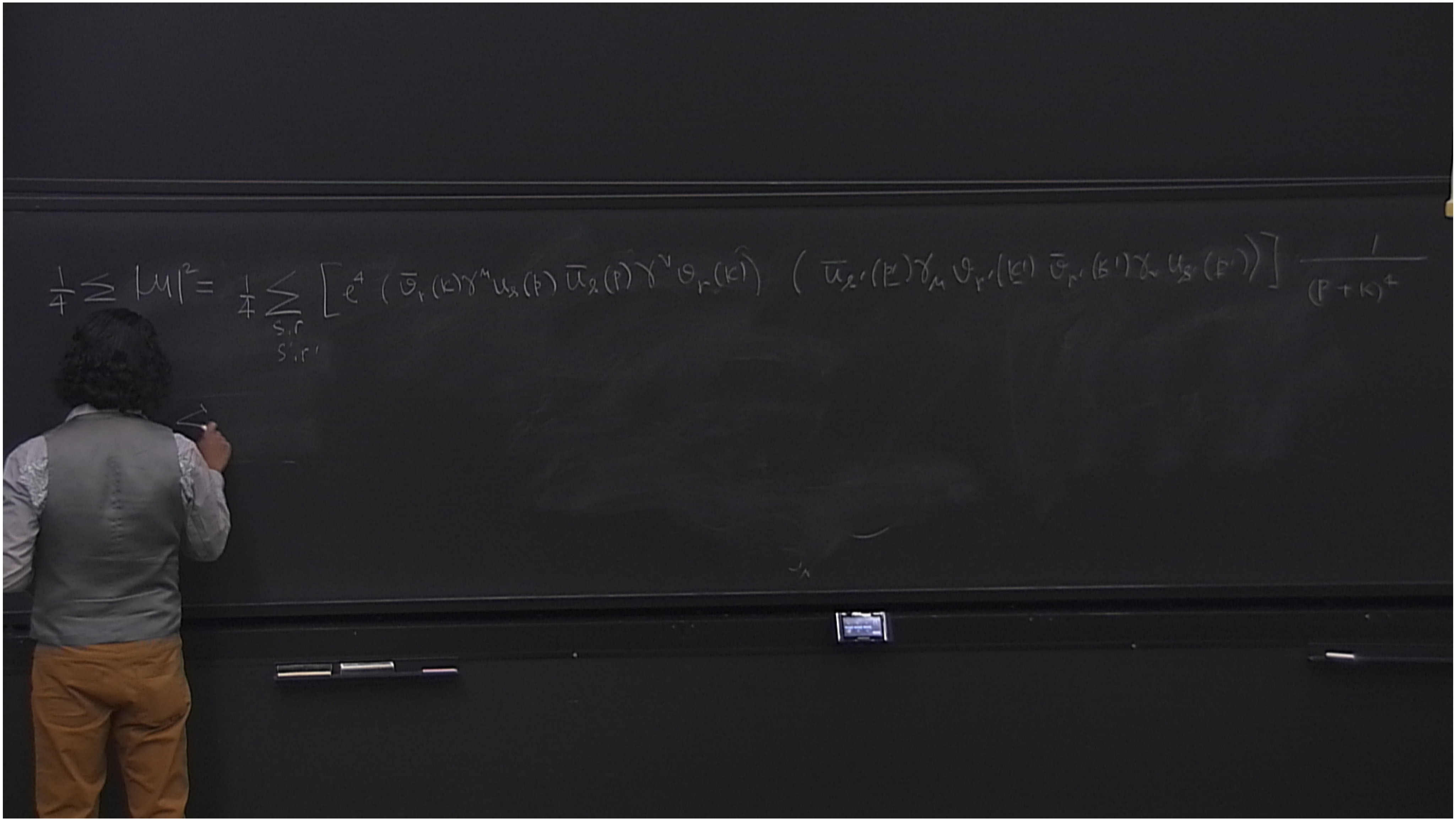
$$\big(\bar{v}_r(k)\gamma^\mu u_r(p)\big)^* = (\ldots\ldots)^\dagger$$

J_A

$$i\mathcal{M} = \underbrace{\bar{v}_r(k)(-ie\gamma^\mu)u_s(p)}_{e^+e^-}\ \frac{-i\eta_{\mu\nu}}{(p+k)^2+i\epsilon}\ \underbrace{\bar{u}_{s'}(p')(-ie\gamma^\nu)v_{r'}(k')}_{\mu^+\mu^-}$$

$$\frac{1}{4}\sum_{\substack{s,r\\ s',r'}} |\mathcal{M}|^2 = \frac{1}{4}\sum \Big[e^4 \left(\underbrace{\bar{v}_r(k)\gamma^\mu u_s(p)}\ \overbrace{\bar{u}_{s'}(p')\gamma_\mu v_{r'}(k')}\right)^* \times \left(\underbrace{\bar{v}_r(k)\gamma^\sigma u_s(p)}\ \overbrace{\bar{u}_{s'}(p')\gamma_\sigma v_{r'}(k')}\right) \times \frac{1}{(p+k)^4} \Big]$$

$$\left(\bar{v}_r(k)\gamma^\mu u_s(p)\right)^* = \left(\cdots\right)^\dagger = \left(u_s^\dagger(p)\gamma^{\mu\dagger}\bar{v}_r^\dagger(k)\right)$$
$$= \bar{u}\gamma^\mu u$$


$\frac{1}{4}\sum|\mathcal{M}|^2 = \frac{1}{4}\sum_{\substack{s,r\\ s',r'}}\Big[e^4\,(\bar{v}_r(k)\gamma^\mu u_s(p)\,\bar{u}_s(p)\gamma^\nu v_r(k))\;(\bar{u}_{s'}(p')\gamma_\mu v_{r'}(k')\,\bar{v}_{r'}(k')\gamma_\nu u_{s'}(p'))\Big]\frac{1}{(p+k)^4}$

$$\frac{1}{4}\sum |M|^2 = \frac{1}{4}\sum_{\substack{s,r\\ s',r'}} \Big[e^4 \left(\bar{v}_r(k)\gamma^\mu u_s(p)\, \bar{u}_s(p)\gamma^\nu v_r(k) \right) \left(\bar{u}_{s'}(p')\gamma_\mu \right.$$

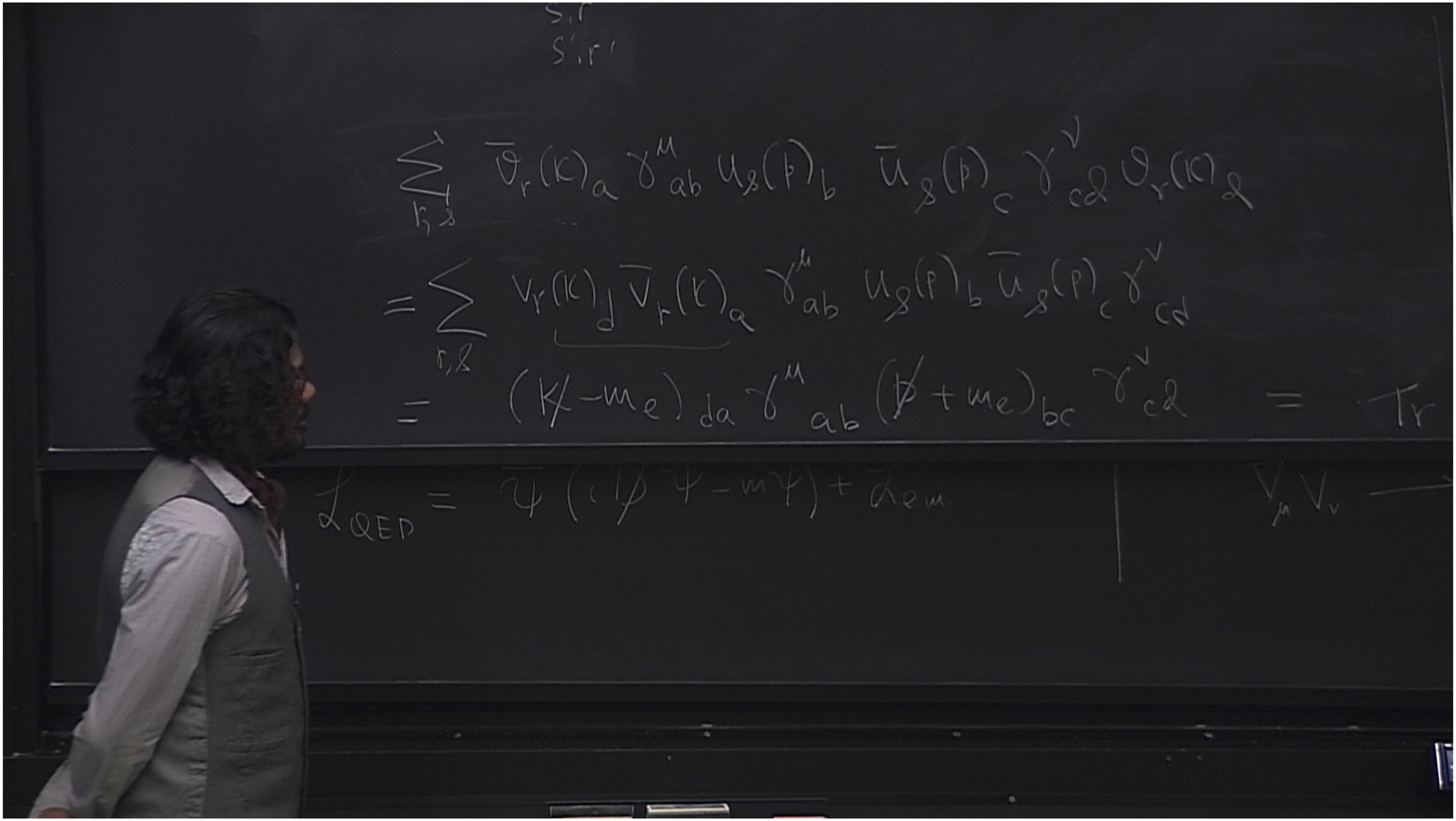
$$\sum_{r,s} \bar{v}_r(k)_a \gamma^\mu_{ab} u_s(p)_b \, \bar{u}_s(p)_c \gamma^\nu_{cd} v_r(k)_d$$

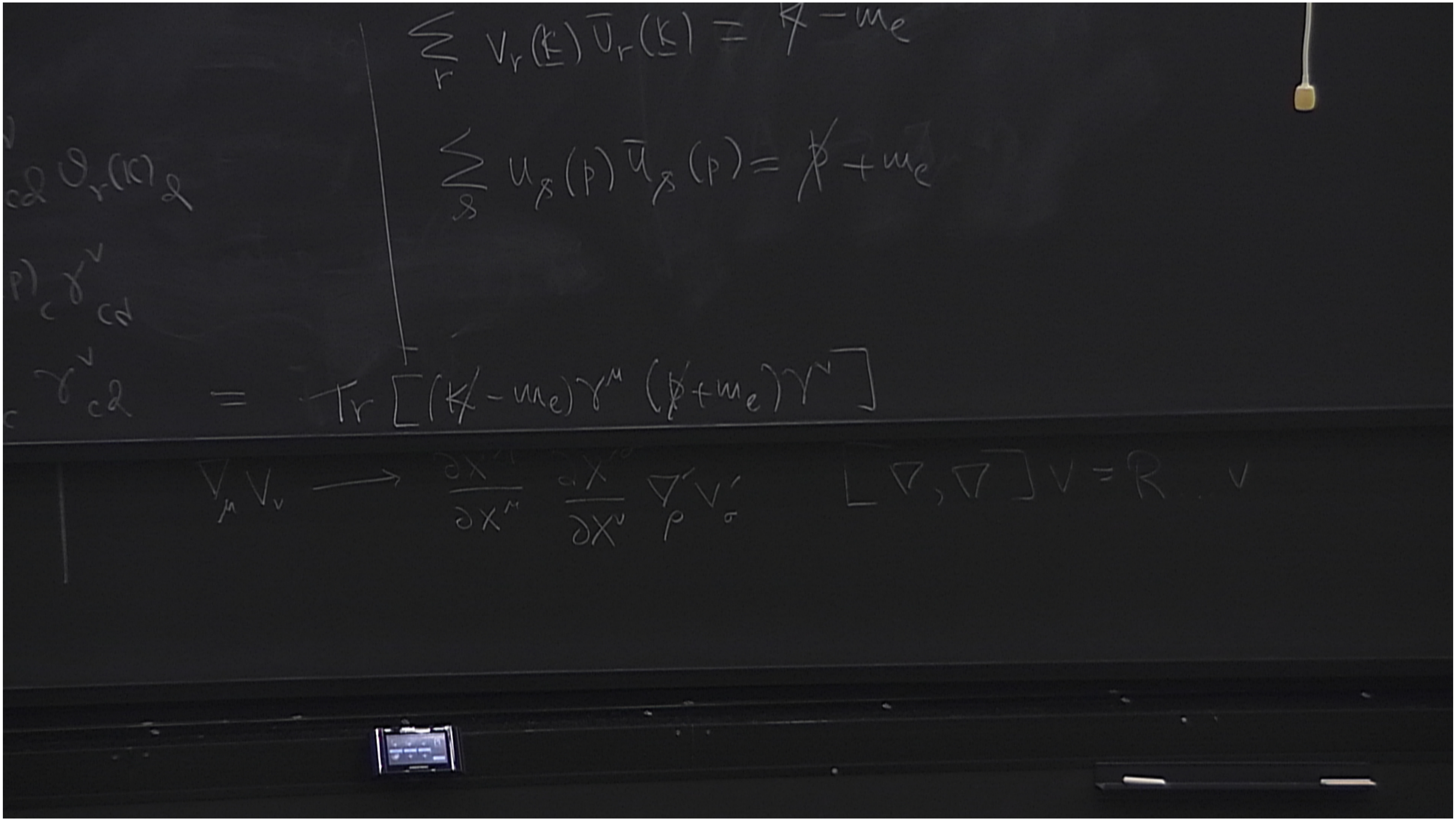
$$= \sum_{r,s} v_r(k)_d \bar{v}_r(k)_a \gamma^\mu_{ab} u_s(p)_b \bar{u}_s(p)_c \gamma^\nu_{cd}$$

$$\left(\bar{u}_{s'}(\underline{p}') \gamma_\mu v_{r'}(\underline{k}') \bar{v}_{r'}(\underline{k}') \gamma_\nu u_{s'}(\underline{p}') \right) \Big] \frac{1}{(p+k)^4}$$

$$\sum_r v_r(\underline{k}) \bar{v}_r(\underline{k}) = \not{k} - m_e$$

$$\sum_s u_s(p) \bar{u}_s(p) = \not{p} + m_e$$


s, r
s', r'
$\sum_{r,s} \bar{v}_r(k)_a \gamma^\mu_{ab} u_s(p)_b \bar{u}_s(p)_c \gamma^\nu_{cd} v_r(k)_d$
$= \sum_{r,s} \underbrace{v_r(k)_d \bar{v}_r(k)_a} \gamma^\mu_{ab} u_s(p)_b \bar{u}_s(p)_c \gamma^\nu_{cd}$
$= (\not{k} - m_e)_{da} \gamma^\mu_{ab} (\not{p} + m_e)_{bc} \gamma^\nu_{cd} = \mathrm{Tr}$
$\mathcal{L}_{QED} = \bar{\psi}(i\not{\partial}\psi - m\psi) + \mathcal{L}_{em}$
$V_\mu V_\nu$



$\sum_r v_r(\underline{k})\,\bar{u}_r(\underline{k}) = \not{k} - m_e$
$\sum_s u_s(p)\,\bar{u}_s(p) = \not{p} + m_e$
$= \mathrm{Tr}\left[(\not{k} - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right]$
$V_\mu V_\nu \longrightarrow \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu}\nabla'_\rho V'_\sigma$
$[\nabla, \nabla] V = R \cdot V$

$$m_e = 0.5 \text{ MeV}$$

$$m_\mu = 105 \text{ MeV}$$

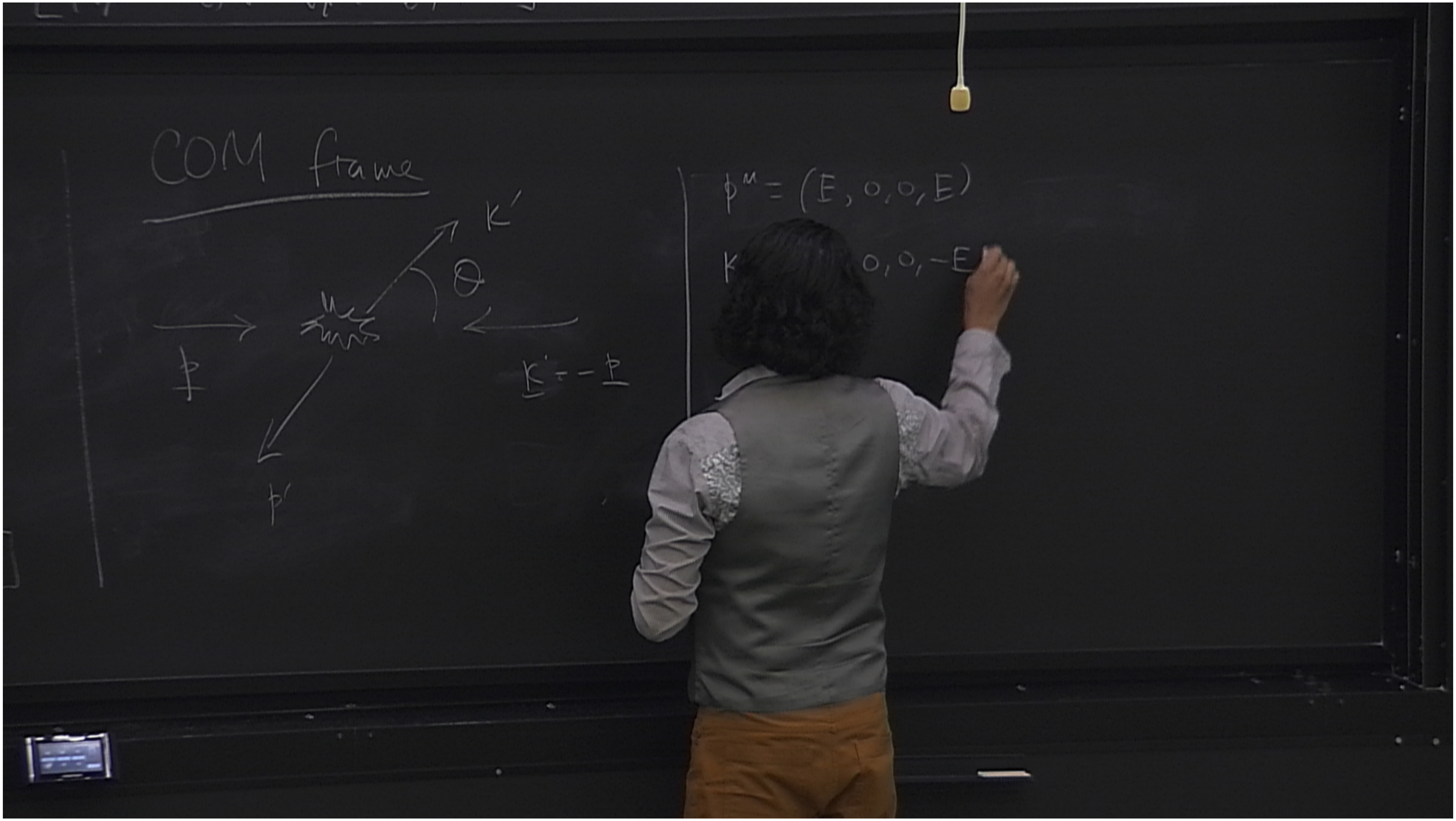
$$X = \frac{e^4}{4}\left[\mathrm{Tr}\left(\not{k}\,\gamma^\mu \not{p}\,\gamma^\nu\right)\mathrm{Tr}\left(\not{p}'\gamma_\mu \not{k}'\gamma_\nu - m^2\gamma_\mu\gamma_\nu\right)\right]$$

$$m_e = 0.5 \text{ MeV}$$

$$m_\mu = 105 \text{ MeV}$$

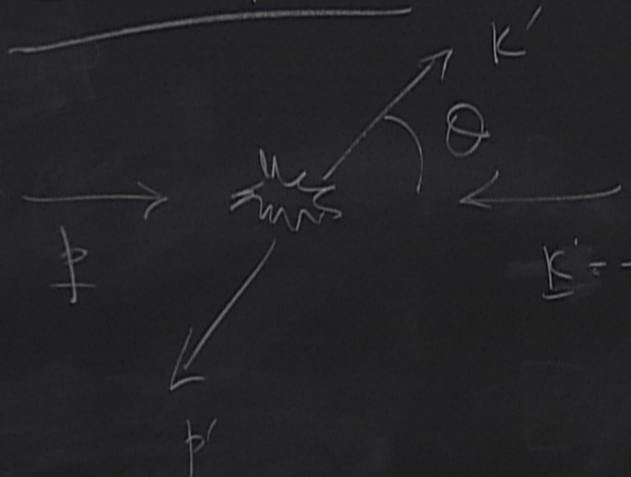
$$X = \frac{e^4}{4}\left[\mathrm{Tr}\left(\not{k}\gamma^\mu \not{p}\gamma^\nu\right)\mathrm{Tr}\left(\not{p}'\gamma_\mu \not{k}'\gamma_\nu - m_\mu^2\gamma_\mu\gamma_\nu\right)\right]$$

$$= \frac{4e^4}{(p+k)^4}\left[2(k\cdot p')(p\cdot k') + 2(k\cdot k')(p\cdot p') + 2(k\cdot p)m_\mu^2\right]$$



COM frame
k'
θ
p
k = − p
p'
p^μ = (E, 0, 0, E)
0, 0, −E

COM frame



$\underline{k} = -\underline{p}$

$p^\mu = (E, 0, 0, E)$

$k^\mu = (E, 0, 0, -E)$

$k'^\mu = (E, \underline{k}')$

$p'^\mu = (E, -\underline{k}')$

$(p+k)^2 = s = 4E^2$

$|\underline{k}'| = \sqrt{E^2 - m_\mu^2}$

$$k \cdot p' = E^2 - |\underline{k}|\, E \cos\theta$$

$$X = e^4 \left[\left(1 + \frac{m^2}{E^2}\right) + \left(1 - \frac{m^2}{E^2}\right) \cos^2\theta \right]$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2E)(2E)\,|\underline{v}_1 - \underline{v}_2|} \; \frac{1}{16\pi^2} \; \frac{|\underline{p}_f|}{E_{CM}} \times \frac{1}{4} \sum |M|^2$$

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$$E_{CM} = 2E \qquad |\underline{v}_1 - \underline{v}_2| = \left| \frac{|\underline{k}|}{E} + \frac{|\underline{k}|}{E} \right| = 2$$

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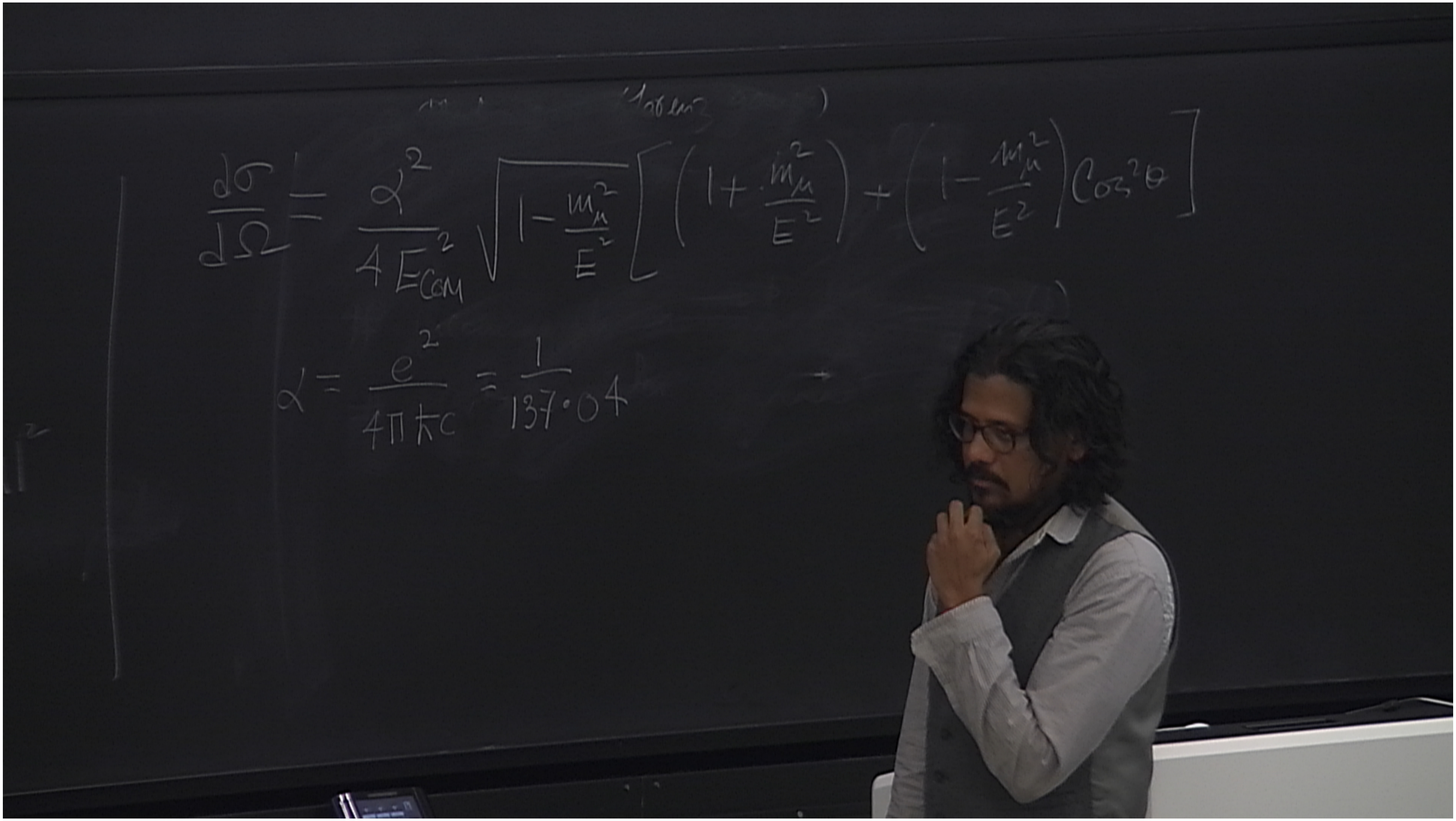
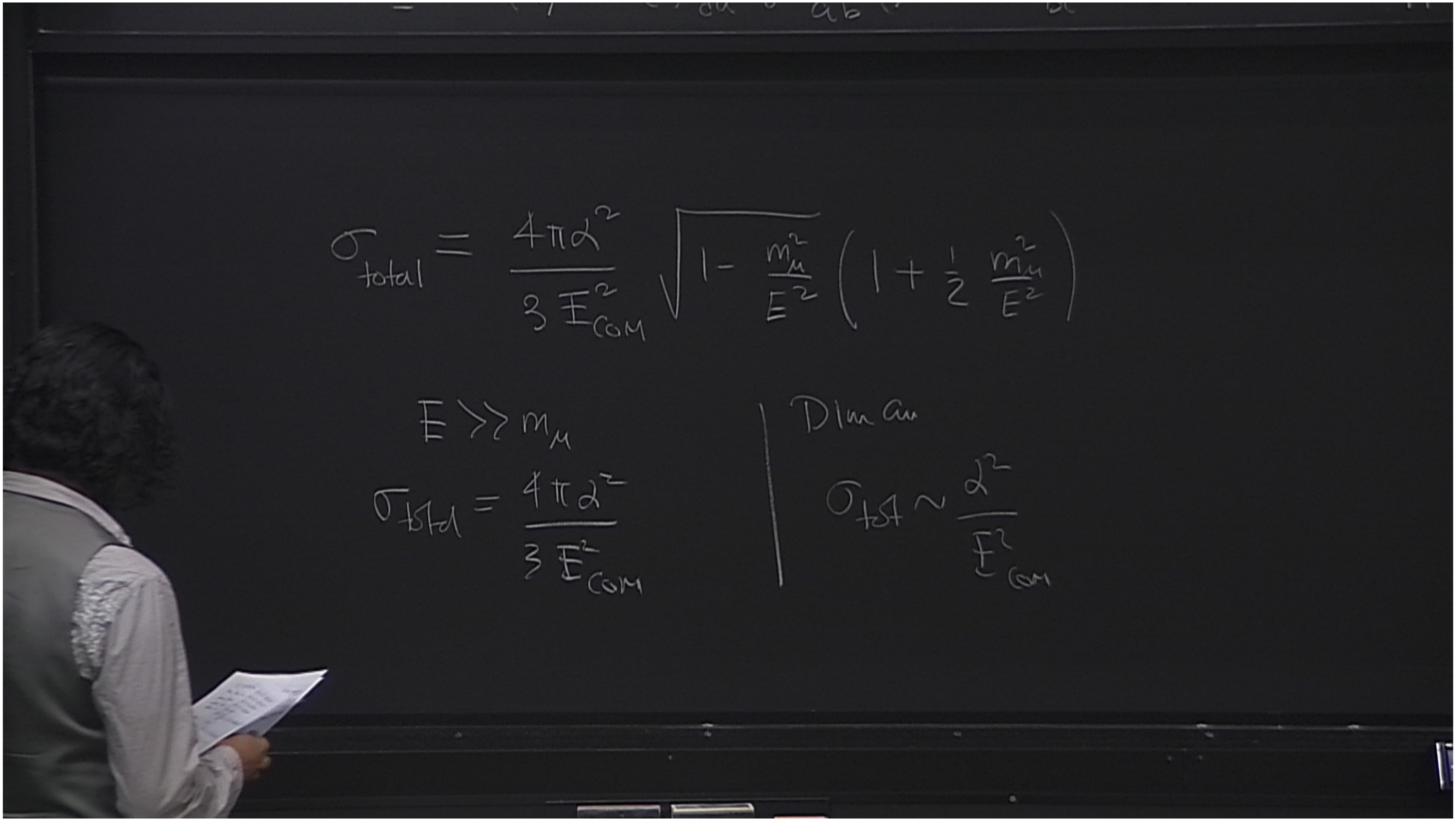
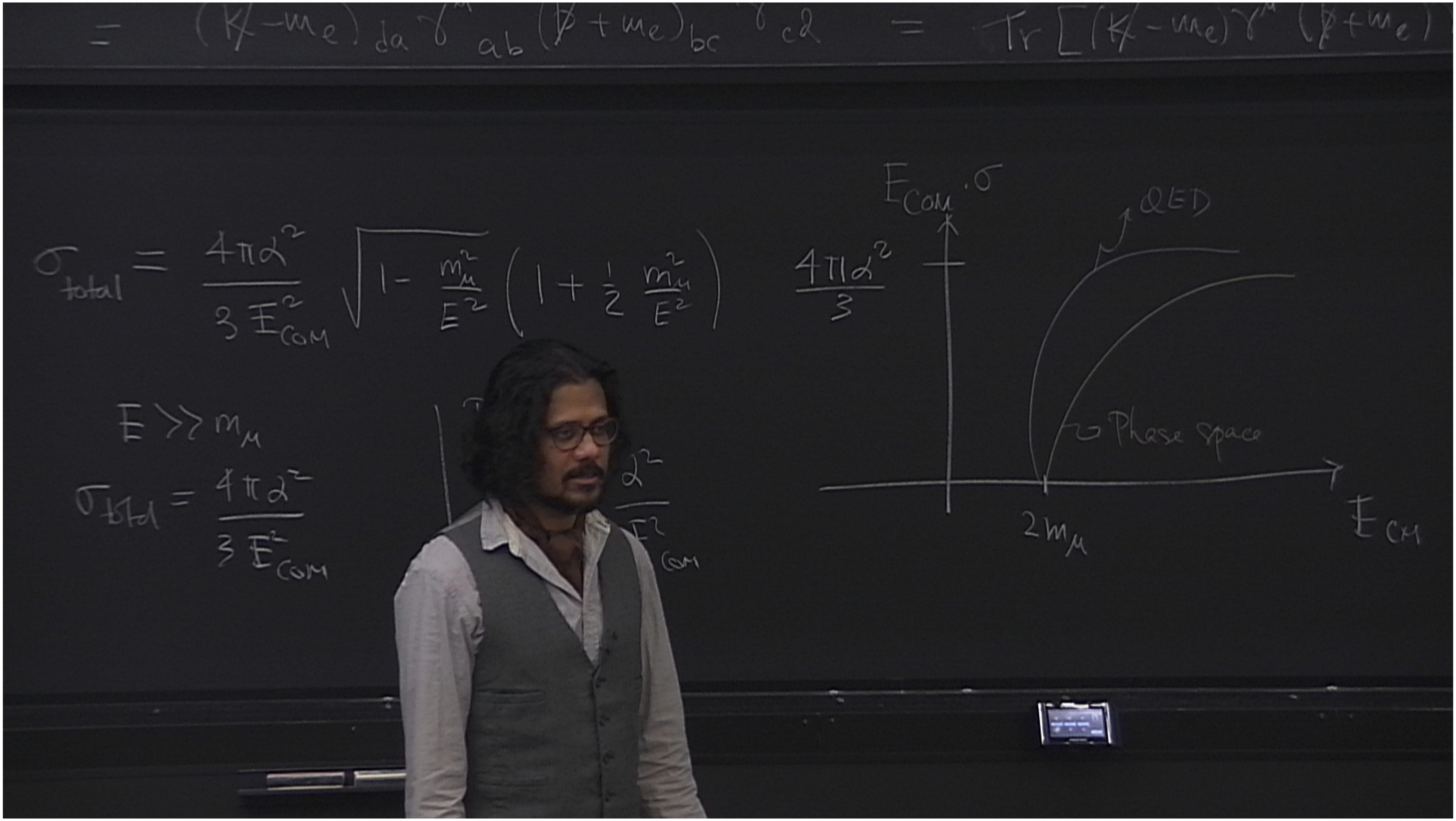
$$E_{CM} = 2E \qquad |\underline{v}_1-\underline{v}_2| = \left|\frac{|\underline{k}|}{E}+\frac{|\underline{k}|}{E}\right| = 2$$

$$\frac{d\sigma}{d\Omega} = $$

$$\alpha = $$


$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_{CoM}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$
$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.04}$


$$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left(1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right)$$
$E \gg m_\mu$
$$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}$$
Dim an
$$\sigma_{tot} \sim \frac{\alpha^2}{E_{COM}^2}$$



$= (\not{k} - m_e)_{da}\, \gamma^\mu{}_{ab}\, (\not{p} + m_e)_{bc}\, \gamma_{cd} = \mathrm{Tr}[(\not{k} - m_e)\gamma^\mu(\not{p} + m_e)$
$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}\sqrt{1 - \frac{m_\mu^2}{E^2}}\left(1 + \frac{1}{2}\frac{m_\mu^2}{E^2}\right)$
$E >> m_\mu$
$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}$
$E_{COM} \cdot \sigma$
$\frac{4\pi\alpha^2}{3}$
QED
Phase space
$2m_\mu$
E_{CM}

$$= (\not{k} - m_e)_{da}\, \gamma^{\mu}{}_{ab}\, (\not{p} + m_e)_{bc}\, \gamma_{cd} = \mathrm{Tr}\left[(\not{k} - m_e)\gamma^{\mu} (\not{p} + m_e) \right.$$

$$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}\sqrt{1 - \frac{m_\mu^2}{E^2}}\left(1 + \frac{1}{2}\frac{m_\mu^2}{E^2}\right)$$

$E \gg m_\mu$

$$\sigma_{total} = \frac{4\pi\alpha^2}{3E_{COM}^2}$$

Dim an

$$\sigma_{tot} \sim \frac{\alpha^2}{E_{COM}^2}$$